ERASMUS UNIVERSITY ROTTERDAM

ERASMUS SCHOOL OF ECONOMICS

MSc Economics & Business

Financial Economics

Hypothetical Inflation-Linked Bonds

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Finish Date: 05.12.2016

Abstract

Hypothetical inflation-linked bonds (ILBs) mitigate the problem of short ILB return series, as these securities have been issued relatively recently compared to ordinary bonds. This thesis studies nine different methods proposed in the literature on datasets of 21 countries. Particular attention is paid to US data, as most studies rely on that data exclusively. The methods range from random walks, time series analysis, linear regressions, backfilling, moving averages and surveys. I find that hypothetical ILBs created with surveys and moving averages to model inflation expectations match actual ILB returns closest. Other popular models in the literature such as the VAR or Kothari and Shanken's regression method only show an average performance. I use these hypothetical ILBs to extend actual ILB returns of eight countries. With the extension I find that ILBs expand the efficient frontier of five countries.

Keywords: hypothetical inflation-linked bonds, inflation expectations, asset allocation

Table of Content

1 Introduction

Inflation-linked bonds (ILB) are a relatively new asset class, exhibiting the usual bond features and protecting against inflation risk. The most commonly known inflation-linked bonds are the so-called “Treasury Inflation Protection Securities” (TIPS) first issued by the US government in 1997. However, modern ILBs have existed since 1981 when the UK first introduced this type of securities. Since then, many countries have followed and more than 25 countries all over the world have raised capital through ILBs (Barclays Capital, 2014).

Unlike common bonds and stocks, the time series of ILBs are rather short. From the perspective of investors this is worrying as questions such as whether ILBs should be added to a portfolio are difficult to answer. A convenient but questionable solution is to trim longer timer series to the size of the short ILB sample. Thereby common procedures like mean-variance portfolio optimization become feasible. This thesis follows the opposite way. Instead of trimming the longer series, hypothetical ILBs are modelled in a systematic way to extend the short sample.

Prior research such as Campbell and Shiller (1996), Chen and Terrien (2001) and Kothari and Shanken (2004) have addressed the question on how to create hypothetical ILBs and have each proposed a very different method. The methods range from random walks and time series models to linear regression based methods. In addition, there is a considerable body of literature on modelling inflation expectations. To my knowledge, there is no paper on which method works best for hypothetical ILBs, no comparison of the proposed methods in the literature and no out-of-sample testing on other countries, beyond the US and UK market.

The purpose of this thesis is to fill this research gap and to create the best possible hypothetical inflation-linked bond series for many countries. Consequently, the research question is: "How can hypothetical inflation-linked bond series be modelled best?" Starting from the regression based method of Kothari and Shanken (2004), this method is studied in detail, fully replicated and updated so that the assessment of the method is possible. Furthermore, I include several alternative models such as VAR, ARMA, backfilling, random walk and surveys discussed in the literature. In addition, I evaluate and test these models out-of-sample on 21 different countries which have issued ILBs. As a last step, I show how hypothetical ILBs can be applied to answer questions on asset allocation. The research goal is of interest to governments (deciding whether debt linked to inflation should be issued), investors (asset allocation) and academic researchers (pricing of ILBs).

2 Literature Review

In this section, I systematically review the literature on different methods used to construct hypothetical ILBs, how inflation expectations are modelled and the asset allocation with ILBs.

2.1 Hypothetical ILBs in the Literature

Campbell and Shiller (1996) were the first using a hypothetical ILB approach to systematically analyse the effects of issuing indexed debt on borrowing costs and answer the question whether governments should issue ILBs, as experience and data for this debt instrument were limited. For their analysis, Campbell and Shiller assume that the rational expectations hypothesis holds for the real term structure and that the inflation risk premium inherent in nominal bonds is not time-varying and does not follow a systematic behaviour. With these assumptions, they estimate a vector autoregression (VAR) model on quarterly inflation and nominal bonds data for the United States and the United Kingdom. The fitted output of the VAR model is the hypothetical ILB yield and is used to construct bond returns. The authors conclude that governments should foster the issuance of ILBs as they are an important financial instrument that help governments reduce the real borrowing costs by eliminating the inflation risk premium. A follow-up paper was done by Campbell et al. (2009), in which the VAR model was re-estimated and validated with new data. The working hypothesis is again the rational expectations hypothesis, where short term real interests are time-varying and risk and liquidity premiums are constant over time. The estimated yields have a high correlation with the observed ILB yields but are more stable and lower in absolute terms whereas observed yields are more volatile. This implies non-negative risk premiums. Moreover, the VAR model works better on UK data as the spread between fitted and observed ILB yields is smaller and more stable over time. This can be explained by a more persistent process for the ex-ante real interest rate. For US data the spread between fitted and observed yields is high at the beginning of the sample period in 1998 and the gap is closing over time until 2008, when the gap is widening again due to high risk and liquidity premiums most likely caused by institutional investors unwinding positions after the collapse of Lehman Brothers. Looking at the risk premiums and bond risk itself Campbell et al. use models based on the stochastic discount factor approach to derive a constant-covariance, a persistent-risk and an unrestricted full model with changing risk. A changing covariance between TIPS and stock returns as a measure for changing risk indicates that it plays only a minor role when modelling hypothetical ILB yields. Another paper using the same VAR methodology to analyse liquidity was conducted by Auckenthaler et al. (2015).

Chen and Terrien (2001) published a paper with a much simpler method in order to calculate hypothetical ILB returns as the previously covered VAR system. As the authors came from practice, they were looking for a pragmatic approach to mitigate the problem of having a short TIPS sample. The basis of their approach is modelling inflation expectations as random walk and back out real yields from the Fisher equation. In addition, a time-varying inflation risk premium is introduced depending on past volatility. Together with stocks and nominal bonds their analysis shows that including TIPS leads to more efficient portfolios in the mean-variance framework. Unfortunately, the authors did not test their method on real data, due to the short sample period as TIPS were introduced in 1998.

A much richer and more complex model was proposed by Kothari and Shanken (2004). The authors use regression analysis to model inflation expectations and incorporate more information in the cross-sectional dimension as the previously covered VAR system. The variables included are spot interest rates, spreads based on the yield curve, lagged inflation and T-bill returns. The real ILB yields are derived in the same fashion as Chen and Terrien did by applying the Fisher equation. A significant difference is that Kothari and Shanken focus on constructing hypothetical zero-coupon bond returns with a maturity of five years instead of constructing short term yields. Consequently, they model inflation expectations up to five years ahead and as expected the forecasting quality drops with increased forecasting horizon. Moreover, the authors show that an investor is better off including ILBs into his portfolio as ILBs provide a diversification benefit. The benefit was greater in the extended sample period from 1953 to 2000 than in the period from 1997 to 2003 with real data. This is due to the relatively low and stable inflation rate in the 1990s, whereas inflation rates before 1990 fluctuated widely and average inflation was higher. Depending on the expected returns of the three asset classes of ILBs, nominal bonds and stocks, ILBs should have a considerable weight into a portfolio, ranging from 5% when the return differential between the bonds is highest to over 75% when there is no return differential and nominal bonds should be entirely substituted with ILBs. Unfortunately, the same methodological weakness of Chen and Terrien also applies to the paper of Kothari and Shanken. Their model is not tested and validated on real data, though the validation sample still would have been rather too short considering the widely backwards extended sample of hypothetical returns.

Another more recent method for reconstructing ILB returns is maximum-likelihood backfilling described in Page (2013). Page did not apply the method directly to ILBs but provides a general method on how to deal with the short sample problem of returns. Generally, the method exploits correlation between two or more assets and uses the properties of the long sample to expand the short sample. The higher the correlation between the assets the less uncertainty is involved in the retrieving process. Additionally, the backfilling method is also able to mimic higher moments by using recycled noise instead of normally distributed noise.

Some important caveats are presented in Fleckenstein et al. (2014). The authors do not reconstruct hypothetical yields but document one of the largest mispricing in the literature. The TIPS-Treasury anomaly is exploited by shorting treasury bonds and replicating the cash flows of the treasury bonds with a combination of TIPS, inflation swaps and STRIPS. The result of this arbitrage strategy shows that treasury bonds are always overvalued compared to TIPS. On average, the mispricing is 54.5 basis points but can exceed over 200 basis points in terms of yields. The anomaly is even greater than the already known on-the-run/off-the-run yield spread anomaly. Fleckenstein et al. argue that only treasury bonds are perceived as liquid safety havens and not their inflation-linked counterparts. As a result, investors are willing to demand a lower return for treasury bonds and the price increases. Other reasons include market frictions and supporting evidence is found for the slow-moving-capital hypothesis as the anomaly narrows down with increasing capital flows. None of the usual reasons such as transaction costs, taxation and liquidity can explain the anomaly alone.

2.2 Inflation Expectations and Risk Premiums

Models in the literature start with the Fisher hypothesis and model inflation expectations to calculate hypothetical ILB yields. Although the Fisher equation is a sound theoretical concept, there are some limitations. Earlier papers such as Evans (1998) already find negative evidence and point to violations of the Fisher hypothesis and a non-zero inflation risk premium. Roll (2004) shows that real yields do not only react to jumps in nominal yields but also to expected inflation, as there is a positive empirical relationship between real yields and anticipated inflation. Aside from the formal link to the CPI, taxation may also distort the relationship between expected inflation and real yields.

Moving on to inflation expectations themselves, Ang et al. (2008) show that about 80% of the variation in nominal yields can be explained by variation in the expected inflation and in the inflation risk premium. The term structure of real rates is fairly flat to downward sloping in most regimes, making the inflation compensation (expected inflation and risk premium) the major determinant of nominal rates on longer maturities. One of the most comprehensive and complete overviews of out-of-sample inflation forecasting power is provided by Ang et al.

(2007). These methods can be used to model inflation expectations and range from time series models to the Phillips curve, term structure models and surveys. In addition, the authors test combined forecasts out-of-sample to make the evaluation complete. The dataset includes four measures of inflation in the United States ranging from 1952 to 2002 and was obtained by the Bureau of Labor Statistics. The models in detail are: ARMA models, random walk and regime-switching for time series models; OLS models based on the Phillips curve taking into account macro data like growth and unemployment; term structure models based on Fama-Bliss (1987) bond rates, VAR and no arbitrage models; surveys such as the Livingston survey, Michigan survey and the Survey of professional forecasters. Ang et al. show that an ARMA model or a simple AR model beats every other time series model. The OLS Phillips curve model is not better or worse compared to a random walk model. Term structure models perform even worse than models based on the Phillips curve. Most explanatory power in term structure model comes from short rates and not as typically believed from term spreads. However, surveys perform surprisingly well and have almost always a lower forecasting error than the ARMA model. The Livingston and the Survey of professional forecasters show better results than the Michigan Survey as the latter is conducted among consumers whereas the participants in the former are professionals. The results also hold when a rolling forecasting window is used. Combining forecasts provide very little to no additional predictive power against using the best stand-alone model or single surveys. All in all, surveys do best in order to forecast inflation, while a simple ARMA(1,1) model is a close competitor. The ARMA model also offers a nice interpretation: inflation expectations follow an AR(1) model subjected to MA(1) shocks. More complexity in terms of richer models does not add much predictive power. Surveys work well because they aggregate information among (sophisticated) participants and thus can already be viewed as a combined forecast. Another explanation may be that surveys adapt to changes in the economy very quickly and can incorporate qualitative insights.

A paper using Kothari and Shanken's method on daily data for inflation forecasting was written by Andonov et al. (2010). They compared the method to the "Survey of Professional Forecasters" in order to get trading signals against the so-called breakeven strategy, capturing break even inflation when going long into ILBs and short into nominal bond with the same maturity. Although the breakeven strategy performed better, the study also shows that breakeven inflation is a poor predictor of future inflation. The authors even observed negative breakeven inflation which could be interpreted as expecting deflation if breakeven inflation is a good measure of inflation expectations. Technically, TIPS are protected against deflation, however this protection covers the entire life of a bond from issuance to maturity. This means that expecting temporary deflation is possible as long as the overall inflation over the entire life remains positive. Andonov et al. find deflationary expectations to be unlikely and attribute the cause of negative break even inflation to a sharp increase of the liquidity premium as the breakeven inflation not only captures inflation expectations but also everything that influences the yield spread between nominal bonds and TIPS. The best method for measuring inflation expectations was the survey, which is publicly available. The results also confirm the outcomes of Ang et al. (2007). As the trading strategy incorporating the information in the survey is profitable, the trading strategy is also an example of inefficiency in the TIPS market.

A well-known violation of the Fisher equation is the inflation risk premium that is usually embedded within the yield of bonds. Buraschi and Jiltsov (2005) thoroughly analysed the magnitude and time-variation of the inflation risk premium. They do so by estimating structural parameters of a stochastic equilibrium model. The average inflation risk premium is about 25 basis points for short term TIPS and about 70 basis points for ten year TIPS and the term structure of the premium is positive. Thus, the risk premium moves with the level and volatility of inflation. Similar results are obtained by Chen et al. (2010) who used an empirical term structure model and by Ang et al. (2008) who used a regime switching term structure model. More recent studies, however, show that there is still little agreement on the absolute size of the inflation risk premium and sometimes even negative results are found (see Bekaert and Wang (2010), Swinkels (2012)). Aside from an inflation risk premium, ILBs often contain (il-)liquidity premiums. There is still no agreement on how to measure this premium and what the average size is. Auckenthaler et al. (2015) use hypothetical yields estimated from a VAR model and compare them to observed yields for three countries. The difference between the two yields is shrinking when liquidity is rising showing that liquidity clearly influences ILB yields. The liquidity premium was especially a problem during the first years of TIPS until 2004, when the TIPS market became clearly more liquid and trading volume doubled, although it still remains a relatively small market representing only 10% of outstanding debt. The liquidity premium for TIPS was about 2% at the beginning of the programme and declined below 0.5% in 2007 before the crisis. Until 2004, the premium was estimated to be higher than the inflation risk premium leading to unnecessary high costs for the US government and to questions of whether TIPS issuance should be continued (Dudley et al. (2009)). Coroneo (2016) shows that the liquidity premium can make up to 22% of the variation in TIPS yields and is Granger-caused by financial stress such as widening corporate spreads. A flight-to-quality effect was observed during the financial crisis when the liquidity premium and hence TIPS yields spiked. By excluding the Quantitative Easing (QE2) program in her joint factor model and comparing the counterfactual model to the realisation, Coroneo also shows that QE did not affect the liquidity premium in a significant way.

2.3 Asset Allocation with ILBs

Ultimately, the goal of constructing hypothetical ILBs is to use the time series for further analysis. One such application is asset allocation and the question whether ILBs should be added to a portfolio. Contrary to Kothari and Shanken (2004), Brière and Signori (2009) derive a quite controversial conclusion about the diversification power of ILBs to nominal bonds. They look at US and French bonds data from 1997 to 2007 and conclude that ILBs are no longer useful for diversification as ILBs and nominal bonds have too similar statistical properties resulting in similar volatility profiles and high correlation. The analysis involves a DCC-MVGARCH model in order to capture time variation of volatility and conditional correlation. In 2003 the behaviour of ILBs altered remarkably both in France and US approaching a correlation to nominal bonds of 0.9. Brière and Signori hypothesise that stable inflation expectations may explain this change in behaviour and as a result real yields move in the same fashion as nominal yields. The dynamic portfolio optimization by maximizing the Sharpe ratio on a monthly basis supports this explanation. Optimal weights for ILBs drop sharply after 2003 and hence the conclusion of no diversification benefits. Swinkels (2012) however, disagrees with the general conclusion of Brière and Signori and argues in another direction. Instead of looking solely on US or Euro data, Swinkels analysed ILBs issued in emerging markets and concludes that ILBs do add value both for both local and international investors, as in most cases they expand the efficient frontier in the mean-variance framework. The correlation of ILBs to nominal bonds, however, varied widely between countries making ILBs more or less useful.

In addition to portfolio diversification, hedging inflation risk is also an investor's concern. According to Bekaert and Wang (2010), hedging inflation with traditional assets remains a quite difficult endeavour. By computing "inflation betas" they measured the sensitivity of nominal yields when inflation is moving. Ideally, the inflation beta should be one, which means that inflation and nominal return move in the same way. The main conclusion after comparing data of 45 countries is that neither stocks nor bonds, nor real estate and gold provide a perfect hedge. It is more difficult to hedge in inflation in developed countries than in developing countries. Consequently, the study provides a theoretical underpinning of why ILBs should be included in a portfolio. Swinkels (2012) uses the same regression on excess returns and shows that for many countries ILBs are a better hedge against inflation than nominal bonds. A simulation study of Brière and Signori (2012) shows that the current macroeconomic regime is the key determinant of the asset correlation structures and inflation hedging abilities. During the period from 1970 to 1990 the US experienced mostly supply shocks, high macroeconomic volatility and countercyclical inflation, whereas the period from 1990 to 2010 is characterized by demand shocks, lower macroeconomic volatility and pro-cyclical inflation. In the former case nominal bonds reacted with a negative correlation to inflation, as variations in inflation expectations affect variation in nominal yields. TIPS, however, are linked to the CPI and exhibit a positive correlation to inflation. In the latter case central bank actions were taken more credible and the markets had more trust in the ability of central banks to target inflation. Inflation changed its behaviour to pro-cyclical and affected the correlation structure to all assets. In this regime, the correlation between US treasury bonds and inflation switched from negative to positive and on top of that provided better hedging opportunities than ILBs as their correlation with inflation lowered.

3 Data

In this section, I explain which data I use to construct hypothetical ILBs and from where I obtained the data. I also provide some basic descriptive statistics and show cross-country differences. Returns are calculated as one-year holding returns on a monthly frequency, unless stated otherwise (see explanation in Section 4 Methodology):

$$\text{Return}_t = \ln P_{\text{t}} - \ln P_{t-12} \tag{1}$$

Over 25 countries have issued inflation-linked bonds (see Barclays Capital (2014)). For this thesis, I construct an individual dataset for 21 countries that are covered in one of the inflation-linked bond indices of Barclays Capital. Bond markets are fragmented and it is difficult to compare bonds on an individual level. Therefore, I use total return indices for ILBs and nominal bonds of each country provided by Barclays Capital and calculate returns as in equation (1). Only return series are used that have at least 12 observations.

Table 1 Summary of ILB returns

Country	**Start**	**N**	**Duration**	**Mean (%)**	**St.Dev. (%)**
Australia	Dec-97	209	9.0	7.43	4.66
Brazil	Sep-04	128	6.4	13.50	7.74
Canada	Dec-97	209	16.0	7.61	7.05
Chile	Sep-03	140	5.6	6.43	3.20
Colombia	Dec-03	137	4.9	10.48	5.83
Denmark	May-13	24	9.9	0.38	7.11
France	Sep-99	188	9.0	5.30	4.60
Germany	Mar-07	98	6.1	4.05	3.50
Israel	Nov-99	186	7.4	5.99	3.80
Italy	Sep-04	128	7.9	5.08	8.30
Japan	Mar-05	122	6.8	2.15	5.62
Mexico	Jan-04	136	8.6	9.18	6.23
New Zealand	Jan-98	208	8.6	7.10	5.35
Poland	Aug-05	117	6.8	6.37	5.75
South Africa	Mar-01	170	9.6	11.88	5.89
South Korea	Mar-08	86	6.8	6.30	5.61
Sweden	Dec-97	209	9.7	5.72	4.52
Thailand	Jul-12	34	8.9	0.63	5.90
Turkey	Feb-08	87	4.2	16.15	11.42
UK	May-82	396	13.6	7.42	5.57
USA	Feb-98	207	8.6	6.05	5.61

Note: Nominal ILB returns are calculated out of total return indices as monthly one-year holding period returns. Last month is April 2015. *Source*: Barclays Capital.

Table 1 summarises the available times series of ILB returns for each country until April 2015. Notably, the UK was the first country issuing ILBs (gilts) and have the longest available time series with 396 monthly observations. Other Anglican countries and Sweden started to issue linkers about 10 years later. The shortest samples are from Denmark and Thailand as they started issuing ILBs only recently. Almost all countries link the bonds to the national Consumer Price Index (CPI), except for the UK which links it to the Retailer Price Index (RPI). There is quite a dispersion in annual mean returns and volatility. On average, emerging countries have a higher return and are more volatile than their developed counterparts. Figure 1 and Figure 2 depict the time series of ILB returns of developed countries and emerging markets. South Korea showed an impressive track record of growth and development during the recent past, however I put it in the group of emerging countries as it is part of the Emerging Markets Government Inflation Linked index (EMGILB) of Barclays Capital. All in all, these ILB returns are used to validate the simulating returns and the goal is to expand these time series.

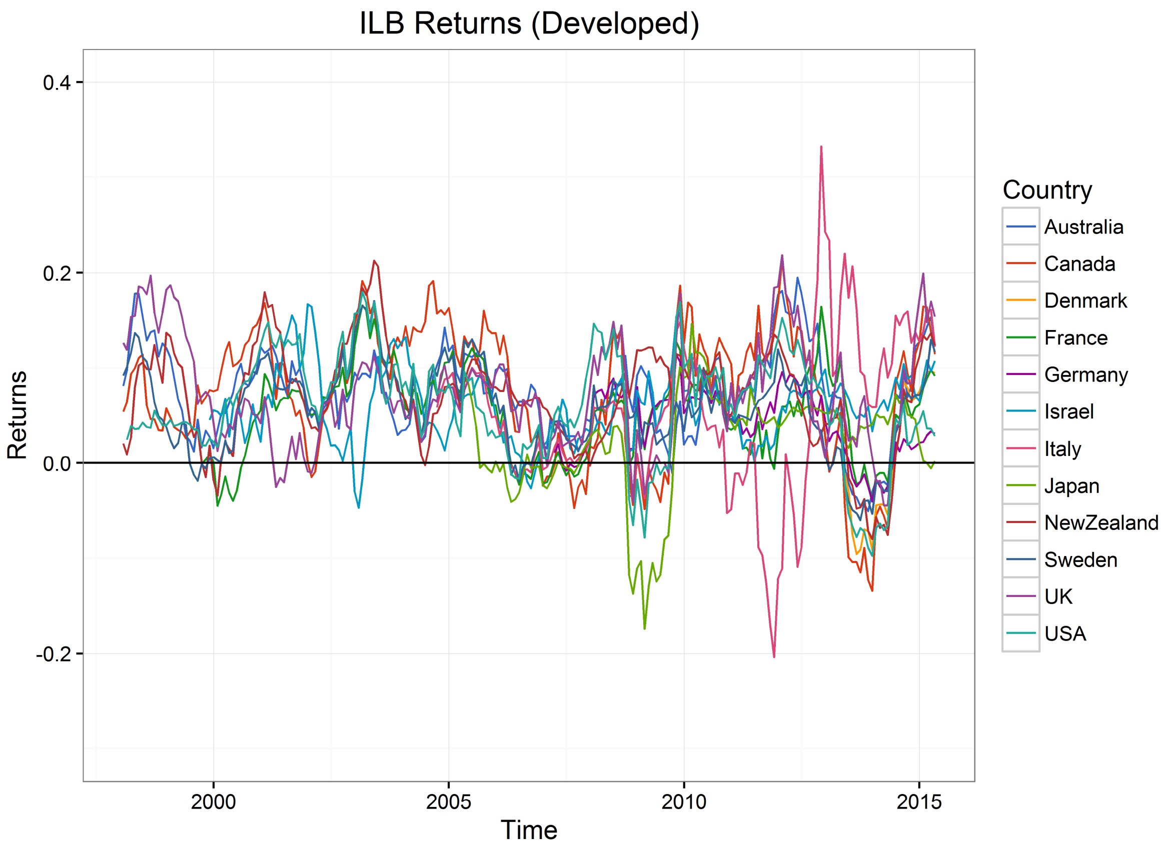


Figure 1 Monthly one-year overlapping ILB returns (developed countries)

The ingredients of simulated ILB returns are inflation rates and nominal yield curves. The data was obtained mainly from Datastream. For the United States the data on treasury rates and TIPS are from CRSP as artificial zero coupon bond prices calculated with the Fama-Bliss (1987) method were directly available. Datastream provides stored yield curve data for most countries up to 25 years. The yield curve is fitted by fifth degree polynomial splines on actual bond data. I use these stored yields with constant maturities from six months to five years in order to create hypothetical bonds as described in the paper of Kothari and Shanken (2004).

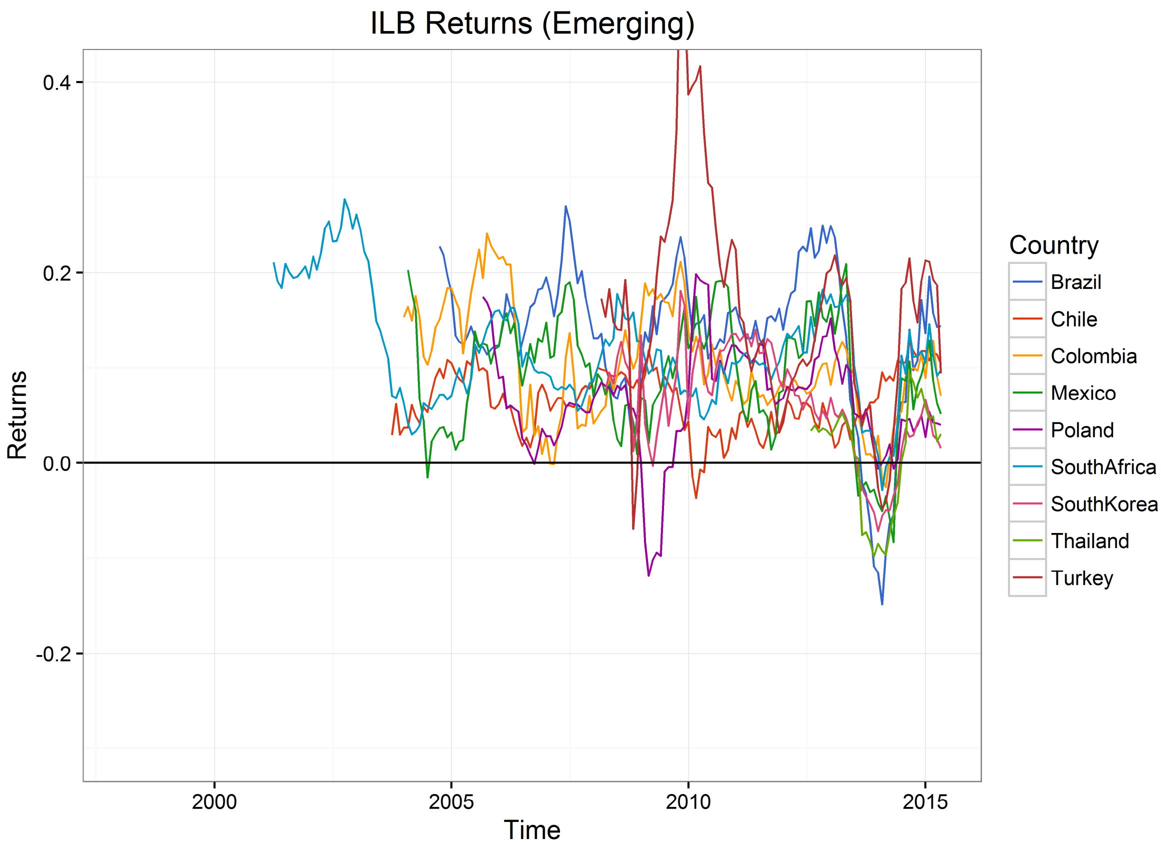


Figure 2 Monthly one-year overlapping ILB returns (emerging markets)

Generally, the main bottlenecks of this thesis are yields and interest rates data because they limit the potential length of the hypothetical series. Only for the US it was possible to obtain historical yield curve data beyond 25 years of length. A serious problem is yield curve data on emerging markets as these markets are still developing and have not issued many bonds during the past or had restricted access to capital markets at the time. As a result, yield curve data from emerging markets are considerably shorter than their developed counterparts. To calculate zero coupon yields out of the par yields from Datastream, I use bootstrapping as described in Hull (2014, p. 100-109). In principle, the idea is to start with the lowest maturity yield such as the six-month rate that does not include any coupons. By discounting the first coupon of a one-year bond trading at par and paying semi-annual coupons with the six-month zero rate it is possible to extract the one-year zero rate. The same procedure is done for all maturities up to five years to derive the zero coupon yield curve for each country. The risk fee rate is assumed to be the lowest below a year maturity zero rate. If not available, the interest rate on savings is used as a proxy.

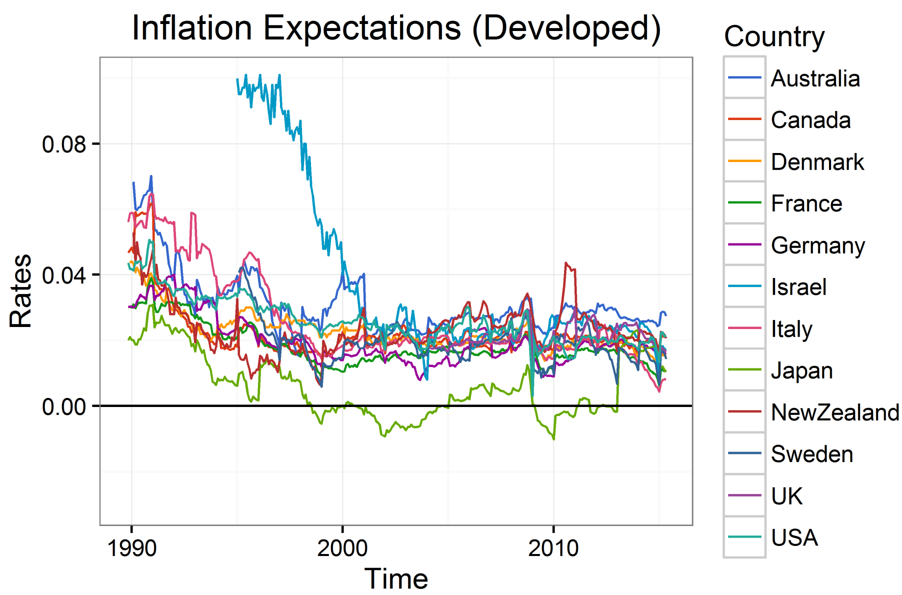
Inflation rates are calculated out of CPIs in the same way as returns by using equation 1. Table 2 shows the descriptive statistics of inflation for each country up to April 2015. The time series are trimmed to the maximum number of observations of available yields as only this data can be used in the retrieving process. Many emerging markets still exhibit higher inflation on average compared to developed countries, although the time series are shorter and only include the more recent years. Inflation rates in developing countries are slightly more volatile and have more pronounced maxima than in developed countries. However, not the whole history is reflected in the numbers of the emerging group in which inflation was sometimes excessively high. Some countries even experienced short periods of deflation except for Japan which had prolonged periods of deflation. Plots of inflation rates are located in Appendix A.

Table 2 Descriptive statistics of inflation

Country	**Start**	**N**	**Mean (%)**	**St.Dev.(%)**	**Min (%)**	**Max (%)**
Australia	Sep-91	284	2.51	1.24	-0.45	5.95
Brazil	Sep-10	56	6.02	0.70	4.60	7.85
Canada	Sep-91	284	1.82	0.94	-0.95	5.31
Chile	Sep-12	32	3.21	1.24	1.47	5.53
Colombia	Aug-09	69	2.80	0.69	1.74	4.53
Denmark	Sep-91	284	1.96	0.73	-0.10	4.29
France	Jan-94	256	1.39	0.73	-0.76	3.50
Germany	Sep-91	284	1.75	0.95	-0.50	4.69
Israel	Mar-08	86	2.32	1.52	-1.02	5.38
Italy	Sep-91	284	2.59	1.38	-0.60	5.92
Japan	Sep-91	284	0.30	1.10	-2.45	3.34
Mexico	Aug-11	45	3.76	0.48	2.96	4.66
New Zealand	Sep-91	284	2.12	0.98	-0.15	4.50
Poland	Mar-06	110	2.49	1.61	-1.35	4.90
South Africa	Sep-01	164	5.61	2.56	0.17	12.16
South Korea	Mar-03	146	2.70	1.12	0.36	5.74
Sweden	Sep-91	284	1.47	1.56	-1.89	7.79
Thailand	May-07	96	2.46	2.16	-4.51	8.78
Turkey	Apr-08	85	7.77	1.74	3.91	11.39
UK	Sep-91	284	2.82	1.25	-1.58	5.44
USA	Jun-52	768	3.64	2.68	-2.13	13.80

Note: Inflation rates are calculated out of the corresponding CPI of a country as monthly one-year rates. Last month is April 2015. The amount of observations (N) is trimmed to the maximum number of observations of nominal yields. *Source*: Datastream.

Equity indices (MSCI) are collected with Datastream as well. Inflation expectations on a monthly frequency are available through Datastream provided by the survey firm Consensus Economics. According to its website[1], the firm polls 700 economists monthly on macroeconomic variables since 1989. The expectations are available as point forecasts of the actual and next calendar year. For this thesis, I use the forecasts of next calendar year's inflation as expectations directly without any pre-processing of the data. Figure 3 illustrates the evolution of inflation expectations through time. In general, the expectations for developed countries are far more stable and change by a lower amount than those in emerging markets.



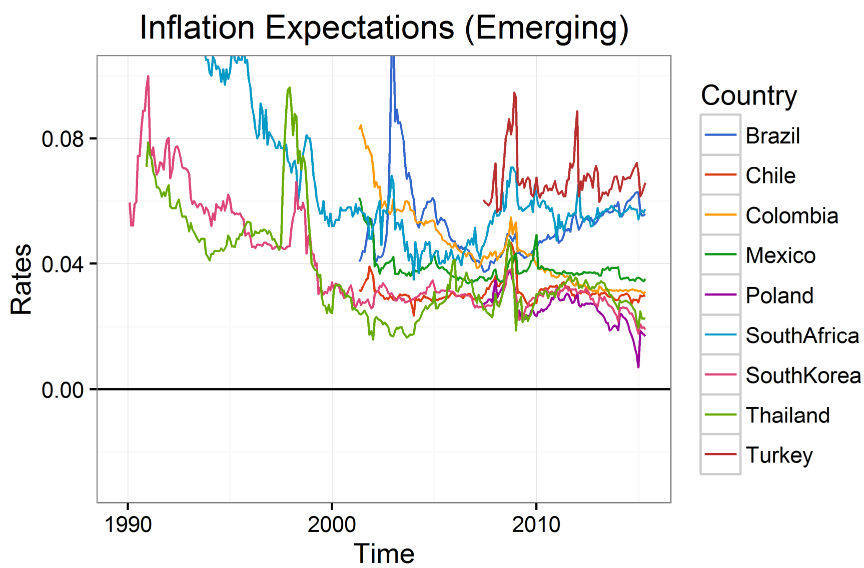


Figure 3 Inflation expectations

[1] www.consensuseconomics.com

4 Methodology

This section outlines the methods employed to construct hypothetical ILB returns.

4.1 Bond Math

The relationship between the fair market price P and the yield Y_T of a zero coupon bond with a maturity T and a face value of 1$ is (see Alexander (2008, p.1-24)):

$$P = \frac{1}{(1+Y_T)^T} \tag{2}$$

The formula assumes discrete compounding on an annual basis. For continuous compounding, the relationship can be written as:

$$P = \exp(-T\,y_T) \qquad \Leftrightarrow \qquad y_T = -\frac{1}{T}\ln P \tag{3}$$

where $y_T = \ln(1+Y_T)$. In this study, I use continuous compounding when dealing with bond prices and yields to avoid dealing with changing discrete compounding frequencies. The theoretical relationship between the nominal yield Y^N, real yield Y^R and inflation Π is described by the Fisher equation:

$$(1+Y^N) = (1+Y^R)(1+\Pi)$$

In logarithmic terms, and for small values the equation simplifies to a sum. Moreover, when looking ex-ante one has to take inflation expectations instead of inflation, as the yields are promised yields and received in the future, where the exact level of inflation is unknown. The ex-ante Fisher equation with logarithmic returns is:

$$y^N = y^R + \mathrm{E}(\pi) \tag{4}$$

Prior research often rejected the Fisher equation and introduced a time-varying risk premium RP in form of an inflation risk and liquidity premium. Depending on which effect currently outweighs the other, the empirical relationship between real and nominal yields may change through time. This can be seen when looking at the decomposition of the covariance:

$$\begin{aligned} \operatorname{cov}(y^N,\ y^R) &= \operatorname{cov}(y^N, y^N - \mathrm{E}(\pi) \pm RP) \\ &= \operatorname{var}(y^N) - \operatorname{cov}\Big(y^N, E(\pi)\Big) \pm \operatorname{cov}(y^N, RP) \end{aligned}$$

The consequence of the existence of risk premiums is that breakeven inflation defined as $\mathrm{BEI} = y^N - y^R$ is a biased market-based measure of inflation expectations. Positive risk premiums in ILB yields bias the expectations of the market downwards (see Auckenthaler et al. (2015)).

4.2 Kothari and Shanken

Kothari and Shanken (2004) create hypothetical zero coupon ILB bonds with an assumed maturity of five years. As already indicated in equation (1), they use monthly overlapping annual observations to avoid noisy time series and short-run positive autocorrelation. Returns and inflation are calculated year-over-year, whereas bond yields and prices are annualized. The log returns of a five-year constant-maturity hypothetical zero coupon bond can be modelled as

$$r_{t+12} = p_{t+12}^{[4]} - p_t^{[5]} \tag{5}$$

where $p_t^{[5]}$ is the log price of a five-year bond at month t and $p_{t+12}^{[4]}$ is the log price of a four-year bond twelve months ahead. In other words, an investor buys a five-year bond and pays the price $p_t^{[5]}$ and one year later, he sells the bond with a remaining maturity of four years for the price $p_{t+12}^{[4]}$. The difference between the two prices is the return received in a year. The price series themselves are

$$\begin{aligned} p_t^{[5]} &= v - 5y_t^{R[5]} \\ p_{t+12}^{[4]} &= v - 4y_{t+12}^{R[4]} \end{aligned}$$

where v is the face value received at the end of the maturity, $y_t^{R[5]}$ is the real yield of a five-year bond at month t and $y_{t+12}^{R[4]}$ is the real yield of a four-year bond at month $t + 12$. With the price equations above, equation (5) can be rewritten as:

$$r_{t+12} = p_{t+12}^{[4]} - p_t^{[5]} \ = \ 5y_t^{R[5]} - \ 4y_{t+12}^{R[4]} \tag{6}$$

Equation (6) can be generalized to create hypothetical returns from a zero coupon ILB with any maturity $m \geq 1$:

$$r_{t+12} = my_t^{R[m]} - (m-1)y_{t+12}^{R[m-1]} \tag{7}$$

Equation (6) and (7) are used throughout this paper to calculate hypothetical returns. For Kothari and Shanken the problem of creating hypothetical linkers now boils down to modelling inflation expectations. The connection between nominal yields and hypothetical real yields is given with the Fisher equation described in (4) and the risk premium is assumed to be zero. Kothari and Shanken use linear regressions with interest rates and past inflation as dependent variables to create inflation forecasts for the next five years. The justification of the model is based on the implicit assumption that investors have rational expectations. The specification for the one-year ahead inflation is

$$\pi_{t+12} = b_0 + b_1 i_t + b_2\left(y_t^{N[5]} - i_t\right) + b_3\pi_t + b_4 RB_t + e_{t+12} \tag{8}$$

with the following variables

- π_t as current inflation rate at month t
- i_t as nominal spot interest rate (proxy: one-year nominal zero coupon bond yield)
- $y_t^{N[5]} - i_t$ as the spread of a five-year nominal bond compared to the spot interest rate
- RB_t as the sum of the past 12 months' real returns on one-month T-bills
- e_{t+12} as residuals

Due to the overlapping sample, Newey-West standard errors with 11 lags are used to correct autocorrelation and heteroscedasticity in residuals of each regression (see Verbeek (2014)). The specification for the change of the inflation rate two years ahead $\Delta\pi_{t+24}$ is

$$\Delta\pi_{t+24} = \pi_{t+24} - \pi_{t+12} = b_0 + b_1\left(f_t^{[2]} - i_t\right) + b_2\pi_t + b_3 RB_t + e_{t+24} \tag{9}$$

where $f_t^{[2]}$ is the forward rate for the second year at month t. Generally, the forward rate can be calculated with the formula given in Hull (2014, p. 87) with any continuously compounded yields y_1, y_2 and any maturity T_1, T_2:

$$f_{T_1,T_2} = \frac{y_2 T_2 - y_1 T_1}{T_2 - T_1}$$

The changes of the inflation rate in the third year $\Delta\pi_{t+36}$ is specified as

$$\Delta\pi_{t+36} = \pi_{t+36} - \pi_{t+24} = b_0 + b_1\left(f_t^{[3]} - f_t^{[2]}\right) + b_2\pi_t + b_3 RB_t + e_{t+24}. \tag{10}$$

Analogously to (9) and (10), the changes of the inflation rate in the fourth and fifth year are calculated in the same manner. Finally, the inflation expectations at month t for each of the next five years are the sum of the fits of (8) and the fitted changes of (9) and (10):

$$
\begin{aligned}
E_t(\pi_{t+12}) &= \hat{\pi}_{t+12|t} \\
E_t(\pi_{t+24}) &= \hat{\pi}_{t+12|t} + \Delta\hat{\pi}_{t+24|t} \\
E_t(\pi_{t+36}) &= \hat{\pi}_{t+12|t} + \Delta\hat{\pi}_{t+24|t} + \Delta\hat{\pi}_{t+36|t} \\
E_t(\pi_{t+48}) &= \hat{\pi}_{t+12|t} + \Delta\hat{\pi}_{t+24|t} + \Delta\hat{\pi}_{t+36|t} + \Delta\hat{\pi}_{t+48|t} \\
E_t(\pi_{t+60}) &= \hat{\pi}_{t+12|t} + \Delta\hat{\pi}_{t+24|t} + \Delta\hat{\pi}_{t+36|t} + \Delta\hat{\pi}_{t+48|t} + \Delta\hat{\pi}_{t+60|t}
\end{aligned}
$$

These inflation expectations are put into equation (4) together with the nominal yields to get the real ILB yields, which are used in equation (6) to calculate hypothetical returns of a five-year zero coupon ILB.

4.3 Vector Autoregression (VAR)

The VAR approach to simulate ILB returns was formulated first by Cambell and Shiller (1996). I use the specification of Auckenthaler et al. (2015) which is the latest paper applying the VAR method and modify it for the use on monthly overlapping annual data. A general first-order VAR model with three variables has the following form (see Verbeek (2012)):

$$
\begin{bmatrix} Y_t \\ X_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} + \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ X_{t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \tag{11}
$$

The VAR system for hypothetical ILB yields is

$$
\begin{aligned}
y_t^R &= \delta_1 + \theta_{11} y_{t-1}^R + \theta_{12} y_{t-2}^N + \theta_{13}\pi_{t-13} + u_t^R \\
y_{t-1}^N &= \delta_1 + \theta_{21} y_{t-1}^R + \theta_{22} y_{t-2}^N + \theta_{23}\pi_{t-13} + u_t^N \\
\pi_{t-1} &= \delta_1 + \theta_{11} y_{t-1}^R + \theta_{12} y_{t-2}^N + \theta_{13}\pi_{t-13} + u_t^\pi
\end{aligned} \tag{12}
$$

where y_t^N is the nominal yield of a zero-coupon shortest maturity security in a given country (e.g. T-Bills in the US) for the month t. π_t is the year-over-year inflation rate each measured monthly and $\boldsymbol{u}_t$ are the residuals. The additional lag in nominal yields and inflation rates is to avoid multicollinearity, as the real yields y_t^R of a short-term bill are approximated with Fisher equation in (4). Additionally, it is assumed that inflation follows a random walk and the best predictor for inflation expectations are the past values of inflation.

The VAR system in equation (12) specifies the shortest maturity bond. A crucial assumption to derive the real yields with the VAR model is the rational expectations hypothesis of the term structure. The term premium is assumed to be zero and the long term rates can be derived from expected short term rates in the future. The theoretical underpinning is that rolling over a short term bond should give the same returns as holding a long term bond (see Cochrane (2005)). Therefore, I use (12) recursively to forecast the expected future short term rates up to five years in the future:

$$\begin{aligned} y^R_{t+1|t} &= \delta_1 + \theta_{11} y^R_t + \theta_{12} y^N_{t-1} + \theta_{13}\pi_{t-12} \\ y^R_{t+2|t} &= \delta_1 + \theta_{11} y^R_{t+1|t} + \theta_{12} y^N_t + \theta_{13}\pi_{t-11} \\ &\vdots \\ y^R_{t+60|t} &= \delta_1 + \theta_{11} y^R_{t+59|t} + \theta_{12} y^N_{t+58|t} + \theta_{13}\pi_{t-47} \end{aligned}$$

The hypothetical n-year zero coupon yield follows from the recursive forecasts and is therefore:

$$y_t^{R[n]} = \frac{1}{n}\sum_{i=0}^{n} y^R_{t+i|t} \tag{13}$$

With equation (13), I calculate hypothetical yields for a four and a five-year zero coupon bond and apply (6) to get hypothetical returns.

4.4 Chen and Terrien

The unique feature of Chen and Terrien's (2001) method is to model the inflation risk premium. Inflation is approximated and assumed to follow a random walk. The logic behind is that inflation is subjected to normally distributed random shocks with an expected value of zero. As a result, the best predictor for the future inflation rate $\mathrm{E}(\pi_{t+12})$ is the actual inflation rate π_t.

$$\pi_{t+12} = \pi_t + u_{t+12} \qquad u_t \sim \mathrm{N}(0,\sigma) \tag{14}$$

The inflation risk premium IRP_t for a given month t is modelled with

$$IRP_t = IRP^C \frac{\sigma_t}{\sigma} \tag{15}$$

where IRP^C is the initial or current value of the inflation risk premium, σ is the standard deviation of inflation of the most recent 36 months, and σ_t is the three-year rolling standard deviation of inflation. In other words, the initial value of the risk premium is scaled proportionally to the volatility of inflation itself. The initial value can be taken from the literature or estimated with the data. Chen and Terrien use the most current value by applying the Fisher equation on nominal and TIPS yields and calculate a value of 0.17% for the US. To simplify the analysis, I use 0.5% as initial value of the IRP for all countries. Finally, the real yields are calculated by subtracting the inflation expectations and IRP from the nominal yields.

4.5 Backfilling

Backfilling as presented in Page (2013) is a generic method based on maximum likelihood estimation (MLE) and bootstrapping to extend time series samples. The general idea is to extract the statistical relationship between a short and a long sample. This relationship is assumed to be stable and that it also would have held if the short sample would not have been limited. Accordingly, it is possible to backfill the short sample by exploiting this statistical relationship using MLE. As the relationship is not perfect and involves uncertainty, the retrieved returns are subjected to noise either drawn from a normal distribution or by resampling (bootstrapping) actual noise.

Applied to bonds, the short sample represents the actual ILB returns and the long sample the nominal bond returns. The subscripts S and L stand for short and long sample. Let I be the ILB returns and N the returns of nominal bonds, then the statistical relationship between nominal and actual ILB returns can be quantified with

$$\beta = \frac{\operatorname{cov}(I_s, N_s)}{\operatorname{var}(I_s)}. \tag{16}$$

In the next step, the hypothetical mean $\hat{\mu}_{I,L}$ of the new long ILB sample is created by updating the mean of short ILB sample $\mu_{I,S}$ with the beta-adjusted difference between the long and short sample mean of the nominal bond returns:

$$\hat{\mu}_{I,L} = \mu_{I,S} + \beta(\mu_{N,L} - \mu_{N,S}). \tag{17}$$

Consequently, the second statistical moments are also updated for the new long ILB sample:

$$\widehat{\operatorname{var}}(I_L) = \operatorname{var}(I_S) + \beta^2\big(\operatorname{var}(N_L) - \operatorname{var}(N_S)\big) \tag{18}$$

$$\widehat{\operatorname{cov}}(I_L, N_L) = \operatorname{cov}(I_S, N_S) + \beta\big(\operatorname{var}(N_L) - \operatorname{var}(N_S)\big) \tag{19}$$

The previously stated relationship β in (16) is also updated with (18) and (19):

$$\hat{\beta} = \frac{\widehat{\operatorname{cov}}(I_L, N_L)}{\widehat{\operatorname{var}}(I_L)} \tag{20}$$

With (20) and (17) it is possible to estimate the missing expected ILB returns conditional upon nominal bond returns for each month t:

$$\mathrm{E}(I_t|N_t) = \hat{\mu}_{I,L} + \hat{\beta}(N_t - \mu_{N,L}) \tag{21}$$

Equation (21) basically states that conditional returns are calculated by summing the new hypothetical mean of the ILB sample with the nominal beta-adjusted deviations from the mean. The conditional variance $\sigma^2_{I|N_t}$ of the backfilled returns reflects the uncertainty in $(1 - \hat{\rho}^2)$:

$$\sigma^2_{I|N_t} = \widehat{\mathrm{var}}(I_L)(1 - \hat{\rho}^2) = \widehat{\mathrm{var}}(I_L)\left(1 - \frac{\widehat{\mathrm{cov}}(I_L, N_L)^2}{\widehat{\mathrm{var}}(I_L)\,\mathrm{var}(N_L)}\right) \tag{22}$$

The crucial part of the (22) is $\widehat{\mathrm{cov}}(I_L, N_L)$, which is stated in (19). Fundamentally, the higher the correlation between nominal bond and actual ILB returns, the lower the conditional variance $\sigma^2_{I|N_t}$ will be. In the extreme case of $\widehat{|\rho|} = 1$ there is no uncertainty involved in the backfilling process and conditional volatility is zero as the ILB returns perfectly follow the behaviour of nominal bond returns, whereas in the case of $\hat{\rho} = 0$ there is maximum uncertainty and no information in the long sample can be used. In this particular case, the conditional variance is simply the estimated variance of the long ILB sample and no variance reduction takes place. Similarly, the conditional expected return in (21) for each month is always the hypothetical long sample average.

Backfilled ILB returns for each month t consist of the conditional expected return and a noise term incorporating uncertainty:

$$r_t^{ILB} = I_t = \mathrm{E}(I_t|N_t) + u_t \tag{23}$$

The noise term u_t has a mean of zero and a variance of $\sigma^2_{I|N_t}$. The distribution of the noise term can be either assumed or the empirical distribution is used. The latter can be obtained by recycling and resampling the error terms in the short sample with $u_{S,t} = I_{S,t} - \mathrm{E}(I_{S,t}|N_{S,t})$. The advantage of resampling is that higher moments like skewness and kurtosis are also considered and modelled within (23) and the backfilled returns are more realistic. I use both the resampling method and the normal distribution assumption. Thus, as the error term is stochastic and to ensure replicability I also use a Monte Carlo simulation to evaluate the impact of randomness on the returns.

4.6 ARMA

According to Ang et al. (2007), the simple ARMA(1,1) model is one of the best performing models to forecast inflation. Due to overlapping inflation rates, I apply an ARMA(1,1) model with a twelfth month lag to model inflation (see Verbeek (2014)):

$$\pi_t = \theta\pi_{t-12} + \gamma u_{t-12} + u_t \tag{24}$$

I apply recursive forecasts of inflation over the period of 60 months to model inflation expectations, conditional upon information at month t. The forecasts are applied in the Fisher equation to obtain real ILB yields.

$$\begin{aligned}\pi_{t+1|t} &= \theta\pi_{t-11} + \gamma u_{t-11}\\ \pi_{t+2|t} &= \theta\pi_{t-10} + \gamma u_{t-10}\\ &\vdots\\ \pi_{t+60|t} &= \theta\pi_{t+48|t} + \gamma u_{t+48|t}\end{aligned}$$

4.7 Moving Average (MA) and Surveys

The moving average method serves as a benchmark to other models both in terms of simplicity and accuracy. It uses inflation of the past two, the current and one year ahead in the future (see equation (25)). The small look-ahead bias is introduced by design. First, the goal is to extend historical ILB yields backwards as accurately as possible and the look-ahead bias can be justified by means of accuracy and utilization of more information in the data. Second, inflation expectations are basically unobservable and the moving average method may come closer to investors' inflation expectations. To a certain extent, the MA mimics investors' expectations forming process by incorporating past data and some ability to accurately predict inflation.

$$\mathrm{E}(\pi_{t+h}) = \frac{1}{4}(\pi_{t-24} + \pi_{t-12} + \pi_t + \pi_{t+12}) \qquad \forall h \geq 0 \tag{25}$$

Inflation expectations from surveys are available from the firm Consensus Economics, which provide this year and next year's expected inflation from a panel of economists.

$$\mathrm{E}(\pi_{t+h}) = \pi_{t+12}^{CE} \qquad \forall h \geq 0 \tag{26}$$

For the moving average and the consensus economics survey I make the assumption that expected inflation does not change with forecasting horizon. Again, the Fisher equation is used to reconstruct ILB yields and (6) to calculate hypothetical ILB returns.

4.8 Duration Matching and Evaluation Measures

The hypothetical ILB returns are compared to the actual returns of Barclays Capital ILB total return indices in Table 1. Actual inflation is added to the hypothetical ILB returns to obtain nominal returns, as Barclays' ILB returns are also in nominal terms. However, there is one big caveat when making a direct comparison: the simulated returns have a constant maturity and duration, while the indexed returns have a different maturity and duration for each country depending on the ever-changing composition of the indices. Moreover, real inflation-linked securities are not zero-coupon but make predefined coupon payments. Using the total return index data alleviates the problem with the coupons. The solution to different maturities and duration is to match the duration of the simulated returns to the duration of indexed returns for each month t. Zero coupon bonds have the advantage that maturity and duration are the same, hence equation (7) can be used to calculate duration matched hypothetical ILB returns. All the methods discussed before can be generalized to produce simulated returns of every maturity.

Nominal yields beyond five-years maturity $y_t^{N[m]}$ are approximated with the difference Δy_t between the four-year and five-year yield:

$$\Delta y_t = y_t^{N[5]} - y_t^{N[4]}$$

$$y_t^{N[m]} = y_t^{N[5]} + (m-5)\,\Delta y_t \quad \forall m \geq 5 \tag{27}$$

The argument for using an approximation for higher maturity nominal yields are data constraints. Especially emerging markets yields data are scarce and there are often no long enough time series available for each maturity bracket, resulting in a loss of observations. Furthermore, the method is very simple to implement, can produce interannual yields and is not limited to integer maturity. The disadvantage is the assumption of a linear distance between yields, which is questionable especially for longer maturities. I use this approximation method for each country to not distort individual countries disproportionally.

Another method would be to use so-called "comparator" bond indices from Barclays Capital. These indices consist of regular bonds without inflation protection and try to match their inflation-linked indices as close as possible. The idea is to use comparator bond returns with the inflation forecasts of the models described before. However, there are quite often significant differences in duration and maturity, because either there is no perfect match or sometimes a bond with a lower maturity is chosen due to liquidity reasons (see Barclays Capital (2004)).

Likewise, using comparator bonds instead of yield approximations is difficult or even not compatible with some methods like VAR or Backfilling, since these methods estimate yields and returns of hypothetical ILBs directly compared to other methods where inflation expectations are produced.

Evaluation is done primarily with two measures: mean squared error (MSE) and correlation. Simulated nominal ILB returns are compared to indexed returns from Barclays Capital. The MSE is tracking first moments, as it is punishing deviations from the benchmark returns, especially larger ones. With the MSE it is possible to track the level of simulated returns compared to the benchmark. The definition of the MSE is

$$MSE = \frac{1}{n-1} \sum_{t} (r_t^{SIM} - r_t^B)^2 \tag{28}$$

where r_t^{SIM} are the hypothetical (simulated) ILB returns of each method and r_t^B are the benchmark returns. The procedure is repeated for each method and each country over the sample of available benchmark returns. Another alternative to compare returns is the so-called tracking error. In contrast to the MSE, the tracking error emphasises the variance of active returns, which are the difference between the simulated and the benchmark return. It is therefore a measure for second moments and does not track the level of returns. In the extreme case of a series perfectly tracking the benchmark but being off by a constant, the tracking error is zero. The second measure is the correlation between simulated returns and benchmark returns, which makes the tracking error for this study redundant. With the correlation it is possible to assess whether the simulated returns exhibit the same statistical patterns as the indexed returns. The definition for the correlation is:

$$cor(r_t^{SIM}, r_t^B) = \frac{\sum_t (r_t^{SIM} - \overline{r^{SIM}})(r_t^B - \overline{r^B})}{\sqrt{\sum_t (r_t^{SIM} - \overline{r^{SIM}})^2 \sum_t (r_t^B - \overline{r^B})^2}} \tag{29}$$

The judgment of which method performs best is done via a combined ranking (CR):

$$CR_{i,c} = 0.5\,\mathrm{rank}(MSE_{i,c}) + 0.5\,\mathrm{rank}(correlation_{i,c}) \tag{30}$$

Each method i is ranked according its MSE and correlation relative to other methods on the same country c. Thereby, each method receives a combined rank on each country, similar to a horserace. The CR of each method is then averaged across all countries weighted by the number of observations. The method i with the lowest average CR is the best performing method.

5 Empirical Results

The empirical results are divided into two parts: First, the simulated ILB yields and returns are investigated for the United States only as most studies use US data almost exclusively. Second, the aggregate empirical results are evaluated to find the best method for creating hypothetical ILB returns.

5.1 The Case of the USA

Figure 4 depicts hypothetical ILB returns of each method against the indexed returns of Barclays Capital. The series spans over 17 years in total for the period from 1998 to April 2015. The returns are adjusted to match the duration of the benchmark in each month. In general, the generated return series vary substantially and there are considerable differences in levels and statistical behaviour. Using a random walk for inflation expectations or Chen and Terrien's method is not recommended for US data. Table 3 shows that both methods have the highest MSE and lowest correlation to the benchmark. The VAR(1) method suffers from relatively low correlation and misses the mean of the benchmark. Several reasons may explain the flatness. First, the four and five-year yields are constructed out of a one-month T-Bill assuming the rational expectations hypothesis holds. However, there is considerable evidence in the literature that the expectations hypothesis does not hold in practice and that longer maturity yields have term premiums embedded. Second, the VAR model is only able to explain about 24% of the variation in real yields and the coefficients of the real yield equation are low and indicate that information deceases quickly (see Table B3 in Appendix B).

Table 3 Comparison of simulated returns (USA)

USA ('98-'15)	Mean	St.Dev.	Skew.	Kurt.	MSE	Cor.
Random Walk	6.50	16.45	-0.55	3.04	24.93	0.28
Kothari & Shanken	6.26	10.40	-0.19	2.93	6.23	0.66
VAR	3.29	2.28	1.31	4.77	3.58	0.34
Chen & Terrien	5.92	16.32	-0.56	2.99	24.57	0.28
Backfill Normal	5.78	6.20	-0.36	2.98	4.25	0.39
Backfill Recycled	5.70	5.98	0.07	2.70	4.72	0.30
ARMA	8.30	12.82	-0.32	3.13	9.88	0.70
Moving Average	6.63	8.74	-0.08	2.77	4.05	0.69
Survey	6.86	8.70	-0.26	2.39	3.35	0.76
Benchmark	6.07	5.62	-0.51	3.23	0.00	1.00

Note: Returns are calculated as overlapping annual returns in percent. The corresponding benchmark is provided by Barclays Capital US ILB index. Returns are duration-matched. Data: USA, 1998-2015.

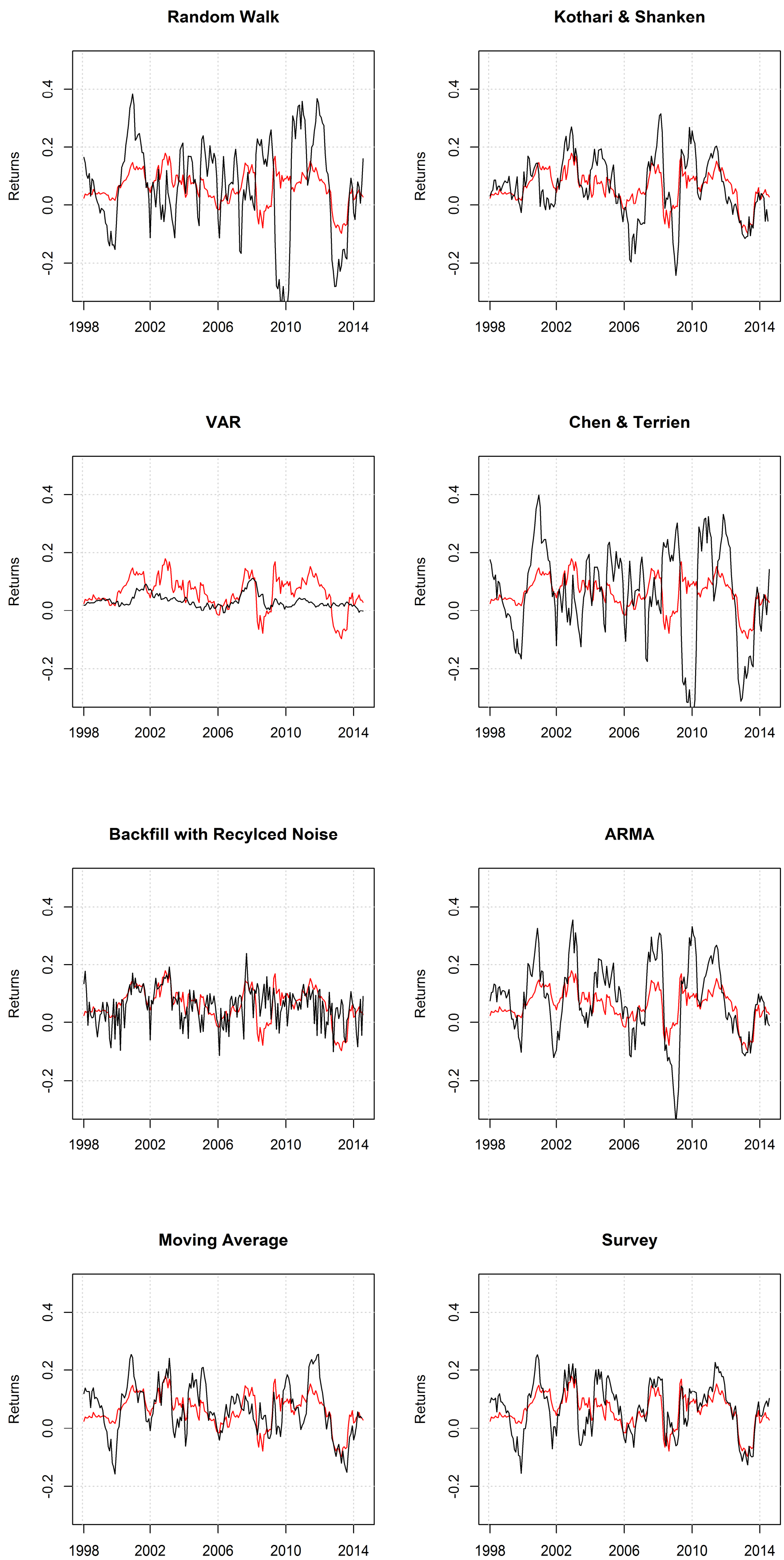


Figure 4 Comparison of simulated ILB returns against benchmark returns (US data)
(Black curve: Hypothetical ILB returns, red curve: Barclays Capital ILB index returns)

Third, the VAR model uses long forecasting horizons of which the forecasts are then averaged to obtain the hypothetical ILB yields. In combination with the low coefficients there is a low variation between yields of different maturities. The ARMA(1,1) model shares the same long forecast horizons with the VAR model but, in contrast, is much better in terms of correlation. The reason for the better correlation is that the ARMA model is used only to forecast inflation (see Table B4 in Appendix B) and hypothetical ILB returns are modelled indirectly via the Fisher equation. However, the ARMA model seems to undershoot and overshoot the benchmark several times leading to more pronounced spikes and a high MSE (quadratic punishment). As only a one-year lag $(t-12)$ is used, the ARMA model is slow in reaction and it is likely that inflation expectations are not simple ARMA forecasts. The Kothari and Shanken approach is slightly worse in terms of correlation compared to ARMA but the MSE is better. The mean level of the hypothetical returns is also closer to the benchmark returns. Kothari and Shanken's method disappoints with its performance as it is the data-hungriest of all methods but the additional data does not make it superior at all. Furthermore, it is also not grounded in theory that well to justify the additional need for data. A full replication of Kothari and Shanken's results can be found in Appendix B. MLE backfilling is another method with similar performance and drawing from the normal distribution instead of recycled noise yields better results. The MSE and correlation of backfilling are obtained from a Monte Carlo simulation with 10,000 runs. Backfilled returns had a lower MSE in each simulation run compared to ARMA and Kothari and Shanken but the correlation is rather volatile as it ranges from about 0.1 to 0.5 depending on the draw of the pseudo random number generator (see Appendix C). Therefore, when using backfilling, it is recommended to run a Monte Carlo simulation to analyse how noise is affecting the hypothetical ILB returns. There is a temptation to generate thousands of simulated runs to pick the best performing backfilled series. However, this is not recommended since the correlation measure is partly misleading for backfilled returns. The noise term is drawn randomly from a distribution and consequently affects the correlation between simulated and benchmarked returns. This is precisely why the correlation might be elusive as there is no certainty that the statistical relationship continues when extrapolating backwards. In other words, correlation is stochastic and averages of simulation runs should be taken.

Surprisingly, the second best and the best method on US data are the moving average and the survey inflation expectations. The moving average is a highly competitive and superior method, despite its simplicity. It involves a small look-ahead bias by design which – as it seems – comes much closer to unobservable real inflation expectations than theoretical models. The look-ahead bias in this case is not a distortion of the rankings since the goal was not to forecast future inflation accurately but to extrapolate ILB returns. The "champion" both in terms of MSE and correlation is the Consensus Economics survey. Surveys are quantitative as well as non-quantitative by nature. The participating economists are allowed to use quantitative methods for prediction, but they are also allowed to use subjective judgements, political reasoning and personal opinions. Adding subjective insights and averaging out over many participants captures a much richer part of reality and information which is not easily quantified with models can be incorporated. Surveys may also be the best method because the financial markets either use these surveys by themselves or investors form inflation expectations in a similar way as the surveyed economists do.

Hypothetical ILB yields are depicted in Figure A2 of Appendix A. The plots show a comparison of simulated yields to TIPS yields, both yields with a constant maturity of five years.

5.2 Cross-sectional Comparison

The next step is to check whether the conclusions drawn from the US data also hold for 20 other countries out-of-sample. About 700 observations belong to emerging countries and about 1700 observations to developed countries. Panel A of Table 4 displays the combined rank based on equation (30), which is the average rank of MSE and correlation. In general, there is considerable variation in average ranks within each method. For some countries like, for example Germany and France, there is more than one method with the top ranking, whereas for other countries like Australia the best combined rank is only three, indicating that there is no superior method both in terms of MSE and correlation. The caveat of the combined rank, however, is that it is a relative and ordinal measure. This makes it difficult to interpret the quality of the fit directly without referring to MSE and correlation. Panel B of Table 4 shows the aggregate results. For each method the weighted average of CR is calculated and the results are ranked again to make interpretation easier. For both the full and the divided samples the surveys win the contest. In this sense, the out-of-sample test supports the conclusions made previously with the US data. The second best method on the full sample is to use backfilling with normally distributed errors. Average MSE and correlation are determined with a Monte Carlo Simulation (see also Table C2 and C3 in the Appendix). Backfilling in general is better suited for developed countries than emerging markets. This can be seen by the abrupt drop in rankings for the recycled version in the emerging sample. Another caveat of backfilling is the implicit look-ahead bias as the benchmarked returns are used for both simulation and evaluation. The injected uncertainty through a noise term alleviates this problem.

Table 4 Cross-country comparisons

	N	K&S	VAR	C&T	BFR	BFN	ARMA	MA	Survey	RW
		A. Combined Rank (CR)								
Australia	207	3.5	6	8.5	3	4	5.5	3.5	3.5	7.5
Brazil	44	8	8	3.5	8	3	6	3.5	2.5	2.5
Canada	208	5	5	6	5	5	4	6	3	6
Chile	32	7	3	6.5	5	3	5.5	6	1	8
Colombia	69	4.5	7.5	6	6.5	4	5.5	3	1	7
Denmark	23	3.5	7.5	8	7	3	2.5	2.5	3.5	7.5
France	187	1.5	8	8.5	5	4	5	4	1.5	7.5
Germany	98	1	5.5	8	4.5	5.5	6.5	2.5	2.5	9
Israel	85	7	3	8	4	2	8	3	3.5	6.5
Italy	128	1	8	7.5	5	4	6	2.5	2.5	8.5
Japan	121	5.5	2	9	6	6	3.5	3	2	8
Mexico	44	6	5	5.5	9	7	4	1.5	2.5	4.5
New Zealand	207	5.5	6	8	3.5	5	4	3	4	6
Poland	96	4	5.5	8	4	3	5	7.5	1	7
South Africa	163	5.5	4	7	5	6	4	4	3	6.5
South Korea	86	6	6	7	7	6	4	2	1	6
Sweden	208	1	7	6.5	5	6	5	5	2	7.5
Thailand	34	7	6.5	7	7.5	6	2	3	1	5
Turkey	84	3	7.5	7.5	8	3	5	1	3	7
UK	123	6	6	6.5	3.5	4.5	3.5	6	4	5
USA	206	5	3.5	8.5	6	5	4.5	3	1	8.5
		B. Summary Rankings								
Full		4	7	9	4	2	4	3	1	8
Emerging		5	6	8	7	3	4	2	1	6
Developed		5	7	9	3	2	6	3	1	8

Panel A: Combined rank = average rank of MSE and correlation. The lower the combined rank (CR), the better the method. Each series generated by each method is compared to its country specific Barclays Capital ILB benchmark. All countries differ in the number of available observations. Last month is April 2015.
Panel B: For each method a weighted CR average is calculated, taking the number of observations (N) of each country into account. The resulting CR averages are ranked again, whereas "1" represents the best and "9" the worst method. See Figure 1 and 2 for the categorisation of countries.

The moving average method has a solid third place in the full sample and performs even better for emerging markets. Once again this supports the previously made conclusion on US data. The moving average method is on average superior to other econometric methods. Kothari and Shanken's method is about of the same quality as the ARMA model. Campbell and Shiller's proposed VAR model is worse. To conclude, the last ranks are occupied by the random walk and Chen and Terrien's approach. It appears that the attempt of modelling the inflation risk premium reduces the quality of hypothetical ILB returns.

Table 5 Cross-sectional comparison of MSE

	K&S	VAR	C&T	BFR	BFN	ARMA	MA	Survey	RW	N
Australia	6.97	4.03	23.88	2.30	2.43	12.81	4.49	5.04	19.57	207
Brazil	23.49	12.88	9.32	14.01	2.87	9.64	4.50	4.10	9.13	44
Canada	13.56	8.71	67.94	9.86	9.88	17.02	16.20	11.30	65.30	208
Chile	4.77	1.76	9.41	2.12	0.84	30.16	2.56	0.29	8.26	32
Colombia	6.39	2.73	7.85	2.61	1.13	1.86	1.76	0.80	8.32	69
Denmark	2.88	5.76	8.54	7.36	1.49	2.72	1.99	2.24	10.64	23
France	1.11	3.44	12.68	1.98	1.96	2.77	2.56	1.78	11.48	187
Germany	0.69	1.23	5.19	1.26	1.27	2.39	0.88	0.98	5.39	98
Israel	4.86	3.16	35.01	1.80	1.16	9.79	3.37	2.91	33.84	85
Italy	2.60	7.52	16.16	4.32	3.86	5.30	3.23	3.15	16.37	128
Japan	4.27	3.28	13.80	5.95	5.98	4.82	3.74	3.98	13.38	121
Mexico	8.03	6.21	7.00	10.27	7.23	7.63	3.16	4.92	6.82	44
New Zealand	5.89	3.89	12.27	2.79	2.87	7.24	3.91	4.11	8.91	207
Poland	7.15	5.69	25.81	6.38	5.98	6.79	7.20	3.86	24.89	96
South Africa	20.43	4.87	113.48	6.27	6.32	31.16	14.45	12.98	116.91	163
South Korea	3.68	3.90	17.85	5.30	5.21	4.47	3.00	2.18	14.88	86
Sweden	2.30	3.25	31.81	2.51	2.65	7.32	5.49	2.31	32.68	208
Thailand	26.02	4.20	6.07	5.92	4.69	1.15	3.88	0.80	5.40	34
Turkey	6.17	16.42	19.89	19.50	6.10	8.83	5.26	5.90	18.78	84
UK	8.01	7.40	144.09	5.03	5.11	18.09	28.82	14.48	135.36	123
USA	6.26	3.60	24.69	4.58	4.44	9.93	4.07	3.37	25.05	206
Average	7.16	5.13	35.38	5.11	4.22	10.25	6.86	5.08	34.09	
Rank	6	4	9	3	1	7	5	2	8	

Note: The values in the table is the MSE for each method compared to Barclays Capital ILB indexes. The MSE is scaled by 1000 due to interpretation. The average MSE is weighted by the number of observations available for each country. Last month April 2015.

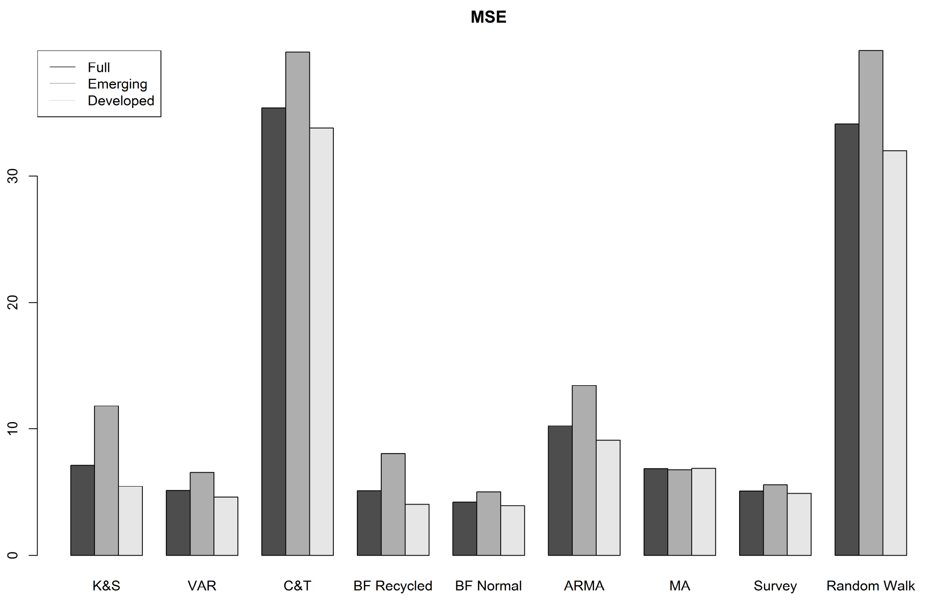


Figure 5 Weighted average of MSE for each method

Table 6 Cross-sectional comparison of correlation

	K&S	VAR	C&T	BFR	BFN	ARMA	MA	Survey	RW	N
Australia	0.71	-0.15	0.28	0.46	0.45	0.55	0.69	0.69	0.29	207
Brazil	0.78	0.46	0.93	0.58	0.85	0.84	0.92	0.92	0.93	44
Canada	0.10	-0.12	0.22	0.02	0.02	0.29	0.04	0.22	0.20	208
Chile	0.26	0.80	0.46	0.38	0.57	0.80	0.33	0.89	0.25	32
Colombia	0.76	0.42	0.70	0.51	0.67	0.65	0.75	0.84	0.68	69
Denmark	0.94	0.17	0.17	0.53	0.82	0.96	0.89	0.86	0.58	23
France	0.78	0.38	0.40	0.54	0.54	0.67	0.72	0.78	0.45	187
Germany	0.74	0.43	0.40	0.51	0.51	0.51	0.70	0.73	0.35	98
Israel	-0.08	0.44	0.28	0.28	0.37	-0.19	0.49	0.36	0.28	85
Italy	0.79	-0.02	0.38	0.68	0.70	0.54	0.78	0.76	0.35	128
Japan	-0.12	0.16	-0.41	0.00	0.00	0.23	0.16	0.29	-0.38	121
Mexico	0.83	0.52	0.62	0.28	0.35	0.88	0.86	0.84	0.68	44
New Zealand	0.53	0.26	0.50	0.50	0.50	0.75	0.73	0.67	0.61	207
Poland	0.13	-0.19	-0.15	0.03	0.03	-0.09	-0.15	0.30	-0.12	96
South Africa	0.36	0.13	0.35	0.06	0.05	0.74	0.51	0.62	0.36	163
South Korea	0.10	0.13	0.25	0.15	0.15	0.48	0.54	0.64	0.33	86
Sweden	0.70	0.05	0.38	0.36	0.35	0.53	0.49	0.64	0.37	208
Thailand	0.45	-0.12	0.35	0.21	0.26	0.88	0.56	0.89	0.47	34
Turkey	0.79	-0.19	0.63	0.55	0.77	0.63	0.83	0.76	0.62	84
UK	0.14	-0.18	0.22	0.21	0.21	0.36	0.22	0.23	0.24	123
USA	0.66	0.34	0.28	0.31	0.32	0.70	0.69	0.76	0.28	206
Average	0.48	0.13	0.31	0.33	0.35	0.53	0.53	0.61	0.33	
Rank	4	9	8	7	5	3	2	1	6	

Note: The values in the table is the correlation for each method compared to Barclays Capital ILB indexes. The average correlation is weighted by the number of observations available for each country. Last month April 2015.

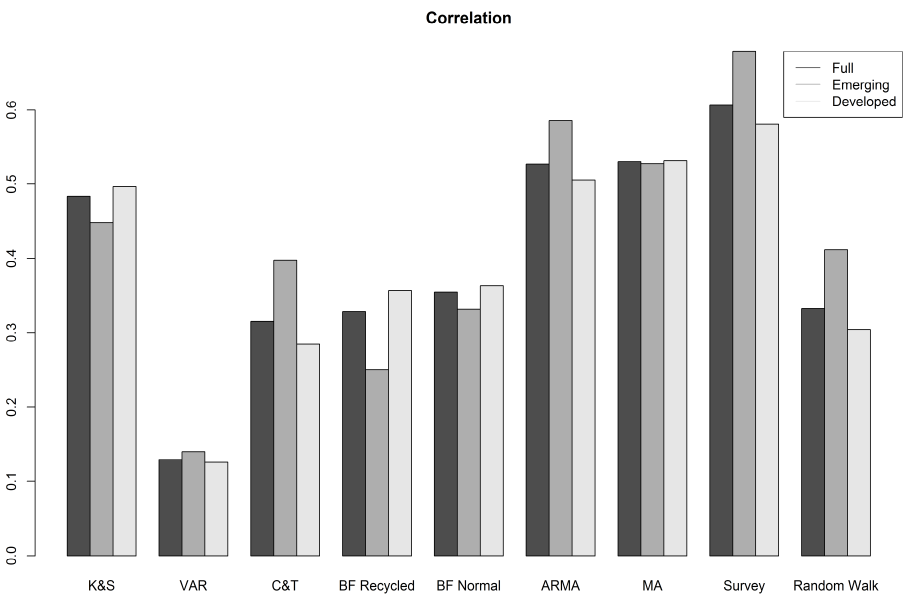


Figure 6 Weighted average of correlation for each method

Table 5 and 6 summarise the cross-sectional results for the MSE and correlation. On average, the best method in terms of MSE is MLE backfilling with normally distributed errors. As already mentioned, backfilling is predestined to have a low MSE as the benchmarked returns are used for both simulation and evaluation. The random noise injected has a larger impact on correlation, in which both backfilling methods are at the lower end of the rankings. Although the recycled version is worse at MSE and correlation, it is debatable whether the normally distributed version is preferable. According to Page (2013), resampling from empirical noise makes the retrieved returns far more realistic since skewness and kurtosis are also modelled. The simulated returns created with surveys have the highest correlation and the second lowest MSE. Strikingly, the correlation for UK and Canada are particularly low in contrast to the other 19 countries. This is due to the fact that the nominal yields are linearly approximated with equation (27) by taking the difference between the fourth and fifth year yield and projecting it linearly for longer maturities. Both the UK's and Canada's benchmarked returns have the highest average duration with 13.6 and 16 years whereas the other countries' durations are below 10 years (see Table 1). With higher duration, which is also the maturity for zero coupon bonds, it is more likely that the approximated nominal yields of (27) are less reliable because usually the slope of the yield curve is decreasing with maturity. Replacing the approximated nominal yields with so-called "comparator" yields from Barclays Capital confirms the explanation (see Table C1 in the appendix).

The moving average method has the second highest correlation numbers, although Kothari and Shanken's method and ARMA are very close. However, the moving average is also beating both methods with a lower MSE. Kothari and Shanken's method is better at MSE and a bit worse at correlation compared to ARMA. Nevertheless, it also has to be noted that it consumes much more data and the results are only marginally better than a simple ARMA(1,1) model for inflation expectations. The VAR model does not seem to work well with monthly data. The average MSE is relatively low whereas the average correlation is the lowest of all methods. The VAR approach gets the mean right explaining the low MSE but stays flat most of the time or reacts only modestly which explains the low correlation. The plain random walk model is not a good model for inflation expectations and has below average MSE and correlation. Adding Chen and Terrien's inflation risk premium scaled with inflation volatility to a random walk model is not recommended either. The model performs worse than the plain random walk inflation expectations. All in all, the out-of-sample tests support the results found with the US data. The three best performing methods are surveys, the moving average, and backfilling with normal distribution.

6 Application: Asset Allocation

This section illustrates the use of hypothetical ILBs in a portfolio setting. Surveys and the moving average method are used to model inflation expectations to extend actual ILB returns with hypothetical returns. Although backfilling with normal noise placed second in the overall rankings, it involves a stochastic element and makes reproducibility difficult. Table 7 shows the number of observations of actual ILB returns and simulated bonds for each country. The maturity of the hypothetical bonds is chosen to be 5 years. The extension is done by appending the survey series to the actual ILB return series. If the resulting time series is shorter than the moving average series the remaining observations of the moving average series are appended. E.g. the actual US ILB return series includes 207 months, is extended with surveys to 295 and with the moving average method to a total of 719 months.

Table 7 Extension of ILBs with simulated bonds

Country	ILB	Survey	MA	Total	Extended (%)
Australia	209	284	282	284	35.89
Brazil	128	56	56	128	0.00
Canada	209	284	283	284	35.89
Chile	140	32	32	140	0.00
Colombia	137	69	69	137	0.00
Denmark	24	284	283	284	1083.33
France	188	284	255	284	51.06
Germany	98	284	284	284	189.80
Israel	186	86	85	186	0.00
Italy	128	284	284	284	121.88
Japan	122	284	283	284	132.79
Mexico	136	45	44	136	0.00
New Zealand	208	284	283	284	36.54
Poland	117	97	109	117	0.00
South Africa	170	164	163	170	0.00
South Korea	86	146	146	146	69.77
Sweden	209	232	283	284	35.89
Thailand	34	96	96	96	182.35
Turkey	87	84	85	87	0.00
UK	396	124	283	396	0.00
USA	207	295	718	719	247.34

Note: "ILB" is the actual ILB return series (Barclays Capital ILB index). "Survey" and "Moving Average" (MA) are hypothetical bonds with five-year maturity. The values are the number of available observations.

Unfortunately, most emerging countries are lacking yield curve data and could not be extended by hypothetical bonds. For these countries, it is recommended to work only with the actual return series. Sampling from the empirical distribution is possible but comes with a loss of the correlation structure to other variables. For further analysis, I focus only on countries where the series could be extended by at least 50%. These countries are Denmark, France, Germany, Italy, Japan, South Korea, Thailand and the USA.

Table 8 shows the correlation between the different asset classes in each country. There is no substantial change in the correlation between ILBs and nominal bonds when the series are extended except Japan and Thailand. Japan experienced long periods of low inflation and deflation which resulted in low inflation expectation. When inflation expectations are low in absolute terms, hypothetical real yields will match the nominal yields more closely. This may explain the increased correlation in Japan's case. Thailand's correlation decreased as the inflation was much more dynamic during the appended period. The reasons for the relatively stable correlation for the remaining countries are twofold: First, hypothetical real yields are derived from the Fisher equation subtracting inflation expectations from nominal yields. Second, most extended time series start in 1991 when inflation was not too high. The only country with a longer series are the US. However, the average correlation between ILBs and nominal bonds remains near-constant.

Table 8 Correlation analysis

	without extension				with extension			
Country	**N**	**ILB, NOM**	**ILB, EQ**	**NOM, EQ**	**N**	**ILB, NOM**	**ILB, EQ**	**NOM, EQ**
Denmark	24	0.97	0.64	0.58	284	0.92	-0.03	-0.07
France	188	0.83	-0.34	-0.48	284	0.89	-0.13	-0.16
Germany	98	0.71	-0.35	-0.73	284	0.85	-0.21	-0.33
Italy	128	0.93	0.44	0.27	284	0.92	0.37	0.32
Japan	122	0.03	0.40	-0.57	284	0.54	0.18	-0.33
South Korea	86	0.43	0.31	0.22	146	0.65	-0.12	-0.25
Thailand	34	0.57	0.66	0.09	96	0.24	0.03	-0.34
USA	207	0.61	-0.15	-0.49	532	0.65	0.02	0.05

Note: "ILB" = Inflation-linked bonds, "NOM" = Nominal bonds, "EQ" = Equities. Data: Barclays Capital, Datastream, MSCI.

Figure C1 in Appendix C illustrates the relation between the inflation volatility and the correlation of nominal and ILB returns which is essentially the diversification benefit. As expected, the diversification benefit is larger (the correlation is lower) with more volatile inflation. The reason can be seen indirectly with a covariance decomposition:

$$\operatorname{cov}(y^N, y^R) = \operatorname{cov}\Big(y^N, y^N - \mathrm{E}(\pi)\Big) = \operatorname{var}(y^N) - \operatorname{cov}\Big(y^N,\, \mathrm{E}(\pi)\Big)$$

The covariance of the nominal yield and inflation expectations can be written to

$$\operatorname{cov}\Big(y^N,\, \mathrm{E}(\pi)\Big) = \sigma_{y^N}\, \sigma_{\mathrm{E}(\pi)}\, \rho\Big(y^N, E(\pi)\Big)$$

In principle, the decomposition shows that the covariance between nominal bond yields and hypothetical real yields decreases when the volatility of expected inflation $\sigma_{\mathrm{E}(\pi)}$ increases ceteris paribus as long as expected inflation and nominal yields are not negatively correlated. The same applies to returns since they are linear transformations of yields. Return correlations between countries can be found in Table C5 in the appendix.

The inflation hedging abilities of ILBs can be tested with the following regression

$$r_t^{ILB} - r_t^N = \alpha + \beta\pi_t + u_t \tag{31}$$

in which the excess returns of ILBs to nominal bonds are regressed on realized inflation. The standard errors are corrected with the method of Newey-West up to 11 lags (see Swinkels (2012)). β measures the sensitivity of excess returns to inflation. Ideally, the beta is positive and significant which means that ILBs provide a better hedge against inflation than nominal bonds. The results are reported in Table 9. Generally, there is no clear pattern when extending actual ILB returns with hypothetical ones. For some countries ILBs do provide a better hedge against inflation whereas an insignificant coefficient implies equal hedging ability. No significance is found for the USA when taking the full sample dating back to 1955. The cause is due to averaging as the ten-year rolling estimate of β indicates (see Figure C2). Further economic explanations are found in Brière and Signori (2012). They argue that changing economic regimes and the perception of central banks' ability to target inflation determine the correlation structure among assets and their inflation hedging ability.

Table 9 Inflation risk betas

	without extension			with extension		
Country	**Beta**	**p-value**		**Beta**	**p-value**	
Denmark	1.33	0.419		2.17	0.000	***
France	1.36	0.001	***	1.05	0.003	**
Germany	1.61	0.005	**	0.74	0.112	
Italy	-0.27	0.735		-0.11	0.882	
Japan	0.45	0.724		-0.01	0.989	
South Korea	2.65	0.000	***	2.43	0.000	***
Thailand	2.85	0.028	*	2.42	0.000	***
USA	1.78	0.001	***	0.66	0.247	

Note: *($p \leq 0.05$), **($p \leq 0.01$), ***($p \leq 0.001$)

To test whether ILBs should be considered when allocating assets, I conduct mean-variance spanning tests and check whether ILBs expand the efficient frontier of each country. For this purpose, the whole sample of ILB returns including hypothetical returns will be considered without further analysing certain periods, regimes or conditional variances and correlations. The mean-variance spanning test for each country is conducted by applying the following regression with Newey-West standard errors (11 lags) to account for heteroscedasticity and autocorrelation:

$$r_t^{ILB} = \alpha + \beta_1 r_t^N + \beta_2 r_t^{EQ} + u_t \tag{32}$$

The ILB returns are regressed on the returns of nominal bonds and equities. With the hypothesis of $\alpha = 0$ and $\beta_1 + \beta_2 = 1$ it is possible to test with a joint Wald test whether ILB returns can be replicated with the other two existing asset classes. In other words, the assumption behind the null hypothesis is that ILB returns are a linear combination of nominal bonds and equity returns as each point of the efficient frontier can be reached with the two assets. Shorting assets is not allowed and a risk-free asset is assumed to be non-existent. The two-funds theorem is met when no weight is assigned to the tested asset in the tangency portfolio ($\alpha = 0$) and the global minimum variance portfolio ($\sum_i \beta_i = 1$). Further details can be found in Kan and Zhou (2012).

Table 10 reports the results of the mean-variance spanning tests. Using only the actual ILB returns provided by Barclays Capital, there are three countries with significant p-values for both restrictions. However, this significance is caused by negative alphas. This indicates that it is better to short ILBs for these countries as they are dominated by nominal bonds (see Table C4). Appending hypothetical bond returns adds value to five out of eight countries as the p-value of both hypotheses combined is below 0.05. The results support the insights of Brière and Signori (2009). They conclude that French ILBs and US TIPS do not diversify because the correlation patterns have changed dramatically since 2003. Their data, however, ended 2007. The spanning tests cannot reject the null hypothesis and approve the conclusion for actual ILB returns. However, looking at the extended series with hypothetical ILBs for both France and the USA the efficient frontier is expanded and the results are significant. The test results also support the outcomes of Kothari and Shanken (2004), who tested optimal asset allocations with hypothetical ILBs. They found that about half of the weights are assigned to ILBs with variations depending on the assumed risk premium and expected returns in general. The optimal asset allocation of minimum variance portfolios for all countries is reported in Table 11. For seven out of eight countries the weight of ILBs increased substantially after including the hypothetical ILB returns. Mostly, nominal bonds have been substituted for ILBs. A caveat has to be considered: mean-variance optimization is sensitive to expected returns.

Table 10 Mean-variance spanning tests

Country	N	α	β Nom	β Eq	P(α = 0)	P(Σβ = 1)	P(Both)
			without extension				
Denmark	24	-0.06	0.98	0.10	**0.00**	0.07	**0.00**
France	188	0.01	0.85	0.01	0.51	0.16	0.36
Germany	98	-0.01	0.97	0.05	0.75	0.95	0.29
Italy	128	-0.02	1.14	0.07	**0.02**	**0.03**	**0.04**
Japan	122	-0.01	1.08	0.14	0.86	0.76	0.95
South Korea	86	0.03	0.62	0.06	0.51	0.57	0.81
Thailand	34	-0.06	0.61	0.32	**0.00**	0.80	**0.00**
USA	207	0.01	0.72	0.06	0.67	0.49	0.70
			with extension				
Denmark	284	0.00	0.87	0.01	0.96	0.21	**0.05**
France	284	0.00	0.82	0.00	0.56	**0.00**	**0.00**
Germany	284	0.00	0.83	0.02	0.69	0.06	**0.02**
Italy	284	0.00	0.86	0.03	0.91	0.12	0.21
Japan	284	0.00	0.81	0.09	0.93	0.68	0.85
South Korea	146	0.01	0.93	0.01	0.82	0.83	0.97
Thailand	96	0.02	0.32	0.03	0.71	0.12	**0.01**
USA	532	0.03	0.60	0.00	0.22	**0.01**	**0.02**

Note: See equation 32 for the regression. The three rightmost columns are p-values of the Wald test. Bold, when $p \leq 0.05$. The hypothesis $\alpha = 0$ tests improvements in the tangency portfolio, whereas $\sum_i \beta_i = 1$ tests improvements in the minimum variance portfolio.

Table 11 Minimum variance portfolios

Country	N	Mean (%)	St.Dev. (%)	w ILB	w NOM	w EQ
			without extension			
Denmark	24	9.87	6.31	0.00	0.68	0.32
France	188	5.32	3.65	0.29	0.61	0.10
Germany	98	4.84	2.14	0.00	0.90	0.10
Italy	128	5.81	6.41	0.00	1.00	0.00
Japan	122	2.10	1.58	0.00	0.96	0.04
South Korea	86	5.59	3.33	0.13	0.87	0.00
Thailand	34	4.48	4.69	0.14	0.77	0.09
USA	207	6.08	3.96	0.18	0.67	0.15
			with extension			
Denmark	284	6.59	5.21	**0.76**	0.18	0.06
France	284	6.35	4.63	**0.76**	0.17	0.08
Germany	284	6.08	3.79	**0.47**	0.46	0.07
Italy	284	8.00	8.15	**0.73**	0.27	0.00
Japan	284	3.48	3.55	**0.10**	0.83	0.07
South Korea	146	5.77	3.59	0.05	0.89	0.07
Thailand	96	5.02	3.66	**0.34**	0.61	0.05
USA	532	7.70	5.50	**0.57**	0.33	0.10

Note: The prefix "w" stands for the weight assigned to obtain the minimum variance portfolio. All weights add up to 1. Short selling is not allowed and no risk-free asset is included. Bold, when ILB weight has increased.

For the portfolio optimization I use the sample means as expected returns (see also Table C4) while the previous cited papers make assumptions about expected returns. Figure 7 plots the efficient frontier of each country and how it is affected when ILBs are introduced as an additional asset. The whole sample is used, which means that the efficient frontier without ILBs includes both nominal bond returns of the comparator bond index and the stored yield curve. For most countries, the ILBs have a lower volatility and expected returns than nominal bonds, and hence expand the frontier to the left bottom. The plot of Italy is quite interesting, as ILBs clearly expand the frontier in the plot, whereas the spanning test failed to reject the null hypothesis. Italian bonds, however, had the highest variances of all countries. Thus, the standard errors of the coefficients in the regression (equation 32) were also larger, which explains the insignificant test results.

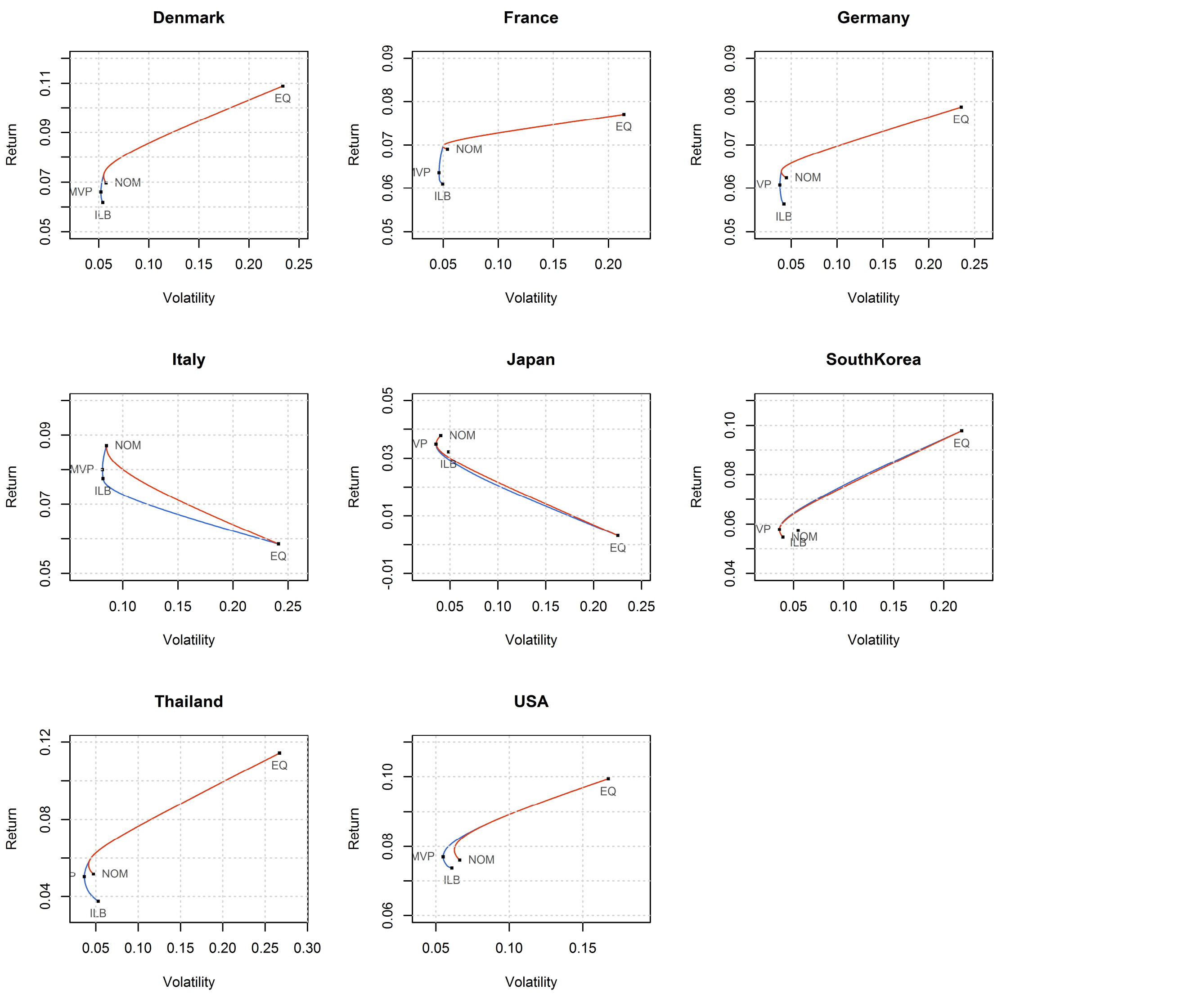


Figure 7 Efficient frontier with (blue) and without ILBs (red)

7 Conclusion

In this thesis, I compared several proposed and self-developed methods to create hypothetical inflation-linked bonds. The purpose of hypothetical ILBs is to mitigate the problem of short time series and data constraints. The study involves individual data sets of 21 countries consisting of nominal yields and inflation rates. Particular attention is paid to the US, as many studies use this data exclusively. Nine different methods are studied in detail and compared against each other in order to find the best fitting hypothetical ILBs.

In principle, there are two ways to create simulated ILBs. The direct approach uses actual ILB returns to create simulated ILB returns directly, whereas the indirect approach uses nominal yield curves and inflation expectations to derive hypothetical returns with the Fisher equation. I construct five-year zero coupon ILBs in the same way as Kothari and Shanken (2004) do and generalize their calculations to match the durations of the benchmark. For the evaluation, ILB total return indices from Barclays Capital are used as benchmark. MSEs and correlations to the benchmark are computed for each country and method, and a final ranking is done based on the combination of the two measures.

The least performing methods are the random walk model and Chen and Terrien's (2001) approach. Both methods model inflation expectations as a random walk, with the latter also modelling the inflation risk premium. However, both methods suffer from a very high MSE and low correlation and are not recommended for further use. The VAR(1) model proposed first by Campbell and Shiller (1996) and used in several other papers also belongs in the lower performing group. Most of the time, VAR hypothetical returns stay flat or react only modestly. The problem is most likely caused by the relatively long forecasting horizon and monthly frequency as the original VAR approach was conducted on quarterly data. In addition, the assumption of the rational expectations hypothesis is necessary, which is often refuted in empirical studies. Another time series model is the ARMA(1,1) model that is used to model inflation expectations. Despite its simplicity, it is recommended by Ang et al. (2007) as it clearly beats other more complex models when forecasting inflation. In this case, the ARMA(1,1) model belongs to the middle performing group. Due to its time lags it reacts slowly to changes and often undershoots or overshoots the benchmark. This often leads to a high MSE. Kothari and Shanken's regression approach is the "hungriest" method for data, as involves inflation rates, yield spreads and forwards. For each year a separate regression is necessary which leads to a quite complex structure. In terms of performance, it is about of the same quality as the

ARMA(1,1) model. Kothari and Shanken's approach is slightly better at MSE, but has a lower correlation compared to the ARMA model. The very same conclusion also holds true for US data alone. Disappointingly, the additional data does not lead to a better performance.

The top three performing methods are backfilling, the moving average and surveys. As it seems, backfilling works better with a normally distributed error term instead of recycling empirical noise. This makes sense as in short and very dynamic ILB return samples large error terms would be re-used again and again. The weakness of this study, however, is that the skewness and the kurtosis of the simulated returns are not evaluated, which Page (2013) cites as the major reason of for using recycled noise. Furthermore, it is problematic that the benchmark returns are used for both simulation and evaluation. As a result, the evaluation of backfilled ILB returns is partly biased and probably needs more sophisticated evaluation strategies, e.g. cross-validation as explained in Varian (2014). Therefore, and due to reasons of replication, the two best methods for hypothetical bonds are the moving average and surveys. The moving average method beats every other econometric model, it is the simplest of all methods, and it does not need any further data. It contains a look-ahead bias, as it incorporates future inflation. However, it is not entirely clear as to why it works so well as inflation expectations are basically unobservable. One explanation might be that it reflects how market participants form inflation expectations by using past data and having some ability to accurately predict inflation. The indisputable winner of the rankings are the Consensus Economics surveys, which polls professional economists. Surveys provide a very rich and informative forecast as they may not only involve quantitative model predictions but also expert insights and opinions. In addition, the forecasts of each participant are combined by averaging. Hypothetical ILBs with survey expectations come closest to actual ILB returns which implies two possible explanations: Either market participants use these surveys directly or they form inflation expectations similarly to the polled economists. The results of this thesis also confirm previous findings such as in Ang et al. (2007) and in Andonov et al. (2010) that use several surveys.

As a last step, I show how hypothetical ILBs can be applied in order to answer questions about the asset allocation. For eight countries the time series of ILB returns could be extended by more than 50%. These are Denmark, France, Germany, Italy, Japan, South Korea, Thailand and the USA. The correlation structure to nominal bonds and equities does not change substantially. Moreover, the inflation risk betas do not show a clear pattern for ILBs compared to nominal bonds. It appears that extending actual bond returns with hypothetical returns weakens the inflation risk betas. This might be attributable to the fact that hypothetical bond returns do not perfectly match actual ILB returns and do not have any risk premiums embedded. It might also reflect that inflation risk betas are not constant but time-varying as the rolling estimates on the US sample indicate. In general, the diversification benefits of ILBs are greater when inflation is more volatile. The economic rationale is that only ILBs offer protection against unexpected inflation shocks (see Bekaert and Wang (2010)) which is more likely when inflation is more volatile. Turning to mean-variance spanning tests, ILBs provide value to five out of eight countries when hypothetical ILBs are included. The plots and test statistics indicate that ILBs improve the global minimum variance portfolio and expand the efficient frontier to the left bottom in the mean-variance space. Seven out of eight countries have a substantial weight increase in ILBs to achieve the global minimum variance portfolio. These findings support the conclusions of Kothari and Shanken (2004), who showed that ILBs are dominating nominal bonds in the asset allocation for a US portfolio. Only when a positive inflation risk premium is introduced more weight is allocated towards nominal bonds.

The main bottleneck of hypothetical ILBs lies in the available nominal yield curve data. Many countries did not have access to international capital markets, did not issue a lot of bonds in the past, or historical data is simply not publicly available. Despite these constraints, extending data to its potential hard limit is ultimately better than dealing with very short or no samples. This is especially useful for recent or future ILB issuances.

This thesis contributes to the literature by comparing different methods on how to create hypothetical ILBs, testing the methods out-of-sample on 21 countries and finding the best performing method. Potential future research may focus on updating the results with new data, further analysing the moving average method and testing other surveys. Moreover, hypothetical ILBs provide opportunities to do further research on risk premiums, asset allocation and policy.

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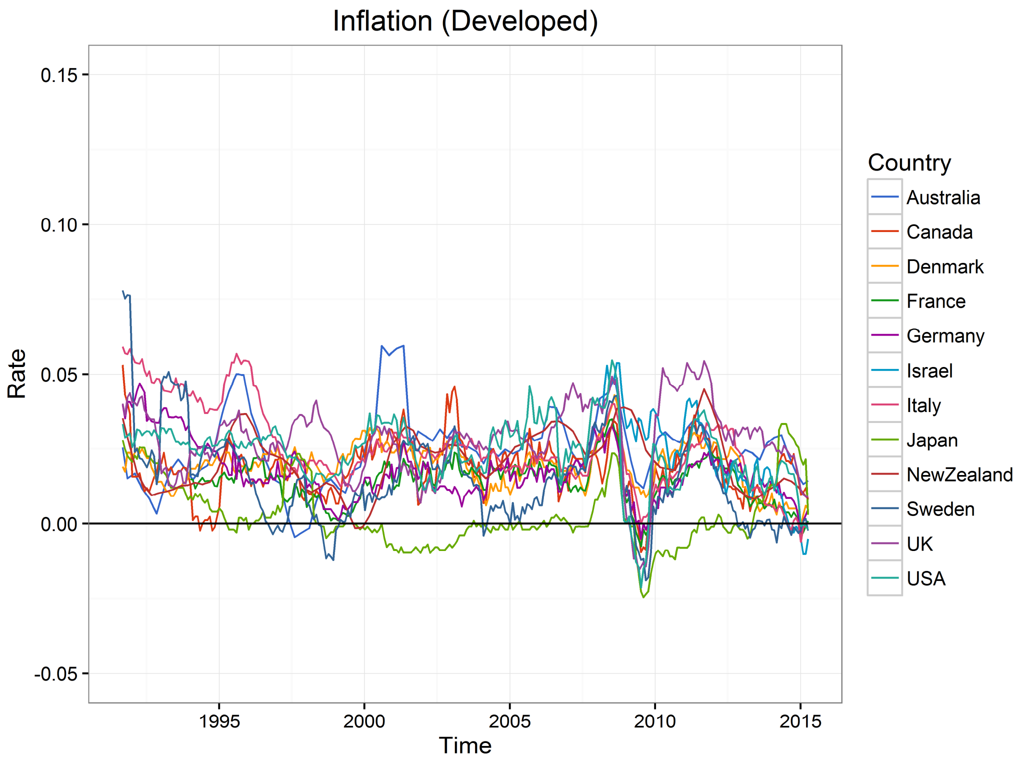
Appendix A: Plots



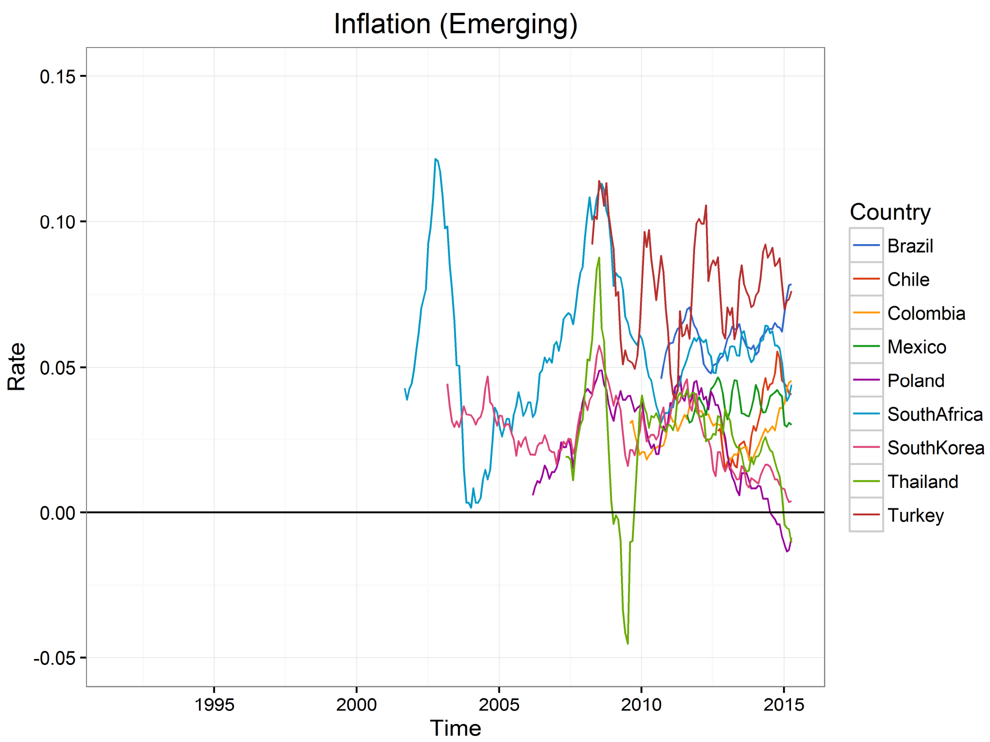


Figure A1: Inflation rates in developed and emerging countries from 1992 to 2015

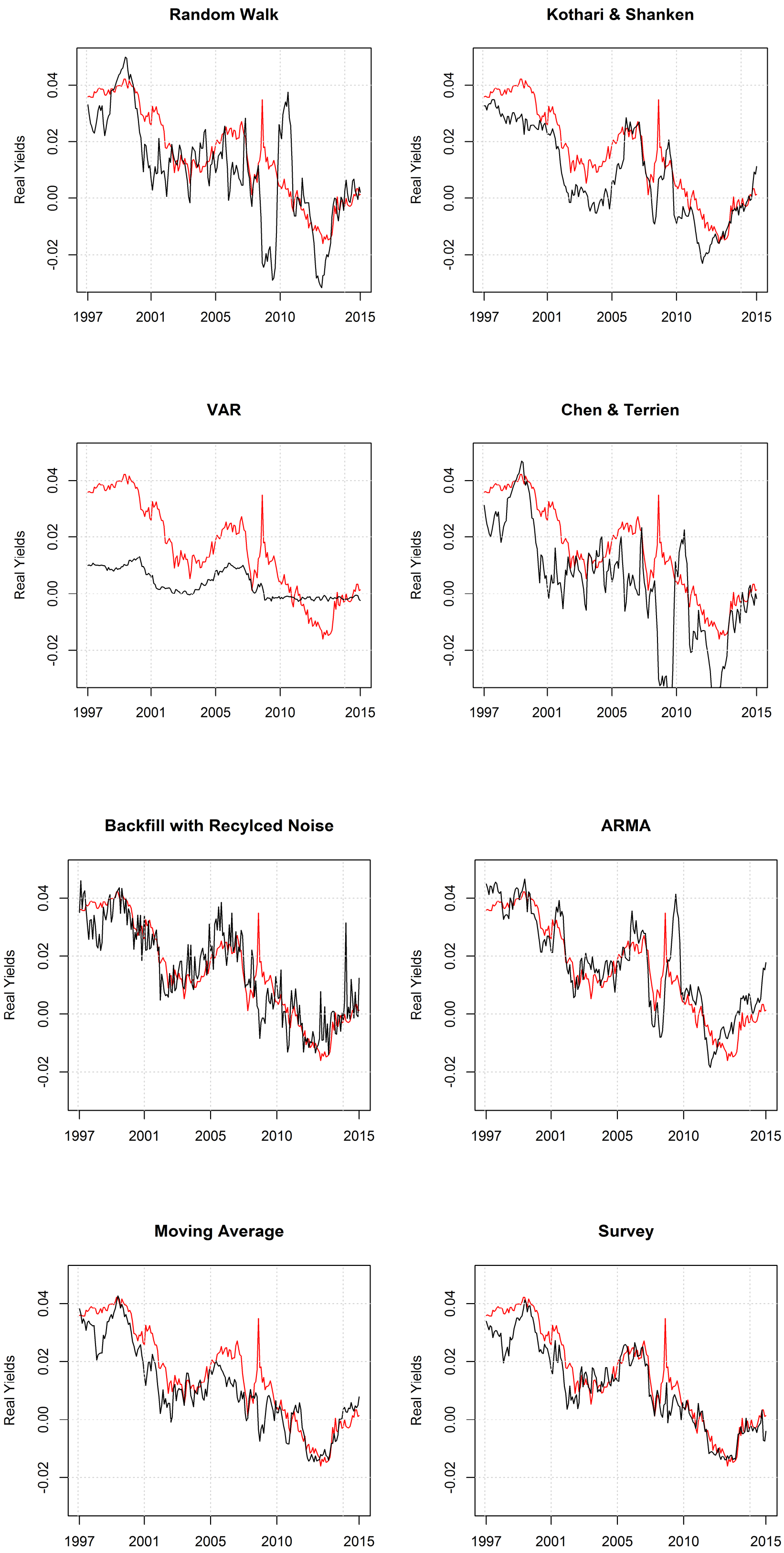


Figure A2: Five-years constant maturity real yields.
(Black curve: hypothetical ILB yields, red curve: TIPS yields)

Appendix B: The case of the USA

B.1. Replication of Kothari and Shanken (2004):

Table B1: Replication of Kothari and Shanken's results

	Kothari and Shanken		Own Results	
Variable	Coeff.	t-statistic	Coeff.	t-statistic
A. Forecast of 1-year ahead inflation				
Intercept	0.98	2.22	0.01	1.66
1yr spot	0.53	3.00	0.53	2.40
Spread	-0.21	-0.94	-0.25	-0.97
Lagged Inflation	0.17	1.10	0.19	0.89
Real Bill	-0.63	-3.18	-0.65	-2.27
Adjusted R^2	71.0%		70.3%	
B. Forecast of 2-year ahead inflation change				
Intercept	0.85	2.36	0.01	1.50
Spread	0.80	3.17	0.77	2.43
Lagged Inflation	-0.25	-3.91	-0.27	-2.66
Real Bill	-0.13	-1.09	-0.16	-0.65
Adjusted R^2	25.3%		26.2%	
C. Forecast of 2-year ahead inflation change				
Intercept	0.10	0.26	0.00	-0.02
Spread	0.83	2.25	0.91	2.32
Lagged Inflation	-0.07	-0.93	-0.06	-0.51
Real Bill	0.04	0.36	0.02	0.11
Adjusted R^2	9.5%		8.5%	

Note: The t-statistic is corrected for heteroscedasticity and autocorrelation

Table B2.1: Comparison of bond returns (Kothari and Shanken 2004, Table 3)

Return Type	Nominal Return	Real Return	Nominal Return	Real Return
Overlapping annual returns, 1953-2000, US Data				
	Hypothetical ILB		Nominal Bonds	
Mean	6.69	2.56	6.78	2.73
Median	5.76	2.02	5.07	2.04
St. Dev.	6.58	5.88	7.15	7.55
Skewness	0.33	0.29	1.28	0.84
Kurtosis	3.04	4.45	5.34	4.70
		Correlation Matrix		
Nominal (Hyp.)	1.00	0.89	0.52	0.34
Real (Hyp.)		1.00	0.59	0.58
Nominal (Nom.B.)			1.00	0.93
Real (Nom.B.)				1.00

Table B2.2: Comparison of bond returns (own results)

Return Type	Nominal Return	Real Return	Nominal Return	Real Return
Overlapping annual returns, 1953-2000, US Data				
	Hypothetical ILB		Nominal Bonds	
Mean	6.89	2.69	6.84	2.72
Median	5.84	2.05	5.20	1.98
St. Dev.	6.86	6.21	7.20	7.62
Skewness	0.34	0.26	1.26	0.84
Kurtosis	3.15	4.50	5.24	4.62
		Correlation Matrix		
Nominal (Hyp.)	1.00	0.89	0.54	0.36
Real (Hyp.)		1.00	0.60	0.59
Nominal (Nom.B.)			1.00	0.93
Real (Nom.B.)				1.00

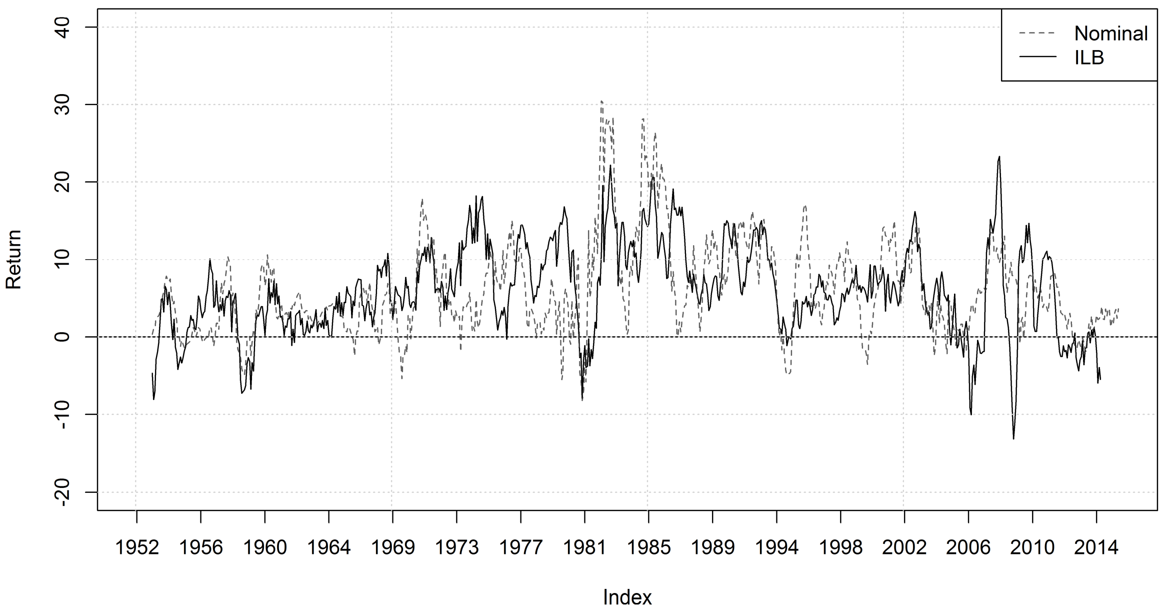


Figure B1: Replication of Kothari and Shanken's approach (US Data)
(Kothari and Shanken's original period: 1953-2000; extended up to 2015)

B.2. VAR

Table B3: Summary statistics of VAR(1) model, US-Data

Dependent Variable	Real	Nom	Infl
Const	0.00	0.00	0.00
(t-stat)	*1.29*	*3.49*	*-0.12*
Real Lag1	0.30	0.04	-2.93
	7.31	*3.82*	*-14.10*
Nom Lag1	0.23	0.88	5.47
	3.30	*45.29*	*15.60*
Infl Lag12	-0.01	0.01	0.48
	-2.33	*3.99*	*16.99*
adj. R^2	24.1%	97.8%	92.3%

Note: Monthly US Data, 1952-2015.

B.3. Chen and Terrien (2001)

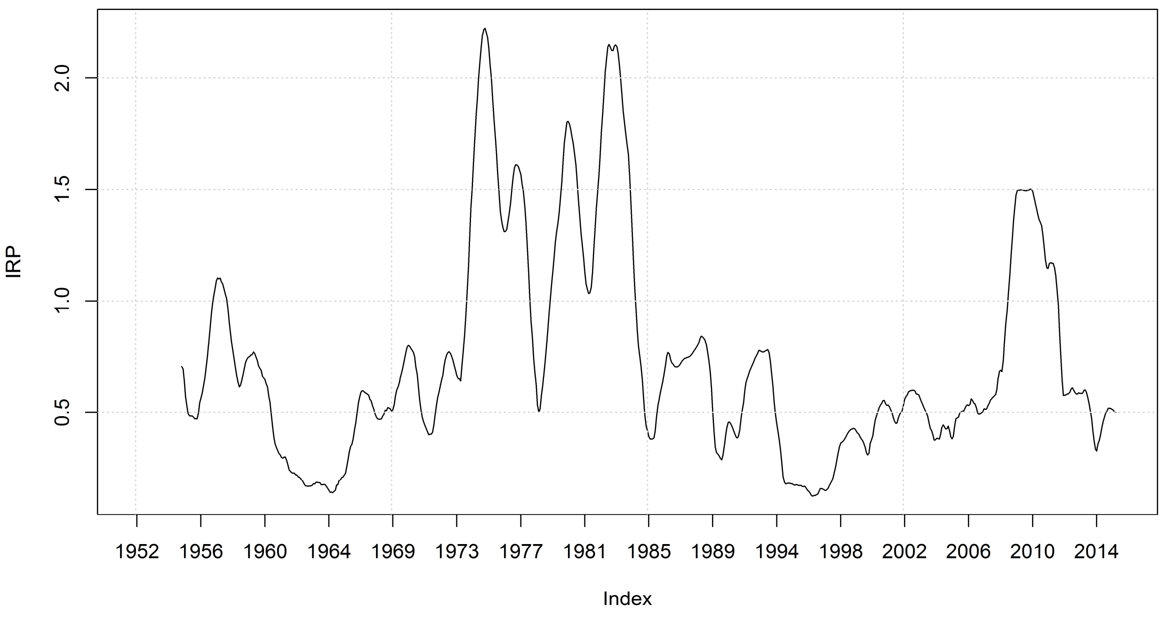


Figure B2: Modelled inflation risk premium (US Data)

B.4. Backfilling

Recycled noise is the difference between the benchmark returns from Barclays Capital and the backfilled conditional returns, see Figure B3.

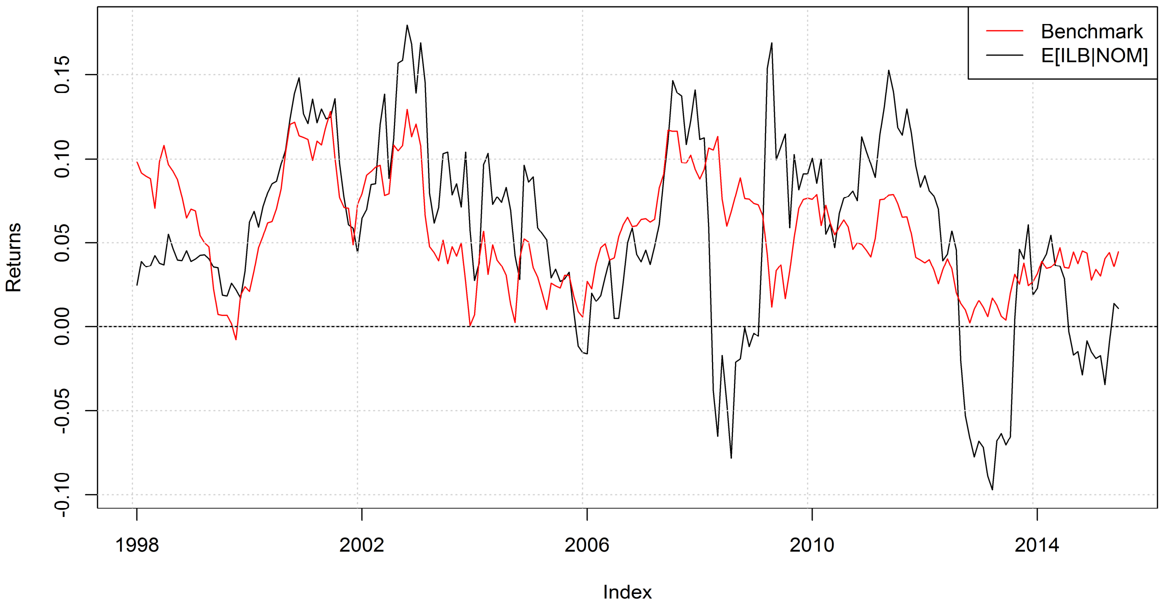


Figure B3: Benchmark returns and backfilled returns without noise (USA data)

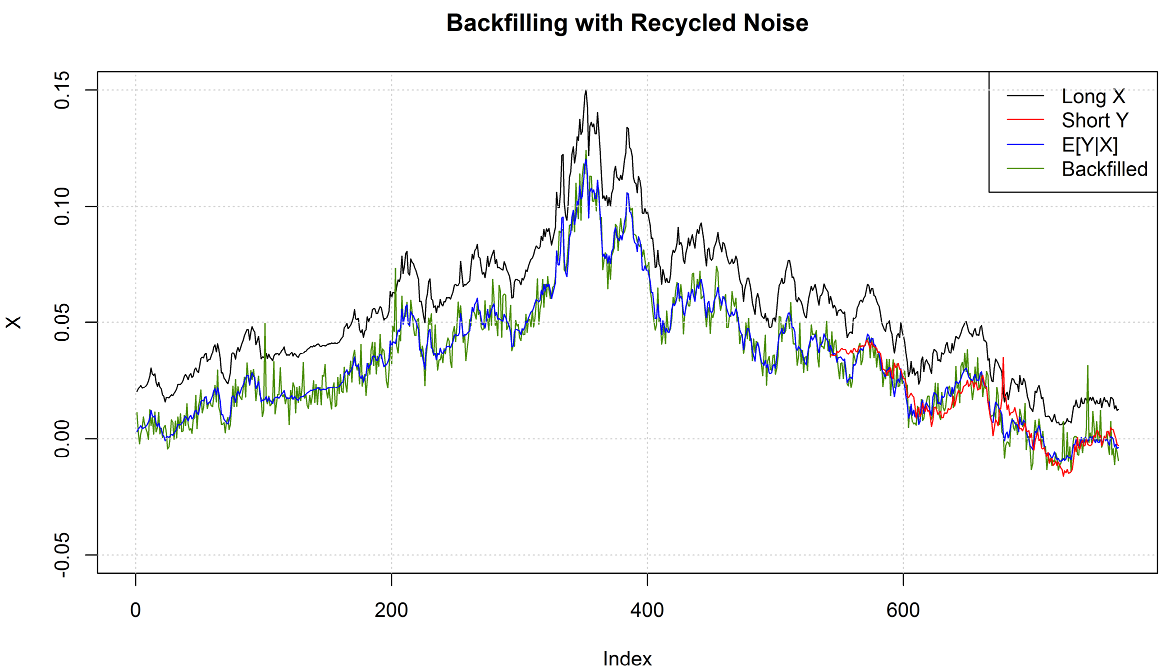
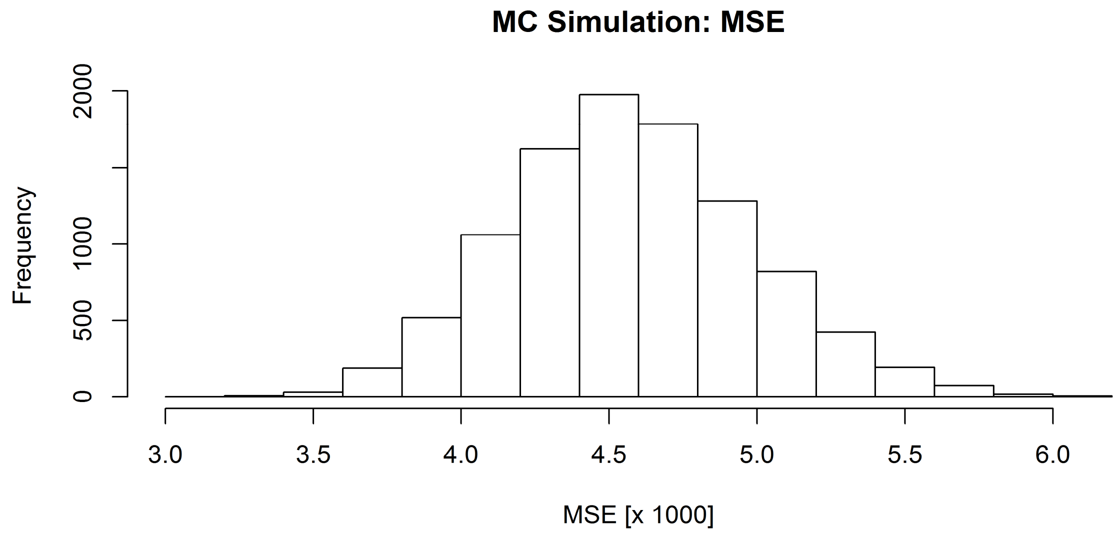


Figure B4: General illustration of the backfilling process over a sample

Backfilled returns exhibit a stochastic element since the noise term is either drawn from a normal distribution or bootstrapped from the actual noise sample (see Figure B3). The plots of Figure B5 show the distribution of the evaluation measures for backfilled returns with recycled noise on a Monte Carlo simulation with 10,000 runs on US data.



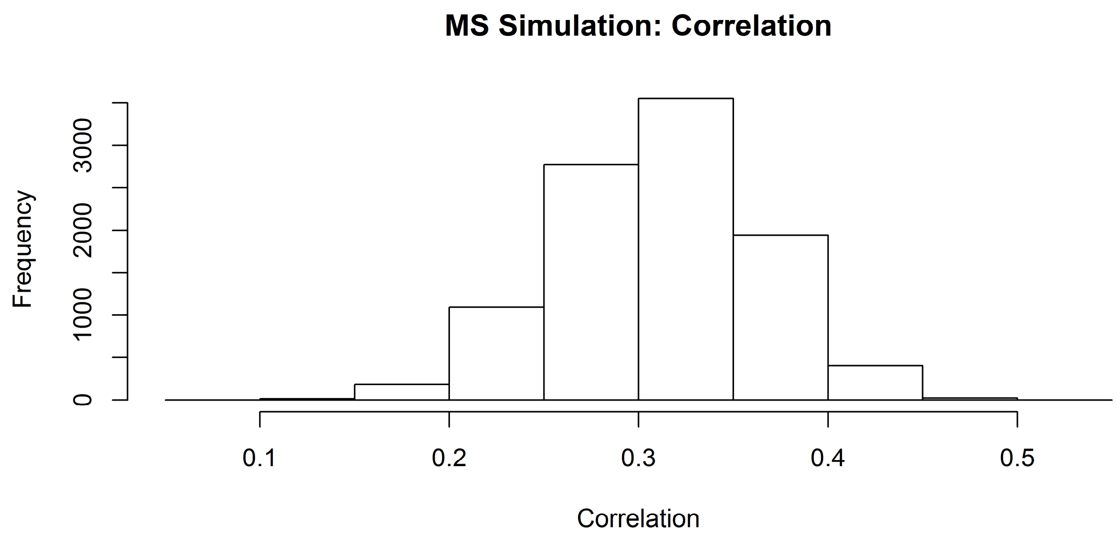


Figure B5: Monte Carlo simulation of backfilled returns with 10,000 runs on US Data

B.5. ARMA

Table B4: ARMA model for inflation expectations (US Data)

Dependent: Inflation	*Coefficient*	*Standard Error*
AR (12)	0.901	0.017
MA (12)	0.002	0.056
R^2	59.02%	

Note: Monthly US Data, 1952-2015.

B.6. Rolling Correlation

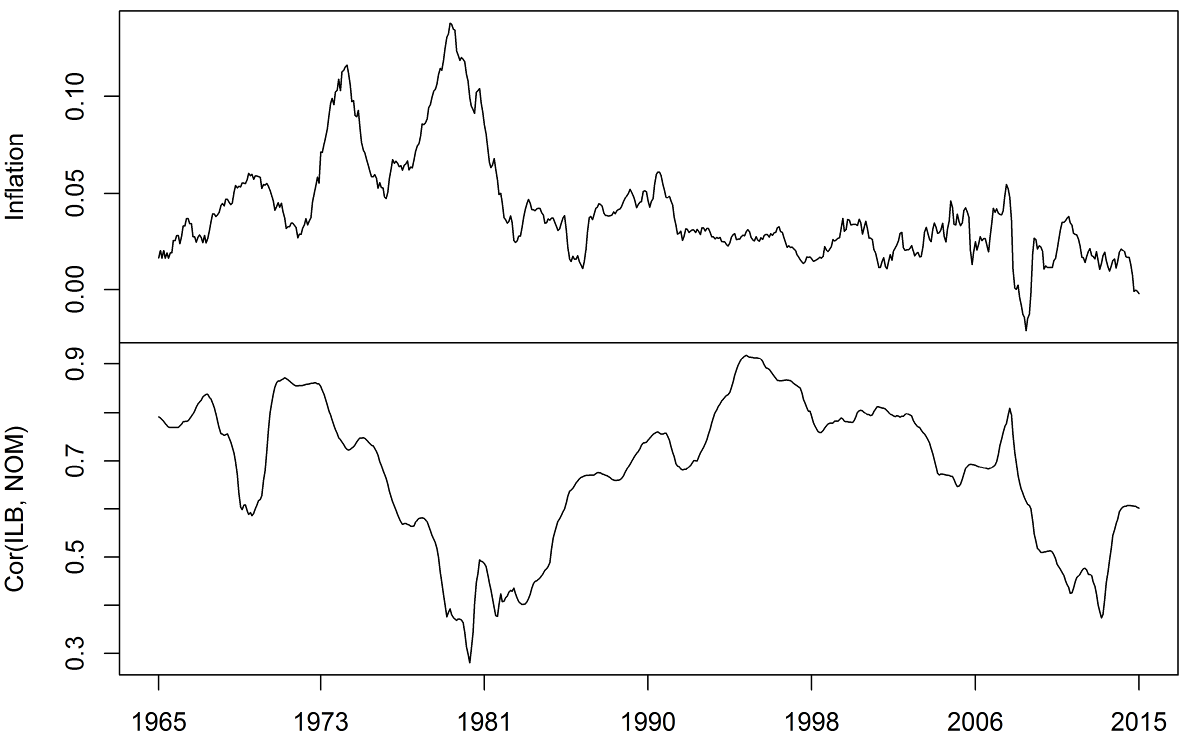


Figure B6: Ten-year rolling correlation between ILBs and nominal bonds, US Data.

Appendix C

C.1. Comparator Bond Yields

The duration matching method for yields explained in equation (27) is compared with actual yields of the comparator bond index from Barclays Capital. Unfortunately, some developing countries are not available. Generally, the approximation method captures the patterns of actual yields and hypothetical returns are similar. For UK and Canada however, there are differences in the correlation of hypothetical returns. Due to the high mean duration, the approximation method is not as precise in the level of yields (see high MSE).

Table C1: Comparison of comparator and approximated yields

			Yields			Hyp. Returns	
Country	N	Duration	Cor(I, C)	Cor(I, A)	MSE	Cor(I, C)	Cor(I, A)
Australia	223	8.92	0.90	0.92	0.98	0.76	0.70
Canada	226	16.00	0.96	0.90	14.78	0.66	0.22
Denmark	42	9.69	0.83	0.74	0.80	0.96	0.87
France	206	8.95	0.96	0.93	9.48	0.82	0.77
Germany	116	6.18	0.96	0.96	0.41	0.76	0.73
Israel	89	7.39	0.94	0.93	1.34	0.55	0.37
Italy	146	7.91	0.88	0.69	12.43	0.81	0.76
Japan	140	6.78	0.62	0.61	0.15	0.27	0.29
New Zealand	226	8.70	0.93	0.92	1.38	0.62	0.67
Poland	128	6.74	0.83	0.74	2.90	0.69	0.30
South Africa	182	9.63	0.84	0.79	4.50	0.59	0.62
South Korea	104	6.74	0.79	0.75	2.94	0.77	0.64
Sweden	226	9.60	0.96	0.94	4.11	0.70	0.63
Thailand	52	8.87	-0.18	0.21	0.82	0.90	0.89
Turkey	103	4.31	0.95	0.94	16.33	0.85	0.76
UK	226	13.77	0.95	0.92	14.04	0.60	0.23
USA	225	8.62	0.93	0.92	1.10	0.72	0.76

Note: "I" = ILB bond index, "C" = comparator bond index; "A" = approximation method (see section 4.8.).
N is the number of observations available in the ILB bond index. Duration is the average duration of the ILB index. The MSE tracks the mean squared error between the comparator bond yields and the approximated yields. In addition, it is scaled by 1000. Hypothetical ILB returns are calculated with survey inflation expectations and the correlation to the actual ILB returns is measured. Source: Barclays Capital.

C.2. Backfilling Monte Carlo Simulation

Table C2: Monte Carlo simulation of Backfilling with recycled noise (10,000 runs)

Country	Correlation						MSE					
	Min	*Mean*	*Max*	*St.Dev.*	*Skew.*	*Kurt.*	*Min*	*Mean*	*Max*	*St.Dev.*	*Skew.*	*Kurt.*
Australia	0.26	0.46	0.60	0.04	-0.10	0.09	1.54	2.30	3.21	0.19	0.11	0.09
Brazil	0.24	0.58	0.81	0.07	-0.29	0.03	5.81	14.01	22.94	2.16	0.19	0.03
Canada	-0.22	0.02	0.30	0.07	0.02	-0.09	7.01	9.86	12.74	0.78	0.11	0.02
Chile	-0.14	0.38	0.81	0.13	-0.27	0.05	0.97	2.12	3.54	0.37	0.20	-0.02
Colombia	0.09	0.51	0.74	0.08	-0.29	0.17	1.30	2.61	4.49	0.45	0.31	0.15
Denmark	-0.06	0.53	0.87	0.12	-0.41	0.21	2.85	7.36	12.32	1.39	0.22	-0.06
France	0.38	0.54	0.67	0.04	-0.17	-0.04	1.45	1.98	2.65	0.17	0.17	-0.05
Germany	0.30	0.51	0.69	0.05	-0.19	0.04	0.76	1.26	1.89	0.14	0.18	0.04
Israel	-0.04	0.28	0.63	0.09	-0.10	-0.04	0.99	1.80	3.10	0.26	0.28	0.05
Italy	0.53	0.68	0.80	0.04	-0.23	0.08	2.61	4.32	6.26	0.51	0.21	0.03
Japan	-0.30	0.00	0.34	0.09	0.08	-0.02	3.16	5.95	9.22	0.72	0.25	0.18
Mexico	-0.22	0.28	0.65	0.12	-0.13	-0.06	4.86	10.27	16.89	1.66	0.17	-0.05
New Zealand	0.34	0.50	0.68	0.04	-0.12	0.01	1.80	2.79	3.79	0.22	0.14	0.11
Poland	-0.35	0.03	0.41	0.09	-0.01	0.02	3.73	6.38	10.29	0.82	0.25	0.17
South Africa	-0.19	0.06	0.36	0.08	0.06	-0.02	4.13	6.27	8.65	0.60	0.17	0.09
South Korea	-0.19	0.15	0.52	0.09	-0.05	-0.10	2.96	5.30	8.18	0.66	0.18	0.03
Sweden	0.12	0.36	0.53	0.05	-0.08	0.09	1.81	2.51	3.48	0.20	0.11	0.11
Thailand	-0.35	0.21	0.71	0.14	-0.10	-0.04	2.40	5.92	10.23	1.07	0.18	0.00
Turkey	0.26	0.55	0.75	0.07	-0.30	0.05	10.14	19.50	34.74	3.29	0.35	0.18
UK	0.02	0.21	0.38	0.05	-0.08	-0.02	3.79	5.03	6.49	0.36	0.15	0.03
USA	0.12	0.31	0.54	0.05	-0.07	0.01	2.99	4.58	6.19	0.41	0.20	0.06

Table C3: Monte Carlo simulation of Backfilling with normally distributed noise (10,000 runs)

Country	Correlation						MSE					
	Min	*Mean*	*Max*	*St.Dev.*	*Skew.*	*Kurt.*	*Min*	*Mean*	*Max*	*St.Dev.*	*Skew.*	*Kurt.*
Australia	0.28	0.45	0.61	0.04	-0.08	-0.01	1.70	2.43	3.21	0.20	0.14	-0.04
Brazil	0.73	0.85	0.93	0.02	-0.27	0.16	1.43	2.87	5.28	0.44	0.27	0.17
Canada	-0.24	0.02	0.30	0.07	-0.03	-0.02	6.90	9.88	13.38	0.82	0.14	-0.01
Chile	0.16	0.57	0.82	0.08	-0.34	0.23	0.34	0.84	1.58	0.17	0.36	0.20
Colombia	0.43	0.67	0.83	0.05	-0.22	0.06	0.59	1.13	1.90	0.16	0.22	0.03
Denmark	0.62	0.82	0.94	0.04	-0.40	0.26	0.46	1.49	3.14	0.34	0.41	0.25
France	0.38	0.54	0.67	0.04	-0.16	0.07	1.39	1.96	2.70	0.17	0.14	0.01
Germany	0.31	0.51	0.72	0.05	-0.12	-0.01	0.71	1.27	1.99	0.15	0.15	0.11
Israel	0.08	0.37	0.66	0.08	-0.10	0.00	0.62	1.16	1.92	0.15	0.21	0.23
Italy	0.56	0.70	0.80	0.03	-0.19	0.10	2.57	3.86	5.49	0.40	0.20	0.09
Japan	-0.29	0.00	0.31	0.09	0.02	0.00	3.88	5.98	8.43	0.63	0.15	-0.03
Mexico	-0.04	0.35	0.71	0.10	-0.19	-0.02	3.04	7.23	12.92	1.24	0.31	0.13
New Zealand	0.35	0.50	0.66	0.04	-0.11	-0.06	1.97	2.87	3.71	0.24	0.10	-0.08
Poland	-0.42	0.03	0.35	0.09	-0.06	0.05	3.51	5.98	9.79	0.67	0.25	0.22
South Africa	-0.26	0.05	0.35	0.08	-0.01	-0.04	4.22	6.32	8.90	0.60	0.18	0.02
South Korea	-0.24	0.15	0.47	0.10	-0.07	-0.08	3.13	5.21	7.67	0.66	0.16	0.02
Sweden	0.16	0.35	0.51	0.05	-0.07	-0.03	1.90	2.65	3.51	0.22	0.12	-0.05
Thailand	-0.32	0.26	0.73	0.13	-0.17	0.01	1.71	4.69	9.17	0.91	0.35	0.21
Turkey	0.66	0.77	0.87	0.03	-0.19	-0.06	3.44	6.10	9.32	0.79	0.19	0.03
UK	0.01	0.21	0.39	0.05	-0.03	-0.10	3.92	5.11	6.52	0.37	0.11	-0.03
USA	0.12	0.32	0.53	0.05	-0.06	-0.09	3.21	4.44	6.04	0.37	0.11	-0.08

C.3. Asset Allocation

Table C4: Descriptive statistics of ILBs, nominal bonds, equities and inflation

Country	Variable	Start	N	Duration	Mean (%)	Volatility (%)
Denmark	*ILB*	May-13	24	9.94	0.38	7.11
	Hyp. ILB Survey	Sep-91	284	5.00	6.11	5.13
	Hyp. ILB Mov. Avg.	Sep-91	284	5.00	6.21	5.83
	Comparator Bonds	Apr-13	24	8.85	4.50	6.66
	Nominal Bonds	Sep-91	284	5.00	6.73	5.61
	Inflation	Sep-91	284	NA	1.96	0.73
	Equities	Sep-91	284	NA	10.87	23.39
France	*ILB*	Sep-99	188	8.98	5.30	4.60
	Hyp. ILB Survey	Sep-91	284	5.00	5.06	4.64
	Hyp. ILB Mov. Avg.	Jan-94	255	5.00	4.48	4.50
	Comparator Bonds	Aug-99	188	7.73	5.52	4.69
	Nominal Bonds	Sep-91	284	5.00	5.79	5.16
	Inflation	Jan-94	256	NA	1.39	0.73
	Equities	Jan-94	256	NA	7.22	22.28
Germany	*ILB*	Mar-07	98	6.13	4.05	3.50
	Hyp. ILB Survey	Sep-91	284	5.00	5.70	4.25
	Hyp. ILB Mov. Avg.	Sep-91	284	5.00	5.76	4.63
	Comparator Bonds	Feb-07	98	5.51	4.72	3.46
	Nominal Bonds	Sep-91	284	5.00	6.23	4.49
	Inflation	Sep-91	284	NA	1.75	0.95
	Equities	Sep-91	284	NA	7.87	23.54
Italy	*ILB*	Sep-04	128	7.92	5.08	8.30
	Hyp. ILB Survey	Sep-91	284	5.00	7.08	6.91
	Hyp. ILB Mov. Avg.	Sep-91	284	5.00	6.78	7.46
	Comparator Bonds	Aug-04	128	6.74	5.81	6.41
	Nominal Bonds	Sep-91	284	5.00	7.97	8.35
	Inflation	Sep-91	284	NA	2.59	1.38
	Equities	Sep-91	284	NA	5.85	24.14
Japan	*ILB*	Mar-05	122	6.78	2.15	5.62
	Hyp. ILB Survey	Sep-91	284	5.00	3.07	4.05
	Hyp. ILB Mov. Avg.	Sep-91	283	5.00	3.15	3.85
	Comparator Bonds	Feb-05	122	6.60	2.03	2.00
	Nominal Bonds	Sep-91	284	5.00	3.47	4.07
	Inflation	Sep-91	284	NA	0.30	1.10
	Equities	Sep-91	284	NA	0.31	22.55
South Korea	*ILB*	Mar-08	86	6.84	6.30	5.61
	Hyp. ILB Survey	Mar-03	146	5.00	5.06	4.31
	Hyp. ILB Mov. Avg.	Mar-03	146	5.00	4.74	4.52
	Comparator Bonds	Feb-08	86	5.79	5.49	3.39
	Nominal Bonds	Mar-03	146	5.00	5.62	3.57
	Inflation	Mar-03	146	NA	2.70	1.12
	Equities	Mar-03	146	NA	9.77	21.79
Thailand	*ILB*	Jul-12	34	8.92	0.63	5.90

	Hyp. ILB Survey	May-07	96	5.00	4.28	3.82
	Hyp. ILB Mov. Avg.	May-07	96	5.00	3.58	4.71
	Comparator Bonds	Jun-12	34	8.05	4.45	5.00
	Nominal Bonds	May-07	96	5.00	5.27	3.82
	Inflation	May-07	96	NA	2.46	2.16
	Equities	May-07	96	NA	11.43	26.70
USA	*ILB*	Feb-98	207	8.62	6.05	5.61
	Hyp. ILB Survey	Oct-90	295	5.00	5.77	4.77
	Hyp. ILB Mov. Avg.	Jun-55	718	5.00	6.28	5.58
	Comparator Bonds	Jan-98	207	7.53	5.97	5.42
	Nominal Bonds	May-53	743	5.00	6.12	6.07
	Inflation	Jun-55	719	NA	3.64	2.68
	Equities	Jan-71	532	NA	9.94	16.75

Note: All returns and rates are one-year overlapping on monthly frequency and a nominal basis. "ILB" is the actual ILB return series of each country (Barclays Capital ILB Total Return Index). "hyp" is abbreviated for hypothetical and surveys and moving average inflation expectations are used to model hypothetical ILB returns. Comparator bonds represent the country-specific nominal comparator index from Barclays Capital, whereas nominal bond returns are calculated out of the yield curve, analogous to hypothetical ILBs. Equity returns are calculated with the corresponding MSCI index of each country. Inflation rates are calculated with CPIs available on Datastream.

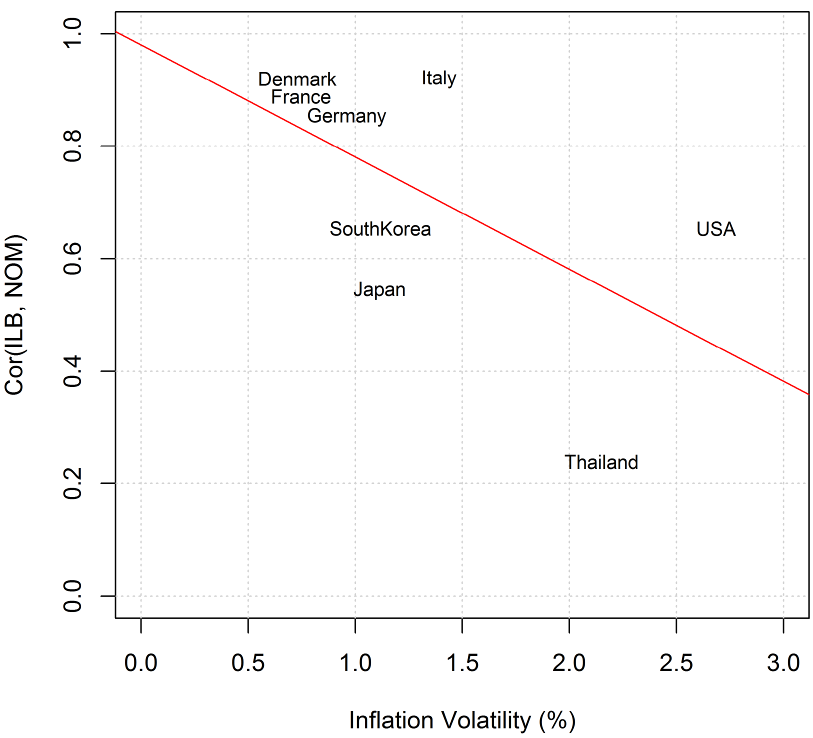


Figure C1: Correlation of ILB and nominal bonds against inflation volatility

Table C5: Cross-country correlation matrix (extended series of ILB returns)

	Denmark	*France*	*Germany*	*Italy*	*Japan*	*S. Korea*	*Thailand*	*USA*
Denmark	1.00	0.76	0.76	0.48	0.27	0.59	0.66	0.48
France	0.76	1.00	0.79	0.66	0.39	0.51	0.31	0.56
Germany	0.76	0.79	1.00	0.43	0.43	0.61	0.45	0.64
Italy	0.48	0.66	0.43	1.00	0.35	-0.12	-0.19	0.07
Japan	0.27	0.39	0.43	0.35	1.00	0.29	-0.10	0.46
South Korea	0.59	0.51	0.61	-0.12	0.29	1.00	0.51	0.70
Thailand	0.66	0.31	0.45	-0.19	-0.10	0.51	1.00	0.51
USA	0.48	0.56	0.64	0.07	0.46	0.70	0.51	1.00

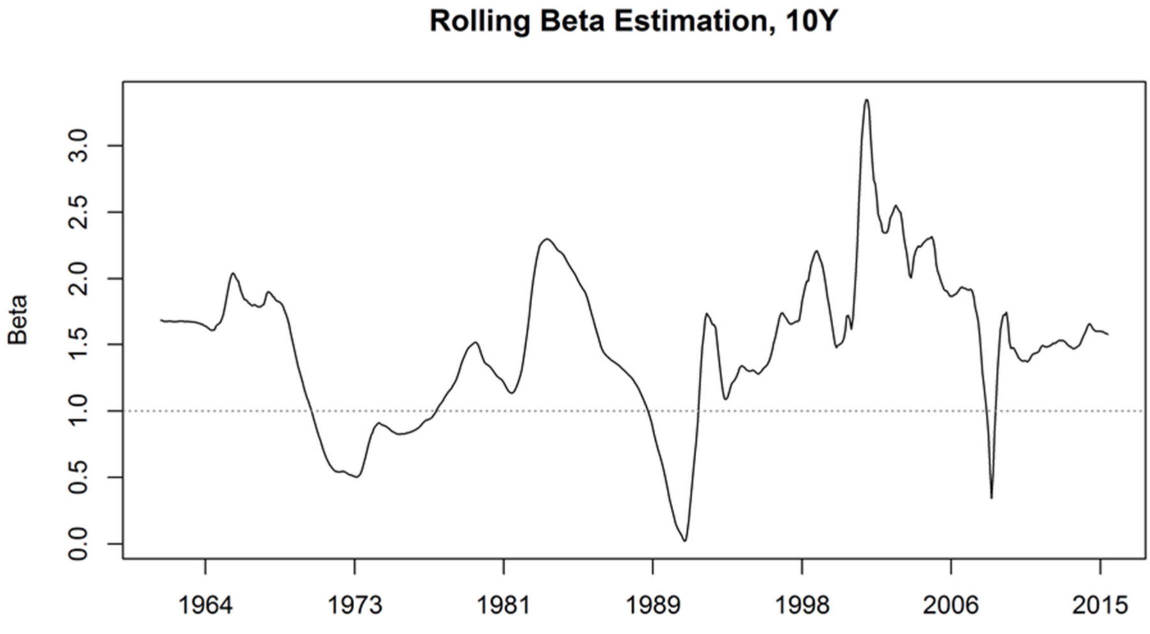


Figure C2: Ten-year rolling estimate of inflation risk beta on US data