The effectiveness of leniency programs

Introducing uncertainty about the fine reductions

Abstract

I study the effectiveness of leniency programs when there exists uncertainty about the fine reduction that the firms will receive. The study is an extension to the model of Motta and Polo (2003). The aim is to find a solution for a problem observed in, among others, Korea, where firms abuse the leniency program by repeatedly colluding and subsequently applying for leniency. The study shows that adding extra uncertainty to the model leads to a situation in which firms are less likely to reveal information to the antitrust authority and therefore it reduces the effectiveness of the leniency programs.

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1. Introduction

Firms have incentives to behave in a certain way to increase their profits. This behaviour could be desirable since it could lead to a better deal for consumers. This is the case when firms increase their efficiency, or when they lower their profit margin. There are also other ways to increase profits. Firms could behave predatorily to deter market entry as well as making collusive agreements. Predatory behaviour and collusion are detrimental to welfare and therefore not allowed under most of the competition laws. Even though collusion is forbidden by law, collusion still does occur. Therefore, making collusive agreements must be profitable for firms despite the possible punishment of behaving against the law.

Antitrust authorities aim to ensure fair competition between firms and they aim to protect consumer interests. Most countries have their own antitrust authority but there are also antitrust authorities on the supranational level. In the European Union, the European Commission oversees the supervision of the competition on the European markets. On national level, the competition authorities tend to be independent from the government. The authorities analyse the market and have the competence and the right instruments to enforce competition policy and to ensure a fair market. Antitrust authorities are therefore also the authorities trying to control predatory behaviour and collusion. They detect cartels through several instruments and if they indeed find that firms are colluding, they have the authority to impose a fine.

One of the instruments of antitrust authorities to detect cartels is the leniency program. In this paper, the leniency programs will be reviewed and a solution will be sought to the problem that firms might abuse the leniency program. This will be done by extending the model of Motta and Polo (2003) with uncertainty about the fine reduction.

Leniency programs are a relatively new phenomenon. In the United States, the U.S. Antitrust Division of the Department of Justice introduced a leniency program in 1978, which allowed for avoiding criminal sanctions given that specific conditions were fulfilled. However, these early leniency programs were unsuccessful. A former Director of Criminal Enforcement of the Antitrust Division of the U.S. Department of Justice, stated that the program lacked transparency, was unpredictable, and failed to provide the necessary incentives and induce self-reporting and cooperation. This among other things led to only one leniency application per year. Also, it did not lead to the detection of international cartels. The leniency program was therefore not appropriate. The U.S. Antitrust Division revised the program and launched the revised program in 1993. The revised program led to nearly twenty times as much applications per year and it also led to the detection of dozens of large international cartels (Hammond, 2004).
The European Union introduced a leniency program in 1996. The reduction of the fine depended on when the firm revealed information, whether this was before or after the opening of an investigation. An important feature of this program was that the firm that revealed the information, must be the first firm. If the other firm already revealed the information, then the second firm did not get a fine reduction. The leniency program was unsuccessful mainly because the leniency was not automatic but depended on the decision of the European Commission. It was uncertain for firms whether they would receive immunity. Also, if the investigation had already begun, the firms could not apply for immunity. The European Commission felt the necessity to optimize the program and they did in 2002. The new program was more transparent and less uncertain and became therefore also more successful (Motta, 2004).

Both programs, of the United States and the European Union, were very sensitive to small changes in the program. Overall, there are general ideas about the implications of a leniency program. In the short-run, leniency programs facilitate the detection of cartels. This leads to a cost reduction for the antitrust authority. The antitrust authority saves time, effort and other resources necessary to investigate a cartel, because the firms share information that is enough to come to a conviction. In the long run, leniency programs should deter firms from collusive behaviour (Brenner, 2009). Often, it is hard for antitrust authorities to detect a cartel. Firms have incentives to hide everything that could lead to information about the cartel and information about a cartel is hard to gather considering that the authorities are outsiders. Therefore, it is easier if the information comes from inside the cartel. This information is more reliable and often more complete. The leniency program functions as an instrument to receive information from inside the cartel. Under the leniency program, whistleblowers tell about the cartel and when they are the first to reveal about the cartel, they receive a fine reduction or full leniency. If the other firm already told the authorities about the cartel, then the second whistleblower does not receive a fine reduction under most of the programs. Leniency programs typically work if the expected fine reduction is high enough and offsets the costs of the cartel being broken down.

A problem of leniency programs is that some firms abuse the programs by repeatedly colluding and then applying for leniency. For example, this was observed in Korea (Choi and Hahn, 2014). This leads to a situation in which firms profit from cartels but do not have to pay a fine or they pay a reduced fine. The antitrust authority aims to maximize social welfare and therefore considers the situation of repeatedly colluding and applying for leniency inferior. A solution could be to introduce uncertainty into the leniency program. This can be done through making the content of the leniency program different every few years, or through making it
uncertain whether the program will exist at all in the future. Another solution is to introduce uncertainty about the fine reduction. I will extend the model of Motta and Polo (2003) to a model with a policy parameter that describes the uncertainty about the fine reduction.

I will define collusion conform the definition given by Motta (2004), as a situation where a firm’s price is higher than some competitive benchmark. It can also be regarded as a situation in which the prices are nearly similar to the monopoly price. There are several market characteristics that can facilitate collusion and we will discuss a few main characteristics that are relevant for this paper. First, collusion might be facilitated by the concentration of the market. When there are less firms in the market, collusion is more likely to occur. When there are more rival firms, there is a greater chance that one of the rival firms deviates from the collusive strategy. A cartel with more firms is therefore less likely to be stable. Another market characteristic that influences the collusive behaviour of firms is the difficulty of entering a market. When it is easy to enter a market, the cartels are less stable. New firms enter the market frequently and disturb the stability of the cartels. Symmetry among firms is also an important feature of markets that influences cartel stability. Symmetry leads to the situation where agreements more easily can be formed. Also, because the firms are symmetric, they have the same incentives and strategy. Asymmetric firms have different incentives, which leads to a situation in which deviation is more likely to occur (Motta, 2004). All these features influence the likeliness of collusion to occur and should be considered when reading this paper.

First, the paper starts with a literature review. Leniency programs have received a lot of attention during the past twenty years and it is therefore worthwhile to review this literature and summarize the most important and most relevant conclusions. The main and most important part will be the introduction of a model in which there exists uncertainty about the fine reduction. I found a two-sided effect of adding uncertainty about the fine reduction. When the firm plays the strategy of colluding and revealing, then by introducing uncertainty about the fine, the incentive to collude can be reduced. This is the case when the reduced fine is accompanied by a lower probability that the firm receives the reduced fine. However, I also find that firms are less likely to reveal information when the uncertainty about the fine reduction becomes more significant.

The conclusion of this paper is consistent with previous literature about the previous leniency programs in the United States and the European Union that failed. I found that by adding uncertainty, firms are less likely to reveal information to the antitrust authority and thus the leniency program is less successful. Whether adding uncertainty is effective to prevail firms
from repeatedly colluding and applying for leniency remains unclear and is a subject for further research.

2. Literature review

The rise of leniency programs drew the attention of mostly industrial economists this led to a lot of literature that has been written in the past ten years. An influential paper is a more general paper about self-reporting by Kaplow and Shavell (1994) in which they add self-reporting to a model in which they control for negative externalities through probabilistic law enforcement. Through this modelling, they found two main advantages of self-reporting. First, they found that resources are saved because through the self-detection, the authorities do not have to invest their own resources. Second, risks are reduced. The firms or individuals that report about violations bear a certain instead of an uncertain sanction. Through this model, Kaplow and Shavell showed why enforcement should take advantage of self-reporting and why leniency programs have advantages. By introducing uncertainty about the fine reduction in this paper, the second advantage of self-reporting will not be present. Firms or individuals profited from self-reporting because the risks were reduced. They bore a certain sanction instead of an uncertain sanction. By remodelling the model of Motta and Polo (2003), the sanction will be uncertain and therefore the second advantage of self-reporting does not apply in the new and extended model.

Leniency programs exist because self-detection has advantages for the antitrust authorities and the antitrust authorities believe collusion is detrimental to social welfare. This implies that collusion has social costs. Spagnolo (2008) states that cartels do not necessarily have to be bad for the society. In most cases cartels negatively affect welfare, but not in all cases. As an example, Spagnolo points out that there are situations where competition harms consumers and consequently, agreements are necessary to limit the negative effect on social welfare. It is important to keep this in mind for the model in this paper that collusion does not always have to be undesirable. At the same time, the social costs that do appear show that the subject of this paper is still a relevant subject and it is necessary to reduce the social costs as much as possible.

The studies of Kaplow and Shavell (1994) and Spagnolo (2008) showed the advantages of self-reporting and the possibility of social costs of collusion. Most of the literature of industrial economists consist of models about the effectiveness of leniency programs. The most important paper about leniency programs and their effectiveness in general comes from Motta and Polo (2003). Motta and Polo signal a two-sided effect of the leniency program. On the one
hand, leniency programs make enforcement of competition law more effective. There is a
greater chance that the antitrust authority receives information about a cartel because of the
leniency program. On the other hand, leniency programs may induce collusion because the
expected punishment is lower. Motta and Polo made a simple model in which they study which
effect prevails. An important feature of the model is that firms can apply for leniency by
revealing information even after the investigation is already opened. In the model, if there is a
leniency program, it depends on the probability that the antitrust authority will successfully
complete an investigation whether firms will reveal information. Motta and Polo found that a
generous leniency policy might stimulate collusion. A leniency program reduces the expected
value of the fine and therefore gives an incentive to collude. This is represented by area 1 in
figure 1. They also found that collusion would break down when firms reveal information to the
antitrust authority which makes collusion less likely. This is represented by area 2 in figure 1.
It follows from the figure that the second effect, leniency programs making enforcement more
effective and breaking down cartels, usually dominates. This suggests that leniency programs
are effective and should always be part of the instruments of antitrust authorities.

![Figure 1: Equilibrium solutions for given policy parameters.](image)

*Source: Motta and Polo (2003)*

Motta and Polo also modelled a different situation in which firms only get a fine reduction when
they reveal information before an investigation is started, contrary to the situation in which
they still get a fine reduction when they cooperate with the antitrust authorities during the
investigation. In the new situation, there does not change anything for the firms between the
decision to collude and the moment that the antitrust authority starts an investigation. Therefore, this does not lead to cartels breaking down. Motta and Polo conclude that the benefits from a leniency program should be extended to the period after the investigation is opened to make the leniency program more effective. The simple model presented by Motta and Polo will be the starting point for this thesis. The model will be extended with uncertainty about the fine reduction when firms reveal their information.

By extending the model of Motta and Polo (2003), there are a few concepts that need to be taken into account. For each of the concepts, I will state the relevance for the model introduced in this paper. One important concept of leniency programs is ‘The Race to the Courthouse Effect’, introduced by Harrington (2008). This effect causes a milder program to increase the expected present value of the punishment. When the leniency program is not favourable for firms, for example when the fine reductions are very low, then it could be a dominant strategy for all firms not to apply for leniency. If the program is favourable, then all the firms have an incentive to apply for leniency as soon as possible, because only the first firm receives leniency. Harrington concludes that ‘The Race to the Courthouse Effect’ leads to a more favourable policy, which makes colluding less attractive because it lowers the expected payoff from collusion. There is a great chance that rival firms will apply for leniency, and therefore a firm wants to apply for leniency too and this starts a competition among the firms. They all want to apply for leniency first. Choi and Hahn (2014) state that ‘The Race to the Courthouse Effect’ exists when every colluding firm has an incentive to apply for leniency as soon as possible, because the penalty increases depending on how many rivals applied before the application of the firm. The effect found by Harrington will not be something to consider in this model since both the firms are entitled to leniency. The timing of the revelation of information and the action of the other firm does not influence the probability of receiving leniency for the other firms. However, this effect is relevant when this model is used in further research and the rules of the game are changed.

Another important concept is the result from a study by Livernois and McKenna (1999). They extended theories about enforcement of standards with a self-reporting option. More specifically, the theory is about enforcement of pollution standards, but the outcome of the paper also has a certain value for leniency programs. Livernois and McKenna observed that firms complied with the standards, even though the expected punishments were low. In the extended model, they proof that under certain conditions, it is possible to have lower fines while maintaining or increasing a high compliance rate. The implications for leniency programs would be that if the antitrust authorities would lower the fine, then under certain conditions, more firms would comply with the competition laws. Another result is that to minimize the costs of the antitrust authorities, the fine for violating the laws should be equal to zero. Lowering the
fine has two implications. There will be less firms that comply with the competition laws, because the expected fine will be lower. This lowers the compliance rate. In the contrary, firms that were already violating the laws are more likely to report more truthfully to the authorities. This increases the compliance rate and the overall effect is an increase of the compliance rate. Violators are detected in an earlier stage and this also reduces the costs of the investigation. In this paper, the effect found by Livernois and McKenna will not be the focus, since the main focus will be the model of Motta and Polo (2003). However, the results found by Livernois and McKenna have influenced the design of leniency programs and they are interesting to consider by redesigning leniency programs. In the discussion, this subject will be considered. The results imply that less uncertainty, and therefore less expected fines, would lead to a higher compliance rate. The conclusion of this paper is that less uncertainty leads to more firms revealing about a cartel, but it is complicated to draw conclusions about the compliance rate since it is not known how many cartels there are.

Besides the general and mostly theoretical literature on leniency programs and other forms of self-reporting, there also exists literature about the revision of leniency programs. Empirical studies are prone to sample selection biases because only firms that were detected can be considered. Therefore, there are not so many empirical studies on leniency programs. Another difficulty to consider with empirical studies is the two-sided effect of the detection rate. A higher detection rate increases the number of successfully discovered cartels compared to the total number of cartels. However, the higher detection rate lowers the formation rate since firms have less incentives to collude, and this decreases the number of cartels that are possible to discover (Choi and Hahn, 2014).

One of the empirical studies on leniency programs is done by Brenner (2009) with data from the leniency program from 1996 of the European Union. Brenner compared cartels that were detected after the implementation of the leniency program with cartels detected in the period without the program. In the short-run, evidence of the study suggests that more information is revealed and legal costs of the antitrust authorities are reduced after the implementation of the leniency program. The average duration of investigations decreased with 1.5 years and this also led to a substantial cost reduction. An interesting result is the result of the model of Brenner in the long-run. The hazard model does not support the view that leniency programs destabilize cartels. The results show that stopping collusion does not significantly change after adopting the new policy instrument. Brenner explains the rise in detected cartels after the adoption of the new program by referring to economic and political events. This explanation shows that the models about leniency programs are sensitive to circumstances on economic and political levels.
Choi and Hahn (2014) studied empirical data from Korea about the introduction and revision of the leniency program. Korea is a good example because they introduced a leniency program in 1995 and ever since the beginning, the program went through a lot of revisions. The revisions led to a more transparent and certain leniency program. The most important revision came in 2005 in which full exemption was offered for the first applicant in both the pre-investigation phase and the post-investigation phase. Choi and Hahn studied the effectiveness of the leniency programs with empirical data and found that the revision in the short-run leads to longer cartel durations, but in the long-run, it decreases the cartel duration. The program increased the ability to detect and deter cartels and therefore could be considered successful. However, Choi and Hahn made some remarks that are relevant for this paper. They observed that some firms in Korea abused the leniency program by repeatedly joining cartels and then applying first for full exemption. They profited from the cartel and did not receive the punishment. This suggests that the successful leniency program with full exemption, as it is seen in the literature, can be abused and is not as effective as thought. Korea solved the problem by not giving leniency to firms twice within five years.

A problem with this solution is that the firms are aware of this rule and will not reveal information when they are in a cartel for the second time within five years. This paper aims to find a solution for this problem of the leniency programs. The empirical results reveal a problem of the programs and by remodelling the model of Motta and Polo (2003), this paper aims to reduce the incentive to repeatedly collude. In the literature, the problem that occurred in Korea is signaled, but there is no solution given for this problem. There are numerous possible solutions that should be considered and this paper will provide one of those possible solutions. The model will be consistent with the models developed by the industrial economists, but will be extended with a more realistic feature which is uncertainty. Therefore, this paper is a valuable addition to the existing literature.

3. The model

Consider an infinitely repeated game with two players, firm 1 and firm 2, in a certain industry. The firms are perfectly symmetric, hence, the firms will choose the same strategy, collusion or deviation. The firms aim to maximize their profits, and this profit-maximization determines the choice of strategy. Consider also an antitrust authority (AA) that aims to maximize the utilitarian welfare function. At $t = 0$, the AA chooses an enforcement policy and the AA is able to commit to this policy. By choosing the enforcement policy, the AA determines the policy parameters and chooses which markets they are going to investigate. There are five policy parameters that the AA sets at the beginning of the game. The budget of the AA is exogenous,
which means that there is a trade-off between two of the parameters, the resources invested in the monitoring of markets and the prosecution stage. In the monitoring stage, the AA does not make type I errors, which means that when the AA monitors an industry and there is no collusion in the industry, then the AA does not enter the prosecution stage.

3.1. Enforcement choices
The first parameter that the AA will choose is $F \in [0, \bar{F}]$, representing the full fines. Firms receive a full fine if they are proven guilty and chose not to reveal information. Also, if a firm did not reveal information, but the other colluding firm did, then the first firm receives the full fine $F$. The second parameter, $R \in [0, F]$, represents the reduced fines. It is above zero, because there still is a punishment for colluding, but it is below the full fine since firms need to be rewarded for revealing information. If this was not the case, they would never reveal information to reduce the chance of the cartel being proven. In this model, for simplicity, firms can receive the reduced fine, even if they reveal information after the AA started an investigation. This is an important feature of this model. The third parameter, $\alpha \in [0,1]$ represents the probability that the AA starts an investigation. The fourth parameter, $p \in [0,1]$ represents the probability that the AA proves the existence of a cartel when both the firms did not cooperate. The fifth policy parameter is the extension to the model of Motta and Polo (2003). The parameter, $\theta \in [0,1]$, represents the probability that the firm receives a reduced fine when the firm revealed information.

The extension of the model is an uncertainty about the fine reduction. In this model, the probability of a fine reduction, $\theta$, depends on the content of the information that the firm reveals to the AA. If the information revealed is already known by the AA, or the information does not contribute to the case, then $\theta$ will be close to 0 and therefore the firms will almost for sure receive the full fine. The information given by the firm is not valuable enough to give the firm a fine reduction. If the information revealed to the AA has a significant value for the investigation, then $\theta$ will be close to 1 and therefore the firms will almost for sure receive the reduced fine. When the firms decide to reveal their information to the AA, they do not know how valuable their information is. It is also possible that some third party already revealed information to the AA or the AA themselves have found out about some details of the cartel. This results in the uncertainty about the outcome and payoff of the game. To conclude, the probability that the AA is able to use the information in the prosecution is equal to $\theta \in [0,1]$, and then the firm will receive a reduced fine.

3.2. Possible strategies of the firms
Given the policy choice of the AA, the firms choose their best strategy. If the AA does not start an investigation in period $t = 0$, both firms earn the profits from being in a cartel, $\pi_M$. These
profits are above the profits with a punishment and below the profits from deviating. This is summarized in the following condition: \( \pi_N < \pi_M < \pi_D \). If the AA does start an investigation in period \( t = 0 \), the firms decide whether to reveal information. If both firms do not reveal information, the outcome depends on the investigation of the AA. With probability \( p \), the AA proves the cartel. The expected payoff in period 2 the firms will then be:

\[
p \left( \frac{\pi_N}{1 - \delta} - F \right)
\]

With probability \( 1 - p \), the AA cannot prove the cartel. The expected payoff for the firms in period 2 will then be:

\[
(1 - p) \frac{\pi_M}{1 - \delta}
\]

If there is one firm that does reveal information, then the AA is able to prove the cartel. The other firm that did not reveal information will therefore receive the full fine for sure. The firm that cooperated might receive leniency. This depends on the value of the information revealed to the AA. The expected payoff in period 1 for the firm that revealed the information will then be:

\[
\theta(\pi_N - R) + (1 - \theta)(\pi_N - F)
\]

The payoff for the firm that did not reveal information is equal to the profits with punishment minus the full fine:

\[
\pi_N - F
\]

If both firms reveal information, then for both firms the payoff depends on the value of the information revealed to the AA. If the information is not valuable because for example, the AA already received the information from a third party, then the AA will not reduce the fine. If however the information is valuable, the AA might reduce the fine. The expected payoff in period 1 is the following for both firms:

\[
\theta(\pi_N - R) + (1 - \theta)(\pi_N - F)
\]

3.3. Timing of the game

Under the strategy of Collude and Reveal (CR), the game starts at \( t = 0 \), when the AA decides whether to start an investigation and decides on the policy. Both the firms collude from \( t = 1 \) onwards, as long as no deviation occurs. If there is no investigation started by the AA, the firms realize the maximum profit \( \pi_M \). If however the AA starts an investigation in period \( t \), then firms reveal information and pay a fine in that period. They are forced to non-cooperative pricing with profit \( \pi_N < \pi_M \). In the next period, the firms return to the collusive strategy. When
a deviation occurs in the marketplace or in the revelation policy, the firms use Nash punishment forever and receive the punishment profit $\pi_N$.

Under the strategy of Collude and Not Reveal (CNR), the game starts again at $t = 0$, when the AA decides whether to start an investigation and sets the policy. Both the firms collude from $t = 1$ onwards, as long as no deviation occurs. If an investigation is opened in period $t$, the firms do not reveal information. The AA needs more time to investigate and during that period, the firms receive the cartel profit $\pi_M$. In period $t + 1$, the firms have to pay a fine if they are proved to be guilty and the firms are forced to non-cooperative pricing. They therefore receive the punishment profit $\pi_N$. In period $t + 2$, the firms return to collusion. If the firms are not proven to be guilty, they continue their collusive behaviour. When there is a deviation in the marketplace or in the revelation strategy, then again, the firms use Nash punishment forever with profits $\pi_N$. If the AA did not start an investigation at all, the firms are not reviewed for the next two periods. The timing of the game and the payoffs are summarized in figure 2.

Figure 2: Game tree with payoffs at $t = 2$.

3.4. Subgame perfect equilibria
In this section, we will derive the conditions for a Collude and Reveal (CR) and a Collude and Not Reveal (CNR) strategy to be preferred to the strategy of deviating.

3.4.1. Collude and Reveal (CR)
The first strategy we will review is the situation in which firms collude and consequently reveal information to the AA if the AA opens an investigation. It is uncertain for the firms whether they
receive a full fine or a reduced fine. After the firms received and payed the fine, the game starts again. The value of the CR-strategy, $V_{CR}$, can be obtained, given the payoffs in figure 2.

$$V_{CR} = \alpha(\theta(\pi_N - R) + (1 - \theta)(\pi_N - F)) + (1 - \alpha)\pi_M + \delta V_{CR}$$

$$= \frac{\pi_M}{1 - \delta} - \alpha \frac{\pi_M - \pi_N + \theta R + (1 - \theta)F}{1 - \delta}$$

(1)

The derivation can be found in appendix A.

The first term contains the profits when there is no investigation. The second term contains the profits when there is an investigation and the leniency program applies. The condition shows that the value of this CR-strategy reduces when the AA enforces their policy. The reduction occurs because firms have a lower profit $\pi_N$ instead of $\pi_M$ and the firms are obliged to pay a fine $R$ with probability $\theta$, or $F$ with probability $1 - \theta$. Firms would benefit the most when the AA does not start an investigation. If it does start an investigation, it reduces the profits of firms, which is exactly what the investigation and the leniency program should do.

It is important to consider whether firms have an incentive to deviate from the strategy to reveal information. If a firm deviates from the revelation strategy, then the other firm will reveal the information. The AA will find out about the cartel and the firm that did not reveal information will receive the full fine $F$ for sure. The other firm that revealed the information receives the fine $R$ with probability $\theta$ and the fine $F$ with probability $1 - \theta$.

Another reason for firms not to break the cartel is that breaking the cartel would lead to further losses in the future. The cartel breaks down and collusion will never arise again. It is better for firms to reveal their information and not break down the cartel and therefore firms have no incentive to deviate. Another condition therefore must be that from the beginning, it is not profitable to deviate. The value of deviating can be obtained:

$$V_D = \pi_D + \delta \frac{\pi_N}{1 - \delta}$$

(2)

Where $\pi_D > \pi_M$ and when the deviation occurred, there will be a Nash punishment forever.

**Lemma 1.** A subgame perfect equilibrium exists where firms collude and reveal information when they are monitored, given the policy parameters $(F, R, \alpha, p, \theta)$ if

$$\alpha < \alpha_{CR}(\theta, R, F) = \frac{\pi_M - (1 - \delta)\pi_D - \delta \pi_N}{\pi_M - \pi_N + \theta R + (1 - \theta)F}$$

(3)

**Proof.** The condition follows from $V_{CR} > V_D$.

How I obtained equation 1 and 3 can be found in appendix A.

**Implication.** The minimal condition shows that $\alpha_{CR}(R, F)$ is decreasing in the expected fine, and we can also see that $\alpha_{CR}(0,0) < 1$. The intuition is such that granting more generous
discounts increases the threshold value, it relaxes the constraint for a CR-equilibrium, and it makes collusion more attractive. However, with this strategy, leniency programs do not automatically increase the incentive to collude, because it also depends on the probability that the reduced fine will be granted. In this extended model, the AA is able to influence the incentive of collusion. A more generous discount $R$ can be compensated by a lower probability $\theta$ that the AA grants the discount. In the old situation, the firms received a fine with a reduction $R$, for sure. In the extended model, with a chance $1 - \theta$, they do not receive the reduced fine but the full fine $F$. The pro-collusive effect of the leniency programs under the CR-strategy can therefore be reduced in this extended model. If the more generous program is not compensated by a lower probability $\theta$, then it must be compensated by a higher probability $\alpha$ that the AA starts an investigation. If this is not the case, then the more generous program increases the incentives to collude. In figure 3, the first constraint is depicted.

Figure 3: Incentive Compatibility Constraint for the Collude and Reveal strategy.

3.4.2. Collude and Not Reveal (CNR)

The second strategy we will review, is the situation in which firms collude and do not reveal information to the AA if the AA opens an investigation. In this situation, the AA will investigate and with a probability $p$ find the firms guilty. The firms then receive the full fine $F$. After the firms received and payed the fine, the game starts again. If the AA does not start an investigation, the firms will not be investigated in the next two periods, because the AA will
investigate other industries. The timing is equal to the timing of the original model. However, the timing of the game is complicated and might not be realistic. In the discussion, I will review the timing of the game.

The value of this CNR-strategy, $V_{CNR}$, can again be obtained, given the payoffs in figure 2. For this strategy, it is important to pay special attention to the timing of the game. We obtain the following value of the strategy CNR:

$$V_{CNR} = \alpha(\pi_M + \delta(p(\pi_N - F) + (1 - p)\pi_M)) + (1 - \alpha)(1 + \delta)\pi_M + \delta^2V_{CNR}$$

(4)

If the AA starts an investigation, then in that current period, the firms still earn a collusive profit of $\pi_M$. With a probability $p$, the cartel is discovered and the firms earn the punishment profit and must pay the full fine. With a probability $1 - p$, the cartel is not discovered and the firms earn a cartel profit. When the AA does not start an investigation, the firms earn the cartel profit. Also, the firms are not reviewed for the next two periods, which is denoted by the term $(1 + \delta)\pi_M$. Rearranging equation 4 gives us the following result:

$$V_{CNR} = \frac{\pi_M}{1 - \delta} - \alpha p \frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2}$$

(5)

More details about the rearrangement of equation 4 can be found in appendix B.

As we also saw under the CR-strategy, the cartel profits are reduced by the expected losses through the investigation of the AA. To find the subgame perfect equilibrium from this strategy, we consider two constraints. The first constraint is that the firms prefer colluding and revealing instead of deviating from the beginning. The second constraint is when the AA reviews the market, they choose not to reveal information to the AA. The first constraint can be found in the same way as we found the constraint under the CR strategy, by requiring $V_{CNR} \geq V_D$. This leads to the following condition:

$$\frac{\pi_M}{1 - \delta} - \alpha p \frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2} \geq \pi_D + \delta \frac{\pi_N}{1 - \delta}$$

After rearranging and simplifying we get:

$$\alpha < \alpha_{NC}(p) = \frac{(1 + \delta)(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)}{\delta p(\pi_M - \pi_N + F)}$$

(6)

More details about the rearrangement and simplification can be found in appendix B, and the constraint is graphically depicted in figure 4.
The constraint is equal to the constraint in the original model since the CNR-strategy remains the same. The constraint for this equilibrium to exist does not depend on $R$, since we are in a situation in which the firms do not reveal information and are therefore not entitled to a fine reduction. However, the constraint does depend on $p$, the probability that the AA can complete the investigation successfully. When $p$ is higher, then $\alpha$ must be lower. In other words, when the probability that the AA can successfully complete the investigation is higher, then it must be balanced by a lower probability $\alpha$ that the AA starts an investigation to keep the firms indifferent between deviation and colluding and not revealing information.

The second constraint for a CNR-equilibrium to exist, is that the firms prefer not to reveal information when the AA starts an investigation. To find the condition, we first consider the value of revealing information, which is a deviation from the CNR-equilibrium:

$$V_R|\alpha = \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta)\left( \frac{\pi_N}{1 - \delta} - F \right)$$ (7)

The value is equal to the expected payoff of revealing information when the AA opens an investigation. If the firm does play the equilibrium strategy and therefore does not reveal information, the value is equal to:

$$V_{NR|\alpha} = \pi_M + \delta(p(\pi_N - F) + (1 - p)\pi_M) + \delta^2 V_{CNR}$$ (8)
As before, when the AA is not able to prove the cartel during the investigation, then the firms are not reviewed for the next two periods. Therefore, the firms earn the maximum cartel profits again in those periods. Rearranging equation 8 and substituting equation 5 gives us the following value of this strategy:

\[ V_{NR} | \alpha = \frac{\pi_M}{1 - \delta} - \frac{\delta p(1 - \delta^2 (1 - \alpha))(\pi_M - \pi_N + F)}{1 - \delta^2} \]  

(9)

More details about the rearrangement and substitution can be found in appendix B.

We know that not revealing information can only be optimal if \( V_{NR} | \alpha \geq V_R | \alpha \). Therefore, we can find the second constraint for a CNR equilibrium to exist:

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p(1 - \delta^2 (1 - \alpha))(\pi_M - \pi_N + F)}{1 - \delta^2} \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \alpha < \alpha_R (p, \theta, R, F) \]

\[ = \frac{(1 + \delta) \left( (\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F)) - (\delta p(1 - \delta)(\pi_M - \pi_N + F)) \right)}{\delta^3 p(\pi_M - \pi_N + F)} \]  

(10)

More details about the rearrangement of equation 10 can be found in appendix B.

This second incentive compatibility constraint gives us the \( \alpha_R (p, \theta, R, F) \)-curve. The curve shifts down if \( R \) is lowered, hence, when the leniency program is more generous. When \( R \) is close to \( F \) and \( \theta \) is close to 1, then the \( \alpha_R \)-curve is always above the \( \alpha_{NC} \)-curve, which means that revealing information always leads to the cartel breaking down, and therefore there is no reason to reveal information. When the probability \( \theta \) of receiving a reduced fine \( R \) is close to 0, then the firms have no incentive to reveal information and the constraint for a CNR-equilibrium to exist is the \( \alpha_{NC} \)-curve. When \( R \) is sufficiently low and \( \theta \) is sufficiently high, then \( \alpha_R \)-curve becomes smaller than the \( \alpha_{NC} \)-curve and the \( \alpha_R \)-curve becomes binding. The \( \alpha_R \)-curve is depicted in figure 5.
Lemma 2. A CNR equilibrium exists for given policy parameters \((F, R, \alpha, p, \theta)\) if \(\alpha < \min[\alpha_{NC}(p), \alpha_R(p, \theta, R, F)]\).

Proof. The condition follows from the two constraints for a CNR subgame perfect equilibrium.

Implication. The first constraint gives the values of \(\alpha\) and \(p\) for which the strategy CNR is preferred above the strategy of deviating. The second constraint gives the values for which not revealing is preferred above revealing. The second curve is binding when \(R\) is sufficiently low or \(\theta\) is sufficiently high. The curve is not binding when \(R\) is close to \(F\) or \(\theta\) is sufficiently low. The AA is able to change the incentives of the firms by changing the value of the fines. All the constraints are depicted in figure 6, and it shows when \(p\) is high, the no revelation constraint becomes binding for any cartel to exist.

3.4.3. Incentive compatibility constraints

In the previous sections, we found the conditions for which the CR and the CNR strategies dominate the strategy of deviation and deviation from a revelation policy. These conditions are:

\[
\alpha < \alpha_{CR}(\theta, R, F) = \frac{\pi_M - (1 - \delta)\pi_D - \delta\pi_N}{\pi_M - \pi_N + \theta R + (1 - \theta)F}
\]

\[
\alpha < \alpha_{NC}(p) = \frac{(1 + \delta)(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)}{\delta p(\pi_M - \pi_N + F)}
\]
\[ \alpha < \alpha_R(p, \theta, R, F) \]

\[
= (1 + \delta) \left( (\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F)) - (\delta p(1 - \delta)(\pi_M - \pi_N + F)) \right) / \delta^3 p(\pi_M - \pi_N + F)
\]

In figure 6, the incentive compatibility constraints are depicted.

3.4.4. CNR vs. CR

We are now able to find the condition for which the CNR-strategy dominates the CR-strategy.

\[ V_{CNR} > V_{CR} \]

\[
\frac{\pi_M}{1 - \delta} - \alpha p \frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2} > \frac{\pi_M}{1 - \delta} - \alpha \frac{\pi_M - \pi_N + \theta R + (1 - \theta)F}{1 - \delta}
\]

\[
p < p_{CNR}(\theta, R, F) = \frac{(1 + \delta)(\pi_M - \pi_N + \theta R + (1 - \theta)F)}{\delta(\pi_M - \pi_N + F)}
\]

The complete rearrangement can be found in appendix C.

The condition shows that not revealing information is preferred above revealing when the probability \( p \) is sufficiently low. This means that the AA must be relatively unsuccessful in the prosecution stage. We can rearrange condition 11 and then we find:

\[
p \frac{\delta(\pi_M - \pi_N + F)}{1 + \delta} = \pi_M - \pi_N + \theta R + (1 - \theta)F
\]
The left side of the expression represents the expected average losses with probability \( p \) when playing a CNR-strategy. In equilibrium, they must equal the right-hand side of the expression, which represents the average losses from playing the CR-strategy. These average losses solely depend on the value of \( \theta \), since the value of \( R \) and \( F \) are known.

The last step before being able to identify the subgame perfect equilibria is to define the intersections of the conditions. When those intersections are defined, we can also define the areas for which equilibria exist. In lemma 1 I found that for a subgame perfect equilibrium to exist where firms collude and reveal information when monitored, \( \alpha \) must be smaller than \( \alpha_{CR} \).

To find the intersections, \( \alpha_{CR} \) must equal the condition stated in lemma 2 where I found that \( \alpha \) must be smaller than the binding minimum of \( \alpha_{NC} \) and \( \alpha_{R} \). Setting \( \alpha_{CR} = \alpha_{NC} \) and solving for \( p \) gives us the following intersection:

\[
\frac{\pi_M - (1 - \delta)\pi_D - \delta\pi_N}{\pi_M - \pi_N + \theta R + (1 - \theta)F} = \frac{(1 + \delta)(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)}{\delta p(\pi_M - \pi_N + F)}
\]

\[
p = \frac{(1 + \delta)(\pi_M - \pi_N + \theta R + (1 - \theta)F)}{\delta(\pi_M - \pi_N + F)}
\] (13)

\[
p = p_{CNR}
\]

Setting \( \alpha_{CR} = \alpha_{R} \) and solving for \( p \) gives us the following intersection:

\[
p_R
\]

\[
= \frac{\delta^2(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)(\delta^2(\pi_M - \pi_N + F))}{(1 + \delta)\delta^2(-\theta F + F + \theta R + \pi_M - \pi_N)(\delta\theta F - \delta\theta R - \delta F - \theta F + F + \theta R + \pi_M - \pi_N)}
\]

\[
+ \frac{(1 + \delta)(\pi_M - \pi_N + F)}{(1 + \delta)\delta^2(\pi_M - \pi_N + F)(-\delta\theta R + \delta\theta F - \delta F + \theta R + \pi_M - \pi_N - \theta F + F)}
\] (14)

The derivations of equations 12, 13 and 14 can be found in appendix C.

With equation 13 and 14 we found the intersections \( p_{CNR} \) en \( p_R \), which is where the downward sloping curves \( \alpha_{NC} \) en \( \alpha_{R} \) intersect the flat curve \( \alpha_{R} \). When \( p_{CNR} < p_R \), then when \( \alpha = \alpha_{CR} \), \( p_{CNR} \) is more to the left in figure 6 compared with \( p_R \). For that to happen, we can see from equations 13 and 14 that \( R \) must be sufficiently low or the probability of receiving the reduced fine must be sufficiently low. All the curves, except for \( \alpha_{NC} \), depend on \( R \) and \( \theta \) and therefore, when \( R \) changes, it changes how the curves are situated with regard to each other. We saw that when \( R \) is lowered, the \( \alpha_{CR} \)-curve increases. From equations 13 and 14 we can conclude that when \( R \) becomes smaller, the intersections move to the left. This leads to conditions for the CNR, CR and Not Collude equilibrium to exist.
Proposition 1. CR-equilibria exists if $\alpha < \alpha_{CR}$, CNR-equilibria exists when $\alpha < \min[\alpha_{NC}(p), \alpha_R(p, \theta, R, F)]$ and Not Collude exists if $\alpha \geq \alpha_{CR}$ and $\alpha \geq \min[\alpha_{NC}(p), \alpha_R(p, \theta, R, F)]$. When the inequality $p < p_{CNR}$ holds, firms prefer CNR above CR, and vice versa. The inequality holds if the probability of receiving a reduced fine, $\theta$, is smaller. Firms are less likely to reveal information when the uncertainty about the fine reduction is more substantial.

Proof. See Lemma 1 and 2.

Implication. Proposition 1 identifies the regions in which the different equilibria exist. The implications are discussed under lemma 1 and 2. We now are able to compare the results from this paper with the results from Motta and Polo (2003). The only difference between the models is the $\theta$, which is a policy parameter that the antitrust authority can use to influence the incentives of the firms. We saw with equation 13 and 14 that the equilibria depend on the value of $R$ and $\theta$. The AA commits to a certain policy, which means that once $R$ is set before the beginning of the game, the AA is not able to change this during the game. If $\theta = 1$, we are in the situation of the model of Motta and Polo (2003) in which the firms received a reduced fine for sure when they reveal information. When $\theta$ becomes smaller, the overall positive effect of revealing information decreases since the firms have a higher expected fine. When $\theta$ becomes smaller, the uncertainty is greater and the probability of receiving a reduced fine is reduced. The expected value of revealing information is smaller and the equilibrium strategy is to collude and not to reveal information. More uncertainty leads to a situation in which firms are less likely to reveal information.

There are many possible changes that could be implemented into the leniency model and that might change the effectiveness of the leniency programs. Motta and Polo (2003) investigated the change in results when it would only be possible to receive a reduced fine when firms revealed before an investigation was opened. It is possible to change the rules of the game and with that change the effectiveness of the program. For example, it would be possible to grant a higher probability $\theta$ on a reduced fine when only one of the firms revealed. This might very well lead to a situation in which ‘The Race to the Courthouse’-effect, as stated by Harrington (2008), occurs. Both firms want to reveal as soon as possible in order to receive the reduced fine. There is more certainty for firms and this would lead to a situation in which firms are more likely to reveal information. There is only more certainty when the firms reveal as soon as possible, and therefore ‘The Race to the Courthouse’-effect would occur. This
example illustrates that a small change in the rules of the game can have a significant effect on the subgame perfect equilibria and the strategies of the firms.

4. Conclusion

In this extended model, we were searching for a way to decrease the incentive to collude and consequently reveal information about the cartel to the authorities in order to get a fine reduction. To decrease the incentive to collude and consequently reveal information, we introduced uncertainty about the fine reduction. The firms do not know if the information they reveal is enough or if it has a good enough value in order to receive a fine reduction. The antitrust authority decides whether the firms receive a full fine or a reduced fine after they revealed information. There are two possible strategies for firms to follow, Collude and Reveal, or Collude and Not Reveal. To find the subgame perfect equilibria, these strategies must prevail above the strategy of deviation, which the firm can choose before the game starts. The constraints we found for these strategies to prevail, are the incentive compatibility constraints.

Under the first strategy, CR, we saw that the AA could grant more generous discounts, but they could compensate the generosity with a lower probability $\theta$. By changing the probability of receiving the reduced fine, the AA can reduce the pro-collusive effect. However, we also saw that more uncertainty, through a lower value of $\theta$, leads to a situation in which firms are less likely to reveal information. The expected value of revealing information is lower. This result is consistent with the literature about the earliest leniency programs in the United States and in the European Union. Those programs failed because of uncertainty about the fine. Through adding extra uncertainty in this model, we can see why those programs failed, since firms are less likely to reveal information and thus are less likely to use the leniency program.

Motta and Polo (2003) found a two-sided effect of leniency programs. Leniency programs stimulated collusion as well as breaking down cartels and the last effect dominated. With this study we can conclude that adding extra uncertainty would reduce the effectiveness of the leniency program in the model used. However, in the real world, it might well be that adding uncertainty leads to a better situation for consumers. This all depends on the compliance rate and the difficulty is that the compliance rate is unknown. The overall effect of adding uncertainty remains unclear.
5. Discussion

The model used in this paper relies on a few assumptions and some specific conditions with regard to timing and profits. These features originated from the model of Motta and Polo (2003) and for simplicity as well as to be able to compare with the model, I used the same features in the extension. However, a few of these assumptions should be reviewed and might not be realistic. For further research, it is useful to review the assumptions and change especially the timing of the game. One of the issues with the timing appears under the CNR-strategy. First, when the AA starts an investigation and the firms do not reveal information, then the firms still earn cartel profits during that period. This period is before the period where the prosecution takes place. I found the equilibria for when not revealing information is preferred to deviating before the games started. However, when the AA started the investigation during the game, the firms know that the prosecution stage will start after the current period, and therefore, they might have a strong incentive to deviate in the current period and earn deviation profits that are greater than the cartel profits. Therefore, in further research, it should be investigated whether it is realistic that the firms still earn cartel profits in period $t = 1$. Another assumption is that the AA will not review the market for two periods when they cannot prove a cartel during the investigation. The firms earn cartel profits during two periods with certainty. There is no justification for this assumption and the choice of two periods seems to be chosen arbitrarily. Therefore, this is also an important feature that should be taken into account in further research.

Another important consideration is the antitrust law. When adding uncertainty to the model, it must be in compliance with the law. The uncertainty about the fine may not arise from decisions that are arbitrarily taken by the AA since this would be in violation of legal certainty. Therefore, it must be possible to capture the uncertainty by law.

Another important consideration is the compliance rate. The old leniency programs in the United States and the European Union failed because there were only a few leniency applications in a year. After the revisions, more firms applied for leniency and this was considered to be successful. This would imply that the model of this paper is inferior compared to the existing leniency programs. However, we do not know the total number of cartels during the old and new leniency programs. Under the old program, when there were not many applications, this could also mean that the total number of cartels was smaller. The new model could have led to a situation in which collusion is more attractive due to the option of leniency and therefore, the greater number of applications cannot be seen as more successful. The compliance rate remains difficult to deal with when modelling leniency programs. Empirical research might be a solution to gain more insights into the compliance rate.
To conclude, in this model, I investigated the effect of adding an extra parameter representing uncertainty on the incentive compatibility constraints and the existence of subgame perfect equilibria. This model can be further extended with an investigation after the optimal enforcement policy with regard to the parameters, including the parameter that was added in this paper. By a more advanced study, it can be determined what the optimal values of the policy parameters should be to make the leniency program more effective and to reduce the incentive to repeatedly collude.
Appendix

A. Collude and Reveal

(1) Value of the strategy 'Collude and Reveal':

\[ V_{CR} = \alpha(\theta(\pi_N - R) + (1 - \theta)(\pi_N - F)) + (1 - \alpha)\pi_M + \delta V_{CR} \]

\[ V_{CR} - \delta V_{CR} = \alpha(\theta \pi_N - \theta R + \pi_N - F - \theta \pi_N + \theta F) + \pi_M - \alpha \pi_M \]

\[ (1 - \delta) V_{CR} = \alpha \theta \pi_N - \alpha \theta R + \alpha \pi_N - \alpha F - \alpha \theta \pi_N + \alpha \theta F + \pi_M - \alpha \pi_M \]

\[ (1 - \delta) V_{CR} = \alpha(-\theta R + \pi_N - F + \theta F - \pi_M) + \pi_M \]

\[ = \frac{\pi_M}{1 - \delta} + \alpha \frac{F\theta - F - \theta R + \pi_N - \pi_M}{1 - \delta} \]

\[ = \frac{\pi_M}{1 - \delta} - \alpha \frac{\pi_M - \pi_N + \theta R + (1 - \theta)F}{1 - \delta} \]

(3) The CR constraint:

\[ V_{CR} > V_D \]

\[ \frac{\pi_M}{1 - \delta} - \alpha \frac{\pi_M - \pi_N + \theta R - \theta F + F}{1 - \delta} > \frac{\pi_D}{1 - \delta} \]

\[ \frac{\pi_M - \pi_D - \delta \pi_D + \delta \pi_N}{1 - \delta} > \frac{\alpha}{1 - \delta} \frac{\pi_M - \pi_N + \theta R - \theta F + F}{1 - \delta} \]

\[ \frac{\pi_M - \pi_D + \delta \pi_D - \delta \pi_N}{1 - \delta} * \frac{1 - \delta}{\pi_M - \pi_N + \theta R - \theta F + F} > \alpha \]

\[ \alpha < \frac{\delta \pi_N + (1 - \delta)\pi_D - \pi_M}{\theta F - F - \theta R - \pi_M + \pi_N} \]

\[ \alpha < \alpha_{CR}(R, F) = \frac{\pi_M - (1 - \delta)\pi_D - \delta \pi_N}{\pi_M - \pi_N + \theta R + (1 - \theta)F} \]

B. Collude and Not Reveal

(5) Value of the strategy 'Collude and Not Reveal', rearranging (4):

\[ V_{CNR} = \alpha(\pi_M + \delta(p(\pi_N - F) + (1 - p)\pi_M) + (1 - \alpha)(1 + \delta)\pi_M + \delta^2 V_{CNR} \]

\[ V_{CNR} = \alpha(\pi_M + \delta(p(\pi_N - F) + (1 - p)\pi_M) + (-\alpha + \delta - \alpha \delta + 1)\pi_M + \delta^2 V_{CNR} \]

\[ V_{CNR} = \alpha(\pi_M + \delta(p(\pi_N - F) + (1 - p)\pi_M) - \alpha \pi_M + \delta \pi_M - \alpha \delta \pi_M + \pi_M + \delta^2 V_{CNR} \]

\[ V_{CNR} = \alpha(\pi_M + \delta(p\pi_N - pF + \pi_M - p\pi_M)) - \alpha \pi_M + \delta \pi_M - \alpha \delta \pi_M + \pi_M + \delta^2 V_{CNR} \]
\[ V_{\text{CNR}} = \alpha(\pi_M + \delta\rho\pi_N - \delta\rho F + \delta\pi_M - \delta\rho\pi_M) - \alpha\pi_M + \delta\pi_M - \alpha\delta\pi_M + \pi_M + \delta^2 V_{\text{CNR}} \]

\[ V_{\text{CNR}} = \alpha\delta\rho\pi_N - \alpha\delta F - \alpha\delta\rho\pi_N - \alpha\pi_M + \delta\pi_M + \delta\pi_M + \pi_M + \delta^2 V_{\text{CNR}} \]

\[ V_{\text{CNR}} - \delta^2 V_{\text{CNR}} = \alpha\delta\rho\pi_N - \alpha\delta F - \alpha\delta\rho\pi_N + \delta\pi_M + \pi_M \]

\[ V_{\text{CNR}} - \delta^2 V_{\text{CNR}} = (1 + \delta)\pi_M - \alpha\rho\delta(\pi_M - \pi_N + F) \]

\[ (1 - \delta^2) V_{\text{CNR}} = (1 + \delta)\pi_M - \alpha\rho\delta(\pi_M - \pi_N + F) \]

\[ V_{\text{CNR}} = \frac{(1 + \delta)\pi_M}{1 - \delta} - \alpha\rho\frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2} \]

\[ V_{\text{CNR}} = \frac{\pi_M}{1 - \delta} - \alpha\rho\frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2} \]

(6) The first CNR constraint:

\[ \frac{\pi_M}{1 - \delta} - \alpha\rho\frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2} \geq \pi_D + \delta\frac{\pi_N}{1 - \delta} \]

\[ -\alpha\rho\frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2} \geq \pi_D + \delta\frac{\pi_N - \pi_M}{1 - \delta} \]

\[ -\alpha\rho\frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2} \geq \frac{(1 - \delta)\pi_D}{1 - \delta} + \delta\pi_N - \pi_M \]

\[ -\alpha\rho(\pi_M - \pi_N + F) \geq \frac{(1 - \delta)\pi_D}{1 - \delta} + \delta\pi_N - \pi_M \]

\[ -\alpha \geq \frac{\pi_N - \pi_M + (1 - \delta)\pi_D}{1 - \delta} * \frac{(1 - \delta^2)}{\rho\delta(\pi_M - \pi_N + F)} \]

\[ -\alpha \geq \frac{(1 - \delta^2)(1 - \delta)\pi_D + \delta\pi_N - \pi_M}{(1 - \delta)\rho\delta(\pi_M - \pi_N + F)} \]

\[ -\alpha \geq \frac{(\delta + 1)(-\pi_M + (1 - \delta)\pi_D + \delta\pi_N)}{\rho\delta(\pi_M - \pi_N + F)} \]

\[ \alpha \geq \frac{(1 + \delta)(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)}{\rho\delta(\pi_M - \pi_N + F)} \]

\[ \alpha < a_{NC}(p) = \frac{(1 + \delta)(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)}{\rho\delta(\pi_M - \pi_N + F)} \]
(9) Rearranging (8) and substituting (5) gives the value if the firm reveals once monitored:

\[ V_{NR|\alpha} = \pi_M + \delta (p (\pi_N - F) + (1 - p) \pi_M) + \delta^2 V_{CNR} \]

\[ V_{NR|\alpha} = \pi_M + \delta (p (\pi_N - F) + (1 - p) \pi_M) + \delta^2 \left( \frac{\pi_M}{1 - \delta} - \alpha p \frac{\delta (\pi_M - \pi_N + F)}{1 - \delta^2} \right) \]

\[ V_{NR|\alpha} = \pi_M + \delta (p (\pi_N - F) + (1 - p) \pi_M) + \frac{\delta^2 \pi_M}{1 - \delta} - \alpha p \frac{\delta^2 (\pi_M - \pi_N + F)}{1 - \delta^2} \]

\[ V_{NR|\alpha} = \pi_M + \delta (p (\pi_N - p F + \pi_M - p \pi_M) + \frac{\delta^2 \pi_M}{1 - \delta} - \alpha p \frac{\delta^3 (\pi_M - \pi_N + F)}{1 - \delta^2} \]

\[ V_{NR|\alpha} = \pi_M + \delta (p (\pi_N - p F + \pi_M - p \pi_M) - \alpha p \delta^3 (\pi_M - \pi_N + F)) \]

\[ V_{NR|\alpha} = \frac{(1 - \delta) \pi_M + \delta^2 p \pi_N - \delta p F + \delta \pi_M - \delta p \pi_M - \delta^3 p \pi_N + \delta^3 p F - \delta^3 \pi_M + \alpha p \delta^3 \pi_M + \alpha p \delta^3 \pi_N - \alpha p \delta^3 F}{1 - \delta^2} \]

\[ V_{NR|\alpha} = \frac{(1 - \delta) \pi_M + \delta^2 \pi_M}{1 - \delta} + \frac{\delta p \pi_N - \delta p F - \delta p \pi_M - \delta^3 p \pi_N + \delta^3 p F + \delta^3 p \pi_M - \alpha p \delta^3 \pi_M + \alpha p \delta^3 \pi_N - \alpha p \delta^3 F}{1 - \delta^2} \]

\[ V_{NR|\alpha} = \frac{(1 - \delta) \pi_M + \alpha p \delta^3 \pi_M}{1 - \delta} + \frac{\delta p \pi_N - \delta p F - \delta p \pi_M - \delta^3 p \pi_N + \delta^3 p F + \delta^3 p \pi_M - \alpha p \delta^3 \pi_M + \alpha p \delta^3 \pi_N - \alpha p \delta^3 F}{1 - \delta^2} \]

\[ V_{NR|\alpha} = \frac{(1 - \delta) \pi_M + \delta^2 \pi_M}{1 - \delta} + \frac{\delta p \pi_N - \delta p F - \delta p \pi_M - \delta^3 p \pi_N + \delta^3 p F + \delta^3 p \pi_M - \alpha p \delta^3 \pi_M + \alpha p \delta^3 \pi_N - \alpha p \delta^3 F}{1 - \delta^2} + \frac{(1 - \delta) \delta^2 \pi_M}{1 - \delta} \]
\[ V_{NR|\alpha} = \frac{\pi_M}{1 - \delta} + \frac{\delta p \pi_N - \delta p - \delta p \pi_M - \delta^3 p \pi_N + \delta^3 p F + \delta^3 \pi_M - \alpha \delta^3 \pi_M + \alpha \delta^3 \pi_N - \alpha \delta^3 F}{1 - \delta^2} \]

\[ V_{NR|\alpha} = \frac{\pi_M}{1 - \delta} + \frac{\delta p (\pi_N - F - \pi_M - \delta^2 \pi_N + \delta^2 F + \delta^2 \pi_M - \alpha \delta^2 \pi_M + \alpha \delta^2 \pi_N - \alpha \delta^2 F)}{1 - \delta^2} \]

\[ V_{NR|\alpha} = \frac{\pi_M}{1 - \delta} - \frac{\delta p (1 - \delta^2 (1 - \alpha))(\pi_M - \pi_N + F)}{1 - \delta^2} \]

(10) The second CNR constraint:

\[ V_{NR|\alpha} \geq V_R|\alpha \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p (1 - \delta^2 (1 - \alpha))(\pi_M - \pi_N + F)}{1 - \delta^2} \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p (\pi_M - \pi_N + F - \delta^2 \pi_M + \delta^2 \pi_N - \delta^2 F + \alpha \delta^2 \pi_M - \alpha \delta^2 \pi_N + \alpha \delta^2 F)}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p \pi_M - \delta p \pi_N + \delta p F - \delta^3 \pi_M + \delta^3 \pi_N - \delta^3 p F + \alpha \delta^3 \pi_M - \alpha \delta^3 \pi_N + \alpha \delta^3 p F}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p \pi_M - \delta p \pi_N + \delta p F - \delta^3 \pi_M + \delta^3 \pi_N - \delta^3 p F}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p \pi_M - \delta p \pi_N + \delta p F - \delta^3 \pi_M + \delta^3 \pi_N - \delta^3 p F}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p \pi_M - \delta p \pi_N + \delta p F - \delta^3 \pi_M + \delta^3 \pi_N - \delta^3 p F}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p \pi_M - \delta p \pi_N + \delta p F - \delta^3 \pi_M + \delta^3 \pi_N - \delta^3 p F}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p \pi_M - \delta p \pi_N + \delta p F - \delta^3 \pi_M + \delta^3 \pi_N - \delta^3 p F}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p \pi_M - \delta p \pi_N + \delta p F - \delta^3 \pi_M + \delta^3 \pi_N - \delta^3 p F}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p \pi_M - \delta p \pi_N + \delta p F - \delta^3 \pi_M + \delta^3 \pi_N - \delta^3 p F}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]

\[ \frac{\pi_M}{1 - \delta} - \frac{\delta p \pi_M - \delta p \pi_N + \delta p F - \delta^3 \pi_M + \delta^3 \pi_N - \delta^3 p F}{1 - \delta^2} \]

\[ \geq \theta \left( \frac{\pi_N}{1 - \delta} - R \right) + (1 - \theta) \left( \frac{\pi_N}{1 - \delta} - F \right) \]
\[ + \alpha \delta^3 p_M - \alpha \delta^3 p_N + \alpha \delta^3 p_f \]
\[ \frac{1}{1 - \delta^2} \]
\[ \geq \frac{\pi_N - \pi_M}{1 - \delta} - \theta R - (1 - \theta)F + \frac{\delta p_M - \delta p_N + \delta p_f - \delta^3 p_M + \delta^3 p_N - \delta^3 p_f}{1 - \delta^2} \]
\[ - \alpha \delta^3 p_M - \alpha \delta^3 p_N + \alpha \delta^3 p_f \]
\[ \frac{1}{1 - \delta^2} \]
\[ \geq \frac{\pi_N - \pi_M + (1 - \delta)(-\theta R + \theta F - F)}{1 - \delta} \]
\[ + \frac{\delta p_M - \delta p_N + \delta p_f - \delta^3 p_M + \delta^3 p_N - \delta^3 p_f}{1 - \delta^2} \]

\[ \alpha \]
\[ \geq \left( \frac{\pi_N - \pi_M + (1 - \delta)(-\theta R + \theta F - F)}{1 - \delta} \right) \left( \frac{1 - \delta^2}{-\delta^3 p_M + \delta^3 p_N - \delta^3 p_f} \right) \]
\[ + \left( \frac{\delta p_M - \delta p_N + \delta p_f - \delta^3 p_M + \delta^3 p_N - \delta^3 p_f}{1 - \delta^2} \right) \left( \frac{1 - \delta^2}{-\delta^3 p_M + \delta^3 p_N - \delta^3 p_f} \right) \]

\[ \alpha \geq \frac{(\delta + 1)(\pi_N - \pi_M + (1 - \delta)(-\theta R + \theta F - F))}{1 - \delta} \]
\[ \left( -\delta^3 p_M + \delta^3 p_N - \delta^3 p_f \right) \]
\[ + \left( \frac{\delta p_M - \delta p_N + \delta p_f - \delta^3 p_M + \delta^3 p_N - \delta^3 p_f}{1 - \delta^2} \right) \left( -\delta^3 p_M + \delta^3 p_N - \delta^3 p_f \right) \]

\[ \alpha \geq \frac{(\delta + 1)(\pi_N - \pi_M + (1 - \delta)(-\theta R + \theta F - F))}{1 - \delta} \]
\[ \left( -\delta^3 p_M + \delta^3 p_N - \delta^3 p_f \right) \]
\[ + \left( \frac{\delta p_M - \delta p_N + \delta p_f - \delta^3 p_M + \delta^3 p_N - \delta^3 p_f}{1 - \delta^2} \right) \left( -\delta^3 p_M + \delta^3 p_N - \delta^3 p_f \right) \]

\[ \alpha \geq \frac{(\delta + 1)(\pi_N - \pi_M + (1 - \delta)(-\theta R + \theta F - F))}{1 - \delta} \]
\[ + \left( \frac{\delta p_M - \delta p_N + \delta p_f - \delta^3 p_M + \delta^3 p_N - \delta^3 p_f}{1 - \delta^2} \right) \left( -\delta^3 p_M + \delta^3 p_N - \delta^3 p_f \right) \]

\[ \alpha \geq \frac{(\delta + 1)(\pi_N - \pi_M + (1 - \delta)(-\theta R + \theta F - F)) + (\delta + 1)(\delta p_M - \delta p_N + \delta p_f - \delta^3 p_M + \delta^3 p_N - \delta^3 p_f)}{\delta^3 p(\pi_N - \pi_M + \pi_N - F)} \]
\[ \alpha \geq \frac{(1 + \delta)\left(\left(\pi_N - \pi_M + (1 - \delta)(\theta R + \theta F - F)\right) + (\delta p(1 - \delta)(\pi_M - \pi_N + F))\right)}{\delta^3 p(-\pi_M + \pi_N - F)} \]

\[ \alpha < \alpha_r(p, R, F, \theta) \]
\[ = \frac{(1 + \delta)\left(\left(\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F)\right) - (\delta p(1 - \delta)(\pi_M - \pi_N + F))\right)}{\delta^3 p(\pi_M - \pi_N + F)} \]

C. CNR vs. CR

(11) Condition for CNR to dominate CR:

\[ V_{\text{CNR}} > V_{\text{CR}} \]
\[ \frac{\pi_M}{1 - \delta} - \alpha p \frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2} > \frac{\pi_M}{1 - \delta} - \alpha \frac{\pi_M - \pi_N + \theta R + (1 - \theta)F}{1 - \delta} \]
\[ -\alpha p \frac{\delta(\pi_M - \pi_N + F)}{1 - \delta^2} > -\alpha \frac{\pi_M - \pi_N + \theta R + (1 - \theta)F}{1 - \delta} \]
\[ p \frac{-\alpha \delta(\pi_M - \pi_N + F)}{1 - \delta^2} > -\alpha (\pi_M - \pi_N + \theta R + (1 - \theta)F) \]
\[ p > \frac{(1 + \delta)(\pi_M - \pi_N + \theta R + (1 - \theta)F)}{(1 - \delta)(-\alpha \delta(\pi_M - \pi_N + F))} \]
\[ p > \frac{(1 + \delta)(\pi_M - \pi_N + \theta R + (1 - \theta)F)}{\delta(\pi_M - \pi_N + F)} \]
\[ p < p_{\text{CNR}}(\theta, R, F) = \frac{(1 + \delta)(\pi_M - \pi_N + \theta R + (1 - \theta)F)}{\delta(\pi_M - \pi_N + F)} \]

(12) Rearranging (11):
\[ p = \frac{(1 + \delta)(\pi_M - \pi_N + \theta R + (1 - \theta)F)}{\delta(\pi_M - \pi_N + F)} \]
\[ p(\delta(\pi_M - \pi_N + F)) = (1 + \delta)(\pi_M - \pi_N + \theta R + (1 - \theta)F) \]
\[ p \frac{\delta(\pi_M - \pi_N + F)}{1 + \delta} = \pi_M - \pi_N + \theta R + (1 - \theta)F \]

(13) Finding the intersection of \( \alpha_{\text{CR}} \) and \( \alpha_{\text{NC}} \):
\[ \alpha_{\text{CR}} = \alpha_{\text{NC}} \]
\[ \frac{\pi_M - (1 - \delta)\pi_D - \delta \pi_N}{\pi_M - \pi_N + \theta R + (1 - \theta)F} = \frac{(1 + \delta)(\pi_M - (1 - \delta)\pi_D - \delta \pi_N)}{\delta p(\pi_M - \pi_N + F)} \]
\[ \delta p(\pi_M - \pi_N + F) = \frac{(\pi_M - \pi_N + \theta R + (1 - \theta)F)(1 + \delta)(\pi_M - (1 - \delta)\pi_D - \delta \pi_N)}{\pi_M - (1 - \delta)\pi_D - \delta \pi_N} \]
\[
\frac{\delta p(\pi_M - \pi_N + F)}{p} = \frac{(1 + \delta)(\pi_M - \pi_N + \theta R + (1 - \theta)F)(1 + \delta)}{\delta(\pi_M - \pi_N + F)}
\]

\[
p = p_{CR}
\]

(14) Finding the intersection of \(\alpha_{CR}\) and \(\alpha_R\):

\[
\frac{\pi_M - (1 - \delta)\pi_D - \delta\pi_N}{\pi_M - \pi_N + \theta R + (1 - \theta)F} = \frac{(1 + \delta)((\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F)) - (\delta p(1 - \delta)(\pi_M - \pi_N + F))}{\delta^3p(\pi_M - \pi_N + F)}
\]

\[
\frac{\pi_M - (1 - \delta)\pi_D - \delta\pi_N}{\pi_M - \pi_N + \theta R + (1 - \theta)F} = \frac{(1 + \delta)((\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F))}{\delta^3p(\pi_M - \pi_N + F)} - \frac{(1 + \delta)(\delta p(1 - \delta)(\pi_M - \pi_N + F))}{\delta^3p(\pi_M - \pi_N + F)}
\]

\[
\frac{\pi_M - (1 - \delta)\pi_D - \delta\pi_N}{\pi_M - \pi_N + \theta R + (1 - \theta)F} = \frac{(1 + \delta)((\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F))}{\delta^2(\pi_M - \pi_N + F)} - \frac{(1 + \delta)((1 - \delta)(\pi_M - \pi_N + F))}{\delta^2(\pi_M - \pi_N + F)}
\]

\[
\frac{\pi_M - (1 - \delta)\pi_D - \delta\pi_N}{\pi_M - \pi_N + \theta R + (1 - \theta)F} = \frac{(1 + \delta)((\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F))}{\delta^2(\pi_M - \pi_N + F)} + \frac{(1 + \delta)((1 - \delta)(\pi_M - \pi_N + F))}{\delta^2(\pi_M - \pi_N + F)}
\]

\[
\frac{(\pi_M - \pi_N + \theta R + (1 - \theta)F)((1 + \delta)((\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F)))}{\pi_M - (1 - \delta)\pi_D - \delta\pi_N} + \frac{(\delta^2(\pi_M - \pi_N + F))((1 + \delta)((\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F)))}{(1 + \delta)((1 - \delta)(\pi_M - \pi_N + F))}
\]

\[
= \delta^3p(\pi_M - \pi_N + F)
\]
\[ p = \frac{(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)\left(\delta^3(\pi_M - \pi_N + F)\right)}{(\pi_M - \pi_N + \theta R + (1 - \theta)F)\left((1 + \delta)(\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F))\right)} \]

\[ p \]

\[ = \frac{(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)\left(\delta^3(\pi_M - \pi_N + F)\right)}{(\pi_M - \pi_N + \theta R + (1 - \theta)F)\left((1 + \delta)(\pi_M - \pi_N + (1 - \delta)(\theta R - \theta F + F))\right)} + \frac{\left((1 + \delta)((1 - \delta)(\pi_M - \pi_N + F))\right)\left(\delta^3(\pi_M - \pi_N + F)\right)}{(1 + \delta)\delta^2(\pi_M - \pi_N + F)(-\delta \theta R + \delta \theta F - \delta F + \theta R + \pi_M - \pi_N - \theta F + F)} \]

\[ p \]

\[ = \frac{(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)\left(\delta^3(\pi_M - \pi_N + F)\right)}{(1 + \delta)(-\theta F + F + \theta R + \pi_M - \pi_N)(\delta \theta F - \delta \theta R - \delta F - \theta F + F + \theta R + \pi_M - \pi_N)}\]

\[ + \frac{\left((1 + \delta)((1 - \delta)(\pi_M - \pi_N + F))\right)\left(\delta^3(\pi_M - \pi_N + F)\right)}{(1 + \delta)\delta^2(\pi_M - \pi_N + F)(-\delta \theta R + \delta \theta F - \delta F + \theta R + \pi_M - \pi_N - \theta F + F)} \]

\[ p \]

\[ = \frac{\delta^2(\pi_M - (1 - \delta)\pi_D - \delta\pi_N)\left(\delta^3(\pi_M - \pi_N + F)\right)}{(1 + \delta)\delta^2(-\theta F + F + \theta R + \pi_M - \pi_N)(\delta \theta F - \delta \theta R - \delta F - \theta F + F + \theta R + \pi_M - \pi_N)}\]

\[ + \frac{\left((1 + \delta)((1 - \delta)(\pi_M - \pi_N + F))\right)\left(\delta^3(\pi_M - \pi_N + F)\right)}{(1 + \delta)\delta^2(\pi_M - \pi_N + F)(-\delta \theta R + \delta \theta F - \delta F + \theta R + \pi_M - \pi_N - \theta F + F)} \]
References


