

ERASMUS UNIVERSITY ROTTERDAM

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BACHELOR THESIS

AN INTEGER PROGRAMMING MODEL FOR  
THE LINER-SHIPING NETWORK DESIGN  
PROBLEM

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### **Abstract**

This thesis looks into the paper of Brouer et al. (2013). The liner shipping network design problem (LSNDP) looks at the flow of containers over a complex network of ports in order to minimize the costs and maximize the revenue. Optimization in this field can yield large advantages in the competitive market of today. This paper will reproduce the base mixed integer programming model for the LSNDP that is discussed in Brouer et al. (2013). In addition to this, we will look at the running time to solve various instances. The problem is strongly NP hard, which has the result that large instances will be very difficult to solve. In order to be able to solve more instances, efficient ways to decrease the number of rotations that is used in the model is discussed. The data that are used have been taken from Brouer et al. (2013).

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# Chapter 1

## Introduction

Liner shipping is a very important business for trade between countries. Due to globalization, and the increasing trade in the world, this business has a substantial influence on the world's economy. Container shipping is the most efficient way to transport large amount of quantities efficiently. Operations research is an important aspect in liner shipping network design. Since the competition in between shipping companies is quite high, it is essential for such a company to have a service network which is as efficient as possible. However, research in this field has shown that it is quite difficult to create a model which takes every aspect of the Liner-Shipping Network Design Problem (LSNDP) into account. The problem seems to be too complex to be solved optimally for big instances. The closer one wants to get to the real world, the more complex models become. However, in the last years, models with a close resemblance to the real world have been created.

### 1.1 Problem definition

In this paper we would like to construct and expand on the base integer programming model created by Brouer et al. (2013). Their model takes most general aspects of liner shipping into account to make the model a basic yet sophisticated model for the liner shipping network. The goal of the LSNDP is to minimize the costs and maximize the revenue for transporting certain cargo throughout a specific area. However, since the problem is NP-hard, solving the problem optimally is difficult. Different approaches need to be considered for larger instances to get a good solution. For simplicity, external factors like weather and currents are omitted from the model. Besides this, the model looks at six fictive but realistic vessel classes. Of these vessels, all daily running costs, fuel costs, canal costs, port-calling fees for active vessels, revenues from transporting cargo and charges from loading and unloading containers.

## 1.2 overview

In Chapter 2, previous research will be examined. Chapter 3 will look into the data that is available and which will be used in the mixed-integer programming model. Chapter 4 will give an overview of the MIP, with a complete explanation of how it works. Next to this, we will look at rotations and in what way we can most effectively limit these without getting too far away from the optimal solution. In Chapter 5 we will look at the outcomes of the model. Specifically, we will look at the time it took to solve an instance and the consequences of limiting rotations to the objective value. In Chapter 6 we will summarize our findings and discuss the limitations of the model as well as suggestions for further research.

## Chapter 2

# Literature review

The work in Brouer et al. (2013) is an extension to that of Álvarez (2009). It extends the work in the paper by adding butterfly routes and imposing (bi)weekly frequency on the rotations. Their paper is interesting for further research in the same field, since it functions as a benchmark suite. Next to this, the paper is interesting for practical applications. With some modifications, it can be used by real shipping companies to optimize their service network. For this reason, the paper has chosen to use real ports and their corresponding characteristics. The Suez and Panama canals are also taken into account.

When looking at previous research on this topic one can see that the research community looks at the problem from different angles. Researchers use different constraints. For example, Rana and Vickson (1991) look at the capacity of vessels and make sure the planning horizon is not exceeded. However, the paper does not look into transshipments. With transshipments being an important factor in international modern trade, this makes the model less useful. Another example is Reinhardt and Pisinger (2012). They take into account everything what Rana and Vickson (1991) did, and included the transshipments. They were also able to solve the problem to optimality with up to 15 ports. Next to this, they implemented butterfly cycles, which is also discussed thoroughly in Brouer et al. (2013). Another big factor in shipping is bunker costs. The velocity at which a vessel is cruising has a large effect on the fuel costs. Shintani et al. (2007) consider this in their model. However, they did not incorporate various factors the previous papers did incorporate in their model. All in all, when you look at different contributions to the LSNDP, you can see that there are a lot of different types of contributions, but a comprehensive base model was still lacking. This is what Brouer et al. (2013) wanted to construct.

# Chapter 3

## Data

In general, there are 4 different data classifications. Data for ports, the fleet, cargo demand and distance. In Table 3.1, all data classifications with their corresponding attributes are listed.

Table 3.1: Data classifications

Port Data	Fleet Data
UNLOCODE	Vessel class
Name	Capacity
Country	TC rate
Cabotage	Min speed
Region	Max speed
Latitude	Design speed
Longitude	Fuel consumption (design speed)
Draft	Fuel consumption (idle)
Move cost	Quantity of vessels
Transshipment cost	Suez fee
Fixed port call cost	Panama fee
Variable port call cost	

  

Cargo demand Data	Distance Data
Origin port	Origin port
Destination port	Destination port
Quantity	Draft
Freight rate	Distance
Max transit time	Suez traversal
	Panama traversal

The data for ports and fleets will generally be used to calculate all appropriate costs.

Table 3.2: Example of port instances. Port: UNLOCODE of the port; Long: longitude; Lat: latitude; Draft: maximal draft to enter the port; Move: move: load and unload costs; Trans: transshipment costs; Fixed: fixed port call costs; Var: variable port call costs.

Port	Name	Country	Cabotage	Region	Long	Lat	Draft	Move	Trans	Fixed	Var
NLRTM	Rotterdam	NL	NL	North Continent Europe	4.5	51.92	13.5	195	148	19187	16
CNSHA	Shanghai	China	China	Central China	121.5	31.22	13.5	150	62	6497	6

An example for the port data is given in Table 3.2. The ports are based on real world data. Every port has a specific UNLOCODE. For example, the UNLOCODE of the Port of Rotterdam is NLRTM. The first couple of instances establish the location of the port. Draft gives the maximum draft of a vessel (in meters) to be able to enter the port. The move cost represent the cost of loading and unloading containers. The transshipment cost represent the cost of transshipping a container from one vessel to another. Finally, there are the fixed and the variable port call fees. The variable cost is per forty feet equivalent unit (FFE). All costs are in USD.

Table 3.3: The fleet. Vessel class denotes the name of the vessel. Cap FFE: max capacity of FFE of the vessel; TC: total cost rate; Draft: draft of the vessel; Fuel\*: fuel consumption at design speed; Fuel0: fuel consumption when idle; Pan: Panama canal fee; Suez: Suez canal fee.

Vessel class	Cap FFE	TC	Draft	Min Speed	Max Speed	Des Speed	Fuel*	Fuel0	Pan	Suez
Feeder_450	450	5000	8	10	14	12	18.8	2.4	64800	175769
Feeder_800	800	8000	9.5	10	17	14	23.7	2.5	115200	218445
Panamax_1200	1200	11000	12	12	19	18	52.5	4	172800	267217
Panamax_2400	2400	21000	11	12	22	16	57.4	5.3	345600	413533
Post_panamax	4200	35000	13	12	23	16.5	82.2	7.4		633007
Super_panamax	7500	55000	12.5	12	22	17	126.9	10		1035376

In Table 3.3, all possible vessels are shown. A vessel contains the following information: the capacity of the vessel; the total cost rate per day; the draft of the vessel; the minimum speed (in knots); the maximum speed (in knots); the design speed (in knots); the daily fuel consumption when cruising at the design speed (denoted by Fuel\* in the table); the daily fuel consumption when idle at port (denoted by Fuel0 in the table); The fee for crossing the Panama canal (impossible for Post\_panamax and Super\_panamax); The fee for crossing the Suez canal. The fuel costs is an important factor of the total costs of the model, to calculate these fuel costs F, the following formula is used.



$$F(s) = (s/v^F)^3 \cdot f^F, \quad s_{min}^F \leq s \leq s_{max}^F \quad (3.1)$$

Here,  $s$  represents speed,  $v^F$  represents the design speed,  $f^F$  represents the fuel costs at the design speed and  $s_{min}^F$  and  $s_{max}^F$  represent the min and the max speed respectively.

Table 3.4: Example cargo file

Origin	Destination	FFE Per week	Revenue	Transit Time
DEBRV	CNDLC	320	780	45

The next data instance is the cargo data. In Table 3.4, an example for a cargo file is shown. There is an origin and a destination. Next to this, there is demand of FFE per week and the revenue which one FFE yields. Lastly, there is the maximum number of days that the demand can be in transit. For simplicity, we will not take the transit time into account for our model.

Table 3.5: Example of a distance file

Origin	Destination	Distance	Draft	Panama traversal	Suez traversal
CNDLC	DEBRV	11129		No	Yes
CNDLC	DEBRV	14524		No	No

The last data instance is the distance data. In Table 3.5 an example of the distance file is shown. First of all, there can be multiple edges from one port to another. This is also shown in 3.5. Because it might not be economically interesting to enter either the Suez or Panama canal, there is also the possibility to take the long way round. A data file consists of the origin and destination of an edge, the distance of this edge, the maximum draft of the edge (if appropriate) and whether the edge traverses through either the Panama or the Suez canal. If an edge has a maximum draft, vessel which exceed this draft will not be able to travel over this edge.

# Chapter 4

## Methodology

The formulation of our model is similar to the formulation of the model of Brouer et al. (2013) since this model is reproduced. However, there is one flaw in the formulation of the model in Brouer et al. (2013). This will be explained in more detail when all the constraints will be discussed in depth. In the following pages, a short summary of all the sets, parameters, decision variables and the mathematical model will be discussed. At the end we will look into what the minimization function consists of and why all the constraints need to be taken into account.

### Sets

Firstly, there are several sets which need to be considered. These include:

- *R*: A rotation is a possible route of a vessel. A rotation includes an ordered list of all the ports it passes, the vessel which is considered for this route and the speed at which the vessel will be cruising at.
- *V*: Vessels are all the ships which are taken into account by the model. *V* consists of all the fleet data discussed in Chapter 3. For all the different instances, the only thing that can change for a vessel is the quantity of the vessels that can be deployed. All the other data is essentially linked to a certain vessel and has no variability.
- *P*: This set includes all the ports which will be taken into account for a certain instance. the ports set consists of all the port data from Chapter 3.
- *G*: This set indicates all available FFE that can be transported in the model. This set consists of the demand data. An origin of the FFE, a destination, the quantity of FFE and the revenue per FFE.

- $E$ : This set consists of all possible edges in the model. The distance data in the previous Section has all appropriate data for this set. If a Suez or Panama traversal is desirable for a shorter route, there will be at least 2 possible routes. one through the Suez/Panama channel and one without crossing those. It is possible that there are three possible edges with the same ports. This would be the case if an edge through the Panama channel is shorter than without crossing it, but also a traversal through the Suez channel is shorter than without crossing either of them.
- $E_r$ : edges in rotation  $r$ . There are a total of  $|R|$  different sets of  $E_r$ .
- $\Omega_r$  : set of ordered port triplets  $(h, i, j)$  in rotation  $r$ . Just as with  $E_r$ , there is a total of  $|R|$  different sets. Next to this, the port triplets are visited the same way as the vessel would visit them.

### Parameters

The following parameters will be used in the model.

- $a_{ij}^v$  : Costs of traversing either the Panama or Suez canal.
- $c^v$  : Capacity of a vessel of type  $v$  (in FFE).
- $d_j^v$  : port call costs for vessel  $v$  when entering port  $j$  (equal to the fixed costs plus the variable costs times  $c^v$ ).
- $e$  : fuel price per ton (\$600 for all ships).
- $f^v$  : daily running costs for vessel  $v$  over the entire planning horizon.
- $\tilde{f}^v$  : Cost or revenue for not operating vessel  $v$ . Depending on the markets condition this can be the revenue obtained from chartering the vessel out over the entire planning horizon, or the cost of laying up the vessel. For our model, we consider  $\tilde{f}^v$  to be equal to  $-f^v$ .
- $g^{vs}$  : Fuel consumption (tons per nautical mile) for vessel  $v$  steaming at speed  $s$  (this parameter is calculated using Equation 3.1).
- $h^v$  : Fuel consumption or vessel  $v$  when idle at a port.
- $k_{od}$  : The total demand from port  $o$  to port  $d$  over the entire planning period (in FFE).
- $l_{ij}^v$  : The distance in nautical miles from port  $i$  to port  $j$  using vessel  $v$ . (the length between these ports can differ for different vessels because either some vessels are not able to traverse through the Panama canal, or a traversal through either the Panama or Suez canals is beneficial for all vessels).

- $m_r$  : Number of round trips that can be done within the planning period. If the trip would not be finished at the end of the planning period, we assume that it will be finished in the following period. Therefore, this parameter does not have to be an integer.
- $p_j^v$  : The time spent at port  $j$  for vessel  $v$ . In our model, we assume this to be 24 hours for every vessel at every port.
- $q_{od}$  : The revenue from transporting one FFE from port  $o$  to port  $d$ .
- $\tilde{q}_{od}$  : The penalty when a certain FFE is not transported from port  $o$  to port  $d$ . In our model, we take this to be \$1000 USD for all cases.
- $s_r$  : The Speed a vessel is cruising at in rotation  $r$ .
- $u_j$  : The cost of lifting one FFE at port  $j$ .
- $t_j$  : The cost of transshipping one FFE at port  $j$ .
- $v_r$  : The vessel type used in rotation  $r$ .
- $z^v$  : Number of available vessels of type  $v$ .

### Decision variables

Lastly, these decision variables will be used in the model.

- $X_{(ij)d}^r$ . For each rotation  $r$ , this decision variable looks stores containers traveling along edge  $(i, j)$  which are headed for port  $d$ .
- $U_{(hi)d}^{rs}$ . This decision variable looks at the transshipments. the number of containers which are traveling to port  $d$  that arrive in port  $i$  via edge  $(h, i)$  of rotation  $r$  for transshipment to rotation  $s$ .
- $W_{id}^r$ . For each rotation  $r$ , this variable stores the containers arriving to their final destination port  $d$  via edge  $(i, d)$ .
- $V_{od}^r$ . For every rotation  $r$ , this variable stores the demand from port  $o$  to port  $d$  that enters the network for the first time because it is loaded to a vessel.
- $O_{od}$ . This variable stores the demand from port  $o$  to port  $d$  that will not be serviced by the liner company.
- $Y^r$ . Number of vessels assigned to rotation  $r$ .

## MIP

At this point, the MIP is presented. After the mathematical formulation, an explanation of the objective function and the constraints will be given.

$$\begin{aligned} \min \quad & Z_{mp} \\ = \quad & \sum_{r \in R} f^{v_r} Y^r + \sum_{v \in V} \tilde{f}^v (z^v - \sum_{r \in R: v_r=v} Y^r) \end{aligned} \quad (4.1)$$

$$+ \sum_{r \in R} m_r Y^r \sum_{(i,j) \in E_r} (eh^{v_r} p_j^{v_r} + eg^{v_r s_r} l_{ij}^{v_r} + d_j^{v_r} + d_{ij}^{v_r}) \quad (4.2)$$

$$+ \sum_{(o,d) \in G} (\tilde{q}_{od} O_{od} - q_{od} \sum_{r \in R} V_{od}^r) \quad (4.3)$$

$$+ \sum_{r \in R} \sum_{(h,i,j) \in \Omega_r} (u_i(W_{(hi)}^r) + \sum_{\substack{d \in P \\ d \neq i}} V^r id) + t_i \sum_{\substack{d \in P \\ d \neq i}} \sum_{\substack{s \in R \\ s \neq r}} U_{(hi)d}^{rs} \quad (4.4)$$

s.t.

$$X_{(hi)d}^r + V_{id}^r + \sum_{\substack{s \in R: \\ (k,i,l) \in \Omega_s \\ s \neq r}} U_{(ki)d}^{sr} = X_{(ij)d}^r + \sum_{\substack{s \in R \\ s \neq r}} U_{(hi)d}^{rs} \quad (4.5)$$

$$r \in R, \quad (h, i, j) \in \Omega_r, \quad d \in P, \quad i \neq d, \quad h \neq d,$$

$$V_{id}^r + \sum_{\substack{s \in R \\ (k,i,l) \in \Omega_s \\ s \neq r}} U_{(ki)d}^{sr} = X_{(ij)d}^r \quad (4.6)$$

$$r \in R, \quad (h, i, j) \in \Omega_r, \quad d \in P, \quad i \neq d, \quad h = d,$$

$$X_{(ij)d}^r = W_{(ij)}^r \quad (i, j) \in E_r, \quad r \in R, \quad (4.7)$$

$$O_{od} + \sum_{r \in R} V_{od}^r = k_{od} \quad o, d \in G, \quad (4.8)$$

$$\sum_{d \in P} X_{ijd}^r \leq c^{v_r} \cdot m_r \cdot Y^r \quad r \in R \quad (i, j) \in E_r, \quad (4.9)$$

$$\sum_{r \in R: v_r=v} Y^r \leq z^v \quad v \in V, \quad (4.10)$$

$$X_{(ij)d}^r, U_{(ij)d}^{rs}, W_{id}^r \in \mathbb{R}^+ \quad (4.11)$$

$$r, s \in R, \quad r \neq s \quad (i, j) \in E_r \quad d \in P,$$

$$O_{od}, V_{od}^r \in \mathbb{R}^+ \quad r \in R \quad o, d \in G \quad (4.12)$$

$$Y^r \in \mathbb{Z}^+ \quad r \in R. \quad (4.13)$$

The objective function is given by (4.1)-(4.4). The first term (4.1) establishes the daily running costs and the cost or revenue from excluding some vessels from operations. Term (4.2) establishes the fuel costs (both when idle at port and

when cruising), canal costs and port-calling fees. The third term (4.3) captures the revenue from transshipping a certain container but also the penalty for not transshipping a certain container. Finally, the last term (4.4) captures the costs of loading and unloading containers at the origin and at the destination. Next to this, the transshipment costs at transshipment points are determined.

Just as the objective function, all constraints have been taken from Brouer et al. (2013). However, Constraints (4.6) are not in Brouer et al. (2013). Ultimately Constraints (4.5) and (4.6) jointly balance the flow of containers at nodes other than their final destination. If only Constraints (4.5) would be considered, the occasions where  $h = d$  would never be taken into account. This leads to the possibility that when  $h = d$  containers whose origin is port  $i$  and who need to travel to port  $d$ , are not included in the flow of the containers. Therefore, the Variables  $V_{od}^r$  (where  $d$  is the final port before going back to port  $o$  in this rotation) in Constraints (4.8) can become whatever number they like without having to influence Constraints (4.5). This means that the model doesn't work properly if Constraints (4.6) wouldn't be taken into account. To go more in detail to what Constraints (4.5) and (4.6) actually do, when a certain number of containers is at port  $i$ , they can only continue to the next port in the same rotation, or transship to a different rotation.

Constraints (4.7) represent the number of containers that arrive to their final destination at port  $j$ . Constraints (4.8) account for the demand from port  $o$  to port  $d$ . If it is economically convenient and feasible to take this demand on for the liner company, the variables  $V_{od}^r$  will increase. If this is not the case,  $O_{od}$  will increase.  $O_{od}$  therefore ensures that a feasible solution is always possible in the MIP.

Constraints (4.9) impose restrictions on the total number of containers that can be transported on each edge of the rotation. The total number of containers that can be transported on a certain edge is calculated by the capacity of a vessel times the number round trips times the total number of vessels assigned on this rotation. Constraints (4.10) ensure that no more vessels than available are used in the model. Finally, Constraints (4.11)-(4.13) make sure that the variables are nonnegative.  $Y^r$  needs to be integer as well.

## 4.1 Rotations

A rotation includes an ordered number of ports and the edges in between these ports, a vessel and the speed at which the vessel is cruising. To be entirely sure that the MIP reaches the optimal allocation of rotations, all possible rotations of the ports, vessels and possible speeds should be taken into account. This does mean that already some rotations get removed. A vessel cannot enter a port which doesn't allow vessels with their draft. For example, a vessel with a draft of 12.5 cannot enter a port which has a maximum draft of 9.5. However, with so many rotations, the MIP becomes too large to run quite quickly. Brouer

et al. (2013) proved that the problem is strongly NP-hard. Therefore, we need to find ways in which we can decrease the number of rotations considered by the model.

Firstly, Brouer et al. (2013) have described that it is important that vessels arrive at a port on a weekly or biweekly basis. Due to the increased competitiveness in markets where vessels have a capacity of above 1200 FFE, these vessels should have a weekly frequency. The other vessels should have a biweekly frequency. To impose this on the rotations, we only include rotations which have a frequency of a week for vessels with a capacity of above 1200 FFE and a biweekly frequency of the vessels below this capacity. We do not only include rotations which precisely adhere to the (bi)weekly frequencies. For larger ships the frequency should be in between 6.3 and 7 days. For the smaller ships, the frequency should be in between 12.7 and 14 days. We do this because it is quite likely that there are rotations of commercial value which miss the frequency by a couple of hours. Next to this, due to changes in tides and weather, it might be a good idea to schedule some slack in the rotation. Ultimately, the constraints would look like the following:

$$6.3\tau_r \leq \frac{d_r}{24s_r} \leq 7\tau_r \quad \text{for } c^{v_r} \geq 1200 \quad (4.14)$$

$$12.7\tau_r \leq \frac{d_r}{24s_r} \leq 14\tau_r \quad \text{for } c^{v_r} < 1200 \quad (4.15)$$

In (4.14) and (4.15)  $d_r$  is the total distance of a certain rotation (in nautical miles),  $s_r$  is the speed the vessel is cruising at (in knots). The term in the middle will then be the number of days it takes to do one round trip.  $\tau_r$  is equivalent to the number of vessels needed to make this rotation have the required frequency. For example, if it takes 27 days for a vessel with more than 1200 FFE to make a round trip, four vessels will be needed to make this rotation have a weekly frequency. This means that for this instance,  $\tau_r$  will be equal to four. Note that we need a different interpretation of the  $Y^r$  variable now. we replace  $Y^r$  with  $\tilde{Y}^r$ .  $\tilde{Y}^r$  can best be described like this:

- $\tilde{Y}^r$ . This variable is not zero if the rotations is used in the objective value. The total number of vessels used in this rotations is given by  $\tau_r \cdot \tilde{Y}^r$ .

If a rotation is able to adhere to the appropriate equation, the rotation will be added to the set of  $R$  to be used in the MIP. However, if we want to have these rotations to have (bi)weekly frequency, some alterations need to be made in the MIP. Constraints (4.9) and (4.10) need to be altered to:

$$\sum_{d \in P} X_{ijd}^T \leq c^{v_r} m_r \tau_r \tilde{Y}^r \quad r \in R(i, j) \in E_r, \quad (4.16)$$

$$\sum_{r \in R: v_r = v} \tau_r \tilde{Y}^r \leq z^v \quad v \in V, \quad (4.17)$$

Next to this, in the objective function, all  $Y^r$  decision variable will be replaced by  $\tilde{Y}^r$  and they all need to be multiplied with  $\tau_r$ . This needs to be done since  $\tau_r \tilde{Y}^r$  is equivalent to  $Y^r$  in the original formulation. Lastly,  $\tilde{Y}^r$  replaces  $Y^r$  in (4.13). When all this is altered in the MIP, the model makes sure the (bi)weekly frequency will be adhered to.

The second way we decrease the number of rotations is the following. If there are two rotation that have the same ports but in a different order, we will only choose the rotations which is the shortest. Because we do not take transit time into account, this will have no implications to the objective value in the MIP. However, if transit times would be taken into account, it might be more valuable to have a rotation included which will be discarded here.

Lastly, there is one more way in which we limit the number of rotations. If several rotations have nearly all ports in common, one could choose to use only one of these rotations in the model instead of all of them. If a rotation is of a length of more than two, we will look at rotations which differ at most one port. If a there is a rotation which satisfies the fact that only one port differs in the rotation, we will choose the rotation which has the shortest length. We execute this heuristic once the other two methods of limiting routes have already been executed. This will remove a substantial number of rotations. Therefore, it might be the case that various useful rotation will not be taken into account in the MIP. However, the decrease in rotations which this heuristic causes could have the consequence that several instances become solvable within reasonable time.



## Chapter 5

# Results

In this section we will look into the results of the two instances. Firstly, we look into a single-hub instance around the Baltic sea. This instance includes eight ports, of which one is a hub (Bremerhaven). All the cargo is either directed to Bremerhaven or originates from Bremerhaven in this instance. Next to this, the available vessels will be four Feeder\_450's and two Feeder\_800's. For simplicity we will only look at rotation where the vessels cruise at the designed speed. The second instance is a combination of ports from Europe and Asia. This instance also has eight ports; four in Asia and four in Europe. We will look at five vessel types in this instance. They are: Feeder\_450, Feeder\_800, Panamax\_1200, Panamax\_2400 and Post\_panamax. For this model as well, we will only look at the designed speed for simplicity. The ports which were considered in both models can be found in Table 5.1. Next to this, for both models, we will not take rotations with more than five ports into account. The MIP would otherwise become too large to solve. In particular, we will look at the cost differences when which technique and limitations are used. Next to this, we will look into the computing time of the particular instances. To create the model and obtain the results, Cplex and java were used. To generate all possible rotations, we have used MATLAB. For further limitations of the number of possible rotations, Java was used. The computer which was used to perform the tests has Intel Core i3, 2.40 GHz with a RAM of 4 GB.

In Table 5.2, the objective value of various instances can be seen. This value is preferred to be negative since we are minimizing. A negative value means a profit. The Eurasia instance in Table 5.2 uses a maximum of 100 vessels; twenty vessels of the five used vessel classes. This has been done such that the MIP is able to use a large possibility of vessel allocation. This means that, ultimately, this instance receives a large amount of revenue from the chartering out of vessels it doesn't need. Only the small instance from EurAsia is able to be solved. The number of rotations in the larger instances is simply too big to be solved by the computer that is used. If we look at the Baltic instances, it is possible to see that medium and large instances do not differ in their optimal value. The only

Table 5.1: The used ports for the two instances. The location of the ports is included next to the maximal draft per port

(a) Baltic

Port	Name	Country	Draft
DEBRV	Bremerhaven	Germany	13.5
DKAAR	Aarhus	Denmark	12.5
FIKTK	Kotka	Finland	9.5
FIRAU	Rauma	Finland	9.5
NOSVG	Stavanger	Norway	12.5
NOAES	Alesund	Norway	9.5
NOKRS	Kristiansand	Norway	8
NOBGO	Bergen	Finland	9.5

(b) Eurasia

Port	Name	Country	Draft
DEBRV	Bremerhaven	Germany	13.5
DKAAR	Aarhus	Denmark	12.5
FIKTK	Kotka	Finland	9.5
FIRAU	Rauma	Finland	9.5
NOSVG	Stavanger	Norway	12.5
NOAES	Alesund	Norway	9.5
NOKRS	Kristiansand	Norway	8
NOBGO	Bergen	Finland	9.5

difference between them is that multiple rotations which have the same ports have been taken into account. It also shows that for this instance, one does not need to take these rotations into account to achieve a better objective value. However, the difference in computation time is quite large. It took 60 times the amount of time to solve the large instance instead of the medium instance with a little more than double the rotations as the medium instance. Table 5.3 illustrates this as well. The possibility for transshipments in the model creates a lot more complexity. Variable  $U_{(hi)d}^{rs}$  is used for the transshipments. As you can see, this variable is larger than all the other variables combined. Next to this, this variable grows the quickest. This has the result that rotations larger than 100 become very difficult to run on the computer that is used.

Table 5.2: Objective values and running times. A size of the instance refers to how many heuristics for limiting the total number of rotations have been used. With small, all heuristics have been used, with medium, only the final heuristic hasn't been used and with large only the first heuristic has been used. The running time is the total time the computer has been running, the solving time is the time cplex needed to solve the MIP. the objective value is the costs/revenue obtained in the best solution. Rot. stands for rotations used in the instance, Max rot. stands for the maximum number of rotations allowed.

	Instance	run time (in sec)	solving time (in sec)	opt. val. (in USD)	Rot.	Max rot.
Baltic	small	2.44	0.66	-2013453	6	5
	medium	262	58.70	-4451649	47	5
	large	15960	1538.92	-4451649	98	5
Eurasia	small	1163	161.37	-299625036	79	5
	medium	NA	NA	NA	1051	5
	large	NA	NA	NA	2312	5

Table 5.3: Number of variables for two instances. The decision variables used in the model. Refer to 4 for further explanation.

Instance	$X_{(ij)d}^r$	$Y^r$	$U_{(hi)d}^{rs}$	$O_{od}$	$W_{id}^r$	$V_{od}^r$
Baltic	1589	47	42456	56	227	1589
medium						
Baltic	3374	98	193548	56	482	3372
large						

Table 5.4: Separation of the costs of the objective value. All values are in USD and are in are to be multiplied with 1000. Z is the optimal value of the best solution obtained. R corresponds to the total revenue obtained from transporting the cargo.  $c_r$  is the total running costs,  $c_f$  the total fuel costs,  $c_p$  the port call costs,  $c_c$  the total canal costs,  $c_t$  the total transshipment costs and  $c_l$  is the total (un)loading costs.  $L_v$  is the revenue obtained from chartering out vessels. P is the penalty costs of not transporting cargo.

Instance	Z	R	$c_r$	$c_f$	$c_p$	$c_c$	$c_t$	$c_l$	$L_v$	P
Baltic small	2013	36497	2340	4796	8321	0	0	18795	4140	4371
Baltic medium	4452	38891	2700	6368	6689	0	0	20188	3780	2276
Baltic large	4452	38891	2700	6368	6689	0	0	20188	3780	2276
Eurasia small	299625	278508	41220	69210	6162	50225	8600	48625	246780	1620
Eurasia medium	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
Eurasia large	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

In Table 5.4 the objective value and the separation of the costs and revenues are shown. If we look at the Baltic instance, we see that the majority of the costs come from the loading and unloading costs. It seems reasonable that these costs are higher since the amount of traveling is relatively low. Furthermore, transshipments are not used in the Baltic model. Again, this seems reasonable since there is only one hub port and the ports are not the far apart from each other. If we look at the small Eurasian instance, we can see that the loading and unloading costs have a smaller influence on the total costs. More time is spent cruising and the Suez canal is crossed several times. We can also see that transshipments are being used.

## Chapter 6

# Conclusion

In this paper, we have replicated the MIP from Brouer et al. (2013). We have altered one thing in the formulation such that the formulation is now mathematically correct. We have looked at the differences in running time of instances with a certain number of rotations. It is possible to see that the running times grow very quickly, which should be the case since the problem is strongly NP hard. This has the result that large instances will be very hard to solve. Brouer et al. (2013) proposed a base heuristic which can solve larger instances heuristically, however not necessarily optimally. Next to this we have looked at what the influences are of these limitations on the possible rotations on the costs. Firstly, to limit the possible rotations, we have made sure all rotations are to a certain extent (bi)weekly. After this all rotations which have all the same ports as another but are longer in total length are deleted. Lastly, rotations which have all but one port in common are investigated. The one with the shortest distance will be considered by the MIP, the other will be discarded. We have seen that the largest and the medium instance of Baltic do not have a difference in optimal value. Therefore, the second route limitation might not have such a negative effect on the optimal value whilst still being able to discard several rotations. The last limitation has a negative effect on the optimal value, but is able to decrease the number of rotations drastically. However, this isn't necessarily the case for all instances. The last limitation might sometimes be needed because otherwise we wouldn't have been able to solve the Eurasia instance.

If we look at further limitations of our research, we can see that we are not yet able to solve the problem efficiently. Next to this, not every aspect of the liner shipping has been taken into account. Cargo usually does not have an indefinite transit time. Therefore, to make a complete model, one should take transit times into account. However, this is likely to increase the complexity. Because of this high complexity, it might be a good idea to look at improvements to solve the problem heuristically.

All in all, the LSNDP is a very hard problem to solve and further research on this aspect could turn out very fruitful. In a world where the competition is increasing rapidly, efficient routes of vessels is paramount.

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