### Cognitive Abilities & Ambiguity Attitudes

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#### Abstract

This thesis investigates the relationship between ambiguity attitudes and cognitive abilities. The question *Can some measures of cognitive abilities partially explain the ambiguity attitudes of an individual, and if so, what is the relationship between ambiguity attitudes and cognitive abilities?* is investigated in this research. This research uses survey data from a representative panel of respondents for the Netherlands. Two components of ambiguity attitudes, ambiguity aversion and ambiguity generated likelihood insensitivity, are measured through matching probabilities following Dimmock, Kouwenberg, and Wakker (2016). I considered two measures of cognitive abilities: the Cognitive Reflection Test and the Numeracy test. There does not seem to be a relationship between elicited ambiguity attitudes and the measures of cognitive abilities, the measures of cognitive abilities are not explanatory for the ambiguity attitude components. Further research should point out whether the results are robust and can be generalised to other populations.

**Keywords**: ambiguity attitudes, a-insensitivity, ambiguity aversion, cognitive abilities, numeracy, cognitive reflection

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## 1 Introduction

For many of our daily decisions, the exact probabilities of potential outcomes are unknown. In many situations, probabilities of some outcomes can only be guessed at and it is impossible to know what the future holds with certainty (S. T. Trautmann and Van De Kuilen, 2015). Such uncertainties, where probabilities of events are unknown, are ambiguous. Ambiguity seems to make agents uncomfortable and they are willing to pay a premium to avoid ambiguity (Ellsberg, 1961).

Agents thus have certain attitudes towards ambiguity, defined as their *ambiguity attitudes*. In this research, ambiguity attitudes are defined as a way of thinking or feeling towards ambiguity. Ambiguity attitudes have been found to be comprised of two components; *ambiguity aversion & ambiguity-generated likelihood insensitivity (a-insensitivity)* (Abdellaoui et al., 2011).

The first component, ambiguity aversion, indicates that an agent is willing to pay a premium to avoid ambiguity (Ellsberg, 1961). Agents are found to be averse to the options for which probabilities are unknown (the ambiguous option), and prefer options for which the probabilities are known (the unambiguous option). This preference is referred to as *ambiguity aversion* (Ellsberg, 1961). Agents differ in their preferences however, where some are more averse to ambiguity than others (Fox and Tversky, 1995).

When an agent is presented with an ambiguous and unambiguous option, ambiguity aversion is also dependent on the likelihood of the unambiguous option. The second component, a-insensitivity, captures agents' inability to discriminate between different levels of likelihoods, which implies the erroneous weighting of extreme events (events with a very low or very high probability of occurrence). Specifically, this means that for low probabilities of success (low likelihoods) agents are actually ambiguity seeking, while for high probabilities of success (high likelihoods), agents are ambiguity averse, relative to a 50-50 probability (0.5). In fact, agents often seem to transform subjective likelihoods towards 50-50 (Abdellaoui et al., 2011). Thus, a-insensitivity is mostly relevant for probabilities closer to 0 or 1, where the effect comes into play. Ambiguity aversion alone, as defined in this research, does not take different likelihoods into account.

It is important to note that ambiguity aversion is distinct from risk aversion. Risk aversion comes into play when agents are presented with two options and probabilities of outcomes are known for both. Risk aversion is widely researched (Allen, Weeks, and Moffitt, 2005; Allen, Weeks, and Moffitt, 2005; Kühberger, 1998), while research on ambiguity aversion and ambiguity attitudes in general is more limited, even though agents are more often confronted with ambiguity.

Given the importance of ambiguity attitudes in (economic) decision making, it is desirable to better understand ambiguity attitudes and its implications. It is useful to know what the underlying causes of the attitudes are to better understand the decision making process. The (economic) decision making process is influenced by many personal characteristics of an agent, alongside the agents' ambiguity attitudes. It could be useful to know which of these characteristics influence ambiguity attitudes directly as well. One of these characteristics could be the cognitive abilities an agent possesses, which is also an important factor in the decision making process (Frederick, 2005).

Cognitive abilities are defined here as the cognitive skills which are used to carry out tasks. Cognition is defined as the act or process of knowing, i.e. perception. Of course, cognitive ability is a broad concept with many components, in this research I focus on two specific components, which I suspect are related to ambiguity attitudes.

One component of an agents' cognitive ability is *cognitive reflection*. In tests for

cognitive reflection, it is measured in what way and in what degree agents might be able to override their initial 'intuitive' reaction and proceed more rationally (and take more deliberate time) in answering certain questions (Kahneman, 2011). The ambiguity attitudes of these individuals might then also reflect a more rational agent. It could be true that agents who score higher on some cognitive reflection test are somewhat more 'rational' in their conscious objective decision making process (Toplak, West, and Stanovich, 2011). One of the implications is that they might be less averse to ambiguity, as ambiguity aversion is inherently irrational. It could also imply that these agents show less a-insensitivity.

Another component of cognitive ability is the understanding of chance, proportions, risks, percentages and the ability to convert percentages to proportions and probabilities and vice versa. This rate of mathematical and statistical intuition, including the ability to attach meaning to numbers is referred to as *Numeracy* (Nelson et al., 2008; Kovas et al., 2013; Lipkus, Samsa, and Rimer, 2001). Intuitively, it seems plausible that especially a-insensitivity could be related to Numeracy as both are concerned with the understanding and evaluation of probabilities. In my opinion, this is the most interesting of the two components in terms of ambiguity attitudes being related to cognitive abilities. In fact, a-insensitivity can be seen as a cognitive bias (Aurélien Baillon, Bleichrodt, et al., 2013) as according to (Dimmock, Kouwenberg, and Wakker, 2016), a-insensitivity "undervalues preventive measures that reduce uncertainty without eliminating it, while overvaluing the complete elimination of uncertainty".

Higher Numeracy skills could lead to less a-insensitivity, because the meaning of probabilities are then more clear. With a more clear understanding of proportions and probabilities, agents might be better at differentiating between different chances and understand the real difference between e.g. a 0.5 and a 0.9 probability on an intuitive level. This should then become part of the agents' decision making process, making the agent less likely to misunderstand high or low probabilities and then less likely to transform subjective likelihoods towards 50-50.

I have noticed that literature regarding this relationship is scant, while it intuitively makes sense that the specific components cognitive abilities previously mentioned might influence ambiguity attitudes, at least when making deliberate decisions aimed at maximising utility. Thus, some ambiguity attitude measures could be explained by some measures of cognitive abilities.

If indeed there is a relationship between ambiguity attitudes and cognitive abilities, that insight could add to the general understanding of the decision making process under ambiguity and aspects which affect that process. When e.g. policy makers seek to influence behaviour under ambiguity, they might then also use the fact that agents with different cognitive abilities handle ambiguity differently and change policies accordingly. This research could also help further the understanding of cognitive abilities and their effects on economic behaviour of agents when they face uncertainty, which happens often for many agents.

Note that for the scope of this research, I only focus on ambiguity attitudes in the gain domain.

This thesis investigates the following research question:

Can some measures of cognitive abilities partially explain the ambiguity attitudes of an individual, and if so, what is the relationship between ambiguity attitudes and cognitive abilities?

The rest of this thesis is structured as follows; I will first give a review of the current literature surrounding this topic of cognitive abilities and ambiguity attitudes in Section 2. This section will also present the hypotheses which are tested to answer the research question. Afterwards, I go into detail regarding the data I use in Section 3. Specifically, this section elucidates how ambiguity attitudes are elucidated from survey questions by Dimmock, Kouwenberg, and

Wakker (2016). Finally, the results are presented in Section 4, followed by the conclusion and a discussion of possible further research in Sections 5 & 6.

### 2 Literature

In this section, the current literature related to the research question is discussed. To the best of my knowledge, there is no research done yet concerning the relationship between ambiguity attitudes as defined by Dimmock, Kouwenberg, and Wakker (2016) and cognitive abilities. Thus, the literature behind ambiguity attitudes and behind cognitive abilities, specifically, the two measures of cognitive ability mentioned in section 1, are explored separately.

This section starts with a discussion of the literature behind ambiguity attitudes and its specific components. This is followed by an exploration of the literary background regarding the relationship of cognitive abilities to preferences & (economic) decision making and two specific measures of cognitive abilities I aim to use to answer the research question. Finally, the hypotheses which aid in answering the research question are formalised.

#### 2.1 Ambiguity Attitudes

As mentioned earlier in Section 1, ambiguity attitudes affect many decisions, which are often made under uncertainty. The effect ambiguity attitudes have, have been confirmed to hold outside the 'laboratory' as well (Aurelien Baillon et al., 2016; Dimmock, Kouwenberg, Mitchell, et al., 2013). In that regard, as also mentioned earlier, it is an important topic, as it represents a systematic deviation from established economic theory based on rational agents (standard expected utility theory) (Machina and Schmeidler, 1992). Initially research was focused on ambiguity aversion in general. In recent years, the a-insensitivity component became more recognized.

Research regarding ambiguity aversion as defined today specifically started with the often cited groundwork in Ellsberg (1961). Further research has shown ambiguity aversion to be a real meaningful effect (MacCrimmon and Larsson, 1979; Maccheroni, Marinacci, and Rustichini, 2006; Fox and Tversky, 1995; Goldsmith and Sahlin, 1983) which holds for more than 50% of the respondents in the American Life Panel (Dimmock, Kouwenberg, Mitchell, et al., 2013), which is claimed to be representative for the US population. Note that the effect differs across different populations however (Rieger and Wang, 2012). The effect of ambiguity aversion holds for both the gain and loss domain with different types of framing according to Keren and Gerritsen (1999).

I find Literature regarding a-insensitivity specifically, to be very limited. Only in recent years researchers have begun paying attention to this component, with Abdellaoui et al. (2011) as an early example, introducing a tractable method of measurement. The current definition of a-insensitivity stems from the research in Einhorn and Hogarth (1985) and Fox and Tversky (1995). Previously, the sensitivity to different likelihoods (generated by ambiguity) was not taken into account often in literature. However, the general idea that people underweigh high probabilities and overweigh small probabilities has been around since Kahneman and Tversky (1979). According to Dimmock, Kouwenberg, Mitchell, et al. (2013), a-insensitivity is prevalent in both Dutch (75%) and US citizen (78%) populations. When an agent does not show a-insensitivity, that agent is able to discern different likelihoods appropriately when making decisions (under uncertainty).

A-insensitivity is quite robust and it increases when more information is received about a choice (Zimper, 2013). Note that a-insensitivity only holds when gains are considered (Ghosh and Ray, 1997); as mentioned in Section 1, this research is focused on the gain domain.

Regarding ambiguity aversion, its traits correlate with risk attitude as has been concluded by Maafi (2011), Ghosh and Ray (1997), Butler, Guiso, and Jappelli (2014), and Huang (2012) among others. These authors claim that generally, low ambiguity aversion correlates with risk seeking behaviour. However, there does not seem to be a scientific consensus regarding this relationship. Wilde, Krahnen, and Ockenfels (2014) and Kocher, Lahno, and S. Trautmann (2015) among others find that there is no close relationship between risk aversion and ambiguity aversion that holds on average when introducing e.g. lower likelihoods.

Aurélien Baillon, Koellinger, and Treffers (2014) showed that affective states matter for the level of a-insensitivity, a state of sadness is associated with low a-insensitivity, compared to states of joy and fear. This again indicates that ambiguity attitudes are not static and can also vary throughout a day.

Note that ambiguity attitudes are not static and the attitudes can be more pronounced, less pronounced or even reverse (from aversion to seeking or vice versa) over a certain period. One thing that can cause this is repeated sampling, agents seem to learn from repeated observations and they update their beliefs according to Ert and S. T. Trautmann (2014) who suggests that repeated experience causes changes in motivation. This is explained in a prospect theory framework, where the weighting of probability beliefs changes with repeated sampling from more pessimistic to more optimistic, creating a higher preference for ambiguity. Similarly, repeated sampling decreases a-insensitivity.

Dimmock, Kouwenberg, Mitchell, et al. (2013) showed that ambiguity aversion (but not a-insensitivity) in US citizen respondents is negatively related to financial market participation (e.g. the stock market). According to Dimmock, Kouwenberg, and Wakker (2016), a-insensitivity is also negatively related to stock market participation in Dutch respondents, agents who show more ainsensitivity are less likely to be active in financial markets. This implies that results and effects can vary, depending on the (sub)population.

As also pointed out by Heath and Tversky (1991), Fox and Weber (2002), and Chow and Sarin (2002), the knowledge of subjects towards a source of uncertainty matters. For instance, if an agent is well informed on the stock market, the agent is expected to be less averse to ambiguity regarding stock market decisions. The research in this thesis focuses on ambiguity attitudes in 'simple' gambling tasks, thus, knowledge does not play a role. As will become clear in Section 3, each agent has the same knowledge/information regarding the ambiguity attitude elicitation survey questions, thus 'expertise' is irrelevant.

#### 2.2 Cognitive Abilities and Decision Making

In this research I try to find a relationship between ambiguity attitudes and cognitive abilities. In this subsection, the literature regarding cognitive abilities and (economic) decision making is explored.

When the Intelligence Quotient (IQ) is used as a cognitive measure, it has been shown that agents who score higher live longer (Arden et al., 2015). A higher IQ score is also related to a better working memory (Engle et al., 1999) and a higher income (Zagorsky, 2007). Not many authors have researched the relationship of cognitive abilities (i.e. measures of cognitive abilities) and (economic) decision making abilities. Specifically, regarding decision making under ambiguity, the relationship with cognitive abilities is rarely explored.

Cognitive abilities could be explanatory for certain types of decision making behaviour (Frederick, 2005; Waraich, 2016). Zamarian, Weiss, and Delazer (2011) and Gleichgerrcht et al. (2010) showed that agents with mild cognitive impairment show higher ambiguity aversion and risk aversion based on the 'Iowa Gambling Task' and the 'Probability-Associated Gambling' measure. The cognitive impairment mentioned, concerns problems with memory, language, thinking and judgement that are greater than normal age-related changes.

I find that in literature, the relationship between cognitive abilities and risk attitudes is explored more often than the relationship to ambiguity aversion. Benjamin, Brown, and Shapiro (2013) find that risk aversion is less prevalent in students with higher standardised test scores, compared to other students, in a sample of Chilean high-school students. Based on a sample of trainee truck drivers, Burks et al. (2009) finds that higher cognitive abilities is related decision making, agents (the trainee truck drivers) are more prone to make decisions that favour their economic success when they have higher cognitive abilities. The results are based on a version of a culture fair non-verbal IQ test, Raven's progressive matrices (Raven, 1998).

In general, risk aversion is negatively related to cognitive ability (Frisell, Pawitan, and Långström, 2012; Jaeger et al., 2010), though authors such as Andersson et al. (2013) contend that the relationship is spurious. In their own experiments, by using choice tasks that vary the bias induced by random choices, the authors are able to generate both a negative and a positive correlation. The authors then state that cognitive abilities are not related to risk aversion, but rather related to random decision making. Factors besides cognitive ability that affect risk taking behaviour are in some cases difficult to control for.

Stanovich and West (2000) argues that some measures of cognitive ability such as the SAT score (standardised test used for college admissions in the USA) correlates with some behavioural biases. These are, among others: denominator neglect, belief bias, and matching bias on the 4-card selection task.

To make cognitive abilities quantifiable, two measures are used in this research. Both measures are based on answers to survey questions and they focus on specific components of cognitive ability. As will become clear in the next subsections, they are both popular measures of cognitive abilities in literature, and have been shown to correlate with many traits related to decision making. Note that the captured component of cognitive abilities is very much dependent on the measure of cognitive abilities (Liberali et al., 2012).

The first measure is the *Cognitive Reflection Test (CRT)* (Frederick, 2005). The second measure is the *Numeracy test* (Lipkus, Samsa, and Rimer, 2001). These measures are explored in the next two subsections. Though both the CRT results and Numeracy test results are correlated (Cokely and Kelley, 2009), they do not measure the same things and are not substitutes for each other

(Szaszi et al., 2017; Welsh, Burns, and Delfabbro, 2013; Liberali et al., 2012).

#### 2.2.1 Cognitive Reflection Test

The CRT could be seen as a measure of the ability to activate system 2 as described in Kahneman (2011). This means that it measures the tendency to override an initial immediate 'intuitive' or 'impulsive' response with *reflection*, a slower more deliberate response, to determine the correct answer (Toplak, West, and Stanovich, 2011), as mentioned earlier in Section 1. The CRT questions are designed such that the initial intuitive response is often wrong.

The CRT as introduced by Frederick (2005) comprises of three short questions, presented in the list below:

#### **Cognitive Reflection Test Questions**

- A bat and a ball cost \$1.10 in total. The bat costs \$1.00 more than the ball. How much does the ball cost?
- If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?
- In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?

As can be seen, the CRT is a quite 'simple' measure, for a specific component of cognitive abilities (reflection). According to Campitelli and Gerrans (2014) and Pennycook, Cheyne, Koehler, et al. (2016), the CRT can adequately evaluate (cognitive) reflection. Again, that is the cognitive process which is slower and more reflective compared to the cognitive process which is fast and intuitive. These processes might also be referred to as system 2 and system 1 respectively

(Stanovich and West, 2000; Kahneman, 2011). The CRT questions are designed such that a system 1 type answer springs to mind immediately, which is often wrong. The CRT then measures the ability to suppress the 'intuitive' response, as mentioned before.

According to Oechssler, Roider, and Schmitz (2009), results of the Cognitive Reflection Test are related to two cognitive biases, the conjunction fallacy and the tendency to be conservative when updating probabilities. Hoppe and Kusterer (2011) finds the CRT scores to be related to the base rate fallacy, the conservatism bias, overconfidence, and the endowment effect. A higher CRT score also reduces susceptibility to anchoring according to Bergman et al. (2010).

The CRT score is also related to more rational behaviour when dealing with the 'Newsvendor problem' (Moritz, Hill, and Donohue, 2013), 2013), where an agent must determine the quantity of inventory: ordering too much creates leftover stock while ordering too little creates lost sales (Schweitzer and Cachon, 2000). Taylor (2013) shows that cognitive abilities as measured by the CRT is negatively related to risk aversion for hypothetical choices, but not for real choices.

High CRT scores have also been found to correlate with better performance on reasoning tasks (Toplak, West, and Stanovich, 2011; Lesage, Navarrete, and De Neys, 2013; Sirota, Juanchich, and Hagmayer, 2014), scientific understanding (Shtulman and McCallum, 2014) and religious belief (Gervais and Norenzayan, 2012; Pennycook, Cheyne, Seli, et al., 2012; Shtulman and McCallum, 2014; Shenhav, Rand, and Greene, 2012).

It could be concluded that CRT results are predictive of many traits and many aspects of the decision making process. Generally it could be said that low cognitive ability (in terms of cognitive reflection measures) are associated with higher susceptibility to behavioural biases. The CRT is related to other measured of cognitive abilities, such as the Intelligence Quotient, though it captures different dimensions. The CRT specifically is a better predictor of susceptibility to behavioural biases in decision making than other measures of cognitive abilities, such as the Wechlser Abbreviated Scale of Intelligence (Frederick, 2005; Bjöörn and Karlsson, 2015).

#### 2.2.2 Numeracy and the Numeracy Test

According to Dehaene, Piazza, et al. (2003), Numeracy is separate from other types of intelligence. With its own specialised neurological circuitry, it could be said that Numeracy is a component of cognitive ability. Dehaene and Cohen (1997) showed that when areas of the brain concerning Numeracy are damaged, other brain functions related to cognitive abilities are unaffected. Different components of cognitive ability can be developed somewhat separately (Blakemore and Bunge, 2012; Roediger and Pyc, 2012; Zamarian, Ischebeck, and Delazer, 2009). Though again, Numeracy is not completely separate from other brain functions related to cognitive abilities. Its connection to literacy has been shown by Steen (2001).

Numeracy is very much related to decision making and greater numeracy skills reduces susceptibility to framing effects according to Levin, Schneider, and Gaeth (1998) and Ellen Peters, Daniel Västfjäll, et al. (2006). Greater Numeracy also reduces the influence of emotional states or 'moods' on decision making according to D Västfjäll, E Peters, and Starmer (2011). When controlled for other intelligence measures such as the SAT test, Numeracy still has a significant effect (Ellen Peters, Hart, and Fraenkel, 2011).

The specific component of cognitive abilities that Numeracy captures ties into a-insensitivity as both deal with interpreting probabilities, which makes it an interesting measure regarding the research question. I suspect that a-insensitivity is a direct result of lacking Numeracy skills. When properly understanding what probabilities and proportions mean, high and low probabilities can be more accurately assessed. A-insensitivity represents a basic intuitive view of probabilities, with Numeracy skills, this view should shift into a view closer to reality (the view of a rational agent).

Petrova, Pligt, and Garcia-Retamero (2014) finds that numerical abilities are related to affective responses to risks. If outcomes are affect-rich, and cognitive abilities are low, then small probabilities are more overweighted by respondents in a population of Dutch university students. These authors did not consider choice under ambiguity however.

Similar to cognitive reflection, Numeracy can also be deduced via questionnaires/surveys. For my research question specifically I choose to employ the 11 questions pertaining to Numeracy as proposed by Lipkus, Samsa, and Rimer (2001). These authors add eight questions to the original three Numeracy elicitation questions proposed by Schwartz et al. (1997). Other (objective) Numeracy elicitation tests include the Berlin test by Cokely and Kelley (2009). The test by Lipkus, Samsa, and Rimer (2001) is chosen because of its proven track record (Mejia, 2015).

Used in many studies since its introduction, the aforementioned 11 question Numeracy test results have been shown to be related to medical decision making, insurance decision making, stock market behaviour and many other fields (Lipkus and Ellen Peters, 2009). Some have criticised the test for not being difficult enough to differentiate agents who are highly educated (Cokely and Kelley, 2009), however, the test is sufficient in differentiating between high and low Numeracy skills in the general population (Lipkus and Ellen Peters, 2009).

The original three questions are based on hypothetical dice rolls and lottery ticket winning odds. The additional eight are framed in the health domain, however the context does not matter in assessing global Numeracy skills (Lipkus, Samsa, and Rimer, 2001). The extension with eight questions seems to be

useful for a higher 'resolution' in the answers given.

The 11 questions, taken directly from Lipkus, Samsa, and Rimer (2001), are presented in the list below:

- Which of the following numbers represents the biggest risk of getting a disease? 1 in 100, 1 in 1000, 1 in 10
- Which of the following represents the biggest risk of getting a disease?
   1%, 10%, 5%
- If the chance of getting a disease is 10%, how many people would be expected to get the disease out of 100?
- If the chance of getting a disease is 10%, how many people would be expected to get the disease out of 1000?
- If the chance of getting a disease is 20 out of 100, this would be the same as having a \_% chance of getting the disease.
- If Person A's risk of getting a disease is 1% in ten years, and Person B's risk is double that of A's, what is B's risk?
- If Person A's chance of getting a disease is 1 in 100 in ten years, and Person B's risk is double that of A, what is B's risk?
- The chance of getting a viral infection is .0005. Out of 10,000 people, about how many of them are expected to get infected?
- In the BIG BUCKS LOTTERY, the chances of winning a \$10.00 prize are 1%. What is your best guess about how many people would win a \$10.00 prize if 1,000 people each buy a single ticket from BIG BUCKS?
- Imagine that we roll a fair, six-sided die 1,000 times. Out of 1,000 rolls, how many times do you think the die would come up even (2, 4, or 6)?
- In the ACME PUBLISHING SWEEPSTAKES, the chance of winning a car is 1 in 1,000. What percent of tickets of ACME PUBLISHING SWEEPSTAKES win a car?

Question 1-7 are the additional questions from Lipkus, Samsa, and Rimer (2001), the final three questions are the original Numeracy elicitation questions from Schwartz et al. (1997).

#### 2.3 Demographics

There are possible many variables explanatory for ambiguity attitudes. These variables can be added to remove biases from the independent variables (i.e. the omitted variable bias). This makes it more clear that the dependent variable can be explained by the independent variable and not some other underlying variable influencing the main independent variables. When omitted, the underlying variable might be strongly related to the independent variable, this means that the seemingly significant effect of the independent variable is actually due to the underlying variable.

It has been shown that there are gender differences concerning ambiguity attitudes (Borghans et al., 2009), in terms of activity on financial markets, females are more ambiguity averse than males. When insurance is concerned, males are more ambiguity averse than males. It is unclear what the gender differences are when there is no real context, as is the case in this research, see Section 3.

According to Dimmock, Kouwenberg, and Wakker (2016) when a gender variable is regressed together with many other variables such as financial literacy, income, total financial assets, household size, education, gender is not significant in explaining any of the ambiguity measures. This means that gender does not explain any of the ambiguity attitude components mentioned earlier. The same holds for age (of the respondent), except for a-insensitivity, where it is significant on the 10% level. This is tested again in Section 3 for this research.

Tolerance for ambiguity has been shown to be related to age (Tymula et al., 2012). Age is also very much related to cognitive abilities (Albert, Duffy, and Naeser, 1987), However, according to Toplak, West, and Stanovich (2011), age is not related to CRT answers. Age is related to Numeracy answers in the Numeracy test however, according to Mejia (2015).

As mentioned in Sutter et al. (2013), the ambiguity indices are only weakly

related to many other demographic variables. For that reason, only age and gender are added in this research.

#### 2.4 Hypotheses

Taking the main research question and the available research in mind I arrive at eight different hypotheses. The hypotheses are based on the relationship between the ambiguity indices and the cognitive ability measures. Specifically, they state the direction of the relationship. Ambiguity aversion and a-insensitivity can be both negative and positive. Agents might deviate from a (rational) neutral position by e.g. being more ambiguity seeking or more ambiguity averse when ambiguity aversion measures are concerned. I also want to investigate the deviation from a neutral (rational) position only, disregarding the actual direction of the effect. This could aid in answering the research question, as such hypotheses are less strict.

The first four hypotheses concern Numeracy. The first hypothesis in this research is that agents who perform bad in Numeracy tests, also have a higher a-insensitivity score. This, because it seems plausible that an agent might underor over estimate probabilities towards 50-50 when an agent has a limited grasp of Numeracy. Further, Numeracy has been shown to be related to the decision making process and to various behavioural biases. The second hypothesis states that higher Numeracy indicates lower ambiguity aversion, agents with higher Numeracy might handle ambiguity more rationally. It is also interesting to investigate the deviation from ambiguity neutrality, disregarding the distinction of agents being ambiguity seeking or ambiguity averse, or agents showing a-insensitivity, or the opposite of a-insensitivity. This leads to hypotheses four and five. These hypotheses should help answer whether or not there is a relationship at all (regardless of direction) between Numeracy and the two ambiguity attitude components.

More formally the first four hypotheses are stated as follows:

- H1: Numeracy score is negatively correlated to a-insensitivity.
- H2: Numeracy score is negatively correlated to ambiguity aversion.
- H3: Numeracy score is related to deviation from ambiguity neutrality.
- H4: Numeracy score is related to deviation from the absence of a-insensitivity.

The last four hypotheses concern the CRT score. Hypothesis five states that agents who score higher on cognitive tests are less ambiguity averse. Earlier in this section it was pointed out that cognitive abilities are related to a myriad of behavioural biases. This is an indication that cognitive abilities could also be related to ambiguity aversion and a-insensitivity. The reasoning is that having non-neutral ambiguity attitudes is an irrational tendency and agents who have higher cognitive abilities (i.e. cognitive reflection ability) might have a lower tendency to show the attitudes captured by the two ambiguity attitude components, as these agents have been shown to avoid other biases. Again, the last two hypotheses are less strict and help in finding out of there is any relationship between CRT score and deviation from ambiguity neutral attitudes (i.e. having non neutral attitudes).

More formally the last four hypotheses are stated as follows:

H5: CRT score is negatively related to ambiguity aversion.

- H6: CRT score is negatively related to a-insensitivity.
- H7: CRT score is related to deviation from ambiguity neutrality.
- H8: CRT score is related to deviation from the absence of a-insensitivity.

With the acceptance or rejection of these hypotheses, the research question can be answered.

## 3 Data & Variables

In this section I will delve further into the data used to answer the research question. This section starts with explaining the source of the data for ambiguity attitudes in detail. To help understand in what way they provide the information needed, this part essentially also contains a literature review. New variables are constructed from the answers of ambiguity attitude related survey questions, these variables are elaborated on in this subsection. Afterwards the cognitive ability related variables used are described in detail. Finally, the control variables and timing of the surveys are touched upon.

As mentioned before, the data comprises of survey question answers. The group of respondents in the surveys is the 'Longitudinal Internet Studies for the Social Sciences (LISS)' panel which comprises 5000 diverse households (with 7000 respondents) in the Netherlands. The organisation behind the LISS panel, CentERdata, has been collecting data through new surveys since 2007. One of the goals of this organisation is that the panel is representative of the dutch population, this representativeness has been verified by Knoef and Vos (2009). CentERdata also makes sure to minimise selection bias, e.g. the surveys are filled in by respondents at home digitally and respondents who do not have a computer are provided with one.

After extracting the data from the database, transformations had to be made, and missing values had to be dealt with. This is discussed in Appendix Subsection 7.1. After the transformations, the data set consists of 213 observations. The variables include results on ambiguity attitude elicitation questions, cognitive ability related questions and demographic variables.

Note that the sample size is 'sufficiently' large (with 213 observations). Based on a statistical power of 0.95, a minimum of 89 observations is needed (calculated using G\*Power software). However, the sample size is still somewhat limited and some caution is warranted when extrapolating results to the whole population. However, in my opinion, the results can at least be extrapolated to the respondents of the whole LISS panel, as most observations are based on a (completely) random subset of the LISS respondents.

#### 3.1 Measures of Ambiguity Attitudes

In Section 2 it was mentioned that literature behind ambiguity attitudes (and its components) is somewhat scarce. Some authors such as Dimmock, Kouwenberg, and Wakker (2016) also noted the lack of empirical research concerning ambiguity attitudes and economic decision making. These authors opted to research ambiguity attitudes using a large group of respondents, which was also the LISS panel. The results are in the same direction (i.e. show correlation with the same sign) of many other studies (Noussair et al., 2013).

To quantify ambiguity attitudes, respondents from the LISS panel group were presented with an ambiguity attitude elicitation survey and they were rewarded with real (monetary) incentives. This research was carried out in 2010. The survey was initially proposed to a large group of respondents, of which 666 provided usable results. The reduction in number (of respondents) stems from removing observations where agents e.g. spent 3 seconds or less on a question or agents giving the exact same answer on each question. These observations might have biased the results and they are assumed to be non-truthful.

The survey answers from respondents who failed the (two) control questions are not taken into account here, because they also might bias the results. The first control question is based on the matching probability of the respondent and making the unambiguous box even more attractive. If the respondent switches, the respondent is inconsistent because it goes against their matching probability (assuming that more utility is better). In this research I then assume that this respondent did not pay attention when filling in the survey, as the switch is inconsistent with preferences elicited from the other questions. Control question two makes the ambiguous box more attractive and checks for inconsistencies in the opposite direction.

The survey was executed by Dimmock, Kouwenberg, and Wakker (2016) and the survey results were added to the LISS database, this data is used in this research. That is, the answers to the ambiguity attitude related questions from the LISS panel. Thus, it is important to know how the data is constructed, therefore the methods used are explained in detail in the following paragraphs.

Dimmock, Kouwenberg, and Wakker (2016) constructed indices for ambiguity aversion and a-insensitivity using elicited *matching probabilities*.

Respondents went through three games with a maximum of six rounds each (preceded by an example question) for games two and three and a maximum of five for game one, where each game is a choice problem presented to the respondent. The questions are all essentially binary choice problems but also include a third choice option labelled 'indifferent', which would end the game. Both choices are gambles, one of which has unknown probabilities (ambiguous) and the other known probabilities (unambiguous). Both options are visualised as boxes containing 100 balls. To illustrate, the first game (game one), based on initial winning probability of p = 0.5 for the unambiguous option, is elaborated on in the next paragraph. See also Figure 1 below, which presents the survey question for game one as presented to the LISS panel by Dimmock, Kouwenberg, and Wakker (2016).

In game one, respondents must choose between two boxes in each round. The boxes contain 100 balls in two different colours. The proportion of the different colours is known for one box (the unambiguous box), and unknown for the other (the ambiguous box). As mentioned before, in the first game all boxes contain 100 balls. For the unambiguous box, the distribution is known, e.g. 50 purple balls and 50 yellow balls. This of course gives a 0.5 winning probability for the

known box. For the ambiguous box, the distribution of colours is unknown. Figure 1 shows exactly how the question was presented to respondents.

One colour is designated beforehand as the 'winning colour' by the respondents. The freedom to choose the winning colour is meant to increase trust in the experiment (and remove suspicion). Note that very few respondents (<2%) chose to change the default winning colour (purple). This indicates that trust in the experiment was already present for most respondents, which is important because distrust might increase ambiguity aversion, which would bias the results (Pulford, 2009).

The respondent would receive a monetary reward  $(15 \in)$  when a ball with the winning colour is 'grabbed from' the box (this happens virtually).

The difference between each of the three games is the initial (objective) chance of winning for the known risk option (p) in the first round, which are 0.1, 0.5, 0.9respectively for games 2, 1, 3. This is actualised by introducing 8 additional ball colours for games two and three. Note that for game three, purple is no longer the winning colour, each of the other nine colours are the winning colours instead.

Note that a rational agent would assume that the ambiguous box has a similar distribution of colours as the unambiguous box. This is because, without any information, the distribution of purple balls (subjective win probability) can be assumed to be uniform, with an average of 50, 10 or 90 for game 1, 2 and 3 respectively. This means that rational agents are indifferent right away because the win probabilities are exchangeable, which is of course by design (Abdellaoui et al., 2011). Again, an assumption made here is that respondents 'trust' the experiment and assume that the distribution of the ambiguous probability is not biased.

For each round in a game, the chance of winning with the known probability changes, and the change is based on the previous answer, using the following rule:

new chance of winning = previous chance of winning (e.g. 50%) + ceiling (100%) / 2 (= e.g. 75%).

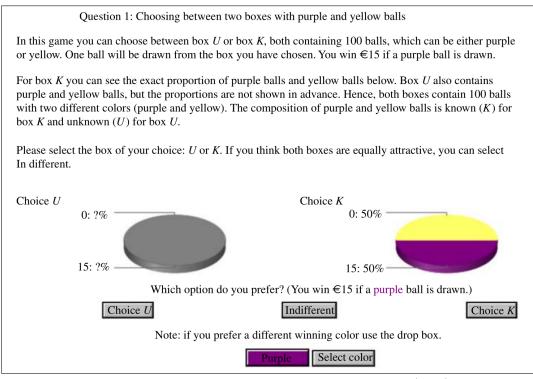
When the respondent chooses the known probability option (the unambiguous box), for the next question this known probability choice would then be made less attractive (e.g. lower number of purple balls 'X'). This process continues until the respondent chooses the 'indifferent' option, or until all six questions are answered. This means that of the six questions only the final answer contains the information needed. From this final answer the authors deduce the *matching probability* m(p). This is the 'X', the number of purple balls in the unambiguous box, for which the respondent is indifferent between boxes. This X is then divided by 100: m(0.5) = X/100. Here 'X' can also be seen as the chance of drawing a purple ball (in percentages), as the total number of balls is 100, or the chance of not drawing a purple ball when game three is concerned.

Note that the respondent is ambiguity averse when m(0.5) < 0.5. Where m(p) entails the specific probability (of a positive outcome) of the known option, which makes the respondent indifferent (between the known and unknown option). Thus, ambiguity aversion can be seen as how much 'value' a respondent is willing to give up in exchange for more certainty of the risk distribution (i.e. the chances of winning and losing).

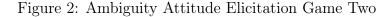
Generally, when respondents are indifferent (m(0.5) = 0.5), they are ambiguity neutral. When they prefer the ambiguous box (m(0.5) > 0.5) they are ambiguity seeking and when they choose the unambiguous box, they are ambiguity averse (m(0.5) < 0.5).

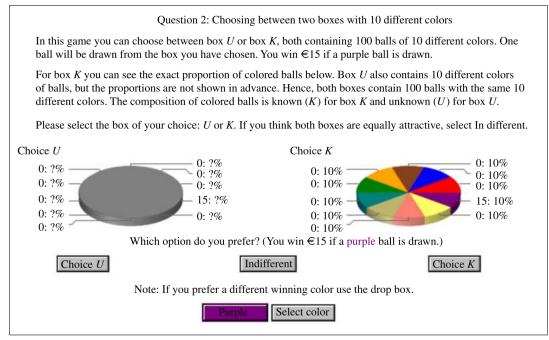
The results of the games where p = 0.1 and p = 0.9 (game two and three) can be used to deduce a-insensitivity. These games are constructed similarly as for p = 0.5, only now there are 100 balls and 10 different colours, as mentioned earlier. When p = 0.1, for the known proportions option, there are 10 balls in each colour, this means 10 purple balls, making the winning probability 0.1. When p = 0.9 again, there are 10 purple balls, however, in this case, purple means losing. Figure 2 below shows exactly how the game with p = 0.1 was presented to respondents, for clarity.

#### Figure 1: Ambiguity Attitude Elicitation Game One



This figure is an excerpt from Dimmock, Kouwenberg, and Wakker (2016), presenting a screenshot of survey question one, the first round in game one. The winning probability for the unambiguous option in game one is 0.5, represented by the purple ball.





This figure is an excerpt from Dimmock, Kouwenberg, and Wakker (2016), presenting a screenshot of survey question two, the first round in game two. The winning probability for the unambiguous option in game two is 0.1, represented by the purple ball.

#### 3.1.1 Ambiguity Indices

Now for the actualisation of the research, the results of the choice question survey are used to construct a specific ambiguity index for each game and each respondent. These indices capture all the information from the survey questions for each game into a few variables, which is helpful when performing regression analysis (see Section 4).

Note that for this research, these ambiguity index values have to be constructed based on the 213 respondents. These are the respondents who filled in the ambiguity elicitation game survey as well as the surveys related to cognitive abilities.

Define Ambiguity Index = p - m(p) for each respondent, where m(p) (the

matching probability) is derived from the answer in the final iteration of each round. This final iteration is again, the first iteration for which the respondent chooses the 'indifferent' option, or the last possible round of the game. This could of course be the first of all rounds for some respondents. It might be somewhat problematic when indifference is not reached in 6 rounds. This would imply that more rounds are needed to reach the matching probability. The authors opt to take the average of the lowest and highest probability for the unambiguous box in that case, this should approximate the matching probability.

The proportions of the 213 respondents who were not indifferent in the final round are 68% for game one, 65% for game two and 70% for game three. The proportions of the 213 respondents who immediately indifferent in the first round are 15% for game one, 19% for game two and three.

The ambiguity indices mentioned earlier are the same indices used by Dimmock, Kouwenberg, and Wakker (2016), based on Kahn and Sarin (1988) and Jaffray (1989).

The Ambiguity Indices created are dubbed AA01, AA05, AA09 for initial probabilities p = 0.1, p = 0.5, p = 0.9 respectively. Note that, when respondents are ambiguity seeking when p = 0.9 and ambiguity averse when p = 0.1, then this is an indication of a-insensitivity for these respondents. Also note that, positive values for these indices indicate ambiguity aversion. Negative values indicate ambiguity seeking attitudes and zero values indicate ambiguity neutral attitudes. Note again that p denotes the initial win probability for the unambiguous box.

Dimmock, Kouwenberg, and Wakker (2016) also construct overall indices, combining information from all three games. This is done by using *Ordinary Least Squares (OLS)*, to find the best fitting line between m(p) and p. The indices together define the *neoadditive weighting function* as mentioned in Abdellaoui et al. (2011). Note that these authors also mention that the best fitting line should be on an open interval (0,1) as the values regard probabilities. That means that the sum of the slope and the intercept are  $\leq 1$ , it should also hold that the regression coefficient and slope are always positive. This is not taken into account in this research for feasibility reasons, that means that some bias is present in the values of the overall indices, especially a-insensitivity might be somewhat understated. This should be noted as a caveat of this research, however, the indices are still very much similar to the AI values in Dimmock, Kouwenberg, and Wakker (2016), see Table 1.

To construct the overall indices, OLS estimates  $m(p) = \alpha + \beta p$  with  $\alpha$  the intercept and  $\beta$  a coefficient for p. That is, with p = 0.10.5 & 0.9 (thus, within a 3x2 matrix) for each respondent. Then,

$$b = 1 - \beta - 2\alpha \tag{1}$$

can be used as general index of ambiguity aversion, combining the information generated from all three games, for each respondent (Abdellaoui et al., 2011). This general index, referred to as index b, is interpreted the same as the specific indices (AA01, AA05, AA09).

A-insensitivity is also a component of ambiguity attitudes, while different from index b. The aforementioned authors calculate the *a-insensitivity* index a, defined as,

$$a = 1 - \beta. \tag{2}$$

Values greater than zero imply a-insensitivity. Values equal to zero indicate no a-insensitivity and values below zero imply the opposite effect of a-insensitivity. Thus, from the survey question answers, five variables are constructed, which are used as dependent variables to answer the research question of this thesis. The indices a, b, and the three Ambiguity Indices AA01, AA05, AA09. The main dependent variables are indices a and b, which proxy a-insensitivity and ambiguity aversion respectively. These are the main components of an agents' ambiguity attitude.

This research focuses on results based on real incentives, the aforementioned authors do this as well. Results based on real incentives might be more close to real world behaviour, compared to results based on hypothetical incentives (Read, 2005).

In Table 1 all the Ambiguity Indices, based on the answers of the 213 respondents in the data set, are presented. To reiterate, m(p) entails the specific probability (of a positive outcome) of the known option, which makes the respondent indifferent between the known and unknown choice option. The indexes AA01, AA05 and AA09 are the ambiguity indexes calculated as p - m(p). Index a and b are general indices based on OLS estimates, as described earlier. The mean, median, min and max values are quite similar to the values presented in Dimmock, Kouwenberg, and Wakker (2016), adding to their validity.

Figure 3 presents histograms for the main dependent variables, the indices a and b. For index a, it seems that a large number of respondents (30) show no a-insensitivity, with values close to zero, or zero. Some respondents display a negative value, this would indicate the reverse effect of a-insensitivity, where the slope  $\beta$  for the best fitting line as described earlier is larger than one. The largest part of the respondents however show some a-insensitivity. Also, note that negative values for index b indicate ambiguity aversion, and positive values indicate ambiguity seeking behaviour. Index b shows a large concentration of values around zero. A value of exactly zero for b would imply neither ambiguity aversion or seeking. Figure 4 shows (absolute) deviations from ambiguity neutrality and (absolute) deviation from absence of a-insensitivity. Figure 2 presents the distribution of behaviour in the three different games. Here it is clear that respondents are more ambiguity seeking when p = 0.1for the unambiguous box, compared to the game where p = 0.9. A sizeable (15 20%) number of respondents are ambiguity neutral in each game.

	Min	Max	Median	Mean	SD
m(0.1)	0.03	0.97	0.10	0.22	0.26
m(0.5)	0.03	0.97	0.41	0.40	0.23
m(0.9)	0.03	0.98	0.90	0.73	0.31
AA01	-0.87	0.08	0.00	-0.12	0.26
AA05	-0.47	0.47	0.09	0.10	0.23
AA09	-0.08	0.87	0.00	0.17	0.31
index a	-0.19	1.69	0.21	0.37	0.42
index b	-0.94	0.94	0.04	0.10	0.42
Numeracy	0	11	9.00	8.46	2.61
Age	18	88	53	50.24	14.74

 Table 1: Descriptive Statistics Continuous Variables

This table shows descriptive statistics of the ambiguity index(b) and ambiguity–generated likelihood insensitivity (a). The ambiguity indexes AA01, AA05, AA09 represent p - m(p) for p = 0.1, 0.5, 0.9, the differences between the accepted ambiguity for indifference and the objective probability. The accepted ambiguity levels per respondent are m(0.1), m(0.5), m(0.9). Also presented are descriptive statistics for age and Numeracy. Numeracy is based on answers to 11 numeracy related questions, the numeracy value denotes the number of right answers per respondent, see Subsection 2.2.2

Table 2: Ambiguity Aversion in Different Games

	Seeking	Neutral	Averse
Game 1 (p= $0.5$ ) Game 2 (p= $0.1$ ) Game 3 (p= $0.9$ )	48.36%	18.78%	32.86%

Presented in this figure are distributions of recorded ambiguity behaviour in the three different games. Each has has different initial win probabilities for the unambiguous choice option.

To better understand the different components of ambiguity attitudes and how they work together, see Figure 5 below. It represents 'best fitting lines' (i.e. neoadditive weighting function) between m(p) and p, as described earlier in this section, under different conditions for indices a and b. The figure is based on the findings in Dimmock, Kouwenberg, and Wakker (2016) and Abdellaoui

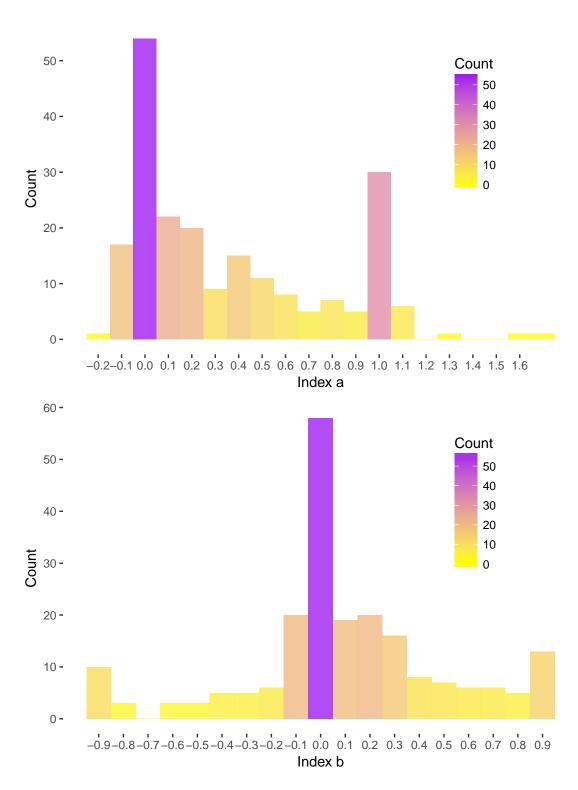


Figure 3: Histograms Index a & Index b

Presented in this figure are histograms for index a and index b. Index a represents a-insensitivity as described in Section 2. Index b represents ambiguity aversion.

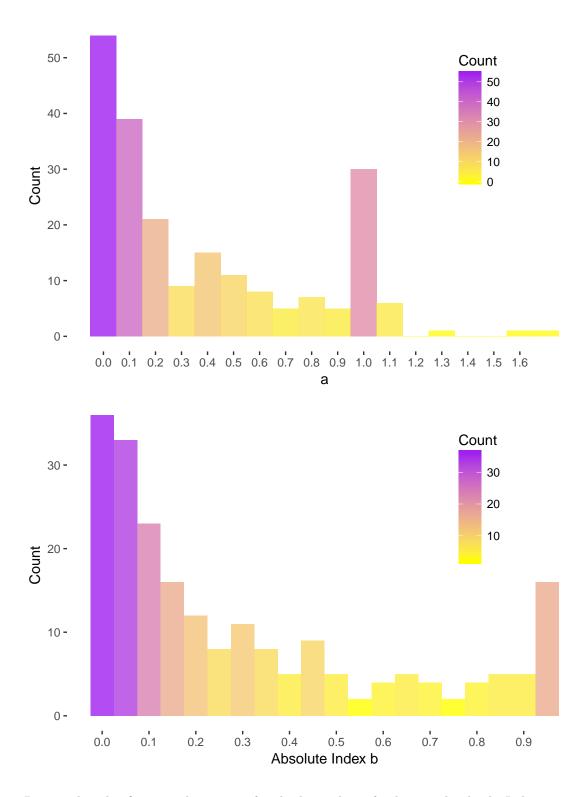


Figure 4: Histograms Absolute Index a & Absolute Index b

Presented in this figure are histograms for absolute values of index a and index b. Index a represents a-insensitivity as described in Section 2. Index b represents ambiguity aversion.

et al. (2011). Shapes should be similar for the indices used in this research.

Figure 3a represents a purely rational agent, with m(p) = p. In figure 3b it is clear that the fitted line is below the 45° line, this implies ambiguity aversion, with a value for index b larger than zero. In this figure, the a-insensitivity index a is an index of the flatness of the curve. This flatness indicates the lack of discrimination between levels of likelihood. It is clear in Figure 3c that the bold line moves towards the 50-50 probability (the centre of the graph). Figure 3d shows the joint effect of the ambiguity attitude components.

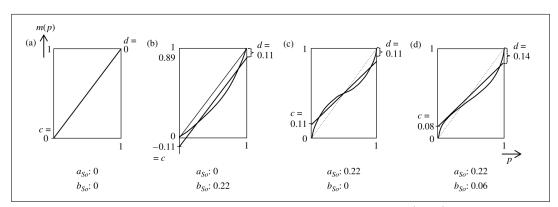


Figure 5: General Ambiguity Indices Together

This figure is an excerpt from Dimmock, Kouwenberg, and Wakker (2016) and Abdellaoui et al. (2011), presenting a situation with ambiguity neutrality (a), only ambiguity aversion (b), only a-insensitivity (c) and both ambiguity aversion and a-insensitivity (d). The y-axis denotes the matching probability m(p) as described in Subsection 3.1. The X-axis represent the real probability of winning. A 45 degree line indicates ambiguity neutral attitudes.

In Table 3, correlations between the indices are presented. Naturally, indices a & b are highly correlated to indices AA01, AA05 & AA09, as a & b are derived from the latter indexes. It seems that the correlation between index AA05 and a is non-significant. Naturally I would expect that they are not related, as index a is constructed from AA01 and AA09. In the game on which AA05 is based, because probabilities are already close to 50-50, a-insensitivity should not play a large role.

	AA01	AA05	AA09	a	b
AA01	1.00	0.54	0.33	-0.47	0.77
AA05		1.00	0.47	0.01*	0.81
AA09			1.00	0.67	0.80
a				1.00	0.14
b					1.00

Table 3: Correlation Ambiguity Indices

This table shows correlations between ambiguity index variables as defined in section 3.  $(\star)$  signifies correlations which are not significant on the 10% level.

### 3.2 Measures of Cognitive Abilities

It was mentioned earlier in Subsection 2.2.1 that I utilise two different measures of cognitive abilities to answer the research question and to test the specific hypotheses mentioned in Subsection 2.4. The first of these measures is the Cognitive Reflection Test. The three questions of this test were presented to the LISS panel directly in 2012. These answers are converted to binary responses per respondent in the form of a 'correct' or 'wrong' outcome (0 and 1 respectively). A new variable is constructed, which contains the number of correctly answered questions per respondent. This is often done in research using the CRT (Pennycook, Cheyne, Koehler, et al., 2016). This variable is named *CRT score* and it is a single independent variable which contains all the information from the CRT survey. It has a range of 0–3. A higher score on the index would imply higher cognitive reflection ability. The respondents were not incentivised in the CRT survey (there was no monetary reward), however this does not reduce accuracy of the results according to (Brañas-Garza, Kujal, and Lenkei, 2015).

Note that I do not distinguish intuitively wrong answers from other wrong answers. The intuitively wrong answers would be the answer expected, when a respondent gives the intuitive (system 1) answer. Other wrong answers would be answers that are neither expected as the intuitive answer, nor the actual correct answer. These types of answers are not distinguished mostly due to time constraints and the limited number of respondents. Also, note that this distinction is mostly important when the focus is on measuring the intuition, instead of reflection of respondents (Brosnan et al., 2014).

The second cognitive ability measure used is the Numeracy Test. This test involves 11 questions related to numeracy as described in Subsection 2.2.2, presented to the panel in 2008. The answers contain data on a continuous scale and data with string values. These answers are again converted to 1 for 'correct' and 0 for 'wrong'. The Numeracy variable is then constructed by adding up all the correct answers, thus this variable is on the scale 0-11, where higher values indicate higher Numeracy. This Numeracy variable becomes an independent variable which proxies the Numeracy test, similar to the CRT score. This constructed Numeracy variable is used by other authors as well, such as Ellen Peters, Daniel Västfjäll, et al. (2006), Schwartz et al. (1997), and Taha, Sharit, and Czaja (2014) with success.

Thus, both the information from the CRT and Numeracy test are summarised into one variable, one for each test. As can be seen in figure 6 below, the Numeracy questions seem to have been less challenging for the 213 LISS respondents than the CRT questions, where for Numeracy, a larger proportion of respondent answered most questions correctly. For the CRT, questions one (the bat and ball question, see Subsection 2.2.1) seems to be the most difficult for respondents, as can be gleaned from Table 4. The correlation between the CRT score and Numeracy index has a significant (5% level) value of r = 0.44. This indicates that they are indeed related though also capture different things, as already mentioned in Subsection 2.2.

The Numeracy variables has a tau-equivalent reliability (also known as Cronbach's alpha) of  $\rho_T = 0.81$ . Typically, this indicates excellent internal consistency (Graham, 2006). The CRT score has a tau-equivalent reliability of  $\rho_T = 0.7$ , which is also acceptable. See Table 5 for descriptive statistics on the two cognitive ability related constructed variables. Means and medians are close together for both variables, indicating a somewhat symmetric distribution.

	correct	wrong
bat and ball question $(1)$	55	158
widget question $(2)$	81	132
lily pad question $(3)$	88	125

 Table 4: Cognitive Reflection Test Answer Distribution

This table shows descriptive statistics of the three cognitive reflection test answers. See Subsection 2.2.1 for a description of the questions.

Table 5: Descriptive Statistics Constructed Cognitive Ability Variables

	Mean	Median	SD
Numeracy	9	8.48	2.62
CRT	1	1.05	1.13

This table shows descriptive statistics of the two constructed independent variables, the

Cognitive Reflection Test score and the Numeracy index. These variables contain the number of correct answers to the specific group of questions (Cognitive Reflection Test and Numeracy test). The questions aim to elicit certain components of cognitive abilities, see Section 2.

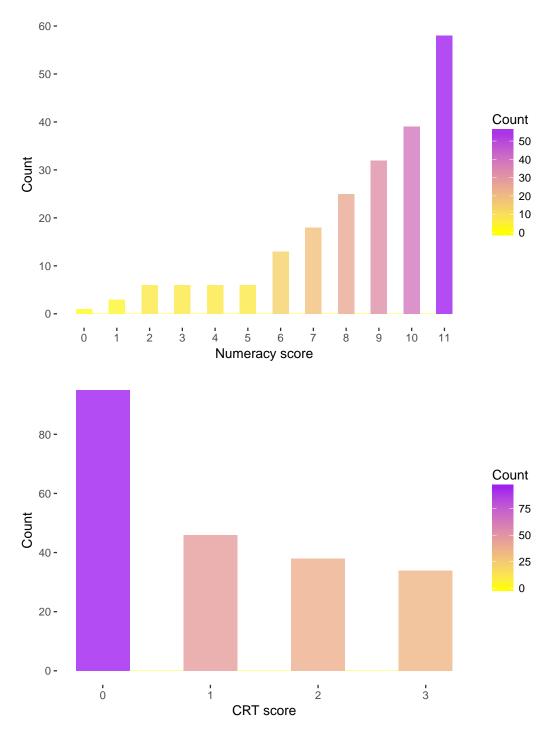


Figure 6: Distribution Numeracy & CRT scores

Presented in this figure are the counts for the Numeracy and CRT test variables. The numeracy score is the sum of correct answers to the 11 numeracy related questions. The CRT score is the number of correct answers to the three cognitive reflection questions.

### 3.3 Control Variables

Age and gender are added as control variables in this research, see Subsection 2.3. See Table 6 for insights into the relationship between the control variables and the dependent variables (i.e. the five ambiguity attitude indices) used in this research. When index a is concerned, age and gender are not explanatory, this also holds for index AA09. For index b, only age is explanatory, this also holds for index AA05. Gender is only explanatory for index AA01.

Note that, there are 103 males and 110 females in the data set, thus the distribution for gender is quite even. Note that females tend to score lower than males on the CRT, based on a meta analysis with over 40000 respondents combined (Brañas-Garza, Kujal, and Lenkei, 2015). In Table 7 the correlations between all the independent variables (including the control variables) are presented. Note that gender is only significantly correlated with CRT score and not with Numeracy. Age is only significantly correlated with Numeracy, but not CRT score.

 Table 6: Regression Results Control Variables

	a		b		AA01		AA05		AA09	
	Coefficient	P-value								
Intercept	0.337	***0.00	0.329	***0.00	0.001	0.99	0.223	***0.00	0.270	***0.00
Age	0.000	0.92	-0.004	**0.04	-0.002	0.15	-0.002	**0.02	-0.002	0.19
Gender	0.079	0.17	-0.059	0.30	-0.071	*0.05	-0.010	0.74	-0.008	0.86
adj. $R^2$	0.000		0.016		0.020		0.016		-0.001	

This table shows regression results of the control variables, with the different ambiguity attitude indices as dependent variables. Index a indicates the rate of a-insensitivity for each respondent. Index b indicates the the general rate of ambiguity aversion for each respondent. The indices represent the answers of survey questions which try to elicit ambiguity aversion based on different initial win probabilities; AA01, AA05, AA09 represent p - m(p) for p = 0.1, 0.5, 0.9, the differences between the accepted ambiguity for indifference and the objective probability. The accepted ambiguity levels per respondent are m(0.1), m(0.5), m(0.9), see Subsection 3.1.1. These results are based on 213 observations. \* indicates significance at the 10% level, \*\* indicates significance at the 5% level, \*\*\* indicates significance at the 1% level.

	CRI	Numeracy	Age	Gender
CRT	1	$0.44\star$	-0.05	-0.16*
Numeracy		1	$-0.14 \star$	-0.1
Age			1	
Gender				1

Table 7: Correlation Independent Variables

### 3.4 Waves

Some survey questions are asked to the LISS panel on multiple occasions (i.e. points in time). These occasions are referred to as *waves*. It should be noted that not every respondent replies to each survey all of the time. The CRT questions are asked in a single wave (February 2012). However, the Numeracy questions were also asked in a single wave, in a different year (September 2008). The questions used to elicit ambiguity attitudes were presented to the panel in 2010. One caveat for this research is that I assume that respondents give similar answers to the questions, regardless of year. This however seems warranted as the years are not that far apart. For simplicity, the age variable is sourced from July 2010, the age of the respondents in the ambiguity attitude elicitation survey, the respondent age in 2010 is also closest to both 2008 and 2012.

## 4 Results

### 4.1 Preliminary Analysis

To answer the research question, a linear model, or specifically *Ordinary Least Squares (OLS)* is employed. OLS is often used to find linear relationship between variables (Heij et al., 2004). The OLS model and its coefficients are viable when its model assumptions are met, this is further explored in Appendix Subsection 7.2.

Note that for each regression the adjusted  $R^2$  is presented based on Yin and Fan (2001). The adjusted  $R^2$  is an estimation of the proportion of variance in the population explained by the regression. The adjusted  $R^2$  can have negative values because it adds penalty terms for the number of independent variables and the sample size. This is because adding independent variables often increases the  $R^2$  inherently.

### 4.2 Regression Results

Presented in this Subsection are the regression results with the five ambiguity indices mentioned in Section 3 as dependent variables. The independent variables are the two cognitive ability variables CRT score & Numeracy, and two control variables Gender & Age. Three models (regression results) are presented for each dependent variable labelled (1), (2) and (3). (1) showing results for a multiple regression including CRT score and Numeracy (and control variables), (2) showing results for a regression with Numeracy and control variables as independent variables only. Finally (3) shows results for a regression with CRT score and control variables as independent variables.

Table 8 shows regression results based on index a as the dependent variable. This is an index indicating a-insensitivity, where a higher value indicates higher insensitivity. Negative values indicate the opposite effect of a-insensitivity. As mentioned earlier in Section 3, the index a is defined as 1 minus the slope of the best fitting line between m(p) and p for a respondent. With the slope being equal to 1 for a strictly rational respondent, the index a value would be zero.

Both the CRT score and Numeracy measures are not significant in explaining a-insensitivity. The adjusted  $R^2$  also indicates that the independent variables do not explain index a in any meaningful way. For all three models, the fit is not different from an intercept only model according to the F-tests. Note that the residuals in this regression do not comply with the assumption of normality. A regression is performed with transformed data which does follow all OLS assumptions, however, conclusions are the same, see Appendix Subsection 7.2.

In Table 9 results are presented with index b as the dependent variable. As mentioned before, index b is a general measure of ambiguity aversion. It seems that both cognitive ability measures separately are not significant in explaining ambiguity aversion in this form. As expected, Age is explanatory for index b. With CRT score and Numeracy together, Numeracy is significant on the 10% level. As mentioned in Appendix Subsection 7.2, there is no meaningful multicollinearity present in the independent variables, thus it is not likely that this is the cause of Numeracy being significant in (1). There must be some bias (e.g. omitted variable bias, confounder bias) present however, because when Numeracy is regressed separately on the dependent variable index b, it is not explanatory (Woolley, 1997). Thus I choose to disregard the result in (1).

Further, though the adjusted  $R^2$  values are much higher than for index a, they are still quite low. Models (1) and (2) do not perform better than an intercept only model according to an F-test with a 5% significance level, they do perform slightly better however when a significance level of 10% is regarded. The performance can not be attributed to the Numeracy or CRT variable only of course, as age is a significant variable as well. When regressed without control variables, F-tests are not significant on the 10% level for (1) and (2) either (results not shown for brevity). Note that all OLS assumptions are met and the coefficients can be deemed reliable as such.

For the results based on the three different ambiguity indices AA01, AA05 & AA09 directly based on three games as mentioned in Subsection 3.1.1, see Table 10. Also here, CRT score and Numeracy individually are not significant in explaining the ambiguity indices. In the combined regressions (1), Numeracy is significant on the 10% level for AA01 and AA05. Again, this could be because of some bias.

The significant effect of Numeracy on AA05 is unexpected additionally, as low Numeracy scores are expected to correlate with high a-insensitivity, and a-insensitivity is not prevalent when probabilities are around 0.5. This is of course the case for the first of the three games, on which index AA05 is based. Models (1) and (2) are significantly better than an intercept only model on the 10% significance level, this does not hold on the 5% level.

For AA01, all three models perform better than an intercept only model when regarding the F-test on a 10% significance level, on the 5% level this only holds for model one. This is peculiar because this is not the case for model (2) which also contains the Numeracy variable but excludes the CRT score variable. It could be said that model (1) has some merit but it is not clear which predictors cause this and in what way.

If assumed that the combined regressions, labelled as (1) in each table, are correct, then for indices AA01 and AA05, when Numeracy goes up with 1 point, the index value goes up with 0.014 (AA01) or 0.012 (AA05). That means that there is a positive correlation, higher Numeracy scores indicate higher ambiguity aversion. However, again I suspect that the results in (1) for both AA01 and AA05 are incorrect.

Also note that adjusted  $R^2$  values are similar for indices AA01 and AA05. In both cases, the values are quite low. However, for index AA09 the values are much lower and it could be said that there is no real relationship between the index and the independent variables (including the control variables) at all. Of course, when regarding a 5% significance level this holds for all models in Table 10 except for model (1) for AA01.

When only deviation from ambiguity neutrality is regarded using absolute values of indices a & b, thus not looking at the direction of deviation from neutrality, the results are also clear, see Tables 11 & 12. The results indicate that there is no relationship between the absolute value of the indices a & b and the independent variables, CRT score and Numeracy. The models do not explain the indices better than models with just an intercept, according to the F-tests. Thus, it could be said that there is no meaningful relationship found.

	Regression 1		Regression 2		Regression 3	
	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
intercept	0.432	0.01	0.427	**0.01	0.338	***0.00
CRT	0.010	0.73			-0.001	0.97
Numeracy	-0.011	0.37	-0.009	0.41		
Age	0.000	0.83	0.000	0.83	0.000	0.92
gender	0.077	0.19	0.074	0.20	0.078	0.17
F-test	0.682	0.60	0.873	0.45	0.644	0.59
adj. $R^2$	-0.006		-0.002		-0.005	

 Table 8: Regression Results Index a

This table shows regression results with index a as the dependent variable. Index a indicates the rate of a-insensitivity for each respondent. These results are based on 213 observations.
\* indicates significance at the 10% level, \*\* indicates significance at the 5% level, \*\*\* indicates significance at the 1% level. The F-test results are based on the hypothesis of an intercept only model having equal fit as the presented model.

	(1)		(2)		(3)	
	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
intercept	0.182	0.23	0.200	0.19	0.360	***0.00
CRT	-0.043	0.13			-0.022	0.40
Numeracy	0.021	*0.09	0.013	0.24		
Age	-0.004	*0.06	-0.004	*0.06	-0.004	**0.04
Gender	-0.064	0.27	-0.052	0.36	-0.067	0.25
F-test	2.305	*0.06	2.3	*0.08	2.071	0.11
adj. $R^2$	0.024		0.018		0.015	

Table 9: Regression Results Index b

This table shows regression results with Index b as the dependent variable. Index b indicates the the general rate of ambiguity aversion for each respondent. These results are based on 213 observations. \* indicates significance at the 10% level, \*\* indicates significance at the 5% level, \*\*\* indicates significance at the 1% level. The F-test results are based on the hypothesis of an intercept only model having equal fit as the presented model.

AA01						
	(1)		(2)		(3)	
	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
intercept	-0.101	0.29	-0.090	0.34	0.017	0.81
$\operatorname{CRT}$	-0.025	0.15			-0.012	0.47
Numeracy	0.014	*0.07	0.009	0.18		
Age	-0.002	0.21	-0.002	0.21	-0.002	0.14
Gender	-0.073	**0.04	-0.066	0.06	-0.075	**0.04
F-test	2.546	**0.04	2.684	*0.05	2.254	*0.08
adj. $R^2$	0.028		0.023		0.017	

Table 10: Regression Results Indices AA01, AA05 & AA09

AA05
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	(1)		(2)	(2)		(3)	
	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	
intercept	0.129	0.12	0.138	*0.09	0.235	***0.00	
CRT	-0.021	0.17			-0.009	0.53	
Numeracy	0.012	*0.06	0.009	0.15			
Age	-0.002	**0.04	-0.002	0.04	-0.002	**0.02	
Gender	-0.011	0.71	-0.006	0.85	-0.013	0.67	
F-test	2.366	*0.05	2.49	*0.06	1.92	0.12	
adj. $R^2$	0.025		0.021		0.013		

### AA09

	(1)		(2)		(3)		
	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	
intercept	0.244	**0.03	0.252	**0.03	0.288	***0.00	
CRT	-0.018	0.41			-0.012	0.52	
Numeracy	0.005	0.58	0.002	0.82			
Age	-0.002	0.21	-0.002	0.20	-0.002	0.18	
Gender	-0.011	0.79	-0.007	0.88	-0.012	0.78	
F-test	0.627	0.64	0.611	0.60	0.735	0.53	
adj. $R^2$	-0.007		-0.006		-0.004		

This table shows regression results based on Indices AA01, AA05, AA09 as dependent variables respectively. The indices represent the answers of survey questions which try to elicit ambiguity aversion based on different initial win probabilities; AA01, AA05, AA09 represent p - m(p) for p = 0.1, 0.5, 0.9, the differences between the accepted ambiguity for indifference and the objective probability. The accepted ambiguity levels per respondent are m(0.1), m(0.5), m(0.9), see Subsection 3.1.1. These results are based on 213 observations. \* indicates significance at the 10% level, \*\* indicates significance at the 5% level, \*\*\* indicates significance at the 1% level. The F-test results are based on the hypothesis of an intercept only model having equal fit as the

presented model.

	(1)		(2)		(3)	
	coefficient	P-value	coefficient	P-value	coefficient	P-value
intercept	0.471	***0.00	0.467	***0.00	0.358	***0.00
CRT	0.010	0.72			-0.004	0.88
Numeracy	-0.013	0.26	-0.011	0.28		
Age	0.000	0.82	0.000	0.82	0.000	0.93
gender	0.070	0.21	0.067	0.22	0.072	0.20
F-test	0.773	0.54	0.993	0.39	0.606	0.61
adj. $R^2$	-0.004		0.000		-0.006	

Table 11: Regression Results Absolute Index a

This table shows regression results with absolute values of index a as the dependent variable. Index a indicates the rate of a-insensitivity for each respondent. These results are based on 213 observations. \* indicates significance at the 10% level, \*\* indicates significance at the 5% level, \*\*\* indicates significance at the 1% level. The F-test results are based on the

hypothesis of an intercept only model having equal fit as the presented model.

	(1)		(2)		(3)	
	coefficient	P-value	coefficient	P-value	coefficient	P-value
intercept	0.368	***0.00	0.367	***0.00	0.260	***0.00
CRT	0.000	0.98			-0.012	0.52
Numeracy	-0.013	0.17	-0.013	0.13		
Age	0.000	0.80	0.000	0.80	0.001	0.66
Gender	0.037	0.39	0.036	0.39	0.039	0.37
F-test	0.907	0.46	1.215	0.31	0.563	0.64
adj. $R^2$	-0.001		0.003		-0.006	

Table 12: Regression Results Absolute Index b

This table shows regression results with absolute values of index b as the dependent variable. Index b indicates the the general rate of ambiguity aversion for each respondent. These results are based on 213 observations. \* indicates significance at the 10% level, \*\* indicates significance at the 5% level, \*\*\* indicates significance at the 1% level. The F-test results are based on the hypothesis of an intercept only model having equal fit as the presented model.

## 5 Conclusion

The question: Can some measures of cognitive ability partially explain the ambiguity attitudes of an individual, and if so, what is the relationship between ambiguity attitudes and cognitive abilities? is researched by testing two hypotheses.

The ambiguity attitudes of agents are built up of two components, ambiguity aversion and ambiguity generated likelihood insensitivity (a-insensitivity). Ambiguity aversion desCRT scores an agent's dislike of ambiguity. A-insensitivity is the rate at which agents overestimate low probabilities and underestimate high probabilities, both towards a 50-50 chance.

Cognition is measured using two scales, each of which measure a certain component of cognition. Both scales are somewhat correlated (r = 0.44). The first of these scales is the Cognitive Reflection Test (CRT) score. This scale is based on the number of correct answers in the 'Cognitive Reflection Test'. This test involves questions, for which the initial 'gut/intuitive' response/answer is often wrong. Through 'reflection' agents might find the actual correct answer. Thus the CRT score measures the rate at which agents can employ reflection.

The second scale is Numeracy, this is also the sum of the number of correct answers, on a test dubbed the 'Numeracy test'. This test contains questions pertaining topics such as: chance, proportions, risks and percentages. The test aims to measure intuition and knowledge of these concepts, and whether or not agents are able to convert proportions to percentages and vice versa.

To reiterate, the following hypotheses were stated in Subsection 2.4:

H1: Numeracy is negatively correlated to a-insensitivity.

H2: Numeracy is negatively correlated to ambiguity aversion.

H3: Numeracy score is related to deviation from ambiguity neutrality.

H4: Numeracy score is related to deviation from the absence of a-insensitivity.H5: CRT score is negatively related to ambiguity aversion.

*H6:* CRT score is negatively related to a-insensitivity.

H7: CRT score is related to deviation from ambiguity neutrality.

H8: CRT score is related to deviation from the absence of a-insensitivity.

Following the results presented in Section 4, all hypotheses are rejected.

The hypotheses aid in answering the research question of this thesis:

Can some measures of cognitive abilities partially explain the ambiguity attitudes of an individual, and if so, what is the relationship between ambiguity attitudes and cognitive abilities?

There does not seem to be a real significant and meaningful relationship between the CRT score and Numeracy cognitive ability measures and both ambiguity aversion and a-insensitivity. It can be said that the trait of ambiguity generated likelihood-insensitivity (a-insensitivity) is not related to Numeracy as measured by the Numeracy Test. That is, understanding the meaning of numbers and proportions does not explain the rate of a-insensitivity of an agent. The results for absolute values of indices a & b are also telling, no relationship is found.

## 6 Further Research

The research presented in this thesis deals with rich topics, which can be explored further. In this section, the possibilities for further research are elucidated. Due to the limited scope (and time constraints) of the research presented in this text, several extensions can be considered. Further research can show whether or not the same results hold for different data sets and with different variables.

Further research can utilise more extensive measures of cognitive ability, or simply investigate the effects of other measures. Further research could also use more data, and compare results for different data sets i.e. different populations. Also, more control variables could be added and more thoroughly tested, to isolate the effects of the cognitive ability measures more.

The general ambiguity attitude indices are based on regressions using the objective and matching probabilities. These regressions should be constrained on the open interval (0,1). The following restrictions should hold: the slopes of the regression coefficient should always be positive; the intercept should always be positive and the sum of the slope and the intercept cannot exceed the value of 1. This was not the case for the research in this thesis, for feasibility reasons. Further research could implement this.

Ideally, all measured independent variables used to explain the dependent variables should be measured at the same point in time. This was not the case in this research, see Subsection 3.4.

In the survey questions aimed to elicit ambiguity attitudes, more rounds (i.e. iterations) can be used as a large number of respondents are still not indifferent in the final round, as mentioned in Subsection 3.1.1.

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# 7 Appendix

### 7.1 Data Preparation

In this research, results from different surveys are merged, that is results from the Numeracy questions, CRT questions and ambiguity attitude elicitation questions. As mentioned in Subsection 3.4 the surveys were proposed to the panel at different points in time. The CRT questions were only presented to a random subset of the panel. When merging the data, I find that only 216 respondents filled in all three surveys, in an adequate manner. It is true that when performing regressions with only one of the cognitive ability measures as an independent variable, more observations could be used. However, these regressions do not present a higher correlation than already found, thus they are not reported, see Section 4. Also, with an equal sample size for all regressions, interpretation and comparisons are more robust, in my opinion.

Upon closer inspection of the initial data set with 216 rows, I find that the data set contains 4536 (216\*21) cells, of which, 16 have NA values. The rows containing these values are deleted (case-wise deletion), with the assumption that the missing values are 'Missing Completely At Random (MCAR)' following Little's MCAR test (Little, 1988). Missing values were present in rows 67, 88, 215. These three rows are removed, which does not bias the results because of the MCAR property.

### 7.2 Model Assumption Checks

For a correct and reliable OLS implementation, there several assumptions which have to be met, here I focus on the five most important assumptions for the purposes of this research (Heij et al., 2004). One assumption, that of no autocorrelation, is not relevant for the type of data used in this research. As such, none of the models violate this assumption based on the Durbin Watson test (Savin and White, 1977).

The second assumption checked is that the values of the dependent variable can be expressed as a linear combination of the values of the independent variables. The third assumption is homoscedasticity, this implies that the variance equal for all the values in the error term. The fourth assumption is that the error term is normally distributed. The fifth assumption regarded is that the error terms are independent and identically distributed with a mean of zero.

To check for multicollinearity, Variance Inflation Factors (VIF) are calculated for each variable in each model. In each case the square root VIF value is well below 2, indicating that multicollinearity is not a problem in these models/regressions. This implies that parameter coefficients are somewhat accurate. The threshold value of 2 is used via a rule of thumb and is very conservative (Dormann et al., 2013).

The other assumptions are check using the R programming language, invoking the 'gvlma' software package available for R.

The assumptions are checked for models based on index a as the dependent variable and index b as the dependent variables, as this is the main focus of this research. This means six models, three for each dependent variable, see Section 4.2. All models based on index b as the dependent variable fit the OLS model assumptions (based on a 5% significance level).

This does not hold for any model with index a as the dependent variable, they meet all assumptions except the third one. The errors can not be seen as normally distributed (Adjusted Jarque-Bera test value of 22.58). The violation of this assumption can influence and distort the p-values. However, there are not too many outliers.

One solution to this is to use a General Linear Model with a different assumption on the error term, one that better reflects the error distribution. The dependent variable index a, has negative values for some respondents. This is somewhat problematic for modelling a GLM. To solve this, I shift all the values of index a with a value of 0.2 (where the minimum value of a is -0.1875), thus all values in the vector are increased with 0.5. This way all values are above zero. This does not change the regression coefficients of the independent variables, it does change the intercept, which can be corrected for after the fact. This shift alone however does not create 'enough normality'.

That is why, to get normalised residuals, the Box Cox transformation is implemented (Box and Cox, 1964). This is implemented using the MASS package available for the R programming language. The maximum Box Cox value *box* is used to transform the dependent variable index a. Altogether index a is transformed as follows:

$$a < -((a+0.2)^{box}-1)/box$$

Now the the residuals of the model are somewhat more acceptably normally distributed and all OLS assumptions are met, this means that 'regular' OLS is applicable to the data now. However, the resulting models still bode the same results as the models using the non-transformed index a. This means that the conclusions made with the non-transformed index a still hold. As such, this model is presented in Section 4.

The results for the model using the two aforementioned transformations are presented in Table 13 below. Again, there is no relation between index a and both cognitive ability variables.

The QQ-plots of the residuals before and after the Box Cox transformation can be seen in figure 7 below. As can be seen, the distribution is still quite heavy tailed, but is 'normal enough'.

	(1)		(2)		(3)	
	coefficient	P-value	coefficient	P-value	coefficient	P-value
intercept	-0.767	0.00386	-0.77764	0.003264	-0.77941	6.63E-05
CRT	0.025	0.616294			0.023153	0.599784
Numeracy	-0.001	0.946182	0.003145	0.8681		
Age	-0.00198	0.557003	-0.00196	0.559712	-0.00195	0.558649
gender	0.163084	0.101527	0.156484	0.112193	0.163317	0.100022
adj. $R^2$	-0.004		-0.001		0.001	

Table 13: Regression Results Transformed Index a

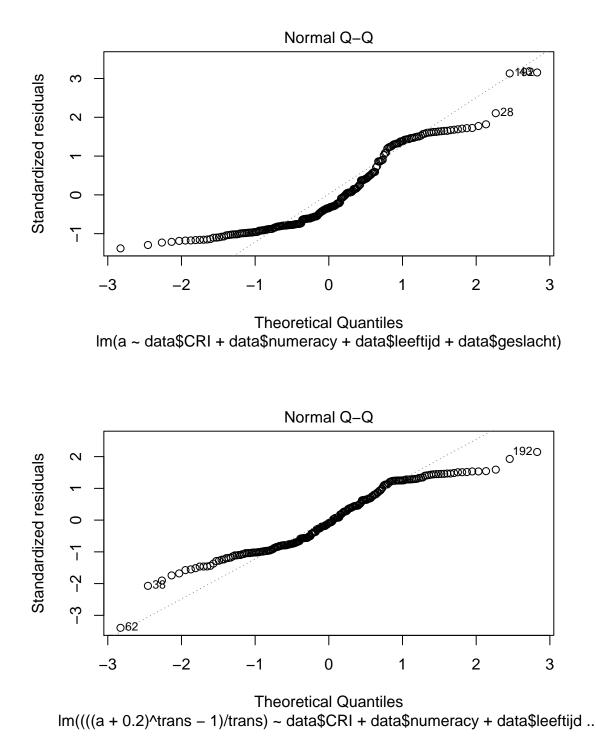
This table shows regression results with index a as the dependent variable transformed with the box cox method and shifted with a value of 0.2. Index a indicates the rate of a-insensitivity for each respondent. These results are based on 213 observations. \* indicates significance at the 10% level, \*\* indicates significance at the 5% level, \*\*\* indicates significance at the 1% level.

	(1)		(2)		(3)	
	coefficient	P-value	coefficient	P-value	coefficient	P-value
intercept	-2.50629	0.001956	-2.58105	0.001404	-1.81159	0.002217
CRT	0.177764	0.234436			0.259949	0.054778
Numeracy	0.08148	0.204158	0.114493	0.04862		
Age	-0.01217	0.23646	-0.01205	0.241723	-0.01382	0.176251
gender	0.48263	0.111202	0.434907	0.147772	0.469247	0.121724
adj. $R^2$	0.023		0.021		0.020	

Table 14: Regression Results Transformed Index a

This table shows regression results with index a as the dependent variable transformed with the box cox method and shifted with a value of 0.2. Index a indicates the rate of a-insensitivity for each respondent. These results are based on 213 observations. \* indicates significance at the 10% level, \*\* indicates significance at the 5% level, \*\*\* indicates significance at the 1% level.





Presented in this figure are histograms for index a and index b. Index a represents a-insensitivity as described in Section 2. Index b represents ambiguity aversion.