Erasmus University Rotterdam

Effect of a recommendation letter on the communication during a job interview

Cheap-talk game job interview

Master thesis

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Abstract

Businesses place importance in finding the right employees for their company. The challenge is hiring the right person based on a brief first impression. An important aspect of the selection process is communication during the job interview. This thesis draws on the Cheap-Talk game, based on the models of Gibbons (1992) and of Crawford and Sobel (1982), to develop a model of strategic communication during the job interview. This thesis contributes to the literature by applying the aforementioned models to a job interview and by looking at the effect of a solution found in real-life, namely a recommendation letter. It is found that the communication during a job interview becomes richer when the Applicant exaggerates less. In addition, adding a recommendation letter gives a better fit for the job and could even give full revelation.

1. Introduction

The end of the financial crisis has prompted an increased demand in labour (CBS). Both companies and prospective employees are continuously searching for the right match. After a first impression from the Curriculum Vitae and a motivational letter, some applicants get invited for a job interview at the company. But how is the communication in this job interview? Do people really get the right job?

This thesis develops a model of strategic communication in a job interview, in which there are two players: the Job Applicant (he) and the Employer (she). The Applicant is the better-informed player who sends a possibly noisy signal, based on his private information, to the Employer during a job interview. The Employer's subsequent action then determines the welfare for both players. This model assumes that the Applicant will be hired. The Employer now has to decide which job to give the Applicant based on his individual ability.

This model is a cheap-talk game based on the theoretical Sender-Receiver models of Gibbons (1992) and of Crawford and Sobel (1982). The cheap talk of the Applicant influences for which job he is being hired. In cheap-talk games the messages of the Sender, in this model the Applicant, are just talk; costless, non-binding, nonverifiable claims. In this model however, a recommendation letter is added, thereby making information more verifiable than in standard cheap-talk games.

During the cheap-talk in the job interview, the Applicant is more informed about his ability than the Employer and the players have partially conflicting preferences. This thesis investigates the effect of the use of a recommendation letter on the communication during the job interview, a phenomenon that is seen in real-life. It examines the effect of this information problem in the cheap-talk game, posing the question: *Does a recommendation letter help the Applicant procure the right job when there is an information problem between himself and the Employer?*

In the previous literature, various remedies for the information problem in a Sender-Receiver model have already been examined, for example: more extensive communication, delegation, contracts, and multiple Senders (Blume & Durlauf, 2010). This thesis considers the effect of a recommendation letter as a specific, tangible solution for the information problem in job interviews. Here we attempt to dissect how job interviews are conducted.

2. The Example

At the job interview the Employer must decide which job to give to the Applicant. The Employer wants the most optimal match, where ability and fit in the company are important factors. She neither wants underqualified workers, nor overqualified workers for each job. The Employer wants to maximize

$$U_E = -(x - \mu)^2$$

where μ is the type/ability of the Applicant and x is the decision of the firm. It is assumed that the Applicant is seeking for a job requiring skills higher than his own. Therefore, the two players have partially conflicting preferences. The Employer wants someone whose ability equals the ability required for the job. Conversely, the Applicant wants a higher skilled job. His hiring however is contingent upon how the Employer perceives him. This is a plausible assumption because a higher job yields a higher salary and prestige. Though, the job should not be too high above his ability, because he might then not be able to do the job. The utility function of the Applicant is:

$$U_A = -\,[x - (\mu + b)]^2$$

where b is how much the Applicant likes to exaggerate.

The timing of the job interview is based on the model of Gibbons (1992) and is as follows:

1. Nature draws an ability μ_i for the Applicant from a set continuous abilities $\mathrm{A} = [0,1]$, according to a probability distribution $p(\mu_i)$, where $p(\mu_i) > 0$ for every i and $p(\mu) = 1$.
2. The Applicant observes μ_i and then chooses a message m_j from a set of feasible messages where the message space is the ability space $M = A$.
3. The Employer observes m_j (but not μ_i) and then chooses which job to give to the Applicant. The action space is the interval from zero to one, $\mathrm{X} = [0,1]$.
4. The Employer's payoff function is $U_E(\mu, x) = -(x - \mu)^2$ and the Applicant's payoff function is $U_A(\mu, x) = -[x - (\mu + b)]^2$.

In this model the message given in the interview has no *direct* effect on neither the Applicant's nor the Employer's payoff. It only has an *indirect* effect since the message can change the Employer's belief about the Applicant's ability, and subsequently the Employer's action.

Since $b > 0$, there is no separating equilibrium. If the Applicant is honest, so $m = \mu$, then the Employer believes him and gives him the job for ability μ. However, the Applicant does not want a job of ability μ, but rather of ability $\mu + b$, and will therefore be incentivized to exaggerate his ability. This exaggerated ability is therefore the message he would send in this cheap-talk game and it is therefore not the equilibrium to be honest. The parameter b measures the similarity of the preferences of the Employer and the Applicant. When b is closer to zero, the preferences are more closely aligned. We can conclude that if the Applicant is even slightly biased, what is assumed in this model since $b > 0$, all equilibria will entail some information loss.

For cheap talk to be informative, Gibbons (1992) claims that the model should hold three conditions. The first necessary condition is that different Applicant types have different preferences over the actions of the Employer since otherwise all Applicants would send the same message. Such is the case in our model, since Applicants with a higher ability prefer higher jobs than Applicants with lower ability. The second condition is that the Employer prefers different actions depending on the type of the Applicant. This condition also holds in our model, since the Employer would be willing to give different jobs to Applicants with different abilities. The third condition is that the Employer's preferences over actions are not completely opposed to the Applicant's. As stated above, this is true in our model as well.

3. Equilibria

In a cheap-talk game, a pooling equilibrium always exists. Gibbons (1992) stated that because the messages have no direct effect on the Applicant's payoff, if the Employer will ignore all messages then pooling is a best response for the Applicant. Also, because messages have no direct effect on the Employer's payoff, if the Applicant is pooling then a best response for the Employer is to ignore all messages (Gibbons, 1992). It is then a pooling perfect Bayesian equilibrium for the Applicant to play any pooling strategy, for the Employer to maintain the prior belief $p(\mu_i)$ after all messages, and for the Employer to take the action x^* after all messages (Gibbons, 1992). This is called the 'babbling' equilibrium. The question is whether nonpooling equilibria also exist.

Crawford and Sobel (1982) show that there is a partially pooling equilibrium. They divided the type space into n intervals: $[0, x_1), [x_1, x_2), \dots, [x_{n-1}, 1]$. In our model these intervals could be seen as messages. All the types in a given interval send the same message, but types in different intervals send different messages. Crawford and Sobel showed that given the value of the preference-similarity parameter b (in our model b is how much the Employer likes to exaggerate) there is a maximum number of intervals that can occur in equilibrium, denoted $n^*(b)$. Henceforth, for a given value of how much the Applicant likes to exaggerate, there are a maximum number of messages he could send. Crawford and Sobel showed that partially pooling equilibria exist for each $n = 1, 2, \ldots, n^*(b)$. A decrease in b increases $n^*(b)$ – this means that more communication can occur through cheap talk when the players' preferences are more closely aligned. In our model that is when the Applicant exaggerates less. They also concluded that $n^*(b)$ is finite for all $b > 0$ but approaches infinity as b approaches zero – perfect communication cannot occur unless the players' preferences are perfectly aligned.

To give an illustration of the partially pooling equilibria, the same illustration is used as Gibbons (1992). A two-step equilibrium ($n = 2$) is shown in figure 1. The midpoint is given by x_1. As stated above, it is assumed that all the types in the interval $[0, x_1)$ send one message while those in $[x_1, 1]$ send another. When the Employer receives the message from the types in $[0, x_1)$, she will believe that the Applicant's type is uniformly distributed on $[0, x_1)$, so the Employer's optimal action will be to give a job with ability $\frac{x_1}{2}$. After receiving the message from the types in $[x_1, 1]$, the Employer's optimal action will be to give a job with ability $\frac{(x_1+1)}{2}$. For the types with ability $[0, x_1)$ to be willing to send their message, it must be that all these types prefer the job with ability $\frac{x_1}{2}$ to the job with ability $\frac{(x_1+1)}{2}$. Likewise, all the types with ability above x_1 must prefer $\frac{(x_1+1)}{2}$ to $\frac{x_1}{2}$. Because the Applicant's preferences are symmetric around his optimal action, the Applicant-type μ (t in figure 1) prefers $\frac{x_1}{2}$ to $\frac{(x_1+1)}{2}$ if the midpoint between these two actions exceeds that type's optimal action, $\mu + b$ (as in Figure 1), but prefers $\frac{(x_1+1)}{2}$ to $\frac{x_1}{2}$ if $\mu + b$ exceeds the midpoint.

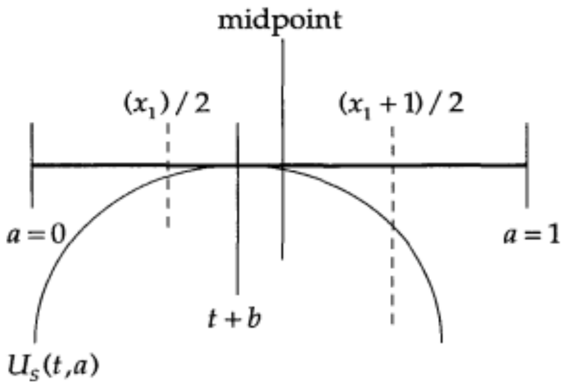

Figure 1, Two-step equilibrium.

Source: Gibbons (1992), pp.216

The two-step equilibrium with two possible messages exists when x_1 is the type μ whose optimal action $\mu + b$ exactly equals the midpoint between the two actions:

$$x_1 + b = \frac{1}{2}\left[\frac{x_1}{2} + \frac{x_1+1}{2}\right],$$

or $x_1 = (1/2) - 2b$. Since the ability of the Applicant is uniformly distributed over $\mathrm{A} = [0,1]$, x_1 must be positive. Therefore a two-step equilibrium exists only if $b < 1/4$; for $b \geq 1/4$ the players' preferences are too dissimilar to communicate.

It is now assumed that the Applicant can choose between N messages. The action (X) the Employer chooses depends on the message she receives from the Applicant. When she receives a certain message, she will use the action that equals the expected payoff of this message. So, $y(X_i, X_{i+1}) = \frac{X_i+X_{i+1}}{2}, \quad i = 0, \dots, N-1.$

The Applicant is then indifferent between choosing the message m_i or m_{i+1} when the payoffs between the two messages equals:

$$(1) \quad -\left(\frac{X_i+X_{i+1}}{2} - (X_i + b)\right)^2 = -\left(\frac{X_{i-1}+X_i}{2} - (X_i + b)\right)^2.$$

Equation 1 only holds if:

$$(2) \quad X_{i+1} - X_i = X_i - X_{i-1} + 4b.$$

The extensive steps of the solution can be found in the appendix 10.1.

Equation 2 states that for the Applicant to be indifferent between message i and message $i+1$, each upper step must be $4b$ longer than the lower one. What is the logic behind this longer length of the upper step? If it is assumed that all the steps are of equal length, then a person who is at the boundary between the two steps or just below the boundary would strictly prefer to send the message of the upper step since this higher job would give more prestige, a higher wage etcetera. Therefore it should be made unattractive to exaggerate his ability and give a higher message. Since the Employer would give the Applicant the job that is the expected value of the next step, $\frac{X_i+X_{i+1}}{2}$, this expected value should be made high enough that it would be too high for the Applicants at or just below the boundary. It is even more than the Applicant wants and therefore he would not want to exaggerate and instead send the message of the lower step (or is indifferent when he is at the boundary).

How many messages does the Applicant have to convince the Employer? This number of messages is a function of b. When it is assumed $b = 0$ the preferences of the Employer and the Applicant are perfectly aligned, and therefore there is a separating equilibrium with an infinite number of messages. But in this model it is assumed that $b > 0$, and therefore the richness of the language of the Applicant can be calculated when b is known. For every b it can be calculated what the richness of the language is. If b gets too high, there will only be one message: "I am fantastic, you should give me the best job". When b becomes smaller and smaller there will be more potential messages to send. Because the Employer would like to get as much precise information as possible, she would prefer more messages.

As an illustration, see figure 2 where it is assumed that there are four messages the Applicant could tell the Employer (where x is the length of one message (or "step")). The length of this line is $4x + 24b = 1$. If b becomes bigger, all boundaries move to the left and therefore there will eventually be a b where $x = 0$ where the left boundary moved all the way to the zero line and there will only be three messages left. At this point, the length of the line is $24b = 1$, which gives $b = \frac{1}{24}$. So if $b > \frac{1}{24}$, the left boundary moves to the left and the number of messages lowers from four to three.

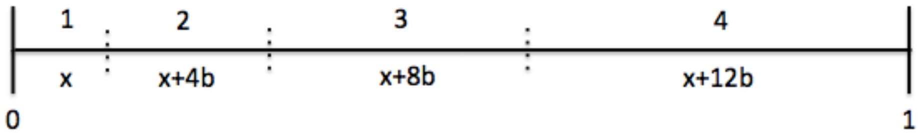

Figure 2. Four possible messages

There is a formula to calculate for which b the number of messages changes (Gibbons, 1992, pp.218). It was concluded above that each step must be $4b$ longer than the last one. Hence if the first step is of length x, then the second step must be of length $x + 4b$, the third of length $x + 8b$, which continues until the n^{th} step. The n^{th} step must end exactly at $t = 1$, so Gibbons (1992) concluded that:

$$x + (x + 4b) + \cdots + [x + (n - 1)4b] = 1.$$

He used the fact that $1 + 2 + \cdots + (n - 1) = n(n - 1)/2$, and therefore the next formula is acquired

$$(3) \qquad n * x + n(n - 1) * 2b = 1$$

"Since the length of the first step must be positive, the largest possible number of steps in such an equilibrium, $n^*(b)$, *is the largest value of* n *such that* $n(n - 1) * 2b < 1$. *Applying the quadratic formula shows that* $n^*(b)$ *is the largest integer less than*

(4) $\frac{1}{2}\left[1 + \sqrt{1 + (\frac{2}{b})}\right]$*."* (Gibbons, 1992, pp.218).

The extensive steps of the solution can be found in the appendix 10.2.

Table 1 shows the number of possible messages for the values of b. There is no communication possible if the players' preferences are too dissimilar (if the value of b is too high). In this stage every Applicant would tell the same message. When the value of b gets smaller, more and more communication is possible through cheap talk, though perfect communication is only possible when the players' preferences are perfectly aligned ($b = 0$).

Value b	**Number of possible messages**
$b > \frac{1}{4}$	1
$\frac{1}{12} < b < \frac{1}{4}$	2
$\frac{1}{24} < b < \frac{1}{12}$	3
$\frac{1}{40} < b < \frac{1}{24}$	4
$\frac{1}{60} < b < \frac{1}{40}$	5
$\frac{1}{84} < b < \frac{1}{60}$	6
$\frac{1}{112} < b < \frac{1}{84}$	7

Table 1: number of possible messages for value b

4. Frustration

An extra element is added to this model, since it is assumed that the Applicant gets frustrated when he gets a job that is below his ability. This is a plausible assumption since Allen and Van der Velden (2001) found that skill underutilization has a strong negative effect on job satisfaction. This frustration is given by d. Therefore, the utility function of the Applicant changes to:

$$\begin{array}{ll} -[x-(\mu+b)]^2 & if\ x > \mu \\ -[x-(\mu+d)]^2 & if\ x < \mu \end{array}$$

where it is assumed that $d > b$.

The Employer still wants to maximize the same utility function as before, $-(x-\mu)^2$. The model and the timing of the job interview are the same as in the previous section. The only differences are that in the timing, the Applicant takes in step two into account that he gets frustrated when the job is lower than his ability and in step three the utility functions for the Applicant are different. In this section we investigate what happens to the partially pooling equilibrium when frustration is added to the model.

For the Applicant to be indifferent between choosing the message m_i or m_{i+1}, the payoffs between the two messages must be equal:

$$(5) \quad -\left(\frac{X_i+X_{i+1}}{2}-(X_i+b)\right)^2=-\left(\frac{X_{i-1}+X_i}{2}-(X_i+d)\right)^2.$$

Equation (5) only holds if:

$$(6) \quad X_{i+1}-X_i=X_i-X_{i-1}+2b+2d.$$

Equation (6) does not differ much from equation (2); the only difference is that $4b$ became $2b+2d$. So for the Applicant to be indifferent between sending message i and message $i+1$, each upper step must be $2b+2d$ longer than the lower one. Since it is assumed $d>b$, this means $2b+2d>4b$, which means that the upper step must be even larger for the Applicant to be indifferent when he gets frustrated when he gets a job lower than his ability. We can conclude that the Applicant would be more inclined to overstate his ability when he gets frustrated when he gets a job below his ability. This seems likely, since this person would exaggerate more to hopefully get a job at or above his ability.

Again we calculate for which values of b or d the number of messages changes. The steps are the same as before, though now each step must be $2b+2d$ longer instead of $4b$. The n^{th} step must end exactly at $t=1$, which gives:

$$x+(x+2b+2d)+\cdots+[x+(n-1)(2b+2d)]=1.$$

Using the fact that $1+2+\cdots+(n-1)=n(n-1)/2$, the next formula is acquired:

$$(7) \quad n*x+n(n-1)*(b+d)=1$$

Since again the length of the first step must be positive, the largest possible number of steps in such an equilibrium, $n^*(b)$, is the largest value of n such that $n(n-1)*(b+d)<1$. Applying the quadratic formula shows that $n^*(b)$ is the largest integer less than

$$(8) \quad n<\frac{1}{2}\left(1+\sqrt{1+\frac{4}{b+d}}\right)$$

The extensive steps of the solution can be found in the appendix 10.3.

Table 2 shows the number of possible messages for the values of $b+d$. If we assume $b=d$, we see, logically, that the values are the same as in table 1. Though, it is assumed $b>d$ and therefore it can be seen when frustration is added to the model, there are fewer possible messages during the job interview. The intuition behind this is that, as stated before, when a person gets frustrated when he gets a job under his ability, he will become more likely to overstate his ability to the Employer. Therefore, there will be fewer messages than when he does not get frustrated.

Value $b + d$	**Number of possible messages**
$b + d > \frac{1}{2}$	1
$\frac{1}{6} < b + d < \frac{1}{2}$	2
$\frac{1}{12} < b + d < \frac{1}{6}$	3
$\frac{1}{20} < b + d < \frac{1}{12}$	4
$\frac{1}{30} < b + d < \frac{1}{20}$	5
$\frac{1}{42} < b + d < \frac{1}{30}$	6
$\frac{1}{56} < b + d < \frac{1}{42}$	7

Table 2: number of possible messages for value $b + d$

It can be shown that both the Employer and the Applicant would be better off if the Applicant would fully reveal his true ability, so $b = 0$ and $d = 0$, and there would be as many messages as possible. This is the case because every Applicant gives a certain message and everybody who gives the same message, will get the job that is average for that message $\left(y(X_i, X_{i+1}) = \frac{X_i+X_{i+1}}{2}\right)$. The Applicants who are under this average ability are happy that they get a job above their ability. However, the Applicants whose abilities are above this do not like that they get a job under their ability. When there is full revelation, the Employer would choose $x = \mu$ and earn a payoff of zero, while the Applicant would earn a payoff of $-b^2$ or $-d^2$. In any other equilibrium the payoff of both players are lower than this. On average, Applicants would be better off if they told the truth and got the job exactly at their ability rather than exaggerating their ability.

5. Recommendation letter

In this section we discuss a practical medium that could enrich the communication between the Employer and the Applicant. In daily life when people are looking for another job, they take a recommendation letter with them. The Applicant decides whom he asks to write this letter and it is the choice of that person to write the letter or not. In this model, the Employer infers information about the ability of the Applicant from the person who wrote the letter rather than from the content of the letter. If the person who wrote the letter is for example a Nobel Prize winner it is known that he will only write a recommendation letter for an Applicant with high ability. If the person who wrote the letter is for example a student assistant, then we can infer that the ability of the applicant is low. So it is assumed that the writer has some strictness for writing a letter, Q, which can be observed by the Employer. If $\mu \geq q$, then the person who is being asked writes a recommendation letter. If $\mu < q$, then he would not write a recommendation letter. There are two stages that could be distinguished: in the first stage the Applicant asks a person for a recommendation letter, in the second stage there is communication between the Employer and the Applicant in a job interview. Does the recommendation letter help the Applicant in getting the right job? We examine this question in this section by integrating the recommendation letter to the model.

When adding the recommendation letter to the model above, the game changes to a dynamic game with subgames. Therefore, backward induction is being used to solve the model. Where the previous model consisted of unverifiable information, the information in this model becomes more verifiable by adding the recommendation letter. The model takes the following form:

1. Nature draws an ability μ_i for the Applicant from a set continuous abilities $\mathrm{A} = [0,1]$, according to a probability distribution $p(\mu_i)$, where $p(\mu_i) > 0$ for every i and $p(\mu) = 1$.
2. The Applicant observes μ_i and decides whom to ask for a recommendation letter. The writer writes a recommendation letter if $\mu \geq q$.
3. The Employer observes who wrote the recommendation letter the Applicant took to the job interview.
4. The Applicant chooses a message m_j from a set of feasible messages where the message space is the ability space $M = A$.
5. The Employer observes m_j (but not μ_i) and then chooses which job to give to the Applicant. The action space is the interval from zero to one, $\mathrm{X} = [0,1]$.

6. The Employer's payoff function is $U_E(\mu, x) = -(x-\mu)^2$ and the Applicant's payoff functions are $U_A(\mu, x) = -[x-(\mu+b)]^2 \; if \; x > \mu$ and $U_A(\mu, x) = -[x-(\mu+d)]^2 \; if \; x < \mu$ (where $d > b$).

The main difference with the previous model is that the Employer gets information about the ability of the Applicant from the recommendation letter. Now we examine whether this information is really informative for the Employer and whether it will help the Applicant obtain the right job.

It will now be examined what $\bar{\mu}$ would be in equilibrium. This equilibrium can be discussed from two points of view, namely from either the Employer or the Applicant.

5.1 Employer

Suppose $\bar{\mu}$ divides the $[0, 1]$ line into two intervals, namely $[0, \bar{\mu})$ and $[\bar{\mu}, 1]$. How would this affect communication between the Employer and the Applicant? Moreover, what is the optimal value of $\bar{\mu}$?

For this extended model we compute again when the Applicant is indifferent between choosing the message m_i or m_{i+1} in the job interview in these intervals (so after he gave the recommendation letter). The calculations are the same as before and we conclude that in the given intervals the upper step must again be $2b + 2d$ longer than the lower one for the Applicant to be indifferent between message i and message $i + 1$.

As an illustration, let us assume that $\bar{\mu} = \frac{1}{2}$ and the Applicant could tell the Employer for example three messages in both intervals, see figure 3. Interestingly, in this model the information becomes more informative for the Employer at the first messages after ability $\bar{\mu}$, since the steps are smaller than in the previous model. Therefore, when the Applicant has a recommendation letter, the information becomes richer for the Employer and is therefore able to make a better-informed choice of action.

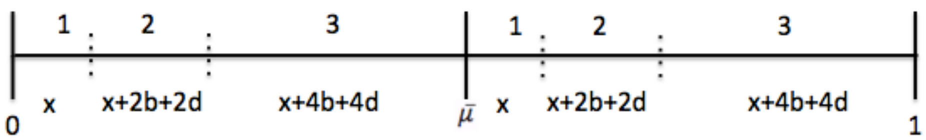

Figure 3. Two intervals $[0, \bar{\mu})$ and $[\bar{\mu}, 1]$, three possible messages per interval

The Employer wants to maximize his expected utility, which is the sum of the utilities of all intervals:

$$(9)\quad \sum_{i=1}^{n} -\left(\frac{1}{2}m_i - \mu\right)^2$$

We will show where $\bar{\mu}$ should be for the Employer to have the most efficient information. If it is assumed that in equilibrium $\bar{u} = \frac{1}{3}$, the intervals are for example:

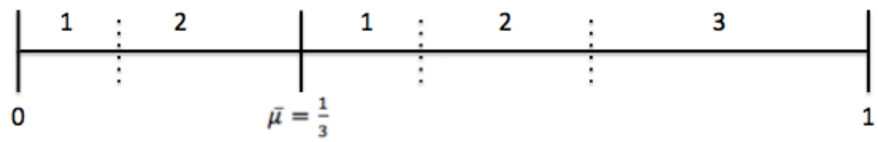

Figure 4. Intervals with $\bar{\mu} = \frac{1}{3}$

On the left-hand side of $\bar{\mu} = \frac{1}{3}$ the messages are smaller than on the right-hand side, making the communication on the left-hand side richer than on the right-hand side. This is not the optimal equilibrium. Though on the left-hand side there are also fewer messages than on the right-hand side, which is also not the optimal equilibrium for the Employer. The Employer would want as many messages as possible and she would want them to be as small as possible. This is because she wants to get the most precise information in order to make the best choice. She gets the most precise information when; (1) the Applicant has as many messages as possible and (2) on the left-hand side and the right-hand side of $\bar{\mu}$ the messages are as equal as possible. This is the case when the optimal ability is $\bar{\mu} = \frac{1}{2}$, which can be seen in the following figure:

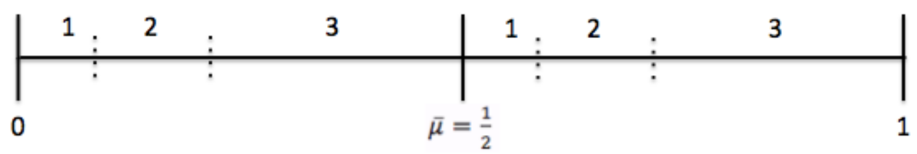

Figure 5. Intervals with $\bar{\mu} = \frac{1}{2}$

In figure 5 we see that when there is a deviation from $\bar{\mu} = \frac{1}{2}$ there could be fewer messages and the messages on the left-hand side and the right-hand side deviate from each other.

Therefore, $\bar{\mu} = \frac{1}{2}$ is the most optimal ability in equilibrium for the Employer where she gets the most efficient information.

5.2 Applicant

From the Applicant's perspective, we see that the communication in the previous section becomes unnecessary. This is the case because every Applicant chooses a different $\bar{\mu}$ and therefore cheap talk is not needed anymore. This will be explained in this section.

Imagine an Applicant with ability μ between $[0, 1]$. He would not get a recommendation letter from a person who has ability higher than μ. Since now $\mu < q$, the person would either not give him a recommendation letter or would write a bad recommendation letter. Although the Applicant wants job $\mu + b$, he would not get it, since he is unable to get a recommendation letter from a person with a q above μ. He would also not bring a letter from a person with ability below his own ability. This is not the equilibrium. Therefore, there is only one equilibrium: an Applicant with ability $\bar{\mu}$ would get a recommendation letter from a person with strictness q and therefore there is full revelation. In equilibrium the Employer knows the ability of the Applicant and she can therefore give him a job exactly at his ability. This applies to all abilities μ between $[0, 1]$.

If in this model it is allowed that the Applicant does not bring a recommendation letter, there is an out-of-equilibrium belief, which is plausible in this model. The Applicants who do not bring a recommendation letter to the job interview, are those who (1) cannot get one or (2) who can receive one, but where the letter is from a person with a low ability and therefore the letter would indicate he has a low ability. It is therefore known that only persons with an ability of zero would not bring a letter to the job interview, which is in line with the equilibrium.

We conclude that bringing a recommendation letter to the job interview enriches the information for the Employer, even giving full revelation. Is this, however, in the advantage of the Applicant? When the equilibrium model was not included in the model, he could get a job above his ability, $\mu + b$. But since he got the average job of the message he sent, he could also get a job *below* his ability. In this extension, he will most likely get a job right at this ability, $\bar{\mu}$. Overall the Applicant is better off with richer information and cheap-talk is no longer necessary.

6. Recommendation letter after job interview

In this section we examine the effect a recommendation letter has when given *after* the job interview. After the job interview, the cheap-talk has already taken place, so the Employer knows which message the Applicant sent, and therefore knows between which abilities the ability of the Applicant lies. How would the recommendation letter contribute to the information if it were given after the job interview? Again, let us examine both the Employer's and the Applicant's perspective.

As stated above, if the Employer decides the optimal ability $\bar{\mu}$, he would put it exactly at $\bar{\mu} = \frac{1}{2}$. If the message the Applicant sent was clearly below or above $\bar{\mu} = \frac{1}{2}$, which is the information the recommendation letter could give, it would not add information. It could, however, give an extra control to the Employer that the Applicant was telling the truth. It becomes interesting when the message the Applicant sent, indicated an ability just below or just above the $\bar{\mu}$. See figure 6 for an illustration where $\frac{1}{20} < b + d < \frac{1}{12}$. If the Applicant sent message three in the job interview, this message is below and above the optimal ability $\bar{\mu}$. When after the job interview the Applicant brings a recommendation letter, indicating whether his ability is above or below $\bar{\mu}$, the Employer has more information about the ability of the Applicant and could therefore make a better-informed choice.

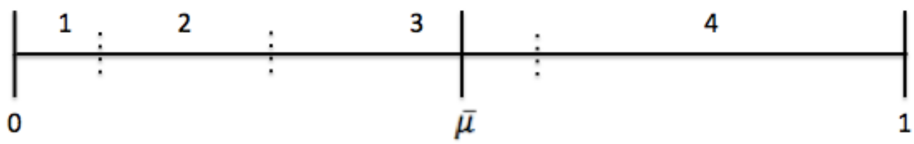

Figure 6. Possible messages for $\frac{1}{20} < b + d < \frac{1}{12}$

In the previous section it was found that the recommendation letter would give full revelation when the Applicant decided where to put $\bar{\mu}$. So what would it add if the recommendation letter were given after the job interview? During the cheap-talk, the Employer knows which message the Applicant sent and she therefore knows in which interval the ability of the Applicant lies. When the recommendation letter is given afterwards, this would reveal the true ability of the Applicant (full revelation) and will therefore also be a control whether the Applicant told the truth during the job interview. It would eventually give the same information as when the recommendation letter was given before the job interview, though in this setting the Employer also knows whether the Applicant is someone who exaggerates or whether he is a trustworthy person.

7. To give or not to give

As stated by Milgrom and Roberts (1986) decision makers (in this model the Employer) often need to make decisions based on information that is provided by individuals (in this model the Applicant) who are affected by the decision he/she eventually makes. Therefore, the interested parties (the Applicant) may try to manipulate the choice of the decision maker (the Employer) by giving manipulated information. In this section we assume that either Applicants might not get a recommendation letter or Applicants might get a recommendation letter, but they manipulate the Employer by not giving the letter.

In this extension the following situation is being examined. The Applicant is invited to a job interview and asked to bring a recommendation letter with him. He may then ask only one person to write the recommendation letter, due to time constraints. With probability α he receives the recommendation letter. As before, the Employer knows that the writer would only write a recommendation letter when the ability equals or is above the strictness (Q) of the writer and the Applicant would ask someone to write a recommendation letter only when the strictness of the writer equals his ability. Now there is a chance with probability $1-\alpha$ that the writer says he will write a letter, but he eventually does not write one. Reasons for this could be that he forgot or he did not have the time. So with probability $1-\alpha$ the Applicant did not get a recommendation letter. The Employer knows this probability exists and knows that this does not mean that the Applicant did not get a recommendation letter because his ability is low. The expected ability of a person who did not get a recommendation letter is therefore $\frac{1}{2}$, since it could happen to any Applicant and the ability is uniformly distributed between $[0,1]$. This brings the question for the Applicant: Is it also advantageous for him to bring a recommendation letter when his ability is $\mu < \frac{1}{2}$? No, since not bringing the recommendation letter would give a higher expected ability. An Applicant with ability $\mu < \frac{1}{2}$ is therefore incentivized not to bring the recommendation letter to the job interview, even though he did get one. Thus, with probability α the recommendation letter is given and the Employer learns the ability of the Applicant, though there is a chance that the Applicant either did not receive a letter or he did receive a letter but did not give it to the Employer.

In this section we calculate the threshold at which the Applicant is indifferent between giving the recommendation letter and not giving it to the Employer. If the ability of the Applicant is above this threshold, he asks someone to write a letter with exactly the same strictness as his ability. But if the ability of the Applicant is below this threshold, he does not give the recommendation letter. The threshold is given by t (and therefore the ability of the Applicant is in this section also given by t). At this threshold, the utility of giving the letter is the same as the utility of not giving the letter to the Employer. The utility of giving the letter equals t. There are two reasons why the Applicant would not give a recommendation letter: (1) he did not receive one, or (2) he did receive one, but he does not give it (because his actual ability is below the expected ability of not giving it). The utility of not giving the letter therefore consists of two terms. The total probability that the Applicant does not give the letter is the probability that he did not receive one (probability $1-\alpha$) plus the probability that he did get one, but does not give it (probability αt). The last probability is αt since he gets a letter with probability α, though he does not give it when his ability is below the threshold t. The probability that he is below this threshold is t. Therefore the total probability that he does not give the letter is: $(1-a)+\alpha t.$ When the Applicant does not give the letter because he did not receive one with probability $1-\alpha$ (reason (1)), his expected ability is $\frac{1}{2}$. When he does not give the letter after receiving one (reason (2)) with probability αt, his expected ability is $\frac{1}{2}t$. This is because only Applicants with ability between $[0, t]$ do not give the letter after receiving it. The expected ability is therefore $\frac{1}{2}t$. The Applicant is indifferent between giving the recommendation letter and not giving it, when:

$$(10) \quad \frac{(1-\alpha)}{(1-\alpha)+\alpha t} * \frac{1}{2} + \frac{\alpha t}{(1-\alpha)+\alpha t} * \frac{1}{2} t = t$$

Consequently, the threshold equals:

$$(11) \ t = 1 - \frac{1}{\alpha} + \frac{\sqrt{1-\alpha}}{\alpha}$$

The extensive steps for the solution can be found in appendix 10.4. The threshold can be computed for every α, see table 3. We see that when α equals zero, which is the point where there is no recommendation letter given, the threshold where the Applicant is indifferent between giving the letter or not is approximately $\frac{1}{2}$. This is because the expected ability is $\frac{1}{2}$ when there is no recommendation letter given. The threshold becomes smaller when the probability of a recommendation letter increases. This is because when there is more chance that a recommendation letter is given, the Applicant is less likely to not get a recommendation letter. Therefore, when he does not bring a recommendation letter, the chance increases that he did receive a letter, though his ability is low and therefore does not bring the letter rather than because he did not get one. This is the reason that the threshold becomes smaller when the probability α increases. When the writer always writes a recommendation letter ($\alpha = 1$), the threshold equals 0. This is the same case as described in section five and there would be full revelation.

Probability α	**Approximate value threshold t**
0	≈0.5
0.1	≈0.49
0.2	≈0.47
0.3	≈0.46
0.4	≈0.44
0.5	≈0.41
0.6	≈0.39
0.7	≈0.35
0.8	≈0.31
0.9	≈0.24
1	0

Table 3: Approximate value threshold t for probability α

When the Employer knows what the probability α is, she knows the threshold t and therefore knows whether the ability of the Applicant is above or below $t.$ It is assumed that the Applicant gives the recommendation letter when he is indifferent between giving and not giving. After the recommendation letter is given or not, communication can occur in the intervals $[0, t)$ or $[t, 1]$. Communication in the intervals is the same as in section 4.

8. Discussion and conclusion

In this thesis it has been explored whether a recommendation letter enriches the information during the strategic communication in a job interview. To this end, we developed a model in which two players, an Applicant and an Employer with information asymmetry about the ability of the Applicant, communicate. In this model an extra dimension is added, since it is assumed that the Applicant gets frustrated if he gets a job under his ability. There are five partially pooling equilibria in this model: the basic equilibria, equilibria when frustration is added, equilibria when the recommendation letter is given before the job interview, equilibria when the recommendation letter is given after the job interview and equilibria when there is also a chance that the writer does not give a recommendation letter.

In the basic model there are partially pooling equilibria with a maximum number of messages for each value of b. Since the maximum number of messages, $n^*(b)$, decreases in b, more communication can occur through cheap talk in the job interview when the Applicant exaggerates less. Therefore, communication improves when b gets smaller, and it is optimal when $b = 0$. For the Applicant to be indifferent between choosing the message m_i or m_{i+1}, each message m_{i+1} must be $4b$ longer than message m_i.

When frustration is added to the model, there are fewer messages that could be sent during the job interview. This is because the Applicant would be more likely to overstate his ability and lie to the Employer in order to avoid getting a job below his ability. Both the Employer and the Applicant would be better off if the Applicant does not exaggerate but rather would fully reveal his true ability since this yields the maximum number of possible messages.

The next equilibria are when a recommendation letter is given before the job interview. This can be seen from two points of view: the Employer and the Applicant. From the Employer's perspective, she would set $\bar{\mu} = \frac{1}{2}$, because this is the most optimal ability in equilibrium for the Employer, where she gets the most efficient information. From the Applicant's perspective, there is only one equilibrium with full revelation since an Applicant with ability $\bar{\mu}$ would get a recommendation letter from a person with strictness q (observable by the Employer).

When the recommendation letter is given after the communication has taken place during the job interview, there would again be full revelation. Although in this setting the Employer has the added benefit of knowing whether the Applicant is a trustworthy person.

The last equilibria are more realistic ones, since here it was assumed that there is also a chance that the Applicant does not receive a recommendation letter. It is computed for every probability α that the recommendation letter is given vs. probability $(1-\alpha)$ that the letter is not given, when the Applicant is indifferent between giving the recommendation letter and not giving it. When there are no recommendation letters given, the threshold where the Applicant is indifferent between giving the letter or not is approximately $\frac{1}{2}$. The threshold becomes smaller when the probability receiving a recommendation letter increases and becomes zero when recommendation letters are always given.

We conclude that the communication during a job interview becomes richer when the Applicant exaggerates less. In addition, adding a recommendation letter gives a better fit for the job and could even yield full revelation. Despite a probability of no recommendation letters given, the information is richer than without the recommendation letter. In the real world, Employers would therefore be better off if they would also look at the recommendation letters brought (or not brought) to the job interview and then look at whom wrote the letter. This enriches the communication during the job interview and would give a better fit for the jobs at the company. For the Applicants it would also be better if the communication becomes richer, since at the ideal setting they would then get a job at their ability, and otherwise they could also get a job below their ability.

This thesis assumes in section 7 that there is a chance with probability $1-\alpha$ that the writer says he will write a letter, but he eventually does not write one. Reasons for this could be that he forgot or he did not have the time. It could be that especially people with strictness above a certain q are more inclined to not write the recommendation letter. The effect of this on the communication could be further investigated in future research. Delegation might give some interesting insights for the communication problem in job interviews. What is the effect when the Applicant decides himself which job he should get? It could be investigated whether this would benefit the Employer and the Applicant.

9. References

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10. Appendix

10.1 Equations (1) and (2)

$$(1) \quad -\left(\frac{X_i+X_{i+1}}{2} - (X_i + b)\right)^2 = -\left(\frac{X_{i-1}+X_i}{2} - (X_i + b)\right)^2.$$

First I will work out the <u>left-hand side</u> of the above equation.

$$-\left(\frac{X_i + X_{i+1}}{2} - (X_i + b)\right)^2 = -\left(\frac{X_i + X_{i+1}}{2} - (X_i + b)\right) * \left(\frac{X_i + X_{i+1}}{2} - (X_i + b)\right)$$

$$= -\left(\frac{(X_i + X_{i+1})^2}{4} - \frac{X_i(X_i + X_{i+1})}{2} - \frac{b(X_i + X_{i+1})}{2} - \frac{X_i(X_i + X_{i+1})}{2} + {X_i}^2 + bX_i \right.$$

$$\left. - \frac{b(X_i + X_{i+1})}{2} + bX_i + b^2\right)$$

$$= -\left(\frac{(X_i + X_{i+1})^2}{4} - X_i(X_i + X_{i+1}) - b(X_i + X_{i+1}) + {X_i}^2 + 2bX_i + b^2\right)$$

$$= \frac{-(X_i + X_{i+1})^2}{4} + {X_i}^2 + X_iX_{i+1} + bX_i + bX_{i+1} - {X_i}^2 - 2bX_i - b^2$$

$$\frac{dU}{dR_i}: -\frac{1}{2}X_i - \frac{1}{2}X_{i+1} + 2X_i + X_{i+1} + b - 2X_i - 2b = 0$$

$$= -\frac{1}{2}X_i + \frac{1}{2}X_{i+1} - b = 0$$

or: $\frac{1}{2}X_i - \frac{1}{2}X_{i+1} + b = 0$

<u>Right-hand side</u>

$$-\left(\frac{X_{i-1}+X_i}{2} - (X_i+b)\right)^2 = -\left(\frac{X_{i-1}+X_i}{2} - (X_i+b)\right) * \left(\frac{X_{i-1}+X_i}{2} - (X_i+b)\right)$$

$$= -\left(\frac{(X_{i-1}+X_i)^2}{4} - \frac{X_i(X_{i-1}+X_i)}{2} - \frac{b(X_{i-1}+X_i)}{2} - \frac{X_i(X_{i-1}+X_i)}{2} + {X_i}^2 + bX_i - \frac{b(X_{i-1}+X_i)}{2} + bX_i + b^2\right)$$

$$= -\left(\frac{(X_{i-1}+X_i)^2}{4} - X_i(X_{i-1}+X_i) - b(X_{i-1}+X_i) + {X_i}^2 + 2bX_i + b^2\right)$$

$$= \frac{-(X_{i-1}+X_i)^2}{4} + X_iX_{i-1} + {X_i}^2 + bX_{i-1} + bX_i - {X_i}^2 - 2bX_i - b^2$$

$$\frac{dU}{dR_i}: -\frac{1}{2}X_{i-1} - \frac{1}{2}X_i + X_{i-1} + 2X_i + b - 2X_i - 2b = 0$$

$$= \frac{1}{2}X_{i-1} - \frac{1}{2}X_i - b = 0$$

<u>Combined:</u>

$$\frac{1}{2}X_i - \frac{1}{2}X_{i+1} + b = \frac{1}{2}X_{i-1} - \frac{1}{2}X_i - b$$

$$\frac{1}{2}X_i - \frac{1}{2}X_{i-1} + 2b = \frac{1}{2}X_{i+1} - \frac{1}{2}X_i$$

(2) $\quad X_{i+1} - X_i = X_i - X_{i-1} + 4b.$

10.2 Equations (3) and (4)

(3) $\quad n * x + n(n-1) * 2b = 1$

Since the length of the first step must be positive, the largest possible number of steps in such an equilibrium, $n^*(b)$, is the largest value of n such that

$n(n-1) * 2b < 1$

For simplicity, it is assumed it equals 1:

$(n^2 - n)2b = 1$

$2bn^2 - 2bn - 1 = 0$

Solve for n by applying the abc-formula:

$a = 2b, b = -2b, c = -1$

$$n = \frac{2b + \sqrt{4b^2 + 8b}}{4b}$$

$$n = \frac{1}{2} + \frac{\sqrt{4b(b+2)}}{4b}$$

$$n = \frac{1}{2} + \frac{\sqrt{\frac{1}{4} * 4b(b+2)}}{2b}$$

$$n = \frac{1}{2} + \frac{\sqrt{b(b+2)}}{2b}$$

$$n = \frac{1}{2} + \frac{1}{2}\frac{\sqrt{b(b+2)}}{b}$$

$$n = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{b(b+2)}{b^2}}$$

$$n = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{b+2}{b}}$$

$$n = \frac{1}{2}\left(1 + \sqrt{1 + \frac{2}{b}}\right)$$

(4) $\quad n < \frac{1}{2}\left[1 + \sqrt{1 + (\frac{2}{b})}\right].$

10.3 Equations (7) and (8)

(7) $\quad n * x + n(n-1) * (b+d) = 1$

$n(n-1)(b+d) < 1$

For simplicity, it is assumed:

$n(n-1)(b+d) = 1$

$(n^2 - n)(b+d) = 1$

$bn^2 + dn^2 - bn - dn - 1 = 0$

$a = (b+d), b = (-b-d), c = -1$

abc-formula:

$$n = \frac{(b+d) + \sqrt{(b+d)^2 + 4(b+d)}}{2(b+d)}$$

$$n = \frac{1}{2} + \frac{1}{2}\frac{\sqrt{(b+d)(b+d+4)}}{(b+d)}$$

$$n = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{(b+d)(b+d+4)}{(b+d)^2}}$$

$$n = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4}{b+d}}$$

$$n = \frac{1}{2}\left(1 + \sqrt{1 + \frac{4}{b+d}}\right)$$

So, (8) $n < \frac{1}{2}\left(1 + \sqrt{1 + \frac{4}{b+d}}\right)$

10.4 Equations (10) and (11)

(10) $\frac{(1-\alpha)}{(1-\alpha)+\alpha t} * \frac{1}{2} + \frac{\alpha t}{(1-\alpha)+\alpha t} * \frac{1}{2} t = t$

$$\frac{(1-\alpha)}{2(1-\alpha)+2\alpha t} + \frac{at^2}{2(1-\alpha)+2at} = t$$

$$\frac{(1-\alpha)+at^2}{2(1-\alpha+at)} = t$$

$$1 - \alpha + at^2 = 2t(1-\alpha+\alpha t)$$

$$1 - \alpha + \alpha t^2 = t(2-2\alpha) + 2\alpha t^2$$

$$\alpha t^2 + (2-2\alpha)t - 1 + a = 0$$

$$t = \frac{-2 + 2\alpha + \sqrt{4-4\alpha}}{a}$$

$$t = 1 - \frac{1}{\alpha} + \frac{\sqrt{4-4\alpha}}{2\alpha}$$

(11) $t = 1 - \frac{1}{\alpha} + \frac{\sqrt{1-\alpha}}{\alpha}$