A game-theoretical model of a financial statement audit

Master Thesis - Economics and Business

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Abstract: In this paper we developed a game theoretical model in order to examine the equilibria when preparing a financial statement about the firm's performance. The model is a dynamic signalling game in which the firm sends a message about its performance to the auditor. The auditor receives the message but does not know the actual performance of the firm. The auditor has the possibility to investigate the message in case he expects this message is exaggerated. When he finds exaggeration he is rewarded with a bonus and the firm is punished with a fine. When the auditor has a large incentive to investigate a message the firm is likely to play a strategy where the probability he gets caught exaggerating is low. When the auditor has no incentive to investigate a message the firm simply chooses the strategy that is most aligned with its desire to exaggerate.

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Introduction

The audit profession is under a lot of pressure as a result of various scandals in financial statements over the last decades (Pierce, 2007). The variety and intensity of those scandals is best described by Lovell (1995) as follows:

> *'...ranging from the failure of accounting documents to reveal a more accurate reflection of the financial well-being/ill health of organizations and the collusion of accountants in the preparation and validation of those documents, to the failure of the accountancy profession satisfactorily to take account of the public interest in the determination of the future of accounting and auditing practice.'*

The Enron scandal is seen as the biggest scandal the auditing practice has ever faced. Enron was a large Texas-based commodities, energy and service company. It declared bankruptcy on the second of December 2001. Enron left out major debts from its balance sheet. The profit of the company was therefore significantly overestimated (The Economist, 2002a). Enron's auditor, Andersen, admitted an error of judgement during the audit process. Andersen eventually had to stop its audit activities completely (The Economist, 2002b).

For managers in firms there are various reasons why they want the external world to believe the firm is performing better than it actually is. Often the manager receives a bonus, which depends on the company's performance. In case he owns stocks he has an incentive to inflate the price of his own stocks before selling them (Beneish, 1991). Some researchers find that managers do not overstate earnings for their own enrichment, they do it to create certain benefits for the firm. Examples of these benefits include enjoying lower costs of capital and avoiding operational restrictions caused by high debt (Dechow, Sloan, & Sweeney, 1996). Over 70% of the users of financial statements believe that auditors should completely eliminate incorrect information in all statements (Wooten, 2003). With the eliminations of incorrect information, future investors are able to make well grounded and informed decisions (source). Their argument is that the modern financial markets perform best if investors are correctly and completely informed when deciding where to allocate their capital. The high number of audit scandals over the past decades indicates that auditors do not fully eliminate incorrect information in financial statements.

Previous literature blames the development of a more commercial ethos of the accounting firms as an underlying cause for the audit scandals (Hanlon, 1994).
Instead of focussing on their duty to serve the public interest, the auditor's focus has shifted towards profit maximization and growth by increasing revenues and lowering costs (Briloff, 1986). Firms tried to increase revenues by adopting various non-audit services or management advisory services. However, in most western countries, audit firms are no longer allowed to perform both audit and non-audit services for the same client due to a conflict in auditor's independence (European Commision, 2010). To reduce costs the audit firms decreased the budget per audit (Bedard, Ettredge, & Johnstone, 2008). This development puts substantial pressure on the auditors, since they are expected to expand the client base, satisfy the current clients and perform a good audit all at the same time. Previous literature finds a negative relationship between budget limitations and reported audit hours (DeZoort & Lord, 1997) (Kelley & Margheim, 1992) (Raghunandan, 1991). The likelihood that incorrect information is not eliminated in the financial statement increases when the auditor spends less time per audit. Ettredge, Fuerherm, & Li (2014), when controlling for a client's risk and increased fee pressure, find that auditors do not sufficiently increase effort for high-risk clients. Financial statements of riskier firms are therefore more likely to be incorrect.

Several critics argue that auditors should follow a set framework forcing them to fulfil a high quality audit. Others, however, do not see this as a sustainable solution. The framework would decrease the need for professional judgement since the auditor would just go through a standardized list of questions. They fear auditors will loose the skills to adjust the framework if needed in the future, this could eventually result in low quality audits (West, 2003) (Staubus, 2004) (Briloff, 1986)

This paper analyses, with the help of a signalling game, the strategies of the auditor and the firm in the audit market. In this market the firm has a specific performance and sends a message to the auditor about this performance. The auditor is able to investigate this message, however, an investigation demands costly effort from the auditor. In this paper we assume the auditor is only able to overrule the firm's performance message after investigating it, which is in line with the vision of several audit firms. PWC, for example, states the auditor uses his experiences and skills to conduct several tests to form an opinion on the performance, this opinion is clearly stated in a separate paragraph in the financial statement (PWC, 2013). In case this opinion is 'clean' the auditor regards the financial statement, thus the signalled performance, to be correct. This analysis separates itself from previous research by focussing on the strategy of the auditor and the strategy of the firm and how those strategies influence each other. The objective of this paper is to find all possible equilibria in this model and to explain the intuition behind the results. In this paper we are going to answer the following question:

'What is the strategy of the firm and what is the strategy of the auditor when preparing a financial statement about the firm's performance?

Solving the model provides us with the following insights. In all equilibria the strategy of the auditor is a best response to the strategy of the firm. Furthermore the firm takes the auditor's best response into account. The firm pays more attention to the strategy of the auditor when he has a large incentive to investigate a message and when the firm needs to pay a fine when getting caught exaggerating. Consequently, the firm is more likely to choose a strategy with a lower probability of getting caught exaggerating. When the auditor has no incentive to investigate a message the firm simply chooses the strategy that is most aligned with its desire to exaggerate.

In the next chapter the model used in this paper is discussed. Then, the results are presented. This part is divided into two sections, the first section provides all possible equilibria in our model and the second section contains the economic intuition. Lastly, the concluding remarks are provided, including the limitations and recommendations following from this theoretical paper.

The Model

In this paper we consider a signalling game where a firm needs to prepare a financial statement about its performance. This report represents the performance of the previous accounting period the firm recently completed. The performance of the firm is a random variable v that can take on the value of $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}$ or $\frac{7}{8}$. The first player in this model is the firm. The firm observes the exact value v. After observing its performance, the firm sends out a signal. This signal can take on the value of $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}$ or $\frac{7}{8}$ represented by variable m. The firm is driven by its desire to exaggerate about its performance. This desire to send a message higher compared to the actual performance is included in the model by parameter x, $x \geq 0$. Below the distribution of the performance v is presented:

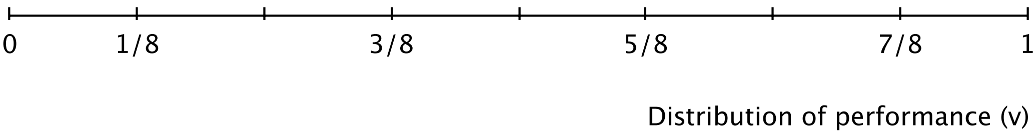


Figure 1.

The likelihood that a firm exaggerates is included in the model by the variable μ_m. Here $\mu_m(m \neq v|m)$ represents the probability that $m \neq v$ for a given message. To clarify, for instance, $\mu_{\frac{5}{8}}$ expresses the likelihood that a firm has exaggerated when the auditor has received $m = \frac{5}{8}$.

The second player in this model is the auditor, who does not directly observe the performance of the firm. However, the auditor does always observe m. After observing m he can choose whether to start an investigation to find the exact value v of the firm.
The choice of the auditor whether to investigate a message is included in the model by the conditional variable q_m. Here $q_m(m|m \neq v)$ represents the probability that the auditor finds exaggeration, given the fact that the firm has exaggerated. An example for clarification, $q_{\frac{5}{8}}$ expresses the probability that a firm of type $v = \frac{1}{8}$ or $v = \frac{3}{8}$ gets caught exaggerating when it has chosen $m = \frac{5}{8}$.

Finding $m \neq v$ is what drives the auditor, since in this case he receives a bonus of value b. The cost of investigation is a quadratic function of the probability of finding exaggeration, given that there is exaggeration. The parameter λ measures the cost of effort input, $\lambda > 0$.

$$\textbf{\textit{Cost of investigation}} = \frac{\lambda}{2}\boldsymbol{q_m}^{\boldsymbol{2}}$$

The utility of the firm depends on the performance stated in the financial statement, represented by S. In case the firm gets caught exaggerating by the auditor the signal about its performance is no longer believable and therefore the auditor uses the actual performance in the financial statement. When the firm gets caught exaggerating it also receives a fine of costs f. In case the firm tells the truth it never has to pay the fine, therefore we introduce the binary variable p:

$$m - v = 0 \rightarrow p = 1$$

$$m - v > 0 \rightarrow p = 0$$

In case the firm exaggerates p equals zero. The performance stated in the financial statement is either equal to the message or equal to the actual performance:

In case the auditor investigates and $p = 0 \rightarrow S = v$

In case the auditor investigates and $p = 1 \rightarrow S = m = v$

In case the auditor does not investigate $\rightarrow S = m$

The firm's utility function is as follows:

$$\boldsymbol{U_{Firm}} = -\big(\boldsymbol{S} - (\boldsymbol{v} + \boldsymbol{x})\big)^{\boldsymbol{2}} - \boldsymbol{f}\boldsymbol{q_m}(\boldsymbol{1} - \boldsymbol{p})$$

The first part of the formula measures the squared difference between the performance stated in the financial statement and the performance the firm would like to be stated in the financial statement. The firm needs to pay a fine in case it has exaggerated and the auditor has investigated the message.

The auditor's utility function depends on the income of the auditor, represented by Y. in case he finds exaggeration he receives the bonus otherwise his income is equal to zero:

In case the auditor investigates and $p = 1 \rightarrow Y = 0$

In case the auditor investigates and $p = 0 \rightarrow Y = b$

In case the auditor does not investigate $Y = 0$

The auditor's utility function is as follows:

$$\boldsymbol{U_{Auditor} = Y - \frac{\lambda}{2} q_m{}^2}$$

Equilibrium definition

In all equilibria in this model the strategy of the auditor is an optimal response to the strategy of the firm and the strategy of the firm is an optimal response to the strategy of the auditor. In equilibrium the players do not have an incentive to deviate. The model contains a dynamic game with incomplete information since the auditor does not know the actual performance of the firm at the beginning of this game. The auditor therefore forms a believe about the firm's performance. This believe is based on the message the auditor has received from the firm. Consequently, the equilibria in our model are Perfect Bayesian Equilibria (PBE) (Gibbons, 1992). To solve this model we use backward induction, in the next chapter we start by examining the strategy of the auditor.

In the rest of this paper we use the following notations:

A firm of type $v = \frac{1}{8}$ is referred to as $V_{\frac{1}{8}}$ and a message of type $m = \frac{1}{8}$ is referred to as $M_{\frac{1}{8}}$.

A firm of type $v = \frac{3}{8}$ is referred to as $V_{\frac{3}{8}}$ and a message of type $m = \frac{3}{8}$ is referred to as $M_{\frac{3}{8}}$.

A firm of type $v = \frac{5}{8}$ is referred to as $V_{\frac{5}{8}}$ and a message of type $m = \frac{5}{8}$ is referred to as $M_{\frac{5}{8}}$.

A firm of type $v = \frac{7}{8}$ is referred to as $V_{\frac{7}{8}}$ and a message of type $m = \frac{7}{8}$ is referred to as $M_{\frac{7}{8}}$.

Results.

Auditor

The auditor chooses his strategy after he has observed the message of the firm. He has a positive income when he investigates an exaggerated message, in that case he receives the bonus.

$$\boldsymbol{U_{Auditor} = \mu'_m q_m b - \frac{\lambda}{2} q_m{}^2}$$

μ'_m represents the equilibrium likelihood of exaggeration for a given message.

By using backwards induction we are able to find the optimal response of the auditor as a function of the strategy of the firm. We arrive at this optimal response by optimising the auditor's utility with respect to the variable q_m. The auditor has a direct influence on the variable q_m because he decides whether he investigates a message or not.

This leads to:

$$q_m = \frac{dU_{auditor}}{dq(m)} \rightarrow \frac{\mu\prime_m\, b}{\lambda} \qquad (1)$$

In all pure and mixed equilibria in the model this is the unique optimal response for the auditor. The equation above shows the ratio of the bonus to the costs incurred by investigation influences the auditor's strategy. In the rest of this paper we often discuss the influence of the bonus, which actually refers to the influence of this ratio. The auditor knows only $V_{\frac{1}{8}}$ chooses $M_{\frac{1}{8}}$, consequently $\mu'_{\frac{1}{8}} = 0$. The best response of the auditor to $M_{\frac{1}{8}}$ is to never investigate this message.

Firm

The firm chooses a message based on its desire to exaggerate, its actual performance, the fine and the strategy of the auditor. The firm knows the strategy of the auditor from the outcome of equation (1), as a result we can take S out of the utility function of the firm.

This leads to:

$$\boldsymbol{U_{Firm} = -(1 - q_m)\big(m - (v + x)\big)^2 - p q_m (-x)^2 - (1 - p) q_m (f + (-x)^2)}$$

The first part of the formula measures the firm's utility when the auditor does not investigate. The second part measures the firm's utility when the firm does not exaggerate. The third part measures the firm's utility when the firm is caught exaggerating.
In the next section we explain which equilibrium holds for a given value of x, starting with $x = 0$. Every equilibrium has a lower and an upper bound restriction on x. These restrictions on the firm's desire to exaggerate depend on the f, b and λ. $V_{\frac{7}{8}}$ always plays the pure strategy $M_{\frac{7}{8}}$, and therefore we do not have to examine the restrictions on x for this type of firm.

Intuition check
We check the intuition behind all restrictions in the pure strategy equilibria by setting the bonus equal to zero, in that case the auditor never investigates a messages. The sum of the firm's desire to exaggerate plus the actual performance must therefore be closest to the equilibrium message. In the mixed strategy equilibrium we examine whether the influence of the bonus and the fine is in line with our expectations. When the bonus or the fine increases exaggeration becomes more risky. The auditor has a higher incentive to investigate a message and the firm is less willing to take the risk of paying this high fine. The firm's incentive to choose a safer message should increase. Whether a message is more or less risky compared to another message is only influenced by μ_m. A low μ'_m decreases, and a high μ'_m increases the auditor's incentive to investigate the message.

The rest of this chapter is divided in two sections. In the first section we discuss all equilibria with their lower and upper bound restrictions, and in the second section we explain why these equilibria occur in this order and provide further economic intuition.

Equilibria

Pure 1: The pure strategy equilibrium in which all firm types choose $m = v$, and therefore every μ'_m equals zero. The auditor's best response in this case is $q_m = 0$ for every message.

The lower bound restriction in this case is equal to zero, no desire to exaggerate results in truth telling. The upper bound restriction occurs when one type of firm includes an exaggerated message in its strategy. The corresponding upper bound restriction on x is calculated as follows:

$$-\left(1-q_{\frac{1}{8}}\right)\left(\frac{1}{8}-\left(\frac{1}{8}+x\right)\right)^2-q_{\frac{1}{8}}(-x)^2>-\left(1-q_{\frac{3}{8}}\right)\left(\frac{3}{8}-(v+x)\right)^2-q_{\frac{3}{8}}((-x)^2+f)$$

$$x\left(\frac{1}{2}-\frac{1}{2}q_{\frac{3}{8}}\right)<\frac{1}{16}-\frac{1}{16}q_{\frac{3}{8}}$$

$$x<\frac{1}{8}$$

Although the calculation above is done for $V_{\frac{1}{8}}$ the outcome also holds for $V_{\frac{3}{8}}$ and $V_{\frac{7}{8}}$. In this separating equilibrium every type of firm tells the truth and therefore the auditor never examines a message. As a result, the bonus and the fine do not have any influence in this equilibrium. For values of x above $\frac{1}{8}$ every type of firm includes an exaggerated message in its strategy, leading to the mixed strategy equilibrium below.

Mix 1: This mixed strategy equilibrium occurs when every type of firm includes an exaggerated message in its strategy.

$V_{\frac{1}{8}}$ mixes between $M_{\frac{1}{8}}$ and $M_{\frac{3}{8}}$.

$V_{\frac{3}{8}}$ mixes between $M_{\frac{3}{8}}$ and $M_{\frac{5}{8}}$.

$V_{\frac{5}{8}}$ mixes between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$.

$V_{\frac{7}{8}}$ never mixes and always chooses $M_{\frac{7}{8}}$.

In this equilibrium the strategies of both the firm and the auditor changes with x. We first calculate the strategy of the auditor and the corresponding strategies of $V_{\frac{5}{8}}$, $V_{\frac{3}{8}}$ and $V_{\frac{1}{8}}$. After we calculate with what probability each type of firm exaggerates, represented by δ_v. Lastly we calculate the lower and upper bound restrictions of this mixed strategy equilibrium.

$V_{\frac{5}{8}}$ plays a mixed strategy in case its utility from sending either $M_{\frac{5}{8}}$ or $M_{\frac{7}{8}}$ is equal to each other, resulting in the following $q_{\frac{7}{8}}$:

$$-(-x)^2=-\left(1-q_{\frac{7}{8}}\right)\left(\frac{2}{8}-x\right)^2-q_{\frac{7}{8}}f-q_{\frac{7}{8}}(-x)^2$$

$$q_{\frac{7}{8}}\left(\frac{1}{2}x-\frac{1}{16}+f\right)=\frac{1}{2}x-\frac{1}{16}$$

$$q_{\frac{7}{8}} = \frac{x-\frac{1}{8}}{x-\frac{1}{8}+2f}$$

By replacing the expression for $q_{\frac{7}{8}}$ in equation (1) we find $V_{\frac{5}{8}}$'s strategy:

$$\mu_{\frac{7}{8}} = q_{\frac{7}{8}}\frac{\lambda}{b}$$

$$\mu'_{\frac{7}{8}} = \frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)}$$

The expressions for $q_{\frac{7}{8}}$ and $\mu'_{\frac{7}{8}}$ also hold for respectively $q_{\frac{5}{8}}$ and $\mu'_{\frac{5}{8}}$ and $q_{\frac{3}{8}}$ and $\mu'_{\frac{3}{8}}$, thus the strategies of $V_{\frac{3}{8}}$ and $V_{\frac{1}{8}}$ are similar to $V_{\frac{5}{8}}$'s strategy. If the fine is zero each type of firm plays a pure strategy. When looking at $V_{\frac{5}{8}}$, only when $q_{\frac{7}{8}}$ equals one $V_{\frac{5}{8}}$ is indifferent between $M_{\frac{5}{8}}$ or $M_{\frac{7}{8}}$. The indifference point of x is a knife-edge point. In that point $V_{\frac{5}{8}}$ is set back to truth telling for sure and therefore $M_{\frac{7}{8}}$ is set back to $M_{\frac{5}{8}}$. In case the fine is zero and the $V_{\frac{5}{8}}$'s desire lies above the knife-edge point $V_{\frac{5}{8}}$ would gain a higher utility by choosing $M_{\frac{7}{8}}$, in that scenario $q_{\frac{7}{8}} < 1$. A high fine lowers the incentive for each type of firm to exaggerate, none of the types want to pay this high fine. The auditor knows about the decrease in the firm's incentive to exaggerate. Consequently, each type of firm is more honest and the auditor decreases the probability that he examines a message. As a result $V_{\frac{5}{8}}$, $V_{\frac{3}{8}}$ and $V_{\frac{1}{8}}$ all increase the probability that they exaggerate. This continues until we are in a stable mixed equilibrium where both the auditor and $V_{\frac{5}{8}}$, $V_{\frac{3}{8}}$ and $V_{\frac{1}{8}}$ have no incentive to change their strategy.

Next we calculate with what probability $V_{\frac{5}{8}}$, $V_{\frac{3}{8}}$ and $V_{\frac{1}{8}}$ exaggerate in this equilibrium, starting with $V_{\frac{5}{8}}$. $V_{\frac{7}{8}}$ never exaggerates, thus $\delta_{\frac{7}{8}} = 0$.

$$\mu_{\frac{7}{8}} = \frac{\delta_{\frac{5}{8}}}{\delta_{\frac{5}{8}}+\left(1-\delta_{\frac{7}{8}}\right)}$$

$$\mu_{\frac{7}{8}}\left(1-\delta_{\frac{7}{8}}\right) = \delta_{\frac{5}{8}}\left(1-\mu_{\frac{7}{8}}\right)$$

$$\delta_{\frac{5}{8}} = \frac{\mu_{\frac{7}{8}}*\left(1-\delta_{\frac{7}{8}}\right)}{\left(1-\mu_{\frac{7}{8}}\right)} \qquad \left(\delta_{\frac{7}{8}} = 0\right)$$

We use this formula again for calculating $\delta_{\frac{3}{8}}$ and $\delta_{\frac{1}{8}}$.

$$\delta_{\frac{5}{8}} = \frac{\frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)}}{\left(1-\frac{\lambda(x-\frac{1}{8})}{b(x-\frac{1}{8}+2f)}\right)}$$

$$\delta_{\frac{5}{8}} = \frac{\lambda(x-\frac{1}{8})}{b(x-\frac{1}{8}+2f)} * \frac{b\left(x-\frac{1}{8}+2f\right)}{b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)}$$

$$\delta_{\frac{5}{8}} = \frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)}$$

An increase in the fine decreases the probability of exaggeration since exaggeration becomes more risky, a low bonus means the fine has little effect.

This outcome allows us to calculate with what probability $V_{\frac{3}{8}}$ exaggerates:

$$\delta_{\frac{3}{8}} = \frac{\mu_{\frac{5}{8}} * \left(1-\delta_{\frac{5}{8}}\right)}{\left(1-\mu_{\frac{5}{8}}\right)}$$

$$\delta_{\frac{3}{8}} = \frac{\frac{\lambda(x-\frac{1}{8})}{b(x-\frac{1}{8}+2f)} * \left(1-\frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)}\right)}{\left(1-\frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)}\right)}$$

$$\delta_{\frac{3}{8}} = \frac{\frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)} - \frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)} * \frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)}}{\left(\frac{b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)}\right)}$$

$$\delta_{\frac{3}{8}} = \frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)} - \frac{\left(\lambda\left(x-\frac{1}{8}\right)\right)^2}{\left(b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)\right)^2}$$

We can rewrite this expression as a function of $\delta_{\frac{5}{8}}$:

$$\delta_{\frac{3}{8}} = \delta_{\frac{5}{8}} - \delta_{\frac{5}{8}}{}^2$$

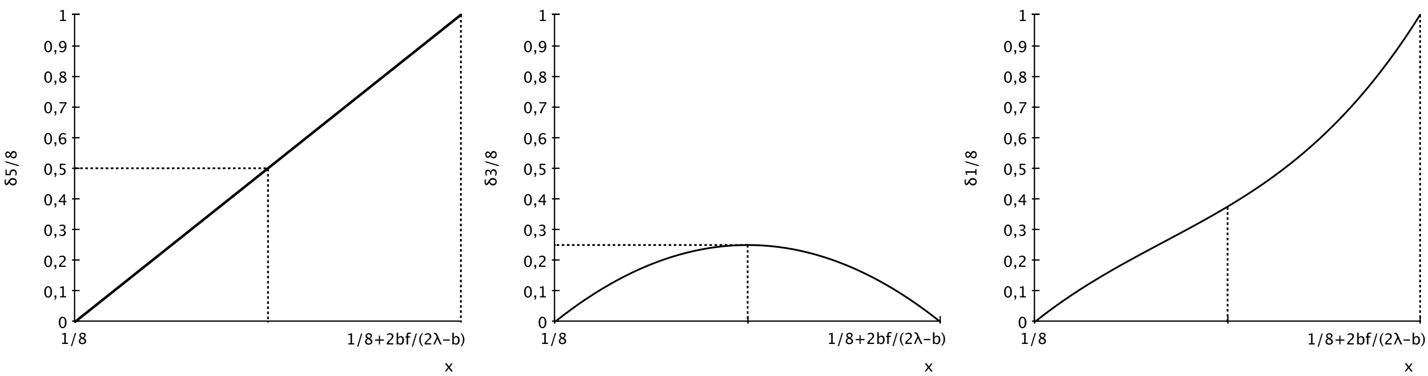
Finally we calculate with what probability $V_{\frac{1}{8}}$ exaggerates, using the same method:

$$\delta_{\frac{1}{8}} = \frac{\frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)} * \left(1-\left(\frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)} - \frac{\lambda\left(x-\frac{1}{8}\right)^2}{\left(b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)\right)^2}\right)\right)}{\left(1-\frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)}\right)}$$

Again we can rewrite this expression as a function of $\delta_{\frac{5}{8}}$:

$$\delta_{\frac{1}{8}} = \delta_{\frac{5}{8}} - \delta_{\frac{5}{8}}{}^2 + \delta_{\frac{5}{8}}{}^3$$

Next we discuss the intuition behind these outcomes. $V_{\frac{7}{8}}$ always chooses $M_{\frac{7}{8}}$, therefore the probability that $M_{\frac{7}{8}}$ is an honest signal is relatively large. In the first graph below we see $V_{\frac{5}{8}}$ is more likely to exaggerate when his desire to exaggerate is higher. Consequently, $M_{\frac{5}{8}}$ is chosen less by $V_{\frac{5}{8}}$ when increasing x, which is know by the auditor. As a result $M_{\frac{5}{8}}$ becomes a relatively unsafe message for $V_{\frac{3}{8}}$. Graphically we see this as well, first $V_{\frac{3}{8}}$ increases the probability it exaggerates, but from the x point where $\delta_{\frac{5}{8}}$ equals 0,5 $M_{\frac{5}{8}}$ becomes overly risky and $\delta_{\frac{3}{8}}$ decreases. The third graph shows us $V_{\frac{1}{8}}$ starts exaggerating slowly, $M_{\frac{3}{8}}$ becomes more risky as $\delta_{\frac{3}{8}}$ grows. After $\delta_{\frac{3}{8}}$ starts to decrease, $\delta_{\frac{1}{8}}$ grows exponentially because $M_{\frac{3}{8}}$'s riskiness decreases.



Figures 2,3 and 4.

By setting $\delta_{\frac{5}{8}}$ equal to zero we obtain the lower bound restriction of this equilibrium:

$$\delta_{\frac{5}{8}} = \frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)}$$

$$\lambda\left(x-\frac{1}{8}\right) = 0$$

$$x = \frac{1}{8}$$

By setting $\delta_{\frac{5}{8}}$ equal to one we obtain the upper bound restriction of this equilibrium:

$$\delta_{\frac{5}{8}} = \frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)-\lambda\left(x-\frac{1}{8}\right)}$$

$$x = \frac{1}{8} + \frac{2bf}{2\lambda-b}$$

This mixed strategy equilibrium holds when the value of x lies between:

$$\frac{1}{8} \leq x \leq \frac{1}{8} + \frac{2bf}{2\lambda - b}.$$

In case the lower bound restriction takes on a higher or similar value compared to the upper bound restriction this mix does not occur. Therefore, this equilibrium only holds when:

$$bf > 0$$

In case the bonus multiplied by the fine equals zero this mix does not hold. When reaching the upper bound restriction of this equilibrium both $V_{\frac{1}{8}}$ and $V_{\frac{5}{8}}$ exaggerate and $V_{\frac{3}{8}}$ tells the truth, guiding us to the next pure strategy equilibrium.

Pure 2: The pure strategy equilibrium in which $V_{\frac{1}{8}}$ and $V_{\frac{5}{8}}$ exaggerate with respectively $M_{\frac{3}{8}}$ and $M_{\frac{7}{8}}$ and $V_{\frac{3}{8}}$ chooses $M_{\frac{3}{8}}$. In this equilibrium $\mu'_{\frac{3}{8}} = \frac{1}{2}$ and $\mu'_{\frac{7}{8}} = \frac{1}{2}$. The auditor's best responses in this equilibrium are $q_{\frac{3}{8}} = \frac{b}{2\lambda}$ and $q_{\frac{7}{8}} = \frac{b}{2\lambda}$.

The lower bound restriction of this equilibrium occurs in case $V_{\frac{1}{8}}$ or $V_{\frac{5}{8}}$ wants to include truth telling in their strategy, below we show the calculation of $V_{\frac{1}{8}}$'s lower bound restriction:

$$-(-x)^2 < -\left(1 - q_{\frac{3}{8}}\right)\left(\frac{2}{8} - x\right)^2 - q_{\frac{3}{8}}(f + (-x)^2)$$

$$\frac{1}{8} + \frac{2bf}{2\lambda - b} < x$$

The same lower bound restriction holds for $V_{\frac{5}{8}}$. Moreover, this restriction is exactly equal to the upper bound restriction of the previous mixed strategy equilibrium. $V_{\frac{3}{8}}$ is still telling the truth, for that reason this firm has no lower bound restriction.

Previously the upper bound restriction for each type of firm was similar, in this equilibrium that is not the case. We only show the lowest upper bound restriction, all other restrictions are redundant. In the Appendix A1.1 all other restrictions are presented.

The upper bound restriction in this equilibrium occurs when $V_{\frac{3}{8}}$ wants to deviate to $M_{\frac{5}{8}}$:

$$-(-x)^2 > -\left(1 - q_{\frac{5}{8}}\right)\left(\frac{3}{8} - \left(\frac{1}{8} + x\right)\right)^2 - q_{\frac{5}{8}}(f + (-x)^2)$$

$$\frac{1}{8} + \frac{2fq_{\frac{5}{8}}}{1 - q_{\frac{5}{8}}} > x$$

No type of firm includes $M_{\frac{5}{8}}$ in its strategy, as a result $\mu'_{\frac{5}{8}}$ is not determined in this equilibrium and is therefore an out of equilibrium believe.

This pure strategy equilibrium holds when the value of x lies between:

$$\frac{1}{8}+\frac{2bf}{2\lambda-b}<x<\frac{1}{8}+\frac{2fq_{\frac{5}{8}}}{1-q_{\frac{5}{8}}}.$$

This equilibrium does hold in case $bf>0$. When the auditor expects $M_{\frac{5}{8}}$ is only chosen by $V_{\frac{3}{8}}$ or $V_{\frac{1}{8}}$ the expected value of $\mu_{\frac{5}{8}}$ equals one, this would result in an upper bound restriction which is equal to $\frac{1}{8}+\frac{2bf}{\lambda-b}$. That value of x is the only x value where $V_{\frac{3}{8}}$ is indifferent between $M_{\frac{3}{8}}$ and $M_{\frac{5}{8}}$, this knife-edge point guides us towards the next pure strategy equilibrium.

Pure 3: The pure strategy equilibrium in which $V_{\frac{1}{8}}$ chooses $M_{\frac{3}{8}}$, $V_{\frac{3}{8}}$ chooses $M_{\frac{5}{8}}$ and $V_{\frac{5}{8}}$ and $V_{\frac{7}{8}}$ choose $M_{\frac{7}{8}}$, and therefore $\mu'_{\frac{3}{8}}=1$, $\mu'_{\frac{5}{8}}=1$ and $\mu'_{\frac{7}{8}}=\frac{1}{2}$. The auditor's best responses are $q_{\frac{3}{8}}=\frac{b}{\lambda}$, $q_{\frac{5}{8}}=\frac{b}{\lambda}$ and $q_{\frac{7}{8}}=\frac{b}{2\lambda}$.

The lower bound restriction of this equilibrium occurs when $V_{\frac{5}{8}}$, $V_{\frac{3}{8}}$ or $V_{\frac{1}{8}}$ wants to include truth telling in its strategy. The lower bound restriction of $V_{\frac{5}{8}}$ is redundant since this type of firm is playing a relatively safer message in equilibrium, meaning $V_{\frac{5}{8}}$ is less likely to deviate. As a result the lower bound restriction of $V_{\frac{5}{8}}$ is higher compared to the one of $V_{\frac{3}{8}}$ and $V_{\frac{1}{8}}$. The lower bound restrictions for $V_{\frac{1}{8}}$ and $V_{\frac{3}{8}}$ are similar and calculated as follows:

$$-\left(1-q_{\frac{1}{8}}\right)\left(\frac{1}{8}-\left(\frac{1}{8}+x\right)\right)^2-q_{\frac{1}{8}}(-x)^2<-\left(1-q_{\frac{3}{8}}\right)\left(\frac{3}{8}-\left(\frac{1}{8}+x\right)\right)^2-q_{\frac{3}{8}}(f+(-x)^2\ )$$

$$-(-x)^2<-(1-q)\left(\frac{2}{8}-x\right)^2-\ q_{\frac{3}{8}}(f+x^2)$$

$$0<-\frac{1}{8}+x-2q_{\frac{3}{8}}f+\frac{1}{8}q_{\frac{3}{8}}-xq_{\frac{3}{8}}$$

$$x\left(q_{\frac{3}{8}}-1\right)<\left(\frac{1}{8}q_{\frac{3}{8}}-\frac{1}{8}\right)-2q_{\frac{3}{8}}f$$

$$x>\frac{1}{8}+\frac{2q_{\frac{3}{8}}f}{1-q_{\frac{3}{8}}}$$

This is de lower bound restriction for $V_{\frac{1}{8}}$, when replacing $q_{\frac{3}{8}}$ by $q_{\frac{5}{8}}$ it becomes the lower bound restriction of $V_{\frac{3}{8}}$. Lastly we substitute the outcome from equation (1) for $q_{\frac{3}{8}}$ or $q_{\frac{5}{8}}$:

$$x > \left(\frac{1}{8} + \frac{2bf}{\lambda - b}\right)$$

The upper bound restriction occurs when $V_{\frac{3}{8}}$ chooses to include $M_{\frac{7}{8}}$ in its strategy. $M_{\frac{7}{8}}$ is a relatively safer message compared to $M_{\frac{5}{8}}$ because $V_{\frac{7}{8}}$ always chooses it. As a result the upper bound restriction of $V_{\frac{1}{8}}$ is redundant.

The upper bound restriction of this pure strategy equilibrium:

$$-\left(1 - q_{\frac{5}{8}}\right)\left(\frac{2}{8} - x\right)^2 - q_{\frac{5}{8}}(f + (-x)^2) > -\left(1 - q_{\frac{7}{8}}\right)\left(\frac{4}{8} - x\right)^2 - q_{\frac{7}{8}}(f + (-x)^2)$$

$$x < \frac{3}{8} - \frac{b + 8bf}{8\lambda}$$

This pure strategy equilibrium holds when the value of x lies between:

$$\left(\frac{1}{8} + \frac{2bf}{\lambda - b}\right) < x < \frac{3}{8} - \frac{b + 8bf}{8\lambda}.$$

This equilibrium holds even when b or f is equal to zero. Furthermore this upper bound restriction guides us to the next equilibrium where $V_{\frac{3}{8}}$ includes $M_{\frac{7}{8}}$ in its strategy.

Mix 2: This mixed strategy equilibrium occurs when $V_{\frac{3}{8}}$ no longer wants to play $M_{\frac{5}{8}}$ **purely.**

$V_{\frac{1}{8}}$ chooses $M_{\frac{3}{8}}$.

$V_{\frac{3}{8}}$ mixes between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$.

$V_{\frac{5}{8}}$ chooses $M_{\frac{7}{8}}$.

$V_{\frac{7}{8}}$ never mixes and always chooses $M_{\frac{7}{8}}$.

Since $\mu'_{\frac{3}{8}} = 1$ and $\mu'_{\frac{5}{8}} = 1$, the auditor's best responses are $q_{\frac{3}{8}} = \frac{b}{\lambda}$ and $q_{\frac{5}{8}} = \frac{b}{\lambda}$.

Similar to mixed strategy equilibrium 1 we first calculate the strategy of the auditor and the corresponding strategy of $V_{\frac{3}{8}}$, thus $\mu_{\frac{7}{8}}$. Next we calculate with what probability $V_{\frac{3}{8}}$ exaggerates by choosing $M_{\frac{7}{8}}$, respresented by $\delta_{\frac{3}{8}}$. After we calculate the lower and upper bound restrictions of this equilibrium. Finally we discuss the restrictions for this equilibrium to hold.

$V_{\frac{3}{8}}$ plays this mixed strategy in case its utility from sending either $M_{\frac{5}{8}}$ or $M_{\frac{7}{8}}$ is equal to each other. The expression for $q_{\frac{7}{8}}$ is as follows:

$$-\left(1-q_{\frac{5}{8}}\right)\left(\frac{2}{8}-x\right)^2-q_{\frac{5}{8}}(f+(-x)^2)=-\left(1-q_{\frac{7}{8}}\right)\left(\frac{4}{8}-x\right)^2-q_{\frac{7}{8}}(f+(-x)^2)$$

$$q_{\frac{7}{8}}=\frac{\frac{3}{16}-\frac{1}{2}x+\frac{1b}{16\lambda}-\frac{fb}{\lambda}-\frac{xb}{2\lambda}}{\frac{1}{4}-x-f}$$

By replacing the expression for $q_{\frac{7}{8}}$ in equation (1) we find $V_{\frac{3}{8}}$'s strategy:

$$\mu_{\frac{7}{8}}=q_{\frac{7}{8}}\frac{\lambda}{b}$$

$$\mu'_{\frac{7}{8}}=\frac{\frac{3\lambda}{16}-\frac{x\lambda}{2}+\frac{1b}{16}-fb-\frac{xb}{2}}{\frac{1}{4}b-xb-fb}$$

Next we calculate with what probability $V_{\frac{3}{8}}$ wants to exaggerate by choosing $M_{\frac{7}{8}}$:

$$\mu_{\frac{7}{8}}=\frac{\delta_{\frac{3}{8}}+1}{\delta_{\frac{3}{8}}+2}$$

$$\delta_{\frac{3}{8}}=\frac{\frac{3\lambda}{8}-x\lambda-\frac{b}{8}-fb}{\frac{3b}{16}-\frac{3\lambda}{16}+\frac{x\lambda}{2}-\frac{xb}{2}}$$

In the numerator the negative signs in front of the fine and the bonus are counter intuitive since $M_{\frac{7}{8}}$ is a less risky message compared to $M_{\frac{5}{8}}$. Increasing the punishment of exaggeration or likelihood of investigation should increase $V_{\frac{3}{8}}$'s incentive to play a safer message, $M_{\frac{7}{8}}$ in this case. To be able to explain this we first need to calculate the lower bound restriction of this mixed strategy equilibrium by setting $\delta_{\frac{3}{8}}$ equal to zero:

$$\frac{3\lambda}{8}-x\lambda-\frac{b}{8}-fb=0$$

$$x=\frac{3}{8}-\frac{b+8fb}{8\lambda}$$

Every value above this value of x leads to a negative outcome in both the numerator and denominator in the formula for $\delta_{\frac{3}{8}}$. Consequently, increasing the fine or the bonus corresponds with an increase in the probability that $\mathrm{V}_{\frac{3}{8}}$ exaggerates by choosing $\mathrm{M}_{\frac{7}{8}}$, this is in line with the intuition provided before.

By setting $\delta_{\frac{3}{8}}$ equal to one we obtain the upper bound restriction of this equilibrium.

$$1 = \frac{\frac{3\lambda}{8} - x\lambda - \frac{b}{8} - fb}{\frac{3b}{16} - \frac{3\lambda}{16} + \frac{x\lambda}{2} - \frac{xb}{2}}$$

$$\frac{3\lambda}{8} - x\lambda - \frac{b}{8} - fb = \frac{3b}{16} - \frac{3\lambda}{16} + \frac{x\lambda}{2} - \frac{xb}{2}$$

$$x = \frac{\frac{9\lambda}{8} - \frac{5b}{8\lambda} - 2fb}{3\lambda - b}$$

This mixed strategy equilibrium holds when the value of x lies between:

$$\frac{3}{8} - \frac{b + 8fb}{8\lambda} \leq x \leq \frac{\frac{9\lambda}{8} - \frac{5b}{8} - 2fb}{3\lambda - b}$$

In case the lower bound restriction takes on a higher or similar value compared to the upper bound restriction this mixed strategy equilibrium does not exist. Consequently, the following restrictions occur:

$$\frac{3}{8} - \frac{b + 8fb}{8\lambda} < \frac{\frac{9\lambda}{8} - \frac{5b}{8} - 2fb}{3\lambda - b}$$

$$\frac{b}{\lambda} < 1 \text{ and } b \neq 0$$

Whether or not this equilibrium holds does not depend on the value of the fine. Furthermore, the value of the bonus is not allowed to exceed the value of λ.

All strategies in this equilibrium are explained above, however, $V_{\frac{1}{8}}$ might include $M_{\frac{5}{8}}$ in his strategy while $V_{\frac{3}{8}}$ is mixing. Before moving on to the next equilibrium we first check when $V_{\frac{1}{8}}$ includes $M_{\frac{5}{8}}$ in its strategy. The upper bound restriction for $V_{\frac{1}{8}}$ is represented by:

$$-\left(1 - q_{\frac{3}{8}}\right)\left(\frac{3}{8} - \left(\frac{1}{8} + x\right)\right)^2 - q_{\frac{3}{8}}(f + (-x)^2) > -\left(1 - q_{\frac{5}{8}}\right)\left(\frac{5}{8} - \left(\frac{1}{8} + x\right)\right)^2 - q_{\frac{5}{8}}(f + (-x)^2)$$

$$x < \frac{3}{8}$$

We therefore check whether the upper bound restriction of this mix lies below or above $\frac{3}{8}$:

$$\frac{\frac{9\lambda}{8} - \frac{5b}{8} - 2fb}{3\lambda - b} > \frac{3}{8}$$

$$f < -\frac{1}{8}$$

It is easy to see $V_{\frac{1}{8}}$ does not deviate to $M_{\frac{5}{8}}$ during this equilibrium, the figure on the next page shows this graphically:

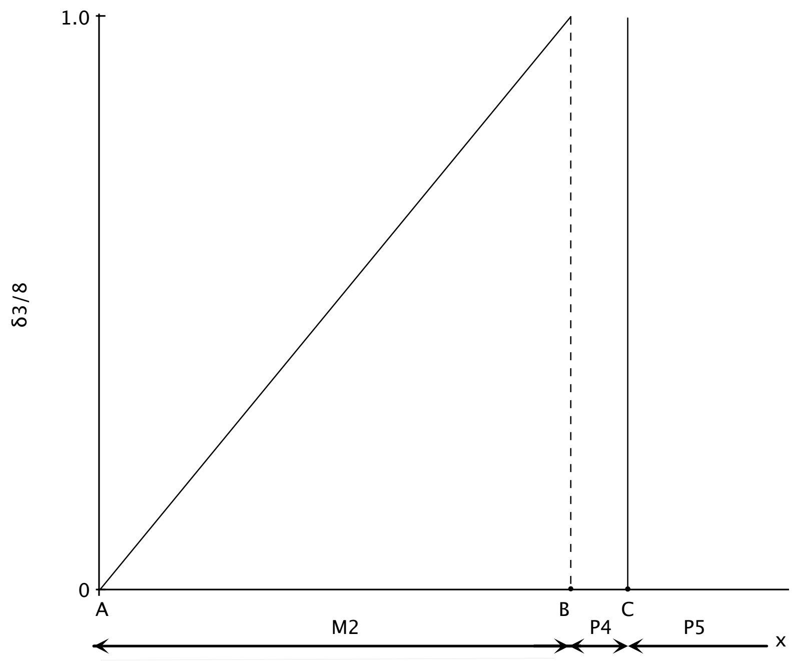


Figure 5.

Point A represents the lower bound restriction of mixed strategy equilibrium 2.

Point B represents the upper bound restriction of mixed strategy equilibrium 2.

Point C represents the value of x where $V_{\frac{1}{8}}$ is indifferent between $M_{\frac{3}{8}}$ and $M_{\frac{5}{8}}$.

This outcome guides us to the next pure strategy equilibrium 4, where $V_{\frac{3}{8}}$ plays $M_{\frac{7}{8}}$ purely and $V_{\frac{1}{8}}$ continues to play $M_{\frac{3}{8}}$.

Pure 4: The pure strategy in which $V_{\frac{1}{8}}$ chooses $M_{\frac{3}{8}}$ and $V_{\frac{3}{8}}$, $V_{\frac{5}{8}}$ and $V_{\frac{7}{8}}$ choose $M_{\frac{7}{8}}$, and therefore $\mu'_{\frac{3}{8}} = 1$ and $\mu'_{\frac{7}{8}} = \frac{2}{3}$. The auditor's best responses are $q_{\frac{3}{8}}=\frac{b}{\lambda}$ and $q_{\frac{7}{8}}=\frac{2b}{3\lambda}$.

The lower bound restriction of this equilibrium occurs when $V_{\frac{3}{8}}$ wants to include $M_{\frac{5}{8}}$ in its strategy:

$$-\left(1-q_{\frac{5}{8}}\right)\left(\frac{2}{8}-x\right)^2 - q_{\frac{5}{8}}(f+(-x)^2) < -\left(1-q_{\frac{7}{8}}\right)\left(\frac{4}{8}-x\right)^2 - q_{\frac{7}{8}}(f+(-x)^2)$$

$$-\frac{1}{16}+\frac{1}{16}q_{\frac{5}{8}}+\frac{1}{2}x-\frac{1}{2}xq_{\frac{5}{8}}-q_{\frac{5}{8}}f < -\frac{1}{4}+\frac{1}{4}q_{\frac{7}{8}}+x-xq_{\frac{7}{8}}-q_{\frac{7}{8}}f$$

$$\frac{\frac{3}{8}+\left(\frac{1}{8}-2f\right)q_{\frac{5}{8}}+\left(\frac{4f}{3}-\frac{1}{3}\right)\frac{b}{\lambda}}{1+q_{\frac{5}{8}}-\frac{4b}{3\lambda}} < x$$

No type of firm includes $M_{\frac{5}{8}}$ in its strategy, as a result $\mu'_{\frac{5}{8}}$ is not defined in this equilibrium and is therefore an out of equilibrium believe. In case the auditor expects $M_{\frac{5}{8}}$ is only chosen by either $V_{\frac{3}{8}}$ or $V_{\frac{1}{8}}$ the expected value of $\mu_{\frac{5}{8}}$ equals one, this would result in a lower bound restriction which is equal to $\frac{\frac{9\lambda}{8}-\frac{5b}{8\lambda}-2fb}{3\lambda-b}$. This outcome is similar to the upper bound restriction of the previous mixed strategy equilibrium 2.

The upper bound restriction occurs when $V_{\frac{1}{8}}$ chooses to include $M_{\frac{5}{8}}$ in its strategy:

$$-\left(1-q_{\frac{3}{8}}\right)\left(\frac{3}{8}-\left(\frac{1}{8}+x\right)\right)^2-q_{\frac{3}{8}}(f+(-x)^2)>-\left(1-q_{\frac{5}{8}}\right)\left(\frac{5}{8}-\left(\frac{1}{8}+x\right)\right)^2-q_{\frac{5}{8}}(f+(-x)^2)$$

$$-\frac{1}{16}+\frac{1}{2}x+\frac{1}{16}q_{\frac{3}{8}}-\frac{1}{2}q_{\frac{3}{8}}x-q_{\frac{3}{8}}f>-\frac{1}{4}+x+\frac{1}{4}q_{\frac{5}{8}}-xq_{\frac{5}{8}}-q_{\frac{5}{8}}f$$

$$x<\frac{\frac{3}{8}+\left(2f-\frac{1}{2}\right)q_{\frac{5}{8}}+\left(\frac{1}{8}-2f\right)\frac{b}{\lambda}}{1+\frac{b}{\lambda}-2q_{\frac{5}{8}}}$$

As before $q_{\frac{5}{8}}$ is not defined in this equilibrium. When the auditor expects $M_{\frac{5}{8}}$ is only chosen by either $V_{\frac{3}{8}}$ or $V_{\frac{1}{8}}$ the expected value $\mu_{\frac{5}{8}}$ equals one, this would result in an upper bound restriction which is equal to $\frac{3}{8}$.

This pure strategy equilibrium holds when the value of x lies between:

$$\frac{\frac{3}{8}+\left(\frac{1}{8}-2f\right)q_{\frac{5}{8}}+\left(\frac{4f}{3}-\frac{1}{3}\right)\frac{b}{\lambda}}{1+q_{\frac{5}{8}}-\frac{4b}{3\lambda}}<x<\frac{\frac{3}{8}+\left(2f-\frac{1}{2}\right)q_{\frac{5}{8}}+\left(\frac{1}{8}-2f\right)\frac{b}{\lambda}}{1+\frac{b}{\lambda}-2q_{\frac{5}{8}}}$$

In case the bonus is equal to zero this equilibrium does not hold.

Pure 5: The pure strategy equilibrium in which $V_{\frac{1}{8}}$ chooses $M_{\frac{5}{8}}$ furthermore $V_{\frac{3}{8}}$, $V_{\frac{5}{8}}$ and $V_{\frac{7}{8}}$ all choose $M_{\frac{7}{8}}$, and therefore $\mu'_{\frac{5}{8}}=1$ and $\mu'_{\frac{7}{8}}=\frac{2}{3}$. The auditor's best responses are $q_{\frac{5}{8}}=\frac{b}{\lambda}$ and $q_{\frac{7}{8}}=\frac{2b}{3\lambda}$.

The lower bound restriction of this equilibrium occurs when $V_{\frac{1}{8}}$ wants to include $M_{\frac{3}{8}}$ in its strategy.

$$-\left(1-q_{\frac{3}{8}}\right)\left(\frac{2}{8}-x\right)^2-q_{\frac{3}{8}}(f+(-x)^2)<-\left(1-q_{\frac{5}{8}}\right)\left(\frac{4}{8}-x\right)^2-q_{\frac{5}{8}}(f+(-x)^2)$$

$$x > \frac{\frac{3}{8}+\left(\frac{1}{8}-2f\right)q_{\frac{3}{8}}-\left(\frac{1}{2}-2f\right)\frac{b}{\lambda}}{1+q_{\frac{3}{8}}-\frac{2b}{\lambda}}$$

No type of firm includes $M_{\frac{3}{8}}$ in its strategy, as a result $\mu'_{\frac{3}{8}}$ is not defined in this equilibrium and is therefore an out of equilibrium believe. In case the auditor expects $M_{\frac{3}{8}}$ is only chosen by $V_{\frac{1}{8}}$ the expected value of $\mu_{\frac{3}{8}}$ equals one. This would result in a lower bound restriction equal to $\frac{3}{8}$, which is equal to the upper bound restriction in pure strategy equilibrium 4.

The upper bound restriction occurs when $V_{\frac{1}{8}}$ wants to include $M_{\frac{7}{8}}$ in its strategy:

$$-\left(1-q_{\frac{5}{8}}\right)\left(\frac{4}{8}-x\right)^2-q_{\frac{5}{8}}f-q_{\frac{5}{8}}(-x)^2 > -\left(1-q_{\frac{7}{8}}\right)\left(\frac{6}{8}-x\right)^2-q_{\frac{7}{8}}f-q_{\frac{7}{8}}(-x)^2$$

$$\frac{5}{8}-\frac{1b}{4\lambda}-\frac{2fb}{3\lambda} > x$$

This pure strategy equilibrium holds when the value of x lies between:

$$\frac{\frac{3}{8}+\left(\frac{1}{8}-2f\right)q_{\frac{3}{8}}-\left(\frac{1}{2}-2f\right)\frac{b}{\lambda}}{1+q_{\frac{3}{8}}-\frac{2b}{\lambda}} < x < \frac{5}{8}-\frac{1b}{4\lambda}-\frac{2fb}{3\lambda}$$

This pure strategy equilibrium even holds when the bonus is equal to zero. A high bonus and fine decrease the range of x, making this equilibrium less likely to occur. In case the upper bound restriction is violated we end up in the next mixed strategy equilibrium.

Mix 3: This mixed strategy equilibrium occurs when $V_{\frac{1}{8}}$ no longer wants play $M_{\frac{5}{8}}$ purely.

$V_{\frac{1}{8}}$ mixes between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$.

$V_{\frac{3}{8}}$ chooses $M_{\frac{7}{8}}$.

$V_{\frac{5}{8}}$ chooses $M_{\frac{7}{8}}$.

$V_{\frac{7}{8}}$ never mixes and always plays $M_{\frac{7}{8}}$.

Since $\mu'_{\frac{5}{8}} = 1$ the auditor's best response is $q_{\frac{5}{8}}=\frac{b}{\lambda}$.

As before, we first calculate the strategy of the auditor and the corresponding strategy of $V_{\frac{1}{8}}$. Next we calculate with what probability $V_{\frac{1}{8}}$ exaggerates by choosing $M_{\frac{7}{8}}$. After, we calculate the lower and upper bound restrictions of this mixed strategy equilibrium. Finally we discuss when this equilibrium holds.

$V_{\frac{1}{8}}$ plays this mixed strategy equilibrium when its utility from sending either $M_{\frac{5}{8}}$ or $M_{\frac{7}{8}}$ is equal to each other. The expression for $q_{\frac{7}{8}}$ is as follows:

$$-\left(1-q_{\frac{5}{8}}\right)\left(\frac{4}{8}-x\right)^2-q_{\frac{5}{8}}(f+(-x)^2)=-\left(1-q_{\frac{7}{8}}\right)\left(\frac{6}{8}-x\right)^2-q_{\frac{7}{8}}(f+(-x)^2)$$

$$q_{\frac{7}{8}}=\frac{\frac{5}{16}-\frac{x}{2}+\frac{b}{4\lambda}-\frac{bx}{\lambda}-\frac{fb}{\lambda}}{\frac{9}{16}-\frac{3x}{2}-f}$$

By replacing the expression for $q_{\frac{7}{8}}$ in equation (1) we obtain the firm's strategy:

$$\mu'_{\frac{7}{8}}=\frac{\frac{5\lambda}{16}-\frac{x\lambda}{2}+\frac{b}{4}-bx-fb}{\frac{9b}{16}-\frac{3xb}{2}-fb}$$

Next we calculate the likelihood that $V_{\frac{1}{8}}$ exaggerates by choosing $M_{\frac{7}{8}}$:

$$\mu_{\frac{7}{8}}=\frac{\delta_{\frac{1}{8}}+2}{\delta_{\frac{1}{8}}+3}$$

$$\delta_{\frac{1}{8}}=\frac{\frac{15\lambda}{16}-\frac{3}{2}x\lambda-\frac{6b}{16}-bf}{\frac{5b}{16}-\frac{xb}{2}-\frac{5\lambda}{16}+\frac{x\lambda}{2}}$$

Similar to the probability of exaggeration in mixed strategy equilibrium 2 the signs in front of the bonus and the fine in the numerator are negative, which is counterintuitive. To explain this we first calculate the lower bound restriction of this mixed strategy equilibrium by setting $\delta_{\frac{1}{8}}$ equal to zero:

$$\frac{5}{8}-\frac{1b}{4\lambda}-\frac{2bf}{3\lambda}=x$$

Every value of x above the lower bound restriction results in a negative outcome in both the numerator and denominator in the formula for $\delta_{\frac{1}{8}}$. Consequently, increasing the fine or the bonus results in an increase of the probability that $V_{\frac{1}{8}}$ exaggerates by choosing $M_{\frac{7}{8}}$. $M_{\frac{7}{8}}$ is less risky compared to $M_{\frac{5}{8}}$ because the $\mu_{\frac{7}{8}}$ contains a lower value compared to $\mu_{\frac{5}{8}}$. $V_{\frac{1}{8}}$'s incentive to choose $M_{\frac{7}{8}}$ should indeed increase for an increase in the bonus or the fine. This lower bound restriction on x is equal to the upper bound restriction of the previous pure strategy equilibrium 5.

By setting $\delta_{\frac{1}{8}}$ equal to one we obtain the upper bound restriction of this equilibrium:

$$\frac{20\lambda-11b-16bf}{32\lambda-8b}=x$$

This mixed strategy equilibrium holds when the value of x lies between:

$$\frac{5}{8}-\frac{1b}{4\lambda}-\frac{2bf}{3\lambda}\leq x\leq\frac{20\lambda-11b-16bf}{32\lambda-8b}$$

In case the lower bound restriction takes on a higher or similar value compared to the upper bound restriction this equilibrium does not hold. Consequently, the following restrictions occur:

$$\frac{5}{8}-\frac{1b}{4\lambda}-\frac{2bf}{3\lambda}<\frac{20\lambda-11b-16bf}{32\lambda-8b}$$

$$\frac{b}{\lambda}<1 \text{ and } b\neq 0$$

Whether or not this equilibrium holds does not depend on the value of the fine. Furthermore, the value of the bonus is not allowed to exceed the value of λ. These findings are similar to our findings in mixed strategy equilibrium 2.

Pure 6: Pure strategy equilibrium in which every type of firm chooses $M_{\frac{7}{8}}$, and therefore $\mu'_{\frac{7}{8}}=\frac{3}{4}$. The auditor's best response is $q_{\frac{7}{8}}=\frac{3b}{4\lambda}$.

The lower bound restriction of this equilibrium occurs when $V_{\frac{1}{8}}$ wants to deviate to $M_{\frac{5}{8}}$:

$$-\left(1-q_{\frac{5}{8}}\right)\left(\frac{4}{8}-x\right)^2-q_{\frac{5}{8}}f-q_{\frac{5}{8}}(-x)^2<-\left(1-q_{\frac{7}{8}}\right)\left(\frac{6}{8}-x\right)^2-q_{\frac{7}{8}}f-q_{\frac{7}{8}}(-x)^2$$

$$\frac{\frac{5}{8}+\left(\frac{1}{2}-2f\right)q_{\frac{5}{8}}+\left(\frac{3}{2}f-\frac{27}{32}\right)\frac{b}{\lambda}}{1-\frac{9b}{4\lambda}+2q_{\frac{5}{8}}}<x$$

No type of firm includes $M_{\frac{5}{8}}$ in its strategy, as a result $\mu'_{\frac{5}{8}}$ is not defined in this equilibrium and is therefore an out of equilibrium believe. When the auditor expects either $V_{\frac{1}{8}}$ or $V_{\frac{3}{8}}$ chooses $M_{\frac{5}{8}}$, the value of $\mu'_{\frac{5}{8}}$ equals one. In that case this lower bound restriction equals:

$$\frac{20\lambda-11b-16bf}{32\lambda-8b}<x$$

This outcome is equal to the expression of the upper bound restriction from the previous mixed strategy equilibrium 3. Since all firms choose $M_{\frac{7}{8}}$ as a pure strategy no upper bound restriction exists in this equilibrium.

Until now we have always reasoned each type of firm exaggerates more when its desire to exaggerate increases. However, we should also wonder whether a type of firm prefers to deviate to a safer message, since $M_{\frac{7}{8}}$ becomes more risky as x increases. In pure strategy equilibrium 6 all firm types have the same probability of getting fined and set back to truth telling. As a result they all have the same incentive to deviate. In case the auditor receives a signal containing $M_{\frac{5}{8}}$ he therefore has no idea which of the three types, $V_{\frac{5}{8}}$ $V_{\frac{3}{8}}$ or $V_{\frac{1}{8}}$, sends this message. Two of the three types of firm would exaggerate when sending this message, for that reason we assume $\mu_{\frac{5}{8}} = \frac{2}{3}$.

Below we calculate the lower bound restrictions for $V_{\frac{1}{8}}$ and $V_{\frac{5}{8}}$, using $\mu_{\frac{5}{8}} = \frac{2}{3}$.

For $V_{\frac{1}{8}}$:

$$-\left(1-q_{\frac{5}{8}}\right)\left(\frac{4}{8}-x\right)^2 - q_{\frac{5}{8}}f - q_{\frac{5}{8}}(-x)^2 < -\left(1-q_{\frac{7}{8}}\right)\left(\frac{6}{8}-x\right)^2 - q_{\frac{7}{8}}f - q_{\frac{7}{8}}(-x)^2$$

$$\frac{60\lambda-49b+16bf}{96\lambda-88b} < x$$

In case $V_{\frac{1}{8}}$ wants to deviate to a mix between $M_{\frac{7}{8}}$ and $M_{\frac{5}{8}}$ we return to mixed strategy equilibrium 3.

For $V_{\frac{5}{8}}$:

$$-(-x)^2 < -\left(1-q_{\frac{7}{8}}\right)\left(\frac{2}{8}-x\right)^2 - q_{\frac{7}{8}}f - q_{\frac{7}{8}}(-x)^2$$

$$\frac{1}{8}+\frac{6bf}{4\lambda-3b} < x$$

In case the lower bound restriction of $V_{\frac{5}{8}}$ contains a higher value compared the lower bound restriction of $V_{\frac{1}{8}}$ it is possible $V_{\frac{5}{8}}$ deviates while $V_{\frac{1}{8}}$ does not, the figure below graphically shows that situation.

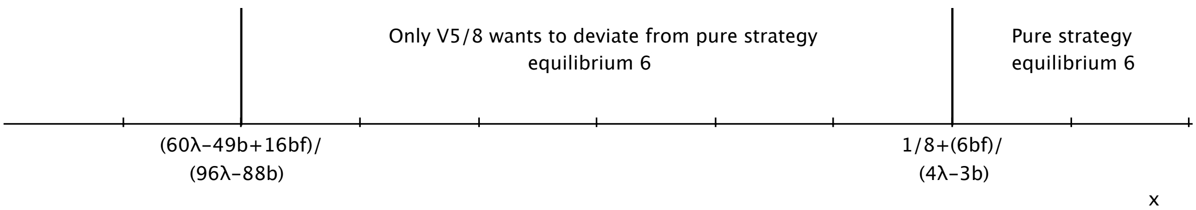


Figure 6.

$V_{\frac{5}{8}}$ is the only type of firm that deviates from pure strategy equilibrium 6 when:

$$\frac{60\lambda-49b+16bf}{96\lambda-88b} < x < \frac{1}{8}+\frac{6bf}{4\lambda-3b}$$

A value of x in this range only exists when:

$$\frac{72\lambda^2-111\lambda b+42\frac{3}{4}b^2}{192\lambda b-180b^2} < f$$

In case the bonus is equal to zero no outcome occurs, $V_{\frac{5}{8}}$ does not prefer $M_{\frac{5}{8}}$. The fine and the bonus both need to be high for this restriction to hold. A value of x within this range possibly leads to the mixed strategy equilibrium below.

Mix 4.a: This mixed strategy equilibrium occurs when $V_{\frac{5}{8}}$ wants to include $M_{\frac{5}{8}}$ in its strategy while $V_{\frac{1}{8}}$ and $V_{\frac{3}{8}}$ purely choose $M_{\frac{7}{8}}$.

$V_{\frac{1}{8}}$ chooses $M_{\frac{7}{8}}$.

$V_{\frac{3}{8}}$ chooses $M_{\frac{7}{8}}$.

$V_{\frac{5}{8}}$ mixes between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$.

$V_{\frac{7}{8}}$ never mixes and always chooses $M_{\frac{7}{8}}$.

Since $\mu'_{\frac{5}{8}} = 0$, the auditor's best response is $q_{\frac{5}{8}} = 0$.

This equilibrium only holds in case both $V_{\frac{3}{8}}$ and $V_{\frac{1}{8}}$ purely choose $M_{\frac{7}{8}}$ over every other strategy. The outcomes below show this is **<u>not</u>** the case for both types of firm, each type prefers to include $M_{\frac{5}{8}}$ in its strategy.

$V_{\frac{5}{8}}$ plays this mixed strategy in case its utility from sending either $M_{\frac{5}{8}}$ or $M_{\frac{7}{8}}$ is equal to each other, resulting in the following expression for $q_{\frac{7}{8}}$:

$$-(-x)^2 = -\left(1-q_{\frac{7}{8}}\right)\left(\frac{2}{8}-x\right)^2 - q_{\frac{7}{8}}f - q_{\frac{7}{8}}(-x)^2$$

$$q_{\frac{7}{8}} = \frac{x-\frac{1}{8}}{x-\frac{1}{8}+2f}$$

By replacing the expression for $q_{\frac{7}{8}}$ in equation (1) we obtain $V_{\frac{5}{8}}$'s strategy:

$$\mu'_{\frac{7}{8}} = \frac{\lambda\left(x-\frac{1}{8}\right)}{b\left(x-\frac{1}{8}+2f\right)}$$

$V_{\frac{3}{8}}$ wants to include $M_{\frac{5}{8}}$ in its strategy when:

$$-\left(1-q_{\frac{5}{8}}\right)\left(\frac{2}{8}-x\right)^2 - q_{\frac{5}{8}}f - q_{\frac{5}{8}}(-x)^2 \nless -\left(1-q_{\frac{7}{8}}\right)\left(\frac{4}{8}-x\right)^2 - q_{\frac{7}{8}}f - q_{\frac{7}{8}}(-x)^2$$

$$-\frac{1}{16}+\frac{1}{2}x \not< -\frac{1}{4}+x+\frac{\left(\frac{1}{4}-x-f\right)\left(x-\frac{1}{8}\right)}{x-\frac{1}{8}-f}$$

$$\frac{1}{128}+\frac{1}{4}f \not< \frac{1}{8}x-\frac{1}{2}x^2$$

This outcome provides us with a few insights. First, in case the fine is nonzero $V_{\frac{3}{8}}$ prefers $M_{\frac{5}{8}}$ in its strategy. Second, in case the fine is equal to zero and x equals $\frac{1}{8}$, $V_{\frac{3}{8}}$ is indifferent between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$. No value for exists x where $V_{\frac{3}{8}}$ strictly prefers $M_{\frac{7}{8}}$.

<u>This result already tells us the mixed equilibrium 4.a does not hold.</u>

$V_{\frac{1}{8}}$ wants to include $M_{\frac{5}{8}}$ in its strategy when:

$$-\left(1-q_{\frac{5}{8}}\right)\left(\frac{4}{8}-x\right)^2-q_{\frac{5}{8}}f-q_{\frac{5}{8}}(-x)^2 \not< -\left(1-q_{\frac{7}{8}}\right)\left(\frac{6}{8}-x\right)^2-q_{\frac{7}{8}}f-q_{\frac{7}{8}}(-x)^2$$

$$-\frac{1}{4}+x \not< -\frac{9}{16}+\frac{3}{2}x+\frac{\left(\frac{9}{16}+\frac{3}{2}x-f\right)\left(x-\frac{1}{8}\right)}{x-\frac{1}{8}-f}$$

$$\frac{1}{32}+\frac{9}{16}f \not< \frac{3}{8}x-x^2$$

The graph below shows all combinations of f and x for which $V_{\frac{1}{8}}$ prefers $M_{\frac{7}{8}}$ over $M_{\frac{5}{8}}$. This graph shows that $V_{\frac{1}{8}}$ prefers $M_{\frac{7}{8}}$ over $M_{\frac{5}{8}}$ only when x contains a value below $\frac{2}{8}$. However, below a value of x equal to $\frac{2}{8}$ $V_{\frac{1}{8}}$ prefers $M_{\frac{1}{8}}$ over $M_{\frac{5}{8}}$.

<u>This result again tells us the mixed equilibrium 4.a does not hold.</u>

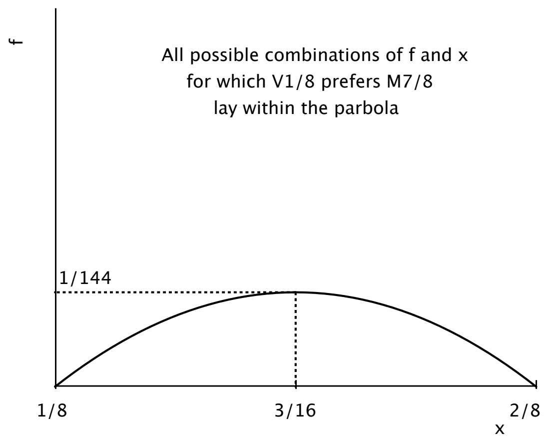


Figure 7.

In the last two equilibria we have investigated whether $V_{\frac{5}{8}}$ wants to include $M_{\frac{7}{8}}$ in its strategy while $V_{\frac{1}{8}}$ purely chooses $M_{\frac{7}{8}}$. Obviously, it is not very likely $V_{\frac{5}{8}}$ deviates from purely choosing $M_{\frac{7}{8}}$ exactly at the point where $V_{\frac{1}{8}}$ stops mixing between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$, it is more likely $V_{\frac{5}{8}}$ starts mixing between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$ while $V_{\frac{1}{8}}$ is also still mixing. In that case, $V_{\frac{5}{8}}$ would deviate from the pure strategy $M_{\frac{7}{8}}$ when x lies in the range of mixed strategy equilibrium 3, guiding us to the last mixed strategy equilibrium.

Mix 4.b: This mixed strategy equilibrium occurs when $V_{\frac{5}{8}}$ wants to include $M_{\frac{5}{8}}$ in its strategy while $V_{\frac{1}{8}}$ is mixing between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$.

$V_{\frac{1}{8}}$ mixes between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$.

$V_{\frac{3}{8}}$ chooses $M_{\frac{7}{8}}$

$V_{\frac{5}{8}}$ mixes between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$.

$V_{\frac{7}{8}}$ never mixes and always chooses $M_{\frac{7}{8}}$.

First we calculate when $V_{\frac{5}{8}}$ decides to include $M_{\frac{5}{8}}$ in its strategy while $V_{\frac{1}{8}}$ is still mixing between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$. To be able to do that we need some information from mixed strategy equilibrium 3:

$$q_{\frac{7}{8}} = \frac{\frac{5}{16}-\frac{x}{2}+\frac{b}{4\lambda}-\frac{bx}{\lambda}-\frac{fb}{\lambda}}{\frac{9}{16}-\frac{3x}{2}-f}$$

$$\mu'_{\frac{7}{8}} = \frac{\frac{5\lambda}{16}-\frac{x\lambda}{2}+\frac{b}{4}-bx-fb}{\frac{9b}{16}-\frac{3xb}{2}-fb}$$

Now we calculate below which value of x $V_{\frac{5}{8}}$ starts mixing:

$$-(-x)^2 \nless -\left(1-q_{\frac{7}{8}}\right)\left(\frac{2}{8}-x\right)^2 - q_{\frac{7}{8}}(f+(-x)^2)$$

$$x \ngtr \frac{-\left(\frac{6}{32}-\frac{3b}{16\lambda}+\frac{3fb}{16\lambda}\right)+\sqrt{D}}{\frac{b}{\lambda}-1}$$

Where:

$$D = \frac{1}{256} - \frac{2b}{256\lambda} + \frac{14fb}{32\lambda} + \frac{2fb^2}{32\lambda^2} + \frac{b^2}{256\lambda^2} + \frac{8f^2b^2}{32\lambda^2} - \frac{f}{2} + \frac{2f^2b^2}{32\lambda}$$

See Appendix A1.2 for the complete calculation.

Above this value of x $V_{\frac{5}{8}}$ does not include $M_{\frac{5}{8}}$ in its strategy. When the bonus or the fine is equal to zero, this x value equals 1/8. From pure strategy equilibrium 6 we know $V_{\frac{5}{8}}$'s decision to include $M_{\frac{5}{8}}$ in its strategy results from a desire to escape a possible fine. This is in line with our findings above. A higher fine increases the value of x in the expression above, meaning $V_{\frac{5}{8}}$'s is more likely to include $M_{\frac{5}{8}}$ in its strategy.

Next we calculate the strategies in this equilibrium, $V_{\frac{5}{8}}$ mixes between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$ when:

$$-(-x)^2 = -\left(1 - q_{\frac{7}{8}}\right)\left(\frac{2}{8} - x\right)^2 - q_{\frac{7}{8}} f - q_{\frac{7}{8}}(-x)^2$$

$$q_{\frac{7}{8}} = \frac{x - \frac{1}{8}}{x - \frac{1}{8} + 2f}$$

By replacing the expression for $q_{\frac{7}{8}}$ in equation (1) we obtain the firm's strategy:

$$\mu'_{\frac{7}{8}} = \frac{\lambda\left(\frac{1}{8} - x\right)}{b\left(\frac{1}{8} - x - 2f\right)}$$

Consequently, $V_{\frac{1}{8}}$ mixes between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$ when:

$$-\left(1 - q_{\frac{5}{8}}\right)\left(\frac{4}{8} - x\right)^2 - q_{\frac{5}{8}} f - q_{\frac{5}{8}}(-x)^2 = -\left(1 - q_{\frac{7}{8}}\right)\left(\frac{6}{8} - x\right)^2 - q_{\frac{7}{8}} f - q_{\frac{7}{8}}(-x)^2$$

$$\frac{1}{4} + x + \left(\frac{1}{4} - x - f\right) q_{\frac{5}{8}} < -\frac{9}{16} + \frac{3}{2}x + \frac{\left(\frac{9}{16} + \frac{3}{2}x - f\right)\left(x - \frac{1}{8}\right)}{x - \frac{1}{8} - f}$$

$$q_{\frac{5}{8}} = \frac{-x^2 + \frac{3}{8}x - \frac{1}{32} - \frac{1}{2}f}{-x^2 + \frac{3}{8}x - \frac{1}{32} + \frac{5}{8}f - 3xf - 2f^2}$$

When fine is equal to zero $q_{\frac{5}{8}}$ and $q_{\frac{7}{8}}$ both equal one, which means $V_{\frac{5}{8}}$ is only indifferent between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$ when the auditor will always investigate. In that case this mixed strategy equilibrium does not exists.

Lastly we calculate with what probability $V_{\frac{5}{8}}$ exaggerates, this also determines with what probability $V_{\frac{1}{8}}$ will choose $M_{\frac{5}{8}}$ or $M_{\frac{7}{8}}$. It is calculated with the help of the following formula:

$$\mu_{\frac{7}{8}} = \frac{\delta_{\frac{5}{8}} + \delta_{\frac{1}{8}}}{2 + \delta_{\frac{5}{8}} + \delta_{\frac{1}{8}}}$$

$$\delta_{\frac{5}{8}} = \frac{\delta_{\frac{1}{8}} - 2\mu_{\frac{7}{8}} - \delta_{\frac{5}{8}}\mu_{\frac{7}{8}}}{\mu_{\frac{7}{8}} - 1}$$

$$\delta_{\frac{5}{8}} = \frac{\delta_{\frac{1}{8}}\left(b\left(\frac{1}{8}-x-2f\right)-\lambda\left(\frac{1}{8}-x\right)\right)-2\lambda\left(\frac{1}{8}-x\right)}{\lambda\left(\frac{1}{8}-x\right)-b\left(\frac{1}{8}-x-2f\right)}$$

Increasing the fine and the bonus decreases the probability that $V_{\frac{5}{8}}$ exaggerates. As a result $V_{\frac{5}{8}}$ is more likely to choose $M_{\frac{5}{8}}$, making this message safer for $V_{\frac{1}{8}}$ to choose. Consequently, $V_{\frac{1}{8}}$ is more likely to choose $M_{\frac{5}{8}}$ since this message is less likely to be investigated.

Intuition

In many of the equilibria described in the previous section the firm chooses a strategy where it exaggerates. A firm can be punished when it exaggerates, punishment in the form of a fine and by setting the message back to truth telling. To be able to fine a firm, the auditor first needs to find exaggeration. The auditor has the possibility to investigate the firm's message. Whether or not he investigates depends on the ratio of the bonus to the costs incurred by investigation and the likelihood that a message is exaggerated.
Below we explain the intuition behind all equilibria in three different cases. First we discuss the case where the bonus is equal to zero, then the case where the bonus is not equal to zero, but the fine is. Finally, we discuss the case where both the fine and the bonus are not equal to zero.

The bonus is equal to zero
In case the bonus is equal to zero the auditor never examines a message, as a result the fine has no effect either. Consequently, the firm chooses the message closest to the sum of its actual performance and its desire to exaggerate. This specific case has the feature that most equilibria described before do not exist, only the pure equilibria 1, 3, 5 and 6 do hold for the values of x given below:

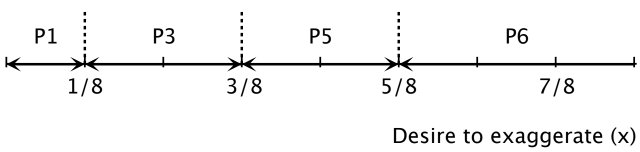


Figure 8.

In the remainder of this paper we refer to these four strategies as the basic strategies. In case the x value lies above $\frac{5}{8}$ pure equilibrium 6 occurs, where each type of firm chooses $M_{\frac{7}{8}}$ as a pure strategy. The auditor knows it is quite likely this message is exaggerated, yet he has no incentive to investigate the message.

This scenario clearly shows policy makers have no power in case the bonus for the auditor is equal to zero, the fine has no effect on its own in that case.

The fine is equal to zero

Next we provide the intuition in case the bonus is not equal to zero, but the fine is. Now the auditor has an incentive to investigate a message when expecting exaggeration. For the firm the only possible punishment is getting set back to truth telling. In this case the pure strategy equilibria 1, 3, 4, 5 and 6 and mixed-strategy equilibria 2 and 3 do hold for specific ranges of x, these ranges of x are shown in the figure below:

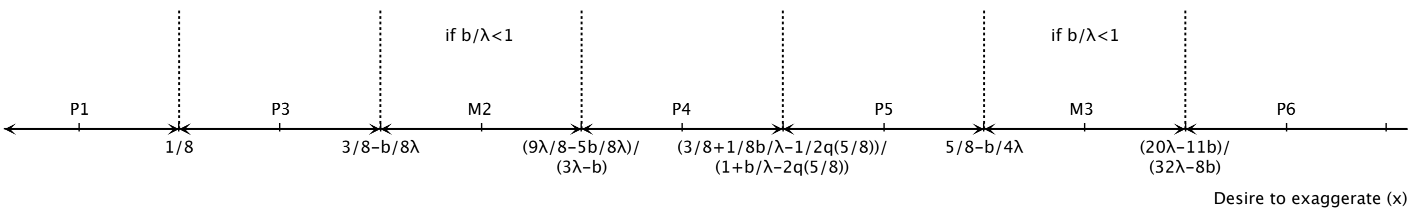


Figure 9.

This scenario shows that an increase in the likelihood that the auditor investigates a message, thus an increase of the bonus, has two effects. First it leads to more possible equilibria, and second, it decreases the likelihood that one of the basic strategies is played. These effects occur because each type of firm tries to lower the probability of getting caught exaggerating. The firm therefore chooses a strategy that corresponds with the lowest likelihood of exaggerating, given its desire to exaggerate. Therefore the firm's incentive to choose a less risky strategy increases. The fine is equal to zero, as a result $V_{\frac{5}{8}}$ fully exaggerates in case x lies above $\frac{1}{8}$, partly for this reason mixed strategy equilibria 1 and 4 and pure-strategy equilibrium 2 do not occur.

Both the fine and the bonus are nonzero.

In the last case both the fine and the bonus are not equal to zero, and therefore all equilibria described above could hold. The auditor has an incentive to investigate since the bonus is nonzero. Getting caught exaggerating is least attractive for the firm in this scenario, consequently it gets fined and is set back to truth telling. Adding the fine results in several extra possible equilibria that could hold compared to the previous case, these are shown below:

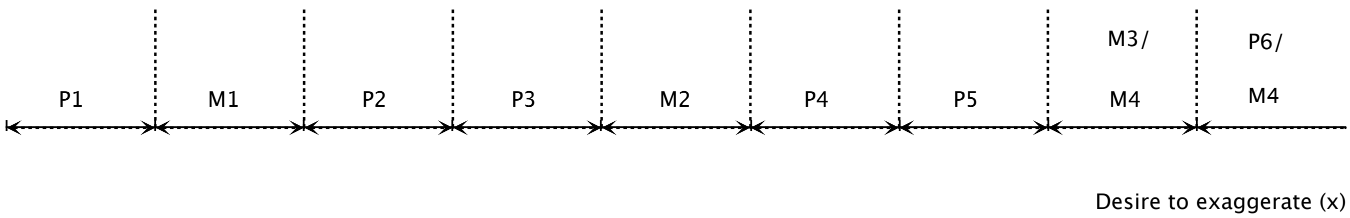


Figure 10.

We did not include the upper and lower bound restriction in this figure as it would compromise its clarity. The effect of the fine always depends on the ratio of the bonus to the costs incurred by investigation, a small value of this ratio implies a small influence of the fine.

As before, each type of firm exaggerates more when increasing its desire to do so, however, in this scenario the firm is more conscious about which strategy it chooses. A high fine and bonus corresponds with a high incentive for each type of firm to escape getting caught exaggerating. Consequently, the firm's incentive to play a strategy including a relatively safe message or messages is high. This high incentive is shown in two different ways. Firstly each type of firm is more likely to mix with truth telling. This is shown in our model by an increase of the upper bound restriction in mixed strategy equilibrium 1. Aside from that, each type of firm decreases the probability it exaggerates in the first mixed strategy equilibrium. Second, it is more likely the firm chooses $M_{\frac{7}{8}}$, since our model shows a decrease of the upper bound restrictions for deviating to $M_{\frac{7}{8}}$ when increasing the fine and the bonus. In pure strategy equilibrium 3 and 5 and mixed strategy equilibrium 2 and 3, $M_{\frac{7}{8}}$ is the upper bound deviation strategy. In all those cases $M_{\frac{7}{8}}$ is less risky compared to the equilibrium strategy since in the equilibria μ'_m equals one. $V_{\frac{7}{8}}$ always plays $M_{\frac{7}{8}}$, and therefore $M_{\frac{7}{8}}$ is relatively safer since $\mu'_{\frac{7}{8}}$ is always lower than one. Moreover, the lower bound restriction for $V_{\frac{1}{8}}$ in pure-strategy equilibrium 6 decreases just like the lower bound restrictions in mixed strategy equilibria 2 and 3. In those cases the firm with the dominant lower bound restriction chooses a riskier strategy when deviating from the equilibrium strategy, which contains $M_{\frac{7}{8}}$. A decrease of the lower bound restriction reduces the likelihood that the firm deviates from the equilibrium strategy, which includes $M_{\frac{7}{8}}$.

Besides the decrease of the lower bound restriction in pure strategy equilibrium 6, this equilibrium has another feature that is important to discuss. In this equilibrium choosing $M_{\frac{7}{8}}$ is quite risky since the auditor knows he finds exaggeration with a probability of 0,75 when examining a message. When the fine and the bonus are high enough $V_{\frac{5}{8}}$ could decide to include $M_{\frac{5}{8}}$ in its strategy, and therefore increase its probability of truth telling and decrease the risk of getting fined. $V_{\frac{5}{8}}$ could decide to include $M_{\frac{5}{8}}$ in its strategy when $V_{\frac{1}{8}}$ starts to play $M_{\frac{7}{8}}$ as a pure strategy, in that case $V_{\frac{1}{8}}$ revises its decision and includes $M_{\frac{5}{8}}$ again in its strategy. It is more likely $V_{\frac{5}{8}}$ includes $M_{\frac{5}{8}}$ in its strategy while $V_{\frac{1}{8}}$ is still mixing between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$, thus during mixed strategy equilibrium 3. In both cases $V_{\frac{1}{8}}$ and $V_{\frac{5}{8}}$ mix between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$ at the same time.

Concluding remarks

The objective of this paper is to derive all possible equilibria in our model. In these equilibria the strategy of the auditor depends on the bonus he gets when he finds exaggeration, the probability that the firm exaggerates and the costs the auditor incurs when investigating. A high bonus and high a probability that the firm exaggerates increases the incentive for the auditor to investigate. High costs for the auditor to investigate have the opposite effect. In equilibrium the strategy of the firm depends on the fine, its actual performance, its desire to exaggerate and the best response of the auditor. In case the firm's desire to exaggerate is zero the equilibrium in the model is a separating equilibrium, each type of firm chooses the message closest to its actual performance. When increasing the firm's desire to exaggerate we find semi-pooling equilibria and mixed strategy equilibria. In case the firm's desire to exaggerate is very high we end up in a pooling equilibrium where all firm types choose the same message. If the fine is high and the auditor has a high incentive to investigate a message, the firm is more focussed on escaping this high fine. The firm is therefore more likely to choose a safe strategy, where μ_m is low.

In this paper we assumed a society with four types of firm and four different possible performances. In reality, each type of firm could have a unique performance. To make this model more in line with reality, future research could extend it by adding more types of firm and corresponding messages. Furthermore, we assumed all firm types have the same desire to exaggerate, which needs to be examined. In case empirical research suggests this assumption is incorrect, our model could be improved by assigning different levels of desire to exaggerate to different types of firm. Finally, we also assumed a constant fine, this fine does not depend on the level of the exaggeration. In reality a large difference between the signal and the actual performance results in a high value of the fine.

Since financial statements play an important role in current financial markets it is important incorrect information is filtered out. Incorrect information in a financial statement can seriously harm the financial system. One of the main problems is the fact that various investment and capital injection decisions are based on a firm's performance stated in a financial statement. This paper advises investors to be critical when judging a financial statement before investing in a firm or a project. Lets assume firms have a high desire to exaggerate about their performance. In that case when the bonus for the auditor and fine for exaggeration are rather high and the costs for the auditor to investigate are relatively low financial statements are still likely to be correct. However, in case the bonus and fine are low and the costs for the auditor to investigate are relatively high, financial statements are likely to be incorrect. In that case the auditor isn’t likely to investigate. When the bonus is equal to zero a financial statement has no meaning. In that case the performance in the financial statement purely depends on the desire of the firm to exaggerate.

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Appendix

A1.1) Below we state all upper and lower bound restrictions in every pure strategy equilibrium for each type of firm, the blue restrictions are dominant and used in the main part of the paper.

The upper and lower bound restrictions of the second pure strategy equilibrium.
Pure 2 holds for $V_{\frac{1}{8}}$ when the value of x lies between:

$$\frac{1}{8} + \frac{2bf}{2\lambda - b} < x < \frac{\frac{3}{8} + \left(2f - \frac{1}{2}\right) q_{\frac{5}{8}} + \left(\frac{1}{16} - f\right)\frac{b}{\lambda}}{1 + \frac{b}{2\lambda} - 2q_{\frac{5}{8}}}$$

Pure 2 holds for $V_{\frac{3}{8}}$ when the value of x lies below:

$$x < \frac{1}{8} + \frac{2fq_{\frac{5}{8}}}{1 - q_{\frac{5}{8}}}$$

Pure 2 holds for $V_{\frac{5}{8}}$ when the value of x lies above:

$$x > \frac{1}{8} + \frac{2bf}{2\lambda - b}$$

The upper and lower bound restrictions of the third pure strategy equilibrium.
Pure 3 holds for $V_{\frac{1}{8}}$ when the value of x lies between:

$$\left(\frac{1}{8} + \frac{2*b*f}{\lambda - b}\right) < x < \frac{3}{8}$$

Pure 3 holds for $V_{\frac{3}{8}}$ when the value of x lies between:

$$\left(\frac{1}{8} + \frac{2*b*f}{\lambda - b}\right) < x < \frac{3}{8} - \frac{b + 8bf}{16\lambda}$$

Pure 3 holds for $V_{\frac{5}{8}}$ when the value of x lies above:

$$x > \left(\frac{1}{8} + \frac{2b*f}{2\lambda - b}\right)$$

The upper and lower bound restrictions of the fourth pure strategy equilibrium.
Pure 4 holds for $V_{\frac{1}{8}}$ when the value of x lies between:

$$\left(\frac{1}{8} + \frac{2*b*f}{\lambda - b}\right) < x < \frac{\frac{3}{8} + \left(2f - \frac{1}{2}\right) q_{\frac{5}{8}} + \left(\frac{1}{8} - 2f\right)\frac{b}{\lambda}}{1 + \frac{b}{\lambda} - 2q_{\frac{5}{8}}}$$

Pure 4 holds for $V_{\frac{3}{8}}$ when the value of x lies above:

$$x > \frac{\frac{3}{8} + \left(\frac{1}{8} - 2f\right) q_{\frac{5}{8}} + \left(\frac{4f}{3} - \frac{1}{3}\right)\frac{b}{\lambda}}{1 + q_{\frac{5}{8}} - \frac{4b}{3\lambda}}$$

Pure 4 holds for $V_{\frac{5}{8}}$ when the value of x lies above:

$$x > \frac{1}{8} + \frac{4bf}{3\lambda - 2b}$$

The upper and lower bound restrictions of the fifth pure strategy equilibrium.
Pure 5 holds for $V_{\frac{1}{8}}$ when the value of x lies between:

$$\frac{\frac{3}{8}+\left(\frac{1}{8}-2f\right)q_{\frac{3}{8}}-\left(\frac{1}{2}-2f\right)*\frac{b}{\lambda}}{1+q_{\frac{3}{8}}-\frac{2b}{\lambda}} < x < \frac{5}{8} - \frac{1b}{4\lambda} - \frac{2fb}{3\lambda}$$

Pure 5 holds for $V_{\frac{3}{8}}$ when the value of x lies above:

$$x > \frac{\frac{9}{8}-\frac{5b}{8\lambda}-\frac{2fb}{\lambda}}{3-\frac{b}{\lambda}}$$

Pure 5 holds for $V_{\frac{5}{8}}$ when the value of x lies above:

$$x > \frac{1}{8} + \frac{4bf}{3\lambda - 2b}$$

The lower bound restrictions of the sixth pure strategy equilibrium.
Pure 5 holds for $V_{\frac{1}{8}}$ when the value of x lies above:

$$x > \frac{\frac{5}{8}+\left(\frac{1}{2}-2f\right)q_{\frac{5}{8}}+\left(\frac{3}{2}f-\frac{27}{32}\right)*\frac{b}{\lambda}}{1-\frac{9b}{4\lambda}+2q_{\frac{5}{8}}}$$

Pure 5 holds for $V_{\frac{3}{8}}$ when the value of x lies above:

$$x > \frac{\frac{3}{8}+\left(\frac{1}{8}-2f\right)q_{\frac{5}{8}}+\left(\frac{6}{4}-\frac{3}{8}\right)*\frac{b}{\lambda}}{1+q_{\frac{5}{8}}-\frac{3b}{2\lambda}}$$

Pure 5 holds for $V_{\frac{5}{8}}$ when the value of x lies above:

$$x > \frac{1}{8} + \frac{6bf}{4\lambda - 3b}$$

<u>A1.2) When does $V_{\frac{5}{8}}$ include $M_{\frac{5}{8}}$ in its strategy during mixed strategy equilibrium 3:</u>

$$x^2 \nless -\left(1-q_{\frac{7}{8}}\right)\left(\frac{2}{8}-x\right)^2 - q_{\frac{7}{8}}\left(f+x^2\right)$$

$V_{\frac{5}{8}}$ continues to play $M_{\frac{7}{8}}$when:

$$\frac{\frac{1}{16}-\frac{x}{2}}{\frac{1}{16}-\frac{x}{2}-f} < \frac{\frac{5}{16}-\frac{x}{2}+\frac{b}{4\lambda}-\frac{bx}{\lambda}-\frac{fb}{\lambda}}{\frac{9}{16}-\frac{3x}{2}-f}$$

$$\left(\frac{1}{2}-\frac{b}{2\lambda}\right)x^2 + \left(\frac{3b}{16\lambda}-\frac{6}{32}-\frac{3bf}{2\lambda}\right)x + \frac{4}{256} + \frac{f}{4} - \frac{b}{64\lambda} + \frac{5bf}{16\lambda} - \frac{bf^2}{\lambda} < 0$$

Divide by minus 1:

$$\left(\frac{b}{2\lambda}-\frac{1}{2}\right)x^2-\left(\frac{3b}{16\lambda}-\frac{6}{32}-\frac{3bf}{2\lambda}\right)x-\frac{4}{256}-\frac{f}{4}+\frac{b}{64\lambda}-\frac{5bf}{16\lambda}+\frac{bf^2}{\lambda}>0$$

Use the ABC-Formula to find x:

$$(1)\ \frac{-\left(\frac{6}{32}-\frac{3b}{16\lambda}+\frac{3fb}{16\lambda}\right)+\sqrt{D}}{\frac{b}{\lambda}-1}\ \text{ or } (2)\ \frac{-\left(\frac{6}{32}-\frac{3b}{16\lambda}+\frac{3fb}{16\lambda}\right)-\sqrt{D}}{\frac{b}{\lambda}-1}$$

The discriminant:

$$D=B^2-4AC$$

$$D=\left(\frac{3b}{16\lambda}-\frac{6}{32}-\frac{3bf}{2\lambda}\right)^2-\left(\frac{2b}{\lambda}-2\right)\left(-\frac{4}{256}-\frac{f}{4}+\frac{b}{64\lambda}-\frac{5bf}{16\lambda}+\frac{bf^2}{\lambda}\right)$$

$$D=\frac{1}{256}-\frac{2b}{256\lambda}+\frac{14fb}{32\lambda}+\frac{2fb^2}{32\lambda^2}+\frac{b^2}{256\lambda^2}+\frac{8f^2b^2}{32\lambda^2}-\frac{f}{2}+\frac{2f^2b^2}{32\lambda}$$

In case the bonus equals zero, $V_{\frac{5}{8}}$ is indifferent between $M_{\frac{5}{8}}$ and $M_{\frac{7}{8}}$ when $x=\frac{1}{8}$. We know this from pure strategy equilibrium 1. This outcome only appears when using formula (1):

$$x \ngtr \frac{-\left(\frac{6}{32}-\frac{3b}{16\lambda}+\frac{3fb}{16\lambda}\right)+\sqrt{D}}{\frac{b}{\lambda}-1}$$