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The Significance of the Vega and the Volatility Skew in the Hedging and Pricing of Index Options

Abstract

This study focuses on the inverse relationship between the price changes of an asset and its volatility. The fact that volatility is time-varying and correlated with the movement of the asset is of great significance for an investor. This inverse relationship can be captured by the skew-adjusted term. The skew-adjusted term is a combination of the volatility skew, the phenomena that the option's implied volatility depends in practice upon the option's moneyness and time to maturity, and the vega, the sensitivity of an option price to changes in the underlying volatility. This thesis shows that the delta-hedging performance of the Black & Scholes model can significantly be improved by taking the skew-adjusted term into account. Empirical test based on both actual market data and simulated data conclude that the skew-adjusted delta is superior to the standard Black & Scholes delta in terms of hedging performance. In addition, this study examines whether the skew-adjusted term is a potential risk factor that is a source of risk premium. After investigating the returns of a self-financing portfolio based on the skew-adjusted term, it is possible to conclude that the skew-adjusted term is not a source of mispricing.

Keywords: Delta-hedging, Black & Scholes model, Volatility skew, Vega, Skew-adjusted delta

JEL Classification: G10, G11, G12, G13 and G31

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Finish date:	4th of August, 2016
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1. Introduction

Many investors believe that the Black & Scholes model is indispensable in the pricing and hedging of index options. This is despite the fact that numerous empirical studies conclude that the model does not always describe price dynamics correctly and has a number of flaws. In particular, the assumption that the volatility of the underlying index is constant does not hold in practice and is violated. Instead of being constant, it has been acknowledged that volatility tends to be negative and inversely related to the price changes of the underlying index and, in addition, time-varying (Black, 1976). This misspecification in the Black & Scholes model manifests itself as the volatility skew. The volatility skew indicates that the implied volatility of an index option tends to decrease with the height of the strike price (Rubinstein, 1994).

This paper focuses on the inverse relationship between the price changes of an underlying index and its volatility, and the consequences of this relationship for the delta-hedging process. Since volatility is a vital determinant of a hedge ratio, an incorrect volatility assumption may result in an incorrect delta and, ultimately, hedging error. Because of this inverse relationship, the delta must account for both the direct effect of the price change of the underlying on the price of the option and for the indirect impact of the simultaneous volatility change on the price of the option. This inverse relationship is thoroughly investigated by authors such as Coleman (2000), Engle and Rosenberg (2000) and Mixon (2002).

It is possible to capture this relationship between the price changes of an underlying index and its volatility by a so-called skew-adjusted term. The skew-adjusted delta uses the volatility skew in combination with the vega of the option to address the inverse relationship (Derman et al, 1996). If the volatility skew is downward sloping, the standard Black & Scholes delta needs to be adjusted downwards. This is achieved by decreasing the implied volatility of each strike price with an amount defined by the slope of the current skew (Derman, 1994). The vega of an option measures the impact of these changes in the underlying volatility on the option prices. Therefore, the volatility skew and the vega interfere with each other and both have an effect on the price of an option. In order to determine the size of the aggregate effect on the price of the option, we compute the skew-adjusted term, which is the vega of an option multiplied by the slope of the volatility skew.

The purpose of this study is to examine whether investors should incorporate the skew-adjusted term in their decision-making process. This is done by investigating the importance of the skew-adjusted term from a hedging- and a trading perspective.

This thesis starts with investigating whether it is possible to improve the delta-hedging[1] performance of the standard Black & Scholes model by taking the skew-adjusted term into consideration. Large amounts of empirical research have been done on option pricing models beyond the standard Black & Scholes model.

[1] Delta-hedging is the process of adjusting a portfolio in such a way that is remains delta-neutral. A delta-neutral portoflio is a portfolio that is unaffected by small changes in the value of the underlying security.

Nowadays, practitioners are focusing on option pricing models that are becoming more and more complex. Examples of these developments are stochastic- and deterministic volatility models. However, this paper focuses on the standard Black & Scholes model and contributes to the existing literature by examining whether the hedging performance of the Black & Scholes can be improved with a simple modification of the standard Black & Scholes delta.

This simple modification, the skew-adjusted term, allows an investor to capture the inverse relationship between the price changes of the underlying index and the volatility of the underlying index in a relative straightforward way. Investors only need to obtain the vega and the slope of the volatility skew of an option. Both characteristics are directly observable and easily measurable in the market.

Besides the hedging-part, this study examines whether trading- and pricing performance can be improved by taking the skew-adjusted term into consideration. Although, a large number of studies have been devoted on the relationship between the pricing of options and the underlying assets, only a limited amount of researchers have focused on understanding the nature of option returns.

It has been widely accepted that index options are regularly mispriced. Essentially, certain returns generated by options are excessive in relation to their underlying risks. Undoubtedly, there has been done research on the mispricing of options. However, so far no research has investigated whether the skew-adjusted term is a source of this mispricing. Therefore, remarkably little is known about whether investors view or should view the skew-adjusted term as a source of risk premium.

The remainder of this paper is organized in the following way. Section 2 gives a description of the Black & Scholes model. Section 3 describes the data that is investigated in this paper. Section 4 focuses on the hedging part of this study. This section provides a theoretical framework, a description of how the skew-adjusted delta is derived and the methodology and the final results. Section 5 covers the trading-related part of this study. This section is organized in a similar manner as Section 4, but now from a trading-perspective. Finally, Section 6 summarizes the paper and offers concluding remarks.

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2. Black & Scholes

2.1 Intuition and assumptions

The Black & Scholes model is one of the most significant concepts in today's financial theory in terms of both methodology and applicability. The underlying idea behind the Black & Scholes model is that the market consists of a risky asset and a riskless asset, the risk-free rate. In an ideal scenario investors can hedge away the risk of holding an option by an offsetting position in the underlying stock and the risk-free rate. Basically, the model represents a mathematical description of a financial market consisting of different investment instruments.

From this model it is possible to construct the Black & Scholes formula, which gives an estimated and theoretical price of an option. Black & Scholes (1973) make the following assumptions when deriving the Black & Scholes formula:

- The price of the underlying instrument S_t, follows a Wiener process W_t, with constant drift μ and volatility σ and the price changes are lognormally distributed
- Short selling the underlying stock is possible
- No arbitrage opportunities
- Continuous trading in the stock is possible
- No transaction costs or taxes
- It is possible to borrow and lend cash at a constant risk-free interest rate r_f

2.2 Derivation of the Black & Scholes option pricing formulae

In the Black & Scholes world, a generalized Wiener process called a geometric Brownian motion generates the price of a security. This process describes the price change d_s in terms of a constant drift μ_s of the security, the standard deviation σ_s of the security, the period of time d_t, and a stochastic term γ, which is drawn from a standard normal distribution.

$$d_S = \mu_s d_t + \sigma_s \gamma \sqrt{d}_t \tag{1}$$

This process is also known as a ''random walk''. It lacks a statistical property and consequently it is statistically impossible to generate an expected return other than μ_s, or the risk-free rate plus a risk premium. The magnitude of the risk premium depends on σ_s. Assuming that the earlier mentioned assumptions hold in practice, it is possible to derive the Black & Scholes model with the help of Hull (2003).

The price of a derivative, *c*, depends on the price of the underlying security, *S*. The parameters μ, expiration date *T* and exercise price *K* are all fixed. As a result the change of *c* must be a function of *S* and time *t*. Using Ito's lemma, the following equation can be retrieved from Equation (1).

$$d_c = \left(\frac{\partial c}{\partial S}\mu_s + \frac{\partial c}{\partial t} + \frac{\partial^2 c}{2\partial S^2}\sigma_S^2\, S^2\right)\delta\partial t + \frac{\partial c}{\partial S}\,\sigma_s S_\gamma \sqrt{t} \tag{2}$$

In this equation, both *S* and *c* experience the same underlying Wiener process. Therefore, it is possible to cancel out this process by selecting a portfolio consisting of the underlying stock and the derivative. Thus, a portfolio consisting of one derivative short and $\delta C/\delta S$ shares long is constructed. The value Π of this portfolio is equal to:

$$\Pi = -c + \frac{\partial c}{\partial S}\, S \tag{3}$$

The value of this portfolio changes as time (δt) passes by:

$$\delta\Pi = -d_c + \frac{\partial C}{\partial S}\delta t\, S \tag{4}$$

Substituting Equations (1) and (2) into Equation (4) gives:

$$\delta\Pi = \left(-\frac{\partial c}{\partial t} - \frac{\partial\partial^2 c}{2\partial S^2}\,\sigma_S^2\, S^2\right)\delta t \tag{5}$$

As a result, γ is removed from the equation and consequently the equation is not stochastic anymore. This implies that the portfolio is risk-less during time δt and can only earn the risk-free rate r_f. This is a direct result of the assumption that there are no arbitrage opportunities. Therefore, the change in value of the portfolio during δt is:

$$\delta\Pi = r_f \Pi\, \delta t \tag{6}$$

Combining Equation (3) and (5) gives the following:

$$\left(\frac{\partial c}{\partial t} - \frac{\partial^2 c}{2\partial S^2}\,\sigma_S^2\, S^2\right)\delta t = r_f\left(c - \frac{\partial C}{\partial S}S\right)\delta t \tag{7}$$

Rearranging Equation (7) results in:

$$\frac{\partial c}{\partial t} + r_f S \frac{\partial c}{\partial S} + \frac{\partial^2 c}{2 \partial S^2} \sigma_S^2 S^2 = r_f c \tag{8}$$

This equation is also known as the Black & Scholes partial differentiation equation. Noteworthy is that the earlier terms involving μ_s are cancelled out of the equation. This means that there is no longer a dependence on the drift of *S*. This enables investors to perfectly hedge the option by buying and selling the underlying asset and consequently "eliminate risk". The market only prices risk that can't be diversified away. As a result, there is only one right price for the option:

$$f(S,t) = \max(S_t - K, 0) \text{ at time t} \tag{9}$$

Ultimately, this leads to the following equation for the Black & Scholes price of a call option:

$$c = SN(d_1) - Ke^{-r(T-t)}N(d_2) \tag{10}$$

where *c* represents the call price, *S* is the price of the underlying security, $Ke^{-r(T-t)}$ is a cash position discounted using the relevant risk-free interest rate r_f for the time-to-maturity *(T-t)*, $N(x)$ represents a cumulative distribution, d_1 and d_2 are defined as:

$$d_1 = \frac{\ln(\frac{S}{K})(+\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \text{ and } d_2 = d_1 - \sigma\sqrt{T-t} \tag{11}$$

The term d_1 can be used to calculate the Black & Scholes delta:

$$\delta_{BS} = N(d_1)$$

So, the standard Black & Scholes delta of an option is a function of the underlying security price, *S*, the strike price of the option, *K*, the risk-free interest rate, r_f, the time-to-maturity, *(T-t)* and the volatility of the underlying security, σ. Simply stated, the delta of an option is the amount by which the option will increase or decrease in value if the underlying security moves by one point.

2.3 Shortcomings in the Black & Scholes model and the volatility skew phenomena

One of the most well-known shortcomings of the Black & Scholes model is related to the manner the volatility term is computed. The Black & Scholes implied volatility is a volatility parameter that, inserted into the Black & Scholes formula for that specific option price, results in the Black & Scholes price of the option being equal to the market price of the option. Some investors describe this method as using "the wrong number in the wrong formula to get the right price." So, the Black & Scholes model assumes that the implied volatility skew of an option is flat and constant. However, in reality the implied volatility skew is neither flat nor constant.

This misspecification is most clearly visible in the so-called volatility skew, the phenomena that the option's implied volatility depends in practice upon the option's moneyness and its time-to-maturity. An example of such a volatility skew can be found in Figure 2 in the Appendix.

Obviously, the presence of the volatility skew is of great significance for an investor and should be integrated into his decision-making process. In this study, the volatility skew will be taken into consideration throughout hedging decisions via the skew-adjusted term. In addition, the skew-adjusted term will be tested on profitability potential and whether it is possible to exploit this model imperfection.

In summary, this study will be centered on the skew-adjusted term and its possible added value for an investor both hedging- and trading wise.

3. Data

The data set consists of the prices of the Eurostoxx50 index options on all trading days that occurred between the 2nd of January and the 28th of December in 2015. The EuroStoxx50 Index (SX5E), Europe's leading Blue-chip index for the Eurozone, provides a Blue-chip representation of super sector leaders in the Eurozone. The index consists of 50 of the largest and most liquid stocks in Europe. The SX5E index options are among the most liquid financial contracts in Europe and are traded on the Eurex Exchange.

The expiration date of an option is the last date on which the holder of the option may exercise the option. Option contracts with the four nearest calendar months as expiration date are always available. In addition, the SXE options have additional delivery months in March, June, September and December. The SX5E options expire on the third Friday of the month. Furthermore, there is a wide range of strike prices available for each option serie.

In 2015, the average return of the EuroStoxx50 index was 0,051% and the standard deviation of the daily returns was 0,015. This standard deviation can also be computed as an annualized volatility of roughly 23,4%, which is close to the average value of the at-the-money implied volatility of 21,8%. The inverse relationship between the Eurostoxx50 index and at-the-money implied volatility is observable in Figure 1 in the Appendix and is confirmed by the correlation between the two entities of -0,67.

The risk-free interest rate is proxied by the 10-year Germany Bund. All data is obtained with the support of Bloomberg. This includes the price of the underlying, the last price of each option serie, the delta of each options serie, the vega of each option serie, the implied volatility of each option serie and the risk-free interest rate.

Two exclusionary criteria are applied to the data set. Firstly, options with fewer than 10 or more than 120 trading days are excluded from the data set. As a result, any expiration-related price effects are eliminated from the data set. Additionally, liquidity problems are minimalized as these regularly occur with long-term options. Secondly, options with a moneyness greater than 1.15 or less than 0.85 are excluded from the data set. Moneyness is defined as the ratio of the price of the underlying to the strike price of the option. On average, the trading volume of out-of-the-money and in-the-money options is relatively small. Therefore, these type of options are excluded from the data set. The final sample contains 9665 observations on options with 10 to 120 trading days to maturity and a moneyness between 0.85 and 1.15.

This sample consists of the most actively traded option contracts and, therefore, represents the SX5E options market accurately. The sample is divided into three moneyness categories and two time-to-maturity categories. An option is defined as out-of-the-money if it's moneyness is 0,97 or lower, at-the-money if it's moneyness ratio is more than 0.97 and less than 1.03, and in-the-money if it's moneyness ratio is 1.03 or higher. An option is said to be short-term if it has less than 40 trading days to expiration and long-term otherwise. This dividing based on time-to-maturity and moneyness produces a total of 12 categories.

4. Hedging

4.1 Fundamental theory

As mentioned earlier, an option is not an independent asset and is perfectly replaceable by other investment instruments such as stocks. If an investor owns a stock worth S, he profits if the stock goes up and he loses if it goes down. In other words, an investor has a linear position in ΔS, which simplifies the hedging process. However, an option is an instrument that exhibits convexity. If an investor buys a call option and knows the derivative's delta, $\frac{\partial c}{\partial S}$, he is able to hedge away the linear term in ΔS by shorting that many shares of the stock. In dynamic hedging, hedge parameters are determined at each rebalancing time and the portfolio is adjusted accordingly. Clearly, hedging with an accurate hedge parameter results in a smaller hedging error. The hedging error from applying a delta hedge factor Δ_t for a time period of δt can be measured by the value $\Pi_{t+\delta t}$ of the self-financing portfolio at time $t + \delta t$ .The value of the portfolio $\Pi_{t+\delta t}$ is the same as $\delta\Pi_t$ where:

$$\delta\Pi_t = \Delta_t \delta S_t + r(c_t - \Delta_t S_t)\delta t - \delta c_t \tag{12}$$

According to Coleman (2000) the instantaneous hedging error can be written as:

$$\delta\Pi_t = \sigma S_t \left(\Delta_t - \frac{\partial c}{\partial S}\right) \delta Wt + \mu St \left(\Delta_t - \frac{\partial c}{\partial S}\right) \delta t \tag{13}$$

where W_t is a standard Brownian motion and μ is the drift. Hence, the instantaneous hedging error is determined by the accuracy of the employed delta. In order to minimize the hedging error, the hedging parameter Δ_t should be equal to $\frac{\partial c}{\partial S}$. Only in this scenario, the mean and standard deviation of the instantaneous hedging error $\frac{\delta \Pi_t}{S_t}$ will be equal to zero. Logically, an investor wishes to hedge his portfolio through the most accurate delta as hedging parameter.

Option hedging error can arise from two sources. The first source is that the option value is a continuous nonlinear function of the underlying security and the Black & Scholes hedging process is instantaneous. Logically, this will result in some divergence as only discrete hedging can be achieved. High transaction costs can result in even more infrequent rebalancing and a larger divergence between the delta hedge factor and the actual delta.

Secondly, errors in the underlying pricing model can lead to poor hedging performances. Implied volatility typically exhibits a dependence on both the option strike and maturity, referred to as the volatility skew. The constant volatility assumption can result in a significant model specification error. As this assumption is indisputable violated in practice, an inconsistency between the Black &Scholes model and the real world arises.

In order to take the volatility skew into account, investors can use the Black & Scholes model with an extension. This extension allows the volatility to be a deterministic function of the underlying price and time. In this paper, two different deltas will be examined and compared:

1. The standard Black & Scholes delta
2. The skew-adjusted Black & Scholes delta

4.2 Derivation of the skew-adjusted delta

An extensive amount of empirical work has been dedicated to the development of option pricing models that takes time-varying volatility into consideration. The Black & Scholes constant volatility assumption is relaxed in deterministic volatility models of authors such as Dupire (1994), Dermand and Kani (1994) and Rubinstein (1994). These models incorporate the stylized fact that volatility appears to be time-varying, and in addition, tends to be inversely and negatively correlated with changes in price of the underlying.

Dupire (1994) starts with the original Black & Scholes delta formula, Equation 11, and assumes that volatility is a deterministic function of *S*, *K*, and *T*. This enables us to write the skew-adjusted delta as:

$$\delta_{\text{skew}} = \frac{\partial c}{\partial S} + \frac{\partial c}{\partial \sigma}\frac{\partial \sigma}{\partial S} \tag{14}$$

where $\frac{\partial c}{\partial \sigma}$ is the vega of an option and the vega of a long call option position is always positive. This, together with the negative correlation between the underlying returns and volatility changes, $\frac{\partial \sigma}{\partial S}$, implies that the skew-adjusted delta should always be smaller than the Black & Scholes delta. Still, the dependence of volatility on the underlying stock price, $\frac{\partial \sigma}{\partial S}$, is very difficult to measure. However, Derman et al (1996) argues that this term can be approximated by the slope of the volatility skew, $\frac{\partial \sigma}{\partial K}$. See the Appendix for further evidence on this statement. After substituting $\frac{\partial \sigma}{\partial K}$ into Equation (2), the skew-adjusted delta can be expressed as:

$$\delta_{\text{skew}} = \frac{\partial c}{\partial S} + \frac{\partial c}{\partial \sigma}\frac{\partial \sigma}{\partial K} = \delta_{\text{BS}} + \text{v}_{\text{BS}}\frac{\partial \sigma}{\partial K} \tag{17}$$

By assuming that $\frac{\partial \sigma}{\partial S}$ is more or less equal to $\frac{\partial \sigma}{\partial K}$, we believe that when *S* changes with one unit, there is a parallel shift of approximately $\frac{\partial \sigma}{\partial K}$ units in the volatility skew. According to Derman (1994) all the fixed-strike volatilities will decrease by the amount defined by the slope of the skew if the volatility skew is shifted downwards. The vega of an option measures the impact of these changes in the volatility on the price of the option.

The term $\frac{\partial \sigma}{\partial K}$ represents the slope of the volatility skew and can be determined by the "slope"-function in Excel. This function is the equivalent of fitting a linear model with the independent variable volatility as a function of the strike price based on least squares criterion.

4.3 Earlier academic research

Practioners such as Dumas et al (1998), Coleman et al (2000), Crepey (2004) and Vähämaa (2003) compare the two deltas and investigate the differences in the two deltas in their empirical work. The results of these researches are in conflict with each other.

Dumas et al (1998) conclude that the standard Black & Scholes delta provides a more accurate hedge than the skew-adjusted delta, while Coleman et al (200) and Vähämaa (2003 argue that the skew-adjusted delta outperforms the standard Black & Scholes delta. Crepey (2004) conclude that on average the skew-adjusted delta provides a better hedge than the standard Black & Scholes delta in negatively skewed markets. Dumas et al (1998) conclude that the Black & Scholes delta outperforms the skew-adjusted hedging-wise. Coleman et al (2000) perform hedging experiments and analyze the results in terms of the standard deviation of the final profit & loss. They find that the skew-adjusted delta outperforms the Black & Scholes delta as long as the hedging horizon is longer than two weeks.

Noteworthy is the limited amount of research that has been devoted to the size of the delta. Derman (1998) argues that market conditions influence the size of the delta. For example, the correct delta should be approximately equal to the Black & Scholes delta in stable markets as the volatility seems to be unaffected by the underlying index level. In trending markets, volatility appears to be dependent of and positively related to the index. As a result the correct delta should be larger than the standard Black & Scholes delta. In addition, Derman (1999) argues that in highly volatile market conditions volatility and the index are likely to be negatively related. Consequently, a delta that is smaller than the standard Black & Scholes should be applied in the hedging process. Mixon (2002) also believes that the delta should be smaller than the standard Black & Scholes due to the stylized facts of volatility.

Moreover, in the empirical studies of Bakshi et al (2002) and Coleman et al (2000) deltas that are significantly different from the Black & Scholes have been thoroughly documented. Bakshi et al (2002) inspect deltas that are produced by stochastic volatility models and conclude that these types of deltas are smaller than the Black & Scholes delta for out-of-the-money put options, but larger for in-the-money put options. Coleman et al (2000) compares the deltas that are produced by deterministic volatility models and conclude that these deltas are consistently smaller than the Black & Scholes delta.

4.4 Methodology

In this part of the thesis a comparison between the two deltas will be made and it will be determined which delta provides the best hedge. Firstly, the hedging results of the two deltas will be empirically tested. These

tests will be based on actual data. Real-life hedging experiments will be conducted using a self-financing portfolio and the real trajectory effectively followed by the underlying index, the SX5E.
Secondly, the hedging results and performance of the two deltas will be examined based on the simulated data. Monte-Carlo simulations will provide hedging simulations and the related hedging performance of the deltas will be closely studied.

The quantitative fit of the Black & Scholes delta and skew-adjusted delta is investigated with the help of the following two regression specifications, respectively:

$$\Delta c = \beta_0 + \beta_1 \left[\frac{\partial c}{\partial S}\Delta S\right] + \varepsilon \quad (18)$$

$$\Delta c = \beta_0 + \beta_1 \left[\frac{\partial c}{\partial S}\Delta S\right] + \beta_2 \left[\frac{\partial c}{\partial \sigma}\frac{\partial \sigma}{\partial K}\Delta S\right] + \varepsilon \quad (19)$$

where *c* represents the price of the option, *S* denotes the price of the underlying security, *K* denotes the strike price of the option and Δ represents the first difference operator.

The regression specification of Equation (20) captures the inverse relationship between the price changes of an underlying index and its volatility as β_2 represents the skew-adjusted term. Since Equation (19) is an extended version of Equation (18), a Wald chi-square statistic is examined to investigate whether the incorporation of the skew-adjustment term into Equation (18) is statistically significant.

Ideally the deltas should generate a value equal to zero for β_0 and equal to one for β_1. In addition, the adjusted R^2 should also be equal to one. These values combined would imply that the estimated delta is accurate. The adjusted R-squared is a figure that indicates the proportion of the variance in the dependent variable, i.e. the option price change that is predictable from the independent variable, the delta. The most accurate delta produces the adjusted R^2-value that is the closest to one.

Next, we shall examine the numerical results obtained by a Monte Carlo procedure of 500 simulations. A Monte Carlo simulation is a procedure for sampling random outcomes for a stochastic process. Earlier we assumed that the process followed by the underlying variable can be written as:

$$d_S = \mu_s d_t + \sigma_s dz \quad (20)$$

In order to simulate the path followed by *S*, it is possible to divide the life of the variable into *N* shorts intervals of length Δ_t and approximate the equation as:

$$S(t + d_t) - S(t) = \mu_s(t)\Delta t + \sigma_s(t)\varepsilon\sqrt{\Delta t} \quad (21)$$

where $S(t)$ denotes the value of S at time t, ε is an error term drawn from a normal distribution with a mean of zero and a standard deviation of one.

The magnitude of the hedging error at time Δ_t can be computed from the initial value of S. Consequently, the magnitude of the hedging error at time $2\Delta_t$ can be calculated from the value of S at time Δ_t, and so on. In total, 500 simulations will occur. One simulation trial involves constructing a trajectory for S using N random samples from a normal distribution. Each trial will result in a final total hedging error, allowing us to determine which delta produces eventually the lowest hedging error. An example of a Monte-Carlo simulation for the standard Black & Scholes- and skew-adjusted delta can be found in Figure 3, 4 and 5 in the Appendix.

The method of Vahamaa (2004) is used to assess the delta-hedging performance of the two deltas based on actual market data. This method involves constructing a self-financed delta-hedged portfolio. The portfolio consists of one unit short in a call option, δ units of the underlying asset, and B units of a risk-free asset. The portfolio has, at time t, a value of:

$$\Pi_t = \delta_t S_t + B_t - c_t \tag{22}$$

At the start of the hedging horizon, the following holds:

$B_0 = c_0 - \delta_0 S_0$ and thus, $\Pi_0 = 0$.

The delta-hedging performance of the two deltas is examined based on 1-day, 2-days, 5-days and 10-days hedging horizons, using daily rebalancing of the hedge portfolio.

At each time the portfolio is hedged, the delta hedging error ε from time $t-1$ to t can be computed as:

$$\varepsilon_t = \delta_{t-1} S_t - c_t + e^{rt} B_{t-1} \tag{23}$$

The total hedging error during the hedging horizon, $\Pi_{T,}$ is equal to the value of the portfolio at the end of the hedging horizon τ:

$$\varepsilon_\tau = \sum_{t=1}^{T} \varepsilon_\tau = \Pi_t \tag{24}$$

By using two error statistics, the mean absolute hedging error and the root mean squared hedging error, the delta-hedging performance of the two deltas is analyzed. These error statistics can be computed as:

$$\text{MAHE} = \frac{1}{n}\sum_{i=1}^{n}|\varepsilon| \text{ and } \text{RMSHE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\varepsilon_t^2} \quad (25)$$

Finally, the effectiveness of a skew-adjusted hedge will be determined by considering the percentage reduction in the sum of squared residuals resulting from the hedge. According to Hull and White (2016), the "Gain" from a skew-adjusted hedge can be determined by:

$$Gain = 1 - \frac{SSE[\Delta f - \delta_{SAD}\Delta S]}{SSE[\Delta f - \delta_{BS}\Delta S]} \quad (26)$$

4.5 Empirical Results

We make a comparison of the average size of the Black & Scholes delta and skew-adjusted delta for different time-to-maturity- and moneyness categories. This comparison is based on the findings of Table 1.

As the volatility skew is generally downward sloping and the vega of an option is always positive, the skew-adjusted delta is always consistently smaller than the corresponding standard Black & Scholes delta. As the moneyness increases the, the difference between the two deltas decreases.

A simple t-test shows that the differences between the means of the two deltas are statistically significant. For each category, the differences in the mean of the two are at least statistically significant at a 5%-confidence level. The differences in the mean of the two deltas is the largest for the out-of-the-money category.

A simple F-test that tests the equality of variance of the deltas indicates that the same conclusion holds for the differences in the variance of the two deltas. The differences in the variance of the two deltas is the smallest for the in-the-money category.

Table 1: Comparison of Black & Scholes - and skew-adjusted delta

** = significant at a 1% level, ** = significant at a 5% level*

Moneyness	Time to Maturity	Average Size				Tests		
		δ_{bs}	δ_{skew}	**Difference**	**%**	**F-test**	**Wald-test**	**Gain**
Full Sample	All	0.307	0.264	0.043	16.3% *	1.12 *	0.00 *	0.22%
	Long-term	0.309	0.259	0.05	19.3% *	1.15 *	0.00 *	3.11%
	Short-term	0.306	0.269	0.037	13.8% *	1.11 *	0.00 *	0.85%
In-the-money	All	0.722	0.671	0.051	7.6% *	0.82 *	0.00 *	3.13%
	Long-term	0.649	0.589	0.06	10.2% *	0.85 *	0.01 *	1.71%
	Short-term	0.768	0.723	0.045	6.2% **	0.81 **	0.00 *	2.25%
At-the-money	All	0.47	0.41	0.06	14.6% *	1.01 *	0.03 *	0.15%
	Long-term	0.461	0.397	0.064	16.1% *	1.04 *	0.00 *	1.65%
	Short-term	0.476	0.42	0.056	13.3% *	1.02 *	0.04 8	0.05%
Out-of-the-money	All	0.16	0.126	0.034	27.0% **	1.41 *	0.00 *	0.59%
	Long-term	0.201	0.158	0.043	27.2% *	1.37 *	0.00 *	3.61%
	Short-term	0.123	0.097	0.026	26.8% *	1.39 *	0.00 *	4.91%

Table 2 reports the regression results, for different moneyness- and maturity categories, that are based on Equations (19) and (20). The adjusted R^2 reports the explanatory power of each regression model. The standard errors are presented in parentheses below the coefficient estimates.

The R^2 values under the Black & Scholes delta are slightly, but consistently lower than under the skew-adjusted delta. This means that the skew-adjusted model is a better quantitative fit and is a better predictor. Both the standard Black & Scholes and skew-adjusted delta can explain approximately 81% of the observed option price changes in the full sample.

Table 2: Quantitative fit of the Black & Scholes - and skew-adjusted delta

		Black & Scholes delta			Skew-adjusted Black & Scholes delta			
Moneyness	**Time to Maturity**	**β_0**	**β_1**	**R^2**	**β_0**	**β_1**	**β_2**	**R^2**
Full Sample	All	-1.597	0.814	0.814	-1.600	0.784	0.026	0.816
		(0.07)	(0.00)		(0.01)	(0.01)	(0.01)	
	Long-term	-1.409	0.887	0.897	-1.446	1.038	-0.110	0.901
		(0.01)	(0.00)		(0.07)	(0.01)	(0.01)	
	Short-term	-1.856	0.761	0.756	-1.864	0.716	0.047	0.759
		(0.11)	(0.00)		(0.11)	(0.01)	(0.01)	
In-the-money	All	0.210	0.805	0.807	-0.178	0.666	0.209	0.813
		(0.35)	(0.01)		(0.35)	(0.02)	(0.03)	
	Long-term	-0.985	0.930	0.924	-0.918	1.029	-0.111	0.925
		(0.35)	(0.01)		(0.35)	(0.03)	(0.03)	
	Short-term	0.409	-0.745	0.751	0.047	0.618	0.231	0.757
		(0.53)	(0.01)		(0.53)	(0.03)	(0.05)	
At-the-money	All	-2.552	0.817	0.808	-2.568	0.764	0.044	0.809
		(0.17)	(0.01)		(0.17)	(0.02)	(0.02)	
	Long-term	-1.677	0.883	0.882	-1.632	1.101	-0.161	0.884
		(0.20)	(0.01)		(0.20)	(0.04)	(0.03)	
	Short-term	-3.342	0.863	0.762	-3.379	0.741	0.028	0.763
		(0.26)	(0.01)		(0.26)	(0.03)	(0.03)	
Out-of-the-money	All	-1.563	0.796	0.803	-1.582	0.695	0.053	0.804
		(0.05)	(0.00)		(0.05)	(0.01)	(0.01)	
	Long-term	-1.471	0.836	0.867	-1.411	1.119	-0.143	0.871
		(0.07)	(0.01)		(0.07)	(0.03)	(0.01)	
	Short-term	-1.673	0.735	0.707	-1.715	0.473	0.144	0.722
		(0.08)	(0.01)		(0.08)	(0.02)	(0.01)	

Overall, the quantitative fit of both regression models is relatively high for all moneyness- and time-to-maturity-categories. However, the quantitative fit is significantly lower for all short-term categories in comparison to the other two maturity categories. The R^2 values of the Black & Scholes delta range from 70,7% to 92,4% whereas the R^2 values of the skew-adjusted delta range from 72,2% to 92,5%. The most accurate delta should produce the highest R^2 value. However, the upper bounds of the R^2 values are roughly equal. Yet, the lower bound R^2 value of the skew-adjusted delta is noteworthy higher than that of the standard Black & Scholes delta.

Table 2 indicate that the difference of the quantitative fits of the models is the largest for short-term out-of-the-money options and the smallest for all-maturity at-the-money options. Both the standard Black & Scholes delta and skew-adjusted delta regression model seem to explain the in-the-money and at-the-money options better than the out-of-the-money options.

The β_0 estimates of both models should in theory have a very small negative value, preferably almost equal to zero. In reality, this is not the case with outliners in the full sample of the at-the-money options of -2,55 and -2,57. Both models have β_1 estimates that are relatively close to one. In the Black & Scholes model the β_1 estimates are all below one. However, in the skew-adjusted model the β_1 estimates of options with a long time-to-maturity are all above one. In almost all cases, there are meaningful differences in β_1 estimates between the two regression models.

Additionally, Table 1 provides information about various tests that were conducted on the two deltas. This table shows that the coefficient of the skew-adjusted term in Equation (20) seems statistically significant. The Wald's chi-square test concludes that the regression models of the Black & Scholes and skew-adjusted delta are statistically significantly different from each other and the latter is a significant improvement over the original Black & Scholes delta. The null hypothesis in the reported Wald chi-square test is $\beta_2 = 0$ and is rejected.

Furthermore, Table 1 indicates that short-term out-of-the-money options "gain" a lot if an investor uses the skew-adjusted delta to hedge instead of the standard Black & Scholes delta. Overall, the "gain" is positive for each category, indicating that the skew-adjusted delta is the superior delta of the two deltas.

Table 3: Hedging error of Black & Scholes - and skew-adjusted delta for a 1-day hedging horizon
** = significant at a 1% level, ** = significant at a 5% level*

		Mean Absolute Hedging Error				Root Mean Squared Hedging Error			
Moneyness	**Time to Maturity**	δ_{bs}	δ_{skew}	**Difference**	**%**	δ_{bs}	δ_{skew}	**Difference**	**%**
Full Sample	All	0.278	0.276	0.003	0.95%	0.421	0.419	0.001	0.35%
	Long-term	0.323	0.321	0.001	0.40%	0.472	0.471	0.001	0.21%
	Short-term	0.242	0.238	0.004	1.50%	0.374	0.372	0.002	0.49%
In-the-money	All	0.764	0.76	0.003	0.44%	0.881	0.878	0.003	0.33%
	Long-term	0.814	0.811	0.004	0.44%	0.986	0.982	0.004	0.37%
	Short-term	0.732	0.73	0.003	0.36%	0.812	0.81	0.002	0.23%
At-the-money	All	0.41	0.41	0.000	-0.07%	0.502	0.502	0.000	-0.04%
	Long-term	0.795	0.792	0.003	0.32%	0.962	0.959	0.003	0.30%
	Short-term	0.343	0.344	-0.001	-0.20%	0.409	0.409	0.000	-0.04%
Out-of-the-money	All	0.127	0.123	0.004	2.87% **	0.189	0.186	0.003	1.61% *
	Long-term	0.175	0.173	0.002	0.97%	0.237	0.236	0.002	0.74%
	Short-term	0.081	0.075	0.005	6.65% **	0.127	0.121	0.006	4.46% **

The results of the delta hedging experiment of Vahamaa (2004) with a hedging horizon of one day are reported in Table 3. Table 2, 3 and 4 in the Appendix show the results of the hedging experiment for respectively, two, five and ten days as hedging horizon. The reported error statistics are the mean absolute hedging error (MAHE) and the root squared mean hedging error (RSME). The results are bootstrapped in order to test whether the differences in the hedging errors are statistically significant. Bootstrapping is a technique that allows estimation of the sampling distribution of almost any statistic using random sampling methods (Varian, 2005). The sampling technique can improve the estimate of a statistic by reusing the same sample over and over again.

Table 3 provides evidence that the absolute hedging error and root squared mean hedging error is the largest for out-of-the-money options. The differences of the hedging error for this particular category are also highly significant, while this is not the case in the other moneyness-categories. The difference in the mean absolute hedging error in this moneyness-category can become as large as 6.65% and the difference in the root mean squared hedging error has a maximum value of 4.46%.

Therefore, the skew-adjusted delta outperforms the standard Black & Scholes delta in this category from a hedging perspective. These results are consistent with the observations of Bakshi et al (1997) who concluded that the hedging improvement of the skew-adjusted delta is the greatest for out-of-the-money options.

Table 2, 3 and 4 in the Appendix shows that as the hedging horizon is extended to two, five and ten days the difference in the hedging error increases greatly in favor of the skew-adjusted delta. These results indicate that the hedging performance of the skew-adjusted delta is superior to that of the standard Black & Scholes model. Consequently, the skew-adjusted delta is substantial improvement over the standard Black & Scholes delta.

Next, the delta-hedging performance of the two deltas is compared based on the output of a Monte Carlo simulation. The following variables are used in the Monte Carlo procedure: S = 100, K = 100, r_f = 0,05, μ = 0,20, σ = 0,20, N = 100, t = 1, Δ_t = 0,01. With these parameters we can easily construct the Black & Scholes delta of the option, $N(d_1)$. In order to calculate the skew-adjusted delta we determine the vega of the option, $S\sqrt{T}\,N'(d_1)$ and we use the mean and standard deviation of the observed volatility skews from our real-market data sample. Along each simulated trajectory of the underlying asset, we delta-hedge both portfolios by using the correct deltas to rebalance the position every time step. Figure 1 and Figure 2 display the distribution of the hedging error in the Monte Carlo procedure.

Figure 1: The aggregated distribution of the hedging error based on the delta-hedging performance of the Black & Scholes delta for 500 simulations

(average hedging error = -0,066, standard deviation = 0,516, skewness = -0,597, kurtosis = 1,604)

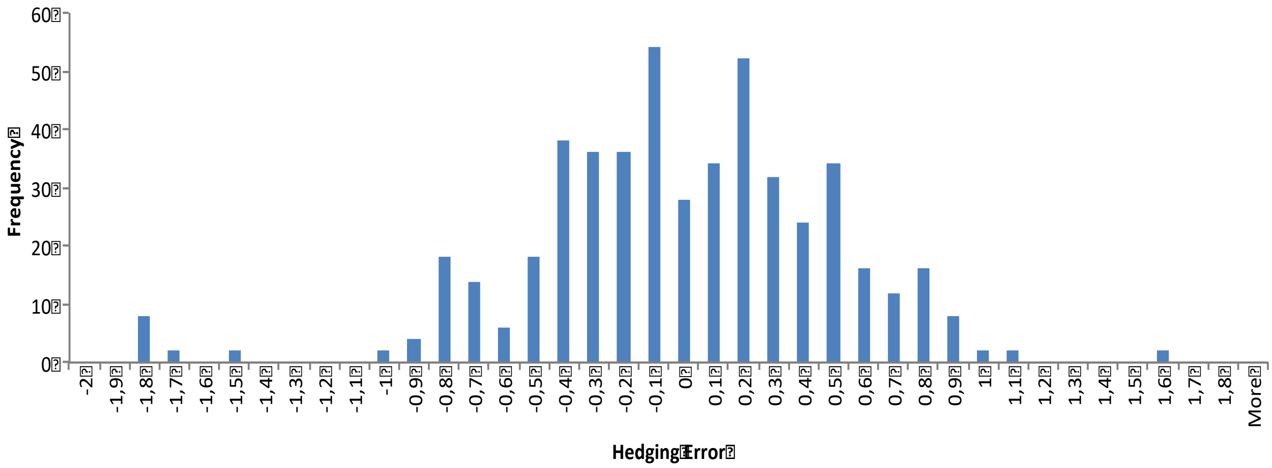
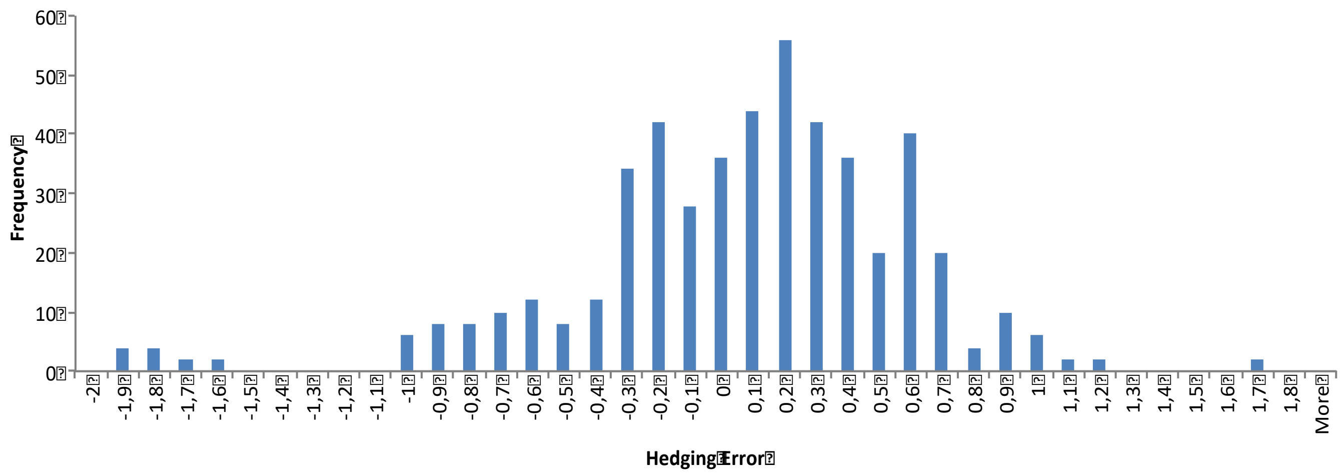


Figure 2: The aggregated distribution of the hedging error based on the delta-hedging performance of the skew-adjusted delta for 500 simulations

(average hedging error = 0,012, standard deviation = 0,531, skewness = -0,881, kurtosis = 2,138)



The distribution of the hedging error shows that skew-adjusted delta outperformed its counterpart in the Monte Carlo procedure. Hedging a portfolio by using the skew-adjusted delta resulted in a profit of $0,012 on average. If a portfolio was hedging according to the Black Scholes delta, on average, a loss of $0,066 was incurred. So, the skew-adjusted delta resulted in an average hedging error that was the closest to zero.

A t-test concludes that the means of the hedging error of both deltas are statistically significant different from each other, on a 5% confidence level. The same applies for the variance of the two deltas according to a simple F-test.

5. Trading

5.1 Fundamental theory

Since their introduction on the financial markets, options contracts have become popular instruments that offer investors the opportunity to precisely customize their risks to their preferences. Taking this into consideration, an in-depth study of the returns of options would provide an opportunity to investigate what kinds of risks the market prices in an option and if the skew-adjusted term is such a risk factor.

Similar to all other financial assets, option contracts must produce returns that are in balance with their risk. In order to evaluate the attractiveness of an option a trader needs to determine whether or not the returns of an option are excessive in terms of their underlying risks. Therefore, it is important for investors to understand option returns as options have remarkable risk-return characteristics.

The first component of options risk is the leverage effect. Options provide the possibility to undertake much of the risk of the option's underlying asset with a relatively small investment. This implies that options have characteristics that are similar to the leveraged position in the underlying asset. The Black & Scholes model assumes that this leverage effect is reflected in the beta of an option and is priced. As options are effectively leveraged positions in the underlying, they should earn an expected return that is superior to that of the underlying.

The second remarkable risk-return characteristic of options is the curvature of the payoff of an option Since the value of an option is a nonlinear function of the value of the underlying, the return of the option is sensitive to the higher moments of the underlying's returns. The Black & Scholes model assumes the returns of the underlying's asset follows a geometric Brownian motion, which is a two-parameter process. As a result, any risks associated with higher moments of the underlying asset returns are not priced and options are believed to be redundant assets. If options are redundant assets, option positions that are delta-neutral should earn an average return of zero.

5.2 Earlier academic research

It is widely acknowledged that options are mispriced, in the sense that certain option returns are excessive in relation to the risk they bear. Constantinides et al. (2009) focus on mispricing of out-of-the-money S&P 500 call options and conclude that options market are not becoming more rationale or less mispriced over time.

Other manifestations of the mispricing of options are the extremely large returns generated by writing put options on S&P 500. According to Bondarenko (2003), the returns of at-the-money puts are, on average, 40% per month. The average returns of deep out-of-the-money options are -95% per month. Jackwerth (2000) focuses also on put options and discovers that strategies involving writing put options produce substantial high returns, in both absolute and risk-adjusted levels. The author believes that the most likely explanation for these returns is the mispricing of the options.

The Black & Scholes model claims that the expected return of delta-hedged option should be equal to zero and unpredictable as these positions have a market beta or delta of zero (Bertsimas, Kogan and Lo, 2001). If an investor can perfectly replicate an option by buying and selling the underlying asset, a delta-hedged option is riskless and should earn no return on average.

Consistent with the findings of Bakshi and Kapadia (2003) and Cao and Han (2013), Zhan et al. (2015) discovers that the average delta-hedged options returns are negative for both call- and put options. For example, the average return of a delta-hedged at-the-money call option is -1.03% over the next month and -1.26% if the option is held until expiration date. The average return of a monthly-rebalanced delta-neutral call writing strategy is 3.67% and the daily-rebalanced delta-neutral call writing strategy has an average return of 1.55%. Coval and Shumway (2000) analyze weekly options and straddle [2] positions and conclude that delta-hedged at-the-money straddles positions produce average losses of approximately 3% per week.

In theory, the returns of these delta-neutral positions should have been equal to zero. However, empirical research shows that is generally not the case. Therefore, it is likely that there is at least one additional factor that is significant in the pricing of options. In this thesis I will investigate whether this unknown risk factor is the skew-adjusted term.

5.3 Methodology

The Black & Scholes model states that the value of an option is determined by the price of the underlying asset and assumes that there are no arbitrage opportunities between the option and the underlying asset. This would imply that the mispricing of an option is a direct result of the option's dependence on the mispriced underlying asset. So, an option inherits its misvaluation through its relationship with the underlying asset. Following this logic, the raw return of an option should be predictable. However, there is a way to examine expected option returns and the predictability of these returns beyond the dependence on the underlying asset. This assessment is possible by focusing on delta-hedged call option returns.

The delta-hedged call option return is measured by following the methodology of Cao and Han (2013). The return of a delta-hedged option is the change in the value of a self-financing portfolio. This self-financing portfolio consists of a short call position, hedged by a long position in the underlying asset in such a manner that the portfolio is not sensitive to movement of the underlying asset. At the end of each specific period, we sell one call option contract with a long position in the underlying asset, and the return of the delta-hedged call option over this period is:

[2] A straddle is an options strategy in which an investor holds a long position in both a call- and put option with the same strike price and expiration date

$$\frac{\Pi_{t+1}}{\Pi_t} - 1 = \frac{\delta_t * S_{t+1} - c_{t+1}}{\delta_t * S_t - c_t} - 1 \quad (27)$$

The leverage effect, the option's exposure to the underlying asset, needs to be taking into consideration. The most straightforward way to do this is to compute the betas of options by using a CAPM-style specification. This approach is closely related to the hedging arguments that are used to derive the Black & Scholes model (Coval and Shumway, 2000). According to this approach, the link between the instantaneous return of an option and the excess returns of the underlying asset is:

$$\frac{\delta c_t}{c_t} = r_f \delta t + \frac{S_t}{c_t}\frac{\partial c_t}{\partial S_t}\left[\frac{\delta S_t}{S_t} - r_f \delta t\right] \quad (28)$$

This implies that the instantaneous changes in the price of an option is linear in the return of the underlying asset, dS_t/S_t, and the instantaneous return of an option return is normally distributed. Coval and Shumway (2000) argue that this instantaneous CAPM can be viewed as an approximate linear factor model for the returns of options:

$$\frac{c_{t+T} - c_t}{c_t} = \alpha_{t,T} + \beta_{t,T}\left(\frac{S_{t+T} - S_t}{-S_T} - rT\right) + \varepsilon_{t,T} \quad (29)$$

These linear factors are used as a pricing factor and, in theory, the alpha should be equal to zero.
A situation in which $\alpha_T \neq 0$ is generally considered to be evidence of mispricing. However, this argument has a potential problem since the CAPM does not hold over finite time. Option prices are convex functions and a linear regression of the returns of options and their underlying asset is generally slightly misspecified. The degree of bias depends on the length of the holding period. Since we only use daily returns, the degree of error is minimal and it is possible to reject this critique.

A convenient method of magnifying the potential source of this mispricing and risk premium is the procedure of portfolio sorting. First, all available options are ranked according to their skew-adjusted term and portfolios of options with similar values are formed. Next, there is a self-financing portfolio constructed. A long position is taken in the options that should have the highest return according to the skew-adjusted term and this position is financed by a short position in the options that should produce the lowest return.

Logically, the slope of the volatility skew of a particular day is the same for each option serie. In our data set, we observe each day a volatility skew that is downward sloping and hence has a negative slope, meaning that the skew-adjusted term is always negative. Vega measures the impact of changes in the underlying volatility on the option prices. Therefore, we will go long in the option with the least negative skew-adjusted term, i.e. smallest vega, as the price of this particular option will be the least sensitive to the volatility decrease as the expected skew flattens.

The self-financing portfolio has no net investment and, thus, should not be profitable. In order to find whether the alpha is statistically significant different from zero, the returns of the self-financing portfolio are evaluated with the GRS-test statistic as suggested by Gibbons et al. (1989). In theory, the alphas are equal to zero, but some deviation will be the result of randomness. Gibbons et al. (1989) show that if the squared alphas are added together and standardized properly, the sum of the alphas follows a F-distribution. This F-distribution, and the threshold deriving from the related F-statistic, allows empirical tests. If we have a model that explains all risk, then there should be no alphas that are (jointly) significant.

Table 4: Summary of descriptive statistics of SX5E call option returns

** = significant at a 1% level, ** = significant at a 5% level*

		Mean Return			Call Options		
Moneyness	**Time to Maturity**	**Call Options**	**EuroStoxx50**	**t-Statistic**	**Std. Dev.**	**Minimum**	**Maximum**
Full Sample	All	2.11%	0.15%	(5.71) **	0.345	-87.34%	400.00%
	Long-term	4.21%	0.24%	(8.29) **	0.005	-76.67%	218.75%
	Short-term	0.43%	0.08%	(0.82)	0.366	-87.34%	400.00%
In-the-money	All	9.67%	0.59%	(9.37) **	0.329	-66.67%	169.23%
	Long-term	8.70%	0.78%	(5.28) **	0.325	-60.00%	164.29%
	Short-term	10.27%	0.48%	(7.76) **	0.332	-66.67%	169.23%
At-the-money	All	3.07%	0.29%	(4.93) **	0.3	-66.67%	218.75%
	Long-term	3.99%	0.43%	(3.99) **	0.309	-66.67%	218.75%
	Short-term	2.42%	0.21%	(3.05) **	0.294	-63.54%	196.88%
Out-of-the-money	All	0.27%	0.01%	(0.54)	0.364	-87.34%	400.00%
	Long-term	0.00%	0.09%	(0.00)	0.363	-86.96%	388.89%
	Short-term	0.51%	-0.07%	(0.74)	0.364	-97.34%	300.00%

However, if there is a source of risk premium, there will be returns that are larger than the fair compensation for the risk they bear. In this case, the large returns can't be explained by the risk factors of the model. When the factor, that is assumed to cause the mispricing, is included in the model, the alpha should be zero. If this is the case, the cause of the mispricing is identified and the alleged risk factor is indeed a source of risk premium.

5.4 Empirical results

Table 4 displays the descriptive statistics of the daily call option returns for different moneyness- and time-to-maturity- categories. With the exception of out-of-the-money long-term options, the mean returns of the SX5E call options are all larger than the return of the underlying asset, the SX5E index. Differences between call option returns and index returns are statistically significant and can be as large as 2.000% as is the case in the short-term in-the-money category. In other words, the leverage effect is evident. Moreover, the average return of options increases as the moneyness of options increases. As expected, the statistics show a substantial degree of positive skewness in the distribution of call option returns.

Table 5: Summary of descriptive statistics of SX5E delta-hedged call option returns

** = significant at a 1% level, ** = significant at a 5% level*

Moneyness	Time to Maturity	Mean return	t-Statistic	Std. Dev.	Minimum	Maximum
Full Sample	All	0.09%	(6.34) **	0.014	-5.34%	4.68%
	Long-term	0.18%	(9.66) **	0	-4.19%	4.02%
	Short-term	0.04%	(1.20)	0.015	-5.34%	4.68%
In-the-money	All	0.45%	(11.71) **	0.012	-2.93%	3.64%
	Long-term	0.41%	(6.35) **	0.013	-2.93%	3.54%
	Short-term	0.47%	(9.95) **	0.012	-2.83%	3.64%
At-the-money	All	0.12%	(5.91) **	0.01	-2.92%	2.61%
	Long-term	0.13%	(4.06) **	0.01	-2.72%	2,50%
	Short-term	0.12%	(4.30) **	0.011	-2.92%	2.61%
Out-of-the-money	All	0.13%	(0.66)	0.015	-5.34%	4.68%
	Long-term	0.00%	(0.61)	0.015	-5.34%	4.68%
	Short-term	0.04%	(1.50)	0.015	-5.33%	4.66%

Next, we focus on the returns of delta-hedged call options in order to determine whether the call options returns are excessive. In addition, this enables us to test the assumption that options are redundant assets. A delta-hedged straddle is a position that has a net delta and/or beta of zero. Therefore, we can abstract from the leverage effect and concentrate on the pricing of higher moments of an asset's returns. According to the Black & Scholes and the closely-related CAPM model, the returns of a delta-hedged straddle position should be equal to zero.

Table 5 provides evidence that the return of the delta-hedged call options is, on average, overall statistically significant different from zero. With the exception of the out-of-the-money options, all the categories provide p-values that are statistically significant. Although, the out-of-the-money delta-hedged call option returns are approximately equal to the zero, the same holds for the out-of-the-money call option returns. Therefore, it is possible to argue that, despite these out-of-the-money delta-hedged call option returns being equal to zero, there is some additional risk factor priced in the options. This is certainly the case for the options in the full sample, the in-the-money options and the at-the-money options. Thus, these results show that options can not considered as redundant assets. Therefore, it is necessary to investigate what kinds of risks are priced in an option and, in particular, if investors value and price the skew-adjusted term.

The relationship between the skew-adjusted term and expected returns is investigated by using sorted portfolios. This approach gives the possibility to check whether the skew-adjusted term is indeed a factor that affects the price of an option. This is done by examining the profitability of a self-financing portfolio. The portfolio consists of a long- and a short position in options based on the size of their skew-adjusted term. The return of self-financing portfolio is the mean return differential between the top and bottom portfolio. These specific periods will be 1-day, 2-days, 5-days and 10-days

Table 6: Summary of statistics of long/short skew-adjusted term-based portfolio sorting strategy
* = significant at a 1% level, ** = significant at a 5% level

Moneyness	Time to Maturity	Average Return	t-Statistic	Average P&L	Total P&L	Std. Dev.	Skewness	Kurtosis
Full Sample	All	0.399	7.78 **	-198.00	-31,284.00	4,595.79	-3.6	22.68
	Long-term	0.606	5.38 **	-267.57	-40,135.50	2,258.75	-2.56	11.99
	Short-term	0.504	7.95 **	-130.38	-21,773.29	4,486.60	-3.52	23.19
In-the-money	All	1.302	12.94 **	110.66	6,860.77	715.12	-0.28	4
	Long-term	1.113	18.26 **	-7,426.19	-363,883.31	10,123.28	-0.64	1.53
	Short-term	1.695	5.64 **	-5,825.58	-512,651.04	10,084.14	-1.15	2.51
At-the-money	All	0.774	6.11 **	57,72	6,522.82	790.25	-0.73	5.79
	Long-term	0.978	13.95 **	-460.07	-41,406.03	2,998.33	-6.4	42.59
	Short-term	0.739	6.15 **	-220.93	-30,708.63	1,126.80	-4.26	34.55
Out-of-the-money	All	0.412	5.64 **	1,166.02	122,432.42	2,243.83	-0.66	4.57
	Long-term	0.767	4.66 **	570.5	57,050.03	837.33	-0.24	5.85
	Short-term	0.588	6.00**	881.49	96,964.09	2,115.10	-0.17	3.33

Table 6 displays the results of the 1-day portfolio-sorting strategy. The results of the 2-days, 5-days and 10-days portfolio-sorting strategy can be found in respectively Table 5, 6 and 7 in the Appendix. For the full sample, the self-financing portfolio is unprofitable and incurs large losses. This is caused by the average increase in the slope of the volatility skew of 4,0% per day, while we expected it to actually flatten. Our trading strategy aimed to benefit from the supposed flatting of the volatility skew.

Table 6 indicates that all average returns are statistically significant different from zero. The average return increases as the moneyness of the option series increases. It is pretty clear that the extraordinary nature of the option returns slightly disturbs the profit and loss results of the long/short strategy. In particular, the results of the long-term at-the-money options long/short strategy experiences non-normal behavior with skewness of -6,40 and kurtosis of 42,59. There are some inconsistencies between the results of the portfolio sorting strategy. The average P&L of the in-the-money options of the full sample is $110,66. Yet, the average P&L of the long-term and short-term in-the-money portfolio sorting returns is respectively $-7.426,19 and $5825,58. A comparable contradiction can be observed if we compare the average P&L of the full sample with the complete in-the-money-, at-the-money and out-of-the-money sample. The full sample generates a return of -$198.00, while the different moneyness categories all produce positive average returns.

These inconsistencies can be explained by the fact that for each category the self-financing portfolio, and hence the long- and short position, consists of different options. Therefore, the P&L of the self-financing portfolio in the full sample is not the sum of the P&L of the self-financing portfolios in the in-the-money-, at-the-money and out-of-the-money category Thus, there is no conclusive answer whether the trading strategies based on the skew-adjusted term are consistently profitable or not.

Table 7: Description regression output of CAPM-model and CAPM-model with an additional skew-adjusted term

		CAPM		CAPM and the Skew-Adjusted Factor		
Moneyness	**Time to Maturity**	**α**	β_{bs}	**α**	β_{bs}	β_{skew}
Full Sample	All	0.081	16.540	0.093	16.570	-0.005
		(0.02)	(0.15)	(0.09)	(0.21)	(0.01)
	Long-term	0.078	14.561	0.056	13.209	0.015
		(0.00)	(0.00)	(0.00)	(0.17)	(0.00)
	Short-term	0.085	17.899	0.101	18.418	-0.010
		(0.00)	(0.24)	(0.01)	(0.34)	(0.01)
In-the-money	All	0.061	11.762	0.029	10.820	0.020
		(0.00)	(0.21)	(0.01)	(0.46)	(0.01)
	Long-term	0.043	10.888	0.039	11.236	0.002
		(0.00)	(0.23)	(0.01)	(0.41)	(0.01)
	Short-term	0.069	11.501	0.072	9.882	0.000
		(0.00)	(0.33)	(0.01)	(0.66)	(0.00)
At-the-money	All	0.086	15.140	0.083	14.701	-0.009
		(0.00)	(0.26)	(0.01)	(0.38)	(0.00)
	Long-term	0.072	13.210	0.095	13.455	-0.020
		(0.00)	(0.21)	(0.01)	(0.30)	(0.00)
	Short-term	0.097	16.594	0.093	15.889	0.001
		(0.01)	(0.35)	(0.01)	(0.54)	(0.01)
Out-of-the-money	All	0.089	17.640	0.140	20.150	-0.012
		(0.00)	(0.21)	(0.01)	(0.39)	(0.01)
	Long-term	0.092	15.533	0.063	13.948	0.014
		(0.00)	(0.20)	(0.01)	(0.29)	(0.00)
	Short-term	0.088	19.169	0.144	23.063	-0.007
		(0.01)	(0.35)	(0.01)	(0.73)	(0.01)

In order to give a conclusive answer to whether the skew-adjusted term is a risk factor that could and should be priced, we incorporate the factor in a Gibbons-Ross-Shanken test. We know there is an unidentified source of risk premium present in our dataset present. Not only generates the delta-hedged call option returns that are excessive, but there are also jointly significant alphas. In order to check if the skew-adjusted term is the unknown risk factor, the excess returns of the call options are regressed on the excess return of the market and the excess return of the skew-adjusted term:

$$r_{i,t} - r_f = \alpha_i + \beta_{market}\left(r_{mkt} - r_f\right) + \beta_{skew}\left(r_{skew} - r_f\right) + \varepsilon_{i,t} \tag{30}$$

Table 8: Description of the regression results of the time-series returns

		Model						GRS-Test	
Moneyness	**Time to Maturity**	**α**	**t-Statistic**	**β_{bs}**	**t-Statistic**	**β_{skew}**	**t-Statistic**	**F-statistic**	**p-value**
Full Sample	All	0.093 (0.09)	16.540	16.570 (0.21)	79.470	-0.005 (0.01)	-0.950	0.903	0.342
	Long-term	0.056 (0.00)	14.561	13.209 (0.17)	79.120	0.015 (0.00)	7.940	3.644	0.097
	Short-term	0.101 (0.01)	17.899	18.418 (0.34)	54.190	-0.010 (0.01)	-1.540	2.372	0.124
In-the-money	All	0.029 (0.01)	11.762	10.820 (0.46)	23.360	0.020 (0.01)	3.010	9.060	0.00 **
	Long-term	0.039 (0.01)	10.888	11.236 (0.41)	27.130	0.002 (0.01)	0.200	0.040	0.841
	Short-term	0.072 (0.01)	11.501	9.882 (0.66)	14.980	0.000 (0.00)	-0.210	0.044	0.833
At-the-money	All	0.083 (0.01)	15.140	14.701 (0.38)	38.960	-0.009 (0.00)	-2.050	4.203	0.061
	Long-term	0.095 (0.01)	13.210	13.455 (0.30)	43.880	-0.020 (0.00)	-4.290	18.404	0.00 **
	Short-term	0.093 (0.01)	16.594	15.889 (0.54)	29.310	0.001 (0.01)	0.310	0.096	0.757
Out-of-the-money	All	0.140 (0.01)	17.640	20.150 (0.39)	51.100	-0.012 (0.01)	-1.410	1.988	0.158
	Long-term	0.063 (0.01)	15.533	13.948 (0.29)	48.470	0.014 (0.00)	5.190	0.269	0.643
	Short-term	0.144 (0.01)	19.169	23.063 (0.73)	31.650	-0.007 (0.01)	-0.660	0.436	0.511

Thus, we test whether the skew-adjusted term is responsible for the unexplained risk premium. This is done by including the factor in the model of the GRS-test. If the skew-adjusted term is causing the mispricing, there shouldn't be any significant alpha left. The F-statistic helps us to test the null hypothesis that the alpha for each time-series regression is equal to zero. Table 8 shows that exclusively the in-the-money- and long-term at-the-money category have an alpha that is not equal to zero. This means that in these categories the skew-adjusted term is a risk factor that should be included in the risk-return relationship. In the other categories, the skew-adjusted term does not prove to be of added value for an investor as the market does not price this risk.

Strictly speaking, the GRS-test will not confirm that exclusively our factor is responsible for the alphas. It is possible that another risk factor is actually responsible for the mispricing. As mentioned earlier, the GRS-test rejects the skew-adjusted term as a statistically significant risk factor for all categories except the long-term at-the-money option series and the full in-the-money series. The GRS-test of the full in-the-money sample considers the skew-adjusted term statistically significant, while the GRS test on both the long-term and short-term of this moneyness category rejects the skew-adjusted term as risk factor. Therefore, it appears that there is a discrepancy within this moneyness-category. We observed this earlier with the results of self-financing portfolio and the same explanation holds.

Table 6 shows that the results of the portfolio sorting strategy are all statistically significant different from zero. However, the remarkable behavior of the options slightly disturbs the profit and loss results of the long/short strategy and hence is not clear whether the trading strategies based on the skew-adjusted term are consistently profitable or not. Still, the average P&L of all the different out-of-the-money categories is positive and relatively large. However, the GRS-test concludes that within this category there are no jointly significant alphas and the skew-adjusted factor cannot be described as a risk factor.

Overall, the evidence in favor of the skew-adjusted term as a significant risk factor and as a source of risk premium is lacking and not convincing. Hence, we can conclude that the skew-adjusted term is not of interest for an investor from a pricing-perspective.

6. Conclusion

Hedging a position with the support of the Black & Scholes model may be prove to be very misleading for investors. For example, it is not unusual for investors to see their delta-neutral positions damaged when the market moves. According to the Black & Scholes model, this is highly unlikely as the position is delta-neutral. However, this occurrence is a consequence of the misspecification of the Black & Scholes model.

This misspecification is clearly visible in the so-called volatility skew, the phenomena that the option's implied volatility depends in practice upon the option's moneyness and time-to-maturity. The Black & Scholes implied volatility is a volatility parameter that, inserted into the Black & Scholes formula for that specific option price, results in the Black & Scholes price of the option being equal to the market price of the option. The implied volatility skew of an option would be flat and constant if the option prices observed in the market would be equal to the theoretical Black & Scholes option prices. In reality, this is not the case and the volatility skew is generally downwards sloping.

Still, the Black & Scholes is the most widely used framework for the pricing and hedging of option contracts, primarily due to its straightforwardness. This thesis extends the standard Black & Scholes model by incorporating the inverse relationship between the movement of the underlying and volatility with a rather simple extension. Following Derman et al. (1996) and Coleman (2000), the volatility skew is utilized to adjust the Black & Scholes delta to account for the inverse relationship between the underlying returns and volatility changes.

This thesis examines whether the delta-hedging performance of Black & Scholes model can be improved by taking this inverse relationship into account. Empirical tests on the Eurostoxx50 index options market show that the delta hedging performance of the Black & Scholes model can be substantially improved by adjusting the Black & Scholes delta. The skew-adjusted delta consistently leads to smaller hedging errors, and thus, outperforms the Black & Scholes delta hedging-wise. This conclusion is valid for both numerical simulations through the Monte-Carlo procedure as actual real-market data. The outperformance of the skew-adjusted delta is most distinct for short-term out-of-the-money option contracts and becomes more significant as the hedging horizon lengthens. In general, the findings of this thesis have implications for risk management as they indicate the importance of the correlation between the returns of the underlying asset and its volatility changes. Furthermore, the empirical results demonstrate that due to the skew-adjusted term, the correct delta is actually smaller than the Black & Scholes delta.

While the pricing of option contracts is the subject of a substantial amount of empirical work, relatively less research has been done on the returns of these options. The Black & Scholes model claims that options risks should be priced similar to that of other financial assets. We find that option returns largely conform to most asset pricing implications such as the leverage effect. However, options are not redundant assets as delta-hedged call options earn returns that are statistically significant different from zero. These results strongly suggest that something besides market risk is important for the pricing of options.

In this thesis we examine the likelihood of the skew-adjusted term as this unknown risk factor. We capture this skew-adjusted term by using sorted portfolio and measuring the return of this self-financing portfolio. The portfolio has no net investment, so in theory it should get no return at all. If we do have a source of risk premium, there will be returns that are bigger than the fair compensation for risk according to the model and there will be a significant alpha. Finally, we incorporate the skew-adjusted term in the model of the GRS test and conclude that there is still a jointly significant alpha present in the model. Hence, we can conclude that the skew-adjusted term is not a source of risk premium.

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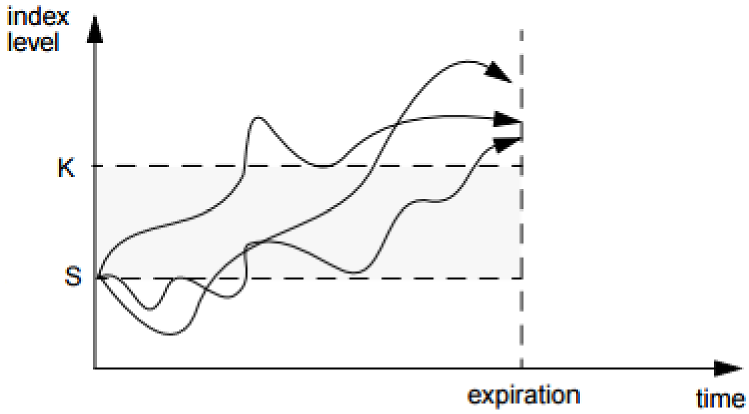
8. Appendix

Derivation of the skew-adjusted term

According to Derman et al. (1996) the change in implied volatility of an option for a change in the market level is approximately equal to the change in implied volatility of an option for a change in the strike level. Consider the implied volatility σ(S,K) of a call option with strike *K* when the index is at *S*. This implies that the call option is out-of-the-money. Any path that contributes to the price of option must go through the area between *S* and *K*.



The volatility of these paths is determined by the local volatility in this specific area. Therefore, it is possible to view the implied volatility of the option as the average of the local volatilities over this particular region between *S* and *K*. This can be written as:

$$\sigma(S, K) \approx \frac{1}{K-S} \int_S^K \sigma(S')dS'$$

Substituting $\sigma(S) = \sigma_0 + \beta S$ into this equation leads to:

$$\sigma(S, K) \approx \sigma_0 + \frac{\beta}{2}(S + K)$$

This equation shows that, if implied volatility varies linearly with strike *K* at a fixed market level *S*, then it also varies linearly at the same rate with the index *S* itself. Therefore, the change in implied volatility of an option for a change in the market level is approximately equal to the change in implied volatility of an option for a change in the strike level.

Figure 1: Development of the Eurostoxx50 index and the at-the-money implied volatility of the Eurostoxx50 index

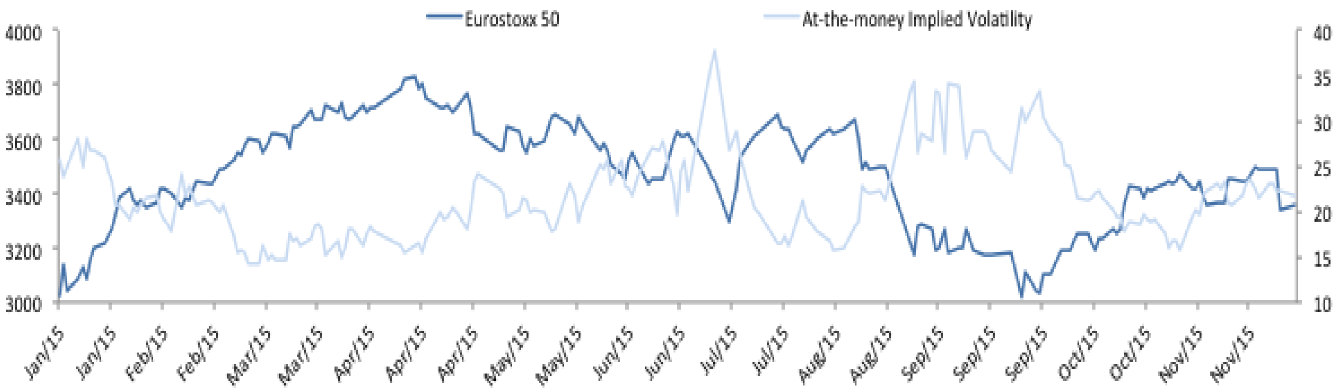
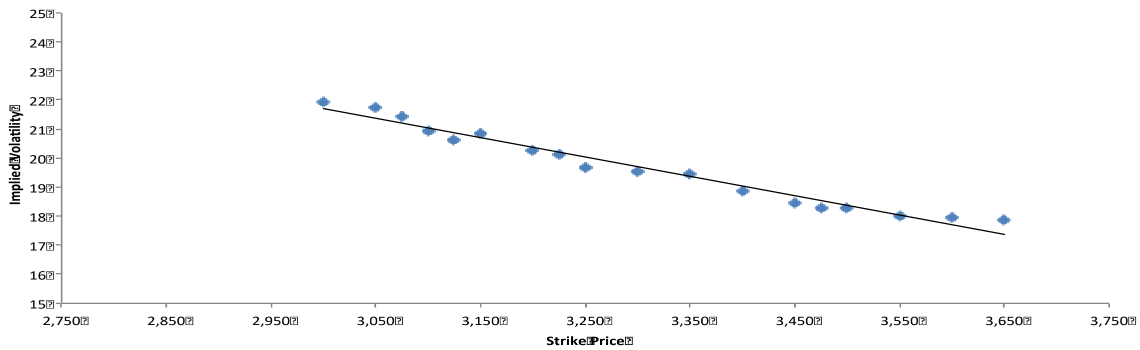


Table 2: Summary of statistics of the Eurostoxx50 and the at-the-money implied volatility for financial year 2015

Moneyness	**EuroStoxx50**	**At-the-money Implied Volatility**
Mean	3,460.57	21.79
Minimum	3,019.34	14.24
Maximum	3,828.78	37.69
Median	3,468.26	21.14
Standard Deviation	191.72	4.81
Kurtosis	-0.70	0.59
Skewness	-0.37	0.90
Correlation SX5E	1.00	-0.67

Figure 2: Observed volatility skew on 08-01-2016



Example of a Monte-Carlo simulation for the standard Black & Scholes- and skew-adjusted delta

Figure 3: Simulated development of the price of the underlying asset used in Monte-Carlo procedure (S_0 = 100, σ = 20% and μ_s= 0,2)

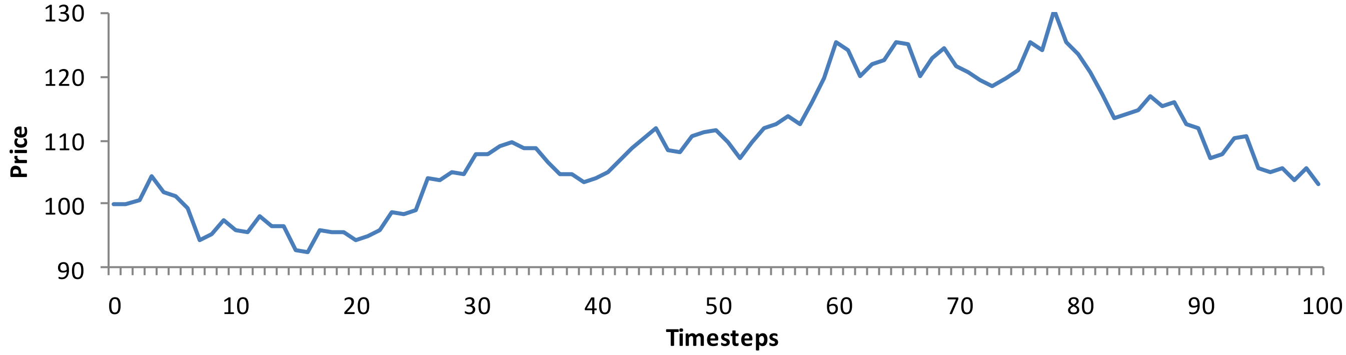


Figure 4: Simulated development of the deltas used in Monte-Carlo procedure **(**K = 100, r_f = 0,05)

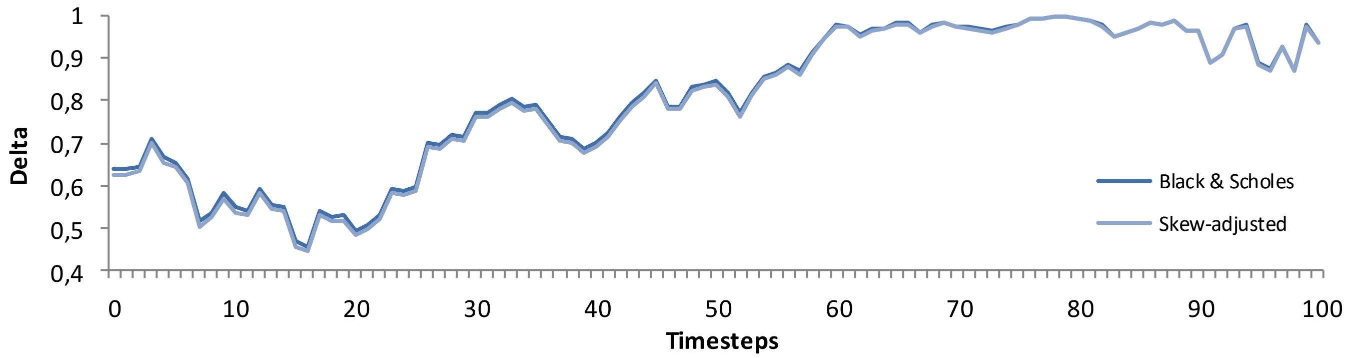
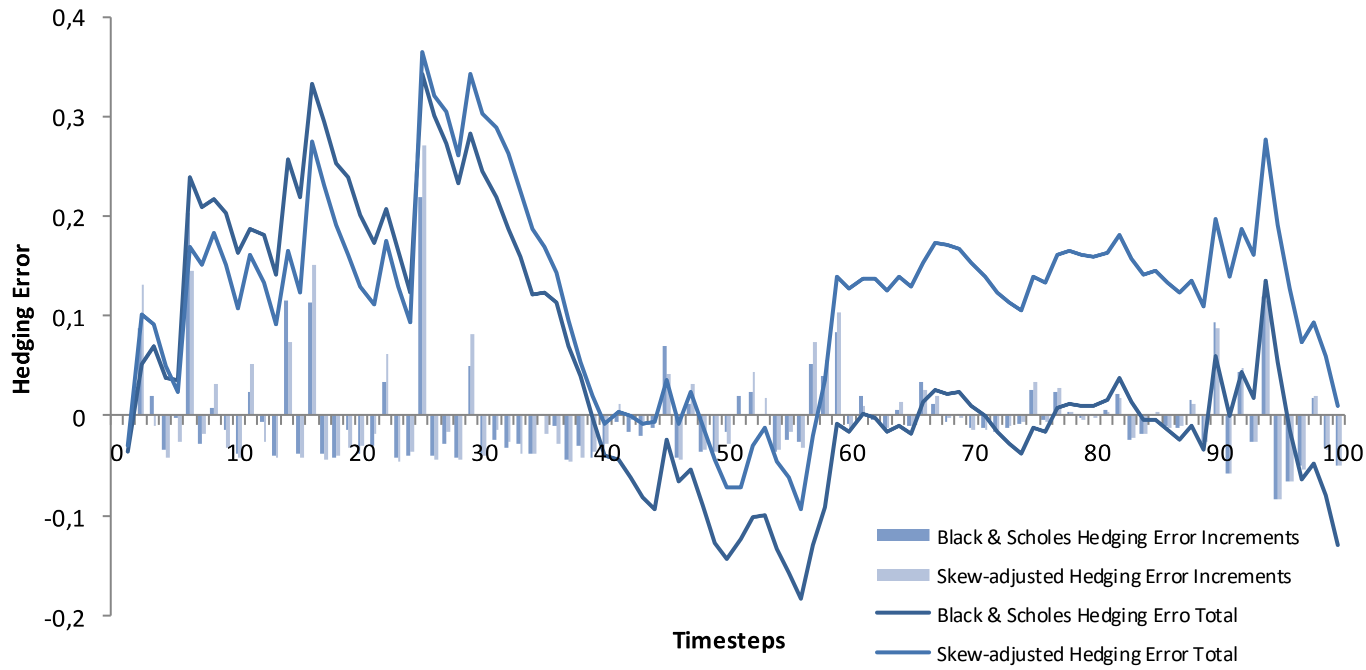


Figure 5: Simulated development of hedging error of the deltas used in Monte-Carlo procedure



Hedging error of the standard Black & Scholes- and skew-adjusted delta for real-market data

Table 2: Hedging error of both deltas for a 2-day hedging horizon

		Mean Absolute Hedging Error				Root Mean Squared Hedging Error			
Moneyness	**Time to Maturity**	δ_{bs}	δ_{skew}	**Difference**	**%**	δ_{bs}	δ_{skew}	**Difference**	**%**
Full Sample	All	5.58	5.58	0.566	10.12% **	8.34	7.76	0.581	6.96% **
	Long-term	5.51	4.95	0.563	10.22% **	7.53	7.06	0.476	6.32% **
	Short-term	5.57	5.05	0.524	9.39% **	8.86	8.28	0.578	6.52% **
In-the-money	All	11.12	10.91	0.209	1.88% **	13.49	13.37	0.121	0.89%
	Long-term	9.85	9.70	0.149	1.51% *	11.16	11.17	-0.011	-0.10%
	Short-term	12.02	11.73	0.288	2.39% *	14.84	14.63	0.208	1.40%
At-the-money	All	7.77	7.13	0.635	8.17% *	10.19	9.64	0.548	5.48% **
	Long-term	7.52	6.94	0.578	7.69% **	9.37	9.03	0.347	3.70**
	Short-term	7.95	7.28	0.672	8.45% **	10.72	10.06	0.660	6.16%*
Out-of-the-money	All	3.18	2.59	0.592	18.60% **	5.08	4.23	0.851	16.77% **
	Long-term	3.79	3.13	0.656	17.32% **	5.46	4.63	0.827	15.16% **
	Short-term	2.48	2.01	0.469	18.89% **	4.37	3.66	0.709	16.23% **

Table 3: Hedging error of both deltas for a 5-day hedging horizon

		Mean Absolute Hedging Error				Root Mean Squared Hedging Error			
Moneyness	**Time to Maturity**	δ_{bs}	δ_{skew}	**Difference**	**%**	δ_{bs}	δ_{skew}	**Difference**	**%**
Full Sample	All	9.64	8.55	1.089	11.30% **	13.26	12.23	1.034	7.79% **
	Long-term	9.48	8.37	1.107	11.67% **	12.07	11.19	0.881	7.30% **
	Short-term	9.58	8.58	1.002	10.46% **	13.93	12.91	1.020	7.32% **
In-the-money	All	20.32	19.92	0.392	1.93% **	22.75	22.47	0.276	1.21% *
	Long-term	17.43	17.11	0.318	1.82% **	18.80	18.70	0.096	0.51%
	Short-term	21.93	21.38	0.556	2.54% **	24.75	24.29	0.462	1.88% **
At-the-money	All	13.71	12.41	1.298	9.47% **	16.09	14.96	1.135	7.05% **
	Long-term	13.34	11.92	1.419	10.64% **	14.95	13.85	1.093	7.31% **
	Short-term	13.94	12.71	1.231	8.83% **	16.78	15.63	1.150	6.85% **
Out-of-the-money	All	5.28	4.19	1.094	20.70% **	7.73	6.39	1.348	17.43% **
	Long-term	6.37	5.12	1.242	19.51% **	8.40	7.02	1.379	16.42% **
	Short-term	4.04	3.18	0.852	21.11% **	6.44	5.38	1.058	16.43% **

Table 4: Hedging error of both deltas for a 10-day hedging horizon

Moneyness	Time to Maturity	Mean Absolute Hedging Error δ_{bs}	δ_{skew}	Difference	%	Root Mean Squared Hedging Error δ_{bs}	δ_{skew}	Difference	%
Full Sample	All	20.07	17.70	2.366	11.79% **	25.76	23.27	2.489	9.66% **
	Long-term	19.90	17.64	2.257	11.34% **	24.11	21.68	2.425	10.06% **
	Short-term	19.76	17.49	2.265	11.47% **	26.54	24.18	2.362	8.90% **
In-the-money	All	47.55	46.79	0.758	1.59% **	49.37	48.49	0.878	1.77% *
	Long-term	39.44	39.12	0.321	0.81% **	40.60	40.10	0.501	1.23% *
	Short-term	51.54	50.34	1.194	2.32% **	53.55	52.09	1.246	2.34 **
At-the-money	All	31.16	28.07	3.095	9.93% **	33.15	29.93	3.221	9.72% **
	Long-term	30.00	26.50	3.498	11.66% **	31.67	28.06	3.602	11.37% **
	Short-term	31.89	29.01	2.885	9.05% **	34.06	31.02	3.037	8.92% **
Out-of-the-money	All	10.25	7.95	2.302	22.45% **	13.71	10.69	3.020	22.04% **
	Long-term	12.72	10.30	2.419	19.01% **	15.78	12.58	3.208	20.32% **
	Short-term	7.57	5.62	1.951	25.76% **	10.62	8.11	2.503	23.58% **

Profit and loss analysis of skew-adjusted term-sorted long/short portfolio strategy

Table 5: P&L analysis of skew-adjusted term-sorted long/short portfolio strategy for a 2-days investment horizon

Moneyness	Time to Maturity	Average P&L	Total P&L	Std. Dev.	Skewness	Kurtosis
Full Sample	All	-207.96	-22,667.53	7,318.91	-4.23	27.54
	Long-term	-547.14	-60,732.19	4,325.39	-4.64	31.47
	Short-term	-227.43	-28,655.74	6,932.57	-3.61	22.92
In-the-money	All	92.23	3,412.62	810.86	-0.39	6.19
	Long-term	-7,880.82	-212,782.06	10,989.95	-0.70	1.70
	Short-term	-8,069.21	-363,114.63	11,949.54	-0.87	2.20
At-the-money	All	40.60	2,841.69	1,138.55	-1.54	9.99
	Long-term	-954.83	-43,922.13	4489.85	-4.48	21.30
	Short-term	-244.92	-22,532.55	1,182.20	-2.43	15.32
Out-of-the-money	All	2,343.89	110,162.83	2,450.35	-0.24	3.31
	Long-term	840.94	48,774.51	1,014.77	-0.14	3.25
	Short-term	1,566.77	90,872.49	2,631.53	0.15	3.81

Table 6: P&L analysis of skew-adjusted term-sorted long/short portfolio strategy for a 5-days investment horizon

Moneyness	Time to Maturity	Average P&L	Total P&L	Std. Dev.	Skewness	Kurtosis
Full Sample	All	2,582.83	136,889.94	8,678.99	-4.66	28.51
	Long-term	448.57	22,876.90	2,562.04	-1.39	6.43
	Short-term	2,380.59	166,641.51	5,650.77	-1.69	7.82
In-the-money	All	230.41	1,612.86	610.01	0.23	2.06
	Long-term	-10,141.28	-141,977.27	13,290.22	-0.77	2.26
	Short-term	-10,583.80	-95,254.20	10,823.39	0.09	1.14
At-the-money	All	375.70	9,016.87	771.04	0.88	3.08
	Long-term	-2,970.57	-32,676.27	6,322.78	-1.67	3.84
	Short-term	9,56	353,72	1,564.55	-1.28	5.31
Out-of-the-money	All	3,572.47	32,152.19	3,071.35	-0.12	1.57
	Long-term	1,396.41	25,135.42	1,273.63	0.58	3.25
	Short-term	2,479.79	29,757.51	2,338.84	0.71	2.31

Table 7: P&L analysis of skew-adjusted term-sorted long/short portfolio strategy for a 10-days investment horizon

Moneyness	Time to Maturity	Average P&L	Total P&L	Std. Dev.	Skewness	Kurtosis
Full Sample	All	6,318.13	107,408.21	7,345.02	-1.52	4.97
	Long-term	-234.62	-5,396.26	5,548.00	-1.84	5.61
	Short-term	5,568.24	150,342.56	8,069.36	-1.91	7.14
In-the-money	All	-	-	-	-	
	Long-term	-15,348.86	-46,046.58	13,860.60	0.70	1.5
	Short-term	-	-	-	-	
At-the-money	All	62.75	250.98	325.71	0.25	1.337
	Long-term	-39.98	-39.98	-	-	-
	Short-term	1,221.25	9,770.03	502.66	-0.18	1.96
Out-of-the-money	All	-	-	-	-	-
	Long-term	1,877.30	11,263.82	914.12	1.46	3.64
	Short-term	644.75	644.75	-	-	-

Figure 6: Incremental P&L of full sample options
(1-day: 158 observations, 2-days: 109 observations, 5-days: 53 observations, 10-days: 17 observations)

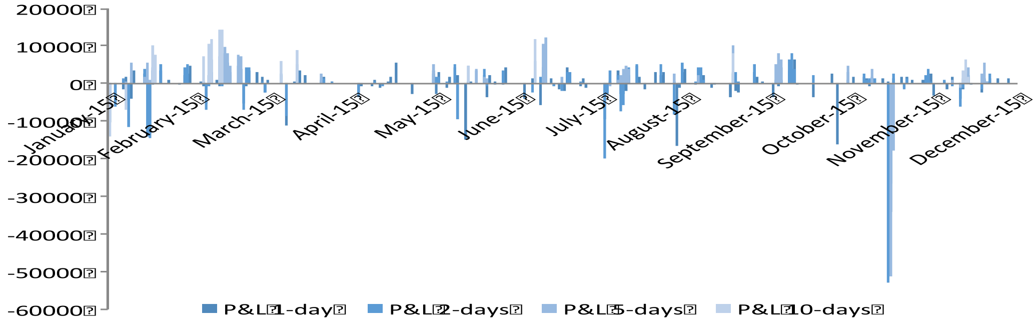


Figure 7: Cumulative P&L of full sample options
(1-day: 158 observations, 2-days: 109 observations, 5-days: 53 observations, 10-days: 17 observations)

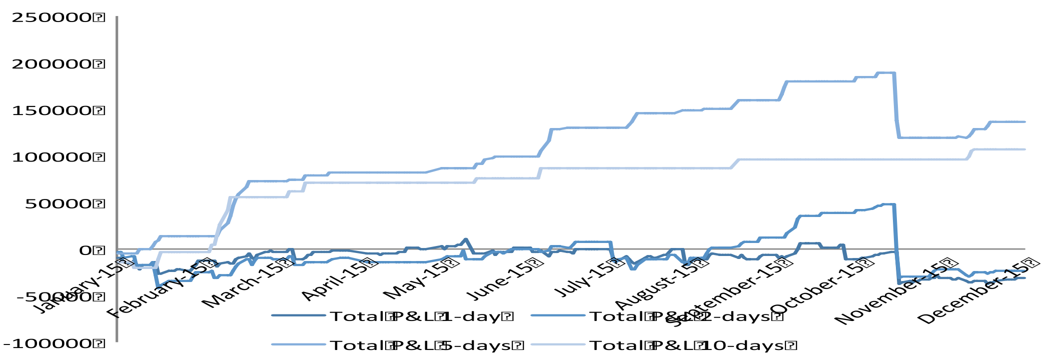


Figure 8: Incremental P&L of long-term options
(1-day: 150 observations, 2-days: 111 observations, 5-days: 51 observations, 10-days: 23 observations)

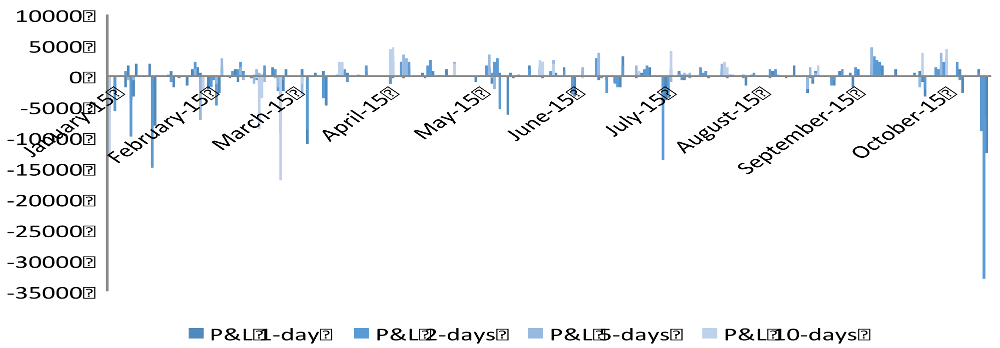


Figure 9: Cumulative P&L of long-term options
(1-day: 150 observations, 2-days: 111 observations, 5-days: 51 observations, 10-days: 23 observations)

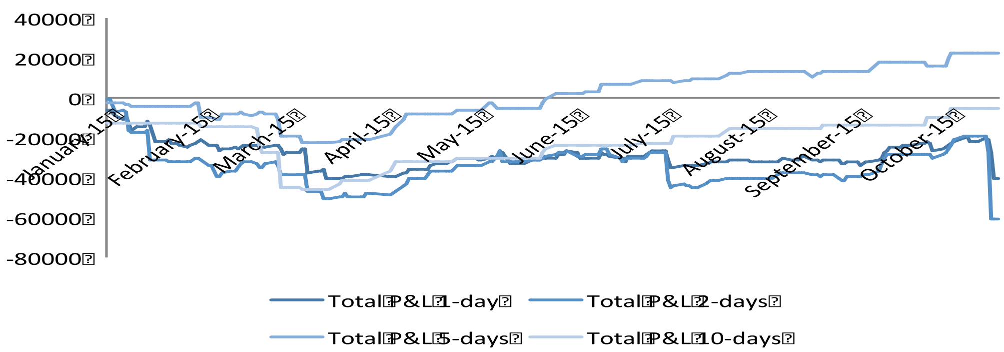


Figure 10: Incremental P&L of short-term options
(1-day: 167 observations, 2-days: 126 observations, 5-days: 70 observations, 10-days: 27 observations)

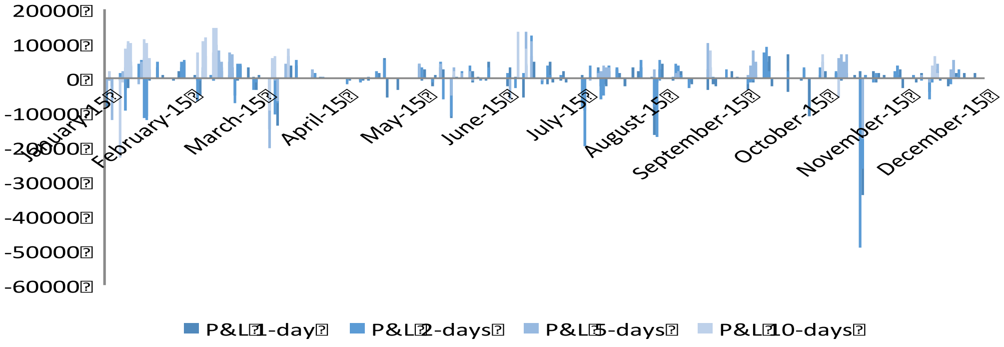


Figure 11: Cumulative P&L of short-term options
(1-day: 167 observations, 2-days: 126 observations, 5-days: 70 observations, 10-days: 27 observations)

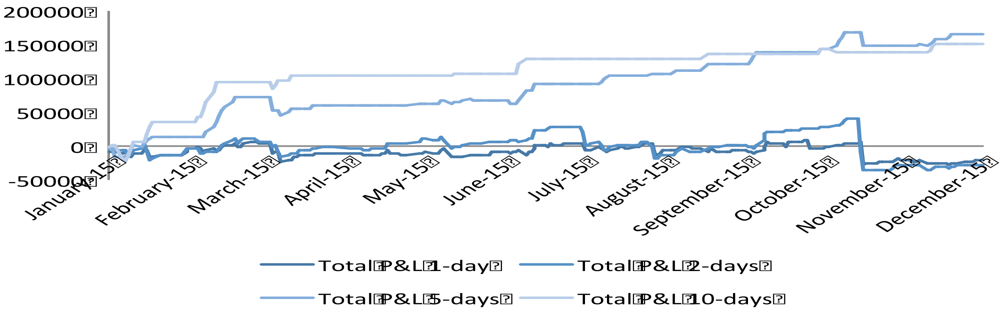


Figure 12: Incremental P&L of in-the-money options
(1-day: 62 observations, 2-days: 37 observations, 5-days: 7 observations, 10-days: 0 observations)

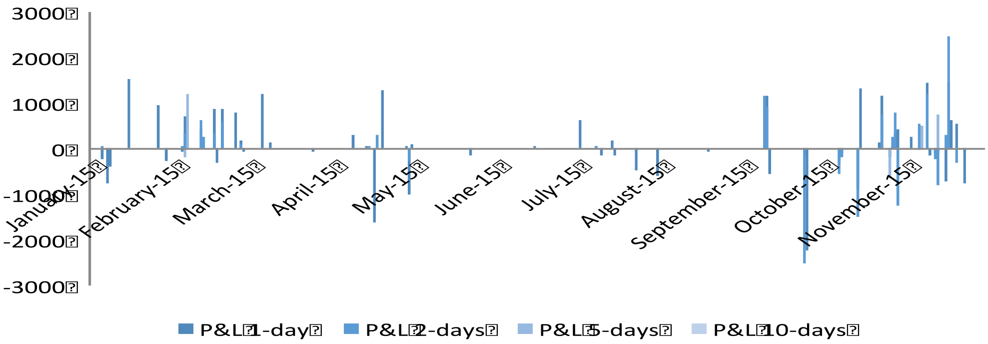


Figure 13: Cumulative P&L of in-the-money options
(1-day: 62 observations, 2-days: 37 observations, 5-days: 7 observations, 10-days: 0 observations)

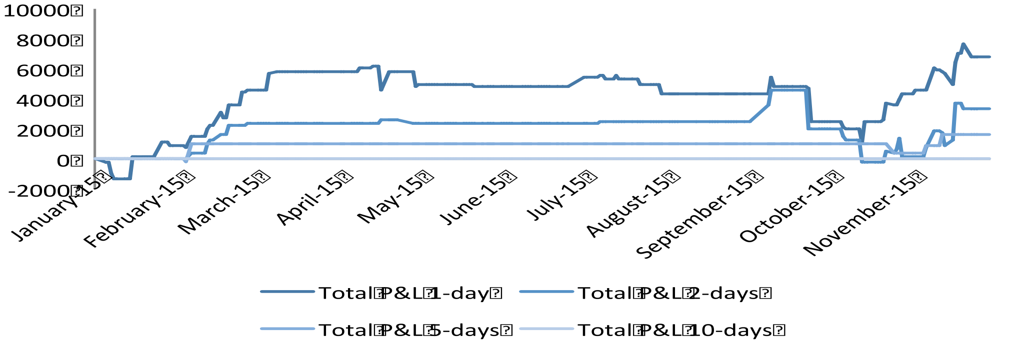


Figure 14: Incremental P&L of at-the-money options
(1-day: 113 observations, 2-days: 70 observations, 5-days: 24 observations, 10-days: 4 observations)

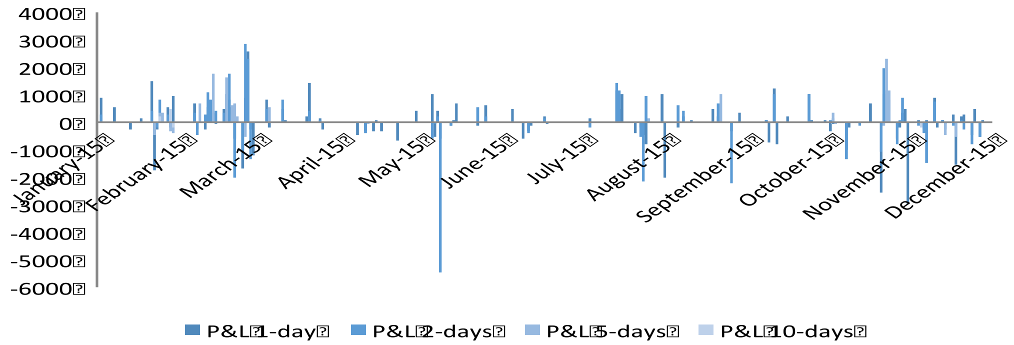


Figure 15: Cumulative P&L of at-the-money options
(1-day: 113 observations, 2-days: 70 observations, 5-days: 24 observations, 10-days: 4 observations)

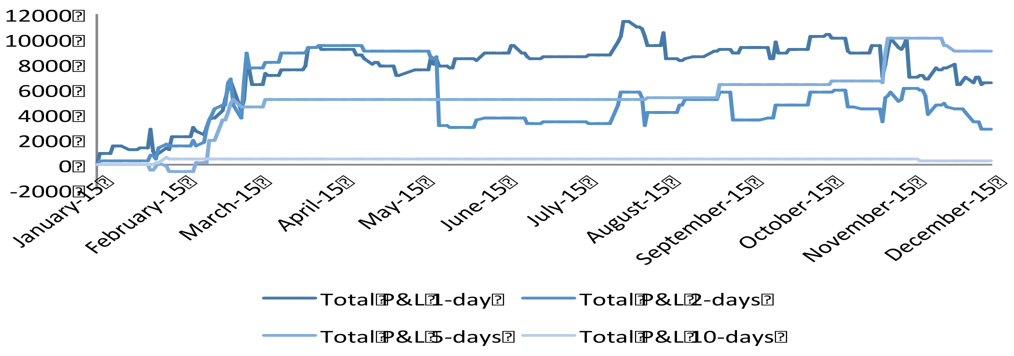


Figure 16: Incremental P&L of out-of-the-money options
(1-day: 105 observations, 2-days: 47 observations, 5-days: 9 observations, 10-days: 0 observations)

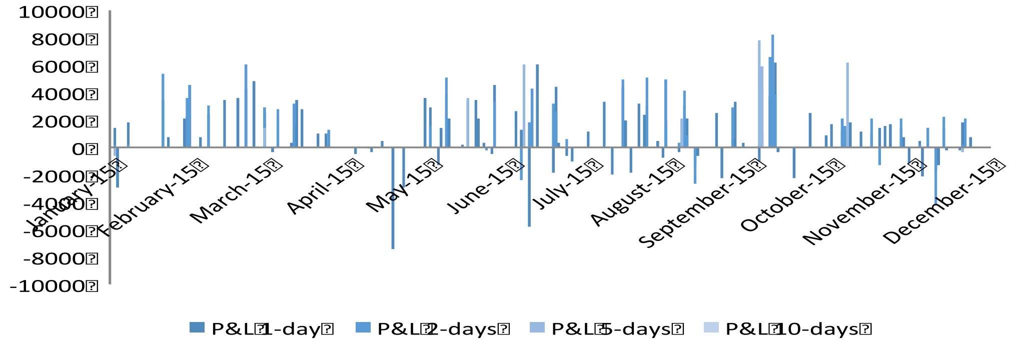


Figure 17: Cumulative P&L of out-of-the-money options
(1-day: 105 observations, 2-days: 47 observations, 5-days: 9 observations, 10-days: 0 observations)

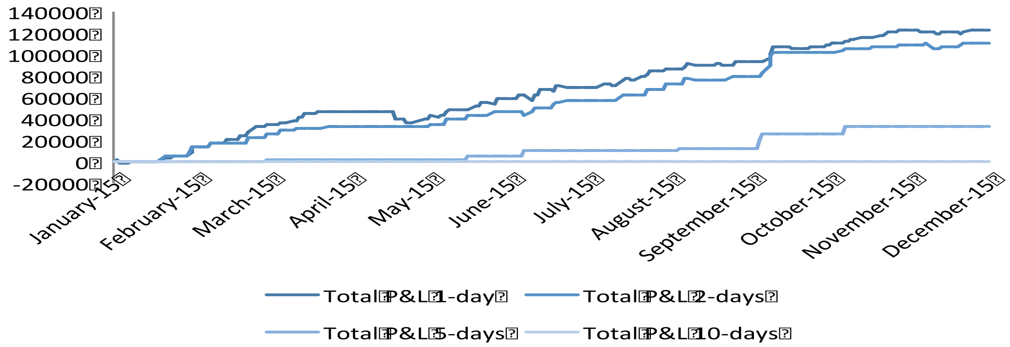


Figure 18: Incremental P&L of long-term in-the-money options
(1-day: 49 observations, 2-days: 27 observations, 5-days: 14 observations, 10-days: 3 observations)

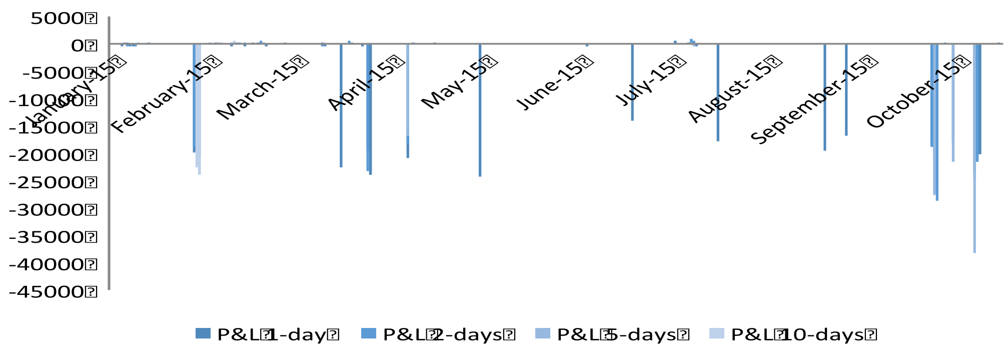


Figure 19: Cumulative P&L of long-term in-the-money options
(1-day: 49 observations, 2-days: 27 observations, 5-days: 14 observations, 10-days: 3 observations)

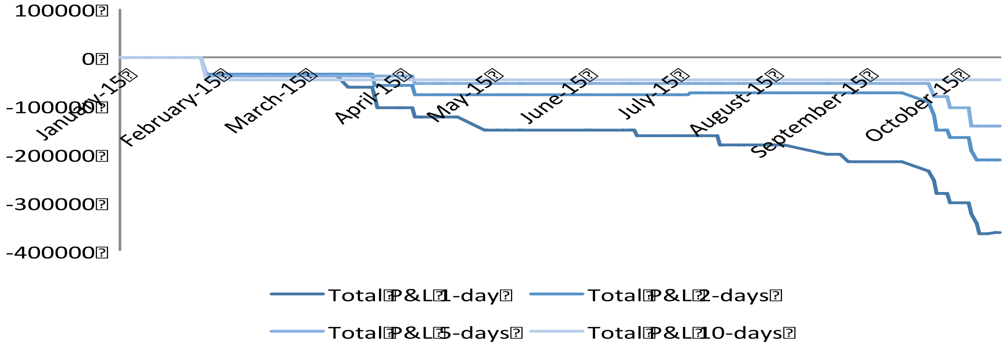


Figure 20: Incremental P&L of long-term at-the-money options
(1-day: 90 observations, 2-days: 46 observations, 5-days: 11 observations, 10-days: 1 observation)

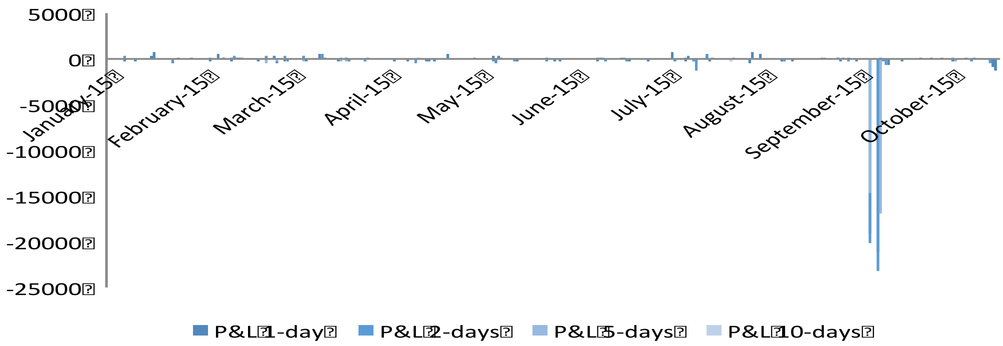


Figure 21: Cumulative P&L of long-term at-the-money options
(1-day: 90 observations, 2-days: 46 observations, 5-days: 11 observations, 10-days: 1 observation)

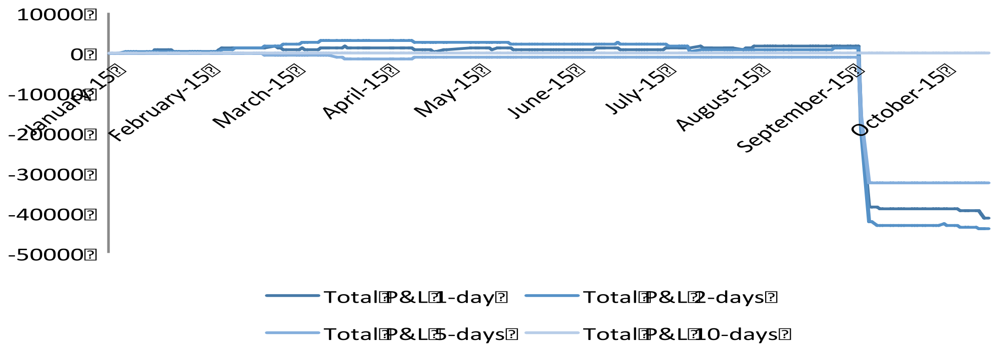


Figure 22: Incremental P&L of long-term out-of-the-money options
(1-day: 100 observations, 2-days: 58 observations, 5-days: 18 observations, 10-days: 6 observations)

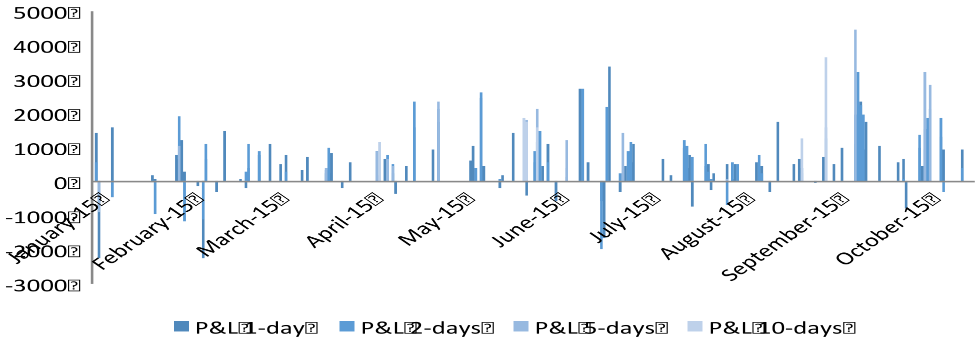


Figure 23: Cumulative P&L of long-term out-of-the-money options
(1-day: 100 observations, 2-days: 58 observations, 5-days: 18 observations, 10-days: 6 observations)

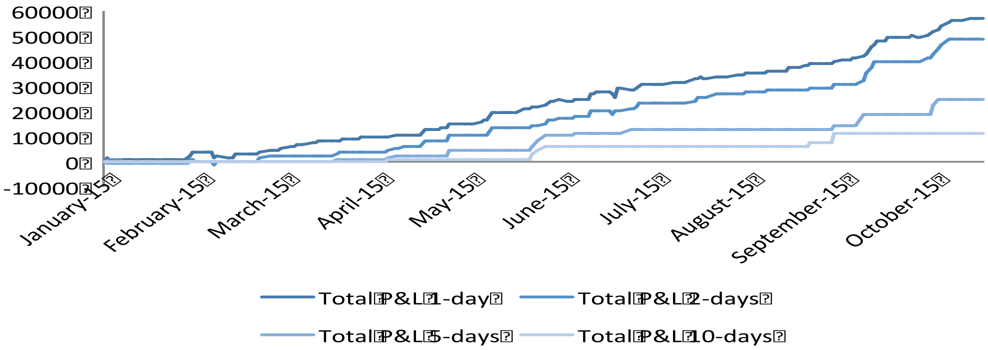


Figure 24: Incremental P&L of short-term in-the-money options
(1-day: 88 observations, 2-days: 45 observations, 5-days: 9 observations, 10-days: 0 observations)

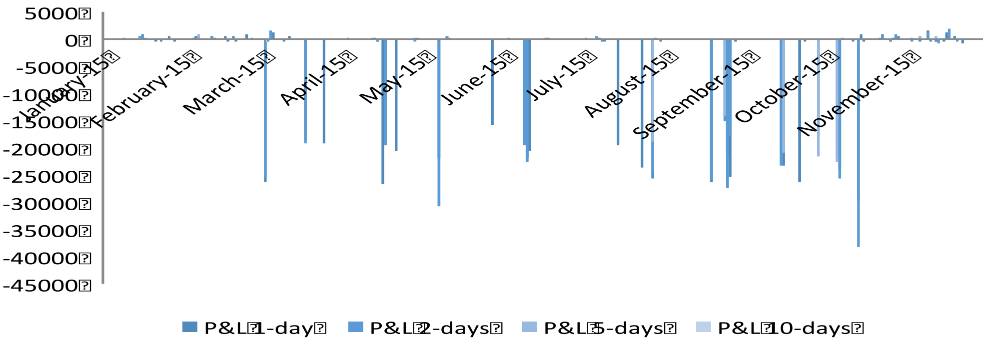


Figure 25: Cumulative P&L of short-term in-the-money options
(1-day: 88 observations, 2-days: 45 observations, 5-days: 9 observations, 10-days: 0 observations)

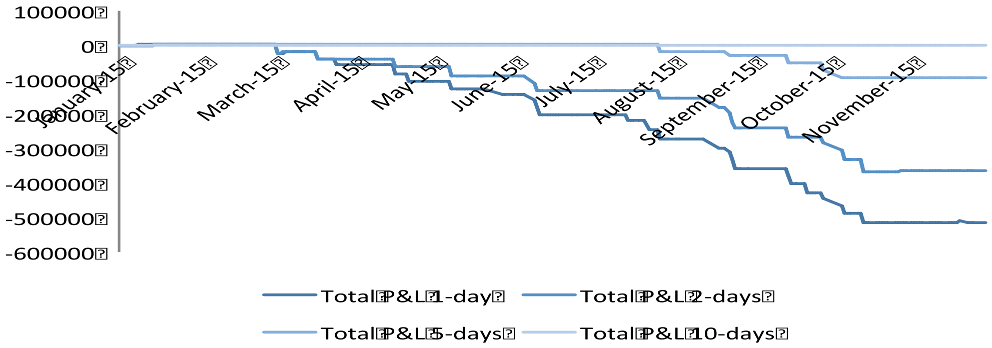


Figure 26: Incremental P&L of short-term at-the-money options
(1-day: 139 observations, 2-days: 92 observations, 5-days: 37 observations, 10-days: 8 observations)

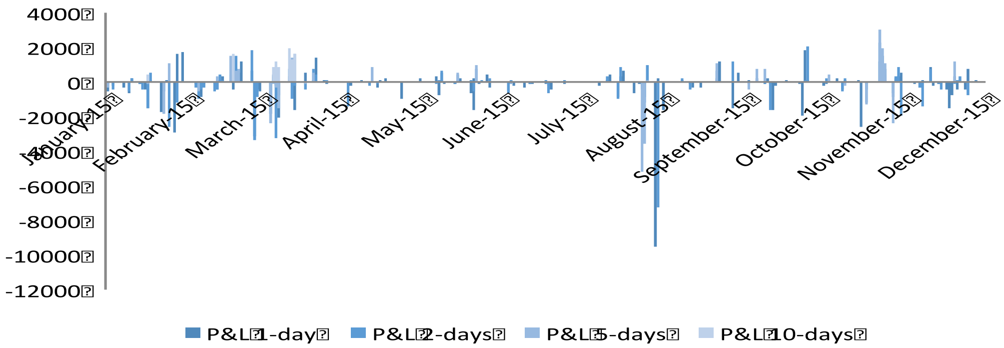


Figure 27: Cumulative P&L of short-term at-the-money options
(1-day: 139 observations, 2-days: 92 observations, 5-days: 37 observations, 10-days: 8 observations)

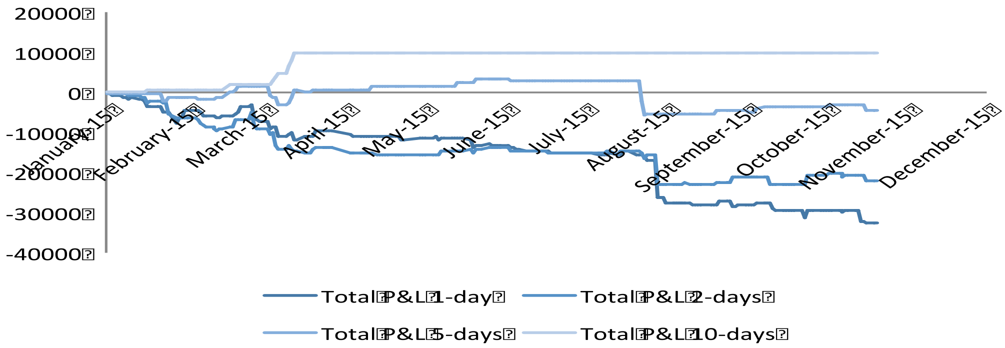


Figure 28: Incremental P&L of short-term out-of-the-money options
(1-day: 110 observations, 2-days: 58 observations, 5-days: 12 observations, 10-days: 1 observation)

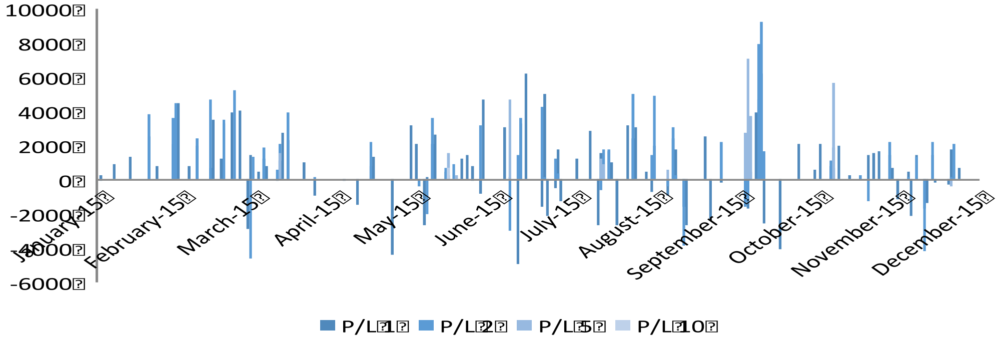


Figure 29: Cumulative P&L of short-term out-of-the-money options
(1-day: 110 observations, 2-days: 58 observations, 5-days: 12 observations, 10-days: 1 observation)