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The Profitability of the Freemium Pricing Strategy for Private Monopolies in Two-Sided Markets

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Abstract Freemium is a dominant pricing strategy in the software industry that especially seems emerging in two-sided markets. Existing models in the literature have already shown why firms have an incentive to practice the free-pricing strategy in two-sided markets. In this paper models are presented for both the free-pricing strategy and the freemium pricing strategy. By comparing the two models the paper shows that a high network externality between the two market sides contributes to the profitability of the freemium pricing strategy for a private monopoly platform.

1. Introduction

The freemium pricing strategy is a strategy which can be largely observed nowadays among software firms such as Spotify, Dropbox and LinkedIn. In a freemium business model, a product or service is provided free of charge, but money is charged for the premium version of the product or service (Schenck, 2011). For instance, Spotify is a free music streaming service, but to get additional features such as advertisement-free streaming and offline music downloads, consumers need to get a paid subscription. Another example is the business oriented social networking service LinkedIn. The basic functionalities of LinkedIn, which include creating a profile and making connections, are offered for free. However, users need to buy a premium subscription to have additional features such as more visibility into who has viewed their profile, deeper search functionality and better e-mail capability (LinkedIn, 2017).

The freemium pricing strategy especially seems emerging in two-sided markets (Baden-Fuller & Haefliger, 2013). Two-sided markets are markets in which a platform has two distinct user groups that provide each other with network benefits (Hagiu & Wright, 2015). Existing models for two-sided markets have already shown that firms have an incentive to practice a free-pricing strategy if the cross-price elasticity of demand in one market side is high and/or the external benefit enjoyed by other market side is large (Armstrong, 2006).

By an extension to the model of Parker and van Alstyne (2005), this paper shows when it is optimal for private monopolies in two-sided markets to practice the freemium pricing strategy. A private monopoly is a privately-owned firm that lacks competition and strives to maximize its profits (Hazlett, 1986). The focus in this paper is on private monopolies to emphasize the profitability of the freemium pricing strategy even in a situation without competition. In existing models of monopolies in two-sided markets there are two markets and a platform firm defined. By making a distinction between low-valuation agents and high-valuation agents in one of the two market sides, Parker and Van Alstyne's model (2005) is extended such that the platform firm sets three prices instead of two: the prices for the low-valuation and high-valuation agents in the first market and the price for the agents in the second market. This paper examines under which conditions a freemium pricing strategy is optimal for private monopolies in two-sided markets.

The main finding from the paper is that two requirements must be met for the freemium pricing strategy to be optimal for a monopoly platform. First, the internetwork externality from the freemium market to the producer market should be large enough. Second, the cross-price elasticity between the demanded quantity in the producer market with respect to the price in the freemium market must be positive and large enough. The intuition behind these requirements is that the increase in the freemium market due to the free goods also increases demand in the producer market due to network externalities. If the increased demand in the producer market is high enough, it is profitable to maintain a freemium pricing strategy.

The rest of the paper is structured as follows. Section 2 provides a review of related literature in two-sided markets and freemium pricing strategies. In section 3, first a simplified model of the free-pricing strategy for private monopolies in two-sided markets is given, then the freemium model is developed by extending the simplified model with a distinction between low-valuation consumers and high-valuation consumers in one of the two market sides. Section 4 contains an analysis of the results of the model. The discussion and the conclusion are provided in section 5.

2. Related literature

This section provides a review of related literature in two-sided markets and freemium pricing strategies. In the first part, the main findings in theoretical papers about two-sided markets are discussed. The focus is on findings that are relevant for private monopolies. In the second part, business strategical literature and their economic implications about freemium pricing strategies are discussed.

2.1 Two-sided markets

Positive network externalities increase the utility that a user derives from the consumption of a good with the number of other agents consuming the good (Katz & Shapiro, 1985). Rochet and Tirole's paper (2003) recognized that many markets with network externalities can be regarded as two-sided markets. Two-sided markets consist of two distinct market sides that interact through a common platform. In two-sided markets, the enjoyed benefits of one market side depends on the size of the other market side. It is important to notice that end-users in two-sided markets do not internalize the external benefit of their platform usage to the other end-users.

A distinction can be made between two-sided markets based on how the platform charges the market sides (table 1). A platform could either choose to charge the users a variable fee (indicated with V), a fixed fee (indicated with F) or a combination of a variable fee and a fixed fee. One example of a two-sided market is the market of credit cards. In this market, the consumers have a higher valuation for a credit card that is accepted widely by retailers, while retailers have a higher valuation to accept credit cards that are carried by many consumers. Credit card companies usually charge cardholders annual fixed fees, whereas the merchants are charged per transaction. Another example is the market for videogames. Videogame platforms need games to attract gamers to buy their video game console, and they need gamers to persuade game developers to design games for their gaming platform. In the videogame market the platform firm charges game developers a fixed fee for development kits and royalties per sold copy (variable fee). Gamers are only charged a fixed fee for the videogame by the platform firm (Rochet & Tirole, 2006).

Platforms often treat one market side as the profit-making segment (subsidizing segment), whereas the other market side is treated as the loss leader (subsidized segment). In the table, the subsidized market sides are indicated with an asterisk. The market side that exerts the largest positive externality on the other market side, is often targeted as the subsidized market side. Moreover, the price in market side 1 is determined by how much benefit that market size exerts to market side 2 instead of how much market side 1 benefits from market side 2 (Armstrong, 2006). The example of the nightclubs and bars illustrates how platform firms apply cross-subsidization. A nightclub is usually only successful if it can manage to attract both men and women. Asymmetry in the interaction effect from men and women makes it possible to use one market to subsidize the other. As it is often assumed that men gain more from interacting with women than vice versa, men are used as the subsidizing market side. By subsidizing the market side of women with free entrance, a nightclub could manage to attract more women because the price barrier disappears, and more men because of the positive network externality.

Table 1: Illustrations of two-sided markets

Market 1	Platform intermediary	Market 2
Readers* (*V*)	Newspapers	Advertisers (*F*)
Cardholders* (*F*)	Credit cards	Merchants (*V*)
Gamers (*F*)	Videogames	Game developers* (*V+F)*
Women* (*V*)	Bars, clubs	Men (*V*)
Consumers* (*V*)	Shopping malls	Shops (*F*)

In the paper of Parker and Van Alstyne (2000) a model of cross-market externalities is introduced to understand the free-pricing strategy that is observed in the information goods market. The model showed that a firm can rationally invest in a product that it tends to give away into perpetuity even in the absence of competition. The reason is that the increase in one market side due to the free goods also increases demand in the other market side due to network externalities. If the increased demand in the complementary market covers the cost of investment in the free goods market, it is profitable to maintain a free-pricing strategy for information goods.

Rochet and Tirole's paper (2003) recognized that two-sided markets are not only present in information goods but in many more markets. In their model, network externalities are combined with multi-product pricing to explain firm behaviour in the credit card markets. The findings also apply to other two-sided markets (see table 1). The main finding from their paper is that both monopoly and competitive platform firms in two-sided markets design their price structure so that they get both sides on board. The presented private monopoly model shows that the total price in both markets chosen by the platform monopoly is given by the standard Lerner formula for elasticity equal to the sum of the two elasticities.

Armstrong's paper (2006) also presents models of two-sided markets. Compared to Rochet and Tirole (2003) there are modelling differences concerning the specification of agents' utility, the structure of platforms' fees, and the structure of platforms' costs. Armstrong's model assumes that platform costs are incurred on a per-agent basis, whereas Rochet and Tirole's model assumes that platform costs are incurred on a per-transaction basis. The difference in assumptions is to make the model more suitable for markets such as nightclubs, shopping malls and newspapers, whereas the model of Rochet and Tirole is more suitable for the credit card market. In the analysis of the monopoly platform, Armstrong finds that it is possible that in the profit-maximizing outcome the platform firm offers one market side a subsidized service in which the price is lower than the per-agent cost of the platform. If the cross-price elasticity of demand is high and/or the external benefit enjoyed by the subsidizing market side is large, the optimal subsidy might be so large that the optimal price is zero or negative.

2.2 Freemium pricing strategy

The freemium business model is a combination between 'free' and 'premium'. It has become a dominant business model among internet start-ups over the past decade. In this business model, a product's basic functionality is given away for free, whereas money is charged for product-specific benefits (Schenk, 2011). The freemium pricing strategy applies price discrimination by charging different prices to different classes of consumers (Schmalensee, 1981). The purpose of the freemium business model is the distribution of a product to the largest possible group of potential users with the expectation that users will upgrade to the paid premium version (Lee et al., 2013). In Seufert's book *Freemium Economics: Leveraging Analytics and User*

Segmentation to Drive Revenue (2003) the four components of the freemium business model are described.

1. The first component is the potential for scale. As only a part of the user base in the freemium business model contributes to the revenue stream, the freemium product must have the potential to reach and be adopted by a large number of people. Therefore, premium products should require product characteristics that facilitate massive scale, such as low marginal distribution and production costs.
2. The second component of the freemium model is insight into the user base. It is important to gain insight into the user base as this provides information on how to best serve the needs of users. This information can be used to attract more revenue contributing users (premium users).
3. The third component is monetization, which is a component in every business model. In the freemium business model, only a low proportion spends money on the paid products, namely the users that upgrade to the premium version.
4. The fourth component is optimization. Optimization is the adapting of the product to the needs and tastes of its users. This is important to prevent the loss of users.

From the freemium business model, we can derive a profit function that captures the importance of the four mentioned components. First, a simple profit function is given for a firm that charges the same price to all its consumers.

$$\pi_1 = U_1 \cdot (P_1 - MC_1)$$

In this profit function U denotes the size of the user base, P the price of the product, and MC the marginal costs. Now price discrimination is applied such that premium users are charged price P_2, whereas the other users get the regular product for free. Assume that a proportion x from the user base contributes to the revenue stream such that a proportion $(1 - x)$ of the user base is charged nothing for the product. The profits of the firm are expressed by,

$$\pi_2 = x \cdot U_2 \cdot (P_2 - MC_2) + (1 - x) \cdot U_2 \cdot (0 - MC_2)$$

$$\pi_2 = U_2 \cdot (xP_2 - MC_2).$$

If we compare the two profit functions, the importance of the business model components becomes clear. The likelihood of π_2 to be greater than π_1 increases if MC_2 is lower than MC_1 (first component) and the higher proportion x (second component). The third component stresses the importance to set an optimal price because the proportion of free users ($x < 1$) lowers the profits compared to the first situation (where you can think of $x = 1$). The fourth component is targeted at preventing a decrease in U. All the four components of the freemium business model contribute to the profitability of a firm.

Several factors contribute to the appeal of the freemium pricing strategy. One factor is that the free features work as a marketing tool that attracts users without costly advertising expenditures. Another factor that makes the freemium pricing strategy appealing is the permanent free feature. The indefinite free access makes freemium products more attractive for consumers compared to limited-term offers (Kumar, 2014).

Besides the mentioned appealing factors, the freemium pricing strategy also brings difficulties with it. One difficulty is finding the right balance between which features should be offered for free and which features should be paid for. Such a balance is important for the following reason. If the free features are not appealing enough, a firm is not able to attract many users. A low user base also means that there are fewer users that potentially could upgrade to the premium version. However, if the free features are too appealing, few people will pay for an upgrade, which means that not much revenue is generated. Another difficulty is that the conversion rate, the percentage of free uses that upgrade to a premium plan, tends to decline over time. The users that are less price-sensitive are the ones that will upgrade early to the premium version. Over time this means that the free user base will consist of more price-sensitive users or users that have a lower valuation for the premium features (Kumar, 2014).

3. Model

In this section the models for the analysis of the free-pricing and freemium pricing strategy for private monopoly platforms are provided. The models are based on the standard externality models of Parker and Van Alstyne (2005). The markets for the models consist of one monopoly platform that sells information goods to two markets: the consumer market C and the producer market P. As discussed under section 2.1, there is usually a subsidizing market and a subsidized market in two-sided markets. In the model, the producer market is used as the subsidizing market, whereas the consumer market is treated as the subsidized market. This implies that the consumer market is the market in which the free-pricing strategy (section 3.1) and the freemium pricing strategy (section 3.2) are implemented. Between the two markets there is a positive two-sided network externality, which means that a purchase in one market increases the value for the product in the other market, leading to an increase in demand in that other market (Katz & Shapiro, 1985). The demand in each market is denoted by

$$D_i(p_i) = \int_{p_i}^{\overline{V}} f(v)dv \; for \; i \in \{C, P\}, \qquad (1)$$

where v denotes the arbitrary willingness to pay, and $\bar{V}$ denotes the maximum valuation (Willig, 1976).

3.1 Monopoly platform with free-pricing

In the free-pricing model the monopoly platform sets two separate prices for the two different markets: the consumer market *C* and the producer market *P*. Therefore, the choice parameters for the platform firm are the prices in both markets. The prices and quantities are denoted by p_i and q_i for $i \in \{C, P\}$. As the monopoly platform sells information goods, we assume that the marginal costs are negligible. Thus, the profits of the monopoly platform are denoted as follows,

$$\pi = \pi_C + \pi_P = p_C q_C + p_P q_P. \qquad (2)$$

The profit function is twice differentiable in both choice parameters such that the first-order conditions yield prices in both markets. The internetwork externality measures the effect that purchases in one market have on purchases in the other market. The effect of purchases in the producer market on purchases in the consumer market is denoted by e_{PC}. Similarly, e_{CP} measures the effect of purchases in the consumer market on purchases in the producer market. By adding the internetwork externality effect to the demand (equation 1), we get the following demand equations:

$$q_C(p_C, p_P) = D_C(p_C) + e_{PC} D_P(p_P), \qquad (3)$$
$$q_P(p_P, p_C) = D_P(p_P) + e_{CP} D_C(P_C). \qquad (4)$$

By substituting these demand equations into the profit equation (equation 2), we obtain the following expression for the profit function:

$$\pi = p_C[D_C(p_C) + e_{PC} D_P(p_P)] + p_P[D_P(p_P) + e_{CP} D_C(p_C)]. \qquad (5)$$

To find the optimal monopoly price for the consumers, we set the partial derivative of the total profits from equation 5 with respect to the price in market C equal to zero,

$$\frac{\partial \pi}{\partial p_C} = D_C + p_C D_C' + e_{PC} D_P + e_{CP} p_P D_C' = 0. \qquad (6)$$

$D_i(p_i)$ is written as D_i to simplify the notification. Now definitions for price elasticities are introduced which are useful for the interpretation of the optimal consumer price equation. We define the own-price elasticity, which measures the change in the demanded quantity in market *C* to a change in the price in market *C*, and the cross-price elasticities, which measures the change in the demanded quantity in one market to the change in the price in the other market (Marshall, 1890).

Lemma 1 *The own-price elasticity is defined as* $\varepsilon_C = -\frac{p_C D_C'}{D_C + e_{PC} D_P}$ *and the cross-price elasticities are defined as* $\varepsilon_{CP} = \frac{e_{PC} p_P D_P'}{D_C + e_{PC} D_P}$ *and* $\varepsilon_{PC} = \frac{e_{CP} p_C D_C'}{D_P + e_{CP} D_C}$.

The proof for the Lemma 1 is provided in the appendix. By using Lemma 1, we can rearrange the optimal consumer price equation (equation 6).

Lemma 2 *The optimal consumer price equation is defined as* $\varepsilon_C - \varepsilon_{PC} \frac{\pi_P}{\pi_C} = 1$.

The proof for Lemma 2 is provided in the appendix. For a free-pricing strategy to be optimal, the optimal price in the consumer market should be lower than or equal to zero.

Proposition 1 *The condition* $\varepsilon_{PC} \frac{\pi_P}{\pi_C} \leq -1$ *must be satisfied for a free-pricing strategy to be profit-maximizing.*

The proof for Proposition 1 is provided in the appendix. From proposition 1 we derive that the cross-price elasticity of the demanded quantity in market P with respect to the price in market C times the profit ratio should be smaller than or equal to -1 for a free-pricing strategy to be profit-maximizing for a monopoly platform.

3.2 Monopoly platform with freemium pricing

The model for a monopoly platform with a freemium pricing strategy also consists of a platform firm, the consumer market C, and the producer market P. Again, the consumer market is treated as the subsidized market, which implies that this is the market side where the freemium pricing strategy is implemented. In the consumer market C a distinction is made between low-valuation consumers (L) and high-valuation consumers (H) to make it possible to maintain a freemium pricing strategy. By applying price discrimination in the consumer market, the monopoly platform firm has to set three prices instead of two: the prices for the low-valuation and high-valuation consumers in market C (p_L and p_H) and the price for the producers in market P (p_P). A freemium pricing strategy is profit-maximizing if it is optimal to offer the low-valuation consumers the product for free. The demands in the consumer and producer markets are denoted as follows,

$$D_C(p_L, p_H) = D_L(p_L) + D_H(p_H) = \int_{p_L}^{p_H} f(v)dv + \int_{p_H}^{\overline{V}} f(v)dv, \quad (7)$$

$$D_P(p_P) = \int_{p_P}^{\overline{V}} f(v)dv. \qquad (8)$$

In equation 7 the consumer demand is separated in the demand for low-valuation consumers and high-valuation consumers. It is assumed that the low-valuation consumers have a valuation between p_L and p_H and that the high-valuation consumers have a valuation that is higher than p_H (Varian, 1987). The prices and quantities are denoted by p_i and q_i for $i \in \{L, H, P\}$. We denote the profits of the monopoly platform as follows:

$$\pi = \pi_C + \pi_P = \pi_L + \pi_H + \pi_P = p_L q_L + p_H q_H + p_P q_P. \qquad (9)$$

The effect of purchases in the producer market on purchases in the consumer market is denoted by $e_{PC} = e_{PL} + e_{PH}$. Similarly, $e_{CP} = e_{LP} + e_{HP}$ measures the effect of purchases in the consumer market on purchases in the producer market. By adding the internetwork externality effect to the (partial) demand from equation 7, we get the following demand equations:

$$q_L(p_L, p_P) = D_L(p_L) + e_{PL} D_P(p_P), \qquad (10)$$
$$q_H(p_H, p_P) = D_H(p_H) + e_{PH} D_P(p_P), \qquad (11)$$
$$q_P(p_P, p_L, p_H) = D_P(p_P) + e_{LP} D_L(P_L) + e_{HP} D_H(p_H). \qquad (12)$$

By substituting the demand equations into the profit equation (equation 9), we obtain the following expression for the profit function:

$$\pi = p_L(D_L + e_{PL} D_P) + p_H(D_H + e_{PH} D_P) + p_P(D_P + e_{LP} D_L + e_{HP} D_H). \qquad (13)$$

To find the optimal monopoly price for the low-valuation consumers, we take the partial derivative of the total profits (equation 13) with respect to the price for the low-valuation consumers in market C and set it equal to zero,

$$\frac{\partial \pi}{\partial p_L} = D_L + p_L D_L' + e_{PL} D_P + e_{LP} p_P D'_L = 0. \qquad (14)$$

We introduce definitions for price elasticities which are useful for the interpretation of the equation for the optimal low-valuation consumers price (equation 14).

Lemma 3 *The own-price elasticity is defined as* $\varepsilon_L = -\frac{p_L D_L'}{D_L + e_{PL} D_P}$ *and the cross-price elasticities are defined as* $\varepsilon_{LP} = \frac{e_{PL} p_P D_P'}{D_L + e_{PL} D_P}$ *and* $\varepsilon_{PL} = \frac{e_{LP} p_L D_L'}{D_P + e_{LP} D_L}$.

The proof of Lemma 3 parallels the proof of Lemma 1 in the appendix. By using the definitions of the price elasticities from Lemma 3, we can rearrange the optimal low-valuation consumers price equation (equation 14).

Lemma 4 *The optimal low-valuation consumers price equation is defined as* $\varepsilon_L - \varepsilon_{PL}\frac{\pi_P}{\pi_L} = 1.$

The proof of Lemma 4 is similar to the proof of Lemma 2 and is therefore omitted from the paper. For a freemium pricing strategy to be profitable, the optimal price for low-valuation consumers in market C should be lower than or equal to zero.

Proposition 2 *The condition* $\varepsilon_{PL}\frac{\pi_P}{\pi_L} \leq -1$ *must be satisfied for a freemium pricing strategy to be profit-maximizing.*

The proof of Proposition 2 is similar to the proof of Proposition 1 and is therefore omitted from the paper.

4. Analysis

In the free-pricing model under section 3.1 we found the following condition for a private monopoly firm to maintain a profit-maximizing free-pricing strategy:

$$\varepsilon_{PC}\frac{\pi_P}{\pi_C} \leq -1.$$

If the cross-price elasticity of the demanded quantity in market *P* with respect to the price in market *C* is high enough and the profit ratio is negative and low enough, it is optimal for a private monopoly platform to maintain a free-pricing strategy. A free-pricing strategy requires the optimal price in one of the two markets to be lower than or equal to zero. As discussed in section 3, the consumer market is used as the subsidizing market, and therefore we look at solutions where the optimal consumer price is lower than or equal to zero ($p_C^* \leq 0$).

For the platform firm to be profitable, an optimal consumer price of lower than or equal to zero makes it necessary to set the optimal producer price higher than zero ($p_P^* > 0$). These prices lead to losses in the consumer market and profits in the producer market ($\pi_C \leq 0$ and $\pi_P > 0$) so that the profit ratio becomes negative ($\frac{\pi_P}{\pi_C} < 0$). The greater the profits in the producer market and the smaller the losses in the consumer market, the more likely it is for the free-pricing condition to be satisfied. If we rewrite the profit ratio as follows,

$$\frac{\pi_P}{\pi_C} = \frac{p_P q_P}{p_C q_C} = \frac{p_P[D_P(p_P)+e_{CP}D_C(P_C)]}{p_C[D_C(p_C)+e_{PC}D_P(p_P)]},$$

we see that the greater the positive effect of purchases in the consumer market on purchases in the producer market (e_{CP}), the more negative the profit ratio becomes. Thus, a higher internetwork externality leads to a greater likelihood to satisfy the free-pricing condition.

Furthermore, the negative profit ratio requires a positive cross-price elasticity of the quantity in market *P* with respect to the price in market *C* to satisfy the free-pricing condition. A positive cross-price elasticity is intuitively explained as follows, a decrease in the price in the consumer market requires the price in the producer market to increase. If the own-price elasticity in the producer market is negative, the caused price increase leads to a decrease in the demanded quantity in the producer market, and hence the cross-price elasticity is positive. According to the law of demand there is an inverse relationship between the price and quantity demanded of a good. Therefore it is plausible to assume that the own-price elasticity in the producer market is negative (Marshall, 1890).

The same reasoning applies for the freemium pricing strategy condition under section 3.2. It is shown that the following condition must be satisfied to make a freemium pricing strategy the optimal pricing strategy for a monopoly platform firm:

$$\varepsilon_{PL}\frac{\pi_P}{\pi_L} \leq -1.$$

This condition is satisfied if two requirements are met. First, the cross-price elasticity of the demanded quantity in market *P* with respect to the price charged to the low-valuation consumers in market *C* (the freemium market) must be high enough. Second, the negative profit ratio between the profits in the producer market and the profits in the freemium market must be low enough. Notice that the demanded quantity in the producer market increases with the internetwork externality (e_{LP}) and that a greater demanded quantity in the producer market relative to the demanded quantity in the freemium market leads to a more negative profit ratio. Hence, it is more likely that the freemium pricing condition is satisfied if the positive effect of purchases in the freemium market to the producer market is high. The idea behind these findings is that the increase in the freemium market due to the free product also increases the demand in the producer market due to positive network externalities. If the increased demand in the producer market is high enough to compensate the costs of the freemium consumers, it is optimal to maintain a freemium pricing strategy.

If we compare the two pricing conditions, we must look at two components. The first component is the cross-price elasticity. Rewrite the cross-price elasticities of the free-pricing model and the freemium pricing model as follows (see Lemma 1),

$$\varepsilon_{PC} = \frac{e_{CP}p_C D_C'}{D_P + e_{CP}D_C},$$

$$\varepsilon_{PL} = \frac{e_{LP} p_L D_L'}{D_P + e_{LP} D_L}.$$

As discussed above, the profit ratios in the pricing conditions are negative. Therefore, the higher the positive cross-price elasticity, the greater the likelihood that the pricing condition is satisfied. If we compare the denominators, we conclude that the denominator of the cross-price elasticity in the free-pricing model is higher than the denominator in the freemium pricing model because $e_{CP} = e_{LP} + e_{HP}$ and $D_C(p_C) = D_L(p_L) + D_H(p_H)$. Therefore, the cross-price elasticity in the freemium model is unambiguously greater than the cross-price elasticity in the free-pricing model if the marginal cross-price contribution from the consumer market to the producer market ($e_{CP} D_C'$) is not greater than the marginal cross-price contribution from freemium market to the producer market ($e_{LP} D_L'$). This requires the partial derivative from the demand of the consumers with respect to the price to be lower than the partial derivate from the demand of the low-valuation consumers with respect to the price ($\frac{\partial q_C}{\partial p_C} < \frac{\partial q_L}{\partial p_L} \rightarrow D_C' < D_L'$).

The second relevant component is the profit ratio. Rewrite the profit ratios as follows for the free-pricing model and the freemium pricing model:

$$\frac{\pi_P}{\pi_C} = \frac{p_P q_P}{p_C q_C} = \frac{p_P [D_P(p_P) + e_{CP} D_C(P_C)]}{p_C [D_C(p_C) + e_{PC} D_P(p_P)]},$$

$$\frac{\pi_P}{\pi_L} = \frac{p_P q_P}{p_L q_L} = \frac{p_P [D_P(p_P) + e_{CP} D_C(P_C)]}{p_L [D_L(p_L) + e_{PL} D_P(p_P)]}.$$

As $q_C(p_C, p_P) = q_L(p_L, p_P) + q_H(p_H, p_P)$, the denominator in the freemium pricing model is smaller than the denominator of free-pricing profit ratio, which leads to a more negative profit ratio in the freemium pricing condition. As the profit ratio in the freemium model tends to be lower than the profit ratio in the free-pricing model, the freemium pricing condition is more likely to be satisfied compared to the free-pricing strategy if we look at the profit ratio component.

If we combine the findings from the two components, we conclude that the freemium pricing strategy is more likely to be satisfied than the free-pricing strategy. First, the profit ratio of the freemium pricing strategy is more negative than the profit ratio in the free-pricing strategy. Second, the positive cross-price externality is likely to be higher in the freemium model than in the free-pricing model. The only possibility for the free-pricing strategy to be sooner satisfied than the freemium pricing strategy is when the cross-price externality is higher in the free-pricing model. This is the case when low-valuation consumers are more price-sensitive than the consumer market in general, and the difference between price-sensitivity outweighs the difference between the denominators of the cross-price elasticity and the difference between the profit ratios.

The reason that the freemium pricing strategy condition is easier satisfied is as follows. Both the free-pricing and freemium pricing strategy are considered optimal if the increase in the subsidizing market (the producer market) compensates the investment in the subsidized market. In the freemium model, the subsidized market decreases with the high-valuation consumers (premium consumers) as they do not get the product for free anymore. Therefore, a smaller investment cost needs to be compensated compared to the free-pricing model. Moreover, as the high-valuation consumers are charged a price and it is assumed that the marginal costs are negligible, the premium consumers contribute to the compensation of the investment costs for the freemium consumers.

5. Discussion and conclusion

In this paper we studied under which conditions the freemium pricing strategy is profitable for a private monopoly platform in two-sided markets. By applying price discrimination to the consumer market in the free-pricing model from Parker and Van Alstyne (2005) a freemium pricing model was developed. We found that if the positive effects of purchases from freemium consumers on purchases in the producer market are large enough and if there is a large enough positive cross-price elasticity, it is optimal for a monopoly platform in a two-sided market to practice the freemium pricing strategy. By comparing the free-pricing condition with the freemium pricing condition, we found that the freemium pricing condition is more likely to be satisfied than the free-pricing condition. This could be explained by the fact that in the freemium model not only the producers contribute to the revenue stream, but also the high-valuation consumers. As the free users decrease compared to the revenue contributing users, it is easier to compensate the investment in the freemium market.

In this paper, we assumed that the marginal costs are zero, which makes the model suitable for information goods. For future research, it could be interesting to extend the model with marginal costs to make the model suitable for other markets as well. Another assumption was the lack of platform competition. In the real world, platform firms in two-sided markets often face competition. Therefore, it is suggested to study a situation with platform competition in future research.

Appendix

Proof of Lemma 1 The own-price elasticity is defined as $\varepsilon_i = -\frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i}$, which can be rewritten as $-p_i\frac{\partial q_i}{\partial p_i}\frac{1}{q_i}$. If we then substitute the demand equation $q_i(p_i, p_j) = D_i(p_i) + e_{ji}D_j(p_j)$ (equation 3) and $\frac{\partial q_i}{\partial p_i} = D_i'$ into the denotation we obtain $\varepsilon_i = -\frac{p_i D_i'}{D_i + e_{ji}D_j}$.

The cross-price elasticity is defined as $\varepsilon_{ij} = \frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i}$, which can be rewritten as $p_j\frac{\partial q_i}{\partial p_j}\frac{1}{q_i}$. If we then substitute the demand equation $q_i(p_i, p_j) = D_i(p_i) + e_{ji}D_j(p_j)$ (equation 3) and the network externality $\frac{\partial q_i}{\partial p_j} = e_{ji}D_j'$ into the denotation we obtain $\varepsilon_{ij} = \frac{e_{ji}p_j D_j'}{D_i + e_{ji}D_j}$.

Proof of Lemma 2 Rewrite he optimal low-valuation consumers price equation (equation 6) as follows,

$$D_C + p_C D_C' + e_{PC}D_P + e_{CP}p_P D_C' = 0$$

$$P_C D_C' + e_{CP}P_p D_C' = -D_C - e_{PC}D_P$$

$$-\frac{P_C D_C' + e_{CP}P_p D_C'}{D_C + e_{PC}D_P} = 1$$

$$-\frac{p_C D_C'}{D_C + e_{PC}D_P} - \frac{e_{CP}p_P D_C'}{D_C + e_{PC}D_P} = 1.$$

By using the own-price elasticity denotation in Lemma 1 we rewrite the equation to the following,

$$\varepsilon_C - \frac{e_{CP}p_P D_C'}{D_C + e_{PC}D_P} = 1$$

$$\varepsilon_C - \frac{e_{PC}p_P D_P'}{D_C + e_{PC}D_P}\frac{e_{CP}D_C'}{e_{PC}D_P'} = 1.$$

By using the cross-price elasticity denotation in Lemma 1 we rewrite the equation further as follows,

$$\varepsilon_C - \varepsilon_{CP}\frac{e_{CP}D_C'}{e_{PC}D_P'} = 1$$

$$\varepsilon_C - \varepsilon_{CP}\frac{e_{CP}p_C D_C'}{D_P + e_{CP}D_C}\frac{D_C + e_{PC}D_P}{e_{PC}p_P D_P'}\frac{p_P(D_P + e_{CP}D_C)}{p_C(D_C + e_{PC}D_P)} = 1$$

$$\varepsilon_C - \varepsilon_{CP}\frac{e_{CP}p_C D_C'}{D_P + e_{CP}D_C}\frac{D_C + e_{PC}D_P}{e_{PC}p_P D_P'}\frac{p_P q_P}{p_C q_C} = 1$$

$$\varepsilon_C - \varepsilon_{CP}\frac{\eta_{PC}}{\eta_{CP}}\frac{p_P q_P}{p_C q_C} = 1$$

If we then substitute the profit equation $\pi = p_C q_C + p_P q_P$ (equation 5) into the equation we obtain the final expression for the optimal consumer price equation $\varepsilon_C - \varepsilon_{PC}\frac{\pi_P}{\pi_C} = 1$.

Proof of Proposition 1 In Lemma 2 we rearranged the optimal consumer price equation to $\varepsilon_C - \varepsilon_{PC}\frac{\pi_P}{\pi_C} = 1$. This can be rewritten as $\frac{p_C D_C'}{D_C + e_{PC} D_P} - \frac{e_{CP} p_C D_C'}{D_P + e_{CP} D_C}\frac{p_P q_P}{p_C q_C} = 1$. In the $\varepsilon_{PC}\frac{\pi_P}{\pi_C}$ part, p_C cancels each other out, so that p_C only remains in the own-price elasticity part. For a free-pricing strategy to be optimal we need the optimal consumer price to be equal to or lower than zero ($p_C^* \leq 0$). This requires the own-price elasticity to be less than zero ($\varepsilon_C \leq 0$). If the own-price elasticity is less than zero, $\varepsilon_{PC}\frac{\pi_P}{\pi_C}$ must be smaller or equal to -1. This gives us the following free-goods market condition: $\varepsilon_{PC}\frac{\pi_P}{\pi_C} \leq -1$.

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