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Testing Multi-factor Models Internationally:

Developed and Emerging Markets

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Abstract: Previous literature has extensively examined the relation between risk and return in U.S. stock markets. International evidence, on the other hand, is far more scarce. This paper tests to what extent multi-factor models explain the variation in average stock returns in developed and emerging markets over the period 1996-2016. The main findings indicate that these models better explain stock returns for developed markets than for emerging markets. Furthermore, incorporating more risk factors does not necessarily increase the explanatory power of the multi-factor models for both market segments.

Keywords: Multi-factor models, international stocks, developed markets, emerging markets.

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SECTION I: INTRODUCTION

The central theorem in asset pricing theory focuses on the relation between expected return and risk. Consequently, economists and academics have put in substantial effort to identify various risk factors which add to the explanation of expected stock returns. Sharpe (1964), Lintner (1965), and Black (1972) introduce the Capital Asset Pricing Model (CAPM) which implies the simple relation that a stock's expected returns is a positive linear function of its market beta, which describes the cross-section of expected returns. There are several contradictions to the CAPM, however, as Rosenberg, Reid and Lanstein (1985) and Stattman (1980) find that stocks with a high book-to-market ratio earn higher returns, on average, than would be warranted by their CAPM betas. Banz (1981) introduces the size effect, which implies that small stocks earn on average higher returns than large stocks. Fama and French (1993) merge the market factor and the additional size and value factors to form the Fama and French three-factor model that accurately explains the cross-sectional variation in U.S. stock returns. Jegadeesh and Titman (1993) provide evidence for short-term persistence among U.S. stock returns, i.e. stocks that have performed well in the recent past tend to perform well in the near future. The momentum effect is included in the Fama and French three-factor model by Carhart (1997) to form the Carhart four-factor model. The inclusion of the momentum factor contributes to explaining the cross-sectional variation in average stock returns. Fama and French (2015) introduce their five-factor model which takes on a different view as it leaves out the momentum factor and incorporates a profitability and investment factor. Using the dividend discount model, Fama and French (2015) argue that stocks with robust profitability and low levels of investment tend to generate higher expected returns, on average, than stocks with weak profitability and high levels of investment.

The models described above are widely accepted in academic literature, however, predominantly apply to U.S. stock markets. In addition, most research dates from the eighties, nineties, and early 2000s. This paper adds to existing literature in a twofold manner. Firstly, it tests multi-factor models internationally, more specifically on developed and emerging markets, by examining the Morgan Stanley Capital International (MSCI) index. The MSCI index is a major international stock index provider, focusing on a large array of developed and emerging markets. The index covers approximately 85 percent of the global investable equity opportunity set. Secondly, it thereby uses a recent time period (1996 to 2016). The goal is to examine the differences between developed and emerging markets by testing how the multi-factor models perform, i.e., to what extent the models explain stock returns.

The next section discusses the multi-factor models in more detail. Section III provides an overview of the regression analyses, hypotheses, and explains how the portfolios are constructed. Section IV describes the sample data. Section V presents and interprets the results. Lastly, Section VI and VII conclude and discuss further research.

SECTION II: LITERATURE REVIEW

The Capital Asset Pricing Model of Sharpe (1964), Lintner (1965), and Black (1972) has been the leading model on how academics think about average returns and risk. The main concept of the Sharpe-Lintner-Black (SLB) model is that investors only care about the mean and variance of their one-period investment return. Consequently, investors choose mean-variance efficient portfolios, in the sense that the portfolios (1) maximize expected return of the portfolio, given the variance of portfolio return, and (2) minimize the variance, given expected return. The expected return on a security is a positive linear function of its market beta and the market beta describes the cross-section of expected returns (Fama and French, 2004).

The strength of the CAPM is that it provides a clear illustration of how to measure risk and of the relation between risk and expected return. Unfortunately, the empirical record of the model is insufficient to such a degree that applications of the model are invalid. The CAPM's empirical issues may be traced back to theoretical shortcomings as a result of the many simplifying assumptions that were made during the construction of the model, or the empirical issues could be caused by difficulties in implementing valid tests of the model. E.g., the model implies that the risk of a specific stock is measured relative to a comprehensive market portfolio. However, the CAPM assumes that the market portfolio only consists of tradable financial assets, but it can principally also include human capital, consumer durables, real estate, etc. Ultimately, whether the model's issues reflect shortcomings in the theory or weaknesses in its empirical implementation, the CAPM's empirical failure implies that most applications of the model are invalid (Fama and French, 2004).

From the late 1970s onwards, academics start to uncover variables like size, price ratios, and momentum which add to the explanation of expected stock returns. Banz (1981) argues that market equity, ME (the number of shares outstanding times the stock's price), adds to the explanation of the variation in average returns. Small stocks (low ME) tend to earn higher average returns than large stocks (high ME). Although Banz (1981) provides valid empirical evidence for the size effect, he fails to propose an underlying theoretical explanation for such an effect. The size factor might even be just a proxy for one or several unknown factors which are correlated with firm size. A possible explanation is given by Klein and Bawa (1977), who argue that investors will not hold a certain subset of securities if only limited information about these securities is available, because these securities are subject to estimation risk, i.e., the uncertainty about the true parameters to determine the optimal portfolio choice. Investors tend to invest relatively more in securities of which the most information is available, and it is most likely that the amount of information available is related to firm size as the availability of information about large stocks is generally greater than for small stocks. Another explanation for the size effect might be that small stocks carry an illiquidity premium. Small stocks are traded less frequently and their tradable amount is smaller than for large stocks. As a result, small stocks tend to be more risky and their expected returns are higher. In summary, whilst theoretical foundation about the size effect is lacking, empirical evidence on the existence of the effect is robust.

Rosenberg, Reid, and Lanstein (1985) and Stattman (1980) find that U.S. stocks with a high book-to-market equity ratio (BE/ME) earn on average higher returns than would be warranted by their CAPM betas, with the underlying methodology that a firm's stock would be undervalued if the BE/ME ratio is high and overvalued if the BE/ME ratio is low. High BE/ME (> 1) stocks are considered value stocks and trade at prices lower than they should be, based on their fundamental value, and on average generate high returns. Growth stocks (BE/ME < 1), on the contrary, trade at prices higher than their fundamental value. As a result, growth stocks tend to underperform in comparison to value stocks, on average. Chan, Hamao, and Lakonishok (1991) find, furthermore, that the value factor also has a strong role in explaining the cross-section of average returns on Japanese stocks. Hence, there are several other risk factors besides the stock's market beta which add to the explanation of expected stock returns.

Fama and French (1993) introduce a three-factor model which incorporates the size and value effect mentioned above. In this model, the expected return of the asset depends on the sensitivity of its return to the market return and the return on two portfolios meant to mimic the additional two risk factors. The mimicking portfolios are *SMB* (small minus big) and *HML* (high minus low). The former is the difference between the return on a portfolio of small stocks and a portfolio of big stocks, the latter is the difference between the returns on a portfolio of high BE/ME stocks and a portfolio of low BE/ME stocks. Fama and French (1993, 1996) show that the three-factor model captures much of the variation in the cross-section of average returns on portfolios formed on size and BE/ME in U.S. stock markets between 1963 and 1991. More specifically, they show that indeed small stocks (low ME) and value stocks (low BE/ME) persistently outperform large stocks (high ME) and growth stocks (high BE/ME) (Fama and French, 1997).

In addition to the size and value factor, Jegadeesh and Titman (1993) firstly present evidence for the existence of a momentum effect among U.S. stocks. They show that a momentum strategy, i.e. forming a zero-investment portfolio which takes long positions in stocks that have performed well in the past (winners) and takes short positions in stocks that have performed poorly in the past (losers), generates significant positive returns over 3- to 12-month holding periods. However, part of the abnormal returns generated in the first year dissipates in the next two years. De Bondt and Thaler (1985), moreover, show a momentum reversal effect that occurs after three years. They suggest that contrarian strategies, i.e. taking a long positions in past losers and shorting past winners, generate abnormal returns by showing that stocks that performed poorly over the previous three to five years generate higher returns than stocks that performed well over the equivalent period. In order to prevent the reversal effect from occurring, the portfolios formed on momentum should be rebalanced frequently.

Although a substantial amount of research provides evidence for the momentum effect across various markets, the source for this effect is still a subject of debate. Intuitively, one could argue that from an economic point of view, momentum has got little do to with risk because one is simply buying past winners and selling past losers and what is in the past is irrelevant for what is going to happen in the future. Lesmond, Schill, and Zhou (2004) suggest, however, that winning stocks are less liquid and more volatile – hence riskier – than losing stocks, such that a higher reward for risk is captured. Furthermore, in terms of market beta, size and BE/ME, their explanation is that stocks that performed well in the past are small value stocks with a high market beta. Hong, Lim, and Stein (2000) support this statement by showing that momentum declines substantially with firm size and that stocks with low analyst coverage are more susceptible to momentum strategies. Following this line of thought, momentum is considered a risk factor.

Novy-Marx (2012) provides a different view on momentum, namely that momentum is primarily driven by the stock’s performance twelve to seven months prior to portfolio formation. The article provides evidence that stocks that appreciated most in value over the past six months, but performed poorly over the first half of the preceding year, significantly underperform to stocks that have depreciated most in value over the past six months but performed well over the first half of the preceding year. Hence, the intermediate horizon – twelve to seven months prior to portfolio formation – appears to better explain average returns than recent past performance. This is contradictory to the traditional view on momentum that rising stocks keep on rising, while falling stocks keep on falling. In this paper the momentum factor will be tested on twelve month momentum and on intermediate past performance.

Carhart (1997) uses Fama and French’s (1993) three-factor model and adds Jegadeesh and Titman’s (1993) one year momentum anomaly to form a four-factor model which Carhart employs to explain average returns of U.S. mutual funds. The results indicate that the four-factor model performs better than the CAPM in explaining mutual funds returns. Moreover, the size and momentum factor account for most of the explanation. Buying last year's top ten percent mutual funds and selling the bottom ten percent yields a return of 8 percent annually, between 1963 and 1993. Of this percentage, the momentum effect explains 4.6 percent. These findings support the one year momentum effect in U.S. stock returns found by Jegadeesh and Titman (1993). The findings presented by Carhart (1997) imply that funds that performed well last year are expected to generate higher-than-expected returns next year and that one should avoid funds that persistently underperform.

More recently, Fama and French (2015) suggest that a better explanation would be offered by a five-factor model that also incorporates profitability and investment (hence, drops momentum). On average, firms with robust profitability generate significantly higher returns than firms with weak profitability. By using a measure of gross profit (revenue minus cost of goods sold) to assets, Novy-Marx (2013) illustrates that profitable firms generate significantly higher returns that unprofitable firms between 1963 and 2010. Higher rates of investments, on the other hand, imply lower expected returns since earnings that are reinvested in the firm cannot be paid out to shareholders. Titman, Wei, and Xie (2003) and Fama and French (2006) provide evidence on the negative relation between capital investments and expected stock returns. The mathematical methodology behind the two factors will be explained in more detail in the next section. Fama and French (2015) test whether the five-factor model better explains average stock returns than their previous three-factor model by examining the alpha of both models and Gibbons, Ross, and Shanken (1989) (GRS) statistics, thereby covering the period 1963-2013. Alpha's and GRS statistics (detailed explanation in the next section) are lower for the five-factor model than for the three-factor model. Although the alpha's are significant (indicating that a portion of the portfolio's return is not explained by the factors incorporated in the model, hence are explained by other factors) and the GRS test easily rejects the five-factor model capturing all patterns in stock returns, the alpha's and GRS statistics are substantially lower for the five-factor model than for the three-factor model. Thus, the five-factor model better explains stock returns than the three-factor model. Interestingly, they furthermore conclude that the *HML* factor is a redundant factor in the sense that the returns related to *HML* are fully captured by the other factors (market, size, profitability, and investment factor). A four-factor model which excludes *HML* would, as they suggest, perform just as well as the five-factor model. These findings are in line with the findings of Hou, Xue, and Zhang (2014) that examine a four-factor model which also drops the *HML* factor. They do not explicitly comment on why *HML* is excluded, but one can assume it is because of the same reason.

From what is previously discussed, one can argue that the academic literature provides sufficient evidence of the existence of value, size, momentum, profitability, and investment premiums. However, their existence is less clear when it comes to market based portfolios. I.e., theoretically demonstrating the existence of the abovementioned factor premiums does not explicitly mean that investors are able to exploit these premiums by trading on it in stock markets. The results of the papers mentioned thus far exclude limits to arbitrage (such as transaction costs and short selling constraints) which substantially decrease the potential premiums. E.g., Chan, Chen, and Lakonishok (2002) find that U.S. growth mangers outperform value managers by approximately 1.2 percent per year on average, suggesting that a value premium could not be captured. Likewise, Houge and Loughran (2006) find that small-cap value funds generated slightly smaller returns than small-cap growth funds (14.10 percent compared to 14.52 percent, respectively) between 1965 and 2001, and hence conclude that a value premium does not exist in managed mutual fund returns. They use value and growth equity style sub-indices of the S&P 500/BARRA and the Russell 3000 index. Scislaw and McMillan (2012) were also unable to capture a statistically significant value premium. They examined a large array of U.S. small-cap and large-cap indices, but only found very limited evidence in the index series examined. Contrary to these results, Dhatt, Kim, and Mukherji (1999) do find a significant value premium, however, in the small-cap Russell 2000 index.

Most of the research done on multi-factor models focuses on U.S. stock markets only. E.g., Jegadeesh and Titman (1993) find momentum premiums using NYSE, NASDAQ, and S&P 500 data. Similarly, Novy-Marx (2012) captures intermediate momentum using all stocks in the Center for Research in Securities Prices (CRSP) database, i.e. the NYSE, NASDAQ, and S&P 500 indices. Carhart (1997) uses a diversified database of U.S. listed funds when testing his four-factor model and successfully captures a momentum premium. The Fama and French three- and five-factor models are tested on NYSE, AMEX, and NASDAQ stocks. From an international perspective, the evidence is more scarce. Fama and French (2010) examine four regions separately (Europe, Japan, North America and Asia Pacific) and capture value premiums in average stock returns in all regions, except for Japan, that decrease with size. Moreover, they find momentum premiums everywhere, again except for Japan. Furthermore, as mentioned above, Chan, Hamao, and Lakonishok (1991) find that the value factor has a strong role in explaining the cross-section of average returns on Japanese stocks (contradictory to the Fama and French (2010) findings). Griffin (2002) examines the usefulness of country-specific, international, and world versions of the Fama and French three-factor model on stock returns and concludes that the country-specific model better explains stock returns than the international and world model.

SECTION III: METHODOLOGY

This section first discusses and defines the theoretical factor model equations and their respective time series regressions. Afterwards, it discusses the construction of the factor premiums, empirical model, and assumptions. Lastly, the hypotheses are presented.

Under the assumption of risk-free borrowing and lending, and when looking at a single security, the relation between expected return and beta then becomes the well-known Capital Asset Pricing Model (CAPM),

$$\mathrm{E}(R_i) = R_f + \beta_i\, [\mathrm{E}(R_M) - R_f], \tag{1}$$

where $\mathrm{E}(R_i)$ denotes the expected return on security i, R_f is the risk-free rate, β_i is the regression coefficient of R_i on R_M, and $\mathrm{E}(R_M)$ is the expected return on the market. Hence, the expected return on security i is the risk-free rate, R_f, plus the security's beta, β_i, times the market risk premium, $\mathrm{E}(R_M) - R_f$. The Fama and French three-factor model incorporates, as mentioned in the previous section, a size and value factor. The expected return on security i is:

$$\mathrm{E}(R_i) - R_f = b_i\, [\mathrm{E}(R_M) - R_f] + s_i\, \mathrm{E}(SMB) + h_i\, \mathrm{E}(HML), \tag{2}$$

where $\mathrm{E}(R_i)$ is the return on security i, R_f is the risk-free rate, $E(R_M) - R_f$ is the expected market premium, $\mathrm{E}(SMB)$ is the expected size premium, and $\mathrm{E}(HML)$ is the expected value premium. *SMB* (small minus big) represents the return on a diversified portfolio of small stocks minus the return on a diversified portfolio of large stocks and *HML* (high minus low) represent the return on a diversified portfolio of high book-to-market stocks and the return on a diversified portfolio of low book-to-market stocks. Factor loadings b_i, s_i, and h_i are the slopes in the time series regression (Fama and French, 1996):

$$R_{it} - R_{ft} = \alpha_i + b_i\, (R_{Mt} - R_{ft}) + s_i\, SMB_t + h_i\, HML_t + e_{it} \tag{3}$$

Carhart's (1997) four-factor model includes the one year momentum factor in addition to the other three factors. Using the four-factor model, the security's expected return is calculated as

$$\mathrm{E}(R_i) - R_f = b_i\, [\mathrm{E}(R_M) - R_f] + s_i\, \mathrm{E}(SMB) + h_i\, \mathrm{E}(HML) + u_i\, (UMD), \tag{4}$$

where the additional risk factor, E(*UMD*) (up minus down), is the expected return on a zero-investment, mimicking portfolio that represents the difference between the return on the highest momentum portfolio (best performing winners over the past twelve months) and the lowest momentum portfolio (worst performing losers over the past twelve months). Factor loadings b_i, s_i, h_i, u_i and are the slopes in the time series regression:

$$R_{it} - R_{ft} = \alpha_i + b_i (R_{Mt} - R_{ft}) + s_i\, SMB_t + h_i\, HML_t + u_i\, 1y\_UMD_t + e_{it} \quad (5)$$

This paper makes a distinction between one year momentum as presented by Carhart (1997), and the intermediate past performance approach presented by Novy-Marx (2012) as described in the previous section. According to Novy-Marx (2012), intermediate past performance should better explain returns than the one year momentum approach by Carhart (1997). Hence, the time series regression for intermediate past performance is:

$$R_{it} - R_{ft} = \alpha_i + b_i (R_{Mt} - R_{ft}) + s_i\, SMB_t + h_i\, HML_t + i_i\, Intermediate\_UMD_t + e_{it} \quad (6)$$

Thus, *1y UMD* in (5) represent the return on the highest one year momentum portfolio minus the return on the lowest one year momentum portfolio. *Intermediate UMD* in (6) is the return on the highest intermediate past performance portfolio minus the lowest intermediate past performance portfolio (i.e., looking at seven to twelve month prior to portfolio formation).

The two additional factors besides the market, size, and value factor in the Fama and French (2015) five-factor model, profitability and investment, are based on the dividend discount model,

$$\mathrm{M_t} = \sum_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}\,, \quad (7)$$

where $Y_{t+\tau}$ is total equity earnings for period $t + \tau$, $dB_{t+\tau}$ is the change in total book equity, and r is the long-term average expected stock return or, more precisely, the internal rate of return on expected cash flows to shareholders. If we divide the dividend discount model by time t book equity then,

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \quad (8)$$

This equation makes two statements about expected stock returns. Firstly, if we hold constant B_t, M_t, and expected growth in book equity (future investments, $dB_{t+\tau}$), higher expected earnings ($E(Y_{t+\tau})$) – profitability – imply higher expected return on the stock (r). Secondly, for fixed values of B_t, M_t and expected earnings ($E(Y_{t+\tau})$), higher expected growth in book equity – investments – implies a lower expected return because reinvested earnings cannot be paid out to shareholders (Fama and French, 2015). The five-factor model is defined as:

$$E(R_i) - R_f = b_i\,[E(R_M) - R_f] + s_i\,E(SMB) + h_i\,E(HML) + r_i\,E(RMW) + c_i\,E(CMA), \qquad (9)$$

where the profitability factor, $E(RMW)$, represents the difference between the expected return on a portfolio which consists of stocks with robust profitability and a portfolio which consists of stocks with weak profitability. The investment factor, $E(CMA)$, is the difference between the expected return on a portfolio which consists of stocks of low investment (conservative) firms and a portfolio which consists of stocks of high investment (aggressive) firms. The factor loadings b_i, s_i, h_i, r_i, and c_i are the slopes in the time series regression:

$$R_{it} - R_{ft} = \alpha_i + b_i\,(R_{Mt} - R_{ft}) + s_i\,SMB_t + h_i\,HML_t + r_i\,RMW_t + c_i\,CMA_t + e_{it} \qquad (10)$$

The factor premiums are formed as follows. For each year, quintile portfolios on market value, book-to-market, momentum, profitability, and investment are constructed using stock price and company fundamentals. Market value is calculated as stock prices times shares outstanding. Book value per share is retrieved from Datastream and Worldscope, multiplied by number of shares outstanding and subsequently divided by the company's market value to create the book-to-market ratio. The one year and intermediate momentum returns are calculated with the use of stock price. I.e., returns over the past twelve months and returns over the past seven to twelve months prior to portfolio formation. Profitability is calculated somewhat differently than the Fama and French (2015) approach. They calculate profitability as revenue minus costs of goods sold, minus selling, general, and administrative expenses, minus interest expenses all divided by book equity. However, data on many of the expenses is missing in Datastream and Worldscope. Therefore, this paper uses net income divided by book equity as indicator for profitability. Investment is calculated as the change in total asset from year zero to one, divided by year zero total assets.

Next, the factor premiums are calculated. The *SMB* factor premium is the return on the portfolio which contains the 20 percent smallest stocks minus the return on the portfolio which contains the 20 percent largest stocks. The *HML* factor premium is the return on the portfolio which contains the 20 percent highest book-to-market stocks minus the return on the portfolio which contains the 20 percent lowest book-to-market stocks. As explained in the literature section, two approaches for momentum are tested. The *1y UMD* factor is the return on the 20 percent highest one year momentum stocks minus the return on the lowest 20 percent one year momentum stocks. *Intermediate UMD* is the return on the 20 percent highest intermediate past performance stocks minus the return on the 20 percent lowest intermediate past performance stocks. Furthermore, the *RMW* factor is the return on the 20 percent highest profitability stocks minus the return on the 20 percent lowest profitability stocks. Lastly, *CMA* is the difference between the returns on the 20 percent lowest and highest investment portfolios.

Since the data is structed as cross-sectional stock data of multiple stocks over time, a multivariate regression framework is applied. Several assumptions about the parameters and error terms are made. Firstly, this paper assumes linearity, implying that the relation between the dependent and independent variables is of linear nature. Secondly, the residuals are identically and independently distributed. Thirdly, homoskedasticity of the residuals' variance must apply. I.e., there should be no patterns in the variance of the residuals. Fourthly, the assumption of no multicollinearity must hold. Lastly, the assumption of no autocorrelation applies, i.e., there should be no correlation between an observation and a lagged version of itself of sequential time intervals. The next section describes the assumptions in more detail.

This paper tests the following hypotheses regarding the risk factors on developed markets, emerging markets, and developed plus emerging markets:

H_0 : The risk factors SMB, HML, 1y UMD, Intermediate UMD, RMW, and CMA have, in their respective factor models, a significant effect on stock returns.

H_1 : The risk factors SMB, HML, 1y UMD, Intermediate UMD, RMW, and CMA do, in their respective factor models, not have a significant effect on stock returns.

To examine whether the additional risk factors add to the explanation of stock returns, two additional parameters are considered as well. Firstly, the alpha indicates how much of the variation in stock returns is left unexplained by the model. Hence, if the additional risk factor(s) adds to the explanation of stock returns, the alpha of four-factor models (5) and (6) and five-factor (10) should be lower than the alpha of the three-factor model (3). Similarly, the alpha of the five-factor model (10) should be lower than the alpha of four-factor models (5) and (6). In addition, the GRS test is used to examine the performance of the models. The GRS test tests whether the intercepts in multivariate time series regression models are jointly equal to zero and is used to test how well factor models explain stock returns. I.e., it tests whether the factor model completely explains stock returns. The GRS test is computed as follows,

$$\frac{T-N-K}{N}\left(1+E_T(f)'\widehat{\Omega}^{-1}E_T(f)\right)^{-1}\hat{\alpha}'\widehat{\Sigma}^{-1}\hat{\alpha} \sim F_{N,\,T-N-K}, \tag{11}$$

with an F distribution with N and $T-N-K$ degrees of freedom, and assumes that the errors are homoskedastic and uncorrelated. T in (11) is the total number of observations, N is the number of assets (in this case, stocks), K is the number of factors, $E_T(f)$ is the sample mean of the factor returns, $\widehat{\Omega}$ is the sample variance matrix of the factors returns, $\hat{\alpha}$ are the estimated alpha's of the multivariate regression, and $\hat{\Sigma}$ is the covariance matrix of the residuals of the multivariate regression. The GRS test results are obtained from the software package STATA after running the GRS test command.

If the alpha's are jointly equal to zero, the GRS statistic will be zero which implies that the factor model completely explains stock returns. Of course, any asset pricing model will never completely explain stock returns, since the error term will include idiosyncratic factors which remain unobservable (hence, any asset pricing model is imperfect and the GRS test will indicate that the alpha's are jointly significantly different from zero). However, the focus of this paper is to examine whether adding risk factors improves the explanation of stock returns rather than testing whether the competing models are rejected. Therefore, given that the GRS test will always reject any asset pricing model completely explaining stock returns, this papers tries to identify whether the four- and five-factors models are less imperfect than the three-factor model, and whether the five-factor model is less imperfect than the four-factor models, thereby applying the approach of Fama and French (2015). If the additional risk factor(s) adds to the explanation of stock returns, the GRS statistic should be lower.

Thus, besides testing the above hypotheses, the alpha's and GRS statistics of each model are considered in the examination of the performance of the multi-factor models. If four-factor models (5) and (6) and five-factor model (10) better explain the variation in stock returns than the three-factor model (3), the alpha and GRS statistic for the four- and five-factor models should be lower than those of the three-factor model. Additionally, the alpha and GRS statistic of the five-factor model should be lower than those of the four-factor models and three-factor model.

Lastly, this paper examines the time-variation of the factor loadings, since literature suggest they are rather unstable over time. In order to do so, both the developed markets and emerging markets samples are split up in 21 years (1996-2016) using dummy variables. The parameters are subsequently estimated for each year and their year-to-year variation is displayed in a graph, as well as their fitted values. The fitted values indicate how the year-to-year variation of the factor loadings fit in a linear relation.

SECTION IV: DATA DESCRIPTION

The data covers the MSCI All Country World Index (ACWI) which is comprised of 23 developed markets and 24 emerging markets. The 23 developed markets are:

- Americas: Canada, United States
- Europe and Middle East: Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom
- Pacific: Australia, Hong Kong, Japan, New Zealand, Singapore

The 24 emerging markets are:

- Americas: Brazil, Chile, Colombia, Mexico, Peru
- Europe, Middle East, and Africa: Czech Republic, Egypt, Hungary, Poland, Qatar, Russia, South Africa, Turkey, United Arab Emirates
- Asia: China, India, Indonesia, Korea, Malaysia, Pakistan, Philippines, Taiwan, Thailand

Unfortunately, the databases at Erasmus University do not have access to the MSCI ACWI constituents list, merely to various MSCI indices. The MSCI website provides a constituents list which contains all the stocks that together form the MSCI ACWI. Consequently, the companies on the list are matched with ISIN numbers from Datastream and Worldscope after which the needed variables can be retrieved. The downside of this approach is that survivorship bias is not accounted for. The MSCI site only provides the constituents list of the current year, thus the number of stocks examined decreases each year moving back in time since not all companies are in the MSCI ACWI for the entire time horizon. Regardless, for both developed and emerging markets data on a substantial amount of stocks is available. The data covers the period 1996 to 2016 and monthly data is used. Table I in the Appendix describes the data in more detail.

The correlations between the risk factors for the three- and five-factor models and for the four-factor models are illustrated in table II and table III below, respectively. Some of the correlations seem rather high, e.g. the correlation between *HML* and *SMB*, *RMW*, and *CMA* and between *CMA*, and *RMW*. Therefore, the variance inflation factors (VIF) are determined after each regression to test for multicollinearity. As a rule of thumb, one should be concerned about multicollinearity if VIF values exceed 4. For each regression performed on all three samples

Table II

Correlation table of the risk factors in the three- and five-factor model.

	$R_M - R_f$	SMB	HML	RMW	CMA
$R_M - R_f$	1				
SMB	0.00247	1			
HML	0.0803	0.624	1		
RMW	-0.286	-0.369	-0.473	1	
CMA	-0.157	0.0480	0.429	-0.467	1

Table III

Correlation table of the risk factors in the four-factor models.

	$R_m - R_f$	SMB	HML	1y UMD	Intermediate UMD
$R_m - R_f$	1				
SMB	0.00823	1			
HML	0.0796	0.634	1		
1y UMD	-0.128	-0.277	-0.492	1	
Intermediate UMD	-0.0649	-0.209	-0.366	0.741	1

(developed plus emerging markets, developed markets, and emerging markets) the VIF values are determined. The lowest VIF value is 1.01 and the highest 2.23. Thus, there is no multicollinearity between the independent variables.

Furthermore, the standard errors of the results presented in the next section are robust to heteroskedasticity and autocorrelation. Thus, the assumptions of constant variance of the residuals and no autocorrelation in the error terms hold.

SECTION V: RESULTS

The regressions in section III are performed on developed plus emerging markets and developed markets and emerging markets separately. Table IV shows the results for developed plus emerging markets.

Table IV

This table reports the results of regressions (3), (5), (6), and (10) in section III. The independent variable, $R_i - R_f$, is the return on security *i* in excess of the one month U.S. Treasury bill rate. $R_M - R_f$ is the value-weighted return on the market portfolio of all sample stocks minus the risk free rate. *SMB* (small minus big) is the size factor, *HML* (high minus low) is the value factor, *1y UMD* and *Intermediate UMD* (up minus down) are the two momentum factors, *RMW* (robust minus weak) is the profitability factor, and *CMA* (conservative minus aggressive) is the investment factor. Alpha is the regression intercept. These results are for developed plus emerging markets from January 1996 to December 2016 (monthly data).

	(1)	(2)	(3)	(4)
	$R_i - R_f$	$R_i - R_f$	$R_i - R_f$	$R_i - R_f$
$R_M - R_f$	1.000***	1.000***	1.000***	1.014***
	(0.0107)	(0.0106)	(0.0107)	(0.0102)
SMB	0.330***	0.325***	0.325***	0.353***
	(0.0124)	(0.0125)	(0.0126)	(0.0114)
HML	0.0967***	0.106***	0.105***	0.0831***
	(0.0128)	(0.0137)	(0.0136)	(0.0133)
1y UMD		0.00877**		
		(0.00520)		
Intermediate UMD			0.0130**	
			(0.00567)	
RMW				0.0507***
				(0.0172)
CMA				0.0606***
				(0.0171)
Alpha	0.00205***	0.00217***	0.00217***	0.00187***
	(0.000308)	(0.000316)	(0.000317)	(0.000307)
Observations	510,249	494,338	494,338	510,249
R-squared	0.142	0.144	0.144	0.142
GRS statistic	111.01***	117.86***	118.96***	81.28***

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Firstly, all factors have a significant effect on the dependent variable with p-values lower than 0.01 except for *1y UMD* and *Intermediate UMD*, which have a p-value lower than 0.05. The factor loadings (coefficients) indicate how the stocks in the sample are tilted towards size, book-to-market, momentum, profitability, and investment and roughly take a value between zero and one. E.g., a coefficient of one for *SMB* indicates that the sample contains mostly small stocks and a coefficient of zero would indicate that it contains mostly large stocks. A factor loading of one for *HML* would indicate that the sample contains mostly high book-to-market stocks, whereas a value of zero would indicate low book-to-market stocks, etc. Hence, the factor loadings 0.330 and 0.097 for *SMB* and *HML* for regression (1) in table IV, for example, indicate that the total sample (developed plus emerging markets) contains relatively large stocks with low book-to-market ratios. This paper, however, is less interested in how the stocks in the samples are tilted towards size, book-to-market, momentum, profitability, and investment, but more in the alpha's and GRS statistics which indicate how well the factor models explain the variation in stock returns.

Secondly, the alpha's are significantly different from zero for all models, indicating that the variation in stock returns is not fully explained by the factors (which is expected). For the three-factor model, the alpha is 0.205 percent per month, which accounts for 2.488 percent annually. For the four-factor models with one year momentum and intermediate momentum, the annual alpha is 2.635 percent for both models. The annual alpha for the five-factor model is equal to 2.267 percent. The alpha's indicate that between 2.267 and 2.635 percent stock return is explained by other factors than those incorporated in the respective model. It furthermore indicates that the five-factor model better explains stock returns than the three- and four-factor models since the alpha for the five-factor model is lower (hence, more of the variation in stock returns is explained by the factors incorporated in the model). Moreover, the alpha of the three-factor model is lower than the alpha of the four-factor models, indicating that the three-factor model performs better than the four-factor models. There is no difference in alpha's between the one year momentum and intermediate momentum models. Novy-Marx's (2012) intermediate momentum, therefore, does not better explain returns than Carhart's one year momentum for developed plus emerging markets.

Lastly, the GRS statistics supports the statement made about the alphas that, although the GRS test easily rejects the alpha's being jointly equal to zero, the five-factor model better explains stock returns than the three- and four-factor models (GRS statistic is lower). Thus, the five-factor model is less imperfect than the four- and three-factor models, and the three-factor model is less imperfect than the four-factor models. This implies that the profitability and investment factor add to the explanation of stuck returns compared to the three-factor model, however, the momentum factors in the four-factor models do not add to the explanation compared to the three-factor model.

The results for developed markets are presented in table V below. The results show that for developed markets the *HML* factor in the five-factor model does not have a significant effect on stock returns. Fama and French (2015) argue that the two additional factors to the three-factor model (profitability and investment) absorb the explanatory power of the value factor, which could be one of the reasons why *HML* loses its significance. The other factors remain significant. Furthermore, the alpha's indicate that for developed markets Carhart's four-factor model (incorporating one year momentum) best explains stock returns with 1.547 percent annual stock return left unexplained by the model. The five-factor model performs worst, most likely due to the insignificance of the value factor. On an annual basis, 3.190 percent stock return is left unexplained. The GRS statistics again support these statements since its value is the lowest for Carhart's four-factor model. Moreover, the difference in explanatory power between the two four-factor models is rather small. The alpha's differ only 0.122 percent annually.

Table V

This table reports the results of regressions (3), (5), (6), and (10) in section III. The independent variable, $R_i - R_f$, is the return on security *i* in excess of the one month U.S. Treasury bill rate. $R_M - R_f$ is the value-weighted return on the market portfolio of all sample stocks minus the risk free rate. *SMB* (small minus big) is the size factor, *HML* (high minus low) is the value factor, *1y UMD* and *Intermediate UMD* (up minus down) are the two momentum factors, *RMW* (robust minus weak) is the profitability factor, and *CMA* (conservative minus aggressive) is the investment factor. Alpha is the regression intercept. These results are for developed markets from January 1996 to December 2016 (monthly data).

	(1) $R_i - R_f$	(2) $R_i - R_f$	(3) $R_i - R_f$	(4) $R_i - R_f$
$R_M - R_f$	0.990***	0.987***	0.989***	1.030***
	(0.0131)	(0.0130)	(0.0131)	(0.0126)
SMB	0.142***	0.138***	0.136***	0.234***
	(0.0122)	(0.0124)	(0.0125)	(0.0117)
HML	0.127***	0.112***	0.126***	0.00452
	(0.0147)	(0.0160)	(0.0159)	(0.0148)
1y UMD		-0.0291***		
		(0.00554)		
Intermediate UMD			-0.0169***	
			(0.00633)	
RMW				0.0508**
				(0.0202)
CMA				0.289***
				(0.0169)
Alpha	0.00130***	0.00128***	0.00138***	0.00262***
	(0.000367)	(0.000378)	(0.000381)	(0.000358)
Observations	360,163	347,965	347,965	360,163
R-squared	0.159	0.161	0.161	0.162
GRS statistic	40.16***	37.40***	42.95***	131.72***

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The results for emerging markets are presented in table VI below. The value factor for emerging markets is significant in the five-factor model, contradictory to the developed markets results. Interestingly, one of the alpha's is not significantly different from zero. This indicates that, for emerging markets, the five-factor model completely explains the variation in stock returns, which is a rather unrealistic finding. As discussed in section III, asset pricing models will never completely explain stock returns, since the error term will include idiosyncratic factors which remain unobservable. The insignificant alpha is, therefore, an uninterpretable finding. The GRS test furthermore indicates that the alpha's are jointly indifferent from zero for the five-factor model. These findings are similarly unrealistic, since it is impossible that absolutely no other factors than the market, size, value, profitability, and investment factor influence stock returns. The three-factor model has a lower alpha and GRS statistic than the four-factor models, implying that the additional momentum factors do not add to the explanation of stock returns. Moreover, the alpha's for the three- and four-factor models in the emerging markets regressions are substantially higher than the alpha's of the developed markets regressions. E.g., the three-factor model alpha for developed markets is 1.571 percent annually, whereas for emerging markets the three-factor model alpha is 5.510 percent annually. These findings, therefore, suggest that the multi-factor models better explain stock returns for

Table VI

This table reports the results of regressions (3), (5), (6), and (10) in section III. The independent variable, $R_i - R_f$, is the return on security *i* in excess of the one month U.S. Treasury bill rate. $R_M - R_f$ is the value-weighted return on the market portfolio of all sample stocks minus the risk free rate. *SMB* (small minus big) is the size factor, *HML* (high minus low) is the value factor, *1y UMD* and *Intermediate UMD* (up minus down) are the two momentum factors, *RMW* (robust minus weak) is the profitability factor, and *CMA* (conservative minus aggressive) is the investment factor. Alpha is the regression intercept. These results are for emerging markets from January 1996 to December 2016 (monthly data).

	(1) $R_i - R_f$	(2) $R_i - R_f$	(3) $R_i - R_f$	(4) $R_i - R_f$
$R_M - R_f$	1.011***	1.021***	1.013***	0.966***
	(0.0178)	(0.0178)	(0.0178)	(0.0171)
SMB	0.778***	0.770***	0.773***	0.629***
	(0.0216)	(0.0218)	(0.0219)	(0.0235)
HML	0.0476**	0.117***	0.0792***	0.300***
	(0.0254)	(0.0265)	(0.0262)	(0.0268)
1y UMD		0.102***		
		(0.0109)		
Intermediate UMD			0.0834***	
			(0.0114)	
RMW				0.0493**
				(0. 03317)
CMA				-0.507***
				(0.0342)
Alpha	0.00448***	0.00495***	0.00473***	0.000582
	(0.000559)	(0.000571)	(0.000571)	(0.000543)
Observations	150,086	146,373	146,373	150,086
R-squared	0.129	0.132	0.131	0.134
GRS statistic	101.02***	120.47***	110.17***	1.63

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

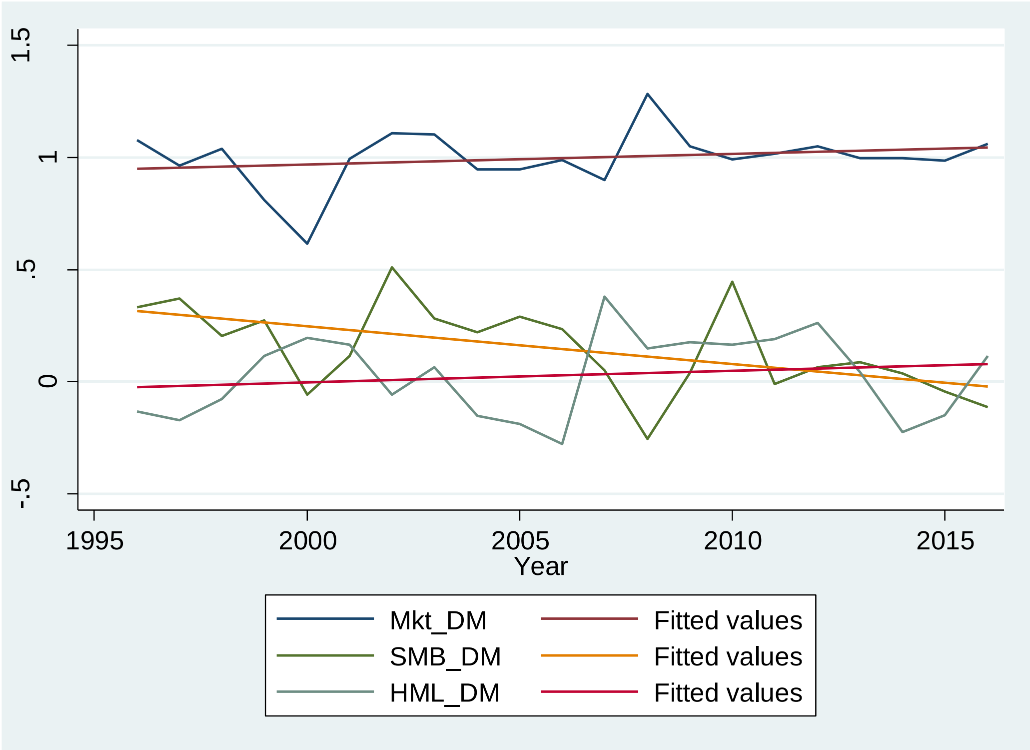
developed markets than for emerging markets. At last, in contradiction to the previous two samples, the intermediate momentum factor does explain stock returns slightly better than one year momentum although the difference is still small. The monthly alpha's for one year and intermediate momentum are 0.495 and 0.473 percent per month, respectively, which account for 6.104 and 5.826 percent annually. Hence, the intermediate momentum factor explains 0.278 percent more of the variation in stock returns than one year momentum for emerging markets.

Overall, several conclusions can be drawn from the results presented above. The null-hypothesis regarding the significance of the factors is accepted for all models in all three samples, with exception of the *HML* factor in the developed markets five-factor model. In addition to the hypotheses, the alpha's and GRS statistics are examined. The alpha's and GRS statistics for the developed plus emerging markets results indicate that the five-factor model performs better than the four- and three-factor models, and that the three-factor model performs better than the four-factor models. For developed markets, they indicate that the five-factor model performs worst (highest alpha and GRS statistic), most likely due to the insignificance of the *HML* factor. Furthermore, the four-factor model which incorporates one year momentum performs best. For emerging markets, the insignificant alpha for the five-factor model seems to be an unrealistic finding as it would imply that the five-factor model completely explains the variation in stock returns, however, there will always be idiosyncratic factors which remain unobservable that influence stock prices. Disregarding the five-factor model results, the alpha and GRS statistic indicate that the three-factor model performs best for emerging markets. Additionally, the multi-factor models examined better explain stock returns for developed markets than for emerging markets, due to the lower alpha's and GRS statistics for the former. At last, the results suggest that the difference in explanatory power between the two four-factor models (one year momentum and intermediate momentum) is negligible.

This paper furthermore examines whether the factor loadings are stable over time, as discussed in section III. Graphs I and II below illustrate how the factors loadings for the three-factor models for both samples fluctuate over time. The fitted values are included as well, which indicate how the year-to-year variation of the factor loadings fit in a linear relation. The fitted values in graph I illustrate that the market and value factor loadings are quite stable over the sample period for developed markets, despite the rather high year-to-year volatility. The size factor loading decreases slightly over the years.

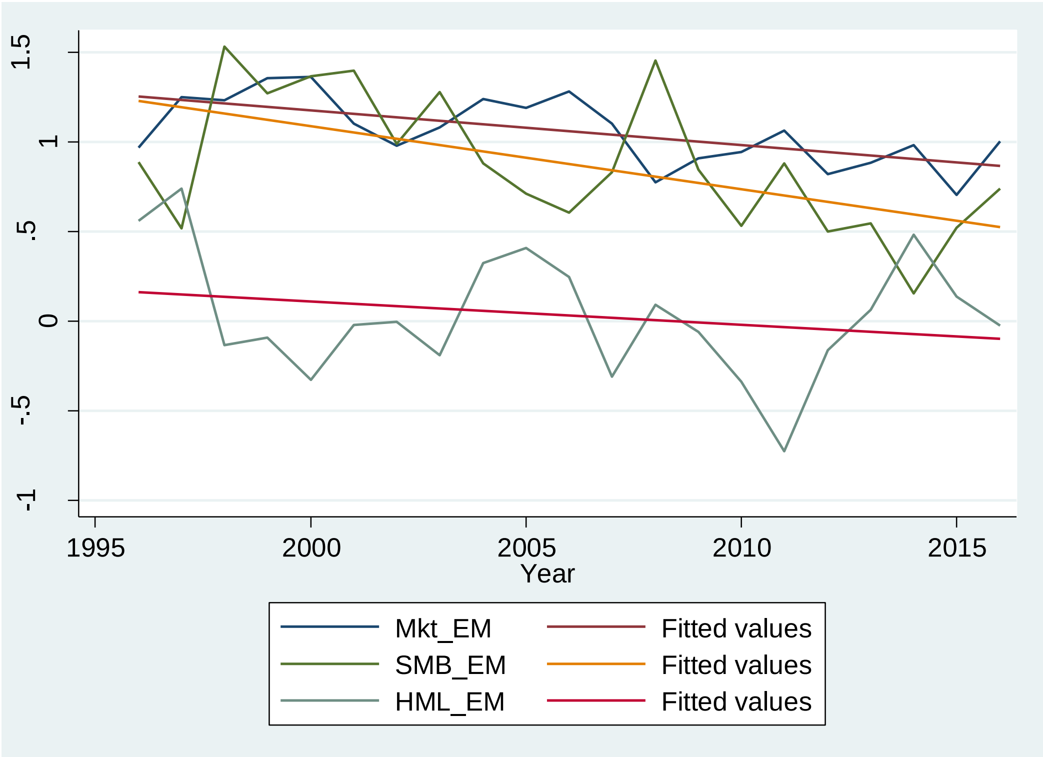
Graph I

This graph illustrates how the three-factor model factor loadings on the market (*Mkt*), size (*SMB*), and value (*HML*) factor move over time for developed markets. Dummy variables for each year are applied over the sample period 1996-2016.



Graph II

This graph illustrates how the three-factor model factor loadings on the market (*Mkt*), size (*SMB*), and value (*HML*) factor move over time for emerging markets. Dummy variables for each year are applied over the sample period 1996-2016.



In addition, the factor loadings on all three factors fluctuate heavily around the Dotcom bubble and financial crisis of 2007-2008. Graph II illustrates the stability of the three-factor model factor loadings for emerging markets. The fitted values indicate that the factor loadings seem to be more unstable for emerging markets than for developed markets. The size factor loading drops from approximately 1.25 to 0.5 over time, whereas for developed markets it drops from approximately 0.3 to 0. The market factor loading drops from approximately 1.25 to 0.8, whereas for developed markets it remains fairly stable. The value factor loading is rather stable over time for emerging markets.

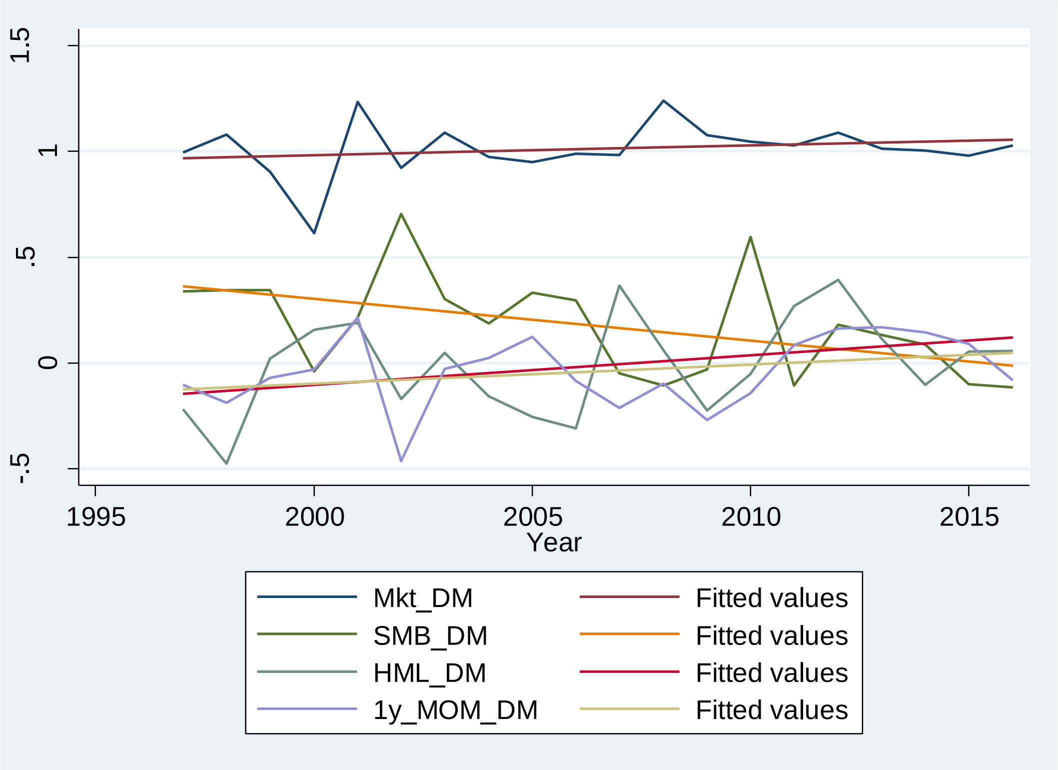
Graphs III and IV below illustrate how the Carhart four-factor model factor loadings, which incorporates one year momentum, vary over time for both samples. The fitted values indicate that for both developed and emerging markets, the momentum factor loadings tends to move in line with the value factor. The momentum factor loading for developed markets is reasonably stable, whereas for emerging market it slightly decreases over the years. Graphs V and VI further below present the results for the four-factor model which incorporates intermediate momentum. The inclusion of the intermediate momentum instead of the one year momentum factor flattens the value and the intermediate momentum loadings for both market segments. Although the year-to-year volatility remains rather high, the fitted values indicate that the value and intermediate momentum loadings tend to be stable over the entire period.

The factor loadings for the five-factor model are presented in graphs VII and VIII on page 25. For developed markets, the fitted values in graph VII demonstrate that the profitability factor moderately increases over time, whereas the investment remains stable and hoovers around zero. However, both factor loadings fluctuate heavily from year to year, especially considering the peaks around the financial crisis. For emerging markets (graph VIII), the fitted valued indicate that the profitability and investment factor remain fairly stable over time. Again, the year-to-year volatility is high.

Overall, the year-to-year variation in factor loadings is high for both market segments. However, the fitted values indicate that the factor loadings are rather stable for developed markets with the exclusion of the size factor loading, which slightly decreases over time, and the small increase of the profitability factor loading for the five-factor model. The factor loadings for emerging markets are more unstable than for developed markets, thereby also fluctuating more from year to year. The emerging markets market and size loadings in the three- and four-factor model and the value factor in the five-factor model slightly decrease over time.

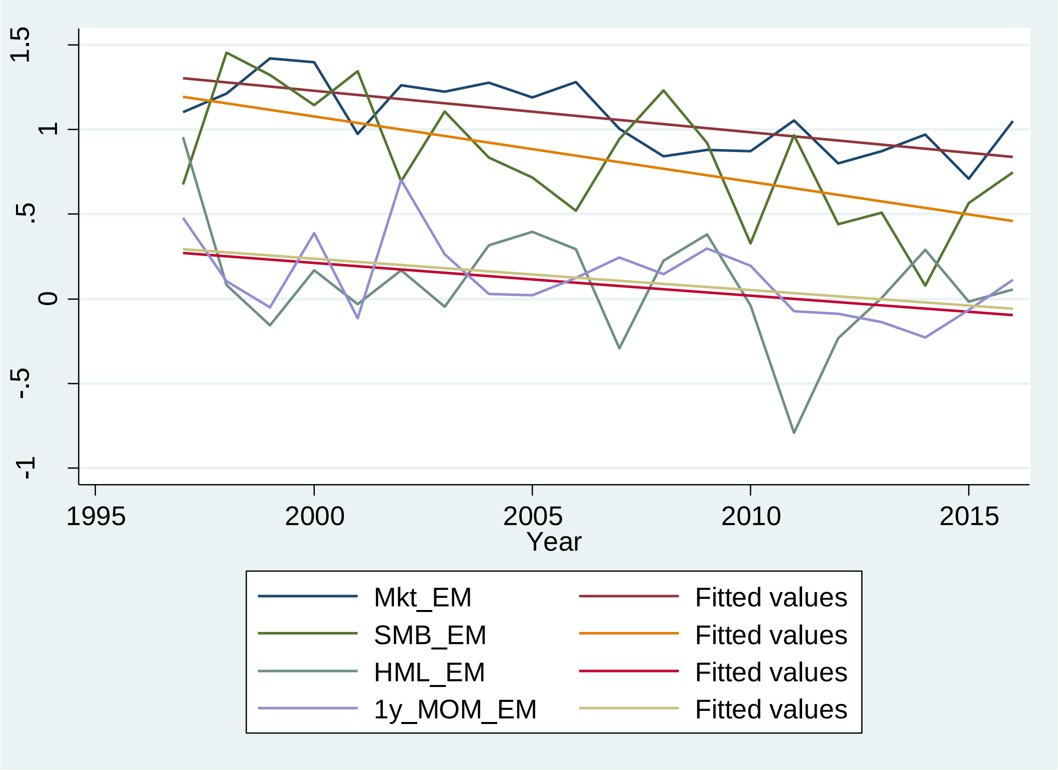
Graph III

This graph illustrates how the Carhart four-factor model factor loadings on the market (*Mkt*), size (*SMB*), value (*HML*), and one year momentum (*1y_MOM*) factor move over time for developed markets. Dummy variables for each year are applied over the sample period 1996-2016.



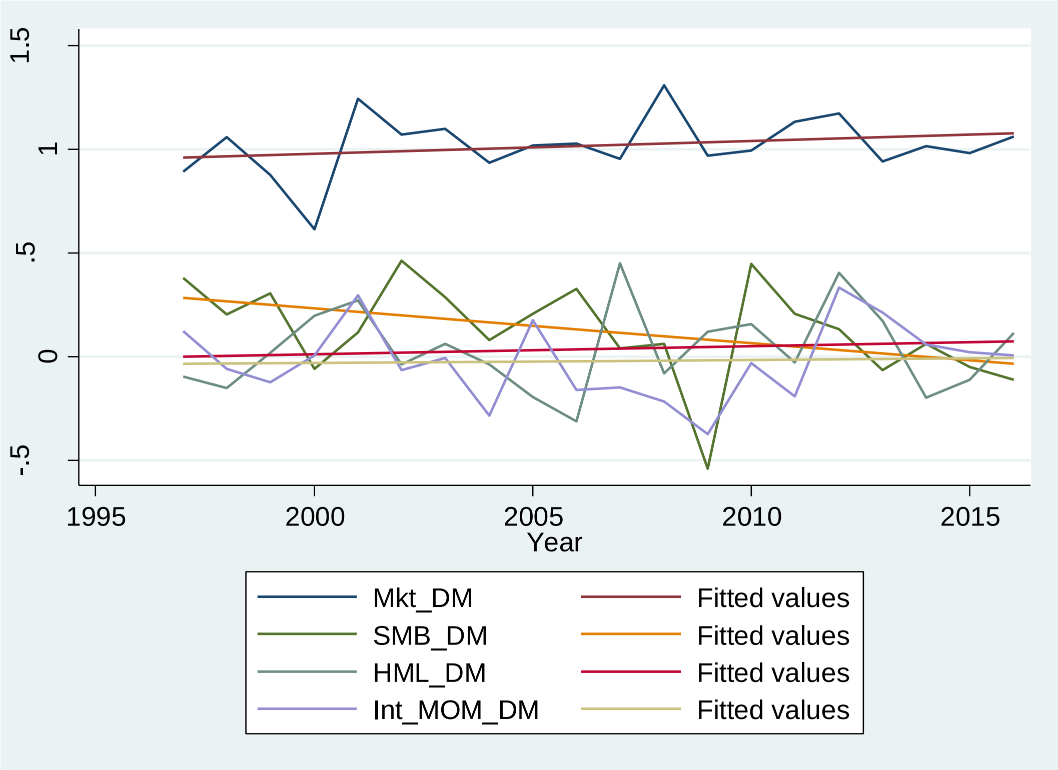
Graph IV

This graph illustrates how the Carhart four-factor model factor loadings on the market (*Mkt*), size (*SMB*), value (*HML*), and one year momentum (*1y_MOM*) factor move over time for emerging markets. Dummy variables for each year are applied over the sample period 1996-2016.



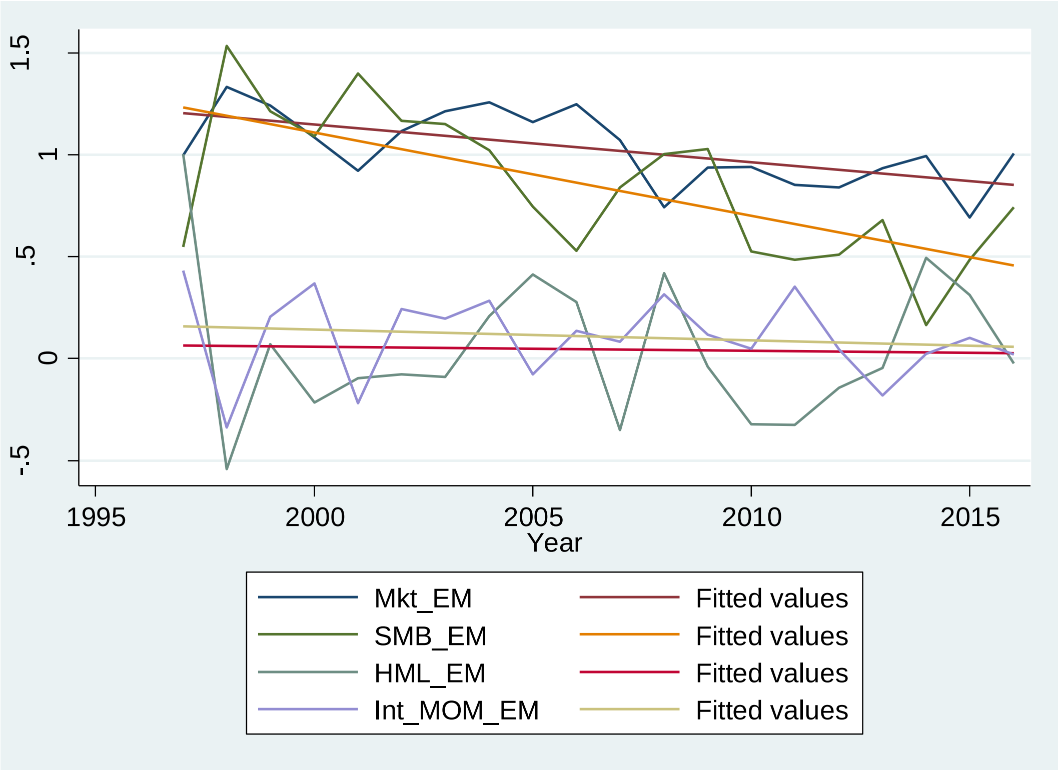
Graph V

This graph illustrates how the four-factor model factor loadings on the market (*Mkt*), size (*SMB*), value (*HML*), and intermediate momentum (*Int_MOM*) factor move over time for developed markets. Dummy variables for each year are applied over the sample period 1996-2016.



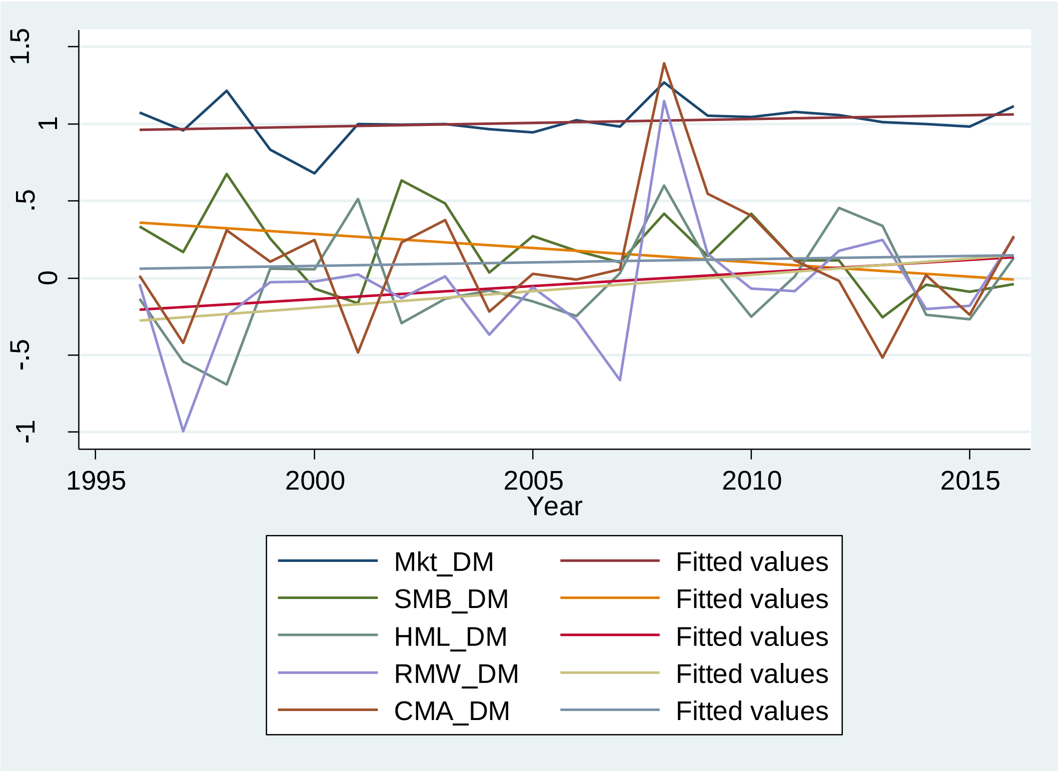
Graph VI

This graph illustrates how the four-factor model factor loadings on the market (*Mkt*), size (*SMB*), value (*HML*), and intermediate momentum (*Int_MOM*) factor move over time for emerging markets. Dummy variables for each year are applied over the sample period 1996-2016.



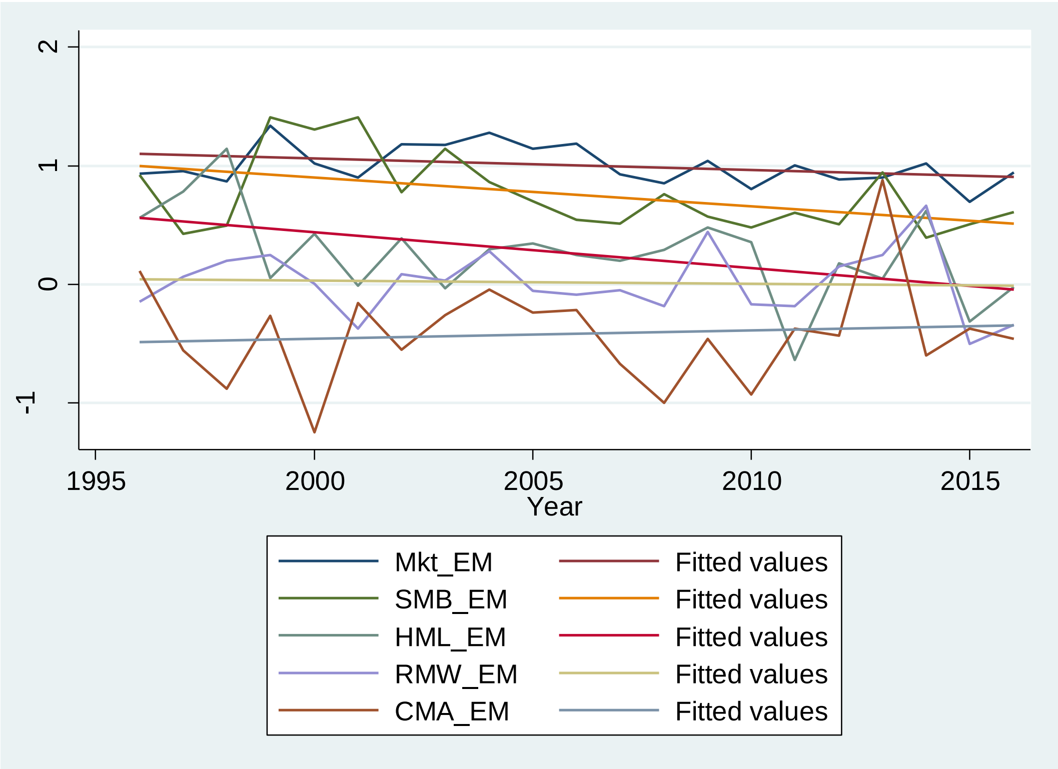
Graph VII

This graph illustrates how the five-factor model factor loadings on the market (*Mkt*), size (*SMB*), value (*HML*), profitability (*RMW*), and investment factor (*CMA*) move over time for developed markets. Dummy variables for each year are applied over the sample period 1996-2016.



Graph VII

This graph illustrates how the five-factor model factor loadings on the market (*Mkt*), size (*SMB*), value (*HML*), profitability (*RMW*), and investment factor (*CMA*) move over time for emerging markets. Dummy variables for each year are applied over the sample period 1996-2016.



SECTION VI: CONCLUSION AND DISCUSSION

The relation between risk and return has been thoroughly examined over the past decades, however, with the focus predominantly on U.S. stock markets. This paper sheds light on how well these multi-factor models perform internationally, thereby specifically focussing on developed and emerging markets. By examining a large array of stocks covering the MSCI ACWI, several conclusions can be drawn. Firstly, the risk factors have a significant effect on average stock returns in all models, with exclusion of the *HML* factor in the five-factor model for developed markets. The value factor is in this case a redundant factor as its explanatory power is absorbed by the market, size, profitability, and investment factor. Secondly, the multi-factor models better explain stock returns for developed markets than for emerging markets since the alpha's are substantially lower, which implies less variation in stock returns is left unexplained, and GRS statistics are lower as well. Thirdly, for developed markets, Carhart's four-factor model performs best, whereas for emerging markets the three-factor model performs best (when one disregards the unrealistic five-factor model results for emerging markets). These results suggest that adding more risk factors does not necessarily increase the explanatory power of multi-factor models, especially when the difference between the three-factor model and Carhart's four-factor model for developed markets is extremely small (0.130 and 0.128 percent alpha per month, respectively). Fourthly, the distinction between one year and intermediate momentum does not add much to the explanation of stock returns. For developed plus emerging markets and developed markets, the difference in explanatory power is negligible and the difference in the emerging markets sample is only 0.275 percent annually. Finally, the factor loadings fluctuate heavily from year to year for both market segments. However, they are fairly stable over the entire period examined for developed markets, whereas for emerging markets the market and size factor loadings in the three- and four-factor model and the value factor in the five-factor model slightly decrease over time.

As mentioned previously, the data used in this paper is prone to survivorship bias. Due to the fact that the databases at Erasmus University do not have access to the MSCI ACWI constituents list, the author had to use the constituents list from the MSCI website and match these companies with Datastream and Worldscope data. Unfortunately, only the constituents of year 2016 were provided. As a result, the results in this paper are prone to survivorship bias. Companies that were part of the index but dropped out anywhere before 2016, for whatever reason, are not included in the sample. However, since this paper examines how factor models explain stock returns, the results might not even be biased because of the survivorship bias issue. This paper does not examine some sort of performance trading strategy of which the results would be biased upwards if survivorship bias is present because one is mostly looking at top performing stocks. Bad performing stocks that drop out the index would probably have some sort of tilt in size, book-to-market, momentum, profitability, and investment which would explain their poor performance. Either way, further research should test the models on a complete dataset which is robust to survivorship bias.

APPENDIX

Table I

Number of stocks

	Developed markets	Emerging markets	Total
1996	1111	340	1451
1997	1148	371	1519
1998	1192	399	1591
1999	1230	414	1644
2000	1276	437	1713
2001	1321	464	1785
2002	1357	492	1848
2003	1379	511	1890
2004	1402	550	1952
2005	1429	582	2011
2006	1459	612	2071
2007	1489	648	2137
2008	1515	684	2200
2009	1525	697	2222
2010	1547	718	2265
2011	1574	743	2318
2012	1596	759	2354
2013	1619	775	2394
2014	1649	787	2436
2015	1668	795	2462
2016	1669	797	2466

REFERENCES

Banz, R. W. (1981). The relationship between return and market value of common stocks, *Journal of Financial Economics*, 9(1), 3–18.

Black, F. (1972). Capital market equilibrium with restricted borrowing, *Journal of Business*, 45, 444-455.

Carhart, M. M. (1997). On persistence in mutual fund returns, *Journal of Finance*, 52(1), 57-83.

Chan, L. K. C., Chen, H. L., Lakonishok, J. (2002). On mutual fund investment styles, *Review of Financial Studies*, 15, 1407-1437.

Chan, L. K.C., Hamao, Y., Lakonishok, J. (1991). Fundamentals and stock returns in Japan, *Journal of Finance*, 46(5), 1739-789.

De Bondt, W. F. M., Thaler, R. (1985). Does the stock market overreact? *Journal of Finance*, 40, 793-805.

Dhatt, M., Kim, Y. and Mukherji, S. (1999). The value premium for small-capitalization stocks, *Financial Analyst Journal*, 55(5), 60-68.

Fama, E. F., French, K. R. (1993). Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 33(1), 3-56.

Fama, E. F., French, K. R. (1996). Multifactor explanations of asset pricing anomalies, *Journal of Finance*, 51(1), 55-84.

Fama, E. F., French, K. R. (1997). Industry costs of equity, *Journal of Financial*

Economics, 43(2), 153-93.

Fama, E. F., French, K. R. (2004). The Capital Asset Pricing Model: theory and evidence, *Journal of Economic Perspective*, 18(3), 25-46.

Fama, E. F., French, K. R. (2006). Profitability, investment, and average returns. *Journal of Financial Economics*, 82, 491-518.

Fama, E. F., French, K. R. (2010). Size, value, and momentum in international stock returns, *Journal of Financial Economics*, 105(3), 457-472.

Fama, E. F., French, K. R. (2015). A five-factor asset pricing model, *Journal of Financial Economics*, 116, 1-22.

Gibbons, M. R., Ross, S. A., Shanken, J. (1989). A test of efficiency of a given portfolio, *Econometrica*, 57(5), 1121-1152.

Griffin, J. M. (2002). Are the Fama and French factors global or country specific? *The Review of Financial Studies*. 15(3), 783-803.

Hong, H., Lim, T., Stein, J. (2000). Bad news travels slowly: size, analyst coverage, and the profitability of momentum strategies. *The Journal of Finance*, 55(1), 265-295.

Hou, K., Xue, C., Zhang, L. (2014). Digesting anomalies: an investment approach. *The Review of Financial Studies.* 28(3).

Houge, T., Loughran, T. (2006). Do investors capture the value premium? *Financial Management*, 5-19.

Jegadeesh, N., Titman, S. (1993). Returns to buying winners and selling losers: implications for stock market efficiency, *Journal of Finance*, 48(1), 65-91.

Klein, R. G., Bawa, V. S. (1977). The effect of limited information and estimation risk on optimal portfolio diversification, *Journal of Financial Economics,* 5, 89-111.

Lesmond, D. A., Schill, M. J., Zhou, C. (2004). The illusory nature of momentum profits, *Journal of Financial Economics*, 71, 349-380.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics*, 47, 13-37.

Novy-Marx, R. (2012). Is momentum really momentum? *Journal of Financial Economics*, 103, 429-453.

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of Financial Economics*, 108, 1-28.

Rosenberg, B., Reid, K., Lanstein, R. (1985). Persuasive evidence of market inefficiency, *Journal of Portfolio Management*, 11, 9-17.

Scislaw, K. E., McMillan, D. G. (2012). The search for an exploitable value premium in market indices, *Journal of Asset Management*, 13, 253-270.

Sharpe, W. F. (1964). Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance*, 19, 425-442.

Stattman, D. (1980). Book values and stock returns, *The Chicago MBA: A Journal of Selected Papers*, 4, 25-45.

Titman, S., Wei, K. C., Xie, F. (2003). Capital investments and stock returns. *National Bureau of Economics* Research, 9951.