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Econometrics and Operational Research

Rounding Probabilistic Expectations in Surveys: A Comparison of the Subjective Probabilities of Living between U.S. and European Residents

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Abstract

In this paper, I investigate the rounding behavior of U.S. and European residents when they are asked to state their subjective probabilities of living till certain ages. By interpreting subjective probabilities as rounded values, I transform them into interval data and further perform an empirical analysis examining the differences of the predictions of expected survival probabilities between U.S. and European residents. The results show that compared to U.S residents, Europeans tend to have larger extent of rounding when reporting probabilities, but their extent of rounding narrows down overtime. Besides, European residents are more optimistic about living a long life than U.S. residents with higher estimated subjective probabilities of living, and the predictions of expected probability of living are consistent with the actual life expectancies in U.S and European Union.

1 Introduction

Rounding is frequently observed when people report a numerical value that falls in an interval. In some professional fields, rounding follows strict standards. For example, Office of the Federal Coordinator for Meteorology (OFCM) in U.S. published the Federal Meteorological Handbooks which stipulated rigorous standards for rounding figures when reporting meteorological parameters (OFCM, 2005). However, rounding is less normative in daily life. Consider a person reporting the current local time as "8 a.m.". It is hard to infer whether this person is reporting the accurate local time or is rounding to the nearest minute. Assuming that s/he rounds the time, others might infer that s/he rounds to the nearest quarter or to the nearest hour. The uncertainty about the extent of rounding is also observed in survey responses. Respondents are frequently asked about their annual income, working hours per week etc., but often the survey questions do not specify to what extent the respondents should round their answers to. Thus, it is often difficult to tell whether the respondents report the exact value or use rounding for the simplicity of communication. Moreover, as Manski and Molinari (2010) pointed out, the survey responses are often taken as face value in most empirical studies, despite the concern of the inaccuracy caused by rounding in data. Therefore, the survey responses would be more credible if they could be interpreted as intervals rather than face values. By examining a person's rounding behavior when answering survey questions and properly transforming the data points into intervals, one may get more convincing and accurate results.

In this paper, I investigate the rounding behavior of U.S. and European residents when they are asked to state the percentage chance that they will live to a certain age, using data from *Health and Retirement Study* (HRS) [1] and *Survey of Health, Aging and Retirement in Europe* (SHARE)[2]. Taking rounding into consideration, I further examine the impact of two factors, age and gender, on people's subjective survival probabilities, using the regression method for interval data described in the Manski and Molinari (2010) paper. Subjective probability has been studied by many previous researchers (e.g. Honig, 1996; Dominitz and Manski, 1996), and it has been shown to give reasonably accurate predictions on future events. In particular, Hurd and McGarry (1995) found that the subjective probabilities of living from survey responses aggregate to the population survival rate and that "they covary with other variables in the same way actual outcomes vary with the variable". Similarly, one of the results of this paper shows that European residents state higher probabilities of living than American residents, and this is consistent with the fact that the life expectancy in Europe (78.968 in 2006) is longer than the one in U.S. (77.688 in 2006) (Data.worldbank.org, 2017). Besides, another result shows that European residents tend to have a wider extent of rounding than Americans, but their extent of rounding narrows down overtime.

This paper follows a "replication-extension" structure. In other words, I first replicate the results of the Manski and Molinari (2010) paper, *"Rounding Probabilistic Expectations in Surveys"*, using the same HRS2006 data set and then apply the their methods to SHARE2006 and SHARE2014 data sets as an extension. The comparison of the results derived from HRS2006 and SHARE2006 reveals the differences in rounding behaviors and subjective life expectancies between European and American residents. Similarly, by comparing results derived from SHARE2006 and SHARE2014, I intend to obtain insights in the changes of the rounding behavior and subjective life expectancy of European residents overtime. The review of the relevant literature is presented in Section 2. Section 3 provides detailed descriptions of the data sets used in this paper. In Section 4, I perform the empirical analysis examining the impact of age and gender on subjective probabilities of living among European and American residents. Note that I use the same method and therefore keep the notations as Manski and Molinari did in their paper. Then in Section 5 I make a cross-country comparison (i.e. a comparison between HRS2006 and SHARE2006) and longitudinal comparison (i.e. a comparison between SHARE2006 and SHARE2014) of the results from empirical analysis.

[1]This paper uses data from HRS Wave 8. The HRS (Health and Retirement Study) is sponsored by the National Institute on Aging (grant number NIA U01AG009740) and is conducted by the University of Michigan.

[2]This paper uses data from SHARE Waves 2 and 6 (DOIs: 10.6103/SHARE.w2.600, 10.6103/SHARE.w6.600), see Börsch-Supan et al. (2013) and Börsch-Supan, A. (2017) for methodological details. (1)
The SHARE data collection has been primarily funded by the European Commission through FP5 (QLK6-CT-2001-00360), FP6 (SHARE-I3: RII-CT-2006-062193, COMPARE: CIT5-CT-2005-028857, SHARELIFE: CIT4-CT-2006-028812) and FP7 (SHARE-PREP: N°211909, SHARE-LEAP: N°227822, SHARE M4: N°261982). Additional funding from the German Ministry of Education and Research, the Max Planck Society for the Advancement of Science, the U.S. National Institute on Aging (U01_AG09740-13S2, P01_AG005842, P01_AG08291, P30_AG12815, R21_AG025169, Y1-AG-4553-01, IAG_BSR06-11, OGHA_04-064, HHSN271201300071C) and from various national funding sources is gratefully acknowledged (see www.share-project.org).

2 Literature Review

The accuracy of subjective probabilities in predicting actual occurrence of events has been extensively studied. Dominitz and Manski (1996) discovered that respondents are clearly willing to answer subjective probability questions. Hurd and McGarry (1995) compared the population mortality rate with the one that was derived from respondents' stated subjective probabilities, and they found out that the subjective mortality rate is representative of the actual one among the population. Similarly, Perozek (2008) asked respondents to provide their subjective life expectations. He suggested that since the individual has private information such as their own genetic background, environmental and behavioral risk factors that are not known to others, their predictions of their true mortality are reasonably accurate. Besides, Michael Hurd (2009) concluded that subjective probabilities have considerable prediction power for events, such as retirement and survival, in which respondents have sufficient private information. He also observed that subjective probabilities for stock market return vary substantially among individuals, and compared to historical averages, subjective probabilities for stock market returns are considerably lower. Moreover, Bassett and Lumsdaine (2001) observed the existence of the high correlations among answers to subjective probability questions. They discovered the presence of unobserved heterogeneity throughout the answers, such as systematic (under)overestimation of probabilities by individuals. This suggests that there exist some behavioral patterns when respondents provide probabilistic expectations, and it is meaningful to further examine those patterns in different contexts. Although the above papers did not take into account the fact that respondents may round their answers when stating probabilities, the rounding behavior in survey responses also has been studied by several previous scholars.

Over two decades ago, the rounding in answers of subjective probabilities survey questions has already aroused interests among researchers. Dominitz and Manski (1996) observed that respondents round their answers to 1% when reporting extreme values (i.e. 0%-2% and 98%-100%), and round to 5% elsewhere (i.e. 5%, 10%, ..., 95%) with even more responses of 50% compared to adjacent values (i.e 40%, 45%, 55%, 60%). However, they did not provide any methods to identify to which extent the respondents round the values to. Similarly, Fischhoff and Bruine de Bruin (1999) suggested that some respondents use 50% as synonyms of "not sure" about the answer rather than believing that the chance of the event occurring is indeed 50%. Kleinjans and Van Soest (2014) developed a panel model that explicitly accounts for the rounding behavior: the focal point "50%" answers and non-responses, in order to explain the probabilistic expectations of binary events. They concluded that including rounding behavior does not significantly influence the estimated distribution of true subjective probabilities. However, Manski and Molinari (2010) pointed out that it is meaningful to examine the rounding behavior of the respondents and interpret their stated probability as interval data so that the researcher could get more accurate results for empirical analysis.

Despite the advantage of obtaining more accurate results, in order to incorporate the rounding in the empirical analysis more extensively, researchers have to deal with the inference of regressing partially identified parameters on regressors or outcomes. This requires methodological work in deriving unbiased interval estimators and constructing corresponding confidence intervals. Horowitz and Manski (2000) and Manski and Tamer (2002) contributed to the methodology work in deriving unbiased interval estimators, and the method they developed were incorporated in the Manski and Molinari (2010) paper. Regarding the confidence interval of the interval estimators, Horowitz and Manski (2000) summarized that recent literature proposes two ways of construction. One may choose confidence interval that (asymptotically) uniformly covers all the points in the identification region with at least a specified probability. This approach is studied and supported by Imbens and Manski (2004) and Stoye (2009). Another approach suggested by researchers such as Horowitz and Manski (2000), Chernozhukov, Hong, and Tamer (2007), Beresteanu and Molinari (2008) etc. is to construct a confidence interval that asymptotically covers the whole identification region with at least the specified probability. Following Manski and Molinari (2010), I adopt the confidence intervals suggested by Imbens and Manski (2004) in this paper.

3 Data

3.1 General description

The data used in this paper are obtained from *the Health and Retirement Study* (HRS) and *the Survey of Health, Aging and Retirement in Europe* (SHARE). Both of them are longitudinal studies of the non-institutionalized population aged 51 or older in U.S. and Europe respectively. Every two years, surveys are administered to the target population who answers questions encompassing a wide range of topics including health condition, employment status, cognitive capability etc.. In particular, I use HRS2006 to replicate the results in the Manski and Molinari (2010) paper, and then make a cross-country comparison (i.e. a comparison between U.S. and European respondents) after applying the same analysis using SHARE2006 data. Moreover, I conduct the analysis using SHARE2014 data in order to make a longitudinal comparison of the changing rounding behavior of European respondents. Questions that are particularly interesting to this paper are the ones from Module P (Expectations) in HRS2006 and Module EX (Expectations) from SHARE2006 and SHARE2014. In those modules, respondents are encouraged to think in probabilistic terms, and they are asked to state the percentage chance that a future event will occur. For example, in HRS2006 questionnaire, the respondents got the following instructions before answering probabilistic questions:

- "Next we would like to ask your opinion about how likely you think various events might be. When I ask a question I'd like for you to give me a number from 0 to 100, where '0' means that you think there is absolutely no chance, and '100' means that you think the event is absolutely sure to happen.
  For example, no one can ever be sure about tomorrow's weather, but if you think that rain is very unlikely tomorrow, you might say that there is a 10 percent chance of rain. If you think there is a very good chance that it will rain tomorrow, you might say that there is an 80 percent chance of rain."

Then, the respondents got questions such as "What do you think are the chances that your income will keep up with the cost of living for the next five years?", "And what are the chances that you [and your] [you/husband/wife/partner] will leave an inheritance totalling $100,000 or more?" etc. Note that the probabilistic questions asked in HRS2006, SHARE2006 and SHARE2014 surveys are not identical. Besides, not all respondents got the same probabilistic questions since both HRS and SHARE extensively used skipping frequency such that some questions were only presented to respondents if they answered the previous question. The exact rules for skipping frequency are recorded in the documentation of Module P for HRS questionnaires and Module EX for SHARE questionnaires. Appendix A provides detailed descriptions of probabilistic questions used in this paper. Besides, the basic demographic characteristic, age and gender, for each respondent answering HRS and SHARE questionnaires are also used.

3.2 Responses to specific questions

Following Manski and Molinari (2010), I first examine the proportion of responses on three types of probabilistic questions in HRS2006: questions on personal finance (P4, P5, P14, P15, P18, P70, P30 and P31), questions on personal health (P28, P103 and P32) and questions on general economic conditions (P34, P110, P47 and P114). The detailed descriptions of the above questions is presented in Appendix A.1. Table 1 shows the replicated results of the Table 1 in the Manski and Molinari (2010) paper. For each question presented, column 3-10 present the proportions of responses satisfying the following categories:

- NR: respondents did not respond or refused to answer.
- 0: respondents answered 0.
- 1-4: the responses falls in the interval [1, 4].
- 50: respondents answered 50.
- 96-99: the responses falls in the interval [96, 99].
- 100: respondents answered 100.
- M10: the responses were multiples of 10 other than (0, 50, 100), e.g. 20, 30 or 60.
- M5: the responses were multiples of 5 other than multiples of 10, e.g. 15, 65 or 75.
- Other: the responses were not multiples of 5 and did not fall in the interval $[1, 4] \cup [96, 99]$, e.g. 7, 23 or 92.

I keep the results to the fourth decimal while Manski and Molinari (2010) rounded to the third decimal. Except for minor rounding errors, the replicated results are the same as the original ones.

Table 1: Responses by questions in the HRS2006 data

		proportion of responses								
Percent chance	N	NR	0	1-4	50	96-99	100	M10	M5	Other
Questions										
P3	17,191	0.0290	0.2183	0.0148	0.1503	0.0036	0.0468	0.4681	0.0657	0.0033
P4	17,191	0.0688	0.1734	0.0124	0.1825	0.0017	0.0628	0.3871	0.1085	0.0028
P5	17,191	0.0529	0.1588	0.0045	0.0665	0.0080	0.4472	0.2089	0.0517	0.0014
P14	4,797	0.0200	0.4609	0.0261	0.1067	0.0013	0.0179	0.2741	0.0905	0.0025
P15	4,797	0.0167	0.1726	0.0136	0.1518	0.0042	0.1432	0.3832	0.1124	0.0025
P18	5,148	0.0157	0.2756	0.0202	0.1257	0.0025	0.0952	0.3485	0.1144	0.0021
P28	6,713	0.0404	0.0526	0.0042	0.2223	0.0051	0.1521	0.3752	0.1445	0.0037
P103	2,558	0.0152	0.0117	0.0043	0.2143	0.0035	0.1360	0.4328	0.1798	0.0023
P70	16,754	0.0640	0.2543	0.0106	0.1375	0.0006	0.0559	0.3574	0.1178	0.0019
P30	16,754	0.0273	0.3816	0.0079	0.1142	0.0019	0.1185	0.2627	0.0845	0.0013
P31	16,754	0.0273	0.6461	0.0200	0.0438	0.0002	0.0162	0.1732	0.0722	0.0011
P32	10,044	0.0746	0.4628	0.0207	0.1013	0.0000	0.0073	0.2310	0.1003	0.0022
P34	16,754	0.0777	0.0661	0.0057	0.2383	0.0021	0.0599	0.4042	0.1422	0.0039
P110	16,754	0.0649	0.0483	0.0027	0.2310	0.0049	0.1199	0.3867	0.1391	0.0024
P47	16,754	0.2398	0.0416	0.0033	0.2312	0.0005	0.0356	0.3390	0.1056	0.0033
P114	16,680	0.2810	0.0681	0.0034	0.1825	0.0005	0.0282	0.3343	0.0993	0.0028

Similarly, I examine the proportion of responses to 8 probabilistic questions in SHARE2006 and 5 probabilistic questions in SHARE2014, and the results are presented in Table 2 and Table 3 respectively. The detailed descriptions of the probability questions can be found in Appendix A.2 and Appendix A.3.

Table 2: Responses by questions in the SHARE2006 data

		proportion of responses								
Percent chance	N	NR	0	1-4	50	96-99	100	M10	M5	Other
Questions										
EX001	36,964	0.0368	0.0429	0.0083	0.1961	0.0093	0.1201	0.5381	0.0421	0.0063
EX002	36,964	0.0378	0.7143	0.0044	0.0435	0.0011	0.0425	0.1410	0.0145	0.0009
EX004	36,963	0.0641	0.2695	0.0025	0.0744	0.0025	0.3772	0.1945	0.0136	0.0017
EX007	15,323	0.0861	0.2518	0.0025	0.1663	0.0020	0.0808	0.3740	0.0350	0.0016
EX008	15,325	0.0763	0.2709	0.0018	0.1421	0.0022	0.0993	0.3722	0.0331	0.0021
EX009	36,960	0.1190	0.0508	0.0027	0.2068	0.0044	0.1354	0.4322	0.0442	0.0046
EX010	36,963	0.0654	0.3288	0.0050	0.2041	0.0005	0.0304	0.3395	0.0241	0.0022
EX011	36,964	0.0727	0.2228	0.0037	0.2251	0.0006	0.0643	0.3810	0.0267	0.0029

Table 3: Responses by questions in the SHARE2014 data

		proportion of responses								
Percent chance	N	NR	0	1-4	50	96-99	100	M10	M5	Other
Questions										
EX001	65,138	0.0118	0.0259	0.0037	0.1720	0.0148	0.1828	0.5300	0.0480	0.0111
EX007	4,066	0.0376	0.0817	0.0054	0.1763	0.0123	0.2012	0.4390	0.0398	0.0066
EX008	4,066	0.0327	0.1008	0.0034	0.1414	0.0081	0.2371	0.4383	0.0344	0.0037
EX009	65,130	0.0629	0.0365	0.0049	0.1834	0.0079	0.1459	0.4885	0.0634	0.0067
EX025	3,694	0.0246	0.1716	0.0041	0.1372	0.0087	0.2112	0.4023	0.0365	0.0038

In all three data sets, there are considerable proportions of responses that are rounded to multiples of five (the aggregate categories of 0, 50, 100, M5 and M10). Take the results of SHARE2006 for example. The proportion of responses that falls in category "1-4" ranges from 0.0018 to 0.0083 across all questions presented, and the proportion of the response category "96-99" ranges from 0.0005 to 0.0093. The proportion of responses categorized as "Other" does not exceed 0.0065 for all questions. Overall, 90% of the responses are multiples of five. Similarly, the overall proportions of responses that are multiples of five for SHARE2006 and SHARE2014 are both above 90%.

Manski and Molinari (2010) proposed that the comparison of 50, M10 and M5 proportions of responses should indicate the extent of rounding for different response categories. Using HRS2006 data set, they pointed out that the proportion of M10 is always twice as large as the proportion of M5 and the proportion of 50 is also larger than the proportion of M5. Based on the above observation, they concluded that "respondents who reported 50 rounded to the nearest 50% while many of those who reported M10 round to the nearest 10%". A similar phenomenon is observed in SHARE2006 and SHARE2014 data sets. The proportion of M10 is almost ten times as large as the proportion of M5 for most of the questions in both SHARE2006 and SHARE2014, and the proportion of 50 is at least three times larger than the proportion of M5. This indicates that European respondents, similar to Americans, also differ in the extent of rounding when they answered 50, M10 and M5. Therefore, in this paper, I use the same rounding inferences for responses in SHARE2006 and SHARE2014 questionnaires as Manski and Molinari did for responses in HRS2006 questionnaires.

3.3 Response patterns

Following Manski and Molinari (2010), I further analyze the response pattern across 38 probability questions in HRS2006 data set. Note that there are in total 41 probability questions asked in the HRS2006 survey. However, for some reason Manski and Molinari (2010) chose to discard questions P003, P100 and P101 for analyzing response patterns. In order to obtain the same results, I also exclude those three questions. According to Manski and Molinari (2010), the responses to probabilistic questions are defined as:

- All NR: the respondent did not response to any of the questions;
- All 0 or 100 : the respondent answered 0 or 100 to all of the questions;
- All 0, 100 or 50: the respondent answered 0, 50 or 100 to all of the questions, and s/he answered at least one 50;
- Some M10: the respondent answered multiples of ten to all of the questions, and s/he answered at least one multiple of ten that is not one of the (0, 50, 100);
- Some M5: the respondent answered multiples of five to all of the questions, and s/he answered at least one multiple of five that is not a multiple of ten;
- Some 1-4 or 96-99: the respondent answered at least one value that falls in the interval of 1-4 or 96-99;
- Some other: the respondent answered at least one value that is not a multiple of five and not falls in the interval of 1-4 or 96-99.

However, the above categories are not mutually exclusive. There exists an overlap between the category "Some 1-4 or 96 and 99"and the category "Some other". For example, if a respondent answered "10, 4, 6, 100", his/her response satisfies both definitions of "Some 1-4 or 96 and 99"and "Some other". Therefore, I change the definition of "Some 1-4 or 96-99" as:

- Some 1-4 or 96-99: the respondent answered at least one value that is not a multiple of five, and that value falls in the interval of 1-4 or 96-99.

Table 4 presents the response pattern across 38 probability questions in the HRS2006 Module P. The table separates the effect of different age groups and gender for respondents aged 50 or older. The total number of respondents in the sample is 16,674.

The sample size for each age group is not completely identical to the one in the Table 2 of Manski and Molinari (2010) paper. However, the biggest difference in sample size is only 2. Besides, compared to the Table 2 in Manski and Molinari (2010) paper, the differences of the response pattern are less than 0.001. This may be caused by the differences of sample size in certain age groups and the rounding of software when reporting numbers. Nevertheless, as the difference is rather small, it is reasonable to conclude that the replicated results are close the ones in Manski and Molinari (2010) paper. Table 5 presents the same response pattern using SHARE2006 data set that contains 14 probability questions and 36,107 respondents aged 50 or above. Table 6 presents the same response pattern using SHARE2014 data set that contains 5 probability questions and 67,214 respondents aged 50 or above.

Table 4: Responses tendencies in the HRS2006 data, 38 probablity questions included

	Sample size	Mean items: Asked per person	Mean items: Responded per person	Response patterns: All NR	All 0 or 100	All 0, 50 or 100	Some M10	Some M5	Some 1-4 or 96-99	Some other
				MALES						
All	6,774	23	22	0.0227	0.0136	0.0195	0.2521	0.5323	0.1182	0.0415
Age 50-54	595	25	24	0.0101	0.0034	0.0101	0.2235	0.5563	0.1445	0.0521
Age 55-59	967	25	24	0.0145	0.0155	0.0134	0.2347	0.5502	0.1303	0.0414
Age 60-64	858	24	23	0.0117	0.0117	0.0198	0.2296	0.5594	0.1247	0.0431
Age 65-70	1,394	22	21	0.0194	0.0115	0.0179	0.2683	0.5187	0.1241	0.0402
Age 70-74	1,147	22	21	0.0200	0.0131	0.0192	0.2711	0.5371	0.1037	0.0357
Age 75-79	824	22	21	0.0243	0.0133	0.0121	0.2731	0.5243	0.1104	0.0425
Age 80-84	590	21	20	0.0339	0.0153	0.0424	0.2322	0.5407	0.1051	0.0305
Age 85+	399	20	18	0.0852	0.0351	0.0351	0.2607	0.4336	0.0927	0.0576
				FEMALES						
All	9,900	22	20	0.0276	0.0234	0.0261	0.2702	0.5013	0.1152	0.0363
Age 50-54	987	24	23	0.0132	0.0142	0.0132	0.2715	0.5015	0.1489	0.0375
Age 55-59	1,442	24	23	0.0153	0.0187	0.0146	0.2455	0.5257	0.1415	0.0388
Age 60-64	1,435	24	22	0.0098	0.0160	0.0105	0.2307	0.5491	0.1359	0.0481
Age 65-70	1,835	22	20	0.0262	0.0169	0.0278	0.2910	0.4817	0.1248	0.0316
Age 70-74	1,511	22	20	0.0258	0.0192	0.0251	0.2733	0.5162	0.1039	0.0364
Age 75-80	1,068	22	19	0.0197	0.0281	0.0272	0.2949	0.5103	0.0871	0.0328
Age 80-84	817	21	18	0.0477	0.0355	0.0453	0.2852	0.4761	0.0759	0.0343
Age 85+	805	19	16	0.0957	0.0609	0.0671	0.2820	0.4025	0.0658	0.0261

Table 5: Responses tendencies in the SHARE2006 data, 12 probability questions included

	Sample size	Mean items: Asked per person	Mean items: Responded per person	Response patterns: All NR	All 0 or 100	All 0, 50 or 100	Some M10	Some M5	Some 1-4 or 96-99	Some other
				MALES						
All	16,303	8	8	0.0236	0.0286	0.0636	0.6661	0.1546	0.0439	0.0197
Age 50-54	2,368	10	10	0.0148	0.0118	0.0359	0.6571	0.2073	0.0519	0.0211
Age 55-59	3,163	10	9	0.0177	0.0206	0.0493	0.6604	0.1812	0.0506	0.0202
Age 60-64	2,969	9	8	0.0182	0.0256	0.0593	0.6700	0.1640	0.0445	0.0185
Age 65-70	2,487	7	7	0.0161	0.0338	0.0663	0.6828	0.1363	0.0426	0.0221
Age 70-74	2,194	7	7	0.0219	0.0310	0.0747	0.6851	0.1299	0.0360	0.0214
Age 75-80	1,606	7	7	0.0342	0.0411	0.0934	0.6737	0.1040	0.0342	0.0193
Age 80-84	995	7	6	0.0482	0.0462	0.0955	0.6392	0.1196	0.0402	0.0111
Age 85+	521	7	6	0.0921	0.0634	0.0883	0.5893	0.1132	0.0384	0.0154
				FEMALES						
All	19,804	8	8	0.0245	0.0366	0.0693	0.6969	0.1168	0.0400	0.0159
Age 50-54	3,290	10	9	0.0116	0.0125	0.0456	0.7064	0.1617	0.0483	0.0140
Age 55-59	3,810	9	9	0.0139	0.0205	0.0533	0.7173	0.1336	0.0446	0.0168
Age 60-64	3,454	8	8	0.0116	0.0278	0.0637	0.7148	0.1262	0.0400	0.0159
Age 65-70	2,851	7	7	0.0116	0.0354	0.0814	0.7254	0.0975	0.0323	0.0165
Age 70-74	2,308	7	7	0.0234	0.0485	0.0771	0.7036	0.0914	0.0386	0.0173
Age 75-80	1,959	7	6	0.0388	0.0587	0.0893	0.6733	0.0888	0.0327	0.0184
Age 80-84	1,296	7	6	0.0640	0.0787	0.1049	0.6127	0.0895	0.0347	0.0154
Age 85+	836	7	5	0.1292	0.0945	0.0933	0.5634	0.0682	0.0431	0.0084

Table 6: Responses tendencies in the SHARE2014 data, 5 probability questions included

		Mean items		Response patterns						
	Sample size	Asked per person	Responded per person	All NR	All 0 or 100	All 0, 50 or 100	Some M10	Some M5	Some 1-4 or 96-99	Some other
				MALES						
All	29,586	2	2	0.0567	0.0702	0.1046	0.6169	0.1053	0.0306	0.0158
Age 50-54	2,428	3	3	0.0404	0.0540	0.0774	0.6400	0.1297	0.0375	0.0210
Age 55-59	4,380	2	2	0.0409	0.0742	0.0913	0.6160	0.1242	0.0356	0.0178
Age 60-64	5,434	2	2	0.0379	0.0878	0.0904	0.6194	0.1128	0.0368	0.0149
Age 65-70	5,546	2	2	0.0415	0.0772	0.1066	0.6289	0.1026	0.0272	0.0160
Age 70-74	4,417	2	2	0.0512	0.0634	0.1257	0.6296	0.0944	0.0242	0.0115
Age 75-80	3,523	2	2	0.0605	0.0590	0.1186	0.6273	0.0974	0.0204	0.0167
Age 80-84	2,309	2	2	0.1035	0.0606	0.1243	0.5834	0.0879	0.0277	0.0126
Age 85+	1,549	2	2	0.1846	0.0562	0.1072	0.5216	0.0710	0.0407	0.0187
				FEMALES						
All	37,628	2	2	0.0482	0.0735	0.1134	0.6347	0.0870	0.0286	0.0146
Age 50-54	3,879	3	3	0.0180	0.0693	0.0887	0.6574	0.1052	0.0436	0.0178
Age 55-59	5,960	2	2	0.0228	0.0794	0.0975	0.6539	0.0998	0.0314	0.0153
Age 60-64	6,565	2	2	0.0239	0.0879	0.1045	0.6495	0.0909	0.0294	0.0139
Age 65-70	6,367	2	2	0.0239	0.0732	0.1131	0.6619	0.0870	0.0261	0.0149
Age 70-74	5,124	2	2	0.0375	0.0613	0.1306	0.6532	0.0802	0.0222	0.0150
Age 75-80	4,274	2	2	0.0508	0.0700	0.1357	0.6231	0.0810	0.0271	0.0124
Age 80-84	3,054	2	2	0.1103	0.0632	0.1333	0.5835	0.0717	0.0259	0.0121
Age 85+	2,405	2	1	0.2295	0.0719	0.1168	0.4852	0.0590	0.0216	0.0158

Manski and Molinari noticed that there exist small but negligible proportions of respondents who did not respond to any of the questions in HRS2006 questionnaire (male 0.0227 and female 0.0276) and those proportions increased substantially for both male and female when the respondents became 80 or older. A similar pattern is found in both SHARE2006 and SHARE2014 in which the non-response rate is 0.0236 and 0.0567 for males and 0.0245 and 0.0428 for females, respectively. When the respondents were aged 85 or above, the proportions of "All NR" become even more prevalent in both SHARE2006 (male 0.0634 and female 0.0945) and SHARE2014 (male 0.1846 and female 0.2295). Besides, in HRS2006 data set, the proportions of "All 0 or 100" and "All 0, 50 or 100" increase as age of the respondents increases, and females are more likely to have such pattern than males. This pattern reappears in SHARE2006 data set but it is not obvious in SHARE2014 data set. Also, compared to the responses in SHARE2006, the responses in SHARE2014 tend to have more extreme values (such as 0, 50 or 100). Moreover, given that respondents provide numerical answers for the probabilistic questions, it can be observed that among all three response patterns, females tend to give answers that belong to categories "All 0 or 100", "All 0, 50 or 100" and "Some M10" in most of the age groups while males tend to answer more "Some M5", "Some 1-4 or 96-99" and "Some other". This shows that in general males may have a more precise understanding of probability and have a narrower extent of rounding, regardless the nationality.

Another major difference between response patterns in the HRS2006 and SHARE2006 (SHARE2014) data sets lies in the category that has the largest proportion of responses: in HRS2006, M5 has the largest proportion of responses among all age groups, (over 0.5 for most of the age groups) while in SHARE2006 and SHARE2014, M5 has much smaller proportion and M10 takes the largest proportion of responses among all age groups (over 0.6 for most of the age groups). This may indicate that the extent of rounding among European respondents is in general larger than that among American respondents. However, it is worth noting that the number of probabilistic questions asked in SHARE2006 (SHARE20014) is much smaller than in HRS2006. This may influence the proportions of responses categorized as "Some M5" "Some 1-4 or 96-99" and "Some other", since the chance that the respondent gives at least one "special" answers among all questions is smaller when there are less probabilistic questions asked. Thus, due to the differences in the structure of questionnaires, the proportion of categories "Some M5" "Some 1-4 or 96-99" and "Some other" tend to be smaller in SHARE2006 (SHARE2014) than in HRS2006 in theory.

Table 7: Responses tendencies in the RDHRS2006 data, 12 random probability questions included

	Sample size	Mean items: Asked per person	Mean items: Responded per person	Response patterns: All NR	All 0 or 100	All 0, 50 or 100	Some M10	Some M5	Some 1-4 or 96-99	Some other
MALES										
All	6774	7	6	0.0251	0.0523	0.0679	0.4103	0.3729	0.0562	0.0153
Age 50-54	595	7	7	0.0116	0.0219	0.0437	0.4161	0.4210	0.0665	0.0193
Age 55-59	967	7	7	0.0156	0.0404	0.0446	0.4020	0.4198	0.0632	0.0145
Age 59-64	858	7	7	0.0138	0.0402	0.0584	0.3986	0.4050	0.0675	0.0166
Age 65-69	1394	6	6	0.0217	0.0477	0.0645	0.4319	0.3593	0.0592	0.0157
Age 70-74	1147	6	6	0.0221	0.0594	0.0704	0.4216	0.3680	0.0461	0.0125
Age 75-79	824	6	6	0.0267	0.0533	0.0749	0.4278	0.3519	0.0494	0.0160
Age 80-84	590	6	6	0.0378	0.0763	0.1070	0.3837	0.3354	0.0481	0.0118
Age 85+	399	6	5	0.0912	0.1109	0.1135	0.3434	0.2786	0.0438	0.0187
FEMALES										
All	9900	6	6	0.0318	0.0766	0.0737	0.3985	0.3496	0.0561	0.0137
Age 50-54	987	7	7	0.0149	0.0438	0.0526	0.4330	0.3666	0.0755	0.0136
Age 55-59	1442	7	7	0.0172	0.0498	0.0503	0.4019	0.3964	0.0693	0.0149
Age 59-64	1435	7	6	0.0121	0.0503	0.0476	0.3959	0.4069	0.0688	0.0184
Age 65-69	1835	6	6	0.0285	0.0636	0.0755	0.4233	0.3374	0.0594	0.0124
Age 70-74	1511	6	6	0.0294	0.0753	0.0763	0.4101	0.3447	0.0495	0.0147
Age 75-79	1068	6	6	0.0259	0.0938	0.0875	0.3960	0.3414	0.0434	0.0120
Age 80-84	817	6	5	0.0566	0.1171	0.1102	0.3749	0.2941	0.0357	0.0113
Age 85+	805	5	5	0.1080	0.1799	0.1237	0.3034	0.2470	0.0283	0.0097

3.3.1 Comparisons of response patterns between HRS2006 and SHARE2006

In order to separate the effect of the different number of probabilistic questions asked in different questionnaires when comparing the responses patterns, I randomly select [3] a subset of probabilistic questions from HRS2006 that has the same number of probabilistic questions as in SHARE2006 and then examine the response pattern of this subset. The response pattern of the subset (denoted as RDHRS2006) is presented in Table 7. Compared to Table 4, the proportions of "Some M10" increase dramatically and the values of "Some M5" decrease among all gender and age groups. Despite all this, the proportions of "Some M10" in RDHRS2006 are still much smaller than the ones in SHARE2006 while "Some M5" has larger values in RDHRS2006 than in SHARE2006, regardless of the age and gender. Given that the proportions of other categories in RDHRS2006 are reasonably similar to the ones in SHARE2006, it can be concluded that compared to American respondents, European respondents are more likely to round to M10 than M5. This indicates that European respondents may have larger the extent of rounding and they are less precise in interpreting probabilities than American respondents.

3.3.2 Comparisons of response patterns between SHARE2006 and SHARE2014

Similar to the previous section, in order to eliminate the effect of different numbers of probabilistic questions asked, I randomly select a subset of 5 probabilistic questions from SHARE2006 and then examine the response pattern of this subset of questions (denoted as RDSHARE2016). Compared to the response pattern of RDSHARE2016 (see Table 8), SHARE2014 (see Table 6) has increased proportions in "All NR", "Some M10", "Some M5", "Some 1-4 or 96-99" and "Some other" but decreased proportions in "All 0 or 100" and "All 0, 50 or 100". This suggests that the European respondents develop a more precise understanding of probabilities and give a lower extent of rounding overtime.

[3]Note that the random selection process is repeated for at least 792 times (depending on the total number of combinatotrial options. For example, the number of combinatorial options of selecting 5 questions out of 12 is 792), and the relevant results are the average of the results derived from all random sub-samples. Same holds for any following random selection process mentioned in this paper.

Table 8: Responses tendencies in the RDSHARE2006 data, 5 random probability questions included

	Sample size	Mean items: Asked per person	Mean items: Responded per person	Response patterns: All NR	All 0 or 100	All 0, 50 or 100	Some M10	Some M5	Some 1-4 or 96-99	Some other
MALES										
All	16303	4	3	0.0303	0.1455	0.1308	0.5756	0.0859	0.0226	0.0093
Age 50-54	2368	4	4	0.0181	0.0632	0.1011	0.6542	0.1275	0.0265	0.0095
Age 55-59	3163	4	4	0.0223	0.0955	0.1102	0.6302	0.1060	0.0258	0.0099
Age 60-64	2969	4	3	0.0233	0.1386	0.1290	0.5871	0.0904	0.0233	0.0083
Age 65-69	2487	3	3	0.0222	0.1761	0.1395	0.5574	0.0714	0.0225	0.0109
Age 70-74	2194	3	3	0.0310	0.1791	0.1504	0.5437	0.0667	0.0185	0.0105
Age 75-79	1606	3	3	0.0441	0.2064	0.1573	0.5141	0.0519	0.0170	0.0092
Age 80-85	995	3	3	0.0601	0.2205	0.1586	0.4769	0.0589	0.0202	0.0048
Age 85+	521	3	2	0.1106	0.2449	0.1410	0.4211	0.0551	0.0205	0.0069
FEMALES										
All	19804	3	3	0.0334	0.1612	0.1384	0.5765	0.0632	0.0197	0.0075
Age 50-54	3290	4	4	0.0159	0.0773	0.1164	0.6667	0.0933	0.0239	0.0064
Age 55-59	3810	4	4	0.0179	0.1149	0.1272	0.6347	0.0757	0.0217	0.0079
Age 60-64	3454	3	3	0.0186	0.1545	0.1358	0.5960	0.0681	0.0193	0.0076
Age 65-69	2851	3	3	0.0205	0.1864	0.1534	0.5650	0.0501	0.0170	0.0076
Age 70-74	2308	3	3	0.0341	0.2014	0.1525	0.5406	0.0445	0.0184	0.0086
Age 75-79	1959	3	3	0.0539	0.2204	0.1557	0.5004	0.0447	0.0164	0.0086
Age 80-85	1296	3	3	0.0806	0.2487	0.1556	0.4457	0.0450	0.0168	0.0076
Age 85+	836	3	2	0.1550	0.2602	0.1288	0.3956	0.0363	0.0207	0.0034

4 Empirical analysis

In this section, I investigate incorporate the rounding in probabilities in an empirical analysis. The probabilistic response of individual i to question k, v_{jk}, is precieved as a rounded value and the true value is assumed to fall in the interval $[v_{jkL},\, v_{jkU}]$. Thus, instead of taking the data of probabilistic responses at face value, I first transform them into interval data based on the response pattern and then regress the interval data on other precisely measured variables (constant, age and gender). Section 4.1 describes the detailed rules and algorithm for transforming v_{jk} to interval $[v_{jkL},\, v_{jkU}]$, and section 4.2 presents the model of the regression with interval data. Both transformation and regression are applied to HRS2006, SHARE2006 and SHARE2014 data sets, and Section 5 compares the regression results derived from different data sets.

4.1 Transformation of the potential rounding responses

Following the illustration example in Manski and Molinari (2010), I focus on question P28 in HRS2006: "What is the percent chance that you will live to be 75 or more.", and transform the answers to that question into interval data. The formation of the interval data $[v_{jkL},\, v_{jkU}]$ given the data point v_{jk} is discussed in detail by Manski and Molinari (2010), and I adopt the same method in this paper.

Let r_{jm} denote person j's response to probabilistic questions in class m. Formally, quoted from Manski and Molinari (2010), the transformation algorithm is:"

1. if v_{jk} = NR, then $[v_{jkL}, v_{jkU}]$ = [0, 100].
2. if r_{jm} = (all 0 0r 100), then $[v_{jkL}, v_{jkU}]$ = [max(0, v_{jkL} - 50), min(v_{jkU} + 50, 100)].
3. if r_{jm} = (all 0, 50 or 100), then $[v_{jkL}, v_{jkU}]$ = [max(0, v_{jkL} - 25), min(v_{jkU} + 25, 100)].
4. if r_{jm} = M10, then $[v_{jkL}, v_{jkU}]$ = [max(0, v_{jkL} - 5), min(v_{jkU} + 5, 100)].
5. if r_{jm} = M5, then $[v_{jkL}, v_{jkU}]$ = [max(0, v_{jkL} - 2.5), min(v_{jkU} + 2.5, 100)].
6. if r_{jm} = (some 1-4 or 96-99) and $v_{jk} \in [0, 5] \cup [95, 100]$, then $[v_{jkL}, v_{jkU}] = v_{jk}$.
7. if r_{jm} = (some 1-4 or 96-99) an $v_{jk} \in [10, 90]$, then $[v_{jkL}, v_{jkU}] = [v_{jkL} - 2.5, v_{jkU} + 2.5]$.
8. if r_{jm} = other, then $[v_{jkL}, v_{jkU}] = v_{jk}$. "

Table 9 is the replication of the Table 3 in the Manski and Molinari (2010) paper and Figure 1 presents the corresponding histograms of $v_U - v_L$. Note that the frequencies of 0 and 5 are different from the ones in Manski and Molinari (2010) for both distributions of $v_U - v_L$ based on questions about personal health and all expectation questions. This is due to a possible mistake in the transformation of the data that satisfies the sixth if condition ("if r_{jm} = (some 1-4 or 96-99) and $v_{jk} \in [0, 5] \cup [95, 100]$, then $[v_{jkL}, v_{jkU}] = v_{jk}$.") in the Manski and Molinari (2010) paper. It appears that the responses that satisfied "r_{jm} = (some 1-4 or 96-99) and $v_{jk} = 5$ or $v_{jk} = 95$" were transformed as "$[v_{jkL}, v_{jkU}] = [v_{jkL} - 2.5, v_{jkU} + 2.5]$" by Manski and Molinari, while the correct way to transform those responses (according to the above algorithm) should be "$[v_{jkL}, v_{jkU}] = v_{jk}$". This has an influence on the results of the empirical analysis (see Section 5) since some of the interval data are not correctly specified. In order to replicate the regression results in Manski and Molinari (2010) paper, I first fit the regression model with the wrongly specified interval data to see whether I obtain the same results and to verify that the method I use is consistent with the one used by Manski and Molinari. Then I use the correctly transformed data set and repeat the analysis to see whether such mistake has a substantial influence on the regression results.

Table 9: Distribution of $v_U - v_L$ in the HRS2006 data

	Based on questions about personal health			Based on all expectation questions		
	Freq.	Percent	Cum.	Freq.	Percent	Cum.
0	189	2.82	2.82	540	8.04	8.04
2.5	306	4.56	7.37	653	9.73	17.77
5	2,637	39.28	46.66	3,991	59.45	77.22
10	2,496	37.18	83.84	1,129	16.82	94.04
25	203	3.02	86.86	45.0	0.67	94.71
50	611	9.10	95.96	84.0	1.25	95.96
100	271	4.04	100.00	271	4.04	100.00
N	6,713			6,713		

Figure 1: Distribution of $v_U - v_L$ in the HRS data

(a) $m = health$

(b) $m = all$

In the next step, I use SHARE2006 and SHARE2014 data set to transform the probabilistic answers to question EX009 into interval data. This question asked respondents their probabilistic expectations of living 10 years more: "What are the chances that you will live to be age [75/80/85/90/95/100/105/110/120] or more?". Although EX009 is not identical to P28, it is the only probabilistic questions asking about life expectancy in both SHARE2006 and SHARE2014 which is reasonably comparable to question P28 in HRS2006. Moreover, if the age of respondents in SHARE2006 data set is restricted to 52 to 64, those respondents actually received question EX009 read as "What are the chances that you will live to be age 75 or more?", and this question is literally the same as P28 in HRS2006. Therefore, in order to make a fair comparison of the rounding behavior between European and American respondents, I select a sub-sample of respondents whose aged between 52 and 64 from SHARE2006, denoted as AGESHARE2006, for further empirical analysis and the cross-sectional comparison with HRS2006. Note that the empirical analysis is also applied to the full sample of SHARE2006 in order to make a longitudinal comparison with SHARE2014.

With respect to the interval data transformation, I use probabilistic questions EX001, EX002, EX003, EX004, EX005, EX006, EX007, EX008, EX009, EX010, EX011 and EX025 in Module EX (Expectations) of SHARE2006 and questions EX001, EX007, EX008, EX009 and EX025 in Module EX (Expectations) of SHARE2014. Due to the small number of probabilistic questions in both SHARE2006 and SHARE2014, I do not separate the above mentioned questions into subgroups. Table 10.A and Table 10.B summarize the frequencies of $v_U - v_L$ taking certain values. It can be concluded that the extent of rounding is smaller than 10% for a large proportion of responses, and this is also reflected in Figure 1a till Figure 2b. However, compared to HRS2006 whose proportion of the extent of rounding smaller than 10% is over 90%, this proportion is much lower in both SHARE2006 (81.18%) and SHARE2014 (77.30%).

Table 10: Distribution of $v_U - v_L$ in the SHARE data

Table 10.A SHARE2006

	Based on all expectation questions		
	Freq.	Percent	Cum.
0	1,103	2.98	2.98
2.5	665	1.80	4.78
5	9,367	25.34	30.13
10	18,863	51.04	81.16
25	916	2.48	83.64
50	1,648	4.46	88.10
100	4,398	11.90	100.00
N	36,960		

Table 10.B SHARE2014

	Based on all expectation questions		
	Freq.	Percent	Cum.
0	2,040	3.13	3.13
2.5	513	0.79	3.92
5	12,101	18.58	22.50
10	35,690	54.80	77.30
25	1,848	2.84	80.14
50	8,844	13.58	93.71
100	4,094	6.29	100.00
N	65,130		

Figure 2: Distribution of $v_U - v_L$ in the SHARE data

(a) SHARE2006

(b) SHARE2014

It may be tempting to conclude that European respondents are more uncertain in reporting probabilities than Americans. However, this may not be true since there were much more expectation questions asked in the HRS2006 questionnaire than in the SHARE2006 (SHARE2014) questionnaire. Therefore, in order to obtain a fair comparison between results derived from HRS2006 and AGESHARE2006 (or SHARE2006 and SHARE2014), I randomly selected 11 (or 4)[4] probabilistic questions from HRS2006 (or SHARE2006) and denote it as RDHRS2006 (or RDSHARE2006).

Due to the averaging of results of all random samples, $v_U - v_L$ becomes continuous and thus the first column of the distribution table is changed to intervals for RDHRS2006 and RDSHARE2006. By comparing Table 11.A and Table 11.B, it can be observed that AGESHARE2006 has larger proportions in both small and large extent of rounding. In other words, compared to RDHRS2006, AGESHARE2006 has a larger proportion of $v_U - v_L$ less than 10 and also a larger proportion of $v_U - v_L$ more than 25. This indicates that European respondents may either have very small or very large extent of rounding probabilities while American respondents are more homogeneous in terms of rounding. Besides, the extent of rounding among European respondents shrinks overtime since the

[4]Note that the transformation target is the answers for EX009 and therefore EX009 is always included in any sub-samples. Thus, the number of random selection is one less than the total number of probabilistic questions.

distribution of $v_U - v_L$ is more concentrated to smaller values in SHARE2014 than in RDSHARE2006 (see Table 10.B and Table 12).

Table 11: Distribution of $v_U - v_L$ in the RDHRS2006 and AGESHARE2006 data, 12 probability questions included

Table 11.A RDHRS2006

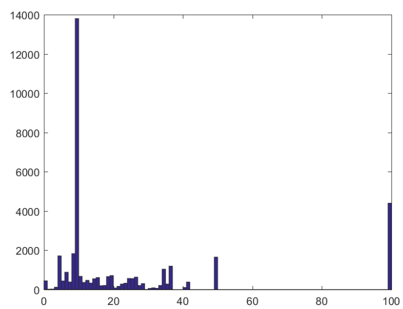
	Based on 12 expectation questions		
	Freq.	Percent	Cum.
0	87	1.296	1.296
(0, 2.5]	153	2.279	3.575
(2.5, 5]	1445	21.525	25.101
(5, 10]	3696	55.057	80.158
(10, 25]	800	11.917	92.075
(25, 50]	261	3.888	95.963
(50, 100]	271	4.037	100.000
N	6,713		

Table 11.B AGESHARE2006

	Based on 12 expectation questions		
	Freq.	Percent	Cum.
0	524	3.077	3.077
2.5	408	2.396	5.473
5	4847	28.465	33.938
10	8717	51.192	85.130
25	400	2.349	87.479
50	531	3.118	90.598
100	1601	9.402	100.000
N	17028		

Figure 3: Distribution of $v_U - v_L$ in the RDSHARE2006 and AGESHARE2006 data

(a) RDSHARE2006



(b) AGESHARE2006

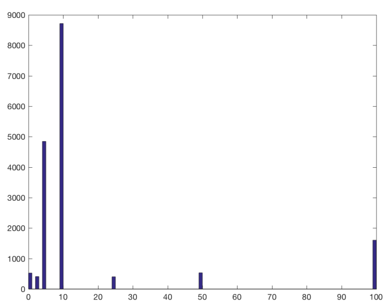
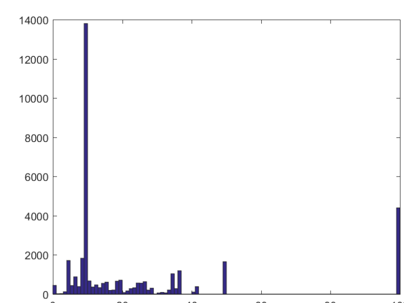


Table 12: Distribution of $v_U - v_L$ in the RDSHARE2006 data, 5 probability questions included

	Based on 5 expectation questions		
	Freq.	Percent	Cum.
0	431	1.166	1.166
(0, 2.5]	42	0.114	1.280
(2.5, 5]	1843	4.986	6.266
(5, 10]	17323	46.870	53.136
(10, 25]	6164	16.677	69.813
(25, 50]	6759	18.287	88.101
(50, 100]	4398	11.899	100.000
N	36,960		

Figure 4: Distribution of $v_U - v_L$ in the RDSHARE2006 data



4.2 Linear regression model for interval data

After formulating the interval data, I regress them on two explanatory variables, Age and Male (a dummy variable indicating whether the respondent is male), as well as a constant in order to obtain interval estimators. The methodological research of regressions with interval data on regressors was conducted by Horowitz and Manski (2000) and Manski and Tamer (2002), and the method they developed was used in the Manski and Molinari (2010) paper. In this section, I first briefly introduce the methods for deriving interval estimators in the context of the best linear prediction (BLP) with square loss. For detailed derivation or proofs for the methods, please refer to the above mentioned papers.

Consider BLP of v given covariates $\mathbf{x}$ using the ordinary least square (OLS). The best linear estimator β is obtained by solving

$$\boldsymbol{\beta} = \arg\min_{\mathbf{b}} E[(v - \mathbf{x}\mathbf{b})^2] \tag{1}$$

And the solution β is well known as:

$$\boldsymbol{\beta} = \left[E\left(\mathbf{x}'\mathbf{x}\right)\right]^{-1} E\left[\mathbf{x}'v\right] \tag{2}$$

However, if v is unobserved, but falls in an interval $[v_L, v_U]$, Beresteanu and Molinari (2008) showed that the best linear estimator β also falls in an identification region (denoted as H[β]):

$$H[\boldsymbol{\beta}] = \left\{\mathbf{b} \in R^k : \mathbf{b} = \left[E\left(\mathbf{x}'\mathbf{x}\right)\right]^{-1} E\left[\mathbf{x}'\tilde{v}\right], \tilde{v} \in [v_L, v_U] \text{ with probability } 1\right\} \tag{3}$$

And the identification regions for $BLP(v|\mathbf{x})$ is:

$$H[BLP(v|\mathbf{x})] = \{\mathbf{x}\mathbf{b}, \mathbf{b} \in H[\boldsymbol{\beta}]\} \tag{4}$$

In the context of BLP with OLS, Beresteanu and Molinari (2008, corollary 4.5) proved that

$$\hat{H}_n[\beta_d] = [\hat{\beta}_{dL}, \hat{\beta}_{dU}] = \frac{1}{\sum_{i=1}^{n} \tilde{x}_{id}^2} \left[\sum_{i=1}^{n} \min\left\{\tilde{x}_{id} v_{iL}, \tilde{x}_{id} v_{iU}\right\}, \sum_{i=1}^{n} \max\left\{\tilde{x}_{id} v_{iL}, \tilde{x}_{id} v_{iU}\right\}\right] \tag{5}$$

where $\tilde{x}_{id}$ is the residual obtained after regressing x_d on other covariates in $\mathbf{x}$. Similarly, the estimated identification regions for $BLP(v|\mathbf{x})$ can be derived as

$$\begin{aligned} \hat{H}_n[BLP(v|\mathbf{x} = \mathbf{x_0})] =& [\bar{v}_{nL|\mathbf{x}}, \bar{v}_{nU|\mathbf{x}}] \\ =& \left[\frac{1}{n}\sum_{i=1}^{n}\left(\min\left(\mathbf{x_0}\hat{\Sigma}_n^{-1}\mathbf{x}' v_{iL}, \mathbf{x_0}\hat{\Sigma}_n^{-1}\mathbf{x}' v_{iU}\right)\right),\right. \\ & \left.\frac{1}{n}\sum_{i=1}^{n}\left(\max\left(\mathbf{x_0}\hat{\Sigma}_n^{-1}\mathbf{x}' v_{iL}, \mathbf{x_0}\hat{\Sigma}_n^{-1}\mathbf{x}' v_{iU}\right)\right)\right] \end{aligned} \tag{6}$$

The construction of confidence interval is also discussed by recent literature (Imbens and Manski, 2004; Stoye, 2009; Horowitz and Manksi, 2000; Chernozhukov, Hong and Tamer, 2007 etc.) Following Manski and Molinari (2010), I report the Imbens-Manski confidence interval that covers the whole identification regions with at least 95% probability. The object of interest is the bound in Equation 5, and the Imbens-Manski confidence interval (Imbens and Manski, 2004)is defined as:

$$\overline{CI}_\alpha = \left[\hat{\beta}_{dL} - \overline{C}_N \cdot \hat{\sigma}_L/\sqrt{N}, \hat{\beta}_{dU} + \overline{C}_N \cdot \hat{\sigma}_U/\sqrt{N}\right] \tag{7}$$

where $\overline{CI}_\alpha$ satisfies:

$$\Phi\left(\overline{C}_N + \sqrt{N} \cdot \frac{\hat{\Delta}}{\max(\hat{\sigma}_L, \hat{\sigma}_U)}\right) - \Phi(-\overline{C}_N) = \alpha \tag{8}$$

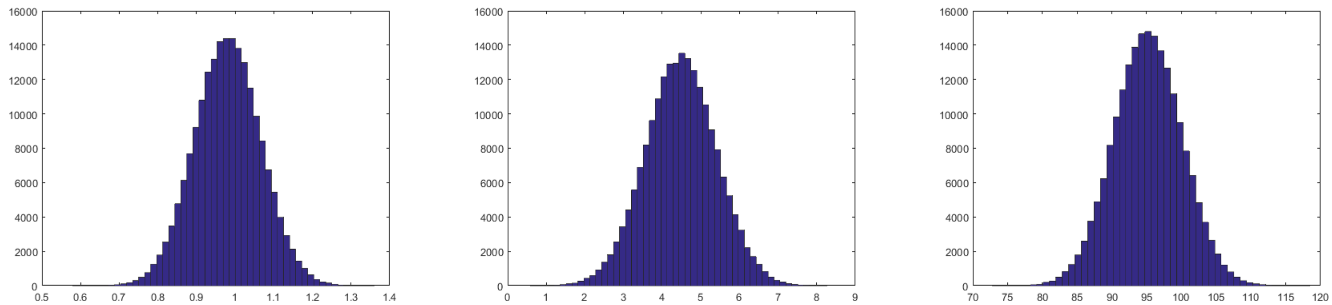
and $\hat{\sigma}_L$ and $\hat{\sigma}_U$ are the consistent estimations of the variance of $\hat{\beta}_{dL}$ and $\hat{\beta}_{dU}$, assuming that:

$$\sqrt{N}\begin{pmatrix}\hat{\beta}_{dL} - \beta_{dL} \\ \hat{\beta}_{dU} - \beta_{dU}\end{pmatrix} \xrightarrow{d} \mathcal{N}\left(\begin{pmatrix}0 \\ 0\end{pmatrix}, \begin{pmatrix}\sigma_L^2 & \rho\sigma_L\sigma_U \\ \rho\sigma_L\sigma_U & \sigma_U^2\end{pmatrix}\right) \tag{9}$$

Manski and Molinari mentioned in the note of the Table 4 that they used bootstrap to obtain the 95% Imbens-Manski confidence sets. However, they did not record explicitly how they performed bootstrapping in the paper. Therefore, I construct the bootstrap method myself, and use simple pair-wise bootstrapping by re-sampling from the original sample and then re-estimate β_{dL} and β_{dU}. Note that the re-sample size is the same as the size of the original sample. Then I repeat the process of re-sampling and re-estimation for 200,000 times in order to obtain the estimated distribution as well as the estimated variance of β. Figure 5a, Figure 5b and Figure 5c show three examples of bootstrapped distribution of β from which it can be verified that the assumed normal distribution Equation 9 is reasonably valid.

Figure 5: Simulated distribution of the coefficient UB for Age, Male and Constant in Table 13.D panel (B)

(a) Age (b) Male (c) Constant



Both HRS and SHARE questionnaires used several sampling methods, such as re-sampling and minority oversampling when administrating questionnaires. Therefore, in order to deduct conclusions for the entire target population, it is important to take the sampling weight in to account when conducting empirical analysis. In this paper, I use respondent weight “kwgtr” from HRS2006, calibrated cross-sectional individual weight “cciw_w2” from SHARE2006 and calibrated cross-sectional individual weight “cciw_w6” from SHARE2014. Since Manski and Molinari did not mention whether they incorporated sampling weights or not, I repeat the regression analysis with and without sampling weights for HRS2006 and then make a inference of whether sampling weight was used in their model after comparing the regression results.

5 Regression results of interval probability responses

5.1 Estimation of BLP parameters

In this section, the estimations of BLP parameters using five different data sets[5] are reported from Table 13.A to Table 15.B. The basic structure of those tables is as follows:

The column labeled as “Point estimates” presents the OLS results of taking the probabilistic responses at face value. Panel A-E reports the upper and lower bounds on each of the three variables (i.e. Age, Male and Constant). The data used for panel A-E are as follows:

- Panel A: $m = health$; all respondents included.
- Panel B: $m = all$; all respondents included.
- Panel C: $m = all$; respondents in (all NR) are dropped.
- Panel D: $m = all$; respondents in (all NR) and (all 0 or 100) are dropped.
- Panel E: $m = all$; respondents in (all NR), (all 0 or 100) and (all 0, 50 or 100) are dropped.

Then I make a cross-sectional comparison between the estimations of BLP parameters derived from RDHRS2006 and AGESHARE2006 as well as a longitudinal comparison between the BLP parameters derived from RDSHARE2006 and SHARE2014.

5.1.1 Replicated results of Table 4 in Manski and Molinari (2010) paper

As mentioned before, it is not clear whether Manski and Molinari incorporated sampling weight when conducting empirical analysis and there may exist an error in interval data transformation in their paper. Therefore, in order to verify the application of the linear regression model for interval data based on the replicated results, I control the above mentioned factors (i.e. sampling weight and transformation error) and conduct four interval data linear regressions (see Table 13.B till Table 13.C).

[5]The five data sets are: HRS2006, RDHRS2006, AGESHARE2006, RDSHARE2006 and SHARE2014.

After comparing Table 13.Aand Table 13.C (or Table 13.B and Table 13.D), it can be observed that the differences of the estimated lower bounds (upper bounds) in each panel are less than 1%. This shows that the transformation error has little influence on the estimation of BLP parameters. However, the sampling weights indeed modify the estimation to a large extent, especially the estimation of the parameters for “Age” and “Constant”. This is due to the design of the sampling weight in both SHARE and HRS such that zero weights are assigned to the age ineligible respondents (i.e. respondents younger than 50 in SHARE and younger than 51 in HRS). The results in the Table 4 of the Manski and Molinari (2010) paper are much closer to the replicated results derived from linear regression in which sampling weights are not incorporated, and therefore it can be reasonable elicited that Manskin and Molinari did not take sampling weights into account when conducting their empirical analysis. However, since both HRS and SHARE used rather complicated sampling methods, it is necessary to incorporate sampling weights in the model in order to extrapolate the conclusion to the whole population. Therefore, in the following sections, I only report the results with sampling weights incorporated.

Table 13: Replicated Table 4 in Manski and Molinari (2010) paper

Table 13.A Using “wrongly” transformed HRS2006 data, without sampling weights.

		Set estimates**									
	Point estimates* MCAR imposed	(A)		(B)		(C)		(D)		(E)	
		LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
Age	0.2510	-0.9758	1.3745	-0.5467	0.9834	-0.2247	0.6879	-0.2100	0.6459	-0.1904	0.6155
	(0.1129, 0.3891)	(-1.0877, 1.4881)		(-0.6637, 1.0992)		(-0.3386, 0.8017)		(-0.3246, 0.7595)		(-0.3049, 0.7291)	
Male	-5.3976	-19.6445	10.1392	-14.6026	4.5236	-11.3769	0.5198	-11.1408	-0.0064	-10.8172	-0.3446
	(-6.8757, -3.9195)	(-20.8237, 11.3215)		(-15.8376, 5.7657)		(-12.6013, 1.7431)		(-12.3666, 1.2151)		(-12.0454, 0.8806)	
Const.	50.9895	-13.8222	119.3865	8.7546	95.3188	26.0611	77.8889	28.6679	77.1855	30.5638	76.2179
	(43.1232, 58.8558)	(-20.3831, 125.7179)		(2.0973, 101.9594)		(19.5553, 84.3776)		(22.1809, 83.7157)		(24.0752, 82.7529)	
N	6,442	6,713		6,713		6,442		6,385		6,313	

Table 13.B Using “wrongly” transformed HRS2006 data, with sampling weights.

		Set estimates**									
	Point estimates* MCAR imposed	(A)		(B)		(C)		(D)		(E)	
		LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
Age	0.1767	-1.5576	1.8364	-0.9188	1.2182	-0.5269	0.8692	-0.5014	0.8428	-0.4631	0.8089
	(-0.0579, 0.4113)	(-1.6936, 1.9748)		(-1.0608, 1.3588)		(-0.6652, 1.0073)		(-0.6405, 0.9806)		(-0.6019, 0.9466)	
Male	-5.3513	-18.6141	9.2246	-13.7897	3.7645	-11.1515	0.5075	-10.9566	0.1778	-10.6268	-0.0955
	(-7.0853, -3.6173)	(-20.0471, 10.6596)		(-15.2916, 5.2770)		(-12.6362, 1.9905)		(-12.4409, 1.6571)		(-12.1132, 1.3872)	
Const.	55.3718	-41.9636	154.9295	-5.7212	118.1407	14.9946	96.0939	16.6550	94.6650	18.7884	92.6054
	(4.7445, 68.9991)	(-49.9570, 162.6243)		(-13.7989, 126.2038)		(7.1003, 103.9695)		(8.7855, 102.5879)		(10.9277, 100.5211)	
N	5,550	5,779		5,779		5,550		5,507		5,447	

Table 13.C Using “correctly” transformed HRS2006 data, without sampling weights.

		Set estimates**									
	Point estimates* MCAR imposed	(A)		(B)		(C)		(D)		(E)	
		LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
Age	0.2510	-0.9743	1.3730	-0.5435	0.9802	-0.2213	0.6846	-0.2066	0.6425	-0.1870	0.6120
	(0.1129, 0.3891)	(-1.0860, 1.4864)		(-0.6605, 1.0958)		(-0.3355, 0.7983)		(-0.3210, 0.7559)		(-0.3018, 0.7258)	
Male	-5.3976	-19.6259	10.1206	-14.5657	4.4867	-11.3385	0.4813	-11.1020	-0.0452	-10.7779	-0.3838
	(-6.8757, -3.9195)	(-20.8089, 11.3048)		(-15.7993, 5.7278)		(-12.5640, 1.7063)		(-12.3302, 1.1787)		(-12.0099, 0.8452)	
Const.	50.9895	-13.7361	119.3003	8.9357	95.1377	26.2514	77.6986	28.8600	76.9935	30.7592	76.0225
	(43.1232, 58.8558)	(-20.2862, 125.6184)		(2.2889, 101.7727)		(19,7503, 84.2011)		(22.3870, 83.5152)		(24.2638, 82.5731)	
N	6,442	6,713		6,713		6,442		6,385		6,313	

Table 13.D Using “correctly” transformed HRS2006 data, with sampling weights.

		Set estimates**									
	Point estimates* MCAR imposed	(A)		(B)		(C)		(D)		(E)	
		LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
Age	0.1767	-1.5552	1.8340	-0.9136	1.2130	-0.5215	0.8639	-0.4961	0.8374	-0.4577	0.8035
	(-0.0579, 0.4113)	(-1.6909, 1.9719)		(-1.0555, 1.3534)		(-0.6597, 1.0017)		(-0.6347, 0.9749)		(-0.5964, 0.9410)	
Male	-5.3513	-18.5956	9.2061	-13.7501	3.7248	-11.1105	0.4665	-10.9153	0.1365	-10.5851	-0.1371
	(-7.0853, -3.6173)	(-20.0304, 10.6417)		(-15.2468, 5.2308)		(-12.5943, 1.9496)		(-12.4011, 1.6167)		(-12.0779, 1.3520)	
Const.	55.3718	-41.8237	154.7897	-5.4235	117.8430	15.3022	95.7863	16.9640	94.3560	19.1005	92.2932
	(4.7445, 68.9991)	(-49.7826, 162.4675)		(-13.4871, 125.8959)		(7.4269, 103.6567)		(9.1161, 102.2603)		(11.2515, 100.2071)	
N	5,550	5,779		5,779		5,550		5,507		5,447	

5.1.2 Comparison of BLP parameter estimations between RDHRS2006 and AGESHARE2006

Table 14.A and Table 14.B report the estimation of BLP parameters using RDHRS2006 and AGESHARE2006 data sets. Judging from the point estimates, age has a positive influence on the expected chance of living over 75 among both European and American respondents. Males tend to give lower expected chances of living regardless of the nationality. The set estimates show wider ranges of BLP parameters in AGESHARE2006 than in RDHRS2006 regardless the treatment of dropping specific responses ((All NR), (All 0 or 100) and (All 0, 50 or 100)). While the interval estimates for "Age" in both RDHRS2006 and AGESHARE2006 are approximately in the same range, the interval estimates for "Male" are more inclined to be negative in RDHRS2006 than in AGESHARE2006. This is consistent with the point estimate in which the coefficient of "Male" has a larger magnitude in RDHRS2006.

Table 14: Point estimator and set estimates conditioning on age and gender, using RDHRS2006 and AGESHARE2006 data

Table 14.A Using RDHRS2006 data

	Point estimates* MCAR imposed	Set estimates** (A) LB	(A) UB	(B) LB	(B) UB	(C) LB	(C) UB	(D) LB	(D) UB	(E) LB	(E) UB
Age	0.1767	-	-	-1.2214	1.5043	-0.8407	1.1654	-0.8146	1.1388	-0.7733	1.1031
	(-0.0579, 0.4113)	-	-	(-1.3614, 1.6425)		(-0.9766, 1.3010)		(-0.9509, 1.2738)		(-0.9095, 1.2384)	
Male	-5.3513	-	-	-16.1987	6.2205	-13.6385	3.0477	-13.4449	2.7185	-13.1153	2.4126
	(-7.0853, -3.6173)	-	-	(-17.6610, 7.6944)		(-15.0833, 4.4907)		(-14.8943, 4.1637)		(-14.5682, 3.8633)	
Const.	55.3718	-	-	-22.3613	163.4113	-2.2268	114.0561	-0.5550	112.5894	1.6711	110.3565
	(4.7445, 68.9991)	-	-	(-30.3217, 143.4140)		(-9.9923, 121,7930)		(-8.2803, 120.3542)		(-6.0665, 118.1214)	
N	5,550	-	-	5,779		5,550		5,507		5,447	

Table 14.B Using AGESHARE2006 data.

	Point estimates* MCAR imposed	Set estimates** (A) LB	(A) UB	(B) LB	(B) UB	(C) LB	(C) UB	(D) LB	(D) UB	(E) LB	(E) UB
Age	0.0811	-	-	-2.0001	2.0986	-1.1142	1.2132	-1.0554	1.1583	-0.8652	0.9694
	(-0.0322, 0.1944)	-	-	(-2.1451, 2.2401)		(-1,2490, 1.3478)		(-1.1911, 1.2935)		(-1.0035, 1.1079)	
Male	-0.7905	-	-	-18.0466	16.7001	-10.5904	9.0848	-10.0372	8.6467	-8.5379	6.9605
	(-1.6137, 0.0327)	-	-	(-19.0949, 17.7431)		(-11.5803, 10.0713)		(-11.0336, 9.6407)		(-9.5568, 7.9765)	
Const.	64.7087	-	-	-54.3378	182.6902	-1.4858	132.9301	1.6484	129.5515	12.8357	118.9146
	(58.1412, 71.2763)	-	-	(-62.6633, 191.0178)		(-9.3331, 140.7486)		(-6.2389, 137.4207)		(4.7725, 126.9404)	
N	15,403	-	-	16,983		15,403		15,221		14,475	

5.1.3 Comparison of BLP parameter estimations between RDSHARE2006 and SHARE2014

Table 15.A and Table 15.B present the point estimation and the set estimation of BLP parameters derived from RDSHARE2006 and SHARE2014 data sets. Unlike the results in Section 5.1.2, the point estimates in Table 15.A and Table 15.B show that "Age" has a negative influence on the expected chances of living 10 years more while males tend to give higher expected chances of living, no matter whether the respondents are Europeans or Americans. Moreover, the negative influence of "Age" is reflected in the set estimates when responses of (All NR), (All 0 or 100), (All 0, 50 or 100) are dropped. This seemingly contradicting results are very likely to be caused by the sample selection of different age group. In Section 5.1.2, the sample of respondents are aged between 52 and 64 while the results in Table 15.A and Table 15.B are obtained from respondents aged between 50 and 100. Therefore, the contradicting results actually reveal the changing influence of age and gender on expected probability of living when respondents grow older. When respondents are in the early age of retirement (such as younger than 65), they are more optimistic about their chances of living longer as they become older, and during this period, females are more optimistic than males in terms of expecting chances of living. However, as they become older, respondents are more pessimistic and give lower expected chances of living 10 years more, and compared to males, females are more negative about the chances that they would live long. Besides, the widths of interval estimates in RDSHARE2006 are wider than the ones in SHARE2014, which is consistent with the previous results in Section 4.1 that the extent of rounding among European respondents shrinks overtime.

Table 15: Point estimator and set estimates conditioning on age and gender, using SHARE2014 and RDSHARE2006 data

Table 15.A Using RDSHARE2006 data

	Point estimates* MCAR imposed	Set estimates**									
		(A)		(B)		(C)		(D)		(E)	
		LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
Age	-1.0802	-	-	-1.9409	0.2662	-1.6420	-0.2655	-1.6097	-0.2931	-1.5446	-0.4024
	(-1.1321, 1.7342)	-	-	(-1.9757, 0.3061)		(-1.6788, -0.2278)		(-1.6470, -0.2547)		(-1.5838, -0.3626)	
Male	0.7771	-	-	-25.2098	25.8676	-15.9538	16.9100	-15.2754	16.2197	-13.3507	14.0309
	(-0.1799, 1.7342)	-	-	(-25.9666, 26.6094)		(-16.6782, 17.6345)		(-16.0048, 16.9488)		(-14.1-93, 14.7886)	
Const.	129.3290	-	-	39.2816	185.5291	74.6071	166.3796	76.4184	164.3066	83.7888	160.0870
	(125.9866, 132.6706)	-	-	(36.6955, 187.8041)		(72.2032, 168.7582)		(73.9799, 166.7069)		(81.2710, 162.5923)	
N	31,519	-	-	35,774		31,519		30,949		29,003	

Table 15.B Using SHARE2014 data

	Point estimates* MCAR imposed	Set estimates**									
		(A)		(B)		(C)		(D)		(E)	
		LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
Age	-0.8633	-	-	-1.5306	0.0331	-1.3620	-0.2159	-1.2791	-0.3034	-1.1567	-0.4798
	(-0.9206, -0.8061)	-	-	(-1.5686, 0.0746)		(-1.4004, -0.1762)		(-1.3181, -0.2628)		(-1.1991, -0.4368)	
Male	0.9825	-	-	-18.1858	20.0704	-13.4808	15.3550	-11.3171	13.0746	-7.7504	9.2167
	(-0.0166, 1.9816)	-	-	(-18.9427, 20.8166)		(-14.2124, 16.0820)		(-12.0690, 13.8195)		(-8.5410, 10.0053)	
Const.	119.9816	-	-	57.8626	163.4113	74.7978	152.9177	80.7329	146.8844	93.4876	139.4853
	(116.0017, 123.9614)	-	-	(55.1143, 165.9323)		(72.1931, 155.4544)		(78.0588, 149.4578)		(90.6920, 142.2734)	
N	59,913	-	-	63,959		59,913		56,124		49,365	

5.2 Parametric and non-parametric prediction of expectations

In this section, I apply the parametric and non-parametric prediction methods described in the Manski and Molinari (2010) paper to the same five data sets used in Section 5.1. Data set HRS2006 is used to get the replicated prediction results, and the other four data sets are used for cross-sectional and longitudinal comparison of the predicted expectations on chances of living conditioning on age and gender. As explained in the Manski and Molinari (2010) paper, due to the joint identification issue of the BLP parameters, the feasible values for all BLP parameters cannot be obtained from the results of the previous empirical analysis. Therefore, the parametric estimation is obtained from Equation 6. The non-parametric results are obtained using simple cell means given certain age. For simplicity, I only report the predictions using all questions in both parametric and non-parametric replication and comparisons for males. The predictions (for males) with certain dropped responses (i.e. (All NR), (All 0 or 100) and (All 0, 50 or 100)) can be found in Appendix B.1 B.2 and B.3.

5.2.1 Replicated prediction of expectations using HRS2006 data

Figure 6: v_L and v_U Constructed Using All Questions in HRS2006 data

(a) Non-parametric Predictions

(b) Parametric Predictions

Figure 6a and Figure 6b present the replicated Figure 1 in the Manski and Molinari (2010) paper. The set estimates are marked as light blue and 95% confidence sets correspond to dark blue area. The replicated prediction of expectations are the same as the original results in the Manski and Molinari (2010) paper.

5.2.2 Comparision of prediction of expectations between RDHRS2006 and AGESHARE2006 data

Next, I make a cross-sectional comparison of the prediction of expectations among American respondents (using RDHRS2006 data) and European respondents (using AGESHARE2006 data).

Figure 7: Non-parametric Predictions of Expectations

(a) Using All Questions in RDHRS2006 data (b) Using All Questions in AGESHARE2006 data

Figure 7a and Figure 7b present the non-parametric (simple cell means) set estimates and 95% confidence sets using RDHRS2006 and AGESHARE2006 data. The set estimates take a wider range and relatively larger values in AGESHARE2006. This shows that European respondents tend to have higher predicted subjective probability of living over 75, but such predictions are more volatile than the ones among American respondents.

Figure 8: Parametric Predictions of Expectations

(a) Using All Questions in RDHRS2006 data (b) All Questions in AGESHARE2006 data

Similar to the non-parametric prediction, the parametric prediction (shown in Figure 8a and Figure 8b) on subjective probability of living over 75 has larger value and wider range in AGESHARE2006 than in RDHRS2006. Note that the above two figure narrow towards different mean ages and then spread again, which is a special property of the BLP identification region (Beresteanu and Molinari, 2008).

5.2.3 Comparision of prediction of expectations between RDSHARE2006 and SHARE2014 data

Figure 9: Non-parametric Predictions of Expectations

(a) Using All Questions in RDSHARE2006 data (b) Using All Questions in SHARE2014 data

Figure 9a and Figure 9b illustrate the set estimates and 95% confidence sets in RDSHARE2006 and SHARE2014 data. Compared to RDSHARE2006, the set estimates in SHARE2014 shifts up to higher predicted subjective probabilities and takes a narrower range across different ages. This is consistent with the previous finding that European respondents develop a more precise interpretation of probabilities overtime and their subjective life expectancy is increasing. It is also interesting to note that the prediction of expected chances of living is rather stable when responses are 65 or younger, but it drops dramatically after age 65. This is consistent with the conflicting effects of age and gender on subjective probability of living discussed in Section 5.1.3.

Figure 10: Parametric Predictions of Expectations

(a) Using All Questions in RDSHARE2006 data (b) Using All Questions in SHARE2014 data

Similar results can be obtained from parametric predictions (shown in Figure 10a and Figure 10b) as from non-parametric predictions. Note that the lower bound of parametric predictions drifts faster than the upper bound after the mean age and it is truncated in Figure 10a as the linear regression model specified in Section 4.2 does not impose any restrictions on prediction of expectations.

6 Conclusions

In this paper, I investigate the subjective probabilities of living among U.S. and European respondents using HRS2006, SHARE2006 and SHARE2014 data sets. Following Manski and Molinari (2010), I first examine the patterns of the responses to probabilistic questions, compare the extent of rounding between U.S and European respondents and then examine the extent of rounding among European respondents overtime. Later I conduct empirical analysis after transforming the reported probabilities of living till certain ages into interval data and investigate the impact of age and gender on respondents' subjective probabilities of living. In the end, I make both parametric and non-parametric predictions of expectations, compare the predicted expectations between U.S. and European respondents and also investigate the predicted expectations of European respondents overtime. The results show that European respondents tend to have a wider extent of rounding than American respondents, and the extent of rounding among European respondents decreases overtime. This suggests that European respondents may have less precise understanding on probabilities than American respondents. But their understanding is improving overtime. One explanation of such improvement is that European respondents are encouraged to think in probability when answering SHARE questionnaires, and they are more used to answering probabilistic questions as the SHARE questionnaire distributed overtime. Beside, it is also found that European respondents are more optimistic about their life expectancy than American respondents. European respondents stated a higher expectation of chance of living above age 75 than American respondents did. In the meantime, European respondents are more optimistic about their life expectancy in 2014 compared to 2006 (with a higher estimated expected probability of living 10 years more in 2014 compared to the one in 2006). The predicted expectations of living is consistent with the life expectancy in Europe and U.S. According to the World Bank (Data.worldbank.org, 2017), in 2006, the life expectancy of European Union is 78.968 and the life expectancy of U.S. is 77.688. It increases to 80.922 for European Union in 2014. This result is also consistent with the conclusions from previous literature (e.g Hurd and McGarry, 1995; Hurd, 2009) that subjective probabilities are reasonably representative to the actually population probabilities and they have substantial prediction power in inferring life expediencies. Thus, the respondents' subjective probabilities, after corrected for rounding inaccuracies, can be of great potential in modeling economic phenomenons (such as differences in life expectancy, and changing retirement age etc.).

There are a few points that future researches could continue investigating on. This paper only concludes that European respondents have lager extent of rounding than Americans but it does not say anything about the significance level of such difference. Future researches could test whether such difference is significant and whether the difference changes overtime. Same holds for the difference of extent of rounding among European respondents in SHARE2006 and SHARE2014 data sets. Besides, the graph of non-parametric predictions show that there may exist a structural change of the impact of age on the chances of living, and future researches could determine the point where such change happens. Moreover, investigations could be done in examining the changing rounding patterns of the responses of U.S. (European) residents using more waves of HRS (SHARE) data sets.

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Appendix A Descriptions of Relevant Questions

A.1 Questions Descriptions in Module P HRS2006

Table 16: Questions Descriptions in Module P HRS2006, 38 Questions Included

Questions	Descriptions
P004	What do you think are the chances that your income will keep up with the cost of living for the next five years?
P005	Including property and other valuables that you might own, what are the chances that you (and your [husband/wife/partner]) will leave an inheritance totalling $10,000 or more?
P006	What are the chances that you [and your] [you/husband/wife/partner] will leave an inheritance totalling $100,000 or more?
P059	What are the chances that you [and your] [you/husband/wife/partner] will leave an inheritance totalling $500,000 or more?
P007	What are the chances that you [and your] [you/husband/wife/partner] will leave any inheritance?
P008	What are the chances that you [or your] [you/husband/wife/partner] will receive an inheritance during the next 10 years?
P014	What are the chances that you will lose your job during the next year?
P015	Suppose you were to lose your job this month. What do you think are the chances that you could find an equally good job in the same line of work within the next few months?
P016	What are the chances that you will be working for pay at some time in the future?
P017	[Thinking about work in general and not just your present job, what/What] do you think the chances are that you will be working full-time after you reach age 62?
P018	And what about the chances that you will be working full-time after you reach age 65?
P020	On this 0 to 100 scale, what are the chances that you will find a job like the one you're looking for within the next few months?
P028	What is the percent chance that you will live to be 75 or more?
P103	Assuming that you are still living at 75, what are the chances that your health will allow you to live independently, that is, to live at home without help and to manage your own affairs?
P104	Assuming that you are still living at 75, what are the chances that you will be free of serious problems in thinking, reasoning or remembering things that would interfere with your ability to manage your own affairs?
P029	What is the percent chance that you will live to be [80/85/90/95/100] or more?
P106	Assuming that you are still living at [85/90/95/100] what are the chances that your health will allow you to live independently, that is, to live at home without help and manage your own affairs?
P107	Assuming that you are still living at [85/90/95/100] what are the chances that you will be free of serious problems in thinking, reasoning or remembering things that would interfere with your ability to manage your own affairs?
P108	What are the chances that your health will [still be excellent/still be very good or better/still be good or better/still be fair or better/have improved significantly] four years from now?
P109	What are the chances that your health will [have declined to very good or worse/have declined to good or worse/have declined to fair or poor/have declined to poor/have improved significantly] four years from now?
P070	What do you think are the chances that medical expenses will use up all your [and your you/husband/wife/partner's/] savings in the next five years?
P030	(Using a number from 0-100) What are the chances that you [and your] [you/husband/wife/partner] will give financial help totalling $5,000 or more to grown children, relatives or friends over the next ten years?
P071	What are the chances that you [and your] [you/husband/wife/partner] will give financial help totalling $1,000 or more to grown children, relatives or friends over the next ten years?
P072	What are the chances that you [and your] [you/husband/wife/partner] will give financial help totalling $10,000 or more to grown children, relatives or friends over the next ten years?
P073	What are the chances that you [and your] [you/husband/wife/partner] will give financial help totalling $20,000 or more to grown children, relatives or friends over the next ten years?

Table 16: Questions Descriptions in Module P HRS2006, 38 Questions Included

Questions	Descriptions
P031	What are the chances that you [and your] [you/husband/wife/partner] will receive financial help totalling \$5,000 or more from your children, relatives or friends over the next 10 years?
P074	What are the chances that you [and your] [you/husband/wife/partner] will receive financial help totalling \$2,500 or more from your children, relatives or friends over the next ten years?
P075	What are the chances that you [and your] [you/husband/wife/partner] will receive financial help totalling \$1,000 or more from your children, relatives or friends over the next ten years?
P076	What are the chances that you [and your] [you/husband/wife/partner] will receive financial help totalling \$10,000 or more from your children, relatives or friends over the next ten years?
P032	[(What is the percent chance) that you will ever have to move to a nursing home?/(What is the percent chance) that you will move to a nursing home in the next five years?]
P034	What do you think are the chances that the U.S.economy will experience a major depression sometime during the next 10 years or so?
P110	Thinking of the Social Security program in general and not just your own Social Security benefits: On a scale from 0 to 100, (where 0 means no chance and 100 means absolutely certain,) what is the percent chance that Congress will change Social Security sometime in the next 10 years, so that it becomes less generous than now?
P111	On a scale from 0 to 100, (where 0 means no chance and 100 means absolutely certain,) what do you think is the percent chance that the benefits you yourself are receiving from Social Security will be cut some time over the next 10 years?
P112	On a scale from 0 to 100, what do you think is the percent chance that over the next 10 years there will be changes to Social Security that will reduce your future benefits compared to what you would get under the current system?
P047	By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?
P114	What are the chances the price of these mutual fund shares will increase faster than the cost of living over the next 10 years?
P115	Over the next 10 years what are the chances the price of these mutual fund shares will increase by 8 percent or more per year on average?
P116	Over the next 10 years what are the chances the cost of living will increase by 5 percent or more per year on average?

A.2 Questions Descriptions in Module EX SHARE2006

Table 17: Questions Descriptions in Module EX SHARE2006, 12 Questions Included

Questions	Descriptions
EX001	What do you think the chances are that it will be sunny tomorrow? For example, '90' would mean a 90 per cent chance of sunny weather. You can say any number from 0 to 100.
EX002	Thinking about the next ten years, what are the chances that you will receive any inheritance, including property and other valuables?
EX003	Within the next ten years, what are the chances that you will receive an inheritance worth more than 50,000 [local currency]?
EX004	Not only thinking about the next 10 years, including property and other valuables, what are the chances that you [or/or/or/or/empty/empty] [your/your/your/your/empty/empty] [husband/wife/partner/partner/empty/empty] will leave an inheritance totaling 50,000 [local currency] or more?
EX005	What are the chances that you [or/or/or/or/empty/empty] [your/your/your/your/empty/empty] [husband/wife/partner/partner/empty/empty] will leave any inheritance?
EX006	What are the chances that you [or/or/or/or/empty/empty] [your/your/your/your/empty/empty] [husband/wife/partner/partner/empty/empty] will leave an inheritance totaling 150,000 [local currency] or more?
EX007	What are the chances that before you retire the government will reduce the pension which you are entitled to?

Table 17: Questions Descriptions in Module EX SHARE2006, 12 Questions Included

Questions	Descriptions
EX025	Thinking about your work generally and not just your present job, what are the chances that you will be working full-time after you reach age 63?
EX008	What are the chances that before you retire the government will raise your retirement age?
EX009	What are the chances that you will live to be age [75/80/85/90/95/100/105/110/120] or more?
EX010	What are the chances that five years from now your standard of living will be better than today?
EX011	And what are the chances that five years from now your standard of living will be worse than today?

A.3 Questions Descriptions in Module EX SHARE2014

Table 18: Questions Descriptions in Module EX SHARE2014, 5 Questions Included

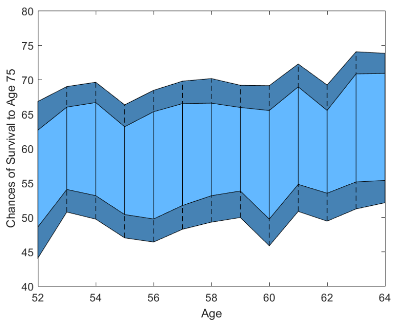
Questions	Descriptions
EX001	What do you think the chances are that it will be sunny tomorrow? For example, '90' would mean a 90 per cent chance of sunny weather. You can say any number from 0 to 100.
EX007	What are the chances that before you retire the government will reduce the pension which you are entitled to?
EX025	Thinking about your work generally and not just your present job, what are the chances that you will be working full-time after you reach age 63?
EX008	What are the chances that before you retire the government will raise your retirement age?
EX009	What are the chances that you will live to be age [Current age rounded up to 5 fold] or more?

Appendix B Parametric and Non-parametric Predictions of Expectations

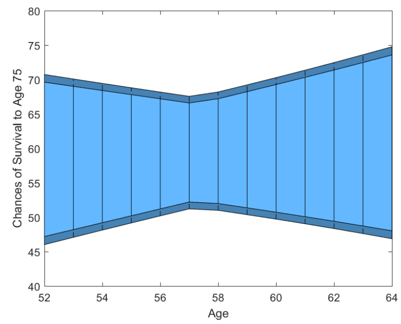
B.1 Replicated prediction of expectations using HRS2006 data set

Figure 11: Parametric (right) and Non-parametric (left) Prediction of Expectations

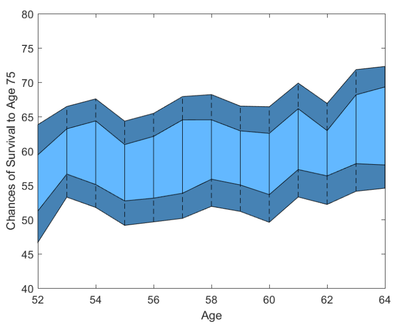
(a) v_L and v_U Constructed Using Only Personal Health Questions



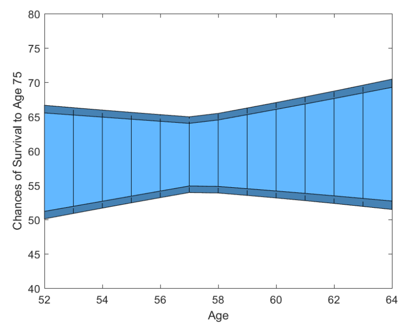
(b) v_L and v_U Constructed Using Only Personal Health Questions



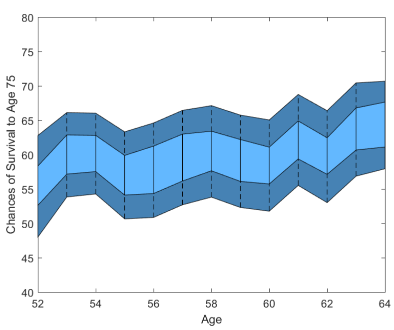
(c) v_L and v_U Constructed Using All Questions



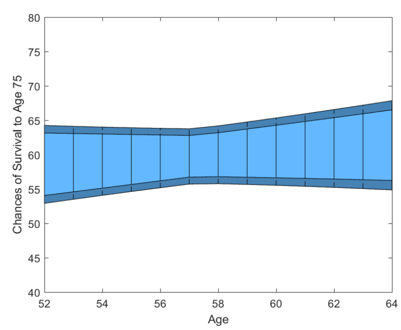
(d) v_L and v_U Constructed Using All Questions



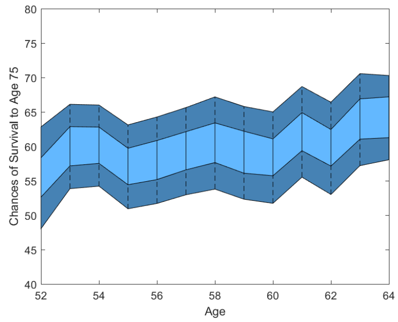
(e) v_L and v_U Constructed Using All Questions, Excluding r_{jm} = NR



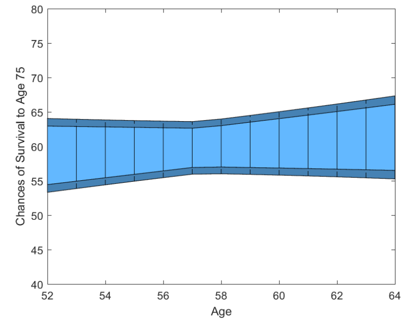
(f) v_L and v_U Constructed Using All Questions, Excluding r_{jm} = NR



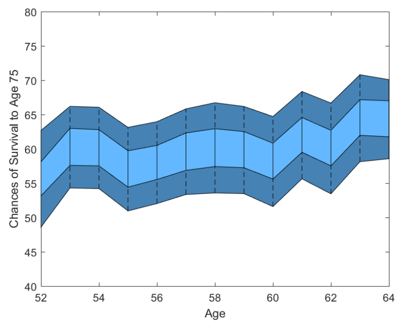
(g) v_L and v_U Constructed Using All Questions, Excluding r_{jm} = NR or (all 0 or 100)



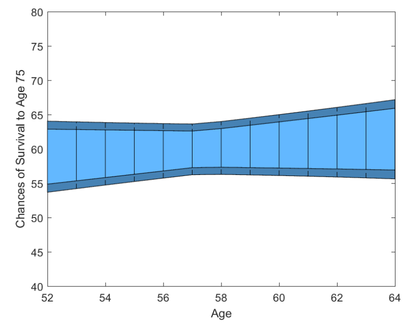
(h) v_L and v_U Constructed Using All Questions, Excluding r_{jm} = NR or (all 0 or 100)



(i) v_L and v_U Constructed Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100)



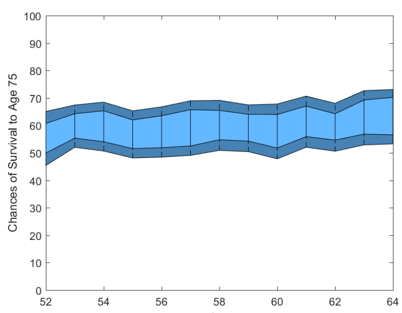
(j) v_L and v_U Constructed Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100)



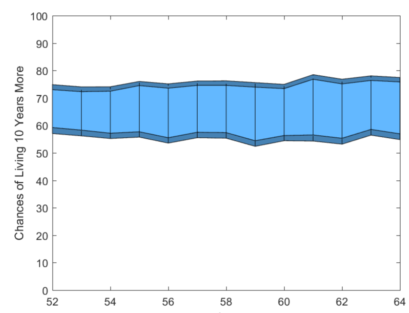
B.2 Comparison between RDHRS2006 and AGESHARE2006

Figure 12: Non-parametric Prediction of Expectations, Using RDHRS2006 (left) and AGESHARE2006 (right) data

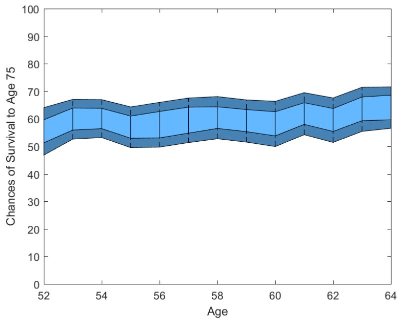
(a) Using All Questions, RDHRS2006



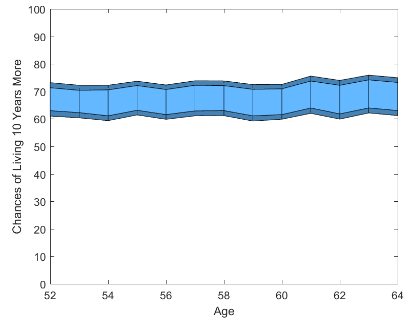
(b) Using All Questions, AGESHARE2006



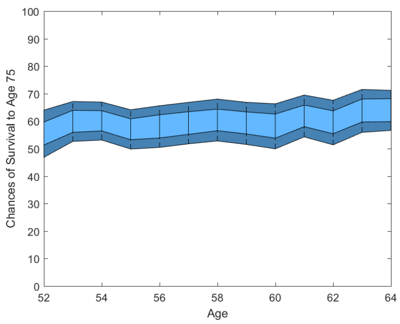
(c) Using All Questions, Excluding r_{jm} = NR, RDHRS2006



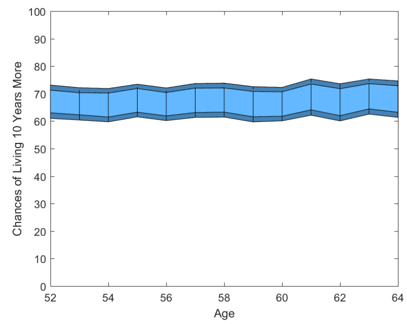
(d) Using All Questions, Excluding r_{jm} = NR, AGESHARE2006



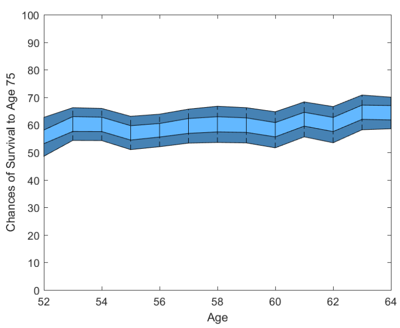
(e) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100), RDHRS2006



(f) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100), AGESHARE2006



(g) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100), RDHRS2006



(h) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100), AGESHARE2006

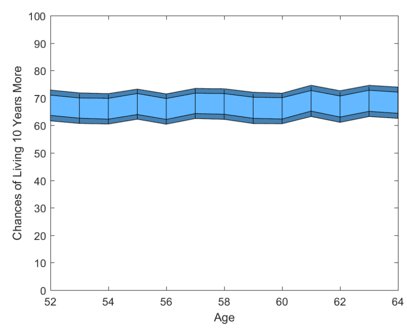
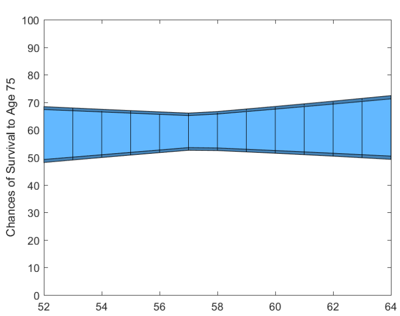
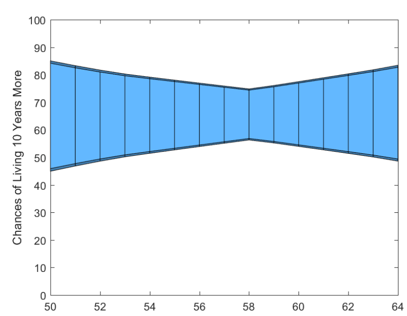


Figure 13: Parametric Prediction of Expectations, Using RDHRS2006 (left) and AGESHARE2006 (right) data

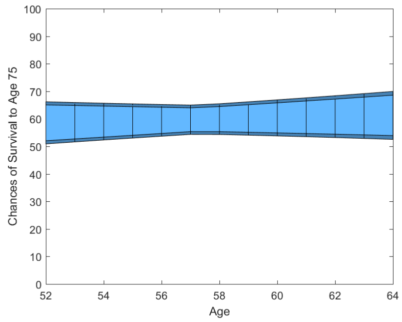
(a) Using All Questions, RDHRS2006



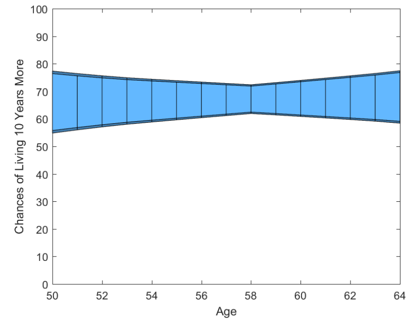
(b) Using All Questions, AGESHARE2006



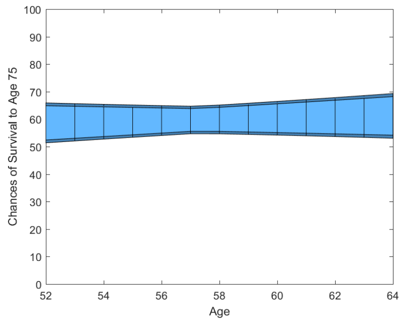
(c) Using All Questions, Excluding r_{jm} = NR, RDHRS2006



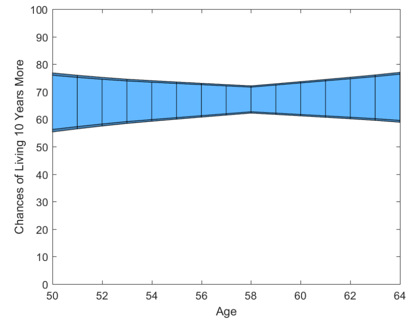
(d) Using All Questions, Excluding r_{jm} = NR, AGESHARE2006



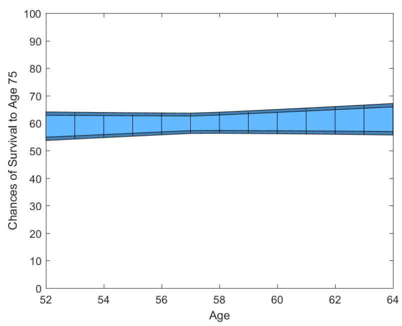
(e) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100), RDHRS2006



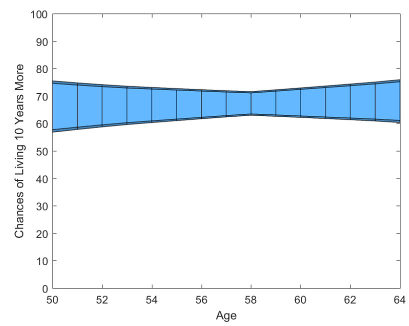
(f) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100), AGESHARE2006



(g) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100), RDHRS2006



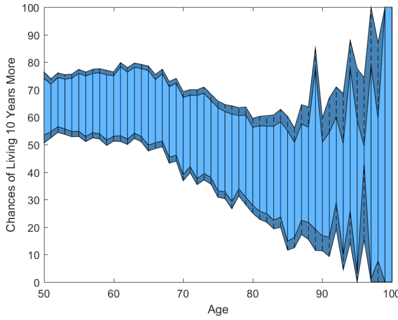
(h) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100), AGESHARE2006



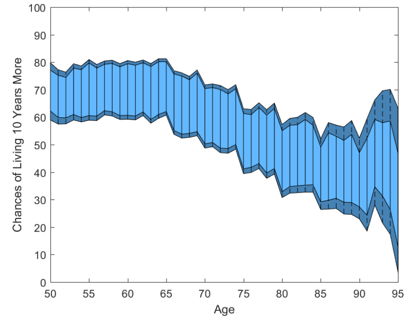
B.3 Comparison between RDSHARE2006 and SHARE2014

Figure 14: Non-parametric Prediction of Expectations, Using RDSHARE2006 (left) and SHARE2014 (right) data

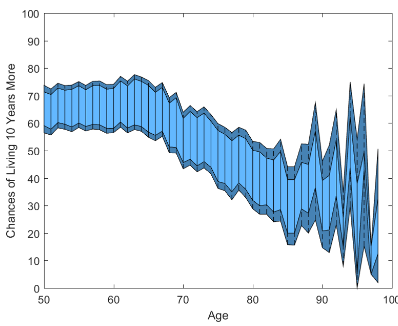
(a) Using All Questions, RDSHARE2006



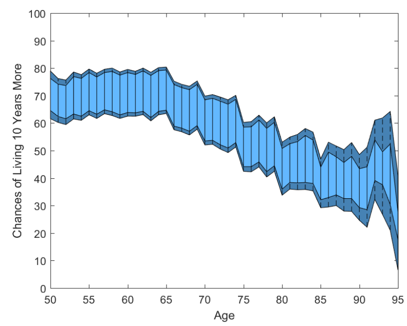
(b) Using All Questions, SHARE2014



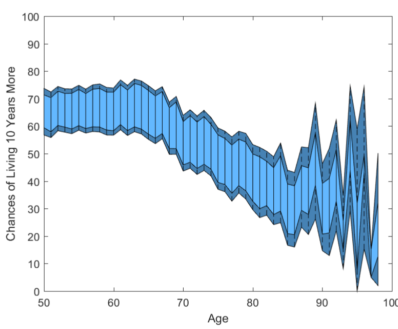
(c) Using All Questions, Excluding r_{jm} = NR, RDHRS2006



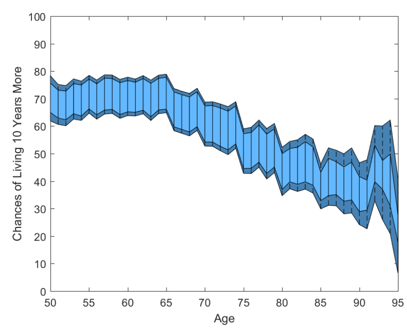
(d) Using All Questions, Excluding r_{jm} = NR, AGESHARE2006



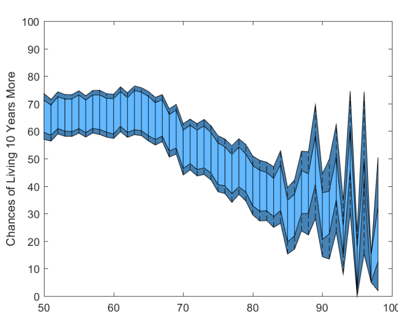
(e) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100), RDHRS2006



(f) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100), AGESHARE2006



(g) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100), RDHRS2006



(h) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100), AGE-SHARE2006

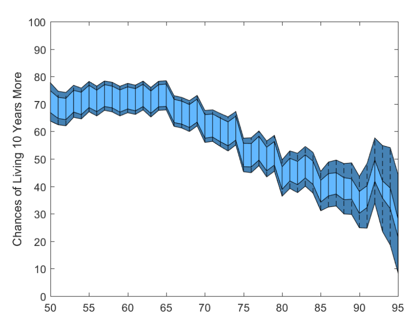
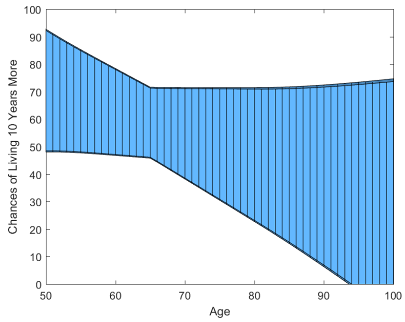
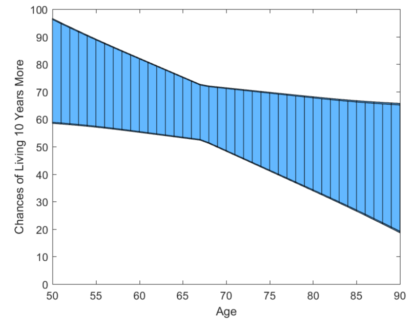


Figure 15: Parametric Prediction of Expectations, Using RDSHARE2006 (left) and SHARE2014 (right) data

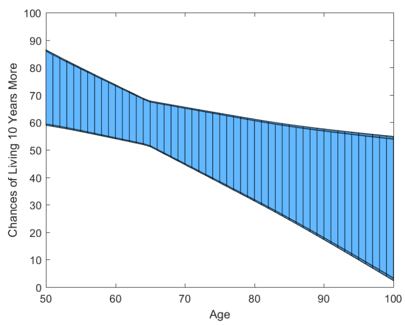
(a) Using All Questions, RDSHARE2006



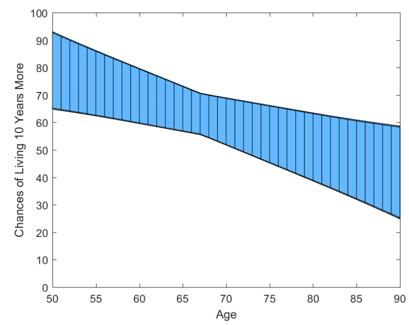
(b) Using All Questions, SHARE2014



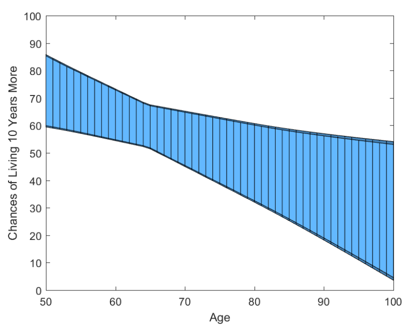
(c) Using All Questions, Excluding r_{jm} = NR, RDHRS2006



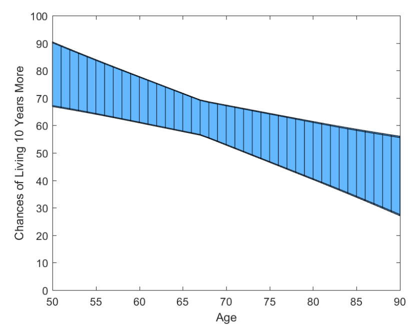
(d) Using All Questions, Excluding r_{jm} = NR, AGESHARE2006



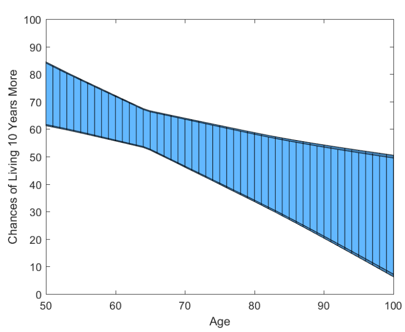
(e) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100), RDHRS2006



(f) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100), AGESHARE2006



(g) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100), RDHRS2006



(h) Using All Questions, Excluding r_{jm} = NR or (all 0 or 100) or (all 0, 50 or 100), AGESHARE2006