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The Private Company Discount

An alternative application to private company valuation

Author: K.F. Olsthoorn
Student number: 429557
Thesis supervisor: drs. H. T. Haanappel
Second reader: dr. J. J. G. Lemmen
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PREFACE AND ACKNOWLEDGEMENTS

Before you lies the master thesis “The Private Company Discount: An alternative application to private company valuation”, the basis of which was set during my time at Maasdael Corporate Finance. It is the conclusion of the Master in Economics and Business with a specialization in Financial Economics at Erasmus University Rotterdam. I was engaged in researching and writing from December 2016 to February 2017.

My profound interest in business and numbers, in particular, has led me along universities that are specialized in economics such as Tilburg University, Universidad del CEMA (Buenos Aires, Argentina) and Erasmus University Rotterdam. The different academic programs have challenged me to strive for results and equated to my desire to continuously learn and improve. More specifically, I learned to critically analyze and describe complex financial problems, setting up and executing a research that is scientifically sound, and to be critical and self-directed.

There are certain people who supported me throughout the process of my research and my studies in general to whom I want to extend my gratitude and thankfulness. First of all, I would like to thank my family, and in particular, my parents, for providing me with the opportunity to obtain this degree and for their continuous support. Second, I would like to distinguish the role of my colleagues at Maasdael Corporate Finance who provided me with practical insights that enabled me to come up with the research topic. Third, I would like to acknowledge my friends and fellow students who gave meaning to my student life and who appeared motivating discussion partners. Finally, I would like to thank my thesis supervisor, drs. H. T. Haanappel, professor of Advanced Corporate Finance & Strategy and the seminar Private Equity at the Erasmus School of Economics (Erasmus University Rotterdam), who supported me throughout the entire process and who was available in case of questions. This support boosted the quality of this thesis and also improved my overall understanding of Corporate Finance.

Koen Olsthoorn

koenolsthoorn@hotmail.com

+31 6 14 50 60 27

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ABSTRACT

Corporate finance practitioners vigorously struggle with the valuation of private, non-marketable, companies. Nowadays, the usage of a small firm premium as surplus in the discount rate is largely debated in academic literature as (i) it is alleged to have dissipated after 1980 and is highly sensitive to specific sub-periods, (ii) is only observable amongst the smallest sized firms and in the month January, and (iii) is prone to large standard errors. Therefore, this study investigates an alternative application that can be used in private company valuation. It examines the differences in acquisition multiples on the market between private and public companies. First, the paper provides evidence for the existence of a so-called private company discount amounting to 15.6% on average using traditional methodology and 19.2% on average using more advanced methodology. Second, the results show that this discount varies significantly across industries. The findings indicate that the healthcare and technology industry have the highest discounts, whereas the wholesale and retail industry have the lowest discounts. Third, this study provides inconclusive results that the discount is negatively related to both size and liquidity. Larger private companies and those with more relative asset liquidity seem to exhibit lower discounts. This finding postulates a partial explanation for the private company discount that adds to preceding explanations observed in literature. Finally, this paper introduces a model that could serve as cross-check in private company valuation.

Keywords: small firm premium, discount for lack of marketability, private company discount, private companies, acquisition multiples

JEL Classifications: G10; G12; G23; G24; G34

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LIST OF TERMS AND ABBREVIATIONS

APT	Arbitrage Pricing Model
CAPM	Capital Asset Pricing Model
DCF	Discounted Cash Flow
DLOM	Discount for Lack of Marketability
EBIT	Earnings Before Interest and Taxes
EBITDA	Earnings Before Interest, Taxes, Depreciation and Amortization
HML	High Minus Low
IPO	Initial Public Offering
M&A	Mergers and Acquisitions
OLS	Ordinary Least Squares
PCD	Private Company Discount
ROA	Return on Assets
SFP	Small Firm Premium
SIC	Standard Industrial Classification
SMB	Small Minus Big
US	United States

CHAPTER 1 Introduction

Many corporate finance practitioners struggle with the valuation of private companies. The valuation is highly subjective and often covered in assumptions, estimates, and industry averages. In general, the process of private company valuation is the same as that of public company valuation where an appraiser evaluates future cash flows and discounts these towards a present value. Therefore, assigning an appropriate discount rate is essential, since it can significantly affect company value. Currently, the discount rate is mostly comprised of assumptions based on historical stock market information. However, the question raised both in literature and in practice is whether this historical information is still the most appropriate measure to discount future cash flows.

Nowadays, the small firm premium (“SFP”) is often added as a constant to the discount rate. It is a practical application for the additional risk premium that investors demand when investing in smaller companies. Historically, smaller companies have outperformed larger companies on the stock markets which proved that risk measures such as the Capital Asset Pricing Model (“CAPM”) and the Arbitrage Pricing Model (“APT”) were incomplete. The SFP could, amongst others, be interpreted by (i) a riskier business environment, (ii) larger capital constraints and (iii) liquidity of assets. The additional required return for investors translates into a higher discount rate and is, therefore, a value-decreasing factor.

A certain risk premium is in line with human intuition, but its usage is not undisputed. From a theoretical perspective, various prominent researchers have expressed serious doubts regarding the practice of incorporating an SFP in the discount rate. Firstly, it is alleged that the premium has disappeared after 1980 (Eleswarapu & Reinaganum, 1993, Dichev, 1998, and Dimson & March, 1999), and, that removal of the smallest firms leads to the belief that the SFP is rather a micro firm premium (Knez & Ready, 1997, Horowitz et al., 2000, and Peek, 2016). Secondly, incorporation of a constant SFP in the discount rate is not accurately reflecting the differences between companies or buyers and across time (Damodaran, 2012).

Therefore, it is interesting to look at alternatives of the SFP. The concept lingers around the topic of illiquidity, which is defined as the inability of selling an asset within a given period that causes reduced marketability. The fact that there is only a limited market for private companies is exposed in the form of a discount (i.e. a lower price). Some literature exists with regards to the topics of the cost of illiquidity, and, existing literature also depicts the private company discount (“PCD”) as a discount for lack of marketability (“DLOM”). The question remains if illiquidity is the only variable causing the discount or whether there are other explanatory variables. Existing literature fails to determine what exactly causes the discount and to provide the link towards private company valuation.

The goal of this research is three-fold: (i) to provide a more thorough insight into the current perspectives on the SFP in private company valuation, (ii) to look at the size of the PCD in general and across different industries, and (iii) to introduce an alternative to the practical application of the SFP in

the form of a PCD that can be used in private company valuation. Consequently, this paper addresses the following research question:

“What is the magnitude of the private company discount and to what extent can it be explained by firm size, the level of liquidity and geography?”

The answer to this research question is of critical importance as the vast majority of businesses are private, and little information is known about the valuation aspect. In 2015, the global market for mergers and acquisitions amounted to 4.5 trillion dollars, which was over 6% of global GDP. Compared to 2014, total deal value rose by 16% when companies announced over 44,000 deals (Institute of Mergers, Acquisitions and Alliances, 2016). It is evident that a significant number of these deals involves private companies. Therefore, the results of this thesis can be used to provide additional understandings within the practice of private company valuation and refine current perspectives that appraisers have with regards to the SFP. Moreover, this thesis contributes to scientific literature as it attempts to fill the research gap with regards to the explanation of the PCD. It can also add to future research as it has a slightly different approach that tries to enhance statistical significance and reliability of the data.

In the first phase of the literature study, the definition and role of the SFP in private company valuation is examined. In more recent literature the existence and computation methods of the SFP persists to be disputed. In the book “Valuation: Measuring and Managing the Value of Companies”, Koller et al. (2010) conclude “The bottom line? It takes a better theory to kill an existing theory, and we have yet to see the better theory. Therefore, we continue to use the CAPM while keeping a watchful eye on new research in the area”. The second phase describes the relation between the SFP and the PCD, which has not clearly been made in literature thus far, both in academic and empirical literature. An important aspect of this relation is portrayed by illiquidity. Private companies are seen as considerably illiquid assets causing discounts on acquisition prices being paid to shareholders. The third and last phase of the literature review focusses on the applicable methodologies used in measuring the PCD and presents comprehensive overviews of all relevant existing literature.

In this paper, two different methodologies are introduced to construct the PCD. First, a traditional method is presented that computes a discount based on conventional, high-over, observations of a discount which is in line with initial papers covering the PCD. Then, a better method is introduced which is further referred to as the portfolio method. The portfolio method computes the PCD by measuring an individual discount for each private acquisition exclusively based on a portfolio of comparable transactions that are defined by the company’s size, the year in which the acquisition took place and the specific industry. The results show that both the traditional method and the portfolio method confirm the existence of a PCD since 1985 that varies significantly across industries. The

variations over the years seem to be related to those of the size effect; the observation that small companies tend to outperform large companies.

In the second part of the paper, the relationship between various predictors (e.g. size, liquidity, and geography) and the PCD is examined. The results show that larger and more liquid companies exhibit lower discounts. A cross-sectional regression analysis also provides a basis for a model that could be used as a tool for private company valuation in practice. However, caution is presented for definite usage as the model is prone to various caveats causing a relatively low level of predictability. Therefore, the model is designed to serve for interpretation of the PCD and provides more feeling behind the research design.

This paper proceeds as follows. Firstly, in Chapter 2, the relevant existing literature is described, which also provides comprehensive overviews of existing studies [*Literature Review*]. Second, in Chapter 3, hypotheses are constructed using existing research as deliberated in the literature review [*Hypotheses*]. Following the hypotheses, Chapter 4 presents the model and a basic outline of the data that was used for this study [*Methodology*]. Subsequently, in Chapter 5, the empirical findings are discussed [*Empirical Findings*]. Chapter 6 includes a discussion that explains how the results could relate to practice [*Practical Implications*]. Finally, Chapter 7 includes the conclusion which provides a concise answer to the research question, describes some limitations to this research, and specifies various recommendations for further research on this topic [*Conclusion*]. A list of the references is enclosed to the end [*REFERENCES*]. The variable definitions are included in *Appendix A Variable Definitions*, the industry definitions in *Appendix B Industry Classification*, an example of the methodologies that are used in *Appendix C Example PCD Computation*, and additional tables and figures in *Appendix D Additional Tests and Statistics*.

CHAPTER 2 Literature Review

This chapter describes the relevant existing literature with regards to the topic of this research. First, it looks at the definition of the SFP and how it can be explained. Second, it talks about which role the SFP currently has in private company valuation, and, third, the different perspectives about the SFP in different studies over the past thirty years. The chapter continues by linking the SFP and the PCD through the topic of illiquidity and private company valuation. By doing this, a basic explanation and interpretation of both concepts are provided. Then, it presents an outline of existing research with regards to the PCD that can be divided into four categories: (i) restricted stock, (ii) initial public offerings (“IPO”), (iii) options, and (iv) acquisition multiples. Finally, the chapter goes into more detail with regards to the explanatory variables of the PCD that can be observed in literature.

2.1 *The small firm premium*

The theoretical framework around the valuation of companies originated from the increasing availability of data on prices of listed companies. Academic research by Sharpe (1964), Lintner (1965), Black (1972) and Black et al. (1972) led to the development of the CAPM some fifty years ago. It is a model that defines the relationship between systematic risk and expected return for securities, such as stocks.

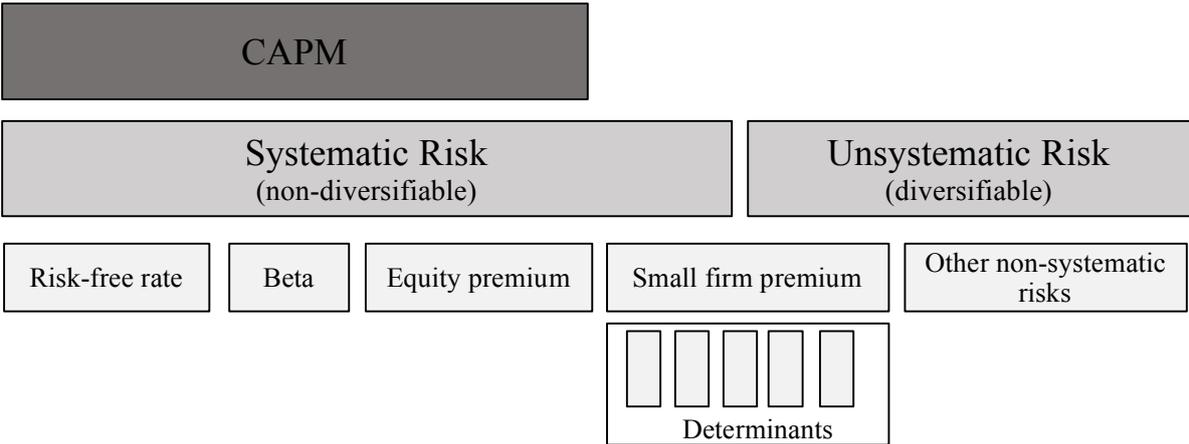
However, there are various empirical contradictions between the findings of preceding researchers. The most eminent contradiction is the size effect as recognized by Banz (1981). He suggests that the CAPM is incorrectly specified because he finds a negative relation between firm size and average return, implying that small companies have experienced significantly higher returns historically than large companies. Thus, return estimations cannot solely be made using a firm’s beta or alternative market risk measures. Therefore, the CAPM is only sufficient to describe security returns in combination with additional, unsystematic, risk factors, such as firm size (Reinganum, 1981 and Lakonishok & Shapiro, 1986). This served as the basis for the so-called SFP, and its existence was later confirmed by many other researchers further set out below.

Reinganum (1980 and 1981) uses both the CAPM model as the APT model to show that market capitalization is an important predictor of average return. Its research recognized that the overperformance of small companies also meant that small companies were riskier than large companies and that risk measures such as the CAPM and the APT were incomplete. Thus, the presence of the size effect is apparent, and many possible explanations exist. Roll (1981), for example, describes that the incorrect assessment of risk and infrequent trading can be a possible explanation of the significant excess returns of low price/earnings ratio firms. Furthermore, Lakonishok & Shapiro (1986) state that the theory of transaction costs in which investors demand a premium as compensation for under-diversification as a result of transaction costs, as posed by Levy (1978) and Mayshar (1979, 1981, 1983), does not hold. They reject both traditional measures of risk (beta) and the alternative risk measures (residual standard deviation or variance) as an explanation for return.

In the period between 1980 and 1995, various prominent researchers confirmed the existence of the SFP. Brown et al. (1983b) demonstrated that small firms have an actual return that is consistently larger than predicted by the CAPM and Keim (1983) further describes the continuous negative relationship between abnormal returns and firm size. Moreover, it was shown by Lakonishok & Shapiro (1986), that different risk measures (e.g. variance or residual standard deviation) neither explain this over-performance of small firms. Lamaroux & Sanger (1989) confirm the SFP by proving a positive relation between average firm size in a portfolio and the average share price, and a negative relation between average firm size in a portfolio and the bid-ask spread. Chan & Chen (1991) consider some of the different structural characteristics (e.g. lower production efficiency and higher financial leverage) between small and large firms in their research. Fama & French (1992) finally prove that size, E/P, leverage and book-to-market equity all explain average returns, and, that beta has little explanatory power. Figure 2.1.1 depicts the position of the SFP graphically.

Figure 2.1.1 Schematic representation of CAPM and the SFP

This figure presents a schematic representation of the CAPM and the SFP which is derived from literature.



The possible economic explanations of the SFP can be divided into rational and irrational explanations. Literature states taxes (Roll, 1983, and Reinganum, 1983) and transaction costs (Schultz, 1983, and Blume & Stambaugh, 1983) as possible rational justifications for the small firm effect. These studies show that the observed higher returns for small firms might be caused by the fact that not all relevant costs are included in return calculations. The significances of the SFP decreases when the calculations correct for taxes and transaction costs. Thus, the effects of taxes and transaction costs may differ between large and small companies. Moreover, studies have emerged that connect economically irrational behavior (psychology-based) of investors as a possible cause of the small firm effect. A number of researchers suggested explanations of momentum that rely on irrational market participants who underreact to news. However, Lakonishok et al. (1994) state that these models are hard to resolve with psychology-based models of overreaction theorized to explain the SFP. However, the extent to which irrational behavior and imperfect functioning markets are determining the SFP has not been established theoretically or empirically.

2.2 The small firm premium in private company valuation

In general, the process of valuing a private company is not any different from that of a public company. The financing of a company requires capital, which is made available by the providers of certain financial assets, also referred to as investors (i.e. equity and debt investors). The corresponding costs to obtain capital of a company are determined by the returns that a particular investor could have generated in the market in a business with a similar risk profile (the so-called opportunity costs of capital). The present value of a future sum of money is determined by discounting it using the cost of capital. Therefore, the cost of capital is also referred to as the discount rate. In the end, the sum of the present value of each expected future cash flow is the company or enterprise value. Thus, the higher the discount rate, the lower the present value of the future cash flows and the lower the company value. A wrong belief of the discount rate leads to misvaluation. Therefore, in valuation, a correct interpretation and assumption regarding the discount rate are key. An over (under) estimated discount rate leads to an under (over) valued asset (company) and thus to under (over) investment. Therefore, the discount rate is a critical element in company valuation.

Nowadays the CAPM model is used as a basis to determine the discount rate of a company. The CAPM is good to use in the valuation of listed companies, but it encounters concerns when applied to private companies as little information is available. This is because the details of acquisitions are usually not announced publicly and there is no market value assigned to either equity or debt of a private company. Therefore, practitioners often add the SFP to the CAPM model to compute the discount rate of a private company (Damodaran, 2012). The SFP is annually determined by renowned organizations such as Duff & Phelps and Ibbotson Associates. Duff & Phelps is a US based valuation consulting firm that publishes its "Risk Premium Report" annually since 2005 after acquiring Standard & Poor's Corporate Value Consulting division. The data used is obtained through Compustat and CRSP and only includes US-listed companies that have revenues above \$1 million. Also, they only consider a full period starting from 1963. Ibbotson Associates, since 2006 part of Morningstar, Inc., conducts asset allocation research and publishes its "Stocks, Bonds, Bills and Inflation Valuation Yearbook" annually since 2000. Its methodology is similar to that of Duff & Phelps. Peek (2016) recently published a paper that was commissioned by Duff & Phelps in which he found evidence for a small firm premium but only for the smallest firms included in the sample.

2.3 Perspectives on the small firm premium

Nonetheless, there are various issues with regards to the existence of the SFP. These issues were introduced by Fama & French in their 1992 paper in which a synthesis of anomalies is presented. The first issue lingers around trend lines and time periods. Concurrent with the introduction of the SFP, Brown et al. (1983) shows that excess returns attributable to size are not constant over time. Handa et al. (1989) even demonstrate this and proof a negative SFP over the period 1941-1954. Overall, the premium has been very volatile historically, disappearing and re-emerging for different periods (Levis,

1985). Academic research that focuses on more recent time periods shows that the SFP is disappearing. Horowitz et al. (2000) find strong statistical evidence that the size effect has disappeared since 1980. They even find that large firms experienced slightly higher returns than small firms over the period 1980-1996. Moreover, various other papers focusing on periods after 1980 do not find evidence for the existence of the size effect (Eleswarapu & Reinaganum, 1993, Knez & Ready, 1997, Dichev. 1998, and Dimson & March, 1999). Dimson & March (1999) compare the period of 1955-1983 to 1983-1997. Although evidence is presented for the existence of the SFP in the earlier period, it does not hold in the later period.

Figure 2.3.1. depicts the returns on the market with regards to the Fama-French Three Factor Model over the period 1927 to 2010. Fama & French (1993) pointed out that (i) small firms perform better than large firms, also referred to as the small firm effect (Small Minus Big - “SMB”), and (ii) that value stocks (higher book-to-market ratios) outperform growth stocks (lower book-to-market ratios), also referred to as value premium (High Minus Low – “HML”). The figure includes a trend line that shows a decreasing small firm effect over the entire period. Furthermore, table 2.3.1 shows that the simple average of SMB over the period after 1980 is significantly lower than before 1980. Both the figure 2.3.1 as the table 2.3.1. were constructed using French’s online data which are derived from Ibbotson Associates.

Figure 2.3.1 The returns on the market, SMB, and HML portfolios over the period 1927 – 2010

This figure presents the development of the returns of the market, the size effect (SMB), and the value effect (HML) over the period between 1927 to 2010. The y-axis represents the relative return in %, and the x-axis represents the years. The data is derived from the available figures of French (2016), which can be found on his website.

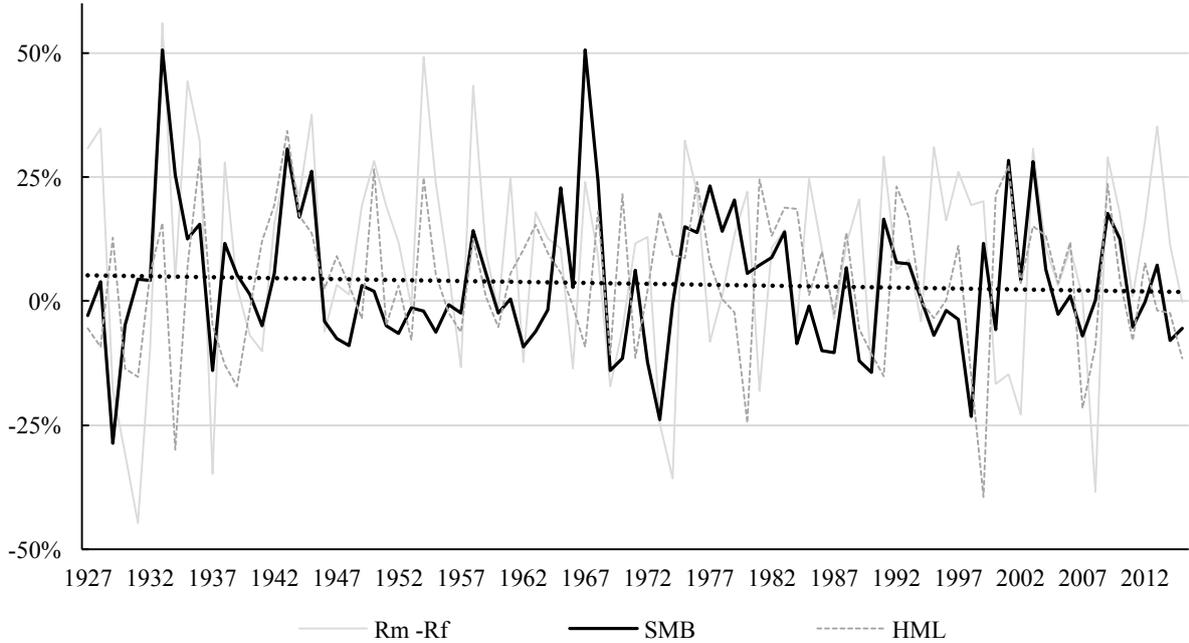


Table 2.1 The simple average returns of the market, SMB, and HML portfolios

This table presents the simple average returns of the market, the size effect (SMB), and the value effect (HML) over the period before 1980 and after 1980. The data is derived from the available figures of French (2016), which can be found on his website.

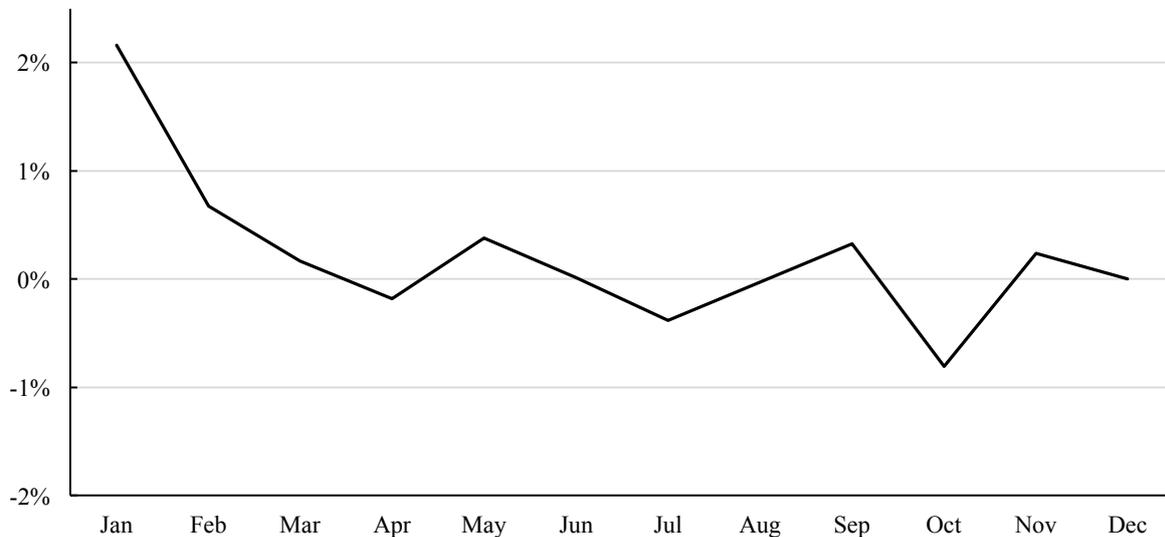
Period	Rm - Rf	SMB	HML
1927 - 1980	8,7%	4,8%	3,9%
1981 - 2015	7,9%	1,7%	3,9%

The second issue elaborates on the nature of the SFP. Knez & Ready (1997) demonstrate that the SFP disappears by eliminating the extreme 1%. Thus, removal of the smallest firms (less than \$5 million market value) eliminates the entire effect, suggesting that it should only be referred to as a premium at a micro level (Horowitz et al., 2000). Peek (2016) confirms the existence of a micro-cap premium (for firms with a maximum market capitalization of \$8 million). Additionally, the majority of literature is based on the United States (“US”). Peek & Crain (2011) find that size premiums are smaller or non-existent in non-US markets. The potential global differences are discussed in Rouwenhorst (1999). Taking the aforementioned contradictions into account, Siegel (1999) argues for downward corrections of the historically calculated premiums (4-6%) and describes the unlikeliness of large premiums in periods of high equity prices relative to earnings. This implies that caution needs to be taken when applying the SFP in non-US markets and across different time periods.

Besides the debate with regards to the existence of the SFP, questions can be raised concerning certain aspects of the size effect. Keim (1983) introduces the January effect as a potential issue, which implies that the most of the excess returns are obtained in January. In other words, daily abnormal return distributions in the first month have great means in comparison to the remaining eleven months. According to Ritter (1988), 46 percent of the year-to-year variation is explained by the early January buy/sell ratio. Lamaroux & Sanger (1989) elaborate on the effect, which they refer to as the turn-of-the-year effect. Subsequently, removal of the month January leaves no statistical evidence for the SFP (Reinganum, 1983, Barry, 1984, Ritter, 1988, and Eleswarapu & Reinganum, 1993). Keim introduces a tax-loss selling hypothesis and information hypothesis as possible explanations for the January Effect. Reinganum (1983) finds evidence for the tax-loss selling hypothesis using US returns. However, Brown et al. (1983) debunk the tax-loss hypothesis using Australian returns and state that tax years and return seasonality are rather correlated. Barry (1984), together with some unpublished memos stated in his paper, finds evidence for the information hypothesis. This hypothesis states that January is a period of more uncertainty and market anticipation leading to higher returns. Figure 2.3.2 depicts the seasonal pattern in the size effect (SMB). The figure was also constructed using French’s online data and clearly shows the presence of the January effect.

Figure 2.3.2 Seasonal patterns in the size effect (SMB) 1927 – 2010

This figure presents the average seasonal pattern of the size effect (SMB) across the various months of the year. The y-axis represents the relative return of the small firms in relation to large firms in %, and the x-axis represents the several months of the year. The data is derived from the available figures of French (2016), which can be found on his website.



Another profound topic is the different methodologies used to obtain results and the corresponding standard errors. Roll (1981) highlights the large differences using different methodologies as arithmetic means show results that are twice as high as buy-and-hold returns. Blume (1974) already documented the biases between the arithmetic and geometric mean, which both do not perfectly estimate returns. The actual predictor lies somewhere in between the arithmetic and geometric mean (Jacquier et al., 2003). Moreover, the standard errors in estimation methods are high and sometimes even as high as the premium itself. If the return interval becomes shorter, the standard error will increase, and the reliability will decrease (Damodaran, 2016). The same holds for a division of the data into more sub-classes, to analyse the premium for certain sizes. Finally, the effect of size on returns can only be measured using market capitalization as a proxy for size (Berk, 1997). He also finds that non-price measures (e.g. revenues or profitability), which are the only ones available for non-listed companies, cannot explain a size effect.

Table 2.3.1 depicts a comprehensive overview of SFP literature. Caution has to be made as the size premiums cannot be directly compared across studies, due to the fact that some studies compute returns in excess of the risk-free rate and others in excess of a market portfolio. Although more recent studies do not find evidence for the existence of the SFP over periods after 1980, Van Dijk (2011) argues that it is too early to conclude that the SFP has gone away. He states that more research is needed to observe robustness of conclusions. Explanations of the premium have not been successfully accepted and are non-sufficient. In the end, still little is known about the movements of a certain premium over time.

Table 2.2 Comprehensive overview of thirty years' small firm premium research

This table presents a comprehensive overview of the most relevant small firm premium studies that exist in literature. These studies intend to measure the systematic outperformance of small firms relative to the market. The size premiums cannot be directly compared across studies, as some studies compute returns in excess of the risk-free rate and others in excess of a market portfolio.

Study	Year	Period	Size premium (p.m.) [*]	Focus	SFP
Banz	1981	1926 - 1975	0.40%	US	Yes
Reinagnum	1981	1963 - 1977	1.77%	US	Yes
Roll	1983	1963 - 1981	0.63%	US	Yes
Keim	1983	1963 - 1979	2.52%	US	Yes
Brown, Kleidon, and Marsh	1983	1967 - 1979	1.85%	US	Yes
Brown, Keim, Kleidon, and Marsh	1983	1958 - 1981	N.A.	AU	Yes
Levis	1985	1958 - 1982	0.40%	UK	Yes
Lakonishok & Shapiro	1986	1962 - 1981	N.A.	US	Yes
Ritter	1988	1970 - 1985	N.A.	US	Yes
Handa, Kothari, and Wasley	1989	1941 - 1982	1.52%	US	Yes
Lamaroux and Sanger	1989	1973 - 1985	2.00%	US	Yes
Chan and Chen	1991	1956 - 1985	0.95%	US	Yes
Fama and French	1992	1963 - 1990	0.63%	US	Yes
Eleswarapu & Reinaganum	1993	1981 - 1990	-	US	No
Strong and Xu	1995	1973 - 1992	0.61%	UK	Yes
Daniel and Titman	1997	1963 - 1993	N.A.	US	Yes
Knez and Ready	1997	1963 - 1990	-	US	No
Doeswijk	1997	1976 - 1994	-	NL	No
Dichev	1998	1980 - 1995	-	US	No
Dimson and March	1999	1988 - 1997	-0.47%	UK	No
Dimson and March	1999	1983 - 1997	-0.20%	US	No
Shumway and Warther	1999	1972 - 1995	0.93%	US	No
Rouwenhorst	1999	1982 - 1997	-	Global	-
Chan, Karceski, and Lakonishok	2000	1986 - 1998	-	US	No
Horowitz, Loughran, and Savin	2000	1980 - 1996	-	US	No
Schwert	2003	1982 - 2002	-	US	No
Fama and French	2006	1955 - 2004	-	US	No
Hou, Karolyi, and Kho	2011	1975 - 2003	-	Global	No
Chaibi, Alioui, and Xiao	2015	2010 - 2012	-	US	No

2.4 The small firm premium and the private company discount

Amihud (2002) and Amihud et al. (2005) prove that at least a portion of premiums obtained by smaller firms can be explained by illiquidity (i.e. reduced marketability). Therefore, the relationship between liquidity and returns seems to get a more prominent role. Expected stock returns bare a cross-sectional relationship with liquidity fluctuations (Pástor & Stambaugh, 2003). Moreover, Brown et al. (2009) show that investors demand a liquidity premium for relatively illiquid stocks. They also confirm that

^{*} Size premium data is taken from Van Dijk (2011) and Hawawini & Keim (1995, 2000).

illiquid stocks are usually smaller and vice versa. Damodaran (2005) elaborates on the discussion by proving positive the relationship between illiquidity and transaction costs. Then, he shows that illiquid stocks have lower prices (i.e. illiquidity discount) and higher returns. Therefore, it seems evident that the SFP strongly depends on the level of liquidity and the size of a firm.

Damodaran (2015) also constructs a method that can be applied to (private company) valuation: value a business as liquid asset first and then apply the illiquidity discount. In his paper, Damodaran (2015) describes that private companies are the least liquid kind of assets and typically smaller than listed companies, causing lower valuations in the case of an illiquidity discount. It was already seen that the SFP is captured within the smallest sized companies that have revenues below \$5 million (or \$8 million depending on data and time period). Thus, in private company valuation, the illiquidity discount as defined by Damodaran (2015) can be explained as a PCD to a certain extent. However, it needs to be interpreted with caution, because Hertzl & Smith (1993) conclude that, although liquidity is a predictor, there must be other factors affecting the PCD. In any case, it seems evident that a clear link seems to exist between the SFP and the PCD, but that it remains unclear what precisely explains the discount for private companies.

2.5 The private company discount

Various methods have been applied in literature to measure the PCD or often also referred to as DLOM. The studies focused on (i) restricted stock, (ii) IPO's, (iii) options, and (iv) acquisition multiples. The former two are older approaches and can be considered more basic than the other two approaches. The studies that focus on options and acquisition multiples deduce their approach from the older approaches.

2.5.1 Restricted stock studies

Restricted stock, also referred to as letter stock or restricted shares, is a form of share-based payment issued by a publicly-traded company. These are conditionally awarded, temporarily non-transferable, shares to an employee (e.g. executives and directors). Whenever the employee does not meet certain conditions on a predetermined date (the vesting date), the employee must return all or some of the awarded shares back to the employer. Certain conditions often include that the employee is still employed and that he meets pre-set performance goals. The restriction period in which the shares cannot often be sold amounts to one or two years. The restricted stock studies are based upon a comparison between the freely traded (liquid) public stock, and the non-freely (illiquid) traded restricted stock. The lower price at which publicly-traded companies issue restricted stock in a private placement represents the discount for lack of marketability. For example, if Apple has freely traded public stocks being traded at \$119 per share and it issues a private placement of \$89 per share, the discount represents 25.2%. This comparison can be made as the restricted stock is considered to be identical to publicly traded stock.

Table 2.5.1 presents various papers between 1970 and 2000 that study the observed private placement discounts and combined they find an average discount of 28.8%. It has to be denoted that more recent studies (after 1990) find lower discounts (23.7%) as compared to older studies (34.7%).

Silber (1991) finds that larger private placements induce large discounts and Johnson (1999) includes a size variable of the issuing publicly-traded firm to explain the size of the discount. Other studies (Moroney, 1973 and Maher, 1976) incorporate general firm and industry characteristics to explain discounts. On the other hand, Hertzels & Smith (1993) discover that the discount also includes compensation for expected monitoring services. Therefore, liquidity (or marketability) does not fully explain the size of a private placement discount. This is also considered to be the biggest critique with regards to restricted stock studies. As the discount includes compensation for due diligence, monitoring, and advice it is seen as an unreliable proxy for the PCD. But the fact that certain activities have an influence on the discount raises questions with regards to the potential effect of information asymmetry in the case of a private company. Another weakness of the restricted stock studies is that they are drawn from publicly traded companies, and, therefore it is undetermined whether the observed discounts relate to private companies. Finally, sample sizes are rather small (below 100 observations). Therefore, it becomes rather difficult to compare different industries with each other.

Table 2.3 Comprehensive overview of restricted stock studies

This table presents a comprehensive overview of the most relevant restricted stock studies that exist in literature. IPO studies look at the discount by comparing the price between freely traded public stock and non-freely traded restricted stock.

Study	Year	Period	# obs.	Avg. Discount
SEC Institutional Investor	1971	1966 - 1969	398	25.8%
Gelman	1972	1968 - 1970	89	33.0%
Trout	1977	1968 - 1972	60	33.5%
Moroney	1973	1969 - 1972	148	35.6%
Maher	1976	1969 - 1973	33	35.4%
Pittock and Stryker (Standard Research Consultants)	1983	1978 - 1982	28	45.0%
Silber	1991	1981 - 1988	69	33.8%
Hertzels and Smith	1993	1980 - 1987	106	20.1%
Hall and Polacek (FMV Opinions)	1994	1979 - 1992	>100	23.0%
Johnson	1999	1991 - 1995	70	20.0%
Oliver and Meyers (Management Planning)	2000	1980 - 1996	53	27.1%
Aschwald (Columbia Financial Advisors)	2000	1996 - 1997	23	21.0%
Bajaj, Denis, Ferris, and Sarin	2001	1991 - 1995	88	20.6%
Total / Average		1966 - 1997	97	28.8%

2.5.2 IPO studies

In an initial public offering, a firm sells shares via the stock exchange for the first time. The company hopes to attract investments by issuing new shares. An IPO has many benefits. First, it gives a firm access to capital which enables growth. This capital can fund growth both via acquisitions and organically. Second, it creates more liquidity and exit opportunities for current owners, as the shares are open to the public and transaction costs are minimized. Third, the IPO maximizes firm value since being

public attracts larger investors and evaluation becomes easier (Ritter & Welch, 2002). In IPO studies, the discount is measured by comparing the IPO offering price (price for which the stock was initially offered to the public) with the historical transaction prices for the same shares that occurred prior to the IPO. The firms that go public disclose a few years of preceding transactions, which are used to compute a discount. Facebook went public in May 2012 for the price of \$38.00 per share valuing the company at \$104 billion (Nasdaq, 2016). Prior to the IPO, Facebook sold shares on private markets like Sharespost and SecondMarket. Auctions at the end of January indicated a valuation somewhere between \$80 and \$85 billion. Therefore, the discount ranged between 18% and 23%. Due to changes in company characteristics (e.g. sales or profitability) that can potentially affect discounts, IPO studies often only look back up to six months prior to the actual IPO date.

Emory (1985 to 2000) is perhaps the most prominent research of pre-IPO discounts, and a summary of his extensive research is presented in table 2.5.2. He finds an average discount of 46%, which is relatively stable throughout the years. His last study, executed in 2000, only represents Dotcom firms. It is not surprising that observed discounts are higher as compared to the eight other studies since it was during a financial bubble (Dotcom bubble) in which prices exceeded fundamental value by a large margin. However, IPO studies suffer from various complications, similar to restricted stock studies. First, it suffers from a selection bias as it looks only at companies that successfully go public and it does not adjust discounts for exit probabilities. The uncertainty of success before an IPO causes prices to be lower, which is eliminated when a company goes public. Second, the private transactions are often executed at attractive prices intended for insiders which enhance discounts (Feldman, 2005). Finally, just like the restricted stock studies, sample sizes are rather small (below 50 observations) as not many successful IPO's take place. While the lack of marketability comprises a majority of the discount in pre-IPO studies, it still does not fully capture the effects of valuation for companies that remain to be private.

Table 2.4 Comprehensive overview of IPO studies conducted by Emory

This table presents a comprehensive overview of the relevant IPO studies that exist in literature conducted by Emory. IPO studies look at the discount by comparing the IPO offering price with historical transaction prices for the same shares that occurred prior to the IPO.

Study	Year	Period	# obs.	Avg. Discount
Emory	1985	1980 - 1981	13	60%
Emory	1986	1985 - 1986	21	43%
Emory	1989	1987 - 1989	27	45%
Emory	1990	1989 - 1990	23	45%
Emory	1992	1990 - 1992	35	42%
Emory	1994	1992 - 1993	54	45%
Emory	1995	1994 - 1995	46	45%
Emory	1997	1995 - 1997	91	43%
Emory	2000	1997 - 2000	53	54%
Total / Average		1980 - 2000	40	46%

2.5.3 Options studies

An option is a financial instrument based on a contract between two parties. The option writer (seller) provides the option holder (buyer) with a right to call (buy) or put (sell) an asset at the strike price (a predetermined price) on a specified date. In the case of a put option, marketability is increased as the buyer has the right, but not the obligation, to sell an, perhaps illiquid, asset. Therefore, the cost of a certain option represents the discount that is taken from the marketable price. This is because the strike price of a put option will never exceed the market price of the underlying asset at the time the option is made available. In that case, exercising the option immediately would be optimal, inferring a direct loss for the option writer. The European option is only exercisable at the end of the option period, providing a model that suffices to measure the discount (Chaffe III, 1993).

Using the Black-Scholes option pricing model, Chaffe (1993) finds minimum applicable discounts ranging from 28% to 41% for private companies. Instead of providing a lower limit of the discount, Longstaff (1995) estimates the upper limit by looking at unrealized gains based on a theoretical lookback put option model. To come to his conclusion, he assumes perfect market timing, single-security portfolios, and certain trading restrictions. The discount is measured as the difference between the maximum value and the current value of the option. Observed discounts range up to 65% depending on the restriction period and standard deviation. Finnerty (2003) builds upon the Longstaff (1995) study in the sense that the option holder (buyer) has no market-timing ability. He measures the discount as the value of an average strike put option. He also finds discounts of around 20% based on private placements of restricted stock. Bruner & Palacios (2004) show how control aspects (e.g. strategic flexibility and private benefits) dominate marketability in discounts using various simulation analyses. Finally, Brooks (2014) combines the Chaffe (1993), Longstaff (1995), and Finnerty (2003) study and provides a general option framework that includes asset maturity, volatility, hedging availability and investor skill. Discounts are highly dependable upon aforementioned variables, but basically ranges from 20% to 45%.

Overall, options studies only consider particular characteristics of private companies, namely holding period and volatility which also have the greatest impact on option pricing. Therefore, these studies may understate the PCD as they ignore certain other aspects of private companies, such as contractual limitations or large levels of illiquid assets. A practical problem is also that it remains difficult to measure the required holding period and level of volatility for a private company. It can thus be concluded that options do not fully represent the characteristics of a private company. Another major flaw in the options studies is that the usage of put options always enables the investor to capture or preserve an asset's upside potential, undoubtedly overstating the discount. Moreover, the options considered in the above-mentioned studies are liquid whereas private companies, as denoted by Damodaran (2015), are the least liquid assets due to the absence of a trading market. A basic conclusion that can be drawn from options studies is that companies that are facing lower levels of risk (volatility) also support using a lower discount.

2.5.4 Acquisition studies

In academic research, the direct concept of the PCD is relatively new. Koeplin et al. (2000) introduce it using a straightforward approach by matching public to private transactions using industry and time as matching criteria. The methodology behind the usage of acquisition multiples dates back to Kaplan & Ruback (1996) who compared the discounted cash flow (“DCF”) method and a method based on multiples. The conclusions showed that estimates were more reliable based on the DCF method, but the usage of multiples ensured the lowest amount of errors in valuation. They include two earnings measures, namely earnings before interest and taxes (“EBIT”) and earnings before interest, taxes, and depreciation and amortization (“EBITDA”), which are both independent of capital structure, and two broader measures, namely sales and book value of assets, as a basis to compute multiples. Based on this approach, Koeplin et al. (2000) found an average discount of 21% based on the EBITDA multiple using data from SDC, covering the period 1984-1998, which was limited to acquisitions of controlling interests in non-financial firms. This resulted in a final set of 192 transactions.

This study provided a solid background for further research towards the PCD. The statistical significance of earnings-based multiples seemed to be greater than revenue based multiples. Also, they identified several value characteristics that are relevant for explaining the PCD. These value characteristics include risk, growth rate, capital structure, the size and timing of cash flows, and liquidity. The discounts appeared to be greater for non-US companies, but statistical significance seems to be lower which is attributed to different accounting standards internationally. An essential problem they identified was the fundamental difference in characteristics of private companies as compared to public companies. The latter seemed to be structurally bigger, showed different growth rates, and had distinctive employment contracts for key personnel. In a private company, key personnel may also receive part (or all in the case of an owner) of the compensation involved in a certain acquisition.

Kooli et al. (2003) elaborate on the Koeplin (2000) study by taking size as matching criteria and making a distinction of PCDs per industry. Using the DoneDeals database, they identify 331 acquisitions between 1995 and 2002 without excluding financial firms. Multiples were used that already existed within the DoneDeals database, namely sales, earnings, and cash flow multiples. An important characteristic of their research lies in the methodology. Built upon research from Brav et al. (2000), the methodology included the development of reference portfolios based on year, industry, and size. Therefore, the structural differences in characteristics between private firms and public firms are accounted for as a carefully selected control portfolio of public deals exists for each private transaction. The Kooli et al. (2003) study finds discounts of 34% based on an earnings multiple, 17% based on the sales multiple, and 20% based on the cash flow multiple. A general conclusion is that the general practice of using a constant discount for different companies across a diverse range of industries is wrong. The PCD should be adjusted for on the basis of company characteristics. For example, they conclude that discounts are smaller for growing companies and conclude that wholesale and retail companies partake the largest discounts.

A particular focus on the size variable was drawn by Block (2007). Larger and more profitable firms have lower discounts than smaller and less profitable firms. The methodology reverts back to the Koeplin (2000) study, in which matches between public and private transactions are constructed. Block focuses on the period between 1999 and 2006 in which he categorizes 91 controlling interest acquisitions over various industries using SDC. The industries are recognized using the Standard Industrial Classification system (“SIC”). He highlights the involvement of information asymmetry for private companies as only limited information is available, potentially causing higher due diligence costs. Therefore, he emphasizes the essential distinction in terminology between the PCD and the DLOM. The argument holds that DLOMs are lower as they only include an illiquidity discount. Whereas the shareholders’ equity of a private company may be highly illiquid, other assets, such as cash, are not illiquid. The justification for an illiquidity discount can be made due to the fact that only a limited market exists for various assets of a private company. This was confirmed by Officer (2007) in the same year. In fact, Officer shows that a greater need for liquidity for a selling party leads to a greater discount often being the case when a parent sells off a subsidiary (i.e. fire sales).

Paglia & Harjoto (2010) use more transactions (431 in between 1994 and 2008 from Pratt’s Stats database) and construct pricing multiples (i.e. market value of invested capital) instead of acquisition multiples. Interesting is the fact that they exclude companies with revenues below \$50 million. The Paglia & Harjoto (2010) study resulted in the following conclusions: (i) large size leads to lower discounts, (ii) positive profits leads to lower discounts, (iii) professional services have highest discounts, (iv) acquisitions driven by strategic motives have lower discounts, and (v) lower probabilities of financial distress leads to lower discounts. Observed discounts are 25% for the EBITDA multiple and 68% for the sales multiple.

Elnathan et al. (2010) continues with the discussion on information asymmetry. They examined the role of financial analysts and financial statement information in company valuation. Public firms comply with regulations causing them to publish extensive financial statements whereas private firms have minimal requirements with regards to their financial statements. Therefore, financial analysts play a crucial role in private company valuation as only limited information is available and they rely on other, non-financial statement information sources irrespective of quality. Due to the large availability of company financials for public firms, both market-based and accounting-based, it is seen that financial analysts have little-added value when the valuated company is public. Using a sample of 147 valuations (i.e. valuation multiples rather than acquisition multiples) over the years from 1991 until 2006 they observe a discount of 22% based on the EBITDA multiple. Interesting is the inclusion of leverage in their multivariate model that demonstrates a significant negative relationship with the PCD. De Franco et al. (2011) expands the previous research by stating that these valuation multiples are significantly higher for private firms that hire one of the big four auditors (i.e. PWC, EY, KPMG, and Deloitte) and find a discount of 33% between 1995 and 2004 covering 664 private stock sales. A more recent research of Klein & Scheibel (2012) finds lower discounts of around 5%. Klein & Schneibel also observe the

influence of buyer type as a predictor for the PCD. Private companies tend to pay lower prices when acquiring another private company than when public companies do.

Table 2.5.3 presents a comprehensive overview of acquisition studies which includes a complete synopsis of the discounts across various multiples. The present figures are based on univariate analysis and computed using median discounts. An overall average PCD of 17.4% is observed. It can be concluded that discounts using the sales multiple are highest. However, the EBITDA multiple seems to be the most consistent even though the researched time periods and methodology differs across the diverse set of completed studies. On the other hand, the book value multiple occasionally shows that a premium is paid for private companies as compared to their public peers. Overall, the studies seem to fail in achieving a significant number of (maximum) private company acquisitions in their analysis.

Table 2.5 Comprehensive overview of acquisition studies

This table presents a comprehensive overview of the most relevant acquisition studies that exist in literature. Acquisition studies look at the discount of private companies as compared to their public peers by comparing various multiples. Their methodologies differ in terms of complexity. The earlier studies use a straightforward approach in which the central tendencies are being compared, whereas more recent studies construct matches or benchmark portfolios for each of the private company acquisitions in which year, industry, and size serve as matching criteria.

Study	Year	Period	# obs.[†]	EBITDA	EBIT	Sales	Book	Other[‡]	Avg.	Focus
Koeplin, Sarin, and Shapiro	2000	1984 - 1998	192	21,1%	17,1%	9,8%	7,6%	-	13,9%	Global
Kooli, Kortas, and L'Her	2003	1995 - 2002	331	34,0%	-	17,0%	-	20,0%	23,7%	US
Block	2007	1999 - 2006	91	22,5%	24,3%	24,5%	14,5%	23,3%	21,8%	US
Officer	2007	1979 - 2003	364	17,2%	-	-	-15,6%	22,9%	8,2%	Global
Paglia & Harjoto	2010	1993 - 2008	431	25,0%	-	68,0%	-	-	46,5%	Global
Elnathan, Gaviious, and Hauser	2010	1991 - 2006	88	21,7%	-	-	-48,0%	-	-13,2%	IL
De Franco, Gaviious, Jin, and Richardson	2011	1995 - 2004	664	33,3%	-	32,8%	-	-	33,1%	US
Klein & Scheibel	2012	1999 - 2009	138	-	-	-	-	-	5,0%	EU
Total / Average		1979 - 2009	287	25,0%	20,7%	30,4%	-10,4%	22,1%	17,4%	

[†] The observations listed are the maximum number of private company acquisitions or valuations observed. For some multiples, such as the EBITDA multiple, the availability of data is much lower and it is therefore possible that discounts were calculated based on a smaller number of observations.

[‡] Other multiples that were used in the listed studies include a cash flow multiple and P/E-based ratios.

2.6 Summary of literature review

Various papers that were published around the beginning of the nineties identified deficiencies in the CAPM model. Smaller sized companies seemed to systematically outperform larger sized companies when controlled for beta, which was referred to as the SFP. However, attempts to explain this premium lingered around rational (i.e. taxes and transaction costs), and irrational (i.e. behavior) explanations remain debated. Besides, thirty years of SFP literature shows that the small firm effect strongly depends on the specific time period or observed company size. Some research even states that the SFP has dissipated after 1980, which seems to be apparent when looking at the trend line over time. Furthermore, the SFP is debated with regards to the January effect (significance of a premium does not hold after removing January returns) and its profound standard error. Therefore, literature has somewhat refocused on alternatives of the SFP in the form of an illiquidity discount, such as the DLOM, and a PCD for companies that are not publicly traded on the market.

Research that looks at certain discounts can be roughly divided into four areas of focus: (i) restricted stock, (ii) IPO's, (iii) options, and (iv) acquisition multiples. The restricted stock and IPO studies ended around 2000 and observed discounts of respectively 29% and 46%. The options studies find discounts between 20% to 45%. Major downsides to these studies are considered to be unreliable proxies of the PCD, because various characteristics of restricted stock, IPO's, and options cannot affect discounts of private companies as they are both related to companies for which a trading market exists. Also, sample sizes for both types of analyses are rather small (below 100 observations). On the other hand, acquisition studies introduced by Koeplin et al. (2000) found a constructive method to measure the PCD using publicly available data of acquisitions of controlling shares in private. Characteristics of private companies can be directly implemented into models and sample sizes are supposedly larger (close to 300 observations on average). Discounts are computed using enterprise value multiples based on earnings (i.e. EBITDA and EBIT) and other accounting data. The characteristics that have a significant effect on the PCD are predominantly firm size and liquidity, but other variables that seemed to affect the discount include growth rate, type of industry, information asymmetry, profitability, type of buyer (public or private), buyer motive (strategic or financial), and the probability of financial distress.

CHAPTER 3 Hypotheses

To further clarify the remainder of this research a few hypotheses are defined that follow from the existing literature in the field. The primary variables explaining the PCD following literature appear to be firm size and the level of liquidity. Therefore, besides the main hypothesis with regards to the existence of a PCD, two hypotheses are formulated. The chapter also includes a few predictions and ideas for remaining variables that potentially affect the PCD.

3.1 Research question

Literature covering the discount from four different aspects observe a substantial discount. Specifically, the acquisition studies are, apart from the book value multiple relatively consistent in observing a PCD that ranges from 20% to 30%. Moreover, a few studies identify that discounts vary significantly across different industries. This leads to the following two related general hypotheses:

H1a: On average, private companies sell at a discount compared to public companies

H1b: On average, the size of the private company discount varies across different industries.

3.2 Size

To investigate the general hypotheses formulated above, a key linkage of interest lies between the size of a firm and the discount observed. The foundation of this variable was set when the SFP was observed. It follows from the acquisition studies (Section 2.5.4) that all papers find a strong significant positive relationship between firm size and the PCD. Firm size is measured by the natural logarithm of book value of total assets. The natural logarithm provides econometric robustness when dealing with outliers in the data and coefficients can directly be interpreted. As a result, the subsequent hypothesis is stated:

H2: Larger sized companies (measured by the natural logarithm of book value of assets) have lower discounts

3.3 Liquidity

Following the intuition of Amihud (2005) and Damodaran (2005) that premiums obtained by smaller firms (or the discounts observed in private company transactions) can be explained by illiquidity, the other key variable is defined as the level of liquidity. The fact that cash is the most liquid asset in a private company leads to the development of a variable cash and marketable securities over book value of assets. This reasoning formalized in the hypothesis below:

H3: Companies with higher levels of liquidity (measured by a proxy of cash over book value of assets) have lower discounts.

3.4 Geography

Koeplin et al. (2003) find lower discounts for non-US targets and Klein & Schneibel (2012) observed a seemingly low PCD in Europe. Moreover, the other acquisition studies that also look at non-US acquisitions find results that mark it interesting to include a dummy whether the target is based in the US. On the other hand, the reports and studies that focus on the SFP merely look at US data. Therefore, the last hypothesis focuses on the geography of the target and is formulated as:

H4: Companies that are based in the United States (measured by a dummy) have lower discounts.

3.5 Control variables

The next step is to identify a limited, but sufficient, set of control variables that adequately describe the PCD. Following the literature review, it can be concluded that, apart from the previously mentioned variables, growth rates, levels of profitability, the probability of financial distress, the type of buyer (public or private), and buyer motive (strategic or financial) all have an effect on observed discounts.

The differences in multiples between private and public companies can be a possible consequence of distinctive growth rates and levels of profitability. Therefore, it is important to control for these characteristics. Furthermore, firms with a higher probability of financial distress are less likely to be acquired. A higher level of risk is, therefore, able to affect discounts. Moreover, the price that is paid for an asset is always a result of negotiations between the buyer and the seller. In this case, the buyer represents the acquirer, and the seller represents the private target. Besides differences in target characteristics, acquirers can also demonstrate fundamental differences in characteristics. A strategic buyer often incorporates foreseen synergies in its valuation indicating a higher price. Also, firms that have little need for liquidity are more likely to pay a higher price. Public firms often have easier access to equity on the market and have deeper pockets. Therefore, it is important to control for acquirer characteristics. Table 3.5.1. presents an overview of the expected relations for the control variables which are further defined in Appendix A.

Table 3.1 Expected signs for the control variables

This table presents the various control variables and their expected signs that are used in this study. The variable definitions are included in Appendix A.

Variable	Proxy	Expected sign
Leverage	Total Liabilities divided by Book Value of Total Assets	Positive
Growth Rate	Cumulative Net Sales growth over the last five years prior to acquisition	Negative
Profitability	EBIT divided by Net Sales	Negative
ROA	EBIT divided by Book Value of Total Assets	Negative
Profitable	Dummy – 1 if the target is profitable, 0 otherwise	Negative
Public Buyer	Dummy – 1 if the acquirer is a public company, 0 otherwise	Positive
Financial Motive	Dummy – 1 if the acquirer is a financial firm, 0 otherwise	Negative
Intercontinental	Dummy - 1 if the target is located in a different continent than the acquirer, 0 otherwise	Negative

Apart from the control variables listed above, there are others that could be identified such as the method of payment and whether the acquisition is hostile or friendly. However, it is not the scope of this research to identify a large set of predictors with regards to the PCD. Besides, this paper intends to focus on the comparability to existing acquisition studies which also do not include certain control variables.

It has to be noted that the hypotheses are based on parts of the findings in the existing literature. The discussion of the empirical results specified to practitioners is presented in Chapter 6 [*Practical Implications*], which could lead to additional reasons and insights into why these relations are expected to hold. The following chapter describes how the aforementioned hypotheses are analyzed.

CHAPTER 4 Methodology

This chapter discusses how the hypotheses are tested using both univariate and multivariate analysis. First, the collection of the data and the selection of the sample is discussed. Then, the methods used for the analysis are explained and a special focus is put on the matching procedure which remains a fundamental part of this research. Finally, the econometric model is described and some necessary robustness checks of significance are highlighted.

4.1 Data

In order to test the hypotheses, data on both private and public company acquisitions is required. The data is extracted from Thomson One (formerly known as SDC Platinum). Thomson One holds data on mergers and acquisitions (“M&A”), IPO’s, and financial information from annual reports. Although its focus is on listed companies, it also has extensive M&A data on private companies. While the coverage of Thomson One with regards to accounting data is relatively small, this paper refrains from merging with accounting data that can be retrieved through Compustat as this creates noise and significantly reduces the number of usable observations. This is also tricky because SIC codes are often assigned differently across databases. Furthermore, global M&A data (thus data covering both US and non-US targets) are retrieved in order to make a geographical comparison of PCD’s. Also, to assess the impact of industrial classification, this study identifies the acquisitions into eight different industries using SIC codes obtained through Thomson One. Finally, similar to existing studies, only acquisitions involving a controlling interest are considered.

Due to the fact that the multiple information as provided by Thomson One is very limited, new multiples were constructed (i.e. EBITDA, EBIT, Net Income, and Sales) similar to existing acquisition studies. Correlations between the multiples given by Thomson One and those self-constructed were very high (above 0.95). In addition to the financial data that was already provided by Thomson One, variables are created for size, leverage, liquidity, profitability, and return on assets (“ROA”). Size is measured by the natural logarithm of book value of total assets. Cash and marketable securities divided by book value of total assets is used as a proxy for liquidity following Damodaran (2015). Leverage is measured as book value of total debt divided by book value of total assets, profitability as EBIT divided by net sales, and ROA as EBIT divided by book value of total assets. Also, dummy variables are created for certain characteristics with regards to deal geography and target or acquirer characteristics. Deal geography dummies include a dummy for whether the deal was intercontinental. This dummy is constructed using both the information with regards to the country of the target and the acquirer. It is expected that intercontinental deals face lower discounts since it entails a chance for the acquirer to enter new markets and to achieve greater geographic diversification opportunities. More comprehensive definitions of variables can be found in Appendix A.

Officer (2007) denotes that SDC only has reliable acquisition data for private companies after 1985. Therefore, the initial extracted sample is comprised of private and public company acquisitions covering the time span between 1985 and 2016. This left 29,773 acquisitions of which 21.8% was private. However, unlike most other acquisition studies, this paper excludes financial firms (firms with one-digit SIC code 6) and regulated utilities (firms with two-digit SIC code 49), because they can typically be characterized by higher levels of leverage and cash, thus potentially troubling the results. More important, earnings-based multiples, which are predominantly used to test the hypotheses, cannot be easily adapted to value financial firms and regulated utilities. Moreover, observations for which the data seemed erroneous (e.g. sales and/or leverage that are negative and liquidity and/or profitability above 100%) were also discarded. The final sample consists of 16,155 acquisitions of which 23.8% is private. The summary statistics of the final sample are further discussed in Section 5.1 [*Descriptive statistics*].

4.2 Method

4.2.1 Overall procedure

This paper uses both univariate and multivariate analysis to test the defined hypotheses and to answer the main research question. Within univariate analysis, this paper specifically focuses on the central tendency (e.g. mean and median) with regards to the PCD and it is used to test the first hypothesis. This includes paired t-tests (mean), Mood's median tests, and Wilcoxon signed-rank tests (both median) for all variables between two groups (i.e. private company acquisitions and public company acquisitions). The t-test is a parametric test and assesses whether the means of two groups are statistically different from each other. The Mood's median test and Wilcoxon rank-sum test (or Mann-Whitney U test) are both non-parametric tests for the comparison of continuous variables between two paired groups. The Mood's median test is a special sort of the Pearson's chi-squared test, whereas the Wilcoxon rank-sum test looks at the sum of all observations below the median and compares these with those above the median. Both are referred to as a non-parametric test and are used as an alternative to the paired t-test. They both do not assume a normal distribution.

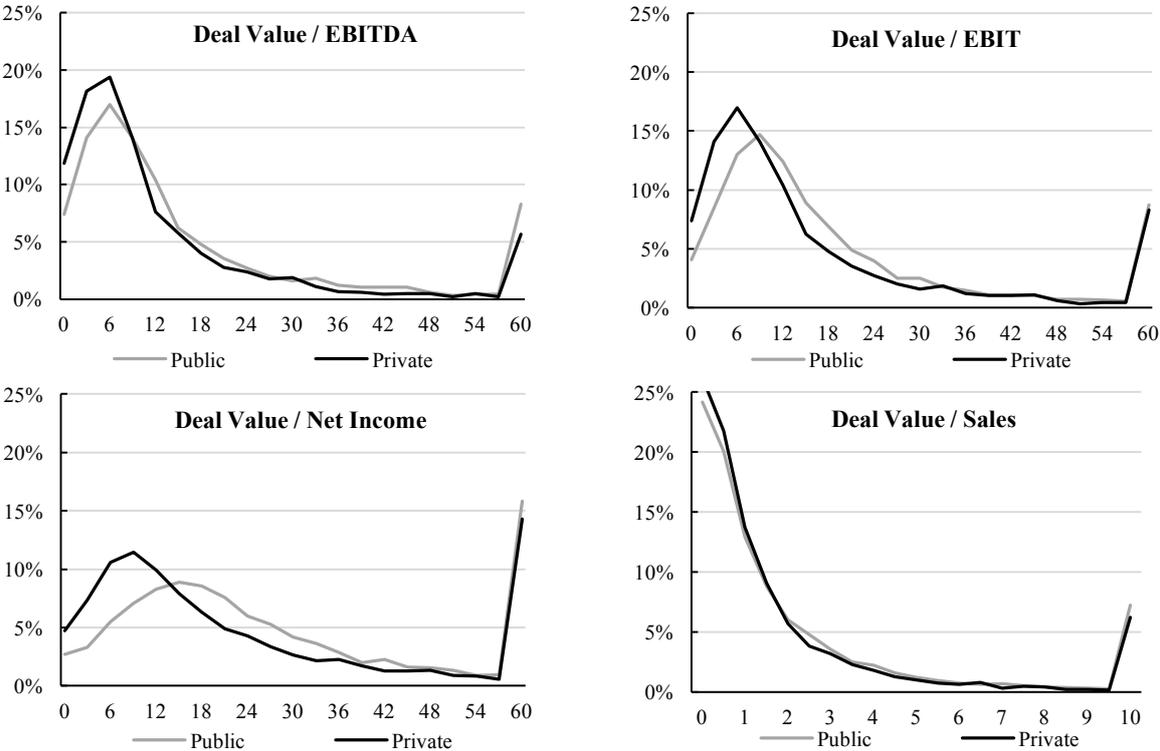
The multivariate analysis includes a cross-sectional multivariate regression model to test hypotheses 2, 3, and 4. The cross-sectional regressions are estimated via quantile regression (or median regression). It is a statistical technique for the analysis of data in which there is potentially some sort of cohesion. In this case, it is the cohesion between the independent variables (i.e. size, liquidity and US-based target) and the dependent variable (i.e. PCD). The parameters are measured in a linear regression model in which the differences (squares) are minimized between the observed discounts and those predicted by a linear function of the independent variables. The difference between Ordinary Least Squares ("OLS") and quantile regression is that the former estimates the mean, whereas the aims at estimating the median or another specified quantile. Nevertheless, both are used to describe the central tendency of the data. This paper focuses on the median due to the fact that the data has many outliers in

the right tail of the distribution (about 15% as can be seen in Figure 4.2.1). These outliers cause the sample to be non-normally distributed. OLS assumes that the data is normally distributed and is therefore referred to as a parametric test. More specifically, the mean (and thus OLS) is very sensitive to outliers and this study aims at keeping a large sample instead of arbitrary truncation of the data.

It seems that former studies that have made use of the Pratt Stats database (Business Valuation Resources, LLC) did not have to cope with large outliers. The observed outliers are mainly located in industries (based on SIC codes) that can be classified as high growth: Pharmaceutical Preparations (2834), Surgical and Medical Instruments and Apparatus (3841), Prepackaged Software (7372), and Information Retrieval Services (7375). These observations should not be discarded due to the fact that PCDs should also be observed across these outliers. On the other hand, OLS regressions are biased as the outliers highly affect observable means. Moreover, growth rates for these companies are often unspecified, potentially because of the early nature of the specific firms. The median regression attempts to correct the sensitivity that results from these outliers. Figure 4.2.1 depicts the frequency distributions and the non-normality of the multiples used in this study.

Figure 4.2.1 Histogram and frequency distributions

This figure presents various histograms in which the frequency distributions of the various multiples (i.e. EBITDA, EBIT, Net Income, and Sales) for 3,843 private company acquisitions (concerning a controlling interest) that occurred between 1985 and 2016.



4.2.2 Acquisition multiples

Equal to other acquisition studies, this study uses a multiple approach in the estimation of the PCD. Multiples are characterized as a ratio between a capital investment (i.e. enterprise value or equity value) and a financial metric associable to the providers of that capital. Using multiples eliminates the potential impact of changing investment rates or earnings over time. For example, a firm with an EBITDA of \$2 million and an EBITDA multiple of 10 would be acquired for \$20 million. If EBITDA grew to \$3 million and the deal value would increase to \$30 million, it would look like the company is valued higher, but this is not the case. It is still acquired at 10 times EBITDA ($\$30/\3). The multiples used in this study (i.e. EBITDA, EBIT, Net Income, and Sales) are all based on deal value. Deal Value is defined as the total value of consideration paid by the acquirer, excluding fees and expenses for all common stock, common stock equivalents, preferred stock, debt, options, assets, warrants, and stake purchases made within six months of the announcement date of the transaction. Therefore, deal value is similar to enterprise value as it is the total consideration paid for both the value of equity as the value of debt. The computation of a net income multiple based on deal value seems arbitrary due to the fact that net income is only available to the providers of equity (as the providers of debt have already been compensated). However, the potential spurious effect of using a certain multiple is mitigated as the multiple is calculated exactly the same for private and public firms. Thus, the troubling effect is eliminated when the PCD is computed. Appendix A depicts a detailed description for each of the multiples that are used in this study and also lists their respective advantages and disadvantages.

Additionally, Koeplin (2000) only found statistical evidence for estimating the PCD with earnings-based multiples. This is potentially because the EBITDA and EBIT multiple normalize for differences in capital structure (the mix of equity and debt) between firms. Koeplin (2000) did not construct a multiple based on net earnings. Also, earnings-based multiples best reflect operational performance which in turn determines value. Therefore, it is expected that earnings-based multiples yield the best results. Besides, the EBITDA multiple has been the most used multiple within former acquisition studies making it the best comparable. On the other hand, since multiples are limited to be positive values, the sales multiple has an advantage because it can be calculated for firms with negative earnings creating a bigger sample. Also, De Franco et al. (2011) denote the potential effect of valuation adjustments for anticipated synergies and abnormal management compensation that can create noise in earnings-based multiples.

4.2.3 Matching technique

In order to measure the discount of private companies as compared to their public peers, it is necessary to construct a careful matching procedure that compares any specific multiple of private companies to that of public companies. Koeplin (2000) and Block (2007) matched every private company transaction to one single public company transaction based on country, year and industry. However, this is a noisy procedure as the firms fundamentally differ in characteristics (e.g. public companies are generally bigger

than private companies and have distinctive growth rates) and the discount is very sensitive to the specific match created. Kooli (2003) controls for this, by introducing a matching procedure that constructs reference portfolios. This paper continues using that procedure by matching every private deal to a reference portfolio of public deals. Officer (2007) also uses a similar matching procedure, but the reference portfolio for each private company is based on public company acquisitions (i) in the same two-digit SIC code, (ii) with a deal value within 20% of the private company deal value, and (iii) occurring within a three-year period surrounding the private company acquisition. However, both studies seem to fail in constructing a large enough sample due to deleting outliers or observations for which the data was incomplete or in which it seemed spurious. Also, their reference portfolios are smaller in size because they define a larger amount of different industries.

The reference portfolios in this paper are constructed on the basis of year, industry, and size. First, breakpoints for size are determined by a division of the private companies into four equal groups (quintiles) using Stata. Second, the public companies are categorized within a specific quantile using the breakpoints observed from the division of private companies. This procedure is fundamentally different from Kooli et al. (2003) that ensures the comparison of companies with the same key characteristics. Finally, any specific private company acquisition is matched to all public company acquisitions that take place within the same year and industry of a similar size. All public company acquisitions that match based on the aforementioned indicators are considered the reference portfolio. Consequently, a benchmark multiple is constructed using the median multiple over all public company multiples for each reference portfolio. This method is repeated for all four multiples that used in this study. It is alleged that this benchmark multiple is a more accurate representation of actual multiples observed in the market and it is less affected by arbitrary aspects caused by direct matching. It also controls for the potential influence of a sample selection bias.

4.2.4 Private Company Discount

The next step in the process is to measure the PCD. The general technique to calculate the PCD was introduced in the Koeplin (2000) study and has been preserved across other acquisition studies. The formula is defined as follows:

$$PrivateCompanyDiscount = 1 - \frac{PrivateCompanyMultiple}{PublicCompanyMultiple} \quad (1)$$

However, the studies differ the methodology that is used in the final stage to determine the size of the PCD. The Koeplin (2000) and Block (2007) study compare the average (median) multiple for all private companies with the average (median) multiple of all public companies. Then, they use these values as input for the aforementioned formula to compute the PCD. In this study, this method will be referred to as the traditional method. On the other hand, Kooli (2003), Officer (2007), and Paglia & Harjoto (2010) compute the PCD for each individual private acquisition using the matching technique as described in

the preceding section. This leads to more reliable conclusions since each transaction is compared to a portfolio of public company transactions with a similar size in the same year and industry. This will be further referred to as the portfolio method.

This paper presents the PCD using both the traditional method and the portfolio method in order to provide a proper comparison with preceding studies. A snapshot of the sample from the third portfolio of the manufacturing industry in 2007 is provided in Appendix C. This table also presents the PCD measured using both the traditional method and the portfolio method with regards to that specific subsample. Basically, the smaller the sample size, the larger the difference between the traditional method and the portfolio method.

4.3 Model

The basic regression model is defined as follows:

$$PrivateCompanyDiscount_j = \beta_0 + \beta_1 * Size_j + \beta_3 * Liquidity_j + \beta_4 * UnitedStates_j + \beta_n * ControlVariables + \sum_{i=1}^5 \delta_i * Industry_i + \sum_{t=1996}^{2016} \phi_t * Year_t + \varepsilon_j \quad (2)$$

Where:

- *PrivateCompanyDiscount_j* is the dependent variable. The PCD per private company is measured using the following formula, which is derived from formula 1:

$$PrivateCompanyDiscount_{k,t,i,s} = 1 - \frac{PrivateCompanyMultiple_{k,t,i,s}}{MedianPublicCompanyMultiple_{k,t,i,s}} \quad (3)$$

- *Size* is measured by the natural logarithm of the book value of total assets;
- *Liquidity* is measured by cash and marketable securities over book value of total assets;
- *UnitedStates* is a dummy variable for whether the target firm is based in the US;
- *ControlVariables* are comprised of deal characteristics (e.g. *Intercontinental*), target characteristics (e.g. *Leverage*, *GrowthRate*, and *Profitability* or *ROA*), and acquirer characteristics (e.g. dummies for *FinancialMotive* and *PublicBuyer*);
- *Industry_i* is a dummy variable that represents a certain industry. The various industries are defined in Appendix B;
- *Year_t* is a dummy variable that represents a certain year between 1985 and 2016;
- β_n are the parameters to be estimated;
- ε_j is the error term;
- *k* represents the applicable valuation multiple. Four basic valuation multiples will be used (EBITDA, EBIT, Income, and Sales), which are further defined in Appendix A;
- *j*, *t*, *i*, and *s* respectively characterize a specific private company, the time period ($t = 1985$ to 2016), the industry ($i = 1$ to 8), and the portfolio size quartiles ($s = 1$ to 4).

4.4 Robustness tests

To check the results of the model, several potential problems need to be addressed. These problems refer to sample selection bias, the robustness of results using several performance measures, serial correlation (or autocorrelation), heteroscedasticity, multicollinearity, and endogeneity. This section states some performed robustness tests that are executed on the data.

First, the robustness of the results could be influenced by the structure of the Thomson One database and, hence, sample selection. If Thomson One systematically has fewer acquisition data on small private firms, this could lead to a sample selection bias, which should be taken into account when interpreting the results. A closer look at the data shows that 96.5% of private company acquisitions involves targets that have sales levels above one million dollars. Therefore, it is likely that many more private company acquisitions occur of firms that have sales levels below one million which are not included in Thomson One. Specifically, Thomson One only indicates up to ten acquisitions annually in which the target has sales levels below one million. These firms denote sales multiples that are about 30 times higher than those for larger firms. It is alleged that the unobserved smaller acquisitions have much lower multiples causing the PCD to be larger. This is in line with findings of Knez & Ready (1997), Horowitz et al., (2000), and Peek (2016) with regards to the SFP. They found that the SFP is captured within the smallest size decile that includes companies with up to eight million revenues (“micro firms”). Therefore, Chapter 5 [*Empirical Findings*] presents the results with and without truncation of the sample to exclude micro firms. The truncated sample also shows evidence for the existence of the PCD. However, the results are somewhat weaker.

Second, if a true effect exists between size and/or liquidity and the PCD, it should not depend on the definitions used in the empirical model. It is important to test if the regression results hold across performance measures. Therefore, different proxies need to be used for both size and liquidity. Besides the natural logarithm for book value of total assets, this paper uses the natural logarithm of sales as a proxy for size following Paglia & Harjoto (2010). It is difficult to define an alternative proxy for liquidity since private firms are not publicly traded on the market. However, as Damodaran (2005) illustrates the importance of controlling shares in private company acquisitions, it is interesting to look at the percentage of shares that is acquired in the transaction. Although this study excludes acquisitions in which a non-controlling interest was acquired, still 76.2% of the observed acquisitions involve an interest that lies between 50.0% and 99.9%. The regression outputs of most of these regressions are not reported due to the number of additional tables, but the results are robust to different proxies.

Third, it is important to address serial correlation in the sample. The standard errors of specific observations can be grouped when firms in the same industry react similarly to economic shocks. This makes the standard errors within an industry correlated. An example is the Dotcom bubble after which information technology firms experienced sudden declining stock prices. The correlation of standard errors also occurs when firms within an industry have similar balance sheet structures (e.g. all services

firms have fewer assets on the balance sheet than manufacturing firms). Industries with similar balance sheet structures can react similarly to economic shocks. The same is true for certain sub-periods or years in which standard errors can be correlated due to periods of crisis. Therefore, this paper adds dummy variables for both the specific year and the corresponding industry in which an acquisition took place. Additionally, in unreported regressions, this paper attempts to correct serial correlation by clustering standard errors at the industry level. The results are not much different to the models in which industry dummies were used.

Fourth, an important robustness check is to test for heteroscedasticity which can be detected using a White-test in Stata. Heteroscedasticity occurs when there are different variabilities across different sub-samples. It needs to be corrected across the sample since the variance of standard errors increases exponentially which in turn biases the results. To correct for heteroscedasticity, this paper uses the robust option in Stata. Finally, this paper tests for multicollinearity between variables by looking at the correlation between all the variables that are included in the various regressions. None of the variables seems highly correlated. The highest correlation for the independent variables is noted between ROA and Profitability (0.53). This is logical as both measures are an indicator to measure profitability and they are not used jointly in the applicable models. The various correlation tables are included in Appendix D (Tables D.1 to D.3).

Usually, another important test is to check for normality in the residuals and the independent variables. Due to the fact that the data has definite outliers, it is clear that the outcome does not follow a normal distribution. A Skewness-Kurtosis test variation on the Jarque-Bera test) that looks at properties of the skewness and kurtosis of the raw data does indeed reject the hypothesis that it is normally distributed. This is supported by four different multivariate normality tests (i.e. Mardia Skewness, Mardia Kurtosis, Henze-Zirkler, and Doornik-Hansen). Nonetheless, the most important tests used in this study (i.e. Mood's Median Tests and quantile regressions) are non-parametric tests. While non-parametric tests are generally less powerful than their parametric peers, they can be used when the data is non-normal. Therefore, the normality of the data is of less importance.

Various robustness tests that are not presented in the main body of the text can be found in Appendix D. The paper continues in the next chapter where the statistics of the sample and various variables are described. It also presents the results of both the univariate and multivariate analysis. These results serve as a basis for the practical implications. Finally, it provides a basic comparison between the development of the SFP and the PCD.

CHAPTER 5 Empirical Findings

In this chapter, the data is described by presenting descriptive statistics and the empirical results of the cross-sectional regressions are discussed. These results are predominantly presented using tables within the text. However, some tables are included in Appendix B. These tables are generally of less importance than those presented within the text. The regression tables are presented with and without the addition of robustness tests such as year and industry dummies (fixed effects), the robust option in Stata, and clustered standard errors. Section 5.2 [*Univariate analysis*] includes tests with regards to the first hypothesis and looks at the development of the PCD over time, whereas tests concerning hypothesis two, three, and four can be found in Section 5.3 [*Multivariate analysis*]. Finally, the last section aims at comparing the development of the PCD to the development of the SFP.

5.1 Descriptive statistics

To test the model, data was collected from Thomson One as further explained in Section 4.1 [*Data*]. The final sample includes 3,843 private company acquisitions that occurred in the time period between 1985 and 2016. Figure 5.1.1 presents the development of activity versus value across the sample over the full period. About 65.2% of the deals and 77.0% of the value took place after 2000. The total sample represents about 2% of the global M&A market which can be derived from Figure D.1. The biggest shocks with regards to M&A activity occurred after 2000 and after 2007. The shock around the change of the century is related to the Dotcom bubble and the other shock has to deal with the global financial crisis that started in 2008. This suggests that, besides their microeconomic situation, private company acquisitions also greatly depend on the macroeconomic environment. It can also be concluded that the average deal value has been increasing steadily after 1995.

Table 5.1.1 and Figure 5.1.2 show splits of the private company acquisitions by industry and geography. The observations are classified into five different industries based on their SIC code. These industries include wholesale (12.2%), retail (11.5%), manufacturing (18.6%), services (15.0%), energy (2.3%), healthcare (8.8%), technology (23.3%), and other (8.3%). Other includes agriculture, construction, and transportation. The division into the various industries was based on their SIC code and corresponding multiples. This meant for example that amongst others computer software (three-digit SIC code 737) is categorized into the technology industry, while this is included in the services category within the SIC directory. This categorization was based on the fact that computer software companies are being acquired for much higher multiples. For a more extensive overview of industry categorization, see Appendix B. The average deal value is highest within the technology sector and smallest in the wholesale sector. Furthermore, over 90% of all private company acquisitions take place in Europe or North America. This seems to be slightly higher than for the public company acquisitions for which 75% takes place in aforementioned continents. Looking at the acquisitions that took place in Europe it seems that more than 50% of the targets are from the United Kingdom.

Figure 5.1.1 Private deal activity versus private deal value 1985 – 2016

This figure presents the private deal activity between 1985 and 2016. The total sample includes 3,843 private company acquisitions and is extracted from Thomson One. The table shows the total completed deals (concerning a controlling interest) and corresponding total deal value.

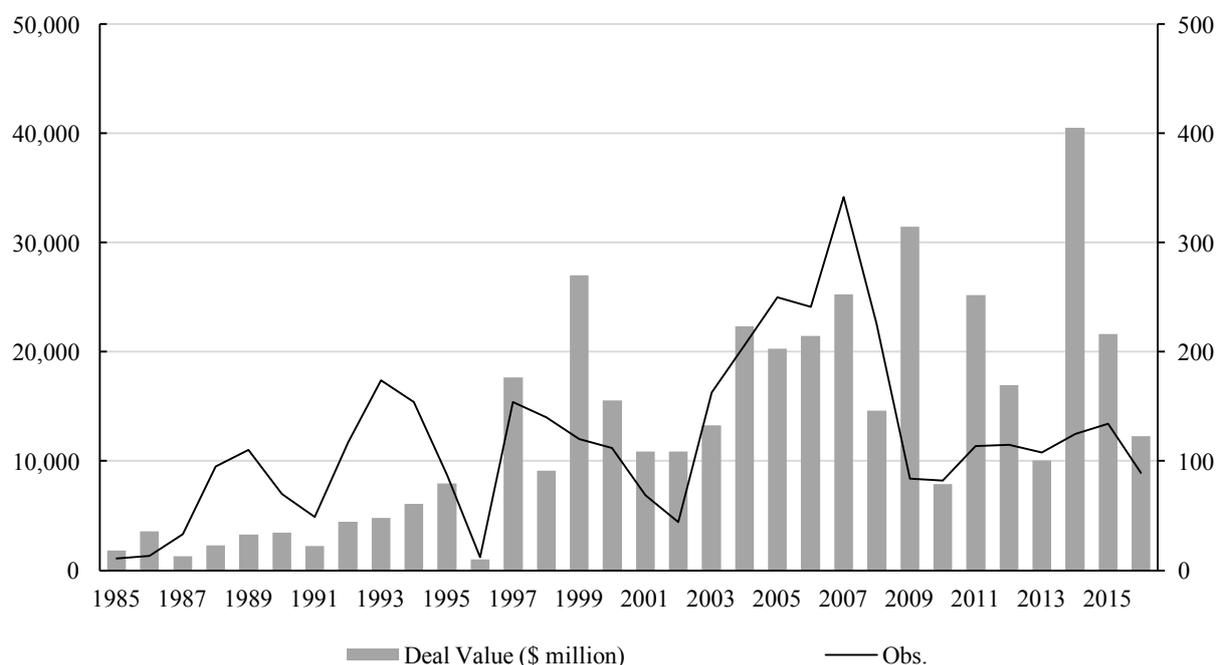


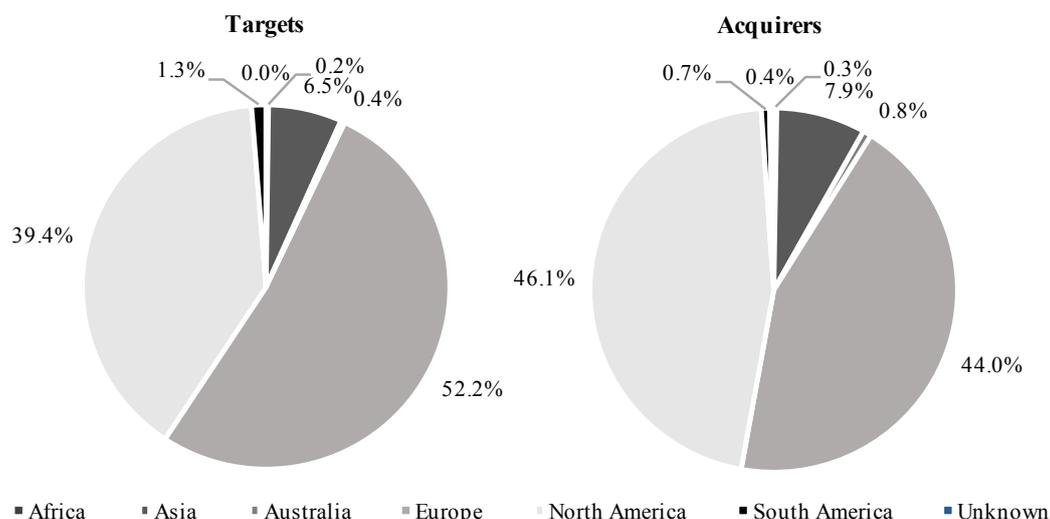
Table 5.1 Private company acquisitions by industry

This table presents the summary statistics for private company acquisitions between 1985 and 2016. The sample includes 3,843 private company acquisitions and is extracted from Thomson One. The table presents the number of observations, total deal value, and average deal value per industry. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%, respectively to the Mood's Median Test that the mean value of private company acquisitions differs significantly from the mean value of public company acquisitions.

Industry	Obs.	Percent	Cum.	Deal Value	Percent	Cum.	Avg. Value
1 Wholesale	470	12.23	12.23	26,970	6.48	6.48	57.4
2 Retail	714	18.58	30.81	67,509	16.21	22.69	94.5
3 Manufacturing	443	11.53	42.34	56,967	13.68	36.36	128.6
4 Services	575	14.96	57.30	35,567	8.54	44.90	61.9
5 Energy	90	2.34	59.64	11,675	2.80	47.71	129.7
6 Healthcare	339	8.82	68.46	39,072	9.38	57.09	115.3
7 Technology	894	23.26	91.73	143,709	34.51	91.59	160.7
8 Other	318	8.27	100.00	35,005	8.41	100.00	110.1
Total	3,843	100		416,474	100		108.4

Figure 5.1.2 Private company acquisitions by geography

This figure presents a geographic split of private company targets and their acquirers between 1985 and 2016. The total sample includes 3,843 private company acquisitions and is extracted from Thomson One.



The descriptive statistics with regards to the deal characteristics are listed in Table 5.1.2. The statistics show that the average stake acquired in private deals is higher than that in public deals. It has to be denoted that only controlling interests are observed. Moreover, the public deals are about ten times the size of a private deal, more often take place in the United States, and are more often executed intercontinental. Interesting is the observation that the acquirer is more often a listed company in the case of a private deal. Also, public deals seem to be backed by financial motives more frequently than strategic motives (i.e. the buyer is less often a corporate).

Table 5.2 Descriptive statistics of deal characteristics

This table presents the summary statistics for both private and public company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. *, **, and *** indicate significance levels of 10%, 5%, and 1%, respectively to the Mood's Median Test that the mean value of private company acquisitions differs significantly from the mean value of public company acquisitions.

Variable	Private Deals	Public Deals	Difference
	Mean	Mean	t-test
Shares Acquired	96.7%	90.2%	6.5%***
Deal Value	108.4	986.5	-878.1***
United States	37.9%	47.2%	-9.3%***
Intercontinental	10.9%	13.2%	-2.3%***
Public Acquirer	71.4%	58.5%	12.8%***
Financial Motive	12.3%	15.3%	-3.0%***
Profitable Target	80.7%	71.4%	9.4%***
Number of deals	3,843	12,312	

Table 5.1.3 lists the summary statistics concerning some of the independent and control variables. Specifically, the statistics list the various characteristics of the private and public deals that are included in the sample. The total number of firms is presented below the table. Many of the variables have some missing variables, causing the specific number of observations to be somewhat lower. Growth variables have the lowest number of observations. Overall, public firms are approximately five times bigger and hold a slightly higher percentage of cash and marketable securities in their books. Also, as the leverage ratio is lower, they tend to have a lower probability of financial distress. Thus, private firms are likely to be riskier because leverage ratios are rather high. On the other hand, private firms seem more profitable, since both ROA and Profitability are significantly higher, and showed larger cumulative growth measures over the years prior to the acquisition whether deliberated in terms of the median or mean. The private firms used in this paper are rather small as those compared to Officer (2007) and Paglia & Harjoto (2010). This is because they respectively delete observations with deal values and sales below \$50 million. On the other hand, the public firms included in this sample seem bigger and more profitable. Following the data, one can see the outliers troubling the means. For some variables (e.g. Sales Growth, EBITDA Growth and Profitability) the detrimental effect of the outliers seems evident.

Table 5.3 Descriptive statistics of private and public firm characteristics

This table presents the summary statistics for both private and public company acquisitions between 1985 and 2016. The total sample includes 16,155 observations and is extracted from Thomson One. The table shows the mean and median for each variable. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%, respectively to the t-test (Mood's Median Test) that the mean (median) value of private company acquisitions differs significantly from the mean (median) value of public company acquisitions.

Variable	Private Firms		Public Firms		Difference	
	Mean	Median	Mean	Median	t-test	Mood-test
Total Assets	132.8	12.1	882.4	128.3	-749.6***	-116.1***
Liquidity	16.2%	8.6%	17.0%	9.4%	-0.7%***	-0.8%***
Shareholder's Equity	32.6	4.2	333.1	60.2	-300.5**	-56.1***
Leverage	60.2%	62.8%	48.5%	48.8%	11.7%***	14%***
Sales	161.0	18.8	778.1	122.3	-617.2***	-103.5***
Sales Growth (5-year)	14.8%	9.1%	16.6%	7.9%	-1.8%	1.2%**
EBITDA	14.4	2.0	102.5	11.1	-88.1***	-9.2***
EBITDA Growth (3-year)	26.7%	13.8%	18.2%	9.9%	8.5%***	3.9%***
EBIT	8.0	1.2	55.8	5.3	-47.8***	-4.1***
ROA	25.5%	13.9%	9.9%	8.2%	15.6%***	5.7%***
Profitability	-74.4%	6.4%	-197.3%	4.8%	122.9%	1.6%***
Net Income	3.6	0.7	27.5	2.3	-23.9***	-1.6***
Employees	1,336.5	128.0	4,128.0	772.0	-2,791.6***	-644***
Acquirer Total Assets	2,659.1	147.0	11,753.5	1,157.3	-9,094.4***	-1,010.2***
Acquirer Total Sales	1,671.5	141.9	7,714.8	910.6	-6,043.2***	-768.6***
Number of firms	3,843		12,312			

5.2 Univariate analysis

5.2.1 Traditional method

In order to test the first hypothesis if the PCD exists and whether it varies across industries two methods have been defined in Section 4.4 [Method]. This section presents results following the traditional method which was introduced by Kooli et al. (2000) and later executed by Block (2007). This method lacks in two ways. First, it provides noisy results as the sample could be unbalanced. This means that a potential overbalance of private company acquisitions positioned in a low multiple industry leads to an overstatement of the PCD. Second, the private and public firms fundamentally differ in characteristics as can be seen in the previous section. Private firms are smaller and have higher leverage ratios. This can possibly influence the results in a similar way as an overbalance of a specific industry. However, the results are also presented using the traditional method in order to make a proper comparison between this study and their studies. Tables 5.2.1 and 5.2.2 show the results.

Table 5.4 Median PCDs measured using traditional method

This table presents the median PCDs for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the traditional method, which indicates that the PCD calculation is based on the median values of the sample as a whole. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%, respectively to the Mood's Median Test that the median value of private company acquisitions differs significantly from the median value of public company acquisitions.

Variable	Private Firms	Public Firms	Difference	PCD
EBITDA Multiple	9.12	9.46	-0.33***	3.53%***
EBIT Multiple	11.29	14.39	-3.10***	21.54%***
Net Income Multiple	17.16	23.22	-6.05***	26.07%***
Sales Multiple	1.06	1.19	-0.14***	11.43%***
Number of firms / Average	3,843	12,312		15,6%

As discussed below, the empirical evidence supports the existence of a PCD, leading to a rejection of the null hypothesis that multiples are the same for private and public firms. It can be concluded that public companies trade at significantly higher multiples than private companies. The discount ranges from 3.5% to 26.1% depending on the multiple that is used. The discount is lowest when calculated using the EBITDA multiple and is highest using the Net Income multiple. For example, a median private company sells at 11.3 times its EBIT, whereas a median public company sells at 14.4 times its EBIT, implying a discount of 21.5%. It also implies that an additional dollar of pre-tax earnings is 27.5% more worth in a public company as compared to a company that is private. However, the sales multiple may have different implications than the earnings-based multiples. This means that the value of an additional

dollar in sales may vary across certain industries. Thus, it is vital to assess the dissimilarities between industries. This is presented in Table 5.2.2.

Table 5.5 Median PCDs measured using traditional method by industry

This table presents the median PCDs per industry for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the traditional method, which indicates that the PCD calculation is based on the median values of the sample as a whole. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A and the industry classifications are further defined in Appendix B. *, **, and *** indicate significance levels of 10%, 5%, and 1%, respectively to the Mood's Median Test that the median value of private company acquisitions differs significantly from the median value of public company acquisitions.

	Industry	EBITDA	EBIT	Net Income	Sales
	<i>Private Deals</i>				
1	Wholesale	7.19	9.45	14.73	0.57
2	Retail	8.93	11.40	18.99	0.76
3	Manufacturing	7.47	9.61	14.94	0.85
4	Services	10.49	12.97	19.45	1.31
5	Energy	7.71	11.61	16.12	2.36
6	Healthcare	10.17	12.38	18.38	1.75
7	Technology	12.14	15.69	20.41	1.91
8	Other	6.36	9.08	14.64	0.60
	<i>Public Deals</i>				
1	Wholesale	7.76	10.53	18.89	0.49
2	Retail	8.82	13.84	22.70	0.64
3	Manufacturing	8.33	12.22	20.61	0.84
4	Services	9.78	14.64	25.14	1.29
5	Energy	7.44	14.49	21.42	3.41
6	Healthcare	13.35	18.33	30.58	2.28
7	Technology	13.25	20.91	29.44	1.85
8	Other	6.84	11.08	18.09	0.62
	<i>Difference</i>				
1	Wholesale	-0.6	-1.1**	-4.2***	0.1*
2	Retail	0.1**	-2.4***	-3.7***	0.1
3	Manufacturing	-0.9	-2.6***	-5.7***	0.0*
4	Services	0.7*	-1.7**	-5.7***	0.0
5	Energy	0.3	-2.9	-5.3	-1.0**
6	Healthcare	-3.2**	-6.0***	-12.2***	-0.5***
7	Technology	-1.1	-5.2***	-9.0***	0.1
8	Other	-0.5	-2.0***	-3.5***	0.0
	<i>PCD</i>				
1	Wholesale	7.4%	10.2%**	22.0%***	-14.4%*
2	Retail	-1.3%**	17.7%***	16.3%***	-19.1%
3	Manufacturing	10.3%	21.3%***	27.5%***	-0.9%*
4	Services	-7.3%*	11.4%**	22.6%***	-1.4%
5	Energy	-3.7%	19.9%	24.7%	30.7%**
6	Healthcare	23.8%**	32.5%***	39.9%***	23.2%***
7	Technology	8.4%	25.0%***	30.7%***	-3.0%
8	Other	7.1%	18.0%***	19.1%***	3.4%

The results in Table 5.2.2. indicate that the second part of the null hypothesis can also be rejected: the PCD differs significantly across different industries. The bottom line multiples (i.e. EBIT and Net Income) compute discounts that are both economically and statistically significant, whereas discounts calculated using the EBITDA and Sales multiple are only significant for some industries. The discount is most eminent and highest in the healthcare industry (29.8% on average). The wholesale, retail and services sector seem to have the lowest discounts or, depending on the multiple, tend to even demonstrate a premium. Discounts in the energy industry are only significant using the sales multiple. This could be because the industry is very capital intensive and determined by size leading to a bigger relative valuation of topline revenues.

The results are similar to Kooli (2003) and Block (2007), especially for the EBIT multiple. They found average discounts of 13.9% and 21.8% as compared to a discount of 15.6% in this study. The much lower discount as observed through the EBITDA multiple is likely to be a byproduct of including non-US firms in the sample as can be observed in Tables D.7 to D.9. Yet, this study was based upon a sample of 3,843 private company acquisitions including non-US transactions, whereas the original studies using the traditional method are based upon 192 and 91 private company acquisitions. Nevertheless, these results should be interpreted with caution because of the noise that is created by the traditional method. The following section shows the results based on the portfolio method.

5.2.2 Portfolio method

This section presents the results with regards to the first hypothesis following the portfolio method. This method has been performed by Kooli (2003), Officer (2007), and Paglia & Harjoto (2010). For a detailed description of the portfolio method, see Section 4.2.3 [*Matching technique*] and Section 4.2.4 [*Acquisition studies*]. The results reported in Table 5.2.3, 5.2.4, and 5.2.4 depicts evidence that support the first hypothesis: private companies sell at an average discount of 19.2% as compared to their public peers based on the full sample of acquisitions between 1985 and 2016. A Pearson's chi-square test with null hypothesis that the private companies do not trade at a discount can be rejected at a significance level of 1% for all multiples as can be seen in Table D.4. Overall, between 60% and 70% of private companies are acquired at a lower price than public companies. The precise number depends on the multiple which can also be found in Table D.4. The discount is highest when constructed using earnings-based multiples. This is indirectly caused by the fact that deal values of private company transactions are lower (after a correction for size), and that, private companies are generally more profitable than public companies. Thus, it can be concluded that they are receiving less for this profitability as compared to their public peers. The results are strikingly similar to former acquisition studies which appeared unable to construct a large sample size. On average, they found a discount of 17.4% for samples ranging anywhere between 88 and 664 observations.

Table 5.6 Median PCDs measured using portfolio method

This table presents the median PCDs for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually. The presented values are the median observed discounts. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

Variable	PCD
EBITDA Multiple	18.1%***
EBIT Multiple	24.3%***
Net Income Multiple	23.8%***
Sales Multiple	10.5%***
Average	19.2%

Furthermore, the results show that discounts are both economically and statistically different across industries. A Pearson's chi-square test with null hypothesis that the private company discount is independent of industry classification can be rejected at a significance level of 1% for the EBITDA Multiple and Sales Multiple, and 5% for the EBIT Multiple and Income Multiple. This is documented in Table D.4. This test is measured by comparing the number of private firms that are sold at a discount with the firms that are sold at a premium within the same industry. The size differences between industries are tested using the Mood's Median Test and shown in Table 5.2.4. The results using the Wilcoxon rank-sum test are similar to those presented. The highest discounts are observed in the construction (49.4%), energy (27.3%), manufacturing (26.1%), and healthcare (23.6%) sector. The lowest discounts can be found in the retail (8.6%) and services sector (10.6%).

Table 5.7 Median PCDs measured using portfolio method by industry

This table presents the median PCDs per industry for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A and the industry classifications are further defined in Appendix B. *, **, and *** indicate significance levels of 10%, 5%, and 1%, respectively to the Mood's Median Test that the median value of private company acquisitions differs significantly from the median value of public company acquisitions.

Variable	EBITDA	EBIT	Net Income	Sales	Average
1 Wholesale	9.3%	15.3%**	22.2%***	11.4%**	14.5%
2 Retail	6.0%	19.9%***	12.1%***	-3.8%*	8.6%
3 Manufacturing	25.8%*	30.8%**	33.0%***	14.6%	26.1%
4 Services	10.0%*	14.8%**	10.3%***	7.4%	10.6%
5 Energy	5.6%	22.4%	31.0%	50.3%**	27.3%
6 Healthcare	30.3%**	22.0%***	19.2%***	22.8%***	23.6%
7 Technology	17.0%	29.8%***	23.6%***	-4.4%	16.5%
8 Other	27.5%	21.7%***	28.9%***	45.4%	30.9%

The results using the portfolio method are somewhat different to those using the traditional method. First of all, the EBITDA multiple deviates significantly (18.1% for the portfolio method compared to 3.5% for the traditional method). Also, the PCD across various industries is more stable when calculated using the portfolio method. Both observations can be attributed to a better comparison as firms are directly matched against a portfolio of public peers. The average portfolio is composed of 10 public company deals while 1,617 portfolios were constructed in total. 2% of the public companies fall in the smallest size portfolio (median book value of total assets \$2.0 million) and 78% fall in the portfolio for the largest size (median book value of total assets of \$214.5 million). The overall average PCD is 15.6% using the traditional method and 19.2% using the portfolio method.

As can be seen in Table 5.2.4, the results are robust and withhold for the most part when micro firms (those with sales below eight million dollars) are deleted from the sample. This is because of two reasons: (i) the SFP is only observed in micro firms and (ii) the sample is likely to be biased towards larger private companies as explained in Section 4.4 [*Robustness tests*]. Based on the three earnings-based multiples (i.e. EBITDA, EBIT and Net Income), the PCD amounts to 16.8%. The Sales Multiple denotes a premium of 9.3% indicating the private companies would sell at higher acquisition multiples than their public peers. This premium could be related to the fact that private companies have sales levels that are considerably larger than private companies. Although a careful matching procedure is conducted to control for size, the public companies within a portfolio are still somewhat bigger than private companies. Also, the median public company is 10.6 times as big as the median private company based on the book value of assets, but the factor is 14.3 based on the level of sales. Additionally, a direct consequence of this truncation is that the average portfolio size decreases slightly, causing results that are somewhat less reliable.

Table 5.8 Median PCD using portfolio method excluding micro firms

This table presents the median PCDs for private company acquisitions between 1985 and 2016 for which sales of the target exceeded \$8 million. The total sample includes 14,015 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually. The presented values are the median observed discounts. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

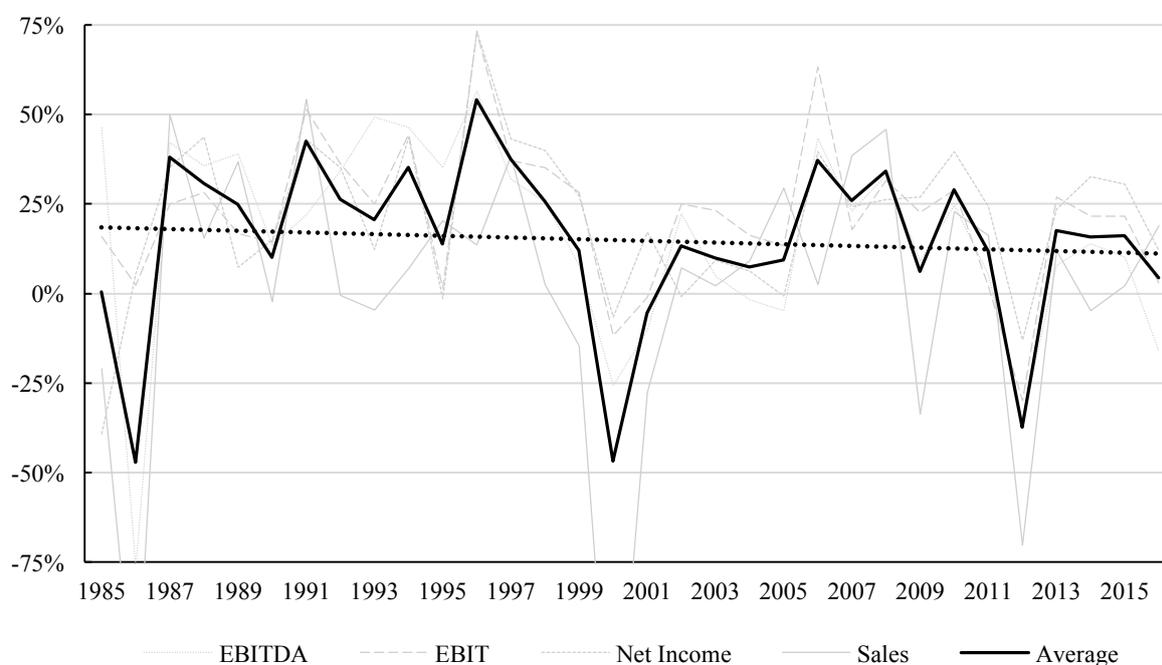
Variable	PCD
EBITDA Multiple	10.5%***
EBIT Multiple	21.2%***
Net Income Multiple	18.7%***
Sales Multiple	-9.3%***
Average	10.3%

5.2.3 Development of the private company discount

It is important to consider the development of the PCD over time. This development is presented for all multiples (including the simple average) in Table 5.2.5 and Figure 5.2.1. Previous acquisition studies found discounts for an average period of 14 years that ranged across different sub-periods between 1979 and 2009. This study finds evidence for the PCD considering a period of 31 years from 1985 to 2016. As can be seen in Figure 5.2.1, the average PCD is highly volatile and shows a decreasing trend. The PCD was 26.7% over the period 1985 until 1999 and 13.0% over the period 2000 to 2016 as depicted in Tables D.4 and D.5 included in Appendix D. This is determined by truncating the sample (deleting the observations from the other period).

Figure 5.2.1 Median PCD development 1985 – 2016

This figure presents the median PCD development between 1985 and 2016. The total sample includes 3,843 private company acquisitions and is extracted from Thomson One. The y-axis represents the size of the PCD, and the x-axis represents the years.



Moreover, Table 5.2.5 shows that the discounts are considerably large in the build-up of a crisis. The PCD averaged 27.9% in the build-up of the Dotcom bubble (1995 to 1999) and 31.8% in the build-up of the global financial crisis (2006 to 2008). As can be observed in Figure 5.2.1, the discount reverted to a premium when the Dotcom bubbles busted. The premium in 2012 could be explained by either (i) the smoothing of the global financial crisis until 2011 and (ii) the peak in the Eurozone debt crisis. The sample only includes 13 observations in 1986. Therefore, the observed premium is likely to be a byproduct of the fact that only large private acquisitions were included in the sample which are biased to denote relatively high multiples. Figure D.2 depicts the PCD per sub-period of five years for each of the multiples included in this study.

Table 5.9 Median PCDs measured using portfolio method by time interval

This table presents the median PCDs for private company acquisitions between 1985 and 2016 in specific periods. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually. The presented values are the median observed discounts. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

Period	Years	EBITDA	EBIT	Net Income	Sales	Average
Pre-internet period	1985 - 1994	31.4%***	25.1%***	25.5%***	22.7%***	26.2%
Dotcom bubble build-up	1995 - 1999	23.9%***	33.7%***	37.7%***	16.2%***	27.9%
Transitional period	2000 - 2005	-2.2%***	15.8%***	7.5%***	1.6%***	5.7%
Financial crisis build-up	2006 - 2008	34.5%***	37.4%***	32.2%***	23.1%***	31.8%
Post-crisis period	2009 - 2016	1.1%***	16.0%***	19.0%***	-18.8%***	4.3%

5.3 Multivariate analysis

This section presents the results of the multivariate analysis. It depicts the cross-sectional quantile regressions that are ran for each multiple. First, the tests are presented and then the results with regards to hypothesis 2, 3, and 4 are discussed. For each multiple, five regressions were executed to test the robustness of the outcome. The models a, b and c depict results for the key variables of interest without the addition of control variables, whereas model d and e include control variables. Furthermore, except for model a, all models include year and industry dummies to control for serial correlation. These intercept dummies capture constant industry-specific and time-specific factors. This paper refrains from adding country intercepts, due to the relatively low number of observations for some countries. In total, the sample includes the acquisitions of targets 93 countries of which half had less than ten observations. Including country dummies would weaken the results and enhance multicollinearity as a dummy variable was included for whether the target was located in the US. Unobserved heterogeneity is controlled for using the robust option in Stata in models c and d. Table 5.3.1 to 5.3.4. respectively show the results for each of the multiples (EBITDA, EBIT, Sales, and Income) used in this study.

Table 5.3.1 shows the results for the EBITDA multiple. It can be concluded that Size, Liquidity, and United States all have a negative effect on the median PCD. The interpretation of the coefficients in model c are as following: (i) a one percent increase in Size leads to a decrease of the PCD by 0.07 percent, (ii) a one percent point higher level of Liquidity leads to a decrease of the median PCD by 0.22 percent point, and (iii) if the target is located in the US, the median PCD is 9.23 percent point lower. Overall, it can be concluded that the test results are as expected. On the basis of model c, all of the null hypotheses with regards the hypothesis 2, 3, and 4 can be rejected at a 1-percent significance level. Thus, companies that are larger, have the highest level of liquidity, and those located in the US have lower discounts. Liquidity seems to be the most consistent and robust variable and its effect seems to become larger when control variables are added. Of the control variables, only Intercontinental and Profitability seem to have a significant effect on the discount, but these are not robust to heteroscedasticity. Leverage and Public Buyer where the only two predictors that were expected to have a positive sign. However,

the results indicate that Public Buyer has a negative sign and Profitability a positive sign. Due to the fact that the results are specific to the median rather than the entire conditional distribution, the Pseudo R-squared cannot be interpreted exactly the same as a conventional R-square in OLS regressions. It represents a calculation of one minus the raw sum of deviations divided by the minimum sum of deviations. It is based on the idea that the median regression is the maximum likelihood approximation for the distribution. However, the basic interpretation that it says something about predictability holds, but focus on the median should be taken into account.

Table 5.10 Cross-sectional results based on the EBITDA Multiple

This table presents the results of the quantile regression model with regards to the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the EBITDA Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. Estimated slope coefficients for Year and Industry dummy variables are not reported to save space. Standard errors of the coefficients are presented in parenthesis. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

	PCD_{EBITDA}				
	(a)	(b)	(c)	(d)	(e)
Size	-8.108*** (1.425)	-7.165*** (1.452)	-7.165*** (0.904)	-3.367 (4.262)	-3.367 (15.838)
Liquidity	-0.334** (0.135)	-0.224* (0.133)	-0.224*** (0.084)	-0.835** (0.415)	-0.835* (0.447)
United States	2.665 (4.738)	-9.232* (5.673)	-9.232*** (3.384)	-15.946 (17.576)	-15.946 (20.112)
Intercontinental				-36.919* (19.431)	-36.919 (108.057)
Leverage				0.313 (0.307)	0.313 (1.878)
Profitability				0.019* (0.010)	0.019 (27.289)
Growth Rate				-0.008 (0.181)	-0.008 (5.285)
Financial Motive				-2.156 (19.167)	-2.156 (169.278)
Public Buyer				-14.824 (15.061)	-14.824 (13.151)
Intercept	51.215*** (6.362)	88.542* (45.652)	88.542*** (12.352)	83.520 (64.330)	83.520 (782.601)
Observations	2,067	2,067	2,067	303	303
Pseudo R²	0.005	0.017	0.017	0.075	0.075
Year & Industry dummies	No	Yes	Yes	Yes	Yes
Robustness	No	No	Yes	No	Yes

Table 5.3.2, 5.3.3 and 5.3.4 depict the same models, but for the PCD measured using other multiples. The results are similar in terms of the signs but show different levels of significance and strengths of the coefficients. Table 5.3.2 model e, shows significant results for both Size and Liquidity when all controls are added. Based on the EBIT Multiple, a one percent increase in Size leads to a decrease in the PCD of 0.08 percent point. This effect is larger than based on the EBITDA Multiple. Besides, a level of liquidity that lies one percent point higher leads to a discount that is almost one percent point lower. Also, acquisitions that are done intercontinental yield discounts that are almost 25 percent point lower as compared to acquisitions in which both the target and the acquirer are from the same continent. The EBIT Multiple yields the best results in terms of significance and is the most robust to serial correlation, heterogeneity and other robustness tests.

The results based on the Income Multiple show very weak levels of significance and reversed signs for Liquidity and Leverage. This is probably related to the crudeness of the multiple as it is prone to variations in capital structure, accounting standards and depreciation policies. It is logical that multiples for firms with higher levels of leverage go up when multiples are calculated after the adjustments for capital structure since interest payments to debt holders are made just before tax payments. A higher level of leverage leads to a higher level of interest payments and a lower level of net income. This results in a higher multiple relative to firms with a lower level of leverage. Thus, leading to lower discounts or even a premium. The same is true for liquidity as more interest payments induce lower levels of cash. Therefore, the results based on the Income multiple are not very reliable. The Sales Multiple leads to similar conclusions as those from the EBITDA Multiple. The results depict a stronger and more significant effect for Size across all models. Liquidity and United States show stronger coefficients across all models, but significance disappears when control variables are included.

Even though the sample is not normally distributed, regular OLS regressions are presented in Appendix D (Tables D.10 to D.13). These regressions yield similar results. Through using a 20% Winsor in both tails, the impact of data outliers of acquisition multiples in the sample is reduced. The p-value for the Winsor is determined following Figure 4.2.1. By Winsorizing the dependent variable (PCD) data is conserved instead of arbitrary sample truncation like the Officer study. Although a 20% Winsor is also arbitrary, a lower p-value does not alter the results to a large degree. The results from these regressions are very similar to those using quantile regression. The coefficients show the same signs and comparable levels of significance. However, the coefficients are less interpretable as they are based on the mean. Although the dependent variable is Winsorized, the mean values are still unconventionally high due to the fact that multiples are ratios and cannot be negative. The R-squared goes up to 0.264 for the Sales Multiple, which means the independent variables in that model explain 26.4% of the PCD. This is similar to existing acquisition studies. For both the quantile regression as the OLS regression, the Sales Multiple has the highest level of predictability, whereas the EBIT Multiple is the most robust.

The results hold when Size is measured as the natural logarithm of Net Sales and when Liquidity is measured using the percentage of shares that is acquired in the deal. The significance of these results

is most trivial in models a, b and c. The results weaken when observations before 2000 are deleted from the dataset, proving the observation that the PCD is less evident in the period between 2000 and 2016 as discussed in Section 5.2.2 [*Portfolio method*]. These regressions are not reported to conserve space. Also, Table D.14 lists the results of model e for all multiples on the basis of a truncated sample in which micro firms are excluded. The results are similar. Apart from the intercepts, all signs are the same and predictability of the model strongly increased. However, the coefficients and their significance slightly decreased. In these models, the acquirer characteristics (i.e. Financial Motive and Public Buyer) seem to play an even larger role.

Table 5.11 Cross-sectional results based on the EBIT Multiple

This table presents the results of the quantile regression model with regards to the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the EBIT Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. Estimated slope coefficients for Year and Industry dummy variables are not reported to save space. Standard errors of the coefficients are presented in parenthesis. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

	PCD_{EBIT}				
	(a)	(b)	(c)	(d)	(e)
Size	-5.864*** (1.178)	-4.958*** (1.473)	-4.958*** (0.826)	-8.137** (3.609)	-8.137** (3.209)
Liquidity	-0.035 (0.112)	-0.071 (0.135)	-0.071 (0.099)	-0.926*** (0.351)	-0.926*** (0.261)
United States	-1.770 (3.989)	-5.259 (5.709)	-5.259 (3.844)	5.228 (14.585)	5.228 (11.780)
Intercontinental				-23.593 (15.433)	-23.593** (11.016)
Leverage				0.097 (0.257)	0.097 (0.172)
Profitability				1.452*** (0.487)	1.452*** (0.401)
Growth Rate				0.044 (0.151)	0.044 (0.160)
Financial Motive				-0.700 (16.973)	-0.700 (14.085)
Public Buyer				-9.721 (12.869)	-9.721 (9.226)
Intercept	45.017*** (4.977)	48.808 (36.706)	48.808 (39.647)	44.959 (57.602)	44.959 (80.530)
Observations	2,388	2,388	2,388	367	367
Pseudo R²	0.002	0.010	0.010	0.052	0.052
Year & Industry dummies	No	Yes	Yes	Yes	Yes
Robustness	No	No	Yes	No	Yes

Table 5.12 Cross-sectional results based on the Income Multiple

This table presents the results of the quantile regression model with regards to the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the Income Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. Estimated slope coefficients for Year and Industry dummy variables are not reported to save space. Standard errors of the coefficients are presented in parenthesis. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

	PCD_{Income}				
	(a)	(b)	(c)	(d)	(e)
Size	-4.176*** (1.535)	-3.202* (1.699)	-3.202* (1.288)	-6.886 (5.616)	-6.886 (4.806)
Liquidity	0.267* (0.138)	0.324** (0.147)	0.324*** (0.111)	0.266 (0.542)	0.266 (0.333)
United States	12.291** (5.016)	6.255 (6.424)	6.255 (5.244)	-13.321 (22.740)	-13.321 (24.091)
Intercontinental				-8.568 (23.634)	-8.568 (15.56)
Leverage				-0.650 (0.410)	-0.650** (0.301)
Profitability				-0.048 (0.101)	-0.048 (0.626)
Growth Rate				0.149 (0.234)	0.149 (0.298)
Financial Motive				12.213 (26.564)	12.213 (19.991)
Public Buyer				-21.394 (20.249)	-21.394 (17.419)
Intercept	28.476*** (6.371)	22.248 (42.599)	22.248 (106.217)	8.963 (87.261)	8.963 (122.132)
Observations	2,270	2,270	2,270	338	338
Pseudo R²	0.002	0.008	0.017	0.055	0.055
Year & Industry dummies	No	Yes	Yes	Yes	Yes
Robustness	No	No	Yes	No	Yes

Table 5.13 Cross-sectional results based on the Sales Multiple

This table presents the results of the quantile regression model with regards to the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the Sales Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. Estimated slope coefficients for Year and Industry dummy variables are not reported to save space. Standard errors of the coefficients are presented in parenthesis. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

	PCD_{Sales}				
	(a)	(b)	(c)	(d)	(e)
Size	-6.399*** (1.68)	-7.407*** (1.909)	-7.407*** (1.389)	-11.124*** (4.078)	-11.124*** (2.308)
Liquidity	-1.294*** (0.149)	-1.448*** (0.168)	-1.448*** (0.205)	-0.294 (0.395)	-0.294 (0.271)
United States	-29.895*** (5.853)	-35.81*** (7.707)	-35.81*** (6.76)	-9.950 (17.050)	-9.950 (9.230)
Intercontinental				-28.297 (17.295)	-28.297** (14.051)
Leverage				0.877*** (0.282)	0.877*** (0.234)
ROA				-1.942*** (0.558)	-1.942*** (0.48)
Growth Rate				-0.135 (0.178)	-0.135 (0.1360)
Financial Motive				-5.564 (19.302)	-5.564 (14.889)
Public Buyer				-20.026 (14.263)	-20.026*** (7.543)
Intercept	59.717*** (6.931)	99.865* (56.759)	99.865** (47.039)	112.035* (67.287)	112.035 (134.595)
Observations	3,368	3,368	3,368	392	392
Pseudo R²	0.000	0.000	0.000	0.116	0.116
Year & Industry dummies	No	Yes	Yes	Yes	Yes
Robustness	No	No	Yes	No	Yes

The results of all models, especially those with regards to model e where all robustness checks are added, support most of the stated hypotheses. Private companies do sell at a discount compared to their public peers. This discount tends to lie between 18% and 25% based on earnings multiples and around 11% based on a sales multiple. The highest discounts are observed in the construction (49.4%), energy (27.3%), manufacturing (26.1%), and healthcare (23.6%) sector, whereas the lowest discounts can be found in the retail (8.6%) and services sector (10.6%). The discounts are lower for larger sized companies and companies that have higher levels of liquidity. The models do not show persistent evidence that supports the fourth hypothesis that states that discounts in the US are lower. This highly depends on the multiple that is used to construct the PCD and the applicable methodology. The significant results of the EBITDA and the Sales Multiple across models a to c support the hypothesis. However, univariate analysis entails a different conclusion. The traditional method (including a Mood's median test) indicates that discounts for US based companies are consistently higher, whereas the portfolio method presents a premium for US based companies using the Sales Multiple and a discount that is higher using the Net Income multiple. The discount is nearly the same for US and non-US companies on the basis of the EBITDA and EBIT Multiple.

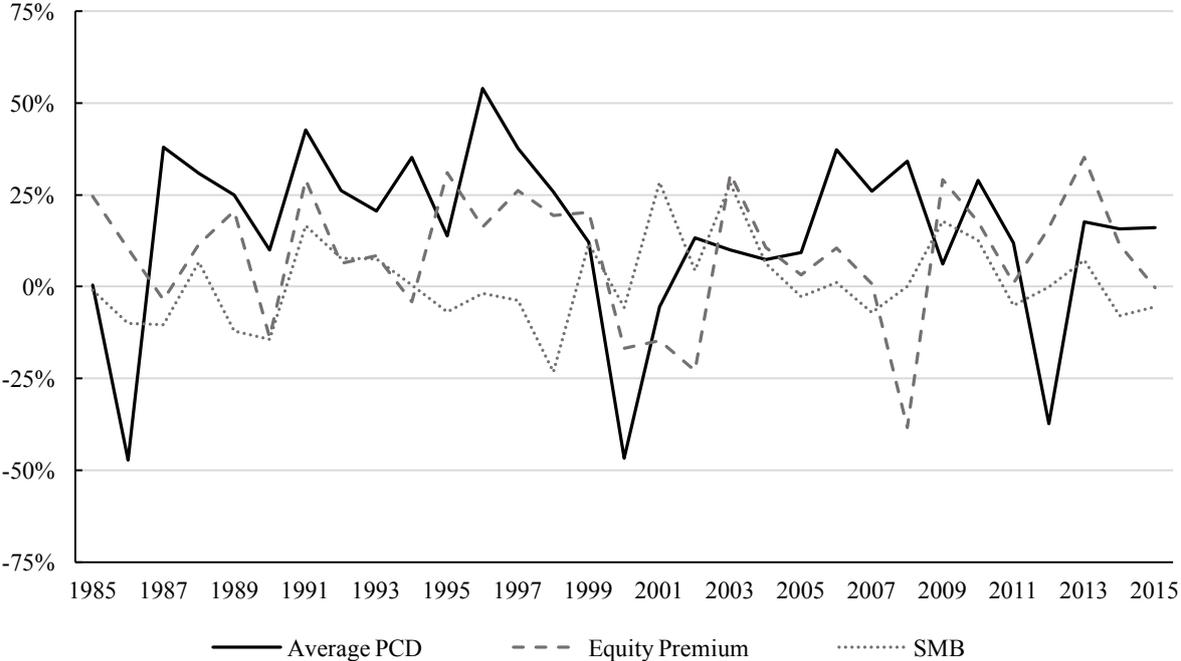
Besides size and liquidity, the discount appears to be lower for deals that are intercontinental and when the acquirer was public (respectively about 30% and 20%). Also, discounts seem to depend on levels of leverage and profitability, but the multiples do not find consistent evidence and the corresponding coefficients vary a lot. Finally, the motive behind the deal and the growth rate do not seem to have an effect on the PCD. Unfortunately, the models did not show a consistent significant effect of the control variables on the PCD. Therefore, this paper can only provide an inconclusive alternative framework for private company valuation.

5.4 The private company discount and the small firm premium

As was shown in Figure 5.2.1, the discount seems to decrease over time and tends to show a peak in which private companies sell at large premiums as compared to their public peers once every ten years. As can be derived from Figure 5.4.1 this peak tends to occur in or right after a period in which market returns became negative. In fact, the development of the median PCD does seem related to that of the size effect (SMB). This seems somewhat inquisitive as its changes over time seem rather countercyclical to that of the size effect. However, the logic behind the opposite movements of the PCD is fairly understandable. If small firms do not outperform large firms in the equity markets, private companies sell at greater discounts and vice versa. Hence, if small firms start outpacing large firms in the equity markets, the PCD tends to disappear. This scrutiny is sometimes lagged by one year, which can be explained by the dispersion of discounts within a year. For example, a grouping of relatively large discounts at the beginning of a year could weaken the cogency of the smaller discounts towards the year-end. Nonetheless, the sample seems too small to look at specific sub-periods within a year.

Figure 5.4.1 Development of median PCD, SFP, and equity premium 1985 – 2015

This figure presents the average PCD development compared to the returns of the market and the size effect (SMB) between 1985 and 2016. The total sample includes 3,843 private company acquisitions and is extracted from Thomson One. The y-axis represents the size of the PCD or the relative return in %, and the x-axis represents the years. The data is derived from the available figures of French (2016), which can be found on his website.



Various tables and figures that are not presented in this chapter, including some of the robustness tests, can be found in Appendix D. The paper continues in the next chapter where the results will be discussed with regards to practical implications. The results of model e for the EBIT multiple are used as they provide results that are the most robust. It presents an inconclusive model which can be used to serve as a basic estimator for the size of the PCD for any private company. This can be used as a discount on value in terms of an alternative to the SFP, or it could be used for more interpretation and a cross check in private company valuation.

CHAPTER 6 Practical Implications

The presented results in the previous chapter are largely robust against serial correlation, heteroscedasticity and other robustness tests. Correspondingly, the fact that many micro firm acquisitions remain unobserved impedes a potential sample selection with regards to larger acquisitions. Thus, the sample is biased towards larger firms. Nonetheless, the effect largely remains when the sample is truncated to delete all micro firms from the sample. This is especially true for the EBIT Multiple. It is alleged that an increase in the number of micro firm acquisitions leads to an increase of the observed discounts. This is because these firms are smaller and relatively less liquid since these firms tend to depend more on their key employees, clients and suppliers. It has been demonstrated that these factors (i.e. size and liquidity) affect the PCD in a negative way. Moreover, due to the fact that the database is prone to have an overbalance of outliers and that the effect of an outlier is unfavorably large on the mean, this paper considers the median to be the crucial determinant of the central tendency in the global M&A market. The findings withhold when other proxies for size and liquidity are used.

Currently, the SFP is added as a surplus on the discount rate. Nowadays, this is largely debated as (i) the SFP appears to be disappearing and is highly sensitive to specific periods, (ii) is only observable amongst micro firms and in the month January, and (iii) is prone to large standard errors. The idea behind the PCD is that a private company is valued using DCF against the discount rate at which a comparable public company would be valued. Consequently, the SFP is not added as a surplus in the discount rate of the private company, but a PCD is calculated based on characteristics of the company and the deal. The valuation of the private company that follows is then a discounted value from its ideal value if the company would be public.

Even though the key areas of research of this paper showed significant results, the model is not robust enough to give a definite alternative to the practical application of the SFP in private company valuation. Therefore, the practical implications below should be interpreted with caution and should primarily be used as benchmark tool instead of a definite alternative to the SFP. Thus, this chapter aims at providing a model of the best results that can serve as cross check for practitioners. Furthermore, this chapter postulates some examples with regards to the computation of the PCD. The model to estimate the PCD is presented below in equation 4 and is based on model e of the EBIT Multiple that has all basic robustness tests included which is presented in Table 5.3.2. This model is available in excel on request.

$$\begin{aligned} PrivateCompanyDiscount_j = & 44.96 - 8.14 * Size_j - 0.93 * Liquidity_j + 5.23 * UnitedStates_j + \\ & 23.59 * Intercontinental + 0.10 * Leverage + 1.45 * Profitability + 0.04 * GrowthRate - 0.70 * \\ & FinancialMotive - 9.72 * PublicBuyer - 34.13 * Wholesale - 8.35 * Retail - 15.95 * \\ & Manufacturing - 11.59 * Services + 15.61 * Energy - 32.62 * Healthcare + 21.95 * Technology + \\ & 0.00 * OtherIndustry \end{aligned} \quad (4)$$

The various variables of the model are defined in Appendix A. The last eight variables are dummy variables representing the specific industry of the target which are further defined in Appendix B. Examples in which the PCD is computed based on example deal characteristics can be found in Table 6.1. Size is measured as the natural logarithm, which implies a base of the mathematical constant e .

Table 6.1 Example PCD's based on specific characteristics

This table presents example computations of the PCD based on several deal characteristics. The model is a result of a quantile regression model with regards to the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the EBIT Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions and industry definitions are included in Appendix A and B.

Example	A	B	C	D	D	E	F
Assets	36.60	666.60	43.57	21.04	2.09	16.79	14.35
Size	3.60	6.50	3.77	3.05	0.74	2.82	2.66
Cash	5.39	49.80	4.18	7.58	0.57	0.47	0.06
Liquidity	14.7%	7.5%	9.6%	36.0%	27.3%	2.8%	0.4%
United States	No	No	No	No	No	Yes	Yes
Intercontinental	No	No	Yes	No	Yes	No	No
Liabilities	20.68	466.60	20.32	5.00	1.77	14.12	1.18
Leverage	56.5%	70.0%	46.6%	23.8%	84.7%	84.1%	8.2%
Sales	39.55	259.46	46.16	36.95	12.04	1.10	28.46
EBIT	10.10	15.96	9.96	8.83	-0.44	-0.08	3.12
Profitability	25.5%	6.2%	21.6%	23.9%	-3.7%	-7.3%	11.0%
Growth Rate	5.0%	10.0%	80.0%	10.5%	15.6%	0%	18.7%
Financial Motive	Yes	No	No	No	No	No	No
Public Buyer	No	Yes	Yes	Yes	Yes	No	No
Industry	3	7	6	5	4	2	8
Discount	28.2%	13.5%	26.0%	30.2%	19.6%	13.9%	45.7%

The most arbitrary coefficients are those related to FinancialMotive and UnitedStates as their standard errors are largest and their economic significance is greatest. This means that those coefficients have a great impact on the PCD. If the model would be based on the truncated sample in which micro firms are excluded, the deal specifics become less eminent and the acquirer characteristics become more eminent. The precise model can be seen in Table D.14.

Furthermore, it can be derived from the presented model and Table 5.4.1 that the PCD is most sensitive to the industry in which a private company is active. As compared to the other industries than those included in the research, the energy and technology industry denote the highest discounts. In contrast, the wholesale industry denotes the lowest discounts. Besides industry, a private company's size and whether the deal is executed internationally play a dominant role in determining the PCD. A larger private company faces a lower discount, whereas an intercontinental deal results in a higher discount. On average, these companies denote a PCD of 25.3%, which is comparable to the median observed in the market.

CHAPTER 7 Conclusion

7.1 Conclusion

Even though it appeared impossible to construct a conclusive and robust model that should be used within private company valuation, this paper finds results that are both statistically and economically significant. The main research question of the paper was defined as:

“What is the magnitude of the private company discount and to what extent can it be explained by firm size, the level of liquidity and geography?”

The results indicate the existence of a significant discount with regards to the acquisitions of private companies that are not listed on the equity markets. This discount typically ranges between 10% and 30% and varies significantly across industries. All methods confirm the same theoretical prediction and the main results of corresponding models support the liquidity and size hypotheses. Larger private companies and those with more relative asset liquidity partake lower discounts. Although discounts seem to depend on geography, the models did not provide consistent statistical evidence that supports a hypothesis which states that US-based companies demonstrate lower discounts. Lastly, the discount also depends on target firm characteristics (i.e. leverage and profitability), deal specifics (i.e. whether the deal is executed intercontinental), and acquirer characteristics (i.e. whether the acquirer is a public company).

7.2 Limitations

This research is predominantly limited to the fact that only about 2% of global M&A activity is included in the sample. It is expected that mostly small private company acquisitions are unobserved which are likely to display larger discount. Therefore, the actual discounts present in the market are potentially higher. Besides the sample in general, the benchmark portfolios that are constructed to estimate the PCD are based on firm size, year of acquisition and industry. The matching procedure is somewhat crude which instigates a sensitivity to portfolios in which few (below 5 observations) are present. Namely, the small size portfolio only contains 2% of all public companies. Besides, a relaxation of the time period to a three-year range around each private company acquisition leads to an increase of all portfolio sizes and increases reliability.

Another limitation is the fact that earnings-based multiples can only be constructed when earnings are positive. This creates a bias towards profitable companies as 26.4% of the firms included in the sample exhibit a negative EBIT. Therefore, these firms do not have EBIT Multiples. The observations of the sales multiple lead to the belief that profitable firms have lower multiples, and depending on the model that is used, have larger discounts. Finally, the sample is non-normal as it includes a large number of definite outliers. This is true for all multiples. Therefore, the non-parametric tests included in this study have less statistical power than would be the case if the data was normally

distributed. The sample is probably biased towards high multiple transactions (the outliers) that are hypothetically driven by growth, synergies or other incidental factors.

7.3 Recommendations

Following the debates with regards to the SFP, the research towards the PCD is still rather young. As many corporate finance practitioners struggle with the valuation of private companies, there is an excessive demand for research on this subject. Based on the various empirical methods that are applied in this research, some recommendations are provided below.

A first recommendation would be to look into which multiple best determines value. This multiple should then be the basis of the entire research. An alternative approach could look into some sort of valuation factor that is based on a combination of various multiples. This valuation factor could include the different advantages of each multiple. On the other hand, an average discount for each private company acquisition could be constructed. However, this procedure is likely to become even more sensitive to outliers. Second, future research could look into additional explanatory variables of the PCD. These additional variables should be added to the models to enhance predictive power and potentially significance. The current levels of R^2 indicate the existence of other predictors that also explain variability in the PCD. Additionally, the application of alternative proxies of liquidity could lead to better results. A start would be to include non-controlling interest acquisitions into the sample.

Finally, further research could address the fact that the PCD seems to be decreasing over time. It should also put a more eminent focus on the link between the PCD and the SFP in their data analysis. Does a large SFP indeed lead to a small PCD? To what extent are the SFP and the PCD interrelated? These are questions beyond the scope of this research but seem trivial in understanding the link between SFP and the PCD with regards to private company valuation. Could the PCD actually be used within private company valuation?

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Appendix A Variable Definitions

This table presents the variable definitions for all variables that are included in this study. All corresponding variables have been taken directly from the Thomson One M&A database and are based on the last 12 months ending on the date of the most current financial information prior to the transaction. Deal Value is defined as the total value of consideration paid by the acquirer, excluding fees and expenses for all common stock, common stock equivalents, preferred stock, debt, options, assets, warrants, and stake purchases made within six months of the announcement date of the transaction. For more variable definitions for Thomson One variables, one can visit the website: <http://mergers.thomsonib.com/DealsWeb/help/def.htm>

Variable Name	Definition	Clarification
<i>Acquisition Multiples</i>		
EBITDA Multiple	Deal Value times the percentage of shares acquired divided by 100 percent divided by EBITDA	The EBITDA multiple is a good proxy for free cash flows which are the fundamental basis of company value. It is an advantage that it ignores variations in capital structure across different firms.
EBIT Multiple	Deal Value times the percentage of shares acquired divided by 100 percent divided by EBIT	The EBIT multiple is similar to the EBITDA multiple, but better allows for variations in capital intensiveness across different firms. This is of particular interest in some industries but remains prone to variations in depreciation policies across different firms.
Income Multiple	Deal Value times the percentage of shares acquired divided by 100 percent divided by Net Income	The Income multiple is a very crude multiple as it is prone to variations in capital structure, accounting standards and depreciation policies. However, it best represents what remains 'bottom line'.
Sales Multiple	Deal Value times the percentage of shares acquired divided by 100 percent divided by Net Sales	The Sales Multiple is the least susceptible to different variations and remains applicable when earnings are negative. This vastly increases sample size and predictability. However, sales are rarely a direct value driver.
<i>Dependent Variables</i>		
Private Company Discount	1 minus the private company multiple divided by the public company multiple	The Private Company Discount is of central interest in this paper. It is defined as the discount at which private companies tend to be acquired as compared to their public peers. It has the potential to become an alternative application in private company valuation.
PCD_{EBITDA}	1 minus the private company EBITDA multiple divided by the median portfolio benchmark multiple	The PCD_{EBITDA} is the Private Company Discount based on the EBITDA Multiple.
PCD_{EBIT}	1 minus the private company EBIT multiple divided by the median portfolio benchmark multiple	The PCD_{EBIT} is the Private Company Discount based on the EBIT Multiple.
PCD_{Income}	1 minus the private company Income multiple divided by the median portfolio benchmark multiple	The PCD_{Income} is the Private Company Discount based on the Income Multiple.
PCD_{Sales}	1 minus the private company EBITDA multiple divided by the median portfolio benchmark multiple	The PCD_{Sales} is the Private Company Discount based on the Sales Multiple.

Independent Variables

Size	Natural logarithm of Book Value of Total Assets	The natural logarithm is taken to be able to directly interpret the coefficients as approximate proportional differences (Gelman & Hill, 2007). In alternative regressions, the natural logarithm of Net Sales is used as a proxy for size to check the robustness of the results.
Liquidity	Cash and Marketable Securities divided by Book Value of Total Assets	In alternative regressions percentage of shares acquired is used as a proxy for liquidity to check the robustness of the results.
United States	Dummy - 1 if the target is located in the US, 0 otherwise	Research from Koeplin et al. (2003) and Klein & Schneibel (2012) indicate lower discounts in the United States. This paper is interested in investigating the potential effect of geography on the Private Company Discount

Control Variables

Leverage	Total Liabilities divided by Total Assets	Leverage is added to control for levels of risk and serve as a proxy for the probability of financial distress following Andrade & Kaplan (1998).
Growth Rate	Cumulative Net Sales growth over the last five years prior to acquisition	Growth Rate controls for a difference in characteristics between private and public targets.
Profitability	EBIT divided by Net Sales	Profitability is added as a control variable whenever Size is measured using Book Value of Total Assets to minimize the potential effect of multicollinearity.
ROA	EBIT divided by Book Value of Total Assets	ROA is added as a control variable for profitability whenever Size is measured using Net Sales to minimize the potential effect of multicollinearity.
Profitable	Dummy – 1 if the target is profitable, 0 otherwise	A target firm is profitable when its Net Income is positive. A private firm that is profitable should be easier to sell than one that is not profitable.
Public Buyer	Dummy – 1 if the acquirer is a public company, 0 otherwise	Public Buyer controls for a difference in characteristics between private and public acquirers.
Financial Motive	Dummy – 1 if the acquirer is a financial firm, 0 otherwise	Financial Motive controls for a difference in characteristics between private and public acquirers.
Intercontinental	Dummy - 1 if the target is located on a different continent than the acquirer, 0 otherwise	An intercontinental deal opens access to new markets and geographies for acquirers and enables them to achieve greater diversification opportunities. This potential synergy is expected to affect the Private Company Discount in a negative way.

Appendix B Industry Classification

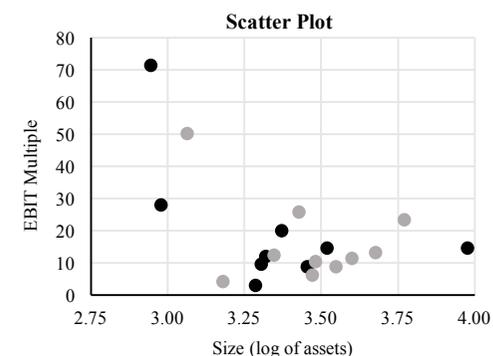
This table presents the industry classification of all targets included in the sample. All industries have been derived from SIC codes as listed in the Thomson One M&A database. Some SIC codes are excluded of those that are listed for the manufacturing and services industry. These are included in the energy, healthcare or technology industry as they have very specific SIC classifications.

Industry / Sector	SIC Code	Examples
1. Wholesale		
Manufacturing	2200 - 2699	Clothing, wood products, furniture, and paper products
Trade	5000 - 5199	Automobiles, furniture, business supplies, and electrical goods
2. Retail		
Manufacturing	2000 - 2199	Food, beverages, and tobacco
Manufacturing	2700 - 2799	Books and newspapers
Trade	5200 - 5900	Restaurants, department stores, grocery stores, and other retail stores
3. Manufacturing		
Soft materials	2800 - 2999	Plastics, rubber, leather, soap, paint, chemicals
Hard materials	3000 - 3499	Stone, concrete, metals
Industrial machinery	3500 - 3599	Engines, agriculture, construction, trucks
Electronics	3600 - 3699	Communications equipment, electronic components
Transportation machinery	3700 - 3799	Motor vehicles and equipment
Miscellaneous	3900 - 3999	Toys, games, sporting, and athletic goods
4. Services		
Services	7000 - 7999	Hotels, employment agencies, security, and amusement
Services	8100 - 9999	Education, accounting
5. Energy		
Mining	1000 - 1499	Metals, oil, and gas
Pipelines	4600 - 4699	Pipelines
Petroleum refining	2900 - 2919	Petroleum, asphalt, and lubricants
6. Healthcare		
Pharmaceuticals	2830 - 2839	Drugs
Medical products	3820 - 3899	Surgical, dental, and photographic equipment
Health services	8000 - 8099	General practitioners, hospitals, dentists, and personal care
7. Technology		
Computer equipment	3570 - 3579	Computers, storage, and peripheral equipment
Semiconductors	3674	Semiconductors
Aerospace	3810 - 3819	Detection, navigation, aeronautical, and nautical
Communications	4800 - 4899	Telephone, radio, and television
Computer technology	7370 - 7379	Computer programming, software, data, and retrieval services
Motion pictures	7800 - 7839	Motion pictures
Engineering	8710 - 8719	Engineering and architecture
Research	8730 - 8739	Physical, biological, economical and sociological research
8. Other		
Agriculture	0100 - 0999	Crops, livestock, forestry, fishing, and hunting
Construction	1500 - 1799	Residences, roads, plumbing, and electrical work
Transportation	4000 - 4599	Trucking, courier, sea freight, airlines, and railroads
Transportation services	4700 - 4799	Arrangement of passenger, cargo, and freight transport

Appendix C Example PCD Computation

This table presents a snapshot of the sample used to determine the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The presented snapshot of acquisitions took place in 2007 in the manufacturing industry. The company sizes range from \$19 million to \$53 million assets, representing the third portfolio. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. It is 17.8% $[(1 - 12.0 / 14.6) * 100\%]$ measured using the traditional method and 17.9% when calculated using the portfolio method. The variable definitions are included in Appendix A.

Private	Country	Target	Acquirer	Shares %	Deal Value	EBIT	Multiple	Benchmark	PCD _{EBIT}
No	United States	Tut Systems Inc	Motorola Inc	100	39.0	-14.0			
No	United States	Reinhold Industries Inc	The Jordan Co LP	100	41.2	13.4	3.1		
No	Canada	TIR Systems Ltd	Koninklijke Philips Elect	100	63.2	-12.4			
No	United States	Wellco Enterprises Inc	Wellco Enterprises Inc SPV	100	18.0	0.6	28.2		
No	South Korea	DigitAll World Co Ltd	Hankook Metal Industrial Co	100	11.6	-0.7			
No	United States	Terayon Communication Sys Inc	Motorola Inc	100	141.8	-17.3			
No	Canada	Macyro Group Inc	Red Holdings Group Inc	100	17.1	0.2	71.5		
No	Hong Kong	MACRO-LINK Intl Hldg Ltd	JLF Investment Co Ltd	57	22.4	2.0	20.1		
No	United States	Rotonics Manufacturing Inc	Rotonics Holding Corp	100	35.4	3.7	9.7		
No	Singapore	Jade Tech Singapore Ltd	Asia Pacific Links Ltd	76	6.4	-0.8			
No	Singapore	Medi-Flex Ltd	Investor Group	66	14.5	1.8	12.0		
No	United States	Farrel Corp	Investor Group	100	18.4	1.3	14.6		
No	United Kingdom	UCM Group PLC	Imerys UK Ltd	100	39.9	2.7	14.7		
No	Sweden	ACSC AB	XPonCard Group AB	97	28.5	3.3	9.0		
Yes	Poland	Zetkama SA	Avallon Sp zoo	100	11.1	2.6	4.3	14.6	70.6
Yes	United States	Thompson/Center Arms Inc	Smith & Wesson Holding Corp	100	102.0	4.3	23.5	14.6	-60.7
Yes	Germany	FEAG	Schuering & Andreas	100	20.2	-0.1		14.6	
Yes	Japan	So-Ken Co Ltd	Silver Seiko Ltd	100	32.1	-1.7		14.6	
Yes	Russian Fed	OAO Volzhsky Abrasive Works	Carborundum Universal Ltd	84	37.6	5.0	9.0	14.6	38.7
Yes	United Kingdom	Aerial Facilities Ltd	Investor Group	100	109.7	4.2	25.8	14.6	-76.4
Yes	United Kingdom	PRD Holdings Ltd	Lone Star Fasteners LP	100	150.0	3.0	50.4	14.6	-244.2
Yes	Taiwan	ABBA Linear Tech Co Ltd	AB SKF	99	60.6	4.6	13.3	14.6	9.5
Yes	United Kingdom	Ferranti Technologies Ltd	Elbit Systems Ltd	100	30.7	2.4	12.6	14.6	13.9
Yes	Spain	Industrias El Gamo SA	MCH Private Equity Asesores SL	80	102.4	11.2	11.4	14.6	21.9
Yes	Denmark	Roug A/S	Hendricks Holding Co Inc	100	17.4	-2.2		14.6	
Yes	United States	Energy Equip Corp	T-3 Energy Services Inc	100	96.4	9.1	10.6	14.6	27.9
Yes	Spain	Iberchem SA	Investor Group	100	51.3	8.2	6.3	14.6	57.1
Public company median				100	25.5	0.9	14.6		
Private company median				100	51.3	4.2	12.0		17.9
Public company mean				93	35.5	-1.2	20.3		
Private company mean				97	63.2	3.9	16.7		-14.2



Appendix D Additional Tests and Statistics

Table D. 1 Pearson Correlations Multiples

This table presents correlation matrix for the four multiples that are constructed to compute the PCD. The sample consists out of private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The multiple definitions are included in Appendix A.

	EBITDA	EBIT	Net Income	Sales
EBITDA	1.000			
EBIT	0.731	1.000		
Net Income	0.737	0.601	1.000	
Sales	0.754	0.623	0.643	1.000

Table D. 2 Pearson Correlations Dependent Variables

This table presents correlation matrix for the various dependent variables that measure the PCD which are constructed using four different multiples. The sample consists out of private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The variable definitions are included in Appendix A.

	PCD_{EBITDA}	PCD_{EBIT}	PCD_{Income}	PCD_{Sales}
PCD_{EBITDA}	1.000			
PCD_{EBIT}	0.764	1.000		
PCD_{Income}	0.149	0.174	1.000	
PCD_{Sales}	0.457	0.312	0.093	1.000

Table D. 3 Pearson Correlations Independent Variables

This table presents correlation matrix for the various independent variables that are considered to be predictors of the PCD. The sample consists out of private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The variable definitions are included in Appendix A.

	Size	Liquidity	United States	Inter-continental	Leverage	ROA	Profitability	Growth Rate	Financial Buyer	Public Buyer
Size	1.000									
Liquidity	-0.161	1.000								
United States	0.083	0.013	1.000							
Intercontinental	0.048	0.011	-0.050	1.000						
Leverage	0.226	-0.398	-0.063	-0.030	1.000					
ROA	-0.152	0.125	0.089	0.060	-0.112	1.000				
Profitability	0.086	0.132	0.010	0.064	-0.196	0.529	1.000			
Sales Growth	-0.038	0.046	0.027	0.030	-0.046	0.083	0.112	1.000		
Financial Motive	0.037	-0.008	0.037	-0.066	0.038	0.005	-0.016	-0.025	1.000	
Public Buyer	0.070	0.012	0.123	0.085	-0.005	0.046	0.050	0.048	-0.448	1.000

Figure D. 1 Observed deal activity versus unobserved deal activity 1985 – 2016

This figure presents the observed deal activity between 1985 and 2016 included in the sample as a percentage of the total deal activity. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The table shows the total number of completed deals (concerning a controlling interest) included in the sample, the total deal activity in the global M&A market, and the corresponding percentage that is observed per year. Data on the total deal activity in the global M&A market is taken from the Institute of Mergers, Acquisitions and Alliances (2016).

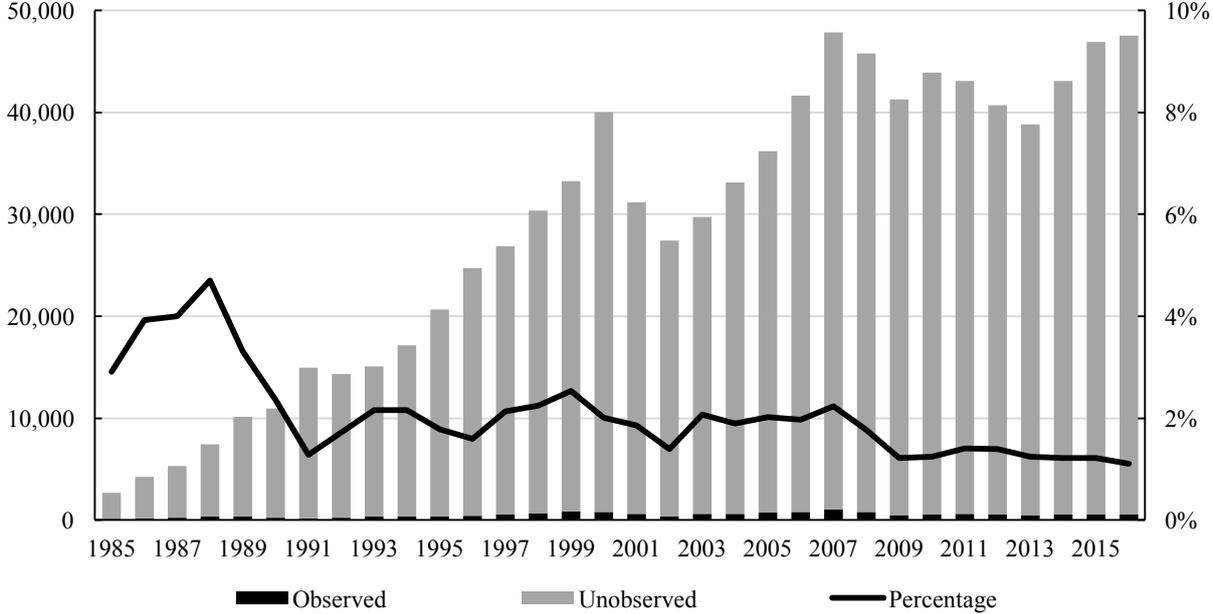


Table D. 4 Pearson's chi-squared tests of independence

This table presents the respective percentage of private company acquisitions that sold at a premium or discount as compared to their public peers between 1985 and 2016. The total sample includes 8,348 private company acquisitions and is extracted from Thomson One. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%, respectively to the Pearson's chi-squared tests of independence.

Industry	EBITDA		EBIT		Net Income		Sales		Average	
	Premium	Discount	Premium	Discount	Premium	Discount	Premium	Discount	Premium	Discount
1 Wholesale	33.1%***	66.9%***	29.8%*	70.2%**	31.3%**	68.7%**	37.4%***	62.6%***	32.9%	67.1%
2 Retail	38.7%***	61.3%***	31.0%**	69.0%**	33.1%**	66.9%**	42.1%***	57.9%***	36.2%	63.8%
3 Manufacturing	33.5%***	66.5%***	27.7%**	72.3%**	30.9%**	69.1%**	42.3%***	57.7%***	33.6%	66.4%
4 Services	30.0%***	70.0%***	28.7%**	71.3%**	32.1%**	67.9%**	37.1%***	62.9%***	32.0%	68.0%
5 Energy	39.7%***	60.3%***	32.9%**	67.1%**	28.1%**	71.9%**	30.3%***	69.7%***	32.8%	67.2%
6 Healthcare	28.7%***	71.3%***	30.5%**	69.5%**	28.9%**	71.1%**	39.6%***	60.4%***	31.9%	68.1%
7 Technology	37.4%***	62.6%***	33.6%**	66.4%**	34.7%**	65.3%**	50.6%***	49.4%***	39.1%	60.9%
8 Other	24.3%***	75.7%***	22.7%**	77.3%**	22.4%**	77.6%**	22.4%***	73.5%***	23.0%	76.0%
Total	33.3%***	66.7%***	29.6%***	70.4%***	31.2%***	68.8%***	41.0%***	59.0%***	33.8%	66.2%

Table D. 5 Median PCD using portfolio method 1985 – 1999

This table presents the median PCDs for private company acquisitions between 1985 and 1999. The total sample includes 5,390 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually. The presented values are the median observed discounts. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

Variable	PCD
EBITDA Multiple	29.8%***
EBIT Multiple	27.2%***
Net Income Multiple	29.7%***
Sales Multiple	20.1%***
Average	26.7%

Table D. 6 Median PCD using portfolio method 2000 – 2016

This table presents the median PCDs for private company acquisitions between 2000 and 2016. The total sample includes 10,765 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually. The presented values are the median observed discounts. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

Variable	PCD
EBITDA Multiple	10.3%***
EBIT Multiple	19.8%***
Net Income Multiple	20.4%***
Sales Multiple	1.3%***
Average	13.0%

Figure D. 2 Median PCDs measured using portfolio method by time interval

This table presents the median PCDs for private company acquisitions between 1985 and 2016 in specific periods. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually. The presented values are the median observed discounts. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A.

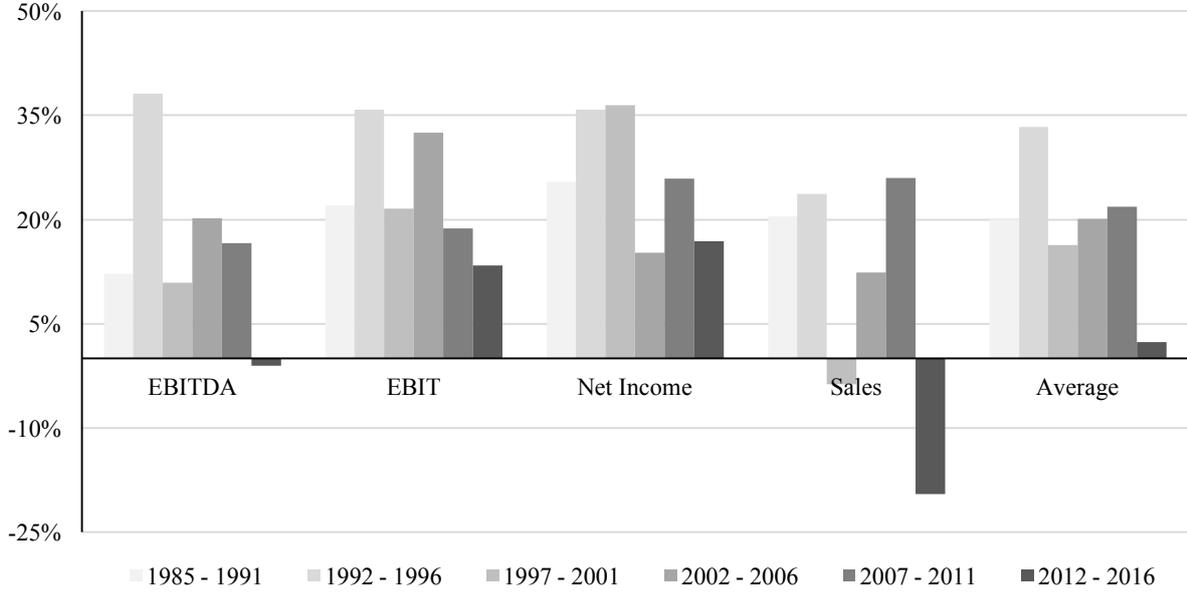


Table D. 7 Median PCD using traditional method by geography

This table presents the median PCDs by basic geography for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the traditional method, which indicates that the PCD calculation is based on the median values of the sample as a whole. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%, respectively to the Mood's Median Test that the median value of private company acquisitions differs significantly from the median value of public company acquisitions. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

Industry	Private Firms	Public Firms	Difference	PCD
<i>US firms</i>				
EBITDA Multiple	8.81	10.34	-1.53***	14.80%***
EBIT Multiple	11.43	15.50	-4.07***	26.23%***
Net Income Multiple	16.88	27.38	-10.51***	38.37%***
Sales Multiple	1.25	1.35	-0.10***	7.56%***
<i>Average</i>				21.74%
<i>Non-US firms</i>				
EBITDA Multiple	9.24	8.59	0.65***	-7.54%***
EBIT Multiple	11.09	13.51	-2.42***	17.91%***
Net Income Multiple	17.29	20.07	-2.78***	13.85%***
Sales Multiple	0.95	1.05	-0.10***	9.60%***
<i>Average</i>				8.46%

Table D. 8 Median PCD using portfolio method US firms

This table presents the median PCDs for private company acquisitions between 1999 and 2016 in which the target was located in the US. The total sample includes 7,262 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually. The presented values are the median observed discounts. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

Variable	PCD
EBITDA Multiple	13.9%***
EBIT Multiple	21.1%***
Net Income Multiple	36.5%***
Sales Multiple	-11.3%***
Average	15.1%

Table D. 9 Median PCD using portfolio method non-US firms

This table presents the median PCDs for private company acquisitions between 1999 and 2016 in which the target was located outside the US. The total sample includes 8,893 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually. The presented values are the median observed discounts. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

Variable	PCD
EBITDA Multiple	11.1%***
EBIT Multiple	20.1%***
Net Income Multiple	14.4%***
Sales Multiple	20.5%***
Average	16.5%

Table D. 10 Cross-sectional OLS results based on the EBITDA Multiple

This table presents the results of the OLS regression model with regards to the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the EBITDA Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. Estimated slope coefficients for Year and Industry dummy variables are not reported to save space. To deal with outliers, the independent is winsorized (top and bottom 20%). Standard errors of the coefficients are presented in parenthesis. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

	PCD_{EBITDA}				
	(a)	(b)	(c)	(d)	(e)
Size	-2.865*** (0.848)	-2.139*** (0.898)	-2.139** (0.918)	-2.413 (2.382)	-2.413 (2.389)
Liquidity	-0.179 (0.080)	-0.096 (0.082)	-0.096 (0.088)	-0.335 (0.232)	-0.335 (0.241)
United States	0.691 (2.819)	-6.249 (3.508)	-6.249* (3.461)	-4.551 (9.816)	-4.551 (9.048)
Intercontinental				-10.999 (10.852)	-10.999 (10.962)
Leverage				0.180 (0.172)	0.180 (0.174)
Profitability				0.007 (0.006)	0.007*** (0.003)
Growth Rate				0.040 (0.101)	0.040 (0.078)
Financial Motive				-10.957 (10.718)	-10.957 (10.946)
Public Buyer				-18.048** (8.418)	-18.048** (8.233)
Intercept	13.908*** (3.785)	72.211*** (28.231)	72.211*** (10.0280)	70.870** (35.987)	70.870*** (19.639)
Observations	2,067	2,067	2,067	303	303
R²	0.007	0.058	0.058	0.195	0.195
Year & Industry dummies	No	Yes	Yes	Yes	Yes
Robustness	No	No	Yes	No	Yes

Table D. 11 Cross-sectional OLS results based on the EBIT Multiple

This table presents the results of the OLS regression model with regards to the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the EBIT Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. Estimated slope coefficients for Year and Industry dummy variables are not reported to save space. To deal with outliers, the independent variable is winsorized (top and bottom 20%). Standard errors of the coefficients are presented in parenthesis. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

	PCD_{EBIT}				
	(a)	(b)	(c)	(d)	(e)
Size	-2.370*** (0.800)	-1.905** (0.860)	-1.905** (0.872)	-5.582*** (2.183)	-5.582*** (2.184)
Liquidity	-0.051 (0.076)	0.011 (0.079)	0.011 (0.084)	-0.214 (0.212)	-0.214 (0.233)
United States	-1.734 (2.710)	-5.602* (3.333)	-5.602* (3.348)	-3.810 (8.820)	-3.810 (9.270)
Intercontinental				-17.633** (9.333)	-17.633** (8.943)
Leverage				0.194 (0.155)	0.194 (0.163)
Profitability				1.304*** (0.294)	1.304*** (0.278)
Growth Rate				0.075 (0.091)	0.075 (0.077)
Financial Motive				-2.968 (10.264)	-2.968 (10.629)
Public Buyer				-8.803 (7.782)	-8.803 (8.103)
Intercept	15.398*** (3.381)	22.517 (21.427)	22.517 (21.69)	44.334 (34.834)	44.334* (26.923)
Observations	2,388	2,388	2,388	367	367
R²	0.004	0.047	0.047	0.210	0.210
Year & Industry dummies	No	Yes	Yes	Yes	Yes
Robustness	No	No	Yes	No	Yes

Table D. 12 Cross-sectional OLS results based on the Sales Multiple

This table presents the results of the OLS regression model with regards to the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the Sales Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. Estimated slope coefficients for Year and Industry dummy variables are not reported to save space. To deal with outliers, the independent variable is winsorized (top and bottom 20%). Standard errors of the coefficients are presented in parenthesis. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

	PCD_{Sales}				
	(a)	(b)	(c)	(d)	(e)
Size	-3.003*** (0.930)	-3.284*** (0.983)	-3.284*** (1.008)	-8.613 (2.953)	-8.613 (3.005)
Liquidity	-0.776*** (0.083)	-0.713*** (0.086)	-0.713*** (0.090)	-0.246 (0.286)	-0.246 (0.303)
United States	-17.013*** (3.239)	-18.927*** (3.967)	-18.927*** (4.040)	-1.421 (12.347)	-1.421 (12.398)
Intercontinental				-30.890** (12.524)	-30.890** (12.795)
Leverage				0.952*** (0.205)	0.952*** (0.212)
ROA				-1.537*** (0.404)	-1.537*** (0.425)
Growth Rate				-0.114 (0.129)	-0.114 (0.171)
Financial Motive				-13.054 (13.978)	-13.054 (15.742)
Public Buyer				-16.654 (10.328)	-16.654 (11.104)
Intercept	7.950** (3.836)	13.391 (29.217)	13.391 (29.211)	-2.283* (48.726)	-2.283* (51.151)
Observations	3,368	3,368	3,368	392	392
R²	0.034	0.075	0.075	0.264	0.264
Year & Industry dummies	No	Yes	Yes	Yes	Yes
Robustness	No	No	Yes	No	Yes

Table D. 13 Cross-sectional OLS results based on the Income Multiple

This table presents the results of the OLS regression model with regards to the PCD for private company acquisitions between 1985 and 2016. The total sample includes 16,155 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the Income Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. Estimated slope coefficients for Year and Industry dummy variables are not reported to save space. To deal with outliers, the independent variable is winsorized (top and bottom 20%). Standard errors of the coefficients are presented in parenthesis. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

	PCD_{Income}				
	(a)	(b)	(c)	(d)	(e)
Size	-1.683*	-1.665*	-1.665*	-2.768	-2.768
	(0.918)	(0.989)	(1.014)	(2.685)	(2.741)
Liquidity	0.241***	0.340***	0.340***	0.177	0.177
	(0.082)	(0.086)	(0.086)	(0.259)	(0.275)
United States	5.656*	3.286	3.286	-12.857	-12.857
	(2.999)	(3.740)	(3.7250)	(10.873)	(11.681)
Intercontinental				-8.040	-8.040
				(11.3)	(11.7050)
Leverage				-0.580***	-0.580***
				(0.196)	(0.202)
Profitability				-0.049	-0.049
				(0.048)	(0.044)
Growth Rate				0.226**	0.226***
				(0.112)	(0.082)
Financial Motive				7.486	7.486
				(12.701)	(13.09)
Public Buyer				-9.299	-9.299
				(9.681)	(9.910)
Intercept	-0.865	-19.323	-19.323	34.847	34.847
	(3.809)	(24.802)	(27.439)	(41.722)	(39.342)
Observations	2,270	2,270	2,270	338	338
R²	0.008	0.041	0.041	0.157	0.157
Year & Industry dummies	No	Yes	Yes	Yes	Yes
Robustness	No	No	Yes	No	Yes

Table D. 14 Cross-sectional results based on all multiples excluding micro firms

This table presents the results of the quantile regression model with regards to the PCD for private company acquisitions between 1985 and 2016 for which sales of the target exceeded \$8 million. The total sample includes 14,015 acquisitions and is extracted from Thomson One. The PCD is measured using the portfolio method, which indicates that the PCD calculation is executed for each acquisition individually and is based on the Income Multiple. The PCD is computed by the following formula: one minus the private company multiple divided by the public company multiple. The variable definitions are included in Appendix A. Estimated slope coefficients for Year and Industry dummy variables are not reported to save space. Standard errors of the coefficients are presented in parenthesis. *, **, and *** indicate significance levels of 10%, 5%, and 1%.

	PCD_{EBITDA}	PCD_{EBIT}	PCD_{Income}	PCD_{Sales}
Size	-0.818 (3.600)	-10.518*** (2.976)	0.299 (4.413)	-3.376 (4.793)
Liquidity	-0.524 (0.393)	-0.243* (0.276)	0.528 (0.38)	-0.32 (0.501)
United States	-16.615 (13.004)	0.1650 (10.792)	6.956 (22.152)	-14.960 (15.993)
Intercontinental	-52.529*** (13.451)	-14.281 (10.859)	-15.768 (17.648)	-57.982*** (18.898)
Leverage	0.454* (0.275)	0.331 (0.209)	-0.639 (0.359)	1.060*** (0.335)
Profitability	0.762 (0.606)	1.175*** (0.350)	0.974** (0.483)	-1.607** (0.638)
Growth Rate	0.111* (0.203)	0.169 (0.316)	0.133 (0.465)	0.077 (0.335)
Financial Motive	-29.862* (16.919)	-12.312 (15.3070)	8.143 (25.012)	-22.811 (19.989)
Public Buyer	-15.359 (10.821)	-19.579* (10.229)	-32.268* (17.435)	-16.064 (13.238)
Intercept	60.594 (140.000)	56.217* (36.995)	-52.737 (92.396)	-64.956 (155.951)
Observations	268	310	276	324
Pseudo R²	0.167	0.187	0.138	0.185
Year & Industry dummies	Yes	Yes	Yes	Yes
Robustness	Yes	Yes	Yes	Yes