

Erasmus University Rotterdam



MASTER'S THESIS

OPERATIONS RESEARCH & QUANTITATIVE LOGISTICS

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# Analyzing Theoretical Capacity of the Metro Network of Rotterdam

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## Abstract

In this thesis a solution method to calculate the theoretical capacity of a metro network is presented. Using this method, the capacity of the metro network of Rotterdam, operated by the RET, is determined. The method consists of a two-step approach, in which in Phase One all possible route-time combinations are generated, referred to as 'paths'. In Phase Two an optimal selection of paths is made, subject to different constraints, such as minimum headway time, line frequencies and endpoint restrictions. A software tool is developed and presented, where the user can configure its desired parameters for the restrictions. Capacity tests under different conditions are performed for the metro network of Rotterdam. The results of the different capacity tests are analyzed and can be used as an indication for the realizable line-frequencies. Furthermore, we present a heuristic approach, as an alternative to the more time-consuming exact approach. Finally, the results of both methods are compared in order to measure the performance of the heuristic method.

**Keywords:** Railway Networks; Metro; Capacity Analysis; Optimization

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# Chapter 1

## Introduction

### 1.1 Company Background

The Rotterdamse Elektrische Tram, commonly known as the RET, is the largest public transport operator in the district of Rotterdam, in the Netherlands. At the moment of writing, the total number of employees is around 3,000. The provided services are carried out by tram, bus, metro and ferry. On a daily basis, over 600,000 passengers are being served. More than 50% of these services are carried out by the metro. The focus of this thesis will be solely on the metro network of Rotterdam. The performed research is done at the subdepartment *Onderzoek Innovatie Vervoerplanketen* (OIV), which is part of the department *Bedrijfsbureau Exploitatie* (BBE). The main tasks of BBE are creating and optimizing crew – and duty-schedules, rostering, training drivers and conductors, functional application management, data management and carrying out new, innovative projects. The subdepartment OIV focuses on the information-systems used by the RET. Furthermore, research is done for the development of new tools, which may contribute to an improvement in information flows in public transport planning processes and in the optimization of crew - and vehicle-schedules and daily assignments.

## 1.2 Motivation of Research

Performing a capacity evaluation of lines and networks is of great importance for transport operators and thereby also for the RET. By calculating the maximum capacity, the physical limits of the metro network are determined.

With the expected increase in the number of passengers in the coming years, the question arises to which extent the amount of rolling stock (i.e. vehicles that can move on the railway network) on the metro network of Rotterdam can be increased, such that passenger's demand will remain to be fulfilled. Moreover, to satisfy passenger's wishes, a higher frequency of arriving trains might be needed, which requires an answer to the very same question.

In the near future the metro network in the district of Rotterdam will be further expanded with the addition of the *Hoekse Lijn*. Currently, the stations in this area are being served by regular trains (operated by the *Nederlandse Spoorwegen, NS*). In the coming period, these stations and the tracks will be adjusted such that they can be accommodated for metro trains as well. With the realization of this project, in total nine stations are adjusted and added to the metro network, making efficient scheduling of rolling stock an even more complex task.

Once we have constructed a model for determining capacity, this can be utilized to determine the effect of the configuration of certain parameters. We perform sensitivity analyses to examine to which extent parameters such as turning time and headway time (both terms are explained in Chapter 2) affect the capacity of the network. By doing so, it may become clear where potential infrastructural bottlenecks in the metro network are located, which might possibly lead to undertaking required measures for improvement.

The capacity of a railway network can either be defined from the transport operator point of view, or from the passenger point of view. The research in this thesis will be performed from the operator point of view, which boils down to determining the maximum amount of rolling stock that can be deployed on the network, such that a smooth moving process is guaranteed and no restrictions (e.g. safety restrictions) are violated. Hence, the capacity examined in this thesis is based on the amount of rolling stock that is capable of moving on the metro network. From the passenger point of view, the maximum number of passengers



that can be transported on a metro network within a certain time window are of interest. This aspect however, is not part of the research done in this thesis.

Since the scientific definition of capacity is not as straightforward as one might think and various definitions are proposed throughout the literature, a suitable unit of measure for capacity has to be found first.

Capacity can be further divided in theoretical and practical capacity. Within this thesis, only the theoretical capacity will be determined. The theoretical capacity is the capacity under 'ideal conditions' [Landex, 2008], that is, no delays, breakdowns or other randomness will occur. Furthermore, it is assumed that the stopping and traveling times are deterministic. Theoretical capacity is restricted by a large number of factors, regardless of the used definition. In order to guarantee safety, restrictions are imposed concerning speed limits, or the minimum distance and time between sequential trains. Theoretical capacity can be used as an upper bound for the practical capacity and can often be calculated by mathematical formulas or by mathematical optimization.

Practical capacity on the other hand is usually obtained by making use of a simulation and is even more complex to determine. This is caused by the fact that besides the restrictions for the theoretical model also restrictions such as electrical power, fleet and crew size and disturbances are taken into account.

The aim of this thesis is to find an answer to the three following research questions:

1. *What is a suitable and applicable definition of capacity for the metro network of Rotterdam?*
2. *Given the metro network and corresponding restrictions, what is the theoretical network capacity of the metro network of Rotterdam?*
3. *Can we develop a software tool that can be used by the RET to perform capacity calculations?*

### 1.3 Thesis Outline

This thesis is organized as follows. Chapter 2 contains a detailed problem description. Furthermore, important terms used throughout this thesis are explained. In Chapter 3 relevant literature is discussed, together with the applicability of each paper to the stated problem. In Chapter 4 the required network data is presented. In Chapter 5 we define the definition for capacity. Subsequently, different models used to determine theoretical capacity are proposed and the modeling of important network restrictions is explained thoroughly. The results of the research are described in Chapter 6. Moreover, the capacity-impact of different network restrictions are analyzed. Finally, conclusions of the research and recommendations for the company are made in Chapter 7.

# Chapter 2

## Problem Description

In the following paragraphs, the problem is further described. For convenience, in Paragraph 2.1 the commonly used terminology is explained, which is frequently referred to throughout this thesis.

### 2.1 Terminology

A brief explanation for all important terms is given in this paragraph.

#### **Section**

A *section* is a part of a track. Each track is divided into multiple sections. This division is mainly made for safety reasons. For instance, restrictions are imposed such that only one train is allowed on a certain section at a certain moment. The time it takes for a train to traverse a section entirely is called *Sectional Running Time* (SRT).

#### **Line**

A line is the route of a train from begin — to endpoint and vice-versa. Each line consists of an ordered number of sections, which are all connected. Every line has a line-duration and line-distance, which is the sum of all sectional running times and section lengths, respectively.

## Headway Time

*Headway time* is the time between two sequential trains on the same track. Headway time is mainly used to impose safety restrictions. For instance, it can be stated that the minimum headway time should be larger than  $X$  seconds at all times. The RET often uses a minimum headway time of 90 seconds in *HASTUS*. *HASTUS* is a software-tool developed by the Canadian company GIRO ([www.giro.ca](http://www.giro.ca)), in which timetables for public transport are created.

## Station

On every station in the network, there are one or multiple *platforms* where can be halted. At the platform, passengers can board and disembark the train. The total stopping time at a platform is referred to as *dwelt time*. Most platforms are matched to a certain direction. On the stations there is either one large platform in the middle or two separate platforms on each track. For each platform it is given which section it adjoins. As one can see in Figure 2.1 below, stations *Delfshaven* (DHV) has two platforms, one at track 2, on the lower side of the figure (black boxes in the figure) serving trains into the direction of *Coolhaven* (CHV) and one at track 1, on the upper side of the figure, serving trains into the direction of *Marconiplein* (MCP). There are exceptions, such as maintenance or interruptions, in which case one platform may be blocked and the other platform serves both directions. However, these disturbances are not part of the research done in this thesis and are therefore omitted. Other cases in which platforms do not serve a certain direction are at the turning points of each line. This will be further explained in the next paragraph.

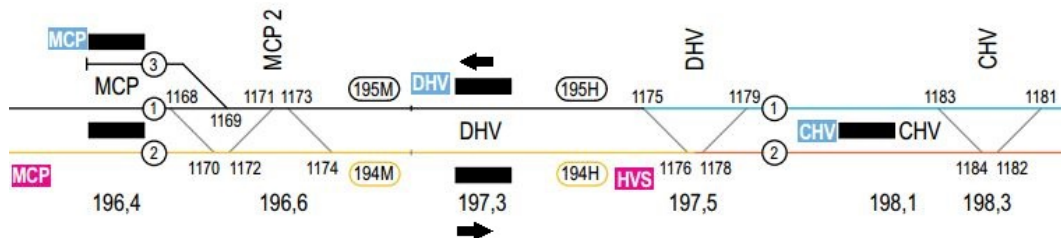


Figure 2.1: *Graphical Representation of the Sections around Station Delfshaven*

## Turning Point

At the turning points, trains end their trip and start a new trip in opposite direction. Turning points are often equipped with a buffer block. In order to make a turn, the train driver has to switch off the train, walk to the other side of the train and switch the train back on again. The time needed to perform these actions is called *turn around time*<sup>1</sup> or *turning time*. In Figure 2.2 station and endpoint *Nesselande* is presented.

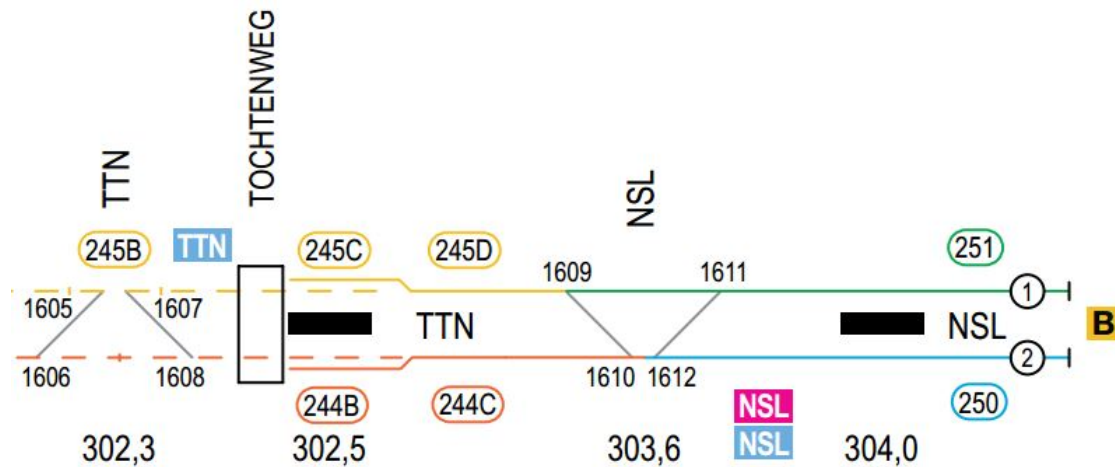


Figure 2.2: *Graphical Representation of the Sections around Station and Endpoint Nesselande*

As one can see, at *Nesselande* (NSL) there is one large platform in the middle, serving both tracks. A train driving line A (different lines are described in Chapter 4) that is coming from station *De Tochten* (TTN) on track 2 has two options for stopping and turning at *Nesselande*. The first option is to go to the opposite track already at the end of the trip (via sections 1610 and 1611). Halting and turning is then performed at track 1 and the train can continue its new trip on track 1 via sections 1611 and 1609, respectively. If the first option will lead to conflicts (e.g. the endpoint of track 1 is occupied), the other option is to stay on the lower side of the figure until the end of the trip. Then halting and turning is done at track 2 and the endpoint is reached via sections 1610 and 1612. In order to start the

<sup>1</sup>*Kopmaaktijd* in Dutch, no accurate translation could be found

trip in the opposite direction again, a switch to track 1 required. This is done via sections 1612 and 1609.

### Intermediate Point

There are some sections in the network that are part of a level-crossing, where head-to-head collisions may occur with trains coming from other directions. These intersections are called *Intermediate Points*. See Figure 2.3 below for the Intermediate point at *Graskruid* (GKD). Head-to-head collisions<sup>2</sup> occur for instance when one train is coming from *Alexander* (ALD) on track 2 in direction of *Romeynshof* (RMH) and at the same time one train is coming from *Hesseplaats* (HSP) track 1 in direction of *Alexander* (ALD).

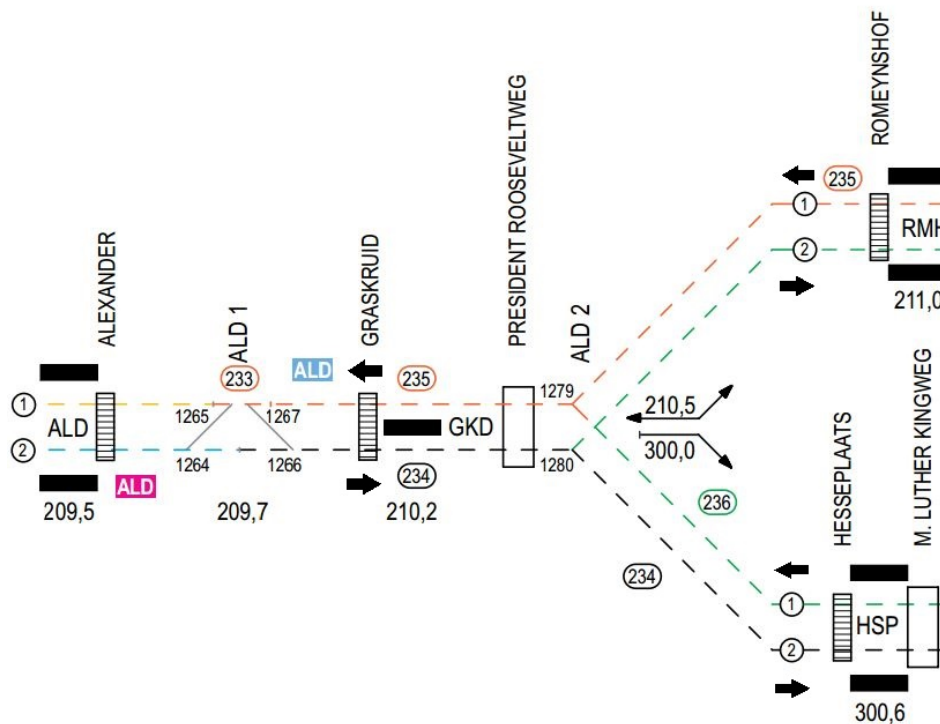


Figure 2.3: *Graphical Representation of the Intermediate Point Graskruid*

Note that collisions may also happen at other locations in Figure 2.3. For

<sup>2</sup>This type of collisions is not necessarily head-to-head. However, these collisions are referred to as head-to-head within the RET

instance, when one train is coming from *Romeynshof* (RMH) on track 1 and at the same time another train is coming from *Hesseplaats* (HSP) on track 1, a collision occurs at 1279 in the figure. Furthermore, this may be the case on endpoints, as can be seen in Figure 2.2. When one train coming from endpoint *Nesselande* (NSL) is going to track 1 via section 1612 and 1609 and one train is coming *Nesselande* (NSL) from section 1611, a collision occurs at section 1609.

Clearly, these collisions must all be prevented and these Intermediate Points have to be taken into account in the calculation of the capacity. There are also level-crossings in the network with other types of traffic, such as pedestrians and road traffic. However, it is assumed that trains always have priority over these types of traffic and therefore these are omitted in the calculations.

### **Safe Haven**

*Safe Haven* is a safety principle used by the RET. This restricts the number of trains between two locations (e.g. stations). *Safe Haven* is currently only used between stations *Blijdorp* and *Rotterdam Centraal*. A train that is coming from *Den Haag Centraal* may only depart from station *Blijdorp* if the foregoing train has departed from station *Rotterdam Centraal*. Hence, all sections between *Blijdorp* and *Rotterdam Centraal* must be free before the train is allowed to depart from station *Blijdorp*. These track restrictions hold in both directions between these two stations.

## **2.2 Problem Statement**

Given the restrictions for minimum headway time, endpoints, Intermediate Points, *Safe Haven* and more explained later, we make an attempt to determine the theoretical capacity of the metro network. In reality more restrictions hold, which unfortunately can not all be modeled, due to time or complexity limitations. We therefore limit the scope of this thesis to a selection of restrictions and determine capacity on this basis. First however, a suitable definition of capacity has to be found.

# Chapter 3

## Literature Review

This chapter surveys previous related work for finding a suitable definition for capacity and determining the theoretical capacity. The amount of available literature for the given problem concerning metro networks is limited. However, for solving the problem for metro networks, assumptions and restrictions can be used that are similar to regular train networks. Therefore, related literature described in this paragraph is mainly focused on trains, but may, after necessary adjustments, also be applied to the aforementioned problem. First, in Paragraph 3.1, various definitions for capacity are proposed, which is discussed among many papers. Subsequently, literature for calculating the theoretical capacity is described in Paragraph 3.2.

### 3.1 Capacity Definition

In the PhD thesis of Landex [2008] the importance of the correct choice of the definition of capacity is discussed. In the thesis, it is stated that the capacity should be expressed in terms of the number of trains moving on the network, rather than measuring the number of passengers that move through the network. The reason is that vehicles with different lengths and characteristics can run on the railway, all having different passenger capacities. This makes determining passenger capacity an even more complex problem than determining vehicular capacity.



Since capacity can be determined using different criteria, such as number of trains, average speed, stability and heterogeneity (the mix of deployed trains), a broad variety of definitions has been proposed throughout the literature. In Landex [2008] a large number of definitions, all proposed by different researchers over time, is presented. These definitions may all be suitable for different networks. The suitability of each definition is highly dependent on the infrastructure itself. For instance, for metro networks covering relatively small distances, the heterogeneity might be of less importance than for a regular train network, which may also ask for high velocity trains, running over larger distances. In Figure 3.1 four important and often used criteria are depicted. As one can see, for metro networks the stability and the number of trains (line-frequencies) will be of larger concern, in order to guarantee fast and reliable transportation over (relatively) short distances, which are in general the main strengths of a metro. Since in regular train networks on average the distances between are longer, the average speed and heterogeneity are more important.

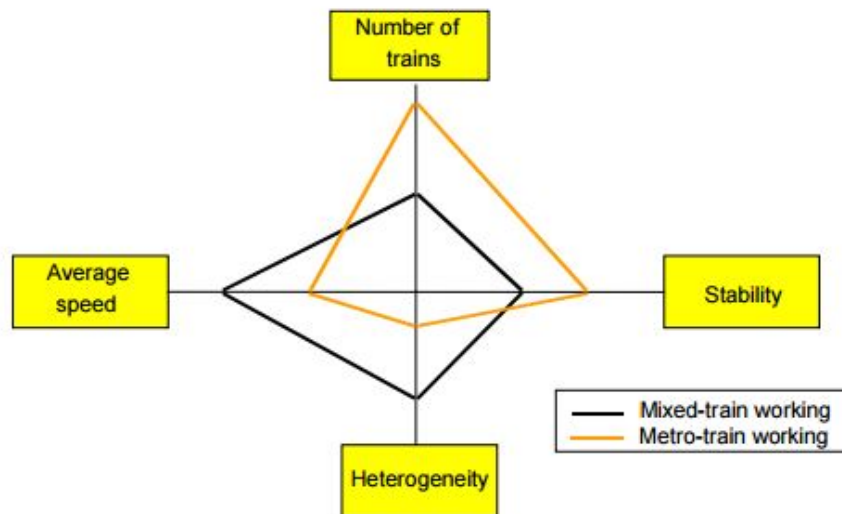


Figure 3.1: *Capacity Balance of Metro Networks and Train Networks, UIC [2004]*

In the paper of UIC [2004] a similar approach for defining capacity is used. The same four characteristics as in the paper of Landex [2008] are used. See again Figure 3.1. The capacities can then be calculated as a function of these

four characteristics. This function can be adjusted for different types of networks. Furthermore, a definition of capacity consumption percentage is proposed, which equals the total track occupation time, divided by the total time window.

Another definition of capacity can be found in the paper of Burdett and Kozan [2006] and is measured as "the maximum number of trains that can traverse on the entire railway or certain sections in a given duration of time". A similar definition can be found in De Kort et al. [2003]. However, in this paper also uncertainty and robustness are taken into account. The definition in this paper therefore becomes: "the maximal number of train movements that can be executed on the particular infra-element in  $T$  time units with probability greater than or equal to  $p$ ", where  $p$  is a certain threshold.

## 3.2 Determining Theoretical Capacity

According to the work of Abril et al. [2008], the theoretical capacity (or often referred to as absolute capacity) can be determined using two global types of methods; analytical and optimization methods. Analytical methods form the most simple approach to calculate theoretical capacity, which is in general obtained by a mathematical formula or algebraic expression.

In the paper of Burdett and Kozan [2006] an analytical method is presented, using a so-called bottleneck approach. This approach is reasoned from the fact that the number of trains able to run on a single track is limited by the most critical section of the line, the bottleneck. Capacity analysis is done by making use of balance equations. The model is then further extended to more complex networks, consisting of multiple lines, which might be applicable to the Rotterdam metro network. Due to the complexity of the multi-line case, it is concluded that the balance equations no longer can be used. Since capacity is based on an efficient schedule and finding an near-optimal schedule is usually not straightforward, optimization models are used instead. The problem is then written as a mathematical program, which is in most cases a Mixed Integer Linear Problem (MILP).

In the paper of Lee and Chen [2009] the train pathing and timetabling problem is written as a mathematical formulation. Since the complexity of the overall

problem is high, also a heuristic-approach is presented. Given the outcome of the pathing and timetabling problem and a suitable definition of capacity (see Paragraph 3.1), the theoretical capacity can be obtained.

Three different capacity evaluation methods are discussed in the paper of Malavasi et al. [2014]; the Potthoff method [Potthoff, 1963 - 1972], the Probabilistic method [Corazza and Musso, 1991] and the Deutsche Bahn method [Bundesbahn, 1979]. The different methods are then compared by doing two case studies, all having their own (dis)advantages.

The same type of problem is discussed in the paper of Harrod [2009], where the capacity of railway-networks is evaluated, also taking heterogeneous velocities of trains into account. The problem is formulated as a multi-commodity flow on a hyper-graph. The impact of adding high-speed trains to the network is analyzed and compared to the situation where only trains with the same velocities are deployed. Finally, the 'loss' in capacity compared to the network with homogeneous velocities is calculated. Capacity here, is measured as 'the count of valid train paths over a fixed time horizon' and is also corrected for delays incurred.

Rather than determining the performance of a railway-network by its capacity, the waiting times (delays) of trains are analyzed in the work of Huisman et al. [2002]. In the paper, the queuing network is modeled as a continuous-time Markov chain. All the stations, junctions and sections are modeled as a queue and the whole network can then be seen as first-come-first-served multi-server queuing systems. It is assumed that trains departing from their starting station follow a Poisson processes. Furthermore, assumptions are made that the service times at stations, junctions and sections are exponentially distributed. The robustness and capacity of the network can then be determined by gradually increasing the number of deployed trains and evaluating the increase of waiting times at different locations (servers). However, these assumptions cannot be justified in our network. For instance, in this case the minimum headway time has to be modeled. Trains can never depart from their starting station within a certain number of seconds of their predecessor. The assumption of the arrival and departure processes following a Poisson process cannot be justified and therefore this solution method is not applicable to the stated problem.

The approach presented in the paper of Burdett and Kozan [2006] seems to fit the described problem better, since also sectional running times are explicitly taken into account. However, in this formulation there is no index incorporated for 'time', which makes modeling some of the network-restrictions impossible. For instance, the requirement that only one train is allowed to be on a certain at a certain moment, cannot be modeled without the use of a time index.

As we have seen, there does not appear to be any comparable and suitable model presented in the literature. Therefore, we decide to develop our own model, while using different components and ideas from the presented literature.

# Chapter 4

## Network Data

In order to find an answer to the proposed research questions, data of the metro network is required. All required data is described in the following paragraphs.

### 4.1 Lines and Stations

In the network data, all relevant information on the metro network itself is included and contains information on number of sections, section lengths, location of stations, maximum allowed speed, etc. In Figure 4.1 on the next page, a simplified map of the current metro network in Rotterdam is shown. The network consists of a total of 62 stations, with a total track length of 78.3km. As one can see, in the network there are multiple lines with overlapping stations, making capacity calculations more complex to perform. In total 32 stations are served by more than one line. On the tracks between *Schiedam Centrum* and *Capelse Brug* trains on three different lines, A, B and C are active, therefore making these tracks potential bottlenecks. Furthermore, there are endpoints that share multiple lines, such as *De Akkers* and *Schiedam Centrum*, which may also restrict the capacity on the corresponding lines.

In 2017, the line B will be further extended with a connection between *Schiedam Centrum* and *Hoek van Holland Strand*, the so-called *Hoekse Lijn*, adding nine new stations to the network. The Hoekse Lijn, however, will not be part of the research.



Figure 4.1: Map of the Metro Network of Rotterdam

Important to note, on line E, the scope of this thesis will be solely on the part *Slinge - Blijdorp* and vice-versa. The part *Blijdorp - Den Haag Centraal* is left out of consideration. The reason for this decision is that on the tracks from *Den Haag Centraal* to *Blijdorp* other train protection systems are being used and therefore a different model is required. Furthermore, on the tracks between *Leidschenveen* and *Laan van NOI* also trams operated by another transport operator — the *HTM* — are driving, which can also be seen a capacity restriction.

We assume that trains on line E coming from *Slinge* do not make a turn at *Blijdorp*. Trains simply 'disappear' from the map after passing the station. By doing so, we prevent that turning at *Blijdorp* becomes one of the bottlenecks, while in reality turning is not performed at this station. Therefore, the theoretical capacity obtained from the model may be somewhat higher than the capacity for the whole network with the tracks *Den Haag Centraal - Blijdorp* included.

The network contains five lines, denoted by A up to E. See Table 4.2 for relevant

statistics of each line.

Line	Origin	Destination	Direction	Stations	Sections
A	Binnenhof	Schiedam Centrum	West	20	135
A	Schiedam Centrum	Binnenhof	East	20	133
B	Nesselande	Schiedam Centrum	West	23	152
B	Schiedam Centrum	Nesselande	East	23	147
C	De Terp	De Akkers	West	26	203
C	De Akkers	De Terp	East	26	202
D	Rotterdam Centraal	De Akkers	West	17	138
D	De Akkers	Rotterdam Centraal	East	17	140
E	Blijdorp	Slinge	West	23	61
E	Slinge	Blijdorp	East	23	60

Table 4.2: *Data of the Five Different Lines of the Metro Network of Rotterdam*

### 4.1.1 Sections

A track is decomposed in sections, all having different lengths. All required information on each section is available in large, officially approved data-sheets. The network consists of a large number of sections, which are all measured with millimeter precision. Each of these sections has its own speed limits which must be obeyed at all times. The length of the sections have a large spread, ranging from 3.692m to 493.351m. Mostly, the smaller sections are special sections, such as sections with a buffer block. A large part of the sections is meant for parking, (un)coupling and other activities and therefore lies beyond the scope of this thesis. Roughly 800 sections are relevant for the the capacity research. Some statistics of all relevant sections can be seen in Table 4.3.

No. Sections	Avg. Length	Max. Length	Min Length.	Avg. Max Speed
801	147.41m	493.35m	12.00m	71.40 <sup>km/h</sup>

Table 4.3: *Statistics of Track Sections*

Using the section data, for each line we create routes, which are ordered sequences of sections corresponding to each of the lines. Those sequences are then matched to a train and added to the model.

### 4.1.2 Line-Distance Comparison

Since the routes are created manually, this task is prone to error. We therefore compare total line-distances of the model with the distances retrieved from a working simulation tool, developed in the thesis of Both [2015]. The distances in this tool are based on a metro simulator, specially developed for the RET. The metro simulator is normally used to educate drivers. The distances are compared in Table 4.4 below.

Line	Dist. Model [m]	Dist. Sim. [m]	Deviation
A Schiedam Centrum - Binnenhof	17,938	17,107	+4.86%
A Binnenhof - Schiedam Centrum	17,901	17,056	+4.95%
B Schiedam Centrum - Nesselande	20,806	19,128	+8.77%
B Nesselande - Schiedam Centrum	20,777	19,990	+3.94%
C De Akkers - De Terp	29,979	29,686	+0.99%
C De Terp - De Akkers	30,122	30,052	+0.23%
D De Akkers - Rotterdam Centraal	21,442	21,390	+0.24%
D Rotterdam Centraal - De Akkers	21,389	21,375	+0.07%
E Blijdorp - Slinge	8,471	8,344	+1.52%
E Slinge - Blijdorp	8,470	8,343	+1.52%

Table 4.4: *Comparison of the Distances in the Model and Distances Retrieved from the Simulation*

As one can see in the table, the distances match the simulation distances quite well. There is one outlier (line B *Schiedam Centrum - Nesselande*) with a deviation over 8%. On average, the model distances are somewhat higher than the distances retrieved from the simulation. An explanation for these differences is that the distances retrieved from the simulation are the distances between the begin — and endstations only, whereas in the model also the distances to the turning points are



included. Especially at *Schiedam Centrum* the distance to the turning point is large, which we see in the table with relatively large deviations for lines A and B.

### 4.1.3 Line-Duration Comparison

The line-durations in our model are computed by taking the sum over all sectional running times of the sections corresponding to each line. It is more complicated to compare the line-durations with those retrieved from the simulation. The reason is that in the model the total line-duration is composed of the sum of all sectional running times and dwell and turning times, which are all dependent on the configuration of the parameters. First, we set the dwell time to regular used value (25 seconds) and compare the total line-durations with those retrieved from the same simulation tool. Furthermore, since the line-durations retrieved from the simulation only consist of the duration from first to the last station and the time it takes to make a turn is excluded, we set the turning time to 0 seconds. The uncorrected durations are presented in Table 4.5 below.

Line	Dur. Model [s]	Dur. Sim. [s]	Deviation
A Schiedam Centrum - Binnenhof	1,748	1,920	-8.96%
A Binnenhof - Schiedam Centrum	1,728	1,905	-9.29%
B Schiedam Centrum - Nesselande	2,035	2,235	-8.95%
B Nesselande - Schiedam Centrum	1,998	2,220	-10.00%
C De Akkers - De Terp	2,461	2,745	-10.35%
C De Terp - De Akkers	2,459	2,775	-11.39%
D De Akkers - Rotterdam Centraal	1,755	1,920	-8.59%
D Rotterdam Centraal - De Akkers	1,755	1,950	-10.00%
E Blijdorp - Slinge	856	975	-12.21%
E Slinge - Blijdorp	871	960	-10.22%

Table 4.5: *Comparison of the Durations in the Model and Durations retrieved from the Simulation*

As we can see in the table, the differences in line-durations between the model and the simulation are significant. This can be explained by the fact that in the

model currently it is assumed that every train traverses each section at maximum speed, which is in reality not possible, considering for instance the comfort of passengers and the finite acceleration and deceleration rates.

In order to obtain a realistic answer to the capacity question, we decide to calculate the fraction of maximum speed for which the total deviation of the durations retrieved from the simulation are minimized. This parameter is then used as a fixed input to the model.

Line	Dur. Model [s]	Dur. Sim. [s]	Deviation
A Schiedam Centrum - Binnenhof	1,905	1,920	-0.78%
A Binnenhof - Schiedam Centrum	1,943	1,905	1.99%
B Schiedam Centrum - Nesselande	2,288	2,235	2.37%
B Nesselande - Schiedam Centrum	2,179	2,220	-1.85%
C De Akkers - De Terp	2,767	2,745	0.80%
C De Terp - De Akkers	2,765	2,775	-0.36%
D De Akkers - Rotterdam Centraal	1,973	1,920	2.76%
D Rotterdam Centraal - De Akkers	1,973	1,950	1.18%
E Blijdorp - Slinge	963	975	-1.23%
E Slinge - Blijdorp	979	960	1.98%

Table 4.6: *Comparison of the Durations in the Model, corrected for the Velocity Factor and Durations retrieved from the Simulation*

The value of the fraction of maximum speed, which minimizes total deviation, is equal to 0.69. Hence, it is assumed that the velocity of each train on a section is equal to 69% of the maximum allowed speed. The line durations corrected for this parameter are presented in Table 4.6. For all lines, the deviations from the durations retrieved from the simulation have decreased significantly, making the model more comparable with the real situation.

# Chapter 5

## Calculating Capacity

### 5.1 Capacity Definition

In accordance with the RET, we have found a suitable definition for capacity, which is as follows.

**Capacity:** *The maximum number of trains that can start their route within a certain time window. Trains must traverse their entire routes without being hindered by other trains.*

Note that this does not necessarily mean that a train completes its assigned trip entirely within the given time window. For instance, when a train starts its trip just before the end of the time window, then it is also counted as being part of capacity. Using this definition, we get insight in the maximum attainable line-frequencies, when the network is at its maximum occupation.

### 5.2 Assumptions for the Theoretical Model

The aim of the model is to calculate the maximum number of trains that can be deployed on the network within a given time window, usually one hour. In order to do so, we develop a tool where the user can set the preferred parameters, such as minimum headway time, train length, turning time, dwell time and the total time

window in which capacity must be calculated. For the lay-out and explanation of the tool, see Appendix B. Given these parameters, the maximum number of trains deployed on the network is calculated. Clearly, this number is subject to some safety and timetable restrictions, which must all be obeyed at any time. For our model we make the following assumptions:

- Dwell and turning times are deterministic and are equal to the configured parameters.
- Dwelling and turning can be performed in parallel. That is, if a train is both dwelling and turning at a section that is both an endpoint and a station, the total time needed for that train to perform both actions is equal to the maximum of both parameters.
- Only one train is allowed to be on a certain section at every point in time.
- On a section, the time between two consecutive trains must always be at least as large as the configured minimum headway time.
- The velocity of a train is constant on the entire section.
- Every train on the network completes its entire line without being hindered by other trains. The distance (or time) between consecutive trains must therefore always be large enough to guarantee that no trains incur waiting time on their lines.
- Trains decelerate with a rate of  $0.50\text{m/s}^2$ . Deceleration is assumed to be linear. The deceleration rate is used for calculating the minimum headway time and is explained later on.
- The velocity of a train is never higher than the maximum allowed speed on the section.
- Only one type of train is deployed on the network, which can traverse all lines. That is, all trains in the network are the same and have the same length.

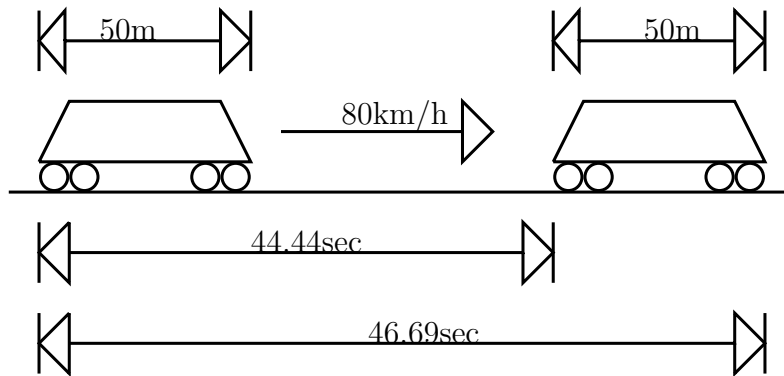
### 5.3 Calculation of Sectional Running Times

As stated in the assumptions, for each section the velocity of a train is assumed to be constant on the entire section it is traversing. The average speed on each section is calculated as the fraction of maximum speed (determined in Paragraph 4.1.3) multiplied by the maximum allowed speed on each section. Using the sectional average speed, we can calculate the sectional running time with the formula:  $SRT^s = L^s / AV^s$ , where  $SRT^s$  is the sectional running time on section  $s$ ,  $L^s$  is the length of section  $s$  in meters and  $AV^s$  is the average speed on section  $s$  in m/s. The sectional running time is then rounded off to seconds in order to fit the model.

### 5.4 Calculation of Minimum Headway Time

In the tool for calculating the capacity, we implement an option for the user to choose between either a global setting for the minimum headway time, or a minimum headway time that is different per section. When using the global setting, the headway time is equal on all sections and is at least as large as the configured parameter (e.g. 90 seconds).

In the section-dependent setting, the headway time is dependent on the velocity of a train on the section itself. Given the velocity on the section and deceleration rate, which was set to  $0.50\text{m/s}^2$ , we can compute the minimum time and distance for the train to safely come to a stop. This time is then used as minimum headway time for this section  $s$ , which we call  $HW^s$ . Since in the mathematical model only the movement of the front of the train is modeled (a train has 0 length) this stopping distance must be corrected for the length of a train, which can also be configured in the tool. This correction is depicted in Figure 5.1 on the next page.

Figure 5.1: *Visualization of Minimum Headway Time*

Assume that the maximum allowed speed on a section is equal to  $80\text{km/h}$  ( $22.22\text{m/s}$ ) and furthermore assume for convenience that the fraction of maximum speed is equal to 1. Using a deceleration-constant of  $0.50\text{m/s}^2$ , the minimum time needed for the train to come to a stop would be 44.44 seconds. However, since the rear of the first train is closer to the front of the second train, this is not sufficient for the second train to stop in time, such that a collision is avoided. We therefore compute the time it takes for a train to drive a distance equal to its length (2.25 seconds in the figure) and add this to the minimum headway time of the section. In the example, the total headway time thus becomes 47 seconds, when rounded off to integers.

## 5.5 Formulation Theoretical Capacity

In this paragraph, the first model used to determine theoretical capacity is presented. The model is partially based on the optimization model developed by Burdett and Kozan [2006]. The model is rewritten such that also 'time' can be modeled explicitly and time-dependent restrictions, such as minimum headway time can be included. First, all required sets, variables and parameters are described on the next page. Subsequently, the model with all the included constraints are explained in detail.

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**Sets & Indices:**

$\tau$	The set of trains, matched to a line
$S(i)$	The set of sections traversed by train $i \in \tau$ .
$S$	The total set of sections. That is, $S = \cup_{i \in \tau} S(i)$
$T$	Time window, with $t$ in seconds, $t \in \{1, \dots, T\}$

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**Variables:**

$x_{i,t}^s$	Binary variable which is equal to 1 if train $i \in \tau$ is present at section $s \in S(i)$ at time $t$ and zero otherwise.
$z_i$	Binary variable which is equal to 1 if train $i \in \tau$ is deployed on the network and zero otherwise.
$C_{network}$	The total capacity of the network. That is, the total number of trains that can start their line within the time window.

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**Parameters:**

$SRT_i^S$	The sectional running time of train $i \in \tau$ section $s \in S(i)$ .
$R_i^s$	Defines the order in which sections are traversed on line $i$ . If $R_i^{s_1} < R_i^{s_2}$ then section $s_1$ is traversed before section $s_2$ . If $R_i^s = 0$ , then section $s$ is not traversed by train $i$ .
$W$	The maximum waiting time at each section.
$HW^s$	The minimum headway time of section $s$
$b(i)$	The first section on the route of train $i \in \tau$ .

---

The formulation becomes:

$$\max \quad C_{network} = \sum_{i \in \tau} z_i \quad (5.1)$$

*s.t.*

$$\sum_{t=1}^T x_{i,t}^{b(i)} \geq z_i, \quad \forall i \in \tau \quad (5.2)$$

$$x_{it}^s \leq x_{i,t+1}^s + x_{i,t+1}^{s'}, \quad \forall i \in \tau, \forall t, \forall s, s' \in S(i) : R_i^{s'} = R_i^s + 1 \quad (5.3)$$

$$x_{it}^{s'} \leq x_{i,t-1}^{s'} + x_{i,t-1}^s, \quad \forall i \in \tau, \forall t, \forall s, s' \in S(i) : R_i^{s'} = R_i^s + 1 \quad (5.4)$$

$$x_{it}^s + x_{i,t+1}^{s'} - 1 \leq x_{i,t-SRT^s+1}^s, \quad \forall i \in \tau, \forall t, \forall s, s' \in S(i) : R_i^{s'} = R_i^s + 1 \quad (5.5)$$

$$x_{it}^s + x_{i,t+SRT_i^s+W}^s \leq 1, \quad \forall i \in \tau, \forall t, \forall s \in S(i) \quad (5.6)$$

$$x_{it}^s + \sum_{i' \neq i} x_{i't'}^s \leq 1, \quad \forall i \in \tau, \forall t, \forall s \in S(i), \forall t', \text{ for } t' \in \{t, \dots, t + HW^s\} \quad (5.7)$$

$$\sum_{s \in S(i)} x_{it}^s \leq 1, \quad \forall i \in \tau, \forall t \quad (5.8)$$

$$\sum_{i \in \tau} x_{it}^s \leq 1, \quad \forall s \in S, \forall t \quad (5.9)$$

$$z_i \geq z_j, \quad \forall j > i : R_i^s = R_j^s, \forall s \in S(i) \quad (5.10)$$

$$z_i \in \{0, 1\}, \quad \forall i \in \tau \quad (5.11)$$

$$x_{it}^s \in \{0, 1\}, \quad \forall t, \forall s \in S(i), \forall i \in \tau \quad (5.12)$$

In this formulation, the aim is to maximize the objective function (5.1), which is the total number of trains that are deployed on the network. Constraints (5.2) state that if a train is deployed on the network, it should always traverse the first section of the route. These can be seen as 'start-up' constraints; they make sure that a train, once chosen to be deployed, will actually start its line. Constraints (5.3) state that if train  $i$  is present on section  $s$  at time  $t$ , at time  $t + 1$  it has to be either on the same section or has to be moved to the next section on its line, section  $s'$ . Constraints (5.4) state that if a train  $i$  is present on section  $s'$  (which



is the successor of section  $s$  on the line of train  $i$ ) on time  $t$ , then at time  $t - 1$  it was either on section  $s$  or already at section  $s'$ . Constraints (5.5) ensure that a train is present on its section for at least the sectional running time, which is calculated as the section's length divided by the given speed (see Paragraph 5.3). Constraints (5.6) ensure that a train occupies the section for at most the sum of the sectional running time and the maximum waiting time. Since we assume that every train completes its route unhindered, we set  $W = 0$ . Constraints (5.7) are constraints imposed on the minimum headway time. If a train is occupying section  $s$  at time  $t$ , then no other trains are allowed to be on the section for at least  $HW$  time units. Constraints (5.8) ensure that a train can occupy at most one section at a time. Here, we assume that a train does not have a length and can therefore never occupy multiple sections at a time. Constraints (5.9) make sure that at most one train is allowed to be on a section at a time. Constraints (5.10) are also included in the model. These constraints do not necessarily have to be included for finding a feasible solution, but are there to avoid symmetric solutions. A solution is symmetric if its variables can be permuted without changing the solution value. For instance, in this case, if train  $i$  and train  $j$  are assigned to the same route, deploying train  $i$  on the network has the same effect as deploying train  $j$ . Including these constraints will reduce the feasible region of the LP-relaxation, which will result in a faster branch-and-bound progress. Finally, Constraints (5.11) and (5.12) ensure that all variables are binary.

### 5.5.1 Model Complexity

As one can see from the formulation and the data, the number of binary-variables and especially the number of constraints becomes very large. The network consists of approximately 800 sections, not taking special sections into account, used for e.g. parking, coupling, etc. Assume 100 trains (rough estimation) are deployed on the network. For one hour, the number of variables would then become approximately  $100 \cdot 3600 \cdot 800 = 288,000,000$ . What is more, the number of constraints exceeds this amount by far. Hence, the model consists of millions of constraints. Clearly, the model is too complex to solve to optimality, within a reasonable amount of time. After running Java with IBM CPLEX 12.6.3 for over 24 hours, no solution

was found at all. In fact, the computer ran out of memory. Next, we attempt to find an upper bound for the theoretical capacity by relaxing the integrality constraints on both the  $x$  – and  $z$ -variables. A check was done on the possibility of the constraint matrix being totally uni-modular. In the case of a totally uni-modular matrix, the integrality gap between the Integer Programming problem (IP problem) and LP-relaxation is equal to zero, such that solving the LP-relaxation is sufficient to find the optimal solution to the IP. However, the constraint matrix turns out not to be totally uni-modular. Moreover, even the LP-relaxation of this Linear Integer Program could not be solved; no solution could be found and again a memory error occurred. Therefore, we either have to develop a stronger formulation, or another solution method has to be found.

### 5.5.2 Complexity Reduction

First, we try rewriting the formulation. An obvious first step would be to increase the time 'step size' from every second to, say, every five seconds, which would already reduce the problem by a factor of five. However, there are a lot of small sections, with sectional running times of only a few seconds. This way, the accuracy of the capacity estimation would decrease significantly. We thus postpone this measure and use it only in case no other method works.

Next, we note that a train must occupy a section for a certain time if and only if the section is on the route of that train. Hence, for train  $i$  and section  $s$  it holds that:

$$\sum_{t=0}^T x_{it}^s > 0 \iff R_i > 0 \quad \forall s \in S, \forall i \in \tau$$

Therefore, it holds that an  $x$ -variable for a certain section and a certain train should only be created if this section is on the route of that train. On average, the route of a train consists of 180 sections. Thus, if we calculate the capacity in one hour, per train roughly  $3600 \cdot (800 - 180) = 2,232,000$  variables are removed from the model. Moreover, this also automatically removes a large number of constraints, which were induced on those removed variables.

The next step to reduce the number of variables is based on the work of Thompson [1995]. In this paper, a formulation is presented to schedule shifts in an optimal

way. Every shift has an earliest starting time, which is the earliest possible time that a shift is allowed to start. The same reasoning can be used in our case. Since for every train we know the order in which sections must be traversed and we know the minimal sectional running time for each section, we know the earliest starting time for each section. Thus, the  $x$ -variables corresponding to a train and a section with a  $t$  smaller than the section's earliest starting time can be removed. Suppose for instance that a train must traverse sections  $1 \rightarrow 2 \rightarrow 3$ , with sectional running times of each section equal to 5. Suppose furthermore that the time window is equal to 15. Then the earliest starting time for each section respectively, are 1, 5 and 10. In the original case, there would be  $1 \cdot 15 \cdot 3 = 45$  variables in the model. Using aforementioned reduction technique would result in a removal of  $0 + 5 + 10 = 15$  variables. Moreover, since we also assumed that each train traverses all sections unhindered, we know that the sectional running time of each section is exactly equal to 5. Therefore, also the variables corresponding to the time later than the finishing time corresponding to the section are removed. What is left is a total of  $5 + 5 + 5 = 15$  variables.

## 5.6 Algorithm for Solving Theoretical Model

After applying all the techniques, we are still dealing with a very large and more importantly, unsolvable IP Problem, for which after testing for several hours, still no solution for the LP-relaxation can be found. Therefore, we conclude that the model is simply too complex to solve at once and thus another solution method has to be developed. For performing capacity tests on smaller networks, or single lines, this model might, however, still be useful.

By far, the majority of the constraints in the original formulation are imposed on only one train per individual constraint. When we look at the formulation, we see that only the constraints on the number of trains occupying a section (Constraints (5.9)) and the minimum headway time (Constraints (5.7)) are imposed on multiple different trains within the same constraint<sup>1</sup>. This sparks the idea of generating routes independently of each other and later solving the whole problem,

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<sup>1</sup>Constraints (5.10) do not necessarily have to be included and can be omitted

with only a binary variable per route.

For every possible line and every possible starting time, we create a sequence of the corresponding line-sections with the section occupation times. To avoid confusion, in the remainder of this thesis, we will refer to these as *paths*. Each path has a certain starting time. The following method can be seen as a two-phase solution approach, in which in *Phase One*, the sub-problems are solved and all possible paths are generated. Then, in *Phase Two* the optimal selection of paths is picked, subject to different sets of constraints.

**Phase One** In the first phase, we generate all possible paths independently of each other, which are all stored. To accomplish this, the original model can be rewritten. All constraints in which multiple trains are involved can be left out.

The formulation for the sub-problem becomes:

$$\max \quad 0 \tag{5.13}$$

*s.t.*

$$x_{i,1}^{b(i)} = 1, \quad \forall i \in \tau \tag{5.14}$$

$$x_{it}^s \leq x_{i,t+1}^s + x_{i,t+1}^{s'}, \quad \forall i \in \tau, \forall t, \forall s, s' \in S(i) : R_i^{s'} = R_i^s + 1 \tag{5.15}$$

$$x_{it}^{s'} \leq x_{i,t-1}^{s'} + x_{i,t-1}^s, \quad \forall i \in \tau, \forall t, \forall s, s' \in S(i) : R_i^{s'} = R_i^s + 1 \tag{5.16}$$

$$x_{it}^s + x_{i,t+1}^{s'} - 1 \leq x_{i,t-SRT^s+1}^s, \quad \forall i \in \tau, \forall t, \forall s, s' \in S(i) : R_i^{s'} = R_i^s + 1 \tag{5.17}$$

$$x_{it}^s + x_{i,t+SRT^s+W}^s \leq 1, \quad \forall i \in \tau, \forall t, \forall s \in S(i) \tag{5.18}$$

$$\sum_{\forall s \in S(i)} x_{it}^s \leq 1, \quad \forall i \in \tau, \forall t \tag{5.19}$$

$$x_{it}^s \in \{0, 1\}, \quad \forall t, \forall s \in S(i), \forall i \in \tau \tag{5.20}$$

In this model there is no real objective function. Since we want all feasible paths, we do not need to maximize or minimize an objective function and finding just a feasible solution suffices. The majority of constraints is the same as in the previous model. In order to obtain only unhindered paths, we again set  $W = 0$ . Note that in the case of  $W = 0$ , the paths can also be generated by an iterative procedure. However, since we have already implemented this formulation, we

decide to utilize this. Constraint (5.14) is added to ensure that exactly one train of each line starts its route at time  $t = 1$ . Hence, for all possible routes we create a path starting at  $t = 1$ . Then, by using an iterative procedure we create all possible paths starting at  $t \in \{2, \dots, T\}$ . In case Phase Two becomes too complex to solve, there is always an opportunity to decrease the precision and only generate paths every, say, 10 seconds. This way, we start their trip only at multiples of 10 seconds, while all constraints are still being obeyed every second. The solutions (paths) are stored and can be seen as input parameters for Phase Two. This finishes Phase One.

**Phase Two** In the second phase, all generated paths together form a set, which we from now on call  $\mathcal{P}(i)$ , with route  $i$  corresponding to the route matched to train  $i \in \tau$ . Every  $p \in \mathcal{P}(i)$  can be seen as a set of parameters, which is a large list of binaries equal to 1 if in path  $p \in \mathcal{P}(i)$  section  $s$  is occupied at time  $t$  and zero otherwise, which we from now on will call  $Q_t^{ps}$ . Furthermore, we introduce a new variable for every path generated in phase one:

$$d_p^i = \begin{cases} 1, & \text{if path } p \in \mathcal{P}(i) \text{ of route } i \in \tau \text{ is used} \\ 0, & \text{otherwise} \end{cases}$$

Next, we must ensure that restrictions on number of trains per section and the minimum headway time are still respected. For restricting the number of trains per section to a maximum of one, we add the following constraints:

$$\sum_{i \in \tau} \sum_{p \in \mathcal{P}} Q_t^{ps} d_p^i \leq 1, \quad \forall t \in \{1, \dots, T\}, \forall s \in S \quad (5.21)$$

The restriction on minimum headway between two trains can be modeled as follows:

$$Q_t^{ps} d_p^i + \sum_{i' \in \tau} \sum_{p' \in \mathcal{P}(i): p' \neq p} Q_{t'}^{p's} d_{p'}^{i'} \leq 1, \quad i \in \tau, \forall p \in \mathcal{P}(i), \forall s \in S(p),$$

$$\forall t \in \{1, \dots, T\}, \forall t' \in \{t \dots t + HW^s\} \quad (5.22)$$

Both sets of constraints are added to the model and form the basic master problem. More sets of constraints are added to the model later on, in order to get a more realistic view on capacity. These are described in the following paragraphs.

The basic master problem becomes:

$$\begin{aligned}
 \max \quad & C_{network} = \sum_{i \in \tau} \sum_{p \in \mathcal{P}(i)} d_p^i \\
 \text{s.t.} \quad & \\
 & \sum_{i \in \tau} \sum_{p \in \mathcal{P}} Q_t^{ps} d_p^i \leq 1, & \forall t \in \{1, \dots, T\}, \forall s \in S \\
 Q_t^{ps} d_p^i + \sum_{i \in \tau} \sum_{p' \in \mathcal{P}(i): p' \neq p} Q_{t'}^{p's} d_{p'}^{i'} \leq 1, & i \in \tau, \forall p \in \mathcal{P}(i), \forall s \in S(p), \\
 & \forall t \in \{1, \dots, T\}, \forall t' \in \{t, \dots, t + HW^s\} \\
 & d_p^i \in \{0, 1\}, & \forall i \in \tau, \forall p \in \mathcal{P}(i)
 \end{aligned}$$

Contrary to the original formulation where a variable was created for each train at each section at every possible time unit, in the new formulation there is only a variable for each path. Therefore, the number of variables is reduced dramatically in comparison with the original formulation. On the other hand, the number of constraints still runs in the millions.

Note that in this formulation, every train completes its entire route, which contradicts the definition for capacity we have agreed upon. In order to get to the correct number, in Phase One we create all paths that can *start* within the time window. However, for paths that end outside the time window, the constraints outside the time window must still be obeyed. For Phase Two we therefore create a larger time window, large enough such that the latest path generated in Phase One can finish.

To illustrate this, suppose we want to know the capacity for the time window of one hour. We then generate all paths starting no later than one hour in Phase One. Trains can still be running outside this time window and therefore conflicts may occur during this period. For solving Phase Two, we therefore create a time window of, say, 2 hours, large enough such that all paths generated in Phase One

are finished.

In order to reduce the computation time of the master problem, we try rewriting Constraints (5.22). To do so, we note that for every time interval with size exactly equal to the minimum headway time, it must hold that only one train is allowed to be present on a certain section. Per section and per time interval with the size equal to the minimum headway time, we create a set which contains all the variables corresponding to the paths, which traverse this section within this time interval. Each of these sets can then be modeled as a so-called *Special Ordered Sets of type One (SOS1)*, which is defined as a set of variables, of which at most one variable can take a strictly positive value, all others being zero. Hence, for every SOS1 corresponding to a certain section and a certain time window, only one path can be chosen (only one variable can be set to one). This way, both Constraints (5.21) and (5.22) are captured simultaneously.

Mathematically, this can be written as follows. We define the sets for all sections and all time intervals:

$$W(s, t) = \{d_p^i | \exists(i, p), s, t' : t' \in \{t, \dots, t + HW^s\} : Q_{t'}^{ps} = 1\} \quad (5.23)$$

In words, all variables corresponding to paths that may cause conflicts<sup>2</sup> on a certain section are collected and put in a single set. This way, we can remove both sets of constraints and include the following constraints:

$$\sum_{(i,p) \in W(s,t)} d_p^i \leq 1, \quad \forall s \in S, \forall t \in \{1, \dots, T\} \quad (5.24)$$

The reduction in computation time on a smaller instance of the problem, using this SOS1-formulation is shown in Table 5.2 on the next page. Clearly, the solution values of both the original formulation and the *SOS1-formulation* are the same. The test was performed on an instance with 10 routes, 12 sections and a total time window of 500 seconds. As one can see, the number of constraints and the running time has reduced dramatically using this new formulation.

---

<sup>2</sup>Conflicts can be multiple trains on a certain section, or multiple trains within the minimum headway boundaries

Method:	Comp. Time [sec.]:	NO. Constraints	Sol. Value:
Master-formulation	2,027.48	2,824,216	15
SOS1-sets	69.32	4,509	15

Table 5.2: *Computation Times comparing two Formulations for Minimum Heady Time*

### 5.6.1 Modeling Turning Movements

On most lines, trains have, upon the completion of their line, multiple possibilities to end their trip and make a turning movement in opposite direction. In the figure below, station and endpoint *Nesselande* (NSL) is depicted. Trains coming from *De Tochten* (the left side of the figure) can halt and turn either at section 750T (coming in via 1610T - 1612T - 748T) or at 749T (coming in via 1610T - 1612T - 1611T - 747T). To model the different opportunities, we create a separate route for every option and add this to the list of routes. For instance, for line B from *Schiedam Centrum* to *Nesselande*, there are two options for starting the route and two for ending the route, which results in four different routes. In total, counting all lines in both directions, there are 36 possible routes.

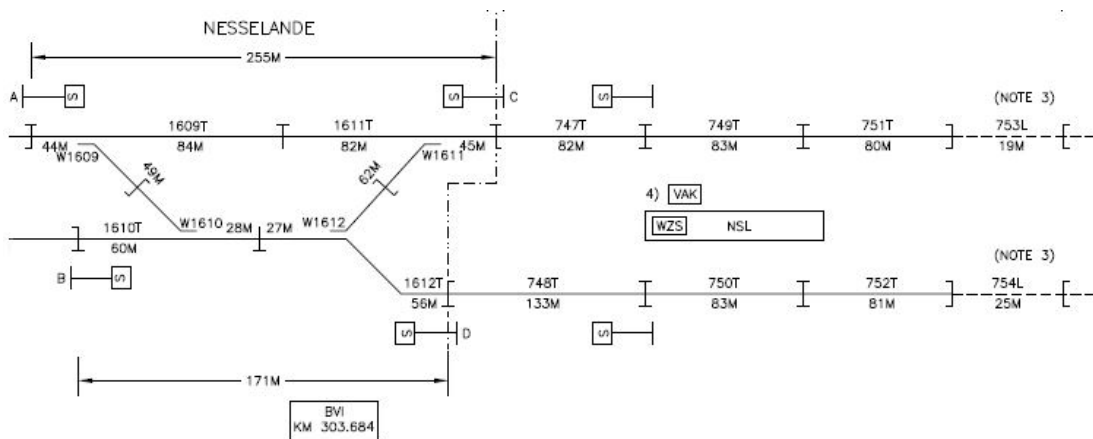


Figure 5.3: *Schematic Representation of Station and Endpoint Nesselande*



### 5.6.2 Modeling Intermediate Points

As reported in Paragraph 4.1, the network consists of multiple Intermediate Points, which all have to be modeled in order to get a realistic view of the capacity. We thus have to add constraints to ensure that no head-to-head collisions will occur and all conflicts on the different sections of Intermediate Points are prevented. To do so, we create *pairs* of sections that are part of an Intermediate Point  $s_1$  and  $s_2$ , on which mutual conflicts may occur. If one of those sections is occupied, then the other section in the pair may not be occupied by another train for a time equal to the headway time corresponding to the Intermediate Point,  $HW^{IP}$ . We model this in a similar way to the set notation we used in Equation (5.23). When section  $s_1$  and section  $s_2$  are potential conflicting sections in an Intermediate Point and therefore form a pair, we get:

$$W(s_1, t) = \{d_p^i | \exists(i, p), s \in \{s_1, s_2\}, t' : t' \in \{t, \dots, t + HW^{IP}\} : Q_{t'}^{ps} = 1\}$$

$$W(s_2, t) = \{d_p^i | \exists(i, p), s \in \{s_1, s_2\}, t' : t' \in \{t, \dots, t + HW^{IP}\} : Q_{t'}^{ps} = 1\}$$

Then we make use of the same type of constraints as Constraints (5.24), where  $I$  is the set of pairs of Intermediate Point sections:

$$\sum_{(i,p) \in W(s_1,t)} d_p^i + \sum_{(i,p) \in W(s_2,t)} d_p^i \leq 1, \quad \forall s_1, s_2 \in I, \forall t \in \{1, \dots, T\} \quad (5.25)$$

This way, conflicts are avoided for each pair of sections that are part of Intermediate Points.

### 5.6.3 Modeling Line Frequencies

When we solve the aforementioned model with the actual data, we see that the solver has a clear preference of choosing trains with relatively short lines over

trains matched to a long line. In the optimal solution, trains that traverse line E outnumber the other lines. Clearly, in a subway network, all lines need to be driven with a certain frequency and every station needs to be served periodically. We therefore need a set of constraints which ensure that all routes are traversed roughly the same number of times. Furthermore, per line the number of trains in the direction of west and the direction of east must be balanced. We include constraints which make sure that at every time unit, the total number of trains that has traversed each line in each direction, may not differ more than one from the other lines. For instance, if at time  $t = 0$  a train starts line A from *Binnenhof* in direction of *Schiedam Centrum*, then no other trains on the same line A from *Binnenhof* to *Schiedam Centrum* are allowed to start their route, until all other routes have been started by a train. This can be modeled as follows:

$$\frac{1}{f_i} \sum_{t'=1}^t \sum_{p \in \mathcal{P}(i)} d_p^i - \frac{1}{f_{i'}} \sum_{t'=1}^t \sum_{p' \in \mathcal{P}(i')} d_{p'}^{i'} \leq 1, \quad \forall t \in \{1, \dots, T\}, \forall i, i' \in \tau : i \neq i'$$

In these constraints  $f_i$  is the integer-valued frequency of line  $i$ . Using these constraints, we can also ensure that other preferred line frequencies are being obeyed. For instance, a usual service-ratio between line D and line E on the stations between *Slinge* and *Rotterdam Centraal* Station is 2:1. That is, in one hour, twice as many trains of line D traverse these sections in comparison with those traversing line E. In order to do so, we set the  $f_i$  corresponding to line D equal to 2 and the parameters  $f_i$  corresponding to the other lines equal to 1. If we want all lines in both directions to be driven with the same frequencies, we set  $f_i = 1, \forall i \in \tau$ .

#### 5.6.4 Modeling Safe Haven

As stated in Paragraph 2.1, between stations *Blijdorp* and *Rotterdam Centraal* Safe Haven restrictions hold. We model this in the same way as we did with the minimum headway restrictions. The minimum headway time for all sections between *Blijdorp* and *Rotterdam Centraal* is set to the total time it takes for a train to traverse all sections between these stations, including dwell times.

### 5.6.5 Modeling Endpoint Restrictions

In the current formulation, trains - or in fact, paths corresponding to a variable - disappear from the network upon the completion of their line. In reality, the endpoints of each line may become occupied, in which case no new trains are able to enter the endpoint until another train has left the endpoint for the beginning of a new trip. Clearly, this is important to model in order to get a more realistic view of the capacity. One way of including this in the model is to make use of balance equations per endpoint. For example, one could think of the following set of constraints. For each time unit  $X$ , the total number of outgoing trains minus the number of incoming trains up to  $X$  must be smaller than or equal to one and vice-versa. The problem with these formulations is, however, that at the beginning of the time window, there are no incoming trains at the endpoints and thus the number of outgoing trains is limited by these restrictions. Therefore, we make a new assumption: a train may always depart from the endpoint (if all other restriction, such as minimum headway and line-frequencies are being respected), independent of the trains ending their trips. On the other hand, trains may enter the endpoint, only if the endpoint is free. An endpoint is set to free when a train departs from the endpoint and is occupied again when a new train arrives. We model this by making use of the following sets per endpoint  $e$  and per time interval  $[t_1, t_2]$ :

$$\Phi_{i,e}^{[t_1,t_2]} = \{d_p^i \mid \text{with path corr. to } d_p^i \text{ starting in interval } [t_1, t_2] \text{ from endpoint } e\}$$

$$\Theta_{i,e}^{[t_1,t_2]} = \{d_p^i \mid \text{with path corr. to } d_p^i \text{ ending in interval } [t_1, t_2] \text{ at endpoint } e\}$$

Accordingly, we ensure that for all endpoints and time intervals the number of incoming trains may never exceed the number of outgoing trains by more than one. By doing so, a train can never enter an occupied endpoint until the endpoint is set to free again by a departing train. We include the following constraints, where  $E$  is the set of endpoints:

$$\sum_{i,p \in \Theta_{i,ep}^{[t_1,t_2]}} d_p^i - \sum_{i,p \in \Phi_{i,ep}^{[t_1,t_2]}} d_p^i \leq 1, \quad \forall t_1, t_2 \in \{1, \dots, T\} : t_1 < t_2, \forall e \in E \quad (5.26)$$

Note that the number of different time intervals is quadratic in  $T$ . More precisely, the number of intervals is equal to  $T(T - 1)/2$ . To reduce the number of restrictions, we only include time intervals of a certain length. This reduces the number of time intervals to a linear number. In this research, we fix the length of each time interval to an arbitrarily chosen length of 500 seconds.

These are the final constraints added to the formulation. The complete formulation together with the corresponding sets are presented in Appendix A. The interface of the developed software-tool can be found in Appendix B.

## 5.7 Heuristic Approach

Apart from the method to solve the stated problem to optimality, we also present a less time-consuming and more general applicable model, which does not require licenses for a solver. We therefore present a heuristic method. Recall that paths, initially generated in Phase One with the use of a solver, can also be generated iteratively and therefore no solver is required in this phase.

The pseudo-code of the heuristic is presented in Algorithm 1. In the heuristic, *Occupation* is a matrix in which the occupation of each section at every time unit is saved. The *Occupation*-matrix thus contains information on when sections are free and when they are occupied. This also includes headway times and Intermediate Point occupations. The function *IsFeasible*( $d_\tau^t$ ) checks if, given the *Occupation*-matrix and the path corresponding to the variable  $d_\tau^t$ ,  $d_\tau^t$  is feasible. That is, no conflicts with other assigned trains may occur.

*ForbiddenLines* temporarily contains different lines with directions (East/West). If a path is chosen, its line is added to *ForbiddenLines*. If a line corresponding to a path is present on the *ForbiddenLines*, the path may not be chosen. The list *ForbiddenLines* is cleared entirely when all lines (with directions) are on the list.

Suppose for instance that a train matched to line A into the direction of *Schiedam Centrum* starts at  $t = 1$ . Then line A (West) is added to the list *ForbiddenLines*. This line with direction becomes temporarily unavailable until all other lines have been started and the list is cleared. This guarantees the required line-frequencies.

The heuristic procedure works as follows. The first iteration starts. For each time unit, starting at  $t = 1$  we check for all lines if there is a feasible path. The path is feasible if the corresponding line is not on the *ForbiddenLines* (line 12) list and the path does not cause conflicts with already assigned paths (line 13). If there is a feasible path, the path is added to the *Occupation*-matrix (line 14) and the corresponding the train is added to *DeployedTrains* (line 15). Furthermore, the line corresponding to this path is added to *ForbiddenLines* (line 17). This is cleared entirely once all lines have been started (line 18 & 19).

For each iteration, the lines are shuffled in random order (line 9). Each order may result in different outcomes. Therefore, for each iteration we keep track of the obtained capacity. If the solution of the current iteration is higher than the incumbent solution, the new solution is stored (lines 25, 26 and 27). After all iterations are carried out, the highest obtained capacity is returned.

Unfortunately, we were unable to model the desired endpoint restrictions. This is caused by the fact that the paths are added chronologically and the path's endpoint-feasibility is dependent on trains leaving and entering the endpoint on later moments. To illustrate this, suppose that we check the feasibility of a path starting at  $t = 1$  ending at section  $X$  at time  $t = 100$ . Then the occupation of the endpoint is dependent on trains coming in and leaving before  $t = 100$ . Since this is not known at  $t = 1$ , we cannot check for endpoint feasibility of the path.

Note that the heuristic is a construction heuristic, which builds a solution chronologically and no improvement steps are carried out. The reason for this decision are the required line-frequencies. For instance, when simple local search improvement heuristics such as 2-opt are implemented, the desired line-frequencies are no longer obeyed, making the solution infeasible.

The performance of the heuristic is tested by comparing the results with the optimal solutions in the next chapter.

**Algorithm 1** Capacity Heuristic

---

```

1: procedure SELECTPATHS(TrainList, Timewindow, Lines, maxIterations)
2:   BestSolution =  $\emptyset$ 
3:   HighestCapacity  $\leftarrow$  0
4:   for Iteration  $\leftarrow$  1 to maxIterations do
5:     DeployedTrains  $\leftarrow$   $\emptyset$ 
6:     NumberOfTrains  $\leftarrow$  0
7:     ForbiddenLines  $\leftarrow$   $\emptyset$ 
8:     Occupation  $\leftarrow$   $\emptyset$ 
9:     Shuffle(Lines)
10:    for t  $\leftarrow$  1 to T do
11:      for each Train  $\tau$  in TrainList do
12:        if  $\tau \notin$  ForbiddenLines then
13:          if isFeasible( $d_\tau^t$ ) = true then
14:            Occupation  $\leftarrow$  Occupation  $\cup$  Path $_\tau^t$ 
15:            DeployedTrains.add( $\tau$ )
16:            NumberOfTrains  $\leftarrow$  NumberOfTrains + 1
17:            ForbiddenLines.append( $\tau$ )
18:            if ForbiddenLines.containsAll(Lines) then
19:              ForbiddenLines  $\leftarrow$   $\emptyset$ 
20:            end if
21:          end if
22:        end if
23:      end for
24:    end for
25:    if NumberOfTrains > HighestCapacity then
26:      HighestCapacity  $\leftarrow$  NumberOfTrains
27:      BestSolution  $\leftarrow$  DeployedTrains
28:    end if
29:  end for
30:  return BestSolution, HighestCapacity
31: end procedure

```

---

# Chapter 6

## Results

In the following paragraphs, the results of the capacity-tests are presented. First of all, we make use of the developed tool to calculate capacity of single lines and the entire network, by setting the parameters as desired by the RET. Furthermore, the impact of the different settings for minimum headway times and other restrictions are examined. Finally, we apply the heuristic and compare the results with the optimal solutions obtained by the CPLEX solver.

### 6.1 Single Line Capacity

In this paragraph, the capacity per hour per single line is presented. By doing so, we identify on which of the lines potential bottlenecks may be. In these calculations, only trains matched to one particular line in both directions are deployed on the network. That is, all other lines are not being used and therefore no conflicts with trains of other lines can occur.

For the single lines, we compare three different configurations for the minimum headway time. First, we use the headway time that is different for each section (see Paragraph 5.4), which is the minimum time needed for each train to come to a safe stop, assuming the deceleration-constant to be equal to  $0.50\text{m/s}^2$ . In the second configuration we use a fixed minimum headway time of 90 seconds. In the third configuration we use a fixed headway time equal to 0 seconds. In this case, there is no required minimum headway. However, the restrictions that only one

train is allowed to be one a section at a time, still holds. This can be seen as an absolute upper bound for the line-frequencies.

We set the length of all trains to 85 meters, which is roughly the length of two coupled train-units of the type *Bombardier SG3*. In reality, trains of type SG3 cannot traverse all lines, due to the absence of a pantograph on this type of train. However, for convenience, we relaxed this restriction (See Paragraph 5.2). The dwell and turning times are set to 25 and 120 seconds, respectively. Headway time on the Intermediate Points is set to be the same as the regular headway time. That is, if the headway configuration is equal to 90 seconds, we also set the headway on the Intermediate Points equal to 90 seconds. Furthermore, the endpoint restrictions are included, which were described in Paragraph 5.6.5. The number of trains of each individual line that can be deployed on the network is presented in Table 6.1 below.

Line / Direction	Headway Configuration		
	Sectional Headway	90 Seconds	0 Seconds
A Schiedam - Binnenhof	33	21	40
A Binnenhof - Schiedam	32	20	39
B Schiedam C - Nesselande	33	21	40
B Nesselande - Schiedam C	32	20	39
C De Akkers - De Terp	35	22	42
C De Terp - De Akkers	36	22	42
D De Akkers - Rotterdam C	30	19	35
D Rotterdam C - De Akkers	30	20	34
E Blijdorp - Slinge	27	19	31
E Slinge - Blijdorp	26	19	32

Table 6.1: *Capacity of the Single Lines when Individual Lines are used and Endpoint Restrictions are included*

All computations are performed using a computer with an Intel Core i5 processor with 4 cores, clocked at 2.20GHz. The computation times are highly dependent on the configurations. For some lines and configurations, the results are obtained



within a few minutes, others lasting a few hours. When the endpoint constraints are excluded from the model, results were obtained significantly faster, i.e., the capacity of each line could be computed under a minute.

Given the number of trains that can be deployed on the network, we can easily derive the maximum realizable driving frequencies for each line. For instance, driving a line 30 times an hour, boils down to driving a 2 minute frequency. Every station on the corresponding line is thereby served every 2 minutes. As expected, we see that minimum headway time has a significant effect on the capacity. Furthermore, we see that the maximum capacities of line D and E are somewhat lower than for the other lines. A possible cause for this observation is that on these tracks there is a section which is exceptionally long, resulting in a longer section occupation time and therefore also a longer headway time. Another explanation might be the presence of the Safe Haven restrictions on these lines.

## 6.2 Network Capacity

In this paragraph we examine the theoretical capacity of the whole network — the main research question of this thesis. The capacity is determined under different parameter settings.

Unfortunately, if all aforementioned restrictions are added to the model and all possible paths are generated, we are unable to obtain near-optimal results for the whole network, even after multiple days of computing. Therefore, we have to make some assumptions to decrease the complexity of the problem. First, for each route, we generate paths only every multiple of a minute. That is, for a time window of one hour, we generate 60 paths per route. Furthermore, per line and direction we assume that there is only one endpoint. For lines that have an endpoint station in common (*Schiedam Centrum* for lines A and B, *De Akkers* for lines C and D), we assume that both lines have a different endpoint section. For instance, we assume that line D always turns at *De Akkers* at section 303 and line C always turns at *De Akkers* at section 308. As a result of these measures, the number of variables has decreased to 600, while we can still provide a good indication for capacity.

First, we set the parameters as we did for determining the single lines, that is:

dwel time 25 seconds, turn around times 120 seconds and we use the three different headway configurations. In this case we make use of endpoints restrictions. All lines are being used and the line-frequencies are all set to be equal to one another.

Line / Direction	Headway Configuration		
	Sectional Headway	90 Seconds	0 Seconds
A Schiedam - Binnenhof	10	8	13
A Binnenhof - Schiedam	10	8	13
B Schiedam C - Nesselande	11	9	13
B Nesselande - Schiedam C	11	9	13
C De Akkers - De Terp	11	9	14
C De Terp - De Akkers	11	9	13
D De Akkers - Rotterdam C	11	8	13
D Rotterdam C - De Akkers	11	9	13
E Blijdorp - Slinge	11	9	14
E Slinge - Blijdorp	11	9	14
<b>Total Capacity</b>	<b>108</b>	<b>87</b>	<b>133</b>

Table 6.2: *Number of Trains able to traverse the Network when all Line-Frequencies are equal and Endpoint Restrictions are used*

On average, 3 hours are needed to find the optimal solution for each setting. In the current timetable of the RET, on all lines, trains are driving with an hourly frequency of six trains, resulting in a total of 60 trains per hour. An exception are the rush hours, on which the frequency of the D line is doubled, yielding a total of 72 trains per hour. As we see from the results, when using sectional headway, the capacity of the network is significantly higher than the current utilized capacity (50%), still taking the constituted assumptions into account. Furthermore, when looking at the 90 seconds headway configuration, which is used in the software to create timetables, and comparing those results to the utilized capacity in the rush hours, we see that there is still considerable room for an increase in line-frequencies.

## 6.3 Effect of Different Network-Configurations

In the following paragraphs, we attempt to find the bottleneck of each line by adjusting relevant parameters. First, the effect of turning time is analyzed by gradually lowering it to zero seconds. Second, we analyze to which extent the safety restrictions at Intermediate Point at *Graskruid* affect capacity.

### 6.3.1 Turning Time

We suspect that the capacity of the network is largely restricted by the endpoints. In particular, the relatively busy endpoints sharing multiple lines (*De Akkers*, *Schiedam Centrum*) are likely to be limiting factors for the capacity of the whole network. In order to test to which extent this is true, we determine capacity in multiple runs, with in each run a different value for the duration of the turning time.

Line / Direction	Turning Time			
	120 Sec	90 Sec	60 Sec	0 Sec
A Schiedam - Binnenhof	10	11	11	12
A Binnenhof - Schiedam	10	11	12	12
B Schiedam Centrum - Nesselande	11	11	11	11
B Nesselande - Schiedam Centrum	11	12	12	12
C De Akkers - De Terp	11	12	12	11
C De Terp - De Akkers	11	11	12	12
D De Akkers - Rotterdam Centraal	11	11	12	12
D Rotterdam Centraal - De Akkers	11	12	13	13
E Blijdorp - Slinge	11	12	13	13
E Slinge - Blijdorp	11	12	12	12
<b>Total Capacity</b>	<b>108</b>	<b>115</b>	<b>120</b>	<b>120</b>

Table 6.3: *Network Capacity with Different Configurations for Turning Time*

For our tests, we make use of sectional-dependent minimum headway and four

configurations for turning time: 120, 90, 60 and 0s, respectively. The configuration with 0 seconds turning time can be seen as driving automatically, i.e., without a driver. Since we made the assumption that dwelling and turning is done in parallel, the total stopping time on endpoints adjoining a platform in this case will be equal to the dwell time and thus be a total of 25 seconds. The results are presented in Table 6.3.

As expected, lowering turning time has a positive effect on capacity. We see a significant increase in capacity when turning times are lowered from 120 seconds to 90 and 60 seconds. However, when the turning time is further decreased from 60 to 0 seconds, there is no gain in capacity anymore. Hence, in this case, the capacity of the network is restricted by other factors. A likely explanation is that there are certain sections with a relatively long sectional running time, restricting the number of trains on these lines.

### 6.3.2 Intermediate Point Graskruid

As mentioned before, at Intermediate Point *Graskruid* line A into the direction of *Binnenhof* and line B into the direction of *Schiedam Centrum* cross on the same level. Currently, a minimum headway time of 90 seconds holds for trains crossing the Intermediate Point. To see to which extent the Intermediate Point restriction affects the capacity of the network, we run our model in two different settings. In both settings only trains of line A and line B are deployed. In the first setting, we set the minimum headway on the Intermediate Point equal to 90 seconds, whereas in the second setting we exclude the Intermediate Point restrictions from the model. In this case it is possible that collisions occur. However, this mimics the situation of a crossing of both lines on a different level (bridge or overpass).

Line / Direction	Intermediate Point Configuration	
	90 Seconds Headway	No Restrictions
A Schiedam - Binnenhof	18	19
A Binnenhof - Schiedam	18	20
B Schiedam C - Nesselande	18	20
B Nesselande - Schiedam C	18	19

Table 6.4: *Capacity of Line A and Line B combined, with Two Settings for Intermediate Point Graskruid*

We see that excluding the Intermediate Point restrictions from the model results in a capacity gain of roughly 10% on lines A and B. Therefore, it can be concluded that the Intermediate Point at *Graskruid* significantly affects the capacity of the A and B line. Lowering minimum headway time or changing the infrastructure at this location may thus further increase the frequencies on these lines.

## 6.4 Heuristic Performance

In this paragraph the analysis of the performance of the heuristic presented in this thesis is addressed. The solutions of the heuristic are compared with the optimal solution obtained by the solver. Since we were unable to model the endpoints properly, we compare the results with the optimal results without the inclusion of the endpoint restrictions. Recall that in the network-results with the use of a solver, only paths starting every minute were included. Due to the fact that the heuristic is less time-consuming, paths starting every second can be used. Therefore, it might be the case that the heuristic yields better results than the optimal solution found by the solver.

The total capacity of each configuration are compared in Table 6.5 below. The gap is calculated as:  $\frac{|z^h - z^s|}{z^s} \cdot 100\%$ , where  $z^h$  and  $z^s$  are the optimal values retrieved from the heuristic and solver, respectively. In order to get a good balance between performance and computation time, the number of iterations is set to 100 for each

calculation. By doing so, the computation times stay below 5 minutes.

Configuration		Heuristic	Solver	Gap
Headway Configuration	Turning Time			
90 Seconds	120 Seconds	78	87	10.34%
90 Seconds	90 Seconds	78	89	12.36%
90 Seconds	60 Seconds	79	89	11.24%
90 Seconds	0 Seconds	83	89	6.74%
Sectional Dependent	120 Seconds	118	135	12.59%
Sectional Dependent	90 Seconds	124	137	9.49%
Sectional Dependent	60 Seconds	134	137	2.19%
Sectional Dependent	0 Seconds	138	138	0.00%

Table 6.5: *Performance Comparison of Heuristic and Solver Results, Endpoint Restrictions are excluded*

From the results, we see that the heuristic performs quite well in general. Especially when the turning time is relatively low, the heuristic tends to perform better, for which we have no real explanation.

# Chapter 7

## Conclusions & Recommendations

In the final chapter of this thesis, we first establish conclusions of the performed research and the limitations of the developed model are determined. Finally, we make recommendations for the RET that might possibly lead to an increase of the realized capacity.

### 7.1 Research Conclusions and Model Limitations

In this thesis we have successfully developed a model that is capable of determining the capacity of the metro network of Rotterdam. Furthermore, we have created a software-tool that can be used by the RET to make desired calculations. This tool can be used when certain parameters change in the future, such as dwell time, turning time or trainlength. Capacity restrictions for minimum headway time, line-frequencies, Intermediate Points, endpoints and Safe Haven are all included in the model.

The solution method used in this thesis consists of a two-step-approach, in which in Phase One all combinations of train routes and starting times are generated. These route-time combinations are referred to as 'paths'. In Phase Two the optimal selection of paths was picked, subject to the aforementioned constraints. By making necessary adjustments, this model can also be applied to other metro or train networks in which similar (safety-)restrictions are being used. One requirement for such a network to be suitable for the model is that every track is

separated in different sections.

As shown, we have already reduced the complexity of the original presented formulation. This is done by reformulating the original formulation to a two-step approach and by making necessary assumptions. Although in the calculation of capacity the computation times are not of great importance (calculations often have to be performed only a couple of times), it might be of interest to do more research on further complexity reduction of the model or on different formulations for the problem. By doing so, the model can be extended by including more paths and including paths that contain waiting time (delays) at certain sections, or it can be applied on larger, more complex networks.

Unfortunately, due to time limitations, certain restrictions could not be modeled. At the moment, the endpoint restrictions do not fully capture the real situation. Trains can always depart from the endpoint, even if in the model no train is present on the endpoint at some point in time. It might be possible to model this by adding balance equations. However, a larger time window is required to get a more realistic view of the capacity, as a result of start-up bias in output. Choosing a larger time window will further increase computation times and therefore, this is not included in this thesis.

Due to the electric infrastructure not all types of trains can drive on all lines in the network. For instance, the tracks *Capelse Brug*  $\leftrightarrow$  *Nesselande*, *Capelse Brug*  $\leftrightarrow$  *Binnenhof* and *Melanchtonweg*  $\leftrightarrow$  *Den Haag Centraal* are powered by overhead wires. Trains that traverse these sections need to be equipped with a pantograph as well as a third rail system, which is used on all the other tracks. The different types of trains all have their own lengths. Hence, another idea for further research is to make trains different per line, even though the effects on capacity may not be too large.

Another assumption we made which might affect the outcome capacity significantly, is that we neglect the part of the Line E from *Blijdorp* to *Den Haag Centraal*. As mentioned before, in 2017 line B is extended with the addition of the *Hoekse Lijn*, which also has to be modeled in order to perform accurate calculations.

Clearly, all assumptions have different effects on the estimate of the theoretical



capacity. The assumptions may either result in an under – or overestimation of the real theoretical capacity. All relevant assumptions together with the effect on the estimate are listed in the table below.

<b>Assumption:</b>	<b>Underestimation/Overestimation of Capacity:</b>
Only unhindered paths are generated	Underestimation
Tracks Blijdorp ↔ Den Haag Centraal are left out of consideration	Overestimation
Endpoint restrictions do not match real situation entirely	Overestimation
Paths are only generated every minute	Underestimation
Only one endpoint per line is used	Underestimation

As can be seen from the table, the assumptions affect estimated capacity in both directions, thus making it difficult to evaluate whether our model under – or overestimates the actual theoretical capacity.

While bearing the limitations of the model in mind, we have concluded that there is still significant room for an increase in line-frequencies, even when the currently used minimum headway time of 90 seconds is being used.

## 7.2 Recommendations for the RET

With the use of our model, an indication for the capacity of the metro network of Rotterdam was given. By changing parameters of interest, the effect of each network setting could be determined. Clearly, as we have already seen in the previous paragraph, some network restrictions that hold in reality could not be modeled, due to time constraints, or are simply too complex to model.

Despite the limitations of our model, we can make some recommendations that may lead to an increase in realized capacity. Obviously, network capacity can be increased by lowering the minimum headway time. However, since lowering minimum headway time may also involve an increase in the probability of collisions to occur, it is difficult to make recommendations concerning this matter. On the

other hand, we have seen that if it is possible to reduce minimum headway time, this will result in a higher realizable capacity.

As mentioned before, the turning time has a significant effect on the capacity of the network. If the turning time at the endpoints can be lowered in some manner, it may be possible to increase the frequencies on each line. Currently, turning at the endpoints is assumed to take 120 seconds. A possible way of lowering turning time is to drive automatically. However, driving automatically requires both the infrastructure and trains to be adapted and therefore it is highly unlikely that this will happen in the near future. A more convenient way to lower turning time is to modify the turning procedure. Currently, it is often the case that upon completion of a line, a driver starts its new trip with the same train in opposite direction. At the endpoint of the line the driver therefore has to walk to the other side of the train, which takes a significant period of time (120 seconds, as assumed by the RET). This time can be lowered by the following procedure. If a train arrives at one of the endpoints, another driver is already waiting at the endpoint. The new driver then enters the opposite side of the train, while the former driver exits the train. The former driver then waits at the endpoint for a new arriving train to start a new trip. Hence, a driver never performs two sequential trips on the same train. This way, the walking time is omitted and the turning procedure can be performed significantly faster. It is outside the scope of this thesis to evaluate whether this is achievable in terms of a possible increase in costs and personnel. As we have concluded from the results, if the turning time can be lowered, the line-frequencies can then be further increased by a considerable amount.

# Bibliography

- M. Abril, F. Barber, L. Ingolotti, M.A. Salido, P. Tormos, and A. Lova. An assessment of railway capacity. *Transportation Research Part E: Logistics and Transportation Review*, 44(5):774 – 806, 2008. ISSN 1366-5545. doi: <http://dx.doi.org/10.1016/j.tre.2007.04.001>. URL <http://www.sciencedirect.com/science/article/pii/S1366554507000579>.
- R. Both. Disruption analysis for the rotterdam metro network. Master’s thesis, Technische Universiteit Eindhoven, 2015.
- Deutsche Bundesbahn. Richtlinien für die ermittlung der leistungsfähigkeit von fahrstraßenknoten, 405, 1979.
- R.L. Burdett and E. Kozan. Techniques for absolute capacity determination in railways. *Transportation Research Part B: Methodological*, 40(8):616 – 632, 2006. ISSN 0191-2615. doi: <http://dx.doi.org/10.1016/j.trb.2005.09.004>. URL <http://www.sciencedirect.com/science/article/pii/S0191261505001037>.
- GR Corazza and A Musso. La circolazione e gli impianti ferroviari. la verifica a lungo termine. *Ingegneria ferroviaria, ottobre*, pages 607–618, 1991.
- A.F. De Kort, B. Heidergott, and H. Ayhan. A probabilistic (max, +) approach for determining railway infrastructure capacity. *European Journal of Operational Research*, 148(3):644 – 661, 2003. ISSN 0377-2217. doi: [http://dx.doi.org/10.1016/S0377-2217\(02\)00467-8](http://dx.doi.org/10.1016/S0377-2217(02)00467-8). URL <http://www.sciencedirect.com/science/article/pii/S0377221702004678>.
- Steven Harrod. Capacity factors of a mixed speed railway network. *Transportation Research Part E: Logistics and Transportation Review*, 45(5):830 – 841,

2009. ISSN 1366-5545. doi: <http://dx.doi.org/10.1016/j.tre.2009.03.004>. URL <http://www.sciencedirect.com/science/article/pii/S1366554509000349>.
- Tijs Huisman, Richard J. Boucherie, and Nico M. van Dijk. A solvable queueing network model for railway networks and its validation and applications for the netherlands. *European Journal of Operational Research*, 142(1):30 – 51, 2002. ISSN 0377-2217. doi: [http://dx.doi.org/10.1016/S0377-2217\(01\)00269-7](http://dx.doi.org/10.1016/S0377-2217(01)00269-7). URL <http://www.sciencedirect.com/science/article/pii/S0377221701002697>.
- A. Landex. *Methods to estimate railway capacity and passenger delays*. PhD thesis, 12 2008.
- Y. Lee and C.H. Chen. Modeling and solving the train pathing problem. *Journal of Systemics , Cybernetics & Informatics*, 7(2):63 – 68, 2009.
- Gabriele Malavasi, Tatiana Molková, Stefano Ricci, and Francesco Rotoli. *Journal of rail transport planning & management*. 2014.
- G. Potthoff. *Verkehrsstnomungslehre 1*. transpress veb verlag für verkehrswesen. 1963 - 1972.
- Gary M Thompson. Improved implicit optimal modeling of the labor shift scheduling problem. *Management Science*, 41(4):595–607, 1995.
- UIC. *Capacity (UIC code 406)*. 2004.

# Appendices

# Appendix A

## Capacity Formulation

$$\max \quad C_{network} = \sum_{i \in \tau} \sum_{p \in \mathcal{P}(i)} d_p^i$$

s.t.

$$\sum_{(i,p) \in W(s,t)} d_p^i \leq 1, \quad \forall s \in S, \forall t \in \{1, \dots, T\}$$

$$\sum_{(i,p) \in W(s_1,t)} d_p^i + \sum_{(i,p) \in W(s_2,t)} d_p^i \leq 1, \quad \forall s_1, s_2 \in I, \forall t \in \{1, \dots, T\}$$

$$\frac{1}{f_i} \sum_{t'=1}^t \sum_{p \in \mathcal{P}(i)} d_p^i - \frac{1}{f_{i'}} \sum_{t'=1}^t \sum_{p \in \mathcal{P}(i')} d_p^{i'} \leq 1, \quad \forall t \in \{1, \dots, T\}, \forall i, i' \in \tau : i \neq i', \forall p \in \mathcal{P}(i)$$

$$\sum_{i,p \in \Theta_{i,ep}^{[t_1,t_2]}} d_p^i - \sum_{i,p \in \Phi_{i,ep}^{[t_1,t_2]}} d_p^i \leq 1, \quad \forall t_1, t_2 \in \{1, \dots, T\} : t_1 < t_2, \forall e \in E$$

$$d_p^i \in \{0, 1\}, \quad \forall i \in \tau, \forall p \in \mathcal{P}(i)$$

In this formulation, the following sets are defined:

$$W(s, t) = \{d_p^i | \exists (i, p), s, t' : t' \in \{t, \dots, t + HW^s\} : Q_{t'}^{ps} = 1\}$$

$$W(s_1, t) = \{d_p^i | \exists (i, p), s \in \{s_1, s_2\}, t' : t' \in \{t, \dots, t + HW^{IP}\} : Q_{t'}^{ps} = 1\}$$

$$W(s_2, t) = \{d_p^i | \exists (i, p), s \in \{s_1, s_2\}, t' : t' \in \{t, \dots, t + HW^{IP}\} : Q_{t'}^{ps} = 1\}$$

$$\Phi_{i,e}^{[t_1,t_2]} = \{d_p^i \text{ with path corr. to } d_p^i \text{ starting in interval } [t_1, t_2] \text{ from endpoint } e\}$$

$$\Theta_{i,e}^{[t_1,t_2]} = \{d_p^i \text{ with path corr. to } d_p^i \text{ ending in interval } [t_1, t_2] \text{ at endpoint } e\}$$

# Appendix B

## Software Tool

The screenshot shows a software window titled "Theoretical Capacity" with the following elements:

- 1**: Input field for "Tijdsvenster:" (Time window) set to 3600 Sec.
- 2**: Input field for "Treinlengte:" (Train length) set to 85 Meter.
- 3**: Radio button for "Oplossingsmethode:" (Solution method) set to "Optimaal".
- 4**: Input field for "Halteertijd:" (Stop time) set to 25 Sec.
- 5**: Input field for "Keertijd:" (Turnaround time) set to 120 Sec.
- 6**: Radio button for "Selecteer Type Opvolgtijd:" (Select type of headway) set to "Globale Opvolgtijd:".
- 7**: Input field for "Fractie Maximum Snelheid:" (Fraction of maximum speed) set to 0.69.
- 8**: Checkboxes for "Restricties:" (Restrictions) including "Intermediate Points" and "Lijn-Frequenties".
- 9**: "Selecteer Lijnen:" (Select lines) section with checkboxes for Lijn A, B, C, D, and E.
- 10**: "Frequentieverhoudingen:" (Frequency ratios) section with input fields for each line, all set to 1.
- 11**: "Opvolgtijd Intermediate Points:" (Intermediate point headway) section with radio buttons for "Sectie Afhankelijk" and "Sectie Afhankelijk" for Graskruid, Rotterdam CS, and Eindpunten.
- 12**: "Berekenen" (Calculate) button.
- 13**: A large empty text area at the bottom of the window.

Figure B.1: Graphical View of the Software Tool Provided for the RET

As mentioned in the thesis, we provide a software tool to the RET in which own calculations can be carried out. In this paragraph a brief explanation of the tool is given. See Figure B.1:

1. Textfield to modify the preferred time window in seconds.
2. Textfield to specify the length of the trains in meters, deployed on the network.
3. Option to choose the preferred solution-method. That is, one can choose between the optimal solution-method, which required the use of a solver and the heuristic method, which is in general non-optimal, but is less time-consuming.
4. Textfield to modify the dwell time in seconds.
5. Textfield to modify the turning time in seconds.
6. Option to choose between global headway time and headway time that is different per section. If the global headway time is chosen, the headway time must be specified in seconds.
7. Textfield to modify the fraction of maximum speed driven on each section, used in the generation of paths.
8. Options to switch on/off the different restrictions for Intermediate Points, line-frequencies, endpoints and Safe Haven, respectively.
9. Options to switch on/off different lines. If a line is switched off, the corresponding line-frequency is automatically set to 0.
10. Options to configure the preferred line-frequencies.
11. Options to toggle between a section-dependent headway time on the different Intermediate Points or a fixed headway time of X seconds, which must then be specified in the textfield.
12. Button to start calculation, after all preferred parameters are configured correctly.
13. Textfield for the outcomes of the calculations: the lengths and distances of the generated paths, the starting time and section of each deployed train, the endpoint of each deployed train, the total theoretical capacity.