Asset allocation under multiple regimes
Master's Thesis in Quantitative Finance

Oscar Liszewski

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Abstract

In this paper we examine the performance of the Markov Switching model with intra-regimes changes such as the bull market correction and bear market rallies. We accommodate this short time rehearsals by imposing restrictions on the transition probability matrix. We compare the model with classic mean-switching and dynamic VAR models in an asset allocation problem with different number of regimes, initial states choices and asset distributions used in the estimation process. In an out-of-sample and bootstrap verification we give evidence that the constrained model outperforms other models in terms of risk-adjusted returns in the long horizon above 2 years.

Keywords: regime switching, transition probability matrix, asset allocation

1 Introduction

Investors and academics agree on the existence of short term trends of opposite direction to the main trend in the stock market. During bear or bull markets the prices of stock can increase or respectively decrease for a short period of time without necessarily meaning a change in the state of the economy. These phenomena called bull corrections and bear rallies might mislead an investor by implying a change in the economy main trend and impact its investments. We want to verify that by accommodating these short time rehearsals through component states of the bear and bull markets in our models, we can improve his allocation choices. We believe that an investor incorporating this knowledge can significantly improve its results. It is in fact the case as we find in an out-of-sample evaluation that this model outperforms classic regime-switching models in terms of risk adjusted returns for almost all tested horizons, especially in the long term (above 2 years).

Regime switching models have been within the scope of financial academic interest for quite some time now. In the past most studies assumed linear dependencies between regressors and dependent variables such as asset returns. More recently, it has been proven that such dependencies are not linear and they do vary across time (Ang and Bekaert (2002a,b, 2004), Ang and Chen (2002), Garcia and Perron (1996), Guidolin and Timmermann (2005a,b, 2006, 2007), Guidolin and Hyde (2012) to cite a few). This is most clearly visible in stock returns where bull markets follow bear markets and so on, but it has been shown that it affects not only equity returns, but also interest rates data (Gray (1996)) or macroeconomic fundamentals (Hamilton (1989)). These different states of economy not only impact expected returns but also returns characteristics such as higher correlation in bear markets or higher correlation Erb et al. (1994), Campbell et al. (2002).

Although the method has been widely propagated due to Hamilton (1989), Ang and Bekaert (2002a) were the first to use a simple 2 regime model to solve an asset allocation problem showing significant improvement compared to the non regime dependent model. Ever since, many authors tried to augment the number of regimes for a better fit of models for the data. Guidolin and Timmermann (2005a) investigate the economic implication of 3 regimes - bear, bull and normal market - on UK stocks and bonds. The effect is particularly large at short investment horizons and if ignored, the presence of such regimes leads to important welfare costs. Finally, Guidolin and Timmermann (2007) show that 4 regimes are required to capture the joint distribution of both equities and bonds. They are characterized as crash, slow growth, bull and recovery states. The authors additionally perform an out of sample forecast test to demonstrate the benefits of accounting for the presence of these regimes in asset returns.

Other authors explore the presence of predictability in asset returns under regime switches. The topic has been widely covered in a linear specification. Authors like Barberis (2000) or Campbell and Thompson (2008) show the evidence for predictability of stock premium by variables such as the dividend yield, term spread or net equity issuance. On the other hand Guidolin and Ono (2006) include many macroeconomic fundamentals in a VAR(1) component and show that it is not regime dependent. The authors explain it by the fact that linkages between the macroeconomy and financial markets are stable over time.

Maheu et al. (2012) is an important paper as it takes a different approach to switching model extensions. Instead of augmenting the number of regimes, they point for the existence of bull market corrections and bear market rallies. Such high-frequency reversals could be modelled by imposing restrictions on the transition probability matrix. Thanks to these modifications they could get a better specification of the 2 general regimes - the bear and bull market. We use a similar model in our work. Although the results of the paper prove that the model fits the data well and gives accurate distribution forecasts the authors do not extend their analysis to any economic evaluation.

A first significant contribution of this paper is to show how the use of a model accommodating intra-regime changes can influence asset allocation framework and then to compare out-of-sample its performance with a set of classic MS models. Second, we test the presence of intra-regimes in monthly data instead of weekly time-frames adopted in Maheu et al. (2012). We believe that monthly data are most frequently used by long term investors. Third, we are sceptical about the priors imposed in the Bayesian estimation of Maheu et al. (2012). Instead, we prefer to let the data speak and estimate the model in a frequentist approach. Finally, we use a joint distribution of stocks and bonds to estimate the regime distribution. This approach is then compared to classic stock only distribution and the distribution including the dividend yield. We perform a thorough analysis estimating 4 models with mean and covariances dependent of regimes and 4 regimes additionally including a VAR(1) component with the dividend yield.

Our work differs from Kole and Dijk (2016) in several aspects. The authors perform a comparison of a set of regime dependent models both with constant and time-varying transition probabilities, but they limit themselves to models with just two and three regimes. In our opinion it is interesting to include 4 regime models into the analysis as Guidolin and Timmermann (2007) prove that they fit the data very well. What is more, the authors only mention the specification of Maheu et al. (2012) but do not include it in their evaluation, this is in fact something we want to explore the most. Next, the authors use rule based semi-parametric (they set a value for minimal changes in prices for the regime to switch) and parametric regime switching models, whereas we use a joint-distribution of both stocks and bonds. Finally, in their analysis the investor can choose from stocks and a risk free rate only, we expand the range of assets with long-term bonds.

We first perform a static allocation based on the model estimates from the whole sample. This test's purpose is to answer the question of many long term investors "what assets should I invest into?". We set an allocation framework similar to the famous paper of Barberis (2000) results of which became a point of reference both for academia and investors. We compare the results with other regime switching models as well. The results are in opposition to the main results of Barberis. In fact, it is not always optimal to increase allocation to stocks with time, even when predictability is taken into account. In a linear framework, predictability from variables such as the dividend yield lowers risk in longer horizons, increasing the allocation to stocks. Regime switching has an opposite effect, return innovations and future expected returns have a negative correlation leading to a reduced allocation. In other words, we know that the economy will not always remain in a given state. Even in a strong bull trend, we should be aware that the economy might finally fall into a bear state. Both effects have an impact on asset allocation resulting in different shapes of the allocation schemes depending on the initial state and time horizon.

We implement these findings into an economic evaluation in order to determine the best model. We choose between 10 models in total. The classical IID model, the linear non switching VAR model, models with regime switching mean returns and volatility in 4 regime configurations - 2,3 and 4 regimes and 4 regimes with bull rallies and bear market corrections and, finally, models in the same regime configurations but with explanatory variables in a VAR(1) configuration.

We find that the performance of regime dependent models often depend on the investment horizon. Adding regimes does not necessarily lead to better results, especially depending on the time horizon. Models with just 2 regimes perform better in short horizon than models with 3 or 4 regimes, which on the other hand give better results in the medium and long term. The main finding of the out-of-sample evaluation is that an elaborate extension of the model, with 4 regimes but the transition matrix modified to accommodate short time-rehearsals, gives the best results of all the models. Our model not only manages to fit the data very well, but also gives a great advantage in asset allocation. In short horizon the investor incorporates brief changes in asset return trends, whereas in the long term, the model give a better specification of the two main regimes - the bull and bear market. Thanks to that, the allocation in this horizon is more stable, especially because of the fact that in the long run a good prediction of returns distribution dominates the model market timing abilities. Interestingly, even in the longest period the asset allocation differ from the investor who ignores regime.

Despite these realistic implications the autoregressive component does not give economic advantages. Mean-switching models dominate their autoregressive counterparts both in testing and in the real life test of out-of-sample verification. Keeping the number of variables at an acceptable level, these models prove to be sufficiently complex to fit the data and are simple enough to be robust on estimation errors.

A common question when increasing the number of regimes is whether the model does not overfit the data. The same issue has been raised by Guidolin and Timmermann (2007). In order to address that problem and perform a final check on our constrained model, we use a historical bootstrap to compare 4 models - the IID model, a 2MS and 4 MS model and the constrained 4 regime model. Unlike the paper of Guidolin and Timmermann (2007) who use a parametric bootstrap based on the estimates of the 4 regime model, we use a historical bootstrap from Politis and Romano (1994) as it does not bias the results in favor of that model.In order to include dependence in the data we use a block bootstrap where the optimal block length is chosen based on the method described in Politis and White (2004). The results show a good performance of both 4 regime models in the short run, however in the long run all models are outperformed in the long run by the constrained 4 regime model, which proves our model to allow a good data fit resulting in a good return distribution forecast.

This paper is organized as follows. Section 2 describes the data. Section 3 covers the methodology of model construction, testing procedures and portfolio calculations. Section 4 presents the results of model estimation, their tests results and the resulting static allocation values. Section 5 evaluates the out-of-sample performance of the models. Section 6 extends the model verification with a bootstrap. Section 7 concludes.

2 Data

Our analysis concentrates on a US investor considering three classes of assets: stocks, bonds and cash. In our research we use the popular data set furnished by Goyal & Welch covering a wide variety of variables. For stock we use the S&P500 Index end of month values from the Center for Research in Security Press. The stock returns are continuously compounded returns. For bond we use the long term return on bonds which is made from a portfolio of long term bonds from Ibbotson's Stocks, Bonds, Bills and Inflation Yearbook - it allows to maintain perpetuity even when some bonds were not issued for some periods of time. Finally for cash and the risk free rate used to obtain excess returns we use the ex post real T-bill rate calculated as the difference between the log return from 3-month T-bill and log inflation. The T-bill rates have been taken from Bloomberg, whereas inflation values from the Federal Reserve Economic Data. We replace that variable from Goyal & Welch, as they do not give precise information on the assets used for their riskfree variable. Stock and Bonds returns are excess returns calculated over the T-bill rate. The dividend yield or the D/P ratio is calculated as the log of the dividend paid by companies from the index through the last 12 months (sum of dividends from $t-11$ to t) divided by the index price. In order for an increased convergence, in the estimation process the D/P ratio is additionally divided by 100. Following the literature we use data after the Treasury Accord from 1951. Therefore our data set covers the period from January 1954 until December 2014 - the latest available update of Goyal & Welch. It gives us a sample o 732 observations. For the out of sample analysis we use a period of 30 years, from January 1985 to December 2014, covering among others the last financial crisis. The table below presents the summary statistics of the data used calculated on the full sample. Data summery statistics are reported in Table 1 below.

Table 1. Summary statistic

The table presents the average, standard deviation, minimum, maximum and the first order correlation of the ex post real T-bill returns, stocks excess returns, long-term bonds excess returns and the D/P ratio. The statistics are calculated on the full sample period from January 1954 to December 2014 and are reported in monthly units.

	T-bills	Stocks	Bonds	D/P
Average	0.0009	0.0049	0.0014	-0.0354
St. dev.	0.0028	0.0425	0.0279	0.0039
min	-0.0107	-0.2476	-0.1194	-0.0452
max	0.0179	0.1489	0.1348	-0.0275
AR(1)	0.5234	0.0592	0.0424	0.9939

We evaluate the stationarity of our variables by conducting for each one an Augmented Dickey-Fuller test. All predictor variables menage to reject the null hypothesis for the presence of the unit root, with the exception of the D/P ratio. However, we decide to not modify the variable. We find evidence in the literature supporting the fact that the variable is globally stationary (Chang et al. (2012)) and follow the same approach as Barberis (2000), Campbell and Viceira (1996) or Brandt (2010). We additionally analyse the impact of the variable on the model stationarity in the estimation results.

3 Methodology

In this chapter, divided into 3 subsections, we present the methodology used in our analysis. The first subsection describes the model construction process. In the second we thoroughly describe the testing procedures and we then describe the portfolio construction methodology in the third.

3.1 Model construction

Regime switching models, due to their elasticity and empirical value have been much researched by academics. They form a wide group of models thanks to their great variety of specifications depending on which parameter is subject to regime related variability. Although we work on several models, we can distinguish in our study two major groups of models' specifications. The first being MSMH(m) models and the second MSIAH(m)-VAR(p) models. The former model can be seen as a constrained form of the latter where p, the parameter determining the number of lags in the autoregressive component, has been set to 0 with only the intercept or mean component remaining. Additionally, in both cases the volatility is also regime-dependent allowing for a regime-switching heteroskedasticity. For models without the VAR component, the mean and intercept are identical and can be used interchangeably, therefore MSI and MSM are identical. It is no longer the case for MSI-VAR and MSM-VAR models. Krolzig (2013) shows that they differ in the dynamics of adjustment after a change in regime. As there are no premises to use a regime-dependent mean model we will follow a MSI framework for the sake of simplicity of model construction.

The first, simpler specification can be written as

$$\boldsymbol{r}_t = \boldsymbol{\mu}_{S_t} + \boldsymbol{\epsilon}_t \tag{1}$$

Where $\mathbf{r}_t$ denotes the $n \times 1$ vector of excess asset returns $(r_{1t}, r_{2t}, ..., r_{nt})'$, $\boldsymbol{\mu}_{S_t}$ is the vector of asset means in state S_t which takes integer value between 1 and k according to the number of regimes. $\boldsymbol{\epsilon}_t$ which is the vector of error component has a distribution $N(0, \Omega_{S_t})$ with Ω_{S_t} being the covariance matrix dependent on the regime S_t. When k=1 this simplifies to a simple linear model, which will often serve as a benchmark.

The second group of models extends our model with an autoregressive component. Recent papers (e.g. Barberis (2000)) have shown strong evidence supporting the predictive power of dividend to price ratio on stock returns. We choose only one predictive variable based on the strong support of the litterature and the fact that other variables would highly increase the number of estimated parameters. Also Guidolin and Ono (2006) find no evidence of dynamic linkages between macroeconomics fundamentals and financial markets. We will implement this variable in our model with a VAR framework of order 1 which can be generally written as:

$$\begin{pmatrix} r_t \\ z_t \end{pmatrix} = \begin{pmatrix} \mu_{S_t} \\ \mu_{zS_t} \end{pmatrix} + \sum_{j=1}^{P} A_{j,S_t} \begin{pmatrix} r_{t-j} \\ z_{t-j} \end{pmatrix} + \begin{pmatrix} \epsilon_{S_t} \\ \epsilon_{zS_t} \end{pmatrix} \tag{2}$$

In this case μ_{S_t} and μ_{zS_t} denote intercepts vectors of R_t and z_t in state S_t, A is a matrix of autoregressive coefficients in state S_t and $(\epsilon_t\ \epsilon_{z_t})' \sim N(0, \Omega_{S_t})$ is the vector of error terms with covariance matrix Ω dependent from the actual regime.

The main idea behind models known as regime-switching (Hamilton (1989)) or Markov-switching, is that the parameters of the model of the observed time series vector r_t depend on a latent process S_t, which represents the probability of being in a given state of the world, although we can never be sure about the regime which actually prevailed.

The main assumption in this type of models is that the switches in the unobservable realizations of regimes are driven by a discrete time and state Markov stochastic process, defined by the transition probability matrix P with elements:

$$p_{ij} = Pr[S_t = i | S_{t-1} = j], \quad \sum_{j=1}^{k} p_{ij} = 1 \tag{3}$$

In our model we allow only for constant transition probabilities. We motivate this choice by the fact, that we wanted to see the performance of the constrained 4 regime model and its advantages in a relatively simple framework. Furthermore, a big part of our work is a long time forecast for the purpose of a long-term investment strategy. In a time-varying transition probabilities model these variations are driven by an information variable such as economic fundamentals. The process of forecasting would require an estimation process for these variables themselves which is far beyond the scope of this paper. Last but not least, the whole estimation process is sufficiently computationally burdensome as already in its current state it requires some trade off in the number of simulations performed.

In order to estimate the parameters of our model we need to make some inference about the state probabilities conditional on the observed returns. We can do so using the Bayes' rule and transition probabilities. The filter derivation is presented in Hamilton (1989), in our work we use the notation from Kole (2010).

$$\xi_{t|t} = \frac{1}{\xi'_{t|t-1} f_t} \xi_{t|t-1} \odot \boldsymbol{f}_t \tag{4}$$

$$\xi_{t+1|t} = \boldsymbol{P} \xi_{t|t} \tag{5}$$

The above recursion allows us to obtain the inferred probabilities $\xi_{t|t} = Pr[S_t | R_t]$ and the forecast probabilities $\xi_{t|t-1} = Pr[S_{t+1}|R_t]$ where $\boldsymbol{f}_t$ is a vector of observation densities conditional on the regimes. A straightforward maximum likelihood estimation would imply maximizing

$$\mathcal{L}(r_1, r_2, ...r_T, \boldsymbol{\theta}) = \sum_{t=1}^{T} log(\boldsymbol{\xi}'_{t|t-1}\boldsymbol{f}_t) \tag{6}$$

However that can be computationally burdensome, especially that we would need to know which regime was relevant at each point in time, that is why we need to get some expectation on the probability $Pr[S_t = s_t|r_1, r_2, ...r_T, \boldsymbol{\theta}]$. We therefore use the Expectation-Maximization (EM) algorithm of Dempster et al. (1977). This method is especially useful as it allows to treat the underlying state variable which is unobservable as latent and iteratively updates the parameter estimates (Hamilton (1989)).

The EM Algorithm is a two-step iterative Maximum likelihood estimation technique. Dempster et al. (1977) and Hamilton (1989) prove that results obtained using that method converge to the maximum likelihood estimation of (6). The estimation rely on a recursion, where the expectation and maximization steps are treated separately. In the Expectation step the unobserved states ξ is estimated with its expectation conditional on the data and the parameters i.e. the smoothed probabilities. In the second step we estimate the parameters θ through the derivation of the first order conditions of the likelihood function with respect to the parameters where the regime probabilities are replaced by their expectations from the last iteration. The method is quite simple for the estimation of the MSMH models, as the number of parameters to estimate is quite low even for a higher number of regimes. However, that is no longer the case for MSIAH-VAR models as the autoregressive component highly increases the number of those parameters. The parameter vector θ can be decomposed to the structural parameters γ itself consisting of the intercept and autoregressive parameters.

We follow the method presented by Krolzig (2013) who presents the estimation procedure in a neat way. The ML estimates are obtained through the differentiation of the likelihood function with respect to the parameters:

$$\frac{\partial \mathcal{L}(r_1, r_2, ...r_T|\boldsymbol{\theta})}{\partial \gamma} = -\frac{1}{2}\frac{\partial \boldsymbol{u}'W^{-1}\boldsymbol{u}}{\partial \gamma} = -u'W^{-1}\frac{\partial \boldsymbol{u}}{\partial \gamma}. \tag{7}$$

Where W^{-1} is the Kronecker product of the smoothed probabilities diagonal vector and covariance matrix and $\mathbf{u}$ is the vector of residuals. When we substitute $\boldsymbol{u} = \mathbf{1}_M \otimes \boldsymbol{y} - \boldsymbol{X}\gamma$, where $\mathbf{X}$ is the matrix of explanatory variables, and set the equation to zero, we obtain

$$\gamma = (\boldsymbol{X}'W^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}'W^{-1}(\mathbf{1}_M \otimes \boldsymbol{y}), \tag{8}$$

which we can see is the GLS estimator weighted with the smoothed probabilities $\xi_{t|T}$. The issue with such function form involves operations on high dimensional matrices (like $MTK \times MTK$) which can be computationally burdensome, especially in out-of-sample analysis, where on each period of time, the whole model has to be reestimated. However, as noted by Krolzig (2013), when all parameters are regime-dependent, meaning there are no mutual parameters, we can estimate separately each vector γ_m specific for each regime m.

$$\gamma_m = ((\boldsymbol{X}'\Xi_m\boldsymbol{X})^{-1}\boldsymbol{X}'\Xi_m \otimes \boldsymbol{I}_K)\boldsymbol{y}. \tag{9}$$

The estimation of the variance follows the same principle of maximizing the likelihood function with respect to the parameter σ giving the following formula:

$$\Sigma_m = T_m^{-1}\boldsymbol{U}_m'\Xi_m\boldsymbol{U}_m \tag{10}$$

The iterations of the above algorithm should be performed until a convergence criterion is met. In our case we perform the estimations until the increase in the loglikelihood is lower than 10^{-8}.

Although simple, the model allows to accommodate many important properties of asset returns like volatility clustering through regime dependent heteroscedasticity or non normal distributions of returns among other stylized facts. These properties are especially important when it comes to asset allocation implications.

The model construction allows for an easy augmentation of regimes, however the literature study shows us that increasing the regimes is not always the solution for a better fit. Therefore we analyse 4 different sets of regimes: (i) the popular two-regime model proposed by Ang and Bekaert (2002a), (ii) a model with 3 regimes similar to Guidolin and Timmermann (2005a) where they however concentrate on UK stocks and bonds, (iii) a four-regime model in the paper of the same authors from 2007, where they argue that 4 regimes are required to capture the joint distribution in both stocks and bonds and (iv) we analyse the performance of the model similar to the one proposed by Maheu et al. (2012). The authors notice that in a normal two-regime model switches between states tend to occur too often although it can be seen from a historical perspective that it was just a short time reversal and not a real change in the state of the economy. Those short periods are called bear market rallies (for bear regimes) and bull market corrections (for bull market) and are common market reactions. The authors of the paper want to deal with this misleading movements. They propose a model which accommodates those short-term reversals. Those inter-regime transitions can be modeled by modifying the transition probability matrix and by imposing parameters restrictions. The authors work on weekly data which can be more susceptible to such movements and use a Bayesian approach to the problem with informative priors. We want to check if such model specifications can be used in monthly intervals and verify it in a frequentist approach. Such model can be constructed with restrictions on the transition probability matrix. Assuming that state 2 is the bull market correction we unable switches to bear regimes i.e states 3,4. The same analogically applies to bear market rallies. The matrix will then have the following form:

$$\begin{bmatrix} p_{11} & p_{12} & 0 & p_{14} \\ p_{21} & p_{22} & 0 & p_{24} \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & p_{43} & p_{44} \end{bmatrix}$$

$$\sum_{j=1}^{k} p_{ij} = 1$$

Each regime - bull and bear markets - have 2 states that allow for periods of positive and negative stock returns within each regime. We therefore impose:

$$\text{Bear regime} \quad \begin{cases} \mu_1 < 0 \quad \text{(bear market state)}, \\ \mu_2 > 0 \quad \text{(bear market rally)}, \end{cases} \tag{11}$$

$$\text{Bull regime} \quad \begin{cases} \mu_3 < 0 \quad \text{(Bull market correction)}, \\ \mu_4 > 0 \quad \text{(Bull market state)}, \end{cases} \tag{12}$$

We do not impose restrictions on variance and bond returns. This specification allows to capture short-time rehearsal. Furthermore due to different variances in all states, it allows for bear and bull regimes to accommodate heteroskedasticity and different higher moments.

Standard errors in the models are calculated by means of numerical differentiation. The covariance matrix is calculated using the first derivatives of the log likelihood (Outer product matrix).

A popular package for MATLAB named MSRegress written by Perlin (2015) allows to estimate regime switching models with constant probability matrix. The package however does not use the EM algorithm and instead uses numerical maximization methods like the fmincon functions. In most cases both algorithms yield to similar values, however the code written for the purpose of this paper is 294 times faster than the MSregress package for a four-regime model and even 1257 time faster for a four-regime VAR model.

A popular method to maximize the chances of finding a global maximum is to begin the estimation with the EM algorithm and then maximize the likelihood instead of its expectation. We recognize the advantages of this method, however for our analysis it is too computationally demanding. Given our hardware limitations we restrain to the EM algorithm. The probability of finding local maxima is minimized by taking different starting points for the estimation and using the one giving the highest likelihood. We check that our models are robust to different initial values and manage to find the global maxima from different values. Furthermore we compare the results with the numerical maximization method of Perlin (2015) and find that our models find the same maxima in different subsets of data. Finally, the negative effects of finding a local maximum are minimized in the out-of-sample evaluation as the model is estimated each month in a slightly different dataset, which limits the chances of getting stuck in one local peak.

3.2 Testing

Although most authors limit themselves to testing the models by comparing information criteria like the BIC or AIC as in Bae et al. (2014) some more advanced specification tests are available. We will be mostly using the Likelihood Ratio as for most hypotheses the distributions remain unchanged. This is not the case when we test for the number of states as we cannot rely on the standard asymptotic distribution theory due to the presence of nuisance parameters under the null hypothesis. For instance when we test an invariant model compared to a two-regime model, and so under null test that $\mu_1 = \mu_2$, we have the parameters of the transition matrix which remain unidentified causing the information matrix to be singular. Most papers rely on Hansen (1992) and Garcia and Perron (1996) tests. By writing the likelihood as a function of the nuisance parameters Hansen elaborated a bound test for the asymptotic distribution of the standardized LR statistic. The test requires demanding simulations, that is why Garcia and Perron (1996) proposes a simplified testing methodology. By limiting the simulations to a grid of transition probabilities whereas all other parameters are ML estimates, the computational burden is significantly decreased. In our work we will use the test of Carrasco et al. (2004). The test is equivalent to a Likelihood ratio test, hence asymptotically optimal. The test says whether the parameters θ_t are constant over time. The hypotheses can be written as:

$$H_0 : \theta_t = \theta_0 \tag{13}$$

against

$$H_1 : \theta_t = \theta_0 + \eta_t \tag{14}$$

where η is the switching parameter and is not observable. First we derive the following expression sticking to the original notations:

$$\mu_{2,t}(\beta,\theta) = \frac{1}{2}\Big(tr((l_t^{(2)} + l_t^{(1)\prime} l_t^{(1)})E(\eta_t\eta_t')) + 2\sum_{s<t} tr(l_t^{(1)} l_s^{(1)\prime} E^{\beta}(\eta_t\eta_s'))\Big) \tag{15}$$

where tr denotes the trace, $l_t(\theta)$ the conditional log-likelihood of the tth observation and $l_t^{(1)}(\hat{\theta})$ and $l_t^{(2)}(\hat{\theta})$ the first and second derivative wrt to the switching parameters calculated at $\hat{\theta}$, the MLE for θ under the null hypothesis, η as the switching parameter which is latent is drawn from a unit sphere. β is as well a nuisance parameter and as it is not identified under H_0, we draw it from the subset B in the interval [-0.7,0.7]. The test statistic TS is calculated as follows:

$$TS_T(\beta) = TS_T(\beta,\hat{\theta}) = \Gamma_T - \frac{1}{2T}\hat{\epsilon}(\beta)'\hat{\epsilon}(\beta) \tag{16}$$

where $\Gamma_T = \frac{1}{\sqrt{T}}\sum_t \mu_{2,t}(\beta,\hat{\theta})$ and $\hat{\epsilon}(\beta)$ is the residual from an OLS regression of $\mu_{2,t}(\beta,\theta)$

on $l_t(\theta)$. As β is unknown under H_0, the authors suggest using a sup-type test i.e. to take the supremum of the test statistic over a subset B of all possible values of the nuisance parameters.

$$supTS = \sup_{\beta \in B} TS_T(\beta), \tag{17}$$

or exponential-type test

$$\int_B \exp\big(TS_t(\beta)\big) dJ(\beta), \tag{18}$$

In order to get critical values and the p-values we perform a parametric bootstrap of Davidson and MacKinnon (2004) with steps neatly presented in the Carrasco et al. (2004) paper. Using the estimated parameters from the previous part θ_0 we generate N independent samples $\{y_1^n, ..., y_T^n\}_{n=1,...,N}$, where each $y_t = \mu + \sigma^2 * \epsilon$ and ϵ is drawn from the normal distribution. We perform 1,000 iterations for the series, which is sufficient to five accurate results. Next, we reestimate for each sample the $\hat{\theta}$ under H_0 by means of ML and compute the resulting expTS and supTS statistics. Finaly, using all the obtained statistics we can get the critical values which are the (1-α) quantile. The p-value from the bootstrap is calculated as $\frac{1}{N}\sum_{n=1}^{N} I(expTS^n > expTS)$.

The CHP test answers the question whether the dependencies between regressors and dependent variables are linear or regime depending. In order to analyse how many regimes are best to fit the model and whether the autoregressive component is needful, we supplement the test by comparing the models in terms of loglikelihood values, Akaike and Schwarz Criteria and perform a likelihood ratio test.

3.3 Portfolio construction

In our work we investigate a buy-and-hold strategy similar to the case proposed by Barberis (2000). As described at the beginning of this paper, we focus on passive investment strategies. It is a common mode of action for many market participants who just want to allocate their savings and earn a safe profit without frequent rebalancing, it is also the reason why we choose only 3 classes of assets as a simplistic choice for such an investor. A buy-and-hold strategy is also enough to prove that our model with constraints is able to outperform other models.

The methodology of calculating portfolio weights rely on the work of Barberis (2000). The investor preferences over terminal wealth are described by constant relative risk-aversion power utility functions of the form:

$$u(W) = \frac{W^{1-A}}{1-A} \tag{19}$$

We can write the cumulative stock excess return as:

$$R_{t+T} = r_{t+1} + r_{t+2} + ... + r_{t+T} \tag{20}$$

Given that the $W_t = 1$ and ω is the vector of portfolio weights the terminal wealth is equal to:

$$W_{t+T} = (1 - \omega\iota)exp(r_f T) + \omega exp(r_f T + R_{t+T}) \tag{21}$$

Therefore the investor solves the following problem

$$\max_{w} E_t \left(\frac{[(1 - \omega\iota)exp(r_f T) + \omega exp(r_f T + R_{t+T})]^{1-A}}{1-A} \right) \tag{22}$$

The subscript t in the expectation denotes the fact that we calculate the expectation conditional on the information set until time t. It is especially important to clarify the notation for the evaluation of the models out of sample.

In order to calculate weights maximizing the investors utility we use Monte Carlo simulations. We simulate N=30,000 paths of returns. Each path is calculated based on the following algorithm:

1. We assume that the current period is t. We use either the inferred, steady or equal probabilities and draw a uniform number. If it is below the probability for state 1 we are in state 1 if below state 2 (the sum of probabilities of state 1 and 2) we are in state 2 etc.

2. We draw again a uniform number. Based on the regime simulated in the previous step we use the probabilities from the transition matrix and simulate the next step in time t+1. We then assign the returns based on the model estimates and draw the error terms from a normal distribution multiplied by the covariance matrix.

3. We draw a value for $\mathbf{r}_{t+1}$ given the state simulated in the previous step and using the model estimates.

4. Repeat steps 2 and 3 for T periods.

5. Simulate all the steps for 30,000 paths.

The number of paths is the result of the trade-off between estimation accuracy and computational limitations related to the economic evaluation. We check that 30,000 is enough to guarantee stable weight repartition and that differences between estimations based on bigger samples have a minor impact on the results. At each step of the simulated path we draw from the estimated transition probability matrix, that allows for shift in regimes. Monte Carlo simulations are particularly useful in our multivariate framework as they do not suffer from the curse of dimensionality. Having the returns paths we calculate cumulative returns and the weights maximizing the utility from equation (22).

4 Results

In this section we will present the estimation results from models described in the previous section. We report the results in 3 subsections analogous to the methodology description. We begin with the presentation of model estimates, then the testing results and finally with the description of portfolios constructed based on model estimates. All the values in this section are calculated on the full sample i.e from January 1954 till December 2014..

4.1 Model estimates

In this section we will present the results of the models estimation. For the sake of clarity we present the results in 2 subsections. First we report the results for 3 mean-switching models and then for the autoregressive models. We give the proof of such selection in the following section and 4.2 describing the results of the testing procedures. We present the estimates in Table 3 in the Appendix in Panels B-E.

4.1.1 Mean-switching models estimates

We see in Panel B the estimates for the two-regime model. The mean excess stock return in the bull market is 0.85% and -0.48% for bear regime. For bonds it is respectively 0.04% and 0.43%. It clearly shows the following relation. When times are bad, the stock market is plummeting as a general manifestation of a recession in the economy. In order to boost the economy, the central bank lowers the interest rates which increase the prices of long term bonds. Furthermore, in times of uncertain markets the investors look for a safe harbor for their money. In a behaviour called "flight to quality" they try to purchase safe assets which are often government backed securities. By increasing the demand on bonds they increase their prices. That is why in the bear market the bond excess returns are high. Also the volatility is properly estimated. As we would expect in bear markets the volatility of assets is much higher than during bull market and the correlation higher.

In case of the four-regime model presented in Panel C of Table 3, we differentiate two regimes with negative returns one which has mean excess stock returns at the very low level of -0.73% and which occurs after regimes with positive returns making of it a crash regime and another with mean -0.5% which we could assign to the bear regime. The regimes differ also in bond mean returns. The crash regime has mean bond returns of 0.16% much lower than the bear regime. This could appear counter-intuitive, as with bad times the bonds prices would be expected to grow. However, as we already said, it is a regime occurring after growth periods, therefore we can assume that the FED would take some time to make changes in interest rates and therefore it is more visible in the bear regime. The third regime has mean excess stock returns of 0.73% and bond 0.07% - it is a normal growth regime also prevailing for most of the time - 66% of the sample. Finally we have a state with high returns on stocks and low or negative for bonds, that would be the bull regime. The volatility behaves in a good way - with the highest values in the crash and bear regimes. Questions might arise whether 4 regimes is not too much and if we do not overfit the data. A similar question has been raised in the paper of Guidolin and Timmermann (2007). We deal with this question in section 6 of the paper.

We follow with the results of the constrained four-regime model, where the transition probability matrix has been restricted in a manner that there is a limited possibility of transition to regimes 2 to 3 which are described as intra-regimes movements. The results are presented in Panel D of Table 3 in the Appendix. Obviously the constraints give a lower log likelihood, but the idea is that those restrictions will allow a better specification of the bull and bear states as they cover the intra-regime changes. It is also hoped that they allow a more robust asset allocation which could be verified economically out-of-sample. It is the model that has not been used in an asset allocation until now.

In this specification the bear regime has a mean stock excess return of 0.38%. It consists of two states, the actual bear state with mean stock returns of -0.25% and a bear market rally with high stock returns at the level of 1.62%. The bear regimes lasts around one third of the total bear regime with the bear states prevailing on its own for around one fourth of the total sample. The bull regime has a total mean of 0.5%. The stock returns in the bull state elevate to 0.79% and in the bull market correction to the very low -1.04%. We see that the means of the complex regimes are closer to each other and the values less extreme than in the case of the normal two-regime model, however, it has to be reminded that each regime is composed of two separate ones which values can be seen in Panel D of Table 3 . We notice that there are periods of very low and very high returns, but they are simply mitigated by other intra-regimes. Also given this specification the bull and bear market lasts longer than in case of a simple two-regime model. In Figure 1 in the Appendix we plot the smoothed probabilities calculated using the 4 regime constrained model as well as the probabilities of the 2 states model. It allows us to see that the constrained model better fits the periods of recession, although we would expect a better differentiation between bear regimes and bull market corrections.

4.1.2 Autoregressive models estimates

We now present the remaining selected models including an autoregressive component which is allowed to vary across regimes. As mentioned in subsection 3.1, the lagged values are the stock and bond market returns and the popular predictive regressor, the dividend to price ratio also called the dividend yield. Although in the majority of cases this component has been proven to be statistically insignificant, we present the only case where it was. In order to give the reader a presentation of the structure of the VAR models, we start by the simple VAR(1) model independent of regimes, which is also one of our benchmarks.

The multivariate least squares (MLS) estimates are as follows:

$$\begin{pmatrix} r_t \\ z_t \end{pmatrix} = \begin{pmatrix} 0.0286 \\ -0.0070 \\ -0.0003 \end{pmatrix} + \begin{bmatrix} 0.0518 & 0.1398 & 0.6834 \\ -0.0958 & 0.0588 & -0.2496 \\ -0.0005 & -0.0014 & 0.9921 \end{bmatrix} \begin{pmatrix} r_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t \\ \epsilon_{S_t} \end{pmatrix},$$

$$\Omega = \begin{bmatrix} 0.0018 & 1.44e^{-04} & -1.77e^{-05} \\ 1.44e^{-04} & 7.60e^{-04} & -1.37^{-06} \\ -1.78e^{-05} & -1.37e^{-06} & 1.82e^{-07} \end{bmatrix},$$

where $\mathbf{r}_t = (r_{1t}, r_{2t}, ..., r_{nt})'$. These estimates show us that higher dividend-to-price ratio forecasts positive stock returns. Also it is worth noting that the D/P ratio is highly persistent with the estimate of 0.9921. We calculated the asset allocation for a range of starting values of the D/P ratio. We notice that the higher the D/P ratio is, the higher will be the allocation to stocks. Also for the unconditional mean we find that the longer the horizon the higher allocation to stocks is predicted, which is in line with earlier findings. As already noticed by Campbell and Viceira (2002) and Guidolin and Timmermann (2007) the D/P ratio is very low after 1993, reaching values of -4.5 in the early 2000s compared to the unconditional mean of -3.4. This results in a very small intercept and close to one coefficient for D/P ratio, resulting in a small impact of the variable especially for short horizon effects. Estimating the same model on a shorter period, when D/P ratio values were much higher until the early 1990, those effects were much more pronounced and therefore the allocation on stocks was much higher. Nevertheless, we will now cover the results estimated on the full sample period until December 2014. The described exercises prove the direct impact of the level of the D/P ratio on the asset allocation. Therefore instead of spanning its mean for simulation on a range of values, we simply use the all sample unconditional mean saving computational time.

The estimates of the four-state model with no constraints on the transition probabilities matrix are given in Table 3 Panel E from the Appendix. We have again a well distinguished bear regime with highly negative excess stock returns which account for 23% of the time. However, we have 3 regimes with positive stock returns. We could differentiate a normal growth regime with 0.43% mean excess stock returns which spans over 56% of the sample period. The two remaining regimes are interesting as they both have high mean stock returns: 1.55% and 4.12%. Nonetheless only regime 3 should be seen as a a bull regime. It has negative bond returns which is typical in time of prosperity on the market and it lasts longer - on average 16 months. The other is a short state lasting on average only 2 months. When we look on the transition probability matrix we see that transitions between regime 1 and 4 have high probabilities showing frequent changes between those states. This results are similar to the recovery state differentiated by Guidolin and Timmermann (2007) with the difference that it lasts for a shorter period and has higher probabilities of returning to the bear regime, which is an outcome similar to bear market rallies described in the previous chapter. One could notice the high parameter estimate for the D/P ratio in the recovery state. Indeed it might suggest that the variable might be explosive, however this regime lasts on average only 2 months, therefore the model remains stationary. This aspect has already been discussed in section 2.

VAR models have a large number of variables highly increasing the estimation errors. It is therefore hard for them to compete in the testing procedures with the mean-switching models. Nevertheless it is interesting to see the performance of the predictability component in the asset allocation exercise.

4.2 Testing results

We present the results from the testing procedures described in section 3.2. We begin with the results of the Carrasco et al. (2004) test presented in Table 4 in the Appendix.

From these results we can unequivocally reject the hypothesis of constant mean and volatility in stock returns. As already mentioned, the test is for the stability of parameters in the framework of MS models where the null hypothesis is linear. Therefore, it is used for answering the question whether the regimes are needed to fit the data, but we do not answer the question how many regimes are best to do it. To answer that question we compare the models under the Akaike and Schwarz Information criteria. Table 5 in the Appendix regroups the log likelihood values and information criteria for the set of our models. The log likelihood values can be compared as both sets of models (mean and VAR models) have the same dependent variables - stock and bonds returns. A likelihood ratio test is additionally performed.

The results in Table 5 show an important feature. Although VAR models increase the likelihood they struggle to beat the MSM models in an Information Criteria comparison especially unnder the Bayesian Information Criterion which strongly penalizes the number of regime. Also when we compare MS and MSVAR models under the Likelihood Ratio, the results show that the autoregressive component does not give statistical improvement, with the only exception being the MS4 VAR model. We therefore concentrate on the MSM analysis in further parts.

4.3 Portfolio allocation results

Before we move to the out-of-sample evaluation of model results, we describe the asset allocation results calculated based on model estimates presented in subsection 4.1 where the full sample was used. These are therefore the portfolios that a buy-and-hold investor would invest in at the end of December 2014. All results are presented in Table 6 in the Appendix. It is important to analyse these results, as they may answer the question of many individual investors, who face an investment dilemma, especially when they do not seek any active investing in the meantime. The results are also interesting, as they may be confronted with the results from Barberis (2000) or Canner et al. (1994) who showed model portfolios recommended by 4 investment advisors in the 1990s. Additionally we see the 6 different horizons for which the weights are calculated. Finally, the analysis is performed for every initial regime and for steady states probabilities, which allows us to analyse the behavior of investments strategies depending on the horizon but most importantly depending on the current state of the economy.

Again we present the results only for the models we have selected in the testing procedures. The Schwarz Criterion strongly penalizes models with high number of parameters and so it was more restrictive for MS VAR models. Thus, we analyze only one VAR model: the four-state VAR model where the tests have proven for the autoregressive component to be significant and which outperforms its counterpart in terms of likelihood and AIC. It is also the model estimated as the best MS specification by Guidolin and Timmermann (2007). Hence we present the results for four models: MSM(2), MSM(4), MSI(4)VAR and MSM(4) under transition constraints.

4.3.1 Asset allocation results of mean-switching models

We will discuss the weights distribution depending on the current regime. We begin with the 2 regime model asset allocation results.

When the economy is in the bull regime, the biggest share is associated to stocks with 100% in 1 month horizon gradually decreasing to 49% in stocks in the longest - 120 month horizon. When allocation to stock decreases, intuitively, the allocation to bonds and T-bills increases, from respectively 0% and 0% in the 1 month horizon to 40% and 11% in the 10 years horizons. When we start in the bear regime allocation to stock starts at the 0% level for growing up to 40% in the longest horizon. Bond allocation starts around 58% and gradually diminishes to 43%. The decrease in T-bills is even more pronounced from 42% to 17%. As for the steady state allocation it allocates 63% in stocks, 36% in bonds and 1% in the T-bill for the 1 month horizon and changes to 45%, 41% and 13% in the equivalent assets. The slow convergences are due to the fact that the regimes are characterised by high persistence. The bear regime persisted 27% of the time and the bull 73%. These results can be contrasted with a 69% allocation to stocks in Barberis (2000) estimated on a similar period, however with only 2 asset classes - stocks and treasury bills - without taking bonds into account. Therefore with bonds being another risky asset, the results from Barberis are less insightful for practitioners.

In case of the four-regime model, both bearish regimes allocate 0% to stocks in the short periods. The allocation then rise to almost 40% in the crash regime and to 31% in the bear state - this is explained by the fact that the crash regime is less persistent. An even bigger difference between these regimes can be observed for bond and t-bills allocation. In the first regime the results suggest to allocate up to 78% in T-bills and 22% in bonds in the 1 month horizon. This allocation gradually diminishes for t-bills settling at around 30% in the 10 year horizon for both bonds and T-bills. In the bear regime the highest allocation is assigned to bonds (100%) falling with time. The normal growth regime starts with a full allocation to stocks in the shortest regime decreasing to 45% in stocks, resulting in a 34% bonds and 21% bills allocation. The bull market state has a very high allocation to stocks for a very long period. The steady state allocation shows a decreasing allocation to stocks starting at 55% and moving to 44%. Bonds allocation is constant around 40% with a slight increase for intermediate horizons.

Finally, we can see the results of the constrained four-regime model. When we are in the bear regime the investor would allocate almost all of his investments in bonds. With the time horizon expanding he would gradually switch to stocks to finish with 41% in stocks, 46% in bonds and 13% in cash. It is interesting to compare this state with the bull correction. Both regimes are characterized by negative stock returns and so in both the investor would start with almost null allocation to stocks, but in the bull correction the investor would rather invest in cash. Given the fact it is a short trend it is smart not to switch to bonds and safely stay with cash and wait to see how the situation will evolve. On the opposite side, in the bull market the investor would invest everything in stocks in the short term gradually moving to cash and bonds. It is interesting to see that in the bear rallies the model would advise to keep for a much longer horizon a full investment in stocks moving to bonds only after 2 years. It is counter intuitive given the lower persistence of that state, but it might be explained by very high stock returns. The steady state probabilities for the constrained model advise a similar allocation to the 4 regime model with 43% in stocks, 32% in bonds and 25% in Cash.

4.3.2 Results for Asset allocation under predictability

When calculating the asset allocation using the dividend yield we come across the problem of choosing the initial value for the paths simulation. We check the allocation for a range of starting values of the D/P ratio. We notice that the higher the D/P ratio is, the higher will be the allocation to stocks. As already noticed by Campbell and Viceira (2002), Barberis (2000) or Guidolin and Timmermann (2007) the D/P ratio is very low after 1993, reaching values of -4.5 in the early 2000s compared to the unconditional mean of -3.4. This results in a very small intercept and close to one coefficient for the D/P ratio, resulting in a small impact of the variable especially for short horizon effects. Estimating the same model on a shorter period, when D/P ratio values were much higher until the early 1990, those effects were much more pronounced and therefore the allocation on stocks was much higher. The described exercises prove the direct impact of the level of D/P ratio on the asset allocation. Having that in mind we keep the initial value fixed at the historical mean of the sample as the most representative.

We will now discuss the only VAR model chosen for further analysis - the 4 regime VAR model. The portfolio weights are given in Table 6 in the Appendix. We notice that in the bear regime at short horizons the allocation to stock is minimal, although because of the high probability of corrections it is not null. Bond allocation is at the level of 55%. With time horizon expanding the allocation to stocks increase at the cost of stock allocation. In other states given positive stock returns at the beginning almost everything is invested in stocks and decrease with time. The only difference is the "recovery" state which despite having the highest stock returns gives not the signal to a full investment in stocks. The reason might be a high probability of returning to the bear regime. All allocations converge to the steady state values around 60% to stocks and 20% to bonds.

It is interesting to note that these results are quite similar to the ones presented in the paper of Canner et al. (1994) in which 4 investment advisers gave their recommendations on asset allocation. The mean weights for a moderately risk averse investor were 50%, 40% and 10% respectively for stocks, bonds and cash. The weights for the steady states give a similar outcome. It is quite different from the results of Barberis (2000) who advise to hold much more in stock for such a long horizon. For a moderate investor, when parameter uncertainty is not taken into account, Barberis suggest a full investment in stock beyond 5 years.

In the following chapters we will cover the results of the forecasting and the out of sample verification of the models.

5 Economic verification

In this chapter, we test the performance of our models in an out-of-sample asset allocation performance. As we estimate the model until December 1984, the out-of-sample period spreads on 30 year i.e. 360 months. We also investigate the impact of the initial state on the strategy returns.

5.1 Initial state

When calculating the return paths it is crucial to assume an initial state of the economy. This is particularly important as we are never sure in which regime we really are. As shown by Veronesi (1999) this uncertainty can explain the asset price dynamics as it is common that investors overreact to bad news in good times and underreact to good news in bad times.

Based on the literature we consider 3 approaches to the initial state selection. The first assumes the state probabilities to be equal, the second is using steady state probabilities and finally, the third is using inferred probabilities from the model. From the out-of-sample evaluation we find that the third method gives the best results and therefore we will present the results for this method only. This conclusion is very intuitive as we would expect that as last period returns should carry important information about the state.

5.2 Out of sample evaluation

In the previous parts we analyzed the model construction and the in-sample results. However, in order to answer the question which model is the most valuable and performs best, we need to test it out of sample. In order to calculate the return of each of the 10 models (including the benchmarks) for each period of the out of sample period, we estimate the model using an expanding window starting from January 1954 to December 1984. The reason we use an expanding window instead of a moving window is that we believe that the information in every period allows to update the model, whereas the ghost effect present in linear models is mitigated by the presence of regimes and in fact makes the regime estimates more accurate. For instance a severe crisis at the beginning of the data set would indeed impact the whole estimation in a linear model, dragging down the estimated stock mean returns, creating a so called ghost effect. However, when regimes are included and the recession state correctly identified, only the bear market estimates would be updated and for instance appropriately lowered, allowing for better forecasts at the brink of a new recession. A moving window would lack this information and could perhaps forecast too optimistic returns in bearish times.

We perform our economic verification in several steps. First we calculate the weights attributed to 3 classes of assets as described in section 3.3 for 6 different time horizons - 1, 6, 12, 24, 60 and 120 months. For a larger sample we use overlapping returns. Next, we calculate the results of each investment strategy based on real returns from the out of sample period and move one month ahead. The first iteration will therefore be estimated on the period from January 1954 till December 1984 and we use the estimates and state probabilities as of December 1984 to calculate the weights and next the portfolio performance for January

1985 (or a period of time spanning on several months depending on the time horizon). Then we repeat the estimation and allocation on a period from 01.1954 to 01.1985 for one month ahead (or other time horizon) and so forth. We iterate those steps for each month, for the whole out-of-sample period until December 2014.

Finally, we compare the performance of the estimated regime switching models with each other and a VAR(1) and a simple IID model with constant mean and variance.

Table 7 regroups the results. For each model we can observe its mean returns across all the data set for different horizons, the standard deviation, the annualized Sharpe ratio (in order to compare the risk-adjusted returns) and the utility which is maximized at each step. The one month Sharpe ratio has been annualized taking into consideration return autocorrelation following Lo (2002). For each regime switching model we evaluated three different results depending on the methodology used to determine the initial state. We present the results only for the initial state chosen based on the inferred probabilities as they proved to be the best setup in the majority of cases, with only a few exception in the shortest time horizons. The analysis relies on regime persistence, and the model investment signal for the next month is taken on actual information. Regime changes cannot be exactly anticipated and so when regimes changes occur frequently the advantages of regime switching models and inferred probabilities might be limited in favor of steady probabilities. It is clearly visible when we perform our analysis in a shorter out-of-sample period concentrating on the period of the financial crisis of 2008. However, these drawbacks are not as severe for all the models as the 4 regimes constrained model is constructed to deal with such frenetic movements. Nevertheless, besides small discrepancies, the inferred probabilities are the best framework based on the performance measures.

We compare the models mainly under the Sharpe ratio criterion and the average power utility. Based on the results we make these observations. First, the mean-changing models in general perform better than their VAR counterparts, especially for longer horizons. Second, the MS4 Constrained model strongly outperforms all others. Third, the VAR models give higher mean returns but these returns are mitigated by higher standard deviation.

It is interesting to note that the models with the autoregressive component do not outperform the mean-switching models, although we would expect them to excel since they use fully the data. However, it goes in line with the tests proving that the autoregressive component is very often not statistically significant. It also appears that VAR models do not benefit much from the increase in the number of regimes. On the contrary, it might be detrimental because of the estimation errors as the number of variables increases quickly. It is true that the VAR models give higher mean returns, however they give an even higher standard deviation. Another observation we can make is that these models tend to change more frequently the allocation to assets (up to 3 times higher turnover). Although we do not perform a transaction cost analysis in our study, based on the high turnover of these models, we would expect very high transaction costs if such where imposed, which would even further drag the performance down, especially for shorter investments horizons and a more active investment strategy. Last but not least, based on Barberis (2000) we know that the VAR component effect is stronger the longer the time horizon. However, when we add regimes, the estimates of the D/P ratio change as well and so we do not obtain the same risk reducing results. Regime mean-reverting effects are stronger than the risk reducing one and so the increase in stock holdings is less pronounced.

Focusing on the first group of models we can see that adding restrictions significantly improves the efficiency of the model both in shorter and longer horizons. In general models with more regimes allow a better allocation. In general models tend to change regimes rather quickly, however the regimes in the 2 regime model are too extreme, therefore when give a wrong signal the losses are bigger compared to model with more regimes. In a 4 regime models the remaining models are better fitted to different economic situations. This allows for a better market timing explaining the higher sharpe ratio in the short horizon, where the autocorrelation of returns is taken into account. Models with more regimes fit the data better in longer periods, but still the constrained model outperforms all of them giving thanks to the additional intra-regimes which allow a better specification of 2 regimes. It all helps in a better asset allocation robust to chaotic movements between regimes, which often might disrupt the construction of a good investment strategy. The constrained model not necessarily gives the best mean returns but it is much more robust and stable giving better risk adjusted returns. Figure 2 in the Appendix show the plotted distribution of weights in the 4 regime constrained model in the 1 month and 10 year horizon. We see that the model is relatively stable but keeps market timing properties. We also add the myopic investor 1 month allocation plot to show the differences in the allocation strategy. The iid model suggests a higher allocation to stocks.

It is surprising to see how well does the VAR(1) model perform. In fact it outperforms all other models in the 1 month horizon. We believe that the explanation is similar to the reason why the initial state choice based on inferred probabilities is not always optimal. When the economy remains in a given regime for some time, regime switching models can fully profit of this situation as they rely on regime persistence, however when the economy changes frequently they might give worse results than the IID or VAR(1) strategies. For the same reason the constrained model performs so well.

5.3 Joint distribution of regimes

So far we have calculated and analysed our regimes under the joint distribution of stocks and bonds. It is a common solution in the literature used in Guidolin and Timmermann (2006) or Guidolin and Hyde (2012), supported by the fact, that we estimate and forecast the values of both stock and bond returns. However, it is worthwhile to ask if the joint distribution is the correct approach to define regimes. Most regime switching models thrive to correctly specify the bear and bull regimes, which are characterized by stock returns. The recession is defined by low or even negative stock returns. Bond returns are dependent in an important part from FED decisions on interest rates taken in response to the actual market situation, they are however inevitably delayed. Perhaps models based only on the distribution of stock returns are more appropriate? On the other side maybe a single or joint distribution is not enough to accurately specify a higher number of regimes. When calculating MS VAR models, we also have the dividend to price ratio as a dependent variable, known for carrying information about future stock returns. Maybe it should be therefore included in the distribution for estimating the models' regimes? That is what we will verify in this chapter. Although we cannot compare the likelihoods of models based on different distributions (as they do not cover the same variables) we could once again resort to the returns each models allowed to gain in the out-of-sample analysis.

5.4 Only stock returns

Models with only stocks return distribution manage to occasionally outperform their joint distribution counterparts especially in shorter horizons. In general, mean variant models without the autoregressive component manage to achieve better results in terms of means, but they also have higher standard deviations. Those high returns suggest that the prediction of a better responsiveness to stock movements is true. As the stock returns are in general more important than those of the bonds, both negative and positive, a model focusing on stocks might give better results. On the other hand information from bonds in the joint distribution might be more insightful in the longer run, as this is suggested by the results. The same can be observed for the models with the autoregressive component which achieve very good results in shorter horizons. For the 1 month horizon the Sharpe ratio value is increased even more by the autocorrelation of model returns.

It is worth stressing that we use the single distribution of stock returns only for finding the regimes. The covariance matrix is calculated for all the dependent variables of the model.

Nevertheless, it appears that the joint distribution of stock and bond returns is optimal for the estimation of models.

5.5 Dividend-to-price ratio

Models including the D/P ratio in the joint distribution are different only for the MS-VAR models, as models based on regime switching means do not include this variable at all. Table 9 in the Appendix regroups the results. We see that again, the constrained model performs very well, but surprisingly in the longer ones it is outperformed by other models. The D/P Ratio is a variable which is rather stable in itself, and so including it in the joint distribution is enough to augment the stability of the model without penalizing the fit of the model by any constraints on the estimation. That is why the normal 4 regime VAR model can beat it in that framework. This would also explain why this observation becomes more clear in the longer period, when the D/P Ratio effects are the strongest. Nevertheless, even in that framework the VAR models do not perform better than the mean-switching counterparts. We will therefore focus on the joint distribution of stock and bonds.

6 Bootstrap

We want to compare the best performing models in a longer period. This would also allow us to answer the question whether the four-state model overfits the data, and because of the estimation errors on parameters, performs worse than a two-state model, which remains the most frequently used in the literature. We want to test also how robust the results of the constrained model are. In order to do that we will simulate a time series of returns to compare our models in an extended analysis similarly to Guidolin and Timmermann (2007). However we will perform this in a slightly different manner. In their paper the authors simulate the time-series using the estimated four-regime model. We believe this approach is strongly biased in favor of the four-regime model. Therefore we use a stationary bootstrap technique which allows to include dependence. The method is described in Politis and Romano (1994). The algorithm can be described as follows: we draw uniformly an observation from the set. It will be denoted as our X_1, then we have two possibilities. With probability p the next observation X_2 will be picked again randomly from the initial set, or, with probability 1-p, it will be the next observation after X_1. It results in a pseudo time-series constituted of blocks $B_{i,b} = \{X_i, X_{i+1}, ..., X_{i+b-1}\}$ where b denotes the length of the block which is random. In order to determine the optimal block length we resort to the method of Politis and White (2004). We choose the length which minimizes the MSE(σ_b^2) using the flag-top window as described in Politis and Romano (1995) which is performed in 3 steps. First, we look for the smallest lag after which the correlation stops being significant. Second, we calculate the estimates of G and D_{SB} which are the functions of flagtop kernel weights and the spectral density of the data calculated as the autocorrelation. Finally, we use the estimates in the following formula (23).

$$b_{opt,SB} = \left(\frac{2G^2}{D_{SB}}\right)^{1/3} N^{1/3} \tag{23}$$

The results of the optimal bloc lengths for each variable are presented in Table 2.

Table 2. Optimal bloc length calculation

Optimal bloc length for the stationary bootstrap calculated based on the work of Politis and White (2004). The value for each variable show to optimal length in months.

	Stocks	Bonds	T-bills	D/P Ratio
$\hat{b}_{opt,SB}$	2.0162	1.6806	46.55	48.2781

Having the estimates of the optimal block length we choose the maximum of them i.e. the length 48.3 estimated for the D/P Ratio. We use it in our bootstrap to generate 1,000 independent samples each containing 732 periods. It will allow us to maintain a similar estimation period (as in the case of our previous models) and to calculate the statistics to compare the results. We compare our results to the IID/myopic model. The results are presented in Table 10 in the Appendix.

The results from the bootstrap support our previous findings. The first question we wanted to answer i.e. whether the four-regime model over-fits the data is overruled as the model performs better, even for shorter periods of time which is a novelty compared to previous results. Again, the simpler model achieves to beat the other model in terms of mean returns, but the advantage is narrowed by the higher standard deviation. Second, the constrained model proves to outperform the other models in terms of risk adjusted returns in almost all horizons. It only struggles to do so in the shortest horizon, which is also new compared to the out-of-sample verification. In general, with the time horizon expanding, the model proves to be more stable and keeps the standard deviation low. Interestingly the model has the lowest average utility. Although we notice this drawback, we support the conclusion that the economic results outweigh this negative point and the constrained model remains the winner of this comparison. It is also worth noting that all the Regime-switching models outperfom the IID model which, although showing good mean return and utility, fails to compete with the other models under the Sharpe ratio criterion. These results prove how powerful predicting instruments these models are, reaffirming the validity of extended research in that field.

7 Conclusions

This paper explores the regime switching models in a stock and bond joint distribution specification. It summarizes and compares different approaches from existing research on regime switching like the analysis of the number of regimes, the VAR specification, the joint or single distribution of assets, as well as the initial state implications. However, what it adds above all is the thorough analysis of regime specification. We found that MS models can be modified not only by adding the number of regimes but also by constraining the transition matrix in order to accommodate short time reversal movements and get a 4 regime model which is in fact a better specified 2 state model. The data supports the existence of bear market rallies and bull market corrections in monthly data.

We found that the constrained 4 regime model not only gives valuable insight in a short horizon framework but it unveils all its power in the long term investment. The asset allocation analysis shows the importance of regime accommodation in investment models. Previous findings and investment rules on increasing allocation to stocks with time horizons have to be conditioned on the regime we are currently observing. It is not always the case especially in times of economic prosperity when it is wiser to lower the allocation in the long run, regardless of additional variables used such as the famous D/P ratio.

Our model gives a wide possibility of extensions as it allows in general a modification of regime switching models which are more and more often used in economics. The current framework could be extended by a better specification of short term interest rates. We assume iid in our cash variable, however many authors point out the presence of regime switching in this variable, which might be a valuable addition to the research. Another extension of the model might by the use of time-varying probabilities which could be as well restricted to accommodate bull market corrections and bear market rallies. Finally, we believe it would be interesting to explore the advantages of our model in a dynamic asset allocation framework.

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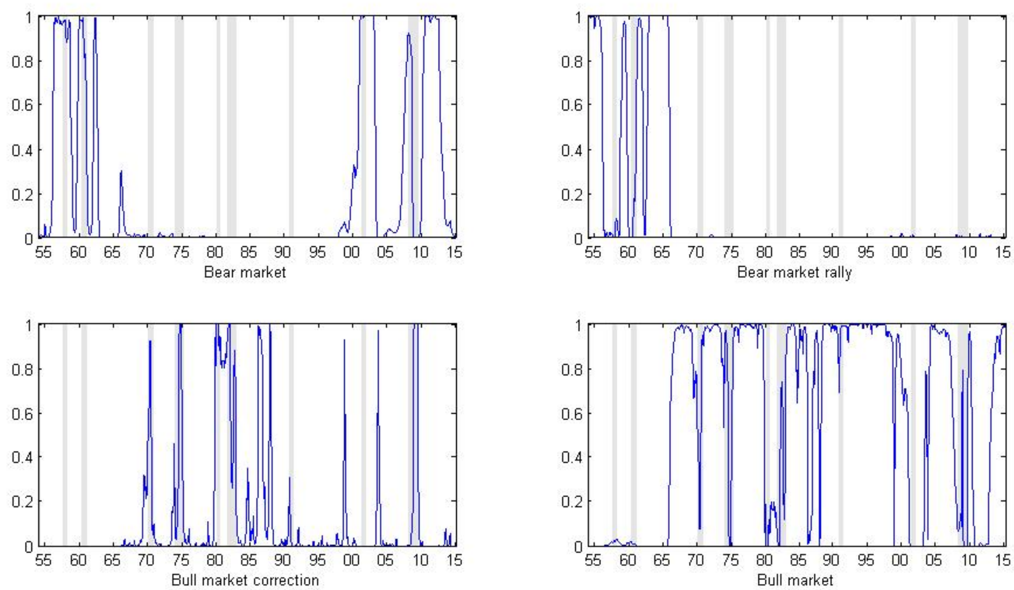
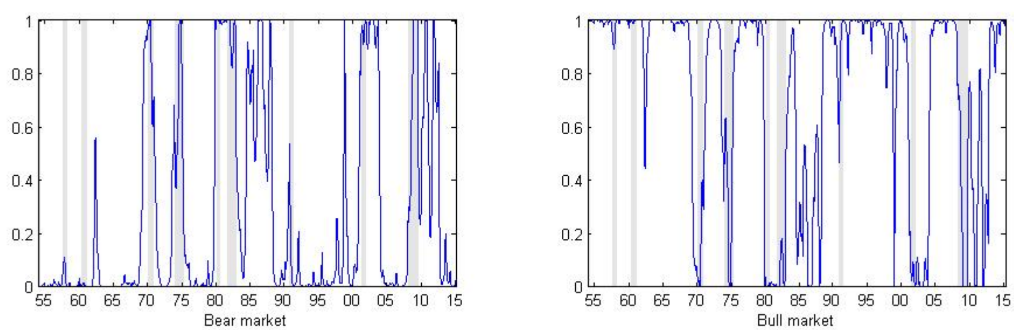
Appendix

Figure 1. Smoothed state probabilities

This figure plots the smoothed state probabilities for regimes 1-4 for the 4 regimes constrained model in the multivariate distribution of stock and bond returns (Panel (a)) and for the 2 regime model in the same multivariate framework. Recession periods are marked in light gray.



Panel (a): 4 Regime Constrained model Smoothed probabilities



Panel (b): 2 Regime model Smoothed probabilities

Figure 2. Out-of-sample portfolio weights

This figure plots the weights distribution in the out-of-sample performance of the 4 regime constrained model in 1 month horizon, the same model in a 10 years investments horizon and 1 month portfolio choices in the IID model. For 1 month ahead the results are presented for 30 years starting from January 1985 to December 2014. For the 10 years investment horizon the results are shown from December 1994 to December 2014

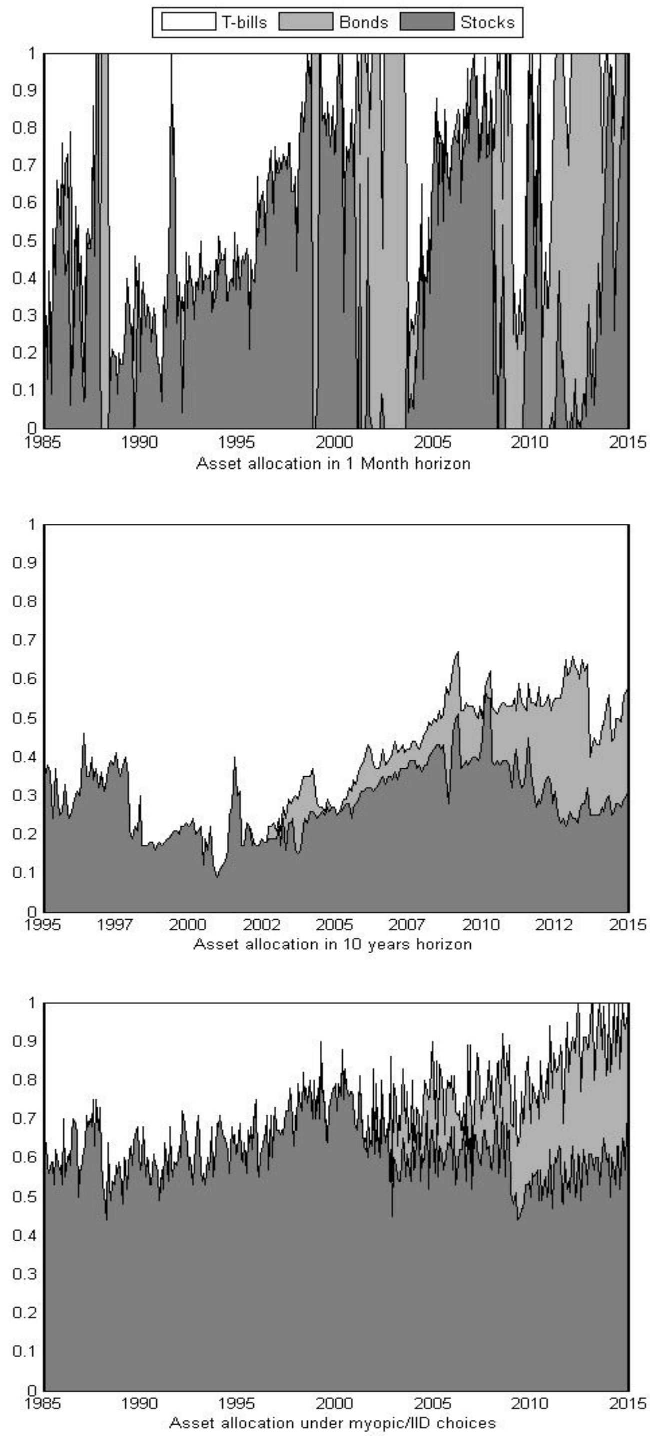


Table 3. Model estimates

In this table we present the result of the estimation of the models we selected in the testing procedures. Panels A-D present the model of the form:

$$\boldsymbol{r}_t = \boldsymbol{\mu}_{S_t} + \boldsymbol{\epsilon}_t$$

and Panel E:

$$\begin{pmatrix} r_t \\ z_t \end{pmatrix} = \begin{pmatrix} \mu_{S_t} \\ \mu_{zS_t} \end{pmatrix} + \sum_{j=1}^{P} A_{j,S_t} \begin{pmatrix} r_{t-j} \\ z_{t-j} \end{pmatrix} + \begin{pmatrix} \epsilon_{S_t} \\ \epsilon_{zS_t} \end{pmatrix}$$

Where μ denotes the intercept vector in state S_t which takes integer value between 1 and k according to the number of regimes, r_t and z_t respectively the vector of returns and dividend yields, $\epsilon \sim N(0, \Omega)$ is the vector of innovations. The dependent variables are the returns of the S&P500 index and a portfolio of long term bonds all in excess of the 3 month T-bill rate. In the VAR model additionally the dividend yield is used. The results are evaluated on the full sample from January 1954 to December 2014. Presented mean excess returns or monthly returns whereas volatilities are annualized. We present the standard errors either in brackets next to the results or with asterisks.

Panel A - Single state model		
1. Mean excess return	Stocks	Bonds
	0.0049*** (0.0009)	0.0014*** (0.0006)
2. Correlation/volatility		
Stocks	0.1471***	0.1171***
Bonds	0.1171***	0.0966***
Panel B - 2 state model		
1. Mean excess return	Stocks	Bonds
Regime 1 *(Bear)*	-0.0048 (0.0044)	0.0043*** (0.0015)
Regime 2 *(Bull)*	0.0085 (0.0044)	0.0004 (0.0004)
2. Correlation/volatility		
Regime 1 *(Bear)*		
Stocks	0.2060***	0.0928
Bonds	0.0928	0.1445***
Regime 2 *(Bull)*		
Stocks	0.1155***	0.1732***
Bonds	0.1732	0.0709***
3. Transition probabilities	Regime 1	Regime 2
Regime 1	0.9174***	0.0826
Regime 2	0.0307	0.9693***
Duration	12.11	32.56

Table 3 (Cont.)

Panel C - 4 regimes				
1. Mean excess return	Stocks		Bonds	
Regime 1 *(Crash)*	-0.0073	(0.0086)	0.0016	(0.0063)
Regime 2 *(Bear)*	-0.0050	(0.0036)	0.0082**	(0.0038)
Regime 3 *(Normal growth)*	0.0073***	(0.0018)	0.0007	(0.0005)
Regime 4 *(Bull)*	0.0127***	(0.0037)	-0.0003	(0.0009)
2. Correlation/volatility				
Regime 1 *(Crash)*				
Stocks	0.2387***		0.2335**	
Bonds	0.2335**		0.0920***	
Regime 2 *(Bear)*				
Stocks	0.1707***		-0.6145***	
Bonds	-0.6145***		0.0097***	
Regime 3 *(Normal growth)*				
Stocks	0.1193***		0.2778***	
Bonds	0.2778***		0.0815***	
Regime 4 *(Bull)*				
Stocks	0.1222***		-0.1735***	
Bonds	-0.1735***		0.0291***	
3. Transition probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1	0.8664***	0.0000	0.1336***	0.0000
Regime 2	0.0309*	0.9545***	0.0147	0.0000
Regime 3	0.0204***	0.0077*	0.9678***	0.0042
Regime 4	0.0000	0.0000	0.0313	0.9687***
Duration	7.48	21.96	31.06	31.97

Table 3 (Cont.)

Panel D - 4 regimes Constrained				
1. Mean excess return	Stocks		Bonds	
Regime 1 *(Bear)*	-0.0025	(0.0058)	0.0043	(0.0033)
Regime 2 *(Bear rally)*	0.0162	(0.0503)	-0.0006	(0.0014)
Regime 3 *(Bull correction)*	-0.0104	(0.0089)	0.0012	(0.0065)
Regime 4 *(Bull)*	0.0079***	(0.0023)	0.0011	(0.0015)
2. Correlation/volatility				
Regime 1 *(Bear)*				
Stocks	0.1570***		-0.4984***	
Bonds	-0.4984***		0.0853***	
Regime 2 *(Bear rally)*				
Stocks	0.1034***		-0.1453***	
Bonds	-0.1453***		0.0269***	
Regime 3 *(Bull correction)*				
Stocks	0.2454***		0.2227**	
Bonds	0.2227**		0.1824***	
Regime 4 *(Bull)*				
Stocks	0.1196		0.3329***	
Bonds	0.3329***		0.0842***	
3. Transition probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1	0.9452***	0.0260	0	0.0288
Regime 2	0.0443	0.9481***	0	0.0076
Regime 3	0.0000***	0	0.8342***	0.1658*
Regime 4	0.0088*	0	0.0321	0.9591***
Duration	18.26	19.26	6.03	24.45

Table 3 (Cont.)

Panel E - 4 regimes VAR				
1. Estimates	Intercept	Stock	Bonds	D/P ratio
Regime 1 *(Bear)*	0.0819**	0.1745**	0.1416	2.7046**
	-0.0382	-0.1543***	0.0817	-1.1431
	-0.0008	-0.0018	-0.0014	0.9721*
Regime 2 *(Normal growth)*	-0.0163	-0.1386**	0.1657**	-0.5918
	-0.0165	-0.1002***	0.1052*	-0.4932
	-0.0017*	0.0020	-0.0050	0.9529
Regime 3 *(Bull)*	0.1946***	-0.2074**	0.4143	5.1666***
	0.0216*	-0.0424	0.0153	0.6481**
	-0.0017	0.0020	-0.0050	0.9529
Regime 4 *(Recovery)*	-0.0016	-0.2424***	0.2478***	-1.2957**
	0.1410***	0.0441	-0.1898*	3.8224***
	-0.0000	0.0024	-0.0023	1.0106
2. Correlation/volatility	Stocks	Bonds	D/P ratio	
Regime 1 *(Bear)*				
Stocks	0.1961***	0.1677***	-0.9938	
Bonds	0.1677***	0.1426	-0.1504***	
D/P Ratio	-0.9938	-0.1504***	0.0020	
Regime 2 *(Normal Growth)*				
Stocks	0.1134***	0.2153***	-0.9827	
Bonds	0.2153***	0.0772***	-0.1982	
D/P Ratio	-0.9827	-0.1982	0.0012	
Regime 3 *(Bull)*				
Stocks	0.0970***	-0.3220***	-0.9801	
Bonds	-0.3220***	0.0234***	0.2921	
D/P Ratio	-0.9801	0.2921	0.0010	
Regime 4 *(Recovery)*				
Stocks	0.0534***	-0.0869*	-0.9019	
Bonds	-0.0869*	0.0695***	0.0243	
D/P Ratio	-0.9019	0.0243	0.0006	
3. Transition probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1	0.7839***	0.0000	0.0000	0.2161
Regime 2	0.0349***	0.9651***	0.0000	0.0000
Regime 3	0.0000	0.0612***	0.9388***	0.0000
Regime 4	0.3512***	0.1488***	0.0821***	0.4179***
Duration	4.63	28.68	16.34	1.72

Table 4. CHP test results

The table reports the results of the Carrasco et al. test. The null hypothesis of the test tells us that the regimes in the stock returns are not observable and the returns are best described with a model without regime-switches. The alternative hypothesis tells us that a 2 regime model better desribes the data

Critical values	supTS	expTS
99th percentile	5.1849	2.4245
95th percentile	3.6950	1.5239
90th percentile	2.9279	1.2411
	Test Statistic	p-value
supTS	10.4863	0.00
expTS	17.5037	0.00

Table 5. Likelihood and information criteria

The table presents the values of the log likelihood, Akaike and Schwarz information criteria calculated for all estimated models

	Likelihood	AIC	BIC
MS2	2958.9	-5891.7	-5832.0
MS3	3002.2	-5958.5	-5852.8
MS4	3024.2	-5978.4	-5817.6
MS4 Constrained	3025.6	-5989.2	-5846.8
MS2 VAR	2969.4	-5872.8	-5721.2
MS3 VAR	3022.0	-5978.1	-5826.5
MS4 VAR	3053.6	-5957.1	-5612.6
MS4 VAR Constrained	3050.0	-5957.9	-5631.7

Table 6. Asset allocation for regimes

The table reports the asset allocation calculates for 6 models - MS2, MS4, MS4 Constrained, MS4VAR and two benchmarks: the VAR(1) model and the myopic strategy. The models are calculated based on estimates calculated on the full sample 01.1954-12.2014 using Monte Carlo simulations to simulate return paths. We use 30 000 iterations. The allocation is calculated for a moderately risk averse investor ($\gamma = 5$). For each model the portfolios are presented for every regime plus the steady states probabilites for 6 time horizons: 1, 6, 12 months and 2, 5 and 10 years

2 Regime

Regime 1 (Bear market)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	0%	0%	9%	22%	35%	40%
Bonds	58%	58%	53%	49%	45%	43%
Cash	42%	42%	38%	29%	29%	17%
Regime 2 (Bull market)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	100%	97%	78%	64%	52%	49%
Bonds	0%	3%	22%	35%	39%	40%
Cash	0%	0%	0%	1%	9%	11%
Steady steates probabilities						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	64%	56%	52%	50%	46%	45%
Bonds	36%	39%	39%	40%	40%	41%
Cash	1%	5%	9%	10%	14%	13%

IID/myopic

Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	60%	62%	63%	61%	63%	62%
Bonds	40%	38%	37%	39%	37%	38%
Cash	0%	0%	0%	0%	0%	0%

VAR(1)

Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	72%	62%	62%	64%	72%	83%
Bonds	28%	38%	38%	36%	28%	17%
Cash	0%	0%	0%	0%	0%	0%

4 Regime

Regime 1 (Crash)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	0%	0%	8%	24%	36%	39%
Bonds	22%	22%	23%	22%	27%	32%
Cash	78%	78%	69%	54%	37%	29%
Regime 2 (Bear Regime)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	0%	0%	0%	5%	23%	31%
Bonds	100%	100%	100%	95%	69%	55%
Cash	0%	0%	0%	0%	8%	14%
Regime 3 (Normal Growth)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	100%	90%	74%	59%	50%	45%
Bonds	0%	10%	17%	23%	34%	34%
Cash	0%	0%	9%	18%	16%	21%
Regime 4 (Bull Regime)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	100%	100%	100%	100%	74%	56%
Bonds	0%	0%	0%	0%	22%	31%
Cash	0%	0%	0%	0%	4%	13%
Steady states probabilities						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	55%	50%	49%	47%	45%	44%
Bonds	37%	40%	36%	39%	37%	37%
Cash	8%	10%	15%	14%	18%	19%

Table 6 (Cont.)

4 Regime Constrained						
Regime 1 (Bear market)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	2%	12%	20%	31%	40%	41%
Bonds	98%	88%	80%	69%	60%	46%
Cash	0%	0%	0%	0%	0%	13%
Regime 2 (Bear rally)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	100%	100%	100%	78%	60%	54%
Bonds	0%	0%	0%	22%	40%	42%
Cash	0%	0%	0%	0%	0%	4%
Regime 3 (Bull correction)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	0%	0%	6%	21%	33%	38%
Bonds	23%	22%	22%	22%	23%	26%
Cash	77%	78%	72%	57%	44%	36%
Regime 4 (Bull market)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	100%	81%	62%	53%	47%	45%
Bonds	0%	5%	15%	19%	25%	27%
Cash	0%	14%	23%	28%	28%	29%
Steady states probabilities						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	55%	52%	49%	47%	44%	43%
Bonds	31%	32%	33%	33%	31%	32%
Cash	14%	16%	18%	20%	25%	25%

4 Regime VAR						
Regime 1 (Bear regime)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	12%	24%	31%	40%	51%	60%
Bonds	55%	57%	53%	46%	33%	25%
Cash	33%	19%	16%	14%	16%	15%
Regime 2 (Normal growth)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	100%	71%	61%	57%	57%	64%
Bonds	0%	29%	35%	36%	29%	21%
Cash	0%	0%	4%	7%	14%	15%
Regime 3 (Bull regime)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	100%	100%	100%	100%	81%	76%
Bonds	0%	0%	0%	0%	15%	15%
Cash	0%	0%	0%	0%	4%	0%
Regime 4 (Recovery)						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	87%	60%	54%	52%	58%	63%
Bonds	12%	38%	38%	40%	31%	21%
Cash	1%	2%	8%	8%	11%	16%
Steady states probabilities						
Horizon	1M	6M	12M	2Y	5Y	10Y
Stocks	86%	59%	55%	55%	59%	63%
Bonds	14%	41%	39%	36%	28%	20%
Cash	0%	0%	6%	9%	13%	17%

Table 7. Out-of-sample performance results - Joint distribution of Stock and Bonds Returns

The table reports the average and standard deviation of returns, annualized Sharpe ratio and average value of the utility function calculated based on the out-of-sample performance for each of the 10 models including the iid and VAR(1) benchmarks. We calculate the performance measures for 6 time horizons: 1, 6, 12, 24, 60 and 120 months. We start in Jan 1986, estimating the models on data from Jan 1954 and iteratively move one month forward reestimating the models. The models are estimated based on the join distribution of Stocks and Bonds returns and the results are presented in monthly units.

	IID	VAR(1)	MS2	MS3	MS4	MS4 Constrained	MS2 VAR	MS3 VAR	MS4 VAR	MS4 VAR Constrained
					1 month horizon					
Mean	0.0068	0.0084	0.0062	0.0058	0.0067	0.0069	0.0075	0.0064	0.0062	0.0081
St. Deviation	0.0285	0.0298	0.0276	0.0284	0.0304	0.0255	0.0299	0.0273	0.0302	0.0335
Sharpe ratio	0.6536	0.9832	0.6287	0.6855	0.7940	0.9665	0.7618	0.7399	0.6265	0.7690
Utility	-0.2474	-0.2453	-0.2470	-0.2474	-0.2465	-0.2474	-0.2451	-0.2458	-0.2447	-0.1376
					6 months horizon					
Mean	0.0404	0.0420	0.0339	0.0323	0.0344	0.0352	0.0360	0.0353	0.0312	0.0462
St. Deviation	0.0715	0.0692	0.0709	0.0612	0.0671	0.0487	0.0712	0.0812	0.0636	0.0829
Sharpe ratio	0.6960	0.7517	0.5710	0.6256	0.6136	0.8687	0.6108	0.5236	0.5773	0.6986
Utility	-0.2351	-0.2344	-0.2344	-0.2368	-0.2327	-0.2384	-0.2322	-0.2279	-0.2354	-0.1351
					12 months horizon					
Mean	0.0802	0.0842	0.0683	0.0656	0.0617	0.0666	0.0750	0.0688	0.0587	0.0915
St. Deviation	0.1078	0.1106	0.1105	0.0894	0.1138	0.0659	0.1130	0.1291	0.0906	0.1261
Sharpe ratio	0.6472	0.6672	0.5244	0.6169	0.4505	0.8518	0.5721	0.4522	0.5333	0.6432
Utility	-0.2217	-0.2210	-0.2225	-0.2264	-0.2201	-0.2303	-0.2199	-0.2141	-0.2273	-0.1324
					2 years horizon					
Mean	0.1545	0.1743	0.1302	0.1268	0.1211	0.1250	0.1537	0.1470	0.1207	0.1797
St. Deviation	0.1658	0.1873	0.1649	0.1356	0.1555	0.0956	0.1806	0.1895	0.1242	0.1997
Sharpe ratio	0.5722	0.5810	0.4711	0.5549	0.4577	0.7732	0.5221	0.4722	0.5710	0.5641
Utility	-0.1994	-0.1967	-0.2047	-0.2103	-0.2011	-0.2178	-0.2013	-0.1952	-0.2144	-0.1287
					5 years horizon					
Mean	0.3669	0.4630	0.3280	0.3219	0.3343	0.2906	0.4355	0.4326	0.3820	0.4348
St. Deviation	0.2565	0.3686	0.2213	0.1992	0.1929	0.1341	0.3407	0.3550	0.2568	0.3646
Sharpe ratio	0.5466	0.4969	0.5550	0.6027	0.6511	0.7909	0.5015	0.4776	0.5721	0.4678
Utility	-0.1547	-0.1394	-0.1702	-0.1781	-0.1639	-0.1993	-0.1592	-0.1544	-0.1825	-0.1248
					10 years horizon					
Mean	0.7290	0.9437	0.6347	0.6455	0.6657	0.5546	0.9359	0.9198	0.9243	0.9061
St. Deviation	0.3765	0.5386	0.2854	0.2934	0.2844	0.1958	0.5375	0.5211	0.5209	0.5299
Sharpe ratio	0.5160	0.4866	0.5761	0.5720	0.6126	0.7105	0.4831	0.4885	0.4915	0.4723
Utility	-0.0893	-0.0603	-0.1140	-0.1196	-0.1084	-0.1567	-0.0850	-0.0809	-0.1088	-0.0905

Table 8. Out-of-sample performance results - Distribution of Stock returns only

The table reports the average and standard deviation of returns, annualized Sharpe ratio and average value of the utility function calculated in the out-of-sample performance for 8 regime switching models. We calculate the performance measures for 6 time horizons: 1, 6, 12, 24, 60 and 120 months. We start in Jan 1986, estimating the models on data from Jan 1954 and iteratively move one month forward reestimating the models. The models are estimated based on the distribution of Stocks returns only. The results are presented in monthly units.

	MS2	MS3	MS4	MS4 Constrained	MS2 VAR	MS3 VAR	MS4 VAR	MS4 VAR Constrained
				1 month horizon				
Mean	0.0070	0.0074	0.0077	0.0077	0.0081	0.0080	0.0082	0.0078
St. Deviation	0.0316	0.0260	0.0311	0.0292	0.0264	0.0278	0.0281	0.0278
Sharpe ratio	0.6421	0.8571	0.8649	0.9415	1.0901	0.9680	1.0293	1.0829
Utility	-0.2463	-0.2460	-0.2455	-0.2456	-0.2458	-0.2455	-0.2455	-0.2457
				6 months horizon				
Mean	0.0366	0.0380	0.0408	0.0396	0.0382	0.0375	0.0392	0.0354
St. Deviation	0.0621	0.0600	0.0623	0.0609	0.0603	0.0629	0.0611	0.0580
Sharpe ratio	0.7146	0.7722	0.8061	0.7981	0.7738	0.7247	0.7844	0.7345
Utility	-0.2340	-0.2319	-0.2304	-0.2314	-0.2358	-0.2346	-0.2350	-0.2366
				12 months horizon				
Mean	0.0725	0.0726	0.0767	0.0745	0.0769	0.0780	0.0786	0.0687
St. Deviation	0.0912	0.0953	0.0989	0.0919	0.0973	0.1027	0.0982	0.0960
Sharpe ratio	0.6813	0.6534	0.6708	0.6974	0.6832	0.6585	0.6944	0.6071
Utility	-0.2231	-0.2197	-0.2174	-0.2202	-0.2233	-0.2216	-0.2224	-0.2257
				2 years horizon				
Mean	0.1285	0.1339	0.1439	0.1346	0.1561	0.1593	0.1566	0.1406
St. Deviation	0.1545	0.1533	0.1657	0.1370	0.1653	0.1688	0.1625	0.1520
Sharpe ratio	0.5895	0.5238	0.4945	0.5271	0.5804	0.5821	0.5930	0.5592
Utility	-0.2054	-0.2012	-0.1978	-0.2030	-0.2009	-0.1986	-0.2003	-0.2065
				5 years horizon				
Mean	0.3263	0.3341	0.3434	0.3208	0.4381	0.4448	0.4336	0.4035
St. Deviation	0.2088	0.2100	0.2308	0.2008	0.3456	0.3412	0.3346	0.3116
Sharpe ratio	0.5843	0.5977	0.5617	0.5954	0.4978	0.5129	0.5081	0.5025
Utility	-0.1699	-0.1639	-0.1582	-0.1693	-0.1482	-0.1460	-0.1493	-0.1610
				10 years horizon				
Mean	0.6458	0.6638	0.6548	0.6240	0.9406	0.9467	0.9447	0.9215
St. Deviation	0.3041	0.3212	0.3003	0.2947	0.5406	0.5249	0.5279	0.5298
Sharpe ratio	0.5522	0.5406	0.5686	0.5465	0.4831	0.5013	0.4972	0.4815
Utility	-0.1085	-0.1061	-0.0999	-0.1145	-0.0670	-0.0674	-0.0690	-0.0804

Table 9. Out-of-sample performance results - Joint distribution of Stocks, Bonds and the dividend yield

The table reports the average and standard deviation of returns, annualized Sharpe ratio and average value of the utility function calculated in the out-of-sample performance for 4 models which include the autoregressive component. The values are calculated for 6 time horizons. The models are estimated based on the join distribution of Dividend yield, Stocks and Bonds returns.The results are presented in monthly units.

	MS2 VAR	MS3 VAR	MS4 VAR	MS4 VAR Constrained
		1 month horizon		
Mean	0.0082	0.0071	0.0088	0.0086
St. Deviation	0.0297	0.0308	0.0318	0.0307
Sharpe ratio	0.9136	0.7282	0.9955	1.1077
Utility	-0.2450	-0.2439	-0.2409	-0.2433
		6 months horizon		
Mean	0.0348	0.0353	0.0390	0.0419
St. Deviation	0.0852	0.0811	0.0878	0.0719
Sharpe ratio	0.4910	0.5236	0.5436	0.7207
Utility	-0.2320	-0.2279	-0.2166	-0.2303
		12 month horizon		
Mean	0.0725	0.0691	0.0818	0.0847
St. Deviation	0.1298	0.1294	0.1309	0.1073
Sharpe ratio	0.4783	0.4531	0.5450	0.6931
Utility	-0.2179	-0.2142	-0.2001	-0.2189
		2 years horizon		
Mean	0.1514	0.1464	0.1605	0.1750
St. Deviation	0.1958	0.1902	0.1892	0.1676
Sharpe ratio	0.4731	0.4686	0.5238	0.6524
Utility	-0.1947	-0.1952	-0.1840	-0.1985
		5 years horizon		
Mean	0.4502	0.4328	0.4468	0.4403
St. Deviation	0.3612	0.3546	0.3426	0.3592
Sharpe ratio	0.4912	0.4784	0.5135	0.4816
Utility	-0.1401	-0.1544	-0.1450	-0.1514
		10 years horizon		
Mean	0.9492	0.9201	0.9449	0.9329
St. Deviation	0.5285	0.5205	0.5002	0.5299
Sharpe ratio	0.4993	0.4893	0.5249	0.4882
Utility	-0.0677	-0.0808	-0.0786	-0.0769

Table 10. Results of the bootstrap asset allocation

The table reports results of the asset allocation test performed on bootstrapped data for 4 models. We use a stationary bootstrap with calculated optimal bloc length of 48 months in order to keep the asset returns properties. We generate 1,000 independent sample 732 periods each. We estimate the models on 610 months and construct strategies for 6 time horizons and then we calculate the realized returns and utility. We use 4 performance measures: the mean returns, the standard deviation, the annualized Sharpe ratio and the average utility

	IID	MS2	MS4	MS4 Constrained
		1 month horizon		
Mean	0.0072	0.0082	0.0085	0.0081
St. Deviation	0.0273	0.0304	0.0298	0.0303
Sharpe ratio	0.8041	0.8403	0.8840	0.8246
Utility	-0.2467	-0.2456	-0.2451	-0.2454
		6 months horizon		
Mean	0.0444	0.0432	0.0438	0.0421
St. Deviation	0.0696	0.0694	0.0695	0.0696
Sharpe ratio	0.7969	0.7726	0.7853	0.7496
Utility	-0.2312	-0.2294	-0.2271	-0.2289
		12 months horizon		
Mean	0.0853	0.0826	0.0811	0.0839
St. Deviation	0.1053	0.0997	0.0975	0.0973
Sharpe ratio	0.7076	0.7206	0.7206	0.7523
Utility	-0.2150	-0.2147	-0.2117	-0.2148
		2 years horizon		
Mean	0.1756	0.1655	0.1666	0.1619
St. Deviation	0.1489	0.1380	0.1384	0.1324
Sharpe ratio	0.7310	0.7374	0.7412	0.7496
Utility	-0.1889	-0.1922	-0.1895	-0.1945
		5 years horizon		
Mean	0.4297	0.4033	0.3996	0.3869
St. Deviation	0.2350	0.2076	0.2007	0.1883
Sharpe ratio	0.7167	0.7543	0.7721	0.7927
Utility	-0.1464	-0.1601	-0.1593	-0.1689
		10 years horizon		
Mean	0.8454	0.7889	0.7749	0.7506
St. Deviation	0.3583	0.3148	0.3000	0.2837
Sharpe ratio	0.6551	0.6889	0.7081	0.7217
Utility	-0.1653	-0.2308	-0.2274	-0.2577