Abstract

In this paper, an extension is studied for the model provided by Kamphorst and Swank (2016), they show that discrimination can be caused by employees expecting their manager to prefer one of the employees. An outside option is added to show that giving employees the option to quit their job to work for another company, can give the manager less reason to discriminate. A stable non-discriminatory equilibrium is found when a manager can be certain that the employee leaves, this is tested by differences in expected competence for the different jobs. This result is extended with parameters for differences in wage and the possibility of switching costs. Wage differences lead to a less discriminatory equilibrium than found in the basic model. When switching costs are added, although the trend is not always clear, the equilibria become more discriminatory when switching costs are included. Adding the outside option gives possible escapes for rational discrimination, this can help in the battle against discrimination.

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Introduction

Intuitively, family businesses are not the most successful way to organize a business, Pérez-González (2006) proves this in his paper. He finds that family businesses tend to make less operational profits and have a lower market-to-book ratio relative to regular businesses. When a company appoints a CEO that is related to an important stakeholder, this firm will underperform compared to a firm with an unrelated CEO. The underperformance indicates that the average related CEO is less compatible to manage a firm compared to the average unrelated CEO. This appears to be irrational, why would a firm choose to have a related CEO over the company’s profitability? When it comes to designating a leader of the firm, it is sensible to act only in the interest of the company. A personal taste for discrimination may not compensate for the loss of profitability when choosing the less able related employee to be CEO. Surely some boards will engage in their taste for promoting a family-member, but not all will value this over the company’s profitability. That logical truth indicates it may have been rational for some boards, acting in the interest of the company, to name the related less compatible CEO.

According to Kamphorst and Swank (2016) this might have to do with the beliefs of the employees. When a board can choose between an unrelated and a related candidate, both candidates will expect the last one to be preferred. The board can decide to promote whoever is best at his job and assuming the unrelated candidate is more compatible; he will be promoted. Since the related candidate expects to be positively discriminated, this will cause him to think poor about himself and this could demotivate him significantly. Simultaneously, unrelated candidates’ motivation is less affected by the task-assignment. The profitability the firm loses due to the demotivation, is possible to be bigger than the profitability loss incurred when the related candidate is promoted. As follows, their model shows that a board can be stimulated to enforce a discriminatory promotion to prevent demotivation. The expectations can be self-fulfilling and preserve the discrimination.

In this paper, the model developed by Kamphorst and Swank (2016) is to be used as the foundation. I will expand the model with an outside option, this implies an employee could decide to leave his current job. In the real world, employees can decide to quit their job, usually because they increase their utility doing so. When you expect to be more competent pursuing another career, you are likely to change careers. Naturally this affects the discriminating equilibrium. If the non-promoted employee leaves for sure, the manager will no longer care for his beliefs and strategies change. As Kamphorst and Swank (2016) point out correctly, discrimination will vanish. Further extensions insert parameters that represent differences in payoff schemes and
switching costs, also introducing uncertainty. These expansions are intended to study ways to escape discriminating equilibria. This can be useful eliminating this type of discrimination in practice. Since expectation can never be completely inhibited, other approaches can be necessary.

The remainder of this paper is arranged as follows, first the theoretical relevance will be discussed. Second, I will explain the basic model written by Kamphorst and Swank (2016), this is followed by the first extension introducing the outside option based on differences in ability. The next extension discusses the possibility of different wages in the outside option and in the last extension, the idea of switching costs is added. With each extension, the corresponding results are illustrated. In the end, a discussion and the concluding remarks will follow.

Theoretical Framework

This paper is an extension of the model build by Kamphorst and Swank (2016). In their model, they point out a possible incentive for managers to discriminate, discriminating can maximize profits. My purpose is to study the managers’ strategy when the model will be extended with an outside option; employees have the possibility to work for another firm. When there is a possibility to increase their expected utility at another job, they will do so. According to Böckerman and Ilmakunnas (2009), perfect competitive labor markets do not occur as such that switching between firms can never increase one’s utility. In their paper, they research numerous significant incentives for employees quitting their job. Among these incentives are experiencing discrimination at the work floor and poor chances on being promoted. Considering these stimuli, the outside option is a relevant extension to the model. Farber (1999) states that heterogeneity between employees is an important factor for mobility in the labor market. Different people are suited for different jobs, this diversity is relevant considering the outside option. When employees expect to excel in another career, according to Farber this can cause them to change careers.

Last century several measures were taken to address discrimination. Gender equality is encouraged and racism is counteracted. Still discrimination is hard to stop, it is complicated to assess and difficult to prove. No general incentives exist, since it is very person-specific. Abrams (1989) writes about the subtleness of discrimination. Implementing formal equality does not always mean everyone is treated equal. A change in attitudes is required for this, which is hard to regulate. Even when a manager has no intentions to discriminate, if employees expect to be preferred or disadvantaged, this could give the manager an incentive to favor one employee. It is a self-fulfilling prophecy. Mishra and Mishra (2015) support the statement that a change in attitudes is needed. The negative environment around certain characteristics, for example obesity,
makes sure the inferior place of these stereotypes is maintained. Due to this, people keep expecting to be discriminated and personal tastes for discrimination continue. This facilitates the discrimination to be maintained. Since discrimination is costly for society (Becker, 1957)\(^1\), it is important to study possible ways to preclude this type of discrimination. Asserting an outside option can be one of the solutions.

Discrimination is expressed in various forms. Men that are preferred over women is a completely different type of discrimination than the son of the boss against an exchangeable employee. Besides this, one can also prefer one employee simply because they share a hobby. Green (2003) discusses the trend that changes the type of discrimination, it has become subtler and thus harder to deal with. The ‘shared hobby’ discrimination has increased while ‘tough’ discrimination has decreased. Talaska, Fiske and Chaiken (2008) discuss this trend in another light, according to their paper emotional prejudices predict discrimination better than general stereotypes or beliefs. Again, this implies a change in the nature of discrimination, for emotional prejudice depends more on the situation, where stereotypes emerge from society. These different types of discrimination are relevant for the outside options. A disadvantage at your current job, because you, unlike your manager, dislike red wine, is unlikely to return another job. This kind of treatment depends on the companies’ culture. On the other hand, there is a significant chance that discrimination because you are a woman will return at your next job, this emerges from the society. Changing jobs has become more relevant in the last decades, since discrimination has become more specific per situation.

Weigelt and Dukerich (1989) have studied the reactions of people on discrimination. They find that people value winning or losing more than just the financial payoff. Winning itself is an extra incentive. This corresponds with the employee’s belief about his ability, when an employee is preferred and still loses, he will think bad about himself. Losing has less impact for employees that are discriminated against, since it is not entirely their own fault. It does affect both types of employees, which can cause them to leave after not being promoted.

Taking all this literature into consideration, I think this paper can be relevant in the battle against discrimination. It is important to find measures that take away incentives to discriminate. This paper proves that emphasizing outside options can be a useful method.

\(^1\) Becker describes how having a taste for discrimination means individuals are willing to forfeit income to avoid certain transactions. When employees expect the manager to discriminate, he will act as if he has a taste for discrimination. This is harmful to the society.
Basic model: ‘Don’t demotivate, discriminate.’
The model published by Kamphorst and Swank (2016) is used as the basic model. In this section, it will be briefly described, for further explanations and proof I refer to their paper. They start by explaining the task allocation. Two tasks are to be executed, a major task, \( m \), and a minor task. Manager, \( M \), decides who gets which task, he can choose between the two employees \( i, i \in \{1,2\} \), where \( m = 1 \) means that employee 1 receives the major task. The manager’s only intention is maximizing the company’s profit. Employees 1 and 2 want to maximize their payoffs, the salary \( y_i \) concludes both ability, \( a_i \), and exerted effort, \( e_i \). Ability and effort are independently drawn from a uniform distribution, \( a_i, e_i \in U \sim [0,1] \). The manager learns the real value of ability, the employees are only familiar with the distribution. It is possible that the major task creates more value, this is given with \( \eta \), where \( \eta \geq 1 \). The employees’ payoff is given by:

\[
y_i = \begin{cases} 
\eta a_i e_i & \text{if } m = i \\
a_i e_i & \text{if } m \neq i 
\end{cases}
\]

With \( E(a_i|m) \) as the expectation of their ability, since the true ability is unknown. Their utility functions are given by:

\[
U_i(e_i) = \begin{cases} 
E(a_i|m) * e_i - \frac{1}{2} e_i^2 & \text{if } m = i \\
\eta E(a_i|m) * e_i - \frac{1}{2} e_i^2 & \text{if } m \neq i 
\end{cases}
\]

The manager wants to maximize the aggregate output:

\[
U_M(m, a_1, a_2) = \sum_{i=1}^{2} y_i.
\]

To compute strategies, the timing of actions and events is quite relevant. At first the competences of the employees, \( a_1 \) and \( a_2 \), are drawn by nature. The next period the manager learns these abilities, the employees do not learn anything this period. Based on the abilities the manager decides which task to assign to each employee, after which the employees form beliefs about their own ability. They choose their effort levels accordingly and in the end the outputs can be realized.

Using backward induction, Kamphorst and Swank determine each agent’s strategy. Employees choose their effort level to maximize their utility level:

\[
e_i = \begin{cases} 
\eta E(a_i|m) & \text{if } m = i \\
E(a_i|m) & \text{if } m \neq i 
\end{cases}
\]
When $\eta = 1$, this model has three equilibria. Two that are discriminatory and stable and one that is neither discriminatory nor stable. For $\eta = 1$, the effort strategy of employee is $e_i = E(a_i|m)$. The manager will decide for the task assignment such that the companies profit is maximized. Employee 1 is assigned to the major task when:

$$a_1 E(a_1|m = 1) + a_2 E(a_2|m = 1) > a_1 E(a_1|m = 2) + a_2 E(a_2|m = 2).$$

This can be reshaped into an indifference equation:

$$a_1 = t \cdot a_2, \text{where } t = \frac{E(a_2|2) - E(a_2|1)}{E(a_1|1) - E(a_1|2)}.$$  

When $a_1 > ta_2$, the manager will prefer giving the major task to employee 1. If $a_1 < ta_2$, employee 2 will be promoted and employee 1 is assigned to the minor task. Equilibria are found after solving this equation, the employees’ beliefs are based on their expectation of $t$. For these beliefs form the manager’s strategy, their expectations can be self-confirming.

The beliefs are established using integrals:

$$E(a_1|1) = \frac{\int_0^1 \int_0^{ta_2} a_1 da_1 da_2}{\int_0^1 \int_0^{ta_2} da_1 da_2} = \frac{3 - t^2}{6 - 3t}$$

$$E(a_2|1) = \frac{\int_0^1 \int_0^{ta_2} a_2 da_1 da_2}{\int_0^1 \int_0^{ta_2} da_1 da_2} = \frac{3 - 2t}{6 - 3t}$$

$$E(a_2|2) = \frac{\int_0^1 \int_0^{ta_2} a_2 da_1 da_2}{\int_0^1 \int_0^{ta_2} da_1 da_2} = \frac{2}{3}$$

$$E(a_1|2) = \frac{\int_0^1 \int_0^{ta_2} a_1 da_1 da_2}{\int_0^1 \int_0^{ta_2} da_1 da_2} = \frac{1}{3} t.$$  

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In this paper only $\eta = 1$ is relevant, for an $\eta > 1$ already decreases discrimination. For this reason, no further attention will be given to the results when $\eta > 1$.  

Don’t Demotivate, Discriminate: Outside Option  

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After entering these beliefs into equation (4), the following strategy is attained:

$$t = \frac{2 - 3 - 2t}{3 - 3t - \frac{1}{3}t}$$

Equation (5) gives two equilibria, $t^* = 1$ and $t^* = 0.5$. These only apply for the assumption that $t > 1$. Since $t \leq 1$ reflects $t > 1$, another solution $t^* = 2$ can be found.

Next, Kamphorst and Swank study the stability of the equilibria to identify the most plausible outcomes. This can be done studying the manager’s strategy in situations that differ from the equilibria. The right-hand-side of equation (5) gives this strategy $\hat{t}$. As seen in figure 1, $\hat{t} > t$ for $0 < t < 0.5$ and $1 < t < 2$ and $\hat{t} < t$ when $0.5 < t < 1$ and $2 < t$. This means for $0 < t < 0.5$, the manager implements less discrimination than expected, it moves to $t = 0.5$. When employees expect $0.5 < t < 1$, it is rational for the manager to discriminate more than expected, again it moves to $t = 0.5$. The same accounts for the situations when the other employee is expected to be preferred, in the end it will move to equilibrium $t = 2$. This means the discriminatory equilibria are stable in oppose to $t = 1$.

At last the model proofs that managers prefer committing to $t = 1$. This maximizes the expected payoff for both employees and is thus optimal.

This was the basic game given by Kamphorst and Swank (2016), in the next part the basic model is extended with an outside option.

**Extension 1: outside option with a correlated ability**

In this section, an outside option is considered. If employees expect to receive a higher payoff when working for another company, it is sensible for them to change jobs. Differences between employees and a diversity of jobs can cause people to switch careers (Mishra & Mishra, 2015). When managers consider this option, it can ensure he will act without discrimination. For all extensions, it is assumed that players are risk neutral. Timing is identical to the one in the basic game, the introduced parameters are drawn by nature in the first period and when learned, this happens in the second period.

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3 Stable equilibria can also be found simplifying equation (5) and checking when $t > \hat{t}$. This is too complicated for the equations in the extensions of the model. That is why only this approach is used in this paper.

Don’t Demotivate, Discriminate: Outside Option

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Differences between jobs are what drives people to quit and switch to another business. This model introduces different abilities as incentive to change jobs. It is doubtful that an excellent secretary will outdo himself as a construction worker. However, when you are competent baking bread, this talent won’t vanish when working at another bakery. Some jobs have a correlated ability whereas other jobs do not. This gives the following equation:

\[(6) \quad E(b_i) = \beta E(x) + (1 - \beta)E(a_i|m).\]

With \(x \in U \sim [0,1]\), this leads to \(E(x) = 0.5\). The ability of employee at the outside option is given by \(b_i\), the correlation is expressed with \(\beta \in [0,1]\). The payoff scheme stays the same and thus the amount of effort an employee will exert, depends on his expected ability. For the payoff is equal to the current job, one will prefer the outside option when his expected ability is better at the other job:

\[(7) \quad E(b_i) > E(a_i|m).\]

Usually employees know what other options they have, along with the corresponding \(\beta\). Ordinarily, managers are aware of these outside options. This custom is implemented in the model by the assumption that both the employee and the manager know the value of \(\beta\) in the outside option.

Three different situations are possible. First, when \(\beta = 0\), this means \(E(b_i) = E(a_i|m)\). Employees are indifferent for both jobs, there is no incentive to leave their present job. Second, \(\beta = 1\) implies that \(E(b_i) = E(x)\). This means that an employee will change jobs when \(E(a_i|m) < 0.5\), because the expected value of \(x\) is 0.5. The last possibility is that \(0 < \beta < 1\), under these circumstances again the employee will leave when \(E(a_i|m) < 0.5\). Mathematics for this can be found in the appendix.

Employees that don’t get promoted and are thus assigned to the minor tasks always expect their ability to be equal or lower than 0.5. Apart from when \(\beta = 0\), this means that every employee that is designated to exert the minor task will leave the firm. The manager knows he will lose this employee and his real or expected ability are no longer relevant. For simplicity, it is assumed the employee will not directly be replaced. The manager’s trade-off is reformed to:

\[a_1 * E(a_1|1) > a_2 * E(a_2|2).\]

This leads to:

\[(8) \quad a_1 > t * a_2, \text{where } t = \frac{E(a_2|2)}{E(a_1|1)}.\]
Using the employees’ belief from the basic model, the equilibrium can be determined from solving:

\[
(9) \quad t = \frac{2}{3 - t^2} = \frac{4 - 2t}{3 - t^2}.
\]

This leads to only one solution, \( t = 1 \). There is no longer an incentive for the manager to discriminate, he will positively promote the employee that is the most competent. As seen in figure 2, the strategy for the manager is to favor the employee that does not expect to be preferred. Since it is optimal for the manager to act according with a \( t > \hat{t} \) when \( 0 < t < 1 \), \( t = 1 \) is a stable equilibrium. For the same reason as in the basic model, this result can be mirrored, for when the other employee is expected to be preferred. This does not lead to more solutions, \( t = 1 \) is the only existing equilibrium.

Figure 2: equilibrium when \( 0 < t < 1 \) and \( \beta = 1 \).

**Extension 2: outside option with a different salary**

In the previous extension, the only relevant difference was ability. It may be obvious that much more differences are essential (Böckerman & Ilmakunnas, 2009). One of these relevant disparities is payoff. Salary is often an incentive to change jobs, what happens when a difference in payoff schemes is realized? The utility function considering the current job remains the same. In the payoff function concerning the other job \( l \) gives the relative difference in payoff, \( l \) is uniformly distributed, \( l \in U\sim[0,2] \). If \( l = 2 \), an employee can double his salary producing the same output at the outside option. The utility functions are:

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Don’t Demotivate, Discriminate: Outside Option

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An employee will quit when $E_i(U_a) < E_i(U_b)$. Using this, a requirement can be composed. An employee will leave when the following applies:

$$E(a_i|m)^2 - 0.5 \cdot E(a_i|m)^2 < l^2 \cdot E(b_i)^2 - 0.5(l \cdot E(b_i))^2$$

This can be boiled down to:

$$\frac{E(a_i|m)}{E(b_i)} < l. \tag{11}$$

Again, there are three possible settings. When $\beta = 0$, there is no difference in abilities between the two jobs. The trade-off becomes $\frac{E(a_i|m)}{E(a_i|m)} < l$, an employee will quit when $1 < l$. The only incentive an employee can have to quit his job is the different wage. When $\beta = 1$, this puts $E(b_i) = 0.5$, employees resign if $2 \cdot E(a_i|m) < l$. This makes sense for when $E(a_i|m) = 0.5$ and wages are equal, employees have no incentive to change jobs. The last possibility is for $0 < \beta > 1$, this setting gives quite complex results. For this reason, this setting is not further illustrated, the other two settings give the basic intuitions.

In the first extension, everything was known for all different players. However, this is not realistic for the introduced variable. Salaries are a private matter, managers are not expected to learn this information. Considering this, I assume only the employee knows the value of $l$, the manager is only informed about the uniform distribution. Due to this uncertainty, the manager must work with chances, he can only determine the chance an employee will maintain working for him. The variables $(o), (p), (q)$ and $(r)$ refer to these probabilities. Again, it is assumed no direct replacement takes place. This gives the following trade-off for the manager:

$$a_1 \cdot E(a_1|1) \cdot o + a_2 \cdot E(a_2|1) \cdot p > a_1 \cdot E(a_1|2) \cdot q + a_2 \cdot E(a_2|2) \cdot r.$$ 

This gives:

$$a_1 > t \cdot a_2, \text{where } t = \frac{E(a_2|2) \cdot r - E(a_2|1) \cdot p}{E(a_1|1) \cdot o - E(a_1|2) \cdot q}. \tag{12}$$

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4 Mathematics and proof are to be found in the appendix.
In the situation that \( \beta = 0 \), the employee will stay when \( 1 > l \), regardless his \( E(a_i|m) \). For \( l \in U \sim [0, 2] \), the chances are all equal to 0.5. The manager’s strategy remains the same. When \( \beta = 1 \), other conditions apply. An employee stays when \( 2 * E(a_i|m) > l \), with this equation we can determine the chances one will stay at the firm after the task assignment. These chances are equal to \( E(a_i|m) \). Using these probabilities and the employees’ beliefs from the basic model, the following equation can be composed:

\[
(13) \quad t = \frac{E(a_2|2) \ast \frac{2}{3} - E(a_2|1) \ast \frac{3 - 2t}{6 - 3t}}{E(a_1|1) \ast \frac{3 - t^2}{6 - 3t} - E(a_1|2) \ast \frac{1}{3} t} = \frac{4 \ast (3 - 2t)^2}{9 - (6 - 3t)^2} - \frac{1}{9} t^2.
\]

The equation has two relevant solutions, \( t = 0.85286 \) and \( t = 1 \). As seen in figure 2, when \( t < 0.85286 \), it is optimal for the manager to discriminate less than expected, while the manager discriminates more than foreseen by the employees when \( 0.85286 < t < 1 \). This means the discriminatory equilibrium is stable. Again, the results can be mirrored, this leads to another discriminatory stable equilibrium \( t = 1.17253 \). These results are coherent with the previous results, in the basic model the manager takes all employees’ beliefs into account, in contrast to extension 1 where only promoted employees are expected to keep working for the company. In this situation, the manager computes the chances employees leave, what leads to a less discriminatory equilibrium. Evidently, this means having an outside option, with a completely unrelated ability and a different payoff scheme, decreases the manager’s incentive to discriminate.

Figure 3: equilibria when \( 0 < t < 1 \) and \( \beta = 1 \).

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5 Mathematics and proof are to be found in the appendix.
6 Mathematics and proof are to be found in the appendix.
Extension 3: outside option with switching costs

Until now there were no costs attached to switching careers. When the payoff at another job was slightly higher than the payoff at the current job, an employee would resign. However, this does not reflect the real world as one would prefer. Switching careers is always concerned with costs, applying for a job takes effort and getting used to your new job is not always easy. Briefly, taking another job requires an investment. Alan Blinder suggests in a paper from Akerlof, Rose and Yellen (1988, p. 593) that these costs of changing jobs are included in the model. All of this can be introduced in the model using a new variable for switching costs, $c$. The utility functions become:

\[
\begin{align*}
E(U_a) &= e_i \cdot E(a_i|m) - 0.5(e_i)^2 & \text{with } e_i = E(a_i|m) \\
E(U_b) &= l \cdot e_i \cdot E(b_i) - 0.5(e_i)^2 - c & \text{with } e_i = l \cdot E(b_i).
\end{align*}
\]

An employee quits when $E(U_a) < E(U_b)$, this is given by:

\[
E(a_i|m)^2 - 0.5E(a_i|m)^2 < l^2 \cdot E(b_i)^2 - 0.5(l \cdot E(b_i))^2 - c.
\]

This can be boiled down to:

\[
\sqrt{\frac{E(a_i|m)^2 + 2c}{E(b_i)^2}} < l
\]

For $E(b_i) = (\beta x + (1 - \beta)E(a_i|m))$ can be applied here, as well as in the other sections, there are three possibilities, of which two will be discussed. First, $\beta = 0$, meaning $E(b_i) = E(a_i|m)$. This gives $\sqrt{1 + \frac{2c}{E(a_i|m)^2}} < l$ as the condition for employees to quit their job. This is plausible, when the switching costs increase, the relative wage must be higher for an employee to quit his job. The second setting is $\beta = 1$, this implies $E(b_i) = E(x) = 0.5$. Employees resign when the rule $2 \cdot \sqrt{E(a_i|m)^2 + 2c} < l$ applies.

The strategy for the manager is built the same way as the manager’s strategy in the previous section. For the relative payoff is again unknown for the manager, he can only derive the chances his employees will keep working for him. These chances are based on the uniform distribution of $l$ and the derivations of the chances are to be found in the appendix.

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7 Mathematics and proof are to be found in the appendix.
8 The setting that $0 < \beta < 1$ is again too complicated, that is why only the limit values are examined.
9 Mathematics and proof for both derived strategies are to be found in the appendix.
The manager’s approach, the same as in extension 2, is determined by the employees’ ability beliefs and the chances they stay to work for him. These expected abilities are equal to those in the basic game.

\[
(12) \quad t = \frac{E(a_2|2) \cdot r - E(a_2|1) \cdot p}{E(a_1|1) \cdot o - E(a_1|2) \cdot q}.
\]

First the manager’s strategy will be discussed when \( \beta = 0 \). When the manager’s strategy is completed with beliefs and chances\(^{10} \), this gives:

\[
(16) \quad t = \frac{2}{3} \left( \sqrt{0.25 + \frac{c}{2 \cdot \left( \frac{2}{3} \right)^2}} - \frac{3 - 2t}{6 - 3t} \left( \sqrt{0.25 + \frac{c}{2 \cdot \left( \frac{3 - 2t}{6 - 3t} \right)^2}} \right) \right).
\]

Important is to keep in mind that chances can never exceed 1. In this formula, the individual chances can get higher than 1 when switching costs are high. Obviously, this is not desirable, for the manager will value the corresponding expectations too much. For the complexity of the equation and because the chances must be verified for every approach, only numerical solutions will be discussed.

If the switching costs equal 0, the equilibria will be identical to the results found by Kamphorst and Swank in their model; \( t = 0.5 \) and \( t = 1 \), where only the discriminatory equilibria is a stable one. For each tested value of \( c \), the chances are determined. When these exceed 1, this chance is replaced with 1 and the other chances become more important. An interesting observation is that the chances \( q \) and \( p \) increase faster, thus unpromoted employees are more eager to stay. They think worse about their ability; a higher wage influences their payoff less and therefore higher switching costs have more influence. After inserting several values for \( c \) and adjusting the formula when chances would otherwise exceed 1, the equilibria be determined. These solutions could be mirrored for when \( t > 1 \) and the final results are to be found in table 1.

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\(^{10}\) Derivations of the chances are found in the appendix.
Switching costs (c) | Solutions | Stable equilibria
---|---|---
0.000 | 0.5 / 1 / 2 | 0.5 / 2
0.005 | 0.52778 / 1 / 1.89473 | 0.52778 / 1.89473
0.01 | 0.54962 / 1 / 1.81944 | 0.54962 / 1.81944
0.05 | 0.64705 / 1 / 1.54548 | 0.64705 / 1.54548
0.1 | 0.28697 / 1 / 3.48468 | 0.28697 / 3.48468
0.2 | 0.16997 / 5.88339 | 0.16997 / 5.88339
0.3 | 0.12373 / 8.08211 | 0.12373 / 8.08211
0.4 | 0.15169 / 1 / 6.59239 | 0.15169 / 6.59239
0.5 | 0.25756 / 1 / 3.88259 | 0.25756 / 3.88259
0.6 | 0.39824 / 1 / 2.51105 | 0.39824 / 2.51105
0.7 ≤ | 0.5 / 1 / 2 | 0.5 / 2

Table 1: equilibria for different levels of switching costs when $0 < t < 1$ and $\beta = 0$.

The discriminatory results are stable, which makes them most likely to happen. Multiple trends can be found in these stable equilibria, no clear strategy can be determined. When all probabilities are below 1, the equilibrium is discriminating less as the switching costs increase. After the switching costs become higher than 0.1, chance $q$ is turned into 1 and increasing switching costs cause the stable equilibrium to become a lot more discriminatory. This effect is turned around after $c \geq 0.3$, change $p$ is set to 1 and the stable equilibrium approaches 0.5 as the switching costs increase. These different trends are likely to be caused by the different courses of the chances, employees react very different from each other. This means unique approaches for each situation are necessary, conclusions can only be drawn on an individual level of switching costs.

Finally, the equilibria are determined when $\beta = 1$. The manager’s strategy is described by:

$$t = \frac{1}{3} \left( \frac{2^2}{3} + 2c \right) - \frac{3 - 2t}{6 - 3t} \left[ \sqrt{\frac{3 - 2t^2}{6 - 3t} + 2c} \right] - \frac{3 - t^2}{6 - 3t} \left[ \sqrt{\frac{(3 - t^2)^2}{6 - 3t} + 2c} \right].$$

The individual chances can again go beyond 1, hence the chances are individually tested and are replaced by 1, when they exceed it. While solving the equations and testing the values of the chances, some interesting things were noticed. In oppose to when $\beta = 0$, chances $r$ and $o$

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11 Derivations of the chances are found in the appendix.
increased faster with the switching costs, this means the promoted employees are more eager to stay at their current job. These employees think higher of their current competence and are less likely to increase their payoff at the outside option. Of course, this influences the manager's strategy, instead of focusing on the non-promoted employees, he focuses on the promoted employees. In table 2, the equilibria for several values of $c$ can be found.

<table>
<thead>
<tr>
<th>Switching costs (c)</th>
<th>Solutions</th>
<th>Stable equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.85286 / 1 / 1.17253</td>
<td>0.85286 / 1.17253</td>
</tr>
<tr>
<td>0.005</td>
<td>0.85240 / 1 / 1.17316</td>
<td>0.85240 / 1.17316</td>
</tr>
<tr>
<td>0.01</td>
<td>0.85120 / 1 / 1.17481</td>
<td>0.85120 / 1.17481</td>
</tr>
<tr>
<td>0.05</td>
<td>0.82963 / 1 / 1.20536</td>
<td>0.82963 / 1.20536</td>
</tr>
<tr>
<td>0.1</td>
<td>0.79669 / 1 / 1.25519</td>
<td>0.79669 / 1.25519</td>
</tr>
<tr>
<td>0.2</td>
<td>0.74114 / 1 / 1.34927</td>
<td>0.74114 / 1.34927</td>
</tr>
<tr>
<td>0.3</td>
<td>0.63060 / 1 / 1.58579</td>
<td>0.63060 / 1.58579</td>
</tr>
<tr>
<td>0.4</td>
<td>0.47060 / 1 / 2.12495</td>
<td>0.47060 / 2.12495</td>
</tr>
<tr>
<td>0.5 ≤</td>
<td>0.5 / 1 / 2</td>
<td>0.5 / 2</td>
</tr>
</tbody>
</table>

Table 2: equilibria for several levels of switching costs when $0 < t < 1$ and $\beta = 1$.

Again, the discriminatory equilibria are stable and are thus most probable. For this setting, it is a lot easier to determine one trend. If the switching costs increase, the manager will discriminate more. Interesting is that when $c = 0.4$, the stable equilibrium is more discriminatory than $c \leq 0.5$, this irregularity is caused by the fact that when $c = 0.4$, only $q$ is lower than 1. Apparently only the employee that is expected to be preferred, but is not promoted, can possibly increase his utility when switching jobs. That causes the manager to discriminate more, for he is less interested in the beliefs of this employee. Overall the outcomes are consistent, when the costs of switching increase it is sensible for the manager to act more discriminatory, for the chances that the employees leave the company, are smaller.

**Discussion and Concluding Remarks**

In this paper, multiple approaches for outside options are added to the basic model. The objective was to illustrate possible situations in which the manager would not discriminate despite expectations of the employees. Such a situation can be stimulated by adding an outside option. A stable non-discriminatory equilibrium is found when the manager can be sure that the non-promoted employee leaves after the task-assignment. The job change was based on differences in the employee’s competence for the both jobs. This setting is not very realistic, there are a lot more
circumstances that influence both the employee’s and manager’s strategy. For this reason, wage differences and switching costs are considered. After adding the difference in wage, the manager’s strategy copes with uncertainty. He cannot be sure when an employee leaves, for he does not know what his employees can earn at another company. Again, the manager’s strategy is only affected when the two abilities are uncorrelated. The outcome is a discriminatory stable equilibrium that discriminates less than the basic model. The last extension illustrates the situation when switching costs are incurred, this decreases the chance that employees leave the firm. This also effects the setting where the ability for the outside option is equal to the current ability. It is hard to draw a clear conclusion for this setting, different trends occur because of the diversity in the courses of the chances. Some employees tend to leave sooner than others. The manager can be stimulated to discriminate when the switching costs increase, but for some values the discrimination will decline. When the current ability does not tell anything about the ability in the outside option, there is one obvious trend. When the switching costs increase, the manager is tended to discriminate more.

These results can be used in the fight against discrimination. When job switching is a more obvious choice for an employee, this may cause the manager to discriminate less. Job switching can be encouraged and promoted by several policy measures. For example, when better facilities are introduced to find job openings, outside options become more relevant for both the employee and his manager. Overall, lower switching costs stimulate the manager to discriminate less. These costs can be reduced when it is easier for employees exclude themselves from their employment contract.

References


Appendix

Extension 1:

employees’ strategy when $0 < \beta < 1$

$$\beta \cdot 0.5 + (1 - \beta)E(a_i|m) > E(a_i|m)$$

$$\beta \cdot 0.5 + E(a_i|m) - \beta \cdot E(a_i|m) > E(a_i|m)$$

$$\beta \cdot 0.5 > \beta \cdot E(a_i|m)$$

$$0.5 > E(a_i|m)$$

Extension 2:

simplification employees’ strategy

$$E(a_i|m)^2 - 0.5 \cdot E(a_i|m)^2 < l^2 \cdot E(b_i)^2 - 0.5(l \cdot E(b_i))^2$$

$$E(a_i|m)^2 - 0.5 \cdot E(a_i|m)^2 < l^2 \cdot E(b_i)^2 - 0.5(l \cdot E(b_i))^2$$

$$0.5 \cdot E(a_i|m)^2 < 0.5 \cdot l^2 \cdot E(b_i)^2$$

$$E(a_i|m)^2 < l^2 \cdot E(b_i)^2$$

$$E(a_i|m) < l \cdot E(b_i)$$

$$\frac{E(a_i|m)}{E(b_i)} < l$$

manager’s strategy

$$t = \frac{E(a_2|2) \cdot r - E(a_2|1) \cdot p}{E(a_1|1) \cdot o - E(a_1|2) \cdot q}$$

$$t = \frac{E(a_2|2) \cdot 0.5 - E(a_2|1) \cdot 0.5}{E(a_1|1) \cdot 0.5 - E(a_1|2) \cdot 0.5}$$

$$t = \frac{0.5 \cdot (E(a_2|2) - E(a_2|1))}{0.5 \cdot (E(a_1|1) - E(a_1|2))}$$

$$t = 1 \cdot \frac{E(a_2|2) - E(a_2|1)}{E(a_1|1) - E(a_1|2)}$$
chances that an employee stays when $\beta = 1$

Employee stays when $2 * E(a_i|m) < l$

$$\Pr(2 * E(a_i|m) < l) = \frac{2 * E(a_i|m) - 0}{2 - 0} = E(a_i|m)$$

$$E(a_i|1) = \frac{3 - t^2}{6 - 3t}$$

$$o = \Pr\left(\frac{6 - 2t^2}{6 - 3t} > l\right) = \frac{3 - t^2}{6 - 3t}$$

$$E(a_2|1) = \frac{3 - 2t}{6 - 3t}$$

$$p = \Pr\left(\frac{6 - 4t}{6 - 3t} > l\right) = \frac{3 - 2t}{6 - 3t}$$

$$E(a_1|2) = \frac{1}{3}t$$

$$q = \Pr\left(\frac{1}{6}t > l\right) = \frac{1}{3}t$$

$$E(a_2|2) = \frac{2}{3}$$

$$r = \Pr\left(\frac{4}{3} > l\right) = \frac{2}{3}$$

Extension 3:

simplification employees’ strategy

$$E(a_i|m)^2 - 0.5E(a_i|m)^2 < l^2 \cdot E(b_i)^2 - 0.5(l * E(b_i))^2 - c$$

$$0.5E(a_i|m)^2 < 0.5(l * E(b_i))^2 - c$$

$$E(a_i|m)^2 < (l * E(b_i))^2 - 2c$$

$$\frac{E(a_i|m)^2 + 2c}{E(b_i)^2} < l^2$$

$$\sqrt{\frac{E(a_i|m)^2 + 2c}{E(b_i)^2}} < l$$
simplification employees’ strategies

\[ \beta = 0 \text{ gives } E(b_i) = E(a_i|m) \]

\[ \sqrt{\frac{E(a_i|m)^2 + 2c}{E(a_i|m)^2}} < l \]

\[ \sqrt{\frac{E(a_i|m)^2}{E(a_i|m)^2 + 2c}} < l \]

\[ \sqrt{1 + \frac{2c}{E(a_i|m)^2}} < l \]

\[ \beta = 1 \text{ gives } E(b_i) = E(x) = 0.5 \]

\[ \sqrt{\frac{E(a_i|m)^2 + 2c}{0.5^2}} < l \]

\[ \sqrt{\frac{E(a_i|m)^2 + 2c}{\sqrt{0.25}}} < l \]

\[ \sqrt{\frac{E(a_i|m)^2 + 2c}{0.5}} < l \]

\[ 2 \sqrt{E(a_i|m)^2 + 2c} < l \]

chances that an employee stays when \( \beta = 0 \)

Employee stays when \[ \sqrt{1 + \frac{2c}{E(a_i|m)^2}} > l \]

\[ \Pr \left( \sqrt{1 + \frac{2c}{E(a_i|m)^2}} > l \right) = \frac{\sqrt{1 + \frac{2c}{E(a_i|m)^2}} - 0}{\frac{1}{2}} = \frac{0.25 + \frac{c}{2 * E(a_i|m)^2}}{2} \]
\[ E(a_1|1) = \frac{3 - t^2}{6 - 3t} \]

\[ o = \Pr \left( \sqrt{1 + \frac{2c}{\left(\frac{3 - t^2}{6 - 3t}\right)^2}} > l \right) = \sqrt{0.25 + \frac{c}{2 \cdot \left(\frac{3 - t^2}{6 - 3t}\right)^2}} \]

\[ E(a_2|1) = \frac{3 - 2t}{6 - 3t} \]

\[ p = \Pr \left( \sqrt{1 + \frac{2c}{\left(\frac{3 - 2t}{6 - 3t}\right)^2}} > l \right) = \sqrt{0.25 + \frac{c}{2 \cdot \left(\frac{3 - 2t}{6 - 3t}\right)^2}} \]

\[ E(a_1|2) = \frac{1}{3} t \]

\[ q = \Pr \left( \sqrt{1 + \frac{2c}{\left(\frac{1}{3} t\right)^2}} > l \right) = \sqrt{0.25 + \frac{c}{2 \cdot \left(\frac{1}{3} t\right)^2}} \]

\[ E(a_2|2) = \frac{2}{3} \]

\[ r = \Pr \left( \sqrt{1 + \frac{2c}{\left(\frac{2}{3}\right)^2}} > l \right) = \sqrt{0.25 + \frac{c}{2 \cdot \left(\frac{2}{3}\right)^2}} \]

chances that an employee stays when \( \beta = 1 \)

\[ \text{Employee stays when } 2 \cdot \sqrt{E(a_i|m)^2} + 2c > l \]

\[ \Pr \left( 2 \cdot \sqrt{E(a_i|m)^2} + 2c > l \right) = \frac{2 \cdot \sqrt{E(a_i|m)^2} + 2c - 0}{2 - 0} = \sqrt{E(a_i|m)^2} + 2c \]

\[ E(a_1|1) = \frac{3 - t^2}{6 - 3t} \]

\[ o = \Pr \left( 2 \cdot \sqrt{\left(\frac{3 - t^2}{6 - 3t}\right)^2} + 2c > l \right) = \sqrt{\left(\frac{3 - t^2}{6 - 3t}\right)^2} + 2c \]
\[
E(a_2|1) = \frac{3 - 2t}{6 - 3t}
\]
\[
p = \Pr \left( 2 \sqrt{\frac{3 - 2t}{6 - 3t}^2 + 2c > l} \right) = \sqrt{\frac{3 - 2t}{6 - 3t}^2 + 2c}
\]
\[
E(a_1|2) = \frac{1}{3} t
\]
\[
q = \Pr \left( 2 \sqrt{\frac{1}{3}^2 + 2c > l} \right) = \sqrt{\frac{1}{3}^2 + 2c}
\]
\[
E(a_2|2) = \frac{2}{3}
\]
\[
r = \Pr \left( 2 \sqrt{\frac{2}{3}^2 + 2c > l} \right) = \sqrt{\frac{2^2}{3} + 2c}
\]