Valuation of Multi-currency CSA's

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Abstract

A Multi-currency Credit Support Annex (CSA) is a contract used to document collateral agreements of a derivative contract between two parties where it is allowed to post the collateral in other currencies than the base currency. In this thesis, I construct a blended Cheapest-to-Deliver (CTD) framework in order to value those type of derivatives. I apply this new framework to a portfolio of Interest Rate Swaps (IRS) and compare it with the CTD method of Fujii & Takahashi (2011) and the multi- and single-curve framework of Fujii et al. (2010a). Key findings are that: (i) choosing the cheapest collateral largely increases the present value of the IRS portfolio when the portfolio holder is the collateral payer, (ii) the impact of the considered CTD methods depends on the knowledge of the counterparty (other party that participates in the swap contract) and the type of IRS and (iii) the single-curve framework highly mis-prices the IRS portfolio relative to the multi-curve framework.

Key words: Multi-currency CSA, Cheapest-to-Deliver, Dual-curve Bootstrapping, Interest Rate Swaps.

Contents

1 Introduction

In the early 1970s, financial derivatives were relative unimportant instruments that were barely used in practice (MacKenzie & Millo, 2003). This suddenly changed drastically when Merton (1973) and Black & Scholes (1973) developed and published their theory about option pricing, which formed the basis regarding the pricing of derivatives. In the early 1980s, also swaps became important financial instruments (Bicksler & Chen, 1986). The most simple and also most common type of swaps is the plain vanilla Interest Rate Swaps (IRS), which is a contract where the two parties exchange fixed for floating rate payments. Other examples of swaps that are commonly used in the market are the Tenor Swaps and Cross Currency Swaps (CCS), which is a contract where two parties exchange two floating rate payments of different tenors and currencies, respectively.

Nowadays, there are swap contracts where credit protection is added by means of collateral posting. In other words, the party that has a negative expected cash flow needs to pay that amount immediately to the other party in the form of for example cash instead of paying all the negative cash flows at the end of the swap contracts. These contracts are traded more frequently since the 2008 financial crisis. In addition, there are also derivative contracts where multiple eligible collateral currencies are allowed, which are referred to as Multi-currency Credit Support Annexes (CSA). The collateral payer has in this case the option which collateral currency to post, which means he should choose the cheapest collateral. In other words, suppose that EUR and USD are the eligible currencies for paying the collateral. It is possible that it costs relative less to pay the collateral in USD compared to paying it in EUR in a particular future point in time. This right is also known as the Cheapest-to-Deliver (CTD) option. In the current market it becomes more common to allow this option and therefore it is necessary to come up with a general strategy to deal with this issue. However, there is no clear guidance in the literature on what type of method to use in order to find the cheapest collateral and takes this into account when valuating derivatives. Additionally, even if there exist an optimal strategy regarding Multi-currency CSA's, it is not even always feasible to apply it. For example, suppose that the USD is the cheapest currency and the EUR the second cheapest collateral. It is possible that the collateral payer is not liquid enough in USD in order to pay the collateral. In that case, it is unclear if the collateral payer has to choose the second cheapest currency, which is EUR, or to source for the cheapest one, which is USD in this case.

The main contribution of this paper is to construct a general valuation method for IRS derivatives that takes multiple collateral currencies into account.[1] The main procedure of this approach is as follows. First, I construct all the discounting curves for all the relevant currencies by using the Overnight Index Swap (OIS) market quotes. Second, I convert the discounting curves to the base currency, which is EUR in my example. This can be done by using the particular Foreign Exchange (FX) rates or/and Cross Currency Spreads, depending on which market quotes and spreads are available. Finally, I blend all the implied discounting rates by taking the minimum for every point in time in order to get the desired CTD curve, which is equal to the cheapest collateral currency.

In the existing literature, there is already one method available proposed by Fujii et al. (2010b) and Fujii & Takahashi (2011) that shows how to determine the cheapest collateral. They show under some particular assumptions that one could minimize the difference between the risk-free rate and the collateral rate between the collateral and base currency, that is obtained when discounting future cash flows of a derivative contract that is collateralized in another currency. This can be bootstrapped by using the present values based on the domestic and foreign currency and setting them equal to each other due to the no-arbitrage assumption. Based on a simple one-factor Hull-White model with randomly chosen mean-reversion and volatility parameters, they show in a simulation process that the right to change the currency of collateral can especially be valuable when the CCS market is volatile. However, the big disadvantage of this approach is the fact that it is hard to interpret the obtained results, while the blended CTD method is easier to interpret. For example, if the implied USD discounting curve is smaller than the EUR discounting curve at a particular point in time, which means that it is cheaper to post the collateral in USD than in EUR, it is easy to interpret why this is the case. For instance, the future exchange rate could be small relative to the current exchange rate or the EUR discounting rate could be large relative to the USD discounting rate. When using the

[1] There exist also a set of slides from Numerix, available at `http://nx.numerix.com/rs/numerix2/images/Numerix_Webinar_Slides__Managing_Embedded_Optionality_Multi-Currency_CSAs__Dec_4_2012.pdf`, which provide broad guidelines for a blended CTD approach. I developed this approach independently, and the resulting method follows from the broad guidelines presented.

method proposed by Fujii et al. (2010b) and Fujii & Takahashi (2011), no such statement can be made. Hence, it only shows that the particular collateral currency is cheaper, but it is not interpretable why this is the case. Also, the blended CTD method does not always need CCS basis spreads while the other method does. This is convenient for the blended CTD approach since these spreads are not always available or liquid enough. Besides that, it still provides a solid benchmark to compare the blended CTD curve with.

Furthermore, there is already quite some extensive literature about building multi-curve frameworks when considering only the base currency. For instance, Mercurio (2009), Bianchetti (2010) and Henrard (2010) consider models that are based on a LIBOR Market Model (LMM) framework, while Kijima et al. (2009), Kenyon (2010) and L. Morino & Runggaldier (2014) use a short rate framework with the focus on modeling the additive short rate spread. Also, there is already quite some research done based on the Heath-Jarrow-Morton (HJM) framework by Fujii et al. (2011), Crépey et al. (2012), Cuchiero et al. (2014) and N. Morino & Pallavicini (2014), to name a few. This paper only uses the methodology of Fujii et al. (2010a) and Fujii et al. (2011) for the multi-curve framework with only one currency due to the fact that it is used extensively in the literature and provided decent results. For example, Gunnarsson (2013) applied this framework for the USD and EUR markets by deriving the discounting curve for EUR derivatives that are collateralized in USD. Similarly, Lidholm & Nudel (2014) implemented this framework when USD and SEK are the eligible currencies. Iñigo & Santander (2017) used the HJM framework by analyzing the differences between EUR and MXN markets, such as no existence of OIS in the MXN market and the fact that neither IRS nor CCS are collateralized in the Mexican currency. Fujii & Takahashi (2016) have also extended their work recently by deriving expressions for the funding spread dynamics which are more suitable for the general dependence of the collateral rates and develops a discretization of the HJM framework with a fixed tenor structure in order to make the approach more attractive for practical purposes. The reason why such a framework is considered in this paper is the fact that it provides a benchmark in order to measure the impact of choosing the cheapest collateral compared to ignoring this option and always choose the base currency. The single (LIBOR) curve framework proposed by Fujii et al. (2010a) and Fujii et al. (2011) is also considered, which approximates the discounting curve with same LIBOR rates that are used for building the forwarding curve and therefore neglects the

LIBOR-OIS spread. The reason why this method is included in this paper is because it shows the impact of using only the LIBOR rates relative to the multi-curve approach more clearly. Furthermore, it provides a solid basis for deriving the multi-curve framework.

I apply the methods considered in this thesis in an empirical setting. For the construction of the forwarding and discounting curves for the single-curve and multi-curve framework, I use the OIS and IRS (for several tenors) market quotes of the base currency. The OIS and IRS market rates for the other eligible currencies are also used when building the CTD curves. Also, I use the relevant CCS and FX rates for constructing the CTD curves. All the market quotes and spreads are from 2017 through 2047. I use the natural cubic spline approach proposed by Burden & Faires (1997) if some data points are missing in order to get smooth and continuous curves. Finally, I apply the constructed forwarding and discounting curves by computing the present value of an anonymized portfolio containing fifteen IRS contracts with different maturities and notionals in order to measure the performance of the forwarding and discounting curves.

This research provides three key implications. First, using one of the CTD approaches mentioned before highly influences the valuation process of the considered portfolio of IRS. This empirical application shows that posting the cheapest collateral instead of simply posting the base currency collateral changes the present value of the IRS portfolio largely, which ranges between 3.9%-5%. Also, when looking at the notional spread, which is the difference in PV relative to the multi-curve framework as a percentage of its notional amount, it is shown that overall it ranges between 0.04%-0.4%. Second, the size of the impact of the considered CTD methods depends on the type of IRS and on who the counterparty is and what they know. It is a safe assumption to make that the counterparty does not post the cheapest collateral when they are the collateral payer do to the complexity of the CTD approaches and therefore simply post the base currency. When comparing this lack of information with the assumption that the counterparty does post the cheapest collateral, the difference in present value can be quite large. In the non-perfect information case, the present value is even 6%-7.4% higher when comparing to the multi-curve framework with only a single currency. Furthermore, the difference is also larger for larger notionals and longer maturities. Third, the single-curve framework highly mis-prices the present value of the of the IRS portfolio when compared to the single-currency multi-curve framework, where the difference between the two frameworks is 2.31%. In this case, the notional spread ranges more or less between 0.02%-0.2% This empirical application shows that the single-curve framework, which was frequently used before the financial crisis of 2008 due to its simplicity, is clearly outdated.

The structure of this thesis is as follows. Section 2 gives background information about the derivative market since the 2008 financial crisis. Section 3 provides an explanation of the methodology. Section 4 elaborates on the dataset used for this research and gives an insight on the historical behavior. The performance of the proposed methodology is shown in section 5, while section 6 concludes and discusses several possibilities for further research.

2 Background Information Derivative Market

Since the 2008 financial crisis, the derivative market has significantly changed, which can be explained by two main changes. Before the crisis, it was sufficient to discount future cash flows of derivative contracts with the London Inter Bank Offer Rate (LIBOR) due to the fact that the basis swap spreads, which is the difference between two floating rates, were so small that it could be ignored. However, there has been a significant increase of the basis spread in both the CCS and single currency market (Fujii et al., 2010a) and therefore not negligible anymore. For example, suppose that there is only an IRS and a CCS market and the only available currencies are EUR and USD. Before the financial crisis, it would be the same to exchange one EUR into USD now and invest in USD 3m-LIBOR compared to investing one EUR in 3m-EURIBOR and buy a forward agreement now in order to exchange the resulting EUR after three months into USD. Unfortunately, this parity doesn't hold anymore, since the currency basis spread included in the FX forward deviates significantly from zero. This would imply that there is an arbitrage opportunity. However, in reality there is no arbitrage opportunity present. Hence, the valuation process of derivatives have to be adjusted by including the basis spread in order to correctly reflect the current market conditions. According to Bianchetti & Morini (2010), reasons why the basis spreads have not disappeared since the beginning of the crisis are changed liquidity preferences of financial practitioners and the supply of money in the different currency markets, which are influenced by several central banks.

Secondly, there has been a big increase in the use of collateral agreements for derivative contracts. According to the Margin Surveys of ISDA (International Swaps and Derivatives Association) in 2009 and 2010, the trade volumes for all the Over-the-Counter (OTC) derivatives that were collateralized in 2003 and 2009 were around 30% and 70%, respectively. Fujii et al. (2010b) argue that the reason behind this increase in collateral agreements is the fact that market participants want to reduce the credit risk of the counterparty, which is the other party that participates in the swap contract. In other words, firms expect a higher probability that the counterparty can not pay their obligations or goes bankrupt before the end of the maturity. Therefore, they want an insurance in the form of collateral, stated in a formal CSA agreement. Under this agreement, the firm receives collateral in the form of for example cash or a debt obligation with triple A rating from the counterparty when the present value of the swap contract is positive. In return, the counterparty receives a margin called the collateral rate, such as the Euro OverNight Index Average (EONIA) rate for EUR and the Fed-fund rate for USD. As a consequence, the pricing process of derivatives changes through the change of effective funding costs. Fujii et al. (2010a) show that discounting future cash flows with the commonly used LIBOR is not appropriate anymore, since one should discount with the collateral rate, which is often measured by Overnight Index Swaps (OIS). Before the crisis, the spread between LIBOR and OIS was negligible and therefore the forwarding and discounting procedure was done with the same LIBOR curve (Fujii & Takahashi, 2011). Since this is not the case anymore, it is necessary to change the discounting process by using the OIS curve, which gives rise to a multi-curve framework. According to Bianchetti & Carlicchi (2012), this is one of the most important impacts of the financial crisis on the interest rate market dynamics. For example, Sengupta & Tam (2008) showed that the USD 3m-LIBOR fixing and the three month to maturity OIS reached a peak of 365bps just after the collapse of the Lehman Brothers in September 2008.

3 Model Framework and Methodology

This section provides an overview of the model framework and methodology. First, I discuss the single-curve framework in section 3.1 in order to obtain a basis for the multi-curve framework, which follows after that in section 3.2. The proposed multi-curve framework is divided into the single-currency and multi-currency case. At last, I discuss in section 3.3 how to deal with the case when there are multiple currencies available for the collateral payer.

3.1 Pricing with Single-curve Framework

In this section, I present the single-curve framework with a single currency proposed by Fujii et al. (2010a, 2011) in order to price an IRS, applied to the EUR currency market. Despite the fact that this framework is not applied anymore in the current market due to the developments explained in section 2, it was before the financial crisis a commonly used model and it still provides a solid basis for explaining the multi-curve framework later on in section 3.2. The general idea of this framework is that a single curve is used for forwarding future cash flows and discounting those cash flows to its present value. In other words, the forward and discount curve are both based on the same LIBOR/EURIBOR curve. Since this thesis focuses on the EUR market, the curve that is used is the EURIBOR curve with a tenor of 6 months due to its liquidity.

For now, I assume that there is no collateral agreement for the IRS. Recall that this swap consists of a fixed leg and a floating leg. So one party pays a fixed rate in exchange for the market rate (normally EURIBOR/LIBOR), which is called the receiver, while the other party pays the market rate and receives the fixed rate, which is the payer, at a certain frequency till maturity. The main idea of this valuation framework is to calculate the present value (PV) for both legs and then subtract them from each other. First, compute the fixed and floating future cash flows. Then, discount every future cash flow back to the present for both legs. Following a simple arbitrage condition, it should hold that the fixed leg and floating leg should be equal to each other when the fixed rate is the market par rate. Hence, the PV for both parties should be equal to zero. This yields the (IRS) condition

$$S^I_{t,T_N}\sum_{n=1}^{N}\delta_{f_i,n}Z_{t,T_n}=\sum_{n=1}^{N}\mathrm{E}^I_t[L(T_{n-1},T_n)]\delta_{f_l,n}Z_{t,T_n}, \tag{1}$$

where the left and right side of the equation represents the fixed and floating leg, respectively. S^I_{t,T_N} is defined as the fixed par rate of an IRS I maturing at T_N, $\mathrm{E}^I_t[L(T_{n-1},T_n)]$ is the expected forward floating rate[2] of an IRS I (6m-EURIBOR) between T_{n-1} and T_n with $L(T_{n-1},T_n)$ being the floating rate and Z_{t,T_n} is the risk-free zero coupon bond maturing at T_n. The maturity dates T_n have time grids $n=1,...,N$ where N represents the number of payments of the IRS. For example, N is equal to 60 when T_N is 30 years and the payments are semi-annual. Furthermore, $\delta_{f_i,n}$ and $\delta_{f_l,n}$ are the day count fractions of the fixed leg f_i and floating leg f_l, respectively. Also, note that t is the trade date of the particular IRS.

Hence, the future cash flow of the fixed leg is the fixed IRS par rate S^I_{t,T_N} times the sum of the fixed day count fraction $\delta_{f_i,n}$ and the future cash flow of the floating leg is the sum of the expected 6m-EUIRBOR rates $\mathrm{E}^I_t[L(T_{n-1},T_n)]$ times its day count fraction $\delta_{f_l,n}$ when considering an IRS with maturity date T_N and N payment dates. Then, every future cash flow is discounted with its corresponding discount factor, which is equal to the risk-free zero coupon bond Z_{t,T_n}. These bonds can be used for discounting purposes since there are no interest payments throughout the life of the security.

Since it is assumed that the 6m-EURIBOR is also used for computing the discounting rate, $\mathrm{E}^I_t[L(T_{n-1},T_n)]$ is obtained by using the following no-arbitrage condition[3]

$$\mathrm{E}^I_t[L(T_{n-1},T_n)]=\frac{1}{\delta_{f_l,n}}\left(\frac{Z_{t,T_{n-1}}}{Z_{t,T_n}}-1\right). \tag{2}$$

Substituting equation (2) into equation (1) yields

[2]All expectations in this thesis are taking under the risk-neutral Q-measure unless stated otherwise.

[3]Note that $(1+\delta_{f_i,n}\mathrm{E}^I_t[L(T_{n-1},T_n)])(1+\delta_{f_i,n-1}z_{t,T_{n-1}})^{n-1}=(1+\delta_{f_i,n}z_{t,T_n})^n$ holds when there are no arbitrage possibilities with z_{t,T_n} being the zero rate at T_n. The definition of the zero-coupon bond is given as $Z_{t,T_n}=1/(1+\delta_{f_i,n}z_{t,T_n})^n$, which yields $(1+\delta_{f_i,n}\mathrm{E}^I_t[L(T_{n-1},T_n)])/Z_{t,T_{n-1}}=1/Z_{t,T_n}$. Rewriting this results in equation (2).

$$S^I_{t,T_N}\sum_{n=1}^{N}\delta_{f_i,n}Z_{t,T_n} = Z_{t,T_0} - Z_{t,T_N}, \tag{3}$$

which can be rewritten in such a way that Z_{t,T_n} can be determined iteratively, given as

$$Z_{t,T_n} = \frac{Z_{t,T_0} - S^I_{t,T_n}\sum_{i=1}^{n-1}\delta_{f_i,n}Z_{t,T_n}}{1 + S^I_{t,T_n}\delta_{f_i,n}}, \qquad n = 1, ..., N, \tag{4}$$

where Z_{t,T_0} can be initialized by setting it equal to one since there is no discounting needed in the present tense T_0. Finally, $\mathrm{E}^I_t[L(T_{n-1}, T_n)]$ can be computed by using the results from equation (4) and substitute it into equation (2). This whole procedure is also called single-curve bootstrapping. In other words, the discounting curve Z_{t,T_n} and forwarding curve $\mathrm{E}^I_t[L(T_{n-1}, T_n)]$ are computed by using only the fixed par rates S^I_{t,T_n}.

3.2 Pricing with Multi-curve Framework

This section provides an overview of the multi-curve framework proposed by Fujii et al. (2010a, 2011). First, I explain the general concept of what a CSA contract is for a single and multiple eligible currencies in section 3.2.1. Then I divide the multi-curve framework into the single currency case and the multiple currencies case, presented in section 3.2.2 and section 3.2.3, respectively.

3.2.1 General idea of CSA contract

In this section, I provide a general explanation of how a CSA contract works for a single and multiple eligible currencies. Let us first consider the situation where the IRS trade between party A and counterparty B is not collateralized, which means that there is no CSA contract. Without loss of generality, assume that the PV of party A is positive and therefore the PV of party B is negative. This is equivalent to providing a loan to firm B financed by firm A with the notional being equal to the PV, since firm A has to wait for the payment of firm B till the maturity of the IRS. On the other hand, the funding cost should be reflected in the price of the IRS contracts in order to make the financing possible for firm A. Normally, a firm has EURIBOR/LIBOR credit quality and therefore the funding cost is e.g. equal to EURIBOR in case the funding currency is EUR. Summarized, the IRS contract at this point can be seen as loan provided to firm B financed by firm A with a notional equal to the PV at a interest rate equal to the EURIBOR market rate.

This situation changes when the IRS trade is collateralized. For simplicity reasons, I assume in this thesis that the CSA contract is specified in such a way that the collateral can only be paid in cash and that the minimum transfer amount and threshold are equal to zero. In this case, firm B posts the amount of cash, which is the collateral payment in EUR, equal to the PV of firm A. In return, firm A has to pay a margin to firm B which is equal to the so-called collateral rate, which is often specified in the CSA contract as the OIS market rate. In case of EUR collateralization, the Euro OverNight Index Average (EONIA) rate is often used, which is also used in this thesis. Hence, firm A does not have to finance a loan to firm B anymore. Instead, firm B pays of the loan immediately in return for an interest payment equal to the EONIA rate. This effectively changes the funding cost to the collateral rate. In other words, the future cash flows resulting from an IRS contracts should in this case be discounted by the collateral rate instead of the EURIBOR in order to get the PV of the swap.

Now, let us assume that there are multiple eligible currencies specified in the CSA contract for paying the collateral. This is also called a multi-currency CSA contract. Fujii & Takahashi (2011) argue that this choice of different collateral currencies has a non-negligible impact on the derivative pricing. For example, they say that paying the collateral in USD cash is expensive relative to other currencies due to the so-called save-haven demand. Hence, this gives rise to the CTD option for the payer of the collateral if there is free replacement among the currencies.

3.2.2 Case of Single Currency

This section provides the multi-curve framework for a single currency. In other words, I assume that the collateral currency is the same as the currency of the derivative payment, which is still EUR in this case. Note that this framework is especially useful when there is a collateral agreement in the derivative contract, as explained in the previous section.

The IRS condition that should hold looks the same as in the section 3.1

$$S^I_{t,T_N}\sum_{n=1}^{N}\delta_{f_i,n}Z^c_{t,T_n}=\sum_{n=1}^{N}\mathrm{E}^{c,I}_t[L(T_{n-1},T_n)]\delta_{f_l,n}Z^c_{t,T_n}, \tag{5}$$

where Z^c_{t,T_n} is the risk-free zero coupon bond which is collateralized c maturing at T_n and $\mathrm{E}^{c,I}_t[L(T_{n-1},T_n)]$ is the collateralized (c) forward 6m-EURIBOR between T_{n-1} and T_n where Z^c_{t,T_n} instead of Z_{t,T_n} is used as the numeraire. As explained in the previous section, it is not appropriate anymore to discount with the EURIBOR curve when pricing trades with a CSA contract due to the change in funding costs. Hence, the difference with equation (1) is the zero-coupon bond and the forward rate resulting from the zero-coupon bond. Since the collateral rate, which is often the EONIA rate for the EUR currency and OIS for other currencies, should in this case be used for discounting, the following condition should hold

$$S^O_{t,T_N}\sum_{n=1}^{N}\delta_{f_i,n}Z^c_{t,T_n}=\sum_{n=1}^{N}\mathrm{E}^{O}_t[L(T_{n-1},T_n)]\delta_{f_l,n}Z^c_{t,T_n}, \tag{6}$$

where S^O_{t,T_N} is the fixed swap rate of an OIS O and $\mathrm{E}^O_t[L(T_{n-1},T_n)]$ is the forward OIS rate. Note that the OIS fixed and forward rate can be replaced by the EONIA fixed and floating rate when considering the EUR case. Equation (2) can be used in the same as in section 3.1 due to the fact that the EONIA rate is now considered as the discounting rate. This yield the following

$$S^O_{t,T_N}\sum_{n=1}^{N}\delta_{f_i,n}Z^c_{t,T_n}=Z^c_{t,T_0}-Z^c_{t,T_N}. \tag{7}$$

Rewriting this condition the same way as in section 3.1 results in

$$Z^c_{t,T_n}=\frac{Z^c_{t,T_0}-S^O_{t,T_N}\sum_{i=1}^{n-1}\delta_{f_i,n}Z^c_{t,T_i}}{1+S^O_{t,T_N}\delta_{f_i,n}}, \qquad n=1,...,N. \tag{8}$$

The collateralized forward 6m-EURIBOR curve $\mathrm{E}_t^{c,I}[L(T_{n-1},T_n)]$ can be constructed by rewriting equation (5) equation (5). This results in the following equation

$$\mathrm{E}_t^{c,I}[L(T_{n-1},T_n)] = \frac{S^I_{t,T_N}\sum_{n=1}^{N}\delta_{f_i,n}Z^c_{t,T_n} - \sum_{n=1}^{N-1}\mathrm{E}_t^{c,I}[L(T_{n-1},T_n)]\delta_{f_l,n}Z^c_{t,T_n}}{\delta_{f_l,N}Z^c_{t,T_N}}, \qquad (9)$$

where $\mathrm{E}_t^{c,I}[L(T_0,T_1)]$ is set equal to the fixed swap rate S^I_{t,T_1} and Z^c_{t,T_0} is again initialized to one. Finally, the the collateralized forward 6m-EURIBOR curve $\mathrm{E}_t^{c,I}[L(T_{n-1},T_n)]$ can be constructed by substituting the obtained Z^c_{t,T_n} into equation (9). Note that equation (2) can not be used anymore for calculating $\mathrm{E}_t^{c,I}[L(T_{n-1},T_n)]$ due to the collateralization. This whole procedure is often referred to as dual-curve bootstrapping since it uses besides the IRS fixed swap rates S^I_{t,T_n} also the OIS fixed swap rates S^O_{t,T_n}. Note that the only difference with the previous section is the fact that the discount factors Z^c_{t,T_n} are now based on the OIS/EONIA rate instead of the LIBOR/EURIBOR rate. The rest of the procedure remains the same.

3.2.3 Case of Multiple Currencies

From now on it is assumed that the collateral currency is different from the currency of the derivative payment. The foreign currencies that I use in this thesis are USD, GBP and JPY, since these currencies are the most common used collateral currencies in practice. For illustration purposes, I only consider the USD market for explaining the methodology, but the procedure is similar for all foreign markets. Furthermore, the Mark-to-Market (MtM) notional is considered in this thesis for the CCS market instead of a constant notional, since it is the most popular type. The difference is that the notional on the currency paying EURIBOR or LIBOR flat is adjusted every period based on the forward exchange spot rate and the difference between the notional used in the previous period. Also, the next one is also paid or received at the reset time. The notional for the other currency (EURIBOR/LIBOR plus spread) is kept constant throughout the contract though. Following Fujii & Takahashi (2011), the present value of an IRS based on EUR is in this case given as

$$PV_{€} = - Z^{c,€}_{t,T_0} \mathrm{E}^{c,€}_t [e^{-\int_t^{T_0} y^{(€,\$)}(s)ds}] + Z^{c,€}_{t,T_N} \mathrm{E}^{c,€}_t [e^{-\int_t^{T_N} y^{(€,\$)}(s)ds}] + \sum_{n=1}^{N} \delta^{€}_n Z^{c,€}_{t,T_n} \mathrm{E}^{c,€}_t [e^{-\int_t^{T_n} y^{(€,\$)}(s)ds} (L(T_{n-1}, T_n) + B_N)], \tag{10}$$

where $y^{(€,\$)}(s) = y^{€} - y^{\$}$ with $y^{\{\cdot\}} = r^{\{\cdot\}} - c^{\{\cdot\}}$, which is the difference between the risk-free rate and the collateral rate in the mentioned currencies. Furthermore, B_N is the CCS basis spread, assumed to be available as a market quote. Note that the zero-coupon bond $Z^{c,€}_{t,T_n}$ is the same as Z^{c}_{t,T_n} defined in section 3.2.2, but the change in notation is from now on needed since multiple currencies are in this case allowed for posting collateral. Also note that there are, besides the change in notation of the discount factor, two main changes compared to the previous section, which is the extra term $\mathrm{E}^{c,€}_t [e^{-\int_t^{T_n} y^{(€,\$)}(s)ds}]$ and the CCS basis spread B_N. The first and second term are needed in order to correct for the discount factor en forward rate in EUR, respectively. For example, the first term is smaller than one, and therefore decreases the discount factor, when $y^{€}$ is larger than $y^{\$}$. This implies that the USD spread between the risk-free rate and the collateral rate is relatively smaller, which directly increases the liquidity and therefore makes the USD a cheaper collateral currency (Johannes & Sundaresan, 2007). The second term is needed to correct for the expected USD forward rate since it is normally not equal to the EUR forward rate. Equation (10) can be rewritten as follows

$$PV_{€} = -Z^{c,(€,\$)}_{t,T_0} + Z^{c,(€,\$)}_{t,T_N} + \sum_{n=1}^{N} \mathrm{E}^{c,€}_t [L(T_{n-1}, T_n) + B_N] \delta^{€}_n Z^{c,(€,\$)}_{t,T_n}, \tag{11}$$

when defining a more general discount factor as $Z^{c,(€,\$)}_{t,T_n} = Z^{c,€}_{t,T_n} \mathrm{E}^{c,€}_t [e^{-\int_t^{T_0} y^{(€,\$)}(s)ds}]$. Note that the additional term is equal to one, and therefore disappears, when $y^{(€,\$)}$ is replaced by $y^{(€,€)}$, which is indeed the single currency case. On the other side, the present value based on USD can be given as (Fujii & Takahashi, 2011)

$$PV_{\$} = -\sum_{n=1}^{N} Z^{c,\$}_{t,T_{n-1}} \frac{E^{c,\$}_t[f^{\$,€}_x(T_{n-1})]}{f^{(\$,€)}_x(0)} + \sum_{n=1}^{N} Z^{c,\$}_{t,T_n} \frac{E^{c,\$}_t[f^{\$,€}_x(T_{n-1})(1+\delta^{\$}_n L(T_{n-1},T_n))]}{f^{(\$,€)}_x(0)}, \tag{12}$$

where $f^{\$,€}_x(t)$ is the foreign exchange (FX) rate at time t representing the price of the unit amount of USD in terms of EUR. The discount factor $Z^{c,\$}_{t,T_n}$ can be calculated in a similar way as $Z^{c,€}_{t,T_n}$ is computed by following the methodology of subsection 3.2.2. This can be rewritten in the following way

$$PV_{\$} = \sum_{n=1}^{N} \delta^{\$}_n Z^{c,\$}_{t,T_n} \frac{E^{c,\$}_t[f^{\$,€}_x(T_{n-1})]}{f^{(\$,€)}_x(0)} B^{\$}(T_{n-1},T_n), \tag{13}$$

where $B^{\$}(T_{n-1},T_n)$ is the LIBOR-OIS spread defined as:

$$B^{\$}(T_{n-1},T_n) = E^{c,\$}_t[L(T_{n-1},T_n)] - \frac{1}{\delta^{\$}_n}\left(\frac{Z^{c,\$}_{t,T_{n-1}}}{Z^{c,\$}_{t,T_n}} - 1\right). \tag{14}$$

The discounting curve $Z^{c,(€,\$)}_{t,T_n}$ can be computed by setting $PV_{€} = PV_{\$}$, which should hold due to the assumption that there are no arbitrage opportunities. Hence, this leads to the following result

$$Z^{c,(€,\$)}_{t,T_n} = \frac{PV_{\$} + Z^{c,(€,\$)}_{t,T_0} - \sum_{n=1}^{N-1} \mathrm{E}^{c,€}_t[L(T_{n-1},T_n) + B_N]\delta^{€}_n Z^{c,(€,\$)}_{t,T_n}}{1 + \mathrm{E}^{c,€}_t[L(T_{N-1},T_N) + B_N]\delta^{€}_N}, \tag{15}$$

where $Z^{c,(€,\$)}_{t,T_0}$ is set to one. For longer maturities, it is possible that the FX rates are not always available. This could be resolved by following the approximation of Fujii & Takahashi (2011), which leads to the following present value based on USD

$$PV_{\$} \simeq \sum_{n=1}^{N} \delta^{\$}_n Z^{c,\$}_{t,T_n} \frac{Z^{c,(€,\$)}_{t,T_{n-1}}}{Z^{c,\$}_{t,T_{n-1}}} B^{\$}(T_{n-1},T_n). \tag{16}$$

The approximated discounted curve can be extracted by substituting equation (16) into equation (15). Note that the discount factor in equation (16) has subscribed $n-1$, meaning that $PV_{\$}$ only contains known variables, just as the $PV_{\$}$ in equation (12). The forward EURIBOR curve collateralized in USD can again be obtained by substituting the obtained discount factors in equation (9), shown in subsection 3.2.2.

3.3 Pricing with Multi-curve Framework with Embedded Optionality

This section provides the two methods that I consider when taking the CTD option into account. First, I discuss the CTD framework proposed by Fujii & Takahashi (2011). Second, I provide the methodology of the blended CTD approach. Both methods are explained by using only the USD as foreign currency, but this can be easily extended for more than one foreign currency.

3.3.1 Cheapest-to-Deliver Framework

The difference between section 3.2.2 and 3.2.3 is in what currency the collateral is paid. In section 3.2.2, the currency of the collateral is the same as the currency of the derivative payment, while this differs in section 3.2.3. It is also possible that there are multiple eligible currencies allowed in order to pay the collateral, which is captured in a multi-currency CSA. This gives rise to the Cheapest-to-Deliver option for the collateral payer. In other words, the payer has the option to post the cheapest collateral currency which can also be the base currency. Note that the collateral payer should have a negative present value in order to be the payer and not the receiver. Hence, the goal of the payer is to let the present value of the derivative contract be as large as possible with an upperbound of zero. In order to achieve this, one should make the discount factor as small as possible, which is equivalent to posting the cheapest collateral currency at every future point in time.

One way of taking this into account is by following the methodology of Fujii & Takahashi (2011). In a sense, they combine the two previous sections. More specifically, they define the optimal discount factor by minimizing the two discounting curve that are computed by using equation (8) and (15). Formally stated, this results in the following equation

$$\tilde{Z}^{ctd,(€,\$)}_{t,T_n} = min\{Z^{c,€}_{t,T_n}, Z^{c,(€,\$)}_{t,T_n}\}. \tag{17}$$

Note again that $Z^{c,€}_{t,T_n}$ is the same as the discount factor in equation (8), but the change in notation was necessary in order to clarify the different discount factors. This can be easily extended for more than two currencies. In order to do so, one can use the more general form as follows

$$\tilde{Z}^{ctd,(€,i)}_{t,T_n} = min_{i \in C}\{Z^{c,(€)}_{t,T_n}, Z^{ctd,(€,i)}_{t,T_n}\}, \tag{18}$$

where C is the set of eligible currencies $i =$ USD, GBP, JPY.

3.3.2 Blended Cheapest-to-Deliver Method

Another possibility in order to incorporate the complex CTD option that I consider in this paper is to construct a blended CTD curve. Again, I explain this approach by only considering the domestic (EUR) and foreign (USD) currency. First, one should construct the two relevant discounting (EONIA/OIS) curves given in EUR. In this case, this is the EONIA curve, which is already in EUR, and the implied USD OIS curve in EUR. After that, I pick the cheapest currency throughout the life of the swap in order to construct the blended CTD curve. Picking the cheapest currency means picking the largest discount rate, which is equivalent to the smallest discount factor.

The EONIA curve can simply be obtained by applying the methodology provided in section 3.2.2. The second curve is a bit more complicated to obtain. One possibility is to compute the USD OIS curve $Z^{c,\$}_{t,T_n}$, which can be obtained in a similar way as the EONIA curve $Z^{c,€}_{t,T_n}$, and the Forward Exchange (FX) rates in order to convert the curve into USD at time T_n and convert it back to EUR at time T_0. Then, the blended CTD curve is computed by minimizing the following discounting curves as follows

$$\hat{Z}^{ctd,(€,\$)}_{t,T_n} = min\left\{ Z^{c,€}_{t,T_n}, \frac{FX^{€\Rightarrow\$}_{t,T_n}}{FX^{€\Rightarrow\$}_{t,T_0}} Z^{c,\$}_{t,T_n} \right\}, \tag{19}$$

where $FX^{€\Rightarrow\$}_{t,T_n}$ is the FX rate at time T_n that expresses the amount of USD in terms of EUR. Furthermore, $1/FX^{€\Rightarrow\$}_{t,T_0}$ is used for the FX rate at time T_0 that expresses the amount of EUR in terms of USD, since they are due to arbitrage more or less equal to each other. This is more convenient, since in this case only the FX rates $FX^{€\Rightarrow\$}_{t,T_n}$ are needed.

The intuition behind this method is as follows. Suppose that the collateral payer has to pay a certain amount a halfyear from now and suppose that EUR is the base currency and USD is the other eligible currency. Since the notional amount is then stated in EUR, the present value of that payment can be directly computed by discounting it with the EONIA curve $Z^{c,€}_{t,T_1}$. However, it is also allowed to pay the collateral in USD. By doing so, the future payment has to be converted first into USD with the FX rate $FX^{€\Rightarrow\$}_{t,T_1}$. After that, the USD OIS curve $Z^{c,\$}_{t,T_1}$ can be used in order to discount the payment. In order to decide which of the two present values is higher, one has to convert the present value stated in USD to EUR, which can be done by dividing it by $FX^{€\Rightarrow\$}_{t,T_0}$. This yields two different ways to compute the present value of the future payment. If these two ways provide two different outcomes, one has to choose the smallest discounting curve in order to get the highest present value, which is exactly what equation (19) represents.

Note that this whole procedure is only feasible when there exists OIS (EONIA for EUR), IRS (with different tenors) and CCS markets for the relevant currencies. Furthermore, this approach can be extended for multiple foreign currencies by constructing all the implied foreign OIS curves expressed in the EUR currency and picking the cheapest collateral currency throughout the whole derivative contract. A drawback for this is that the FX rates are sometimes only available for shorter maturities. One could also use the EONIA curve obtained earlier on and Cross-Currency (CC) OIS, but then it has to be assumed that the Cross-Currency OIS market exists and is liquid enough, which is not often the case.

If these approaches are not feasible, then one could imply the FX rates for longer maturities, which results also in a curve. This FX curve can be implied by rewriting the USD/EUR CCS condition derived by Fujii et al. (2010a) in the following way

$$FX_{t,T_n}^{€\Rightarrow\$} = \frac{-Z_{t,T_0}^{c,€} + Z_{t,T_N}^{c,€} + \sum_{n=1}^{N} \delta_n^{€} Z_{t,T_n}^{c,€} (\mathrm{E}_t^{c,€}[L(T_{n-1},T_n)] + B_N)}{-Z_{t,T_0}^{c,\$} + Z_{t,T_N}^{c,\$} + \sum_{n=1}^{N} \delta_n^{\$} Z_{t,T_n}^{c,\$} \mathrm{E}_t^{c,\$}[L(T_{n-1},T_n)]}, \tag{20}$$

where B_N is the CCS basis spread between the 3m-EURIBOR and 3m-LIBOR USD rates, which is given as a market quote. Furthermore, $E_t^{c,€}[L(T_{n-1},T_n)]$ and $E_t^{c,\$}[L(T_{n-1},T_n)]$ are in this case the 3m-EURIBOR curve and 3m-LIBOR USD curve, respectively. These two curves can be constructed applying the methodology provided in section 3.2.2.

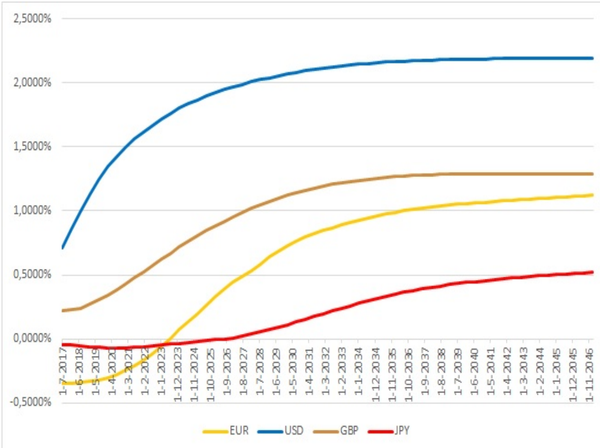
There are two main difference between this approach and the method described in section 3.3.1. The first difference is that for the CTD approach, the CSS basis spread is not needed when the FX rates are available, while these spreads are always needed for the CTD framework. This is convenient since the CCS basis spreads are not always available or not liquid enough. The second difference is that the outcome of the blended CTD approach is easier to interpret. For example, if the implied USD discounting curve in EUR is smaller than the EUR discounting curve at a particular point in time, which means that it is cheaper to post the collateral in USD than in EUR, it is easy to interpret why this is the case. For instance, the future exchange rate could be small relative to the current exchange rate or the EUR discounting rate could differ a lot from the USD discounting rate. No such statement can be made when using the method proposed by Fujii et al. (2010b) and Fujii & Takahashi (2011).

4 Preliminary Data Analysis and Historical Behavior

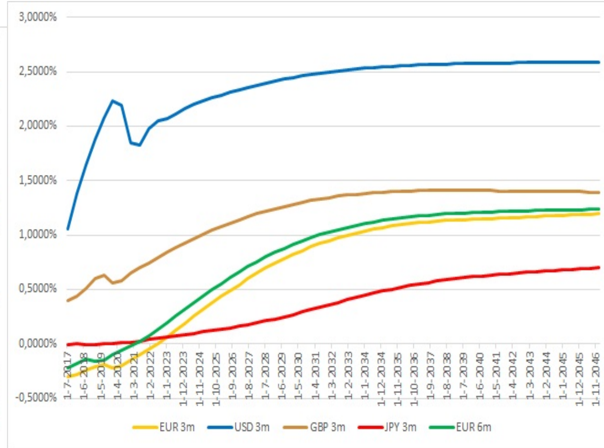
The data that is used in this research regarding the construction of the forwarding and discounting curves are all obtained from Bloomberg with a maturity of 30 years. I consider the EUR market as the domestic market. Hence, I only use the EURIBOR par swap rates for the single-curve framework with a tenor of 6 months, since this is the most commonly used market rate in swap contracts. For the multi-curve framework with only the domestic currency available, I also use the EONIA par swap rates. When allowing a foreign currency to be the collateral currency, I use the foreign OIS rates instead of the EONIA rates. Furthermore, I also use the foreign LIBOR rates with a tenor of 3 months due to its liquidity and several CCS spreads with a tenor of 3 months. The foreign markets that are applied here are USD, GBP and JPY, since these are the most common foreign collateral currencies used in practice.

Several bootstrapping and interpolation techniques can be used in order to apply the methods mentioned earlier in section 3. Ametrano & Bianchetti (2013) provide a detailed paper about multi-curve bootstrapping for several yield curves and delta sensitivities. They derived modern pricing formula's from scratch and have worked out the EUR market case regarding for example the selection of market instruments, synthetic market quotes, possible negative interest rates and the effect of OIS discounting. Furthermore, Hagan & West (2006) and Hagan & West (2008) discuss various interpolation techniques in order to get continuous yield curves. The approach that is used in this thesis is the natural cubic spline proposed by Burden & Faires (1997). There are more advanced and computational intensive spline methods (Hagan & West, 2006), but these are not considered since this is not the focus of this paper.

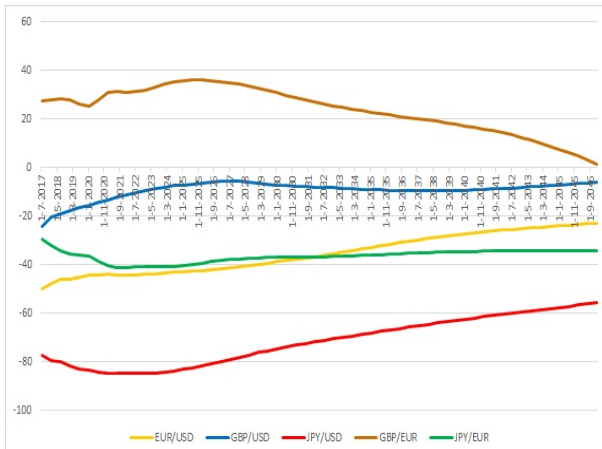
Figure 2 shows the market rates and spreads. Figures 2a and 2b represent the OIS and EURIBOR/LIBOR market rates, respectively, of the EUR, USD, GBP and JPY market. The missing rates are interpolated as mentioned before with the natural cubic spline method over the period 2017-2047. Noticeable is the fact that the OIS and EURIBOR/LIBOR rates of the USD market are remarkably higher than the other rates. The EONIA and EURIBOR are in the first five years the smallest, while the JPY OIS and 3m-LIBOR JPY are significantly the smallest after that. The market rates of the EUR and JPY market are even negative in the beginning of the period. Figure 2c shows the cross currency spreads, where the first three lines have basis currency USD and the last two lines have the EUR as the basis currency. Also for these spreads the natural spline method is used in case some spreads were not available. It is shown that all the spreads based on USD are largely negative, meaning that the USD market is relatively expensive. Especially the JPY/USD spread is largely negative over time. Even the JPY/EUR spread is quite negative, suggesting that the JPY market is probably relatively cheap. Intuitively,



(a) OIS market rates of EUR, USD, GBP and JPY.



(b) 3m/6m LIBOR market rates of EUR, USD, GBP and JPY.



(c) CCS market spreads with USD and EUR as basis currency.

Figure 2: Market rates/spreads based on EUR, USD, GBP and JPY which are interpolated over the period 2017-2047 with the natural cubic spline method.

one could conclude from these large negative spreads that it is likely that JPY provides the cheapest collateral.

Furthermore, EURIBOR/LIBOR and OIS rates have sometimes different spot lags and day count conventions for different currency markets, which is summarized in Table 1. This table shows that the spot lag, which is the amount of days between the trade date and the actual settle date, is zero for the GBP market and two for the other markets. Note that there are different sources used by Bloomberg in order to obtain the particular fixed rates. The cash source is always only used for getting the market quote at the calculation date plus the spot lag. The Future/FRA source is sometimes used for the rates in the

Currency	Rate	Source	Spot lag	Day count convention
EUR	OIS (EONIA)	Cash	2	ACT/360
	OIS (EONIA)	Swap	2	ACT/360
	EURIBOR	Cash	2	ACT/360
	EURIBOR	Future/FRA	2	ACT/360
	EURIBOR	Swap	2	30/360
USD	OIS (Fed Fund)	Cash	2	ACT/360
	OIS (Fed Fund)	Swap	2	ACT/360
	LIBOR	Cash	2	ACT/360
	LIBOR	Future	2	ACT/360
	LIBOR	Swap	2	30/360
GBP	OIS (SONIA)	Cash	0	ACT/365
	OIS (SONIA)	Swap	0	ACT/365
	LIBOR	Cash	0	ACT/365
	LIBOR	Future/FRA	0	ACT/365
	LIBOR	Swap	0	ACT/365
JPY	OIS (MUTAN)	Cash	2	ACT/365
	OIS (MUTAN)	Swap	2	ACT/365
	LIBOR	Cash	2	ACT/360
	LIBOR	FRA	2	ACT/360
	LIBOR	Swap	2	ACT/365

Table 1: Spot lags and day count conventions for the OIS and IBOR rates regarding the EUR, USD, GBP and JPY market.

first few years, while the rest of the quotes are obtained while using swaps. Furthermore, it shows that there are three types of day count conventions that needs to be considered in order to compute the day count fraction δ between two dates. The most common conventions used here are ACT/360 and ACT/365, which simply means the number of days between the two payment dates divided by 360 and 365, respectively. This results in the following (Henrard, 2012) equation

$$\delta_n = \frac{d_n - d_{n-1}}{T}, \qquad n = 1, ..., N, \quad T = 360, 365, \tag{21}$$

where $d_n - d_{n-1}$ is the number of days between the dates. The 30/360 convention is also used in the EUR and USD market. Following Henrard (2012), the day count fraction δ_n is calculated as

$$\delta_n = \frac{360(Y_n - Y_{n-1}) + 30(M_n - M_{n-1}) + D_n - D_{n-1}}{360}, \qquad n = 1, ..., N, \tag{22}$$

where $Y_n - Y_{n-1}$, $M_n - M_{n-1}$ and $D_n - D_{n-1}$ is defined as the amount of years, months and days between the dates, respectively.

When the forwarding and discounting curves are constructed, I use these to determine the present value of an anonymized portfolio containing only IRS contracts with different maturities. The present value of this portfolio is computed by subtracting the present value of the floating leg from the present value of the fixed leg in the following way

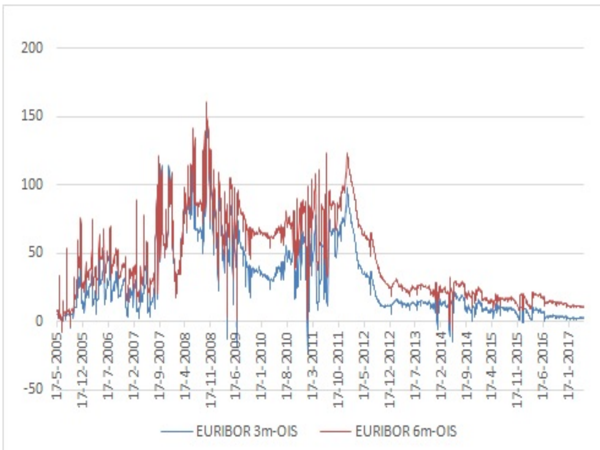
$$PV = S_{t,T_N} \sum_{n=1}^{N} \delta_{f_i,n} D_{t,T_n} - \sum_{n=1}^{N} \mathrm{E}_t[L(T_{n-1}, T_n)] \delta_{f_l,n} D_{t,T_n}, \tag{23}$$

where S_{t,T_N} is the fixed coupon rate, D_{t,T_n} is the discount factor and $\mathrm{E}_t[L(T_{n-1}, T_n)]$ is the 6m-EURIBOR rate.

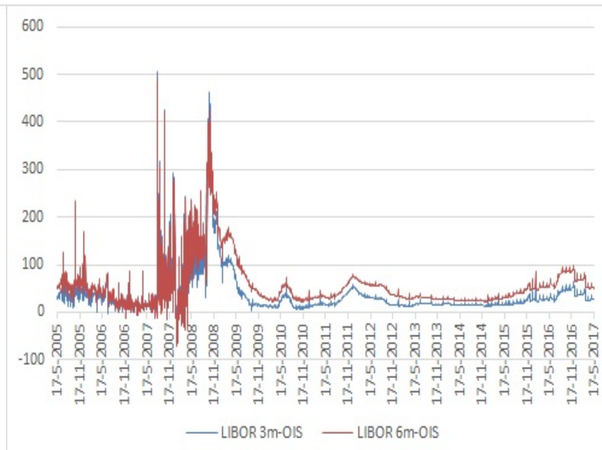
#	Type	Notional (fixed)	Calculation date	Maturity	Fixed coupon	Floating coupon
1	IRS Pay	-70,000,000	31-12-2016	10-7-2043	0.824%	6m-EURIBOR
2	IRS Pay	-110,000,000	31-12-2016	23-3-2041	2.857%	6m-EURIBOR
3	IRS Pay	-120,000,000	31-12-2016	5-10-2046	3.667%	6m-EURIBOR
4	IRS Pay	-45,000,000	31-12-2016	16-8-2045	3.843%	6m-EURIBOR
5	IRS Receive	71,000,000	31-12-2016	30-4-2038	1.037%	6m-EURIBOR
6	IRS Receive	131,000,000	31-12-2016	12-5-2037	1.043%	6m-EURIBOR
7	IRS Receive	43,000,000	31-12-2016	3-12-2043	1.119%	6m-EURIBOR
8	IRS Receive	117,000,000	31-12-2016	11-10-2039	1.125%	6m-EURIBOR
9	IRS Receive	62,000,000	31-12-2016	25-4-2036	1.138%	6m-EURIBOR
10	IRS Receive	54,000,000	31-12-2016	24-5-2036	1.148%	6m-EURIBOR
11	IRS Receive	32,000,000	31-12-2016	13-6-2036	1.174%	6m-EURIBOR
12	IRS Receive	124,000,000	31-12-2016	29-11-2035	1.205%	6m-EURIBOR
13	IRS Receive	145,000,000	31-12-2016	8-7-2045	1.423%	6m-EURIBOR
14	IRS Receive	123,000,000	31-12-2016	7-8-2035	1.579%	6m-EURIBOR
15	IRS Receive	163,000,000	31-12-2016	4-2-2036	1.640%	6m-EURIBOR

Table 2: Portfolio of several IRS contracts.

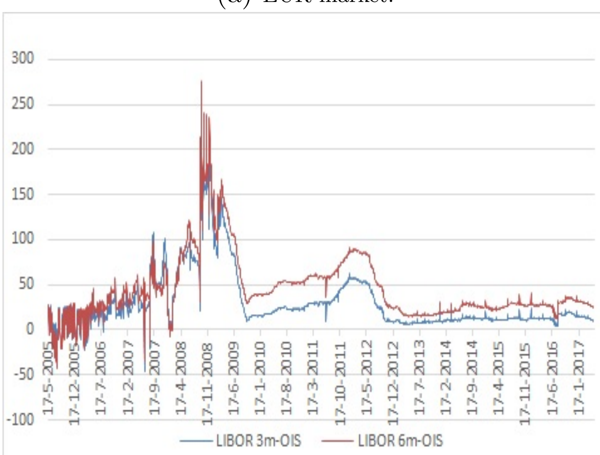
The primary details of the considered portfolio are given in Table 2. The portfolio used here contains 4 IRS contracts where the holder of this portfolio pays the fixed leg and the 11 IRS contracts where the holder receives the fixed leg. The amount of the notional is in the perspective of the fixed leg and is expressed in the EUR currency. Also, the calculation date for every contract starts at the end of 2016 and the maturities range between 2035 and 2046. The frequency of the future cash flows are semi-annual for both the fixed and floating leg. Note that both the fixed and floating coupon rate stated in the table above are yearly rates. Furthermore, the day count convention for both legs are ACT/360. Note that all the considered IRS contracts have a multi-currency CSA with EUR as base currency and USD, GBP and JPY as eligible currencies and only cash collateral is allowed.



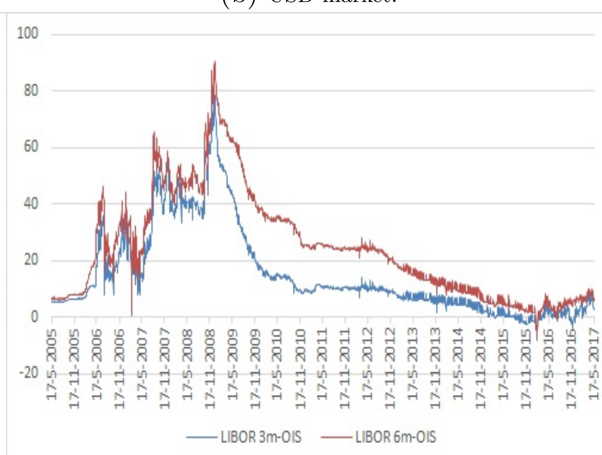
(a) EUR market.



(b) USD market.



(c) GBP market.

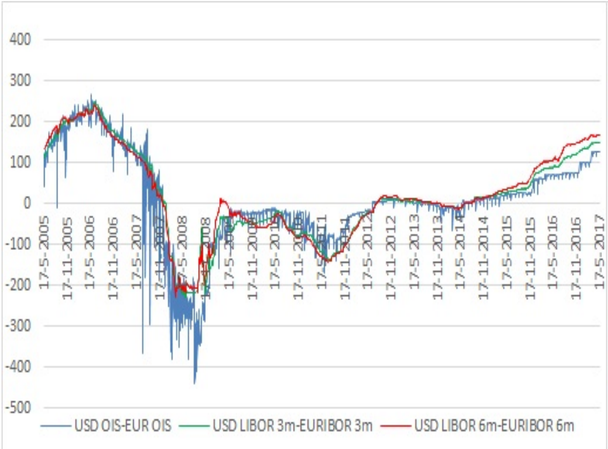


(d) JPY market.

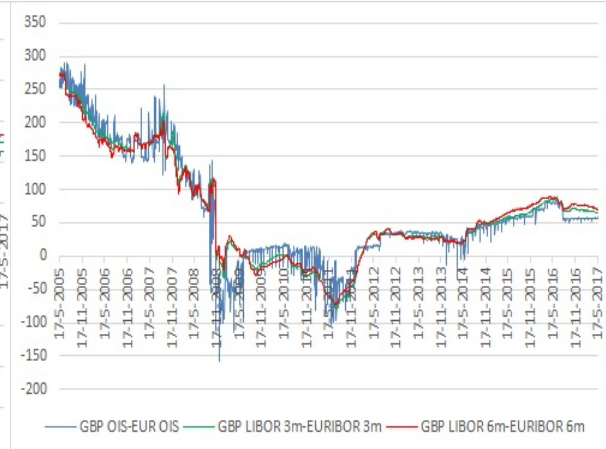
Figure 3: Historical 3m LIBOR-OIS and 6m LIBOR-OIS spreads of the EUR, USD, GBP and JPY market over the period 2005-2017.

Figure 3 shows the historical 3m and 6m LIBOR-OIS spreads for the EUR, USD, GBP and JPY market over the period 2005-2017. This figure shows that the spreads are more or less positive over the whole period for all considered market, especially during the financial crisis of 2008. Focusing on the EUR market, the spreads have been increasing since 2005 with a maximum of 156 and 161 basis points (bps) for the 3m LIBOR-OIS and 6m LIBOR-OIS, respectively. It remains after the financial crisis for quite some time relatively high, suggesting that the single-curve framework used frequently before the financial crisis needs to be adjusted as mentioned before. Since 2012, the spreads are relatively smaller than before, but are still around 15-30 bps which is too large to neglect. The USD market has even higher spreads than the EUR market, with 490 and 507 bps as the largest difference between the OIS and the 6m and 3m LIBOR, respectively. As Figure 3b shows, the spreads are recently ranging between 30-50 and 50-70 bps for the 3m and 6m LIBOR-OIS spread, respectively, meaning that these spreads are also far from neglectable. The GBP and JPY market also have a high peak around 270 and 90 bps, respectively. Noticeable is the fact that the spreads of the JPY market are slowly getting smaller with only a few bps left at the end of the period, On the other hand, the GBP spreads remain quite positive till the end.

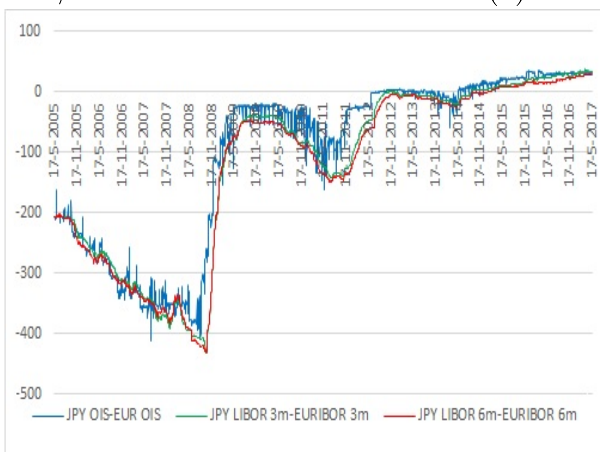
Figure 4 shows the historical CCS spreads based on the EUR market for different tenors over the same period as before. When looking at subfigure 4a, it stand out that the CCS spreads were largely positive till 2008. However, the spreads decreased drastically after that, especially for OIS. This means that the USD rates are lower than the EUR rates, which suggests that the financial crisis may have hit the USD market harder than the EUR market. However, these spreads increase relatively fast and become even positive again 2012. Since these spreads are most of the time different from zero, it is important that these spreads are included in the valuation of derivatives when USD collateral is allowed. The CCS of GBP/EUR show more or less the same pattern as before, meaning that the spreads were negative during the financial crisis and positive before and after. The only difference is that the values are less extreme. However, these spreads are still quite large and therefore not neglectable. The JPY/EUR spreads were, in contrast with the other CCS spreads, before the financial crisis largely negative and even decreased more till the financial crisis. After that, the spreads suddenly increases and sometimes become even positive, especially at the end of the period. Although these spreads are overall the smallest at the end of the period, they are also still too large (25-30 bps) in the sense that it has to be included in the valuation process.



(a) CCS of USD/EUR.



(b) CCS of GBP/EUR.



(c) CCS of JPY/EUR.

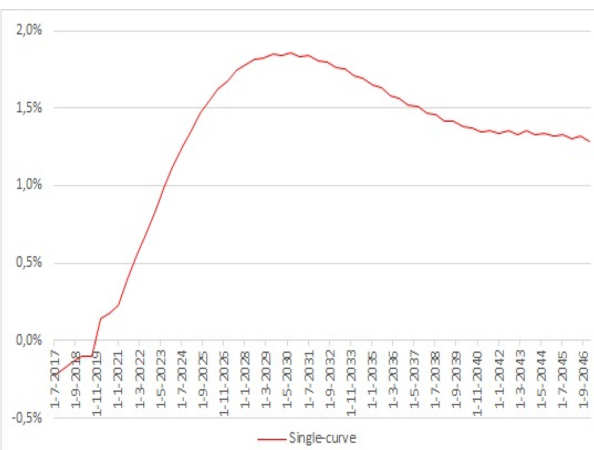
Figure 4: Historical CCS spreads of USD, GBP and JPY market for OIS, 3m LIBOR and 6m LIBOR based on EUR over the period 2005-2017.

5 Results

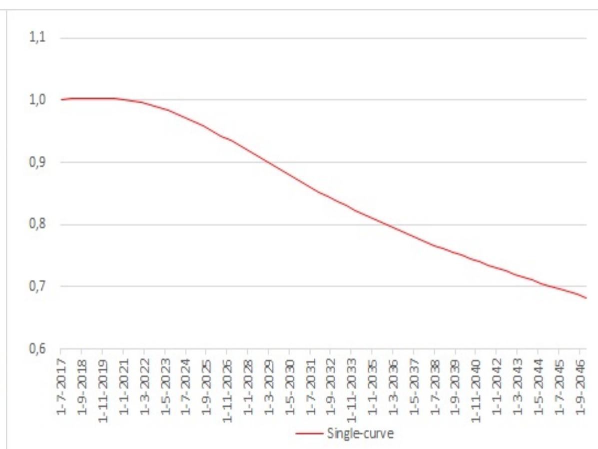
This section provides the performance of the methods provided in section 3 when applied to the data described in section 4. This section is divided into three subsections. First, I present and discuss the performance of the single-curve framework. After that, I add the multi-curve framework for a single currency (EUR) and discuss the importance of this procedure relative to the single-curve framework. Finally, I discuss the results of the three different CTD approaches in two different settings.

5.1 Performance of Single-curve Framework

Figure 5 shows the bootstrapped forwarding curve $\mathrm{E}_t^I[L(T_{n-1}, T_n)]$ and the discounting curve Z_{t,T_n} while using the single-curve framework. This forwarding and discounting curve is calculated by applying equation (2) and (4) in section 3.1, respectively, to the interpolated 6m-EURIBOR market rates described in section 4. Both the forwarding curve $\mathrm{E}_t^I[L(T_{n-1}, T_n)]$ and the discounting curve Z_{t,T_n}, which are shown in the Figures 5a and 5b, respectively, are overall quite smooth. This results from the fact that the natural cubic spline interpolation method is applied. Furthermore, Figure 5a shows that in the beginning of the period, the forward 6m-EURIBOR rate is increasing quite fast towards 2024. After that period, it is slightly descending until the beginning of 2031, where the forward rate is at a maximum of approximately 1.85%. The forward rate is decreasing in the last part of the period, which is the expected behavior of the forward 6m-EURIBOR rate. The discount factor shown in Figure 5b declines at a steady pace which is also expected. Only in the first few years, the discount factor is a bit above one, but this can be explained due to the negative swap rates in the first part of the period between 2017-2021, shown in Figure 2b in section 4.



(a) Forwarding curve



(b) Discounting curve

Figure 5: The forwarding curve $\mathrm{E}_t[L(T_{n-1}, T_n)]$ and the discounting curve Z_{t,T_n} of the EUR market using the single-curve bootstrapping method over the period 2017-2047. The forwarding and discounting curve are the result of applying equation (2) and (4), respectively, formulated in section 3.1.

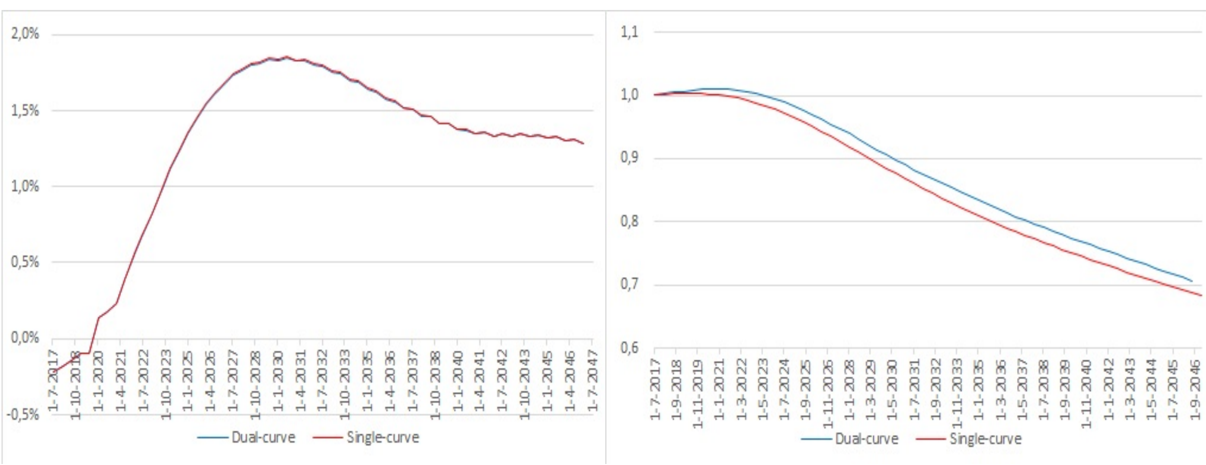
Table 3 shows the Present Value (PV) of the IRS portfolio, divided in the sum of the 4 Pay IRS PV, 11 Receive IRS PV and all the IRS PV when using the single-curve framework, the multi-curve framework and the three CTD approaches, where the latter is also divided into the perfect market information case and non-perfect market information case. Let us first focus on only the first line of the table, which is the PV of the portfolio when using the single-curve framework. Noticeable is the fact that the PV of the Pay IRS is largely negative. This can be explained by the relative large fixed rates for the last three IRS. While comparing this fixed rate with the forward 6m-EUIRBOR rate, the fixed rate is always bigger than the forward rate. Moreover, the difference between the two rates is quite high meaning that the fixed rate is at least 1%-2% higher. This means that the payer of the fixed leg receive a lot less that the payer has to pay, which results in a large negative present value. The actual PV of the last three Pay IRS are stated in table A.2, A.3 and A.4. The fact that the notional of the second and third IRS is quite big is also a factor in the big negative PV. On the other hand, the fixed rates of the Receive IRS are a lot lower relative to the Pay IRS. Hence, it makes sense that the PV of the Receive IRS is not that large compared to the Pay IRS. In total, the PV of the IRS portfolio is approximately $-€112$ mln.

	PV Pay Portfolio	PV Receive Portfolio	PV Total Portfolio
Single-curve	-142,217,195 (2.18%)	30,358,972 (1.71%)	-111,858,224 (2.31%)
Multi-curve	-145,316,318 (-)	30,878,701 (-)	-114,437,616 (-)
CTD	-138,982,667 (4.57%)	29,954,175 (3.09%)	-109,028,492 (4.98%)
CTD approx.	-138,962,647 (4.56%)	29,952,737 (3.09%)	-109,009,910 (4.96%)
CTD blended	-140,227,105 (3.63%)	30,073,091 (2.68%)	-110,154,014 (3.89%)
CTD non-perfect	-138,622,251 (4.85%)	32,067,362 (3.71%)	-106,554,889 (7.42%)
CTD approx. non-perfect	-138,601,101 (4.83%)	32,069,543 (3.71%)	-106,531,558 (7.40%)
CTD blended non-perfect	-139,933,879 (3.85%)	31,953,642 (3.36%)	-107,980,237 (5.98%)

Table 3: PV of a portfolio of fifteen IRS contracts expressed in EUR while using the single-curve framework, multi-curve framework with one currency and three CTD frameworks in case of perfect and non-perfect market information. The methodology of the single-curve framework and multi-curve framework with a single currency are presented in section 3.1 and 3.2.2, respectively. The CTD and approximated CTD framework is described in section 3.3.1, while the blended CTD framework is provided in section 3.3.2. The percentual difference of the PV with respect to the multi-curve framework is given in brackets.

5.2 Performance of Multi-curve Framework

Figure 6 represents the forwarding and discounting curve based on the single-curve framework and the multi-curve framework with a single currency obtained from section 3.1 and 3.2.2, respectively. Noticeable from Figure 6a is the fact that the two forwarding curves resulting from equation (2) and (9) look very similar. The difference between the two curves are most of the time at maximum 1 bps, which is more or less neglectable. On the other side, the two discounting curves shown in Figure 6b are a bit more deviating from each other relative to the two forwarding curves. They start both naturally from the same place, since the discount factor is one at the beginning of the period. After that, the figure shows that the discounting curve based on the multi-curve framework is at every point in time approximately 2%-3% larger than the discounting curve based on the single-curve framework. This results in consistently underestimating the discount factor, which suggests that the single-curve approximation overestimates or underestimates the valuation of derivative contracts when the present value is negative or positive, respectively. In other words, a negative present value is valued smaller and a positive present value is valued larger than it is in reality. Hence, this result suggests that the single-curve framework is outdated and should be replaced by the multi-curve framework.



(a) Forwarding curves (b) Discounting curves

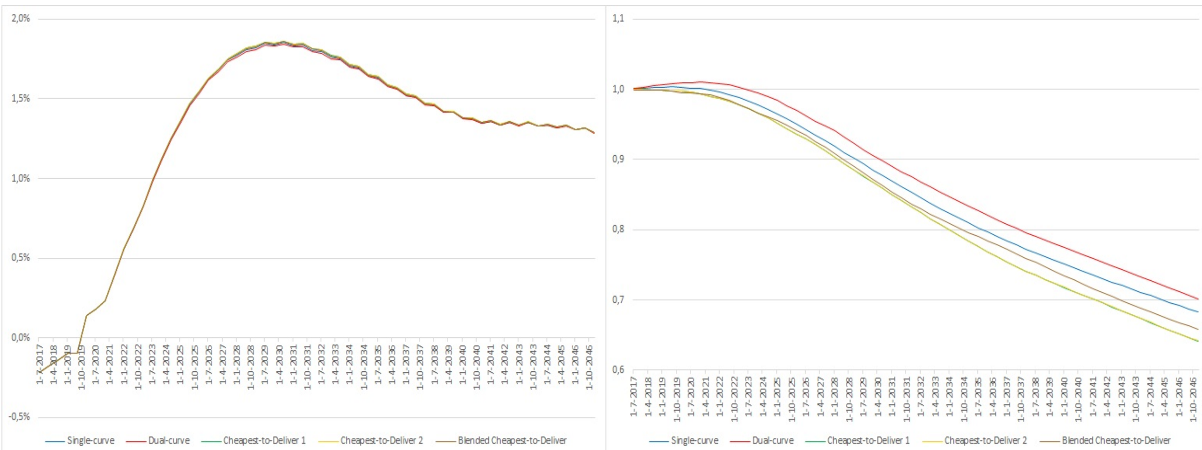
Figure 6: The forwarding and discounting curves of the EUR market using single-curve and dual-curve bootstrapping over the period 2017-2047. Applying equation (9) and (8) are the result of the forwarding and discounting curve based on the multi-curve framework with one currency, which can be found in section 3.2.2

The second line of Table 3 provided in the previous section shows the PV of the Pay

IRS, Receive IRS and Total IRS portfolio based on the multi-curve framework with a single currency which methodology is given in section 3.2.2. What immediately stands out from this table is the PV of the Pay IRS portfolio. The PV is more than €3 mln (2.18%) smaller when applying the multi-curve framework compared to using the single-curve framework. On the other hand, the difference between the two PV of the Receive portfolio is much smaller, which is more or less €0.5 mln (1.71%). This can be explained by the type of IRS that is valued in this thesis. In particular, the amount of the notional and the difference between the fixed and floating rate play an important role in the largeness of the difference between the two valuation methods. For example, the notional of Pay IRS nr.3 and nr.4 is −€120 mln and −€45 mln, respectively. When looking at the difference of the PV of these two IRS, which are shown in Table A.3 and A.4, the difference of the PV between the two frameworks is for Pay IRS nr.3 is much larger than for Pay IRS nr.4, which are more or less €1.7 mln and €0.7 mln, respectively. The difference is even smaller for the Pay IRS nr.1 (approximately €0.1 mln), which PV is stated in Table A.1 due to the lower difference in the fixed and floating rate compared to Pay IRS nr.4. Nonetheless, the PV of the whole IRS portfolio is much lower when using the multi-curve framework compared to the PV when using the single-curve framework. Furthermore, the fourth columns in Tables A.1-A.15 provide the difference in PV relative to the multi-curve framework as a percentage of its notional amount, which is defined here as the notional spread. In other words, the PV of the single-curve framework as a percentage of its notional amount plus the notional spread is equal to the PV of the multi-curve framework as a percentage of its notional amount. For the most IRS, it is shown that this spread ranges between 0.02%-0.2%, which shows again that the difference in PV is not neglectable. There are even some IRS where the spread is larger than 1% due to the relative large fixed rates for these IRS. Hence, the single-curve framework is for this portfolio not longer a good approximation of the multi-curve framework with a single currency.

5.3 Performance with Cheapest-to-Deliver Option

In this subsection I first discuss the outcome of the forwarding and discounting curves while using the different CTD approaches. After that, I distinguish to cases when applying the CTD approaches. In the first case, I simply assume that both parties in the swap contract have full knowledge of these methods and therefore both post the cheapest collateral in case they are the collateral payer. In the second case I assume that the counterparty does not possess this knowledge how to apply these methods. Hence, the counterparty follows the multi-curve framework with one currency, meaning that they simply post the EUR collateral currency when they are the collateral payer. The reason why this distinction is made is because of the fact that not every company knows how to correct for multi-currency CSA's and therefore ignore the CTD option.



(a) Forwarding curves (b) Discounting curves

Figure 7: Forwarding and discounting curves of the EUR market using single-curve and dual-curve bootstrapping with a single currency and multiple currencies over the period 2017-2047. Also, the blended CTD method is included. The forwarding curve is calculated similar to the previous section. The first discounting curve of the CTD framework is computed by using equation (18), which can be found in section 3.3.1. The second curve uses the same equation, but replaces equation (12) by the approximated equation (16). The blended CTD curve is calculated by applying equation (19) stated in section 3.3.2.

Figure 7 illustrates all the forwarding and discounting that are considered and explained earlier on in this thesis. When looking at Figure 7a, it is shown that again all the forwarding curves are very similar to each other. Also in this case, the differences between the curves are at most 1-2 bps. Moreover, the pattern of the forwarding curves of the CTD approaches are even in the first two years very similar to the multi-curve framework with only one currency. The discounting curves shown in Figure 7b are yet again less similar to each other in comparison with the forwarding curves. Recall that the first discounting curve of the CTD framework is computed by using equation (18), which can be found in section 3.3.1. The second curve uses the same equation for calculating the discounting curve, but replaces equation (12) by equation (16). These two curves are only in the first year slightly higher than one. After that, they decline relatively fast compared to the multi-curve framework with a single currency. Overall, these two CTD curves are always far below the multi-curve framework curves, with the largest difference at the end of the period around 9%. This suggests that posting the cheapest collateral yields a far lower PV when it is negative, which is what the collateral payer wants. When comparing the two CTD curves with each other, it is noticeable that they are very similar to each other, meaning that the first curve is approximated quite well with the second curve. This means that the FX rates are not even needed when building this curve, which could be quite convenient if the FX rates are not available, which is likely for some currencies for longer maturities. When looking at the blended CTD curve, which is calculated by applying equation (19) in section 3.3.2, it is illustrated that this curve has in the beginning of the period the pattern as the other two CTD curves. After that, the blended CTD curve becomes a bit larger than the other two CTD curves, but the difference is at most 2% which suggests that the difference in PV when valuating the IRS portfolio is probably not that much. However, the blended CTD curve is also quite lower than the discounting curve of the multi-curve framework with only one currency, with the largest difference being approximately 7%. Hence, this suggests that the blended CTD curve also probably provides a lower PV when it is negative relative to the multi-curve framework, meaning that it pays off not to ignore the CTD option when it is available in a CSA contract.

5.3.1 Case of Perfect Market Information

The third and fourth line of Table 3 given in section 5.1 shows the PV of the Pay IRS, Receive IRS and Total IRS portfolio based on the CTD framework and approximated CTD framework which methodology is given in section 3.3.1. The fifth line shows the PV of the IRS portfolio's based on the blended CTD framework stated in section 3.3.2. These results confirm what is suggested in section 5.3.

All the three CTD approaches provide a much higher PV relative to the multi-curve framework with one collateral currency. The biggest difference is again present for the Pay IRS portfolio. The PV is for that portfolio approximately €5-6 mln (3.6%-4.6%) larger than for the multi-curve PV, which is a big improvement. The PV of the Receive portfolio is in contrast to the Pay portfolio smaller than the PV of the multi-curve framework. This makes sense because it is assumed that the counterparty also post the cheapest collateral. Hence, the PV is lower when the counterparty is most of the time the collateral payer, which was probably the case here. Note that the importance of posting the cheapest collateral is also dependent of the type of IRS, because its effect is smaller when the notional is relatively small. Overall, the PV of the portfolio is more or less €4-5 mln larger when using one of the CTD methods, which is a difference of approximately 4-5%. When looking again at Tables A.1-A.15, it is shown that overall the notional spreads ranges between 0.04%-0.4%. There are even two IRS contracts where the notional spread is around 2.4%-3%, depending on the CTD framework. Hence, the option to post the cheapest currency collateral should be exploited instead of being ignored.

Note that the three CTD frameworks have quite similar outcomes. This result strengthens posting the cheapest collateral, since it proves that the PV is not accidentally much lower for this portfolio, but is holds for multiple CTD approaches. Furthermore, the performances of the three CTD frameworks are more or less quite similar. It only differs a bit for the Pay IRS portfolio when the difference between the fixed and floating rate is high and the notional amount is high, but overall this is neglectable. Hence, in this case it does not matter which CTD framework is used. On the other hand, it still could be beneficial to formally test which CTD framework performs better under different circumstances. One way of testing this is by using simulation processes in different kinds of settings, but this is left for further research. However, if there are insufficient CCS market spreads available, the first two CTD frameworks are not feasible anymore. Also, the blended CTD method is the easiest method to interpret the results from and it is less computational intensive. Therefore, the blended CTD framework is in general preferred over the other two CTD frameworks.

5.3.2 Case of Non-perfect Market Information

Until now I have assumed that the counterparty also knows what the cheapest collateral is. However, this is in reality not always applicable since very few or maybe none know how to apply it. Instead, they still often post collateral in EUR for simplicity reasons. Hence, I assume from now on that if the other party still pays the collateral in EUR if this party is the collateral payer and therefore uses the multi-curve framework for one currency. If the party who holds this IRS portfolio is the collateral payer, then it is payed with the cheapest collateral over time, which could be EUR but also USD, GBP or JPY.

The last three lines of Table 3 in section 5.1 show the PV of the Pay IRS, Receive IRS and Total IRS portfolio based on the same three CTD frameworks as in section 5.3.1, but applied to the non-perfect market information case. Noticeable is the fact that the PV of the Receive IRS portfolio is much larger when the counterparty does not post the cheapest collateral if they are the collateral payer. The difference for all the CTD frameworks is around €2 mln. This is mostly due to the last four Receive IRS, which PV is shown in the tables A.12-A.15, which have a relative larger notional amount and larger fixed rate.

Even the PV of the Pay IRS portfolio is a bit higher. When looking more closely at the individual Pay IRS, which are stated in the Appendix, it is shown that only the first IRS increases in value. This can be explained by the fact that the counterparty only posts collateral for the first IRS due to the relative low fixed rate, which results in an increase in PV. The order fixed rates of the Pay IRS are at every point in time higher than the floating rates, which means that only the party that holds this portfolio posts collateral. Hence, the PV is the same as for the case with perfect market information.

In total, the non-perfect market information case shows substantial differences in PV relative to the perfect market information case, which is around €2-2.5 mln. This shows that the impact of using one of the CTD approaches when valuating derivatives can be even bigger when the counterparty does not possess the same knowledge about the CTD methods. Again, the difference in performance of the three CTD frameworks is neglectable in this scenario. Especially the Receive IRS portfolio has similar valuation outcomes.

6 Conclusion and Further Research

This thesis considers three kinds of valuation frameworks in order to value a portfolio of IRS. First, a single-curve framework stated in section 3.1 which approximates the discounting curve by using the same EURIBOR rates applied for the forwarding curves. Second, a multi-curve framework with only one currency described in section 3.2.2 which uses the collateral (EONIA) rates and EURIBOR rates for constructing the discounting and forwarding curve, respectively. Third, a CTD framework given in section 3.3 that takes into account which currency is the cheapest for posting collateral instead of always choosing the base currency. Three types of CTD frameworks are considered in a perfect and a non-perfect market information setting. The first and second method use the PV based on the relevant currencies by means of the CCS market given in section 3.3.1. The first method also applies FX rates, where the second uses an approximation for this. The third method is based on a blended CTD curve provided in section 3.3.2.

I report three key findings. First, choosing the cheapest collateral largely influences the valuation process of IRS. All three CTD frameworks show that its discounting curve is consistently lower than the discounting curve of the multi-curve framework, while the forwarding curve remains more or less the same. Therefore, the PV of an IRS portfolio becomes higher when its negative and lower when the PV is positive. Second, the impact of the considered CTD methods depends on the knowledge of the counterparty and the type of IRS. When the counterparty post collateral based on EUR instead of the cheapest collateral, the PV of the IRS portfolio is higher. Also, the size of the notional amount and difference between the fixed rate and EURIBOR rate influences how much the PV changes. Third, the single-curve framework highly mis-prices the IRS portfolio in terms of PV relative to the multi-curve framework with only one currency. Hence, the single-curve framework provides a bad approximation for the discounting curve and is therefore outdated.

There are multiple directions to consider for further research. For example, it is assumed that both parties possess all the eligible currencies for posting collateral. However, it is possible that the cheapest currency is not available for the particular party, which makes the CTD method unfeasible. For instance, suppose that the USD is the cheapest collateral to post, but the party that has to pay collateral does not have enough dollars. In that case, it could be better the source for the cheapest currency. On the other hand, it could also be better to choose the second cheapest currency, which is for example EUR. Another direction is to relax particular assumptions regarding CSA contracts of derivatives. For example, it is assumed in this paper that only cash collateral is allowed. However, it is also possible that debt obligation issued by governments of certain countries are allowed as collateral. Also, for simplicity reasons it is assumed that there is no threshold specified in the considered CSA's, which is in reality not always the case. Furthermore, I only consider USD, GBP and JPY as eligible currencies. This could be extended by including other currencies or excluding the currencies that are allowed here. The choice of eligible currencies could have a large impact on the outcome of valuating derivatives when one of the CTD methods are applied. Furthermore, only IRS are considered in this paper. This could be extend by valuating other types of derivatives, such as Overnight Index Swaps, Tenor Swaps, Cross Currency Swaps and different types of Swaptions. Lastly, it still could be an improvement to this thesis to formally test which CTD framework performs better, if there is any. One way of doing this is by apply these frameworks on simulation processes under different circumstances. In this way, one gets a better feeling for the differences between the frameworks in a clean laboratory setting.

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A Appendix

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	-13,870,800	19,960,042	6,089,242 (8.70)	0.198%
Multi-curve	-14,168,783	20,396,409	6,227,626 (8.90)	-
CTD	-13,572,215	19,511,081	5,938,866 (8.48)	0.414%
CTD approx.	-13,570,551	19,508,559	5,938,008 (8.48)	0.413%
CTD blended	-13,686,365	19,681,287	5,994,921 (8.56)	0.332%
CTD non-perfect	-14,022,613	20,321,895	6,299,282 (9.00)	-0.103%
CTD approx. non-perfect	-14,022,074	20,321,629	6,299,555 (9.00)	-0.102%
CTD blended non-perfect	-14,050,004	20,338,151	6,288,147 (8.98)	-0.087%

Table A.1: The first three columns provide the PV of Pay IRS nr.1 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	-69,635,818	28,685,671	-40,950,147 (37.23)	0.774%
Multi-curve	-71,095,350	29,293,665	-41,801,685 (38.00)	-
CTD	-68,264,143	28,111,204	-40,152,939 (36.50)	1.502%
CTD approx.	-68,257,728	28,108,530	-40,149,198 (36.50)	1.499%
CTD blended	-68,780,363	28,327,130	-40,453,233 (36.78)	1.226%
CTD non-perfect	-68,264,143	28,111,204	-40,152,939 (36.50)	1.502%
CTD approx. non-perfect	-68,257,728	28,108,530	-40,149,198 (36.50)	1.499%
CTD blended non-perfect	-68,780,363	28,327,130	-40,453,233 (36.78)	1.226%

Table A.2: The first three columns provide the PV of Pay IRS nr.2 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	-114,869,818	37,806,662	-77,063,155 (64.22)	1.434%
Multi-curve	-117,439,567	38,655,506	-78,784,061 (65.65)	-
CTD	-112,029,558	36,851,810	-75,177,748 (62.64)	3.016%
CTD approx.	-112,010,433	36,845,373	-75,165,060 (62.65)	3.005%
CTD blended	-113,125,248	37,218,387	-75,906,861 (63.26)	2.398%
CTD non-perfect	-112,029,558	36,851,810	-75,177,748 (62.64)	3.016%
CTD approx. non-perfect	-112,010,433	36,845,373	-75,165,060 (62.65)	3.005%
CTD blended non-perfect	-113,125,248	37,218,387	-75,906,861 (63.26)	2.398%

Table A.3: The first three columns provide the PV of Pay IRS nr.3 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	-44,011,003	13,717,868	-30,293,135 (67.32)	1.478%
Multi-curve	-44,982,141	14,023,943	-30,958,198 (68.80)	-
CTD	-42,975,265	13,384,418	-29,590,847 (65.75)	3.048%
CTD approx.	-42,968,692	13,382,294	-29,586,398 (65.76)	3.039%
CTD blended	-43,373,989	13,512,056	-29,861,932 (66.36)	2.436%
CTD non-perfect	-42,975,265	13,384,418	-29,590,847 (65.75)	3.048%
CTD approx. non-perfect	-42,968,692	13,382,294	-29,586,398 (65.76)	3.039%
CTD blended non-perfect	-43,373,989	13,512,056	-29,861,932 (66.36)	2.436%

Table A.4: The first three columns provide the PV of Pay IRS nr.4 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	14,607,875	-16,415,456	-1,807,581 (2.55)	0.052%
Multi-curve	14,897,168	-16,741,646	-1,844,478 (2.60)	-
CTD	14,354,548	-16,124,603	-1,770,055 (2.49)	0.105%
CTD approx.	14,353,705	-16,123,634	-1,769,929 (2.49)	0.105%
CTD blended	14,439,656	-16,222,270	-1,782,614 (2.51)	0.087%
CTD non-perfect	14,432,043	-16,145,965	-1,713,923 (2.41)	0.184%
CTD approx. non-perfect	14,431,194	-16,144,994	-1,713,801 (2.41)	0.184%
CTD blended non-perfect	14,518,459	-16,243,928	-1,725,469 (2.43)	0.168%

Table A.5: The first three columns provide the PV of Receive IRS nr.5 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	26,021,286	-28,882,866	-2,861,581 (2.18)	0.042%
Multi-curve	26,525,308	-29,441,917	-2,916,609 (2.23)	-
CTD	25,591,471	-28,393,704	-2,802,233 (2.14)	0.087%
CTD approx.	25,590,211	-28,392,265	-2,802,054 (2.14)	0.087%
CTD blended	25,724,912	-28,545,723	-2,820,812 (2.15)	0.073%
CTD non-perfect	25,736,806	-28,434,145	-2,697,338 (2.06)	0.168%
CTD approx. non-perfect	25,735,535	-28,432,703	-2,697,168 (2.06)	0.167%
CTD blended non-perfect	25,872,609	-28,586,721	-2,714,112 (2.07)	0.155%

Table A.6: The first three columns provide the PV of Receive IRS nr.6 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	11,513,880	-12,307,909	-794,028 (1.85)	0.044%
Multi-curve	11,766,692	-12,579,669	-812,977 (1.89)	-
CTD	11,257,660	-12,034,215	-776,555 (1.81)	0.085%
CTD approx.	11,256,192	-12,032,661	-776,469 (1.81)	0.085%
CTD blended	11,355,727	-12,139,167	-783,440 (1.82)	0.069%
CTD non-perfect	11,443,658	-12,133,752	-690,094 (1.60)	0.287%
CTD approx. non-perfect	11,442,834	-12,132,520	-689,686 (1.60)	0.286%
CTD blended non-perfect	11,504,208	-12,217,016	-712,808 (1.66)	0.233%

Table A.7: The first three columns provide the PV of Receive IRS nr.7 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	27,655,333	-28,806,570	-1,151,237 (0.98)	0.024%
Multi-curve	28,220,254	-29,399,381	-1,179,127 (1.01)	-
CTD	27,143,137	-28,263,816	-1,120,678 (0.96)	0.050%
CTD approx.	27,141,081	-28,261,646	-1,120,565 (0.96)	0.050%
CTD blended	27,329,031	-28,460,423	-1,131,391 (0.97)	0.041%
CTD non-perfect	27,279,687	-28,297,978	-1,018,291 (0.87)	0.138%
CTD approx. non-perfect	27,277,620	-28,295,805	-1,018,185 (0.87)	0.138%
CTD blended non-perfect	27,467,998	-28,495,073	-1,027,075 (0.88)	0.130%

Table A.8: The first three columns provide the PV of Receive IRS nr.8 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	12,889,609	-12,802,085	87,523 (0.14)	-0.001%
Multi-curve	13,132,089	-13,045,333	86,755 (0.14)	-
CTD	12,689,864	-12,601,162	88,702 (0.14)	-0.003%
CTD approx.	12,689,339	-12,600,635	88,704 (0.14)	-0.003%
CTD blended	12,745,422	-12,657,066	88,356 (0.14)	-0.003%
CTD non-perfect	12,763,849	-12,619,725	144,125 (0.23)	-0.093%
CTD approx. non-perfect	12,763,318	-12,619,196	144,121 (0.23)	-0.093%
CTD blended non-perfect	12,820,681	-12,675,887	144,794 (0.23)	-0.094%

Table A.9: The first three columns provide the PV of Receive IRS nr.9 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	11,315,555	-11,237,698	77,857 (0.14)	0.144%
Multi-curve	11,529,899	-11,451,094	78,805 (0.15)	-
CTD	11,138,542	-11,059,209	79,333 (0.15)	-0.001%
CTD approx.	11,138,072	-11,058,737	79,336 (0.15)	-0.001%
CTD blended	11,188,310	-11,109,401	78,909 (0.15)	-0.000%
CTD non-perfect	11,221,738	-11,090,127	131,610 (0.24)	-0.098%
CTD approx. non-perfect	11,221,308	-11,089,685	131,623 (0.24)	-0.098%
CTD blended non-perfect	11,267,628	-11,137,156	130,472 (0.24)	-0.096%

Table A.10: The first three columns provide the PV of Receive IRS nr.10 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	6,852,320	-6,690,157	162,164 (0.51)	0.010%
Multi-curve	6,982,672	-6,817,203	165,469 (0.52)	-
CTD	6,744,501	-6,583,225	161,276 (0.50)	0.013%
CTD approx.	6,744,213	-6,582,940	161,273 (0.50)	0.013%
CTD blended	6,775,007	-6,613,463	161,544 (0.50)	0.012%
CTD non-perfect	6,814,245	-6,615,490	198,754 (0.62)	-0.104%
CTD approx. non-perfect	6,814,023	-6,615,244	198,778 (0.62)	-0.104%
CTD blended non-perfect	6,838,272	-6,641,665	196,607 (0.61)	-0.097%

Table A.11: The first three columns provide the PV of Receive IRS nr.11 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	26,665,115	-24,884,396	1,780,719 (1.44)	0.023%
Multi-curve	27,163,256	-25,354,222	1,809,034 (1.46)	-
CTD	26,259,320	-24,507,347	1,751,974 (1.41)	0.046%
CTD approx.	26,258,301	-24,506,408	1,751,894 (1.41)	0.046%
CTD blended	26,366,880	-24,607,019	1,759,862 (1.42)	0.040%
CTD non-perfect	26,498,666	-24,608,387	1,890,279 (1.52)	-0.066%
CTD approx. non-perfect	26,497,835	-24,607,573	1,890,262 (1.52)	-0.066%
CTD blended non-perfect	26,587,519	-24,695,113	1,892,405 (1.53)	-0.067%

Table A.12: The first three columns provide the PV of Receive IRS nr.12 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	52,587,504	-44,107,295	8,480,209 (5.85)	0.117%
Multi-curve	53,741,841	-45,091,904	8,649,937 (5.97)	-
CTD	51,360,236	-43,033,089	8,327,147 (5.74)	0.223%
CTD approx.	51,352,490	-43,026,264	8,326,227 (5.74)	0.223%
CTD blended	51,832,804	-43,443,800	8,389,005 (5.79)	0.180%
CTD non-perfect	52,519,137	-43,552,728	8,966,408 (6.18)	-0.219%
CTD approx. non-perfect	52,514,989	-43,547,070	8,967,919 (6.18)	-0.218%
CTD blended non-perfect	52,777,493	-43,877,921	8,899,571 (6.14)	-0.172%

Table A.13: The first three columns provide the PV of Receive IRS nr.13 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	34,777,791	-24,423,050	10,354,741 (8.42)	0.148%
Multi-curve	35,409,750	-24,872,491	10,537,259 (8.57)	-
CTD	34,266,795	-24,044,434	10,222,362 (8.31)	0.256%
CTD approx.	34,265,560	-24,043,525	10,222,035 (8.31)	0.256%
CTD blended	34,396,946	-24,140,798	10,256,148 (8.34)	0.229%
CTD non-perfect	34,953,579	-24,401,247	10,552,332 (8.58)	-0.012%
CTD approx. non-perfect	34,952,886	-24,400,606	10,552,280 (8.58)	-0.012%
CTD blended non-perfect	35,021,853	-24,465,112	10,556,741 (8.58)	-0.016%

Table A.14: The first three columns provide the PV of Receive IRS nr.14 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.

	PV Fixed Leg	PV Floating Leg	PV of IRS	Notional Spread
Single-curve	48,969,066	-32,938,878	16,030,187 (9.83)	0.168%
Multi-curve	49,872,796	-33,568,162	16,304,634 (10.00)	-
CTD	48,229,324	-32,436,422	15,792,902 (9.69)	0.314%
CTD approx.	48,227,425	-32,435,141	15,792,285 (9.69)	0.314%
CTD blended	48,429,802	-32,572,277	15,857,525 (9.73)	0.274%
CTD non-perfect	49,244,674	-32,941,175	16,303,499 (10.00)	-0.001%
CTD approx. non-perfect	49,243,722	-32,940,324	16,303,398 (10.00)	-0.001%
CTD blended non-perfect	49,338,537	-33,026,022	16,312,515 (10.01)	-0.005%

Table A.15: The first three columns provide the PV of Receive IRS nr.15 expressed in EUR when using the single-curve framework, multi-curve framework and CTD approaches for perfect and non-perfect information. The PV as a percentage of its notional amount is given in brackets. The fourth column provides the difference in these percentages relative to the multi-curve framework, denoted as the notional spread.