The Effect of Bonus Caps on Workers' Productivity

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Abstract

This research examined the effect of bonus caps on workers' productivity, and focused on how low-ability and high-ability workers are affected. Therefore, I extended the model of Lazear (1989) with two different regulations of a bonus cap and compared them to the situation without a bonus cap. In addition, I investigated whether it is beneficial for firms, to increase the workers' fixed salaries under a binding and non-binding bonus cap restriction. Finally, I studied whether bonus caps are efficient in terms of social welfare. In all cases, I considered both a binding and non-binding limited liability constraint. The results indicate that bonus caps indeed affect workers' productivity, and lead to lower optimal levels of effort and workers' utilities. However, a high-ability worker always exerts more effort in the optimum than a low-ability worker, when the bonus is capped at an average maximum percentage of the aggregate fixed salary. Also under such a bonus cap, it is not optimal for the firm to reward different bonus height levels under both a binding and non-binding limited liability constraint. Then, the profit will not be maximized and inequality arises as shown by the exchange mechanism. Besides, it is more beneficial for the firm to increase the fixed salaries under bonus caps, when the price level and the workers' abilities are higher, and the bonus cap percentage is lower. Finally, bonus caps can distort workers' contracts and can negatively affect social welfare.

Keywords: *bonus caps, worker's productivity, incentives, performance pay, reward schemes, inequality, social welfare*

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Content

1. Introduction

An important topic in the theory of personnel economics is whether and how the workers' productivity is affected by incentives and rewards (Lazear, 2000). Organisations constantly strive for optimizing workers' resources with the use of performance management (Lewis, 1998). By using performance-related pay and reward schemes, organisations induce employees to increase their effort to produce more output. There is a rich literature that shows how these compensation schemes should be designed, concerning different working environments. For instance, Lazear (1986) uses those theories on compensation schemes, to determine the choice of payment by input (salaries) over payment by output (piece rates). In the financial sector, such compensation schemes including high bonuses are used to induce bankers to improve their performance (Bénabou & Tirole, 2015). However, since the financial crisis that started in 2007, restrictions on bankers' compensation schemes have been an important subject for discussion. As a result, the European Union introduced a new guideline for a bonus cap in 2013. It caps the bankers' bonuses at a maximum percentage of their fixed salaries. The existing literature leaves large gaps on how the employee's performance is influenced by such a bonus cap. For example, Thanassoulis (2012) just focuses on how a bonus cap in the labour market of bankers alters the risk of banks, caused by competition between banks. My research contributes to the field of reward schemes and workers' productivity. It provides a theoretical analysis considering the new restriction on compensation schemes, namely bonus caps, which has not been analysed in such a way regarding employee's productivity.

For many years, there have been lots of discussions going on about the bankers' pay in the financial sector. It is a given that bonus payments in the financial sector are often extraordinary high (Lalkens, 2015). Many economists worldwide see these high rewards and bonuses as a contributing factor for the financial crisis that started in 2007 (Murphy & Jensen, 2011). For that reason, a guideline was introduced by the European Union in Brussels in 2013 and came into force one year later. It implies, that the bankers' bonuses are capped at a maximum of 100 percent of the fixed salary (Jonker & Bökkerink, 2015). Many countries argued that the percentage of the bonus cap was too high. For instance, the countries Belgium, Denmark, Germany and Finland have fixed the bonus cap at a maximum of 50 percent of the fixed salary. In the more extreme case, the Netherlands has fixed the bonus cap at a maximum of only 20 percent of the fixed salary. This has resulted in a lot of dissatisfaction among Dutch bankers, who opine that the bonuses given in foreign countries are unfair with respect to competition

(Financieel Dagblad, 2014). In reaction to this, different Dutch banks increased the fixed salaries of their top employees, to lower the dissatisfaction that top bankers have about the new guideline (De Horde, 2015).

Even though these top bankers worry about their bonus height, it seems that the fixed bonus cap of 20 percent will not be such a big problem for their reward heights (Hordijk, 2014). Hence, this percentage will not be a fraction of the fixed individual salary, but a fraction of the aggregate fixed salary of the entire organisation (Schlingmann & Schutte (2013), Ministerie van Financiën (2014), Van Tuyll van Serooskerken (2015), Bonjer & Op het Veld (2014)). This gives freedom to firms, for rewarding some types of employees a higher fraction and some a lower fraction bonus pay. In fact, this means that it is possible for a top employee to obtain the entire aggregate bonus a bank can give to all employees. Since the height of the aggregate bonus decreases due to the bonus cap regulation, it could be the case that only top workers will be rewarded. Employees, who are not in the top management of a bank, might become less motivated after the introduction of the bonus cap (Huffman & Bognanno, 2014). Banks might even sink the bonuses for these lower-level employees. It could also be the other way around, that the bank cuts the bonuses at the top to increase the bonuses in the middle. When looking at how banks compensate for a bonus cap in real life, it follows that they only increase the fixed salaries of their higher-level workers or only give extra shares to higher-level workers ((Bökkerink & Kooiman (2014), Bonjer & Op het Veld (2014), De Horde (2015), Bökkerink (2015), Financieel Dagblad (2015)). In addition, it seems that lower-level workers will not be compensated or will not even obtain a reward anymore (Bökkerink & Kooiman, 2014).

The dissatisfaction among lower-level employees indeed appeared, as a result of the banks that only compensated their top employees. It seems that the regulation of a bonus cap has led to division and anger among lower-level employees within banks, because they are not covered by the new guideline (Bökkerink & Kooiman, 2014). The lower-level employees namely deem, that they work the same hours and exert the same level of effort as higher-level employees do (Bökkerink & Kooiman, 2014). Therefore, the introduction of the bonus cap might have brought a new economic problem into existence within the firm. The division between lower-level and higher-level employees might become bigger and bigger, and will probably affect the workers' performance. Lower-level employees might exert less effort, because they will not receive a bonus anymore after the introduction of the bonus cap.

Bénabou and Tirole (2015) propose a resolution to the problem of such high bonus payments, affecting pay levels and differentials. They analyse the effect of labour market competition on the structure of the firm's compensation scheme. Bénabou and Tirole especially look at the interaction between the competition for the highest productive workers and the firm's incentive scheme, to undermine work ethics. That is, whether workers are averse to bad behaviour, such as ripping off customers or shareholders (Bénabou & Tirole, 2015). Actually, the idea is that highly competitive markets make it hard for employers, to find an optimal balance between the benefits and costs of high reward incentives for employees. Such a competition for talent results in a 'bonus culture', which leads to efficiency losses and distorted decisions. They also take in account, that market returns to measured and talent performance are both reflected by pay levels and differentials. Considering the political regulation of a bonus cap, they show that it can help employers to find a balance in incentives. It can effectively limit the ability of firms to 'steal' each other's high-skilled workers. Decreasing the worker's bonus until the bonus cap causes a Pareto improvement, for which the firm's profit is used to increase the workers' fixed salaries as compensation. On the other hand, bonus caps could also cause some distortion, when firms consecutively substitute the bonus decrease for inefficient transfers. That is, firms may substitute performance bonuses for other types of rewards, which depend on the worker's type but that are less efficient regarding the screening of workers. However, Bénabou and Tirole (2015) do not consider different forms of bonus caps, how they are determined and what they consist of. This research fills that gap and provides a theoretical model, to determine whether and how different bonus caps affect workers' productivity. Therefore, the research question is: *"What is the effect of bonus caps on workers' productivity?"*

The objective of this research is to investigate the effect of bonus caps on workers' productivity, by expanding the general theoretical model of Lazear (1989). This research provides a specific analysis on how low-ability (lower-level) employees and high-ability (higher-level) employees are differently affected by the introduction of bonus caps. To research the effects, this research focuses on four different cases. The first case includes a normal situation, wherein no bonus cap is present and in which the organisation is not restricted to the determination of the aggregate bonus height. In the second case, the model is expanded with the introduction of a bonus cap. In this situation, the bonus is capped at an average maximum percentage of the aggregate fixed salary. It is therefore possible that only a higher-level employee will receive a bonus. Also in this case, I consider both a binding and non-binding limited liability constraint. The third case also includes a bonus cap, but it differs from the second case in that each worker can now only obtain a maximum percentage of the individual fixed salary as a bonus. Based on the results of the first three cases, the fourth case considers whether it is beneficial for a firm to increase the workers' fixed salaries. Finally, I also investigate if bonus caps are efficient regarding social welfare. In all cases, I consider both a binding and non-binding limited liability constraint.

The findings show that bonus caps indeed have an impact on workers' productivity. They lead to lower levels of effort and workers' utilities in the optimal situation. However, a high-ability worker always exerts more effort in the optimum than a low-ability worker, when the bonus is capped at an average maximum percentage of the aggregate fixed salary. This holds for both situations wherein the bonus cap restriction is binding and non-binding. When the bonus is capped individually, the worker's ability does not affect the optimal effort level. Furthermore, when the bonus is capped at an average maximum percentage of the aggregate fixed salary, it is not optimal for the firm to reward different bonus height levels. In that case, the firm does not maximize his profit and it would lead to inequality as follows from the exchange mechanism. Moreover, firms are more willing to increase the fixed salaries under the different bonus cap regulations, when the bonus cap percentage is lower, and the price level and the workers' ability levels are higher. When the bonus cap restriction is non-binding, it is not beneficial for firms to increase the fixed salaries. In addition, a new kind of competition might have come into existence, since bonus payments have become lower and there is no regulation cap on the fixed salary in addition to the bonus cap, or on the total salary of the workers. Finally, bonus caps can distort workers' contracts and can have a negative impact on social welfare.

Different economic papers that study performance-related pay and reward schemes are considered to answer the research question. These papers will be discussed in the next section that deals with the related literature. Afterwards, the basic theoretical model of Lazear (1989) is elaborated in the first case. In the analysis, the model will be expanded with four different cases. By analysing the results of the five different cases, the research question is answered in the conclusion. Also in this section, any possible shortcomings and recommendations for future research will be discussed.

2. Related Literature

There is rich related literature that discusses the mechanisms of performance-related pay and reward schemes. First of all, many papers study the competition for a bonus between employees. In the existing literature, such a competition is known as the “tournament theory”. Literature on the tournament theory provides useful information on reward schemes, although the model in this research leaves out tournament incentives. The basis of the tournament theory is reflected in a theoretical paper of Lazear and Rosen (1981). They study a different form of incentive pay than piece rates, namely contest and prizes. The main difference between these two forms is that incentive payment by contest and prizes depends on the rank order of employees within an organisation, whereas pay by piece rates is based on the individual performance. One could see in the situation of a bonus cap, that such reward schemes also depend on what level employees are within the firm. Lower-level employees need to be higher in the rank of the firm to receive a higher bonus, since it seems that only higher-level employees are rewarded after the introduction of the bonus cap (Bökkerink & Kooiman, 2014). Lazear and Rosen (1981) analyse the different compensation methods and determine different parameter values that maximize employees’ utility.

Lazear and Rosen (1981) find that under certain conditions in a competitive economy, reward schemes based on the individual relative position within a firm are preferred above reward schemes based on the absolute output. They also mention that it might be less costly to observe the relative position than to determine the individual’s performance, when performance is hard to measure. In this case, the economic effect is actually the same for both. For that reason, some employees would obtain wages that are much higher than their presumed marginal product. This is actually what lower-level workers are complaining about regarding the bonus cap regulation, because they opine that they work the same hours and exert the same effort as higher-level workers do (Bökkerink & Kooiman, 2014). The last important result considers the case, wherein heterogeneity is possible among workers within a firm. It then seems that rewarding workers based on their relative position does not lead to efficiency, because low-quality workers will try to affect an organisation that consists mostly of high-quality workers. That is, low-quality workers enter the firm and will produce low-quality products whereas the firm mostly consists of high-quality workers.

Nalebuff and Stiglitz (1983) expand the theoretical work of Lazear and Rosen (1981) and study the competitive effect of compensation schemes, which are based on relative performance. They take in account the problem of observing workers' effort, because it is not costless and hard to observe input. Therefore, they search for a reward structure that avoids the problem of observing effort. However, the model in this research assumes that effort is observable and contractible for the firm. An employee will therefore receive a fixed payment and an incentive reward depending on the exerted effort. An important finding by Nalebuff and Stiglitz (1983) is that rewarding employees based on their relative performance, causes the "losses" to become continuous, because workers will become unmotivated if they do not get a reward. So they stop with competing their co-workers and will never obtain a reward anymore. In addition to this, Nalebuff and Stiglitz (1983) investigate in another paper what effects competitive compensation schemes have on the market itself. Green and Stokey (1983) also expand the work of Lazear and Rosen (1981). They investigate reward structures based on relative performance and compare them with a principal-agent model, which consists of individual contracts. Furthermore, they add a shock term to the level of effort. Their main goal is to make a comparison between the efficiency of independent contracts and tournaments. Their results indicate that it is better to use individual contracts rather than using an optimal tournament, if the common shock is absent.

Finally, Rosen (1986) studies the incentive properties of prizes in elimination tournaments, wherein prizes become bigger each round. Especially, there is a great emphasize on the rewards given in the top ranks of a firm. Just as with the problem of the bonus cap, the higher-level workers receive more than proportionate shares of compensation. This leads to a greater division between lower-level and higher-level workers. So, Rosen explains why the top rank of a firm gets such high rewards. According to Rosen (1986), this must motivate lower-level workers to compete with other workers, by exerting more effort to become promoted. Murphy (1984) explains that these top-level managers also needed to come up through the ranks in a competitive environment from the beginning, before they earned a promotion.

After all these previous papers designed different theories for reward schemes and tournaments, the emphasis becomes more on the effects of incentive contracts. Lazear (1989) investigates the different forms of incentive contracts more specifically, which therefore provides an important basis for the model in this research. Lazear (1989) shows different forms of incentive contracts and describes them in detail. He concludes that markets overcome incentive problems by using different forms of performance payment, for different competitive environments. Finally, it seems that incentive contracts are not always the first best. My research elaborates the model of Lazear (1989) and expands the model with two different situations of a bonus cap. In another paper, Lazear (1986) studies the fact why workers are sometimes paid by piece rates and sometimes paid by salaries. For both compensation schemes, there are different arguments why to choose for piece rates or salaries. If the costs of monitoring are low, it is better to choose for a payment by piece rates. However, it is better to use a payment by salaries, when measuring output is hard. Lazear looks more specifically at these issues in his paper. He provides an analysis on when to choose for payment by input (salaries) or payment by output (piece rates), and its' consequences.

Lazear (1986) finds that firms are more likely to use piece rates instead of salaries, when; measuring costs of output are low, the output is measured with less error, employees differ in ability, and the wage of the outside option is high relative to the average output at the current firm. Furthermore, low-ability workers are always in the salary firms and high-ability workers are always in the piece rate firms. Though, lower-level workers with a high-ability are willing to bear the monitoring costs, which makes it possible for a firm to separate low-ability workers from high-ability workers. He also finds that employees become unmotivated by exerting less effort, when the rate of next period depends on the output of this period. One could think that the introduction of bonus caps could also lead to unmotivated workers. Someone gets a bonus in a certain period and in the period after that, the reward is smaller or is even omitted. Lazear argues that it is always possible to avoid such a problem, with the use of an optimal reward scheme in the previous periods. Later in an empirical study, Lazear (2000) finds evidence that piece rate pay compensation increases the workers' effort and thus, the output in a firm.

Due to the introduction of a bonus cap, the division between lower-level and higher-level employees might become bigger. Actually, it is very hard to let employees feel that they are fairly paid (Tremblay, St-Onge, & Toulouse, 1997). Earlier evidence found by Dornstein (1991) namely shows that lower-level employees compare their salaries with higher-level employees. This causes inequality, which might also be present after the introduction of a bonus cap (Bökkerink & Kooiman, 2014). For that reason, Cowherd and Levine (1992) research the effect of interclass pay equity on product quality, measured by 102 corporate business units. The theory indicates that product quality is high, when the pay differential between lower-level and higher-level workers is small. By comparing the payment and inputs per hour per worker, and that for lower-level and higher-level workers (interclass pay equity), Cowherd and Levine find that pay equity has a positive relationship with product quality under both forms of measuring. Their results also indicate that a high wage differential between lower-level and higher-level employees lowers product quality, because lower-level employees will exert less effort. Other papers also show, that big wage differentials between lower-level and higher-level workers affect the performance of lower-level workers.

Over the years, researchers also started focusing on the intrinsic motivation instead of extrinsic motivation. In a theoretical model, Bénabou and Tirole (2003) show that the effect of performance pay also depends on psychological variables. Even in the absence of these variables, the influence of performance pay on workers' effort could be greater. Therefore in an empirical paper, Huffman and Bognanno (2014) use a psychological experimental design to implement in a real work setting with paid workers. They investigate the effect of performance pay on the non-monetary motivations of an employee, which is known as the intrinsic motivation from psychology. Although the model in this research leaves out intrinsic motivation, Huffman and Bognanno show a quite interesting finding. They use a control and treatment group for working at a street festival. Only the treatment group obtained a performance pay, but this was only temporary. At the end, they find evidence that the treatment group had a higher output than the control group. However, after the bonus was dropped out, the output of the treatment group was lower than the output of the control group. Huffman and Bognanno thus show, that the removal of the bonus in this period causes this period's intrinsic motivation to depend on the reward in the previous period.

Overall, it is important to determine how the allocation of rewards should be in an aggregated pay system. The introduction of a bonus cap causes a shift from individual rewards toward an aggregated pay system, wherein relative performance is important for the determination of rewards. In their paper, Barber and Simmering (2002) study this issue of allocating rewards. The main question is (Barber & Simmering, 2002): "Should these aggregate rewards be allocated in an equal way to all workers within the firm? Or should the individual performance be used as the determinant for allocation?" They predict the reactions of employees for different ways of allocation. For instance, an equal pay system leaves out individual performance and also causes unequal fairness among workers. Overall, it is a consideration between advantages and disadvantages that indicates what reward scheme should be implemented, like in the situation of a bonus cap.

That bonus caps may have positive and negative effects, also follows from the paper of Bénabou and Tirole (2015). They investigate the effect of labour market competition on how the compensation scheme is structured, by including screening and multitasking in the Hotelling model. Bénabou and Tirole therefore emphasize the interaction between the competition for the highest productive workers and the firm's incentive scheme, to undermine work ethics. That is, whether employees behave neatly instead of behaving badly, such as ripping off customers or shareholders (Bénabou & Tirole, 2015). The thought behind this is that highly competitive markets make it hard for employers, to find an optimal balance between the benefits and costs of high reward incentives for employees. This results in a 'bonus culture', which is inefficient and leads to distorted decisions. They also take in account that high pay levels and differentials are mainly caused by market returns to measured and talent performance. The findings of Bénabou and Tirole (2015) indicate that bonus caps can help employers to find a balance in incentives. It could effectively limit the ability of firms to 'steal' each other's high-skilled workers, leading to a Pareto improvement when the worker's bonus is decreased until the bonus cap. Besides that, remaining profits are used to increase the fixed salaries of workers as compensation, leading to a new kind of competition among firms. However, bonus caps could also result in distortion when firms substitute performance bonuses for other types of rewards, which depend on the worker's type but are less efficient regarding the screening of workers.

Chou and Chen (2015) also investigate the role of bonus caps within the organisation. They analyse the influence of bonus caps on agents, regarding their expected utility. Furthermore, they determine how the structure of incentive schemes is affected. They use a moral-hazard model and extend it with limited liability. In their model, a principal employs an agent whose outside options are not relevant due to limited liability. Furthermore, they assume that the agent's effort is unverifiable. Therefore, the principal is only able to reward the agent by observing the agent's output level. Whereas the purpose of bonus caps is to discipline bankers, Chou and Chen (2015) show a contradicting result. Namely, agents could be better off with a pay cap. For that reason, bonus caps lead to lower incentives for the agent to exert effort, when rewards for the most deserving outcomes are decreased. When the principal also prefers the same level of effort, rewards for other outcomes that also deserve some incentives but less than the most deserving outcomes, should be increased as long as the bonus cap is satisfied. This leads to a higher expected payment by the principal, because a higher bonus cap restriction causes a less effective rent extraction (Chou & Chen, 2015). This will consecutively cause an increase in the risk-neutral agent's expected payoff. However, Chou and Chen (2015) also show that the agent's expected utility under a fixed level of effort might decrease, when the bonus cap becomes more strict. This is the case if the assumption does not hold, that the marginal product of effort is higher than the marginal costs of exerting effort (a binary choice in the model). If the bonus cap decreases to some level below the point at where the firm prefers workers to exert no effort, it leads to a worker's expected utility of zero.

There are also many empirical papers that study relative performance payments and pay inequalities. Bandiera et al. (2005) use personnel data, to determine whether employees adjust their working behaviour on their co-workers. They provide evidence, that the productivity of an average worker is at least 50 percent higher under a reward scheme with piece rates, compared to a reward scheme based on relative performance. This is an interesting finding, because this research considers a reward scheme with piece rates. Bandiera et al. (2005) also show that under relative incentives, the standard for exerting effort is lower. This is especially the case, when working with friends. It seems that working under a reward scheme based on relative performance, lowers the effort in a company and certainly for the lower-level workers who suffer from a wage differential.

Frick et al. (2003) focus on these pay inequalities and team performance. They use a dataset from the North American team sports industry, to determine the effect of wage differentials within a team on team performance. The results are quite different, which implies that a higher pay differential could both have a positive and negative effect on team performance. This actually depends on the environment, such as; the production process, the size of the team and cooperation. However, the role of prize differentials in tournaments is very determining. For instance, Harbring and Irlenbusch (2003) provide evidence that the design of a rank order tournament is quite important. The introduction of a bonus cap causes a lower proportion of rewards, but the variability of effort is expected to be higher (Harbring & Irlenbusch, 2003).

3. The Theoretical Model

3.1 General

The model, that will be described in the next paragraph, is based on Lazear (1989) and can be applied in different working environments. As already mentioned, an example for an appropriate situation is the rewarding of employees in the banking sector. In this sector, the bonus cap regulation has come into force in 2014. It implies that a bonus is capped at an average/individual maximum percentage of the aggregate/individual fixed salary. However, the regulation of a bonus cap could also be implemented in other working environments. Think of industrial firms, that reward their employees based on piece rates. By exerting more effort, they could obtain a higher bonus. Under the restriction of a bonus cap, there would be a maximum bonus they can earn. Thus at some point, exerting more effort does not lead to a higher reward. Just to give a good insight in how the model in this research can be applied in a real work setting, the situation of the banking sector is used as an example.

The model describes the working behaviour of two employees, who both work for the same bank. The bank always strives for the maximization of profits. In the model, I assume that there is a low-ability (lower-level) employee and high-ability (higher-level) employee, who both work for this bank. Since both jobs require different abilities, they may result in different fixed wages. The bank has implemented an incentive contract, wherein both workers are able to obtain a bonus per unit besides their fixed salaries. I also assume that the reward per unit produced, may also differ between both working levels.

Case 1 considers a normal situation without the regulation of a bonus cap. I assume, that the effort exerted by a worker is observable and contractible. An employee can therefore receive a fixed payment and an incentive reward, depending on the exerted effort. The analysis in chapter 4 expands the model with the following four cases. In case 2, a bonus cap regulation is implemented under a binding and non-binding limited liability constraint. The bonus is now capped at an average maximum percentage of the aggregate fixed salary. A worker is thus able to obtain the total reward a firm can give to all his employees. Case 3 includes the regulation of a bonus cap with an individual maximum percentage of the worker's individual fixed salary. In case 4, I study whether the firm wants to increase the workers' fixed salaries. Finally, I study the impact on social welfare caused by the introduction of a bonus cap in case 5. I investigate all cases by considering a binding and non-binding limited liability constraint.

3.2 Case 1: No bonus cap

The first case considers just a normal situation without a bonus cap, which lasts for only one period. In this case, there are two employees working for a firm. For simplicity, these employees are called worker A and B. Each worker exerts some effort and I assume, that each worker maximizes his expected utility. There are two types of workers, who both have a different ability (a_i): a low-ability worker (L) and a high-ability worker (H). In this model, both the firm and employees know the worker's ability. Workers exert effort as to maximize their utility. Their utility level consists of the worker's expected income function ($E(W_i)$) and the worker's cost function ($C(e_i)$), for exerting effort.

The cost function of worker A:

$$C(e_A) = \frac{1}{2}\theta e_A^2$$

The cost function of worker B:

$$C(e_B) = \frac{1}{2}\theta e_B^2,$$

where e_i stands for effort, exerted by the worker. Here, I assume that $e_i \geq 0$. Furthermore, θ is a fixed parameter that reflects the cost of effort and which is the same for both employees. The worker's expected income function $E(W_i)$ and the worker's cost function $C(e_i)$ are now added together, which represents their utility functions.

The utility function of worker A:

$$U_A = E(W_A) - \frac{1}{2}\theta e_A^2$$

The utility function of worker B:

$$U_B = E(W_B) - \frac{1}{2}\theta e_B^2$$

Both workers obtain a certain utility U_0, when working for an outside option. Therefore, the participation constraints for worker A and B must hold for staying in the same firm:

$$E(W_A) - \tfrac{1}{2}\theta e_A^2 \geq U_0 \text{ and } E(W_B) - \tfrac{1}{2}\theta e_B^2 \geq U_0$$

Assumed that these two conditions hold, workers A and B exert effort to produce output for the same firm. The output that each worker produces, also depends on his or her ability. A basic specification is used to represent the production of output for both workers.

The output of worker A:

$$q_A = a_A e_A$$

The output of Worker B:

$$q_B = a_B e_B$$

The total output of both workers:

$$Q = q_A + q_B,$$

where e_i stands for effort and a_i for the ability of the employee. I assume that there is no luck present at both workers' production functions. Besides, the equations show that the production generated by worker A and B depends on their ability ($a_i \in \{\mathrm{L}, H\}$) and the level of effort they exert. The firm uses an incentive contract with piece rates, to induce workers to exert more effort. It follows from the production functions that a higher level of effort will lead to a higher output per worker. As a result, the expected wage of both workers is $E(W_i)$. This expected wage consists of a fixed wage and a variable bonus wage. A bonus is given for every unit that is produced, which thus depends on the amount of effort a worker exerts. The total bonus will now be determined, using the total exerted effort of each worker.

The expected income function of worker A:

$$E(W_A) = \alpha_A + \beta_A q_a$$

The expected income function of worker B:

$$E(W_B) = \alpha_B + \beta_B q_B,$$

where α_i is the fixed wage part and where β_i is the bonus per unit q_i. The model allows for different alpha's and different beta's, because there is a lower-level and higher-level job that both require a different ability level. The production functions of both workers will now be substituted into the expected income functions.

The expected income function of worker A after substitution:

$$E(W_A) = \alpha_A + \beta_A(a_A e_A)$$

The expected income function of worker B after substitution:

$$E(W_B) = \alpha_B + \beta_B(a_B e_B)$$

Based on previous functions, it is now possible to compute the final utility functions for both employees. It stands out that the utility functions of worker A and B depend on a fixed salary and a variable bonus wage minus the total costs of effort.

The utility function of worker A:

$$U_A = \alpha_A + \beta_A(a_A e_A) - \frac{1}{2}\theta e_A^2$$

The utility function of worker B:

$$U_B = \alpha_B + \beta_B(a_B e_B) - \frac{1}{2}\theta e_B^2$$

In case 1, there is no bonus cap present. This means that workers are not restricted by a regulation and that there is no maximum bonus to obtain. Since worker A and B both work for the same firm, they also contribute to the same firm's profit. The firm always strives to maximizing his profit and therefore sets optimal levels of β_i.

The profit function of the firm:

$$\pi = PQ - E(W_{A+B}),$$

where P is the price level, Q the total output of worker A and B, and $E(W_{A+B})$ are the total expected wage costs for the firm. The firm's profit function is rewritten, using substitution.

The profit function of the firm after substitution:

$$\pi = P(a_A e_A + a_B e_B) - \alpha_A - \beta_A(a_A e_A) - \alpha_B - \beta_B(a_B e_B))$$

As follows from the profit function, the profit that the firm obtains from both workers depends on; the effort that each worker exerts, the workers' abilities, the firm's price level, the workers' individual fixed salaries and the bonus heights per unit.

3.2.1 Workers' effort choice under a binding and non-binding limited liability constraint

The optimal level of effort for each worker will now be determined, by maximizing each worker's utility function with respect to e_i.

The utility function of worker A is maximized with respect to e_A (see Appendix A, p.66):

$$U_A = \alpha_A + \beta_A(a_A e_A) - \frac{1}{2}\theta e_A^2$$

$$\frac{dU_A}{de_A} = \beta_A a_A - \theta e_A = 0$$

So, worker A exerts effort until the marginal cost of effort (θe_A) is equal to the marginal benefit per unit and the worker's ability ($\beta_A a_A$).

This results in the optimal level of effort for worker A (see Appendix A, p.66):

$$e_A^* = \frac{\beta_A a_A}{\theta}$$

The utility function of worker B is maximized with respect to e_B (see Appendix B, p.66):

$$U_B = \alpha_B + \beta_B(a_B e_B) - \frac{1}{2}\theta e_B^2$$

$$\frac{dU_B}{de_B} = \beta_B a_B - \theta e_B = 0$$

It follows that worker B also exerts effort until the marginal cost of effort (θe_B) is equal to the marginal benefit per unit and the worker's ability ($\beta_B a_B$).

This results in the optimal level of effort for worker B (see Appendix B, p.66):

$$e_B^* = \frac{\beta_B a_B}{\theta}$$

When analysing and comparing the optimal effort levels of both workers, it follows that the optimal levels of effort depend on the height of the bonus per unit (β_i), the ability of the employee (a_i) and the cost of effort (θ). An increase in β_i, leads to an increase in the optimal level of effort e_i^*. This β_i could be different for both employees and therefore could cause a difference between the workers' optimal levels of effort. Besides that, the effect of an increase in β_i on the optimal level of effort also depends on the cost of effort θ. However, it is assumed that the parameter θ is equal for both workers. So, a difference between the workers' optimal levels of effort will not be affected by θ. Finally, a higher ability a_i leads to a higher optimal level of effort. I assume that two types of ability are present in this model. For that reason, the difference in abilities leads to different optimal levels of effort between the two workers, since effort is positively affected by the worker's ability level. This means that a high-ability worker exerts more effort than a low-ability worker. By contrast, if both workers have the same ability level and reward per unit, they will exert the same level of effort as to maximize their utility. The maximized individual utilities of both workers will now be calculated, by substituting the optimal levels of effort into the individual utility functions.

The utility of worker A (see Appendix C, p.67):

$$U_A^* = \alpha_A + \frac{1}{2}\frac{\beta_A^2 a_A^2}{\theta}$$

The utility of worker B (see Appendix D, p.67):

$$U_B^* = \alpha_B + \frac{1}{2}\frac{\beta_B^2 a_B^2}{\theta}$$

3.2.2 Profit maximization under a binding limited liability constraint

First, I consider a situation under a limited liability constraint ($\alpha_i > 0$), wherein the minimum fixed salary is always positive. For the determination of the firm's expected profit, the optimal levels of effort for worker A and B will be substituted into the firm's profit function.

The total expected profit of the firm realized by the two workers (see Appendix E, p.68):

$$\pi = \frac{P a_A^2 \beta_A}{\theta} + \frac{P a_B^2 \beta_B}{\theta} - \alpha_A - \alpha_B - \frac{a_A^2 \beta_A^2}{\theta} - \frac{a_B^2 \beta_B^2}{\theta}$$

As already mentioned, the firm always strives to maximize his profit. By maximizing the expected profit function of the firm with respect to α_i and β_i, it allows the firm to determine what α´s and β´s must be set to maximize profit. However, due to a binding limited liability constraint ($\alpha_i > 0$), the optimal α_i is always positive. The fixed wages are determined by the firm and therefore, it is not allowed to set negative fixed salaries. This means that a fixed wage results in utility for both employees and that the firm always sets a positive fixed wage.

However, effort is observable and contractible, which allows the firm to write a contract for a fixed salary conditional on some effort level. So the firm needs to reward his workers based on their performance and sets an optimal level of β_i. To calculate the optimal bonus height level, the firm's profit function will be maximized with respect to β_i.

For worker A, the firm's profit function is maximized with respect to β_A (see Appendix F, p.68):

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} - \frac{2a_A^2\beta_A}{\theta} = 0$$

It follows that the variables α_A and β_A are variables that are controlled by the firm, because it is the firm that determines the fixed salary and reward per unit. The first term in the equation includes the effect of an increase in β_A on e_A^* ($\frac{de_A^*}{d\beta_A}$). An increase in β_A namely increases π, via e_A^*. Furthermore, an increase in the incentive β_A increases the firm's profit with price level P. This is quite logical, since the level of effort increases in the incentive height. This results in a higher production level and thus generates a higher revenue. Finally, the firm's profit also increases with a_A^2. Namely, an increase in the bonus height results in a higher effort level and thus a higher production level. Since both the effort and production level depend on the worker's ability, an increase in β_A has a higher effect on the firm's profit for a worker with a higher ability. The second term shows that an increase in β_A leads to higher wage costs. The reason for this, is that firm needs to pay a higher bonus due to a higher effort and production level generated by the worker. The derivative will now be solved for β_A.

This results in the optimal bonus height level of worker A (see Appendix F, p.68):

$$\beta_A^* = \frac{1}{2}P$$

For worker B, the profit function is maximized with respect to β_B (see Appendix G, p.69):

$$\frac{d\pi}{d\beta_B} = \frac{Pa_B^2}{\theta} - \frac{2a_B^2\beta_B}{\theta} = 0$$

The derivative shows that α_B and β_B are again variables that are controlled by the firm, since it is the firm that controls the fixed salary and reward per unit. The first term in the equation also includes the effect of an increase in β_B on e_B^* ($\frac{de_B^*}{d\beta_B}$). An increase in β_B namely increases π, via

e_B^*. Moreover, an increase in the incentive β_B increases the firm's profit with price level P. This is quite obvious, because the level of effort increases in the incentive height per unit. This consecutively results in a higher production level and thus results in a higher revenue. Finally, the firm's profit also increases with a_B^2. Since an increase in the bonus height results in a higher effort level, the production level also increases. Due to the fact that both the effort and production level depend on the worker's ability, an increase in β_B has a higher effect on the firm's profit for a worker with a higher ability. The second term indicates that an increase in β_B results in higher wage costs. Namely, the firm must pay a higher bonus due to a higher effort and production level. It is now possible to solve the derivative for β_B.

The results in the optimal bonus height level of worker B (see Appendix G, p.69):

$$\beta_B^* = \frac{1}{2}P$$

When analysing the bonus heights per unit of both workers, it is optimal for the firm to set the bonus heights at the same level for both ability levels. This must be half of the marginal product for both workers. Finally, the optimal levels of effort will be determined, by substituting the optimal bonus height levels into the optimal levels of effort.

The optimal effort level of worker A:

$$e_A^* = \frac{Pa_A}{2\theta}$$

The optimal effort level of worker B:

$$e_B^* = \frac{Pa_B}{2\theta}$$

3.2.3 Profit maximization under a non-binding limited liability constraint

Previous results are different with a non-binding limited liability constraint. Then, the participation constraints should be taken in account and firms need to offer a certain level of fixed salary, which is required to attract the employee for all wage schemes. The participation constraints for worker A and B are satisfied, which implies that the utility of working for this firm must be larger than or equal to the utility of working for an outside option.

$$\alpha_i + \beta_i(a_i e_i) - \frac{1}{2}\theta e_i^2 \geq U_0$$

This participation constraint must be rewritten to α_i:

$$\alpha_i \geq U_0 - \beta_i(a_i e_i) + \frac{1}{2}\theta e_i^2$$

It thus follows from the participation constraint how the firm must set the workers' fixed salaries, so that both employees want to work for this firm. Therefore, the incentive heights of both workers are the only variables controlled by the firm. To calculate these, the participation constraints and optimal levels of effort will be substituted into the firm's profit function and maximized with respect to β_A and β_B and subject to the participation constraints. This results in the optimal incentive heights (see Appendix H, p.69):

$$\beta_A^* = P$$

$$\beta_B^* = P$$

So when the limited liability constraint is non-binding, the firm needs to set the same level of bonus heights for both working-levels. This must be the full marginal product for each worker. Finally, the optimal levels of effort will be calculated by substituting the optimal bonus height levels into the optimal levels of effort.

The optimal effort level of worker A:

$$e_A^* = \frac{Pa_A}{\theta}$$

The optimal effort level of worker B:

$$e_B^* = \frac{Pa_B}{\theta}$$

At the end, this results in the following workers' fixed salaries, determined by the participation constraints:

$$\alpha_A = U_0 - \frac{1}{2}\frac{a_A^2 P^2}{\theta}$$

$$\alpha_B = U_0 - \frac{1}{2}\frac{a_B^2 P^2}{\theta}$$

Now the optimal bonus height levels and effort levels are known, it is possible to determine whether and how the optimal value of the worker's utility is affected by a change in the optimal bonus height level. The envelope theorem is used to calculate this for both workers, by differentiating the optimal value function of the worker's utility with respect to β_i:

$$U_i^* = \alpha_i + \frac{1}{2}\frac{\beta_i^2 a_i^2}{\theta}$$

$$\frac{dU_i^*}{d\beta_i} = \frac{\beta_i a_i^2}{\theta} > 0$$

The derivative shows that an increase in the bonus height level per unit leads to a higher utility level. More important, it is interesting to see that this effect on the worker's utility is positive and larger for a high-ability worker than for a low-ability worker.

3.2.4 Results case 1: No bonus cap

Table 1: A summary of the results in case 1

1) Optimal bonus height 2) Optimal level of effort		Limited liability constraint			
		Binding		**Non-binding**	
Bonus cap restriction	**Binding**				
	Non-binding	Case 1: Worker A: 1) $\beta_A^* = \frac{1}{2}P$ 2) $e_A^* = \frac{Pa_A}{2\theta}$	Case 1: Worker B: 1) $\beta_B^* = \frac{1}{2}P$ 2) $e_B^* = \frac{Pa_B}{2\theta}$	Case 1: Worker A: 1) $\beta_A^* = P$ 2) $e_A^* = \frac{Pa_A}{\theta}$	Case 1: Worker B: 1) $\beta_B^* = P$ 2) $e_B^* = \frac{Pa_B}{\theta}$

Table 1 (p.22) shows a summary of the results in case 1, wherein no regulation of a bonus cap is present. The results indicate, that it is optimal for the firm to set the bonus heights per unit at the same level for both workers to maximize profit. It also stands out that the bonus levels per unit need to be half of the marginal product under a binding limited liability constraint. When the limited liability constraint is non-binding, the optimal incentive heights must equal the full marginal product. Besides, a higher price level leads to a higher level of effort. For that reason, a higher price level induces and allows the firm to give higher incentives per unit for rewarding his employees. I assume that both employees have the same cost of effort, but only differ in their ability due to a lower-level and higher-level job. This implies, that the worker with the highest ability also exerts more effort in the optimal situation. Comparing the optimal levels of effort of both workers namely indicates, that the ability of the worker is the only different parameter in these expressions. This consecutively means that the worker with the highest ability also obtains the highest total reward, since a high-ability worker is more productive than a low-ability worker.

4. Analysis

4.1 General

The theoretical model in chapter 3 includes a normal situation without the restriction of a bonus cap. In this chapter, I expand the model of Lazear (1989) with the regulation of a bonus cap and elaborate four different cases. First of all, in case 2 the bonus is capped at an average maximum percentage of the aggregate fixed salary under both a binding and non-binding limited liability constraint. In case 3, the bonus is capped at an individual maximum percentage of the individual fixed salary. Based on previous cases, I investigate whether it is beneficial for the firm to increase the workers' fixed salaries in case 4. Finally, I study the influence of a bonus cap on social welfare in case 5.

4.2 Case 2: A bonus cap with an average maximum percentage

In this situation, the regulation of a bonus cap is introduced within the firm, for the restriction of bonuses received by personnel. This restriction implies that the bonus is capped at an average maximum percentage of the aggregate fixed salary of the firm. So, it is possible for an employee, to obtain the total bonus a firm can give to all his employees. This means that an employee could receive a higher individual percentage, than the average percentage of the aggregate fixed salary. This could only be the case, when the average percentage holds according to the restriction that will be explained later on.

Again, the situation lasts for only one period, wherein two employees work for a firm. These employees are called worker A and B, and both have a different ability ($a_i \in \{L, H\}$). Both employees and the firm know the workers' abilities and are willing to exert effort ($e_i \geq 0$). Effort is observable and contractible by the firm and therefore, workers exert effort as to maximize their utility. The height of the utility is determined by the worker's expected income function ($E(W_i)$) and the worker's cost function ($C(e_i)$) for exerting effort.

The firm still uses an incentive contract with piece rates, which gives an opportunity for a worker to receive a bonus per unit produced. However, the restriction of a bonus cap is now present at the firm. As a consequence, the variable bonus wage differs from previous case. For simplicity again, the fixed wage is called α_i and the bonus is called β_i given for every unit produced (q_i). In this case, the bonus is capped at an average maximum percentage of the aggregate fixed salary of the firm. For that reason, the bonus must be divided between the two workers. The regulation of the bonus cap is called t ($\frac{percentage\ level}{100}$) and is set by the authority to cap the total bonus B. As in different European countries, there is the following condition t for the bonus cap: $0 \leq t \leq \infty$. This means that the bonus can be capped at a minimum of zero percent, and must be lower than or equal to a maximum percentage of the firm's aggregate fixed salary, which is determined by the authority.

The aggregate fixed salary of the two workers of the firm is:

$$\alpha_A + \alpha_B$$

Under the absence of a bonus cap, the total expected bonus rewarded by the firm is:

$$B = \beta_A q_A + \beta_B q_B$$

Under the restriction of a bonus cap, the following condition must hold:

$$\frac{(\beta_A q_A + \beta_B q_B)}{(\alpha_A + \alpha_B)} \leq t$$

4.2.1 Workers' effort choice under a binding and non-binding limited liability constraint

The restriction t is rewritten to β_B, so that it is possible to maximize the firm's profit function to determine the optimal level of β_A. Therefore, the optimal levels of effort will be calculated first, to determine the workers' production functions under both a binding and non-binding limited liability constraint. These effort levels of worker A and B are respectively $e_A^* = \frac{\beta_A a_A}{\theta}$ and $e_B^* = \frac{\beta_B a_B}{\theta}$ as was already found in case 1, since both workers maximize their utility again (see Appendix A & B, p.66). Then, the effort levels will be substituted into the production functions of both workers.

The production function of worker A:

$$q_A = \left(\frac{\beta_A a_A}{\theta}\right) a_A$$

$$q_A = \frac{\beta_A a_A^2}{\theta}$$

The production function of worker B:

$$q_B = \left(\frac{\beta_B a_B}{\theta}\right) a_B$$

$$q_B = \frac{\beta_B a_B^2}{\theta}$$

Secondly, the production functions of both workers are substituted into the bonus cap restriction (see Appendix I, p.70):

$$\frac{\left(\beta_A \frac{\beta_A a_A^2}{\theta} + \beta_B \frac{\beta_B a_B^2}{\theta}\right)}{(\alpha_A + \alpha_B)} = t$$

Then, the restriction is rewritten to β_B (see Appendix I, p.70):

$$\beta_B = \sqrt{\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}}$$

4.2.2 Profit maximization under a binding limited liability constraint

I first consider the situation wherein the limited liability constraint is binding. The firm maximizes his profit and so, the rewritten restriction to β_B is used to find the optimal level of β_A. To do so, the firm's profit function is maximized with respect to β_A subject to the rewritten bonus cap restriction β_B.

The total expected profit of the firm realized by worker A and B (see Appendix J, p.71):

$$\pi = \frac{P a_A^2 \beta_A}{\theta} + \frac{P a_B^2}{\theta}\sqrt{\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}} - \alpha_A - \alpha_B - \frac{a_A^2 \beta_A^2}{\theta} - \frac{a_B^2}{\theta}\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}$$

The bonus cap restriction is satisfied when the total bonus equals a minimum of zero, and is lower than or equal to the maximum level determined by the authority ($0 \leq t \leq \infty$). I first consider the situation wherein both the bonus cap restriction and the limited liability constraint are binding. The restriction does only bind when the limited liability constraint of $\alpha_i > 0$ holds, since the firm only sets positive fixed salaries then. The firm's profit function is therefore not maximized to α_A and α_B, because the limited liability assumption of $\alpha_i > 0$ must always hold. Next, the optimal incentive height for worker A will be determined.

The profit function of the firm is maximized with respect to β_A (see Appendix J, p.71):

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\left[\frac{1}{2}\left(\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}\right)^{-\frac{1}{2}}\left(-2\frac{\beta_A a_A^2}{a_B^2}\right)\right] - \frac{2\beta_A a_A^2}{\theta} + \frac{2\beta_A a_A^2}{\theta} = 0$$

The first term in the equation is similar to the first term case 1 with a non-binding bonus cap restriction and with a binding limited liability constraint. It also includes the effect of an increase in β_A on e_A^* ($\frac{de_A^*}{d\beta_A}$). An increase in β_A namely increases π, via e_A^* and therefore captures the parameter of θ in each term. Furthermore, an increase in the incentive β_A increases the firm's profit with price level P. This is quite logical, since the increase in the level of effort leads to a higher production level and thus a higher revenue. Finally, an increase in β_A increases the firm's profit with a_A^2. Namely, an increase in the bonus height results in a higher effort level and so a higher production. Since both the effort and production level increase in the worker's ability, the effect of the ability is quadratic and higher for a worker with a higher ability.

The third term shows that an increase in the incentive β_A, leads to an increase in the wage costs of the firm ($\frac{2\beta_A a_A^2}{\theta}$). On the other hand, the fourth term shows the opposite. For this term, an increase in the incentive β_A decreases the wage costs ($\frac{2\beta_A a_A^2}{\theta}$). At the end, the third and the fourth term are cancelled from the derivative, because they are equal to each other. This can be explained as follows. An increase in the incentive β_A will not lead to any extra wage costs. This is quite obvious since a bonus cap restriction is now present. It is not allowed to exceed the percentage of the aggregate fixed salary, which means that the restriction $\frac{(\beta_A q_A + \beta_B q_B)}{(\alpha_A + \alpha_B)} \leq t$ must always hold. It follows from the restriction that the total bonus B $(t(\alpha_A + \alpha_B))$ always remains the same, when adjusting the bonus heights per unit. The wage costs could only be higher, when the firm increases his workers' fixed salaries. In fact, a lot of banks do this to compensate their workers for a bonus cap (Bökkerink & Kooiman (2014), De Horde (2015), Bökkerink (2015)). For that reason, this will be investigated in case 4.

The second term indicates that it can be simplified:

$$-\frac{Pa_A^2}{\theta}\frac{\beta_A}{\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)}} = -\frac{Pa_A^2}{\theta}\frac{\beta_A}{\beta_B}$$

Since the third and fourth term are cancelled from the derivative, it becomes:

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} - \frac{Pa_A^2}{\theta}\frac{\beta_A}{\beta_B} = 0$$

At the end, the first order condition of the firm's profit function is thus zero for $\beta_A = \beta_B$. Compared to case 1 with a non-binding bonus cap restriction and a binding limited liability constraint, the second term has changed from a positive to negative effect on the firm's profit, when β_A is increased. It now follows that an increase in the incentive β_A leads to a decrease in the firm's profit generated by worker B. The thought behind this can be explained as follows. The term stems from the division of the the total bonus between worker A and B, and describes the effect of incentive inequality on the firm's profit. Hence, the incentive levels of both workers are substitutes as follows from the bonus cap restriction. An increase in the incentive of worker A substitutes the incentive of worker B, which affects the incentive ratio ($\frac{\beta_A}{\beta_B}$). Since effort increases in β_A ($\frac{de_A^*}{d\beta_A} > 0$), it implies that the worker with an increase in his bonus height exerts more effort and that the worker with a decrease in his bonus height exerts less effort.

The incentive ratio indicates that there are two different subcases. In the first subcase if $\beta_A > \beta_B$, an increase in β_A negatively affects the firm's profit since the marginal effect on worker B's generated profit is then higher and negative due to incentive inequality. That is, the marginal increase in productivity by worker A is lower than the marginal decrease in productivity by worker B. In the second subcase if $\beta_A < \beta_B$, an increase in β_A positively affects the firm's profit. The reason for this, is that the marginal effect on worker B's generated profit is then lower and negative due to incentive inequality. This implies, that the marginal increase in productivity by worker A is higher than the marginal decrease in productivity by worker B. The marginal effects stem from the interaction between β_A and β_B, and highly depend on the abilities of both workers. That the abilities are important for the interaction between β_A and β_B, is also reflected by the following derivative of the bonus cap restriction with respect to β_A (see Appendix K, p.72):

$$\frac{d\beta_B}{d\beta_A} = -\frac{a_A^2}{a_B^2}\frac{\beta_A}{\beta_B} < 0$$

The derivative of the bonus cap restriction shows the exchange rate between the incentive levels of β_A and β_B. Since the bonus cap restriction must always hold and the firm must divide the bonus between the two workers, the negative expression shows that an increase in the incentive level of worker A decreases the incentive level of worker B. The workers' abilities are important parameters for the size of this effect. Namely, the derivative indicates that an increase in the incentive level of a high-ability worker will decrease the bonus height level of a low-ability worker more, than an increase in the incentive level of a low-ability worker would decrease the incentive level of a high-ability worker. So, there is an exchange mechanism between the substitutes β_A and β_B, which highly depends on the ability levels.

Due to the fact that the effort levels are influenced by a change in the bonus heights, it is also interesting to analyse how an increase in the bonus heights affects the production levels of both workers ($q_i = \frac{\beta_i a_i^2}{\theta}$). Therefore, the production function q_i is maximized with respect to the bonus height per unit β_i.

The effect of a change in β_i on the production q_i of worker *i*:

$$\frac{dq_i}{d\beta_i} = \frac{a_i^2}{\theta}$$

The derived expression shows that an increase in the incentive level of a high-ability worker will increase his production level more, than an increase in the incentive level of a low-ability worker would increase his production level. So, the substitution effect caused by an increase in a worker's incentive level increases his productivity and decreases the other worker's productivity by contrast. However, such an effect thus highly depends on the ability levels. Now, it is possible to calculate the optimal bonus height levels per unit of both workers.

The derivative results in the optimal level of β_A^* (see Appendix J, p.71):

$$\beta_A^* = \sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}$$

When substituting this optimal level of β_A^* into the rewritten restriction of β_B, it is possible to determine the optimal level of β_B^* (see Appendix L, p.73):

$$\beta_B^* = \sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}$$

The derived optimal levels of the incentive β_i for both workers are exactly the same. Just like the result in case 1 with a non-binding bonus cap restriction and a binding limited liability constraint, it is optimal for the firm to set the incentives per unit at the same height for both workers. This will eventually lead to the maximization of the firm's profit. Hence, the firm must divide the total bonus between the two workers. With different incentive heights, increasing the incentive of worker A would lower the incentive of worker B, which leads to incentive inequality. As was mentioned earlier, the real-life situation shows that banks only increase the fixed salaries or incentive heights of their higher-level workers (Bökkerink & Kooiman (2014), De Horde (2015), Bökkerink (2015)). The results from this case show that this is not optimal, because they indicate that it is better to fix the bonus heights per unit at the same level and not to increase the incentive heights of only higher-level workers. Since the optimal bonus levels for both workers are now determined, it is possible to calculate the optimal level of effort that is exerted by each worker in the optimal situations as to maximize utility. So, I substitute the optimal incentive heights β_A^* and β_B^* into the effort functions e_A and e_B.

The optimal effort level of worker A:

$$e_A^* = \frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}$$

The optimal effort level of worker B:

$$e_B^* = \frac{a_B}{\theta}\sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}$$

Although it is optimal for the firm to set the same bonus height levels to maximize profits and to avoid inequality, a high-ability person exerts more effort than a low-ability person in the optimal situation under a binding limited liability constraint. This follows from the optimal levels of effort, for which the only parameter that differs between the expressions is the worker's ability. This is consistent with case 1 under a non-binding bonus cap restriction, and both a binding and non-binding limited liability constraint. Furthermore, rewarding workers with different levels of bonus heights again leads to different effort levels. Then, the worker with the lowest bonus would also exert less effort. It actually reflects the real-life problem, since lower-level workers might become unmotivated due to substitution of their incentives and the fact that higher-level workers receive higher rewards (Bökkerink & Kooiman, 2014).

4.2.3 Profit maximization under a non-binding limited liability constraint

The results under a non-binding limited liability constraint might differ from the previous situation under a binding limited constraint, since the participation constraints must be satisfied now. This again implies, that the utilities of working for this firm must be larger than or equal to the utilities of working for an outside option.

The participation constraint of worker A rewritten to α_A and by substituting e_A:

$$\alpha_A \geq U_0 - \frac{1}{2}\frac{\beta_A^2 a_A^2}{\theta}$$

The participation constraint of worker B rewritten to α_B and by substituting e_B:

$$\alpha_B \geq U_0 - \frac{1}{2}\frac{\beta_B^2 a_B^2}{\theta}$$

As the participation constraints must be satisfied, these are substituted into the bonus cap restriction in addition to the optimal effort levels (see Appendix M, p.73):

$$\beta_A^2 a_A^2 + \beta_B^2 a_B^2 = \frac{4t\theta}{(2+t)} U_0$$

This bonus cap restriction is rewritten to β_B (see Appendix M, p.73):

$$\beta_B = \frac{1}{a_B}\sqrt{\frac{4t\theta}{(2+t)} U_0 - \beta_A^2 a_A^2}$$

When substituting β_B in the firm's profit function, this results in (see Appendix N, p.74):

$$\pi = \frac{P}{\theta}\left(a_A^2\beta_A + a_B\sqrt{\frac{4t\theta}{(2+t)} U_0 - \beta_A^2 a_A^2}\right) - 2\frac{2+2t}{(2+t)} U_0$$

The profit function of the firm is maximized with respect to β_A and subject to the participation constraints and bonus cap restriction (see Appendix N, p.74):

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} - \frac{Pa_A^2}{\theta}\frac{\beta_A a_B}{\sqrt{\frac{4\theta t}{(2+t)} U_0 - \beta_A^2 a_A^2}} = 0$$

First of all, the first term in the equation is similar to the first term in the situation with both a binding and non-binding bonus cap restriction, and a binding limited liability constraint. The term captures the effect of an increase in β_A on e_A^* ($\frac{de_A^*}{d\beta_A}$). An increase in the incentive height namely leads to a higher profit, via a higher level of effort. For that reason, θ is captured in each term. Moreover, an increase in β_A increases the firm's profit with price level P and a_A^2. A higher incentive height namely increases the firm's profit, which is larger for a high-ability worker than for a low-ability worker as both the effort and production level increase in ability.

Consistent with both a binding bonus cap restriction and binding limited liability constraint, the derivative also shows that an increase in the incentive height does not lead to any extra wage costs as the firm is restricted to the bonus cap. This follows from the fact, that the last term is cancelled from the derivative when maximizing the firm's profit function with respect to β_A. Besides, the second term of the derivative again captures the division of the bonus between the two workers and indicates the effect of incentive inequality. For that reason, the optimal bonus heights are calculated first by solving the derivative for zero (see Appendix N, p.74):

$$\beta_A^* = \frac{1}{(a_A + a_B)} \sqrt{\frac{4\theta t}{(2+t)} U_0}$$

$$\beta_B^* = \frac{1}{(a_A + a_B)} \sqrt{\frac{4\theta t}{(2+t)} U_0}$$

The expressions again indicate, that it is optimal for the firm to set the same bonus height levels for both workers. However, the optimal bonus height levels now depend on the workers' utilities of the outside options. When the utility of the outside option is higher, the firm needs to set a higher fixed salary due to the participation constraint. As the bonus cap restriction depends on the height of the aggregate fixed salary, this leads to a higher optimal bonus height per unit. In paragraph 4.5, the fixed salaries are analysed in more detail. When substituting the optimal bonus height levels into the effort functions, this results in the optimal effort levels:

$$e_A^* = \frac{a_A}{\theta(a_A + a_B)} \sqrt{\frac{4\theta t}{(2+t)} U_0}$$

$$e_B^* = \frac{a_B}{\theta(a_A + a_B)} \sqrt{\frac{4\theta t}{(2+t)} U_0}$$

As the optimal levels of effort indicate, the worker with the highest ability will exert the highest level of effort. This is consistent with previous findings. However, as the bonus height now depends on the outside utility option, a higher outside utility will lead to a higher level of effort. Furthermore, it is interesting to see how the bonus cap percentage affects the optimal effort levels. For that reason, the optimal effort functions are rewritten first:

$$e_A^* = \frac{a_A}{\theta(a_A + a_B)}\sqrt{4\theta U_0\left(1 - \frac{2}{(2+t)}\right)}$$

$$e_B^* = \frac{a_B}{\theta(a_A + a_B)}\sqrt{4\theta U_0\left(1 - \frac{2}{(2+t)}\right)}$$

The expression indicates that workers will exert no effort when $t = 0$, that is $e_i^*(t = 0) = 0$. This results in a firm's profit of $\pi(t = 0) = -2U_0$. By contrast, when $t \to \infty$, workers will exert the following effort level:

$$e_i^* = \frac{a_i}{\theta(a_A + a_B)}\sqrt{4\theta U_0}$$

This is different from the previous situation wherein the bonus cap restriction is binding and the limited liability constraint is also binding. When $t \to \infty$, the firm's profit results in:

$$\pi = \frac{P}{\theta}\left(\frac{a_A^2 + a_B^2}{(a_A + a_B)}\sqrt{4U\theta_0}\right) - 4U_0$$

At the end, the optimal bonus height and optimal level of effort lead to the following fixed and total wage, after substituting and rewriting:

$$\alpha_i = U_0\left(1 - \frac{1}{2}\frac{a_i^2}{(a_A + a_B)^2}\frac{4t}{2+t}\right)$$

$$w_i = U_0\left(1 + \frac{1}{2}\frac{a_i^2}{(a_A + a_B)^2}\frac{4t}{2+t}\right)$$

The results thus indicate, that a higher bonus cap percentage causes lower fixed wages as it is more beneficial for a firm to increase the incentive. Under a non-binding limited liability constraint, the bonus height level per unit is namely the only variable controlled by the firm depending on the bonus cap restriction. Overall, the worker's total wage increases in the bonus cap percentage, as the firm is less restricted to rewarding bonuses.

4.2.4 Results case 2: A bonus cap with an average maximum percentage

Table 2: A summary of the results in case 1 and case 2

<table>
<tr><th colspan="2" rowspan="2">1) Optimal bonus height
2) Optimal level of effort</th><th colspan="4">Limited liability constraint</th></tr>
<tr><th colspan="2">Binding</th><th colspan="2">Non-binding</th></tr>
<tr><td rowspan="2">Bonus cap restriction</td><td>Binding</td><td>Case 2:
Worker A:
1) $\beta_A^* =$
$\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$
2) $e_A^* =$
$\frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$</td><td>Case 2:
Worker B:
1) $\beta_B^* =$
$\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$
2) $e_B^* =$
$\frac{a_B}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$</td><td>Case 2:
Worker A:
1) $\beta_A^* =$
$\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$
2) $e_A^* =$
$\frac{a_A}{\theta(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$</td><td>Case 2:
Worker B:
1) $\beta_B^* =$
$\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$
2) $e_B^* =$
$\frac{a_B}{\theta(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$</td></tr>
<tr><td>Non-binding</td><td>Case 1:
Worker A:
1) $\beta_A^* = \frac{1}{2}P$
2) $e_A^* = \frac{Pa_A}{2\theta}$</td><td>Case 1:
Worker B:
1) $\beta_B^* = \frac{1}{2}P$
2) $e_B^* = \frac{Pa_B}{2\theta}$</td><td>Case 1:
Worker A:
1) $\beta_A^* = P$
2) $e_A^* = \frac{Pa_A}{\theta}$</td><td>Case 1:
Worker B:
1) $\beta_B^* = P$
2) $e_B^* = \frac{Pa_B}{\theta}$</td></tr>
</table>

Table 2 (p.34) shows a summary of the results in case 1 and case 2. When comparing case 2 with case 1, the results show that there are differences in the bonus heights per unit and the optimal levels of effort. The optimal levels of effort in case 2 partly depend on the fixed salaries of both workers under a binding limited liability constraint. This is obvious, because the total bonus is now determined by the aggregate fixed salary of the firm. The higher the aggregate fixed salary is, the more effort a worker exerts to obtain a higher bonus in the optimum. This follows from the fact that a higher aggregate fixed salary allows the firm to reward a higher total bonus. For that reason, firms increase the fixed salaries of their workers in real life (Bökkerink & Kooiman (2014), De Horde (2015), Bökkerink (2015)). In case 2 under a non-binding limited liability constraint, the optimal levels of effort depend on the outside utilities by contrast as the participation constraints must be satisfied. In case 1, the workers' effort levels only depend on the firm's price level, the cost of effort and ability. By contrast, the results in case 2 under both a binding and non-binding limited liability constraints indicate that the choice of effort by a worker also depends on the incentive height of the other worker, as shown by the exchange rate and as the firm must now divide the bonus between the two workers.

Overall, exerting more effort will not always lead to a higher bonus, because the condition of $\beta_A q_A + \beta_B q_B \leq t(\alpha_A + \alpha_B)$ must always be satisfied. Therefore, the expected bonus in case 2 is smaller than the expected bonus in case 1 under both a binding and non-binding limited liability constraint. When considering the individual fixed salaries in case 1 and case 2, it follows that the firm's profit would be higher in case 2 as the total bonus (B) is lower then:

$$\pi^{bonus\ cap\ case\ 2} > \pi^{\text{no}\ bonus\ cap\ case\ 1}$$

$$(PQ - \alpha_A - \alpha_B - B)^{bonus\ cap\ case\ 2} > (PQ - \alpha_A - \alpha_B - B)^{\text{no}\ bonus\ cap\ case\ 1}$$

Both expressions show that $PQ - B \geq \alpha_A + \alpha_B$ is the possible maximum height of the worker's fixed salaries, where the firm will reach his break-even point. So, in the situation of a bonus cap, there is a higher remaining profit left for the fixed salaries $(\alpha_A + \alpha_B)$. As mentioned before, banks therefore increase the fixed salaries of their employees to compensate for a bonus cap (Bökkerink & Kooiman (2014), De Horde (2015), Bökkerink (2015)). Though, it seems that a higher fixed salary would not completely cover the affected bonus. Such an increase still depends on the bonus cap percentage, the inequality and the marginal effect on the firm's profit. Under a non-binding limited liability constraint, the firm must satisfy the participation constraints. However, as the bonus payment is lower, a new competition on fixed salaries might come in existence. Firms could use the higher remaining profits, to pay higher fixed salaries.

When the bonus cap restriction and limited liability constraint are binding in case 2, the bonus heights per unit are lower in the situation of a bonus cap than in the situation without a bonus cap. Therefore, the results indicate that $\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} < \frac{1}{2}P$. The same finding holds for the situation in case 2, wherein the bonus cap restriction is binding and the limited liability constraint is non-binding. The results imply, that $\frac{1}{(\alpha_A+\alpha_B)}\sqrt{\frac{4\theta t}{(2+t)}}U_0 < P$. This consecutively means that the optimal levels of effort under a binding bonus cap restriction are lower than the optimal levels of effort under a non-binding bonus cap restriction, for both a binding and non-binding limited liability constraint.

Overall, lower optimal bonus heights also result in:

$$e_i^{bonus\ cap\ case\ 2} < e_i^{no\ bonus\ cap\ case\ 1}$$

Under a binding bonus cap restriction and binding limited liability constraint:

$$\frac{a_i}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} < \frac{Pa_i}{2\theta}$$

Under a binding bonus cap restriction and binding limited liability constraint:

$$\frac{a_i}{\theta(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0} < \frac{Pa_i}{\theta}$$

As was described in case 1, the envelope theorem shows how an increase in the incentive level per unit affects the worker's utility. This also holds for the bonus cap in case 2:

$$U_i^* = \alpha_i + \frac{1}{2}\frac{\beta_i^2 a_i^2}{\theta}$$

$$\frac{dU_i^*}{d\beta_i} = \frac{\beta_i a_i^2}{\theta} > 0$$

The expression shows that an increase in the bonus height level per unit has a larger and positive effect on the worker's utility of a high-ability worker than on the worker's utility of a low-ability worker. Due to the fact that the bonus payment is lower under the regulation of a bonus cap, the worker's utility is also lower in case 2 than in case 1 keeping the individual fixed salary constant:

$$U_i^{bonus\ cap\ case\ 2} < U_i^{no\ bonus\ cap\ case\ 1}$$

4.3 Case 3: A bonus cap with an individual maximum percentage

A regulation of a bonus cap is still present in this new case, but it differs from the previous case. Case 3 studies the situation wherein the bonus is capped at an individual maximum percentage of the individual fixed salary. The regulation of the bonus cap is still called t ($\frac{percentage\ level}{100}$) and is in this case set by the authority, to cap the individual bonus B_i. For t_i, there is the following condition again: $0 \leq t_i \leq \infty$. So, this percentage t_i is now restrictive to the individual salary. It means that the individual bonus can be capped at a minimum of zero percent, and must be lower than or equal to a maximum individual percentage of the individual fixed salary. Since t_i is now a maximum percentage of the individual fixed salary, the bonus cap is an individual regulation for both workers.

The restriction of t_A for worker A:

$$\frac{\beta_A q_A}{\alpha_A} \leq t_A$$

The restriction of t_B for worker B:

$$\frac{\beta_B q_B}{\alpha_B} \leq t_B$$

The situation in this case differs from case 2, because the firm is not obliged now to divide the total bonus between the two workers. Each worker has his own restriction, which means that the bonus height per unit will not be determined by an average maximum percentage of the aggregate fixed salary. To calculate the bonus height per unit of each worker, the individual restriction will first be rewritten to β_i.

The restriction for worker A rewritten to β_A:

$$\beta_A = \frac{t_A \alpha_A}{q_A}$$

The restriction for worker B rewritten to β_B:

$$\beta_B = \frac{t_B \alpha_B}{q_B}$$

4.3.1 Workers' effort choice under a binding and non-binding limited liability constraint

In case 1 under a non-binding bonus cap restriction, the utility functions of both workers are maximized to determine the optimal levels of effort that both workers exert in the optimal situation. These optimal effort levels of worker A and B are respectively $e_A^* = \frac{\beta_A \alpha_A}{\theta}$ and $e_B^* = \frac{\beta_B \alpha_B}{\theta}$ (see Appendix A & B, p.66). Before it is possible to determine how the effort levels are affected by the individual bonus cap restriction, the production functions of both workers will be calculated first. So, the optimal effort levels in case 1 without a bonus cap are substituted into the production functions of both workers, to calculate the optimal production levels.

The production function of worker A is:

$$q_A = \left(\frac{\beta_A a_A}{\theta}\right) a_A$$

$$q_A = \frac{\beta_A a_A^2}{\theta}$$

The production function of worker B is:

$$q_B = \left(\frac{\beta_B a_B}{\theta}\right) a_B$$

$$q_B = \frac{\beta_B a_B^2}{\theta}$$

4.3.2 Profit maximization under a binding limited liability constraint

I first consider the situation wherein the individual bonus cap restriction is binding and the limited liability constraint is binding. This means that the firm always sets a positive individual fixed salary ($\alpha_i > 0$). However, the individual bonus heights per unit will not be determined by maximizing the firm's profit function with respect to β_i. Hence, it is not possible for the firm to choose the division of the total bonus between the two workers, because two individual bonus cap restrictions must be satisfied now. First, the production functions of q_i will be substituted into the restrictions of t_i and rewritten to β_i, which makes it possible to calculate the optimal bonus height levels for both workers.

The optimal level of β_A (Appendix O, p.76):

$$t_A = \frac{\beta_A \left(\frac{\beta_A a_A^2}{\theta}\right)}{\alpha_A}$$

$$\beta_A^* = \sqrt{\frac{t_A \alpha_A \theta}{a_A^2}}$$

The optimal level of β_B (Appendix P, p.76):

$$t_B = \frac{\beta_B \left(\frac{\beta_B a_B^2}{\theta}\right)}{\alpha_B}$$

$$\beta_B^* = \sqrt{\frac{t_B \alpha_B \theta}{a_B^2}}$$

When analysing the optimal levels of β_i, it follows that the incentive heights per unit are different for both workers. Both optimal incentive levels depend on the individual percentage, the individual fixed salary, the worker's ability level and the cost of effort. The parameter for the cost of effort is assumed to be the same in this model and will not cause a difference between the incentive heights. However, due to the fact that each worker obtains an individual percentage of his fixed salary based on his ability, each worker will also get a different incentive per unit. The expressions show, that an increase in the fixed salary and bonus cap percentage lead to a higher optimal bonus height per unit. However, the effect of an increase in these parameters on the optimal bonus height also depends on the ability level of the worker. The expressions namely show, that the effect of an increase in the fixed salary and/or bonus cap percentage on the optimal bonus height diminishing decreases in the worker's ability level. Due to that, the diminishing decreasing effect of an equal increase in these parameters on the optimal bonus height of both a low-ability and high-ability worker is higher for a low-ability worker than for a high-ability worker.

Since the firm is highly restricted to a binding bonus cap restriction, the individual fixed salary is the only parameter that is completely under control by the firm. However, whether the firm wants to adjust the individual fixed salaries thus depends on if the bonus cap restriction is binding, which will be investigated in case 4. It is now possible to determine how the optimal levels of effort are affected by the regulation of an individual bonus cap, by substituting the optimal bonus heights per unit into the effort functions.

The optimal effort level of worker A:

$$e_A^* = \frac{a_A}{\theta} \sqrt{\frac{t_A \alpha_A \theta}{a_A^2}}$$

The optimal effort level of worker B:

$$e_B^* = \frac{a_B}{\theta} \sqrt{\frac{t_B \alpha_B \theta}{a_B^2}}$$

The results again show, that the effect of an equal increase in the bonus cap percentage and fixed salary on the optimal incentive height for both a low-ability and high-ability worker, is higher for a low-ability worker than for a high-ability worker. However, this becomes clear and logical when analysing the optimal levels of effort. Since the firm must now restrict both workers individually, it is not possible to exceed the individual bonus cap percentage. Thus at some point, exerting effort will not lead to a higher reward and will not be Pareto efficient. The bonus would only be higher, when the firm increases the individual fixed salary. Since the marginal productivity of a high-ability worker is higher than the marginal productivity of a low-ability worker, the bonus height of a high-ability worker is more affected by his ability, than the bonus height of a low-ability worker. An important finding though, is that the optimal effort levels under a binding individual bonus cap and binding limited liability constraint are not affected by the worker's ability level, but only by the individual fixed salary, the bonus cap percentage and the cost of effort. Therefore, an increase in the individual fixed salary and bonus cap percentage always leads to a higher optimal level of effort. This is quite self-evident, because it allows the firm to give a higher individual reward. Furthermore, the optimal incentive height decreases in ability to compensate for a bonus cap, because a high-ability worker is more productive than a low-ability worker. It is thus optimal for a worker to exert effort until the bonus cap is reached, because exerting more effort would not lead to a higher reward.

4.3.3 Profit maximization under a non-binding limited liability constraint

The results of the previous situation under a binding individual bonus cap restriction and binding limited liability constraint could differ from the situation under a binding individual bonus cap restriction and non-binding limited liability constraint. In this situation, the individual participation constraint must be satisfied. Therefore, the participation constraints are substituted into the individual bonus cap restriction. Then, the individual bonus cap restrictions are rewritten to β_A and β_B (see Appendix Q, p.77):

$$\beta_A^* = \frac{1}{a_A}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}$$

$$\beta_B^* = \frac{1}{a_B}\sqrt{\frac{t_B 2\theta}{2+t_B}U_0}$$

The optimal effort level of worker A:

$$e_A^* = \frac{1}{\theta}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}$$

The optimal effort level of worker B:

$$e_B^* = \frac{1}{\theta}\sqrt{\frac{t_B 2\theta}{2+t_B}U_0}$$

The optimal bonus height levels and effort levels show that the utility of the outside option is now important, as the participation constraints must be satisfied now. Therefore, an increase in the utility of the outside option increases the level of effort. Besides, it follows from the expressions that a worker will exert no effort when $t_i = 0$, that is $e_i^*(t_i = 0) = 0$. This results in a profit for the firm of $\pi(t_i = 0) = -2U_0$. When $t_i \to \infty$, the worker exerts more effort, which follows from rewriting the effort function:

$$e_i^* = \frac{1}{\theta}\sqrt{2\theta U_0\left(1 - \frac{2}{2+t_i}\right)}$$

When $e_i^*(t_i \to \infty)$, this results in:

$$e_i^* = \frac{1}{\theta}\sqrt{2\theta U_0}$$

This results in a profit for the firm of $\pi(t_i \to \infty)$:

$$\pi = \frac{P}{\theta}\left((a_A + a_B)\sqrt{2\theta U_0}\right) - 2$$

At the end, the optimal bonus height and optimal level of effort result in the following fixed and total wage, after substituting and rewriting:

$$\alpha_i = \frac{2}{2+t_i}U_0$$

$$w_i = \frac{2+2t_i}{2+t_i}U_0$$

The results for the fixed wage and total salary again imply that a higher bonus cap percentage makes it possible for the firm, to increase the wages as the bonus cap is less strict. However, it is more beneficial to increase the bonus when the bonus cap percentage is increased.

4.3.4 Results case 3: A bonus cap with an individual maximum percentage

Table 3: A summary of the results in case 1, case 2 and case 3

1) Optimal bonus height **2) Optimal level of effort**		Limited liability constraint			
		Binding		**Non-binding**	
Bonus cap restriction	**Binding**	Case 2: Worker A: 1) $\beta_A^* =$ $\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$ 2) $e_A^* =$ $\frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$	Case 2: Worker B: 1) $\beta_B^* =$ $\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$ 2) $e_B^* =$ $\frac{a_B}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$	Case 2: Worker A: 1) $\beta_A^* =$ $\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$ 2) $e_A^* =$ $\frac{a_A}{\theta(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$	Case 2: Worker B: 1) $\beta_B^* =$ $\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$ 2) $e_B^* =$ $\frac{a_B}{\theta(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$
		Case 3: Worker A: 1) $\beta_A^* = \sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}$ 2) $e_A^* = \frac{a_A}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}$	Case 3: Worker B: 1) $\beta_B^* = \sqrt{\frac{t_B\alpha_B\theta}{a_B^2}}$ 2) $e_B^* = \frac{a_B}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}}$	Case 3: Worker A: 1) $\beta_A^* = \frac{1}{a_A}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}$ 2) $e_A^* = \frac{1}{\theta}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}$	Case 3: Worker B: 1) $\beta_B^* = \frac{1}{a_B}\sqrt{\frac{t_B 2\theta}{2+t_B}U_0}$ 2) $e_B^* = \frac{1}{\theta}\sqrt{\frac{t_B 2\theta}{2+t_B}U_0}$
	Non-binding	Case 1: Worker A: 1) $\beta_A^* = \frac{1}{2}P$ 2) $e_A^* = \frac{Pa_A}{2\theta}$	Case 1: Worker B: 1) $\beta_B^* = \frac{1}{2}P$ 2) $e_B^* = \frac{Pa_B}{2\theta}$	Case 1: Worker A: 1) $\beta_A^* = P$ 2) $e_A^* = \frac{Pa_A}{\theta}$	Case 1: Worker B: 1) $\beta_B^* = P$ 2) $e_B^* = \frac{Pa_B}{\theta}$

Table 3 (p.42) shows a summary of the results in case 1, case 2 and case 3. First of all, comparing the optimal bonus height levels in case 3 with the optimal bonus height levels in case 2 under a binding limited liability constraint, indicates that a difference between these levels depends on several parameters. When the bonus is capped at an average maximum percentage in case 2 under a binding limited liability constraint, it is optimal for the firm to set the bonus heights per unit at the same level. In case 3, it is not possible for the firm to divide the total bonus between the two workers, since the bonus is capped at an individual maximum percentage. Therefore, the optimal bonus heights differ between the two cases.

This results in three different possibilities of equations under a binding limited liability constraint:

1. $\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}^{case\ 2} > \sqrt{\frac{t_i\alpha_i\theta}{a_i^2}}^{case\ 3}$

2. $\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}^{case\ 2} < \sqrt{\frac{t_i\alpha_i\theta}{a_i^2}}^{case\ 3}$

3. $\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}^{case\ 2} = \sqrt{\frac{t_i\alpha_i\theta}{a_i^2}}^{case\ 3}$

As indicated by the three equations above, the difference in optimal bonus height levels in both cases depends on the parameters. The denominator under the square root of case 2 is always bigger than the denominator under the square root of case 3, since the bonus cap in case 2 captures both workers' abilities. Moreover, it is assumed that the worker's cost of effort is the same for both employees, which will not cause a difference. Whether the numerators under the square roots of both cases also differ, thus depends on the height of the aggregate and individual bonus cap percentage, and the individual fixed salaries under both cases. It is more likely though, that the maximum aggregate bonus $t(\alpha_A + \alpha_B)$ is larger than the maximum individual bonus $t_i\alpha_i$. This implies that the first equation holds.

When the limited liability constraint is non-binding, this results in the following three different possibilities of equations:

1. $\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}^{case\ 2} > \frac{1}{a_A}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}^{case\ 3}$

2. $\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}^{case\ 2} < \frac{1}{a_A}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}^{case\ 3}$

3. $\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}^{case\ 2} = \frac{1}{a_A}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}^{case\ 3}$

Also for these three possibilities, it depends on the height of the parameters if and what difference there is between the optimal bonus heights per unit under a binding bonus cap restriction and non-binding limited liability constraint. The results of both bonus cap restrictions show, that the optimal levels of effort mainly depend on the optimal bonus heights. However, it depends on the aggregate and individual bonus cap percentage, the individual fixed salaries and individual participation constraints, whether the optimal bonus heights under both cases differ from each other. Also, when comparing the optimal levels of effort under both regulations of a bonus cap, the results show that a difference between the effort levels is determined by the bonus height level.

It is more interesting though, to analyse the effects of an increase in the fixed salaries and bonus cap percentage on the optimal level of effort. In case 3, the optimal levels of effort will not be affected by the worker's ability level under both a binding and non-binding limited liability constraint, but only by a change in the individual fixed salary or the utility of the outside option, the bonus cap percentage and the cost of effort. A high-ability worker will only exert more effort when the fixed salary, utility of the outside option, and individual bonus cap percentage are higher than a low-ability worker. This is different for the optimal levels of effort in case 2. Since the aggregate bonus must be divided between the two workers and since the optimal bonus heights are equal to each other in the optimal situation, a high-ability worker will always exert more effort in the optimum than a low-ability worker. Besides, workers with a high ability will also lead to a higher level of effort and thus a higher production level.

To determine whether and how the individual bonus cap restriction in case 3 affects the optimal bonus height level and the optimal effort level compared to case 1 without a bonus cap, the results of case 3 and case 1 are analysed. First of all, the results show that the individual bonus cap restriction has a great impact on the optimal effort level compared to the situation without a bonus cap regulation. This is due to the optimal bonus height level in case 3, for which the worker is now very restricted to his bonus cap percentage and fixed salary or participation constraint. In case 1, the optimal bonus height levels are both equal to the half and full marginal product under respectively a binding and non-binding limited liability constraint which makes it therefore optimal for a firm to set the bonus heights per unit at the same level. By contrast, the bonus height levels of both workers differ from each other in case 3, wherein the individual bonus cap percentage and individual fixed salary or outside utilities are now important determinants.

As mentioned, the optimal levels of effort mainly depend on the optimal bonus height levels. Since case 1 does not include a restriction, exerting more effort will always lead to a higher reward. However, workers maximize utility and will exert effort until their optimum is reached. This results in a higher optimal level of effort for a high-ability worker than for a low-ability worker. In case 3, the worker's individual percentage and fixed salary or outside utilities are the main determinants for the optimal levels of effort. This means that exerting more effort will not always lead to a higher bonus, since the bonus cap is reached at some point. So, it is not possible for a worker to obtain a higher reward than the restriction allows him to, and exerting more effort is not Pareto efficient then. A worker will only exert more effort, when his fixed salary or bonus cap percentage is increased. At the end, the results show that the firm's profit generated by a worker is higher in case 3 with an individual bonus cap restriction than in case 1 without a bonus cap restriction, as the individual expected bonuses ($\beta_i q_i$) are lower:

$$\pi_i^{bonus\ cap\ case\ 3} > \pi_i^{no\ bonus\ cap\ case\ 1}$$

$$(Pq_i - \alpha_i - \beta_i q_i)^{bonus\ cap\ case\ 3} > (Pq_i - \alpha_i - \beta_i q_i)^{\text{no}\ bonus\ cap\ case\ 1}$$

Also for an individual regulation of a bonus cap, the equation shows that $(Pq_i - \beta_i q_i) \geq \alpha_i$ is the maximum height of the worker's fixed salary, where the firm reaches his break-even point. This means that there is a higher profit left for the individual fixed salary α_i, which is used by banks use to compensate their workers for a bonus cap (Bökkerink & Kooiman (2014), De Horde (2015), Bökkerink (2015)). Again, such an increase in the worker's fixed salary will not completely cover the affected bonus, because this highly depends again on the bonus cap percentage and the marginal effect on the firm's profit. Besides, an individual bonus cap disciplines workers, which leads to a lower optimal bonus pay. That is $\sqrt{\frac{t_i \alpha_i \theta}{a_i^2}} < \frac{1}{2}P$ and $\frac{1}{a_i}\sqrt{\frac{t_i 2\theta}{2+t_i}U_0} < P$, which implies that the optimal levels of effort in case 3 are also lower than the optimal levels of effort in case 1, for a non-binding and binding limited liability constraint:

$$e_i^{bonus\ cap\ case\ 3} < e_i^{no\ bonus\ cap\ case\ 1}$$

Under a binding limited liability constraint:

$$\frac{a_i}{\theta}\sqrt{\frac{t_i \alpha_i \theta}{a_i^2}} < \frac{Pa_i}{2\theta}$$

Under a non-binding limited liability constraint:

$$\frac{1}{\theta}\sqrt{\frac{t_i 2\theta}{2+t_i}U_0} < \frac{Pa_i}{\theta}$$

Also for the individual bonus cap restriction, the envelope theorem indicates how an increase in the bonus height level per unit affects the worker's utility level:

$$U_i^* = \alpha_i + \frac{1}{2}\frac{\beta_i^2 a_i^2}{\theta}$$

$$\frac{dU_i^*}{d\beta_i} = \frac{\beta_i a_i^2}{\theta} > 0$$

It thus follows that an increase in the bonus height level per unit has a larger and positive effect on the utility of a high-ability worker than on the utility of a low-ability worker. Besides, a lower bonus payment due to the bonus cap regulation leads to a lower utility in case 3 than in case 1, keeping the individual fixed salary constant:

$$U_i^{bonus\ cap\ case\ 3} < U_i^{no\ bonus\ cap\ case\ 1}$$

4.4 Case 4: Is it beneficial to increase the workers' fixed salaries?

The purpose of a bonus cap regulation is the same for every country, in that it must discipline the high bonus payments. However, the percentages are different among European countries. The standard guideline that was introduced in Brussels by the European Union, implies that the bonus is capped at a maximum percentage of 100%. Many countries thought that the percentage was too high and changed it to a smaller percentage. For instance, in Belgium, Denmark, Germany and Finland, the percentage of the bonus cap is fixed at 50%, whereas the Netherlands has even set the percentage at 20%. The results of previous cases show, that the individual fixed salaries are very important for the determination of the bonus heights under different bonus cap restrictions. Hence, higher individual fixed salaries allow a firm to reward higher bonuses. For that reason, case 4 investigates whether it is beneficial for a firm to increase the workers' fixed salaries in the previous three cases.

Table 4: An overview of case 4

		Limited liability constraint	
		Binding	**Non-binding**
Bonus cap restriction	**Binding**	Case 2: A bonus cap with an average maximum percentage *First order conditions to:* α_A, α_B, β_A	Case 2: A bonus cap with an average maximum percentage *First order conditions to:* β_A *(α_A and α_B participation constraints)*
		Case 3: A bonus cap with an individual maximum percentage *First order conditions to:* α_A, α_B	Case 3: A bonus cap with an individual maximum percentage *First order conditions to:* *(α_A and α_B participation constraints)*
	Non-binding	Case 1: No bonus cap *First order conditions to:* α_A, α_B, β_A, β_B	Case 1: No bonus cap *First order conditions to:* β_A, β_B *(α_A and α_B participation constraints)*

Table 4 (p.47) shows an overview of case 4. To investigate whether a firm wants to increase the workers' fixed salaries, it is important to take in account whether the bonus cap restriction and limited liability constraint are binding or non-binding. In the first subcase, I start with investigating the situation wherein the bonus cap restriction is binding. In the second subcase, I will investigate the situation wherein the bonus cap restriction is non-binding. For both subcases, I consider a binding and non-binding limited liability constraint.

4.4.1 Increasing fixed salaries under a binding bonus cap restriction

I first start with characterizing the situation wherein the bonus cap restriction in case 2 is binding and the limited liability constraint is binding and a firm is willing to set high rewards, but not allowed to do so due to the strict bonus cap restriction. The profit maximizing firm wants to increase the fixed salary, if and only if it results in a higher profit. So, the profit function in case 2 under a binding limited liability constraint will be maximized to α_A, α_B and β_A (see appendix J, p.71):

$$\pi = \frac{Pa_A^2\beta_A}{\theta} + \frac{Pa_B^2}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2a_A^2}{a_B^2}} - \alpha_A - \alpha_B - \frac{a_A^2\beta_A^2}{\theta} - \frac{a_B^2}{\theta}\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2a_A^2}{a_B^2}$$

First of all, the effect of an increase in the fixed salaries of worker A and B on the firm's profit is determined by maximizing the profit function with respect to α_A and α_B (see Appendix R, p.78):

$$\frac{d\pi}{d\alpha_A} = \frac{Pt}{2\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2a_A^2}{a_B^2}\right)}} - 1 - t = 0$$

$$\frac{d\pi}{d\alpha_B} = \frac{Pt}{2\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2a_A^2}{a_B^2}\right)}} - 1 - t = 0$$

The expressions show that an increase in the workers' fixed salaries are equal to each other. This is quite logical, because in the optimum it does not matter whether the firm increases the fixed salary of a high-ability or the fixed salary of a low-ability worker, as long as the bonus cap restriction is satisfied. The bonus cap restriction namely shows, that a higher aggregate fixed salary allows the firm to reward a higher bonus. It is also important to take in account that the first order condition for the optimal bonus height of worker A must be satisfied, since the firm maximizes profits. For that reason, the firm's profit function will be maximized with respect to β_A and solved, just like in paragraph 4.2.2.

The firm's profit function is maximized with respect to β_A (appendix J, p.71):

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\left[\frac{1}{2}\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)^{-\frac{1}{2}}\left(-2\frac{\beta_A a_A^2}{a_B^2}\right)\right] - \frac{2\beta_A a_A^2}{\theta} + \frac{2\beta_A a_A^2}{\theta} = 0$$

After some rewriting, the optimal bonus height of worker A is (see Appendix J, p.71):

$$\beta_A^* = \sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$$

By substituting the optimal incentive height of worker A into the bonus cap restriction, it follows that the optimal incentive height of worker B is the same (see Appendix L, p.73). Next, the optimal incentive height of worker A is substituted into the first order conditions for the fixed salaries of worker A and B, and rewritten to α_A and α_B, since the optimal bonus heights per unit must be satisfied. At the end, the firm wants to increase the fixed salaries if the following condition holds (see Appendix S, p.79):

$$\alpha_A + \alpha_B \leq \frac{P^2 t(a_A^2 + a_B^2)}{\theta(2t+2)^2}$$

First of all, the condition indicates a positive effect of price level *P* and the workers' ability levels. A higher price level and worker's ability result in a higher revenue and thus a higher remaining profit after the bonus payment, which can be used to pay higher fixed salaries. By contrast, a higher bonus cap percentage makes it less likely for a firm to increase the fixed salaries. This stems from the fact, that the optimal bonus heights per unit increase in the bonus cap percentage and fixed salaries. So, a higher bonus cap percentage and higher fixed salaries would lead to a higher bonus payment. This also means that a higher fixed salary in addition decreases the remaining profit after the bonus payment. Finally, the condition also negatively depends on the cost of effort. Under a non-binding limited liability constraint in case 3, the firm must satisfy the participation constraints and therefore sets the fixed salary based on these.

The bonus cap restriction in case 3 differs from case 2, in that the bonuses are now capped individually. I again start with characterizing the situation wherein both the bonus cap restriction and the limited liability constraint are binding. Since there is no division of the bonus between the two workers and the bonus cap restriction is individually, the firm's profit function is not maximized to β_A and β_B. By substituting the optimal levels of effort and production levels into the individual bonus cap restrictions, this results in the optimal bonus heights under a binding bonus cap restriction (see Appendix O & P, p.76):

$$\beta_A^* = \sqrt{\frac{t_A \alpha_A \theta}{a_A^2}}$$

$$\beta_B^* = \sqrt{\frac{t_B \alpha_B \theta}{a_B^2}}$$

Since the individual bonus cap restrictions must be satisfied, these are substituted into the firm's profit function and maximized with respect to α_A and α_B (Appendix T, p.81):

$$\pi = \frac{P a_A^2}{\theta}\sqrt{\frac{t_A \alpha_A \theta}{a_A^2}} + \frac{P a_B^2}{\theta}\sqrt{\frac{t_B \alpha_B \theta}{a_B^2}} - \alpha_A - \alpha_B - \frac{a_A^2}{\theta}\frac{t_A \alpha_A \theta}{a_A^2} - \frac{a_B^2}{\theta}\frac{t_B \alpha_B \theta}{a_B^2}$$

$$\frac{d\pi}{d\alpha_A} = \frac{P t_A}{2\sqrt{\frac{t_A \alpha_A \theta}{a_A^2}}} - 1 - t_A = 0$$

$$\frac{d\pi}{d\alpha_B} = \frac{P t_B}{2\sqrt{\frac{t_B \alpha_B \theta}{a_B^2}}} - 1 - t_B = 0$$

At the end, the firm wants to increase the fixed salaries if the following conditions hold for worker A and B (see Appendix T, p.81):

$$\alpha_A \leq \frac{P^2 t_A a_A^2}{4\theta(1 + t_A)^2}$$

$$\alpha_B \leq \frac{P^2 t_B a_B^2}{4\theta(1 + t_B)^2}$$

Consistent with an increase in the fixed salaries in case 2, both conditions show a positive effect of price level P and the worker's ability level, and a negative effect of the bonus cap percentage and the cost of effort. Again, under a non-binding limited liability constraint, the firm must satisfy the participation constraints to attract the employees to work for this firm and only adjusts the fixed salaries as long as the participation constraint is satisfied in addition to the optimal bonus heights.

4.4.2 Increasing fixed salaries under a non-binding bonus cap restriction

I first start with characterizing the subcase wherein the bonus cap restriction is non-binding and the firm sets the workers' fixed salaries based on a binding limited liability constraint. The firm always maximizes his profit and so, the firm's profit function under a non-binding bonus cap restriction will be maximized with respect to β_A and β_B.

The firm's profit function under a binding limited liability constraint (see Appendix E, p.68):

$$\pi = \frac{Pa_A^2\beta_A}{\theta} + \frac{Pa_B^2\beta_B}{\theta} - \alpha_A - \alpha_B - \frac{a_A^2\beta_A^2}{\theta} - \frac{a_B^2\beta_B^2}{\theta}$$

So, the optimal incentive heights for worker A and B are (see Appendix F & G, p.68-p.69):

$$\beta_A^* = \frac{1}{2}P$$

$$\beta_B^* = \frac{1}{2}P$$

Since the bonus cap restriction is non-binding in this case, the bonus height levels and effort levels do not depend on the fixed salary. Furthermore, a binding limited liability constraint implies that the optimal α_i is always positive. The fixed wages are determined by the firm and therefore, it is not allowed to have negative fixed salaries in this model. This means that a fixed wage results in utility for both workers and that the firm always sets a certain positive fixed wage. In general, contracting lower fixed salaries results in a higher profit since:

$$\frac{d\pi}{d\alpha_i} = -1 < 0$$

However, due to a binding limited liability constraint, the firm will never set such a fixed salary. This is different for the subcase wherein both the bonus cap restriction and limited liability constraint are non-binding. It is then important that the participation constraints for worker A and B are satisfied, which implies that the utility of working for this firm must be larger than the utility of working for an outside option. This participation constraint is rewritten to α_i:

$$\alpha_i \geq U_0 - \beta_i(a_i e_i) + \frac{1}{2}\theta e_i^2$$

The participation constraint thus shows how the firm must set the workers' fixed salaries, so that both workers want to work for this firm. Besides, the optimal incentive heights of both workers are controlled by the firm and for which the first order conditions must be satisfied, when increasing the workers' fixed salaries. To calculate this, the participation constraints and optimal levels of effort will be substituted into the firm's profit function and maximized with respect to β_A and β_B.

The profit function under a non-binding limited liability constraint (see Appendix H, p.69):

$$\pi = \frac{Pa_A^2\beta_A}{\theta} + \frac{Pa_B^2\beta_B}{\theta} - U_0 + \frac{a_A^2\beta_A^2}{\theta} - \frac{1}{2}\frac{\theta a_A^2\beta_A^2}{\theta^2} - U_0 + \frac{a_B^2\beta_B^2}{\theta} - \frac{1}{2}\frac{\theta a_B^2\beta_B^2}{\theta^2} - \frac{a_A^2\beta_A^2}{\theta} - \frac{a_B^2\beta_B^2}{\theta}$$

This results in the optimal incentive heights (see Appendix H, p.69):

$$\beta_A^* = P$$

$$\beta_B^* = P$$

To determine whether the firm wants to increase the fixed salary, the optimal incentive heights are substituted into the firm's profit function under a non-binding limited liability constraint:

$$\pi = \frac{P^2a_A^2}{\theta} + \frac{P^2a_B^2}{\theta} - \alpha_A - \alpha_B - \frac{p^2a_A^2}{\theta} - \frac{p^2a_B^2}{\theta} = -\alpha_A - \alpha_B$$

The result shows that the firm's profit is only positive when the fixed salaries of worker A and B are negative. This means that the firm is not willing to increase the worker's fixed salaries, when the bonus cap restriction and limited liability constraint are both non-binding.

4.5 Case 5: Social welfare

In this case, I investigate whether bonus caps are efficient regarding social welfare. I define social welfare as the sum of utilities of the firm (employer) and the two workers. First of all, the utilities are added, to determine the social welfare function (see Appendix U, p.82):

$$SW = Pa_Ae_A + Pa_Be_B - \alpha_A - \beta_Aa_Ae_A - \alpha_B - \beta_Ba_Be_B + \alpha_A + \beta_Aa_Ae_A - \frac{1}{2}\theta e_A^2 + \alpha_B + \beta_B(a_Be_B) - \frac{1}{2}\theta e_B^2$$

After some rewriting, this results in (see Appendix U, p.82):

$$SW = Pa_Ae_A + Pa_Be_B - \frac{1}{2}\theta e_A^2 - \frac{1}{2}\theta e_B^2$$

Then, the social welfare function is maximized with respect to e_A and e_B, to determine the optimal levels of effort in the social optimum (see Appendix U, p.82):

$$\frac{dSW}{de_A} = Pa_A - \theta e_A = 0$$

$$e_A^{SW} = \frac{Pa_A}{\theta}$$

$$\frac{dSW}{de_B} = Pa_B - \theta e_B = 0$$

$$e_B^{SW} = \frac{Pa_B}{\theta}$$

Finally, the optimal effort levels of worker A and B are both substituted into the social welfare function, which gives the total social welfare in the social optimum (see Appendix U, p.82):

$$SW^* = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P$$

First of all, I compare social welfare in the social optimum with social welfare in case 1, wherein the bonus cap restriction is non-binding and the limited liability constraint is binding (LL). As already determined, the optimal levels of effort are:

$$e_A^* = \frac{Pa_A}{2\theta}$$

$$e_B^* = \frac{Pa_B}{2\theta}$$

These optimal effort levels lead to a social welfare of (see Appendix V, p.83):

$$SW_{case\ 1\ (LL)}^* = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{3}{8}P$$

Now, it is possible to calculate the welfare loss for case 1, wherein the bonus cap restriction is non-binding and the limited liability constraint is binding (LL) (see Appendix V, p.83):

$$WL_{case\ 1\ (LL)}^* = \left[SW^* - SW_{case\ 1\ (LL)}^*\right]$$

$$WL_{case\ 1\ (LL)}^* = \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{8}P\right]$$

Based on positive values of the corresponding parameters in the expression, the result indicates that there is a welfare loss. To determine whether there is a welfare loss in case 1 when the bonus cap restriction is non-binding and the limited liability constraint is also non-binding (NLL), the optimal effort levels of both workers are considered again:

$$e_A^* = \frac{Pa_A}{\theta}$$

$$e_B^* = \frac{Pa_B}{\theta}$$

These effort levels are substituted into the social welfare function (see Appendix W, p.83):

$$SW_{case\ 1\ (NLL)}^* = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P$$

Therefore, the welfare loss in case 1 under a non-binding limited liability constraint (NLL) and non-binding bonus cap restriction is (see Appendix W, p.83):

$$WL_{case\ 1\ (NLL)}^* = \left[SW^* - SW_{case\ 1\ (NLL)}^*\right]$$

$$WL_{case\ 1\ (NLL)}^* = \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P\right] - \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P\right] = 0$$

This result shows that there is no welfare loss and the worker's contracts are not distorted, when both the bonus cap restriction and limited liability constraint are non-binding. This implies that:

$$WL_{case\ 1\ (LL)}^* = \left[SW^* - SW_{case\ 1\ (LL)}^*\right] > WL_{case\ 1\ (NLL)}^* = \left[SW^* - SW_{case\ 1\ (NLL)}^*\right]$$

The welfare loss in case 1 with a binding limited liability constraint (LL) and non-binding bonus cap restriction is larger than the welfare loss in case 1 with a non-binding limited liability constraint (NLL) and a non-binding bonus cap restriction. This result also indicates that social welfare can be affected, even though the bonus cap restriction is non-binding.

When the bonus cap restriction is binding, the optimal levels of effort differ from the optimal levels of effort in case 1. First of all, when the bonus cap restriction in case 2 is binding and the limited liability constraint is binding (LL), the optimal levels of effort are:

$$e_a^* = \frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}$$

$$e_B^* = \frac{a_B}{\theta}\sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}$$

These optimal effort levels lead to a social welfare of (see Appendix X, p.84):

$$SW^*_{case\ 2\ (LL)} = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}} - t(\alpha_A + \alpha_B)$$

Based on the derived social welfare in case 2, wherein the bonus cap restriction is non-binding and the limited liability constraint is binding (LL), the welfare loss is now calculated (see Appendix X, p.84):

$$WL^*_{case\ 2\ (LL)} = \left[SW^* - SW^*_{case\ 2\ (LL)}\right]$$

$$WL^*_{case\ 2\ (LL)} = \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\left(\frac{1}{2}P - \sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}\right) + t(\alpha_A + \alpha_B)\right]$$

A binding bonus cap restriction leads to lower bonus heights per unit, since the firm must discipline the bonus payments. As paragraph 4.2.3 states, the bonus heights per unit in case 2 are therefore lower than the bonus heights in case 1 under a binding limited liability constraint (LL), and a non-binding bonus cap restriction. That is, $\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} < \frac{1}{2}P$ under a binding limited liability constraint (LL). Therefore, the first term in the expression displays a positive value, if and only if the first part of the first term is positive, which is likely to occur. The second term of the expression indicates, that a higher aggregate bonus payment leads to a higher welfare loss. Since the bonus cap percentage is at least equal to zero and since it is also likely that the fixed salaries are positive as firms now compete on these, the second term is also at least zero. Overall, the welfare loss in case 2 is positive, which means that the bonus cap in case 2 negatively affects social welfare. This also implies that the welfare loss in case 2 is larger than the welfare loss in case 1 under a non-binding bonus cap restriction and non-binding limited liability constraint (NLL):

$$WL^*_{case\ 2\ (LL)} = \left[SW^* - SW^*_{case\ 2\ (LL)}\right] > WL^*_{case\ 1\ (NLL)} = 0$$

The welfare loss in case 2 under a binding bonus cap restriction and binding limited liability constraint (LL) is also larger than the welfare loss in case 1 under a binding limited liability constraint and non-binding bonus cap restriction, when the following condition holds:

$$\left[\left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\left(\frac{1}{2}P-\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}\right)+t(\alpha_A+\alpha_B)\right]>\left[\left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\frac{1}{8}P\right]$$

When the bonus cap restriction is binding and the limited liability constraint is non-binding NLL) in case 2, the optimal levels of effort are:

$$e_A^*=\frac{a_A}{\theta(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$$

$$e_B^*=\frac{a_B}{\theta(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$$

These optimal effort levels lead to a social welfare of (see Appendix X, p.84):

$$SW_{case\ 2\ (NLL)}^*=\left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\left(\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)-2(a_A^2+a_B^2)\frac{t}{(a_A+a_B)^2(2+t)}U_0$$

Therefore, the welfare loss in case 2 under a non-binding limited liability constraint (NLL) is (see Appendix X, p.84):

$$WL_{case\ 2\ (NLL)}^*=\left[SW^*-SW_{case\ 2\ (NLL)}^*\right]$$

$$WL_{case\ 2\ (NLL)}^*=\left[\left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\left(\frac{1}{2}P-\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)+2(a_A^2+a_B^2)\frac{t}{(a_A+a_B)^2(2+t)}U_0\right]$$

The results of case 2 indicate that $\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}<P$. However, it is likely that the welfare loss in case 2 under a binding bonus cap restriction and non-binding limited liability constraint

(NLL) is larger than the welfare loss in case 1 under a non-binding bonus cap restriction and non-binding limited liability constraint. Therefore, the following condition holds:

$$\left[\left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\left(\frac{1}{2}P-\frac{1}{(a_A+a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)+2(a_A^2+a_B^2)\frac{t}{(a_A+a_B)^2(2+t)}U_0\right]>0$$

In case 3, wherein both the individual bonus cap restriction and the limited liability constraint are binding (LL), the optimal levels of effort also differ from the optimal levels of effort in case 1. These optimal effort levels of both workers are:

$$e_A^*=\frac{a_A}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}$$

$$e_B^*=\frac{a_B}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}}$$

The optimal levels of effort (including the optimal bonus heights) are again substituted into the general social welfare function to calculate social welfare (see Appendix Y, p.87):

$$SW_{case\ 3\ (LL)}^*=\frac{Pa_A^2}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}+\frac{Pa_B^2}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}}-\frac{1}{2}t_A\alpha_A-\frac{1}{2}t_B\alpha_B$$

Using the calculated social welfare above, the welfare loss in case 3 under a binding bonus cap restriction and binding limited liability constraint (LL) is (see Appendix Y, p.87):

$$WL_{case\ 3\ (LL)}^*=\left[SW^*-SW_{case\ 3\ (LL)}^*\right]$$

$$WL_{case\ 3\ (LL)}^*=\left[\left(\frac{Pa_A^2}{\theta}\right)\left(\frac{1}{2}P-\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}\right)+\frac{1}{2}t_A\alpha_A+\left(\frac{Pa_B^2}{\theta}\right)\left(\frac{1}{2}P-\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}}\right)+\frac{1}{2}t_B\alpha_B\right]$$

The expression especially indicates that the bonus is now capped individually. The first and second term term reflect the welfare loss caused by the individual bonus cap for worker A, whereas the third and fourth term reflect the welfare loss caused by the individual bonus cap for worker B. Just as the aggregate bonus cap, the firm must discipline the reward height in this case. Therefore, as paragraph 4.3.3 states, the individual bonus heights in case 3 are lower than the bonus heights in case 1 under a binding limited liability constraint (LL) and a non-binding bonus cap restriction. That is, $\sqrt{\frac{t_i \alpha_i \theta}{a_i^2}} < \frac{1}{2}P$. Therefore, the first and third term are both positive, if and only if the first parts of these terms are also positive. The second and fourth term show that higher individual bonus payments lead to a higher welfare loss. As the bonus cap percentage is at least equal to zero and it is also likely that the fixed salaries are positive as a new competition arises due to the bonus cap regulation, the second and fourth term are also at least zero. For that reason, the welfare loss in case 3 is again positive, which indicates that the bonus cap in case 3 also has a negative impact on social welfare. This also means that the welfare loss in case 3 is larger than the welfare loss in case 1 under a non-binding bonus cap restriction and binding limited liability constraint:

$$WL^*_{case\ 3\ (LL)} = \left[SW^* - SW^*_{case\ 3\ (LL)}\right] > WL^*_{case\ 1\ (NLL)} = 0$$

The welfare loss in case 3 under a binding bonus cap restriction and binding limited liability constraint (LL) is also larger than the welfare loss in case 1 under a binding limited liability constraint and non-binding bonus cap restriction, if the following condition holds:

$$\left[\left(\frac{Pa_A^2}{\theta}\right)\left(\frac{1}{2}P - \sqrt{\frac{t_A \alpha_A \theta}{a_A^2}}\right) + \frac{1}{2}t_A \alpha_A + \left(\frac{Pa_B^2}{\theta}\right)\left(\frac{1}{2}P - \sqrt{\frac{t_B \alpha_B \theta}{a_B^2}}\right) + \frac{1}{2}t_B \alpha_B\right] > \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{8}P\right]$$

The optimal effort levels in case 3 under a non-binding limited liability constraint (NLL) also differ from the optimal effort levels in case 1 under a non-binding limited liability constraint:

$$e_A^* = \frac{1}{\theta}\sqrt{\frac{t_A 2\theta}{2 + t_A}U_0}$$

$$e_B^* = \frac{1}{\theta}\sqrt{\frac{t_B 2\theta}{2 + t_B}U_0}$$

The optimal levels of effort (including the optimal bonus heights) are again substituted into the general social welfare function to determine the social welfare (see Appendix Y, p.87):

$$SW^*_{case\ 3\ (NLL)} = \frac{Pa_A}{\theta}\sqrt{\frac{t_A 2\theta}{2 + t_A}U_0} + \frac{Pa_B}{\theta}\sqrt{\frac{t_B 2\theta}{2 + t_B}U_0} - \frac{t_A}{2 + t_A}U_0 - \frac{t_B}{2 + t_B}U_0$$

With determined social welfare, the welfare loss in case 3 under a binding bonus cap restriction and non-binding limited liability constraint (NLL) is (see Appendix Y, p.87):

$$WL^*_{case\ 3\ (NLL)} = \left[SW^* - SW^*_{case\ 3\ (NLL)}\right]$$

$$WL^*_{case\ 3\ (NLL)} = \left[\left(\frac{Pa_A^2}{\theta}\frac{1}{2}P - \frac{Pa_A}{\theta}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}\right) + \left(\frac{Pa_B^2}{\theta}\frac{1}{2}P - \frac{Pa_B}{\theta}\sqrt{\frac{t_B 2\theta}{2+t_B}U_0}\right) + \frac{t_A}{2+t_A}U_0 + \frac{t_B}{2+t_B}U_0\right]$$

The results of case 3 indicate that $\frac{1}{a_i}\sqrt{\frac{t_i 2\theta}{2+t_i}U_0} < P$. However, it depends on the height of the parameters whether the welfare loss in case 3 under a binding bonus cap restriction and non-binding limited liability constraint (NLL) is larger than the welfare loss in case 1 under a non-binding bonus cap restriction and non-binding limited liability constraint. Therefore, the following condition must hold, which is likely:

$$\left[\left(\frac{Pa_A^2}{\theta}\frac{1}{2}P - \frac{Pa_A}{\theta}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}\right) + \left(\frac{Pa_B^2}{\theta}\frac{1}{2}P - \frac{Pa_B}{\theta}\sqrt{\frac{t_B 2\theta}{2+t_B}U_0}\right) + \frac{t_A}{2+t_A}U_0 + \frac{t_B}{2+t_B}U_0\right] > 0$$

5. Conclusion

As the existing literature leaves large gaps on the effect of bonus cap regulations on workers' productivity, I investigated this by using the theoretical model of Lazear (1989). This study provides new theoretical insights and complements the previous literature on bonus caps. To do so, I analysed five different cases under both a binding and non-binding limited liability constraint. In case 1, no bonus cap regulation was present. By contrast, the bonus was capped at an average maximum percentage of the aggregate fixed salary of the firm in case 2. In addition, case 3 included an individual bonus cap regulation, wherein the bonus was capped at an individual maximum percentage of the individual fixed salary. Furthermore, I studied in case 4 whether and when it is beneficial for firms to increase the workers' fixed salaries in the three different cases described above. Finally, I investigated in case 5 whether bonus caps are efficient regarding social welfare. The findings of this research indicate that bonus caps indeed have an impact on worker's productivity, compared to a situation without a bonus cap.

If there is no bonus cap present, the results show that it is optimal for the firm to set the bonus heights per unit at the same level. This holds in both situations under a binding and non-binding limited liability constraint, for which the optimal bonus heights per unit under a binding limited liability constraint are lower than under a non-binding limited liability constraint. Furthermore, the optimal levels of effort under both a binding and non-binding limited liability constraint only differ, in that a high-ability worker exerts more effort in the optimum than a low-ability worker. At the end, the optimal levels of effort under a binding limited liability constraint are lower than under a non-binding limited liability constraint.

When the bonus is capped at an average maximum percentage of the aggregate fixed salary, the firm must choose the distribution of the aggregate bonus between the workers. It is again optimal for the firm, to set the bonus heights per unit at the same level under both a binding and non-binding limited liability constraint. As the firm must discipline the bonus payments, the results also indicate that the optimal bonus heights per unit are lower than without a bonus cap regulation, under both a binding and non binding limited liability constraint. This consecutively results in lower optimal levels of effort exerted by a low-ability and high-ability worker. Overall, a high-ability worker again exerts more effort in the optimum than a low-ability worker under both a binding and non-binding limited liability constraint. On the other hand, when the bonus is capped at an individual maximum percentage of the individual fixed salary, the optimal bonus heights per unit could differ from each other, depending on the individual fixed salary and the individual bonus cap percentage. However, a difference in the optimal levels of effort under an individual bonus cap restriction is not caused by the workers' ability levels. Furthermore, the optimal bonus heights per unit and the optimal levels of effort under an individual bonus cap are also lower than without the regulation of a bonus cap.

More important, the introduction of a bonus cap with an average maximum percentage of the aggregate fixed salary could lead to incentive inequality, when the firm sets different bonus heights per unit. Such a bonus cap makes it possible for a worker to obtain a higher percentage than the average maximum percentage, as long as the bonus cap restriction is binding. However, the results indicate that there is an exchange mechanism between the bonus heights of both workers. This mechanism indicates that an increase in the bonus height per unit of a high-ability worker leads to a higher decrease in the bonus height per unit of a low-ability worker, than an increase in the bonus height of a low-ability worker would decrease the bonus height per unit of a high-ability worker. Therefore, an increase in the bonus height per unit of the worker with the lowest bonus would be more efficient, moving the bonus heights per unit towards each other and reducing inequality.

The main objective of the bonus cap restriction is to discipline bankers, but it does not forbid the firm to increase the workers' fixed salaries. For that reason, a new kind of competition might have come into existence under a bonus cap, which is also mentioned by Bénabou and Tirole (2015). This stems from the fact that the remaining rent of the firm becomes bigger after the bonus payment, since the bonuses are lower under a bonus cap. Therefore, a higher profit is left to increase the workers' fixed salaries or to satisfy the participation constraints. Since bonus caps highly depend on the workers' fixed salaries, an increase in the fixed salary would lead to a higher total wage as both the fixed wage part and variable bonus wage part increase. This could lead to higher optimal levels of effort and higher workers' utilities, but also to higher wage costs for the firm. Therefore, it is only beneficial for firms to increase the fixed salaries when the derived conditions are satisfied, subject to the optimal bonus heights and the participation constraints under a non-binding limited liability constraint. These conditions indicate, that it is more likely for the firm to increase the fixed salaries when the price level and workers' ability levels are higher, and the bonus cap percentage is lower. Hence under bonus caps, an increase in the fixed salaries leads to higher bonuses and a higher rent extraction. In addition, the findings show that an increase could therefore have a negative impact on social welfare and could distort the workers' contracts. Therefore, bonus caps could be less efficient in terms of social welfare.

The results suggest, that it would be interesting to investigate how fixed salary caps in addition to bonus caps, and total salary caps would affect workers' productivity and efficiency in terms of social welfare. As the purpose of bonus caps is to discipline the extraordinary high rewards, firms are still able to partly avoid the bonus cap by increasing the fixed salaries and therefore compete on this with other banks. So, future research can raise the question, whether the government should have used another guideline. As the model in this research is quite basic, it could be hard to conclude whether and how society is affected by the introduction of a bonus cap. For instance, a more specific analysis should be done on the role of consumers and how bankers extract rents from the bank. Furthermore, the model could be adjusted to a dynamic setting with more than one period, wherein the exerted effort under a bonus cap of this period could depend on the exerted effort under no bonus cap in the previous periods. Besides, I made the assumption that the firm knows the workers' ability levels and it is possible to contract on effort. It would be interesting to determine whether and how the results change, if the firm does not know the workers' abilities.

At the end, the findings in this research especially suggest to study whether the theoretical insights of this research hold in real-life. For instance, one could empirically investigate the effect of a bonus cap in the financial sector, by comparing workers' productivity in the situation before and after the introduction of the bonus cap regulation, and controlling for different social and economic demographics. This could for instance be done, by using a regression discontinuity design. Furthermore, such an empirical study makes it possible to analyse how pay differentials between workers and how job satisfaction among workers are influenced by the introduction of a bonus cap. Overall, the theoretical insights of this research could function as a fundament for future research.

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Appendix

(A)

The utility function of worker A needs to be maximized with respect to e_A:

$$U_A = \alpha_A + \beta_A(a_A e_A) - \frac{1}{2}\theta e_A^2$$

$$U_A = \alpha_A + \beta_A a_A e_A - \frac{1}{2}\theta e_A^2$$

$$\frac{dU_A}{de_A} = \beta_A a_A - \theta e_A = 0$$

$$\beta_A a_A = \theta e_A$$

This gives the optimal level of effort for worker A:

$$e_A^* = \frac{\beta_A a_A}{\theta}$$

(B)

The utility function of worker B needs to be maximized with respect to e_B:

$$U_B = \alpha_B + \beta_B(a_B e_B) - \frac{1}{2}\theta e_B^2$$

$$U_B = \alpha_B + \beta_B a_B e_B - \frac{1}{2}\theta e_B^2$$

$$\frac{dU_B}{de_B} = \beta_B a_B - \theta e_B = 0$$

$$\beta_B a_B = \theta e_B$$

This gives the optimal level of effort for worker B:

$$e_B^* = \frac{\beta_B a_B}{\theta}$$

(C)

The utility of worker A:

$$U_A = \alpha_A + \beta_A(a_A e_A) - \frac{1}{2}\theta e_A^2$$

The optimal level of effort for worker A (e_A^*) is substituted into the utility function:

$$U_A = \alpha_A + \beta_A(a_A \frac{\beta_A a_A}{\theta}) - \frac{1}{2}\theta\left(\frac{\beta_A a_A}{\theta}\right)^2$$

$$U_A = \alpha_A + \frac{\beta_A^2 {a_A}^2}{\theta} - \frac{1}{2}\left(\frac{\theta\beta_A^2 a_a^2}{\theta^2}\right)$$

$$U_A = \alpha_A + \frac{\beta_A^2 {a_A}^2}{\theta} - \frac{1}{2}\left(\frac{\beta_A^2 a_a^2}{\theta}\right)$$

Finally, this gives the optimal utility of worker A (U_A^*):

$$U_A^* = \alpha_A + \frac{1}{2}\frac{\beta_A^2 a_a^2}{\theta}$$

(D)

The utility of worker B:

$$U_B = \alpha_B + \beta_B(a_B e_B) - \frac{1}{2}\theta e_B^2$$

The optimal level of effort for worker B (e_B^*) is filled into the utility function:

$$U_B = \alpha_B + \beta_B(a_B \frac{\beta_B a_B}{\theta}) - \frac{1}{2}\theta\left(\frac{\beta_B a_B}{\theta}\right)^2$$

$$U_B = \alpha_B + \frac{\beta_B^2 {a_B}^2}{\theta} - \frac{1}{2}\left(\frac{\theta\beta_B^2 a_B^2}{\theta^2}\right)$$

$$U_B = \alpha_B + \frac{\beta_B^2 {a_B}^2}{\theta} - \frac{1}{2}\left(\frac{\beta_B^2 a_B^2}{\theta}\right)$$

Finally, this gives the optimal utility of worker B (U_B^*):

$$U_B^* = \alpha_B + \frac{1}{2}\frac{\beta_B^2 a_B^2}{\theta}$$

(E)

The profit function of the firm:

$$\pi = P(a_A e_A + a_B e_B) - (\alpha_A + \beta_A(a_A e_A) + \alpha_B + \beta_B(a_B e_B))$$

The optimal levels of effort for both workers are filled into the profit function:

$$\pi = P\left(a_A \frac{\beta_A a_A}{\theta} + a_B \frac{\beta_B a_B}{\theta}\right) - \left(\alpha_A + \beta_A\left(a_A \frac{\beta_A a_A}{\theta}\right) + \alpha_B + \beta_B\left(a_B \frac{\beta_B a_B}{\theta}\right)\right)$$

$$\pi = P\left(\frac{\beta_A a_A^2}{\theta} + \frac{\beta_B a_B^2}{\theta}\right) - \left(\alpha_A + \frac{\beta_A^2 a_A^2}{\theta} + \alpha_B + \frac{\beta_B^2 a_B^2}{\theta}\right)$$

This results in the expected profit of the firm:

$$\pi = \frac{P a_A^2 \beta_A}{\theta} + \frac{P a_B^2 \beta_B}{\theta} - \alpha_A - \alpha_B - \frac{a_A^2 \beta_A^2}{\theta} - \frac{a_B^2 \beta_B^2}{\theta}$$

(F)

The expected profit function of the firm:

$$\pi = \frac{P a_A^2 \beta_A}{\theta} + \frac{P a_B^2 \beta_B}{\theta} - \alpha_A - \alpha_B - \frac{a_A^2 \beta_A^2}{\theta} - \frac{a_B^2 \beta_B^2}{\theta}$$

This function needs to be maximized with respect to β_A:

$$\frac{d\pi}{d\beta_A} = \frac{P a_A^2}{\theta} - \frac{2 a_A^2 \beta_A}{\theta} = 0$$

$$\frac{P a_A^2}{\theta} = \frac{2 a_A^2 \beta_A}{\theta}$$

$$\frac{1}{2}\frac{P a_A^2}{\theta} = \frac{a_A^2 \beta_A}{\theta}$$

$$\frac{\theta}{a_A^2}\left(\frac{1}{2}\frac{P a_A^2}{\theta}\right) = \beta_A$$

$$\beta_A = \frac{1}{2}\frac{\theta P a_A^2}{a_A^2 \theta}$$

The optimal level of the height of the bonus for worker A (β_A^*) is:

$$\beta_A^* = \frac{1}{2}P$$

(G)

The expected profit function of the firm:

$$\pi = \frac{Pa_A^2\beta_A}{\theta} + \frac{Pa_B^2\beta_B}{\theta} - \alpha_A - \alpha_B - \frac{a_A^2\beta_A^2}{\theta} - \frac{a_B^2\beta_B^2}{\theta}$$

This function needs to be maximized with respect to β_B:

$$\frac{d\pi}{d\beta_A} = \frac{Pa_B^2}{\theta} - \frac{2a_B^2\beta_B}{\theta} = 0$$

$$\frac{Pa_B^2}{\theta} = \frac{2a_B^2\beta_B}{\theta}$$

$$\frac{1}{2}\frac{Pa_B^2}{\theta} = \frac{a_B^2\beta_B}{\theta}$$

$$\frac{\theta}{a_B^2}\left(\frac{1}{2}\frac{Pa_B^2}{\theta}\right) = \beta_B$$

$$\beta_B = \frac{1}{2}\frac{\theta P a_B^2}{a_B^2\theta}$$

The optimal level of the bonus height for worker B (β_B^*) is:

$$\beta_B^* = \frac{1}{2}P$$

(H)

The participation constraints rewritten to α_A and α_B:

$$\alpha_A \geq U_0 - \beta_A(a_A e_A) + \frac{1}{2}\theta e_A^2$$

$$\alpha_B \geq U_0 - \beta_B(a_B e_B) + \frac{1}{2}\theta e_B^2$$

Substituting this rewritten participation constraint and the optimal effort level into the firm's profit function results in:

$$\pi = \frac{Pa_A^2\beta_A}{\theta} + \frac{Pa_B^2\beta_B}{\theta} - U_0 + \frac{a_A^2\beta_A^2}{\theta} - \frac{1}{2}\frac{\theta a_A^2\beta_A^2}{\theta^2} - U_0 + \frac{a_B^2\beta_B^2}{\theta} - \frac{1}{2}\frac{\theta a_B^2\beta_B^2}{\theta^2} - \frac{a_A^2\beta_A^2}{\theta} - \frac{a_B^2\beta_B^2}{\theta}$$

Then, the firm's profit function is rewritten and maximized with respect to β_A:

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} + \frac{a_A^2\beta_A}{\theta} - 2\frac{a_A^2\beta_A}{\theta} = 0$$

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} - \frac{a_A^2\beta_A}{\theta} = 0$$

$$\frac{Pa_A^2}{\theta} = \frac{a_A^2\beta_A}{\theta}$$

$$\beta_A = P$$

This result also holds for maximizing the firm's profit function with respect to β_B:

$$\beta_B = P$$

(I)

Both production functions are substituted into the restriction:

$$t = \frac{(\beta_A q_A + \beta_B q_B)}{(\alpha_A + \alpha_B)}$$

$$t = \frac{\left(\beta_A \frac{\beta_A a_A^2}{\theta} + \beta_B \frac{\beta_B a_B^2}{\theta}\right)}{(\alpha_A + \alpha_B)}$$

$$t = \frac{\left(\frac{\beta_A^2 a_A^2}{\theta} + \frac{\beta_B^2 a_B^2}{\theta}\right)}{(\alpha_A + \alpha_B)}$$

$$t(\alpha_A + \alpha_B) = \frac{\beta_A^2 a_A^2}{\theta} + \frac{\beta_B^2 a_B^2}{\theta}$$

$$t\theta(\alpha_A + \alpha_B) = \beta_A^2 a_A^2 + \beta_B^2 a_B^2$$

$$\beta_B^2 a_B^2 = t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2$$

$$\beta_B^2 = \frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}$$

This results in the bonus cap restriction for β_B:

$$\beta_B = \sqrt{\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}}$$

(J)

The expected profit function of the firm after substitution of β_B:

$$\pi = \frac{Pa_A^2\beta_A}{\theta} + \frac{Pa_B^2}{\theta}\sqrt{\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}} - \alpha_A - \alpha_B - \frac{a_A^2\beta_A^2}{\theta} - \frac{a_B^2}{\theta}\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}$$

Then, the firm's profit function is maximized with respect to β_A subject to the bonus cap:

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\left[\frac{1}{2}\left(\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}\right)^{-\frac{1}{2}}\left(-2\frac{\beta_A a_A^2}{a_B^2}\right)\right] - \frac{2\beta_A a_A^2}{\theta} + \frac{2\beta_A a_A^2}{\theta} = 0$$

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\left[-\frac{\beta_A a_A^2}{a_B^2}\left(\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}\right)^{-\frac{1}{2}}\right] = 0$$

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} - \frac{P\beta_A a_A^2}{\theta}\left(\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}\right)^{-\frac{1}{2}} = 0$$

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta} - \frac{Pa_A^2\beta_A}{\theta}\frac{1}{\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)}} = 0$$

$$\frac{Pa_A^2}{\theta} = \frac{Pa_A^2\beta_A}{\theta}\frac{1}{\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)}}$$

$$\frac{Pa_A^2\theta}{P\theta a_A^2} = \beta_A\frac{1}{\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)}}$$

$$1 = \beta_A \frac{1}{\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)}}$$

$$\beta_A = \sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)}$$

$$\beta_A^2 = \frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}$$

$$\beta_A^2 a_B^2 + \beta_A^2 a_A^2 = t\theta(\alpha_A+\alpha_B)$$

$$\beta_A^2(a_A^2 + a_B^2) = t\theta(\alpha_A+\alpha_B)$$

$$\beta_A^2 = \frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2 + a_B^2)}$$

The optimal level of β_A is:

$$\beta_A^* = \sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2 + a_B^2)}}$$

<u>**(K)**</u>

The bonus cap restriction for β_B:

$$\beta_B = \sqrt{\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}}$$

$$\frac{d\beta_B}{d\beta_A} = \frac{1}{2}\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)^{-\frac{1}{2}}\left(-2\frac{\beta_A a_A^2}{a_B^2}\right)$$

$$\frac{d\beta_B}{d\beta_A} = \left(-\frac{\beta_A a_A^2}{a_B^2}\right)\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)^{-\frac{1}{2}}$$

$$\frac{d\beta_B}{d\beta_A} = \left(-\frac{\beta_A a_A^2}{a_B^2}\right)\frac{1}{\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2 a_A^2}{a_B^2}\right)}}$$

$$\frac{d\beta_B}{d\beta_A} = \left(-\frac{\beta_A a_A^2}{a_B^2}\right)\frac{1}{\beta_B}$$

$$\frac{d\beta_B}{d\beta_A} = -\frac{a_A^2}{a_B^2}\frac{\beta_A}{\beta_B} < 0$$

(L)

The rewritten restriction to β_B:

$$\beta_B = \sqrt{\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}}$$

$$\beta_B^2 = \frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}$$

The optimal level of β_A^* will now be substituted:

$$\beta_B^2 = \frac{t\theta(\alpha_A + \alpha_B) - \frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)} a_A^2}{a_B^2}$$

$$a_B^2\beta_B^2(a_A^2 + a_B^2) = t\theta(\alpha_A + \alpha_B)(a_A^2 + a_B^2) - t\theta(\alpha_A + \alpha_B)a_A^2$$

$$a_B^2\beta_B^2(a_A^2 + a_B^2) = t\theta(\alpha_A + \alpha_B)(a_B^2 + a_A^2 - a_A^2)$$

$$a_B^2\beta_B^2(a_A^2 + a_B^2) = t\theta(\alpha_A + \alpha_B)(a_B^2)$$

$$\beta_B^2(a_A^2 + a_B^2) = t\theta(\alpha_A + \alpha_B)$$

$$\beta_B^2 = \frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}$$

$$\beta_B^* = \sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}$$

(M)

The participation constraint of both workers rewritten to α_i and after substituting e_i:

$$\alpha_i \geq U_0 - \frac{1}{2}\frac{\beta_i^2 a_i^2}{\theta}$$

The fixed salary levels are substituted into the bonus cap restriction in addition to the optimal effort levels, and simplified:

$$\beta_A \frac{\beta_A a_A^2}{\theta} + \beta_B \frac{\beta_B a_B^2}{\theta} = t\left(2U_0 - \frac{1}{2}\frac{\beta_A^2 a_A^2}{\theta} - \frac{1}{2}\frac{\beta_B^2 a_B^2}{\theta}\right)$$

$$\beta_A^2 a_A^2 + \beta_B^2 a_B^2 = t\theta\left(2U_0 - \frac{1}{2}\frac{\beta_A^2 a_A^2}{\theta} - \frac{1}{2}\frac{\beta_B^2 a_B^2}{\theta}\right)$$

$$\left(1 + \frac{t}{2}\right)(\beta_A^2 a_A^2 + \beta_B^2 a_B^2) = t\theta 2U_0$$

$$\beta_A^2 a_A^2 + \beta_B^2 a_B^2 = \frac{4t\theta}{(2+t)}U_0$$

This bonus cap restriction subject to the participation constraints is substituted into the firm's profit function together with the optimal effort levels:

$$\pi = \frac{Pa_A^2\beta_A}{\theta} + \frac{Pa_B^2\beta_B}{\theta} - 2U_0 - \frac{1}{2}\frac{\beta_A^2 a_A^2}{\theta} - \frac{1}{2}\frac{\beta_B^2 a_B^2}{\theta}$$

$$\pi = \frac{Pa_A^2\beta_A}{\theta} + \frac{Pa_B^2\beta_B}{\theta} - 2U_0 - \frac{2t}{(2+t)}U_0$$

$$\pi = \frac{Pa_A^2\beta_A}{\theta} + \frac{Pa_B^2\beta_B}{\theta} - 2\frac{2+2t}{(2+t)}U_0$$

Then, the bonus cap restriction subject to the participation constraints is rewritten to β_B:

$$\beta_B^2 a_B^2 = \frac{4t\theta}{(2+t)}U_0 - \beta_A^2 a_A^2$$

$$\beta_B = \frac{1}{a_B}\sqrt{\frac{4t\theta}{(2+t)}U_0 - \beta_A^2 a_A^2}$$

(N)

The profit function of the firm after substituting the bonus cap restriction β_B:

$$\pi = \frac{P}{\theta}(a_A^2\beta_A + a_B^2\beta_B) - 2\frac{2+2t}{(2+t)}U_0$$

$$\pi = \frac{P}{\theta}\left(a_A^2\beta_A + a_B\sqrt{\frac{4t\theta}{(2+t)}U_0 - \beta_A^2 a_A^2}\right) - 2\frac{2+2t}{(2+t)}U_0$$

The profit function of the firm is maximized with respect to β_A and subject to the participation constraints and bonus cap restriction β_B:

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta}\left(1 - \frac{\beta_A a_B}{\sqrt{\frac{4\theta t}{(2+t)}U_0 - \beta_A^2 a_A^2}}\right) = 0$$

$$\frac{Pa_A^2}{\theta} = \frac{Pa_A^2}{\theta}\frac{\beta_A a_B}{\sqrt{\frac{4\theta t}{(2+t)}U_0 - \beta_A^2 a_A^2}}$$

$$\sqrt{\frac{4\theta t}{(2+t)}U_0 - \beta_A^2 a_A^2} = \beta_A a_B$$

$$\frac{4\theta t}{(2+t)}U_0 - \beta_A^2 a_A^2 = \beta_A^2 a_B^2$$

$$\beta_A^2 a_B^2 + \beta_A^2 a_A^2 = \frac{4\theta t}{(2+t)}U_0$$

$$\beta_A^2(a_A^2 + a_B^2) = \frac{4\theta t}{(2+t)}U_0$$

$$\beta_A(a_A + a_B) = \sqrt{\frac{4\theta t}{(2+t)}U_0}$$

$$\beta_A^* = \frac{1}{(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$$

When substituting the optimal bonus height level of worker A into the bonus cap restriction and after some rewriting, this results in the optimal bonus height level of worker B:

$$\beta_B^2 a_B^2 = \frac{4t\theta}{(2+t)}U_0 - a_A^2\left(\frac{1}{(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)^2$$

$$\beta_B^2 a_B^2 = \frac{4t\theta}{(2+t)} U_0 \left(1 - \frac{a_A^2}{(a_A + a_B)^2}\right)$$

$$\beta_B a_B = \sqrt{\frac{4t\theta}{(2+t)} U_0 \left(\frac{(a_A + a_B)}{(a_A + a_B)} - \frac{a_A}{(a_A + a_B)}\right)}$$

$$\beta_B^* = \frac{1}{(a_A + a_B)} \sqrt{\frac{4\theta t}{(2+t)} U_0}$$

(O)

The production function is substituted into the restriction:

$$t_A = \frac{\beta_A q_A}{\alpha_A}$$

$$t_A = \frac{\beta_A \left(\frac{\beta_A a_A^2}{\theta}\right)}{\alpha_A}$$

$$t_A \alpha_A = \beta_A \left(\frac{\beta_A a_A^2}{\theta}\right)$$

$$\beta_A^2 a_A^2 = t_A \alpha_A \theta$$

$$\beta_A^2 = \frac{t_A \alpha_A \theta}{a_A^2}$$

The optimal level of β_A^* is:

$$\beta_A^* = \sqrt{\frac{t_A \alpha_A \theta}{a_A^2}}$$

(P)

The production function is substituted into the restriction:

$$t_B = \frac{\beta_B q_B}{\alpha_B}$$

$$t_B = \frac{\beta_B \left(\frac{\beta_B a_B^2}{\theta}\right)}{\alpha_B}$$

$$t_B \alpha_B = \beta_B \left(\frac{\beta_B a_B^2}{\theta} \right)$$

$$\beta_B^2 a_B^2 = t_B \alpha_B \theta$$

$$\beta_B^2 = \frac{t_B \alpha_B \theta}{a_B^2}$$

The optimal level of β_B^* is:

$$\beta_B^* = \sqrt{\frac{t_B \alpha_B \theta}{a_B^2}}$$

(Q)

The participation constraint of both workers rewritten to α_i and after substituting e_i:

$$\alpha_i \geq U_0 - \frac{1}{2} \frac{\beta_i^2 a_i^2}{\theta}$$

The individual bonus cap restriction for worker A after substitution of the participation constraint:

$$t_A = \frac{\beta_A \frac{\beta_A a_A^2}{\theta}}{U_0 - \frac{1}{2} \frac{\beta_A^2 a_A^2}{\theta}}$$

$$t_A \left(U_0 - \frac{1}{2} \frac{\beta_A^2 a_A^2}{\theta} \right) = \frac{\beta_A^2 a_A^2}{\theta}$$

$$t_A U_0 - t_A \frac{1}{2} \frac{\beta_A^2 a_A^2}{\theta} = \frac{\beta_A^2 a_A^2}{\theta}$$

$$\frac{\beta_A^2 a_A^2}{\theta} + t_A \frac{1}{2} \frac{\beta_A^2 a_A^2}{\theta} = t_A U_0$$

$$\beta_A^2 a_A^2 \left(\frac{1}{\theta} + \frac{t_A}{2\theta} \right) = t_A U_0$$

$$\beta_A^2 a_A^2 \left(\frac{2 + t_A}{2\theta} \right) = t_A U_0$$

$$\beta_A^2 a_A^2 = \frac{t_A 2\theta}{2 + t_A} U_0$$

$$\beta_A a_A = \sqrt{\frac{t_A 2\theta}{2 + t_A} U_0}$$

$$\beta_A^* = \frac{1}{a_A} \sqrt{\frac{t_A 2\theta}{2 + t_A} U_0}$$

For worker B, the optimal incentive height is:

$$\beta_B^* = \frac{1}{a_B} \sqrt{\frac{t_B 2\theta}{2 + t_B} U_0}$$

(R)

The firm's profit function under the bonus cap restriction of case 2:

$$\pi = \frac{P a_A^2 \beta_A}{\theta} + \frac{P a_B^2}{\theta} \sqrt{\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}} - \alpha_A - \alpha_B - \frac{a_A^2 \beta_A^2}{\theta} - \frac{a_B^2}{\theta} \frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2}$$

The firm's profit function is maximized with respect to α_A:

$$\frac{d\pi}{d\alpha_A} = \frac{P a_B^2}{\theta} \left[\frac{1}{2} \left(\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2} \right)^{-\frac{1}{2}} \left(\frac{t\theta}{a_B^2} \right) \right] - 1 - \frac{a_B^2 t\theta}{\theta a_B^2} = 0$$

$$\frac{d\pi}{d\alpha_A} = \frac{P a_B^2 t\theta}{2\theta a_B^2} \left[\left(\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2} \right)^{-\frac{1}{2}} \right] - 1 - t = 0$$

$$\frac{d\pi}{d\alpha_A} = \frac{Pt}{2} \left[\left(\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2} \right)^{-\frac{1}{2}} \right] - 1 - t = 0$$

$$\frac{d\pi}{d\alpha_A} = \frac{Pt}{2\sqrt{\left(\frac{t\theta(\alpha_A + \alpha_B) - \beta_A^2 a_A^2}{a_B^2} \right)}} - 1 - t = 0$$

The firm's profit function is maximized with respect to α_B:

$$\frac{d\pi}{d\alpha_B} = \frac{Pa_B^2}{\theta}\left[\frac{1}{2}\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2a_A^2}{a_B^2}\right)^{-\frac{1}{2}}\left(\frac{t\theta}{a_B^2}\right)\right]-1-\frac{a_B^2t\theta}{\theta a_B^2}=0$$

$$\frac{d\pi}{d\alpha_B} = \frac{Pa_B^2t\theta}{2\theta a_B^2}\left[\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2a_A^2}{a_B^2}\right)^{-\frac{1}{2}}\right]-1-t=0$$

$$\frac{d\pi}{d\alpha_B} = \frac{Pt}{2}\left[\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2a_A^2}{a_B^2}\right)^{-\frac{1}{2}}\right]-1-t=0$$

$$\frac{d\pi}{d\alpha_B} = \frac{Pt}{2\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2a_A^2}{a_B^2}\right)}}-1-t=0$$

The firm's profit function is maximized with respect to β_A:

$$\frac{d\pi}{d\beta_A} = \frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\left[\frac{1}{2}\left(\frac{t\theta(\alpha_A+\alpha_B)-\beta_A^2a_A^2}{a_B^2}\right)^{-\frac{1}{2}}\left(-2\frac{\beta_Aa_A^2}{a_B^2}\right)\right]-\frac{2\beta_Aa_A^2}{\theta}+\frac{2\beta_Aa_A^2}{\theta}=0$$

The optimal level of β_A results in:

$$\beta_A^* = \sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$$

<u>(S)</u>

Substituting the optimal level of β_A into the first order conditions to α_A and α_B, results in:

$$\frac{d\pi}{d\alpha_i} = \frac{Pt}{2\sqrt{\left(\frac{t\theta(\alpha_A+\alpha_B)-\left(\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}\right)a_A^2}{a_B^2}\right)}}-1-t=0$$

$$\frac{d\pi}{d\alpha_A} = \frac{Pt}{2\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}}-1-t=0$$

$$\frac{d\pi}{d\alpha_B} = \frac{Pt}{2\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}} - 1 - t = 0$$

Then, the first order conditions are rewritten to α_A and α_B and must be equal to or larger than zero:

$$\frac{Pt}{2\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}} - 1 - t \geq 0$$

$$\frac{Pt}{2\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}} \geq t + 1$$

$$\frac{Pt}{2} \geq (t+1)\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$$

$$\frac{Pt}{2t+2} \geq \sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}$$

$$\left(\frac{Pt}{2t+2}\right)^2 \geq \frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}$$

At the end, the firm wants to increase the fixed salaries if the following condition is satisfied:

$$\alpha_A + \alpha_B \leq \frac{P^2 t(a_A^2+a_B^2)}{\theta(2t+2)^2}$$

For worker A, the individual condition is:

$$\alpha_A \leq \frac{P^2 t(a_A^2+a_B^2)}{\theta(2t+2)^2} - \alpha_B$$

For worker B, the individual condition is:

$$\alpha_B \leq \frac{P^2 t(a_A^2+a_B^2)}{\theta(2t+2)^2} - \alpha_A$$

(T)

By substituting the bonus cap restrictions into the firm's profit function, this results in:

$$\pi = \frac{Pa_A^2}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}} + \frac{Pa_B^2}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}} - \alpha_A - \alpha_B - \frac{a_A^2}{\theta}\frac{t_A\alpha_A\theta}{a_A^2} - \frac{a_B^2}{\theta}\frac{t_B\alpha_B\theta}{a_B^2}$$

$$\frac{d\pi}{d\alpha_A} = \frac{Pa_A^2}{\theta}\left[\frac{1}{2}\frac{t_A\alpha_A\theta}{a_A^2}\right]^{-\frac{1}{2}}\frac{t_A\theta}{a_A^2} - 1 - \frac{a_A^2}{\theta}\frac{t_A\theta}{a_A^2} = 0$$

$$\frac{d\pi}{d\alpha_A} = \frac{Pt_A}{2}\left[\frac{t_A\alpha_A\theta}{a_A^2}\right]^{-\frac{1}{2}} - 1 - t_A = 0$$

$$\frac{d\pi}{d\alpha_A} = \frac{Pt_A}{2\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}} - 1 - t_A = 0$$

Then, the first order condition is rewritten to α_A and must be equal to or larger than zero:

$$\frac{Pt_A}{2\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}} - 1 - t_A \geq 0$$

$$\frac{Pt_A}{2(1+t_A)} \geq \sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}$$

$$\frac{P^2t_A^2}{4(1+t_A)^2} \geq \frac{t_A\alpha_A\theta}{a_A^2}$$

At the end, the firm wants to increase the fixed salary of worker A if the following condition is satisfied:

$$\alpha_A \leq \frac{P^2t_Aa_A^2}{4\theta(1+t_A)^2}$$

For worker B, the firm wants to increase the fixed salary if the following condition is satisfied:

$$\alpha_B \leq \frac{P^2t_Ba_B^2}{4\theta(1+t_B)^2}$$

(U)

The general social welfare function includes the utilities of worker A and B, and the firm's profit function:

$$SW = Pa_Ae_A + Pa_Be_B - \alpha_A - \beta_A a_A e_A - \alpha_B - \beta_B a_B e_B + \alpha_A + \beta_A a_A e_A - \frac{1}{2}\theta e_A^2 + \alpha_B + \beta_B(a_B e_B) - \frac{1}{2}\theta e_B^2$$

After some rewriting, this results in:

$$SW = Pa_Ae_A + Pa_Be_B - \frac{1}{2}\theta e_A^2 - \frac{1}{2}\theta e_B^2$$

To determine the social optimal level of effort, the general social welfare function is maximized with respect to e_A and e_B:

$$\frac{dSW}{de_A} = Pa_A - \theta e_A = 0$$

$$e_A^{SW} = \frac{Pa_A}{\theta}$$

$$\frac{dSW}{de_B} = Pa_B - \theta e_B = 0$$

$$e_B^{SW} = \frac{Pa_B}{\theta}$$

Substituting the social optimal levels of effort into the general welfare function results in:

$$SW^* = \frac{P^2a_A^2}{\theta} + \frac{P^2a_B^2}{\theta} - \frac{1}{2}\theta\left(\frac{Pa_A}{\theta}\right)^2 - \frac{1}{2}\theta\left(\frac{Pa_B}{\theta}\right)^2$$

$$SW^* = \frac{P^2a_A^2}{\theta} + \frac{P^2a_B^2}{\theta} - \frac{1}{2}\frac{P^2a_A^2}{\theta} - \frac{1}{2}\frac{P^2a_B^2}{\theta}$$

$$SW^* = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)P - \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P$$

$$SW^* = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P$$

(V)

The optimal levels of effort in case 1 under a binding limited liability constraint are:

$$e_A^* = \frac{Pa_A}{2\theta}$$

$$e_B^* = \frac{Pa_B}{2\theta}$$

These optimal levels of effort (including the optimal bonus heights) are substituted into the general social welfare function under a binding limited liability constraint (LL):

$$SW_{case\ 1\ (LL)}^* = Pa_A\frac{Pa_A}{2\theta} + Pa_B\frac{Pa_B}{2\theta} - \frac{1}{2}\theta\left(\frac{Pa_A}{2\theta}\right)^2 - \frac{1}{2}\theta\left(\frac{Pa_B}{2\theta}\right)^2$$

$$SW_{case\ 1\ (LL)}^* = \frac{P^2a_A^2}{2\theta} + \frac{P^2a_B^2}{2\theta} - \frac{1}{2}\theta\left(\frac{Pa_A}{2\theta}\right)^2 - \frac{1}{2}\theta\left(\frac{Pa_B}{2\theta}\right)^2$$

$$SW_{case\ 1\ (LL)}^* = \frac{P^2a_A^2}{2\theta} + \frac{P^2a_B^2}{2\theta} - \frac{P^2a_A^2}{8\theta} - \frac{P^2a_B^2}{8\theta}$$

$$SW_{case\ 1\ (LL)}^* = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{4}{8}P - \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{8}P$$

$$SW_{case\ 1\ (LL)}^* = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{3}{8}P$$

Therefore, the welfare loss in case 1 under a binding limited liability constraint (LL) is:

$$WL_{case\ 1\ (LL)}^* = \left[SW^* - SW_{case\ 1\ (LL)}^*\right]$$

$$WL_{case\ 1\ (LL)}^* = \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{4}{8}P\right] - \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{3}{8}P\right]$$

$$WL_{case\ 1\ (LL)}^* = \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{8}P\right]$$

(W)

The optimal levels of effort in case 1 under a non-binding limited liability constraint are:

$$e_A^* = \frac{Pa_A}{\theta}$$

$$e_B^* = \frac{Pa_B}{\theta}$$

These optimal levels of effort (including the optimal bonus heights) are substituted into the general social welfare function under a non-binding limited liability constraint (NLL):

$$SW_{case\ 1\ (NLL)}^* = Pa_A \frac{Pa_A}{\theta} + Pa_B \frac{Pa_B}{\theta} - \frac{1}{2}\theta\left(\frac{Pa_A}{\theta}\right)^2 - \frac{1}{2}\theta\left(\frac{Pa_B}{\theta}\right)^2$$

$$SW_{case\ 1\ (NLL)}^* = \frac{P^2a_A^2}{\theta} + \frac{P^2a_B^2}{\theta} - \frac{1}{2}\theta\left(\frac{Pa_A}{\theta}\right)^2 - \frac{1}{2}\theta\left(\frac{Pa_B}{\theta}\right)^2$$

$$SW_{case\ 1\ (NLL)}^* = \frac{P^2a_A^2}{\theta} + \frac{P^2a_B^2}{\theta} - \frac{P^2a_A^2}{2\theta} - \frac{P^2a_B^2}{2\theta}$$

$$SW_{case\ 1\ (NLL)}^* = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)P - \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P$$

$$SW_{case\ 1\ (NLL)}^* = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P$$

Therefore, the welfare loss in case 1 under a non-binding limited liability constraint (NLL) is:

$$WL_{case\ 1\ (NLL)}^* = \left[SW^* - SW_{case\ 1\ (NLL)}^*\right]$$

$$WL_{case\ 1\ (NLL)}^* = \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P\right] - \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P\right]$$

$$WL_{case\ 1\ (NLL)}^* = 0$$

(X)

The optimal levels of effort in case 2 under a binding limited liability constraint (LL) are:

$$e_A^* = \frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}$$

$$e_B^* = \frac{a_B}{\theta}\sqrt{\frac{t\theta(\alpha_A + \alpha_B)}{(a_A^2 + a_B^2)}}$$

These optimal levels of effort (including the optimal bonus heights) are substituted into the general social welfare function:

$$SW^*_{case\ 2\ (LL)} = Pa_A \frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} + Pa_B \frac{a_B}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} - \frac{1}{2}\theta\left(\frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}\right)^2 - \frac{1}{2}\theta\left(\frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}\right)^2$$

$$SW^*_{case\ 2\ (LL)} = \frac{Pa_A^2}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} + \frac{Pa_B^2}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} - \frac{1}{2}\theta\left(\frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}\right)^2 - \frac{1}{2}\theta\left(\frac{a_A}{\theta}\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}\right)^2$$

$$SW^*_{case\ 2\ (LL)} = \left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} - \frac{1}{2}\frac{a_A^2 t(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)} - \frac{1}{2}\frac{a_B^2 t(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}$$

$$SW^*_{case\ 2\ (LL)} = \left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} - \frac{1}{2}\frac{a_A^2}{(a_A^2+a_B^2)}t(\alpha_A+\alpha_B) - \frac{1}{2}\frac{a_B^2}{(a_A^2+a_B^2)}t(\alpha_A+\alpha_B)$$

$$SW^*_{case\ 2\ (LL)} = \left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} - t(\alpha_A+\alpha_B)$$

Therefore, the welfare loss in case 2 under a binding limited liability constraint (LL) is:

$$WL^*_{case\ 2\ (LL)} = \left[SW^* - SW^*_{case\ 2\ (LL)}\right]$$

$$WL^*_{case\ 2\ (LL)} = \left[\left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\frac{1}{2}P\right] - \left[\left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}} - t(\alpha_A+\alpha_B)\right]$$

$$WL^*_{case\ 2\ (LL)} = \left[\left(\frac{Pa_A^2}{\theta}+\frac{Pa_B^2}{\theta}\right)\left(\frac{1}{2}P - \sqrt{\frac{t\theta(\alpha_A+\alpha_B)}{(a_A^2+a_B^2)}}\right) + t(\alpha_A+\alpha_B)\right]$$

When the bonus cap restriction is binding and the limited liability constraint is non-binding (NLL), the optimal levels of effort are:

$$e_A^* = \frac{a_A}{\theta(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$$

$$e_B^* = \frac{a_B}{\theta(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}$$

These optimal effort levels are substituted into the general social welfare function:

$$SW^*_{case\ 2\ (NLL)} = Pa_A\frac{a_A}{\theta(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0} + Pa_B\frac{a_B}{\theta(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0} - \frac{1}{2}\theta\left(\frac{a_A}{\theta(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)^2 - \frac{1}{2}\theta\left(\frac{a_B}{\theta(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)^2$$

$$SW^*_{case\ 2\ (NLL)} = \frac{Pa_A^2}{\theta}\frac{1}{(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0} + \frac{Pa_B^2}{\theta}\frac{1}{(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0} - \frac{1}{2}\theta\left(\frac{a_A}{\theta(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)^2 - \frac{1}{2}\theta\left(\frac{a_B}{\theta(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)^2$$

$$SW^*_{case\ 2\ (NLL)} = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\left(\frac{1}{(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right) - 2\frac{a_A^2 t}{(a_A + a_B)^2(2+t)}U_0 - 2\frac{a_B^2 t}{(a_A + a_B)^2(2+t)}U_0$$

$$SW^*_{case\ 2\ (NLL)} = \left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\left(\frac{1}{(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right) - 2(a_A^2 + a_B^2)\frac{t}{(a_A + a_B)^2(2+t)}U_0$$

Therefore, the welfare loss in case 2 under a non-binding limited liability constraint is:

$$WL^*_{case\ 2\ (NLL)} = \left[SW^* - SW^*_{case\ 2\ (NLL)}\right]$$

$$WL^*_{case\ 2\ (NLL)} = \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P\right]$$

$$- \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\left(\frac{1}{(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)\right.$$

$$\left. - 2(a_A^2 + a_B^2)\frac{t}{(a_A + a_B)^2(2+t)}U_0\right]$$

$$WL^*_{case\ 2\ (NLL)} = \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\left(\frac{1}{2}P - \frac{1}{(a_A + a_B)}\sqrt{\frac{4\theta t}{(2+t)}U_0}\right)\right.$$

$$\left. + 2(a_A^2 + a_B^2)\frac{t}{(a_A + a_B)^2(2+t)}U_0\right]$$

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The optimal levels of effort in case 3 under a binding limited liability constraint are:

$$e_A^* = \frac{a_A}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}$$

$$e_B^* = \frac{a_B}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}}$$

These optimal levels of effort (including the optimal bonus heights) are substituted into the general social welfare function:

$$SW^*_{case\ 3\ (LL)} = Pa_A\frac{a_A}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}Pa_B\frac{a_B}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}} - \frac{1}{2}\theta\left(\frac{a_A}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}\right)^2 - \frac{1}{2}\theta\left(\frac{a_B}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}}\right)^2$$

$$SW^*_{case\ 3\ (LL)} = \frac{Pa_A^2}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}} + \frac{Pa_B^2}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}} - \frac{1}{2}\theta\left(\frac{a_A}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}\right)^2 - \frac{1}{2}\theta\left(\frac{a_B}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}}\right)^2$$

$$SW^*_{case\ 3\ (LL)} = \frac{Pa_A^2}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}} + \frac{Pa_B^2}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}} - \frac{1}{2}\frac{a_A^2 t_A\alpha_A}{a_A^2} - \frac{1}{2}\frac{a_B^2 t_B\alpha_B}{a_B^2}$$

$$SW^*_{case\ 3\ (LL)} = \frac{Pa_A^2}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}} + \frac{Pa_B^2}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}} - \frac{1}{2}t_A\alpha_A - \frac{1}{2}t_B\alpha_B$$

Therefore, the welfare loss in case 3 is:

$$WL^*_{case\ 3\ (LL)} = \left[SW^* - SW^*_{case\ 3\ (LL)}\right]$$

$$WL^*_{case\ 3\ (LL)} = \left[\left(\frac{Pa_A^2}{\theta} + \frac{Pa_B^2}{\theta}\right)\frac{1}{2}P\right] - \left[\frac{Pa_A^2}{\theta}\sqrt{\frac{t_A\alpha_A\theta}{a_A^2}} + \frac{Pa_B^2}{\theta}\sqrt{\frac{t_B\alpha_B\theta}{a_B^2}} - \frac{1}{2}t_A\alpha_A - \frac{1}{2}t_B\alpha_B\right]$$

$$WL^*_{case\ 3\ (LL)} = \left[\left(\frac{Pa_A^2}{\theta}\right)\left(\frac{1}{2}P - \sqrt{\frac{t_A\alpha_A\theta}{a_A^2}}\right) + \frac{1}{2}t_A\alpha_A + \left(\frac{Pa_B^2}{\theta}\right)\left(\frac{1}{2}P - \sqrt{\frac{t_B\alpha_B\theta}{a_B^2}}\right) + \frac{1}{2}t_B\alpha_B\right]$$

When the bonus cap restriction is binding and the limited liability constraint is non-binding, the optimal effort levels are:

$$e_A^* = \frac{1}{\theta}\sqrt{\frac{t_A 2\theta}{2 + t_A}U_0}$$

$$e_B^* = \frac{1}{\theta}\sqrt{\frac{t_B 2\theta}{2 + t_B}U_0}$$

By substituting the optimal effort levels into the general social welfare function, this results in:

$$SW^*_{case\ 3\ (NLL)} = \frac{Pa_A}{\theta}\sqrt{\frac{t_A 2\theta}{2 + t_A}U_0} + \frac{Pa_B}{\theta}\sqrt{\frac{t_B 2\theta}{2 + t_B}U_0} - \frac{1}{2}\theta\left(\frac{1}{\theta}\sqrt{\frac{t_A 2\theta}{2 + t_A}U_0}\right)^2 - \frac{1}{2}\theta\left(\frac{1}{\theta}\sqrt{\frac{t_B 2\theta}{2 + t_B}U_0}\right)^2$$

$$SW^*_{case\ 3\ (NLL)} = \frac{Pa_A}{\theta}\sqrt{\frac{t_A 2\theta}{2 + t_A}U_0} + \frac{Pa_B}{\theta}\sqrt{\frac{t_B 2\theta}{2 + t_B}U_0} - \frac{t_A}{2 + t_A}U_0 - \frac{t_B}{2 + t_B}U_0$$

With determined social welfare, the welfare loss in case 3 under a binding bonus cap restriction and non-binding limited liability constraint (NLL) is:

$$WL^*_{case\ 3\ (NLL)} = \left[SW^* - SW^*_{case\ 3\ (NLL)}\right]$$

$$WL^*_{case\ 3\ (NLL)} = \left[\left(\frac{Pa_A^2}{\theta}\frac{1}{2}P - \frac{Pa_A}{\theta}\sqrt{\frac{t_A 2\theta}{2+t_A}U_0}\right) + \left(\frac{Pa_B^2}{\theta}\frac{1}{2}P - \frac{Pa_B}{\theta}\sqrt{\frac{t_B 2\theta}{2+t_B}U_0}\right)\right.$$

$$\left. + \frac{t_A}{2+t_A}U_0 + \frac{t_B}{2+t_B}U_0\right]$$