ERASMUS UNIVERSITY ROTTERDAM
Erasmus School of Economics
Bachelor Thesis Economics (Algemene Economie)

Optimal favoritism within ECB policy

Bas Scheepers (357515)

Abstract

This thesis establishes a framework to justify the perceived regional biases within ECB interest rate setting policy. In order to accomplish this, a Taylor-rule is constructed from which five conditions are derived regarding differences in economic circumstances among member states. These economic circumstances consist of: (1) differences in output persistence, (2) interest rate elasticity, (3) sensitivity to real exchange rate changes, (4) exposure to non-Eurozone trade and (5) inflationary pressure of the output-gap. If one of the conditions derived from the Taylor rule is violated, it indicates that it is optimal for the ECB to favor economic circumstances within a member state more than is merited by its economic size alone. To demonstrate how these conditions justify regional biases within ECB policy, the weights of the four largest Eurozone member states are determined for which it holds that the ECB conducts optimal policy. Indeed, it was found that in some cases, the ECB conducts optimal policy if it under- or overweights some member states. However, due to a lack of data points, the constructed model could not be empirically validated. Therefore, the values of the estimated optimal weights are not completely accurate, but merely serve as an indication that it could be optimal for the ECB to under- or overweigh certain member states.

Supervisor: prof. dr. I.J.M. Arnold
Second assessor: prof. dr. C.G. de Vries

Date Final Version: 21/08/2017

Table of Contents

1 Introduction

The European Central Bank (ECB) has been assigned the difficult task to singlehandedly conduct monetary policy on behalf of all the EMU member states. Part of this task is to set an interest rate, which is used as an instrument to stimulate output or increase price stability. Deciding which interest rate is optimal for an economic area can be challenging, since pursuing one of those targets opposes achieving the other. This is because a low interest rate stimulates output, but may decrease price stability and vice versa. Since all member states experience different economic circumstances, they all have their own preferences regarding the interest rate. Hence, it is possible that economic circumstances in one EMU member state require an interest rate that increases price stability, while economic circumstances in another member state require an interest rate that stimulates output. As the Eurozone consists of 19 member states, it becomes a daunting task to combine all those different preferences into one single interest rate. So how does the ECB make its interest rate decision? The official method employed by the ECB is to consider aggregate Eurozone data only. Through this approach, the economic circumstances of each member state are proportionally reflected within the interest rate decision. In other words, the individual influence of each member state on the interest rate is equal to its economic size; the bigger the economic size of a member state, the more its economic circumstances affect the interest rate. By using this method, the ECB ensures that the possibilities of regional biases or favoritism are limited. In this context, a regional bias exists if more weight is placed on a member state's economic circumstances than is merited by its economic size. However, this method might not be optimal in an economic sense. This is because the sensitivity of national economies to the interest rate do not affect the amount of influence they possess. As a result, the chosen interest rate might not be suitable for each individual member state. Considering that more sensitive economies are relatively more harmed by an unsuitable interest rate than economies which are insensitive, the former states might be underrepresented in the interest rate decision. By increasing the influence of more sensitive economies, the net effect of the ECB's interest rate policy might be more optimal compared to the current policy wherein only aggregates are considered. Therefore, a more optimal method to determine the interest rate could be to proportionally consider both economic size, as well as the resulting effect of the proposed interest rate on national economies.

By employing this method, the individual influence of member states will likely deviate from their respective economic sizes. With the current method employed by the ECB, this indicates that a regional bias exists within ECB policy. However, this is not the case if a member state's sensitivity to the proposed interest rate is also considered. Then, if the individual influence of a member state on the interest rate exceeds its economic size, it does not necessarily indicate the presence of a regional bias; it could also indicate that it is optimal for the aggregate Eurozone if that member state has more influence. A perceived regional bias within ECB interest rate policy could thus be optimal in an economic sense. Such a bias is referred to as an optimal regional bias.

These economic justifications for regional biases will be the focus of this thesis. In particular, this thesis will examine which economic circumstances within member states give rise to optimal regional biases. Therefore, the main research question of this thesis is: "*Which circumstances of national economies of EMU member states justify the existence of regional biases within ECB interest rate policy?*" In order to demonstrate which economic circumstances justify a regional bias, a model of the Eurozone consisting of two member states is constructed. This model, combined with a condition for optimal ECB policy, results in a Taylor-rule which describes the optimal interest rate as dependent on a member state's output, the optimal level of its output and its inflation. If the relative influence of a member state's economy on the optimal interest rate deviates from its economic weight, the ECB should attribute a different weight to economic circumstances in that member state. This suggests that an optimal regional bias is present. From the employed model, it appears that there are five characteristics that induce an optimal regional bias, namely: output persistence, interest rate elasticity, sensitivity to real exchange rate changes, exposure to non-Eurozone trade and inflationary pressure of the output-gap. The employed model is to a great extent derived from Arnold (2006), but while he exclusively focused on the influence of non-Eurozone trade, this thesis examines all of the aforementioned characteristics. This is due to the fact that multiple justifications could exist for both a positive and a negative bias at the same time. This plurality of justifications could make it possible to collectively reinforce or neutralize the bias.

The remainder of this thesis is organized as follows. First, Section 2 reviews some of the literature regarding the ECB's monetary policy. Then, in section 3 a model is constructed to determine in which instances it is optimal for the ECB to deviate from a member state's economic weight. Section 4 determines the optimal weights that the ECB should place on economic circumstances within member states. Finally, in section 5, a summary and a conclusion are presented.

2 Literature Review

A common tool that is employed by authors to analyze ECB policy is the so-called Taylor-rule. This rule, named after its creator John B. Taylor (Taylor, 1993), offers a guideline for central banks to determine how they should respond to changes in output and inflation by altering the nominal interest rate. Taylor-rules offer a simple, mathematical strategy to determine and audit monetary policies. These characteristics have attracted a plethora of authors who apply this instrument to examine central bank policy, including the ECB. See for example Taylor (1998), Gerlach & Schnabel (2000), Sauer & Sturm (2003) and Fourçans & Vianceanu (2004). By employing the same aggregate approach as the ECB takes itself, these authors found evidence that Taylor rules adequately explain ECB policy. However, by taking an aggregate approach to examine ECB policy, the existence of regional biases cannot be ruled out.

Other studies which took a more disaggregate approach, found evidence that economic conditions in Germany and France were overrepresented in the interest rate decision of the ECB. For instance, Kool (2006) found that the interest rate set by the ECB has been consistent with predominantly German preferences, which leads to the impression that a regional bias might be present. Likewise, Von Hagen & Brückner (2001) concluded that ECB interest rate setting could be more accurately estimated by averaging the optimal interest rates for Germany and France compared to employing aggregate Eurozone data. Furthermore, Heineman & Hüfner (2002) found evidence that the interest rate decisions by the ECB could be more effectively explained when regional differences are accounted for, again suggesting that regional biases could be present in ECB policy. In particular, Crowley and Lee (2008) found that if member states could hypothetically determine their own interest rate, most of these rates would be significantly different than what the actual ECB interest rate was. Only the interest rate of Germany and smaller member states with similar economic performances were found to correspond with the actual interest rate.

Instead of examining the possible existence of regional biases from an economic perspective, some authors have opted to take a more politically inspired approach. For example, Hayo & Méon (2013) concluded that the scenario in which at least some of the members of the Governing Council pursue national interests explained the actual interest rates more precise than a scenario in which all the members pursue solely European interests. This is in accordance with previous findings by Dorbusch et al (1998), Berger and De Haan (2002) and Meade and Sheets (2005), who all suggested that it is likely that national economic conditions influence voting by individual members on the ECB council. These findings indicate that regional biases are not necessarily based on economic circumstances, but can also originate from political considerations. While this could help to better explain the ECB's monetary policy, this element of regional biases will not be further investigated in this thesis.

Contrary to most authors, who merely try to prove the existence of regional biases within ECB policy, Arnold (2006) examined whether regional biases could be optimal in an economic sense. For this purpose, Arnold (2006) constructed a Taylor rule by conjoining individual country economies into a single aggregate Eurozone economy. He found that the ECB should, under several specified circumstances, deviate from its practice where a member state's influence on the interest rate corresponds to their economic size. Optimal monetary policy, according to him, implies that the ECB should place more weight on output and inflation of larger member states. Sturm and Wollershäuer (2008) found evidence to support that actual ECB policy does overvalue economic conditions, but in smaller member states. They used the deviation of each member state's individual optimal interest rate from the ECB interest rate, and constructed a measure of deviation for the whole Eurozone. It appeared that Austria, Belgium, Finland, Greece, Ireland, and Portugal were overrepresented in the ECB decision-making process. Their influence on the interest rate was found to be greater than what their economic sizes would suggest. On the other hand, larger member states, namely France, Germany, and Italy, were found to be underrepresented compared to their economic sizes. Drometer, Siemsen, and Watzka (2013) contest these results. They found that between 1999 and 2007 the optimal interest rate of Germany followed the interbank lending rate. However, they found evidence that the ECB placed more weight on weaker economies since the start of debt crisis in 2008. Thus, the interbank lending rate in that period was closer to the optimal rates in the weaker economies than it was to Germany's. A similar conclusion was reached by Bouvet and King (2013) and Olsen and Wohar (2015).

From this literature review, it appears that the ECB is indeed subject to regional biases. However, which member states are favored by the ECB depends on which time period is investigated. Pre 2008, it appears that economic circumstances in Germany were the best indicator of the ECB's interest rate decision. Post 2008, after the start of the sovereign debt crisis, it appeared that economic circumstances in smaller member states were more influential. While most studies support this conclusion, they do not provide an answer for why regional biases exist or whether they are optimal. This is where this thesis contributes to the existing literature through extending and validating the methodology of Arnold (2006). By constructing a framework to determine how much weight should be attributed to economic circumstances within member states, these previous findings can be explained as part of optimal monetary policy.

3 Model construction

In this section, a model is constructed to determine whether, and if so, when the ECB conducts optimal monetary policy if it places a different weight on economic conditions in a member state than its economic size alone. To that end, a model is developed for the individual economies of each EMU member state, after which the individual economies are combined into an aggregate model of the whole Eurozone. Subsequently, it is determined which conditions should be satisfied in order for the ECB to conduct optimal monetary policy. Finally, these conditions for optimal policy are incorporated into the aggregate model of the Eurozone, through which a Taylor rule is formed for optimal policy. Each situation, resulting from this Taylor rule, in which the ECB should deviate from employing just the economic weight of a member state, is discussed. The used model is largely based on the model of Romer (2001) and the subsequent extension of Arnold (2006). It involves a backward looking model in the sense that current economic circumstances are explained by events in the past. However, the model lacks a microeconomic foundation, which limits the ability of this model to produce definite conclusions. Despite these shortcomings, the model does provide a theoretical framework to demonstrate that differences in sensitivity to macroeconomic developments between member states could lead to an optimal regional bias within ECB policy.

3.1 Eurozone Economies

The economies of member states are modulated by employing two equations, namely, an equation that describes a nation's output, and an equation that describes a nation's inflation. For convenience, the Eurozone is initially assumed to consist of just two member states. The resulting conditions for the presence of optimal regional biases will be generalized in a later stage to describe the complete Eurozone, thus including all of its member states.

3.1.1 Output

First, an equation for the output of member states is constructed. The model of Arnold (2006) describes the logarithm of output as dependent on the logarithm of lagged output ($y_{i,t-1}$), the lagged real interest rate ($r_{i,t-1}$) and the lagged change in the real exchange rate ($\Delta q_{i,t-1}$). Here, the change in the real exchange rate serves as an indicator of the change in competitiveness of member states. A negative value for $\Delta q_{i,t-1}$ indicates that prices have fallen relative to other nations, and thus has a positive effect on output. The model is completed by adding the disturbances ($e_{i,t}^{y}$), which are assumed to be normally distributed and have a mean of zero. The output of member state 1 can thus be described as:

$$y_{1,t} = \rho_1 y_{1,t-1} - \beta_1 r_{1,t-1} - \mu_1 \Delta q_{1,t-1} + e_{1,t}^{y} \quad (1)$$

and the output of member state *2* as:

$$y_{2,t} = \rho_2 y_{2,t-1} - \beta_2 r_{2,t-1} - \mu_2 \Delta q_{2,t-1} + e_{2,t}^{y} \quad (2)$$

Here, the real interest rates for each nation are assumed to be the difference between the nominal interest rate in the Eurozone and the national inflation, so:

$$r_{1,t} = i_{\epsilon,t} - \pi_{1,t}, \text{ and } \qquad r_{2,t} = i_{\epsilon,t} - \pi_{2,t}, \quad (3)$$

In addition, three small-country assumptions are made regarding the impact non-Eurozone countries have on the real exchange rate of member states. These assumptions are added for simplicity, since they make it possible to determine the changes in the real exchange rate ($\Delta q_{i,t}$ in equation 1 and 2), without having to construct a model for non-Eurozone countries. These assumptions are:

A1. The non-Eurozone area is large compared to individual member states.
A2. Inflation shocks in individual member states do not affect the (nominal) exchange rate between the euro and other currencies.
A3. No inflation shocks occur outside the Eurozone.

By virtue of these assumptions, changes in the real exchange rate between an Eurozone member state and a non-Eurozone country are solely caused by inflation within that member state. This can be explained as follows. As a result of A1, member states cannot individually influence price-levels of goods and services. This implies that individual member states lack the market power to unilaterally change prices of their goods and services. Any domestic price-level increase within the Eurozone is thus caused by inflation. A2 ensures that the effect of domestic inflation on a member state's real exchange rate state is not partially negated by a simultaneous depreciation of the euro. Therefore, inflation within member states always decreases its competitiveness and results in an equal depreciation of its real exchange rate. Finally, A3 stipulates that no inflation shocks occur outside the Eurozone. Hence, depreciation of the real exchange rate due to inflation within an Eurozone member state is not negated by inflation within non-Eurozone countries. The combination of these three assumptions ensures that the bilateral real exchange rate between a member state and a non-Eurozone country only changes due to domestic inflation. This can be derived from the formula of the bilateral real exchange rate, as presented below:

$$q_{i,j} = \frac{S * P_j}{P_i} \quad \text{(F1)}$$

Here, $q_{i,j}$ represents the bilateral exchange rate between countries *i* and *j*, S represents the (nominal) bilateral exchange rate and P the price-level. Now, if country *i* is an Eurozone member state and country *j* a non-Eurozone country, de denominator of (F1) remains constant due to A2 and A3. Furthermore, as a result of A1, the denominator of (F1) only changes due to inflation within the member state. Thus, the bilateral real exchange rate between a Eurozone member state and a non-Eurozone country only changes because of inflation within that member state. In that case, the change in the real exchange rate between a member state and a non-Eurozone country is equal to that member state's inflation. This is true for all countries within the non-Eurozone area.

However, equations (1) and (2) employ a member state's *unilateral* real exchange rate (Δq_i), and not a bilateral real exchange rate ($\Delta q_{i,j}$). Therefore, a member state's real exchange rate is not only determined by its competitiveness with the non-Eurozone area, but also by its competitiveness with other member states. Since assumptions A2 and A3 only apply to non-Eurozone countries, the numerator of F1 does not have to remain constant when two Eurozone member states are compared. Then, a price-level increase in one member state (P_i) can be accompanied by a simultaneous price-level increase in another member state (P_j). Because of this, a change in a member state's real exchange rate is no longer solely dependent on its own inflation, but also dependent on inflation within other member states. In short, inflation within an Eurozone member state causes an equal depreciation of its real exchange rate compared to the non-Eurozone area, but not necessarily compared to other Eurozone members states. Then, a change in the real exchange rate depends on both member state's their inflation.

Now that is clear how a member state's real exchange rate changes due to its own inflation and due to inflation within other member states, it is possible to construct an equation for $\Delta q_{i,t}$ as used in equations (1) and (2).This is based on the fact that a member state's *unilateral* real exchange rate ($\Delta q_{i,t}$) is simply a weighted average of all of its bilateral exchange rates. This weighted average is based on trade weights. If inflation occurs within a member state, then the depreciation of its real exchange rate compared to the non-Eurozone area will be equal to its inflation. However, if other member states also experience inflation, then the depreciation of the unilateral real exchange rate will be suppressed. This is expressed in the equation below, wherein the change of the real exchange rate of member state *1* is presented:

$$\Delta q_{1,t} = \pi_{1,t} - \theta_{2,1}\pi_{2,t} - \theta_{3,1}\pi_{3,t} - \ldots - \theta_{n,1}\pi_{n,t}$$
$$\Delta q_{1,t} = \pi_{1,t} - \sum_{i=2}^{n}\left[\theta_{i,1}\pi_{i,t}\right] \tag{4}$$

Here, $\theta_{i,1}$ represents the share of member state 1's total trade that it conducts with member state i. By employing the trade weights, a simultaneous price-level increase in another member state only suppresses the real exchange rate for the part of trade that is competitive with a member state's own trade. Thus, a member state's change in its real exchange rate should be interpreted as its inflation minus the share of the inflation of all other member states which compete with its own trade. Since, for now, it is assumed that the Eurozone consists of only two member states, the change in the real exchange rate of member state 1 and member state 2 simplifies to:

$$\Delta q_{1,t} = \pi_{1,t} - \theta_{2,1}\pi_{2,t} \quad \text{and} \quad \Delta q_{2,t} = \pi_{2,t} - \theta_{1,2}\pi_{1,t} \tag{5}$$

The next step is to combine the individual output of both member states into an aggregate model for the output of the Eurozone, whereby, for the purposes of this thesis, it is crucial that a member state's individual contribution to the aggregate output can be precisely distinguished. However, this requisite gives rise to a complication, since the logarithm of output is used within the model. Simply adding the logarithm of both member states' output in order to create an aggregate output variable distorts both the share member states individually contribute to the aggregate output, as well as the absolute value of the aggregate output.[1] The method Arnold (2006) uses to solve this problem, is to define the aggregate output as the GDP-weighted sum of the national output, so:

$$y_{\epsilon,t} = \omega_1 y_{1,t} + (1 - \omega_1) y_{2,t} \tag{6}$$

Here, ω_1 represents the GDP-share of member state *1*. This approach causes the share of individual member states in the aggregate output to remain intact, but the absolute value of the aggregate output to not correspond with reality, since it differs from $y_{\epsilon,t} = \log(Y_1 + Y_2)$. However, it is impossible to construct a model where both the absolute value of the aggregate output, as well as the individual share of member states' in the aggregate output are both mathematically correct. Thus, a choice needs to be made, and since this thesis's purpose is to find a justification for the ECB deviating from employing the economic weight of member states, the relative contribution of member states to the aggregate output is more important than its absolute value. Furthermore, by employing this definition of the aggregate output, the aggregate values of a member state's real interest rate and its change in its real exchange rate are correctly represented when the model for individual output is inserted. This is because the aggregate values of these variables are, per definition, the GDP-weighted averages of their national equivalents, so:

$$\begin{gathered} r_{\epsilon,t} = \omega_1 r_{1,t} + (1 - \omega_1) r_{2,t} \\ \Delta q_{\epsilon,t} = \omega_1 \Delta q_{1,t} + (1 - \omega_1) \Delta q_{2,t} \end{gathered} \tag{7}$$

[1] $\log Y_1 + \log Y_2 \neq \log(Y_1 + Y_2)$

By utilizing the aggregate variables of equations (6) and (7), a model for the aggregate output of the Eurozone can be constructed, which strongly resembles the models of individual output:

$$y_{\epsilon,t} = \rho_\epsilon y_{\epsilon,t-1} - \beta_\epsilon r_{\epsilon,t-1} - \mu_\epsilon \Delta q_{\epsilon,t-1} + e^{y}_{\epsilon,t} \quad (8)$$

Equation (8) is assumed to be the model through which the ECB assesses the effect of its monetary policy on the aggregate output. While the ECB thus does not consider individual economic circumstances within member states, each individual member state does influence its policy, since the aggregate variables can all be disintegrated into individual variables of member states. Therefore, the aggregate coefficients (denoted by subscript-ϵ) depend on their national counterparts the economic size of other member states. Thus, for example, $\rho_{\epsilon,t}$ depends on both $\rho_{1,t}$ as well as $\rho_{2,t}$, since equation (8) can be rewritten to:

$$y_{\epsilon,t} = \omega_1[\rho_1 * y_{1,t-1} - \beta_1 r_{1,t-1} - \mu_1 \Delta q_{1,t-1}] + (1-\omega_1)[\rho_{2,t} * y_{2,t-1} - \beta_2 r_{2,t-1} - \mu_2 \Delta q_{2,t-1}] + e^{y}_{\epsilon,t} \quad (9)$$

3.1.2 Inflation

The second equation that is used to describe national economies is a member state's inflation, which is assumed to depend on the lagged inflation ($\pi_{i,t-1}$) and the lagged output-gap ($y_{i,t-1} - y^*_{i,t-1}$). Romer (2001) defined the output-gap as the difference between output and the optimal level of output, whereby the latter consists of the level that would be achieved in a perfect Pareto-efficient world. Again, the disturbances are added which are assumed to be normally distributed and have a mean of zero ($e^{\pi}_{i,t}$). The inflation of member state *1* is thus described by the following equation:

$$\pi_{1,t} = \pi_{1,t-1} + \alpha_1(y_{1,t-1} - y^*_{1,t-1}) + e^{\pi}_{1,t} \quad (10)$$

and member state *2's*:

$$\pi_{2,t} = \pi_{2,t-1} + \alpha_2(y_{2,t-1} - y^*_{2,t-1}) + e^{\pi}_{2,t} \quad (11)$$

Note that no coefficient is incorporated for the impact of the lagged inflation, which means that inflation remains constant if the output-gap in the previous year was zero. This is due to the fact that, if the optimal level of output is reached, prices are not pressured to rise or fall faster than the previous year. By applying the same principles as with the construction of the aggregate output model, an aggregate model of inflation is constructed whereby the aggregate variables can be disintegrated into their national counterparts, so:

$$\pi_{\epsilon,t} = \pi_{\epsilon,t-1} + \alpha_{\epsilon}(y_{\epsilon,t-1} - y^{*}_{\epsilon,t-1}) + e^{\pi}_{\epsilon,t} \tag{12}$$

which can be rewritten to:

$$\pi_{\epsilon,t} = \omega_1\big[\pi_{1,t-1} + \alpha_1(y_{1,t-1} - y^{*}_{1,t-1})\big] + (1-\omega_1)\big[\pi_{2,t-1} + \alpha_2(y_{2,t-1} - y^{*}_{2,t-1})\big] - e^{\pi}_{\epsilon,t} \tag{13}$$

As before, equation (12) is assumed to be the model through which the ECB determines its monetary policy, whereby equation (13) demonstrates what the individual influence of economic circumstances within member states is on its policy. In addition, it should be noted that the ECB cannot directly control neither individual nor aggregate output. The only instrument available to the ECB is the interest rate, which affects next periods output through the aggregate real interest rate, whereby the adjusted output affects inflation in the ensuing period. Thus, a delay exists, consisting of two periods between the moment the ECB sets its interest rate and the indirect effect it has on aggregate inflation. The next paragraph examines this delay in more depth.

3.2 Optimal ECB Policy

As mentioned in the introduction, the ECB claims that it sets its interest rate based on aggregate Eurozone data only. Therefore, it is assumed that the ECB sets the interest rate at a level that places both the aggregate output and inflation as close to their target levels as possible. When no interest rate exists for which the differences between both the aggregate output and inflation and their target levels are smaller, the ECB is said to conduct optimal policy. For simplicity, the target level of inflation is set to zero. Furthermore, it assumed that the target level of output is the same as the optimal level of output (y_ϵ^* in equation (12)). Thus the ECB conducts optimal policy when it minimizes the following condition:

$$E[(y_\epsilon - y_\epsilon^*)^2] + \lambda E[\pi_\epsilon^2] \tag{14}$$

Here, λ represents a parameter that indicates the weight the ECB attributes to achieving the optimal level of inflation, compared to achieving the optimal level of output. Therefore, if the ECB only focuses on achieving the optimal level of inflation and is not concerned with output, λ will be infinite. However, the ECB is restricted in its capability to actually guide both output and inflation to their optimal levels, since in this model, the only intervention instrument available to the ECB is the nominal interest rate. Through adjustment of the nominal interest rate in period *t*, the ECB indirectly adjusts the aggregate output in period *t+*1, as can be deduced from equation (8). Subsequently, the adjusted output in period *t+1* also adjusts the inflation in period *t+2*. Due to the mechanism of the interest rate instrument of the ECB and the lagged structure of the model, it can be argued that the ECB chooses an interest rate that causes a certain level of output in the next period. Put differently, the ECB has a high degree of control over next period's output, through observing what the current economic conditions are and adjusting the interest rate. The output that the ECB targets in the next period is thus explained by the following equation:

$$E_t[y_{\epsilon,t+1}] = \rho_\epsilon y_{\epsilon,t} - \beta_\epsilon r_{\epsilon,t} - \mu_\epsilon \Delta q_{\epsilon,t} \tag{15}$$

Note that the disturbances do not influence ECB decision making, since these are not known at moment *t*. Although the disturbances do not influence the decision making, they do affect the actually achieved output in period *t+1*, making the actual output:

$$y_{\epsilon,t+1} = E_t[y_{\epsilon,t+1}] + e_{\epsilon,t}^y \tag{16}$$

In addition, it should be noted that, in order to solely achieve the targeted level of the output, the current or future inflation is not relevant for ECB decision making. However, the ECB should, in order to conduct optimal policy, consider inflation in future periods when targeting next period's output, due to the relationship between the output-gap and inflation in the long run. Optimal policy therefore requires that future inflation is taken into consideration when targeting next period's output. However, the level of output that is necessary to achieve the optimal level of inflation might deviate from the optimal level of output. The ECB thus faces a dilemma when setting the interest rate; to what extent should it deviate from targeting the optimal level of output so that the optimal level of inflation can be achieved? Romer (2001) assumed that this relation was linear and proposed the following solution for optimal policy when faced with this dilemma:

$$E_t\left[y_{\epsilon,t+1}\right] - y^*_{\epsilon,t+1} = -\eta\ \pi_{\epsilon,t+1} \tag{17}$$

Here, it is to be determined for what value of η the condition of equation (14) is satisfied. An identical ratio would be achieved if linearity is not assumed, but instead the expected discounted sums of condition (14) are minimized with a discount rate approaching zero (Svensson, 1997). Intuitively, equation (17) describes that the higher inflation is in the next period, the more the ECB already has to start deviating from the optimal level of output in the next period, causing inflation to be lower in the long run. To identify for which value of η condition (14) is minimized, the targeted output of equation (17) is replaced with the actual output of equation (16). This results in:

$$y_{\epsilon,t+1} - y^*_{\epsilon,t+1} = -\eta\ \pi_{\epsilon,t+1} - e^y_{\epsilon,t} \tag{18}$$

Hereafter, the actual output-gap, as described in the equation above, is substituted in the aggregate inflation of equation (12), so:

$$\pi_{\epsilon,t+1} = \pi_{\epsilon,t} + a_\epsilon\left(-\eta\ \pi_{\epsilon,t} - e^y_{\epsilon,t}\right) + e^\pi_{\epsilon,t} \tag{19}$$

The next step is to realize that due to the linear structure of the model and the assumption of normally distributed disturbances with mean zero, inflation will be constant in the long term and is independent of its initial conditions. This is because economic theory dictates that in the long run, a perfect Pareto-efficient market will exist, which causes the actual level of output to be equal to the optimal level of output and therefore, inflation will no longer change in the long run according to equation (12). Thus, based on the fact that $E\left[\pi_{\epsilon,t}\right]$ approaches $E\left[\pi_{\epsilon,t+1}\right]$ given enough time, equation (19) is solved for $E[\pi^2_\epsilon]$[2], which results in:

$$E[\pi^2_\epsilon] = \frac{{a_\epsilon}^2}{a_\epsilon\eta(2 - a_\epsilon\eta)}(\sigma^y_{\epsilon,t} + \sigma^\pi_{\epsilon,t}) \tag{20}$$

[2] See Romer (2001, p. 534) for a more detailed derivation.

Here, $\sigma_{\epsilon,t}^{y}$ and $\sigma_{\epsilon,t}^{\pi}$ represent the variances of the disturbances $e_{\epsilon,t}^{y}$ and $e_{\epsilon,t}^{\pi}$. Equation (20) describes the expected squared inflation as a function of η, which combined with the optimal output-gap/inflation ratio of equation (18), can be inserted into the condition of optimal monetary policy of equation (14). As a result, the condition for optimal monetary policy can be minimized so that the optimal level of η^* is found:

$$\eta^* = \frac{-\lambda a_\epsilon + \sqrt{{a_\epsilon}^2 \lambda^2 + 4\lambda}}{2} \tag{21}$$

The interpretation of $\eta^*$is as follows: it is the weight the ECB should attribute to the inflation of the next period when targeting next period's output-gap, which satisfies the condition for optimal policy of equation (14). This optimal weight depends on two variables, namely λ, which is the relative importance the ECB attaches to achieving the optimal level of output, and a_ϵ, which measures the inflationary pressure of the output-gap on next period's inflation. If the ECB exclusively focuses on achieving the optimal level of inflation (λ=∝), η^* simplifies to $\frac{1}{a}$. This corresponds to a policy that aims to minimize inflation in the next period, since this implies that $E_t[y_{\epsilon,t+1}] = -\frac{1}{a}\,\pi_{\epsilon,t+1}$, which constitutes to an expected inflation of zero according to equation (12). The reverse also applies, when the ECB only desires to achieve the optimal level of output (λ=0), η^* will reduce to 0, resulting in an expected output-gap of $E_t[y_{\epsilon,t+1}] - y_{\epsilon,t+1}^* = 0$.

3.3 Taylor rule

The next step is to apply the condition of optimal monetary policy to the aggregate Eurozone model. For this purpose, the aggregate inflation and output from equation (9) and (13) are inserted into the optimal ratio of the output-gap and inflation from equation (17). This results in the following equation[3], where the time subscripts of both the GDP-shares and the trade weights are suppressed for convenience:

$$\begin{gathered}\omega_1 * \left[\rho_1 * y_{1,t} - \beta_1 * (i_{\epsilon,t} - \pi_{1,t}) - \mu_1 * (\pi_{1,t} - \theta_{2,1}\pi_{2,t}) - y_{1,t}^*\right] + \\ (1-\omega_1) * \left[\rho_{2,t} * y_{2,t} - \beta_2 * \left(i_{\epsilon,t} - \pi_{2,t}\right) - \mu_2 * (\pi_{2,t} - \theta_{1,2}\pi_{1,t}) - y_{2,t}^*\right] = \\ -\eta^* * \left[\omega_1 * \left(\pi_{1,t} + a_1(y_{1,t} - y_{1,t}^*)\right) + (1-\omega_1)\left(\pi_{2,t} + a_2(y_{2,t} - y_{2,t}^*)\right)\right]\end{gathered} \tag{22}$$

Which can be rearranged to:

$$i_{\epsilon,t} = \Phi_1 y_{1,t} - \vartheta_1 y_{1,t}^* + \Phi_2 y_{2,t} - \vartheta_2 y_{2,t}^* + \varphi_1 \pi_{1,t} + \varphi_2 \pi_{2,t} \tag{23}$$

with:

$$\begin{aligned}\Phi_1 &= \frac{\omega_1(\rho_1 + \eta^*\alpha_1)}{\omega_1\beta_1 + (1-\omega)\beta_2} \\ \Phi_2 &= \frac{(1-\omega_1)(\rho_2 + \eta^*\alpha_2)}{\omega_1\beta_1 + (1-\omega_1)\beta_2} \\ \vartheta_1 &= \frac{\omega_1(1 + \eta^* a_1)}{\omega_1 * \beta_1 + (1-\omega_1) * \beta_2)} \\ \vartheta_2 &= \frac{(1-\omega_1)(1 + \eta^* a_2)}{\omega_1 * \beta_1 + (1-\omega_1) * \beta_2)} \\ \varphi_1 &= \frac{\omega_1(\eta^* + \beta_1 - \mu_1) + (1-\omega_1)\mu_2\theta_{1,2}}{\omega_1\beta_1 + (1-\omega_1)\beta_2} \\ \varphi_2 &= \frac{(1-\omega_1)(\eta^* + \beta_2 - \mu_2) + \omega_1\,\mu_1\theta_{2,1}}{\omega_1\beta_1 + (1-\omega_1)\beta_2}\end{aligned} \tag{24}$$

Equation (23) has the form of a Taylor rule, which expresses the optimal nominal interest rate set by the ECB as a linear function of output, optimal level of output and inflation of EMU member states. The coefficients of equation (24) depend on both domestic as well as foreign variables. In order to determine if an optimal regional bias exists, it is examined if the relative weight the coefficients assign to a member state deviates from the economic weight of that member state, whereby the economic weight is considered to be a state's GDP-weight(ω_i). Thus, if the situation arises that $\frac{\Phi_1}{\Phi_1+\Phi_2} > \omega_1$, the ECB should, in order to conduct optimal policy, attribute more weight to the output

[3] See Appendix A for a complete derivation.

of state *1* than merely it economic size. This same principle applies to situations where $\frac{\varphi_1}{\varphi_1+\varphi_2} > \omega_1$ and $\frac{\vartheta_1}{\vartheta_1+\vartheta_2} > \omega_1$ occur, for respectively inflation and the optimal level of output of state *1*. The remainder of this section examines which conditions ensure that these relative weights are equal to a member state's economic weight. This will provide a framework to determine whether an optimal regional bias is present by inspecting when these conditions are violated. The discussed conditions are based on the Eurozone model consisting of two member states but can and will be generalized to include all member states within the Eurozone.[4]

3.3.1 Output

From equation (24), it follows that the relative weight the ECB should attribute to a member state's output is equal to:

$$\begin{aligned}\frac{\Phi_1}{\Phi_1+\Phi_2} &= \frac{\omega_1(\rho_1+\eta^*\alpha_1)}{\omega_1(\rho_1+\eta^*\alpha_1)+(1-\omega_1)(\rho_2+\eta^*\alpha_2)} \\ &= \omega_1 \qquad \text{if } \rho_1=\rho_2=\rho_\epsilon \text{ and } \alpha_1=\alpha_2=\alpha_\epsilon\end{aligned} \tag{25}$$

The equation above demonstrates that no optimal regional bias exists if two conditions are satisfied. First, the inflationary pressure of output on next period's inflation should be equal among member states ($\alpha_1 = \alpha_2$). If this effect is stronger for member state *1* than it is for member state *2*, the relative weight the ECB should place on member state *1*'s output rises above its economic size, and thus an optimal regional bias exists ($\frac{\Phi_1}{\Phi_1+\Phi_2} > \omega_1$). This can be explained due to the fact that as α_i gets higher, the more a member state's output imposes upward pressure on the aggregate inflation (equation (12)). As a result, the ECB has to raise the interest rate to reduce inflation due to economies which inflation reacts strongly to output, compared to economies where this effect is weaker. Intuitively, this means that when two states share the same GDP-weight, the state with the highest α_i will force the ECB to relatively raise the interest rate so that aggregate inflation is pressured downwards. Thus, this effectively means that the ECB should place more weight on the latter state's output compared to the other state. As a result, an optimal regional bias arises when the effect of one state's output on inflation is higher compared to others ($\alpha_1 > \alpha_2$).

Second, output persistence should also be equal among member states in order for optimal biases to be absent ($\rho_1 = \rho_2$). If the situation arises where this effect is stronger for state *1* than for state *2* ($\rho_1 > \rho_2$), the ECB should place more relative weight on state *1*'s output ($\frac{\Phi_1}{\Phi_1+\Phi_2} > \omega_1$). The explanation for this effect runs as follows. As output persistence is stronger for state *1*, output grows

[4] See Appendices B and C for a derivation of the conditions for generalized models.

relatively stronger, and thus, with it inflation. Therefore, states with stronger output persistence pose a relative bigger threat to aggregate inflation than states with weaker output persistence. As a result, the ECB should take the output of the former states more in consideration when setting the interest rate, and thus the relative weight of those states increases.

3.3.2 Optimal level of output

Next, the relative weight that the ECB should place on a member state's optimal level of output is examined. From equation (24) it follows that:

$$\frac{\vartheta_1}{\vartheta_1+\vartheta_2} = \frac{\omega_1(1+\eta^*\alpha_1)}{\omega_1(1+\eta^*\alpha_1)+(1-\omega_1)(1+\eta^*\alpha_2)} \qquad (26)$$
$$= \omega_1 \qquad \text{if } \alpha_1 = \alpha_2 = \alpha_\epsilon$$

This equation demonstrates that the relative weight the ECB should place on a member state's optimal level of output is equal to that member state's economic weight, if the individual deflationary pressure of the optimal level of output is identical among member states ($\alpha_1 = \alpha_2$). If this effect is stronger within the economy of member state *1* than it is in member state 2's economy ($\alpha_1 > \alpha_2$), it appears that the ECB should attribute more weight to the optimal level of output of member state *1* ($\frac{\vartheta_1}{\vartheta_1+\vartheta_2} > \omega_1$). However, note that in the Taylor-rule of equation (23) a negative sign stands in front of a member state's optimal level of inflation. Thus, in terms of the effect on the interest rate, the bias thus works in the opposite way; if ($\alpha_1 > \alpha_2$) than it holds that $\frac{\vartheta_1}{\vartheta_1+\vartheta_2} < \omega_1$. For the explanation of this cause of an optimal bias, it is important to note that the ECB has no influence on the optimal level of output through its interest rate instrument. Therefore, the optimal level of output acts as a natural mechanism through which inflation is suppressed. The stronger this mechanism is, meaning α_i is relatively high, the less the ECB has to intervene in order to decrease inflation. This ensures that the ECB's response to changes in the optimal level of inflation can be more muted, which effectively means that less weight is attributed to that member state's optimal level of output.

3.3.3 Inflation

Finally, the optimal weight of the inflation of member states is examined.

$$\frac{\varphi_1}{\varphi_1+\varphi_2} = \frac{\omega_1(\eta^*+\beta_1-\mu_1)+(1-\omega_1)\mu_2\theta_{1,2}}{\omega_1(\eta^*+\beta_1-\mu_1)+(1-\omega_1)\mu_2\theta_{1,2}+\ (1-\omega_1)(\eta^*+\beta_2-\mu_2)+\omega_1\,\mu_1\theta_{2,1}}$$

$$= \frac{\omega_1(\eta^*+\beta_\epsilon-\mu_\epsilon)+(1-\omega_1)\mu_\epsilon\theta_{1,2}}{(\eta^*+\beta_\epsilon-\mu_\epsilon)+(1-\omega_1)\mu_\epsilon\theta_{1,2}+\omega_1\,\mu_\epsilon\theta_{2,1}} \quad \text{if } \beta_1=\beta_2=\beta_\epsilon \text{ and } \mu_1=\mu_2=\mu_\epsilon \qquad (27)$$

$$= \omega_1 \quad \text{if } \theta_{1,2}=\frac{\omega_1}{1-\omega_1}\theta_{2,1}$$

From equation (24) it follows that the weight the ECB should place on a state's inflation simplefies to its GDP-share if three conditions are met. First, it turns out that the interest rate elasticities should be equal among member states. If this elasticity is stonger for state *1* than for state *2* ($\beta_1 > \beta_2$), the relative weight that the ECB should place on state *1* its inflation increases ($\frac{\varphi_1}{\varphi_1+\varphi_2} > \omega_1$). Intuitively, this makes sense since an adjustment of the interest rate by the ECB will generate different responses from individuel member states, depending on their own interest rate elasticities. The stronger a member state's interest rate elasticty, the more impact the interest rate has on that member state's output. Due to the delayed effect of a member state's output on inflation, this also means that a member state's inflation is more sensitive to the interest rate. Thus effectively, the more weight should be placed upon its inflation due to the sensitivity to the interest rate.

Second, the effect of changes in the real effective exchange rate should be equal among member states ($\mu_1 = \mu_2$). Contrary to the interest rate elasticities, a stronger effect of changes in the real effective exchange rate constitutes to a reduction in the relative weight that should be placed upon a member state's inflation($\mu_1 > \mu_2, \frac{\varphi_1}{\varphi_1+\varphi_2} < \omega_1$). The explanation for this relation runs as follows. As the real effective exchange rate increases, the competitiveness of the member state decreases and thus output decreases, which in turn decreases inflation. When this process happens automatically, the ECB does not have to respond often to changes in the real effective exchange rate to achieve the optimal level of inflation, which is set to zero in this model. However, if this process of adjustment does not happen automatically, meaning μ_i is low compared to other member states, the ECB is forced to intervene. Thus there exists a negative relation between the ability of a member state's economy to automatically correct its inflation due to changes in the real effective exchange rate and the necessity for the ECB to intervene. Hence the optimal weight that should be placed upon a member state's inflation increases if μ_i is low compared to other member states.

Finally, the trade shares among member states should be proportional to their GDP-shares. If nation 1's share in the total trade of nation 2 is higher, proportional to their GDP's, compared to nation 2's share in the total trade of nation 1 ($\theta_{1,2} > \frac{\omega_1}{1-\omega_1}\theta_{2,1}$), the ECB should place relatively more weight on nation 1's inflation ($\frac{\varphi_1}{\varphi_1+\varphi_2} > \omega_1$). This is due to the fact that if the trade share of nation 1 is relatively high, it conducts, per definition, relatively less trade with the non-eurozone area. Therefore changes in the real exchange rate of nation 1 are better negated by inflation in other member states. Consequently, the competitiveness of the member state is less affected by inflation, causing output to also not be less affected bt inflation. Therefore, the automatic adjustment of inflation due to a member state's worsening of its competitiveness, as explained in the previous paragraph, is disrupted. Hence, the ECB should intervene more often to the inflation of a member state if that state trades relatively more within the Eurozone.

3.4 Summary

Table 1 summarizes the conditions that determine whether an optimal regional bias is present in the ECB's monetary policy, whereby these conditions are generalized to include all member states of the Eurozone into the model.[5] These optimal regional biases occur due to differences between member states in sensitivities to macroeconomic adjustments and different levels of trade with the non-Eurozone area. A positive bias indicates that the ECB should attribute more weight to a member state's economic circumstances than just its economic weight. Hereby is examined what the regional bias is, if the circumstance as stated in the first column occurs; all other variables are assumed to satisfy the condition that corresponds with the absence of an optimal regional bias.

Table 1: *Summary of the conditions that cause an optimal regional bias*.

	Output coefficient $\frac{\Phi_1}{\Phi_1+\Phi_2}$	Optimal Output coefficient $\frac{\vartheta_1}{\vartheta_1+\vartheta_2}$	Inflation coefficient $\frac{\varphi_1}{\varphi_1+\varphi_2}$
$\rho_1 > \sum_{i=1}^{n}[\omega_i\rho_i]$	Positive bias	No bias	No bias
$\alpha_1 > \sum_{i=1}^{n}[\omega_i\alpha_i]$	Positive bias	Negative bias	No bias
$\beta_1 > \sum_{i=1}^{n}[\omega_i\beta_i]$	No bias	No bias	Positive bias
$\mu_1 > \sum_{i=1}^{n}[\omega_i\mu_i]$	No bias	No bias	Negative bias
$\sum[\omega_i\mu_i\theta_{j,i}] = \sum\frac{\omega_i\mu_i\theta_{1,i}}{\omega_1}$	No bias	No bias	Positive bias

[5] See Appendices B and C for the complete derivation.

4 Model Application

This section examines how the constructed model can be used to determine whether it is optimal for the ECB to under- or overweigh certain member states in its interest rate decision. To this end, the individual coefficients of equation (24) are estimated for each member state using OLS regression analysis. Hereafter, the optimal weights of the four biggest economies, namely: Germany, France, Italy and Spain, are calculated and discussed in depth. The used data is derived from the Eurostat-Comext database and the Worldbank, whereby the ECB's one-year lending rate is used as the nominal interest rate. The potential output is derived from the OECD's Economic Outlook No 99. The model is estimated based on annual time-series data from the period 1999 to 2015 and is limited to those member states which joined the EMU from the moment of its inception in 1999, with the addition of Greece. This is due to a lack of data points and the relatively negligible economic weight of member states that joined the EMU in a later stage. Furthermore, a dummy variable is added to both the inflation and output models to compensate for the effect of the economic crisis that started in 2008.

Unfortunately, even with the exclusion of member states that joined the EMU at a later stage, there are not enough data points available to perform a thorough empirical validation of the constructed model. The main bottleneck is the availability of trade data, as it is only registered on an annual basis. As a result, only 16 data points are available for regression analysis for each member state, which is insufficient to investigate the reliability and validity of the model. Thus, subjects like stationarity, heteroscedasticity of the residues or omitted/redundant independent variables are not further investigated. Therefore, this section primarily serves as an example of how the constructed model can be used to detect and measure optimal regional biases.

4.1 Regression analysis

The estimated coefficients and the corresponding explanatory powers of the output and inflation models are presented in the Tables 2 and 3. The presented R-squares of the individual models are reported as averages.

Table 2: *Explanatory power individual and aggregate models*

Model Tested	R-squared
Individual[*]*:*	
$y_{i,t} = \rho_i y_{i,t-1} - \beta_i r_{i,t-1} - \mu_i \Delta q_{i,t-1} - \delta_i^y + e_{i,t}^y$	0.83
$\pi_{i,t} = \pi_{i,t-1} + \alpha_i (y_{i,t-1} - y_{i,t-1}^*) + \delta_{i,t} + e_{i,t}^\pi$	0.33
Aggregate:	
$y_{\epsilon,t} = \rho_\epsilon y_{\epsilon,t-1} - \beta_\epsilon r_{\epsilon,t-1} - \mu_\epsilon \Delta q_{\epsilon,t-1} + e_{\epsilon,t}^y$	0.84
$\pi_{\epsilon,t} = \pi_{\epsilon,t-1} + \alpha_\epsilon (y_{\epsilon,t-1} - y_{\epsilon,t-1}^*) + e_{\epsilon,t}^\pi$	0.41

*. The r-squared of the individual models is calculated by averaging the r-squared of each member state.

Table 3: *Estimated coefficients of the model*

Member State i	Output persistence $(+)\rho_i$	Interest rate elasticity $(-)\beta_i$	Change in REER $(-)\mu_i$	Inflationary pressure output-gap $(+)\alpha_i$	Sum of trade shares $\sum[\theta_{j,i}]$
Austria	1.015**	-0.021**	-0.106**	+6.565	0.106
Belgium	1.014**	-0.026*	-0.010**	+15.170	0.403
Finland	1.017**	-0.017	-0.077**	+20.684*	0.042
France	1.013**	-0.011*	-0.124**	+6.842	0.650
Germany	1.010**	-0.021**	-0.100**	+15.141	1.284
Greece	0.998**	+0.039**	+0.042**	+8.620	0.027
Ireland	1.013**	-0.014	-0.020*	-1.119	0.080
Italy	1.008**	+0.042**	-0.058**	+9.182	0.401
Luxembourg	1.033**	-0.005	-0.111**	+0.384	0.031
Netherlands	1.006**	-0.014**	-0.023**	+25.062*	0.480
Portugal	1.006**	-0.019**	-0.019**	+6.174	0.078
Spain	1.002**	+0.025**	+0.008*	+9.120	0.458
Aggregate Eurozone	1.010**	-0.013**	-0.100*	+8.024*	

****.** Coefficient significantly differs from 0 at the 0.01 level
***.** Coefficient significantly differs from 0 at the 0.05 level

A first impression from Table 2 is that both models appear to possess considerably different explanatory powers. The output model is substantially better at explaining the data than the inflation model, with R-squares of 0.83 and 0.84 for the respective national and aggregate models. However, even though these numbers are quite high, this does not necessarily mean that the output model can accurately predict output. It merely indicates that the model is a good fit for the (current) data. Therefore, the high explanatory power does not imply that the model can be used as an effective policymaking instrument. Nonetheless, results from Table 3 corroborate the capability of the output model to predict next period's output. From Table 3, it appears output persistence, the effect of change in the real effective exchange rate, and to a lesser extent the interest rate elasticity, are significant predictors of next period's output. The signs of these three variables also correspond to their anticipated values. The only exceptions are Greece and Spain, who both experience positive effects of an increase in the real interest rate and a depreciation of the real effective exchange rate. A possible explanation of this anomaly could be that the effect of the economic crisis is not sufficiently captured by the crisis-dummy alone, since unemployment rates within these countries are still substantially higher compared to before the crisis.[6] Nevertheless, these predominantly significant coefficients suggest that the used variables could be suitable to predict next period's output.

While the output model shows promising signs that it is able to predict next period's output, the results from Table 2 and 3 should be critically evaluated. Recall that the assumptions (A1-A3) regarding the change in the real effective exchange rate do not reflect actual changes in trade competitiveness. Subsequently, the ECB lending rate might not accurately describe the true cost of capital within the Eurozone in the sense of equation (3). Finally, no additional tests were performed to assess whether the observed relation between the dependent and independent variables actually exists. Especially stationarity could pose a problem, since next period's output is predicted based on the current output. Interpretation of these results should thus be done with some caution.

When turning to the inflation model, the explanatory power appeared to be considerably less than the output model, with R-squares consisting of 0.33 and 0.41 for respectively the individual and aggregate inflation models. Surprisingly, the aggregate model appeared to fit the data slightly better than the individual models. This could be explained due to the fact that the aggregate model uses the GDP weighted average of inflation within the Eurozone. This causes inflation of the aggregate Eurozone to be less volatile, compared to member states' individual inflation. Therefore, the aggregate inflation model could be more suitable for linear OLS regression than (most) individual inflation models. Furthermore, results from Table 3 indicate that the inflationary pressure of the output-gap is not a significant predictor of next period's inflation. While the signs of the coefficients do correspond to their predicted values (with the exception being Ireland), only two coefficients of the individual models appeared to be significant. Moreover, the magnitude of the effect was found to be inconsistent, with values ranging from +0.384 to a massive +25.062 for respectively

[6] The unemployment rates in Q4.2015, derived from the Eurostat database, were determined to be 24.4 percent and 20.9 percent for respectively Greece and Spain. Pre-crisis, the unemployment rate in both countries was 8.2 percent and 8.6 percent respectively.

Luxembourg and the Netherlands. Thus, due to the inconsistency and lack of significance of the coefficients, the inflationary pressure of the output-gap does not seem to accurately predict next period's inflation. This could be caused by three possible reasons. First, the employed optimal level of output data might not be accurate. Since the optimal level of output cannot be measured, nor observed, it is impossible to know if the used data correctly represents the real output-gap, which therefore might skew the results. Second, the inflation model solely employed the output-gap to predict inflation. Adding one or more independent variables to the model might not only increase the explanatory power of the model, but also increase the explanatory power of the inflationary pressure of the output-gap[7]. The third and final reason could be that the output-gap is not related to inflation at all, and therefore was found to not significantly predict inflation. However, other authors have shown that, especially within the EU, the output-gap significantly predicts future inflation (e.g. Bolt & Els, 1998), making this cause less likely.

Finally, the sums of trade shares are estimated for the year 2015 and presented in the last column of Table 3. Note that these values can exceed one, since they consist of the proportion of total trade of nation *j*, which it conducts with nation *i.* The higher these values are, the higher the volume of trade member state *i* conducts with the other member states of the EMU. In turn, the higher this volume is, the more the effect of inflation shocks within member state *i* on its real effective exchange rate is suppressed by simultaneous inflation shocks in other member states, as per equation (4). Perhaps not surprisingly, the two biggest economies, namely Germany and France, appear to have the highest trade volumes within the EMU.

[7] Since five specific economic characteristics are examined to justify the existence of regional biases within ECB policy, it is beyond the scope of this thesis to further investigate the possibility of adding more explanatory variables.

4.2 Relative weights

Now that all the individual coefficients of equation (24) are estimated, it is possible to determine the relative weights of equations (25)-(27) for all member states. Below, the optimal weights of the four largest economies within the Eurozone are examined. These optimal relative weights are estimated for the year 2015, using the individual coefficients of Table 3. Since it is not known how the ECB values achieving the optimal level of inflation compared to achieving the optimal level of output, three possibilities are considered. These possibilities are: the ECB only focuses on achieving the optimal level of output ($\lambda = 0$), the ECB focuses on achieving the optimal level of output and inflation equally ($\lambda = 1$), and the ECB only focuses on achieving the optimal level of inflation ($\lambda = \infty$). Based on those three levels of importance, in combination with the aggregate inflationary pressure of the output-gap (a_{ϵ}) from Table 3, the value of η^* (21) is estimated for which the ECB conducts optimal policy. The estimated optimal weights are presented in Table 4.

Table 4: *Relative weights of Germany, France, Italy and Spain in 2015*

	Member State	$\lambda = 0$	$\lambda = 1$	$\lambda = \infty$	ω_i
Output bias $\frac{\Phi_i}{\sum[\Phi_n]}$	Germany	29.73%	38.17%	38.90%	29.72%
	France	21.51%	13.39%	12.69%	21.45%
	Italy	16.05%	13.02%	12.76%	16.07%
	Spain	10.54%	8.55%	8.37%	10.62%
Optimal level of output bias $\frac{\vartheta_i}{\sum[\vartheta_n]}$	Germany	29.72%	38.17%	38.90%	29.72%
	France	21.45%	13.38%	12.69%	21.45%
	Italy	16.07%	13.02%	12.76%	16.07%
	Spain	10.62%	8.55%	8.37%	10.62%
Inflation bias $\frac{\varphi_i}{\sum[\varphi_n]}$	Germany	52.96%	28.97%	29.72%	29.72%
	France	61.10%	20.17%	21.45%	21.45%
	Italy	-2.59%	16.67%	16.07%	16.07%
	Spain	1.85%	10.90%	10.62%	10.62%

From Table 4, it appears that the optimal relative weights are extremely subject to the relative level of importance (λ) that the ECB places upon inflation. For example, the optimal weight the ECB should place on France's inflation drops from 61.10% for ($\lambda = 0$) to just 21.45% for ($\lambda = \infty$). This is due to the fact that as $\lambda = 0$, η^* also becomes zero (equation (21)). Since it can be deduced from Table 3 that the interest rate elasticity is weaker than the effect of changes in the real effective exchange rate for almost all member states, the first part of the denominator $\omega_i(\eta^* + \beta_i - \mu_i)$ of equation (27) becomes negative ($\eta^* = 0$, $\beta_i < \mu_i$). Therefore, only member states that conduct sufficiently high trade with the other member states ($\sum[\theta_{j,i}]$) are able to compensate for this negative effect due to the second part of the denominator of equation (27) $(+\sum[\omega_j\mu_j\theta_{i,j}])$. As a result, a majority of the member states, including Italy, face negative optimal relative weights. This suggests that inflation shocks in those member states cause such a substantial worsening of their competitiveness that the ECB should actually respond with lower interest rates instead of higher. Also, due to these negative inflation weights of the majority of member states, the relative weight of member states with positive inflation weights becomes higher.[8] This results in the huge disparity between the economic weight of Germany and France (29.72% and 21.45%) and their relative optimal inflation weights (52.96% and 61.10%). However, if λ increases, η^* also increases, causing the first term of the denominator $(\omega_{1i}(\eta^*+\beta_i-\mu_i))$ to become positive for all member states if λ becomes sufficiently high. This results in a rapid decrease of the relative optimal inflation weight of member states which previously had positive inflation weights when $\lambda=0$, due to the adjustment of inflation weights of other member states from negative to positive values.

Contrary to the negative effect of λ on the optimal relative weight of Germany's inflation, λ appears to affect its optimal relative output and optimal level of output weights positively. This is a result of the differences between member states regarding the inflationary pressure from the output-gap. The stronger this effect is, the higher the optimal relative weight that should be attributed to that nation's output. This is due to the fact that as $\lambda=0$, η^* also becomes zero, making the inflationary pressure of the output-gap irrelevant for the denominator of equation (26) $(\omega_i(1+\eta^*\alpha_i)=\omega_1$, if η^*=0). This explains why the optimal weight for the optimal level of output is equal to a member state's GDP-weight when $\lambda=0$. The relative weight of a member state's output differs slightly from its GDP-weight due to the effect of output persistence $(\omega_i(\rho_i+\eta^*\alpha_i)=\omega_1\rho_i$, if η^*=0). However, the differences among member states their output persistence appeared to be almost negligible, causing their relative weights of output to approach their GDP-weight. Now, if the ECB does not focus exclusively on achieving the optimal level of output (λ>0, η^*>0), the inflationary pressure does become relevant, in the sense that the stronger this effect is, the higher the value of the denominator of equations (25) and (26), and thus the higher the relative weight will be. Since this effect is among the strongest within Germany's economy, and among the weakest within France's economy (a_i=15.1 and a_i=6.8, respectively), a positive trend of the relative weight can be observed from Table 4 through higher values of λ for Germany, and a negative trend for France.

Overall, the relative weights of Table 4 suggest that optimal policy requires the ECB to under- or overweigh certain member states. This depends on differences in economic circumstances among member states and the relative importance the ECB places on achieving its objectives. However, the German-French bias at the ECB that other authors have observed, appears to lack a justification,

[8] If instead the absolute values of φ_i are used in the case of η^*=0, the inflation weights of Table 4 will be: Germany 36.17% (52.96%); France 41.73% (61.10%); Italy 1.77% (-3,11%); Spain 1.26% (1,85%)

since a negative optimal bias was found for France in almost all cases, and a negative optimal inflation bias was found for Germany. Although these results do not necessarily contradict previous findings, since optimal policy is examined and not actual ECB policy, the relative weights of Table 4 are quite surprising. Even more so since both Germany and France both face negative inflation biases, which are, contrary to both output biases, estimated through coefficients which were found to be significant predictors in Section 4.1. Thus, the most reliable bias of the three estimated biases, points towards less influence on the interest rate of these member states, instead of higher. On the contrary, it appears to be optimal to favor Italy's and Spain's inflation more than their economic weights, given λ is high enough. This is due to the relatively small sensitivity to changes in the real exchange rate for Italy (-0.058), and an even positive effect for Spain (+0.008). Since the sensitivity to changes in the real exchange rate works to decrease the relative inflation bias (equation (27)), both member states should be subject to a positive regional bias. Furthermore, the influence of the trade weights appears to be confined to the situation where the ECB only focuses on achieving the optimal level of output $(\lambda = 0)$. In that case, the trade shares cause the inflation bias to become positive for member states with sufficiently high trade volumes. As a result, the relative weights of the few member states with positive inflation biases will increase dramatically, compared to member states with negative inflation biases.

While these optimal weights indeed suggest that it could be optimal for the ECB to under-or overweigh certain member states in its interest rate decision, the estimated weights of Table 4 are probably not accurate. This is due to a couple of flaws in the model itself, as well as the estimated coefficients that are used to calculate the optimal weights. Most importantly the estimated coefficients of the inflationary pressure of the output-gap (α_i) have been found to vary widely among member states. Also, none of these coefficients were found to be a significant predictor of next period's inflation. Therefore, using the output-gap coefficients to calculate member states their optimal weight will probably not produce accurate results. Furthermore, the coefficients of Table 3 were not tested for their ability to predict. Because of this, the predicted effect of a certain interest rate is likely to deviate from its actual effect. The actual effect lies on an interval around the predicted effect. The better the model can predict next periods output and inflation, the smaller this interval, and the closer the policy of the ECB resembles optimal policy by using the relative weights. This is where further research might contribute, in the sense that other economic characteristics might be investigated in order for the explanatory power of the models, used for the construction of the Taylor-rule, improves.

5 Conclusion

This thesis attempted to construct a theoretical framework to justify the regional bias within ECB interest rate policy that some authors have observed. For this purpose, the model of Romer (2001) and the subsequent expansion by Arnold (2006) were adapted to construct a Taylor rule. This Taylor rule determined the optimal interest rate based on a member state's output, its optimal level of output and its inflation. As the Taylor rule was constructed, it appeared that there were five different economic circumstances that could cause an optimal bias, namely: output persistence, interest rate elasticity, sensitivity to real exchange rate changes, exposure to non-Eurozone trade and inflationary pressure of the output-gap. Indeed, it was found that if these circumstances are different among member states, the optimal weights could deviate from member states their economic size. Therefore, these circumstances could, at least partially, explain why in some cases it is optimal for the ECB to under- or overweigh certain member states in its interest rate decision.

Unfortunately, a thorough empirical validation of the constructed models was not possible due to a lack of data points. Whether the constructed model can be actually used as a policymaking instrument thus remains unclear. Nonetheless, some promising results have been found. Especially the constructed output model fitted the data well, with output persistence, the interest rate elasticity and sensitivity to real exchange rate changes being significant predictors of next period's output. These results corroborate that differences within these circumstances among EMU member states do, at least partially, explain the presence of regional biases within ECB interest rate policy. This was in particularly true for the weight the ECB should attribute to member states' inflation. However, these findings should be interpreted carefully, due to the assumptions that were applied to construct the models. The absence of these assumptions could alter the mechanism of the optimal regional biases or derogate the investigated economic circumstances of their explanatory power. Therefore, no definite conclusions can be drawn regarding the economic circumstances that give rise optimal regional biases.

While the research question as stated in the introduction can thus not be fully answered, this thesis does present a starting point for further research. It provides a methodology to help explain the perceived regional biases within ECB policy and an indication which economic circumstances might play a role therein. Further research is needed to determine whether the constructed model is able to accurately predict next period's output and inflation. Only when this ability to predict is demonstrated will it be possible to conclude to what extent the investigated circumstances lead to optimal regional biases.

6 References

Arnold, I.J.M., 2004. Optimal regional biases in ECB interest rate setting. European Journal of Political Economy, 22, 307-321.

Berger, H. and de Haan, J., 2002. Are small countries to powerful within the ECB? Atlantic Economic Journal 30-3, 1-20

Bolt, W. and Els, P.J.A. van, (1998), Output gap and inflation in the EU, de Nederlandsche Bank, Working Paper Series 550.

Bouvet, F. and King, S., 2013. Do national economic shocks influence ECB interest rate decisions? The impact of the financial and sovereign debt crises. Journal of Common Market Studies 51-2, 212-231

Crowley, P. and Lee, J., 2008. Do All Fit One Size? An Evaluation of the ECB Policy response to the changing economic conditions in the Euro Area member states', paper presented at the European Union Studies Assoc. Economics Workshop, Federal Reserve Bank of St. Louis, Missouri.

Dornbusch, R., Favero, C., Giavazzi, F., 2002. Immediate challenges for the European Central Bank, Economic Policy 26, 15-64.

Drometer. M., Siemsen, T., Watzk, S., 2013. The monetary policy of the ECB: a Robin Hood approach?. CESifo Working Paper Series 4178.

Fourçans, A., and Vranceanu, R., 2004. The ECB interest rule under the Duisenberg presidency. European Journal of Political Economy 20, 579-595.

Gerlach S., Schnabel, G., 2000. The Taylor rule and interest rates in the EMU area. Economics Letters no. 67, 165-171.

Hayo, B. and Méon, P.-G., 2002. Behind closed doors: Revealing the ECB's decision rule. Journal of International Money and Finance no. 37, 135-160.

Heinemann, F., Hüfner, F.P., 2004. Is the view from the Eurotower purely European? National divergence and ECB interest rate policy. Scottish Journal of Political Economy 51, 544–558.

Kool, C.J.M., 2005. What drives ECB monetary policy? Discussion Paper no. 05-03, Utrecht School of Economics, Utrecht.

Maede, E. and Sheets, D., 2005. Regional influences on FOMC voting patterns. Journal of Money, Credit and Banking 37, 661-677.

Olsen, E. and Wohar, M., 2016. An evaluation of ECB policy in the Euro's big four. Journal of Macroeconomics 48, 203-213.

Romer, D., 2001. Advanced Macroeconomics. McGraw-Hill, New York. 531-536

Sauer, S. and Sturm, J.-E., 2003.Using Taylor rules to understand ECB Monetary Policy, Working Paper no. 110, CESifo, Munich.

Sturm, J. and Wollmershaeser, T., 2008, The stress of having a single monetary policy in Europe. CESifo Working Paper Series 2251.

Svensson, L., 1997. Inflation forecast targeting: implementing and monitoring inflation targets. European Economic Review 41, 1111-1146.

Taylor, J.B., 1993. Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

Taylor, J.B., 1998. The robustness and efficiency of monetary policy rules as guidelines for interest rate setting by the ECB. Working paper no. 58. Sveriges Riskbank, Stockholm.

Von Hagen, J. and Brückner, M., 2001. Monetary policy in unknown territory: the European Central Bank in the early years. ZEI working paper No. B18.

Appendix A

Taylor rule consisting of two member states

Start with the condition for optimal (aggregate) ECB policy from equation (17):

$$E_t(y_{\epsilon,t+1}) - y^*_{\epsilon,t+1} = -\eta^* \, \pi_{\epsilon,t+1}$$

Which disintegrates into:

$$\omega E_t(y_{1,t+1}) + (1-\omega)E_t(y_{2,t+1}) - (\omega y^*_{1,t+1} + (1-\omega)y^*_{2,t+1}) = -\eta^*(\omega\pi_{1,t+1} + (1-\omega)\pi_{2,t+1})$$

Insert equations (1), (2), (10) and (11). The subscript t is omitted for simplicity.

$$\begin{aligned}\omega(\rho_1 y_1 - \beta_1 r_1 - \mu_1 \Delta q_1) + (1-\omega)(\rho_2 y_2 - \beta_2 r_2 - \mu_2 \Delta q_2) - (\omega y^*_1 + (1-\omega)y^*_2) \\ = -\eta^*[\omega(\pi_1 + a_1(y_1 - y^*_1)) + (1-\omega)(\pi_2 + a_2(y_2 - y^*_2))]\end{aligned}$$

Remove all brackets:

$$\begin{aligned}\omega\rho_1 y_1 - \omega\beta_1 r_1 - \omega\mu_1\Delta q_1 - \omega y^*_1 + \rho_r y_2 - \beta_2 r_2 - \mu_2\Delta q_2 - y^*_2 - \omega\rho_2 y_2 + \omega\beta_2 r_2 + \omega\mu_2\Delta q_2 + \omega y^*_2 \\ = -\eta^*\omega\pi_1 - \omega\eta^* a_1 y_1 + \omega\eta^* a_1 y^*_1 - \eta^*\pi_2 - \eta^* a_2 y_2 + \eta^* a_2 y^*_2 + \omega\eta^*\pi_2 \\ + \omega\eta^* a_2 y_2 - \omega\eta^* a_2 y^*_2\end{aligned}$$

Move the REER (r) to the left side:

$$\begin{aligned}-\omega\beta_1 r_1 - \beta_2 r_2 + \omega\beta_2 r_2 \\ = -\omega\rho_1 y_1 + \omega\mu_1\Delta q_1 + \omega y^*_1 - \rho_r y_2 + \mu_2\Delta q_2 + y^*_2 + \omega\rho_2 y_2 - \omega\mu_2\Delta q_2 - \omega y^*_2 \\ + -\eta^*\omega\pi_1 - \omega\eta^* a_1 y_1 + \omega\eta^* a_1 y^*_1 - \eta^*\pi_2 - \eta^* a_2 y_2 + \eta^* a_2 y^*_2 + \omega\eta^*\pi_2 \\ + \omega\eta^* a_2 y_2 - \omega\eta^* a_2 y^*_2\end{aligned}$$

Put the right side into brackets:

$$\begin{aligned}-\omega\beta_1 r_1 + (\omega\beta_2 - \beta_2)r_2 \\ = -\omega(\rho_1 + \eta^* a_1)y_1 - \omega\eta^*\pi_1 - (1-\omega)(\rho_2 + \eta^* a_2)y_2 - (\eta^* - \omega\eta^*)\pi_2 \\ + \omega(1 + \eta^* a_1)y^*_1 + (1-\omega)(1 + \eta^* a_2)y^*_2 + \omega\mu_1\Delta q_1 + (\mu_2 - \omega\mu_2)\Delta q_2\end{aligned}$$

Insert equation (3) $[r_i = i_\epsilon - \pi_i]$:

$$\begin{aligned}-\omega\beta_1 i_\epsilon + \omega\beta_1\pi_1 + \omega\beta_2 i_\epsilon - \beta_2 i_\epsilon - \omega\beta_r\pi_2 + \beta_r\pi_2 \\ = -\omega(\rho_1 + \eta^* a_1)y_1 - \omega\eta^*\pi_1 - (1-\omega)(\rho_2 + \eta^* a_2)y_2 - (\eta^* - \omega\eta^*)\pi_2 \\ + \omega(1 + \eta^* a_1)y^*_1 + (1-\omega)(1 + \eta^* a_2)y^*_2 + \omega\mu_1\Delta q_1 + (\mu_2 - \omega\mu_2)\Delta q_2\end{aligned}$$

Move inflation to the right side:

$$\begin{aligned}-\omega\beta_1 i_\epsilon + \omega\beta_2 i_\epsilon - \beta_2 i_\epsilon \\ = -\omega(\rho_1 + \eta^* a_1)y_1 - \omega(\eta^* + \beta_1)\pi_1 - (1-\omega)(\rho_2 + \eta^* a_2)y_2 \\ - (\eta^* - \omega\eta^* - \omega\beta_2 + \beta_2)\pi_2 + \omega(1 + \eta^* a_1)y^*_1 + (1-\omega)(1 + \eta^* a_2)y^*_2 + \omega\mu_1\Delta q_1 \\ + (\mu_2 - \omega\mu_2)\Delta q_2\end{aligned}$$

Put the interest rate between brackets:

$$
\begin{aligned}
&(-\omega\beta_1 + \omega\beta_2 - \beta_2)i_\epsilon \\
&= -\omega(\rho_1 + \eta^* a_1)y_1 - \omega(\eta^* + \beta_1)\pi_1 - (1-\omega)(\rho_2 + \eta^* a_2)y_2 \\
&- (\eta^* - \omega\eta^* - \omega\beta_2 + \beta_2)\pi_2 + \omega(1 + \eta^* a_1)y_1^* + (1-\omega)(1 + \eta^* a_2)y_2^* + \omega\mu_1\Delta q_1 \\
&+ (\mu_2 - \omega\mu_2)\Delta q_2
\end{aligned}
$$

Multiply both sides with -1:

$$
\begin{aligned}
&(\omega\beta_1 - \omega\beta_2 + \beta_2)i_\epsilon \\
&= \omega(\rho_1 + \eta^* a_1)y_1 + \omega(\eta^* + \beta_1)\pi_1 + (1-\omega)(\rho_2 + \eta^* a_2)y_2 \\
&+ (\eta^* - \omega\eta^* - \omega\beta_2 + \beta_2)\pi_2 - \omega(1 + \eta^* a_1)y_1^* - (1-\omega)(1 + \eta^* a_2)y_2^* - \omega\mu_1\Delta q_1 \\
&- (\mu_2 - \omega\mu_2)\Delta q_2
\end{aligned}
$$

Insert equation (4), which simplifies to equation (5) [$\Delta q_i = \pi_i - \theta_{j,i}\pi_j$]:

$$
\begin{aligned}
&(\omega\beta_1 - \omega\beta_2 + \beta_2)i_\epsilon \\
&= \omega(\rho_1 + \eta^* a_1)y_1 + \omega(\eta^* + \beta_1)\pi_1 + (1-\omega)(\rho_2 + \eta^* a_2)y_2 \\
&+ (\eta^* - \omega\eta^* - \omega\beta_2 + \beta_2)\pi_2 - \omega(1 + \eta^* a_1)y_1^* - (1-\omega)(1 + \eta^* a_2)y_2^* - \omega\mu_1(\pi_1 \\
&- \theta_{2,1}\pi_2) - (\mu_2 - \omega\mu_2)(\pi_2 - \theta_{1,2}\pi_1)
\end{aligned}
$$

Simplify both sides:

$$
\begin{aligned}
&(\omega\beta_1 + (1-\omega)\beta_2)i_\epsilon \\
&= \omega(\rho_1 + \eta^* a_1)y_1 + \left[\omega(\eta^* + \beta_1 - \mu_1) + (1-\omega)\mu_2\theta_{1,2}\right]\pi_1 \\
&+ (1-\omega)(\rho_2 + \eta^* a_2)y_2 + [(1-\omega)(\eta^* + \beta_2 - \mu_2) + \omega\mu_1\theta_{2,1}]\pi_2 \\
&- \omega(1 + \eta^* a_1)y_1^* - (1-\omega)(1 + \eta^*)y_2^*
\end{aligned}
$$

Divide both sides by $(\omega\beta_1 + (1-\omega)\beta_2)$:

$$i_{\epsilon,t} = \Phi_1 y_{1,t} - \vartheta_1 y_{1,t}^* + \Phi_2 y_{2,t} - \vartheta_2 y_{2,t}^* + \varphi_1\pi_{1,t} + \varphi_2\pi_{2,t}$$

With:

$$\Phi_1 = \frac{\omega(\rho_1 + \eta^*\alpha_1)}{\omega\beta_1 + (1-\omega)\beta_2}$$

$$\Phi_2 = \frac{(1-\omega)(\rho_2 + \eta^*\alpha_2)}{\omega\beta_1 + (1-\omega)\beta_2}$$

$$\varphi_1 = \frac{\omega(\eta^* + \beta_1 - \mu_1) + (1-\omega)\mu_2\theta_{1,2}}{\omega\beta_1 + (1-\omega)\beta_2}$$

$$\varphi_2 = \frac{(1-\omega)(\eta^* + \beta_2 - \mu_2) + \omega\,\mu_1\theta_{2,1}}{\omega\beta_1 + (1-\omega)\beta_2}$$

$$\vartheta_1 = \frac{\omega(1 + \eta^* a_1)}{(\omega\beta_1 + (1-\omega)\beta_2)}$$

$$\vartheta_2 = \frac{(1-\omega)(1 + \eta^* a_2)}{(\omega\beta_1 + (1-\omega)\beta_2)}$$

Appendix B

Taylor rule consisting of *n* member states

Adding more member states to the Taylor-rule of Appendix A is a matter of including more GDP weights and national models to the already existing framework. Note that in the two state Taylor rule, the GDP weight of state 1 was denominated ω, which resulted in the GDP-weight of state 2 being $(1-\omega)$. However, it also possible to denote the GDP-weight of member state 2 as ω_2 and the GDP-weight of member state 1 as ω_1. The GDP-weight of an added third state then simply becomes ω_3, and the GDP-weight of member state *n* becomes ω_n. Extending the existing two states Taylor-rule to include *n* states then comes down to repeating the same methodology as was shown in Appendix A:

Start with the condition for optimal (aggregate) ECB policy from equation (17):

$$E_t\left(y_{\epsilon,t+1}\right) - y^*_{\epsilon,t+1} = -\eta^*\,\pi_{\epsilon,t+1}$$

Which disintegrates into:

$$\omega_1 E_t\left(y_{1,t+1}\right) + \omega_2 E_t\left(y_{2,t+1}\right) + \ldots + \omega_n E_t\left(y_{n,t+1}\right) - \left(\omega_1 y^*_{1,t+1} + \omega_2 y^*_{2,t+1} + \ldots + \omega_n y^*_{n,t+1}\right)$$
$$= -\eta^*\left(\omega_1 \pi_{1,t+1} + \omega_2 \pi_{2,t+1} + \ldots + \omega_n \pi_{n,t+1}\right)$$

Note that none of the variables of member states interact with each other; variables with subscript *i* only interact with variables with the same subscript. This indicates that by including more member states to the aggregate model, the Taylor-rule will have the same form as it had when it consisted of just two member states. Therefore, each included member state will add the following three terms to the Taylor rule:

$$+\omega_i(\rho_i + \eta^* a_i) y_i$$
$$+\left[\omega_i(\eta^* + \beta_i - \mu_i) + \sum_j^n \omega_j \mu_j \theta_{i,j}\right]\pi_i$$
$$-\omega_i(1 + \eta^* a_i) y_i^*$$

Alternatively, one can repeat the same steps as were shown in Appendix A with the equation above. This will yield the same result. The Taylor-rule consisting of *n* member states will be:

$$i_{\epsilon,t} = \Phi_1 y_{1,t} - \vartheta_1 y^*_{1,t} + \Phi_2 y_{2,t} - \vartheta_2 y^*_{2,t} + \ldots + \Phi_n y_{n,t} - \vartheta_n y^*_{n,t} + \varphi_1 \pi_{1,t} + \varphi_2 \pi_{2,t} + \ldots + \varphi_n \pi_{n,t}$$

With:

$$\Phi_i = \frac{\omega_i(\rho_i + \eta^* \alpha_i)}{\sum_i^n [\omega_i \beta_i]} \quad \text{(B1)}$$

$$\varphi_i = \frac{\omega_i(\eta^* + \beta_i - \mu_i) + \sum_j^n [\omega_j \mu_j \theta_{i,j}]}{\sum_i^n [\omega_i \beta_i]} \quad \text{(B2)}$$

$$\vartheta_i = \frac{\omega_i(1 + \eta^* a_i)}{\sum_i^n [\omega_i \beta_i]} \quad \text{(B3)}$$

Appendix C

Generalized conditions for which it holds that no optimal regional bias exists

Now that Appendix B made it clear how adding more member states to the model would alter equations (23) and (24), it is possible to determine the optimal weights of equations (25), (26) and (27) for a model consisting of *n* member states. This is done by using the same methodology as was shown in section 3.3, but this time with an unspecified number of member states instead of just two. Again, the optimal relative weight of member state 1 is determined.

C.1 Optimal level of output **[equation (26)]**

From (B2), the relative weight that the ECB should assign to member state 1's optimal level of output is equal to:

$$\frac{\vartheta_1}{\sum \vartheta_i} = \frac{\omega_1(1+\eta^* a_1)}{\sum[\omega_i(1+\eta^* a_i)]} = \frac{\omega_1(1+\eta^* a_1)}{\sum \omega_i + \eta^* \sum[\omega_i a_i]}$$

Note that the sum of all GDP-weights is equal to 1 ($\sum \omega_i = \omega_1 + \omega_2 + \ldots + \omega_n = 1$). Thus:

$$\frac{\vartheta_1}{\sum \vartheta_i} = \frac{\omega_1(1+\eta^* a_1)}{1+\eta^* \sum[\omega_i\, a_i]}$$

Now the condition needs to be found for which it holds that the relative weight the ECB should place on a member state's optimal level of output is equal to that member state's economic weight. In formula form:

$$\frac{\vartheta_1}{\sum \vartheta_i} = \frac{\omega_1(1+\eta^* a_1)}{1+\eta^* \sum[\omega_i\, a_i]} = \omega_1$$

Which is solved as follows:

$$\frac{1+\eta^* a_1}{1+\eta^* \sum[\omega_i\, a_i]} = 1 \qquad \Rightarrow \qquad 1+\eta^* a_1 = 1+\eta^* \sum[\omega_i a_i] \qquad \Rightarrow \qquad a_1 = \sum[\omega_i a_i]$$

Thus, if $a_1 = \sum[\omega_i a_i]$ the optimal weight the ECB should attribute to member state *1*'s optimal level of output is equal to its economic weight, regardless of how many member states the EMU consists of. When this condition is applied to a EMU consisting of just two member states, the found condition simplifies to $a_1 = a_2$, which is exactly the same condition as was found in equation (26).

C.2 Output [equation (25)]

Again, the condition needs to be found for which it holds that the weight (B1) assigns to member state 1's output is equal to its economic weight, relative to the output weights of all member states. In formula form:

$$\frac{\Phi_1}{\sum \Phi_i} = \frac{\omega_1(\rho_1 + \eta^* a_1)}{\sum[\omega_i(\rho_i + \eta^* a_i)]} = \frac{\omega_1(\rho_1 + \eta^* a_1)}{\sum[\omega_i \rho_i] + \eta^* \sum[\omega_i a_i]} = \omega_1$$

<u>Apply the condition as found in (C.1)</u>

$$a_1 = \sum[\omega_i a_i] \qquad \Rightarrow \qquad \frac{\Phi_1}{\sum \Phi_i} = \frac{\omega_1(\rho_1 + \eta^* \sum[\omega_i a_i])}{\sum[\omega_i \rho_i] + \eta^* \sum[\omega_i a_i]} = \omega_1$$

<u>Which is solved as follows:</u>

$$\frac{\rho_1 + \eta^* \sum[\omega_i a_i]}{\sum[\omega_i \rho_i] + \eta^* \sum[\omega_i a_i]} = 1 \qquad \Rightarrow \qquad \rho_1 + \eta^* \sum[\omega_i a_i] = \sum[\omega_i \rho_i] + \eta^* \sum[\omega_i a_i]$$

$$\Rightarrow \qquad \rho_1 = \sum[\omega_i \rho_i]$$

Thus, if both $a_1 = \sum[\omega_i a_i]$ and $\rho_1 = \sum[\omega_i \rho_i]$ are not violated, the optimal weight the ECB should attribute to member state *1*'s output is equal to its economic weight. In a two country model of the EMU, these conditions simplify to $a_1 = a_2$ and $\rho_1 = \rho_2$. These were also the conditions that were presented in equation (25).

C.3 Inflation [equation (27)]

Finally, the conditions for which it holds that the relative weight (B3) assigns to member state 1's inflation are equal to its economic size need to be found. In formula form:

$$\frac{\varphi_1}{\sum \varphi_i} = \frac{\omega_1(\eta^* + \beta_1 - \mu_1) + \sum[\omega_i \mu_i \theta_{1,i}]}{\sum[\omega_i(\eta^* + \beta_i - \mu_i)] + \sum[\omega_i \mu_i \theta_{j,i}]} = \omega_1$$

Which can be rewritten as:

$$\eta^* + \beta_1 - \mu_1 + \frac{1}{\omega_1}\sum[\omega_i \mu_i \theta_{1,i}] = \sum[\omega_i(\eta^* + \beta_i - \mu_i) + \sum[\omega_i \mu_i \theta_{j,i}]$$

$$\eta^* + \beta_1 - \mu_1 + \frac{1}{\omega_1}\sum[\omega_i \mu_i \theta_{1,i}] = \sum[\omega_i \eta^*] + \sum[\omega_i \beta_i] - \sum[\omega_i \mu_i] + \sum[\omega_i \mu_i \theta_{j,i}]$$

Now, since the value of η^* is equal among all member states and $\sum[\omega_i] = 1$, it holds that $\sum[\omega_i \eta^*] = \eta^*$. This results in:

$$\beta_1 - \mu_1 + \frac{1}{\omega_1}\sum[\omega_i \mu_i \theta_{1,i}] = \sum[\omega_i \beta_i] - \sum[\omega_i \mu_i] + \sum[\omega_i \mu_i \theta_{j,i}]$$

Since the interest rate elasticities (βi) do no interact with other variables, a first condition can be derived.

$$\beta_1 = \sum[\omega_i \beta_i] \qquad \text{(C3.a)}$$

This results in:

$$-\mu_1 + \frac{1}{\omega_1}\sum[\omega_i \mu_i \theta_{1,i}] = -\sum[\omega_i \mu_i] + \sum[\omega_i \mu_i \theta_{j,i}]$$

Which can be rewritten as:

$$\mu_1 = \sum[\omega_i \mu_i] + \frac{1}{\omega_1}\sum[\omega_i \mu_i \theta_{1,i}] - \sum[\omega_i \mu_i \theta_{j,i}]$$

$$\mu_1 = \sum[\omega_i \mu_i * (1 - \theta_{j,i} + \theta_i/\omega_1] \qquad \text{(C3.b)}$$

If conditions (C3.a) and (C3.b) are not violated, the relative weight the ECB should attribute to member state *1*'s inflation is equal to its economic weight. In other words, no optimal bias exists for member state *1's* inflation. However, condition (C3.b) is quite difficult to interpret. Even in the simplest possible model, consisting of just two member states, condition (3.C.b) results in:

$$\mu_1(\omega_1 \theta_{2,1} - \omega_1 - 1) = \frac{\omega_1 - 1}{\omega_1} * \mu_2(\omega_1 \theta_{1,2} - \omega_1 - \theta_{1,2})$$

In order to make (C3.b) easier to interpret, it is helpful to derive a more specific condition from (C3.b). This specified condition is presented below:

$$\mu_1 = \sum[\omega_i \mu_i] \quad if \quad \sum[\omega_i \mu_i \theta_{j,i}] = \sum \frac{\omega_i \mu_i \theta_{1,i}}{\omega_1} \qquad \text{(C3.c)}$$

In a two country model, this simplifies to:

$$\mu_1 = \mu_2 \quad if \quad \theta_{1,2} = \frac{\omega_1}{1 - \omega_1} \theta_{2,1}$$

Condition (C3.c) should be interpreted as follows. If $\mu_1 = \sum[\omega_i \mu_i]$ is not violated, condition (C3.b) simplifies to $\sum[\omega_i \mu_i \theta_{j,i}] = \sum \frac{\omega_i \mu_i \theta_{1,i}}{\omega_1}$. Naturally, the opposite also holds. Thus, if $\sum[\omega_i \mu_i \theta_{j,i}] = \sum \frac{\omega_i \mu_i \theta_{1,i}}{\omega_1}$ is not violated, condition (C3.b) simplifies to $\mu_1 = \sum[\omega_i \mu_i]$. Now, if both conditions from (C3.c) and (C3.a) are not violated, the optimal weight the ECB should attribute to member state 1's inflation is equal to its economic size. In that case, no optimal bias exists for member state 1.

However, the conditions of (C3.c) are not exhaustive. This is due to the interaction between the trade shares (θ_{I},j) and the sensitivity to changes in the real exchange rate (μ_i). The conditions of (C3.c) are merely a specification of (C3.b). If (C3.c) holds, then (C3.b) automatically also holds. However, the opposite is not true; if (C3.c) is violated, this does not mean that (C3.b) is also automatically violated. Thus, (C3.b) is the general condition to determine the presence of optimal regional biases, while (C3.c) is a specification of the general rule that is easier to interpret.