EMPLOYEE TRAINING AND DEVELOPMENT: TOKEN OF TRUST OR SIGN OF INCOMPETENCE?

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ABSTRACT

Training and development of employees has become a key practice in modern human resources management, as empirical data indicates positive correlations of training and development on employee motivation and productivity. This thesis proposes that training and development may not have a unilaterally positive effect on employee motivation and productivity, but could also give rise to adverse effects given the circumstances under which the worker is provided with training. We propose a rank-order tournament model in which the relationship of training, worker motivation and worker productivity is examined. By extending this model with a discretionary training decision for the supervising manager we find that the marginal effect of training can be ambiguous for both worker motivation and productivity.

Keywords: Training, differentiation, principal-agent, tournament, manager

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[2]Writing a micro-economic thesis by constructing a model has proven to be challenging to me. I feel very grateful that I was allowed to write a thesis about some of my biggest interests while studying economics: game theory, organizations and human capital. I would like to thank Jurjen Kamphorst for his guidance, as I could not have accomplished writing this thesis without the many insights and suggestions he provided me with during the writing process of this thesis.

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I. Introduction

Investing in human capital through the training and development of employees is a process in which employees learn and improve their skills, resulting in a higher marginal productivity of the employee. Human capital investments have been known to give rise to a broad variety of economic phenomena (Becker, 1962). It has been suggested that the discrete decision of training is always a costly affair, as the demand for training would otherwise be limitless. Therefore, employers looking to improve the performance of their employees are confronted with the dilemma of providing potential, but costly training to the employees, or preserving these costs by abstaining from such training policies. The opportunity costs of training could find an alternative use by the manager, which might be more enticing to the manager over the uncertain and long-term effects of training that could be spurious and vary in time and in employee-specific characteristics (e.g. the employee being already familiar with the skills being lectured). These peculiar features of training and development have given rise to extensive research in multiple academic fields in order to uncover the many underlying dynamics of investing in human capital. Where theoretical literature predicts that training has positive effects on marginal productivity, this seems to be confirmed by empirical works that show a positive correlation to worker performance and worker motivation and training and development in the firm's human capital in the financial sector (Huselid, 1995). However, the causal relationship between training and performance has not been fully uncovered yet (Noe et al., 2014).

In this thesis I will observe the effects of human capital investments in a competitive environment by applying a principal-agent analysis. By implementing a costly training decision for a manager in a tournament game where two workers compete for a prize (e.g. promotion), I will illustrate how training may give rise to both beneficial and detrimental effects on worker performance. These effects will be shown to affect the decisions of the management with regards to the training of employees. This thesis will contribute to the academic literature by considering human capital investments in tournaments as managerial strategies in a game-theoretical setting. The predictions of this model will provide insights in plausible causality in the relationship of training on worker motivation and productivity.

The subject of worker motivation has already been explored extensively in the discipline of behavioural economics. Bénabou and Tirole (2003) model contributions from the psychological literature and provide explanations for the relationship of motivation and productivity by applying these psychological findings in a principal-agent analysis. One of the driving forces behind this relationship is the phenomenon of the 'looking-self glass', which was discovered in several lab studies and field experiments by psychologists. The concept of the 'looking-self glass' is the formation of self-images by the workers, in which their self-image is constructed by reasoning from an expected point-of-view by their peers. This 'looking-self glass' was introduced as a mechanism in a principal-agent analysis in which the players could update their expected beliefs about their abilities as a result of shifts in the information they possess. The main contribution of this mechanism is that low-powered incentives (such as performance appraisals or encouragement by the manager) can highly affect the motivation of employees, as it empowers employees to exert more effort by boosting their self-confidence.

In this thesis the ‘looking-self glass’ will be applied as a mechanism in order to determine the effects of training on employee motivation and productivity. As already stated before I pose the hypothesis that training could give rise to both beneficial and detrimental effects in the motivation and self-confidence of workers in competitive working environments. When performance directly affects of the remuneration of the workers, then any visible imperfections or inequalities in the measurement of the perceived performances will influence the perceived competitiveness of the workers. Especially in organizations where real performance cannot be perfectly verified because of imperfect performance measures, the effect of low-powered incentives could be weakened or even be negative in competitive working environments (Baker et al., 1988). Subjective performance measurement could be a solution for this problem, by delegating a discretionary responsibility for investment decisions in human capital to the direct management of the workers. Bol (2008) finds that subjective performance appraisals by the direct supervisor in determining the eligibility for training, could offer a solution for imperfect measurement of performance.

Subjective performance appraisals and discretionary training decisions have been shown to give rise to strategic behaviour by both managers or workers. Contributions from both psychological and management literature suggest that the ‘looking-self glass’ could give rise to adverse effects in the behaviour of workers and managers in the presence of perceived unfairness and evaluation imprecision. Napier and Latham (1986) finds that employees overwhelmingly hold the opinion that their direct supervisor should conduct performance appraisals, as the direct supervisor can judge their performance on their observed day-to-day performance.

However, fear of the consequences of incorrect performance appraisals have been shown to give rise to several managerial biases, such as the centrality (Moers, 2005) and the leniency bias (Landy and Farr, 1980). The role of feedback in competitive environments has recently seen a large strand of literature with the introduction of the cheap talk games in ranked order tournament models. This literature suggests that feedback mechanisms and other informative signals on the competence of workers can cause managers to be reluctant in stating their evaluation findings completely or truthfully. The introduction of the tournament game partially solved this problem, by forcing the manager to assess and promote the best performing workers to their right positions (Prendergast, 1999). The possibility to make promotion is a powerful incentive that is used in career plans of many firms. The tournament game, in which workers compete to make promotion or to win a prize, was modeled by Lazear and Rosen (1981). By introducing the all-pay auction in a personnel economics setting Lazear and Rosen find that high competition between employees results in high effort exertion. Since the introduction of the rank-order tournament game the role of information transmission in tournament games has been researched in several economic studies. Aoyagi (2010) studies the problem of information revelation through cheap talk communication in a multi-stage tournament. He finds that the manager should reveal no information under a no-feedback policy and that he should reveal all of his information in a feedback policy. Ederer (2010) finds that managers committing to a feedback policy in a multi-stage tournament will find a rise in effort in the first period by its employees, as the employees will anticipate on an effect of their effort in the first period on the output score difference announced at the start of the second period. The workers will therefore work harder in order to receive positive news in the second period.

Their beliefs on ability directly influence their effort choice and the right feedback system can motivate all workers in the first period. Goltsman and Mukherjee (2011) find that feedback mechanisms, in which relative performance is only partially revealed, can be optimal in a multi-stage tournament setting.

By combining the 'looking-self glass' with an information revelation mechanism in the form of a possibly differentiating training decision of the manager in a tournament game I expect a few effects to occur. If the manager conveys information about the skill level by taking a costly training decision in which the skill level itself determines the efficiency of the training, then the workers should learn about their relative performance and chances in the tournament. Differentiation between the workers in the training decisions may then reveal information about their relative expected performance, which will affect the self-confidence and the motivation of the workers. In order to substantiate these claims I will construct a theoretical model in the next section of this thesis. After the introduction of this model I will analyze the equilibrium behaviour of the workers and the manager. I will conclude with a discussion of the key assumptions and findings of the model.

II. The model

i. Players, payoffs and strategies

This model describes the interaction between a manager and two risk-neutral workers A,B. The workers can both be either low- or high-skilled: $s_A \in \{L,H\}$, where $H > L \geq 1$. The workers do not know their skill levels, but they know the ex ante probabilities of their skill levels. The ex ante belief that the worker is high-skilled is α, so that $Pr(H) = \alpha$ and $Pr(L) = (1-\alpha)$. The manager is the supervisor of the workers in the tournament game. It is assumed that the manager perfectly learns the skill levels during the tournament game and that this fact is common knowledge to the workers.

The manager is responsible for the training of both workers A,B and can take the action to provide the respective worker with training (denoted by the dummy variable $m_A = 1$) or to abstain from giving training ($m_A = 0$). When a worker receives training it will boost the skill levels of the worker:

$$s'_A = \begin{cases} \varphi_L s_A \text{ if } m_A = 1 \wedge s_A = L \\ \varphi_H s_A \text{ if } m_A = 1 \wedge s_A = H \\ s_A \text{ otherwise} \end{cases}$$

It is assumed that training has relatively more beneficial for low-skilled workers than for high-skilled workers: $\varphi_L > \varphi_H \geq 1$.

In this rank-order tournament game the workers compete for a fixed prize, which is normalized to a unit of utility of 1 for the winner and zero utility for the loser. The workers can exert effort e_A in order to produce output x_A. The output score, as observed by the manager, is the outcome of effort exerted, the worker's skill level and a stochastic production shock ε_A: $x_A = s_A e_A + \varepsilon_A$. This implies that when a worker receives training it will directly boost the extent to which the worker's skill translates his effort e_A into output x_A. In determining the winner of the tournament the manager can observe the output score x_A, but not e_A and ε_A. This makes it possible that a low-skilled worker wins the tournament.

The workers compete to outperform their rivalling colleague by exerting effort. Worker A wins the tournament when $x_A > x_B$ and loses the tournament if this inequality is reversed. As the tournament may give rise to stochastic production shocks in output this may result in an advantage for one worker over the other, where the difference in shocks is defined as follows: $\varepsilon_A - \varepsilon_B = \Delta\varepsilon$. As only two workers are considered in this tournament I only make assumptions on the properties of $\Delta\varepsilon$ for reasons of simplicity. It is assumed that $\Delta\varepsilon$ is distributed according to the cumulative density function $F(\cdot)$ and the probability density function $f(\cdot)$. For the base model I assume a uniform distribution of the stochastic error term: $\Delta\varepsilon \sim U[-l,l]$. Only interior solutions are considered in this setting, so that the workers will always have a strictly positive probability of winning the tournament.

The utility of the worker is equal to their expected utility of winning the tournament minus the costs of exerting effort $c(e_A) \equiv \frac{1}{2}ke_A^2$, which is convex and rising in effort. The utility function is given by the following expression:

$$E(U_A) = Pr(x_A > x_B) - \frac{1}{2}ke_A^2 \tag{1}$$

The objective of the manager is to optimize the firm's profits π, which is the sum of output scores x_A and x_B minus the costs of training C incurred upon providing training:

$$\pi(\sigma \mid m', s') = x_A + x_B - (m'_A + m'_B)C \tag{2}$$

The strategy of the workers is to decide on how much effort to exert in this tournament game. The strategy of the manager, denoted by σ, is to decide which skill types receive training. The manager can choose to give none of the workers training, to provide only low-skilled workers with training, to provide only high-skilled workers with training or to provide all skill types of workers with training. In this thesis I will be only be looking for Perfect Bayesian Equilibria in pure strategies of the manager in order to avoid convoluting the analysis.

ii. Equilibrium skill beliefs

We find Perfect Bayesian Equilibria when the chosen strategy is sequentially rational and consistent with the beliefs of the players. The workers must not have reason to believe that the manager will deviate from his announced training strategy, as these beliefs would then be inconsistent with the updated beliefs of the players. Given that p_H and p_L represent the probabilities for which the manager gives training to high- and low skilled employees are common knowledge, we can use Bayes' rule to find the conditions under which the beliefs of the workers are consistent with the strategy of the manager. In this pure strategies setting we assume that these probabilities take on the value of 0 or 1: $p_i \in \{0,1\}$. The posterior beliefs are given by the following expressions:

$$Pr(s_A = L \mid m'_A = 1) = \frac{(1-\alpha)p_L}{\alpha p_H + (1-\alpha)p_L} \tag{3}$$

$$Pr(s_A = L \mid m'_A = 0) = \frac{(1-\alpha)(1-p_L)}{\alpha(1-p_H) + (1-\alpha)(1-p_L)} \tag{4}$$

$$Pr(s_A = H \mid m'_A = 1) = \frac{\alpha p_H}{\alpha p_H + (1-\alpha)p_L} \tag{5}$$

$$Pr(s_A = H \mid m'_A = 0) = \frac{\alpha(1-p_H)}{\alpha(1-p_H) + (1-\alpha)(1-p_L)} \tag{6}$$

I will show that the manager can credibly transmit information that are sequentially rational and consistent with these ex post beliefs. As we have two training strategies in which the manager does not differentiate I assume that the worker will form the worst possible out-of-equilibrium skill beliefs upon deviation off the equilibrium path. The out-of-equilibrium belief under deviation of the strategy $(\sigma = (0,0))$ is the lowest possible skill belief for whichever product of training and skill level is the minimal value:

$$E(s'_A \mid m'_A = 1 \wedge \sigma = (0,0)) = \min\{\varphi_L L, \varphi_H H\}$$

The out-of-equilibrium beliefs under deviation of the strategy $(\sigma = (1,1))$ are considered to be low-skilled, because of the assumption that $H > L$. From this assumption it follows that the belief that the worker is low-skilled is always strictly worse than the other option where the worker beliefs that he is high-skilled:

$$E(s'_A \mid m'_A = 0 \wedge \sigma = (1,1)) = L$$

iii. Timing of the game

This tournament game is a dynamic game of incomplete information. The manager learns the skill types of the employees in the beginning stage, after which the tournament begins. This tournament is a one-shot game where the worker with the highest realized output score is declared the winner of the tournament.

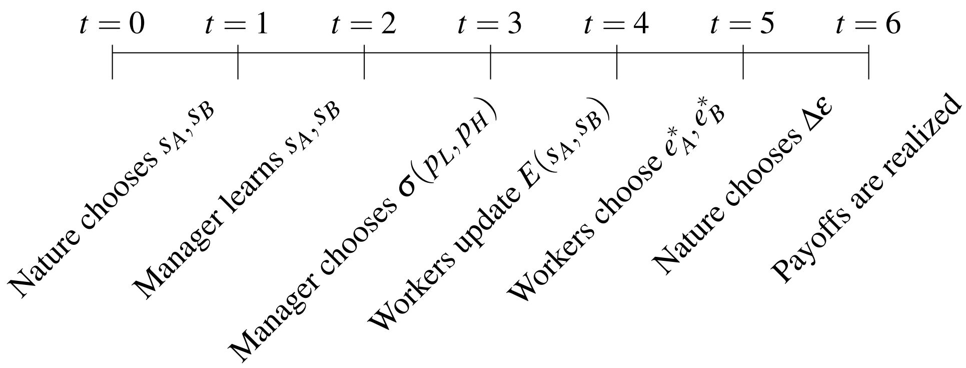

Figure 1: Timeline of the game

Figure 1 gives a summary of the timing of the game. It begins with a move of nature in determining the skill types of the workers. After a trial stage of working the manager perfectly learns the skill types. The manager then decides which employees to give training. After the employees receive training they update their respective beliefs and decide the amount of effort they exert in the tournament. Then, nature determines the stochastic output shock, after which the tournament ends and the winner is announced. Finally, payoffs are realized by the workers and the manager.

III. Analysis

i. Employee behaviour

The objective of the worker is to maximize his utility function:

$$\max_{e_A} E(U_A) = E_{s_A,s_B,\Delta\varepsilon}[F(e_A E(s'_A \mid m'_A) - e_B E(s'_B \mid m'_B) + \Delta\varepsilon)s'_A \mid \sigma'] - c(e_A) \quad (7)$$

The first-order condition is found when the costs of exerting effort is equal to the expected marginal probability of winning the tournament given the corresponding amount of effort:

$$c'(e_A) = E_{s_A,s_B,\Delta\varepsilon}[f(e_B E(s'_B \mid m'_B) - e_A E(s'_A \mid m'_A) \geq \Delta\varepsilon)s'_A \mid \sigma'] \quad (8)$$

This thesis merely seeks to uncover the dynamics of training decisions on employee motivation and performance. The simplicity of interior solutions will suffice in explaining the dynamics of training and motivation, so that strategies with atoms are not considered. In order to find interior solutions I will make the following assumption:

Assumption 1: *The condition $[-l < s'_A e^*_A + \Delta\varepsilon < l]$ holds for all values of s_i, k and $\Delta\varepsilon$.*

Given that Assumption 1 holds and we are in the tournament game where the workers determine their effort level according to Equation 8, I will show that we are in an interior solution under the following Lemma:

Lemma 1: *Consider the tournament game. When the distribution of the stochastic error term $\Delta\varepsilon$ is uniformly distributed and Assumption 1 applies, then the workers will choose their optimal effort levels regardless of the training decision of the manager with respect to the other worker given that effort levels are chosen simultaneously according to Equation 8. Then, we find an interior solution in which there are no atoms in the behaviour of the workers.*

Proof. See appendix. ■

As the workers simultaneously decide their optimal effort levels they will not take the optimal effort decisions of their competitors in consideration in their own optimal effort level decision. Rational workers will not have strategic incentives to try to influence the effort decision of the rivalling workers by exerting more effort than is marginally optimal according to Equation 8. The crucial element here is that the outer bounds of the stochastic error shock are assumed to be larger than the amount of output required for a guaranteed win of the tournament, so that neither of the workers expect to win the tournament with certainty. The uniform distribution of the stochastic error term always implies a constant marginal probability of winning the tournament, so that there is always a positive probability of winning the tournament for all types of workers regardless of the training decisions of the other workers. The probability of a tie is infinitely small as $\Delta\varepsilon$ has full support on the interval of all possible differences of output between workers. Lemma 1 learns us that workers will only regard the training decision m'_A on their own effort, as this is the decision that affects their marginal payoffs. The optimal effort decision of the worker is their best response in this Bayesian Nash Equilibrium. In an interior solution the rational manager can anticipate on the optimal effort levels of the rivalling workers by applying backward induction to find the expected best responses of the workers for all of the four possible training strategies. When the workers update their respective skill beliefs according to the manager's sequentially rational strategies, then we find Perfect Bayesian Equilibria in which the beliefs of the workers are consistent with the equilibrium skill beliefs.

In the rest of this thesis I will utilize Lemma 1 for all the managerial strategies. However, the results of the analysis will likely change when Lemma 1 does not hold anymore (e.g. when the distribution of the stochastic error term $\Delta\varepsilon$ is no longer uniformly distributed). I refer to the Discussion section for an elaboration on how this relaxation of the assumptions on the stochastic error term distribution could affect the results of this thesis.

ii. Equilibria

In this section I will now consider for which ranges of parameters Perfect Bayesian Equilibria exist for all four different training strategies of the manager. This analysis will enable the prediction of the first-best managerial training decision rule, which will be paramount for uncovering the possible effects of training on the behaviour of the worker. The rational manager will look for the most profitable training strategy, i.e. the strategy that results in the highest amount of output produced by the workers. However, the manager might want to deviate from the equilibrium path when he observes that he can motivate the workers by deceiving them about their skill types. By means of backward induction the workers are able to determine for which parameters the manager might be deceiving them. When this is the case, the skill beliefs of the workers are no longer consistent with their strategy beliefs and we find there is no Perfect Bayesian Equilibrium for the strategy given those parameters. In order for the manager to credible convey information about the workers' skill types the following condition will have to hold:

$$\pi(m_i(\sigma_i \mid s_A)) \geq \pi(m_{-i}(\sigma_i \mid s_A)) \; \forall \; s_A \tag{9}$$

When this condition holds, we are able to find Perfect Bayesian Equilibria in pure strategies for each respective skill type. This training strategy can only be a Perfect Bayesian Equilibrium when the training is sequentially rational and consistent with regards to both skill types L and H. We now consider the four pure training strategies for the manager with respect to these skill types.

A. No training

Suppose that the manager never gives training to any type L or H, so that he plays the strategy $\sigma = (0,0)$.

Lemma 2: *Consider the tournament game. The expected profits of the manager are given by the following expressions:*

$$\pi(\sigma = (0,0) \mid s_A = L) = \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L, \tag{10a}$$

$$\pi(\sigma = (0,0) \mid s_A = H) = \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H \tag{10b}$$

Given the condition that:

$$\min\{\varphi_L L, \varphi_H H\} = \varphi_L L \ \wedge\ C \geq \Big[\frac{\varphi_L^2 L^2 - \alpha HL - (1-\alpha)L^2}{2lk}\Big] \wedge C \geq \Big[\frac{\varphi_L \varphi_H HL - \alpha HL - (1-\alpha)L^2}{2lk}\Big] \vee$$
$$\min\{\varphi_L L, \varphi_H H\} = \varphi_H H \ \wedge\ C \geq \Big[\frac{\varphi_L \varphi_H HL - \alpha H^2 - (1-\alpha)HL}{2lk}\Big] \wedge C \geq \Big[\frac{\varphi_L^2 L^2 - \alpha HL - (1-\alpha)L^2}{2lk}\Big]$$

then there exists a Perfect Bayesian Equilibrium in which the manager credibly plays the strategy $[\sigma = (0,0)]$ *and the workers exert their best response effort level according to Lemma 1.*

Proof *(*$\min\{\varphi_L L, \varphi_H H\} = \varphi_L L$*).* This proof will be constructed according to the two possible out-of-equilibrium beliefs. When the out-of-equilibrium belief upon receiving training conveys to the worker that he is low-skilled, then the worker will exert as much effort as he will marginally benefit from in the tournament. Now suppose that worker A is low-skilled. The expected profits of the manager are then given by inserting the low-skilled worker's best response effort level of Equation 8 in the payoff function:

$$\pi(\sigma = (0,0) \mid m'_A = 1 \wedge s_A = L) = \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L - C$$

The manager will not deviate from this strategy if an increase in output from this deviation does not offset the costs C, which is satisfied when:

$$\pi(\sigma = (0,0) \mid m'_A = 0 \wedge s_A = L) \geq \pi(\sigma = (0,0) \mid m'_A = 1 \wedge s_A = L)$$
$$\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L \geq \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L - C$$
$$\Leftrightarrow$$
$$C \geq \Big[\frac{\varphi_L^2 L^2 - \alpha HL - (1-\alpha)L^2}{2lk}\Big]$$

The manager will never deviate from the equilibrium path for any profits of deviation that result in an amount smaller than C, as deviation will then not be profitable for the manager.

Now suppose that worker A is high-skilled. The expected profits of deviating off the equilibrium path are now given by inserting the high-skilled worker's best response effort level of Equation 8 in the payoff function:

$$\pi(\sigma=(0,0)\mid m'_A=1\wedge s_A=H)=\left[\frac{\varphi_L L}{2lk}\right]\varphi_H H-C$$

When the expected gain in profits from training the high-skilled worker are not high enough to offset this costly deviation, then the manager will not deviate given the following condition:

$$\pi(\sigma=(0,0)\mid m'_A=0\wedge s_A=H)\geq\pi(\sigma=(0,0)\mid m'_A=1\wedge s_A=H)$$
$$\left[\frac{\alpha H+(1-\alpha)L}{2lk}\right]H\geq\left[\frac{\varphi_L L}{2lk}\right]\varphi_H H-C$$
$$\Leftrightarrow$$
$$C\geq\left[\frac{\varphi_L\varphi_H HL-\alpha H^2-(1-\alpha)HL}{2lk}\right]$$

The expression above states the values of C for which the manager will not deviate to giving training to a high-skilled worker. When the conditions of Lemma 2 hold, I find that the manager will not train both types of workers according to this equilibrium strategy. ■

Proof *(*$\min\{\varphi_L L,\varphi_H H\}=\varphi_H H$*).* Suppose that the out-of-equilibrium skill belief upon receiving training ($m'_A=1$) implies that he is high-skilled. The manager will receive the following profits by training a low-skilled worker:

$$\pi(\sigma=(0,0)\mid m'_A=1\wedge s_A=L)=\left[\frac{\varphi_H H}{2lk}\right]\varphi_L L-C$$

When the costs of training are very high, then the manager will never deviate from his equilibrium strategy when the net profits of deviation are lower than the expected profits of staying on the equilibrium path by not training the worker. This is given by the following condition:

$$\pi(\sigma=(0,0)\mid m'_A=0\wedge s_A=L)\geq\pi(\sigma=(0,0)\mid m'_A=1\wedge s_A=L)$$
$$\left[\frac{\alpha H+(1-\alpha)L}{2lk}\right]L\geq\left[\frac{\varphi_H H}{2lk}\right]\varphi_L L-C$$
$$\Leftrightarrow$$
$$C\geq\left[\frac{\varphi_L\varphi_H HL-\alpha HL-(1-\alpha)L^2}{2lk}\right]$$

Now consider training high-skilled workers. Upon receiving training high-skilled workers will believe that they are high-skilled trained workers. By examining the conditions for which the manager will not deviate off the equilibrium path with respect to these high-skilled workers, the first step is to identify the expected profits of deviation with regards to the high-skilled worker:

$$\pi(\sigma=(0,0)\mid m'_A=1\wedge s_A=H)=\left[\frac{\varphi_H H}{2lk}\right]\varphi_H H-C$$

The manager will deviate from the equilibrium strategy when the costs of training are low enough to result in a beneficial training decision. In order for this strategy to be a Perfect

Bayesian Equilibrium the costs of training have to be larger than the benefits of training, which is given by the following condition:

$$\pi(\sigma = (0,0) \mid m'_A = 0 \wedge s_A = H) \geq \pi(\sigma = (0,0) \mid m'_A = 1 \wedge s_A = H)$$
$$\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big] H \geq \Big[\frac{\varphi_H H}{2lk}\Big] \varphi_H H - C$$
$$\Leftrightarrow$$
$$C \geq \Big[\frac{\varphi_H^2 H^2 - \alpha H^2 - (1-\alpha)HL}{2lk}\Big]$$

For both out-of-equilibrium beliefs the manager will not deviate from the equilibrium strategy path when the costs of training are high enough. We find that the strategy of the manager to not train the workers is a Perfect Bayesian Equilibrium when the conditions of Lemma 2 hold, as the strategy will only then be sequentially rational and consistent with the beliefs of the workers.

■

The intuition behind training none of the workers is given by the fact that intervening in the tournament imposes training costs C on the manager. As training always has a stimulating effect on the marginal productivity of the workers, the manager will consider giving training to the employees when the expected benefits of providing training exceed the costs of training. For all combinations of H and φ_L where the decision to invest in training will results in an expected rise of worker productivity the manager will provide training, where he will abstain from giving training when this is not the case. For these values the status quo is maintained in which no information is conveyed to the workers by the manager. They will not be able to update their expected skill levels with new information, so that there are no motivation effects present in this equilibrium strategy.

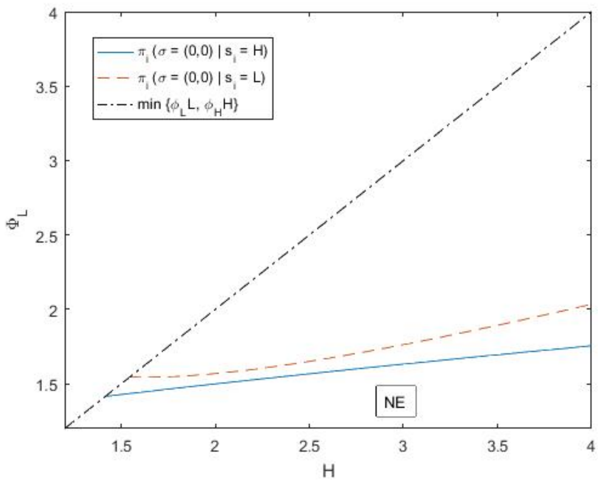

Figure 2: Equilibrium in which none of the workers receive training $(L = 1.2, \varphi_H = 1.2, k = 0.15, l = 8, C = 0.55)$

In Figure 2 the interaction of skill and training has been graphed for which values the strategy of training none of the workers is a Perfect Bayesian Equilibrium. The figure represents the combinations of H and φ_L as interaction variables of the high-skill type and the training strength with respect to low-skilled workers. The other variables of the model (i.e. L, φ_H, k, l, C) are held static. The black line, representing $\min\{\varphi_L L, \varphi_H H\}$, is a straight linear line in H and φ_L as I assume that $L = \varphi_H = 1.2$. This line marks the border of the product of skill and training where the out-of-equilibrium beliefs revert from low-skilled to high-skilled upon receiving training (and vice versa). This border is useful, as it can help to illustrate what the out-of-equilibrium beliefs are given these comparative statics and for which combinations of H and φ_L this strategy is a Perfect Bayesian Equilibrium. The solid blue line represents the combinations of H and φ_L where the manager is indifferent between training a high-skilled worker according to the training strategy or to deviate of the equilibrium path. The striped red line depicts the same indifference for the low-skilled worker. The blue and red line therefore depict the conditions of Lemma 2 and can be used to interpret the effect of skill and training on the training decisions of the manager. In the following figures the same layout will be used as in Figure 2.

When L and φ_H are assumed to take the value 1.2, only the out-of-equilibrium skill belief where trained workers believe they are low-skilled can be used to find Perfect Bayesian Equilibria for this training strategy. Figure 2 shows that the out-of-equilibrium belief where the worker believes he is high-skilled upon receiving training cannot be a Perfect Bayesian Equilibrium, as the incentive compatibility constraints are not visible for the statics I assumed in this Figure. I will not further elaborate on high-skilled out-of-equilibrium skill beliefs, as the intuition of Lemma 2 becomes clear enough from the statics following from low-skilled out-of-equilibrium beliefs. The equilibrium is illustrated in Figure 2 for the combinations of φ_L and H that lie below the blue and black lines, as these combinations satisfy the conditions of Lemma 2 that states that the costs of training must be high enough to offset the potential benefits of training that are increasing in φ_L. The area of combinations H and φ_L that contains the mark 'NE' represents the indifference line of the manager between training according to his equilibrium strategy or deviating of the equilibrium path.

Figure 2 shows that both low- and high-skilled workers will increase their optimal effort levels in a rise of H under the training strategy $\sigma = (0,0)$. When the out-of-equilibrium beliefs imply that a worker that receives training is low-skilled, then a rise in H will result in more combinations of φ_L and H where the strategy $\sigma = (0,0)$ is a Perfect Bayesian Equilibrium, as it increases the expected payoff of the manager of the pure strategy $\sigma = (0,0)$ for workers of both skill types. A rise in H might result in a possible deviation of the manager when the out-of-equilibrium beliefs are that the worker is high-skilled. The combination of H and φ_H may result in a deviation of the equilibrium path, so that the equilibrium skill beliefs are not be consistent anymore. Figure 2 illustrates how the same principle applies to φ_L. A rise in training strength might result in a deviation of the equilibrium path when the benefits of deviation by training the worker have become high enough.

B. Training low-skilled workers

Suppose that the manager always gives training to low-skilled workers and never to high-skilled workers, so that the manager plays the strategy $\sigma = (1,0)$.

Lemma 3: *Consider the tournament game. The expected profits of the manager are given by the following expressions:*

$$\pi(\sigma = (1,0) \mid s_A = L) = \left[\frac{\varphi_L L}{2lk}\right]\varphi_L L - C \tag{11a}$$

$$\pi(\sigma = (1,0) \mid s_A = H) = \left[\frac{H}{2lk}\right]H \tag{11b}$$

Given the condition where:

$$\left[\frac{\varphi_L^2 L^2 - HL}{2lk}\right] \geq C \geq \left[\frac{\varphi_L \varphi_H HL - H^2}{2lk}\right]$$

there exists a Perfect Bayesian Equilibrium in which the manager credibly plays the strategy $\sigma = (0,0)$ and the workers respond by exerting effort according to Lemma 1.

Proof. Upon observing m'_A, the worker will update the skill beliefs accordingly: $E(s_A \mid m'_A = 1) = \varphi_L L$ and $E(s_A \mid m'_A = 0) = \varphi_H H$. When the manager considers not giving training to the low-skilled worker the costs must be high enough in order to offset the benefits of φ_L and the motivation effect of learning the worker is high-skilled when $m'_A = 0$. The expected payoffs of this deviation for a low-skilled worker are given by the following expression:

$$\pi(\sigma = (1,0) \mid m'_A = 0 \wedge s_A = L) = \left[\frac{H}{2lk}\right]L$$

Comparing the payoffs of deviation for the low-skilled worker with Equation 11a gives the conditions for which the manager will not deviate of the equilibrium path:

$$\pi(\sigma = (1,0) \mid m'_A = 1 \wedge s_A = L) \geq \pi(\sigma = (1,0) \mid m'_A = 0 \wedge s_A = L)$$

$$\left[\frac{\varphi_L L}{2lk}\right]\varphi_L L - C \geq \left[\frac{H}{2lk}\right]L$$

$$\Leftrightarrow$$

$$\left[\frac{\varphi_L^2 L^2 - HL}{2lk}\right] \geq C \tag{12}$$

Similarly the manager can incur costs by training high-skilled workers. The high-skilled worker will update his beliefs according to $m'_A = 1$, but receive training with the specific training effects for high-skilled workers. The expected payoffs of this deviation for a high-skilled worker are given by the following expression:

$$\pi(\sigma = (1,0) \mid m'_A = 1 \wedge s_A = H) = \left[\frac{\varphi_L L}{2lk}\right]\varphi_H H - C$$

Comparison of the payoffs of deviation with Equation 11b gives that the manager will not deviate of the equilibrium path when costs are sufficiently high:

$$\pi(\sigma = (1,0) \mid m'_A = 0 \wedge s_A = H) \geq \pi(\sigma = (1,0) \mid m'_A = 1 \wedge s_A = H)$$
$$\left[\frac{H}{2lk}\right] H \geq \left[\frac{\varphi_L L}{2lk}\right] \varphi_H H - C$$
$$\Leftrightarrow$$
$$C \geq \left[\frac{\varphi_L \varphi_H HL - H^2}{2lk}\right] \tag{13}$$

Combining Equations 12 and 13 gives the condition in which the manager always trains the workers according to Lemma 3:

$$\left[\frac{\varphi_L^2 L^2 - HL}{2lk}\right] \geq C \geq \left[\frac{\varphi_L \varphi_H HL - H^2}{2lk}\right]$$

■

The intuition of the existence of an equilibrium strategy in which only low-skilled workers are trained lies in balancing the various training and motivation effects of low- and high-skilled workers. A substantial rise in φ_L will result in the manager deviating off the equilibrium path by also training high-skilled workers, as the high-skilled workers will be highly motivated to exert effort, so that the training costs C are offset by this deviation. Similarly the manager might consider deviating for low-skilled workers when the value of H is very high. Aside from the motivation effect of learning that the low-skilled worker is high-skilled will the manager also save the costs of training. The dynamics of this equilibrium imply a range of parameters for which the manager can credibly train according to this strategy. Outside of this range of parameters the manager will have an incentive to deviate, so that this strategy is not a Perfect Bayesian Equilibrium anymore.

Low-skilled workers will perfectly learn their skill level according to the training strategy in this separating Perfect Bayesian Equilibrium. It follows from transitivity that the manager prefers the workers not knowing their skill levels, as the possibility of being high-skilled will motivate the workers more compared to the case where they learn that the same amount of effort will not result in the same expected marginal probability of winning the tournament. However, because of the effect of training on the worker's skill level the manager might still be enticed to provide the worker with training, even when the worker is demotivated by this training decision. The product of skill and effort might result in a positive change in productivity of the worker, which might be preferred by the manager that has to consider providing training to a possibly struggling employee or giving no training under the status quo decision. This is given by Corollary 1:

Corollary 1: *Consider that strategy $\sigma = (1,0)$ is a Perfect Bayesian Equilibrium. It follows from transitivity that training must be:*

a. Motivating, given that: $\varphi_L L \geq \alpha H + (1-\alpha)L$
b. Demotivating, given that: $\varphi_L L < \alpha H + (1-\alpha)L$

compared to the status quo strategy $\sigma = (0,0)$ *in which no training is given.*

Figure 3 gives an illustration of the range of equilibrium parameters the manager is bound to in order to credibly play this strategy. The red line depicts the combinations of H and φ_L for which the manager is indifferent of playing the equilibrium strategy and deviating of the equilibrium path. The blue line depicts the combinations of H and φ_L for which the manager is indifferent with regards to a high-skilled worker. The figure shows how deviation for the two skill types is interesting for combinations of H and φ_L that are not too far apart from each other. A strong increase in φ_L will result in deviation with regards to high-skilled workers by training them under this strategy, where a strong increase in H will cause the manager to deviate with regards to low-skilled workers by not training them.

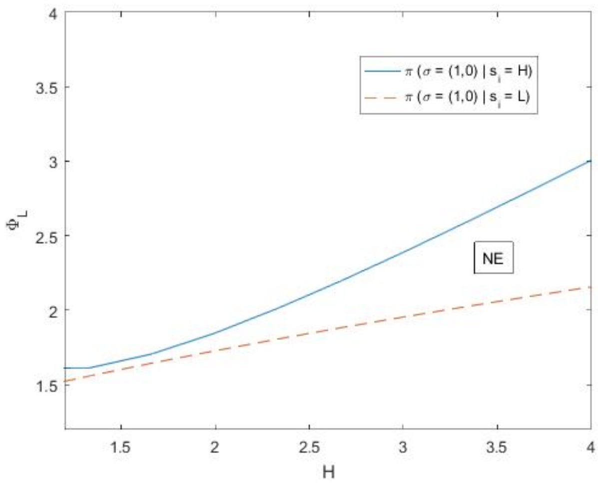

Figure 3: Equilibrium in which low-skilled workers receive training $(L = 1.2, \varphi_H = 1.2, k = 0.15, l = 8, C = 0.55)$

It is a plausible thought that the strategy of training low-skilled workers will result in a more competitive tournament between a rivalling high-skilled and low-skilled worker. The tournament effect that arises under small differences in competitiveness, in combination with the training and motivation effects of this strategy could result in a positive effect on productivity, indicating that training for low-skilled workers could be a powerful tool in the empowerment of the struggling employees within the organization. However, Figure 3 shows how this can only be the case for the correction of small imbalances in the tournament caused by differences in H and φ_L, as the manager will deviate for large differences between the values of these parameters. Furthermore, the tournament effect that is described in this paragraph is not present under a uniform distribution of the stochastic error term $\Delta\varepsilon$, as has been assumed until now. For an elaboration on the potential tournament effects caused by training I refer to the Discussion section.

C. Training high-skilled workers

Suppose that the manager never gives training to low-skilled workers and always gives training to high-skilled workers.

Lemma 4: *Consider the tournament game. The expected profits are given by the following equations:*

$$\pi(\sigma = (0,1) \mid s_A = L) = \left[\frac{L}{2lk}\right] L \tag{14a}$$

$$\pi(\sigma = (0,1) \mid s_A = H) = \left[\frac{\varphi_H H}{2lk}\right] \varphi_H H - C \tag{14b}$$

Given the condition where:

$$\left[\frac{\varphi_H H \varphi_L L - L^2}{2lk}\right] \leq C \leq \left[\frac{\varphi_H^2 H^2 - LH}{2lk}\right]$$

there exists a Perfect Bayesian Equilibrium in which the manager credibly plays the strategy $\sigma = (0,1)$ and the workers respond by exerting effort according to Lemma 1.

Proof. The low-skilled worker will believe he is high-skilled and receive training.

$$\pi(\sigma = (0,1) \mid m'_A = 1 \wedge s_A = L) = \left[\frac{\varphi_H H}{2lk}\right] \varphi_L L - C$$

The manager will deviate of the equilibrium path when the increase in productivity of deviation exceeds the incurred costs of training C, so that the manager will not deviate when the costs of training are higher than the potential benefits of training:

$$\pi(\sigma = (0,1) \mid m'_A = 0 \wedge s_A = L) \geq \pi(\sigma = (0,1) \mid m'_A = 1 \wedge s_A = L)$$
$$\left[\frac{L}{2lk}\right] L \geq \left[\frac{\varphi_H H}{2lk}\right] \varphi_L L - C$$
$$\Leftrightarrow$$
$$C \geq \left[\frac{\varphi_H H \varphi_L L - L^2}{2lk}\right] \tag{15}$$

Suppose that the manager does not give training to a high-skilled worker. The manager does not incur the costs of training C, so that expected payoff of this deviation are given by:

$$\pi(\sigma = (0,1) \mid m'_A = 0 \wedge s_A = H) = \left[\frac{L}{2lk}\right] H$$

Comparison of Equation 14b and the payoff of deviation shows that the manager will not deviate for high-skilled workers when the benefits of training are not offset by the costs of training:

$$\pi(\sigma = (0,1) \mid m'_A = 1 \wedge s_A = H) \geq \pi(\sigma = (0,1) \mid m'_A = 0 \wedge s_A = H)$$
$$\left[\frac{\varphi_H H}{2lk}\right] \varphi_H H - C \geq \left[\frac{L}{2lk}\right] H$$

$$\Leftrightarrow$$

$$\left[\frac{\varphi_H^2 H^2 - LH}{2lk}\right] \geq C \tag{16}$$

Combining Equations 15 and 16 give that the manager will truthfully train the workers according to his strategy when the following condition holds:

$$\left[\frac{\varphi_H H \varphi_L L - L^2}{2lk}\right] \leq C \leq \left[\frac{\varphi_H^2 H^2 - LH}{2lk}\right]$$

■

The differentiating strategy in which only high-skilled workers are trained is motivating for high-skilled workers and strictly demotivating for low-skilled workers. High-skilled workers are both trained and motivated in this strategy, which may result in a large boost in productivity for these workers. However, workers that do not receive training under this strategy will be strictly worse off in the tournament. In this setting it depends on the value of training costs C whether or not the manager might want to both motivate and train a low-skilled worker by deviating from the equilibrium strategy by training the low-skilled worker. For large values of H and φ_L the manager should be tempted to provide the low-skilled worker with training too.

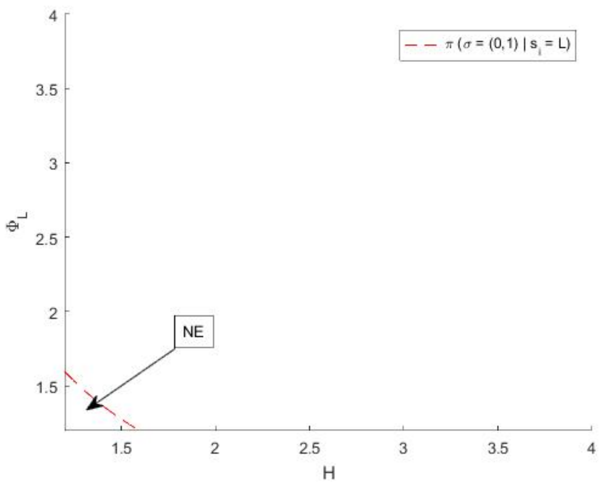

Figure 4: Equilibrium in which high-skilled workers receive training $(L = 1.2, \varphi_H = 1.2, k = 0.15, l = 8, C = 0.55)$

Figure 4 shows the intuition behind this thought process. The strategy in which exclusively high-skilled workers are trained can only be a Perfect Bayesian Equilibrium for a small range of the parameters H and φ_L. The manager will only abstain from giving training to the low-skilled worker for low values of H and φ_L, because the product of training and skill level will be too low to offset the costs of training incurred by the manager. This is depicted by the red striped line in Figure 4, which marks the range of parameters for which this strategy is a Perfect Bayesian Equilibrium. Values exceeding this range of parameters will result in deviation of the manager to training low-skilled workers. The implication of this strategy is that the product of effort and skill, also known as the motivation effect of this model, has to be large enough in order to offset the costs of training. Deviation will not be in the managers interests when the product of L, H, φ_L and φ_H does not exceed the costs of training C. This strategy is only a Perfect Bayesian Equilibrium for low values of H and φ_L. Only when the product of L, φ_L, H and φ_H is low (not substantially higher than 1) will this strategy be a tool for the manager. The intuition in the tournament might be given by the fact that when it is not viable to train all worker the firm might still benefit by focusing all training and development efforts on their best workers, while simultaneously resulting in more self-confidence because of the informative signal.

D. *Training all workers*

Suppose that the manager gives training to both types L and H, so that he plays the strategy $\sigma = (1,1)$.

Lemma 5: *Consider the tournament game. The expected profits of the manager are given by the following expressions:*

$$\pi(\sigma = (1,1) \mid s_A = L) = \Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_L L - C \tag{17a}$$

$$\pi(\sigma = (1,1) \mid s_A = H) = \Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_H H – C \tag{17b}$$

Given the following conditions:

$$C \leq \Big[\frac{\alpha\varphi_H\varphi_L HL + (1-\alpha)(\varphi_L^2 - 1)L^2}{2lk}\Big], \wedge \tag{18a}$$

$$C \leq \Big[\frac{\alpha\varphi_H^2 H^2 + (1-\alpha)\varphi_L\varphi_H H - HL}{2lk}\Big] \tag{18b}$$

we find a Perfect Bayesian Equilibrium in which the manager credibly plays the strategy $[\sigma = (1,1)]$ *and the workers exert effort according to Lemma 1.*

Proof. When the manager chooses to deviate of his strategy for low-skilled workers, then the worker will believe he is low-skilled. The expected profits of deviation are then given by the following equation:

$$\pi(\sigma = (1,1) \mid m'_A = 0 \wedge s_A = L) = \Big[\frac{L}{2lk}\Big]L$$

Comparison of this equation with the strategy expected profits shows that deviation is not profitable when the costs of training C are high enough:

$$\pi(\sigma = (1,1) \mid m'_A = 1 \wedge s_A = L) \geq \pi(\sigma = (1,1) \mid m'_A = 0 \wedge s_A = L)$$
$$\Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_L L - C \geq \Big[\frac{L}{2lk}\Big]L$$
$$\Leftrightarrow$$

$$\Big[\frac{\alpha\varphi_H\varphi_L HL + (1-\alpha)(\varphi_L^2 - 1)L^2}{2lk}\Big] \geq C$$

Similarly the manager will receive the following expected profits by deviating off the equilibrium with respect to high-skilled workers:

$$\pi(\sigma = (1,1) \mid m'_A = 0 \wedge s_A = H) = \Big[\frac{L}{2lk}\Big]H$$

Comparison of the equation above with the expected payoff of the strategy for high-skilled workers shows that the manager will not deviate when the following condition holds:

$$\pi(\sigma = (1,1) \mid m'_A = 1 \wedge s_A = H) \geq \pi(\sigma = (1,1) \mid m'_A = 0 \wedge s_A = H)$$

$$\Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_H H - C \geq \Big[\frac{L}{2lk}\Big]H$$

$$\Leftrightarrow$$

$$\Big[\frac{\alpha\varphi_H^2 H^2 + (1-\alpha)\varphi_L\varphi_H H - HL}{2lk}\Big] \geq C$$

■

In this strategy all workers are trained by the manager. This strategy is an equilibrium strategy when the product of φ_L, φ_H, H and L results in enough productivity for both types of workers to offset the costs of training. Both types of workers are motivated to work harder compared to the status quo strategy, as the expected marginal competitiveness of both workers has increased as a result of their skill improvements.

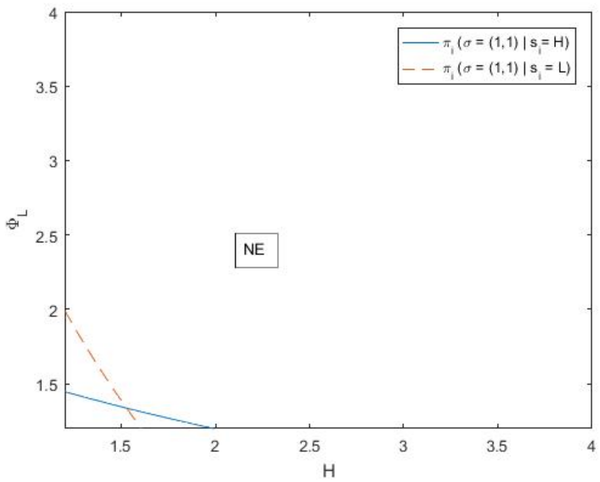

Figure 5: Equilibrium in which all workers receive training ($L = 1.2, \varphi_H = 1.2, k = 0.15, l = 8, C = 0.55$)

Training can result in improvements or deterioration of the workers' tournament competitiveness, where the sign of this change depends on the relative differences of $\varphi_L L$, $\varphi_H H$ and the interaction of training and skill with the workers' beliefs in the case of incomplete information. However, strong deterioration in competitiveness will not lead to a decrease in worker productivity and motivation. The workers will not learn information about their skill levels as the manager does not differentiate in skill level, so that they will not infer whether or not their competitiveness actually improves or deteriorates as a result of the training strategy.

Regardless of the changes in the tournament competitiveness of the workers, this non-differentiation training strategy will result in a marginal positive motivational effect for all workers. In Figure 5 it can be seen how this strategy is feasible for a wide range of parameters. Only for combinations of H and φ_L to the left and below of the blue and red lines will the manager deviate from this equilibrium strategy. This is caused by the fact that there are not a lot of possible downsides to this training strategy, as all employees benefit from the training regardless of their skill level. The training allows all workers to achieve a higher probability of winning the tournament by exerting more effort, regardless of the fact that they do not learn new information about their skill levels. The finding that this equilibrium strategy is credible for a wide range of parameters seems to correspond with the correlations that Huselid (1995) found in his survey study on the relationship of motivation and productivity with regards to training and development of the employees. When the increase in production of the organization is the result of the training and development of the worker's skills and this also results in a net profit for the organization, then it might be in the best interest of the organization to train all of the employees without differentiating between skill types.

iii. Welfare properties of training

In this section I will address the implications and expectations of the equilibria formulated in Lemma's 2-5 for the hypothesis of this paper. The previous section indicates that training can have a wide variety of effects on the motivation and productivity of workers. The relationship that Huselid (1995) found in the correlation between training and productivity is confirmed by the equilibria of this model. However, in addition to this empirical paper I find that that the net effect of training does necessarily move in a unilateral positive direction. The equilibria point towards a large range of effects in motivation and productivity that can result in adverse effects of training with ambiguous signs, when the training decision is capable of influencing the self-esteem of the workers.

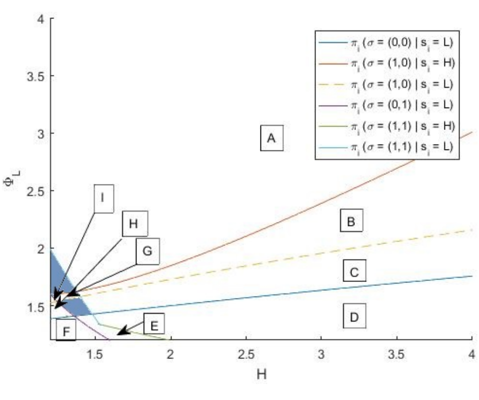

Figure 6: $C = 0.55$

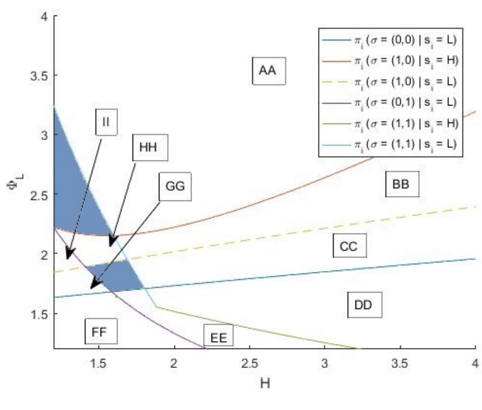

Figure 7: $C = 1$

Table 1: Equilibrium strategies of Figure 6 and 7

Surface	Strategies
A AA	Training all workers
B BB	Training low-skilled workers Training all workers
C CC	Training all workers
D DD	Training no workers Training all workers
E EE	Training no workers
F FF	Training no workers Training high-skilled workers
G GG	Training high-skilled workers
H HH	Training low-skilled workers
I II	Training low-skilled workers Training high-skilled workers

A graphical illustration may help to illustrate the ambiguity in the effect of training on worker motivation and productivity. Figure 6 shows all possible equilibrium strategies of the previous section in a single figure. This figure shows how the strategies can be equilibria given the specific interaction of the training and motivation effects with the costs of training. The lines depict the incentive compatibility constraints we found for the Lemma's 2-5 in the previous section. The figure illustrates that several training strategies can be played as a Perfect Bayesian Equilibrium for the ranges of parameters. The surfaces for which strategies are a Perfect Bayesian Equilibrium are marked with a capital letter, which have have been summarized in Table 1. It can be seen in Figure 6 and Table 1 that the manager can credibly train none of the workers in the surface A, but can only credibly train low-skilled workers or all of the workers as Perfect Bayesian Equilibrium strategies in surface B. This figure shows that the manager can credibly influence the motivation and productivity of the workers in certain directions by training them according to one of the four training strategies.

The dynamics of changes in the costs of training on the equilibria are illustrated in Figure 7. Figure 7 represents the same equilibria of Figure 6 under a rise in the costs of training ($C = 1$, instead of $C = 0.55$). In order to easily identify and compare the differences caused by the rise in costs the surfaces of the incentive compatibility constraints have been rebranded with a double capital letter, so that surface A of Figure 6 becomes surface AA in Figure 7. The first obvious and expected result is that a rise in costs will result in less available combinations of H and φ_L for which the strategy $\sigma = (1,1)$ is a Perfect Bayesian Equilibrium. The lowest ranges of parameters will no longer result in expected beneficial payoffs compared to the same strategies under lower costs of training.

Figure 6 and Figure 7 show that the surfaces of AA, BB, CC and DD have shrunk in comparison with surfaces A, B, C and D. It has become less interesting for the manager to provide training to all workers because of the rise in costs. Similarly we find that a rise in costs results in a higher likelihood that the manager will abstain from giving training. At the same time a rise in costs will make it more likely that the manager will not train any of the workers. This can be seen in Figures 6 and 7 by comparing the surfaces D, E and F with DD, EE and FF. The effect of the costs of training on productivity is more ambiguous under the differentiating strategies, as the decision of the manager to deviate of the equilibrium path under the differentiating strategies by training either of the workers will be more (or less) attractive when the product of training and skill level is small (or large).

By combining Lemma's 2-5 with Corollary 1, I find that the model is capable of predicting the effects of training on worker motivation and productivity in a competitive tournament environment.[3] The findings of the model are summarized in Proposition 1:

Proposition 1: *Consider the tournament game. Given that the manager trains the workers according to conditions of Lemma's 2-5, the expected effects of training on the workers' probabilities of winning the tournament, the motivation of the workers and the productivity of the workers are given by Table 2 and Figures 8-11.*

Proof. This proposition follows from Lemma's 2-5.

Proposition 1 implies that differentiation of employees might result in a possible deterioration of worker motivation. This implies that the effect of training is not unilaterally positive. The signal of receiving training may result in a demotivated worker, as the worker will learn that he is not as competitive as he previously believed to be. The outcomes of this model are driven by the motivation effects of training and differentiation. Where training improves the marginal output of workers, it can also send a signal to the workers about their respective skill levels. The informative signal of receiving training may therefore result in less confidence of the workers, which will reduce their motivation to exert effort. As can be seen in Table 1, the effect of training on productivity and motivation is not solely positive, but also knows negative effects compared to the status quo situation in which none of the workers is trained. According to Lemma's 3 and 4 we find that differentiation by providing training to only one type of workers can only be given under very small ranges of parameters.

Furthermore, I find that training both types of workers may influence the actual tournament probabilities of the workers, but not be reflected in the ex ante effort levels of the workers. Figure 8-11 show how the four training strategies affect the probabilities of winning the tournament for the low-skilled (red surface) and high-skilled workers (blue surface) given the parameters H and φ_L. Figure 11 shows how training all workers may result in low-skilled workers being relatively better off as a result of training than high-skilled workers, compared to the training strategy in which none of the workers are trained. The combination of training and skill may give rise to both positive and negative effects on

[3] In this thesis I will leave the refinement of the equilibria through the application of the Intuitive Criterion of Cho and Kreps (1987) as a subject for further research.

Predicted effects of training on worker effort in the tournament game			
Surface	**Strategies**	s_A	e^*
A	Training all workers	L H	+ +
B	Training low-skilled workers	L H	-/+ +
	Training all workers	L H	+ +
C	Training all workers	L H	+ +
D	Training no workers	L H	. .
	Training all workers	L H	+ +
E	Training no workers	L H	. .
F	Training no workers	L H	. .
	Training high-skilled workers	L H	- +
G	Training high-skilled workers	L H	- +
H	Traning low-skilled workers	L H	-/+ +
I	Training low-skilled workers	L H	-/+ +
	Training high-skilled workers	L H	- +

Table 2: *The table above shows the predicted effects of the training strategies for the effort levels of the workers. In the previous subsections the lemma's provided proofs for the predicted effects of the training strategies. Corollary 1 predicts whether or not the strategy of training low-skilled workers will result in a rise or drop in motivation of the low-skilled worker, which indicates the ambiguous effect of training on worker motivation in this table for training strategies in which only low-skilled workers are trained. The results can then be found in the final column of the table, where the effect of training will increase (+) or decrease (-) for each respective skill level compared to the status quo strategy in which none of the workers are trained (.).*

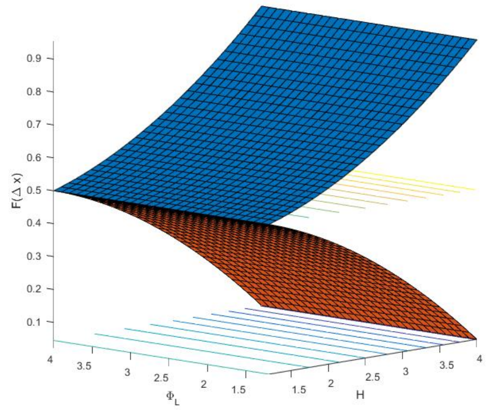

Figure 8: $E(F_i(\sigma = (0,0))$

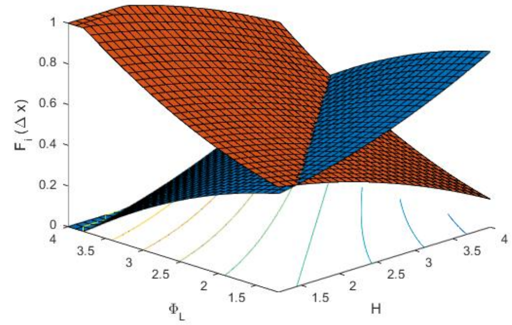

Figure 9: $E(F_i(\sigma = (1,0))$

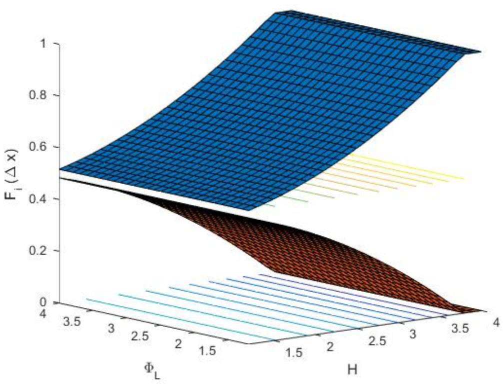

Figure 10: $E(F_i(\sigma = (0,1))$

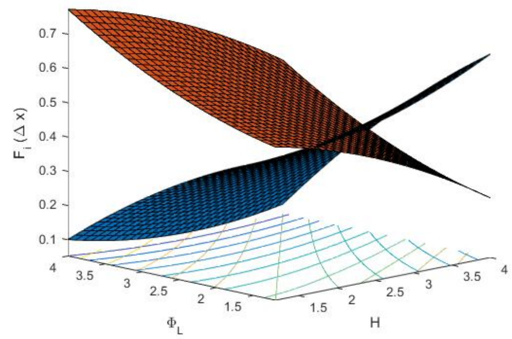

Figure 11: $E(F_i(\sigma = (1,1))$

motivation and the probability of winning the tournament for both types of skill, depending on the training strategy chosen by the manager. The effects of the respective training strategies on the probabilities of winning the tournament is represented by Figures 8-11, which show how the expected probabilities of winning the tournament and expected payoffs of the workers change in the combination of H and φ_L.

Non-differentiating strategies may result in changed probabilities of winning the tournament, but as these strategies do not result in any learning by the workers the expected probabilities of winning the tournaments will stay equal for both skill types from an ex ante perspective. This is not the case for differentiating strategies. Training is strictly positive for high-skilled workers under the strategy $\sigma = (0,1)$, as the high-skilled worker will be both motivated by receiving the training and learning about his skill type. The other separating strategy $\sigma = (1,0)$ also results in improved tournament odds, but the effect of training will be diminished when receiving training is demotivating according to Corollary 1. This explains the inverse shapes of the tournament odds in Figure 9.

The wide variety of effects of this model, especially the outcomes on worker motivation suggested by Proposition 1, pose some interesting questions for the organizational design of the human resources practices of organizations. This theory suggests that tournament incentives may result in adverse effects of training on motivation and productivity, given that this training is visible. Differentiation might actually result in weakened tournament incentives, which indicates that organizations should carefully consider how their training and incentive structures interact with each other. This might differ per firm or segment, which means that it remains a question how these theoretical findings would work out in a field experiment.

IV. Discussion

In this section I will relax some of the assumptions of the base model in order to gain more insight in the dynamics of training and motivation of workers.

i. Managerial tournament bonus

Suppose that the organization employs both low-skilled and high-skilled workers. It could be in the firm's interest to have the best workers win the tournament, for example promotion to a higher position upon winning the tournament. It could then be in the best interest of the firm to stimulate the performance of high-skilled workers in order to promote the best qualified workers to higher positions. For the sake of this argument, assume that the manager is not incentivized through the realization of output, but by the prospect of a tournament bonus δ upon the tournament victory of the high-skilled worker A:

$$\pi(\sigma \mid m', s') = E[F(x_A > x_B)]\delta - (m'_A + m'_B)C \tag{19}$$

The manager will never provide training when the workers A and B are both low- or high-skilled, as the manager will always receive the bonus in the first situation and never receive the bonus in the second situation. In these cases the manager has no incentive to incur costs in order to influence the tournament. In this section I will only focus on the situation where worker A is high-skilled and worker B is low-skilled, so that their respective expected probabilities of winning the tournament are ex ante different from the perspective of the manager. The training decision can then be used by the manager as an incentive to boost the performance of the high-skilled worker in order to increase his own chances of winning the bonus. Then, it follows from intuition that the manager will likely not be interested in stimulating the performance of the low-skilled worker by training the low-skilled worker. This is confirmed by the following Proposition:

Proposition 2: *Consider the tournament game. Assume that the payoffs of the manager are given by Equation 19. Then, the manager will abstain from giving training to low-skilled workers, unless the training of all workers results in a relative improvement of the high-skilled worker's probability of winning the tournament as a result of no information transmission in this pooling equilibrium.*

Proof. Again, assume that the best response effort levels of the workers are given by Lemma 1 and that the tournament bonus is set at unity: $\delta = 1$. The conditions for which each of the four pure training strategies are a Perfect Bayesian Equilibrium are now reviewed.

Training low-skilled workers

This proof starts off with reviewing the possibility of a Perfect Bayesian Equilibrium for the most counterintuitive strategy: training low-skilled workers. The expected payoffs for this strategy are given by the following equation:

$$\pi(\sigma=(1,0)) = F\Big(\Big[\frac{H}{2lk}\Big]H - \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L\Big) - C$$

The fact that this training strategy is counterintuitive also follows from a comparison of the payoffs of this training strategy. The manager will deviate in both cases by definition of the given strategy. This follows with regards to both the high-skilled and low-skilled workers from transitivity. This is proven by comparing the payoffs of the equilibrium strategy truthfully with the payoffs of deviation with regards to the high-skilled player A. The payoffs of deviation for the high-skilled worker is given by:

$$\pi(\sigma=(1,0)\mid s_A = H \wedge m'_A = 1) = F\Big(\Big[\frac{\varphi_L L}{2lk}\Big]\varphi_H H - \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L\Big) - 2C$$

The payoffs of deviation for the low-skilled worker B is given by:

$$\pi(\sigma=(1,0)\mid s_B = L \wedge = m'_B = 0) = F\Big(\Big[\frac{H}{2lk}\Big]H - \Big[\frac{H}{2lk}\Big]L\Big)$$

By comparing the payoffs of this strategy I find the conditions for which parameters the manager is enticed to credibly train the workers according to his strategy. The manager will not deviate from the equilibrium strategy with regards to the high-skilled worker A when the following condition holds:

$$\pi(\sigma=(1,0)) \geq \pi(\sigma=(1,0)\mid s_A = H \wedge m'_A = 1)$$
$$F\Big(\Big[\frac{H}{2lk}\Big]H - \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L\Big) - C \geq F\Big(\Big[\frac{\varphi_L L}{2lk}\Big]\varphi_H H - \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L\Big) - 2C$$
$$\Leftrightarrow$$
$$C \geq F\Big(\Big[\frac{\varphi_L\varphi_H HL - H^2}{2lk}\Big]\Big)$$

The manager will not deviate from the equilibrium strategy with regards to the low-skilled worker B when the following condition holds:

$$\pi(\sigma=(1,0)) \geq \pi(\sigma=(1,0)\mid s_B = L \wedge = m'_B = 0)$$
$$F\Big(\Big[\frac{H}{2lk}\Big]H - \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L\Big) - C \geq F\Big(\Big[\frac{H}{2lk}\Big]H - \Big[\frac{H}{2lk}\Big]L\Big)$$
$$\Leftrightarrow$$
$$F\Big(\Big[\frac{HL - \varphi_L^2 L^2}{2lk}\Big]\Big) \geq C$$

The incentive compatibility constraints show that it is possible for the manager to credibly play this training strategy when training is not beneficial enough with respect to high-skilled workers and demotivating enough with respect to the low-skilled workers.

However, this training strategy will never be played by the manager given the payoff structure of Equation 19, as this training strategy will be strictly payoff dominated by the training strategy $\sigma = (0,1)$. A rational manager will never train the workers according to the given strategy, as a higher payoff can be realized by training the worker according to the training strategy $\sigma = (0,1)$. In anticipation of the subsection in which this particular strategy is regarded as a Perfect Bayesian Equilibrium, I will show that the training strategy where only low-skilled workers are trained is not a Bayesian Nash Equilibrium as a result of iterated elimination as a strictly dominated strategy. The payoff of strategy $\sigma = (0,1)$ is given by the following equation:

$$\pi(\sigma = (0,1)) = F\Big(\Big[\frac{\varphi_H H}{2lk}\Big]\varphi_H H - \Big[\frac{L}{2lk}\Big]L\Big) - C$$

Comparison of these strategies shows the following condition with regards to the payoffs:

$$\pi(\sigma = (1,0)) < \pi(\sigma = (0,1))$$

$$F\Big(\Big[\frac{H}{2lk}\Big]H - \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L\Big) - C < F\Big(\Big[\frac{\varphi_H H}{2lk}\Big]\varphi_H H - \Big[\frac{L}{2lk}\Big]L\Big) - C$$

$$\Leftrightarrow$$

$$F\Big(\Big[\frac{(1-\varphi_H^2)H^2 - (\varphi_L^2 - 1)L^2}{2lk}\Big]\Big) < 0$$

This rewritten equation shows how strategy $\sigma = (1,0)$ is not a Bayesian Nash Equilibrium. From the equation above, and the assumptions of the model, it follows that $(1-\varphi_H^2)$ must be equal or lower than zero and that $(\varphi_L^2 - 1)$ is strictly larger than zero. Therefore, the combination of these two products makes it impossible to conclude that $\pi(\sigma = (0,1)) \leq \pi(\sigma = (1,0))$. Strategy $\sigma = (1,0)$ is therefore a never-best response of the manager, as it is strictly payoff dominated by $\sigma = (0,1)$.

Training high-skilled workers

Now, let us consider the most intuitive strategy given the payoff definition of Equation 19. In this strategy low-skilled workers are demotivated by not receiving training (indicating their low skill) and high-skilled workers are motivated by learning about their skill level and receiving performance-enhancing training. The expected payoffs of this strategy are given by:

$$\pi(\sigma = (0,1)) = F\Big(\Big[\frac{\varphi_H H}{2lk}\Big]\varphi_H H - \Big[\frac{L}{2lk}\Big]L\Big) - C$$

Deviation by not giving training to the high-skilled worker A, who will then believe he is then low-skilled, will result in the following payoff:

$$\pi(\sigma = (0,1) \mid s_A = H \wedge m'_A = 0) = F\Big(\Big[\frac{L}{2lk}\Big]H - \Big[\frac{L}{2lk}\Big]L\Big)$$

Deviation by providing training to the low-skilled worker B, who will then believe he is high-skilled, will result in the following payoff:

$$\pi(\sigma = (0,1) \mid s_B = L \wedge m'_B = 1) = F\Big(\Big[\frac{\varphi_H H}{2lk}\Big]\varphi_H H - \Big[\frac{\varphi_H H}{2lk}\Big]\varphi_L L\Big) - 2C$$

Then, the parameters are found for which the manager will not want to deviate when:

$$\pi(\sigma = (0,1) \mid s_A = H \wedge m'_A = 1)) \geq \pi(\sigma = (0,1) \mid s_A = H \wedge m'_A = 0)$$
$$F\Big(\Big[\frac{\varphi_H H}{2lk}\Big]\varphi_H H - \Big[\frac{L}{2lk}\Big]L\Big) - C \geq F\Big(\Big[\frac{HL - L^2}{2lk}\Big]\Big)$$
$$\Leftrightarrow$$
$$F\Big(\Big[\frac{\varphi_H^2 H^2 - HL}{2lk}\Big]\Big) \geq C$$

This equation shows how $\sigma = (0,1)$ will not be an equilibrium with regards to the high-skilled worker for very high values of C, as the decision to provide training to the high-skilled worker will not result in a net expected profit for the manager.

$$\pi(\sigma = (0,1) \mid s_B = L \wedge m'_B = 0)) \geq \pi(\sigma = (0,1) \mid s_B = L \wedge m'_B = 1)$$
$$F\Big(\Big[\frac{\varphi_H H}{2lk}\Big]\varphi_H H - \Big[\frac{L}{2lk}\Big]L\Big) - C \geq F\Big(\Big[\frac{\varphi_H H}{2lk}\Big]\varphi_H H - \Big[\frac{\varphi_H H}{2lk}\Big]\varphi_L L\Big) - 2C$$
$$\Leftrightarrow$$
$$C \geq F\Big(\Big[\frac{L^2 - \varphi_L\varphi_H HL}{2lk}\Big]\Big)$$

When the manager exclusively trains high-skilled workers, he will never deviate of the equilibrium path with regards to the low-skilled worker. This deviation is stricty payoff dominated by this equilibrium strategy, as a result of transitivy (because of the property $[L^2 - \varphi_L\varphi_H HL < 0]$, this deviation will result in a strictly decreasing tournament winning probability for the high-skilled worker) and the costliness of training.

No training

Now consider the pooling strategies. The training strategy in which none of the workers are trained, $\sigma = (0,0)$, gives the following expected payoffs:

$$\pi(\sigma = (0,0)) = F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H - \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L\Big)$$

The expected payoffs are then given by the following equation, given that the out-of-equilibrium belief is that receiving training indicates high skill ($\min\{\varphi_L L, \varphi_H H\} = \varphi_H H$):

$$\pi(\sigma = (0,0) \mid s_A = H \wedge m'_A = 1) = F\Big(\Big[\frac{\varphi_H H}{2lk}\Big]\varphi_H H - \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L\Big) - C$$

The expected payoffs given the out-of-equilibrium belief that training indicates low skill ($\min\{\varphi_L L, \varphi_H H\} = \varphi_L L$) is given by the following equation:

$$\pi(\sigma = (0,0) \mid s_A = H \wedge m'_A = 1) = F\Big(\Big[\frac{\varphi_L L}{2lk}\Big]\varphi_H H - \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L\Big) - C$$

Comparison of the payoffs of the equilibrium strategy and the payoff of the two out-of-equilibrium skill beliefs give us the parameters for which the strategy $\sigma = (0,0)$ is a Perfect Bayesian Equilibrium:

$$\pi(\sigma = (0,0)) \geq \pi(\sigma = (0,0) \mid s_A = H \wedge m'_A = 1 \wedge \min\{\varphi_L L, \varphi_H H\} = \varphi_H H)$$

$$F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H - \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L\Big) \geq F\Big(\Big[\frac{\varphi_H H}{2lk}\Big]\varphi_H H - \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L\Big) - C$$

$$\Leftrightarrow$$

$$C \geq F\Big(\Big[\frac{\varphi_H^2 H^2 - \alpha H^2 - (1-\alpha)HL}{2lk}\Big]\Big)$$

$$\vee$$

$$\pi(\sigma = (0,0)) \geq \pi(\sigma = (0,0) \mid s_A = H \wedge m'_A = 1 \wedge \min\{\varphi_L L, \varphi_H H\} = \varphi_L L)$$

$$F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H - \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L\Big) \geq F\Big(\Big[\frac{\varphi_L L}{2lk}\Big]\varphi_H H - \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L\Big) - C$$

$$\Leftrightarrow$$

$$C \geq F\Big(\Big[\frac{\varphi_L \varphi_H HL - \alpha H^2 - (1-\alpha)HL}{2lk}\Big]\Big)$$

The conditions above show it is not interesting for the manager to deviate with regards to high-skilled workers from strategy $\sigma = (0,0)$ when the benefits of training are high enough to offset the costs of training. Now the same incentive compatibility constraints are shown with regards to the low-skilled worker for this strategy. The expected payoffs of training the low-skilled worker B are given by the following equation, given that the out-of-equilibrium belief is that receiving training indicates high skill ($\min\{\varphi_L L, \varphi_H H\} = \varphi_H H$):

$$\pi(\sigma = (0,0) \mid s_B = L \wedge m'_B = 1) = F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H - \Big[\frac{\varphi_H H}{2lk}\Big]\varphi_L L\Big) - C$$

The expected payoffs given the out-of-equilibrium belief that training indicates low skill ($\min\{\varphi_L L, \varphi_H H\} = \varphi_L L$) is given by the following equation:

$$\pi(\sigma = (0,0) \mid s_B = L \wedge m'_B = 1) = F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H - \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L\Big) - C$$

Comparison of the payoffs of the equilibrium strategy and the payoff of the two out-of-equilibrium skill beliefs give us the parameters for which the strategy $\sigma = (0,0)$ is a Perfect Bayesian Equilibrium:

$$\pi(\sigma = (0,0)) \geq \pi(\sigma = (0,0) \mid s_B = L \wedge m'_B = 1 \wedge \min\{\varphi_L L, \varphi_H H\} = \varphi_H H)$$

$$F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H - \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L\Big) \geq F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H - \Big[\frac{\varphi_H H}{2lk}\Big]\varphi_L L\Big) - C$$

$$C \geq F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L - \Big[\frac{\varphi_H H}{2lk}\Big]\varphi_L L\Big)$$

$$\vee$$

$$\pi(\sigma = (0,0) \mid s_B = L) \geq \pi(\sigma = (0,0) \mid s_B = L \wedge m'_B = 1 \wedge \min\{\varphi_L L, \varphi_H H\} = \varphi_L L)$$

$$F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H - \Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L\Big) \geq F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]H - \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L\Big) - C$$

$$C \geq F\Big(\Big[\frac{\alpha H + (1-\alpha)L}{2lk}\Big]L - \Big[\frac{\varphi_L L}{2lk}\Big]\varphi_L L\Big)$$

The condition above shows that deviation with respect to the low-skilled worker B might only be in the best interest for the manager, when the probability of winning the tournament bonus increases by demotivatig the low-skilled worker by signalling he is low-skilled given that the out-of-equilibrium belief indicates this skill type upon receiving training. When the out-of-equilibrium belief dictates that receiving training indicates the high skill type, then deviation of this equilibrium strategy is never in the interest of the manager as it follows from transitivity that $[(\alpha H + (1-\alpha)L) - \varphi_L \varphi_H HL < 0]$. This means that the probability of winning the tournament bonus will never rise by training the low-skilled worker for this out-of-equilibrium belief, so that the manager will never deviate with regards to the low-skilled worker when the out-of-equilibrium beliefs indicate a high skill type.

Training all workers

Lastly, consider the training strategy $\sigma = (1,1)$. It might seem illogical to provide training to both the high-skilled and the low-skilled worker, but it could be possible that in the absence of information transmission there are values of φ_L, φ_H, L and H for which this training strategy could be a Perfect Bayesian Equilibrium. Let the payoffs of this strategy be given by the following equation:

$$\pi(\sigma = (1,1)) = F\Big(\Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_H H - \Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_L L\Big) - 2C$$

Therefore, the manager has an incentive to deviate to giving no training to the low-skilled worker. The payoffs of this deviation are then given by the following equations for respectively the low- and high-skilled worker:

$$\pi(\sigma = (1,1) \mid s_A = H \wedge m'_A = 0) = F\Big(\Big[\frac{L}{2lk}\Big]H - \Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_L L\Big) - C$$

$$\pi(\sigma = (1,1) \mid s_B = L \wedge m'_B = 0) = F\Big(\Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_H H - \Big[\frac{L}{2lk}\Big]L\Big) - C$$

By equating the payoffs of the strategy and the payoffs of deviation we find the following equations:

$$\pi(\sigma = (1,1)) \geq \pi(\sigma = (1,1) \mid s_A = H \wedge m'_A = 0)$$

$$F\Big(\Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_H H - \Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_L L\Big) - 2C$$

$$\geq$$

$$F\Big(\Big[\frac{L}{2lk}\Big]H - \Big[\frac{\alpha\varphi_H H + (1-\alpha)\varphi_L L}{2lk}\Big]\varphi_L L\Big) - C$$

$$\Leftrightarrow$$

$$F\Big[\Big[\frac{\alpha\varphi_H^2H^2+(1-\alpha)\varphi_L\varphi_H HL-HL}{2lk}\Big]\Big]\geq C$$

Contrary to $\sigma=(1,0)$, we cannot conclude that the strategy $\sigma=(1,1)$ cannot be a Perfect Bayesian Equilibrium with respect to the high-skilled worker. For low values of C and a high product of φ_L, φ_H, L and H, this strategy will not necessarily be strictly dominated by the strategy $\sigma=(0,1)$. However, when training is considered for worker B we see that training for low-skilled workers is troublesome for the existence of a Perfect Bayesian Equilibrium given strategy $\sigma=(1,1)$:

$$\pi(\sigma=(1,1))\geq\pi(\sigma=(1,1)\mid s_B=L\wedge m'_B=0)$$

$$F\Big(\Big[\frac{\alpha\varphi_H H+(1-\alpha)\varphi_L L}{2lk}\Big]\varphi_H H-\Big[\frac{\alpha\varphi_H H+(1-\alpha)\varphi_L L}{2lk}\Big]\varphi_L L\Big)-2C$$

$$\geq$$

$$F\Big(\Big[\frac{\alpha\varphi_H H+(1-\alpha)\varphi_L L}{2lk}\Big]\varphi_H H-\Big[\frac{L}{2lk}\Big]L\Big)-C$$

$$\Leftrightarrow$$

$$F\Big(\Big[\frac{L^2-\alpha\varphi_H\varphi_L HL-(1-\alpha)\varphi_L^2L^2}{2lk}\Big]\Big)\geq C$$

Because of the property $[L^2-\alpha\varphi_H\varphi_L HL-(1-\alpha)\varphi_L^2L^2]<0$, this strategy cannot be an equilibrium with regards to low-skilled workers, as the manager will always want to deviate of the equilibrium path by not training these workers. There are no benefits of training the low-skilled worker, which will result in the impossibility to offset the costs of training. That means that this strategy is not consistent for all skill types, as the manager will always deviate with respect to low-skilled workers. ■

The proof of Proposition 2 shows how the manager is not enticed to provide training to the low-skilled worker, even when this training decision could result in a demotivating effect for the low-skilled worker. The manager is strictly better off by signalling indications of low- and high-skilled by exclusively training high-skilled workers. The proof of Proposition 2 indicates that the strategy $\sigma=(1,1)$ is not a Perfect Bayesian Equilibrium as the manager will be enticed to deviate with respect to low-skilled workers, but will still be a Bayesian Nash Equilibrium as this training strategy will be more profitable for certain values and products of φ_L, φ_H, L and H than the expected payoffs of the strategy $\sigma=(0,1)$. The partial finding that this strategy can be a Bayesian Nash Equilibrium (in which no restrictions are placed on the beliefs of the workers) is interesting for the purposes of this thesis and this proposition in particular.

Proposition 2 indicates that the combination of several incentives might result in different values of the parameters under which strategies are a Perfect Bayesian Equilibrium. Given the strength of the various incentives in the organization, it is possible that the manager will be induced to provide training to the workers based on motives other than the realization of output I assumed in the base model of this thesis. The management of the organization should be careful in designing the incentive structures of the manager, as the discrete decision to provide training might result in adverse effects on the motivation and productivity of the workers when multiple high-powered incentives co-exist in the sense of Williamson (1989) and Lazear (2000). This particular finding has been readily confirmed in the academic literature, where it has been shown that multiple high-powered incentives can lead to adverse behaviour in principal-agent settings due to the conflicting effects on worker and manager objectives (Holmstrom and Milgrom, 1991). When several objectives are pursued and rewarded simultaneously, this might possibly lead to adverse training decisions of the manager which might not be in the best interest of the firm.

ii. Stochastic error term distribution

In the previous sections we assumed that the distribution of the stochastic error term is uniformly distributed, so that a very large mistake can occur with the same probability as a very small mistake. This has useful properties in determining the best responses of the workers and the manager, as the expected marginal probability of the players will not be affected by the training decision of the manager with regards to the other player under simultaneous decision making. However, in reality the probability of smaller production measurement errors is more likely to occur than very large production measurement errors.

In this first part of the discussion we change the stochastic error term distribution from a uniform to a normal distribution. As we have illustrated before, the tournament model is an applied all-pay auction in a personnel setting, where the expected marginal probability of winning the auction given the expected bids of the other players is equal to the costs of the optimal bid. A change in distribution means that the optimal bid will change due to the differences in the probability density curves of the stochastic error term. In such a setting will the optimal bid no longer be independent of the other players' bids, as the marginal expected probability of winning the tournament will be lower for ex ante observable differences in productivity. As Lazear and Rosen (1981) showed the introduction of handicaps in tournaments can result in large differences in effort exertion.
Let the stochastic error shock be distributed normally, so that:

$$\Delta\varepsilon \sim N[0, Var] \tag{20}$$

Under a normal distribution of the stochastic error term the optimal effort level has to satisfy the following condition:

$$ke_A^* = E_{s_A,s_B,\Delta\varepsilon}[f(e_B E(s'_B \mid m'_B) - e_A E(s'_A \mid m'_A) \geq \Delta\varepsilon) s'_A \mid \sigma'] \tag{21}$$

This condition shows that competitors are pushed to perform harder when they believe that they are tied in the tournament. Ex ante differences in the competitiveness of the players will result in a deterioration of the effort levels of both workers, as the advantaged worker will have to exert less effort for his optimal probability of winning the tournament. At the same time will the disadvantaged worker exert less effort, as it is no longer optimal to exert more effort than it is marginally beneficial for him to do so. The allocation of training is literally a form of handicap as proposed by Rozen and Lazear. Training affects the competition inside a tournament by directly improving the marginal output of the workers. As the allocation also contains information about the skill levels of the workers, this learning effect will result in an even more unbalanced tournament.

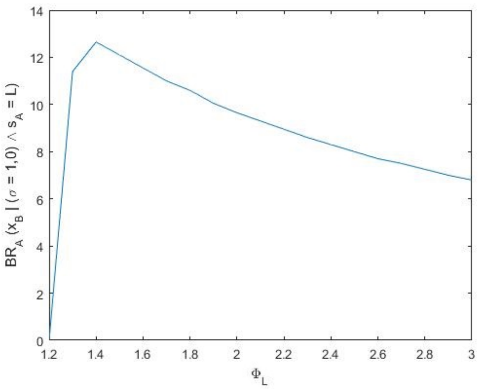

Figure 12: $e_A^*(s_A = L \mid \varphi_L \wedge x_B = 10)$

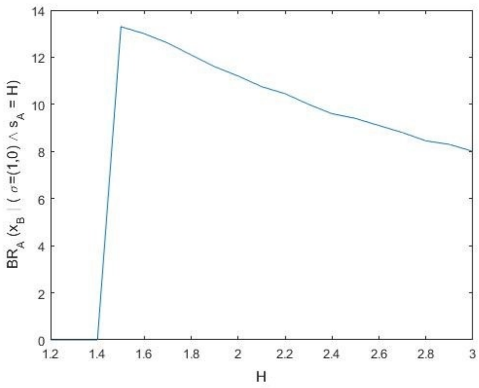

Figure 13: $e_A^*(s_A = H \mid H \wedge x_B = 10)$

In Figures 12 and 13 I illustrate the effect of training φ_L and H under the strategy $\sigma = (1,0)$ on the optimal effort decision for a lower-skilled and a higher-skilled worker. For simplicity I assume that the output score of worker B is fixed at a certain level. Consider a low-skilled and a high-skilled worker in this tournament game, where the manager plays the strategy $\sigma = (1,0)$.

When the product of $\varphi_L L$ is lower than H, the strength of the training will be too low for the low-skilled worker in becoming competitive with the high-skilled worker. Then, differentiation implies that the low-skilled worker is less competitive than the high-skilled worker. The training decision demotivates the lower-skilled worker, which results in a lower effort level for the low-skilled worker as exerting effort is expected to have a lower payoff. When the strength of training increases, the low-skilled worker will find himself becoming increasingly competitive with regards to the other player. When the difference between $\varphi_L L$ and H decreases, the worker will find himself more motivated to exert effort as the marginal probability of outperforming the other worker increases.

Figures 12 and 13 indicate that the highest amount of effort is exerted by the low- and high-skilled workers when the training decision of the manager results in a more balanced competition between the workers. This is shown by the blue line that depict the optimal effort level of the workers, which illustrates a convex shape and is skewed to the right given the strength of H and φ_L. In the previous example we have seen that the effort levels chosen by the workers increase in the strength of training and skill. The highest amount of effort is exerted when the training decision and the information transmission as a result of credible differentiation between the workers leads to a new, more balanced tournament between the workers. Under the condition $\varphi_L L = H$ the workers will expect to be ex ante equally capable of winning the tournament. Under a normal distribution of the stochastic error shock a small increase in effort of worker A over worker B will then result in a large marginal increase in the probability of winning the tournament, as small errors in the stochastic error shock are more likely than large errors. Worker B will anticipate this move and will exert more effort as well in order to balance the scales of this tournament in his favour. This process repeats itself until the condition of Equation 21 is reached, where workers A and B exert the same high amount of effort in which the marginal probability of winning the tournament is equal to the marginal costs of effort.

When the strength of training results in the situation where $\varphi_L L > H$, the tournament becomes more imbalanced as a result of the decision of the manager to give an advantage to the low-skilled worker. Given that we are still in an interior solution where all workers exert a positive amount of effort, then this condition results in a decrease of effort exerted for increases in training strength φ_L. As small errors are more likely than large errors due to the normal distribution of the stochastic error term the marginal probability of winning the tournament still increases but decelerates in a rise in effort exerted. As we have assumed that the cost function of effort is convex, this implies that the tournament advantage of training will result in a lower necessity of exerting a high amount of effort, as the same probability of winning the tournament is reached with a lower amount of effort exerted. This getting-ahead-effect will result in less output produced due to a decrease in effort of both workers as a result of the tournament imbalances. When the objective of the manager is to produce output, then the manager will want to avoid differentiating between the employees in order to keep the tournament competitive.

This process can be illustrated by means of a numerical example. Assume that the output of the rivalling worker is fixed and that the variance of the stochastic error term and cost of effort parameter k are given the following values:

$$x_B = 10$$
$$Var = 5$$
$$k = 0.015$$

These assumptions will enable us to construct an example of the effects of training in isolation. By assuming that the output of the rivalling worker is fixed, the high-skilled will only have to take the changes in the tournament chances in the strength of H or φ_L into account. The other calibration values used in this analysis remain the same as used in the previous figures of the base model. Suppose that the parameters φ_L and H are fixed, so that: $\varphi_L = 1.5$ and $H = 1.5$. Then, a numerical example shows how the expected effort and output levels change in the distribution of the error term.

Tables 4-7 show how the best responses of the workers differ for an uniform distribution and a normal distribution of the stochastic error term. Comparison of the effort levels and productivity in isolation for single workers shows how the statics change given the difference of the normally distributed stochastic error term compared to the uniform distribution of the stochastic error term. The amount of effort exerted by the low-skilled worker has dropped compared to the amount of effort exerted under a uniform distribution. This numerical example shows how differentiation under a normal distribution results in less effort exerted compared to the non-differentiating strategy, while under a uniform distribution differentiation will result in more effort exerted by both skill types of workers.

Non-differentiation results in an ex ante equal probability of winning the tournament, so that the marginal utility of exerting effort is very high when a tie is assumed given that the stochastic error term is normally distributed. Under an uniform distribution the expected marginal probability of effort is not influenced by the effort decision of the other worker, contrary to the case of differentiation under a normal distribution where the effort decision of the other worker is influenced by the effort decision. Therefore, differentiation has large effects on the decision of the manager to provide training to only one of

Table 3: Numerical example

Table 4:

$\Delta\varepsilon \sim U[-l,l]$		
$s_A = L$	$\sigma = (0,0)$	$\sigma = (1,0)$
e_A^*	6.67	10
$E(x_A)$	8	24
C	0	5.5
π	8	18.5

Table 5:

$\Delta\varepsilon \sim U[-l,l]$		
$s_A = H$	$\sigma = (0,0)$	$\sigma = (1,0)$
e_A^*	6.67	11.5
$E(x_A)$	13.33	16.67
C	0	0
π	13.33	16.67

Table 6:

$\Delta\varepsilon \sim N[0,\sigma]$		
$s_A = L$	$\sigma = (0,0)$	$\sigma = (1,0)$
e_A^*	13	11.5
$E(x_A)$	15.6	27.6
C	0	5.5
π	15.6	22.1

Table 7:

$\Delta\varepsilon \sim N[0,\sigma]$		
$s_A = H$	$\sigma = (0,0)$	$\sigma = (1,0)$
e_A^*	13	9.5
$E(x_A)$	26	19
C	0	0
π	26	19

the workers. This example shows that differentiation under a normal distribution can be detrimental for the motivation of the workers to exert large amounts of effort. When the workers believe they are not capable in competing with a positive marginal probability given their effort decision they will exert zero effort. Likewise, we find that strong workers will decrease their optimal effort in the strength of their handicap. Highly skilled workers will exert less effort, as their marginal probability of winning the tournament rises compared to the constant marginal costs of effort. Under a normal distribution we find that observable imbalances between the workers result in less effort exerted by all type of workers, which might make differentiation through training a less attractive option for the manager. In this numerical example it is still beneficial for the manager to provide training, despite the sharp decrease in effort exerted by both the low-skilled and high-skilled workers.

The indications of this section are in line with the academic literature on tournament games. Training and development in tournament schemes can result in a handicap for one of the contestants, which can be detrimental for the probability of winning of the disadvantaged worker and beneficial for the advantaged worker's probability of winning. The management of the organization should consider the distribution of the stochastic error term when considering the possible adverse effects of differentiation in training of the workers. Training decisions in organizations where measurement errors are likely offer an interesting perspective regarding a known phenomenon in the economics of management and organization, because the tendency to avoid differentiation when large differences in the stochastic error shock are irregular will result in the decision of the firm to train all or none of the workers. When the training is effective for both lower-skilled and higher-skilled workers, the optimal decision of the manager could be to avoid differentiation and reward everyone with training. This explains a potential centrality bias in which managers train all workers, even when differentiation could result in higher effort exerted as a result of learning, should all workers keep their exerted effort constant. The differences in behavior we find in this model under a normal distribution therefore adds a new perspective to the literature on the centrality bias, that was introduced by Murphy and Cleveland (1991) and Landy and Farr (1980).

V. Conclusion

In this thesis I proposed a model in which I explore the possible effects of the training decisions of managers on the motivation and productivity of the workers. By extending a ranked-order tournament model with a managerial discrete training decision I formulated several mathematical propositions that indicate possible adverse effects of training. Training might not always result in a positive change in motivation and effort. There is a wide range of effects as a result of training, which may depends on the characteristics of the firm, such as incentives, training characteristics, worker characteristics, managerial behaviour and the level of competition in the organization. I find that there is a large range of parameters in which one of the four training strategies can be an equilibrium strategy, which might predict the expected changes in motivation and producivity of the workers as a result of training.

Another finding of this thesis is that the decision to differentiate in training the workers may depend on the distribution of the stochastic error term. When measurement errors are common and large we find that differentiating strategies are more likely to be played by the manager compared to the situation in which large measurement errors are unlikely. This particular finding contributes to the existing literature on tournament games, as differentiating training strategies may result in uneven balanced tournament games, which can severely affect the motivation of employees to exert high levels of effort. This thesis sheds light on the role of training in the sense of organizational phenomena, such as the centrality bias, leniency bias and the reluctance of supervisors to differentiate between workers. This thesis suggests that the manager can avoid differentiating by training all of the workers, regardless of their skill and impact of the training. Non-differentiation has been given another dimension in a tournament setting by suggesting all workers require training, revealing no demotivating information in the process. It suggests that both employees might benefit from the training, even when the benefits for high-skilled worker are significantly lower than for low-skilled workers.

Naturally, this micro-economic thesis on the impact of training has a very specific game-theoretic focus. A lot of micro-economic research remains to be done in order to strengthen the theoretical basis of this proposition. The equilibria of this thesis have not been thoroughly refined, which makes it hard for this thesis to provide the reader with unique predictions of the effect of training on worker motivation and productivity. Other suggestions for further research could be to take the possibility of the manager into consideration to apply mixed training strategies. The overview of equilibria in this thesis suggest a variety of parameters for which no training strategies are a Perfect Bayesian Equilibrium. The continuity of these pure strategies indicate it should be possible to find several combinations of skill and training in which the manager mixes the training strategies. Given that in real life there could be some noise in the information transmission and observability of skill levels I find it plausible that a mix of strategies could provide additional explanations for the effect of training on motivation and productivity. It will be interesting to see how this extension changes the equilibria. Furthermore, in this thesis I also made some strong assumptions regarding the information distribution of the workers and the manager. I assumed perfect information for the manager in order not to convolute my research. However, one can wonder how observable and measurable skill, talent, emotional quotient, work ethic and self-confidence are. The interaction of all these factors with motivation and productivity can make certain theoretical predictions hard and could be explored further by integrating these factors as extensions of this model. Finally, I believe that the discrete variables of skill form a good start of this training model. However, reality does not make a distinction between high- and low-skilled workers. I have considered constructing this model with a Tullock function of the skill distribution, but I abandoned the Tullock function because of the complexity of the model. I believe that the introduction of a Tullock function will result in a more realistic theoretical model.

The results of this micro-economic theory might be interesting in performing additional research on the motivation and productivity effects of training by means of empirical experiments. Surveys and lab experiments could be used as a start in order to find out the effect of training on motivation and productivity. Once more determinants of motivation and productivity have been uncovered in certain market segments it might be interesting to perform a field experiment in order to uncover the true effects of training on motivation and productivity. Implications of the propositions of this thesis are that the firm will have to carefully tailor its human capital practices and adjust its organization in order to let training have the right effects on the workers.

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A. PROOF OF LEMMA 1

Suppose that Assumption 1 does not hold and that worker A is advantaged over worker B. Given that both the advantaged and the disadvantaged worker in the tournament are aware of the ex ante marginal value of exerting effort in the tournament, the advantaged worker will be able to win the tournament game with certainty when Assumption 1 does not hold: $Pr(x_A > x_B) = 1$ and $Pr(x_B > x_A) = 0$. According to Equation 8 we then expect to find atoms in the following effort levels: $e_A^* = \frac{1}{k}, e_B^* = 0$. Backward inducing workers could then reason how worker A could reduce his effort level by an arbitrarily small amount in order to receive higher payoff. Sequentially worker B will have an incentive to invest an arbitrarily small amount of effort in order to have a marginally positive probability of winning the tournament. A mixed equilibrium remains in which the workers mix the possibility of an atom in zero and an atom in e^*, which is not interesting for the purposes of this thesis. Therefore, the presence of atoms given $s'_A e^*_A + \Delta\varepsilon > l$ result in corner solutions in which the workers might not have incentives to exert effort. Given that the condition $l < s'_A e^*_A + \Delta\varepsilon < l$ holds workers A and B will always have a positive expected probability of winning the tournament, regardless of the (dis-)advantage of the other worker: $f(\cdot) > 0 \; \forall \; s'_A, k, l$. The monotonic properties of the uniform cumulative density functions result in a strictly positive expected payoff of exerting effort where the tournament probabilities cannot be affected by the effort decisions of the other workers. Lemma 1 implies that when the extreme values of k and l are substituted in Assumption 1, we find an interior solution in which both the advantaged and the (dis-)advantaged worker exert the amount of effort that is strictly positive and not dependent on the training decision m' of the manager. The worker will exert the amount of effort that is expected to be marginally optimal as a result of the training strategy of the manager:

$$e^*_A(\sigma \mid m'_A) = \left[\frac{E(s'_A \mid m'_A)}{2lk} \right] > 0 \tag{22}$$