HARD CHOICES

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Akshath Jitendranath

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Supervisor: **Martin van Hees**

Advisor: **Constanze Binder**

Reader: **Jack Vromen**

ERASMUS INSTITUTE FOR PHILOSOPHY AND ECONOMICS,
ERASMUS UNIVERSITY ROTTERDAM,
ROTTERDAM, THE NETHERLANDS

Acknowledgements

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Lord of solutions,
teach us to dissolve
and not to drown

-A. K. RAMANUJAN

(From verse 8 of *Prayers to Lord Murugan*)

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This work is about decision making without drowning. While engaged in it, the threat of the author drowning himself in this work became a real possibility. The following people made sure I did not drown.

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For Ma, Jo, and the Fakir in Shirdi

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INTRODUCTION

The opening couplet of a poem penned by the radical 15th century poet Kabir Das reads;

गुरु गोबिन्द दोउ खड़े काके लागूँ पाँय

My Teacher and my God stand before me now,

To whom, in respect, should I first bow[1]

Kabir's dilemma – to whom should he first bow, his teacher or his god – articulates the nature of a problem addressed in the literature on practical reason, decision theory, and incommensurability of values. This problem is called 'hard choice'[2]. The problem of a hard choice involves decision making without the existence of an easy or straightforward choice from among the alternatives in a choice situation. In this work I will be concerned with this problem. I will be concerned with (i) what is a hard choice; (ii) what are the conditions in which they emerge; and (iii) how does an agent make a justified choice in the conditions that are sufficient for a choice situation to be a hard choice. To investigate these concerns, I have two research questions. I present these questions, and the claims I make while answering them below.

Research Question and Claims

First, "What is a hard choice?" In answering this question I provide a formal definition of a hard choice which identifies two necessary conditions which are jointly sufficient for a choice situation to be a hard choice.

I proceed to discuss one of the necessary conditions – incompleteness of an evaluation – in a more substantive way. In this discussion I present and discuss the reasons for the incompleteness of an evaluation, and the implications of these reasons for a choice situation. In this discussion I identify one sufficient condition for a hard choice.

Second, "Is there a justification for making a hard choice?" In particular, I ask is there a justification for making a choice when the sufficient condition for a hard choice identified above is present in a choice situation. In answering this question I evaluate two different frameworks of practical reasoning that have been proposed. I evaluate them critically and in a

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[1] Translated by the author

[2] See Hsieh (2007a) and Chang (2013) for overviews of the literature.

[3] In particular we will see it is the empty optimal set defined by the transitive closure R* of a relation R, and the incompleteness of this in detailing

[2] See Hsieh (2007a) and Chang (2013) for overviews of the literature.

[4] We will, however, return to this issue below and in subsequent chapters. In particular, when defining a hard

comparative way. I show that both frameworks have problems. The proposal of one framework – comparativism – involves dropping or rejecting the axiom of transitivity of relations. The second framework – maximization – takes practical reason outside the scope of rational agency.

Outline

The two questions specified above will be taken up in three chapters.

In Chapter One – What is a Hard Choice? – we address the first research question of this work that we have stated above. This chapter will provide an answer to the question, "What is a Hard Choice?" This answer gives us a formal definition of a hard choice. Within this chapter I also motivate the need for a definition. This is because the literature on hard choice provides a definition for the binary, or two alternative, case. The definition provided for a binary case does not transpose to choice situations with more than two alternatives. To provide this definition I use the concepts and notation developed in rational choice theory. This definition characterises a hard choice for every choice situation involving a finite number of alternatives in a choice situation. The definition identifies two necessary conditions which are jointly sufficient for a hard choice, viz. a null optimal set and an incomplete binary relation[3].

In Chapter Two – Incompleteness and Incomparability – we discuss one of the necessary conditions for a hard choice. In particular, we ask two questions: (i) what is the reason for the incompleteness of an evaluation in a choice situation; and (ii) what implication does this reason have for a choice situation. We discuss and critically evaluate three different positions in the literature that answer the first question – epistemic limits, incommensurable values, and incomparability. We show that the most plausible reason for incompleteness – incomparability – can be a feature of a choice situation in two ways – partial incomparability and global incomparability. We conclude by discussing one implication of incomparability for a choice situation. In particular, we show that global incomparability is a sufficient condition for a hard choice.

There is an interregnum between chapters two and three. In this interregnum I introduce the "problem" the sufficient condition we identify in chapter two presents to a choice situation. I introduce this problem before proceeding to chapter three because the discussion in chapter three is about overcoming this problem. I show that the 'problem' is that if there is global incomparability of alternatives in a choice situation – which is the sufficient condition for hard choice – then it precludes the possibility of justified choice.

Chapter Three – Comparativism and Maximization – will be concerned with the second research question, "Is there a justification for making a hard choice?" In particular, I ask is there a justification for making a choice when the choice situation is constituted by global incomparability. I consider two responses in the literature to the problem presented by global

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[3] In particular we will see it is the empty optimal set defined by the transitive closure R* of a relation R, and the incompleteness of this relation.

incomparability discussed in the interregnum. In particular I discuss two frameworks of practical reason that claim to provide a solution to the problem of making a justified choice in the presence of incomparable alternatives. One of these frameworks is called comparativism. Comparativists argue that the 'problem' posed by global incomparability, or the incomparability of every alternative in a choice situation, follows from an unjustified assumption we have made in conceptualizing incomparability. This assumption is: if alternatives can be compared, then they can be compared in only three ways, viz. "better than," "worse than," and "equally good". Comparativists drop this assumption and proceed to introduce and argue for a fourth way in which alternatives can be compared. Now, if incomparable alternatives can be compared by this fourth relation, then the problem for the justification of a choice, which exists because alternatives are incomparable, can be overcome. The second response comes from a framework of reasoning developed in rational choice theory. This is called maximization. This response does not see incomparability as a problem for the justification of choice. A 'maximizer' argues that justified choice is possible in the presence of incomparable alternatives by choosing that alternative which is not "strictly worse than" other alternatives. Since incomparable alternatives are not strictly worse than each other, maximization can provide a justification for the choice of incomparable alternatives. In this way they overcome the problem presented by a hard choice situation. However we will see that both responses do not provide a justification. After presenting each of these responses, I will argue that neither the comparativist response nor the maximization view of rationality provide a justification for choice. I show that the comparativist response entails dropping or rejecting the axiom of transitivity. The maximization response is unsatisfactory because it could justify choice based on a randomized procedure like flipping a coin which takes practical reason outside the scope of rational agency.

A conclusion summarises the claims made.

What is a Hard Choice?

In this chapter I take up the first question of this work. "What is a hard choice?" The literature on hard choice, we will see, provides a definition for the binary, or two alternative cases. Here I provide a definition that applies to every choice situation defined by a finite set of alternatives.

This will proceed in three sections. In section II we introduce some basic notation and concepts. In section III we take up the question what is a hard choice? In the first part of this section we motivate the need for a definition of hard choice that goes beyond the definition found in the literature. In the second part of this section I discuss what hard choices are not. In the third and final part of this section I provide a formal definition of a hard choice. A final section concludes.

II.   Notation and Concepts

Let X denote a finite set of alternatives. Let **Z** be the set of all non-empty subsets of X. We call **Z** a set of *agendas*. An *agenda* A in **Z**, or a choice situation as we shall call it, is a non-empty subset of X. Let R stand for a binary relation on X; R is a subset of ordered pairs in the product of X×X. If a pair (x, y) from X×X is an alternative of R then it will be denoted x R y. Conversely, if this is not the case we write ~ (x R y).

In the context of discussing choices, the relation R has been interpreted as articulating value. Here I do not commit to any substantive interpretation[4], and will call the relation R a preference relation[5].

One can think of a number of different properties a binary relation can satisfy. The following have been found to be important to specify.

Reflexive: $\forall\ x \in X: x\ R\ x$

Complete: $\forall\ x,\ y \in X:\ (x \neq y) \rightarrow (x\ R\ y \vee y\ R\ x)$

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[4] We will, however, return to this issue below and in subsequent chapters. In particular, when defining a hard choice and when discussing substantive reasons for why a hard choice is hard, we will see that many authors commit to an interpretation of the binary relation as representing value relations. When I engage with those authors I too will use their interpretations to respond to them. Note however that in this chapter, unless engaging with a specific interpretation in the literature, I do not commit to any interpretation of a binary relation.

[5] Note, here too I do not commit to any substantive interpretation of preference. I am only calling the relation R a preference relation and attach to this nomenclature only its formal meaning, viz. a binary relation on a set of alternatives.

Transitive: $\forall$ *x, y, z* $\in$ *X: (x R y & y R z)* $\rightarrow$ *(x R z)*

Now, with the binary relation or preference relation R, we can define relations of "strict preference" P and of "indifference" I.

Definition 1:   *x P y iff [x R y & ~ (y R x)]*

Definition 2:   *x I y iff [x R y & y R x]*

Further, R is said to be quasi transitive if the strict preference relation P is transitive.

R is said to be acyclical if for all finite sequences $\{x_1, x_2, x_3,....... x_n\}$ of X if $x_1 P x_2$, $x_2 P x_3$,......., $x_{n-1} P x_n$; then ~($x_n P x_1$).

We now define the 'optimal set' of an agenda A in **Z** with respect to a binary relation R

Definition 3: *An alternative x in an agenda A is said to be an optimal alternative of A with respect to a binary relation R iff:*

$$\forall \ y \in A: x \ R \ y$$

The set of optimal alternatives of an agenda is called the optimal set and is denoted B(A,R).

The optimal set captures the intuition that an alternative x can be called an 'optimal' alternative of an agenda A if it is "at least as good as" every other alternative in the agenda A with respect to the relevant binary relation R.

For every agenda A in **Z** we can also define the 'maximal set' with respect to the relevant binary relation R.

Definition 4: *An alternative x in an agenda A is a maximal alternative of A with respect to a binary relation R iff:*

$$\forall \ y \in A: \sim(y \ P \ x)$$

The set of maximal alternatives is called the 'maximal set' and is denoted M(A,R).

The maximal set is constituted by the alternatives of an agenda which are not dominated by, or not "strictly worse than", any other alternatives in that agenda.

A point of clarification is worth making here. An optimal alternative is also a maximal alternative but not generally the converse. That is, an alternative in the optimal set is also an alternative in the maximal set, but an alternative in the maximal set need not be an alternative in the optimal set. To illustrate, if x R y for all y in A, then clearly there is no y in A such that y P x. On the other hand, if neither x R y nor y R x, then x and y are both maximal alternatives of the set {x, y}, but neither is a best alternative.

Thus, B(A, R) $\subseteq$ M(A, R) but not generally the converse.

III. Defining a Hard Choice

With the conceptual and notational preliminaries out of the way, we can now focus on the task of this chapter. In this section I will take up and answer the question, "What is a hard choice?" The answer will consist in providing a formal definition of a hard choice. This discussion will consist of three parts. In Part A we motivate the need to define a hard choice by referring to both, the agenda and the binary relation, as opposed to defining a hard choice *exclusively* in terms of the binary relation. In Part B, we discuss what a hard choice is not. In Part C we define a hard choice.

A. *A definition for all agendas A (and not just the agenda with two alternatives)*

In the literature on hard choice, the definition of a hard choice is given by referring to the case of two alternatives[6]. Readers are asked to consider a choice situation with two alternatives, and the definition that follows is that a hard choice is a choice situation where the relation R is not defined over these two alternatives. That is R is incomplete in the choice situation. To illustrate this, consider the following list of examples from Ruth Chang's discussion of a hard choice. (2012: p. 106-07)

> "You are a single parent unhappy in your current job and have just received your dream job offer in a different city. But your young children are leading happy, fulfilled lives which would be less good were you to move the family. Should you take the job?"

> "You have decided to spend your Saturdays giving back to the community. You can help organize for your favorite candidate's re-election campaign or mentor a disadvantaged child in your neighborhood. How should you spend your Saturdays?"

> "You're getting a bonus in your paycheck and could buy a new car or donate the funds to Oxfam. What should you do?"

In these discussions R is called a positive value relation and R is interpreted to articulate value[7]. In particular, R is interpreted to articulate one of the following three values: "better than," "worse than," and "equally good as"[8]. To illustrate, x P y is interpreted as x is "better than" y. Similarly, y P x is interpreted as x is "worse than" y. Finally, x I y is interpreted as x is "equally good as" y. A hard choice is then defined as follows: if it is not true that either, x is "better than" y, or x is "worse than" y, or x is "equally good as" y; (call this a *valuational conflict*[9]), then there is a hard choice.

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[6] See in particular Chang (2012). See also Chang (1998), Hsieh (2007a)

[7] See Chang (1998), Chang (2002a), Chang (2002b), Chang (2005).

[8] This is an interpretation of the binary relation to represent value, and I am only illustrating how this interpretation has been used in the literature to let the argument proceed. I do not commit to an interpretation for the purpose of providing a definition. We will, however, return to this issue in subsequent chapters.

[9] Note that it is only in this Part that I am calling this a *valuational conflict*. The term valuational conflict should not mean anything for this thesis after Part A of Section III of Chapter One. We will see in the subsequent chapters that what I have just called a valuational conflict is called *incomparability* in the literature See Chang

The disadvantage of this definition is that it is restricted to the set of agendas A in **Z** which consist of only two alternatives. Indeed, this definition does not apply to every agenda A in **Z**. There can be what I have called a *valuational conflict* in the evaluation of two of the alternatives in an agenda, or choice situation, of more than two alternatives. But it does not follow that the choice to be made from this agenda, or choice situation, must necessarily be hard. To illustrate this point, consider the case of John who prefers hot chocolate to tea, and hot chocolate to coffee, and cannot decide whether he likes tea or coffee equally, or prefers one to the other. Here the choice situation involves a *valuational conflict* between tea and coffee. But the choice for John when he wants a hot beverage is clear, and indeed rather easy. When John can choose between tea, coffee, and hot chocolate, John will choose hot chocolate. To make this more precise, consider a choice situation given by agenda A = {x, y, z} if x R y, and x R z and ~(y R z or z R y), then we have an easy choice, viz. x, in the presence of valuational conflict between y and z. This is because x is optimal in this agenda[10]. As this illustrates, definitions of a hard choice where the agenda, or choice situation, consists of only two alternatives cannot be transposed to the choice situation where the agenda, or choice situation, consists of more than two alternatives.

In what follows, I provide a formal definition of a hard choice in terms of the binary relation R. However, I refer to the agenda A which gives the choice situation from which a choice is to be made. I do not assume that the agenda consists of only two alternatives. By dropping this assumption I provide a definition that holds for all agendas A in **Z**, including the agenda that consists of two alternatives.

In what follows, I always assume that R is reflexive.

B. *Straightforward choice and irrational behaviour (Or what is not a hard choice)*

In this part of the section, I separate hard choice from (i) a straightforward choice; and (ii) a type of *behaviour*. To do so, I present a case that represents a straightforward choice.

Case 1:   B(A,R) is non empty;

To get a grip on what is a hard choice, it would be instructive to begin with a seemingly straightforward choice. Case 1 presents such a situation. In case 1, we have a choice situation given by an agenda A, and the optimal set of that agenda B(A,R) is non-empty. The optimal set of an agenda A has been defined in (3) as consisting of alternatives from an agenda A that are at least as good as every other alternative in that agenda. If there is an alternative that is at least as good as every other alternative in an agenda A, or choice situation, then the choice situation given by agenda A is 'straightforward' and not hard. This is because there is a

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(2013) and Hsieh (2007a) for overviews. To avoid confusion with nomenclature, let it be known that I am only calling it a valuational conflict for the purpose of motivating the need for a general definition of a hard choice. Indeed, the first systematic presentation of the problem of hard choices by Levi (1986) uses the term *valuational conflict* to both motivate and describe cases of hard choice. The subtitle of that classic work reads "Decision Making Under Unresolved Conflict". However, the literature has since proceeded and grown, and unresolved conflict is now called *incomparability* in the literature on hard choice. We return to this in chapter two.

[10] The presence of optimal alternatives does not necessarily imply the choice is "easy" as I have implied it is in my illustration. If there are more than one optimal alternatives, for instance, then it does not have to follow that this choice is easy. We return to this issue in Part B of this section.

rational principle to justify the choice, namely optimization[11] and there are alternatives that meet the demands placed by this principle, namely the set of 'optimal alternatives' B(A,R).

To be sure, a 'straightforward choice' does not imply the choice is "easy". Here I am only claiming that the choice is *not hard*. If the optimal set is a singleton set, then the choice is easy[12]. This is because there is a unique best or optimal alternative. However, if the optimal set is not a singleton set, then the choice situation, given by agenda A, has more than one alternative that is at least as good as every other alternative. In this instance, the alternatives in the optimal set are "indifferent". When the alternatives in the optimal set are indifferent, there is symmetry in the preference between them. In their classic discussion Edna Ullmann-Margalit and Sidney Morgenbesser (1977) make a distinction between decision making with symmetric and asymmetric preferences. They call decision making with symmetric preferences "picking", and decision making with asymmetric preferences "choosing". The latter case is an 'easy' choice, because the choice is determined. In the former case though, there is indeterminacy. They write,

> "We speak of choosing among alternatives when the act of taking (doing) one of them is determined by the differences in one's preferences over them. When preferences are completely symmetrical, where one is strictly indifferent with regard to the alternatives, we shall refer to the act of taking (doing) one of them as an act of *picking*." (Ullmann-Margalit and Morgenbesser 1977: p. 757)

To illustrate this with a (classic) example, consider the story of Buridan's ass. This is the story of the donkey who was indecisive over two equally good stacks of hay x and y, that it died of starvation z. Indeed, there is no consideration that favours choosing one stack of hay over the other stack of hay to determine the choice. This is indicated by the two stacks of hay being symmetric, or equally good. However, there is some consideration in favour of choosing of choosing either x or y. This consideration is that both stacks of hay x and y are better than, or strictly preferred to, dying of starvation z. This consideration in favour of either alternative provides sufficient reason to 'pick' either alternative. There is no need to suspend decision making till we find some consideration in favour of choosing one alternative in the optimal set over the other. The case of picking from an optimal set, though not as easy as choosing from a singleton set, remains straightforward.

Symmetry between the alternatives indicates that a choice is not easy. It does not suggest that the choice is not straightforward. It is not 'easy' because among the alternatives in the optimal set there is no consideration that provides most reason for choosing one alternative over the other. But picking, as Ullmann-Margalit and Morgenbesser (1977) call the selection when this is the case, is not a *hard* choice, because it is not a *choice*, i.e., there can be no *reason-based* selection. Indeed, (as is obligatory to say when one uses the example of Buridan's donkey) only an ass would think otherwise.

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[11] This principle can be expressed as choosing, among the available alternatives, a "best" or optimal alternative

[12] A singleton set, also known as a unit set, is a set with exactly one element.

There can, however, be puzzling instances of *behaviour* that can be observed in a choice situation involving optimal alternatives. In what follows I use case 1 to separate a hard choice from this puzzling *behaviour.*

Consider the following: Ram faces a choice situation A = {x, y, z} and Ram evaluates the alternatives in the following way: x P z, x I y, and y P z. In this choice situation x and y are the 'optimal alternatives' and the choice is straightforward. However, Ram selects z instead of x or y. Ram's selection presents a puzzle. When faced with an agenda that has a non-empty optimal set, Ram did not choose the optimal alternatives. Ram's selection, 'z', does not reflect his evaluation. Rationality demands that action coincides with an overall evaluation of the alternatives, but Ram's doesn't. When the overall evaluation of the alternatives diverges from action, as Ram's behaviour does, it is irrational behaviour. We will not be concerned with cases of irrational behaviour in this work. Ram's choice situation is different from hard choice because Ram has a set of optimal alternatives from which to make a choice.

C. A hard choice

We now discuss cases that discriminate a hard choice from a straightforward choice, or an easy choice. At the end of this discussion we define a hard choice.

Consider the following case:

Case 2:   B(A,R) is empty.

This case represents an agenda, or choice situation, where the choice is not straightforward. This is because the optimal set B(A,R) of that agenda A is an empty set or a null set. This means that no alternative from an agenda A, is at least as good as every other alternative in that agenda A. This case presents a curious situation because there appears to be no alternatives for rational choice. Indeed, if someone is only looking at this choice set, then she might be tempted to conclude that we cannot think of this agenda as a *choice* situation. That is to say, when the optimal set of an agenda B(A,R) is empty there appear to be no alternatives in that agenda A from which a *choice* can be made.

Note however that we have not made any reference to the maximal set M(A,R). Indeed, it can be the case that B(A,R) is empty, but M(A,R) is non empty. Thus a *choice* will be possible to make from an agenda A. We will return to this below. However, I want to use Case 2 here as a point of departure. The purpose of this departure is to get a necessary condition for the characterisation, or definition, of a hard choice by referring to the optimal set of an agenda A.

There are many results in social choice theory that are like the agenda, or choice situation, given by Case 2[13]. In what follows I present one of the responses in that literature when faced with an optimal set of an agenda that is empty[14]. Recall that we defined the optimal set and maximal set in (3) and (4) above using the relation R. We can, however, define the optimal

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[13] Indeed, Kenneth Arrow's (1963) classic result generalizes this for all agendas A in Z. Also see Sen (1970) Mas-Colell and Sonnenschein (1972), Schwartz (1986) and Schwartz (2001).
[14] See in particular Schwartz (1970) and Schwartz (1972). See also Duggan (2007)

set and the maximal set of an agenda A in **Z** based on other formal properties of R. Indeed, one of the responses in social choice theory to overcome the problem posed by case 2 has been to define the optimal set and maximal set using the transitive closure of the binary relation R. In what follows I first define this relation, and present two amended versions of case 2 involving the optimal set of this relation.

Before I proceed, let me motivate the concept of the closure of a binary relation. A binary relation R on a set may not have a particular property, such as reflexivity. However it may be possible to extend that relation so that it does have that property. Extending R means finding a larger subset of $X \times X$ that contains R and has the desired property. One such closure is the transitive closure. We denote this relation R*.

Definition 5: *x R* y iff there exist x_1, x_2, x_3,....... x_n in X: $x_1 R x_2$, $x_2 R x_3$,........, $x_{n-1} R x_n$, $x_n R y$*

To illustrate, x R* y means that you can move from x to y in the following way: $x = x_1 R x_2$, $x_2 R x_3$,........, $x_{n-1} R x_n = y$. So x R* y means that we may move from x to y in a number of steps in R.

Note that R=R* when R is transitive

We can now present the first of the two amended versions of Case 2.

Case 2.1: Both B(A,R) and B(A,R*) are empty

We saw above that Case 2 suggests a curious situation where there appears to be no alternatives for *rational choice*. Indeed, we were tempted to conclude that we cannot think of the agendas of case 2 as a *choice* situation. The real test of whether an agenda A presents a straightforward choice situation is to see it together with the optimal set of the transitive closure R*, and Case 2.1 presents just that.

Here we have a case where the optimal set defined using the binary relation R is a null set. Further, the optimal set defined using the transitive closure R* of R is also a null set. Therefore this choice situation is not as straightforward as the choice situation in Case 1. Indeed, we can posit that if B(A,R) and B(A,R*) are null sets, then the choice is not straightforward as there are no optimal alternatives to make a choice from.

However, what if B(A,R) is empty, but B(A,R*) is non-empty. We take this up in the next amendment of case 2.

Case 2.2: B(A,R) is empty, but B(A,R*) is non-empty

We saw above that Case 2.1 suggests there is no optimal choice to be made. Here we have a case where the optimal set defined using the relation R is a null set. This suggests that the case is definitely not an easy choice, because the possibility of singleton optimal sets does not exists in this case. But, the optimal set defined using the transitive closure R* of R is non-empty. Is this case as straightforward as Case 1?

There can be one objection against this being a 'straightforward' choice. In what follows, I take up this objection and argue against it.

The objection to considering a choice from B(A,R*) as not being straightforward is that the choice from B(A,R*) involves making a choice from a set defined by an intransitive preference relation. Indeed, R and R* diverge only when R is not transitive. So B(A,R*) is a set of alternatives with intransitive preference relation when R ≠ R*. Consider for instance the case of a set of alternatives with cyclical preference relations. To illustrate, when R = {(aPb), (bPc), cPa)}, we have R* = {(aI*b), (bI*c), (cI*a)}. In this situation, B(A,R) is a null set, but B(A,R*) = {a, b, c}. The objection to considering the choice from this set being 'straightforward' consists in the claim that transitivity is necessary for a choice to be straightforward[15]. Since B(A,R*) consists of a set of alternatives with intransitive preference relations the choice from B(A,R*) is not straightforward when R is not transitive. In response we can ask, is transitivity necessary for a 'straightforward' choice[16]? I present an argument for the view that transitivity of the preference relation is necessary for a choice to be straightforward and argue against it.

It can be argued that a preference relation is important for determining choice[17]. When an individual makes a choice from a set of alternatives, then the preference relation in that set should determine choice. When choosing from B(A,R*), in the example above R* does not determine choice. This is because we have {(aI*b), (bI*c), (cI*a)}. Either every alternative should be chosen, or none of the alternatives should be chosen. So the argument for the necessity of transitivity for choice in cases as these is the following. A preference relation guides and determines choice. Here, however, the preference relation is not able to guide or determine choice because of its intransitivity. Notice however, that even in the instance of non-singleton optimal sets B(A,R) discussed above, the preference relation R does not guide or determine choice from B(A,R). This is because in non-singleton B(A,R) the preference relation is indifferent. In that instance too the preference relation does not guide or determine choice as we discussed above. Indeed, this was the reason a distinction was made between 'picking' and 'choosing'. 'Picking' there refers to choice situations where the preference relation does not guide, or determine choice. The same argument can be made here. Just because the preference relation does not determine a choice, it does not have to follow that a choice is not straightforward. We can 'pick' from B(A,R*) in just the same way as we pick from a non-singleton B(A,R). In both instances the preference relation does not guide or determine a choice. But it does not follow that they are not straightforward choices.

We have now discussed two cases. Case 1 presented a non-empty optimal set B(A,R) of a choice situation given by an agenda A and showed why this is a straightforward choice. Case 2 presented an empty optimal set B(A,R). However, we did not conclude from this that the case is not straightforward. This is because in case 1 and case 2, we defined the optimal set

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[15] A claim can also be made that the transitivity of the preference relation is a matter of logical consistency. For instance the relation "taller than" has to be transitive from a logical point of view. Here however, I am concerned with the choice aspect of transitivity, so I restrict my argument against transitivity as being necessary for choice to be straightforward.

[16] Note again that I am not claiming that B(A,R*) presents an easy choice. I am arguing that it is straightforward, and not *hard*. For a fuller critique of whether transitivity is necessary for rational choice, see Anand (1993).

[17] See Hansson and Grüne-Yanoff (2012), in particular section 3.2 where they make this argument for choice functions, and not choice sets as I present the argument here.

using the binary relation R. To test if case 2 is indeed a hard choice, we first defined the transitive closure R* of the relation R, and then presented two amended versions of case 2, viz. case 2.1 and case 2.2, where the optimal set is defined using R*. In the former case, both the optimal set B(A,R) of the binary relation R, and the optimal set B(A,R*) of the transitive closure R* of the binary relation R were null sets. This is clearly not a straightforward choice. However, is B(A,R*) being non-empty a straightforward choice? We discussed this in case 2.2. There we defended a non-empty B(A,R*) as constituting a straightforward choice.

The implication of these cases for our task of providing a definition is clear; *it is necessary for the definition of a hard choice that B(A,R) and B(A,R*) are null sets*. This is because there would be nothing "hard" about making a choice when B(A,R) or B(A,R*) are non-empty.

Now, consider the maximal set defined in (4) above. M(A,R*) is non-empty in every agenda A. So I do not present a non-empty M(A,R*) as a special case, because this is always the case. Indeed, a non-empty M(A,R*) allows us to consider an agenda A as a choice situation. This is because there is some *choice* to be made. The question now is, when is this choice hard? To answer this we present a result established by Sen (1970) on the relationship between M(A,R) and B(A,R).

Theorem 1 (Sen 1970)**:** *M(A,R) = B(A,R) for all agendas A in Z iff R is complete.*

The implication of this result is clear. We partition it into two parts. First, if R is transitive, then R = R*, and M(A,R) is non-empty. By theorem 1, if R is complete, then M(A,R) = B(A,R), and since M(A,R) is non-empty the choice situation becomes straightforward as discussed in case 1. Second, in case R is not transitive, then M(A,R*) is non-empty. By theorem 1, if R* is complete, then M(A,R*) = B(A,R*), and since M(A,R*) is non-empty the choice situation becomes a straightforward choice as was discussed in case 2.2.

Therefore, an incomplete R* is a necessary condition for B(A,R) and B(A,R*) to be null sets. Else the choice situation becomes a straightforward choice. Since it is necessary for a hard choice that B(A,R) and B(A,R*) are null sets, it is also necessary or a hard choice that R* is incomplete.

Having discussed these cases, and arrived at necessary conditions for the characterisation, or definition, of a hard choice, we can present this definition now.

Definition 6 (Hard Choice): *Given a set of alternatives X, and a binary relation R, a choice situation given by an agenda A in* **Z** *constitutes a hard choice iff: B(A,R*) is empty*

Conclusion

In this chapter we took up a lacuna in the literature on hard choices. In particular there is no general answer beyond the binary, or two alternative case, to the question: what is a *hard choice*? This chapter provided an answer to this question, using the paradigm of rational choice theory, by providing a formal definition of what a hard choice is. A hard choice is

defined as: *Given a set of alternatives X, and a binary relation R, a choice situation given by an agenda A in **Z** constitutes a hard choice iff: B(A,R*) is empty.*

Incompleteness and Incomparability

In the previous chapter we took up and answered the question, "What is a hard choice?" by providing a formal definition of a hard choice. Our discussion there identified the *hard* aspect of a choice to be associated with the completeness property of both, a binary relation and the transitive closure of that relation. We saw that incompleteness of the binary relation, and the transitive closure of the binary relation, is necessary for a hard choice.

In this chapter we will be addressing two issues that are concerned with incompleteness. The first issue is: what is the reason for incompleteness in a choice situation? Note that while this does not answer why a hard choice is hard, because incompleteness is only a necessary condition for a hard choice. This does inform us on why a choice is not straightforward, because completeness is sufficient for a straightforward choice in a finite set of feasible alternatives. The second issue that we discuss is the following: what implications does this reason for incompleteness have for a choice situation? We will see that this reason can provide a sufficient condition for a hard choice.

To proceed, we first partition incompleteness into two types – resolvable incompleteness and irresolvable incompleteness. I argue that irresolvable incompleteness is the 'type' of incompleteness that is necessary for a hard choice. We then proceed to discuss three different positions in the literature that provide a substantive answer to the question what are the reasons for irresolvable incompleteness. The first position that we consider is epistemic limits. I will show that epistemic limits cannot explain the persistence of irresolvable incompleteness. The second position that we take up is the incommensurability of values. I argue that the presence of incommensurable values does not entail that an evaluation of alternatives in a choice situation will be incomplete. We proceed to present the most convincing reason for or cause of irresolvable incompleteness – "incomparability". Indeed, incomparability is the third position we discuss. I present incomparability as being the most plausible answer to the question what is the reason for irresolvable incompleteness. Finally, we discuss the implications this reason for irresolvable incompleteness has for a choice situation. In particular, I show that incomparability can be a feature of a choice situation in two ways – partial incomparability and global incomparability. Partial incomparability refers to when there are *some* alternatives in a choice situation, given by an agenda A, that are incomparable. Global incomparability refers to when *every* alternative in a choice situation, given by an agenda A, is incomparable.

I then proceed to show one implication of incomparability for a choice situation. In particular, I show that if there is global incomparability in an agenda A, then it is sufficient to make a choice *hard*.

Section II will introduce and discuss the two types of incompleteness, viz. resolvable and irresolvable incompleteness. In this section I will show why irresolvable incompleteness is what is necessary for a hard choice. In Section III we present and critically discuss three positions in the literature that provide reasons for or causes of irresolvable incompleteness. These three positions are: (a) epistemic limits; (b) incommensurable values; and (c) incomparability. In Section IV we discuss the implications of incomparability for a choice situation. Here I first discuss and then define the two ways in which incomparability can be a feature of a choice situation, viz. partial incomparability and global incomparability. I then proceed to argue that if there is global incomparability in an agenda A, then it is sufficient to make a choice situation a hard choice. A final section concludes.

II.   Types of Incompleteness

A binary relation on a set, in this instance a set of feasible alternatives for choice, is complete if every alternative in that set is related to every other alternative in that set. The relation is incomplete when this is not the case. Indeed, completeness, as we saw in the characterisation, or definition, of a hard choice is sufficient to make a choice straightforward when the feasible set is finite, and incompleteness is necessary for a choice situation to be a hard choice. In what follows we discuss the different types of incompleteness, and I argue that one of these 'types' of incompleteness is what is necessary for a hard choice.

Incompleteness of a binary relation or preference relation can be partitioned into two types – resolvable incompleteness and irresolvable incompleteness[18]. Further, resolvable incompleteness can be divided into uniquely resolvable incompleteness and multiply resolvable incompleteness (Hansson & Grüne-Yanoff, 2012). We discuss them in turn[19].

Incompleteness may be *uniquely resolvable.* As the nomenclature suggests, this type of incompleteness can be overcome or resolved in exactly one way (Hansson & Grüne-Yanoff, 2012). The reason for this type of incompleteness is either the lack of a deep analysis about the alternatives in a choice situation, or it can be caused by the lack of access to the information relevant to evaluate the alternatives in a choice situation. What we see as an incomplete ranking can be made a complete ranking by either gaining more facts and information about the alternatives, reflecting deeper and making inferences, or some other means of analysis[20] (Hansson & Grüne-Yanoff, 2012). For example, given that you prefer Italian food to Indian food and if you are undecided between going to restaurant A and restaurant B. Getting to know that restaurant A is an Italian restaurant and restaurant B is an

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[18] They have also been called tentative incompleteness and assertive incompleteness. The distinction is discussed in this way in Sen (1992, pp. 46-49), Sen (1997) Sen (2002) and Sen (2004)

[19] This discussion draws on (Hansson & Grüne-Yanoff, 2012) and Sen (2004: p. 54-55)

[20] See in particular section 1.2 of (Hansson & Grüne-Yanoff, 2012)

Indian restaurant can resolve the incompleteness of the preference relation – in this instance via a strict preference for restaurant A over restaurant B.

Incompleteness may be multiply resolvable. As the name suggests, this type of incompleteness is possible to resolve in many different ways. Hansson & Grüne-Yanoff, (2012) suggest that with this type of incompleteness it is undetermined what the outcome of completing the preference relation will be. To illustrate with an example, consider the situation of a heterosexual couple planning their first baby. The individuals involved have no preference for whether they will love a boy more than they would love a girl[21]. One way this can be resolved is if after the delivery, the couple finds out, rather happily, that the *children* are safe and indeed they are twins – a boy and a girl. The children will be loved equally. Another way this can be resolved is if the couple have a girl child. Then they would love their girl child more than they would the non-existent boy child. Indeed, this sort of preference incompleteness can be resolved in multiple ways as the example suggests, and it is undetermined how the resolution will take place.

Finally, incompleteness may be *irresolvable*. This type of incompleteness does not wait for resolution. Instead, incompleteness, or the lack of a complete ranking or judgement between two alternatives is an assertive statement[22]. The reason for this type of incompleteness is that the two alternatives being compared differ in the values they bear or instantiate and an individual cannot make a comparative ranking between them at the end of an evaluative exercise (Hansson & Grüne-Yanoff, 2012).

To illustrate this with an example, consider the case of Mohammed[23]. Mohammed is an anti-terror operative. Mohammed and his team know that a terrorist group is planning two attacks, but they can stop only one. The first attack is on the ancient city of Palmyra. The city isn't densely populated, but is rich with the heritage of the human civilisation. The second attack is on the central district in the city of Bombay. Bombay is much more densely populated than Palmyra which means Mohammed and his team would save many more lives. There are good reasons to prevent both attacks. Mohammed can choose to save a historical site, or he can choose to save many lives. But Mohammed and his team of commandoes can prevent only one attack, now the choice is which attack should Mohammed and his team prevent?

Our intuitions of what constitutes a hard choice suggest that what Mohammed is facing is a hard choice. Not the deliberation of which restaurant to go to for dinner. In Mohammed's case, incompleteness is irresolvable. No amount of additional information, or deeper reflection, can complete this evaluation and make a choice straightforward.

However, in the definition of a hard choice, we identified the incompleteness of a binary relation as being a necessary condition for a choice situation to be a hard choice. The definition was a formal one. A definition does not worry about whether the incompleteness that exists in a choice situation is of a certain type. If there is incompleteness, whatever type it is, then in a formal sense it is necessary for a choice situation to be a hard choice. In a more

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[21] Apologies for the social construction of gender involved in this example.
[22] See Sen (2004: p. 54-55). There it is called assertive incompleteness.
[23] I am unsure of the provenance of this example, or where I heard it first. I note here that this is not my example. It is an amended version of a story I have heard but cannot place.

substantive sense, our intuitions about what constitutes a hard choice suggest that resolution of incompleteness is not possible. Indeed, it is irresolvable incompleteness that is necessary for a hard choice.

In what follows I will investigate what is the reason for, or cause of irresolvable incompleteness. This is because irresolvable incompleteness gets at our intuitions of a hard choice. We will present three different positions in the literature that provide an answer to this question – epistemic limits, incommensurable values, and incomparability. I will argue that epistemic limits cannot explain the persistence of irresolvable incompleteness. I will show that the incommensurability of values – or the lack of a common scale of measure – does not imply that incompleteness should follow. Finally, we will discuss incomparability, and show why it provides the most plausible argument for irresolvable incompleteness.

III. Why is there Irresolvable Incompleteness?

We have now partitioned incompleteness into two types – resolvable and irresolvable. While making this partition we already alluded to what are the reasons for or causes of incompleteness. In what follows we discuss this in detail and more substantively. We consider and critically evaluate three arguments in the literature which purport to answer this question, viz. epistemic limits, incommensurable values, and incomparability. I take the side of incomparability.

A. *Epistemic Limits*

One argument that answers the question why is there irresolvable incompleteness, attributes the reason for this to epistemic limits (Chang 2012: 108–111). The specific epistemic limit an individual encounters while evaluating alternatives in a choice situation is uncertainty. There is uncertainty about the many factors – normative and non-normative – involved in an evaluation of the alternatives in a choice situation (Chang 2012: 108). This uncertainty about the factors relevant to an evaluation places epistemic limits on an individuals' evaluation of the alternatives, because of which the individual does not have access to these multiple factors when confronted with a choice situation[24]. Since individuals do not have access to the factors relevant to evaluating the alternatives in a choice situation, the evaluation of the alternatives remains incomplete. Thus, the relation that holds among the alternatives for choice is not defined, or incomplete, because of uncertainty which places an epistemic limit that restricts a choice making agents' access to these factors (Chang 2012: 109). On this view, the property of completeness is not satisfied because of epistemic limits in general and uncertainty in particular.

Indeed, in many cases epistemic limits can be the reason for the incompleteness of a ranking of alternatives. Consider the example of comparing two lotteries without information about the probabilities involved. However, in these cases incompleteness can be resolved with more information and facts, or a deeper analysis and reflection. That is epistemic limits can be

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[24] See Sepielli (2014) for a discussion of decision making when faced with normative uncertainty.

overcome to resolve the incompleteness involved. Incompleteness that follows from epistemic limits is not an instance of irresolvable incompleteness we identified above. They are instances of the two types of resolvable incompleteness – singly resolvable and multiply resolvable. Epistemic limits cannot explain why instances of incompleteness seem irresolvable.

This category – irresolvable incompleteness – distinguishes cases where incompleteness is asserted at the end of an all things considered evaluation, yielding statements such as x and y cannot be ranked[25] (Sen 2004: 55). In the case of the lotteries, with information about the probabilities, the lotteries can be ranked. The assertion of incompleteness is different from incompleteness that is accepted due to epistemic limits, while awaiting resolution (Sen 2004: 54-55). Indeed, in Mohammed's case in the example above, no further analysis of the facts of the alternatives or no deeper reflection of the choice situation can resolve how the alternatives are ranked. That is to say, epistemic limits do not explain why there are types of incompleteness that remain unresolved even with full epistemic access. For this reason it seems unwise to point to epistemic limits as a cause of or reason for irresolvable incompleteness. Note however, that I am not arguing that epistemic limits do not cause incompleteness. It would be a claim that is difficult to sustain. I am only arguing that epistemic limits cause a certain class of incomplete orderings or rankings. This class of incomplete rankings can be overcome or resolved, and thus do not explain the type of incompleteness which seem irresolvable.

B. *Incommensurable Values*

Another argument proposed as the reason for irresolvable incompleteness is the incommensurability of values[26]. In this part I argue that the presence of incommensurable values – or the lack of a common scale of measure – does not entail the incompleteness of a ranking[27].

There have been many different ideas that have been discussed using the label incommensurable values[28]. The most common use of the label of incommensurability refers to the lack of a common scale to which different values can be reduced and measured[29] (Chang 2013: 5-6). In this discussion I will be using incommensurability in this sense[30]. I will argue that incommensurability does not imply that a binary relation has to be incomplete.

Central to the idea of commensurability is a common scale of value (Chang 2013: 5-6). In the context of choice, a common scale of value allows the evaluation of different alternatives

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[25] We discuss this further while discussing incomparability as the main reason for incompleteness of an ordering
[26] These arguments are not responding to formal issue of whether incommensurability is what causes incompleteness of an evaluation. These arguments are responding directly to the substantive issue of why some choices are hard. See Raz (1986), Sunstein (1994), Anderson (1993)
[27] This part relies on the discussion of the issue in Sen (2004), Hsieh (2007a) and Chang (2013)
[28] See Hsieh (2007a) and Chang (2013) for illuminating overviews of incommensurability, its history and the different ways in which it has been and continues to be used. See also the enormous literature cited there.
[29] See Wiggins (1997) Chang (1997), Sunstein (1994). See also the other essays in Chang (1997) and the literature cited in Hsieh (2007a) and Chang (2013)
[30] See also the Introduction in Chang (1997)

to be reduced to and expressed in a single unit of value. Incommensurability refers to a situation when there is no common scale. This entails that an evaluation of different alternatives cannot be expressed, or measured, in a single unit of value (Chang 13: 5-6). Incommensurability is present when several dimensions of value are not reducible to a common scale with a single unit of value. It is easy to see why a choice situation that involves commensurable alternatives is not a hard choice. This is because the evaluation of commensurable alternatives is complete, which provides a sufficient condition for a straightforward choice when the feasible set is finite. When evaluating different alternatives that are commensurable, all that is required is to count which alternative has the highest value on the common scale, and choose that alternative. The choice is straightforward[31]. A ranking of commensurable alternatives can be complete because the value of these alternatives can be measured and compared on the common scale to which their value is reduced and expressed. However, to suggest that when this is not the case an evaluation must be incomplete does not necessarily follow. To illustrate, consider Amartya Sen's example of choosing between a mango and buying a record of a good song (Sen 2004). Sen writes,

> "a fine mango may give us nutrition as well as some palatal or olfactory pleasure, whereas buying the record of a good song may offer a very different reward (not immediately reducible into the dimensions of the other), and given a budget constraint we could quite possibly face the choice of having one or the other. This involves choosing between noncommensurable results. And yet we may have no great difficulty in opting for the mango when immensely hungry or starved, and going for the song, when well endowed with tasty food but short of melodious entertainment. The choice need not be hard to make in many situations, despite the noncommensurability involved. The distinct dimensions of values may not be reducible into one another, and yet there may be no problem whatsoever in deciding what one should sensibly do when our priorities or weights over these values are clear enough."

The point Sen is making using this example is that there can be choice situations that involve incommensurable alternatives, and we need not find them *hard*. This is because the presence of incommensurable alternatives in a choice situation does not necessarily mean that these alternatives cannot be compared or ranked. Indeed, completeness demands that every alternative in a set of feasible alternatives can be compared with every other alternative and itself in *some way*. It is tempting to think that if the value of the alternatives in a choice situation cannot reduced to and expressed on a common scale, then these alternatives cannot be compared in any other way (Chang 2013: 7). The lack of a common scale of value however does not entail incompleteness. Two alternatives can lack a common scale on which their value can be measured and still one of these alternatives might be better than the other (Chang 2013: 7). This is because these alternatives can be ordinally ranked. So even when

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[31] Note that this does not entail that a choice will be 'easy'. This is because there can be more than one alternatives with the highest value. If there is more than one alternative that is equally as good as each other, then we return to the problem of picking and choosing which we highlighted in our discussion of non-singleton sets in the previous chapter.

incommensurable alternatives are part of a choice situation, before we claim that the ranking of the alternatives in that choice situation is incomplete, we have to investigate whether it is easy or difficult to weigh these different values and rank these alternatives accordingly[32] (Sen 2004: 44-45).

Such an approach would focus on the *comparability* of alternatives we face in a choice situation. This seems like a more productive line of enquiry to pursue, and indeed it might be the case that the reason for or cause of an incomplete ranking derives from the world having alternatives that cannot be compared in an evaluative exercise. This possibility will be explored next.

C. Incomparability

Instead of focusing on the commensurability or incommensurability of alternatives in a choice situation, a few philosophers[33] call our attention to comparisons that can or cannot be made between alternatives *in the context of choice*[34]. It is the incomparability of alternatives then that is proposed as the reason for an incomplete relation. Before I proceed, let me clarify. It might seem strange to suggest that irresolvable incompleteness, or the lack of a comparative judgement, is caused by incomparability. It can be seen as a circular claim. However, irresolvable incompleteness is just a classification. Incomparability, as it will be discussed here is conceptual. It underpins the classification of a type of incompleteness as being irresolvable. Indeed, as we shall see, it provides an argument for the existence of irresolvable incompleteness. In what follows I discuss incomparability.

Discussions of what is incomparability however begin with specifying comparability, and only then proceed to define incomparability. I follow the same path here[35]. Comparability, or comparative judgements require what is called a *positive value relation*[36] and a *covering consideration* (Chang 2012; 112-13). A positive value relation describes the way in which two alternatives are ranked. A covering consideration is a set of respects on which the ranking is made[37]. To illustrate, Chang says, "X cannot be better than Y, full stop, but it can be better than Y only with respect to, say, well-being, or beauty, or morality, or making one's mother happy" (Chang 2013: 8). Here "better than" is the positive relation, and the considerations with respect to which this relation holds – well-being, beauty, etc – is called the covering consideration. The positive value relations are given by the three standard value

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[32] This is also the line suggested by Ruth Chang in a series of papers. See among others Chang (1997), Chang (2002a), Chang (2012) and Chang (2013).

[33] See Sen (2004), Chang (2002a)

[34] The context of choice is defined by some set of respects and is called the covering consideration. What is precisely meant by this will be explained as we proceed. I note here however the distinction between an "all things considered" comparison and a comparison with respect to the set of respects that define the context of choice. See also Chang (2004) for a discussion on how they relate.

[35] This relies on the discussion in Chang (1998), Chang 2002b) and Chang (2012)

[36] This has also been called a comparative value relation or just a value relation in different parts of the literature. I will use these terms interchangeably.

[37] If this set of considerations exhausts all the considerations in the world, then the set of considerations is said to be "all things considered"

relations "better than", "worse than", and "equally good"[38]. The covering condition defines what we called the context of choice above.

Thus, in the context of choice, or in the light of some set of considerations with respect to which a comparison is made, two alternatives are (i) *comparable* if it is true that a positive value relation holds between the two alternatives; and (ii) *incomparable* if it is not true that a positive value relation exists between the two alternatives[39] (Chang 2002b: 663) (Hsieh 2007b: 68). We know that the *positive value relations* are given by the three standard value relations "better than", "worse than", and "equally good as". So to get a firmer grip on incomparability we can say that two alternatives are *incomparable* if it is not true that either, x is "better than" y, or x is "worse than" y, or x is "equally good as" y[40].

Is it possible for two alternatives to be incomparable in this way? Or to put it in the context of our discussion of the types of incompleteness above; is it possible that incompleteness is irresolvable? The strongest argument for the possibility of incomparability is what has been called the "Small- Improvement Argument" (Chang 1997: 23–6; Chang 2002b: 667–73).

To illustrate this argument I use Qizilbash's (2002: 143) discussion of an example presented by Joseph Raz[41] (1986: 330-31). Raz asks us to consider an individual facing a choice situation between a good career in law x, and a good career in music y. We are drawn by the intuition that two good careers are not "worse than" each other. If this is the case, then both good careers must be "equally good"[42]. The Small Improvement Argument claims that this does not follow. This claim establishes the possibility of incomparability, or irresolvable incompleteness. To show this, Raz introduces a slightly better good career in law x+. Indeed, a small improvement on the initial good career in law. It is clear that the slightly better good career in law x+ is "better than" the initial good career in law x. But, if the initial good career in law x is "equally good as" the good career as a musician y, then it would follow that the slightly better good career in law x+ is "better than" the good career as a musician y. However, it intuitively feels incorrect to judge a slightly better good career in law x+ to be "better than" the good career as a musician y (Hsieh 2007b: 69). Hence, the initial good career in law and the good career in music are not "equally good", they are incomparable.

Ruth Chang (2002b) states the general form of the Small Improvement Argument in the following way: "if (1) *A* is neither better nor worse than *B* (with respect to *V*), (2) *A+* is better than *A* (with respect to *V*), (3) *A+* is not better than *B* (with respect to *V*), then (4) *A* and *B* are not related by any of the three standard relations (with respect to *V*)", where *V* represents the

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[38] Note here that a positive value relation is an interpretation of the binary relation as a bearer of value. Indeed, they can be articulated using the concept of strict preference and indifference. So x is "better than" y iff x P y, x is "worse than" y iff y P x, and x is "equally good as" y iff x I y.

[39] See also Chang (2012: -112-13) Chang (2002b: 663) and Chang (2002a) for a discussion this. For overviews, see Chang (2013: 6-7) and Hsieh (2007a)

[40] We will see in the next chapter that making the assumption that the evaluative space of comparability is exhausted by the set of three value relations – "better than", "worse than", and "equally god as" – to define incomparability will be critiqued. Indeed, notice that we are assuming the three relations exhaust the space of positive relations that can hold between the alternatives.

[41] The example was first introduced by Ronald de Sousa and later employed by Raz. The illustration of the example relies on its account in Qizilbash (2002) and Hsieh (2007b)

[42] We assume the transitivity of the relation "equally good" here.

relevant set of considerations for purposes of the comparison, or the covering consideration (Chang 2002b, 667-668).

This argument does seem convincing. It explains why incompleteness can be irresolvable – there are alternatives which cannot be compared on a set of considerations. To be sure, this can be *caused* by the incommensurability of values. But the reason incomparability is a better answer to the question, what is the reason for irresolvable incompleteness, is that it identifies the specific reason rather than point to a general background condition of the world. Indeed, the claim that the incommensurability of values is the reason for incomparability of alternatives is a like saying what caused the fire was oxygen[43]. Incommensurable values, like oxygen in the earth's atmosphere, are always there. The presence of incommensurable values does not imply that there is irresolvable incompleteness. Just like the presence of oxygen in the earth's atmosphere does not imply that there will be a fire. The argument for incomparability provides a reason for why the conflicts between alternatives that bear these values cannot be resolved[44].

We now know the incomparability position. The reason for an incomplete ranking is the failure of one of three positive relations – "better than", "worse than", or "equally good as" – to describe the relationship between two alternatives with respect to a set of considerations that matters in a choice (the covering consideration). If there is no positive value relation that describes how two alternatives relate with respect to a set of considerations that matters in a choice (the covering consideration), then there is incompleteness that is irresolvable[45] (Chang 2012: 113).

Indeed, this position can be seen by referring to the definition of a hard choice in the previous chapter, and our discussion of irresolvable incompleteness above. We saw in those discussions that a 'hard choice' refers to those choice situations where it is necessary that the binary relation does not satisfy the completeness property, and this feature maybe durable, or irresolvable[46]. In light of those discussions, the incomparability position can be seen as follows: if the binary relation R is interpreted as a positive value relation, then the 'incomparabilist' asserts that the failure to satisfy the completeness property is a durable and definitive part of the end product of either an all things considered evaluation or even an evaluation of the alternatives with respect to a set of considerations that matters in a choice (the covering consideration). If so, the incompleteness will not await resolution at a later stage, and will yield such statements as: x and y cannot be ranked or compared. On this view, there is a need to see incomparability leading to irresolvable incompleteness as a conceptual category of its own.

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[43] As Amartya Sen (2004; p. 45) says with characteristic wit, "it would be like saying that we feel hungry primarily because we have a stomach."
[44] See also Sen (2004; p. 45)
[45] The question remains as to the relationship between an "all things considered" comparison and a comparison with respect to the covering consideration. For purposes of this discussion, I leave open this question and assume that if an alternative is positively related to another alternative with respect to the covering consideration, then it is also, all things considered, positively related in the same way with another alternative.
[46] In that discussion we specified the binary relation to be the transitive closure R^* of the relation R. Philosophers engaged in substantive discussions however do not refer to which relation is incomplete.

Notice however, that the three positions we have discussed here are concerned with identifying the reasons for or causes of irresolvable incompleteness. How do they relate to a choice situation? In the following section we will be concerned with this issue. In particular, we discuss an implication of incomparability for a choice situation given by an agenda A. I will first show that incomparability can be a feature of a choice situation in two ways – partial incomparability and global incomparability. I then proceed to show one implication of incomparability: if there is global incomparability in a choice situation given by agenda A, then it is sufficient for a hard choice.

IV.   The Implications of Incomparability for a Choice Situation

If it is incomparability that is the reason for irresolvable incompleteness, then what implication does this have for a choice situation? We have thus far only seen that incomparability is the reason for the irresolvable incompleteness of an evaluation. In this section I show one implication of incomparability for a choice situation. To show this, we first need to make a distinction in the way incomparability can be a feature of a choice situation. Indeed, incomparability we will see can be a feature of a choice situation given by an agenda A in two ways. In what follows we first discuss and then define the two ways in which incomparability can be a feature of a choice situation, viz. partial incomparability and global incomparability. We then proceed to show that if there is global incomparability in a choice situation A, then it is sufficient for a hard choice.

A. *Partial incomparability and global incomparability*

In this part I make a distinction between two ways in which the incomparability of alternatives can be a feature of a choice situation given by an agenda A. Let us first restate what is meant by incomparability. Two alternatives are *incomparable* if it is not true that either, x is "better than" y, or x is "worse than" y, or x is "equally good as" y. The first way in which incomparability can be a feature of a choice situation is if *there are some alternatives* in an agenda A which are incomparable. To define this I will call this "partial incomparability".

Definition 7: $\forall A \in Z,$ *there is 'partial incomparability' in A iff* $\exists x, y \in A: \sim(x\,R\,y\ or\ y\,R\,x)$

Note that in this definition I am interpreting *R* to be a positive value relation.

Another way in which incomparability can be a feature of a choice situation is when *every alternative* in an agenda A is said to be incomparable. We call this "global incomparability" to define it.

Definition 8: $\forall A \in Z,$ *there is 'global incomparability' in A iff* $\forall x, y \in A: \sim(x\,R\,y\ or\ y\,R\,x)$

In this definition too we interpret *R* to be a positive value relation.

Note that "global incomparability" implies "partial incomparability". However, depending on which incomparability – partial or complete – is a feature of the choice situation given by an agenda A, the implication for a choice situation A can vary. I take this up next.

B. The Implications of Partial and Global incomparability

In this part I will provide an argument for the following claim: if there is global incomparability in a choice situation A, then it is sufficient for a hard choice. I conclude by discussing how this implication bears on our analysis of hard choice.

If there is global incomparability in an agenda A, then it is sufficient to make a choice situation given by agenda A, a hard choice. To proceed, consider the agenda A = {w, x, y, z,}. If every alternative in this set is incomparable to every other alternative, then (i) the binary relation R is incomplete *and* (ii) B(A,R*) is empty. Therefore global incomparability is sufficient to make a choice hard.

What does this implication mean for our analysis of hard choice? The meaning is clear, if there is global incomparability in a choice situation, then there is a hard choice. Indeed, this accords with our intuitions of a hard choice. Our intuitions suggest that a hard choice involves a choice situation that we face where *all* the alternatives in that choice are incomparable. Note however, that 'global incomparability' is sufficient but not necessary for a choice situation to be a hard choice. That is, the set of agendas that constitute a hard choice is not exhausted by those choice situations or agendas constituted by global incomparability.

However, we still have not discussed the *problem* this implication presents for practical reason and decision making. In chapter three we take up responses to a 'problem' that global incomparability in a choice situation presents for practical reason. It would be unwise to move directly to responses to a 'problem' without seeing what the problem is. Therefore the next part of the thesis will not consist of chapter three, but an interregnum between chapter two and chapter three which discusses the 'problem' that a choice situation' with global incomparability presents for practical reason and decision making.

Conclusion

In this chapter we discussed two issues. First, what are the causes of or reasons for incompleteness, and second, what implication does this reason have for a choice situation. We took up the first discussion in two sections. One, we began by introducing and discussing two types of incompleteness, viz. resolvable and irresolvable incompleteness. Two, we proceeded to discuss the reasons for or causes of irresolvable incompleteness where we considered and critically evaluated three positions in the literature which answer this question, viz. epistemic limits, incommensurable values, and incomparability. We identified incomparability as the reason for irresolvable incompleteness.

The second issue that we discussed was what implications does incomparability have for a choice situation. We first made a distinction between two ways in which incomparability can be a feature of a choice situation – partial incomparability and global incomparability. Partial

incomparability refers to when there are some alternatives in a choice situation A which are incomparable. Global incomparability refers to when every alternative in a choice situation is incomparable. We then proceeded to show one implication of incomparability for a choice situation. If there is global incomparability in a choice situation given by an agenda A, then it is sufficient to make a choice situation a *hard choice*. Indeed, the bearing this implication has for our discussion of hard choice is clear. When there is global incomparabilityin a choice situation, there is a hard choice.

In the interregnum between chapter two and three we will discuss the 'problem' this presents for practical reason and decision making. The interregnum sets up the discussion in chapter three. In chapter three we will see the responses to this 'problem' which we discuss next.

Why Is (Global) Incomparability a Problem?

We concluded our discussion in the previous chapter by identifying a sufficient condition for a hard choice. In particular; if there is 'global incomparability' in a choice situation given by an agenda A, then there is a hard choice in agenda A. Note however, that the set of agendas that constitute a hard choice is not exhausted by those choice situations or agendas constituted by global incomparability. This is because global incomparability is a sufficient and not necessary condition for a hard choice. However, in the rest of this work, we restrict our discussion to the class of hard choices constituted by global incomparability.

In this interregnum we ask the question, "Why is global incomparability a problem?" The reason we take up this question here is because the next chapter deals with responses to the 'problem' the class of hard choices constituted by 'global incomparability' presents. Indeed, chapter three will discuss arguments in the literature to overcome the 'problem' presented by a hard choice constituted by 'global incomparability'. Therefore, we use this interregnum to *see* what this problem is that the arguments in chapter three are responding to.

Here we introduce and discuss two problems that a hard choice constituted by global incomparability presents. The first of these is what we will call decisional inescapability; there are unranked alternatives from which a choice *has* to be made (Sen 1997: 745). The second problem that I discuss is the possibility of justifying a choice in a hard choice situation with global incomparability.

To proceed, consider the amended case of Mohammed whom we met in the previous chapter. Mohammed as we know is an anti-terror operative. But Mohammed and his team know that a terrorist group is planning not two, but three attacks. Just like before, Mohammed and his team of commandoes can only prevent one of these attacks. The first attack is on Palmyra. The second attack is on Bombay. The third attack is on the embassy of Mohammed's country in Turkey. The embassy isn't densely populated, but is populated by individuals Mohammed shares an identity with, namely his nationality. There are good reasons to prevent every attack. Mohammed can choose to save a historical site, or he can choose to save many lives, or he can choose to save the lives of people he shares a common identity with, viz. citizenship. But Mohammed and his team of commandoes can prevent only one attack, now the choice is which attack should Mohammed and his team prevent?

The first problem to notice here is decisional inescapability. Mohammed *has* to make a decision at the end of his evaluation, whatever the result of that evaluation may be. Now, given that there is decisional inescapability in every choice situation, here it combines with

'global incomparability'. That is, assuming Mohammed[47] cannot rank any of the alternatives, *every alternative* in the choice situation is incomparable to every other alternative in the choice situation. To illustrate, let A = {Palmyra, Baghdad, Turkey}; for every x and y in A, we have ~(x R y or y R x). Indeed, this is a hard choice.

Therefore, decisional inescapability and 'global incomparability' combine to produce a serious problem for practical reason, viz. how to make a justified choice?

This is the problem a choice situation with 'global incomparability' presents to the framework of practical reason that has been called *comparativism*[48]. On the comparativist view of practical reason, comparative judgements based on the facts, reasons, or values about alternatives in a choice situation "is not only a necessary condition for the existence of a justified choice, but it also *determines* which alternative is justified. If, with respect to what matters in the choice, for example, one alternative is better or supported by more reason than the other, then choosing it is justified"[49] (Chang 2012: 114). In the previous chapter we saw that these comparative judgements are described by the positive value relation. So the positive value relation is necessary for the justification of a choice, and *determines* what one rationally ought to do.

But, if the feature of 'global incomparability' is present in a choice situation, then every alternative in that choice situation is incomparable to every other alternative in that choice situation. The implication of this is that there is no positive value relation that describes a comparative judgement. Since this comparative judgement described by a positive value relation is necessary for both the justification of choice, and to determine what one rationally ought to do (Chang 2012: 114). The absence of a positive value relation that describes a comparative judgement in a choice situation precludes the possibility of a justified choice (Hsieh 2002b: 66). Indeed, this is the 'problem' global incomparability presents – the justification of choice is precluded.

Now that we have an understanding of the problem posed by global incomparability, we can proceed to examine the responses that have been proposed to overcome this problem. In the next chapter we see two responses that have been made in the literature to overcome this problem.

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[47] It can be the case that the choice might actually be straightforward because saving more human lives trumps any other consideration. If unconvinced, grant the assumption that there is global incomparability to let the argument proceed.

[48] For a detailed account of this position, see (Chang 1997), (Chang 1998), (Chang 2002a), (Chang 2002b). For an overview of this account, see Chang (2013) Hsieh (2007a)

[49] Emphasis on the word 'determines' has been reproduced as it is in the original text.

Comparativism and Maximization

We have now provided a definition of what a hard choice is, and towards the end of chapter two we presented a sufficient condition for a choice situation to be a hard choice – 'global incomparability' is sufficient for a hard choice. In the interregnum before this chapter we saw the *problem* that a hard choice constituted by 'global incomparability' presents. In particular, we saw that in a hard choice constituted by 'global incomparability' the possibility of a justified choice is precluded.

This chapter will be concerned with responses in the literature to this problem. In this chapter I take up the question: Is there a justification for making a hard choice constituted by 'global incomparability'?

Here I investigate the responses made to justify a choice when there is 'global incomparability' in a choice situation. I introduce and discuss two frameworks of practical reason that have been proposed as responses to overcome the problem presented by 'global incomparability'. These two frameworks articulate how we can make a justified choice when there is global incomparability of alternatives in a choice situation.

Since a hard choice constituted by 'global incomparability' presents a problem for the comparativist view of practical reason, we introduce and discuss the comparativist response to this problem first. To avoid confusion with nomenclature, I clarify, a hard choice constituted by 'global incomparability' presents a problem for comparativists, and we discuss their response to this problem first. The comparativist response consists of two parts[50]. One, comparativists argue that the concept of incomparability as we have presented it in the previous chapter is conceptualized on an unjustified assumption. This assumption is that if two alternatives can be compared, then they can be compared only by the three positive value relations – "better than", "worse than", and "equally good" (Chang 2002b: 660). Building this substantive assumption about the number of ways in which alternatives can be compared into our conception of incomparability is mistaken, or so the comparativists' argue (Chang 2012: 111). Two, comparativists proceed to argue that once you abandon this assumption, alternatives that are not comparable by these three positive value relations can be compared by a fourth value relation. They proceed to introduce and argue for the existence of a fourth value relation that makes incomparable alternatives comparable (Griffin 1986: 80–1, 1997: 38–9; 2000: 285–9) (Parfit 1984: 431) (Chang 2002b). Since comparativists require a

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[50] See Chang (1997), Chang (2002b), Chang (2002a), Chang (2005), Griffin (1986), Griffin (1997), Griffin (2000) for the literature associated with this response. This discussion relies on the accounts in Chang (2002b) Chang (2012) Chang (2013) Hsieh (2007a), Hsieh (2007a), Hsieh (2007b) and Qizilbash (2002)

comparative judgement described by a positive value relation to justify a choice, this response makes both the comparison of alternatives, and thus the justification of choice possible (Hsieh 2007b: 66).

The second response to the problem of hard choices constituted by 'global incomparability' comes from what Amartya Sen[51] (1997, 2000, 2004) calls the maximization view of rationality. This framework of reasoning has been developed in the paradigm of rational choice theory. Maximization does not see 'global incomparability' as a problem for the justification of choice. This is because a 'maximizer' argues that a justified choice is possible in the presence of incomparable alternatives by choosing that alternative which is not "strictly worse than" other alternatives[52]. This approach to decision making is called maximization. Sen contrasts this with optimization, which posits like the comparativists, that for a choice to be justified it has to be shown to be "at least as good as" every other alternative in the choice situation[53]. Incomparable alternatives are not strictly worse than each other. So choosing any of the incomparable alternatives is justified. In this way maximization has been proposed as a justification for choice in a choice situation with 'global incomparability'[54] (Hsieh 2007b).

After presenting each of these responses, I will argue that neither the comparativist response nor the maximization view of rationality provide a justification for choice in a choice situation constituted by 'global incomparability'. I claim that the comparativist response entails dropping or rejecting the axiom of transitivity[55]. While transitivity of a relation is not necessary for rationality[56], comparativists demand that a positive value relation determine choice. If the fourth value relation that comparativist propose is supposed to determine a choice, then if it is an intransitive relation it could justify a sequence or series of determined choices that leave an individual worse off (Hsieh 2007a). The maximization response is unsatisfactory because it could justify a choice based on a randomized procedure, like flipping a coin or rolling a die. This takes practical reason outside the scope of rational agency (Chang 2012: 118).

To proceed, Section II introduces and discusses the response to the problem presented by 'global incomparability' from those who hold a comparativist position on practical reason. Section III introduces and discusses the response to the problem from the maximization view of rationality. A final section concludes.

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[51] See Sen (1970), Sen (1997), Sen (2000), Sen (2002), Sen (2004) for its development in rational choice theory. But for an illustration of its use in philosophy in discussions of incomparability see Qizilbash (2002) and Hsieh (2007b)

[52] I note that maximization is possible when the alternatives are ranked as indifferent in the choice situation as well. But here I will concern myself only with the maximization response to the incomparability of all alternatives in a choice situation.

[53] In this chapter we will use optimization and comparativism interchangeably. Optimization is similar to the comparativist view in the sense that it demands a comparative relation between alternatives for the alternatives to be justified as choice. Optimizers demand that this comparative relation, or the positive value relation be 'at least as good as'.

[54] See in particular Hsieh (2007b) and Qizilbash (2002) for a discussion of how the two responses to the 'problem' of 'global incomparability' are related.

[55] See Qizilbash (2002) and (Hsieh 2007b) on this.

[56] I have made this argument in chapter one.

II.   Comparativism

We saw in the interregnum with the example of Mohammed that a choice situation where there is 'global incomparability' presents a problem for the comparativist view of practical reason. In particular, we saw that the possibility of a justified choice is precluded in the choice situation where there is 'global incomparability'. In this section we introduce and critically discuss the comparativist response to this problem for the justification of choice[57]. This response consists of two parts[58]. In this section I present both parts of this response and proceed to discuss one implication of this response. The first part of the response consists in asking the question, "Are there only three positive value relations?" and answering this in the negative. The second part of the response consists in proposing and arguing for the existence of a fourth positive value relation, or a fourth way in which alternatives can be compared. I present these two parts and proceed to show that the comparativist response entails dropping the axiom of transitivity. While the transitivity of relations is not necessary for rational choice, as I discussed in chapter one, comparativists demand that positive value relations guide and indeed determine a choice. For this reason the transitivity of preference relations is necessary for their response to provide a justification, else an intransitive positive value relation could justify a sequence of determined choices that leave an individual worse off (Hsieh 2007a).

A. *Are there only three value relations?[59]*

We saw in the interregnum that the problem presented by a choice situation where there is 'global incomparability' is that if comparative judgements are necessary for the justification of choice, then 'global incomparability' implies that justification is not possible. The first part of the comparativist response to this problem is to ask the following question: are the number of ways in which two alternatives can be compared exhausted by the three positive relations – "better than", "worse than", and "equally good"? (Chang 2012: 112).

Comparativists' proceed to answer this question in the negative. The claim they make is that it is mistaken to *assume* that if two alternatives can be compared, then the two alternatives can only be compared by the three positive value relations – "better than", "worse than", and "equally good" (Chang 2002b: 660). Indeed, comparativists are critiquing the concept of incomparability as we discussed and presented it in chapter two[60]. Over there we saw that incomparability refers to the failure of one of the three positive value relations – "better than", "worse than", and "equally good as" – to describe the relationship between two alternatives being compared. On the comparativist view, incomparability conceptualized in this way assumes that the three positive value relations "exhaust[s] the conceptual space of

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[57] The presentation of the comparativist response relies on the discussion of the comparativist position found in Chang (2002b), Chang (2012), Hsieh (2005), Hsieh (2007a) Hsieh (2007b) Qizilbash (2000) and Qizilbash (2002)
[58] See Chang (1997) Chang (1998) Chang (2002a) Chang (2002b), Chang (2005) Chang (2012), Griffin (1986), Griffin (1997) and Griffin (2000)
[59] This part draws on the presentation in Chang (2002b) and Chang (2012)
[60] See in particular Chang (2002b)

comparability" (Chang 2012: 112). Chang (2002b: 660) calls this assumption the "trichotomy thesis". On Chang's account, "[a]ccording to this thesis, the conceptual space of comparability between two items is spanned by the trichotomy of relations "better than," "worse than," and "equally good": if none of those relations holds, the items are incomparable" (Chang 2002b: 660-61). Comparativists like Chang go on to argue that the set of three positive value relations do not exhaust the number of ways in which two alternatives can be related or compared. The argument Chang makes is that the intuitive notion of incomparability does not presuppose the number of ways in which alternatives can be compared, and we should desist from placing this assumption – in this case three – into our conception of incomparability (Chang 2012: 111-12).

To argue her case Chang provides the following thought experiment.

> "Imagine 'dichotomists' who define incomparability as the failure of 'better than' and 'worse than' to hold between two items. When the 'trichotomist' comes along and says, 'You've overlooked the relation of 'equality'', and the dichotomist demurs, the disagreement is substantive. Both are trying to characterize the intuitive idea of being incomparable, not simply stipulating the use of a term to mean the failure of some favored set of value relations. Both are maintaining that their set of value relations exhausts the conceptual space of comparability between items, and they substantively disagree about which set of relations exhausts that space. In just the same way, we might imagine a 'tetrachotomist' who defines incomparability as the failure of four basic relations, one in addition to the usual trichotomy. She and the trichotomist would similarly have a substantive debate over which account best captured the intuitive notion of incomparability." (Chang 2012: 111-12)

The intuition this thought experiment brings out for Chang is that the concept of incomparability allows disagreement – like the disagreement between the 'dichotomist' and 'trichotomist' and in turn between the 'trichotomist' and 'tetrachomist' – over the number of ways in which two alternatives can be compared. As Chang puts it, "What this shows is that the intuitive notion of incomparability permits substantive disagreement over which relations exhaust the conceptual space of comparability" (Chang 2012: 112). Therefore, when we conceptualize incomparability, we should not place into this conceptualization an assumption, like the trichotomy thesis does, about the number of ways in which alternatives can be compared[61] (Chang 2012: 112).

If comparativists abandon this assumption – the trichotomy thesis – then the alternatives in a choice situation, or agenda A, which are not comparable by the three positive value relations can be comparable by a fourth positive value relation (Hsieh 2007b: 66). Indeed, comparativists reject this assumption and proceed to introduce this fourth positive value relation that makes alternatives that were incomparable in the choice situation given by an

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[61] I must confess that I am unconvinced by this thought experiment. In particular, I would be hard pressed to find an individual who is a 'dichotomist' as Chang presents it. Indeed, the only substantive disagreement I envisage, and as the literature bears out, is between the 'trichotomist' and the 'tetrachotomist'.

agenda A, comparable via this fourth relation. This is the second part of the comparativist response, and we discuss it below.

B. *Parity, Roughly Equal and Comparativism*[62]

We have seen the first part of the comparativist response to the problem presented by global incomparability in a choice situation. The first part of their rejoinder consists in rejecting the "trichotomy thesis". The trichotomy thesis we saw claims that if two alternatives can be compared, then they can only be compared in three ways – "better than", "worse than", and "equally good as" (Chang 2002b: 660). The rejection of this thesis is made by arguing that our intuitions about the concept of incomparability allows disagreement on the question how many positive value relations exhaust the conceptual space of comparability (Chang 2012: 111-12). Proposing that alternatives in a choice situation given by an agenda with global incomparability can in fact be compared rescues the comparativist view of practical reason. This is because it is now possible to make comparative judgements, and thus justify choice. However, rescuing comparativism in this way requires another step. This step involves proposing and arguing for the existence of a fourth positive value relation which can make the alternatives that were incomparable by the three positive value relations, comparable via this fourth relation (Hsieh 2007b: 66). This is the second part of the comparativist response. Comparativists propose and argue for the existence of this fourth positive value relation. Alternatives thought incomparable are then presented to be comparable via this relation.

Arguments for two additional positive value relations have been advanced. We discuss them in turn[63].

The first of these proposals for a fourth positive value relation is the argument for "roughly equal" (Griffin 1986: 80–1, 1997: 38–9; 2000: 285–9) (Parfit 1984: 431). Derek Parfit argues that two alternatives A and B are "roughly equal" if there is an equivalence between them, but an imprecise or rough equivalence. To illustrate, A and B are evaluated as "roughly equal", or A is "roughly equal" to B if: neither A "better than" B, nor B "better than" A, and C "better than" B. But from this evaluation, Parfit says, it does not follow that, C "better than" A (Hsieh 2007b: 69). To make this more accessible, consider Parfit's (1984: 431) example of comparing two poets A and B and a novelist C for a literary prize reported by Hsieh (2007b: 69). In this illustration, the poet A and the novelist C are said to be "roughly equal", or A "roughly equal" to C, when both A and C are not better (or worse) than each other, and the poet B is only marginally better than the poet A. "If the first poet and the novelist were equally good, it would follow that the second poet is better than the novelist. This judgement, according to Parfit, need not follow" (Hsieh 2007b: 69). Parfit posits that the first poet is "roughly equal" to the novelist[64]. The idea that motivates arguments for "roughly equal" is that even though these three literary figures can be compared based on their literary qualities, the comparison that can be made between them cannot be precise but is rough or imprecise (Hsieh 2007b: 69).

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[62] This part draws on Chang (2002b), Hsieh (2007a) Hsieh (2007b) and Hsieh (2005) and Qizilbash (2002)
[63] The discussion of the two value relations draws on the account in Hsieh (2007b).
[64] See also Hsieh (2005) and Qizilbash (2000) for a discussion of roughly equal.

The plausibility of "roughly equal" being a genuine fourth relation which is both independent from and an addition to the three standard relations has come under scrutiny. Indeed, those who posit its existence do not see it as a genuine fourth relation that is independent of the three standard relations. Chang (2002b: 661) reports that Derek Parfit is of the view that "roughly equal" is just a roughed up version of "equally good as". Indeed, Parfit, Chang reports, is of the view that the trichotomy thesis is true, but it can have precise and rough versions[65].

Another argument for a fourth positive value relation, which is independent from and an addition to the three standard relations we have seen is Ruth Chang's argument for "on a par" or "parity" (Chang 1997; Chang 2002b). If two alternatives are not better (or worse) than each other, but there are significant differences between the two alternatives that prevent them being evaluated or compared as "equally good", then they are said to be "on a par" (Hsieh 2007b:69). To make this less abstract, Chang asks us to consider the question, "Who is more creative, Mozart or Michelangelo?" (Chang 2002b: 659). Mozart is a composer, and Michelangelo is, among other things, a sculptor. Both artists' display their creative traits in very different fields that involve creativity. Chang says that neither Mozart nor Michelangelo is better (or worse) than the other. However, it is wrong to infer from this evaluation that they are "equally good". This is because Mozart the composer and Michelangelo the sculptor display their creative traits in very different fields. For Chang, Mozart is "on a par with" Michelangelo (Hsieh 2007b: 69 - 70).

We have now seen two proposals for a fourth way in which two alternatives can be related and compared. How does this bear on the problem that a choice situation with global incomparability presents for the comparativist view of practical reason. Or, how does the comparativist response provide an argument for a justified choice in a choice situation with global incomparability?

To illustrate this, I use an example. In this example I assume the choice situation is given by an agenda A with two alternatives x and y. If A = {x, y} and neither xRy, nor yRx, then there is what we have called global incomparability[66] in the previous chapter. That is, if alternatives are not related by R, there is no comparative judgement that can be made[67]. In the interregnum we saw that this absence of a comparison between alternatives in the choice situation presents a problem to the comparativist view of practical reason. In particular, if a comparison is necessary for a choice to be justified, then a justified choice is precluded because of global incomparability in a choice situation.

The comparativists response is able to overcome this problem. This is because they claim that alternatives in a choice situation which cannot be compared by the three relations expressed by R – "better than", worse than", and "equally good as", can be compared by a fourth relation: like "roughly equal" or "on a par". So even if ~(xRy or yRx), the alternatives can be compared as follows: x is "on a par with" y, or x is "roughly equal" to y. This additional

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[65] See Chang (2002b: p. 661 footnote 5)
[66] Note that there is also partial incomparability.
[67] The binary relation R is interpreted as one of the following three positive value relations here: "better than", "worse than" and "equally good as"

relation make a comparison possible, and thus make a justified choice possible. Proposing and arguing for comparative relations beyond those expressed by the three standard relations by invoking a fourth relation, like 'on a par' makes possible the justification of choice. I will call this "Parity-Optimization"[68] and state it as follows: for the choice of an alternative in a choice situation with global incomparability to be justified, the alternative chosen must be "at least on a par with" all other alternatives in a choice situation.

To make this example more concrete, let x and y in A be the two careers of Joseph Raz's example we saw in chapter two. So x is a career in law, and y is a career as a musician. Neither is better than the other career, and we saw that nor are they equally good. An individual is faced with the choice of these two careers. Indeed, there is 'global incomparability' – every alternative in this choice situation is not comparable to every other alternative with the three standard relations. On the comparativist account we have discussed above, an individual can justify the choice she makes by evaluating the two careers as "on a par" with each other. That is, if either career is "at least on a par with" each other, then the choice the individual makes in this choice situation is justified.

C. What is the catch?

Indeed, here we can ask, if the comparativist response to the problem can provide an argument for justifying choice, then what is left to say about justifying the hard choice constituted by the choice situation with 'global incomparability'? To be sure, there is a catch. An implication of this response – arguing for a fourth value relation – to the problem presented by choice situation constituted by 'global incomparability' is dropping the axiom of transitivity[69] (Hsieh 2007b: 70) (Qizilbash 2002: 143).

To be sure transitivity of a relation is not necessary for rational choice, as I argued in the first chapter. The argument I made there was that a preference relation, or what can be interpreted as a positive value relation, does not have to guide action. However, on the comparativist view of practical reason comparative judgements based on the facts, reasons, or values about alternatives in a choice situation is necessary for the justification of a choice, and *determines* what one rationally ought to choose (Chang 2012: 114). Since the positive value relation describes these comparisons, the positive value relation *determines* what one rationally ought to choose. Now if this relation is intransitive, then it *determines*, and provides a justification for a series or sequence of choices that can leave an individual worse off, as illustrated by the classic "money pump" argument.

In what follows, I show that the relation "on a par" is intransitive, and how choices determined and justified by this relation can leave an individual worse off.

Consider two alternatives x and y. Let x be on a par with y. Now consider a slightly improved version of x, and let us call this x+. Assume that both x and x+ are neither better nor worse

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[68] Those interested in what is precisely meant by "Parity-Optimization" can see Qizilbash (2002). There it is called A-Optimization.

[69] See Hsieh (2007a), Hsieh (2007b), Qizilbash (2002), Rabinowicz (2008), and Temkin (2000). This discussion draws on Qizilbash (2002) and Hsieh (2007a)

than y. Now, x is on a par with y, similarly, we can say that y is on a par with x+. But we know that x+ is better than x because it is a slightly improved version of x. Hence, defending comparativism by way of "roughly equal" or "on a par" requires dropping the axiom of transitivity (Hsieh 2007b: 70-71).

To make this less abstract and more concrete, consider the following account presented in Qizilbash (2002: 143). Let B be 'on a par' and P be 'better than'. Qizilbash (2002: 143) asks us to consider a set of three alternatives, {x, x+, y} where x is an excellent French meal, x+ a very slightly better excellent French meal, and y is an excellent Italian meal. Assume that any excellent French meal is neither better, nor worse, than any excellent Italian meal.

In the above example, we have xBy and yBx+ and x+Px. This implies that B is not transitive.

Now I show how a sequence of justified choices determined by this relation can leave an individual worse off[70]. Consider again Joseph Raz's example of the choice between the career in music and career as a lawyer discussed in chapter two. Let us say that the individual evaluated both careers to be 'on a par with' each other. Now, assume she made the decision to become a musician. In the future if she is presented with the opportunity to switch professions and pursue a career in law that is marginally worse than the initial career in law, she will evaluate the marginally worse career in law and her career in music to be 'on a par with' each other as well, then on the comparativist view, the choice of the marginally worse career in law is determined and justified. Now, if in the future this individual in the marginally worse career in law is presented with the option of a marginally worse career in music, she will evaluate the marginally worse career in law to be 'on a par with' the marginally worse career in music, then she will be justified in choosing the marginally worse career in music. This can go back and forth till an individual ends up in a very bad career in law, or a very bad career in music. But she reaches there through a series or sequence of apparently justified choices determined by the relation 'on a par with' (Hsieh 2007a). In this way the comparativist response provides a justification for a series or sequence of determined choices that leave an individual worse off.

Comparativists can of course respond by making the same distinction between picking and choosing that I invoked in chapter one to defend why a choice from B(A,R*) is straightforward. But that would require giving up the claim that the choice with the relation 'on a par with' is determined.

We have now seen the first response to the 'problem' that a choice situation with 'global incomparability' presents. In what follows, we see the second response.

III.   Maximization and the Justification of Choice

A second response to the problem for the justification of a choice with incomparable alternatives, or a choice situation constituted by 'global incomparability' of alternatives, has been to present a framework of reasoning developed in the paradigm of rational choice

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[70] This illustration draws on Hsieh's(2007a) discussion of the problem. See in particular section 4.2

theory. This framework is called maximization[71]. In what follows, we first present this framework. We then proceed to show that in a choice situation with global incomparability, maximization takes practical reason outside the scope of rational agency.

In the paradigm of rational choice theory, a distinction is made, most notably by Amartya Sen, between maximization and optimization (Sen 1970; 1997; 2000). Optimization requires the choice of an alternative that is "at least as good as" other alternatives in a choice situation[72]. The requirement optimization demands rules out the possibility of a justified choice when there is global incomparability in a choice situation. Indeed, it is because the optimal set is empty that we are in a choice situation that is a hard choice. Within rational choice theory, in particular in social choice theory, there emerged a problem of the non-existence of optimal sets, or rather, the possibility of optimal sets which were null. This was presented as a general problem by Arrow (1963). We noted this in chapter one. There we saw one of the responses to this problem, defining the choice set using the transitive closure of a relation. Maximization was another response[73].

Maximization, unlike optimization, only requires the choice of an alternative that is not "strictly worse than" other alternatives. The maximal set, by definition[74], consists of alternatives that are not dominated by, or not strictly preferred by, any other alternatives in that set. That is to say, the maximal set consists of the alternatives we face in a choice situation which is not strictly preferred by any other alternative in the choice situation we are in[75]. Note that these alternatives might be "equally good" or "indifferent" or they might be "incomparable" (Qizilbash 2002: 149). However, we have assumed here that maximization is responding to the problem presented by a choice situation with the global incomparability of alternatives. Incomparable alternatives are not strictly worse than each other, or not strictly preferred by any other alternative. Therefore, it is justified to choose any of the incomparable alternatives in the choice situation. In this way maximization provides for the possibility of a justified choice[76] (Hsieh 2007b: 72).

Indeed, maximization provides an argument for the justification of choice in a choice situation with global incomparability by arguing that every alternative in the choice situation is justified for choice. The justification follows from each alternative not being worse than every other alternative[77]. The maximization view of rationality does not demand that *all* alternatives be comparable, and does not even require that a best alternative be identifiable. This is a demand made by the comparativists. Maximization only requires that we do not choose an alternative that is worse than another alternative that can be chosen instead (Sen

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[71] This section relies on the account of maximization presented by Amartya Sen in Sen (1997), Sen (2000) and Sen (2004). It also draws on how maximization has been proposed as an account of justified choice in the presence of incomparable alternatives in Hsieh (2007b)

[72] See definition 3 in chapter one

[73] See Sen (1970)

[74] See definition 4 in chapter one

[75] Note however that this requires the binary relation to be acyclic. On the formal properties of maximization see Sen (1997). See in particular section 5

[76] See Hsieh (2007b) for a defense of maximization against the comparativist view.

[77] See Sen (1997) Sen (2000) Sen (2004) Hsieh (2007b) Qizilbash (2002)

2004: 49-50). Since incomparable alternatives are not strictly worse than each other, they are all justified choices to make (Hsieh 2007b: 71-2).

To be sure, there is one attractive feature to the maximization view of rationality that allows us to view maximization as a defensible account of justified choice[78]. This attractive feature of maximization is that it gets at an intuition about the justification of choice, namely the identification and rejection of those alternatives which cannot be justified because they are strictly worse than some other alternative (Hsieh 2007b: 72). If an alternative is evaluated to be strictly worse than another alternative, then choosing this alternative is unjustified. Further, this establishes the relative 'goodness' of those alternatives in the maximal set. You can point to those alternatives in the choice situation which are worse than the alternatives in the maximal set and establish the 'goodness' of alternatives in the maximal set in this relative way. However, in a choice situation with global incomparability, every alternative in a choice situation, or an agenda A, is incomparable to every other alternative in that agenda. Thus, you are unable to identify those alternatives which are unjustified, or establish the relative 'goodness' of those alternatives in the choice situation by showing they are better than some other alternative in the choice situation. The agenda and the maximal set are equivalent when there is global incomparability in a choice situation. Nevertheless, this has been proposed as an account of justified choice in a choice situation with 'global incomparability'. To make this argument, Hsieh (2007b: 72) asks us to consider Joseph Raz's example of the case involving the legal career and the music career. Maximization, he says,

> "[a]ppears to accord with the justification of choice in such cases. Imagine giving advice to the individual facing such a choice. One natural form of advice to give her would be to point out that she cannot go wrong in choosing either career. Both are good careers. This advice expresses the judgment that the choice of either alternative would be a justified choice because none of the alternatives are strictly worse than the other. Notice however, that it does so not by reference to the fact that each alternative is at least as good as the other. Instead the justification is that both choices are good careers and neither alternative is worse than the other and neither alternative is worse than the other." (Hsieh 2007b: 72)

There are two things to be said about this. First, it is not obvious that either alternative are good careers, because there is no alternative in the choice situation which is demonstrably worse. If the judgment that they are good is made by referring to other careers, then it is not a case of global incomparability. Because then every alternative in the choice situation, or agenda, is not incomparable to every other alternative in the choice situation, or agenda. To establish that either career is a good career, we have to establish the existence of some alternative in the choice situation which is worse than either career. This is not possible here because we are referring to agendas with global incomparability. Indeed, we can ask if this argument is a justification for an agenda with global incomparability or what we called partial incomparability in the previous chapter.

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[78] See for instance Hsieh (2007b), Qizilbash (2002) and Sen (2000)

Second, assuming that this is a choice situation constituted by global incomparability, then in what way is this response a justification? Notice that our advice stops when we say that either careers are good. This is of little help to someone who is actually confronted with this choice. The person making this choice might as well suspend her rational agency and flip a coin or roll a die to make a choice. Indeed, in choosing between incomparable alternatives, maximization would justify a choice based on randomized procedure, like the flipping of a coin[79].

However, it can be argued that a suspension of our rational agency is involved even when we are "picking" an alternative from a non-singleton optimal set B(A,R). In chapter one, we defended 'picking' as being straightforward. Note that when the selection from a choice situation involves 'picking' an alternative, and not "choosing" an alternative, we can make a selection by flipping a coin, or base our selection on some other arbitrary procedure. In chapter one, we argued that this was a straightforward choice. Does 'picking' not involve a suspension of our rational agency?

No it does not. Let me explain. When we 'pick' an alternative we do not have *most* reason to choose that alternative *over* another alternative. This is articulated by the symmetric relation between the two alternatives. However, when we 'pick' an alternative arbitrarily, we have some reason, or some set of considerations, that favour choosing that alternative. In particular, we can show that there are alternatives which are demonstrably worse. This provides a sufficient reason to 'pick' and keeps 'picking' within the scope of rational agency.

When you make a choice from an agenda with global incomparability, or when you make a selection from a choice situation where all the alternatives are incomparable. Then there is no set of considerations or reasons that favour the 'picking' of one alternative or the other (Chang 202: 117). However, someone can respond that in the example above, if both careers are good careers and if neither career is worse than the other, then this provides a sufficient reason to 'pick' either career. But notice that in this choice situation there is no other alternative in the agenda which is demonstrably worse than the two careers. So how is the "goodness" of the career established? You can establish that both careers are good careers only by referring to the choice situation and show that there is a demonstrably worse career. With 'global incomparability', every alternative is incomparable to every other. We are unable to provide some set of considerations for 'picking' one alternative from the maximal set in this choice situation. This, it can be argued is problematic because when we make a choice based on randomized procedures without sufficient reason to make that choice, then we suspend our rational agency and take practical reason outside the scope of rational agency (Chang 2012: 117).

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[79] Note that I am assuming, like everyone else does in the literature that only *two* alternatives are incomparable. This as is not an innocuous assumption, but I make it to let the argument proceed. Alternately, we can also imagine other randomized acts that choose an alternative among incomparable alternatives – like rolling a die.

Conclusion

In this chapter we took up the question, "Is there a justification for making a hard choice constituted by 'global incomparability'?" We introduced and discussed the two responses to the problem we identified in the interregnum that an account of justification has to overcome. The first of these responses from the comparativist position was discussed. We saw that this response consists of two parts. The first part consists in claiming that the concept of incomparability which we discussed and introduced in chapter two is conceptualized on an unjustified assumption. This assumption is that the evaluative space of comparability is exhausted by the three standard relations – better than, worse than, and equally good as. Building a substantive assumption about the number of ways in which alternatives can be compared into our conception of incomparability, it is argued, is mistaken. The second part of this response involved presenting and arguing for a fourth comparative relation which make incomparable alternatives comparable. Accounts for two such relations have been proposed viz. 'on a par' or 'parity', and 'roughly equal'. We saw however that this response entails dropping the axiom of transitivity. Thus one problem was overcome by introducing another problem, and the task of justifying choice remains.

The second response that we took up comes from the maximization view of rationality. This response does not see global incomparability as a problem for the justification of choice. A 'maximizer' argues that justified choice is possible in the presence of incomparable alternatives by choosing that alternative which is not "strictly worse" than other alternatives. Since incomparable alternatives are not strictly worse than each other, maximization can provide a justification for a choice in a hard choice situation. We saw however that this response too was unconvincing. This is because maximization can justify a choice based on randomized procedure, like the flipping of a coin. When we make a choice based on randomized procedures we suspend our rational agency.

CONCLUSION

This work began with the question "What is a hard choice?" It motivated the need for an answer to this question by showing that definitions found in the literature are not satisfactory. In particular, they are restricted to choice situations with two alternatives. In the first chapter we provided a definition for every choice situation among a universal set of finite alternatives. While making the case for this definition we distinguished what hard choices are not, viz. straightforward choices and irrational behaviour. A hard choice was defined as: *Given a set of alternatives X, and a binary relation R, a choice situation given by an agenda A in **Z** constitutes a hard choice iff: B(A,R*) is empty.*

We proceeded to discuss one of these necessary conditions – incompleteness – more substantively. In particular, we discussed what are the reasons for incompleteness, and what implications does this reason have for a choice situation. To have that discussion we first partitioned incompleteness into two types – resolvable incompleteness and irresolvable incompleteness. We argued that the type of incompleteness that is necessary for a hard choice is irresolvable incompleteness. We then discussed three different positions in the literature that provide a substantive answer to the question what are the reasons for irresolvable incompleteness, viz. epistemic limits, incommensurable values, and incomparability. We showed that epistemic limits cannot explain the persistence of irresolvable incompleteness. We argued that the incommensurability of values does not entail the incompleteness of an evaluation. Finally, we presented and discussed the most convincing reason for irresolvable incompleteness – incomparability. We concluded this chapter by showing incomparability can be a feature of a choice situation in two ways. – partial incomparability and global incomparability. Partial incomparability refers to when there are some alternatives in a choice situation given by an agenda A are incomparable. Global incomparability refers to when every alternative in a choice situation A is incomparable. We then proceeded to show one implication of incomparability for a choice situation. If there is global incomparability in choice situation given by an agenda A, then it is sufficient to make a choice situation a hard choice.

Between chapter two and chapter three there was an interregnum. This interregnum was introduced because the discussion in chapter three was about the responses to a problem that the hard choice constituted by 'global incomparability' presents to a choice situation. We used this interregnum to see what this problem is. In particular we saw that global incomparability precludes the possibility of a justified choice.

The next question this work took up was in chapter three. "Is there a justification for making a hard choice?" This was the 'problem' identified in the interregnum. In particular, we asked is there a justification for making a choice when the hard choice is constituted by 'global

incomparability'. In answering this question we presented and critically discussed two responses in the literature to this problem. The first response we discussed comes from the framework of practical reason called comparativism. This response we showed consists of two claims. The 'problem' posed by hard choices, or the incomparability of alternatives in a choice situation, follows from an unjustified assumption in the way incomparability has been conceptualized. This assumption is: if alternatives can be compared, then they can be compared in only three ways, viz. "better than," "worse than," and "equally good". The second part of the comparativist response that we saw consists in dropping this assumption and proceeding to introduce and argue for a fourth way in which alternatives can be compared. Proposals for two additional relations were discussed. Comparativist posit that if alternatives that are incomparable by the three standard positive relations can be made comparable by this fourth relation, then the problem for the justification of a choice, which exists because alternatives are incomparable, can be overcome. Or so the comparativists argue. But we saw that this response entails rejecting the axiom of the transitivity of relations. While transitivity is not necessary for rationality, as our definition of a hard choice defends, comparativists demand that a positive value relation determine choice. If the fourth value relation that comparativist propose is supposed to guide and indeed determine action, then we showed that it could justify a series or sequence of determined choices that leave an individual worse off. For this reason their response is unsatisfactory.

The second response that we considered comes from a framework of reasoning developed in rational choice theory called maximization. We saw that the maximization response does not see incomparability as a problem for the justification of a choice in a hard choice situation constituted by 'global incomparability'. This is because a maximizer argues that justified choice is possible in the presence of incomparable alternatives by choosing that alternative which is not "strictly worse" than other alternatives. Since incomparable alternatives are not strictly worse than each other, maximization can provide a justification for the choice of any of the alternatives in the agenda with global incomparability. In this way they overcome the problem presented by a hard choice situation constituted by global incomparability. However, we saw that the maximization response is unsatisfactory because it could justify a choice based on a randomized procedure like flipping a coin which takes practical reason outside the scope of rational agency.

In conclusion, we began this work with three concerns. First, what is a hard choice? Second, what are the conditions in which they emerge? Third, how does an agent make a justified choice in the presence of the condition that is sufficient for a choice situation to be a hard choice? Our analysis has addressed all these concerns. We first provided a definition which identified two necessary conditions which are jointly sufficient for a choice situation to be a hard choice. We proceeded to identify a sufficient condition in a choice situation for that choice situation to be a hard choice. Finally, we discussed how to make a justified choice in the presence of the condition that is sufficient for a choice situation to be a hard choice.

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