An audit game: optimal response strategies of a firm and its auditor

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1. Introduction

During the last few decades, many accounting scandals occurred which lead to major shareholders losses, job losses and bankruptcies. Illegal accounting techniques are used to increase short term profits or meet short term market expectations. Examples of such creative accounting are overstating earnings, understating losses or underreporting the existence of liabilities. Bonuses and share prices are often positively linked to reaching particular goals which gives incentives for executives to commit this kind of fraud (Farrell, 2015).

A restatement of a report is a revision of a former publication of a report. This occurs when accounting errors happened or fraud has been committed. When these accounting scandals come to light in the form of a restatement by the firm, confidence of shareholders in the company is shaken. After such a restatement, shareholders will reassess the credibility of the management of a firm and its future cash flows. In the case of fraud, the announcement is rapidly followed by a decrease in share prices. This effect on share prices and the lack of confidence in the company by investors often lead to bankruptcies (Agrawal & Chadha, 2005).

To show the magnitude of accounting scandals occurring at large companies, one of the most disastrous scandals in history will be described. In 2000, energy company Enron was one of the biggest and most innovative companies in the US. Governance and incentive problems were the factors which ultimately led to its bankruptcy.

During the booming years, up to 2001, Enron was able to attract a lot of capital through questionable business models and multiple accounting and financing manoeuvres. At that time, its market value was six times its book value. This shows high future expectations and therefore Enron seemed to be a good investment. Incentive problems occurred within the management of Enron because they were heavily compensated with stock options. This gave incentive to meet high short term profits rather than to take future value into account (Hall & Knox, 2002).

Enron’s external auditor, Arthur Andersen, was paid huge fees and this has very likely had a big impact on Andersen’s local staff during meetings with Enron’s management. Andersen’s auditors ‘failed’ to judge certain transactions as financial reporting while these transactions had clearly been used for business purposes. Internal auditors should have protected Enron against these local incentives but they failed as well. Enron had $63.4 billion in assets and the bankruptcy resulted in a $74 billion loss for investors. CEO J. Skilling was sentenced to 24 years in prison and was fined $45 million. Enron’s bankruptcy was the largest corporate bankruptcy in US history up to that time (Healy & Palepu, 2003).
Since the Enron scandal, several other accounting scandals of similar magnitude have occurred. Another famous accounting scandal is the Worldcom scandal (2002) which lead to $180 billion in losses for investors, 30,000 job losses and a 25 year prison sentence for CEO B. Ebbers. These cases show the important role auditing firms have towards investors and therefore the whole capital market.

Some academic research on the incentive problems between a firm and its auditor has already been done. Border & Sobel (1987) came up with a theoretical model to illustrate the difficulties that arise during a principal agent problem. Such a problem occurs because of information asymmetry since the agent is fully informed about the agent’s wealth but the principal is not. An example of such a situation is when a tax payer is the agent and a tax authorities is the principal. The principal can expand resources by hiring an auditor to determine the tax needed to be paid by the tax payer, however this is costly. The example that Border & Sobel created, to give a clearer view, is referred to as Samurai Accounting. It shows a situation where a band of brigands has to determine whether to plunder a peasant village. It might be the case that there live samurai warriors in the village which would lead to a very costly plunder for the brigands. Assumptions are that the brigands know the probability distribution of which the agent is drawn but he does not know the accurate wealth of the agent. The principal can choose to commit to a verification strategy. In the model, honest agents receive a rebate to incentivise them to be honest thus these agents prefer to be audited. Results show that in the case of agents being honest, taxes are monotonically increasing and audit probabilities are monotonically decreasing in reported wealth.

A normal cheap talk game consists of a sender and a receiver. The sender observes private information that is relevant for both player and sends a message to the receiver. After the message is received, the receiver makes a decision which affects welfare of both players. The message is called cheap talk because the players only care about the informational content, not about the message per se. Therefore the sender does not find lying costly and he cannot be punished afterwards. (Crawford, 1997)

Bijkerk et al. (2014) extended a cheap talk game by giving the receiver the option to verify the message of the sender. They showed that if the receiver has to cover for most of the verification costs then this option to verify drives all cheap-talk communication away. The sender can be disciplined as well if he has to cover a substantial part of these costs.

In this paper a model is studied which consists of a firm and an auditor who both can incur costs if the auditor decides to verify the message sent by the firm. The model contains two payoff functions, one for the firm and one for the auditor. These functions consist of some key variables during an audit game: the willingness of the firm to exaggerate its report, a potential fine for the firm in case of a found error, a bonus for the auditor if it found an error and the costs of searching for the auditor. Comparing this
model with the extended cheap talk model of Bijkerk et al., the sender’s cost is the potential fine for the firm and the verification costs for the receiver are the search costs for the auditor.

2. The model

This model describes a situation where a firm S delivers its report to an auditor A. The value of the report \((v)\) of the firm represents the value of the firm. This situation is simplified in such a way that \(v = 1\) equals the highest possible value and \(v = 0\) is the lowest possible value of the firm. Theoretically there could be \(n\) different values of \(v\) but since we are interested in strategies played by both parties, a few values are be sufficient for this model. The interval of \(v\) is evenly distributed and we choose four possible values for the report. The average value of each quarter is taken because this is a good representation of the average value within a quarter given \(v\) is evenly distributed: \(v \in \{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\}\).

The firm sends its report, which is equal to a certain value, to the auditor. The value of the report which is sent is called the message \(m\). Irrespective of the real value \(v\), the firm can send any message but it must be equal to one of the four possible values of \(v\). So the message can have the values \(m \in \{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\}\).

When the report is sent to the auditor, there is a probability \(p\) that the message from the firm does not contain the real value of the firm, so \(m \neq v\). This means that the firm has put an error within the report and therefore the message is not equal to the real value of the report. Given that there is an error within the report, a probability \(q\) is assigned to the probability that the error is found by the auditor: \(q(error|error)\). For the auditor to be incentivized to find an error, he receives a bonus \(b\) if he manages to find it. However, it also costly for the auditor to search for the error in terms of effort and therefore searching influences its payoff function negatively. Search costs are denoted by \(\lambda\). The expected payoff function of the auditor is given by the following equation:

\[
\text{Expected payoff auditor} = p(m \neq v) \cdot q(error|error) \cdot b - \frac{\lambda}{2} q^2(error|error)
\]

The firm has to pay a fine \(f\) if the auditor finds the error so this negatively influences the payoff function of the firm. This fine is multiplied by \(q\) which gives the expected value of the fine. Without the \(q\), we would obtain the payoff function since if no error is found the fine is zero and otherwise the fine is \(f\). The parameter \(x\) is a fixed parameter which represents the intrinsic preference of the firm. The larger \(x\), the less risk averse a firm is and therefore the more it wants to exaggerate its real value. The firm will never send a message which is worse than the actual value since the firm does not benefit from outsiders
valuing the firm less than it actually is, so \( x \geq 0 \). The following equation is given and represents the expected payoff for the firm:

\[
\text{Expected payoff firm} = -[m - (v + x)]^2 - f \cdot q(\text{error} | \text{error})
\]

As the example of the Enron scandal has shown, an accounting scandal occurs when there is enough incentive for the firm to apply creative accounting and/or the governance by the auditing firm is not exercised correctly. A game theoretical model, used in a simplified situation, can give more clarity about the incentives between a firm and its auditor. This model helps the firm to determine its game theoretical optimal actions.

If a firm hands its report over to an auditor, the auditor has to decide whether he will search for a possible error within the report and if he will, how much effort he will put into searching. The firm will try to anticipate the intensity of searching by the auditor and will use this information to determine whether and how big of an error it puts in the report. An error is refers to the difference between the value of the report and the real value of the firm. An assumption that has to be made about the error is that if the report contains an error, it has always has been put there deliberately by the firm so the report will never contain a lower value than the real value.

The characteristics of the model have to be specified so that the model can be solved correctly. The game consists of incomplete information because not all the characteristics of the players are known thus a Bayesian game will be played which will lead to Bayesian equilibria. A perfect Bayesian equilibrium concept puts structure on the assessments with consistency conditions that are formulated without reference to strategy trembles. (Watson, 2016) Within this model, multiple equilibria can be established. These equilibria consists of strategies of the firm and the auditor. Such a strategy is an optimal response (the strategy that yields the highest payoff) to the other player’s strategy. Given these strategies within an equilibrium, a player could decide to deviate from the equilibrium by choosing a different strategy which yields himself a higher payoff.

As a start, the pure strategies will be determined. There are four situations one can think of regarding the pure strategies in this game. There is an equilibrium where the firm has no incentive to lie so it sends \( m = v \). Besides that, there are equilibria where the firm exaggerates its real value (with \( m = \frac{7}{8} \) being the highest possible message) which are three possibilities namely where the firm sends \( m = v + \frac{1}{4}, m = v + \frac{1}{2} \) and \( m = v + \frac{3}{4} \). So for example, if \( v = \frac{5}{8} \) and the firm decides to apply the strategy to send \( m = v + \frac{1}{2} \) for every \( v \), it will send \( m = \frac{7}{8} \).
If it appears that not all equilibria have been found via pure strategies, the other equilibria can be determined via mixed strategies according to Nash. A mixed strategy is a combination of pure strategies each assigned with a certain probability. So for example, a $\sigma\%$ of the time a player will choose to play a strategy A and $(1-\sigma)\%$ of the time he will choose to play a strategy B. (Erev & Roth, 1998)

3. Pure strategies

3.1 Equilibrium where firm has no incentive to lie

At first we will look whether an equilibrium exists where $m = v$ and $q = 0$. This means that the firm will always send the correct value of the firm to the auditor and the auditor will never try to find the error. As a result, the payoff of the auditor will be zero since $q = 0$ and therefore he can never receive its bonus. If $m = v$, the payoff of the firm is the following:

$$\text{Payoff Firm} = -[v - (v + x)]^2 = -x^2$$

We will look for what value of $x$ the firm has no incentive to send $m \neq v$ given the auditor will not search for the error. Therefore, for such a value of $x$ the auditor will not search for the error. Assume $v = \frac{1}{8}$ and the firm sends $m = \frac{3}{8}$, then the following equation will give the restriction of $x$ for which the firm will send $m = v$:

$$-x^2 > -[\frac{3}{8} - (\frac{1}{8} + x)]^2$$

Which yields: $x < \frac{1}{8}$

This is a game of pure strategies so if $x < \frac{1}{8}$ then the firm will always send $m = v$ given the auditor will not search for the error. Therefore, given that the firm is honest if $x < \frac{1}{8}$, the auditor has no incentive to search for the error.

If we now take a look what happens for $v = \frac{3}{8}$ and $m = \frac{5}{8}$ and given the auditor will not search for the error, it makes sense that the outcome is the same when we put these two values in the same equation as above. This same reasoning applies to the situation where $v = \frac{5}{8}$ and $m = \frac{7}{8}$.

So the equilibrium outcome for sending $m = v$ and the firm exaggerates one quarter given the auditor will not search for the error is $x < \frac{1}{8}$. 
3.2 Equilibrium where firm, if possible, exaggerates one quarter

*Initial equilibrium*

This is a game of pure strategies where in this situation the firm exaggerates its own value by a quarter. Given that the firm will always send a message which is $\frac{1}{4}$ bigger than its actual value, the message will never be equal to $\frac{1}{8}$. This situation gives the auditor incentive to search for the error since the firm wants to exaggerate its real value. The payoff function of the auditor is maximized by taking the derivative with respect to $q$ and setting it equal to zero, which yields:

$$\lambda \cdot q = b \cdot p(m \neq v)$$

The effort of searching for the error has to be high enough because if $\lambda$ would be small, there would be a big probability of finding the error. Therefore the firm would not have enough incentive to exaggerate since its payoff is negatively influenced by the potential fine.

We will use the probability that the message is not equal to the real value to determine $q$. If we look at the real values $v = \frac{1}{8}$ and $v = \frac{3}{8}$, the messages will respectively be $m = \frac{3}{8}$ and $m = \frac{5}{8}$. In these two situations it is clear that the probability $p(m \neq v) = 1$. When this probability is put in the maximizing function of the auditor it follows that the effort for the auditor is $q = \frac{b}{\lambda}$.

If the message is $v = \frac{7}{8}$, it can have two reasons. Firstly, the real value is $v = \frac{5}{8}$ and the firm exaggerates its value by one quarter. Secondly, the real value is $v = \frac{7}{8}$ and thus this will be the message as well since this is the highest value possible. Therefore, if the auditor receives $m = \frac{7}{8}$, the probability that the message is not the real value is $p(m \neq v) = \frac{1}{2}$ since the range of $v$ is evenly distributed. Hence, the effort that the auditor will exercise is $q = \frac{b}{2\lambda}$. To give a good overview, the results are shortly summarized on the next page.
\(v = \frac{1}{8} \& m = \frac{3}{8}\) 
Gives \(p(m \neq v) = 1\) and therefore \(q = \frac{b}{\lambda}\).

\(v = \frac{3}{8} \& m = \frac{5}{8}\) 
Gives \(p(m \neq v) = 1\) and therefore \(q = \frac{b}{\lambda}\).

\(v = \frac{5}{8} \& m = \frac{7}{8}\) 
Gives \(p(m \neq v) = \frac{1}{2}\) and therefore \(q = \frac{b}{2\lambda}\).

\(v = \frac{7}{8} \& m = \frac{7}{8}\)

**Incentives to deviate**

Given this equilibrium we will check if the firm has incentive to deviate. We will start with \(v = \frac{3}{8}\) and see whether it benefits the firm to send \(m = \frac{7}{8}\) instead of \(m = \frac{5}{8}\). The parameter \(q\) in the payoff function of the firm represents the probability that an error has been found. The following equation will give the \(x\) for which the firm has incentive to deviate and therefore sends \(m = \frac{7}{8}\):

\[ -\left[\frac{7}{8} - \left(\frac{3}{8} + x\right)\right]^2 - f \cdot \frac{b}{2\lambda} > -\left[\frac{5}{8} - \left(\frac{3}{8} + x\right)\right]^2 - f \cdot \frac{b}{\lambda} \]

\[ x > \frac{3}{8} - \frac{fb}{\lambda} \quad (2) \]

For \(v = \frac{3}{8}\), the firm will not exaggerate at all if sending \(m = v\) yields a higher payoff than deviating by one quarter. This is the case for the following restriction:

\[ -x^2 > -\left[\frac{5}{8} - \left(\frac{3}{8} + x\right)\right]^2 - f \cdot \frac{b}{\lambda} \]

\[ x < \frac{1}{8} + \frac{2fb}{\lambda} \quad (3) \]

The same method as above is applied to find whether there is enough incentive for the firm to send \(m = \frac{7}{8}\) when \(v = \frac{1}{8}\). The following restriction shows for what \(x\) there is incentive to send \(m = \frac{7}{8}\):

\[ -\left[\frac{7}{8} - \left(\frac{1}{8} + x\right)\right]^2 - f \cdot \frac{b}{2\lambda} > -\left[\frac{3}{8} - \left(\frac{1}{8} + x\right)\right]^2 - f \cdot \frac{b}{\lambda} \]

\[ x > \frac{1}{2} - \frac{fb}{2\lambda} \quad (4) \]
These three restrictions for $x$ are equilibria as well since they describe for what intrinsic value the firm has enough incentive to deviate from the initial equilibria. Notable is that two of the three restrictions above, (2) and (4), are negatively influenced by $f$ and $b$. This makes sense since these are the situations where the firm wants to exaggerate more than the initial quarter. A higher fine negatively influences the firm and a higher bonus gives the auditor more incentive to search for the error which increases the probability of the error being found. Therefore, the fine and the bonus restrain the firm from exaggerating too much.

Regarding restriction (3), this $x$ is positively influenced by the bonus and the fine. This again makes sense because this is the restriction where the firm deviates from the initial equilibrium by choosing a less aggressive strategy. The higher the fine and the bonus, the more often the firm will choose to send $m = v$.

### 3.3 Equilibrium where firm, if possible, exaggerates two quarters

#### Initial equilibrium

The same reasoning as the previous part applies to the situation where the firm exaggerates its real value by two quarters, with $m = \frac{7}{8}$ being the highest message possible. If the real value is $v = \frac{1}{8}$, the firm will send $m = \frac{5}{8}$. Therefore $p(m \neq v) = 1$ so the effort of the auditor will be $q = \frac{b}{\lambda}$.

In this situation, for the real values $v = \frac{3}{8}$, $v = \frac{5}{8}$ and $v = \frac{7}{8}$ the message will be the same, namely $m = \frac{7}{8}$. Two of these three real values are not equal to the message and therefore $p(m \neq v) = \frac{2}{3}$. It can be derived that the effort of the auditor is equal to $q = \frac{2b}{3\lambda}$. This leads to the following equilibrium:

- $v = \frac{1}{8}$ and $m = \frac{5}{8}$ gives $p(m \neq v) = 1$ and therefore $q = \frac{b}{\lambda}$.
- $v = \frac{3}{8}$ and $m = \frac{7}{8}$
- $v = \frac{5}{8}$ and $m = \frac{7}{8}$ gives $p(m \neq v) = \frac{2}{3}$ and therefore $q = \frac{2b}{3\lambda}$.
- $v = \frac{7}{8}$ and $m = \frac{7}{8}$
Incentives to deviate

In the equilibrium for $v = \frac{1}{8}$ & $m = \frac{5}{8}$, the $x$ for which the firm has incentive to deviate by sending $m = v$ is determined. The firm will deviate via this strategy for the following $x$:

$$-x^2 > -\left[\frac{5}{8} - \left(\frac{1}{8} + x\right)\right]^2 - f \cdot \frac{b}{\lambda}$$

$$x < \frac{1}{4} + \frac{fb}{\lambda}$$  \hspace{1cm} (5)

In the same equilibrium, the $x$ for which the firm has incentive to exaggerate even more is found by comparing the payoff for $v = \frac{1}{8}$ & $m = \frac{5}{8}$ with $v = \frac{1}{8}$ & $m = \frac{7}{8}$:

$$-\left[\frac{7}{8} - \left(\frac{1}{8} + x\right)\right]^2 - f \cdot \frac{2b}{3\lambda} > -\left[\frac{5}{8} - \left(\frac{1}{8} + x\right)\right]^2 - f \cdot \frac{b}{\lambda}$$

$$x > \frac{5}{8} - \frac{2fb}{3\lambda}$$  \hspace{1cm} (6)

Now the incentive to deviate from the equilibrium $v = \frac{3}{8}$ & $m = \frac{7}{8}$ is investigated. If the firm deviates from this equilibrium by sending $m = v$, it will do it for the following $x$:

$$-x^2 > -\left[\frac{7}{8} - \left(\frac{3}{8} + x\right)\right]^2 - f \cdot \frac{2b}{3\lambda}$$

$$x < \frac{1}{4} + \frac{2fb}{3\lambda}$$  \hspace{1cm} (7)

In the equilibrium $v = \frac{3}{8}$ & $m = \frac{7}{8}$, the firm can also decide to deviate by sending $m = \frac{5}{8}$. The firm will deviate for the following $x$:

$$-\left[\frac{5}{8} - \left(\frac{3}{8} + x\right)\right]^2 - f \cdot \frac{b}{\lambda} > -\left[\frac{7}{8} - \left(\frac{3}{8} + x\right)\right]^2 - f \cdot \frac{2b}{3\lambda}$$

$$x < \frac{3}{8} - \frac{2fb}{3\lambda}$$  \hspace{1cm} (8)

The firm decides to deviate from the equilibrium $v = \frac{5}{8}$ & $m = \frac{7}{8}$, if the payoff for sending $m = v$ is higher than the payoff in the equilibrium. This is the case for the following $x$:

$$-x^2 > -\left[\frac{7}{8} - \left(\frac{5}{8} + x\right)\right]^2 - f \cdot \frac{2b}{3\lambda}$$
The equilibria (5), (7) and (9) are all positively influenced by the fine and the bonus. This is because a higher fine or bonus results in less incentive to exaggerate and therefore more incentive to send \( m = v \). Since (7) and (9) send the same message in the initial equilibrium we can look at their restriction more closely. Restriction (9) has a higher constant but puts a higher weight on the fine and the bonus as well. This means that if there would be a very small fine or bonus, the firm would more often send \( m = v \) when \( v = \frac{3}{8} \) than \( v = \frac{5}{8} \). However, for a very big fine or bonus the firm would be more inclined to send a message which equals the real value for \( v = \frac{5}{8} \) than \( v = \frac{3}{8} \).

The restriction for \( x \) at the equilibrium (6) shows that the higher the punishment, the more incentive the firm has to exaggerate even more. Since a higher punishment leads to a lower required value of \( x \) to deviate from the initial equilibrium.

### 3.4 Equilibrium where firm, if possible, exaggerates three quarters

**Initial equilibrium**

Once again, the same reasoning as in the previous parts is applied. Irrespective of the value, the firm will send \( m = \frac{7}{8} \) which leads to the following equilibrium:

\[
\begin{align*}
    v &= \frac{1}{8} & m &= \frac{7}{8} \\
    v &= \frac{3}{8} & m &= \frac{7}{8} & \text{Gives } p(m \neq v) = \frac{3}{4} \text{ and therefore } q = \frac{3b}{4\lambda}. \\
    v &= \frac{5}{8} & m &= \frac{7}{8} \\
    v &= \frac{7}{8} & m &= \frac{7}{8}
\end{align*}
\]

**Incentives to deviate**

If we want to investigate whether the firm has incentive to deviate from the above equilibrium, it is obvious that in this case we have an out of equilibrium belief. This means that when the firms deviates by sending a different message, we have to assign a certain probability of the error being found \( q \) to this situation.
When looking at \( v = \frac{1}{8} \), it might benefit the firm to deviate from the equilibrium by sending \( m = \frac{5}{8} \). To be able to determine the \( x \) for which it would yield the firm a higher payoff, an assumption about \( q \) has to be made. This is because given the above equilibrium, \( m = \frac{5}{8} \) is not expected to be sent so it is not known how much effort the auditor will exert to find an error when he receives this message. The \( q \) has to be lower than the probability of the error being found by the auditor in the equilibrium above since the probability that \( m = v \) increases when \( m = \frac{5}{8} \) is sent. Therefore, the \( q \) will be assumed to be \( q = \frac{2b}{3\lambda} \) since this was the probability in the equilibrium where three out of four messages where equal. This results in the following restriction for \( x \):

\[
-\left[ \frac{5}{8} - \left( \frac{1}{8} + x \right) \right]^2 - \frac{2b}{3\lambda} \geq -\left[ \frac{7}{8} - \left( \frac{1}{8} + x \right) \right]^2 - \frac{3b}{4\lambda}
\]

\[
x < \frac{5}{8} + \frac{fb}{6\lambda}
\]  \hspace{1cm} (10)

Restriction (10) shows the highest constant up till now, with the variables being positive related to \( x \). This means a high incentive for the firm to deviate from the initial equilibrium. However, since the last part of the equation is relatively heavily discounted by the constant \( \frac{1}{\lambda} \), the fine and bonus have a less big impact on the decision whether to deviate.

Under the same assumption regarding \( q \), the following equation shows for what \( x \) the firm has incentive to send \( m = \frac{3}{8} \) while \( v = \frac{1}{8} \):

\[
-\left[ \frac{3}{8} - \left( \frac{1}{8} + x \right) \right]^2 - \frac{2b}{3\lambda} \geq -\left[ \frac{7}{8} - \left( \frac{1}{8} + x \right) \right]^2 - \frac{3b}{4\lambda}
\]

\[
x < \frac{1}{2} + \frac{fb}{12\lambda}
\]  \hspace{1cm} (11)

Comparing restriction (10) and (11), it shows that under the assumption of \( q = \frac{2b}{3\lambda} \) the firm will more often send \( m = \frac{5}{8} \) instead of \( m = \frac{3}{8} \).

4. Mixed strategies

Not all equilibria have been determined with the pure strategies. Whenever no equilibrium in pure strategies exists, then there must exist an equilibrium in mixed strategies, according to Nash. To illustrate such a situation we will look at a model where the firm can only have two values, assume \( v = \frac{1}{4} \) or \( v = \frac{3}{4} \).
A mixed strategy in this situation is a strategy where the firm is indifferent between sending \( m = \frac{1}{4} \) and sending \( m = \frac{3}{4} \) when \( v = \frac{1}{4} \). He will send \( m = \frac{3}{4} \) with a probability \( \rho \) and thus sends \( m = \frac{1}{4} \) with a probability \((1-\rho)\). We have to determine for what \( q^* \) the auditor will search for the error, this is the equilibrium value of \( q \) given this equilibrium. This is done by solving the equation of the mixed strategy of the firm. The left hand side of the equation below equals the payoff of the firm when sending \( m = \frac{1}{4} \), which means that the auditor will never search for an error. The first part of the right hand side represents the times that the auditor does not find the error, multiplied with that payoff for the firm. The last part represents the times that the auditor does find an error and therefore the firm has to pay a fine.

\[
-(x)^2 = -(1-q) \left( \frac{3}{4} - \left( \frac{1}{4} - x \right) \right)^2 - q \left( \frac{3}{4} - \left( \frac{1}{4} - x \right) \right)^2 - q \cdot f
\]

Which yields: \( q^* = \frac{4x-1}{4f} \)

So when the auditor receives \( m = \frac{3}{4} \), he will search for the error with a probability of \( q^* = \frac{4x-1}{4f} \).

This is the equilibrium value of \( q \) given the equilibrium above. Given this \( q^* \), we will look what payoff this gives the auditor. The firm sends \( m = \frac{3}{4} \) with a probability \( \rho \), thus \( p(m \neq v) = \rho \). When maximizing the payoff function of the auditor and replacing \( p(m \neq v) \) by \( \rho \), it gives:

\[
\rho \cdot b - \lambda \cdot q^* = 0
\]

Which yields: \( \rho = \frac{\lambda q^*}{b} \)

This equation of \( \rho \) shows that the higher the search costs for the auditor (\( \lambda \)), the higher \( \rho \) will become. This makes sense since search costs give the auditor negative utility thus it gives less of an incentive to search for the error. This stimulates the firm to exaggerate. The probability of searching, \( q^* \), depends on \( x \) and the fine. The higher the intrinsic preference of the firm to exaggerate, the more often the auditor will search for the error since the probability of the firm sending a higher message than its value increases. The fine restrains the firm from exaggerating too often. Therefore it also decreases the search probability of the auditor since he knows that the higher the fine, the less inclined the firm is to exaggerate. At last, the equation of \( \rho \) shows that \( \rho \) decreases with an increase in the auditor’s bonus. This is because a higher bonus incentivizes the auditor to search for the error and therefore the firm will less often send \( m = \frac{3}{4} \) in this mixed strategy.
The approach in this simplified model can be used to determine mixed equilibria in the situations where the firm exaggerates one, two or three quarters. The results regarding the variables will most likely be similar to the results above, however constants will vary depending on which mixed strategy is determined.
5. Conclusion

The amount of exaggeration by the firm (x) influences the message and the search probability of the auditor. The more the firm exaggerates the more often a high message will be sent. Therefore the search probability (q) of the auditor increases since less different messages will be received by the auditor.

For every situation where the firm decides to deviate by sending \( m = v \), the x has to be smaller than a certain value. This makes sense since it means that the situations where the firm does so, the firm does not have a high preference to exaggerate. This implies a low value of x. Whenever the firm has exaggerated and has to decide whether to deviate by sending \( m = v \), the more the firm has exaggerated initially, the more inclined the firm is to deviate by \( m = v \). Moreover, in such situations, the bonus and the fine become less important but the effort for the auditor becomes more important. This can be concluded by comparing the constants in these restrictions.

The decision to deviate by exaggerating even more requires at least a certain value of x. This is logical since this means a high willingness to exaggerate. These restrictions are negatively influenced by the fine and the bonus. The higher the bonus, the more incentive it gives to the auditor to search for the error which increases the probability of the firm getting caught. A higher fine restrains the firm from exaggerating since this results in a lower expected payoff.

A notable result is that when the firm had sent \( m = v + \frac{1}{2} \) and decided to deviate by sending a smaller message, as restriction (8) shows, the bonus and fine were negatively related to the required x. On the other hand, restriction (10) and (11) show a (small but) positive relationship between the x and the fine and bonus.

A simplified model is used to show how a mixed strategy has to be determined. This method can be applied to the model used in this paper. Results will very likely be similar to the results we have obtained via this model. The mixed strategy showed that the equilibrium search probability of the auditor is positively linked to the intrinsic preference of the firm and negatively related to the fine for the firm. Regarding \( \rho \), it showed that the probability of the firm sending a higher message increases in the search costs for the auditor (\( \lambda \)) and the equilibrium search probability (\( q^* \)) but decreases in the bonus for the auditor (\( b \)).
6. Bibliography


