

# FEM21030-15: Master's Thesis Econometrics & Management Science

MASTER'S PROGRAM QUANTITATIVE FINANCE

## Volatilities and Correlations of the largest Banks of Europe and America with Financial Conditions Indexes

R. Budgenhagen\* 366391  
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Supervisor: Dr. Michel van der Wel  
Second assessor: Sander Barendse



### Abstract

Modelling covariances across financial asset returns is important for financial management. This research models the influence of financial conditions on the European and American largest banks volatilities and correlations. Using daily stock returns of both EU and US largest banks during the period 2001 to 2016 and proxying financial conditions by the European and American Bloomberg Financial Conditions Indexes, I find that a block structure in the (c)DCC models is needed to separate the influences of the EU and US FCI on the respective correlations. I also find that incorporating EU and US financial conditions indexes has a significant affect on the respective variance processes. Specifically, variances go up when financial conditions get worse. Another contribution is using a data sampling technique to incorporate different frequency data in the models. The Log-Garch-Midas-X and Spline-Garch-X are the most preferred variance processes. I forecast Value-at-Risk using different variance and correlation processes. Various statistical tests and performance measures are considered to obtain the statistically preferred processes.

**Keywords:** Variance processes, Correlation processes, Financial Conditions Indexes, Value-at-Risk.

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## 1 Introduction

Modelling volatilities and covariances across financial asset returns is important for portfolio decision-making, risk management and other purposes in financial management. Volatilities and correlations are used for risk management or for the determination of hedge ratios and leverage factors and thus studying them are key for financial institutions. Volatilities and covariances of the asset returns vary over time and estimating models for covariances in large dimensions is a challenge.

In this paper I focus on both the European and American equity market and introduce correlation models with a block structure to best describe the variances in largest European and American banks and their correlations. These correlation models vary over time and can be estimated in large dimensions. A lot of research has been done on volatility and correlation models in the American market. Boudt et al. (2012) introduced a regime switching model to characterize the dynamics in the volatilities and correlations, Connor & Suurlaht (2012) models dynamic correlations of returns. In Karanasos et al. (2014) they model stock volatilities using a bivariate GARCH model. Opschoor et al. (2014a) use a modification of the Spline-GARCH model to study the relationship between order flow and volatility. In Rangel & Engle (2012) a Factor-Spline-GARCH model is discussed. Literature tries to link macroeconomic and financial fundamentals to volatility and correlations of asset returns, where evidence motivates that financial conditions are an important driver of the economy<sup>2,3</sup>. Boudt et al. (2012) found that financial conditions affect transition probabilities of volatility and correlation states which I further investigate in this research.

The main contribution is the extension and modification of the research of Bilio et al. (2006) by investigating the block structure in the European and American comovements and also by adding an explanatory variable. This block structure is needed to answer the question of how the correlations between the largest European banks are, between the largest American banks and cross-wise. Also, it separates the affects of the FCI on the correlations of the EU and the US. This model can be used in high dimensions and varies over time. It uses the benefits of the (c)DCC by using a small number of parameters, where the (c)DCC model restrict the parameters to be the same for all dimensions. In this setup, the dynamics for Europe and America can be separated and thus be more intuitively interpreted. This model can also be used when comparing different markets, where each market assumes the same dynamics. In a statistical sense, the block structure is also preferred to the correlation models without a block structure. The (c)DCC model is nested in the block correlation models and a simple log-likelihood ratio tests prefers the block-structure. The forecast performances between the two models are compared by using a Diebold-Mariano-West test which concludes that the block structure is preferred.

Another contribution is based on the research of Engle et al. (2013), where I use the modified Log-Garch-Midas-X model with as explanatory variable the EU and US bloomberg financial condition indexes. Incorporating these financial conditions indexes for the Spline-Garch-X and Log-Garch-Mias-X models has a significant affect on the respective variance processes both statistically and economically. Specifically, on average the variances go up when the financial conditions get worse. This confirms the findings for the Spline-Garch-X model of Opschoor et al. (2014b) and is thus also true for Europe and for the Log-Garch-Midas-X model in both Europe and the US. It also confirms Goodhart & Hofmann (2001) which states that financial conditions are an important driver of the economy. The best

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<sup>2</sup>see Goodhart & Hofmann (2001), Guichard & Turner (2008) and Hatzius et al. (2010)

<sup>3</sup>Other macro variables are mentioned in Bracker & Koch (1999), Murinde & Poshakwale (2004), Stavarek (2005) and Ehrmann et al. (2013).

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Log-Midas-X model is the one using monthly observations of the FCI with a lag length of one year. This model is also later used as a first stage for the correlation models. This model is used to combine data that are sampled at different frequencies and is based on mixed data sampling (MIDAS).

The third contribution is also using European banks and European financial conditions index for the research of Opschoor et al. (2014b). Now the behavior of the volatilities of the largest banks of Europe is known and can be compared to the American ones. For the Spline-GARCH-X and GARCH-MIDAS-X models I include the Bloomberg EU and US Area Financial Conditions Indexes as explanatory variables, where for the GARCH-MIDAS-X, monthly data is considered for the explanatory variable(s). These conditions influence economic behavior and the future state of the economy. More explanatory variables are added in these volatility models, for example by including the VDAX and/or EuroStoxx50, which are both components of the EU FCI, VIX and/or S&P 500, which are both components of the US FCI.

I estimate variance and correlation models separately, which is popular since the introduction of the DCC model of Engle (2002). They are modelled separately to decompose the effect of financial conditions on the variability of asset prices and their co-movement. Specifically, I first estimate the well known GARCH of Bollerslev (1986) and the Threshold GARCH (GJR) model as in Glosten et al. (1993), where the second model accounts for asymmetry from bear and bull markets in the returns, often encountered in the financial market. Thereafter the two component variance models are estimated. Spline-Garch model of Engle & Rangel (2008) is explored which measures volatility using two components: the spline part which is a long run deterministic component and a short run mean reverting unit GARCH component. Spline-GARCH-X model first introduced in Opschoor et al. (2014b) and the modified GARCH-MIDAS-X model from Engle et al. (2013) both with financial conditions indexes as explanatory variable to describe economic behavior. These variance models are then used for the Dynamic Conditional Correlation of Engle (2002) and the corrected Dynamic Conditional Correlation models of Aelli (2012). These models are then extended to the Block-DCC and Block-cDCC models, where the Block-DCC model is introduced in Bilio et al. (2006). I extend this model to also capture the covariance of the US and EU FCI as an extra explanatory variable. The models are all positive definite, which is a necessary condition which can be achieved through constraints. The parameters for the models are estimated using maximum likelihood, where the returns are assumed to be Student-t distributed following Rangel & Engle (2012), which better captures fat-tails and is typically observed in time series of the financial sector.

These models are then used for the Value-at-Risk measure over time. Afterwards the models are compared with various performance measures. The variance models are compared using the Akaike and Bayesian information criteria from respectively Akaike (1973) and Schwarz (1978), log-likelihood ratio of Neyman & Pearson (1933) tests between two component variance model and their nested counterpart and forecast performance based on Diebold-Mario-West (DMW) test from Diebold and Mariano (1995) and West (1996). The correlation models are compared using log-likelihood ratio tests between the block correlation models and the models without a block structure and by using the DMW test. The Value-at-Risk estimates are backtested following Christoffersen (1998) with the unconditional coverage, independence and conditional coverage tests.

The data used in this analysis is daily returns of the largest EU and US banks over the past 16 years. I include daily observations of Bloomberg EU and US Area Financial Conditions Indexes as proxy for the financial conditions. These daily indexes show the equally weighted sum of the overall conditions in the EU and US money market, bond market and equity

market. To obtain all the stock prices in euro, exchange rates from US dollar to Euro and from pounds to Euro are also included in this research. For the sub-indexes of the EU FCI, the effect of the VDAX and EuroStoxx50 and their individual explanatory power is investigated. The same holds true for the US FCI, with the sub-indexes being VIX and S&P500. Alternative Financial indexes are mentioned in Kliesen et al. (2012) and Zhen et al. (2014).

The results show that the block structure in the correlation processes is economically and statistically preferred to the (c)DCC model. Adding financial conditions to the variance models affect the variances of the largest EU and US bank returns, where on average the variances go up when the financial conditions get worse. The backtests for the Value-at-Risk estimates show that the block correlation model leads to less violations. Further, the FCI is preferred to the realized variances as explanatory variables, where splitting the FCI in its components adds relevant economic information. The two component variance models are preferred to the one component in a statistical way. In particular, the two component models with the FCI as explanatory variable are preferred the most.

The remainder of the paper is organized as follows: section 2 illustrates the modeling framework including the variance and correlation processes with and without financial conditions, the Value-at-Risk measure and statistical tests. Section 3 describes the data. In section 4 the results are discussed. Section 5 concludes. The analysis and implementations of the different models are done in the R environment and by using Matlab.

## 2 Methodology

In this section the covariances/correlations of the stock returns of the largest European and American banks are modeled. Starting with the basic set up of the stock return, these stock returns are used for the volatility models. The volatility models are then used for the correlation/covariance models. At first, define the  $N \times 1$  vector  $r_t = (p_t - p_{t-1})/p_{t-1}$  as the daily returns on the stocks of the European and American banks at time  $t$ , where  $p_t$  denotes the daily stock price index. Next I follow Engle (2002) and assume that

$$r_t - \mu = \varepsilon_t = D_t^{1/2} z_t, \quad \varepsilon_t | \mathfrak{S}_{t-1} \sim (0, H_t = D_t^{1/2} R_t D_t^{1/2}), \quad (1)$$

where  $H_t = [h_{ijt}]$  and  $D_t = \text{diag}(H_t) = \text{diag}(h_{11t}, h_{22t}, \dots, h_{NNt})$  with  $h_{iit}$  the modelled variance processes which are defined in the next subsection,  $\varepsilon_t$  a  $N \times 1$  vector of unexpected returns and  $z_t$  a  $N \times 1$  vector of standardized residuals which are assumed to follow a conditional Student-t distribution. Further, the  $N \times N$  matrices  $H_t$  and  $R_t$  are respectively the conditional covariance matrix and conditional correlation matrix of  $r_t$ .  $R_t$  is also the conditional covariance matrix of the standardized residuals  $z_t = D_t^{-1/2} \varepsilon_t$ .

In order to model the covariances/correlations of the stock returns, a model for  $H_t$  is needed. In this section I discuss the implementation of the volatility models and the correlation models. To disentangle the variability of the asset returns and their co-movement, the volatility<sup>4</sup> models and correlation models are estimated separately. In subsections 2.1 and 2.1.1 I discuss the implementation of the univariate volatility models  $h_{iit}$  with and without financial conditions, respectively. The correlation models  $R_t$  are discussed in 2.2, 2.2.1 and 2.2.2. These models are used in order to get a model for  $H_t$ .

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<sup>4</sup>The square root of the variance at time  $t$  represents the volatility at time  $t$ .

## 2.1 Variance Processes

At first, the univariate volatility models without financial conditions  $h_{iit}$  are estimated for the  $i^{th}$  stock return on the  $t^{th}$  day. The univariate volatility models are needed in order to model the proposed correlation models  $R_t$  and are the diagonal elements of the covariance matrix  $H_t$ . By estimating the variances and correlations separately, the number of parameters that need to be estimated is reduced and  $H_t$  is guaranteed to be positive definite.

The most common approach to estimate volatility in financial markets is the generalized autoregressive conditional heteroskedasticity model GARCH(1,1) from Bollerslev (1986). This model is defined as

$$h_{iit} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1}, \quad (2)$$

where  $\omega_i > 0, \alpha_i > 0$  and  $\beta_i > 0$  to guarantee  $h_{iit} > 0$  and  $\alpha_i + \beta_i < 1$  for covariance stationarity for all  $i$  and  $t$ . Then we allow for asymmetry in the  $\varepsilon_{i,t}$  by following Glosten et al. (1993) in the Threshold GARCH (GJR)

$$h_{iit} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \gamma_i \varepsilon_{i,t-1}^2 \mathbb{1}(\varepsilon_{i,t-1} < 0) + \beta_i h_{ii,t-1}, \quad (3)$$

where the restriction  $\gamma_i > 0$  is added to ensure  $h_{iit} > 0$  and  $(\alpha_i + \gamma_i)/2 + \beta_i < 1$  for covariance stationarity and  $\mathbb{1}(\cdot)$  is an indicator function.

At last I explore the Spline-GARCH model of Engle & Rangl (2008) which is a less known volatility process

$$h_{iit} = \tau_{iit} g_{iit} \quad (4)$$

$$g_{iit} = \omega_i + \alpha_i \frac{\varepsilon_{i,t-1}^2}{\tau_{i,t-1}} + \gamma_i \frac{\varepsilon_{i,t-1}^2}{\tau_{i,t-1}} \mathbb{1}(\varepsilon_{i,t-1} < 0) + \beta_i g_{ii,t-1} \quad (5)$$

$$\log(\tau_{it}) = \zeta_i + \rho_{0i} + \sum_{k=1}^{K_i} \rho_{ki} ((t - t_{k-1})_+)^2, \quad (6)$$

where  $(t - t_{k-1})_+ = (t - t_{k-1})$  if  $t > t_{k-1}$  and 0 otherwise and  $t_0 = 0, t_1, t_2, \dots, t_K = T$  is a partition of the time horizon  $T$  in  $K$  equally spaced intervals. The GARCH term  $g_{iit}$  captures the typical high-frequency behavior, which is also documented as the leverage effect; the spline part captures the more slowly moving macroeconomic fluctuations. Furthermore, the resulting volatility is always positive because of the following restrictions:  $\omega_i > 0, \alpha_i > 0, \gamma_i > 0, \beta_i > 0$  and  $\zeta_i > 0$  to guarantee  $h_{iit} > 0$ . The number of knots  $K$  are different for each bank  $i$  and are selected via performance measures discussed in 2.1.3.

### 2.1.1 Variance Processes with Financial Conditions

Next, I describe how the financial conditions are added to the univariate volatility models. The relation between stock market volatility and financial conditions is distinguished from short run and secular movements in two component models.

First, the changed Spline-GARCH model which captures the financial conditions measure is used. Therefore I use the Spline-GARCH-X following Opschoor et al. (2014) where the second component now includes the financial conditions index. This model is used instead of just a GARCH model because by including the financial condition variable as replacement of the Spline component, the GARCH term doesn't change compared to its usual form. Later on, more explanatory variables are added and their explanatory power are compared to each

other by means of performance measures and likelihood ratio tests. This is modelled as

$$h_{iit} = \tau_{it}g_{it} \quad (7)$$

$$g_{it} = \omega_i + \alpha_i \frac{\varepsilon_{i,t-1}^2}{\tau_{i,t-1}} + \gamma_i \frac{\varepsilon_{i,t-1}^2}{\tau_{i,t-1}} \mathbb{1}(\varepsilon_{i,t-1} < 0) + \beta_i g_{i,t-1} \quad (8)$$

$$\log(\tau_{it}) = \kappa_{i,0} + \kappa_{i,1} X_{i,t-1}, \quad (9)$$

where I use for the explanatory variable  $X_{i,t-1}$  daily observations of the Bloomberg Area Financial Conditions Index for the specific region<sup>5</sup> as financial condition measure.

Last, the Log-GARCH-MIDAS-X is estimated. This is also a two component variance process where MIDAS stands for mixed data sampling. The first component is the usual fast moving garch component. The second component is based on the financial conditions index computed on monthly basis using the mean of the daily FCI. This is an adaption to the previous work of Engle et al. (2013) who uses realized volatility as explanatory variable. The difference between this model and the Spline-Garch-X model is the frequency of the realisations of the explanatory variable used in the second component and the weighting of each realisation in the second component. So

$$h_{iit} = \tau_{iit}g_{ijt} \quad ,j = \{1, \dots, N_t\} \quad (10)$$

$$g_{ijt} = \omega_i + \alpha_i \frac{\varepsilon_{i,t-1}^2}{\tau_{i,t-1}} + \gamma_i \frac{\varepsilon_{i,t-1}^2}{\tau_{i,t-1}} \mathbb{1}(\varepsilon_{i,t-1} < 0) + \beta_i g_{i,j,t-1} \quad (11)$$

$$\log(\tau_{it}) = \eta_i + \vartheta_i \sum_{k=1}^K \lambda_{i,k}(\rho_i) X_{i,t-k} \quad (12)$$

$$\lambda_{i,k}(\rho_i) = \frac{(1 - k/K)^{\rho_i - 1}}{\sum_{j=1}^K (1 - j/K)^{\rho_i - 1}}, \quad (13)$$

where  $t$  is fixed at a monthly, quarterly or biannual frequency on the  $j^{th}$  day with the weights equation summing up to one,  $K$  denotes the lag length<sup>6</sup>. The  $\tau_{it}$  does not change for the fixed time span. The number of days in month  $t$  are represented as  $N_t$ . The  $\lambda_k(\rho)$  is a beta lag structure with the weights monotonically decreasing over the lags. By looking at information criteria and maximizing the log likelihood, the best  $t$  and  $K$  are chosen.

### 2.1.2 Maximum Likelihood Estimation

The parameters of the variance processes with and without financial condition indices are modeled using maximum likelihood, where  $z_{it}$  is assumed to be Student-t distributed following Rangel & Engle (2012), which better captures fat-tails and is typically observed in time series of the financial sector. The log-likelihood function that is to be maximized<sup>7</sup> is defined as

$$-\log L(\theta) = \frac{1}{2} \sum_{t=1}^T \left[ (v+1) \log \left( 1 + \frac{\varepsilon_t^2}{\tau_{it}g_t(v-2)} \right) + \log(\tau_{it}g_t) \right] - T \log \left( \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}} \right),$$

where  $\theta$  represents the variance process parameters,  $\Gamma$  is the gamma density function,  $v$  is denoted as the number of freedoms.

<sup>5</sup>The European countries are  $i = \{1, 2, \dots, m_1\}$  and for America  $i = \{m_1 + 1, m_1 + 2, \dots, N = m_1 + m_2\}$ .

<sup>6</sup>monthly 1 year means  $K = 12$ . Quarterly 3 years is also  $K = 12$ , etc.

<sup>7</sup>subject to the restrictions mentioned for the variance processes.



### 2.1.3 Performance Measures Variance Processes

Now that all of the volatility processes are estimated, the performances of these models are compared using various performance measures, such as the Akaike and Bayesian information criteria and their log-likelihood. The log-likelihood ratio tests between nested variance processes are also considered. Next, forecast performances of the variance models are defined. At last the economic contribution of the Financial Conditions Indexes are considered.

First I choose the best variance model according to the Akaike information criteria (AIC) from Akaike (1973) and Bayesian information criteria (BIC) from Schwarz (1978). These two criteria are defined as

$$\text{AIC} = -2 \log L + 2k, \text{ and} \quad (14)$$

$$\text{BIC} = -2 \log L + k \ln T, \quad (15)$$

where  $k$  denotes the number of estimated parameters,  $\log L$  be the maximum value of the log-likelihood function for the model and  $T$  denotes the sample size. The model with the lowest AIC and BIC is chosen for each bank and each model.

Second, the log-likelihood ratio test is used following Neyman & Pearson (1933), this is defined as

$$H_0 : \theta = \theta_0, \quad (16)$$

$$H_1 : \theta = \theta_1, \quad (17)$$

$$LR = 2 [\ln L(\theta_1) - \ln L(\theta_0)], \quad (18)$$

where  $\ln L(\theta_1)$  and  $\ln L(\theta_0)$  are respectively the log-likelihood of the alternative model and the likelihood for the model under the null hypothesis and  $\theta$  are the parameters of the variance process. This test statistic is  $\chi^2(df)$  with  $df$  the number of restricted parameters. In this research the following LR-tests are performed:  $H_0 : \gamma_i = 0$  to compare the GJR model to the Garch model for each bank, where the Garch model is the model under the null hypothesis and nested in the GJR model.  $H_0 : \kappa_{i,1} = 0$  to compare the Spline-Garch-X model to the GJR model. This test is justified because the Spline-Garch-X model with  $\kappa_{i,1} = 0$  is  $h_{iit} = e^{\kappa_{i,0}} g_{it}$ , where  $g_{it}$  is equal to the variance process  $h_{iit}$  of the GJR model, meaning that the GJR model is nested in the Spline-Garch-X model. Notice that here  $\kappa_{i,0}$  is a nuisance parameter, meaning that the likelihood function not depend on  $\kappa_{i,0}$ .  $H_0 : \vartheta_i = 0$  to compare the Log-Garch-Midas-X to the GJR model, where  $\eta_i, \lambda_{i,k}(\rho)$  and  $\rho_i$  are nuisance parameters. Another performance test is looking at the variance of the standardized returns  $z_{it}$ . If the model is correctly specified, this value should be equal to one.

Third, forecast performance measures are used for the variance processes of Europe and America as the variance processes are known one-step ahead. Therefore, I compare the estimated variance processes against a proxy for the true conditional variance for the whole data sample. Define the mean absolute error on standard deviations of the variance processes MAE-SD as

$$\text{MAE-SD} = \frac{1}{T} \sum_{t=1}^T \left| \sqrt{\sigma_t^2} - \sqrt{h_t} \right| = \frac{1}{T} \sum_{t=1}^T \text{AE-SD}_t. \quad (19)$$

As the true conditional variance  $\sigma_t^2$  is not observable, I use the squared returns  $r_t^2$  as a proxy<sup>8</sup>. I take the absolute error measure combined with the square root of the variance processes

<sup>8</sup>The squared returns are a noisy proxy for the true conditional variance and an alternative is the 'realised' variance, which is calculated using intra-day data.

to shrink the larger values towards zero, therefore the impact of the most extreme values is reduced. The variance processes with the least forecast errors are favored. To perform a statistical test on the absolute errors, the Diebold-Mariano-West (DMW) test is considered of Diebold and Mariano (1995) and West (1996). Define

$$H_0 : E(d) = 0, \quad (20)$$

$$DM = \frac{\bar{d}}{\sqrt{\hat{V}(d)/T}}, \quad (21)$$

where  $d_t = e_{1t} - e_{2t}$  the difference of the forecast errors,  $\bar{d}$  the mean value of  $d_t$  and  $\hat{V}(d)$  an estimate of the asymptotic variance of  $d_t$ . The DM test statistic is asymptotic standard normally distributed under the null hypothesis. Here, the loss functions are  $e_t = \text{AE-SD}_t$ , with  $e_{2t}$  being the benchmark model, to statistically compare the different variance processes with each other. Specifically, I compare the GJR model to the GARCH model and the second component models to the one component models. The exponentially weighted moving average model (EWMA) is also considered as a benchmark model and is defined as

$$h_t^{EWMA} = 0.04r_{t-1}^2 + 0.96h_{t-1}^{EWMA} \text{ with } h_1^{EWMA} = 0. \quad (22)$$

At last, the contribution of economic sources are measured. This can be done using the variance ratio following Engle et al. (2013)

$$\text{Variance Ratio} = \frac{\text{Var}[\log(\tau_{iit})]}{\text{Var}[\log(h_{iit})]}. \quad (23)$$

This ratio explains how much of the expected volatility can be explained by the financial conditions indexes and can be used for the two component models with Financial Conditions.

## 2.2 Correlation Processes

After estimating the volatility processes, the correlation model(s)  $R_t$  are estimated for the correlations of the stock returns. These are then used to model the covariance matrix  $H_t$ . Therefore, I start to estimate the Dynamic Conditional Correlation model (using  $z_{it}$  of the previously estimated variance processes). The advantage of these correlation models over the usual multivariate GARCH models is that the number of parameters to be estimated in the correlation process is independent of the number of series to be correlated, while still maintaining time-varying correlations. First, define

$$R_t = P_t^{-1/2} Q_t P_t^{-1/2}, \quad (24)$$

where  $Q_t = [q_{ijt}]$  and  $P_t = \text{diag}(q_{11,t}, q_{22,t}, \dots, q_{NN,t})$ . Now for the Dynamic Conditional Correlation model,

$$Q_t = (1 - \psi - \phi)\bar{Q} + \psi z_{t-1} z'_{t-1} + \phi Q_{t-1}, \quad (25)$$

where  $\bar{Q} = E(Q_t)$  a  $N \times N$  matrix. To gain consistent estimators in the DCC model, I implement the cDCC model following Aelli (2013) where  $c$  stands for corrected, with

$$Q_t = (1 - \psi - \phi)\bar{Q} + \psi \left[ P_{t-1}^{1/2} z_{t-1} z'_{t-1} P_{t-1}^{1/2} \right] + \phi Q_{t-1}. \quad (26)$$

The unit diagonal matrix  $\bar{Q} = E(Q_t) = E[z_t z'_t]$ , such that the estimators are consistent. In both models  $\psi$  is a positive and  $\phi$  a non-negative scalar parameter, such that  $\psi + \phi < 1$ . This results in  $Q_t$  to be a positive definite matrix.

### 2.2.1 Block Dynamic Conditional Correlation Processes

In the (c)DCC models, it is assumed that the dynamics in the correlations are all equal to each other. This is a strength of these models, because the numbers of parameters that need to be estimated is low, but can also be seen as weakness when  $N$  is large, because all the correlation processes are then restricted to follow the same dynamic structure. Therefore, both the DCC and cDCC models are extended to enrich the correlation processes by imposing a block-diagonal structure which separates the EU and US dynamics. The dynamics  $\psi$  and  $\phi$  are then only equal among the group of variables and can thus be interpreted for the EU, US and crosswise. The Block DCC model is proposed by Bilio et al. (2006), where the dynamic correlation equation is equal to

$$Q_t = (U - \psi - \phi) \odot \bar{Q} + \psi \odot z_{t-1} z'_{t-1} + \phi \odot Q_{t-1}, \quad (27)$$

where  $\psi$  and  $\phi$  are symmetric square full matrices of  $N \times N$ ,  $U$  is a  $N \times N$  matrix of ones and  $\odot$  indicates the entrywise (Hadamard) matrix product. Here,

$$\psi = \begin{bmatrix} \psi_{11} \iota_{m_1} \iota'_{m_1} & \psi_{12} \iota_{m_1} \iota'_{m_2} \\ \psi_{12} \iota_{m_2} \iota'_{m_1} & \psi_{22} \iota_{m_2} \iota'_{m_2} \end{bmatrix} \text{ and } \phi = \begin{bmatrix} \phi_{11} \iota_{m_1} \iota'_{m_1} & \phi_{12} \iota_{m_1} \iota'_{m_2} \\ \phi_{12} \iota_{m_2} \iota'_{m_1} & \phi_{22} \iota_{m_2} \iota'_{m_2} \end{bmatrix}$$

where  $\iota_{m_1}$  and  $\iota_{m_2}$  are column vectors of ones of dimension  $m_1$  and  $m_2$  (with  $m_1 + m_2 = N$ ), which respectively represent the number of variables in the EU and US groups.

The Block cDCC is implemented in the same way, with

$$Q_t = (U - \psi - \phi) \odot \bar{Q} + \psi \odot [P_{t-1}^{1/2} z_{t-1} z'_{t-1} P_{t-1}^{1/2}] + \phi \odot Q_{t-1}. \quad (28)$$

To yield a positive definite variance-covariance matrix  $H_t$ ,  $Q_t$  needs to be positive definite. This is the case when the minimum eigenvalue of each  $\psi$  and  $\phi$  are greater or equal to zero (both are positive semi definite) and when the minimum eigenvalue of  $(1 - \psi - \phi) \odot \bar{Q}$  is strictly greater than zero (positive definite), see Ding & Engle (2001) and Engle & Sheppard (2001) .

### 2.2.2 Block (c)DCC Models with Explanatory Variable

To add flexibility to the correlation models, I add an exogenous variable and call this extended model Block-cDCC-Y, with

$$Q_t = (U - \psi - \phi - \xi) \odot \bar{Q} + \psi \odot [P_{t-1}^{1/2} z_{t-1} z'_{t-1} P_{t-1}^{1/2}] + \xi \odot Y_{t-1} + \phi \odot Q_{t-1}, \quad (29)$$

where

$$\xi = \begin{bmatrix} \xi_{11} \iota_{m_1} \iota'_{m_1} & \xi_{12} \iota_{m_1} \iota'_{m_2} \\ \xi_{12} \iota_{m_2} \iota'_{m_1} & \xi_{22} \iota_{m_2} \iota'_{m_2} \end{bmatrix} \text{ and } Y_t = \begin{bmatrix} y_{11,t} \iota_{m_1} \iota'_{m_1} & y_{12,t} \iota_{m_1} \iota'_{m_2} \\ y_{12,t} \iota_{m_2} \iota'_{m_1} & y_{22,t} \iota_{m_2} \iota'_{m_2} \end{bmatrix}.$$

Here  $y_{11,t}$  and  $y_{22,t}$  define respectively the variance of the European and United States financial conditions indices till time  $t$ ,  $y_{12,t}$  defines the covariance between the European and American financial conditions indices up to time  $t$ . To yield a positive definite variance-covariance  $Q_t$ , the following restriction is added: the minimum eigenvalue of  $\xi$  also needs to be greater or equal to zero (positive semi definite) and  $\xi \odot Y_t$  needs to be positive semi definite for each  $t$ .<sup>9</sup>

<sup>9</sup>Another way to ensure a positive definite matrix  $Q_t$ , is to consider the Cholesky decomposition of the quasi-correlation matrix  $Q_t$ , see Boudt et al. (2014).

### 2.2.3 Quasi-Maximum Likelihood Estimation

All the models are estimated using quasi-maximum likelihood following Newey & McFadden (1994) and Opschoor et al. (2014b), where a two-stage GMM estimation method is used. For the DCC and cDCC models the composite likelihood is maximized, which is proposed by Engle, Shephard, and Sheppard (2008), where I reduce the number of pairs used in the CL by only considering randomly chosen contiguous pairs  $i, i + 1$  in the data set. This speeds up the estimation procedure, but leads to non-efficient parameter estimates. The CL is equal to the exact QML if all of the pairs are independent.

In the first stage, the parameters of the volatility model are estimated. Then the standardized residuals are constructed:  $\hat{z}_{it} = \hat{\varepsilon}_{it} / \sqrt{\hat{h}_{iit}}$  for  $i = 1, \dots, N$ . After that, I estimate the correlation model parameters for the conditional correlations, where  $\bar{Q}$  is replaced by  $\hat{Q}_n = \frac{1}{T} \sum_{t=1}^T \hat{z}_t \hat{z}_t'$  for the DCC, Block-DCC and Block-DCC-Y models which is the unconditional correlation matrix of  $\hat{z}_t$  and by  $\hat{Q}_n = \frac{1}{T} \sum_{t=1}^T P_{t,n-1}^{1/2} \hat{z}_t \hat{z}_t' P_{t,n-1}^{1/2}$  for the cDCC, Block-cDCC and Block-cDCC-Y models. The  $P_{t,n-1}^{1/2}$  estimated through  $Q_t$ , by using  $\hat{Q}_0 = \frac{1}{T} \sum_{t=1}^T \hat{z}_t \hat{z}_t'$  as initial value, such that the  $(n-1)^{th}$   $P_t$  is used for  $\hat{Q}_n$ , where I assume that the  $(n-1)^{th}$   $P_t$  converges to the true  $P_t$ . This estimation procedure leads to consistent estimators, but they are no longer efficient. For all of the correlation processes, the usual joint log likelihood is also estimated. The CL is not possible for the block correlation models because the parameters are not assumed to be equal for the contiguous pairs between the sectors.

### 2.2.4 Statistical Performance Tests Correlation Processes

Here, the different dynamics of the correlation processes are compared by means of likelihood ratio tests. Specifically, I perform two tests following the idea of Bilio et al. (2006). First, the null hypothesis of a block structure in the correlation processes is tested, which thus means that I test the DCC(1,1) against the Block-DCC (or cDCC against Block-cDCC). Therefore I define the following null hypothesis  $H_0 : \psi_{11} = \psi_{12}, \psi_{12} = \psi_{22}$  and  $\psi_{22} = \psi$  and  $\phi_{11} = \phi_{12}, \phi_{12} = \phi_{22}$  and  $\phi_{22} = \phi$ . This implies that the DCC model is nested in the Block-DCC model. Now the log-likelihood ratio test has a chi-square distribution with six degrees of freedom given the normal asymptotic distribution of the QMLE. Second, the same regime is followed for comparing the Block-DCC-Y with the Block-DCC model. The null hypothesis is  $H_0 : \xi_{11} = \xi_{12}, \xi_{12} = \xi_{22}$  and  $\xi_{22} = 0$ . This implies that the Block-DCC-Y model reduces to the Block-DCC model under the null hypothesis. In this case the asymptotic distribution is chi-square with three degrees of freedom.

Next, the forecast performance measures are used for the correlation processes, which is analog to the univariate approach of section 2.1.3. Specifically, I compare the estimated covariance processes against a proxy for the true conditional covariance for the whole data sample. Define the mean absolute error of the covariance processes MAE as

$$\text{MAE} = \frac{1}{T} \sum_{t=1}^T \sum_{i,j} |\Omega_t - H_t| = \frac{1}{T} \sum_{t=1}^T \text{AE}_t. \quad (30)$$

As the true conditional covariance  $\Omega_t$  is not observable, I use the outer product of the returns  $r_t r_t'$  as a proxy<sup>10</sup>. I again take the absolute error measure to shrink the larger values towards zero, reducing the impact of the most extreme values. The covariance processes with the

<sup>10</sup>The proxy for the true conditional covariance can be improved by using the 'realised' covariance, which is calculated by using intra-day data.

least forecast errors are favored. To perform a statistical test on the absolute errors, the Diebold-Mariano-West (DMW) test is again considered. In this setup, the loss functions are  $e_t = \text{AE}_t$ . I test the correlation models against a multivariate EWMA model and against  $H_t = D_t$ , with  $D_t$  the univariate GJR model with on the diagonal the observations for each bank over time and zero on the off-diagonals. Also, the block-correlation models are compared to the DCC and cDCC models. The EWMA in the multivariate case is defined as

$$H_t^{EWMA} = 0.04r_{t-1}r'_{t-1} + 0.96H_{t-1}^{EWMA} \text{ with } H_1^{EWMA} = \mathbf{0}. \quad (31)$$

### 2.3 Value at Risk

After selecting the statistically best volatility and correlation models, I investigate the Value-at-Risk portfolio based on an equally weighted portfolio. This is interesting because it makes it possible to compare the different variance processes and correlation models, it is useful in risk management. Express the portfolio  $100(1 - \alpha)\%$  Value-at-Risk (VaR) at time  $t$  as

$$\text{VaR}_{1-\alpha,t} = \mu_t^P + z_\alpha \sqrt{w_t' H_t w_t}, \quad (32)$$

with  $\mu_t^P = \mu' w_t$  the conditional mean return of the portfolio, where  $\mu$  is the mean return of the banks defined in (1). Next,  $w_t = w = [1/N, 1/N, \dots, 1/N]'$  is the  $N \times 1$  equally portfolio weight and  $H_t$  is the portfolio covariance matrix based on the correlation models, such that  $\sigma_t^P = \sqrt{w_t' H_t w_t}$  is the portfolio standard deviation. Also,  $D_t = H_t$  is used which makes it possible to estimate the VaR by only considering the variance processes.  $N$  can also be  $m_1$  or  $m_2$ , then  $\mu_t^P$  is the mean of the respective bank returns. Lastly,  $z_\alpha$  defines the estimated  $\alpha^{th}$  empirical quantile of the in-sample standardized portfolio returns  $z_t^P = (r_t^P - \mu_t^P)/\sigma_t^P$  where the portfolio returns are defined as  $r_t^P = r_t' w_t$ . The advantage is that a parametric judgment about the appropriate distribution is avoided. The VaR are estimated for the  $\alpha = 1\%$  and  $5\%$  significance levels for the whole sample period.

After estimating the VaR's, they are backtested following Christoffersen (1998) where they proposed the unconditional coverage test, the independence test and the conditional coverage test. Define the indicator function  $\mathbf{1}(r_t^P < -\text{VaR}_{1-\alpha,t})$ . The unconditional coverage (uc) test tests if the coverage is on average correct and assumes the indicator to be independent over time. This test is defined as

$$\begin{aligned} H_{0,uc} : \pi &= \alpha \\ LR_{uc} &= 2 [\log L(\mathbf{1}, \hat{\pi}) - \log L(\mathbf{1}, \alpha)], \end{aligned}$$

where ML estimate of  $\pi$  is  $\hat{\pi} = T_1/T$  defined as the percentage of that the portfolio returns are smaller than  $-\text{VaR}_{1-\alpha}$ , where  $T_1$  is the number of ones in the sample and  $T$  the sample size. The log-likelihood function  $\log L(\mathbf{1}, \alpha) = T_1 \log \alpha + (T - T_1) \log(1 - \alpha)$  is iid Bernoulli with the unknown probability parameter  $\alpha$ , the  $\log L(\mathbf{1}, \hat{\pi})$  is the same function with  $\alpha$  being replaced by  $\hat{\pi}$ . This test statistic is asymptotically distributed as  $\chi^2(1)$ .

To test if the indicator function is indeed independent over time and not clustered, the independence test is used and defined as

$$\begin{aligned} H_{0,ind} : \pi_{01} &= \pi_{11} \\ LR_{ind} &= 2 [\log L(\mathbf{1}, \hat{\pi}_{01}, \hat{\pi}_{11}) - \log L(\mathbf{1}, \hat{\pi})] \end{aligned}$$

where  $\hat{\pi}_{01} = T_{01}/(T_{00} + T_{01})$  and  $\hat{\pi}_{11} = T_{11}/(T_{10} + T_{11})$  are the ML estimates of  $\pi_{01}$  and  $\pi_{11}$  and where  $\log L(\mathbf{1}, \hat{\pi}_{01}, \hat{\pi}_{11}) = T_{00} \log(1 - \hat{\pi}_{01}) + T_{01} \log \hat{\pi}_{01} + T_{10} \log(1 - \hat{\pi}_{11}) + T_{11} \log \hat{\pi}_{11}$ . The  $T_{ij}$  denotes the number of observations  $i$  in time  $t$  and  $j$  in time  $t + 1$ . This is true when

we consider the indicator function to follow a first order markov switching process<sup>11</sup>. This test statistic is also asymptotically distributed as  $\chi^2(1)$ .

The conditional coverage test is a combination of the two tests and defined as

$$H_{0,cc} : \pi_{01} = \pi_{11}$$

$$LR_{cc} = LR_{uc} + LR_{ind} = 2 [\log L(\mathbf{1}, \hat{\pi}_{01}, \hat{\pi}_{11}) - \log L(\mathbf{1}, \alpha)],$$

with the test statistic being asymptotically distributed as  $\chi^2(2)$ .

### 3 Data

I investigate how European and American Financial conditions affect volatilities and correlations in the European and American financial markets. Therefore, the models discussed in the previous sections are applied on stock returns of the largest EU and US financial companies. Consider daily stock prices of the fifteen largest European and American banks ranked by assets<sup>12</sup> in 2015 during the period December 14, 2001 to March 29, 2016, where the time series are adjusted for dividends and stock splits. This provides a total of 3710 observations.

I look at the fifteen largest banks, because these banks have the most influential power in the European and American financial sector. Both the European and American stock prices are downloaded from Yahoo Finance. Only eleven of the fifteen largest European banks have all the data available.

**Table 1:** The table shows the largest banks of Europe and America, where the banks are ranked by asset in 2015. The ticker of the banks that are used in this research are given.

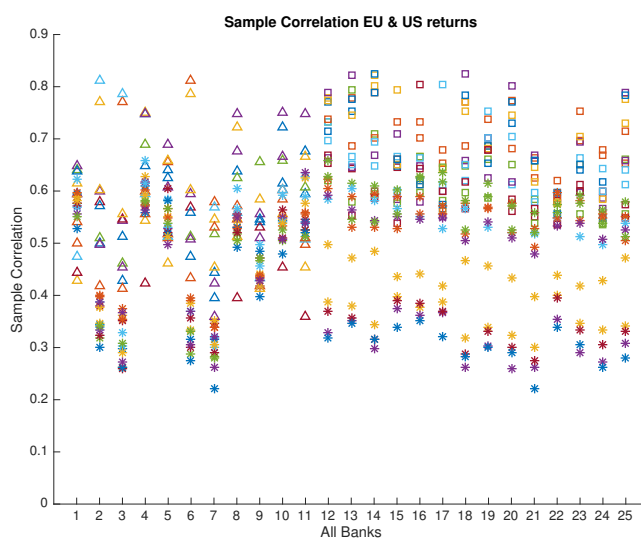
Europe		America	
Name	Ticker	Name	Ticker
HSBC Holdings	HSBC	J.P.Morgan Chase & Co.	JPM
BNP Paribas	BNP.PA	Bank of America	BAC
Credit Agricole Group	ACA.PA	Wells Fargo & Co.	WFC
Deutsche Bank	DBK.DE	Citigroup Inc.	C
Barclays PLC	BCS	Goldman Sachs Group	GS
Societe Generale	GLE.PA	Morgan Stanley	MS
Royal Bank of Scotland Group	RBS.L	U.S. Bancorp	USB
Banco Santander	SAN	Bank of New York Mellon	BK
(Groupe BCPE)	-	PNC Financial Services	PNC
(Lloyds Banking Group)	-	Capital One Financial	COF
(UBS AG)	-	(HSBC North America Holdings)	-
UniCredit S.p.A	UCGMI	TD Group US Holding	TD
ING Group	ING	State Street Corporation	STT
Credit Suisse Group	CS	Charles Schwab Corp.	SCHW
(Banco Bilbao Vizcaya Argentaria)	-	Suntrust Banks	STI

The eleven European banks with full data availability sorted by the number of assets in descending order are given in the table above. For America fourteen of the fifteen largest banks have all the data available and are also given in the table. The names in brackets (and without a ticker) don't have enough data available and are not included in my investigation.

The stock prices are in Euros, US dollars and pounds. I obtain the specific exchange rates from the European Central Bank to convert all the prices to euros. These exchange rates are also in the same time range and thus are converted on a daily basis. The returns are also used to compute the monthly realized volatility, which is defined as the sum of the daily squared returns for each month.

<sup>11</sup>If  $T_{11} = 0$ ,  $\log L(\mathbf{1}, \hat{\pi}_{01}, \hat{\pi}_{11})$  is equal to  $(T_{00} - T_{01}) \log(1 - \hat{\pi}_{01}) + T_{01} \log \hat{\pi}_{01}$ .

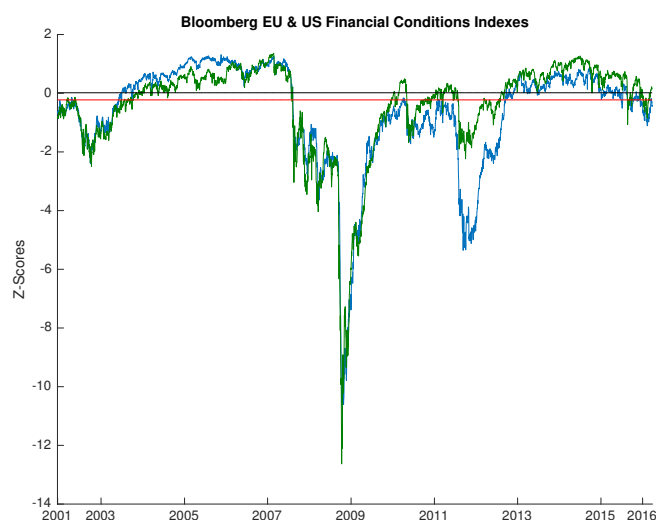
<sup>12</sup>see: <http://www.relbanks.com/top-european-banks/assets> and [www.relbanks.com/top-us-banks/assets](http://www.relbanks.com/top-us-banks/assets)



**Fig. 1:** Sample correlations of the EU and US largest bank returns over the period 14 december 2001 to 29 March 2016. Each column shows the sample correlation with that bank against the remaining banks. Triangles correspond to EU banks, squares to US banks and stars show cross correlations. On the x-axis the banks in consecutive order are first for the EU: HSBC Holdings, BNP Paribas, Credit Agricole Group, Deutsche Bank, Barclays PLC, Banco Santander, Societe Generale, Royal Bank of Scotland Group, UniCredit S.p.A, ING Group and Credit Suisse Group. Then the US banks follow: J.P.Morgan Chase & Co., Bank of America, Wells Fargo & Co., Citigroup Inc., Goldman Sachs Group, Morgan Stanley, U.S. Bancorp, Bank of New York Mellon, PNC Financial Services, Capital One Financial, TD Group US Holding, State Street Corporation, Charles Schwab Corp. and Suntrust Banks.

Figure 1 shows the correlations between the bank stock returns over the complete sample period<sup>13</sup>. The horizontal axis in the graph shows the 25 banks, where the first 11 are the European and the last 14 are the American banks. For each bank the 24 sample correlations with the other banks are shown. The triangles are the European co-movements and vary between 0.36 and 0.81 with a mean of 60%. The correlations for the American largest bank returns are depicted in squares and are on average higher than the European correlations. They vary between 0.54 and 0.82 with a mean of 68%. Lastly, the cross correlations between the US and EU bank returns are shown as stars, they vary between 0.22 and 0.66 with an average of 46.5%. This shows that the cross correlations behave differently between EU and US than within each sector, this raise the idea for a block structure in the correlation models.

<sup>13</sup>Figure 5 in the Appendix plots the correlations separately.



**Fig. 2:** Here the plots of the daily EU and US Bloomberg FCI are shown for the period 14 december 2001 to 29 March 2016. The blue and green line depicts respectively the EU and US FCI with corresponding sample mean of black and red. Positive values correspond on average to better financial conditions of the economy for the respective economy, while negative values mean the opposite. The y-axis are the number of standard deviations above or below its average over the index.

In this research the Bloomberg EU and US Area Financial Conditions Indexes are used as the daily financial conditions index. The FCI and the underlying indicators are Z-scores that indicate the number of standard deviations by which current financial conditions deviate from normal levels. Figure 2 shows a graph of the two indexes for the same time range as the stock prices<sup>14</sup>. The blue and green line depict respectively the EU and US FCI with corresponding mean of black and red. A noticeable difference between the two sectors is the period of 2012 where Europe shows a crisis with a low FCI below -4.

**Table 2:** This table shows Bloomberg’s EU and US FCI sub-indexes, their indicators and weights for both Europe and America.

Europe			America		
		weights			weights
Money market	Euro TED Spread	16.7%	US TED Spread	11.1%	33.3%
	Euribor/OIS Spread	16.7%	Libor/OIS Spread	11.1%	
			Commerical Paper/T-Bill Spread	11.1%	
Bond Market		33.3%	Baa/10Y Treasury Spread	6.7%	33.3%
	EU 10Y Swap Spread	16.7%	US High-Yield/10Y Treasury Spread	6.7%	
	JP Morgan High Yield Europe Index	16.7%	US 10Y Swap/Treasury Spread	6.7%	
			US Muni/10Y Treasury Spread	6.7%	
			Swaption Volatility Index	6.7%	
		33.3%			33.3%
Equity Market	EuroStoxx Index	16.7%	S&P 500	16.7%	33.3%
	VDAX Index	16.7%	VIX Index of S&P 500 Volatility	16.7%	
		33.3%			33.3%
Total		100%			100%

The EU and US Bloomberg FCIs track the overall level of financial stress in Euro and American area money, bond, and equity markets. The FCI is an equal-weighted sum of three sub-indexes for each of these markets. The sub-indexes for the EU equity market are the EuroStoxx50 Index and the VDAX Index, and S&P 500 Index and VIX Index for the US equity market. The specific indicators that form each sub-index are also equally weighted. Table 2 shows the composition of the specific indicators and their sub-indexes for Europe and America. These two indexes for the EU and US are also included in my investigation and

<sup>14</sup>In the appendix figure 6 plots them separately.



are daily available for the same data frame. I follow the approach of Opschoor et al. (2014b) for the decomposition of the equity market components together with the FCI without the effects of these components. These equity components together with the cleaned EU and US FCI are later used for the Spline-Garch-X and Log-Garch-Midas-X models.

At last I test if the data has the skewness and kurtosis matching a normal distribution. Define  $H_0$  : the data follows a normal distribution with the test statistic  $JB$  of the Jarque-Bera test as

$$JB = \frac{T}{6} \left( S^2 + \frac{(K - 3)^2}{4} \right), \text{ under } H_0: JB \overset{a}{\sim} \chi_{(2)}^2, \quad (33)$$

where  $S$  is the skewness and  $K$  the kurtosis. As financial data is investigated, I expect that the data doesn't follow a normal distribution.

	Europe			America			Total		
	Min	Average	Max	Min	Average	Max	Min	Average	Max
$S$	-0.46	0.23	0.88	-1.35	0.84	4.69	-1.35	0.57	4.69
$K$	9.12	13.85	20.19	8.26	35.41	128.40	8.26	25.93	128.40
$JB$	5,850	20,056	46,361	4,453	310,793	2,454,369	4,453	182,869	2,454,369
$p$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

The table above shows clearly a rejection of the null hypothesis. All of the bank returns have a high kurtosis ranging from 8 to 128. The corresponding Jarque-Bera statistics range from 4 to 2 million, all with a  $p$ -value lower than 0.001. Therefore I assume the innovations to follow a student-t distribution. The skewness shows small values for Europe ranging from -0.46 to 0.88. In America, the max skewness is 4.69, so a skewed t-distribution can also be used here. A downside to that approach is that more parameters need to be estimated.

## 4 Results

In this section the results are interpreted. At first, the parameter estimations for the volatility models for the HSBC and JPM banks are discussed which respectively represent the European and American sector. The volatility results of the other banks are in the appendix, but the minimum, average and maximum volatility parameters are discussed for each volatility model. The parameters for the DCC and cDCC correlation models are divided in European, American and a combination of both, where the conditional likelihood is estimated using contiguous pairs by only considering the respective banks. Then I interpret the parameters for the block correlation models, which I compare to the DCC and cDCC model. Next, the Value-at-Risk is backtested with the unconditional coverage, independence and conditional coverage tests. At last, the monthly realized variance and the cleaned FCI are considered as explanatory variables for the variance processes.

## 4.1 European Variance Models Parameter Estimates

**Table 3:** The table shows the parameters of the different variance processes for the HSBC bank. The fields that are blank correspond to parameters that are not used for the variance process. For the explanatory variable X, the EU Bloomberg FCI is used. Further, the log-likelihood, Akaike and Schwarz information criteria, variance of standardized returns and the mean absolute errors between square root of the variance process and true variance proxy are shown. AIC and BIC are divided by T, to get a better comparison with the Log-Garch-Midas-X model. The mean absolute errors are multiplied by 100. Next the LR tests for the variance processes are shown.

HSBC	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X
$\alpha$	0.073 (0.009)	0.045 (0.009)	0.039 (0.013)	0.05 (0.014)	0.045 (0.013)
$\gamma$		0.054 (0.010)	0.069 (0.023)	0.049 (0.022)	0.073 (0.023)
$\beta$	0.921 (0.009)	0.92 (0.015)	0.889 (0.028)	0.895 (0.032)	0.891 (0.025)
$\zeta$			-7.749 (0.017)		
K			13		
$\kappa_0$				-8.701 (0.130)	
$\kappa_1$				-0.325 (0.034)	
$\eta$					-8.659 (0.059)
$\vartheta$					-0.279 (0.521)
$\rho$					2.327 (1.513)
$\nu$	5.615 (0.561)	5.754 (0.582)	6.145 (0.624)	6.014 (0.559)	5.699 (0.160)
LogL	10,921	10,929	10,945	10,945	10,225
AIC	-5.862	-5.865	-5.866	-5.875	-5.918
BIC	-5.857	-5.859	-5.835	-5.865	-5.906
Var( $z_t$ )	0.985	0.987	0.992	0.986	0.986
MAE-SD	0.856	0.852	0.838	0.845	0.839
<b>LR-test</b>					
$H_0$		$\gamma = 0$		$\kappa_1 = 0$	$\vartheta = 0$
LR		14.488		37.192	12.021
p		0.000		0.000	0.001

Table 3 shows the estimated parameters for the variance processes belonging to the HSBC bank, which is the largest European bank in terms of assets in 2015<sup>15</sup>. The standard errors are in brackets. The ARCH effects  $\alpha$  and GARCH effects  $\beta$  show the typical pattern for financial data. The mean of the estimated  $\alpha$  show little variation between the variance processes and is between 0.021 and 0.030 for all the variance processes except for the GARCH process where it is 0.079, this value is larger because it also incorporates the leverage effect. They take values between 0.008 and 0.107. For HSBC bank, this value is around 0.045 for the variance processes (except for GARCH), suggesting that the volatility shock today feeds through into next period's volatility on average more for the HSBC bank than for the others. The estimated  $\beta$  has a mean above 0.900 for all the variance processes and takes on values between 0.838 and 0.948. For the HSBC bank, the two component variance processes have a lower  $\beta$  than the one component models, suggesting that the two component models are less persistence than the one component models. The leverage effects  $\gamma$  exhibit only positive values between 0.033 and 0.159 for all the banks and all the variance processes. This is consistent with the

<sup>15</sup>Table 13 in the appendix shows the summary of the variance models parameter estimates for the European banks.

leverage theory of Black (1976) and Christie (1982). The HSBC bank fits with leverage effects of 0.049 for the Spline-Garch-X model to 0.073 for the Log-Garch-Midas-X model.

The constant  $\zeta$  in the Spline-Garch model takes values between -8.590 and -7.110. The number of knots for the Spline-Garch model have a mean integer value of  $8^{16}$  and takes on the value 13 for the HSBC bank. This suggests that the largest banks in EU show a lot of changes in the curvature of the long-term trend of the variance processes, especially for the HSBC bank. Considering  $\kappa_0$  and  $\kappa_1$  corresponding to the Spline-Garch-X model, it is observed that the mean of  $\kappa_0$  is -7.811. This value can be compared to the constant belonging to the Spline-Garch model. For the HSBC bank, a value of -8.701 is observed, which is lower than the  $\zeta$  estimate. The  $\kappa_1$  estimate ranges from -0.440 to -0.220 with HSBC having a value of -0.325. The coefficient is negative for all of the EU banks, meaning that worse financial conditions in the EU cause the variance of the bank of this sector to increase. Looking at the second component parameter estimates for the Log-Garch-Midas-X model, the constant  $\eta$  has a mean value of -7.651, which can be compared to the constant  $\zeta$  and  $\kappa_0$  for the second component for both of the Spline-Garch models. The constants of the second component models should be interpreted by raising  $e$  to the power of that value first. These exponential values are then all positive resulting in a positive variance process. For the HSBC bank this value is -8.659. So the HSBC bank has for all the two component models a lower constant than the average value.

The scaling parameter  $\vartheta$  estimates vary between -0.523 and 0.136. It is interesting to note that it has positive and negative values. For the HSBC bank, this estimate has a negative value of -0.279 meaning that that an increase in the EU FCI is associated with a decrease in HSBC's long-term variance. The interpretation of this variable can be compared to  $\kappa_1$ . The weighting parameter  $\rho$  varies between 1.776 and 91 and can be interpreted as how fast the weights  $\lambda_k(\rho)$  decay to zero. For the HSBC bank this value is 2.327 suggesting that observations in the distant past still have an important influence. Next the degrees of freedom  $\nu$  of the univariate Student-t distributions are interpreted. They range from 4.680 to 9.751, with a mean value of roughly 6.700, which is again evidence for non-normality and excess kurtosis in the equity returns.

The log-likelihoods range from 8878 to 10945. For the HSBC bank, both Spline-Garch models have the same largest log-likelihoods at 10945, which is 16 and 24 larger than the GJR model and Garch model, respectively. To be more specific, the loglikelihood of the Spline-Garch model is 0.287 lower than that the LogL for Spline-Garch-X for the HSBC bank<sup>17</sup>. For the HSBC bank, both the Akaike and Schwarz information criteria suggest that the Log-Garch-Midas-X model is preferred having the lowest value at -5.918 and -5.906, respectively. AIC and BIC also indicate that the two component models are preferred to the one component models, except for the Spline-Garch, where the BIC is higher than the one component models. The same hold true for the average AIC and BIC values of the other banks. In the appendix table 14 the ranking for each variance model for the AIC and BIC is considered. That table shows that both the Log-Garch-Midas-X and Spline-Garch-X are the most preferred models for each of the largest European bank. Another performance measure is looking at the variance of the standardized returns  $\text{Var}(z_{it})$ . The results indicate that the mean of the variances corresponding to the Log-Garch-Midas-X are the closest to one with 1.001. For the HSBC bank, the Spline-Garch has the closest value to one, with 0.992. The forecast performance measure MAE-SD multiplied by 100 is smallest for the two component

<sup>16</sup>The maximum number of knots considered is equal to 15.

<sup>17</sup>When re-estimating the Spline-Garch-X model using the same time frame as the Log-Garch-Midas-X, the log-likelihood is 10236, which is still 11 values larger than for the Log-Garch-Midas-X, meaning that the AIC and BIC are both in favor of the Spline-Garch-X model.

models. Table 15 in the appendix ranks the mean absolute variance and forecast error for each variance model. The table indicates that the Log-Garch-Midas-X has the smallest forecast errors for most European banks.

The values of the log-likelihoods of the different variance processes are already briefly discussed. To statistically compare the improved performance of the different variance processes, different log-likelihood ratio tests are performed. For the HSBC bank, the GJR model is preferred to the Garch model, with a p-value of 0.000. Further, the two component models are preferred to the one component models for the 1% significance level for both the Spline-Garch-X against GJR and Log-Garch-Midas-X against GJR model.

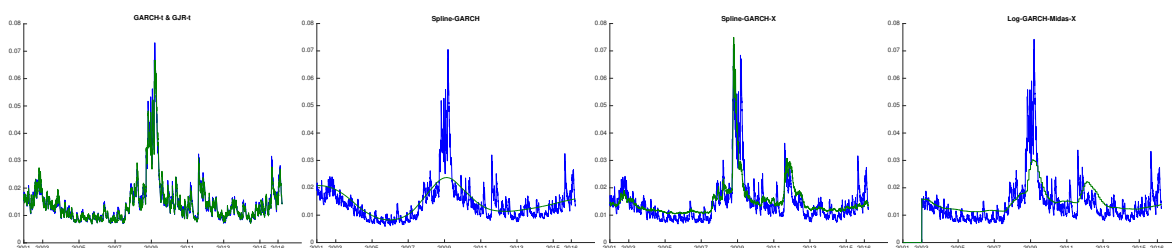
**Table 4:** This table shows the t-statistics of the Diebold-Mariano-West tests with  $H_0$  of equal forecast accuracy over the whole sample. The variance models on the horizontal axis are statistically compared to the benchmark models on the vertical axis.

DM	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X
EWMA	2.996	1.424	-2.085	-0.353	-0.015
Garch		-1.946			
GJR			-6.023	-1.842	-3.521
Spline-Garch-X					-0.185

The DMW test shows the results for statistically comparing the AE-SD of the different variance models. An absolute t-statistic greater than 1.96 indicates a rejection of the null hypothesis of equal forecast accuracy at the 0.05 significance level. A negative sign implies that the variance model produced smaller average losses than the benchmark model and is thus preferred. When comparing the variance processes to the EWMA model, the two component models are preferred. The Spline-Garch model significantly outperforms the EWMA model. The GJR statistically outperforms the GARCH model and the Spline-Garch and Log-Midas-X model statistically outperform the GJR model. The Spline-Garch-X model is favored over the GJR model, but not on a 5% significance level. The Log-Garch-Midas-X is preferred to the Spline-Garch-X model, but the difference is not statistically different from 0.

For the Spline-Garch-X model and Log-Midas-X model the contribution of economic sources are measured using the Variance Ratio defined in section 2.1. For the HSBC bank 54% of the expected volatility can be explained by the European financial conditions index. The contribution of the EU FCI for the European banks ranges from 17% to 64%, indicating that this variable captures a quite significant fraction of variation in expected volatility. For the Log-Midas-X model these values range from almost 0% for the Credit Agricole Group bank to 70%. What is interesting is that the economic contribution is 51% for the Credit Agricole Group bank for the Spline-Garch-X model meaning that the filter in the Midas model absorbs most of the economic contribution.

All in all the performance measures suggest that the two component models are preferred to the one component models, with the Spline-Garch-X being the most preferred variance process. Especially for the Spline-Garch-X and Log-Garch-Midas-X model the Bloomberg FCI has a significant impact on the variances of the bank returns and capture a quite significant fraction of variation in expected volatility. The Log-Garch-Midas-X model is ideal to be used for lower frequency data as it performs almost as good as the Spline-Garch-X model, but using monthly data.



The four plots in the figure above depict the in-sample volatilities  $\sqrt{h_{it}}$  for the HSBC bank returns for the different variance processes. The first plot shows the Garch process in blue line against the GJR with green line. For the other plots, the green line refers to the second component. All of the processes look very much alike. What is interesting is that the Spline component for the Spline-Garch model captures the cycles very well, with the exception for the big peak end 2008, it shows the same pattern as the second component of the Log-Midas-X model. The same period gives also a substantially higher estimate when including the FCI for the Spline-Garch-X. The second component of the Log-Midas-X model captures the second peak around 2012 much better than for the Spline-Garch model.

## 4.2 American Variance Models Parameter Estimates

**Table 5:** The table shows the parameters of the different variance processes for the JPM bank. The fields that are blank correspond to parameters that are not used for the variance process. For the explanatory variable X, the US Bloomberg FCI is used. Further, the log-likelihood, Akaike and Schwarz information criteria, variance of standardized returns and the mean absolute errors between square root of the variance process and true variance proxy are shown. AIC and BIC are divided by T, to get a better comparison with the Log-Garch-Midas-X model. The mean absolute errors are multiplied by 100. Next the LR tests for the variance processes are shown.

JPM	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X
$\alpha$	0.071 (0.009)	0.017 (0.007)	0.021 (0.010)	0.015 (0.014)	0.024 (0.016)
$\gamma$		0.083 (0.007)	0.091 (0.020)	0.056 (0.020)	0.099 (0.029)
$\beta$	0.923 (0.009)	0.939 (0.012)	0.915 (0.021)	0.950 (0.033)	0.900 (0.045)
$\zeta$			-6.473 (0.032)		
K			14		
$\kappa_0$				-7.889 (0.442)	
$\kappa_1$				-0.329 (0.110)	
$\eta$					-8.078 (0.063)
$\vartheta$					-0.469 (0.740)
$\rho$					3.565 (1.471)
$\nu$	5.835 (0.573)	6.656 (0.699)	7.055 (0.742)	6.83 (0.752)	6.588 (0.204)
LogL	9,743	9,767	9,773	9,779	9,229
AIC	-5.229	-5.241	-5.237	-5.249	-5.341
BIC	-5.225	-5.235	-5.203	-5.239	-5.329
$\text{Var}(z_t)$	0.968	0.997	1.009	0.990	0.986
MAE-SD	1.246	1.209	1.191	1.225	1.155
<b>LR-test</b>					
$H_0$		$\gamma = 0$		$\kappa_1 = 0$	$\vartheta = 0$
LR		47.662		28.264	9.061
p		0.000		0.000	0.003

Now that the results for the European variance processes are discussed, I compare them to the American results. Table 5 shows the estimation results for the JPM bank<sup>18</sup>. For the JPM bank, the ARCH and GARCH effects show again the same typical pattern with a small  $\alpha$  ranging from 1.5% for the Spline-Garch-X model to 7.1% for the Garch model and a  $\beta$  estimate ranging from 0.900 for the Log-Midas-X model to 0.950 for the Spline-Garch-X model. The leverage effect for the JPM bank is positive for every model. Here the same scenario as for the HSBC bank occurs: lowest value of 0.056 for the Spline-Garch-X model to 0.099 for the Log-Garch-Midas-X model. The sum of  $\alpha + \beta + 0.5\gamma$  is noticeably smaller than 1 for the two component models. The constants for the second component of the models range from -6.473 for the Spline-Garch to -8.078 for the Log-Garch-Midas-X model. The number of knots  $K$  for the Spline-Garch model is 14 for the JPM bank.

Comparing this to the HSBC bank, I notice that the largest banks of EU and US have many cycles. The  $\kappa_1$  estimate for the JPM bank is -0.329, which is the same value as for HSBC. The scaling parameter  $\vartheta$  has a negative value of -0.469. Both coefficients are negative, meaning that both the Spline-Garch-X and the Log-Garch-Midas-X model suggest that worse US financial conditions cause the variance of the J.P.Morgan bank returns to increase on average. The Spline-Garch-X model even suggests that this impact is the same effect that the EU FCI cause the HSBC bank variance to increase. The weighting parameter  $\rho$  takes on the value 3.565, suggesting that observations in the distant past still have an important influence which is less severe than for the HSBC bank. The degrees of freedom  $\nu$  ranges from 5.835 for the Garch model to 7.055 for the Spline-Garch model which implies non-normality and excess kurtosis in the equity returns for the JPM bank returns.

The log likelihood for the JPM variance processes is smaller than for the HSBC bank, ranging from 9743 for Garch to 9779 for the Spline-Garch-X model<sup>19</sup>. The AIC and BIC again indicate that the Log-Midas-X model is preferred to the other variance processes for the JPM bank. However, for the JPM only the Spline-Garch-X is preferred over the one component models. Table 17 in the appendix ranks the AIC and BIC for each US bank and shows the same as in Europe, namely that both the Log-Garch-Midas-X and Spline-Garch-X are the most preferred models. The variances of the standardized returns indicate that the best two component model is the Spline-Garch-X model with a value of 0.990 for JPM. The MAE-SD takes on the smallest values for the Spline-Garch and Log-Garch-Midas-X model. Table 18 in the appendix ranks the mean absolute variance and forecast error for each variance model and indicates the same as in Europe, namely that the Log-Garch-Midas-X has the smallest forecast errors for most American banks. The log likelihood ratio tests give the same conclusions as for the HSBC, namely that the two component models are preferred to the one component models for the 1% significance level with asymmetry in the unexpected returns.

**Table 6:** This table shows the t-statistics of the DMW tests with  $H_0$  of equal forecast accuracy over the whole sample. The variance models on the horizontal axis are statistically compared to the benchmark models on the vertical axis.

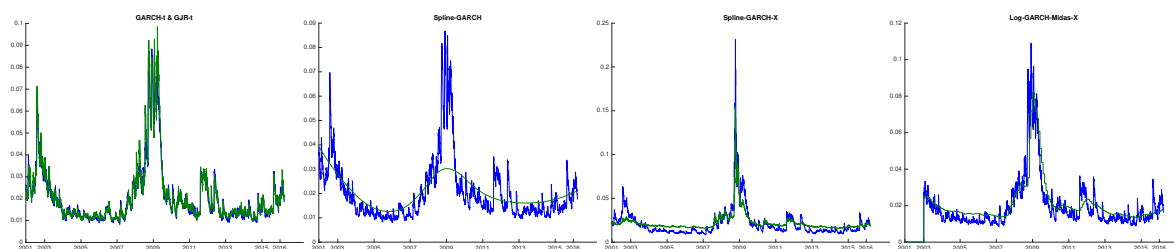
DM	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X
EWMA	3.022	-4.202	-5.664	-0.616	-2.247
Garch		-8.540			
GJR			-6.132	1.493	2.147
Spline-Garch-X					-1.277

The DMW test shows that the two component models outperform the EWMA model, that

<sup>18</sup>Table 16 in the appendix shows the summary of the variance models parameter estimates for the American banks.

<sup>19</sup>9236 when using same time frame as Log-Garch-Midas-X

the GJR is preferred to the EWMA and GARCH models and that the Spline-Garch model outperforms the GJR model. What is shocking that GJR is preferred over Spline-Garch-X and Log-Garch-Midas-X, with the second model being even statistically outperformed. The table also shows that the Log-Garch-Midas-X is preferred to the Spline-Garch-X. The variance ratio using US FCI indicates the same as for the EU case. Here TD Group US Holding shows the same phenomena for the US as Credit Agricole Group for the EU, namely a small economic contribution of almost 0% for the Log-Midas-X model compared to a significant fraction of 66.3% for the Spline-Garch-X model. The Spline-Garch-X is again the most preferred model. The Log-Garch-Midas-X performs very well, despite only using monthly observations.



The four plots in the figure above show the volatilities for the JPM bank returns for the different variance models. The interpretations are the same as for Europe, with the exception for the Spline-Garch-X model where the peak in the crisis period 2009 is much higher than for the other models.

### 4.3 Correlation Results Composite Likelihood EU, US and All

In the two previous subsections the results of the European and American variance processes are discussed and the different variance models are depicted. This subsections interprets the parameters of the DCC and cDCC model using the banks of the EU, US and all of them using all of the two component variance processes as first stage<sup>20</sup>. The composite likelihood is obtained by estimating the parameters with contiguous pairs.

**Table 7:** This table shows the parameters for the different correlation models using the three two component variance processes as first stage. The estimation procedure is composite likelihood with contiguous pairs, where CL is short for composite likelihood. The parameter estimates are based using the whole sample, where for EU all the bank returns in the EU are used. In US, only the US banks are included. Finally ALL uses all of the bank returns.

	Vola.	Spline-Garch		Spline-Garch-X		Log-Garch-Midas-X	
	Corr.	DCC	cDCC	DCC	cDCC	DCC	cDCC
EU	$\psi$	0.033 (0.008)	0.035 (0.008)	0.032 (0.007)	0.035 (0.009)	0.036 (0.009)	0.036 (0.008)
	$\phi$	0.933 (0.021)	0.942 (0.015)	0.932 (0.022)	0.943 (0.021)	0.910 (0.033)	0.927 (0.024)
	CL	19,185	19,184	19,191	19,188	17,919	17,919
US	$\psi$	0.013 (0.004)	0.012 (0.003)	0.015 (0.004)	0.020 (0.02)	0.014 (0.004)	0.015 (0.005)
	$\phi$	0.982 (0.006)	0.988 (0.004)	0.976 (0.007)	0.973 (0.022)	0.979 (0.006)	0.981 (0.006)
	CL	19,185	20,504	20,521	20,520	19,218	19,217
ALL	$\psi$	0.018 (0.007)	0.014 (0.012)	0.019 (0.004)	0.024 (0.008)	0.017 (0.007)	0.019 (0.012)
	$\phi$	0.971 (0.011)	0.984 (0.008)	0.966 (0.008)	0.965 (0.013)	0.970 (0.015)	0.973 (0.019)
	CL	19,925	19,919	19,935	19,933	18,650	18,649

<sup>20</sup>The correlation models using Garch and GJR as first stage are here omitted to preserve space and because we already saw that the two component models are preferred to the one component models.

Table 7 shows the estimated parameters for the correlation models and the corresponding composite likelihood. The estimation for the parameter  $\psi$  ranges in the EU from 0.033 for the DCC-Spline-Garch to 0.036 for the DCC & cDCC-Log-Garch-Midas-X. The lowest values for  $\psi$  are observed in the US, where they range from 0.012 for the cDCC-Spline-Garch to 0.020 for the cDCC-Spline-Garch-X. Inbetween  $\psi$  values are obtained when using all of the bank returns. They range from 0.014 to 0.024, meaning that they are always lower than when only including European bank returns. The parameter  $\psi$  follows the same interpretation idea as the  $\alpha$  from the Garch model, namely that the correlation shock today feeds through into next period's correlation on average less for the American banks than for the European.

The parameter  $\phi$  ranges in EU from 0.910 for the DCC-Log-Midas-X to 0.943 for the cDCC-Spline-Garch-X and is lowest for Log-Garch-Midas-X for both the DCC and cDCC model. The parameters  $\phi$  are higher for the US than for the EU with values ranging from 0.973 for the cDCC-Spline-Garch-X to 0.988 for the cDCC-Spline-Garch. Again, inbetween values are obtained when using all of the bank returns. They range from 0.965 for cDCC-Spline-Garch-X to 0.984, meaning that they are always higher than when only including European bank returns. The interpretation of the high values of  $\phi$  can be compared to that of  $\beta$  in the variance models, namely that the US correlations are more persistence than that of Europe. This table indicates that EU has different comovements than US, indicating that a block structure is needed.

The composite likelihood indicates that when comparing the different variance processes for the different correlation models, the Spline-Garch-X as variance process for the first stage leads to the highest likelihood. For Europe, the CL is 6 larger than DCC-Spline-Garch and 4 larger than for the cDCC-Spline-Garch.<sup>21</sup> For US, the CL for the DCC is much larger than using Spline-Garch or Log-Midas-X as first stage. It is again 4 larger than for the cDCC-Spline-Garch and much larger than cDCC-Log-Midas-X. When using all of the bank returns, the difference is 10 for the DCC and 14 for the cDCC Spline-Garch. A reason that the CL for the US is larger in some cases, could be that the randomly chosen contiguous pairs are not independent. Another reason could be that combining EU and US correlations with randomly chosen pairs can lead to a low CL.

#### 4.4 Results Block Structure in Correlations with Standard QMLE

In this subsection the parameters of the block correlation models are interpreted and compared to the DCC and cDCC model. The estimation procedure is the standard QMLE approach.

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<sup>21</sup>The correlation models using Log-Garch-Midas-X as first stage perform worse because a shorter time period is considered. Therefore I use LR tests to get a better indication which model is preferred with QMLE as estimation procedure in the next subsection.



**Table 8:** This table shows the parameters for the different correlation models using the three two component variance processes as first stage. The parameter estimates are based using the whole sample with all the bank returns included.

Vola. Corr.	Spline-Garch				Spline-Garch-X				Log-Garch-Midas-X			
	DCC	cDCC	Block-DCC	Block-cDCC	DCC	cDCC	Block-DCC	Block-cDCC	DCC	cDCC	Block-DCC	Block-cDCC
$\psi$	0.005 (0.002)	0.005 (0.002)			0.005 (0.001)	0.006 (0.002)			0.005 (0.001)	0.005 (0.001)		
$\psi_{11}$			0.007 (0.003)	0.007 (0.015)			0.007 (0.003)	0.008 (0.003)			0.007 (0.004)	0.007 (0.004)
$\psi_{12}$			0.004 (0.006)	0.004 (0.002)			0.004 (0.003)	0.004 (0.002)			0.004 (0.003)	0.004 (0.003)
$\psi_{22}$			0.006 (0.002)	0.006 (0.002)			0.006 (0.001)	0.007 (0.002)			0.005 (0.001)	0.006 (0.001)
$\phi$	0.983 (0.004)	0.993 (0.002)			0.980 (0.003)	0.987 (0.001)			0.983 (0.001)	0.990 (0.002)		
$\phi_{11}$			0.978 (0.008)	0.978 (0.048)			0.974 (0.012)	0.973 (0.014)			0.974 (0.018)	0.973 (0.021)
$\phi_{12}$			0.981 (0.006)	0.980 (0.03)			0.977 (0.008)	0.976 (0.007)			0.979 (0.011)	0.978 (0.014)
$\phi_{22}$			0.983 (0.001)	0.983 (0.011)			0.981 (0.004)	0.980 (0.005)			0.984 (0.003)	0.982 (0.004)
LogL	277,486	276,409	277,555	277,555	277,614	276,959	277,699	277,717	259,676	258,991	259,748	259,756
MAE	0.333	0.332	0.332	0.332	0.352	0.349	0.351	0.351	0.331	0.331	0.330	0.330

Table 8 shows the estimated parameters for the correlation models. The low values of 0.005 to 0.006 for the estimated  $\psi$  for the DCC and cDCC models can be compared to the values of  $\psi_{11}$ ,  $\psi_{12} = \psi_{21}$  and  $\psi_{22}$  ranging from 0.004 to 0.008 of the block correlation models. The values look like weighted averages, where all of the values are different from eachother. The  $\psi_{11}$  parameter is larger or equal to  $\psi_{22}$ , which is in turn larger or equal to  $\psi_{12} = \psi_{21}$ . This suggests that the EU and US have different correlation regimes and that the block structure was a good extension for the correlation models. The  $\phi$  parameter is lower than when using contiguous pairs.

The  $\phi$  values range from 0.980 to 0.993 for the DCC and cDCC model and can also be compared to the  $\phi_{11}$ ,  $\phi_{12} = \phi_{21}$  and  $\phi_{22}$  values ranging from 0.973 to 0.984 for the block correlation models. These values again look like weighted averages, where all of the values are different from eachother. Now  $\phi_{22}$  is larger or equal to  $\phi_{12} = \phi_{21}$ , which is in turn larger or equal to  $\phi_{11}$ . This again strengthens the suggestion of having different comovements between Europe and America. The  $\psi$  parameter is higher than when using contiguous pairs. The  $\psi$  and  $\phi$  parameters are different than the ones using contiguous pairs, because the parameters are not efficient. Next subsection performs log-likelihood ratio tests to statistically compare these models.

When comparing the log likelihoods, the Block-cDCC-Spline-Garch-X is preferred the most with a LogL of 277,717. The Block-cDCC performs better than the Block-DCC for all of the variance processes. The block correlation models are all preferred to the usual DCC and cDCC correlation models. What is interesting is that the block correlation models with the covariance of the EU and US FCI as explanatory variable perform poorer than without the explanatory variable<sup>22</sup>. This suggests taking another explanatory variable or not using that model to best describe the covariances of the returns of the largest banks.

**Table 9:** This table shows the LR test statistic together with the  $p$ -value for testing if the block structure in the correlation models is necessary.

LR	LR	Spline-Garch		Spline-Garch-X		Log-Garch-Midas-X	
		Block-DCC	Block-cDCC	Block-DCC	Block-cDCC	Block-DCC	Block-cDCC
$H_0$ : No Block structure		136.067	2292.362	170.067	1516.369	144.170	1531.180
	$p$	0.000	0.000	0.000	0.000	0.000	0.000

<sup>22</sup>The results for the block correlation models using explanatory variable can be found in the appendix, table 20 where the elements of  $\xi$  are first multiplied by 1000.

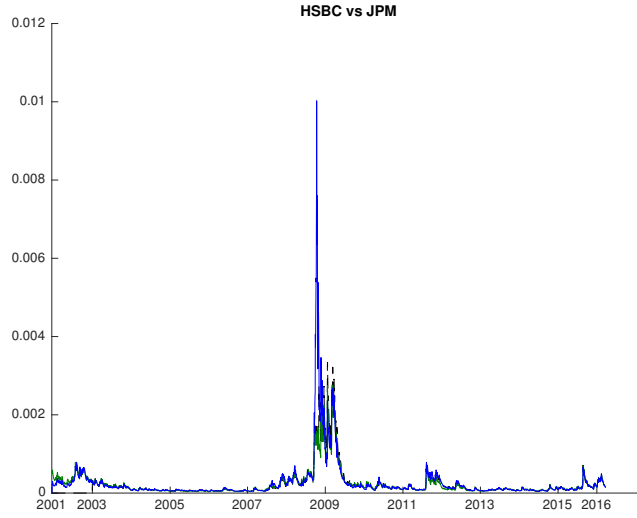
The table above shows the results for the log likelihood ratio test to statistically compare the different correlation models. The first test is:  $H_0 : \psi_{11} = \psi_{12}, \psi_{12} = \psi_{22}$  and  $\psi_{22} = \psi$  and  $\phi_{11} = \phi_{12} = \phi_{22} = \phi$  with  $LR \overset{a}{\sim} \chi^2(6)$ . This test gives p-values of 0.000 for each variance process separately when comparing the Block-DCC against the DCC, meaning that for 1% significance level the block correlation model is preferred. The LR statistic ranges from 136 when using the Spline-Garch variance process at the first stage to 170 for the Spline-Garch-X for the Block-DCC models. The results for comparing the Block-cDCC against the cDCC are more extreme, with p-values again 0.000 for all variance processes, but here the LR statistics range from 1516 for the Spline-Garch-X to 2292 for the Spline-Garch model. The log-likelihood for the Block-DCC-Y is smaller than the Block-DCC, meaning that a LR-test cannot be performed, the same holds true in the cDCC case.

The forecast performance measure of the mean absolute error is smallest when using the Log-Garch-Midas-X model in the first stage and largest when using the Spline-Garch-X model. A Diebold-Mariano-West test is needed to distinguish the results of the absolute errors.

**Table 10:** This table shows the t-statistics of the Diebold-Mariano-West tests with  $H_0$  of equal forecast accuracy over the whole sample. The covariance models on the horizontal axis, with the respective variance models as first stage are statistically compared to the benchmark models on the vertical axis, where No Block structure indicates a test of the block-correlation model against the correlation models without a block structure.

DM	Spline-Garch				Spline-Garch-X				Log-Garch-Midas-X			
	DCC	cDCC	Block-DCC	Block-cDCC	DCC	cDCC	Block-DCC	Block-cDCC	DCC	cDCC	Block-DCC	Block-cDCC
EWMA	-9.511	-10.112	-9.651	-9.632	0.292	-0.060	0.146	0.167	-7.475	-7.659	-7.935	-8.004
GJR	-5.488	-5.215	-5.438	-5.451	-5.234	-4.956	-5.201	-5.212	-5.613	-5.486	-5.530	-5.528
No Block structure			-6.096	-1.283			-6.145	-9.922			-7.324	-3.028

The DMW test shows the results for statistically comparing the absolute errors over time of the different covariance models with the different first stage variance models. An absolute t-statistic greater than 1.96 indicates a rejection of the null hypothesis of equal forecast accuracy at the 0.05 significance level. A negative sign implies that the covariance model produced smaller average losses than the benchmark model and is thus preferred. When comparing the covariance processes to the multivariate EWMA model, both the DCC and cDCC with and without block structure outperform the EWMA model on a 5% level, with the exception when using the Spline-Garch-X as first stage. The DCC and cDCC models with and without a block structure also statistically outperform the diagonal model with GJR processes on the diagonal for each bank on a 5% significance level. The most interesting result of this table is that the block structure in the correlation models is necessary because all of the block correlation models outperform their counterpart without a block structure on a 5% significance level, with the exception of Block-cDCC-Spline-Garch.



**Fig. 3:** The plots depict the correlation processes over the whole sample period for the HSBC bank returns against the JPM returns. Blue line uses the Block-CDCC-Spline-Garch-X, the green line the Block-CDCC-Spline-Garch and in black the Block-DCC-Log-Garch-Midas-X.

The plots in the figure above show the daily in-sample correlations between the HSBC returns and JPM using the Block-CDCC model with all of the two component volatility models as first step separately. The blue line refers to the Block-CDCC-Spline-Garch-X model, green is Spline-Garch and in black the Log-Garch-Midas-X. Two things are observed: the Block-CDCC all follow a very similar path. The second observation is that they only differ a lot in the period of 2009, where the Block-CDCC-Spline-Garch-X shows a large spike. The other two models show also spikes in that period, but less substantial.

### 4.5 Value-at-Risk Backtesting

**Table 11:** The table shows the results for the unconditional coverage, independence and conditional coverage backtests for the VaR95%. The European, American bank returns are shown separately and combined. The second column shows the correlation models using the variance processes shown on the vertical. A minus indicates univariate results. The  $\hat{\pi}$  indicates empirical percentage of violations,  $\hat{\pi}_{01} = T_{01}/(T_{00} + T_{01})$  and  $\hat{\pi}_{11} = T_{11}/(T_{10} + T_{11})$  are the ML estimates of  $\pi_{01}$  and  $\pi_{11}$ , while  $p_{uc}$ ,  $p_{ind}$  and  $p_{cc}$  show the p-values for respectively the  $LR_{uc}$ ,  $LR_{ind}$  and  $LR_{cc}$  tests which are respectively asymptotically chi-distributed with one, one and 2 degrees of freedom.

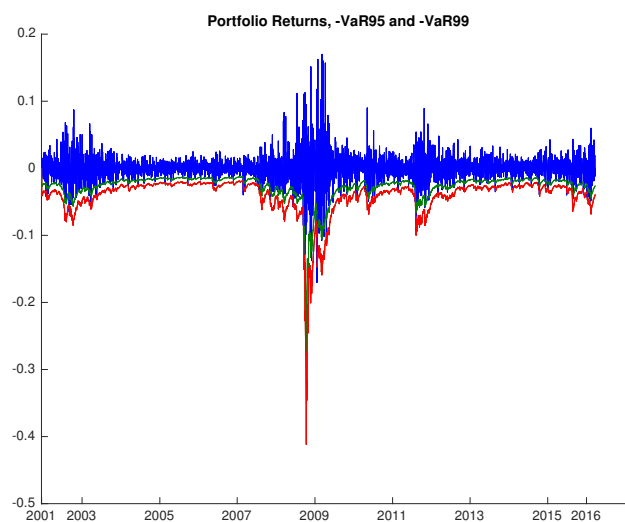
Model	Spline-Garch						Spline-Garch-X						Log-Garch-Midas-X					
	$\hat{\pi}$	$\hat{\pi}_{01}$	$\hat{\pi}_{11}$	$p_{uc}$	$p_{ind}$	$p_{cc}$	$\hat{\pi}$	$\hat{\pi}_{01}$	$\hat{\pi}_{11}$	$p_{uc}$	$p_{ind}$	$p_{cc}$	$\hat{\pi}$	$\hat{\pi}_{01}$	$\hat{\pi}_{11}$	$p_{uc}$	$p_{ind}$	$p_{cc}$
-	0.050	0.043	0.050	0.925	0.673	0.911	0.049	0.033	0.049	0.809	0.266	0.524	0.049	0.036	0.049	0.775	0.384	0.658
EU DCC	0.049	0.038	0.049	0.806	0.469	0.747	0.049	0.038	0.049	0.868	0.448	0.740	0.049	0.036	0.049	0.775	0.384	0.658
cDCC	0.048	0.039	0.048	0.532	0.576	0.706	0.049	0.038	0.049	0.751	0.489	0.749	0.049	0.041	0.049	0.836	0.609	0.859
-	0.046	0.052	0.046	0.314	0.726	0.568	0.045	0.054	0.045	0.142	0.575	0.292	0.046	0.044	0.046	0.280	0.900	0.556
US DCC	0.044	0.061	0.044	0.104	0.322	0.164	0.045	0.059	0.045	0.189	0.398	0.297	0.045	0.045	0.045	0.161	0.987	0.377
cDCC	0.044	0.061	0.044	0.074	0.288	0.116	0.046	0.064	0.046	0.247	0.265	0.276	0.045	0.045	0.045	0.138	0.959	0.335
-	0.048	0.045	0.048	0.532	0.854	0.811	0.048	0.051	0.048	0.486	0.834	0.769	0.048	0.048	0.048	0.657	0.977	0.908
DCC	0.046	0.035	0.046	0.314	0.432	0.444	0.048	0.034	0.048	0.486	0.359	0.516	0.047	0.043	0.047	0.358	0.844	0.645
cDCC	0.046	0.035	0.046	0.314	0.432	0.444	0.046	0.035	0.046	0.247	0.471	0.396	0.047	0.043	0.048	0.496	0.762	0.760
ALL Block-DCC	0.047	0.045	0.047	0.437	0.907	0.736	0.047	0.051	0.047	0.395	0.780	0.672	0.047	0.043	0.047	0.447	0.789	0.725
Block-cDCC	0.047	0.045	0.047	0.437	0.907	0.736	0.047	0.051	0.047	0.395	0.780	0.672	0.047	0.043	0.047	0.447	0.789	0.725
Block-DCC-Y	0.048	0.045	0.048	0.532	0.854	0.811	0.047	0.045	0.047	0.439	0.907	0.738	0.047	0.043	0.048	0.496	0.762	0.760
Block-cDCC-Y	0.048	0.045	0.048	0.483	0.880	0.775	0.047	0.045	0.047	0.439	0.907	0.738	0.047	0.043	0.048	0.496	0.762	0.760

Table 11 shows the backtest results for the VaR95% using all of the variance and correlation processes for separately EU, US and using both combined (ALL)<sup>23</sup>. For the unconditional coverage test the empirical percentage of violations  $\hat{\pi}$ , varies between 4.4% to 5%, which comes close to the 5% which they are compared to. The VaR uc backtests have the lowest

<sup>23</sup>In the appendix table 21 shows the same table for VaR99%

$\hat{\pi}$  for the US with values of 4.4% to 4.6% and fails the unconditional coverage for the cDCC model at 10% significance level using the Spline-Garch model at first stage. The DCC and cDCC are too conservative and lead to more violations than expected especially for the US and ALL. The univariate approach using only the EU returns give percentage violations of 4.9% to 5%. Now these results are compared to the backtest results of VaR99%. The p-values range between 24.8% and 96.7% which is significantly larger than the significance levels. In the EU, the empirical percentage of violations come closest to the 1%, whereas using all of the banks, these values come as low as 0.8%. The block models have the lowest violation percentages, indicating that these models are again preferred. For the independence test, the results show that the difference between  $\hat{\pi}_{01}$  and  $\hat{\pi}_{11}$  vary between 0.000 and 0.017, indicating that the hit sequences are independent in some cases, when the difference is almost 0. On a 5% significance level, non of the differences are significantly different from 0. This means that the null hypothesis of independence cannot be rejected, meaning that there is no volatility clustering. When using the 1% level, the p-values for the independence test are substantially lower, rejecting the null for all of the variance processes for EU and US and for the block correlation models when using Spline-Garch-X or Log-Garch-Midas-X as first stage. Due to the lack of observations at a 1% violation level, the results can be biased or obtained through luck. Finally for the conditional coverage test, the results show that p-values range from 0.116 to 0.911, which is larger than the 5% level, indicating that the correlation models do not violate the independence property and supply the just coverage rate. For the 1% level, the p-values are again much smaller, rejecting the null when the Log-Garch-Midas-X model is used as first stage. In Europe, the p-values for all of the tests are much larger than for the US, indicating that the VaR estimates are preferred for Europe.

The previous results showed that the Block-cDCC-Spline-X is the most preferred model. Therefore I depict the -VaR estimates over the whole sample using the Block-cDCC-Spline-X for the correlation process against the portfolio returns  $r_t^P$  using all of the EU & US bank returns<sup>24</sup>.



**Fig. 4:** This figure shows the portfolio returns using equally weights over all the banks over the full data sample using the Block-cDCC-Spline-X as correlation model. The blue line represents the portfolio returns, the green line the -VaR95% estimates and the red line represents the -VaR99% estimates.

Looking at the figure, the VaR value for the crisis end 2008 show a large outlier, indicating overestimation. Overall the VaR seems to give accurate results of the number of violations.

<sup>24</sup>The VaR plots for the other models are very similar and therefore are left out.

It is not possible to judge whether this model is very good because the true correlations are not observable.

#### 4.6 Realized Variance and FCI Components

In the previous results the Log-Garch-Midas-X model performed almost as good as the Spline-Garch-X model when using the EU/US FCI as explanatory variable. In this section two things are done: at first monthly realized variance as explanatory variable is compared to the monthly FCI for the Log-Garch-Midas-X model. Second, the components of the equity market for the EU and US FCI are used as explanatory variables together with the FCI without the effects of these components. Table 12 shows the parameter estimates for these models for the HSBC and JPM banks representing both EU and US<sup>25</sup>.

**Table 12:** This table shows the parameters for the different correlation models using the variance processes: Log-Garch-Midas-RV, Log-Garch-Midas-BigX and Spline-Garch-BigX as first stage for both the HSBC bank and JPM bank. RV stands for monthly Realized Variance and BigX stands for using the bloomberg components separately.

	HSBC			JPM		
	Log-Midas-RV	Log-Midas-BigX	Spline-BigX	Log-Midas-RV	Log-Midas-BigX	Spline-BigX
$\alpha$	0.048 (0.013)	0.042 (0.024)	0.050 (0.034)	0.023 (0.01)	0.033 (0.014)	0.017 (0.008)
$\gamma$	0.054 (0.018)	0.081 (0.045)	0.023 (0.119)	0.081 (0.016)	0.111 (0.026)	0.037 (0.016)
$\beta$	0.916 (0.018)	0.875 (0.027)	0.901 (0.056)	0.935 (0.012)	0.859 (0.041)	0.962 (0.013)
$\eta$	-8.200 (5.07)	-8.506 (0.13)		-6.602 (1.354)	-8.235 (0.12)	
$\vartheta_1$	-4.720 (0.482)	-0.254 (0.072)		-2.957 (0.662)	-0.507 (0.045)	
$\rho_1$	83.991 (27.36)	1.793 (1.229)		97.348 (15.396)	3.270 (0.733)	
$\vartheta_2$		0.063 (0.083)			-0.216 (0.039)	
$\rho_2$		2.437 (1.18)			33.619 (5.689)	
$\vartheta_3$		0.072 (0.291)			-0.222 (0.069)	
$\rho_3$		21.944 (14.78)			56.624 (7.133)	
$\kappa_0$			-8.674 (0.121)			-7.182 (0.316)
$\kappa_1$			-0.151 (0.119)			-0.250 (0.074)
$\kappa_2$			-0.140 (0.078)			-0.495 (0.22)
$\kappa_3$			0.326 (0.175)			-0.028 (0.024)
$\nu$	5.582 (0.33)	5.804 (0.544)	6.286 (0.628)	6.414 (0.656)	6.954 (0.859)	5.954 (0.488)
LogL	10219	10228	10964	9225	9228	9780
AIC	-5.915	-5.918	-5.884	-5.327	-5.338	-5.248
BIC	-5.902	-5.898	-5.870	-5.327	-5.319	-5.235
Var( $z_t$ )	0.990	0.990	0.990	0.998	0.998	0.980

Comparing the two Log-Garch-Midas-X models with respectively monthly RV and monthly FCI as explanatory variable, the same pattern is observed for Europe and America. The leverage effect  $\gamma$  declines, while the  $\beta$  increases, implying that the RV absorbs some of this impact.

<sup>25</sup>The other banks are available upon request and are left out due to insufficient space while adding little extra information

The scaling parameter  $\vartheta$  estimates show negative values for both HSBC and JPM for both models. The value declines from FCI to RV, indicating that an increase in the RV is associated with a bigger decrease in the long-term variance than the same size increase in FCI. The value of the log-likelihood decreases in the model with the monthly RV as explanatory variable.

For the Log-Midas-BigX model with as explanatory variables: cleaned European FCI, standardized Eurostoxx50 and standardized VDAX, the scaling parameter  $\vartheta$  belonging to the FCI increases, while the parameters for the other components are both positive (parameter for Eurostoxx50 is not significantly different from zero). For the Spline-Garch-BigX model, the same phenomena holds true where the  $\kappa_1$  increases from using EU FCI as explanatory variable to using the three variables. The parameter for the standardized Eurostoxx50 is negative while the parameter for the standardized VDAX is positive, indicating that information is lost when using only the FCI instead of splitting them. For America with cleaned US FCI, standardized S&P500 and standardized VIX, the scaling parameter for cleaned FCI decreases while the parameters for the other two components are both negative. So here the VIX has the opposite effect on the variance as the VDAX for Europe. The  $\kappa_1$  increases from using US FCI as explanatory variable. The parameter for the standardized S&P500 is again negative as in Europe, while the parameter for the standardized VIX is negative for the US as in the Log-Garch-Midas-BigX model for JPM.

## 5 Conclusion

This research studies volatilities and correlation models in the financial sector. The one component models Garch and GJR are compared to the two component models: Spline-Garch, Spline-Garch-X and Log-Garch-Midas-X, where the last two include the EU & US financial condition indexes as explanatory variable. A block structure in the correlation matrices is considered and statistically compared. The data that I use is daily stock prices of the EU & US largest banks during the period 2001 to 2016, where the different currencies are all converted to Euros, by considering the corresponding exchange rates. The financial conditions are proxied by the Bloomberg EU and US FCI.

The results show that a block structure in the correlations is needed in order to capture the EU, US and cross-wise correlations better in both an economic and statistical way. Next, incorporating financial conditions indexes has a significant affect on the variance processes both statistically and economically. Specifically, variances go up when financial conditions get worse. The two component models are preferred to the one component in a statistical way. The second component of the Spline-Garch model which captures the long-term effects, captures the cycles nicely for both EU & US. The second component of the two component models with explanatory variables is also able to capture the effects through the FCI variable. Using the financial conditions is preferred to using realized variances as explanatory variable. Lastly, splitting the components of the FCI adds economic information.

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## A Sample Correlation of EU & US returns

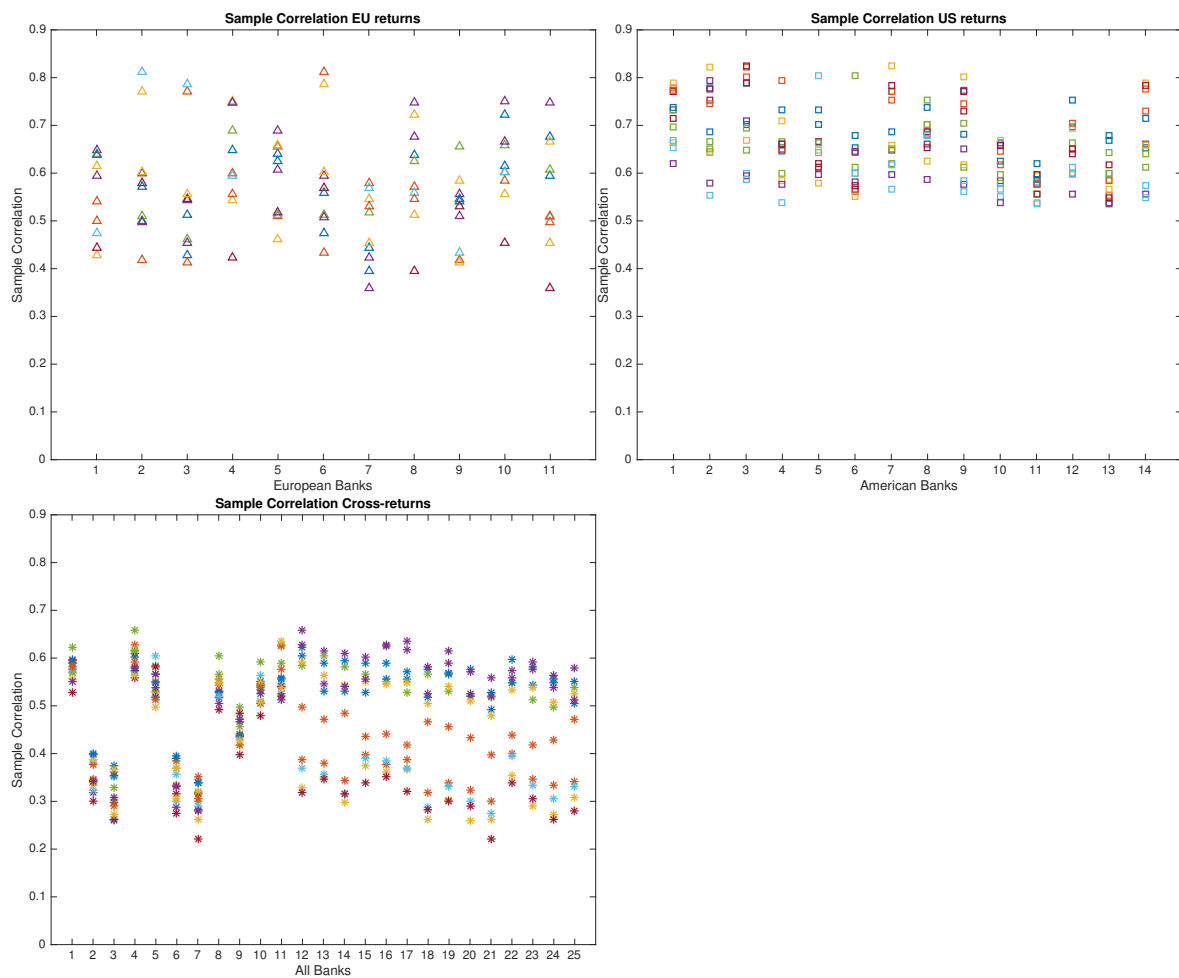


Fig. 5: Here the sample correlations are plotted separately.

## B Bloomberg FCI of EU & US

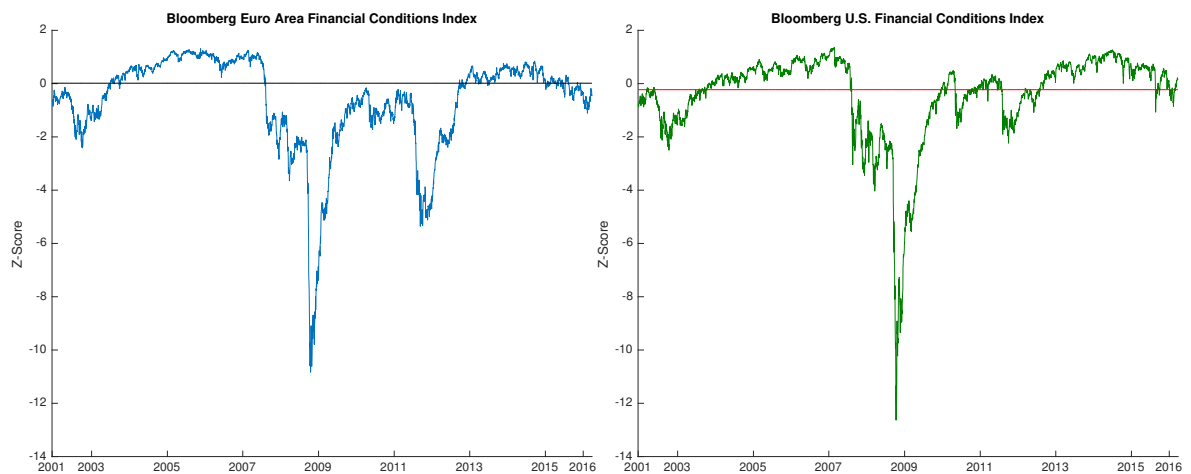


Fig. 6: The EU and US FCI separately.

## C European Variance Processes

**Table 13:** This table shows the summary of parameter estimates and performance measures for the different variance processes using all of the European banks. There are 3 numbers for each parameter/measure which denote from above to below the minimum, average and maximum.

	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X
$\alpha$	0.057	0.008	0.009	0.011	0.006
	0.079	0.023	0.021	0.030	0.022
	0.107	0.045	0.040	0.066	0.045
$\gamma$		0.054	0.069	0.033	0.066
		0.090	0.103	0.076	0.101
		0.132	0.149	0.112	0.159
$\beta$	0.893	0.917	0.874	0.838	0.859
	0.919	0.930	0.911	0.909	0.906
	0.939	0.945	0.941	0.947	0.948
$\zeta$			-8.590		
			-7.638		
			-7.110		
K			2		
			8		
			14		
$\kappa_0$				-8.701	
				-7.811	
				-6.983	
$\kappa_1$				-0.440	
				-0.344	
				-0.220	
$\eta$					-8.659
					-7.651
					-6.579
$\vartheta$					-0.523
					-0.292
					0.136
$\rho$					1.776
					16.854
					90.952
$\nu$	5.114	5.284	4.680	5.516	5.252
	6.937	6.606	6.593	6.898	6.643
	9.751	9.140	9.597	9.607	9.224
LogL	8,878	8,895	8,900	8,912	8,286
	9,351	9,380	9,392	9,391	8,774
	10,921	10,929	10,945	10,945	10,225
AIC	-5.862	-5.865	-5.866	-5.875	-5.918
	-5.019	-5.033	-5.035	-5.040	-5.078
	-4.764	-4.773	-4.771	-4.783	-4.795
BIC	-5.857	-5.859	-5.835	-5.865	-5.906
	-5.014	-5.027	-5.011	-5.030	-5.065
	-4.760	-4.767	-4.748	-4.773	-4.783
Var( $z_t$ )	0.959	0.985	0.936	0.986	0.986
	1.003	0.997	0.982	1.005	1.001
	1.078	1.008	1.018	1.019	1.043

**Table 14:** This table ranks the variance models for each of the largest European bank based on the Akaike and Schwarz information criteria, with 1 being the best model with the lowest AIC or BIC.

	AIC					BIC				
	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X
HSBC	5	4	3	2	1	4	3	5	2	1
BNP.PA	5	4	3	2	1	5	3	4	2	1
ACA.PA	5	4	2	3	1	5	3	4	2	1
DBK.DE	5	3	4	2	1	4	3	5	2	1
BCS	5	3	4	2	1	5	3	4	2	1
GLE.PA	5	4	2	3	1	5	3	4	2	1
RBS.L	5	3	4	2	1	4	3	5	2	1
SAN	5	4	3	2	1	5	3	4	2	1
UCGMI	5	3	4	2	1	4	3	5	2	1
ING	5	4	3	2	1	5	3	4	2	1
CS	5	4	3	2	1	4	3	5	2	1

**Table 15:** This table ranks the variance models for each of the largest European bank based on the absolute distance between the variance of the standardized returns and 1 and based on the MAE-SD. The number 1 indicates the most preferred model, with the lowest absolute distance and/or lowest MAE-SD.

	Var( $z_t$ )-1					MAE-SD				
	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X
HSBC	5	2	1	3	4	5	4	1	3	2
BNP.PA	3	2	5	4	1	5	4	2	3	1
ACA.PA	4	1	3	2	5	5	4	1	3	2
DBK.DE	2	1	5	3	4	5	3	4	2	1
BCS	4	3	5	1	2	3	1	4	5	2
GLE.PA	1	2	3	4	5	5	4	3	2	1
RBS.L	4	1	5	2	3	5	2	4	3	1
SAN	2	3	1	4	5	5	4	2	3	1
UCGMI	5	1	4	3	2	1	4	5	2	3
ING	5	1	4	2	3	5	4	2	3	1
CS	4	3	2	1	5	5	4	2	3	1

## D American Variance Processes

**Table 16:** This table shows the summary of parameter estimates and performance measures for the different variance processes using all of the American banks. There are 3 numbers for each parameter/measure which denote from above to below the minimum, average and maximum.

	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X
$\alpha$	0.039	0.009	0.004	0.000	0.006
	0.077	0.025	0.023	0.019	0.027
	0.144	0.052	0.048	0.050	0.054
$\gamma$		0.039	0.052	0.022	0.054
		0.077	0.092	0.051	0.092
		0.113	0.126	0.099	0.165
$\beta$	0.832	0.912	0.809	0.873	0.840
	0.918	0.932	0.907	0.938	0.898
	0.957	0.955	0.948	0.982	0.959
$\zeta$			-9.191		
			-7.166		
			-6.208		
K			5		
			10		
			15		
$\kappa_0$				-8.799	
				-7.890	
				-6.751	
$\kappa_1$				-0.451	
				-0.370	
				-0.295	
$\eta$					-8.485
					-7.916
					-6.683
$\vartheta$					-0.556
					-0.362
					0.063
$\rho$					1.260
					20.880
					84.498
$\nu$	4.459	4.560	4.717	4.650	4.560
	5.919	6.167	6.052	6.521	6.062
	10.000	7.353	7.500	7.760	7.290
LogL	8,954	8,974	8,981	8,991	8,476
	9,749	9,770	9,783	9,789	9,176
	10,998	11,017	11,035	11,039	10,314
AIC	-5.903	-5.912	-5.917	-5.926	-5.970
	-5.232	-5.243	-5.244	-5.254	-5.310
	-4.805	-4.815	-4.812	-4.826	-4.905
BIC	-5.898	-5.906	-5.892	-5.915	-5.957
	-5.228	-5.237	-5.217	-5.244	-5.298
	-4.801	-4.809	-4.782	-4.816	-4.893
Var( $z_t$ )	0.931	0.977	0.920	0.967	0.955
	0.992	0.998	0.987	1.001	0.994
	1.042	1.043	1.045	1.029	1.036

**Table 17:** This table ranks the variance models for each of the largest American bank based on the Akaike and Schwarz information criteria, with 1 being the best model with the lowest AIC or BIC.

	AIC					BIC				
	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Midas-X	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Midas-X
JPM	5	3	4	2	1	4	3	5	2	1
BAC	5	4	3	2	1	4	3	5	2	1
WFC	5	4	3	2	1	5	3	4	2	1
C	5	4	3	2	1	4	3	5	2	1
GS	5	4	3	2	1	5	3	4	2	1
MS	5	3	4	2	1	4	3	5	2	1
USB	5	3	4	2	1	4	3	5	2	1
BK	5	3	4	2	1	4	3	5	2	1
PNC	5	4	3	2	1	4	3	5	2	1
COF	5	4	3	2	1	4	3	5	2	1
TD	5	4	3	2	1	4	3	5	2	1
STT	5	4	3	2	1	4	3	5	2	1
SCHW	5	3	4	2	1	4	3	5	2	1
STI	5	3	4	1	2	4	3	5	1	2

**Table 18:** This table ranks the variance models for each of the largest American bank based on the absolute distance between the variance of the standardized returns and 1 and based on the MAE-SD. The number 1 indicates the most preferred model, with the lowest absolute distance and/or lowest MAE-SD.

	Var( $\hat{z}_t$ )-1					MAE-SD				
	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Midas-X	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Midas-X
JPM	5	1	2	3	4	5	3	2	4	1
BAC	2	1	3	4	5	5	4	1	3	2
WFC	5	2	4	3	1	5	2	4	1	3
C	4	3	5	1	2	5	4	1	3	2
GS	5	4	3	2	1	5	3	2	4	1
MS	2	1	5	4	3	5	2	3	4	1
USB	3	2	5	4	1	4	2	5	3	1
BK	4	2	5	1	3	4	2	3	5	1
PNC	1	3	2	4	5	5	4	2	3	1
COF	3	4	5	2	1	4	3	2	5	1
TD	3	5	4	1	2	5	4	2	3	1
STT	2	1	5	4	3	5	4	1	3	2
SCHW	1	2	3	4	5	4	3	2	5	1
STI	3	2	5	4	1	4	2	5	1	3

## E Combined Variance Processes

**Table 19:** This table shows the summary of parameter estimates and performance measures for the different variance processes using all of the banks. There are 3 numbers for each parameter/measure which denote from above to below the minimum, average and maximum.

	Garch	GJR	Spline-Garch	Spline-Garch-X	Log-Garch-Midas-X
$\alpha$	0.039	0.008	0.004	0.000	0.006
	0.078	0.024	0.022	0.024	0.025
	0.144	0.052	0.048	0.066	0.054
$\gamma$		0.039	0.052	0.022	0.054
		0.083	0.097	0.063	0.096
		0.132	0.149	0.112	0.165
$\beta$	0.832	0.912	0.809	0.838	0.840
	0.918	0.931	0.909	0.924	0.902
	0.957	0.955	0.948	0.982	0.959
$\zeta$			-9.191		
			-7.347		
			-6.208		
K			2		
			9		
			15		
$\kappa_0$				-8.799	
				-7.850	
				-6.751	
$\kappa_1$				-0.451	
				-0.360	
				-0.220	
$\eta$					-8.659
					-7.795
					-6.579
$\vartheta$					-0.556
					-0.330
					0.136
$\rho$					1.260
					19.034
					90.952
$\nu$	4.459	4.560		4.650	4.560
	6.367	6.360		6.642	6.328
	10.000	9.140		9.607	9.224
LogL	8,878	8,895	8,900	8,912	8,286
	9,574	9,598	9,611	9,610	8,991
	10,998	11,017	11,035	11,039	10,314
AIC	-5.903	-5.912	-5.917	-5.926	-5.970
	-5.138	-5.151	-5.152	-5.158	-5.204
	-4.764	-4.773	-4.771	-4.783	-4.795
BIC	-5.898	-5.906	-5.892	-5.915	-5.957
	-5.134	-5.145	-5.126	-5.148	-5.191
	-4.760	-4.767	-4.748	-4.773	-4.783
Var( $z_t$ )	0.931	0.977	0.920	0.967	0.955
	0.997	0.997	0.985	1.002	0.997
	1.078	1.043	1.045	1.029	1.043

## F Correlation Processes Parameter Estimates

**Table 20:** This table shows the parameters for the different correlation models using the three two component variance processes as first stage. The parameter estimates are based using the whole sample with all the bank returns included.

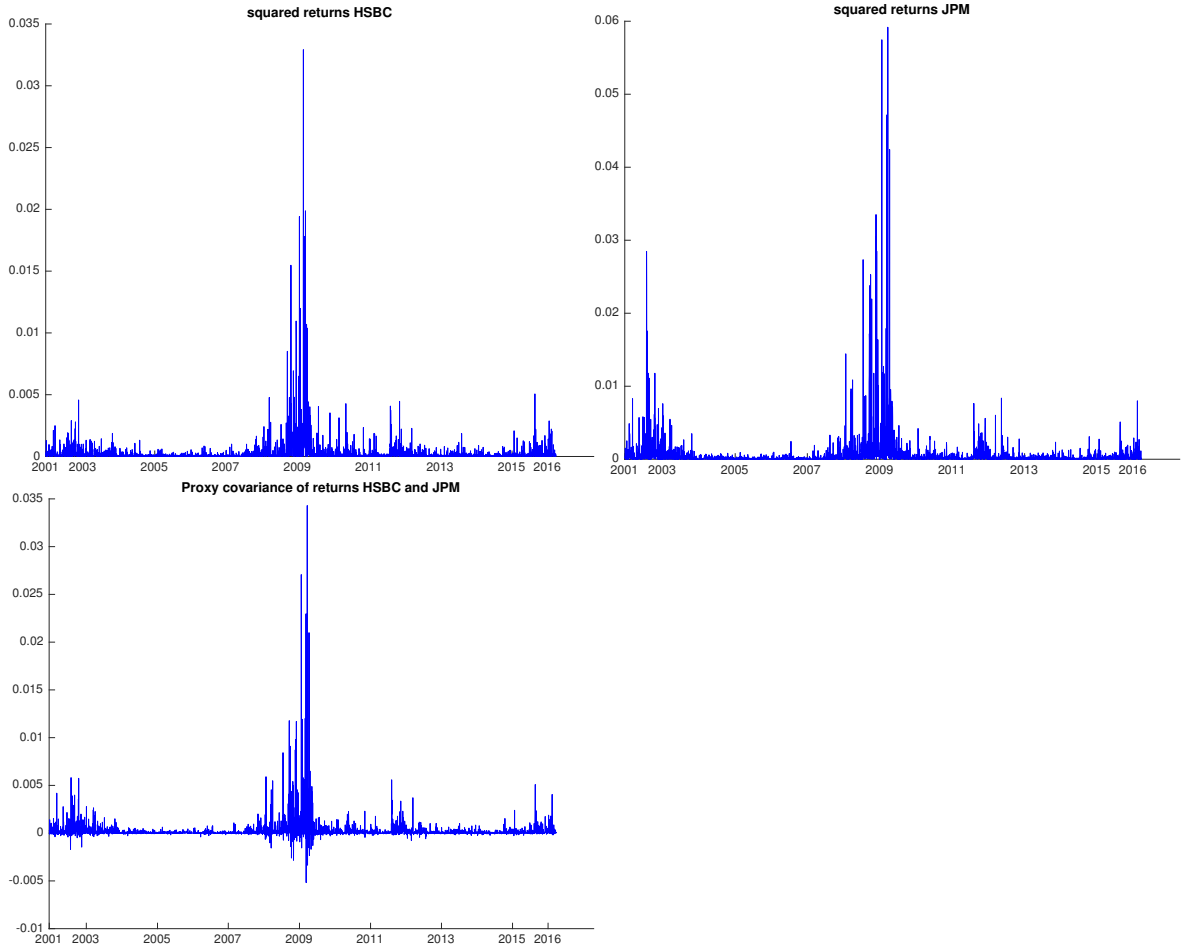
Vola. Corr.	Spline-Garch				Spline-Garch-X				Log-Garch-Midas-X			
	Block-DCC	Block-cDCC	Block-DCC-Y	Block-cDCC-Y	Block-DCC	Block-cDCC	Block-DCC-Y	Block-cDCC-Y	Block-DCC	Block-cDCC	Block-DCC-Y	Block-cDCC-Y
$\psi_{11}$	0.007 (0.003)	0.007 (0.015)	0.005 (0.002)	0.005 (0.005)	0.007 (0.003)	0.008 (0.003)	0.005 (0.003)	0.005 (0.001)	0.007 (0.004)	0.007 (0.004)	0.005 (0.003)	0.005 (0.003)
$\psi_{12}$	0.004 (0.006)	0.004 (0.002)	0.005 (0.002)	0.005 (0.005)	0.004 (0.003)	0.004 (0.002)	0.005 (0.002)	0.005 (0.002)	0.004 (0.003)	0.004 (0.003)	0.005 (0.002)	0.005 (0.003)
$\psi_{22}$	0.006 (0.002)	0.006 (0.002)	0.005 (0.004)	0.005 (0.001)	0.006 (0.001)	0.007 (0.002)	0.005 (0.001)	0.005 (0.003)	0.005 (0.001)	0.006 (0.001)	0.005 (0.002)	0.005 (0.002)
$\phi_{11}$	0.978 (0.008)	0.978 (0.048)	0.984 (0.009)	0.984 (0.03)	0.974 (0.012)	0.973 (0.014)	0.980 (0.011)	0.981 (0.004)	0.974 (0.018)	0.973 (0.021)	0.983 (0.010)	0.983 (0.011)
$\phi_{12}$	0.981 (0.006)	0.980 (0.03)	0.984 (0.017)	0.984 (0.004)	0.977 (0.008)	0.976 (0.007)	0.980 (0.006)	0.981 (0.007)	0.979 (0.011)	0.978 (0.014)	0.983 (0.006)	0.983 (0.009)
$\phi_{22}$	0.983 (0.001)	0.983 (0.011)	0.984 (0.040)	0.984 (0.028)	0.981 (0.004)	0.980 (0.005)	0.980 (0.007)	0.981 (0.01)	0.984 (0.003)	0.982 (0.004)	0.983 (0.010)	0.983 (0.008)
$\xi_{11}$			0.210 (0.594)	0.214 (0.652)			0.001 (0.268)	0.001 (0.257)			0.026 (0.364)	0.050 (0.354)
$\xi_{12}$			0.209 (0.475)	0.214 (0.306)			0.001 (0.313)	0.001 (0.325)			0.026 (0.200)	0.050 (0.207)
$\xi_{22}$			0.208 (0.768)	0.214 (0.536)			0.001 (0.348)	0.001 (0.35)			0.026 (0.337)	0.050 (0.345)
LogL	277,555	277,555	277,513	277,524	277,699	277,717	277,615	277,633	259,748	259,756	259,677	259,688

## G Backtesting VaR 99%

**Table 21:** The table shows the results for the unconditional coverage, independence and conditional coverage backtests for the VaR99%. The European, American bank returns are shown separately and combined. The second column shows the correlation models using the variance processes shown on the vertical. A minus indicates univariate results. The  $\hat{\pi}$  indicates empirical percentage of violations,  $\hat{\pi}_{01} = T_{01}/(T_{00} + T_{01})$  and  $\hat{\pi}_{11} = T_{11}/(T_{10} + T_{11})$  are the ML estimates of  $\pi_{01}$  and  $\pi_{11}$ , while  $p_{uc}$ ,  $p_{ind}$  and  $p_{cc}$  show the p-values for respectively the  $LR_{uc}$ ,  $LR_{ind}$  and  $LR_{cc}$  tests which are respectively asymptotically chi-distributed with one, one and 2 degrees of freedom.

Model	Spline-Garch							Spline-Garch-X						Log-Garch-Midas-X					
	$\hat{\pi}$	$\hat{\pi}_{01}$	$\hat{\pi}_{11}$	$p_{uc}$	$p_{ind}$	$p_{cc}$		$\hat{\pi}$	$\hat{\pi}_{01}$	$\hat{\pi}_{11}$	$p_{uc}$	$p_{ind}$	$p_{cc}$	$\hat{\pi}$	$\hat{\pi}_{01}$	$\hat{\pi}_{11}$	$p_{uc}$	$p_{ind}$	$p_{cc}$
-	0.010	0.000	0.010	0.967	0.389	0.689		0.010	0.028	0.010	0.837	0.362	0.647	0.010	0.000	0.010	0.928	0.411	0.710
EU DCC	0.010	0.000	0.010	0.967	0.389	0.689		0.010	0.027	0.010	0.968	0.384	0.684	0.010	0.000	0.010	0.928	0.411	0.710
cDCC	0.010	0.000	0.010	0.967	0.389	0.689		0.009	0.000	0.009	0.709	0.415	0.670	0.010	0.000	0.010	0.928	0.411	0.710
-	0.010	0.056	0.010	0.836	0.050	0.143		0.010	0.056	0.010	0.837	0.050	0.143	0.009	0.067	0.009	0.428	0.027	0.064
US DCC	0.009	0.029	0.009	0.587	0.320	0.527		0.009	0.059	0.009	0.588	0.039	0.103	0.009	0.065	0.009	0.539	0.031	0.081
cDCC	0.009	0.000	0.009	0.587	0.429	0.631		0.009	0.057	0.009	0.709	0.044	0.124	0.009	0.065	0.009	0.539	0.031	0.081
-	0.009	0.059	0.009	0.587	0.039	0.103		0.009	0.057	0.009	0.709	0.044	0.124	0.009	0.063	0.009	0.661	0.036	0.100
DCC	0.009	0.030	0.009	0.475	0.300	0.453		0.009	0.029	0.009	0.588	0.320	0.527	0.010	0.061	0.010	0.792	0.041	0.119
cDCC	0.009	0.030	0.009	0.475	0.300	0.453		0.009	0.030	0.009	0.476	0.300	0.454	0.009	0.063	0.009	0.661	0.036	0.100
ALL Block-DCC	0.008	0.032	0.008	0.289	0.261	0.304		0.009	0.057	0.009	0.709	0.044	0.124	0.008	0.071	0.008	0.248	0.020	0.035
Block-cDCC	0.008	0.032	0.008	0.289	0.261	0.304		0.009	0.057	0.009	0.709	0.044	0.124	0.008	0.071	0.008	0.248	0.020	0.035
Block-DCC-Y	0.008	0.032	0.008	0.289	0.261	0.304		0.009	0.057	0.009	0.709	0.044	0.124	0.009	0.067	0.009	0.428	0.027	0.064
Block-cDCC-Y	0.008	0.032	0.008	0.289	0.261	0.304		0.009	0.057	0.009	0.709	0.044	0.124	0.009	0.067	0.009	0.428	0.027	0.064

## H Proxy of Covariance HSBC and JPM



**Fig. 7:** The squared returns for respectively HSBC and JPM bank. The third figure represents a proxy for the covariance between the returns of HSBC and JPM.

## I Optimization Method Variance Processes

In this section of the appendix, I derive the optimization method used for the Variance processes: Spline-Garch, Spline-Garch-X and Garch-Midas-X.

Maximum log-likelihood estimation is used where the bank returns are assumed to follow a student-t distribution with the number of freedoms being the number of parameters minus one, this negative log likelihood function is defined as:

$$-\log L(\theta) = \frac{1}{2} \sum_{t=1}^T \left[ (v+1) \log \left( 1 + \frac{\epsilon_t^2}{\tau_t g_t (v-2)} \right) + \log(\tau_t g_t) \right] - T \log \left( \frac{\Gamma((v+1)/2)}{\Gamma(v/2) \sqrt{\pi(v-2)}} \right) \quad (34)$$

where  $\theta$  represents the variance process parameters,  $\Gamma$  is the gamma density function,  $v$  is denoted as the number of freedoms.

Now Newton's Method is used as an optimization approximation method on  $\theta$ :

$$\theta_{k+1} = \theta_k - J^{-1}(\theta_k) \nabla L(\theta_k) \quad (35)$$



with gradient  $\nabla L = \frac{\partial L}{\partial \theta}$  and Fisher Information matrix  $J = E \left( \frac{\partial^2 L}{\partial \theta \partial \theta^T} \right)$ .

Now guess initial starting values for the Garch-Midas-X model:  $\theta_0 = [\omega_0, \alpha_0, \gamma_0, \beta_0, m_0, \vartheta_0, w_0]$ . Next calculate  $\partial \tau_t / \partial \theta$  and  $\partial g_t / \partial \theta$  for every parameter separately. These are then used to calculate the gradient. Specifically:

$$\frac{\partial \tau_t}{\partial \alpha} = 0, \frac{\partial \tau_t}{\partial \gamma} = 0, \frac{\partial \tau_t}{\partial \beta} = 0 \quad (36)$$

$$\frac{\partial \tau_t}{\partial \vartheta} = \tau_t \sum_{k=1}^K \lambda_k (\omega_1 \omega_2) X_{t-k} \quad (37)$$

$$\frac{\partial \tau_t}{\partial m} = \tau_t \quad (38)$$

$$\frac{\partial g_t}{\partial \alpha} = -1 + \frac{\epsilon_{t-1}^2}{\tau_{t-1}} + \beta \frac{\partial g_{t-1}}{\partial \alpha} \quad (39)$$

$$\frac{\partial g_t}{\partial \gamma} = -\frac{1}{2} + \frac{\epsilon_{t-1}^2}{\tau_{t-1}} \mathbb{1}(\epsilon_{t-1} < 0) + \beta \frac{\partial g_{t-1}}{\partial \gamma} \quad (40)$$

$$\frac{\partial g_t}{\partial \beta} = -1 + g_{t-1} + \beta \frac{\partial g_{t-1}}{\partial \beta} \quad (41)$$

$$\frac{\partial g_t}{\partial \vartheta} = \frac{-\alpha \epsilon_{t-1}^2 - \gamma \mathbb{1}(\epsilon_{t-1} < 0) \epsilon_{t-1}^2}{\tau_{t-1}^2} \frac{\partial \tau_{t-1}}{\partial \vartheta} + \beta \frac{\partial g_{t-1}}{\partial \vartheta} \quad (42)$$

$$\frac{\partial g_t}{\partial m} = \frac{-\alpha \epsilon_{t-1}^2 - \gamma \mathbb{1}(\epsilon_{t-1} < 0) \epsilon_{t-1}^2}{\tau_{t-1}^2} \frac{\partial \tau_{t-1}}{\partial m} + \beta \frac{\partial g_{t-1}}{\partial m} \quad (43)$$

$$\frac{\partial g_t}{\partial w} = \frac{-\alpha \epsilon_{t-1}^2 - \gamma \mathbb{1}(\epsilon_{t-1} < 0) \epsilon_{t-1}^2}{\tau_{t-1}^2} \frac{\partial \tau_{t-1}}{\partial w} + \beta \frac{\partial g_{t-1}}{\partial w} \quad (44)$$

where  $\mathbb{1}$  is an indicator function.

$$\frac{\partial L}{\partial \alpha} = \frac{1}{2} \sum_{t=1}^T \frac{\partial g_t}{\partial \alpha} / g_t + \frac{v+1}{2} \sum_{t=1}^T \frac{-\epsilon_t^2 / (\tau_t g_t^2) \cdot \frac{\partial g_t}{\partial \alpha}}{1 + \epsilon_t^2 / [\tau_t g_t (v-2)]} \quad (45)$$

$$\frac{\partial L}{\partial \gamma} = \frac{1}{2} \sum_{t=1}^T \frac{\partial g_t}{\partial \gamma} / g_t + \frac{v+1}{2} \sum_{t=1}^T \frac{-\epsilon_t^2 / (\tau_t g_t^2) \cdot \frac{\partial g_t}{\partial \gamma}}{1 + \epsilon_t^2 / [\tau_t g_t (v-2)]} \quad (46)$$

$$\frac{\partial L}{\partial \beta} = \frac{1}{2} \sum_{t=1}^T \frac{\partial g_t}{\partial \beta} / g_t + \frac{v+1}{2} \sum_{t=1}^T \frac{-\epsilon_t^2 / (\tau_t g_t^2) \cdot \frac{\partial g_t}{\partial \beta}}{1 + \epsilon_t^2 / [\tau_t g_t (v-2)]} \quad (47)$$

$$\frac{\partial L}{\partial \vartheta} = \frac{1}{2} \sum_{t=1}^T \frac{\frac{\partial \tau_t}{\partial \vartheta} g_t + \tau_t \frac{\partial g_t}{\partial \vartheta}}{\tau_t g_t} + \frac{v+1}{2} \sum_{t=1}^T \frac{-\epsilon_t^2 / (\tau_t^2 g_t^2) \cdot (\frac{\partial \tau_t}{\partial \vartheta} g_t + \tau_t \frac{\partial g_t}{\partial \vartheta})}{1 + \epsilon_t^2 / [(\tau_t g_t (v-2))]} \quad (48)$$

$$\frac{\partial L}{\partial m} = \frac{1}{2} \sum_{t=1}^T \frac{\frac{\partial \tau_t}{\partial m} g_t + \tau_t \frac{\partial g_t}{\partial m}}{\tau_t g_t} + \frac{v+1}{2} \sum_{t=1}^T \frac{-\epsilon_t^2 / (\tau_t^2 g_t^2) \cdot (\frac{\partial \tau_t}{\partial m} g_t + \tau_t \frac{\partial g_t}{\partial m})}{1 + \epsilon_t^2 / [(\tau_t g_t (v-2))]} \quad (49)$$

$$\frac{\partial L}{\partial w} = \frac{1}{2} \sum_{t=1}^T \frac{\frac{\partial \tau_t}{\partial w} g_t + \tau_t \frac{\partial g_t}{\partial w}}{\tau_t g_t} + \frac{v+1}{2} \sum_{t=1}^T \frac{-\epsilon_t^2 / (\tau_t^2 g_t^2) \cdot (\frac{\partial \tau_t}{\partial w} g_t + \tau_t \frac{\partial g_t}{\partial w})}{1 + \epsilon_t^2 / [(\tau_t g_t (v-2))]} \quad (50)$$

Then the negative likelihood gradient is  $[\frac{\partial L}{\partial \alpha}, \frac{\partial L}{\partial \gamma}, \frac{\partial L}{\partial \beta}, \frac{\partial L}{\partial \vartheta}, \frac{\partial L}{\partial m}, \frac{\partial L}{\partial w}]$ .

$$\frac{\partial l_t(\theta_k)}{\partial \theta}$$

The Spline-Garch and Spline-Garch-X are analogous with the difference for the Spline-Garch-X being the change of  $\frac{\partial \tau_t}{\partial \vartheta}$ ,  $\frac{\partial \tau_t}{\partial m}$  and  $\frac{\partial \tau_t}{\partial w}$  equations for  $\frac{\partial \tau_t}{\partial \kappa_0} = \tau_t$  and  $\frac{\partial \tau_t}{\partial \kappa_1} = \tau_t \cdot X_t$ . The  $\frac{\partial g_t}{\partial \vartheta}$ ,  $\frac{\partial g_t}{\partial m}$  and  $\frac{\partial g_t}{\partial w}$  are replaced for  $\frac{\partial g_t}{\partial \kappa_0}$  and  $\frac{\partial g_t}{\partial \kappa_1}$ . Lastly,  $\frac{\partial L}{\partial \vartheta}$ ,  $\frac{\partial L}{\partial m}$  and  $\frac{\partial L}{\partial w}$  are replaced for  $\frac{\partial L}{\partial \kappa_0}$  and  $\frac{\partial L}{\partial \kappa_1}$  such that the negative gradient is equal to  $[\frac{\partial L}{\partial \alpha}, \frac{\partial L}{\partial \gamma}, \frac{\partial L}{\partial \beta}, \frac{\partial L}{\partial \kappa_0}, \frac{\partial L}{\partial \kappa_1}]$ .

For the Spline-Garch, change the  $\frac{\partial \tau_t}{\partial \vartheta}$ ,  $\frac{\partial \tau_t}{\partial m}$  and  $\frac{\partial \tau_t}{\partial w}$  equations for

$\frac{\partial \tau_t}{\partial \zeta} = e^{w_0 t + \sum_{k=1}^K w_{ki}((t-t_{k-1})_+)^2}$ ,  $\frac{\partial \tau_t}{\partial w_0} = t \cdot \tau_t$  and  $\frac{\partial \tau_t}{\partial w_k} = ((t-t_{k-1})_+)^2 \cdot \tau_t$  for  $k = \{1, 2, \dots, K\}$ . The  $\frac{\partial g_t}{\partial \vartheta}$ ,  $\frac{\partial g_t}{\partial m}$  and  $\frac{\partial g_t}{\partial w}$  are replaced for  $\frac{\partial g_t}{\partial \zeta}$ ,  $\frac{\partial g_t}{\partial w_0}$  and  $\frac{\partial g_t}{\partial w_k}$  for  $k = \{1, 2, \dots, K\}$ . Lastly,  $\frac{\partial L}{\partial \vartheta}$ ,  $\frac{\partial L}{\partial m}$  and  $\frac{\partial L}{\partial w}$  are replaced for  $\frac{\partial L}{\partial \zeta}$ ,  $\frac{\partial L}{\partial w_0}$  and  $\frac{\partial L}{\partial w_k}$  for  $k = \{1, 2, \dots, K\}$ . Now the negative gradient is equal to  $[\frac{\partial L}{\partial \alpha}, \frac{\partial L}{\partial \gamma}, \frac{\partial L}{\partial \beta}, \frac{\partial L}{\partial \zeta}, \frac{\partial L}{\partial w_0}, \frac{\partial L}{\partial w_1}, \dots, \frac{\partial L}{\partial w_K}]$ .