ERASMUS UNIVERSITY ROTTERDAM

ERASMUS SCHOOL OF ECONOMICS

MASTER THESIS QUANTITATIVE FINANCE

The Optimal Lifecycle when investing contributions in defined contributions pension plans in different asset classes

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Author:
J.E. OWIE

Second Assessor:
B.J.L. KEIJSERS MSc

Supervisor EY:
drs. R. VAN DAALEN AAG

Supervisor EUR:
dr. H.J.W.G. KOLE

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Abstract

Currently there is a huge shift from defined benefit pensions to defined contribution pensions. In defined contribution pensions the contributions are invested following a lifecycle strategy. This paper investigates the optimal lifecycle within a set of different lifecycles when investing contributions in defined contribution pension plans in stocks, real estate and bonds. For this purpose economic state, asset, interest rate, inflation and salary data are analysed for the period April 2005 until April 2017. The analysis and 42 years ahead simulation are done using a Markov switching model in combination with a VARX model and a CIR model. In total 10,000 different paths are simulated and for each path 13 different lifecycles are analysed. The lifecycles result in a for inflation corrected monthly pension and these are analysed using multiple ranking criteria. This research finds that within the 13 analysed lifecycles the optimal lifecycle in terms of expected utility and chance of reaching a pension of at least 75% of the average income is a lifecycle fully invested in stocks.

Contents

1   Problem Description

Currently there is a huge shift from defined benefit (DB) pensions to defined contribution (DC) pensions. This is a result of the financial crisis, which had a major impact on the financial position of many companies. Moreover, the historically low interest rate and the increase in life expectancy, caused the DB pension to become very expensive. This had a huge impact on the pension liabilities and therefore many companies opt for DC (Jansen, 2014). For most internationally operating companies it is mandatory to determine future obligations with respect to employee benefits. This also concerns pension plans. The classification of a pension plan determines whether there are any obligations. If the pension plan can be qualified as a DB plan (i.e. final pay or average pay in the Netherlands) this could have negative consequences for the balance sheet for accounting purposes. If a pension plan can be qualified as a DC plan, no obligations arise for the balance sheet.

In a DC pension plan the pension benefits are uncertain. The predetermined (often a percentage of salary) contributions for pension are paid into an individual account for each employee. The contributions are invested, according to a lifecycle, in multiple assets. Pension benefits depend on the asset allocation and market performance. The employee bears the risk in a DC pension plan. On retirement, the employee's individual account is used to provide retirement benefits. In the Netherlands this is done through the purchase of a pension annuity.

Members of DC plans can choose in what way their pension money is invested. They can choose to invest it themselves or they can choose a structure, a "lifecycle". A lifecycle is a premade asset allocation per age. The purpose of this research is to investigate how lifecycle investment strategies affects retirement income outcomes in DC plans and to find the optimal lifecycle within the set of analysed lifecycles. It requires a thorough understanding of investments and of individual needs and expectations to make a rational decision about which lifecycle to take. Empirical research shows that many people are incapable or unwilling to make such decisions: over 95% of employees participating in lifecycle plans leave it on the default option. For this reason, the design of lifecycle investment options is of critical policy relevance.

The most important aspect in this research is the return distribution of the resulting monthly pension when a given lifecycle is used. Aspects that are taken into account are economic good and bad times, investment risk, interest risk and inflation risk. Besides the return distribution the career path

and development of salary are taken into account.

This research focusses on people following the lifepath of Jan Modaal as described in section 4.4.1, who start investing from the age of 25 and retire at the age of 67. Contributors are for the full period 100% in a DC pension plan, they have no additional pension savings and receive a pension based on a basic state pension and the lifecycle results. Furthermore this research assumes that there can only be invested in going long in stocks, real estate and bonds and that there are no transaction costs.

The analysis and 42 years ahead simulation is done using a Markov switching model for simulating expansion and recession period, a CIR model for simulating the short rate and the corresponding term structure and a VARX model for simulating the asset returns taking the previous period, interest rate, inflation and other asset returns into account. For this purpose the MSCI index for European Stocks, the MSCI index for European Real Estate, the 1 month Euribor, and salary and inflation data, obtained from CBS, are analysed for the period April 2005 until April 2017.

In total 10,000 different paths are simulated and for each path 13 different lifecycles are analysed. The lifecycles result in a for inflation corrected monthly pension and these are analysed using multiple ranking criteria with the most important criteria being expected utility and the chance to reach a monthly pension of 75% of the average salary. This research finds that the optimal lifecycle, within the set of 13 analysed lifecycles, in terms of expected utility and chance of reaching a pension of at least 75% of the average income is a lifecycle fully invested in stocks. This result is useful for pension providers in order to improve their lifecycles. But it also can be a starting point for further research in lifecycle optimization.

As countries have very different pension structures this paper only focusses on the Dutch pension system. Furthermore this paper assumes that members of a DC pension plan will use the pension capital at retirement for buying an annuity. Due to limited time, no attention is paid to the impact of the introduction of variable annuities for DC plans which has come into effect in the Netherlands as of 1 September 2016 ("Wet verbeterde premieregeling").

In the next section, some background information is provided regarding the Dutch pension system and regarding the lifecycle principle in DC pension plans. Then section 3 contains a literature review followed by section 4 which discusses the methods and data. Section 5 discusses the obtained results and section 6 concludes.

2   Pension System & Lifecycles

Before discussing the construction of a lifecycle it is important to understand how the pension system and lifecycles work. This section explains the Dutch pension system and how lifecycles work.

2.1   Pension System

There are three possible ways someone receives his pension benefits: when the pension age is reached, in case of death and when you are unable to work. What happens in each case depends on the pension plan. This paper focusses only on what happens when pension age is reached.

2.1.1   Pension Pillars

The pension system exists out of three different parts, the pension "pillars". The definition used comes from the study by Centre of European Policy Studies (Lannoo et al., 2014).
The first pillar is the publicly managed Pay As You Go(PAYG) system, in the Netherlands that is the AOW(Algemene OuderdomsWet). The purpose is to prevent pensioners to fall into extreme poverty. It guarantees an adequate pension for everyone older than the pension age. First pillar pension plans are based on the DB principle. The AOW part of pensioners is taken into account in this research. The DB and DC principle are discussed in section 2.1.4.
The second pillar contains the pension plans linked to labour market. It aims to ensure a secure and adequate income level, which reflects the income level during the working career. These pension plans come in the form of DB and DC. The DC plan in this pillar is where this research focusses on.
The third pillar is for the individual, voluntary pension plans. Because saving in this pillar is voluntary it is hard to distinguish pension savings from other long- or short-term savings. For this reason these kind of savings are disregarded in this research.

2.1.2   Pension Triangle

In pension plans linked to labour market (second pillar) there is a triangular relation between the employee, employer and the pension provider shown in Figure 1. These relations are:
1. Employer-Employee: The employer and employee engage in a pension agreement as part of the labour contract. A pension agreement is a pension

plan in which the employer and the employee make contributions. The aim is to generate income for the employee after retirement.

2. Employer-Pension provider: According to Dutch law the employer and pension provider must be two strictly separate businesses. The employer and the pension provider have an administration agreement. The administration agreement must be in line with the pension agreement and contains financial and exit conditions between employer and pension provider.

3. Employee-Pension provider: They have a relation as a result of the outsourcing of the execution of the pension agreement to the pension provider. The pension provider makes a pension plan and sends a start letter to the employee. The content of the pension plan is based on the pension agreement.
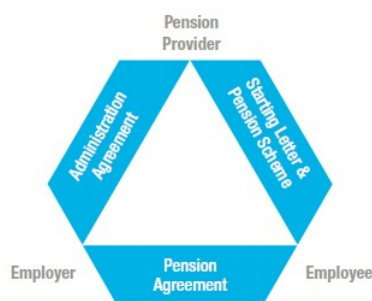

Figure 1: Pension triangle

2.1.3   Legal Framework

While there is no legal obligation for employers to offer a pension to employees, more than 95% of the employees in the Netherlands are in a supplementary pension plan provided by their employer. The reason for this high rate is that in approximately 70% of the cases, a compulsory supplementary pension plan applies for the industry in which the employer operates. Such plans are provided through industry-wide pension funds, managed by employers and employees.

Pension rights are not taxed at the time they are awarded if the particular plan qualifies for wage tax purposes. A pension plan qualifies if:
a. the limits set by the Dutch Wage Tax Act are not exceeded;
b. all formal provisions are complied with;
c. the pension provider is permitted to operate as such under the law.

In the Netherlands, the Dutch Central Bank (DNB) and the Authority Financial Markets (AFM) are responsible for monitoring pension providers.

DNB performs material and prudential supervision over pension funds and considers the financial assessment framework and pension fund governance. The AFM supervises the conduct of pension administrators, covering the provision of information prescribed by law and the duty of care with regard to DC plans.

2.1.4 Pension Plans

There are various systems for accumulating pension benefits in the Netherlands. Every pension plan must take the state pension (AOW) into consideration as pensions can only be built-up on the salary minus the AOW-deductible (Franchise). The different plans are:
Defined Benefit (DB), Defined Contribution (DC) and Cash Balance (CB)

The most common pension plan is the Defined Benefit plan. In this plan the benefits are certain and known in advance, regardless of the returns of the underlying investments. The benefits are based on the number of years in service, the salary history and age.
Other important aspects of DB plans are that:
Individuals don't have influence on the investment strategy. Risk is spread over different years/generations. This means that results of excess return on investments, interest rate movements and changes in longevity risk whether positive or negative can be spread over multiple years. Effectively this causes solidarity between participants of different generations.
There are a few different DB agreements. The two most used are the final pay agreement and the career average agreement. In these two agreements the final salary and average salary are used to determine pension benefits.
In a DB plan the employer pays a premium to the pension provider. This premium is set in such a way that it minimizes the risk that the pension provider cannot pay the benefits. The premium increases if the interest rate drops, the financial market is performing poorly and/or the life expectancy increases, because then a lifetime of pension benefits becomes more expensive.

In Defined Contribution plans companies do not get any obligations on the balance sheet which is good for accounting purposes. This causes the DC plan to grow in popularity. In a DC plan the contributions are predetermined and paid by the employer, the employee or both. Most times the contributions made by the employer are matched to some degree by the contributions made by the employee. At retirement the total build up capital depends on the amount of contributions and the investment return. The total capital is

then used to buy an old age pension. The employee bears the risk in investment, longevity and interest rate.

Then there is the Cash Balance pension plan. A rarely used plan, it is a combination of DB and DC. In the CB plan the capital at retirement is certain and known in advance, but the pension benefits are not. At retirement age the capital will be used to buy an old age pension. This causes the risk to be more spread between employer and employee. The employer bears the investment risk and the employee the longevity and interest rate risk.

2.2 Lifecycles

In a DC pension plan contributions are invested by the pension provider. The pension provider invests according to a predetermined asset allocation per age, a lifecycle. Participants can also choose to invest the contributions themselves but in practice this rarely happens. In the following figure an example of a lifecycle consisting of three different asset classes is given.

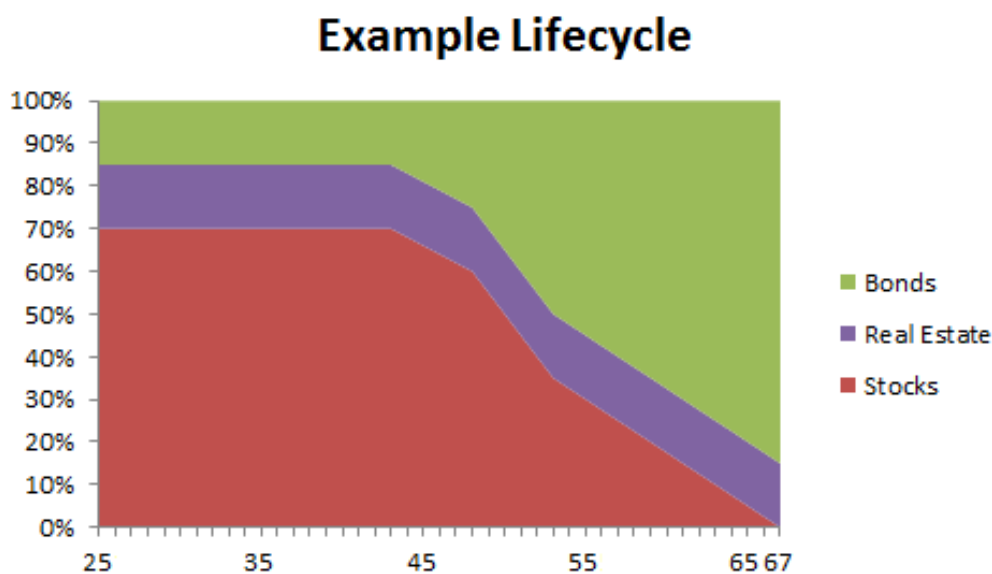Figure 2: Example Lifecycle.

In this lifecycle the percentage invested in stocks remains 70% until the age of 43. After this moment the allocation slowly shifts toward bonds. This can be a linear transition, however in this case and often in practice this is not a linear transition. The percentage invested in real estate remains 15% for the whole period.This is just an example of how a lifecycle can look.

2.2.1   Lifecycle Goals

The resulting pension capital depends on the amount of contributions and on the investment return of the underlying assets. The risk lies with the participant. It is important that the allocation has an appropriate balance between maximizing long-term growth and managing risk. Too much emphasis on returns may cause the portfolio to be too volatile. Too much emphasis on controlling swings may limit growth.

The goal is to obtain the "right" allocations based on theory of what risk tolerance would be. The allocations are typically determined through a model optimization process. Another approach is to design allocations based on investors' observed behaviour. This approach assumes that, on average, investors are able to choose appropriate portfolios, and that valid conclusions about risk tolerance can be derived by observing their choices over time.

To determine which lifecycle type to include, plan sponsors first must understand the target audience for each type. Then sponsors must assess their employee base to see which lifecycle type gives the better fit.

2.2.2   Lifecycle in practice

Pension providers offer multiple lifecycles. However these lifecycles differ for every pension provider. Even the neutral/default lifecycles of different pension providers can have big differences. In the figures below the neutral lifecycles of Amersfoortse and Achmea are shown.
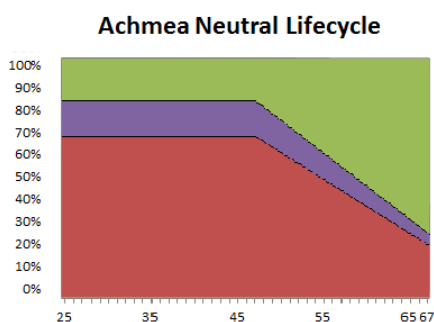

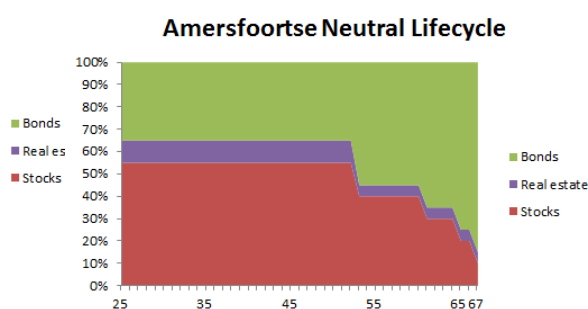Figure 3: Neutral lifecycle Achmea   Figure 4: Neutral lifecycle Amersfoortse

The two graphs show big differences. Achmea has a structural higher asset allocation toward stocks and a more smooth transition. It also shows similarities. They both have a similar asset allocation at pension date. They also

start around 15 or 20 years before pension date with the transition toward bonds. This agrees with the idea to lower risk as individuals get older. Both these lifecycles are in the set of lifecycles that are analysed in this paper.

3 Literature Review

The main focus of this research is to find the optimal lifecycle within the given set of lifecycles. Besides that, this paper aims to bring more insight into the differences of different lifecycles. This section will discuss relevant literature and how this research contributes to this literature.

A very influential model in lifecycle literature is from Merton (1969). His model assumes a two asset market, consisting of a risk free asset and a risky asset. In this model an investor must choose how much to consume and how he allocates his wealth between the two assets to maximize expected lifetime utility. By using dynamic programming Merton (1969) derives closed form solutions under the assumptions of log normal distributed risky assets and Constant Relative Risk Aversion(CRRA), which implies that attitude toward financial risk is independent of wealth level. The resulting lifecycle solutions depend on human capital, financial wealth, on the expected rate of return, volatility of risky asset returns and the relative risk aversion coefficient. The model by Merton (1969) has been an important building block for lifecycle literature. It has been the starting point of a great number of papers. Bodie et al. (1992) extended the model by adding expected benefits from human capital. Then Teulings and de Vries (2006) and Bovenberg et al. (2007) extended the model by adding a more elaborate model for anticipated returns on human capital and additional risk factors in financial markets.

This research will use the suggested extension to more assets from Merton (1969) to find "Merton's optimal lifecycle". This lifecycle will be used in the comparison to find the optimal lifecycle. For full details about the model see section 4.5

An interesting report that looks at differences between various lifecycles is van Ling and van Soest (2016). They show that there are big differences between neutral lifecycles of different pension providers. By using economical simulation (with 1000 scenarios) on three different neutral lifecycles, they find differences rising to 9% in expected return. Moreover they find big differences in how pension providers deal with their risk exposure.

This paper adds to the existing literature by the focus on real monthly pen-

sion results. By looking at pension results instead of total wealth this paper takes into account that pension is more expensive when interest is low and that you can buy less with the same amount of money when inflation is high. This paper also uses other simulation techniques for the asset returns. This is discussed in the next section.

3.1 Literature on Asset Returns

Correct model assumptions are crucial when forecasting asset returns over a long horizon. There is tremendous literature regarding the predictability of asset returns.

However the opinion about return predictability differs. Welch and Goyal (2008) argue that there is very little stock predictability. They show that the historical average excess return forecasts future stock returns better than regressions of excess returns on predictor variables. However there is also literature that suggests the opposite. Ang and Bekaert (2007) argue that stock returns are predictable, but mainly on a short-horizon. Ju and Miao (2012) argue that investors act differently in a recession state. They are more pessimistic and have low continuation values causing the returns to be lower during recession periods.

Guidolin and Timmermann (2006) estimate the optimal number of states for predicting asset returns and finds that a four state regime switching model is optimal to capture the time variation in mean and volatility. They find that the two model adequately captures periods with high volatility. For this reason this paper investigates the state dependency of the data used.The states used are expansion and recession, indicated by the CEPR.

The next step is to determine which assets to take into account for the asset allocation. It is important to simplify the asset allocation to asset classes. Otherwise too many variables are used in the simulation. Most literature only focusses on stocks and bonds. This paper adds real estate. The reason is that the best instruments for hedging against inflation and maintaining purchasing power are real estate. Bond and Seiler (1998), O'Donnell (2009) and Lee et al. (2014). It is important to look at inflation when investing pension contributions. It is essential to not only look at the amount of benefits people receive, more importantly people want to be able to maintain a similar lifestyle. Another important thing to take into account is the height of the interest rate at pension date, as pensions are more expensive when interest rates are low.

4   Methods and Data

In this section first the essential components for lifecycle analysis are discussed. Then there is a discussion and analysis about the data that is used. Followed by a discussion about the simulation methods. And finally the ranking criteria are explained.

4.1   Lifecycle Construction

Lifecycle consist of a few components. These are:
The asset allocation in stocks, real estate and bonds over time, which can be either over age or over time until reaching the pension age. In this research the focus lies on a 25 year old starter who retires at the age of 67. The results are displayed with a focus on age. The distinction between age or time until pension does not matter here since the pension age and starting age are fixed.
Then the lifecycle capital return distribution at pension age is determined by the contributions, asset allocation and asset returns. This research assumes that it is not possible to go short and that there are no transaction costs.
The capital return distribution is then transformed into a monthly pension, taking into account the interest rate at that point of time.
Then the AOW is added to the monthly pension provided by the lifecycle.
The final step is to turn this monthly pension into a monthly real pension by correcting the monthly pension for inflation. By doing this the obtained real monthly pension can be fairly compared across different types of simulation paths, as it says how much the pensioner is able to buy.

4.2   Historical Data

The first step is to determine the lifecycle capital return distribution. The capital return distribution is based on the asset allocation over the asset classes and the return of the asset classes. The asset classes this paper takes into account are stocks, bonds and real estate. The focus is on Dutch pension providers. Dutch pension providers allocate their capital over different funds that invest in each asset classes. The most comparable data are European indices of stocks and real estate. For this reason European MSCI Indices are used as proxies for the asset classes. The proxies that are used are:

The MSCI Europe Index for stocks and the MSCI Europe Real Estate index for real estate. For the bond returns this papers uses a term structure model with the Euribor as proxy for the short rate. This is done to adequately

capture the relation between interest rates and bond yields with different maturities. Using this term structure the yield of a 20 year maturity bond is estimated and used as bond returns. The historical inflation levels are taken from Centraal Bureau Statistiek (CBS).
Then the stock return, real estate return and inflation level are simulated using a Vector Auto Regressive (VAR) model, with the short rate and economic state as exogenous variables.

Asset returns behave differently depending on the stage of the economy. In times of recession stock return and real estate returns are lower than in times of expansion Campbell and Cochrane (1999) and Ju and Miao (2012). The next part shows the asset, interest and inflation data with their summary statistics and performance during different stages of the economy in the period April 2005 - April 2017, which is the longest period available.

4.2.1   Return statistics

In the following graphs and tables stock returns, real estate returns, short rate and inflation level can be found.

In the tables the mean, median and standard deviation(Std. Dev.) can be found for the whole period, the expansion(Expan) and the recession(Reces) periods. In the period from April 2005 until April 2017 there have been two periods of economic recession in the European region. These periods were from January 2008 until March 2009 and from August 2011 until March 2013 as indicated by the Centre for Economic Policy Research (CEPR). In the graphs these recession periods are marked as grey areas.
The mean and standard deviation are then compared to determine whether they are different in expansion and recession periods. The tests used to determine this are,

$$t_{score} = \frac{(\mu_{expan} - \mu_{reces})}{\sqrt{\frac{\sigma^2_{expan}}{obs_{expan}} + \frac{\sigma^2_{reces}}{obs_{reces}}}} \ \ and \ \ F_{score} = \frac{\sigma^2_{expan}}{\sigma^2_{reces}}, \tag{1}$$

with

$$t_{score} \sim t(obs_{expan} + obs_{reces} - 2) \ \ and \ \ F_{score} \sim F(obs_{expan}, obs_{reces}), \tag{2}$$

where μ_{reces} and σ_{reces} are the average return and the std. dev. respectively in all recession periods. obs_{reces} is the number of recession periods.
The results of these tests can be found in the small table containing the Student t-statistic and F-statistic, for the mean and volatility respectively.

Stocks

This research uses the MSCI Europe Index as proxy for stocks. This data is obtained from Bloomberg. The table shows that the stock returns are significantly higher in periods of expansion. It also shows that the volatility is significantly higher in periods of recession. This research assumes that stock returns are dependent on the stage of the economy, which agrees with most literature.
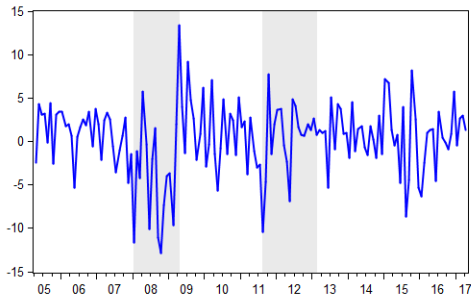

Figure 5: MSCI Europe Index returns

	All	Expan	Reces
Mean	0.32	0.94	-1.64
Median	0.92	1.13	-0.30
Std. Dev.	4.18	3.50	5.44
		Score	prob.
Mean	t-test	2.64	0.00
Volatility	F-test	0.41	0.00

Table 1: MSCI Europe return Statistics

Real Estate

This research uses the MSCI Europe Real Estate Index as proxy for real estate. This data is obtained from Bloomberg. The table shows that the real estate returns are significantly higher in periods of expansion. It also shows that the volatility is significantly higher in periods of recession. For these reasons, this research assumes that real estate returns are dependent on the stage of the economy.
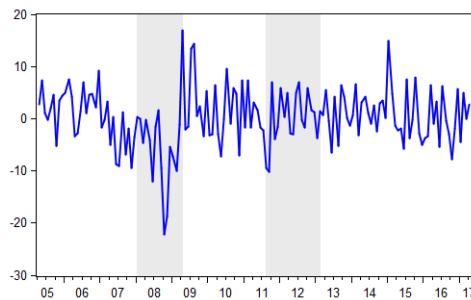
Figure 6: MSCI Europe Real Estate returns

	All	Expan	Reces
Mean	0.19	1.06	-2.55
Median	0.03	0.46	-1.41
Std. Dev.	5.77	5.17	6.73
		Score	prob.
Mean	t-test	2.92	0.00
Std. Dev.	F-test	0.59	0.02

Table 2: MSCI Europe Real Estate return Statistics

Short rate

This research uses the 1 month Euribor as proxy for the short rate. The data shown is the annualized 1 month Euribor yield obtained from Bloomberg. The figure shows that the Euribor has periods of decrease during recessions. However a simple t test on the mean in the two periods indicate the opposite, a higher short rate in periods of recession. The data seems to be non-stationary. For this reason the one period growth of the Euribor should be examined. These can be found in the appendix 8.1.1.

Figure 7: Monthly 1 month Euribor rates

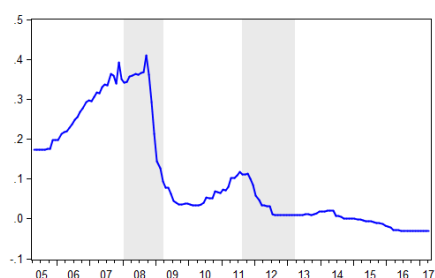|	All	Expan	Reces
Mean	0.11	0.09	0.15
Median	0.04	0.04	0.10
Std. Dev.	0.13	0.12	0.15
		Score	prob.
Mean	t-test	-2.25	0.01
Std. Dev.	F-test	0.67	0.06

Table 3: 1 month Euribor Statistics

Inflation

This research uses the inflation as stated by CBS. The figure and table show that inflation has high volatility. There do not seem to be significant differences in between different stages of the economy.
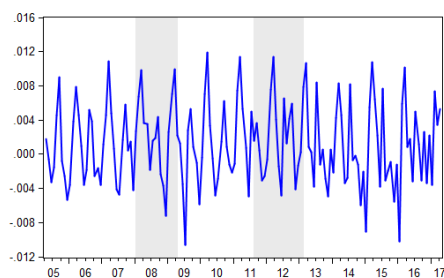
Figure 8: Monthly Inflation Levels

	All	Expan	Reces
Mean	0.13	0.09	0.25
Median	0.09	0.06	0.26
Std. Dev.	0.48	0.47	0.48
		Score	prob.
Mean	t-test	-1.70	0.05
Std. Dev.	F-test	0.99	0.47

Table 4: Inflation Statistics

For stocks and real estate the data shows that returns are significantly lower in recession periods and that the volatility is significantly higher in recession

periods. The data shows that the short rate has a significant difference in level and some doubtful results about the difference in volatility across the two economic stages. This is mainly due to the fact that the time series is non-stationary. For this reason the first difference is examined. These show that the change in the short rate has significant differences in the two economic states. In Appendix 8.1.1 the full return statistics of the first differences can be found.

4.2.2   Comparing Asset Classes and Inflation

In Table 5 a correlation matrix is shown, showing the correlation across the different time series. Observe that stocks and real estate are strongly positively correlated, stocks and bonds are negatively correlated and real estate and bonds have no significant correlation. Furthermore, inflation is most correlated with real estate, as literature suggested, but the correlation is small.

	Stocks	Real Estate	Short Rate	Inflation
Stocks	1.00	0.78	-0.23	0.06
Real Estate	0.78	1.00	-0.27	0.08
Short Rate	-0.22	-0.27	1.00	0.02
Inflation	0.06	0.07	0.02	1.00

Table 5: Correlation Matrix Asset Returns and Inflation

Table 6 shows the different correlation statistics per economic state. The values within brackets are for the recession periods and the values without brackets are for the expansion. The table shows that the correlations are in all cases stronger during recessions. The short rate is in recessions strongly negatively correlated with stocks and real estate whereas this is not the case in expansions. Moreover the correlation between real estate and inflation is a lot more noteworthy in times of recessions. Even though these findings might suggest state dependent regression coefficients (possibly only for the short rate), due to the limited sample size this research chooses not to do so.

Expansion(Recession)	Stocks	Real Estate	Short Rate	Inflation
Stocks	1.00	0.76 (0.78)	-0.01 (-0.48)	0.11 (0.11)
Real Estate	0.76 (0.78)	1.00	-0.10 (-0.47)	0.06 (0.26)
Short Rate	-0.01 (-0.48)	-0.10 (-0.47)	1.00	0.01 (-0.07)
Inflation	0.11 (0.11)	0.06 (0.26)	0.01 (-0.07)	1.00

Table 6: Correlation Matrix Asset Returns and Inflation per economic state

4.3   Simulation

To determine values of stocks, bonds and real estate this paper uses simulation. The first step in simulating asset returns is determining the stage of the economy. This is because the data shows dependence on the economic stage. Then, with these simulation results, the different time series can be estimated and with these, lifecycle returns are calculated. This research does the full simulation 10,000 times, the full simulation code can be found in the Appendix 8.3.

4.3.1   Stage Of The Economy

For the simulation of the stages of the economy this paper uses a discrete Markov model (Puterman, 2014). With two different economic stages. Expansion, when the economy is doing well and recession, when the economy is performing poorly. The Markov transition matrix is given in table 7.

	Expansion	Recession
Expansion	p	1-p
Recession	1-q	q

Table 7: Markov transition states

To find steady state probabilities π_{exp} and π_{rec} one solves the following set of equations,

$$\pi_{exp} = p\pi_{exp} + (1 - q)\pi_{rec}$$
$$\pi_{rec} = (1 - p)\pi_{exp} + q\pi_{rec} \tag{3}$$
$$\pi_{exp} + \pi_{rec} = 1.$$

By analysing the historic stages of the economy, stated by the Center for Economic Policy Research (CEPR),
$\pi_{rec} = 75.9\%$ and $\pi_{exp} = 24.1\%$ are observed. These equations reduce to

$$p = \frac{0.518}{0.759} + \frac{0.241}{0.759}q. \tag{4}$$

Then using the Matlab code found in Appendix 8.3.1 the optimal transition probabilities are calculated, these can be found in Table 8 .

	Expansion	Recession
Expansion	0.9810	0.0190
Recession	0.0598	0.9402

Table 8: Markov transition states used in this Research

4.3.2   Term Structure Model

This paper uses the term structure model by Cox et al. (1985a) to simulate
bond returns. In this model the short rate determines the development of all
interest rates. Further it assumes the short rate has mean reversion and that
interest rates are unable to become negative. Both assumptions are common
in interest rate literature. Even though negative interest rates seem more
and more plausible since the European Central Bank (ECB) has a negative
deposit rate since 2014, this paper assumes that the retail rates will remain
above zero. This is because banks are reluctant to use negative interest rates
as they believe that negative interest rates will scare customers away from
the use of financial institutions. A drawback of this model is that it doesn't
account for different economic stages. To overcome this problem a single
jump factor is added to the model. The short rate dynamics can, following
Cox et al. (1985a), be expressed as:

$$dr = k(\theta + \beta I(rec) - r)dt + \sigma\sqrt{r}dz_t, \tag{5}$$

or the discrete time equivalent studied by Wood (1964):

$$r(t + \Delta t) - r(t) = k(\theta + \beta I(rec) - r)\Delta t + \sigma\sqrt{r(t)\Delta t}Z, \tag{6}$$

where r is the short rate, k the adjustment rate, θ the long-term value, σ the
interest rate volatility, dt the change over time, dz_t a Wiener process, Z a
random normal variable, β the jump factor and $I(rec)$ an indicator function
that equals 1 in the case of a recession. For k and $\theta > 0$ this corresponds to a
first order autoregressive process where the interest rate is elastically pulled
toward a long-term value, θ. k determines the speed of adjustment.

The price of bond at time t maturing at time T is given by

$$P(r,t,T) = A(t,T)e^{-B(t,T)r}, \tag{7}$$

where

$$
\begin{aligned}
A(t,T) &= (\frac{2\gamma e^{(\gamma+k+\lambda)(T-t)/2}}{(\gamma+k+\lambda)(e^{\gamma(T-t)}-1)+2\gamma})^{2k\theta/\sigma^2}, \\
B(t,T) &= \frac{2(e^{\gamma(T-t)}-1)}{(\gamma+k+\lambda)(e^{\gamma(T-t)}-1)+2\gamma},
\end{aligned}
\tag{8}
$$

with

$$\gamma = \sqrt{((k+\lambda)^2 + 2\sigma^2)}. \tag{9}$$

From the price function it is observed that, bond prices are an increasing
function of time and a decreasing function of maturity. Bond prices also are

an increasing concave function of the price of market risk parameter λ. The short rate variance σ^2 and the speed of adjustment parameter k. At last bond prices are a decreasing convex function of the short rate and long term value θ.

The short rate together with the economic stage are used to simulate the bond returns. The price of market risk parameter λ is dependent on the economic stage because people are more risk averse in periods of recession. Due to the lack of literature and data regarding European λ the paper from Graham and Harvey (2016) is used to determine the price of market risk. They use S&P500 return data to determine the market price of risk for each period. This gives $\lambda = 3.98\%$ during recession periods and $\lambda = 3.35\%$ during expansion periods. This research assumes that European recessions have a similar effect on the market price of risk in Europe as U.S. recessions have on the market price of risk in the U.S.

Knowing the price function we derive the yield at time t for a discount bond maturing at T by

$$y(t,T) = \frac{-\log(P(t,T))}{T-t}. \tag{10}$$

Substituting previous P(t,T) gives

$$y(t,T) = \frac{\log(A(t,T)) - B(t,T)r_t}{T-t} \tag{11}$$

Using Cox 1985 the probability density of the short rate at time t, conditional on its value at time t - Δt is given by:

$$f(r(t)|r(t-\Delta t)) = ce^{-u-v}(\frac{v}{u})^{q/2}I_q(2(uv)^{1/2}), \tag{12}$$

where

$$
\begin{aligned}
c &= \frac{2k}{\sigma^2(1-e^{-k\Delta t})}, \\
u &= cr(t-\Delta t)e^{-k\Delta t}, \\
v &= cr(t), \\
q &= \frac{2k\theta}{\sigma^2} - 1,
\end{aligned} \tag{13}
$$

and $I_q(...)$ is the modified Bessel function of the first kind of order q. The distribution function is the noncentral chi-square, $\chi^2[2cr(t); 2q+2, 2u]$, with

$2q + 2$ degrees of freedom and parameter of noncentrality $2u$ proportional to the current short rate.

In order to estimate parameters k, θ, σ Maximum Likelihood Estimation(MLE) is used. MLE selects the set of values of the model parameters that maximizes the likelihood function.

$$(k, \theta, \sigma) = \arg \max_{k, \theta, \sigma} \log(L(k, \theta, \sigma)), \tag{14}$$

$$\log(L(k, \theta, \sigma)) = (N-1)\log(c) + \sum_{i=1}^{N-1}[(-u_i - v_i) + \frac{q}{2}\log(\frac{v_{i+1}}{u_i}) + \log(I_q(2\sqrt{u_i v_{i+1}}))]. \tag{15}$$

The full derivations can be found in the Appendix 8.2.2 .

The Bessel function oscillates around zero, see Figure 22 in the Appendix. This causes the optimization routines to rapidly approach minus infinity as $\log(0) = -\infty$. Since the Matlab optimization routines are not able to handle this problem, Kladívko (2007) suggests to use a scaled version of the Bessel function. This scales version transforms the function to

$$\log(L(k, \theta, \sigma)) = (N-1)\log(c) + \sum_{i=1}^{N-1}[(-u_i - v_i) + \frac{q}{2}\log(\frac{v_{i+1}}{u_i}) + \\ \log(I_q^*(2\sqrt{u_i v_{i+1}}) + 2\sqrt{u_i v_{i+1}})], \tag{16}$$

where $I_q^*(2\sqrt{u_i v_{i+1}})$ is the scaled version of the Bessel function, which does not oscillate around zero. The starting points are crucial for the optimization. Kladívko (2007) suggests to use Ordinary Least Squares (OLS) on the short rate to obtain the starting points. The expression is then solved using a Matlab code, which can be found in Appendix 8.3.5.

4.3.3   Asset Return Model

According to the data, the movements of the stock returns, real estate returns and inflation depend on each other. Moreover they also depend on the economic state and the short rate. The model that captures these elements the best way is a Vector Auto Regressive model with exogenous variables, a VARX model. Comparing the Akaike Information Criterion (AIC) of one to four lags shows that the optimal number of lags equals one. Therefore this research uses the following VARX(1) model for the stock returns, real estate returns and inflation level,

$$Y(t) = \alpha + AY(t-1) + BX(t) + U(t), \tag{17}$$

where $Y(T)$ is a 3x1 vector containing stock return, real estate return and inflation level in period t, $X(t)$ is the 2x1 vector containing the exogenous variables economic period and the short rate, α is a 3x1 vector of intercepts, A is a 3x3 matrix of coefficients, B is a 3x2 matrix of coefficients and $U(t)$ is 3x1 vector of multivariate random distributed errors $\sim MVN(\mathbf{0}, \Sigma)$ with Σ based on the error terms of the VAR regression. The model is estimated using Eviews and the data mentioned in section 4.2. The regression results can be found in Appendix 8.1.5.

4.4   Wealth

In order to determine investment capital two components need to be analysed. These are financial wealth and human capital, together they are seen as an individual's wealth.

4.4.1   Salary Income

Both financial wealth and human capital depend on the income received at starting age and future ages. In order to find a good estimate of the different income levels over age this paper uses data from Centraal Bureau Statistiek (CBS) and Modaal-inkomen.nl. This paper uses the placeholder named Jan Modaal[1] as an example person.
The Assumptions used in this paper are:
-Jan Modaal is the median employee.
-Jan Modaal is not the mode of all men, women are also taken into account.
-Jan Modaal has a different base income dependent on his age.
-Jan Modaal's base income increases yearly with a factor related to the inflation, see Figure 24 in the Appendix.
-Jan Modaal's base income is based on Modaal-inkomen.nl. Since the average income is used from different professions in 2013 and the mode income in 2017 is estimated to be €37000. This paper scales the income at the age of 40 to €37000 but keeping all yearly growth percentages the same. Looking at the salary differences at each age, see Figure 23, it is observed that the salary growth has an exponential function. This comes down to the following Salary function:

$$S(a,t) = S(a-1, t-1) * (1 + g_t(t-1)) * (1 + g_a(a-1)), \qquad (18)$$

with

$$
\begin{aligned}
g_t(t-1) &\sim N(\mu + i(t-1), \sigma^2) \\
g_a(a-1) &= b * g_a(a-2)
\end{aligned}
\qquad (19)
$$

---

[1]The Dutch version of Average Joe

where $S(a, t)$ is the Salary for an individual aged a in year t, $g_t(t-1)$ and $g_a(a-1)$ are growth functions dependent on year and on age respectively, $i(t)$ is the inflation in year t and b is a coefficient. Using simulation and age dependent salary statistics from CBS an optimal age dependent salary function is generated. The function (only dependent on age) that optimally fits the CBS data points is

$$
\begin{aligned}
S(a) &= S(a-1) * (1 + g_a(a-1)) \\
&= S(a-1) * (1 + 0.035 * 0.926^{(a-25)}).
\end{aligned}
\tag{20}
$$

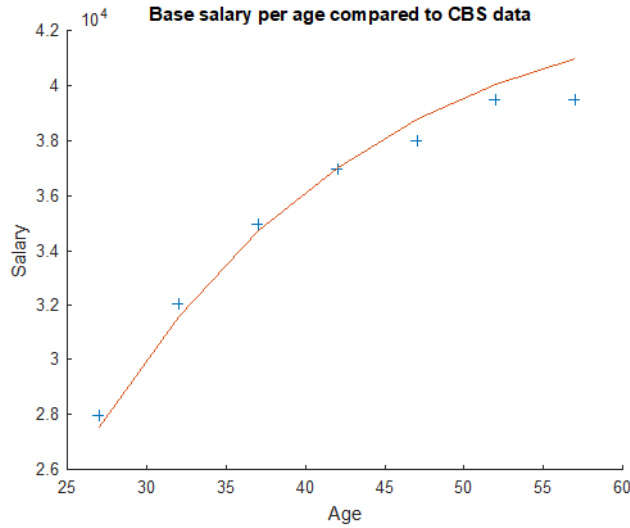Figure 9 shows how the function fits the data points.

Figure 9: Salary generated by formula compared to the CBS mode datapoints.

The code used can be found in Appendix 8.3.2 .
The optimal growth function dependent on time is based on the real salary growth over time, see Figure 25 in the Appendix, and can be represented as

$$
g_t(t-1) \sim N(0.26\% + i(t-1), 0.026\%).
\tag{21}
$$

This ultimately comes down to the following salary function

$$
S(a, t) = S(a-1, t-1) * (1 + 0.035 * 0.926^{a-25}) * (1 + \epsilon_t),
\tag{22}
$$

where ϵ_t is Normal distributed with mean equal to the inflation in period t $+ \, 0.26\%$ and a variance of 0.026% ($\epsilon_t \sim N(inflation_t + 0.26\%, 0.026\%)$).

4.4.2 Contributions

A part of the salary is used to contribute in the pension plan. The height of
the contribution depends on four factors.

The first factor is the franchise, that is the part of the salary that is not
used to build up pension. This is because the pension benefits consist out
of a first pillar part (AOW) and a second pillar part (DC). The franchise is
the part that is expected to come from the first pillar. If the franchise rises
the contributions will decrease and the pension will be lower. In this paper
a franchise of €13,123.00 is used; this is the franchise for a single married
person according to the Belastingdienst (Dutch Tax and Customs Adminis-
tration). The average yearly increase of the franchise is taken into account.
From analysing the data from 2005 to 2017 an average yearly growth of 1.9%
is found and therefore this paper assumes a yearly franchise growth of 1.9%.
The second factor is the pensionable earnings, that is the salary minus the
franchise. It should be positive and besides that it is capped at €103,317.
This maximum value does change every year, but since the salary of an av-
erage/median employer does not come close to this value, the growth of the
pensionable earnings cap is disregarded in this research.

The third factor is the part-time factor, which is a factor of how many hours
someone works compared to a full time job. For example someone who works
28 hours at a company instead of the normal 40 hours, works for 70% which
means a part-time factor of 0.7. In this paper a full time working person
(part-time factor =1) is considered.

The final factor is the contribution percentage, this is the maximum percent-
age of the pensionable earnings that can be contributed to the pension plan.
Which is often the default, but depends on the pension provider. This factor
is dependent on the age of the contributor and the kind of pension. For a
table of contribution premiums see Table 16 in the Appendix.

In short, the pensionable earnings are salary minus franchise. Then multiply
this with the part-time factor and the contribution percentage. The result
will be that years contribution to the pension plan.

4.4.3 Financial Wealth and Human Capital

Financial wealth is the total value of assets in that year. In order to determine
financial wealth the asset allocation of the contributions over time must be
known. This asset allocation, the lifecycle, consists out of three different
classes which all yield returns. With these returns the Financial wealth in
each period can be determined.

This comes down to

$$F(t) = F(t-1) * R(t-1) + C(t), \tag{23}$$

where R(t) is the weighted asset return and C(t) is the contribution in period t.

Human capital is the present value of total future income. That is the sum of all discounted future incomes,

$$H(t) = \sum_{i}^{T} Dsc(i)C(i) \quad with \quad t+1 < i < T-t, \tag{24}$$

where Dsc(i) is the current value of €1 in year i, the discount rate and C(i) is the expected contribution to pension in year i. Human capital is used to determine the optimal lifecycle according to Merton (1969).

4.5   Lifecycles

In this research 13 different lifecycles are compared. Part of them are simple lifecycles that do not change over time, the other lifecycles do change time. It is interesting to see how the simple lifecycles perform compared to the more sophisticated lifecycles. Also it is interesting to see how the practical lifecycles perform.
There are seven simple lifecycles that do not depend on anything and are constant over time. These are, three different lifecycles fully invested in one of the three assets classes, Bonds (Lifecycle Bo), Stocks (Lifecycle St) and Real Estate (Lifecycle RE). Three different lifecycles constant over time consisting for 50% out of two of the three assets and one lifecycle equally weighted over the three different lifecycles, Bonds and Stocks (Lifecycle StBo), Stocks and Real Estate (Lifecycle StRE) and Bonds and Real Estate (Lifecycle REBo).

Next are the more interesting lifecycle. The first three are different lifecycles that depend on age and therefore change over time. The Example lifecycle from section 2.2, (Lifecycle Ex), which is a self invented lifecycle used to explain how lifecycles work. And two lifecycles that are used in practice by Achmea and by Amersfoortse see section 2.2.2, (Lifecycle Ac and Lifecycle Am respectively). Unfortunately I wasn't able to get accurate lifecycle details from other pension providers. Finally this research considers three different implementations of Merton's Optimal lifecycle. The following figures give a visual representation of the models by Merton (1969). These lifecycles depend on the investment return and are for that reason different

in every simulation path. Most lifecycles are the same in each scenario. The exceptions are the different versions of Merton's optimal lifecycle. These are explained in the following parts.

4.5.1   Merton's Model

The optimal lifecycle following Merton (1969) shows that the optimal port-folio is the same for all investors under the assumption of Constant Relative Risk Aversion(CRRA). Following Merton, the fraction of wealth invested in stocks and real estate is equal to

$$w_t = \Omega^{-1}\frac{\mu}{1-\gamma_t}, \tag{25}$$

where w_t is a 2x1 vector of weights, μ is a 2x1 vector of average asset returns, Ω is the covariance matrix of asset returns and γ_t is the relative risk aversion coefficient. In Merton's original model γ_t is constant and one should invest in the same manner whether it is an expansion or recession. γ_t coefficient value is 5 as this is common in most literature. As an addition to this model this research also looks at the optimal lifecycle according to Merton taking into account the economic stage, in this model γ_t is not constant. This additional lifecycle is explained in Section 4.5.2.

Let w_t^* be the sum of w_t, the total fraction invested in stock and real estate. Besides financial wealth, wealth also exists of human capital.
This comes down to $W(t) = F(t) + H(t)$.
Since only financial wealth can be invested only the fraction of financial wealth (w_t^f) that should be invested in each asset must be determined. Now using,

$$\begin{aligned} \Delta W(t) &= W(t) - W(t-1) \\ &= \mu + W(t-1)\epsilon_w(t), \end{aligned} \tag{26}$$

where ϵ is a shock term. The magnitude of the shock depends on W(t-1). With the simplification that human capital is risk free $(\epsilon_h(t) = 0)$, which is not 100% correct but is the best approach since we are looking at the modal employee.

$$\Delta W(t) = \mu + W(t-1)\epsilon_f(t), \tag{27}$$

the fluctuations in total wealth and financial wealth should be the same. This means,

$$w_t W(t-1)\epsilon = w_t^f F(t-1)\epsilon, \tag{28}$$

using this equation it follows that

$$
\begin{aligned}
w_t^f &= w_t \frac{W(t-1)}{F(t-1)} = \Omega^{-1} \frac{\mu}{1-\gamma_t} \frac{W(t-1)}{F(t-1)} \\
&= \Omega^{-1} \frac{\mu}{1-\gamma_t} (1 + \frac{H(t-1)}{F(t-1)}).
\end{aligned}
\tag{29}
$$

With the requirement that the fraction can not be negative and must be smaller than one, that is

$$
\mathbf{0} \leq w_t^f \leq \mathbf{1}.
\tag{30}
$$

A big part of the future pension for a median Dutch employee is AOW. Because Merton (1969) does not take the first pillar (AOW) into account it can be argued that the lifecycle is not aggressive enough. Future research should try to implement AOW in Merton's model.

4.5.2   Merton's optimal lifecycle improved

As an addition to Merton's optimal lifecycle this research also looks at the optimal lifecycle when including economic state information. By making the parameters state dependent this model aims to improve upon Merton's model. Since the economic state is not known before investing, the economic state of the previous period is used to determine the parameter values for the mean and volatility.

The choice of γ_t is difficult, this is because the relative risk aversion coefficient is also assumed to be different depending on the economic stage. And there is little research done on the dependence of the risk aversion on the economic state. Papers as Bovenberg et al. (2007) and Gomes et al. (2008) use a risk aversion coefficient value of 5 for the average investor. Both argue how wrong values of the coefficient can lead to enormous welfare losses, especially too low risk aversion coefficient values (Bovenberg et al. (2007) shows that a false risk aversion of 2 leads to a 30% welfare loss compared to the welfare in the case of a true risk aversion of 5).

Li (2007) argues that the relative risk aversion coefficient should consist out of a constant plus a countercyclical time varying market price of risk component. But he leaves the estimation for future research. Appendix 8.2.3 shows my interpretation of the suggestion by Li (2007) and how this is done. Using the derivation in Appendix 8.2.3, which uses the market price of risk in each economic state, the relative risk aversion coefficient is estimated. This results in γ_t equals 5.546 in case of a recession and 4.827 in case of an expansion.

4.5.3 Merton's optimal lifecycle literature parameters

Another additional lifecycle used is Merton's optimal lifecycle in which the parameters are chosen as values which are common in literature. It is common in literature to take parameters from government committees. The reason is that pension providers have to take these parameters into account when calculating their expected returns and pension benefits. The dutch committee that determines these parameters is the Advies Commissie Parameters. From their documentation the parameters regarding expected returns (4%) and standard deviation (20%) are obtained. However these parameters are only available for stock returns. For this reason Merton's optimal lifecycle with parameters from literature only invests in bonds or stocks. This turns the fraction found in section 4.5.1 into

$$w_t^f = \frac{\mu}{\gamma_t \sigma^2}(1 + \frac{H(t-1)}{F(t-1)}) = \frac{1}{\gamma_t}(1 + \frac{H(t-1)}{F(t-1)}). \tag{31}$$

Because now $\frac{\mu}{\sigma^2} = 1$,while in the dataset this was $\frac{\mu_{data}}{\sigma^2_{data}} = \frac{1}{2}$. In practice this means that the asset allocation slowly turns towards $\frac{1}{\gamma_t}$. Which is a slightly more agressive strategy than the model of Merton (1969) with parameters from the data.

4.6 Lifecycle Criteria & Ranking

With the asset returns and the lifecycles the expected returns are determined, together with the short rate and inflation this will determine the pension result. Analysing pension results from the simulations from each lifecycle will show how each lifecycle performs in terms of expected pension and volatility. This analysis gives a ranking. This part explains how the pension return distributions are used to rank the lifecycles. This is based on expected utility and pension at pension date together with the volatility in the pension resuls. The criterias used are:
Expected utility, Sharpe Ratio, expected return, volatility, median, Value at Risk, Expected Shortfall, the chance they have a monthly pension above 75% of their average salary and chance of a monthly pension above 70% of their final salary. The expected utility will determine the final ranking, but special attention should also be paid to the chance of having a real income above 75% of their average salary, because this is the pension received in defined benefit pension plans and is a good indication on how the lifecycle performs compared other pension plans.

4.6.1 Capital to real monthly pension

This research looks at how much people can buy with their pensions. Therefore the focus is on monthly real pension and not on capital. Therefore the first step is to turn the pension capital into a monthly pension. This is done using the function

$$Pm = (0.045901 * r + 0.0033087) * M, \tag{32}$$

where Pm stands for the monthly pension, r for the interest rate and M for the initial total capital. Derived from 25 year long annuity offers of 15 Dutch banks and Dutch insurance companies. For a more detailed explanation see Appendix 8.1.7. Then the AOW is added and this nominal monthly pension is divided by the simulated inflation factor, to obtain the real monthly pension.

4.6.2 Expected Utility

The most important criterion to determine the optimal lifecycle in this research is the expected utility. This is because it weights the resulting pensions in such a way that it penalizes low pension results more than it rewards high pension results. This causes more volatile lifecycles to have a lower rating than lifecycles that are less volatile, when they both have a similar expected pension. To do this, this paper assumes a power utility function which agrees with the Constant Relative Risk Aversion (CRRA) as discussed in section 4.5.1. For the calculation of the expected utility, the power utility function is integrated. This comes down to the following equation,

$$\int_0^\infty p(K)U(K)dK, \tag{33}$$

which in this research is estimated by

$$\frac{1}{N} * \sum_{i=1}^{N} U(K_i), \tag{34}$$

where N is the number of simulations, K_i is the monthly real pension capital in simulation i, $p(K)$ is the chance of ending on a real monthly pension of K and $U(K)$ is the power utility defined as,

$$U(K) = \begin{cases} \frac{K^{1-\gamma}-1}{1-\gamma} & \gamma \neq 1 \\ ln(K) & \gamma = 1, \end{cases} \tag{35}$$

where γ is a parameter measure of risk aversion.
This research does 10,000 simulations, each simulation is equally weighed to determine the expected utility of each lifecycle.

4.6.3   Expected Return and Volatility criteria

Besides the expected utility, the expected return and volatility are important and give more insight in the differences across lifecycles. One of the most popular criteria in finance which takes both the expected return as the volatility into account is the Sharpe Ratio(SR). The Sharpe Ratio is defined by

$$SR = \frac{r_{lc} - r_f}{\sigma_{lc}}, \tag{36}$$

which is the lifecycle return r_{lc} minus the risk free rate r_f divided by the lifecycle standard deviation σ_{lc}.

Another common criteria mostly used in risk management applications is the Value at Risk (VaR) of $\alpha\%$. Which is the maximum pension you will receive in the worst $\alpha\%$ of the cases. Accompanied with the Expected Shortfall (ES) of $\alpha\%$ which gives the expected pension in the worst $\alpha\%$ of the cases. These two together give good insight in the worst case scenarios for the different lifecycles. In this research $\alpha\%$ is chosen to be 5% which is common in most literature.

4.6.4   Reaching average income or final pay

In defined benefit pension systems the pension is often 70% of the final pay or 75% of the average salary. It is therefore important to compare the lifecycle results with the traditional expected pensions, as most participants are still expecting similar pensions. This research looks at both 70% of the final and 75% of the average pay as in the models used the expected 70% of the final salary is more than 50% more than the expected 75% of the average salary. As the difference is big and these two measures are common in practice, both are shown in the results.

5   Results

This section discusses the simulated data and lifecycle returns. First the focus will be on how well the simulated data compares to the historic data. This is done by analysing the simulation statistics. Then the resulting lifecycles are shown and the differences among lifecycles are discussed.

5.1   Simulated Data

This section contains a discussion about the simulated economic state, short rate, stock return, real estate return and inflation after doing 10,000 simulations.

5.1.1   Economic State

First the focus is on the simulated recession and expansion periods. These are simulated using the Markov switching model described in section 4.3.1. The following figure shows the fraction of periods being recession periods in each simulation run. Figure 10 shows the fraction of recession periods in the
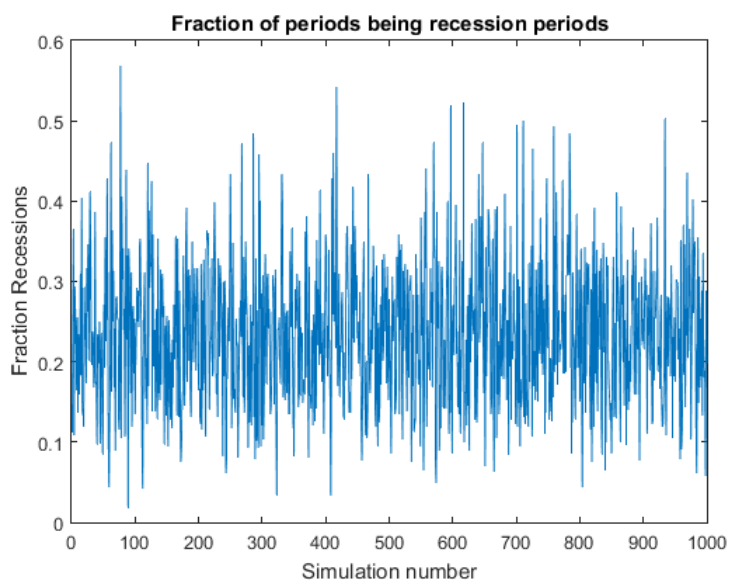
Figure 10: Fraction of periods being recession periods.

first 1000 simulation paths. The fraction of recession periods differs quiet a bit between the different simulations. The biggest fraction of recessions is 0.67 and smallest fraction of recessions is 0, which happened only once. The average number of recession periods is 23.5%, this is just a slight deviation from the observed 24.1% number of recession periods that were present in the data.

5.1.2   Short Rate

Second the model by Cox et al. (1985a) is used to forecast the short rate and the yield curve. Section 4.3.2 and Appendix 8.2.2 explain the full model

estimation. Among other parameters the model estimates a long term short rate average, however this long term average was estimated to be a negative value. This is most likely due to the decreasing interest rate in the past 12 years. Since the model assumes that the interest rates can not go negative, the forecasts rarely changed from being 0%. But because this is unrealistic this research chooses to overrule the estimate by replacing it with a yearly rate of 2% which is a common long-term interest rate average in recent literature. This resulted in more realistic and dynamic interest rates.

Of all 10,000 simulations of 504 months per simulation, the average monthly short rate is $8.8*10^{-4}$ with a standard deviation of $6.8*10^{-4}$. This is a yearly average short rate of 1.1%. The short rate is smaller than in the observed dataset, however in recent years the interest rates have dropped to an all-time low. And interest rates are expected to stay low for a long time. For this reason the obtained interest rates are acceptable forecasts.

5.1.3   Stocks, Real Estate and Inflation

For the simulation of the stock returns, real estate returns and inflation a Vector Auto Regressive (VAR) model is used. Section 4.3.3 and Table 15 in the Appendix give more model details. Table 9 and Table 10 show the resulting simulation averages and standard deviation respectively. The values between brackets show the empirical data averages and standard deviation. The simulation averages show similar results as the empirical data and the simulation standard deviation is almost identical to the empirical data standard deviation. Only inflation has a significant higher standard deviation than empirically found. Excluding inflation the model correctly captures the empirical data statistics. Therefore the model forecasts are an adequate representation of the data as the averages are within a 25% range of the empirical data averages.

Mean	Stock Returns	Real Estate Returns	Inflation
All	0.40 (0.32)	0.25 (0.19)	0.13 (0.11)
Expansion	1.00 (0.94)	1.04 (1.06)	0.08 (0.09)
Recession	-1.56 (-1.64)	-2.31 (-2.55)	0.26 (0.15)

Table 9: Average return statistics model (and historical data)

St. Dev.	Stock Returns	Real Estate Returns	Inflation
All	4.19 (4.18)	5.71 (5.77)	0.49 (0.13)

Table 10: Average standard deviation model (and historical data)

5.1.4   Salary

The simulation of the salaries is on a yearly basis and done by multiplying previous year's salary with two functions; a function that is only dependent on time and a function that depends on the inflation in that period and also contains a random part. See section 4.4.1 for the full description. Figure 11 contains 5 random salary paths and Figure 12 contains the salary paths of the top 5%, bottom 5% and median salary, together with the average salary. The figures show that there is quiet some difference between the salaries in the multiple cases. This is understandable as the accumulated inflation (see figure 14) in top 5% of the cases is also around the factor 2 bigger than the accumulated inflation in the bottom 5%.
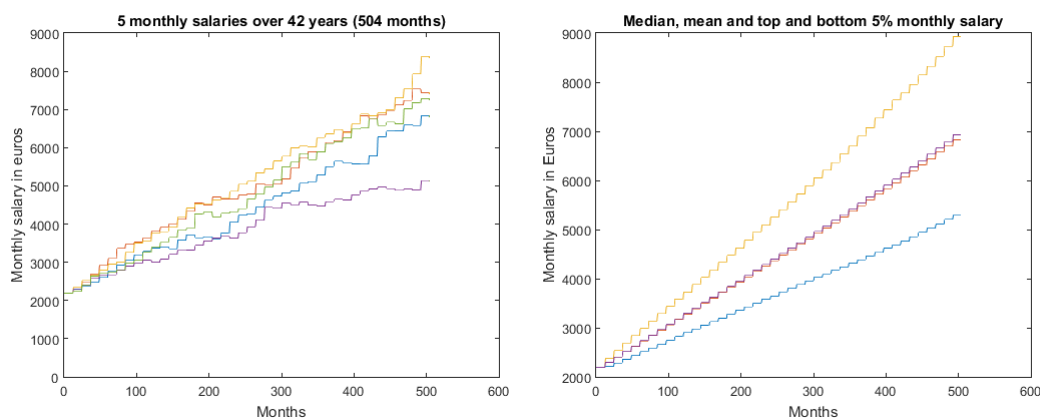

Figure 11: 5 monthly salaries over 42 years.   Figure 12: Median, Mean, Top& Bottom 5%

5.1.5   How does €1 grow in the different asset classes

In this section the grow of €1 is shown for four different paths. The first path is the 5% percentile, the second is the median, third the mean and the last one is the 95% percentile. This gives an idea of the investment return of investing €1 in each asset class.
Figure 13 shows the investment return on investing €1 for 42 years (504 months) in the short rate. Investing in the short rate seems a safe investment, with low returns but none are negative. The worst 5% gives a positive investment return of 20%, which is a yearly return of 0.4%. However the top 5% only gives a return of 130%, which is a yearly return of only 2%. The average investment return is 59%.
Figure 14 shows the investment return on investing €1 for 42 years (504 months) in a hypothetical tracker that perfectly follows the inflation. The inflation shows a similar spread as the short rate. The top 5% shows an
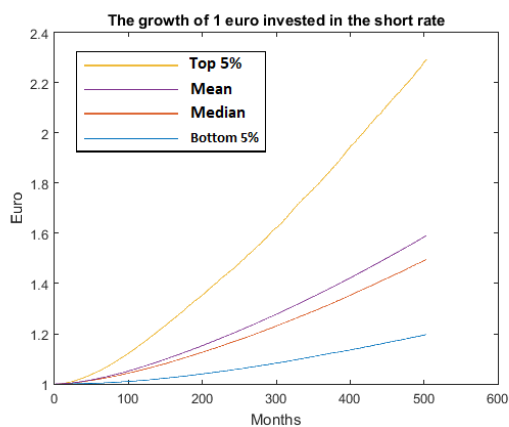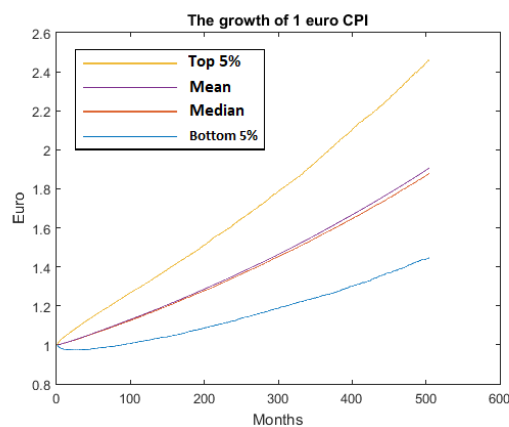
Figure 13: €1 invested in the short rate   Figure 14: €1 invested in a CPI tracker

146% increase and the bottom 5% a 45% increase, a 101% difference which is similar to spread at the short rate. The average investment return is 91%.

Figure 15 shows the investment return on investing €1 for 42 years (504 months) in stocks. Investing in stocks is a risky investment, with huge differences between the top and bottom percentiles. The worst 5% lost more than half of their money and end up with an investment return of -66%. The top 5% made a huge profit, their €1 became €56,70 an investment return of 5570%. They averaged a yearly investment return of 10.1%. The median return is 387% which comes down to a yearly investment return of 3.8%. And the yearly average return is 6.6%.

Figure 16 shows the investment return on investing €1 for 42 years (504 months) in real estate. Investing in real estate is an even riskier investment, with enormous differences between the top and bottom percentiles. The worst 5% lost almost all of their money and end up with an investment return of -96%. The top 5% made a huge profit, their €1 became €43.25 an investment return of 4225%. They averaged a yearly investment return of 9.4%. The median return is 58% which is a yearly investment return of 1.1%. And the yearly average return is 5.8%. Table 11 summarizes these results.
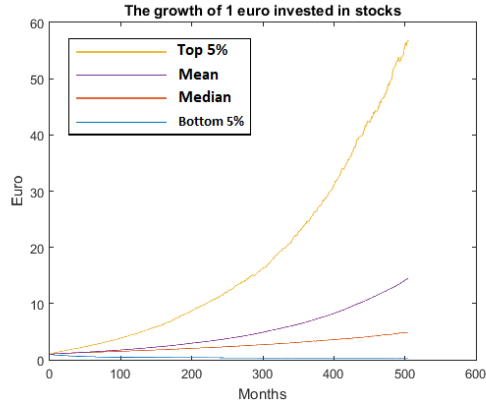
Figure 15: €1 invested in stocks
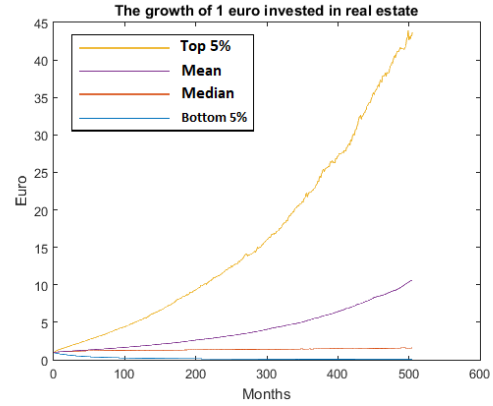
Figure 16: €1 invested in real estate

	Bottom 5%	Median	Mean	Top 5%
Short rate	1.20	1.50	1.59	2.30
Inflation	1.45	1.88	1.91	2.46
Stocks	0.34	4.88	14.43	56.70
Real Estate	0.04	1.58	10.59	43.25

Table 11: The final worth of €1 invested in the different classes

5.2   Lifecycle Results

This section contains the simulated lifecycle results in terms of tables, graphs and a discussion. In the first section the estimated asset allocation over time is discussed. The following sections contains an overview and discussion of the resulting pension returns.

5.2.1   Merton's Lifecycles constructions

This section gives a visual representation of the various versions of Merton's optimal lifecycle.

In the figures below the lifecycles for the three Mertons models are shown for a single simulation run. Appendix 8.1.8 shows the average Merton allocation for all simulation runs. The lifecycles show different patterns. The first model, Lifecycle M, uses the data to estimate the optimal fraction without taking the economic state into account. This lifecycle invest fully in stocks but quickly shifts towards bonds. It does not invest anything into Real Estate.

The second model, Lifecycle Me, does take the economic state into account. After the first few years where the fraction is invested in stocks and real estate, the lifecycle slowly shifts towards bonds. The graph shows how the asset allocation is more invested in bonds than in Merton's lifecycle that doesn't take economic state into account, but it does have sudden drops. This is due to the change from an expansion state into a recession state. The different risk averseness in the states causes the asset allocation to make the jumps it does.

The third model, Lifecycle Mp, uses parameters suggested by Advies Commissie Parameters. Besides that the model does not invest in real estate per definition, but this does not make much of a difference as the other two lifecycles make close to no investment in real estate as well.
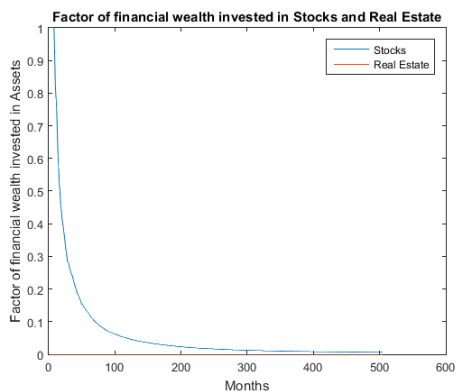
Figure 17: Lifecycle M (first)
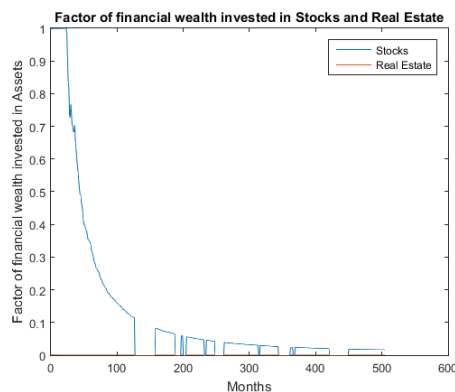
Figure 18: Lifecycle Me (second)
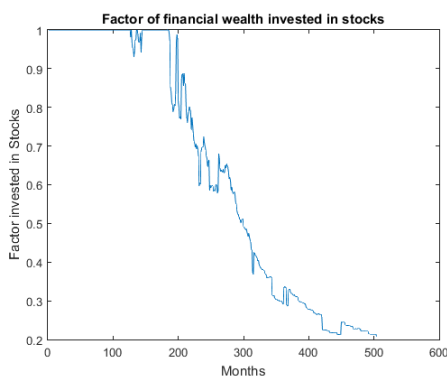
Figure 19: Lifecycle Mp (third)

5.2.2   Lifecycle results

This section contains an overview and discussion of the lifecycle results. The results are in terms of monthly real pension, that is monthly pension corrected for the inflation rate. With the exception of the chance of reaching 75% of the average salary and the chance of reaching 70% of the final salary. This is because these are common pension amounts in defined benefit pension plans, where employees compare their future pension to their salary in nominal terms. And for this reason the salary related criteria are based on the nominal monthly pension. The monthly pension consists out two parts, a part determined by the lifecycle returns and an AOW part. Both parts are subject to risk. The AOW grows each year with a fraction closely related but not similar to the inflation. And the lifecycle returns are subject to inflation risk, investment risk and interest risk. The lifecycle results in terms of expected return and amount of risk can be displayed in various ways. The following figure contains a box plot to easily compare the return distribution of the 13 used lifecycles.
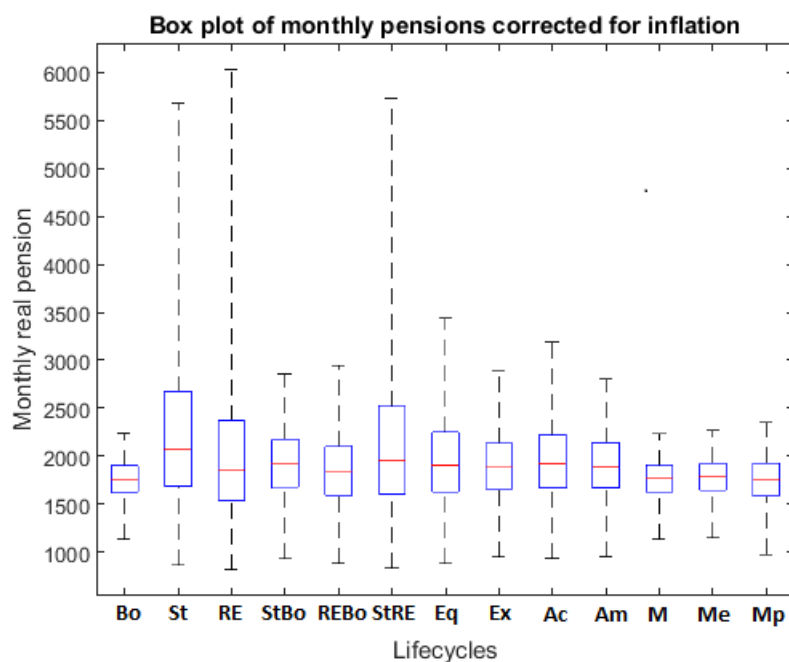
Figure 20: Box plot of the 13 different lifecycles, excluding the top 2.5%

Figure 20 contains the box plot of the 13 different lifecycles, this box plot excludes the top 2.5% in order to keep the figure readable. For the box plot containing all simulated data see Figure 30 in the Appendix. See section 5.2.1 for the abbreviations used for each lifecycle.

The blue boxes in the box plot contain the middle 50% of the simulated monthly real pensions. The red line stand for the median and the top and bottom black line stand for the highest and lowest observed monthly pension. The box plot shows relatively similar median lifecycle returns, but huge differences in lifecycle spread. The top 2.5% of all lifecycle returns are excluded from the box plot mostly due to a few extreme high pension returns. Most coming from the lifecycle fully invested in Real Estate, Stocks and the combination of these two. The box plot gives a good insight in the differences in spread across the set of lifecycles. Together with the other lifecycle criteria this gives a good understanding in the lifecycle performance.

Table 12, found on the following page, shows the ranking criteria statistics of all ranking criteria mentioned in section 4.6 plus a few additional percentiles for extra insight. The Expected Pension is the mean of the simulated monthly real pension lifecycle returns. The lifecycles that perform best in the given criteria are marked bolt for convenience.

Theory suggests that Merton's optimal lifecycle should yield optimal pension results when looking at expected utility. This is not the case however, none of Merton's three different lifecycles make it into the top 5. Most likely due to the fact that the monthly pension does not completely depend on the lifecycle return but also on the AOW. Leaving more room for risk taking in the part of the pension generated by the lifecycle. This idea agrees with the construction of the top ranked lifecycles. When comparing Merton's model that takes economic state into account we see that it scores better on every criteria (except a slightly higher standard deviation). This shows that taking economic state into account increases lifecycle performance. The best performing lifecycles are the lifecycles St, Ac and StBo which are all lifecycles that invest most in stocks. The lifecycle that fully invests in stocks, lifecycle St, performs best in terms of expected real monthly pension, the 25%, 50%, 75% and 95% percentile and on the chance of reaching a pension higher than 75% of the average salary and 70% of the final salary.

Due to the fact that the upward risk is greater than the downward risk in all lifecycles, the standard deviation and Sharpe Ratio are misleading criteria in this context. It is more informative to look at the lower percentiles and Expected Shortfall. For example when comparing Lifecycle Am (Amersfoortse) with Lifecycle Mp (Merton's with parameters from literature), observe that Lifecycle Am performs worse in terms of standard deviation and Sharpe Ratio. However Lifecycle Am has higher monthly real pension results for all percentiles. And naturally should be considered as the better lifecycle.

	Bo	St	RE	StBo	REBo	StRE	Eq	Ex	Ac	Am	M	Me	Mp
Expected Utility	.2214	**.2273**	.2173	.2259	.2204	.2227	.2233	.2250	.2261	.2255	.2216	.2227	.2187
Expected Pension	1783	**2441**	2277	1972	1905	2348	2031	1948	2015	1948	1787	1804	1777
St. Dev.	**216**	1322	1657	397	436	1402	581	400	483	377	217	219	265
Sharpe Ratio	**8.27**	1.85	1.37	4.97	4.37	1.67	3.49	4.87	4.17	5.16	8.22	8.24	6.70
95% percentile	2156	**4705**	4562	2692	2683	4595	3073	2668	2886	2625	2162	2182	2244
75% percentile	1917	**2758**	2450	2204	2128	2605	2286	2158	2249	2164	1922	1941	1945
Median	1769	**2093**	1870	1926	1844	1981	1920	1893	1938	1903	1772	1790	1758
25% percentile	1631	**1696**	1543	1686	1600	1615	1636	1668	1681	1677	1633	1650	1588
5% percentile	1452	1336	1243	1407	1328	1287	1336	1415	1409	1423	1455	**1470**	1373
Expected Shortfall	1385	1231	1143	1309	1232	1184	1236	1317	1307	1325	1385	**1399**	1291
75% avg salary	46.8	**72.0**	55.8	70.5	57.1	64.5	64.6	66.8	69.3	68.9	48.0	55.1	44.5
70% final salary	0.1	**30.4**	22.2	8.0	7.3	26.2	14.7	7.6	12.0	6.6	0.1	0.1	0.4

Table 12: Lifecycle statics

When looking at lifecycle performance in worst case scenarios, Lifecycle Me (Merton's model that takes economic state into account) performs best. It has the highest expected shortfall and 5% percentile. Merton's lifecycle can therefore be the optimal lifecycle for an employee who is more risk averse.

Looking at the lifecycles we can sort them into four different groups. The minimum risk group, with the Merton lifecycles and Bond lifecycle(Bo, M, Me and Mp). These are characterized by having an expected monthly real pension of around €1785, the lowest standard deviations, the highest Sharpe Ratios and the highest 5% percentile monthly real pensions.

The average risk group, with the mixed bond lifecycles (StBo and ReBo), the Example lifecycle (Ex) and lifecycle Amersfoortse (Am) This group is characterized by somewhat higher returns by taking some extra risk. Their expected monthly real pensions is around €1950. And they perform best in the mid-low percentiles (between 5 and 25%).

Then we can categorize the above average risk group containing Lifecycle Achmea (Ac) and the Equally weighted lifecycle (Eq). These lifecycles take on a little additional risk which results in additional higher real monthly pension results. Their expected monthly real pension is around €2020.

The final group would be the extreme risk group, containing the three lifecycles that invest 100% in either stocks or real estate (St, RE and StRE). The lifecycles in this groups have the highest standard deviation. In these lifecycles the allocation toward stocks and real estate does not change when coming close to retiring. And since the pension capital is generally the highest in the final years of contributing, this results in huge jumps in the expected pension just before retiring. Therefore this group can yield the highest and the lowest pensions. This can be seen when looking at the Expected Shortfall and the 95% percentile. The lifecycles in this group have the lowest Expected Shortfal and the highest 95% percentile. The lowest Expected Shortfal of all lifecycles is that of the lifecycle RE which is a monthly real pension of €1243. However the lifecycle RE also has one of the highest 95% percentile, a monthly real pension of €4562. But even if we look at the average and median results these lifecycles are performing best. It pays off to take a more risky lifecycle!

Table 12 shows a huge gap in terms of standard deviation between the above average risk group and the extreme risk group. The lifecycles within the extreme risk group have standard deviations 1320-1660 which are a factor 2 to 3.5 higher than the above average risk group 480-580. This comes from the relative high allocation towards stocks and real estate in lifecycle Eq and the relative small allocation towards stocks and real estate in the last 10 years

in lifecycle Ac. For future research it would be good to look at lifecycles with standard deviation in between the standard deviations of these groups. As currently there is a large gap between the most risky lifecycles and the above average risk group, these lifecycles should be analysed as they might give better results than the lifecycles currently analysed.

This research looks at the real monthly pension based on the final period. It might also be interesting to add restrictions, or alter the ranking criteria to stimulate lifecycles that take the preferences of the contributors into account. For example people do not want their expected pension to change much in the last 5 or 10 years before retiring and people might want to avert risk when a certain expected monthly pension is reached. This could mean that the practical lifecycles rank higher, as these take less risk in the last years before retiring.

When comparing the two practical lifecycles of Achmea (Ac) and Amersfoortse (Am) only a small difference of 3.4% in expected pension is found. This difference is approximately 10% when comparing the accumulated pension capital which is similar to the findings of van Ling and van Soest (2016). The difference between these two pension capitals comes from Achmea's lifecycle being more offensive. That the difference in pension capital and expected pensions has such a big difference is due to the influence the AOW has on the monthly pension. For people with higher income the AOW part is less significant and different results could be found.

The best performing lifecycle is lifecycle St, that fully invests in stocks for the entire career. It has the highest Expected Utility and the highest chance of reaching a monthly pension of 75% of the average salary received over the employees working life.
The second best performing lifecycle is lifecycle Achmea, lifecycle Achmea is slightly more conservative. It has a slightly lower 25% percentile, but higher 5% percentile and Expected Shortfall compared to the lifecycle St. This indicates higher pensions in the lesser scenarios, while also having high pension results in good scenarios and just a slightly lower chance of reaching 75% of the average salary compared to lifecycle St.

The reason that offensive lifecycles perform best is most likely related to the fact that a big part of the monthly pension comes from the almost risk free AOW. Since this part of the pension contains little risk, more risk can be taken in the pension part generated by the lifecycle.

6   Conclusion

This paper looks at the asset allocation of pension contributions in defined contribution (DC) pension plans over the asset classes stocks, bonds and real estate. This paper finds that the optimal lifecycle within the set of analysed lifecycles, for someone following the lifepath of Jan Modaal, in terms of expected utility is to follow a lifecycle that invests fully in stocks for the whole contribution period. It pays off to take a risky lifecycle.

The factors that impact on the lifecycle results in this research are the economic state, interest rate, investment return, inflation and salary development. Factors as life expectancy are out the scope of this research and are left for future research. For the economic state a Markov switching model is used. The interest rates, bond returns and term structure are estimated following CIR model Cox et al. (1985b). The inflation, stock and real estate return are estimated with a VARX model. And the salary development is simulated with an exponential model with a random term. 13 different lifecycles are analysed, including lifecycles that are constant over time, lifecycles seen in practice and a few versions of the theoretic optimal lifecycle by Merton (1969).

The optimal lifecycle within the set of analysed lifecycles according to this research in terms of expected utility, expected pension, median Pension and chance to reach a pension of at least 75% of the average work life salary is to fully invest in stocks. This does not agree with the theory by Merton (1969) and also does not fully agree with the lifecycles observed in practice. This research shows that lifecycles should be more allocated to stocks than they currently do.
The reason that the optimal lifecycle does not agree with the optimal lifecycle suggested by Merton (1969) could be related to the fact that pension exists out of a part determined by the AOW. And that for this reason more risk should be taken, since the AOW part does not contain that much risk. This could be different for people who earn more, have a shorter contribution period or come in with a significant starting pension capital (from another pension plan for example). This should be further investigated in future research.
Another reason could be that the lifecycles by Merton (1969) in fact are optimal but that the employees are more risk averse than estimated by the models used in this research. More risk averse employees would also mean that the lifecycle by Achmea, that currently is ranked second best, would be ranked higher compared to the lifecycle purely invested in stocks. Achmea's

lifecycle and Merton's(taking economic state into account) lifecycle perform better in the worst case scenarios of the simulation results. This could mean that the practical lifecycles are in fact more optimal than purely investing in stocks. More research in this area is beneficial for improving the lifecycles and for determining the optimal lifecycle.

The optimal lifecycle strongly depends on the data and model assumptions. In this research MSCI Indices and 1 Month Euribor of the period April 2005 until April 2017 are used in combination with the CIR model and a VARX model. This period contains the financial crisis followed by recovering period containing all time low interest rates. The financial crisis and the shift to extremely low and even negative interest rates could affect the research. Moreover the estimation uncertainty and the slight deviation of the average return statistics of the assets compared to the historical data could affect the research. Therefore longer or other horizon data, with the possibility of negative interest rates could be investigated in future research.

This research focusses on people contributing their whole working life. An addition to this research would be to look at people coming in at a later age or with a starting pension capital. It might be more beneficial to reduce the risk when a certain amount of pension capital is reached. Furthermore since 2016 September the first, the law "Wet voor verbeterde premieregeling" came in force. With this law it is possible to keep investing a part of the pension capital when receiving the pension. This means that even at pension age the received monthly pension can be dependent on the investment returns. This introduces investment risk for the pensioner. The impact of this law to lifecycle investing in defined contribution plans should be investigated in future research.

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8   Appendix

The Appendix contains some extra descriptive information about the data, functions and models used, the full derivation of models and estimation methods and the Matlab codes that are used to conduct this research.

8.1   Descriptive

This section contains some additional tables and figures for a better understanding of the methods and models used in this paper.

8.1.1   Change in Euribor

The figure shows the change in Euribor 1 month interest compared to the previous month. The table and figure clearly show significant differences in recession and expansion periods. All values shown are percentages.
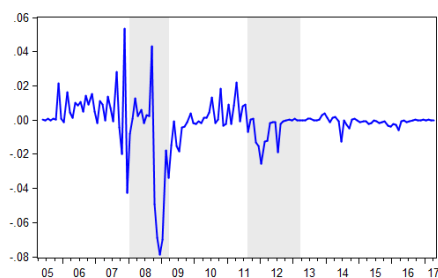
	All	Expan	Reces
Mean	-0.14	0.15	-1.05
Median	-0.02	0.00	-0.12
Std. Dev.	1.54	0.97	2.41
		Score	prob.
Mean	t-test	2.86	0.00
Std. Dev.	F-test	0.16	0.00

Figure 21: Absolute change; 1 month Euribor rates

Table 13: 1 month Euribor growth Statistics

8.1.2   Excess Stock Return

In the table below the excess stock return statistics are shown. All values shown are percentages.

	All	Expansion	Recession
Mean	0.22	0.75	-1.44
Median	0.93	1.14	-0.47
Std. Dev.	4.21	3.29	6.06

Table 14: Stock returns minus the 1 month Euribor

8.1.3   Functions
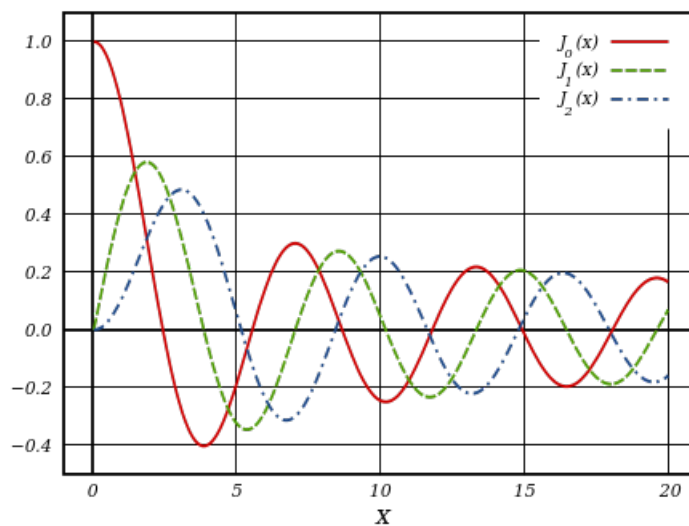
Figure 22: Bessel function $J_q(x)$ for q =0, 1 and 2.

This Figure shows the oscillating characteristic of the Bessel functions that makes it impossible for the fminsearch function to reach an optimum.

8.1.4   Salary

Figure 23: Salary growth over age shows a exponential trend

The salary function is best estimated with an age dependent exponential function.
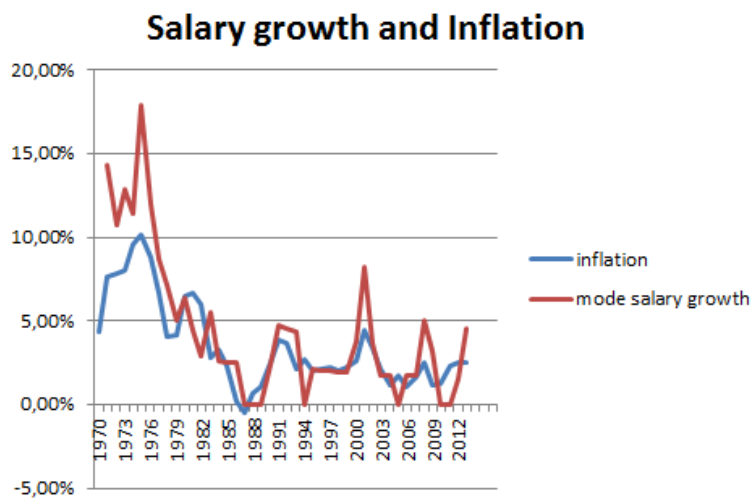
Figure 24: Inflation shows a similar pattern as mode salary growth

This Figure shows that the salary growth follows a similar path as the inflation. This is data obtained from CBS.

Figure 25: Salary growth minus inflation. Period 1993-2013($\mu = 0.26\%$ and $s = 1.61\%$)

The salary growth excess on the inflation, the real salary growth, has a mean of 0.26% with a standard deviation of 1.61%.

8.1.5   VAR Estimation Output

|	R_Stocks	R_RE	Inflation
R_Stocks(-1)	-0.002	-0.086	-0.009
	(0.128)	(0.174)	(0.015)
R_RE(-1)	0.103	0.169	0.014
	(0.093)	(0.127)	(0.011)
Inflation(-1)	-0.320	-1.319	0.251
	(0.707)	(0.962)	(0.082)
C	0.931	1.076	0.057
	(0.468)	(0.637)	(0.054)
Euribor 1Month	-0.394	-0.721	0.000
	(0.216)	(0.294)	(0.025)
Recession Dummy	-2.195	-2.831	0.162
	(0.830)	(-1.131)	(0.096)

Table 15: VAR coefficients matrix (values between brackets are standard errors)

This table shows the regression output of the VAR model in section 4.3.3. The regression period is April 2005 until April 2017. R_RE stands for return Real Estate and (-1) indicates the previous period.

8.1.6   Contribution Percentages

Ages	OP	OP and delayed effective accrued PP	OP and immediately effective accrued PP	OP and immediate obtainable PP
15 until 19	5.7	6.9	7.8	8
20 until 24	6.3	7.7	8.6	9.1
25 until 29	7.3	8.9	10	10.5
30 until 34	8.5	10.3	11.5	12
35 until 39	9.8	12	13.2	13.8
40 until 44	11.4	13.9	15.3	15.9
45 until 49	13.3	16.2	17.6	18.4
50 until 54	15.5	18.9	20.4	21.2
55 until 59	18.2	22.3	23.5	24.5
60 until 64	21.6	26.4	27.3	28
65 until 67	24.9	30.5	30.9	31.1

Table 16: Percentages of pensionable income used to build up pension

This table shows the contribution percentages for the age 15 until 67. OP stands for Old age Pension and PP stands for Partner Pension. This research bases the pension contributions on OP pension (the first column). The information from this table is obtained from Belastingdienstpensioensite.

8.1.7 Interest rate and Pension

This sections reviews the effect different interest rates have on the monthly pension. Using the webpage `http://www.bank-sparen.nu/banksparen/vergelijken/` I find the monthly pension based on different interest rates. These rates are from 15 Dutch banks and/or Dutch insurance companies.[2] The pensions are based on a pay out period of 25 years, a constant monthly payout, when the initial total capital is €100,000. The following figure shows the monthly pension for the different interest rates.
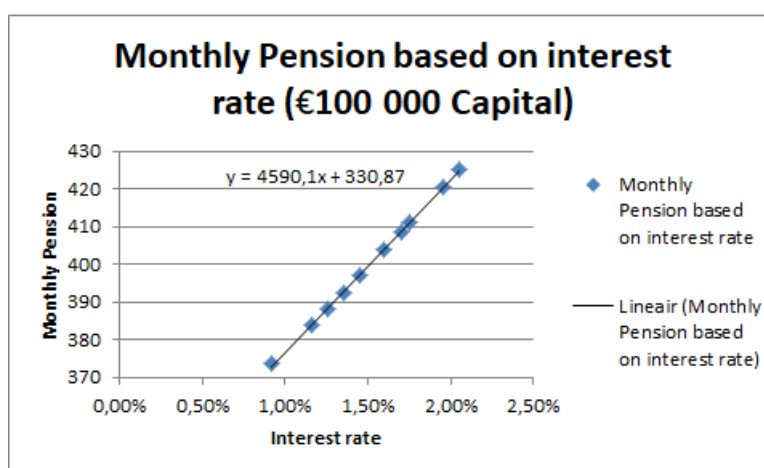

Figure 26: Pension and Interest follow a linear function

Using this information we can determine a function which generates the monthly pension given interest rate and initial capital. Given the initial capital we have a linear function given by

$$Pm = (0.045901 * r + 0.0033087) * C, \qquad (37)$$

where Pm stands for the monthly pension, r for the interest rate and C for the initial total capital.

---

[2]The providers are: Centraal Beheer Achmea, Nationale Nederlanden Bank, BLG Wonen, Delta Lloyd Bank, Regio Bank, OHRA Bank, SNS Bank, Rabobank, ASR Bank, AEGON Bank, Reaal Levensverzekeringen, ABN AMRO Bank, AEGON Verzekeringen, Allianz Nederland Bank en Delta Lloyd Verzekeringen

8.1.8   Average Merton Allocation

In the figures below the average allocation of the Merton models in the 10,000 simulations are shown.
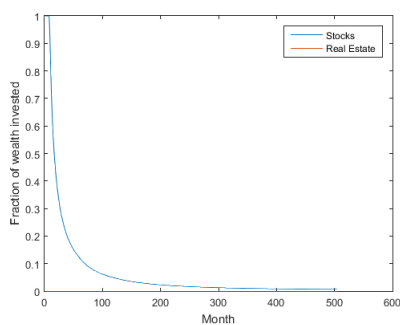

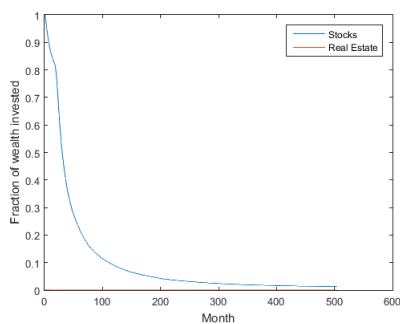Figure 27: The average allocation of Merton's lifecycle

Figure 28: The average allocation of Merton's lifecycle taking economic stage into account
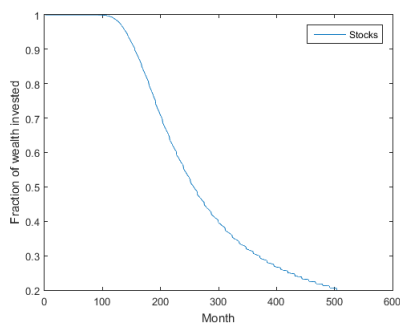

Figure 29: The average allocation of Merton's lifecycle using parameters from literature

8.1.9   Additional Lifecycle Result statistics

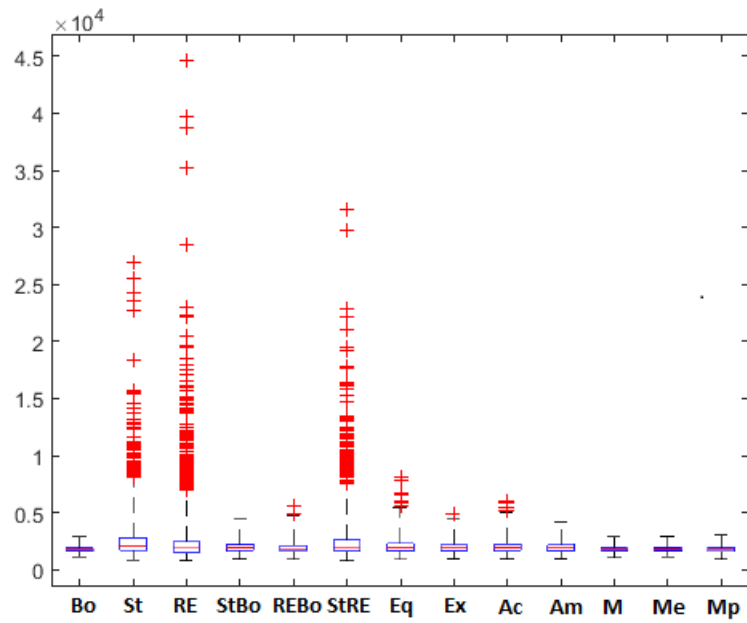This Figure shows the boxplot of all 13 lifecycles including all simulation results.

Figure 30: Boxplot of the 13 different lifecycles

8.2 Derivations

This sections contains additional derivations and explanation of models and parameter values that are used.

8.2.1 Markov states

The equations

$$\pi_{exp} = p\pi_{exp} + (1 - q)\pi_{rec}$$
$$\pi_{rec} = (1 - p)\pi_{exp} + q\pi_{rec} \tag{38}$$
$$\pi_{exp} + \pi_{rec} = 1,$$

with $\pi_{rec} = 75.9\%$ and $\pi_{exp} = 24.1\%$ reduce to

$$p = \frac{0.518}{0.759} + \frac{0.241}{0.759}q \ \ or \ \ q = \frac{-0.518}{0.241} + \frac{0.759}{0.241}p. \tag{39}$$

These equations are used to determine optimal values for p and q. The code used to find these optima are in Appendix 8.3.1 .

By analysing the historic stages of the economy, stated by the CEPR[3], the transition probabilities from table 17 are observed.

	Expansion	Recession
Expansion	0.9725	0.0275
Recession	0.0571	0.9429

Table 17: Markov transition states empirically found

These transition probabilities result in steady state probabilities of 67.5% for expansion and 32.5% for recession. Which is quiet different from the observed 75.9% and 24.1%. For this reason it is better to use the transition probabilities of table 18.

	Expansion	Recession
Expansion	0.98	0.02
Recession	0.06	0.94

Table 18: Markov transition states better to be used

These transition probabilities result in steady state probabilities of 75.0% and 25.0% for Expansion and Recession respectively.

---

[3]Center for Economic Policy Research

8.2.2   Maximum Likelihood

In this part the derivation of the Likelihood function of the short rate is shown. The likelihood function is

$$L(k, \theta, \sigma) = \prod_{i=1}^{N-1} f(r_{i+1}(t)|r_i(t - \Delta t)), \tag{40}$$

with N the total number of observations. In order to simplify the calculations the natural log is taken, the reason is that taking a natural log does not change the arguments for the maximum value. The log-likelihood is given by

$$\log(L(k, \theta, \sigma)) = \sum_{i=1}^{N-1} f(r_{i+1}(t)|r_i(t - \Delta t)). \tag{41}$$

Using that

$$f(r_{i+1}(t)|r_i(t - \Delta t)) = ce^{-u-v}(\frac{v}{u})^{q/2} I_q(2(uv)^{1/2}), \tag{42}$$

we get the following formulae,

$$
\begin{aligned}
\log(L(k, \theta, \sigma)) &= \sum_{i=1}^{N-1} (\log(ce^{-u_i-v_{i+1}}(\frac{v_{i+1}}{u_i})^{q/2} I_q(2(u_i v_{i+1})^{1/2})) \\
&= \sum_{i=1}^{N-1} \log(c) + \sum_{i=1}^{N-1} [(-u_i - v_{i+1}) + \frac{q}{2}\log(\frac{v_{i+1}}{u_i}) + \log(I_q(2\sqrt{u_i v_{i+1}}))] \\
&= (N-1)\log(c) + \sum_{i=1}^{N-1} [(-u_i - v_i) + \frac{q}{2}\log(\frac{v_{i+1}}{u_i}) + \log(I_q(2\sqrt{u_i v_{i+1}}))].
\end{aligned}
\tag{43}
$$

Maximizing this log-likelihood yields the optimum parameter values $(k, \theta, \sigma) = argmax_{k,\theta,\sigma}$ and is done with aid of the Matlab function fminsearch which is incorporated in the code found in
Appendix 8.3.5.

8.2.3   Risk Aversion Parameter Estimation

This section contains the derivation of the γ parameters in each economic stage. Li (2007) argued that γ should be dependent on a constant and negatively with the time varying market price of risk. Using Graham and Harvey (2016) I found a market price of risk that depends on the economic

stage, λ equals 3.98% in expansions and 3.35% in recessions. Combining this information I get the formula

$$\gamma_t = a + b * \lambda_t, \tag{44}$$

where a is a positive constant, b a negative coefficient and λ_t the market price of risk. As it is common on literature to use a γ value of 5 for the average investor and as Bovenberg et al. (2007) states that wrong values for γ can have enormous impact on the welfare loss. I choose to have the expected γ using the steady state probabilities to be equal to 5. This comes down to

$$0.241 * \gamma_{rec} + 0.759 * \gamma_{exp} = 5, \tag{45}$$

$$\gamma_{rec} = \frac{5}{0.241} - \frac{0.759}{0.241} * \gamma_{exp}. \tag{46}$$

Now inserting the first equation results in

$$a + b * 0.0398 = \frac{5}{0.241} - \frac{0.759}{0.241} * (a + b * 0.0335), \tag{47}$$

Solving for a gives

$$a + \frac{0.759}{0.241} * a = \frac{5}{0.241} - \frac{0.759}{0.241} * b * 0.0335 - b * 0.0398, \tag{48}$$

$$\frac{a}{0.241} = \frac{5}{0.241} - \frac{b}{0.241} * (0.759 * 0.0335 + 0.0398 * 0.241), \tag{49}$$

$$a = 5 - b * (0.759 * 0.0335 + 0.0398 * 0.241), \tag{50}$$

finally this comes down to

$$a = 5 - 0.0350b \tag{51}$$

and

$$b = 142.7825 - 28.5565a. \tag{52}$$

Trying out multiple a between 0 and 5 results in γ_{rec} values between 5 and 5.68 and γ_{exp} values between 4.78 and 5. The range is small and the smaller a the bigger the difference, for this reason I choose a equal to 1 which results in $\gamma_{rec} = 5.5462$ and $\gamma_{exp} = 4.8266$.

8.3   Matlab Codes

This section contains the most important Matlab codes that were used in this research.

8.3.1   Markov transition probabilities

This section contains the code for the determination of the markov transition probabilities.

%Code for determining the optimal values for p and q.

%initialising possible vectors
p = zeros(1,100);
q = zeros(1,100);
x = [0.01:0.01:1];
y = [0.01:0.01:1];

%calculating p and q when the other is between 0 and 1
p = 0.518/0.759 + 0.241/0.759*x; %values for p when q equals x
q = -0.518/0.241 + 0.759/0.241*y; %values for q when p equals y

%determining optimal p and q

%calculating p when q is between 0.9 and 1
x2 = [0.9001:0.0001:1];
$p2 = 0.518/0.759 + 0.241/0.759 * x2;$

%calculating errors compared to emperical transition probabilities
%p=0.9725 and q=0.9429
$c2 = [(x2 - 0.9429).^2; (p2 - 0.9725).^2];$ %quadratic error function
$c3 = [abs(x2 - 0.9429); abs(p2 - 0.9725)];$ %absolute error function
d2 = sum(c2,1);
d3 = sum(c3,1);

%determining the smallest error
$[loc\,level] = min(d2);$
$[loc2\,level2] = min(d3);$

$q\_optimal = 0.900 + level/10000$
$p\_optima] = p2(level)$
$\%q2 = -0.518/0.241 + 0.759/0.241 * y2;$

8.3.2   Salary function

This section contains the code for the determination of the salary function.

```
%%finding an optimal salary function dependent on age
%It was already found that salary could best be estimated using an
%exponential function. Optimally fit a line through data points with
%restrictions that at age 42 salary = 37000 and salary growth = 1.1%

%emperical salary per age
good27 = 28000;
good32 = 32100;
good37 = 35000;
good42 = 37000;
good47 = 38000;
good52 = 39500;
good57 = 39500;
good = [good27 good32 good37 good42 good47 good52 good57];

%initializing error vectors
error = zeros(2,1000);
error2 = zeros(2,1000);
temp = zeros(1000,31);
temp2 = zeros(1000,30);
goodtemp = 0;

%starting simulation with 1000 different growth of salary growth factors
for i=1:1000
error(1,i) = i;
error(1,i) = i;
temp(i,16) = 37000; %mode salary at age 42
temp2(i,16) = 1.1; %mode salary growth at age 42


for j=17:30
temp2(i,j) = temp2(i,j-1)*(1/(1+(i/10000))); %salary growth
end
for j=1:15
temp2(i,16-j) = temp2(i,16-j+1)/(1/(1+(i/10000)));
end

for j=1:15
```

```
temp(i,16-j) = temp(i,16-j+1) / (1+temp2(i,16-j+1)/100); %salary age>42
end

for j=17:31
temp(i,j) = temp(i,j-1) * (1+temp2(i,j-1)/100); %salary age<42
end
goodtemp = [temp(i,1) temp(i,6) temp(i,11) temp(i,16) temp(i,21) temp(i,26)
temp(i,31)];

%calculating errors for each growth of salary growth factor
error(2,i) = sum(abs(goodtemp-good)); %absolute error term
error2(2,i) = sum((goodtemp − good).²); %quadratic error term
end

%optimal growth of salary growth factor
[error_value error_point] = min(error2(2,:));
growth_rate_opt = 1/(1 + error_point/10000);

%for visualisation
plot1 = zeros(7,2);
plot2 = zeros(7,2);

plot1(:,1) = 27:5:57;
plot2(:,1) = 27:5:57;

for i=1:7
plot1(i,2) = good(i);
plot2(i,2) = temp(error_point, (1+(5*(i-1))));
end

figure
hold on
plot(plot1(:,1),plot1(:,2),'+')
plot(plot2(:,1),plot2(:,2))
xlabel('Age')
ylabel('Salary')
hold off
```

8.3.3 Simulation

The following sections contain the different scripts and functions used to run the whole simulation.

```
%%%Input Parameters

StartAge = 25;              %must be smaller than 42 for this version
PensionAge = 67;            %must be between 44 and 69 for this version

Franchise = 13123;
FranchiseGrowth = 1.019;

Conperc = zeros(67,1);      %Contribution percentages based on OP(1 to 67)
Conperc(15:19,1) = 5.7;
Conperc(20:24,1) = 6.3;
Conperc(25:29,1) = 7.3;
Conperc(30:34,1) = 8.5;
Conperc(35:39,1) = 9.8;
Conperc(40:44,1) = 11.4;
Conperc(45:49,1) = 13.3;
Conperc(50:54,1) = 15.5;
Conperc(55:59,1) = 18.2;
Conperc(60:64,1) = 21.6;
Conperc(65:67,1) = 24.9;

Workfactor = 1;             %fulltime = 1, not working = 0, parttime = <0,1>

lambdaexp = 0.0335;         %market price of risk in an expansion
lambdarec = 0.0398;         %market price of risk in a recession


%%%Initializing Variables

Years = PensionAge - StartAge;          %number of years
T = 12 * (Years) + 1;                   %number of periods (months)

Age = zeros(T,4);                       %matrix with two columns
Age(:,1) = 25:(1/12):67;                %age with partial months (25.083)
Age(:,2) = floor(Age(:,1));             %integer value of age (25)
Year = Age(:,2)-24;                     %vector of years  (1-42)
Month = (mod(Age(:,1),1)+(1/12))*12;    %vector of months (1-12)
Age(:,3) = Month;
Age(:,4) = 1:T;

Yeary = 1:(Years+1);
```

```
Agey = 25:67;

AOW = (Franchise * FranchiseGrowth^Years)/12; %monthly expected AOW

%%%Loads Asset data (just 3 files)
%input:     -
%output:    S_R, RE_R, I_R
LoadAssetData;


%%%Estimate short rate parameters
%input:     -
%output:    k,k2,theta,theta2,sigma,sigma2,b2
EstimateShortPara;

%%%The Simulation
RealSimulation;

%% Simulation results
Statistics;     %simulation statistics

StatisticsLC;   %lifecycle simulation statistics

%%%The Ranking
Ranking;
```

8.3.4 LoadAssetData

```
%%%Retrieving Asset Data 2005m4- 2017m4
S_R = xlsread('returns.xls',1,'B125:B269');
RE_R = xlsread('returns.xls',1,'C125:C269');
I_R = xlsread('cpi_increase.xls',1,'B125:B269');
```

8.3.5 EstimateShortPara

This section contains the code for the Maximum Likelihood Estimation.

```
%%%Script for Short Rate Parameter estimation
%Maximum Likelihood
%Following Kladivko instructions
%input:      -
%output:    k,k2,theta,theta2,sigma,sigma2,b2
```

```
%%%Initiation
Ebo1 = xlsread('matlabeuribor.xls',1,'A1:A147'); % Euribor 2005m4 until 2017m6
Ebo2 = ((1+ Ebo1/100).^(1/12)-1); %annualized 1month euribor to monthly
Ebo = Ebo2(1:145); % Euribor 2005m4 2017m4
Ebo5 = Ebo;
Ebo3 = Ebo(2:145);
Ebo4 = Ebo3;


Nebo = length(Ebo);

Recdum1 = xlsread('matlabeuribor.xls',1,'C1:C147');
Recdum = Recdum1(1:145); %2005m4 - 2017m4
Recdum4 = Recdum(2:145); %2005m5 - 2017m4 (same length as diff vector)



if Ebo(1) <= 0
    Ebo5(1) = 0.000000001;
end

for i=1:Nebo-1
    if Ebo3(i) <= 0
        Ebo4(i) = 0.000000001; % replace negative interest to 10^-7
        Ebo5(i+1) = Ebo4(i);
    end
end



%%%Starting values for MLE
dEbo1 = diff(Ebo5); %calculates grow in each period
dEbo = dEbo1./Ebo4.^0.5;

%without stage of economy
dt = 1/12;
x = [dt./Ebo4.^0.5, dt*Ebo4.^0.5]; % x * B = dEbo
B = x\dEbo;
resi = x*B - dEbo;

k = -B(2);
theta = -B(1)/B(2);
sigma = sqrt(var(resi,1)/dt);
param = [k theta sigma];
```

```
%with stage of economy
x2 = [dt./Ebo4.^0.5, dt*Ebo4.^0.5, Recdum4./Ebo4.^0.5]; %add dummy stage
B2 = x2\dEbo;
resi2 = x2*B2 - dEbo;

k2 = -B2(2);
theta2 = -B2(1)/B2(2);
b2 = B2(3);
sigma2 = sqrt(var(resi2,1)/dt);
param2 = [k2 theta2 sigma2 b2];

%%%Finding optimal k theta sigma using MLE
%no economic stage
paramstart = param;
paramstart(paramstart<=0)=1/1000000;
opt_para = fminsearch(@(B) LL_COX(dt,Ebo5,B), paramstart);
LL = LL_COX(dt,Ebo5,opt_para);
k = opt_para(1);
theta = opt_para(2);
sigma = opt_para(3);

%%%Finding optimal k theta sigma beta using MLE
%with economic stage
paramstart2 = param2;
paramstart2(paramstart2<=0)=1/1000000;
opt_para2 = fminsearch(@(B) LL_COX(dt,Ebo5,B), paramstart2);
LL2 = LL_COX(dt,Ebo5,opt_para2);
k2 = opt_para2(1);
theta2 = opt_para2(2);
b2 = opt_para2(4);
sigma2 = opt_para2(3);

%temporary overrule estimates
theta2 = 0.0017; %0.0017 => 0.02 on yearly base
theta = theta2;
```

8.3.6 LLCOX

```
function [LL] = LL_COX(dt,Ebo5,param)
%returns the likelihood value given the parameters
```

```
if size(param,2) == 3 %no economic state
    %parameters
    k = param(1);
    theta = param(2);
    sigma = param(3);

    %calculate loglikelihood
    n = length(Ebo5);
    c = (2*k)/((sigma^2)*(1-exp(-k*dt)));
    u = c * Ebo5(1:n-1).*exp(-k*dt);
    v = c * Ebo5(2:n);
    q = (2*k*theta/sigma^2)-1;
    w = 2.*(u.*v).^0.5;

    L = -u-v + (q/2).*log(v./u) + log(besseli(q,w,1)) + w;
    L2 = log((besseli(q,w,1)));

    if k<0 || theta<0 || sigma<0
        LL = 1000^100;
    else
        LL2 = (n-1)*log(c) + sum(L);
        LL = -LL2;
    end
else %=4 => with economic state
    %parameters
    k = param(1);
    theta = param(2);
    sigma = param(3);
    beta = param(4);

    %calculate loglikelihood
    n = length(Ebo5);
    c = (2*k)/((sigma^2)*(1-exp(-k*dt)));
    u = c * Ebo5(1:n-1).*exp(-k*dt);
    v = c * Ebo5(2:n);
    q = (2*k*theta/sigma^2)-1;
    w = 2.*(u.*v).^0.5;

    L = -u-v + (q/2).*log(v./u) + log(besseli(q,w,1)) + w;

    if k<0 || theta<0 || sigma<0
```

```
        LL = 1000^100;
    else
        LL2 = (n-1)*log(c) + sum(L);
        LL = -LL2;
    end
end
end
```

8.3.7 RealSimulation

```
%%Script for Simulating Nsim number of runs
Nsim = 200;                                    % number of simulations

% initializing variables for storing all important variables
ALL_S_dumrec = zeros(T,Nsim);
ALL_S_shortrates = zeros(T,Nsim);
ALL_S_Stock = zeros(T,Nsim);
ALL_S_RE = zeros(T,Nsim);
ALL_S_Inflation = zeros(T,Nsim);
ALL_S_Salary = zeros(T,Nsim);

% all different lifecycles nominal and corrected for inflation
ALL_LC_R_Ac = zeros(T,Nsim);
ALL_LC_N_Ac = zeros(T,Nsim);
ALL_LC_R_Am = zeros(T,Nsim);
ALL_LC_N_Am = zeros(T,Nsim);
ALL_LC_R_Bo = zeros(T,Nsim);
ALL_LC_N_Bo = zeros(T,Nsim);
ALL_LC_R_Eq = zeros(T,Nsim);
ALL_LC_N_Eq = zeros(T,Nsim);
ALL_LC_R_Ex = zeros(T,Nsim);
ALL_LC_N_Ex = zeros(T,Nsim);
ALL_LC_R_M = zeros(T,Nsim);
ALL_LC_N_M = zeros(T,Nsim);
ALL_LC_R_Mer = zeros(T,Nsim);
ALL_LC_N_Mer = zeros(T,Nsim);
ALL_LC_R_RE = zeros(T,Nsim);
ALL_LC_N_RE = zeros(T,Nsim);
ALL_LC_R_REBo = zeros(T,Nsim);
ALL_LC_N_REBo = zeros(T,Nsim);
ALL_LC_R_St = zeros(T,Nsim);
```

```
ALL_LC_N_St = zeros(T,Nsim);
ALL_LC_R_StBo = zeros(T,Nsim);
ALL_LC_N_StBo = zeros(T,Nsim);
ALL_LC_R_StRE = zeros(T,Nsim);
ALL_LC_N_StRE = zeros(T,Nsim);

%[start for loop]
for i=1:Nsim


% Generating Recession and Expansion states
%input:     Age, Year, lambdaexp, lambdarec, T
%output:    dumrec, lambda, inrec
[dumrec, lambda, inrec] = ...
genrecessionstates(Age, Year, lambdaexp, lambdarec, T);

% Estimate short rate and bond prices
%input:     Ebo, T, Nebo, k2, theta2, sigma2, dt, dumrec, lambda
%output:    shortrate, fP, fY, fA, fB, yield
[shortrate, fP, fY, fA, fB, yield] = ShortRateSimulation(Ebo, T, Nebo, ...
k2, theta2, sigma2, dt, b2, dumrec, lambda);

% VAR Estimate Stock returns, Real Estate returns and Inflation
%input:     S_R, RE_R, I_R, ...
Ebo, Recdum, shortrate, dumrec, Nebo, T, Years
%output:    Y_new, Z, Inflation
[Y_new, Z, Inflation] = EstimateAssets(S_R, RE_R, I_R, Ebo, Recdum, ...
shortrate, dumrec, Nebo, T, Years);

% Calculate Salary and Human Capital
%input:     Years, StartAge, Inflation, Conperc, fY, T, shortrate
%output:    Salary3, Contribution, ContributionT, ContributionT2, ...
Human Capital, HumanCapitalT
[Salary, Contribution, ContributionT, ContributionT2, ...
HumanCapital, HumanCapitalT] = SalaryandHumanCapital(Years, StartAge, ...
Inflation, Conperc, fY, T,shortrate );

% Lifecycle Returns
%input:     fY, T, ContributionT2, Y_new, shortrate, Age, PensionAge,
%HumanCapitalT, dumrec
%output:    LC_St, LC_RE, LC_Bo, LC_Eq, LC_StRE, LC_StBo, LC_REBo, LC_Ex,
```

```
%LC_M, LC_Mer, LC_Ac, LC_Am, LC_Mp
[LC_St, LC_RE, LC_Bo, LC_Eq, LC_StRE, LC_StBo, LC_REBo, LC_Ex, ...
LC_M, LC_Mer, LC_Ac, LC_Am, LC_Mp] = Lifecycles(fY, T, ContributionT2, Y_new, ..
shortrate, Age, PensionAge, HumanCapitalT, dumrec);



%filling in results

ALL_S_dumrec(:,i) = dumrec(:,4);
ALL_S_shortrates(:,i) = shortrate;
ALL_S_Stock(:,i) = Y_new(:,1);
ALL_S_RE(:,i) = Y_new(:,2);
ALL_S_Inflation(:,i) = Y_new(:,3);
ALL_S_Salary(:,i) = Salary;
ALL_S_yield1(:,i) = yield(:,4);
ALL_S_yield2(:,i) = yield(:,5);
ALL_S_Contribution(:,i) = ContributionT2;
ALL_S_InflCPI(:,i) = Z(:,3);
ALL_S_StockZ(:,i) = Z(:,1);
ALL_S_REZ(:,i) = Z(:,2);

% all different lifecycles nominal and corrected for inflation
ALL_LC_R_Ac(:,i) = LC_Ac./Z(:,3);
ALL_LC_N_Ac(:,i) = LC_Ac;
ALL_LC_R_Am(:,i) = LC_Am./Z(:,3);
ALL_LC_N_Am(:,i) = LC_Am;
ALL_LC_R_Bo(:,i) = LC_Bo./Z(:,3);
ALL_LC_N_Bo(:,i) = LC_Bo;
ALL_LC_R_Eq(:,i) = LC_Eq./Z(:,3);
ALL_LC_N_Eq(:,i) = LC_Eq;
ALL_LC_R_Ex(:,i) = LC_Ex./Z(:,3);
ALL_LC_N_Ex(:,i) = LC_Ex;
ALL_LC_R_M(:,i) = LC_M./Z(:,3);
ALL_LC_N_M(:,i) = LC_M;
ALL_LC_R_Mer(:,i) = LC_Mer./Z(:,3);
ALL_LC_N_Mer(:,i) = LC_Mer;
ALL_LC_R_RE(:,i) = LC_RE./Z(:,3);
ALL_LC_N_RE(:,i) = LC_RE;
ALL_LC_R_REBo(:,i) = LC_REBo./Z(:,3);
ALL_LC_N_REBo(:,i) = LC_REBo;
```

```
ALL_LC_R_St(:,i) = LC_St./Z(:,3);
ALL_LC_N_St(:,i) = LC_St;
ALL_LC_R_StBo(:,i) = LC_StBo./Z(:,3);
ALL_LC_N_StBo(:,i) = LC_StBo;
ALL_LC_R_StRE(:,i) = LC_StRE./Z(:,3);
ALL_LC_N_StRE(:,i) = LC_StRE;
ALL_LC_R_Mp(:,i) = LC_Mp./Z(:,3);
ALL_LC_N_Mp(:,i) = LC_Mp;
end

%%monthly pension (nominal and real monthly lifecycle + AOW)
%nominal
ALL_LC_mn_Ac = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_Ac(T,:);
ALL_LC_mn_Am = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_Am(T,:);
ALL_LC_mn_Bo = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_Bo(T,:);
ALL_LC_mn_Eq = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_Eq(T,:);
ALL_LC_mn_Ex = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_Ex(T,:);
ALL_LC_mn_M = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_M(T,:);
ALL_LC_mn_Mer = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_Mer(T,:);
ALL_LC_mn_RE = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_RE(T,:);
ALL_LC_mn_REBo = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_REBo(T,:);
ALL_LC_mn_St = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_St(T,:);
ALL_LC_mn_StBo = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_StBo(T,:);
ALL_LC_mn_StRE = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_StRE(T,:);
ALL_LC_mn_Mp = AOW + (0.045901.*ALL_S_shortrates(T,:) + ...
 0.0033087) .*ALL_LC_N_Mp(T,:);

%real
ALL_LC_m_Ac = AOW./ALL_S_InflCPI(T,:) + ...
```

```
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_Ac(T,:);
ALL_LC_m_Am = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_Am(T,:);
ALL_LC_m_Bo = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_Bo(T,:);
ALL_LC_m_Eq = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_Eq(T,:);
ALL_LC_m_Ex = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_Ex(T,:);
ALL_LC_m_M = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_M(T,:);
ALL_LC_m_Mer = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_Mer(T,:);
ALL_LC_m_RE = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_RE(T,:);
ALL_LC_m_REBo = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_REBo(T,:);
ALL_LC_m_St = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_St(T,:);
ALL_LC_m_StBo = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_StBo(T,:);
ALL_LC_m_StRE = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_StRE(T,:);
ALL_LC_m_Mp = AOW./ALL_S_InflCPI(T,:) + ...
 (0.045901*ALL_S_shortrates(T,:) + 0.0033087) .*ALL_LC_R_Mp(T,:);
```

8.3.8 genrecessionstates

```
function [dumrec lambda inrec] = ...
genrecessionstates(Age, Year, lambdaexp, lambdarec, T)
%This function generates recession and expansion periods.
%input:     Age, Year, lambdaexp, lambdarec
%output:    dumrec, lambda, inrec
prec = 0.9402;              %probability from recession to recession
pexp = 0.0190;              %probability from expansion to recession
dumrec = zeros(T,4);
dumrec(:,1) = Year;         %year indicator 1:untill pension age reached
dumrec(:,2) = Age(:,1);     %age indicator startage until pension age
dumrec(:,3) = Age(:,4);     %recession indicator 0 or 1
```

```
lambda = ones(T,1);
lambda = lambdaexp*lambda;

temp = zeros(T,2);

%compare random draw from uniform distribution[0,1], if below prec in case
%previous period was recession or below pexp if previous period was
%expansion => next stage is recession.
for i=1:T-1                           %choosing the right probability
    temp(i,1) = pexp^(abs(dumrec(i,4)-1)) *(prec^(dumrec(i,4)));
    temp(i,2) = rand(1);              %taking random value from uniform(0,1)
    if temp(i,2) < temp(i,1)          %if smaller next state is recession
        dumrec(i+1,4) = 1;
        lambda(i,1) = lambdarec;
    end
end

inrec = mean(dumrec(:,4)); % percentage periods in recession

end
```

8.3.9 ShortRateSimulation

```
function [shortrate, fP, fY] = ShortRateSimulation(Ebo, T, Nebo, ...
k2, theta2, sigma2, dt, b2, dumrec, lambda)
%This function generates Shortrates and the functions to price bonds
%input:     Ebo, T, Nebo, k2, theta2, sigma2, dt, b2, dumrec, lambda
%output:    shortrate, fP, fY
Nss = 1;                          %Number of Short rate Simulations
shortrate = zeros(T,Nss);
shortrate(1,:) = Ebo(Nebo);

Shocks = randn(T,Nss);

for i=2:T
    shortrate(i-1,:) = max(0,shortrate(i-1,:));
    shortrate(i,:) = max(0, shortrate(i-1,:) + k2*(theta2-shortrate(i-1,:)) *dt
        + (sigma2 * sqrt(shortrate(i-1,:).*dt)).* Shocks(i,:) + b2*dumrec(i,4));
end
```

```matlab
%function for gamma, A and B
fgamma = @(tt)(sqrt((k2+lambda(tt))^2 + 2*sigma2^2));
fA = @(tt,TT) (((2*fgamma(tt)*exp((fgamma(tt)+k2+lambda(tt))*(TT-tt)/2))/ ...
    ((fgamma(tt)+k2+lambda(tt))*(exp(fgamma(tt)*(TT-tt))-1)+2*fgamma(tt)))^...
    (2*k2*theta2/sigma2^2)) ;
fB = @(tt,TT) ((2*(exp(fgamma(tt)*(TT-tt))-1))/ ...
    ((fgamma(tt)+k2+lambda(tt))*(exp(fgamma(tt)*(TT-tt))-1)+2*fgamma(tt)));

%Price function of a bond and the yield function
fP = @(shortr,tt,TT) (fA(tt,TT) * exp(-fB(tt,TT)*shortr));
fY = @(shortr,tt,TT) ((-log(fP(shortr,tt,TT)))/(TT-tt));


yield = zeros(T-1,6);
for i=1:T-1
    yield(i,2) = fA(1,i+1);
    yield(i,3) = fB(1,i+1);
    yield(i,4) = fP(shortrate(100,1),1,i+1);
    yield(i,5) = fY(shortrate(100,1),1,i+1);
    yield(i,6) = yield(i,2) - yield(i,3)*shortrate(100,1);
    yield(i,1) = ((yield(i,5)+1)^12)-1;
end

end
```

8.3.10 EstimateAssets

```matlab
function [Y_new, Z, Inflation] = EstimateAssets(S_R, RE_R, I_R, Ebo, ...
Recdum, shortrate, dumrec, Nebo, T, Years )
%This function does the estimation of the Stock and Real Estate returns
%together with the estimation of the inflation.
%This estimation is done with an VAR model.
%input:    S_R, RE_R, I_R, Ebo, Recdum, shortrate, dumrec, Nebo, T, Years
%output:   Y_new, Z, Inflation

%%Matlab2015 unable to perform VARX regression use Matlab2017 or Eviews
% Spec = vgxset('n',3,'nAR',1,'nMA',0,'nX',2,'Constant',true);
% %3 times series, 1 auto regressive term, 0 moving average terms, 2
% %exogenous variables, and with a constant
% Spec2 = vgxset('n',1,'nAR',1,'nMA',0,'nX',1,'Constant',true);
% ESTmodel = vgxvarx(Spec,Y_old,X_old);
```

```
%%Model Y(t) = A + Y(t-1)*B + X(t)*C + E
% Y(t) = [Stock_returns Real_Estate_returns Inflation] in period t
% X(t) = [Shortrate Recession_dummy] in period t
% E = Residuals normal distributed N(0,Cov), Cov is estimated from model
% A[1x3], B[3x3], C[2x3] are constants
Y_old = [S_R RE_R I_R];
X_old = [Ebo Recdum];
X_new = [shortrate(:,1) dumrec(:,4)];

Y_new = zeros(T,3);
Z = zeros(T,3);

% initiate coefficients from Eviews VAR regression results
VAR_A = [1.33 1.88 0.06];
VAR_B = [0.00 -0.09 -0.01;0.10 0.17 0.01;-0.32 -1.32 0.25];
VAR_C = [-0.39 -0.72 0;-1.90 -2.53 0.16];

% error terms based on statistics of error terms from Eviews VAR regression
VAR_mu1 = [0 0 0];
VAR_Sigma = [15.4 15.8 0.19;15.8 28.5 0.34;0.19 0.34 0.21];
VAR_E = mvnrnd(VAR_mu1,VAR_Sigma);

Y_new(1,:) = VAR_A + Y_old(Nebo,:)*VAR_B + X_new(1,:)*VAR_C + VAR_E;

Z(1,:) = ones(1,3);

for i=2:T
    VAR_E = mvnrnd(VAR_mu1,VAR_Sigma);
    Y_new(i,:) = VAR_A + Y_new(i-1,:)*VAR_B + X_new(i,:)*VAR_C + VAR_E;
    Z(i,1) = Z(i-1,1)*(1+Y_new(i,1)/100); %growth of 1 euro invested Stocks
    Z(i,2) = Z(i-1,2)*(1+Y_new(i,2)/100); %invested in Real Estate
    Z(i,3) = Z(i-1,3)*(1+Y_new(i,3)/100); %invested in Inflation
end

Inflation = zeros(Years,1);
for i=1:Years
    Inflation(i) = (Z(1+i*12,3) - Z(1+(i-1)*12,3))/Z(1+(i-1)*12,3);
end


end
```

8.3.11 SalaryandHumanCapital

```
function [Salary3, Contribution, ContributionT, ContributionT2, ...
HumanCapital, HumanCapitalT] = SalaryandHumanCapital(Years, StartAge, ...
Inflation, Conperc, fY, T, shortrate)
%this function returns the Contributions for each period, the salary for
%each period and the amount of Human Capital in each period.
%% Calculate Salary and Human Capital
%input:     Years, StartAge, Inflation, Conperc, fY, T, shortrate
%output:    Salary3, Contribution, ContributionT, ContributionT2,
%HumanCapital, HumanCapitalT
Salary = zeros(Years,1);
Saltemp = 42-StartAge+1;
Salary(Saltemp) = 37000; %42-25+1 = 18

Franchise = 13123;
FranchiseGrowth = 1.019;
FranchiseM = zeros(T,1);

%Salary based purely on age
for j=1:Saltemp-1                %salary age<42
    Salary(Saltemp-j) = Salary(Saltemp-j+1)/(1+0.035*0.926^(Saltemp-j-1));
end

for j=Saltemp+1:Years+1         %salary age>42
    Salary(j) = Salary(j-1) * (1+0.035*0.926^(j-1));
end

% inflation correction and time correction
Salary2 = zeros(Years,1);
Salary3 = zeros(T,1);
Salary2(1) = Salary(1);
Saltemp3 = ones(12,1);
Salary3(1:12) = (Salary(1)*Saltemp3)/12;
for i=2:Years
    Saltemp2 = normrnd(Inflation(i-1)+0.0026,0.00026);
    Salary2(i)= Salary2(i-1) * (1+0.035*0.926^(i-1)) * (1+Saltemp2);
    Salary3(1+(i-1)*12:i*12) = (Salary2(i)*Saltemp3)/12;
end

% Franchise growth
```

```
for i =1:T
    FranchiseM(i,1) = Franchise * FranchiseGrowth^(floor((i-1)/12)+1);
end

% Human Capital
SalExp = zeros(Years,1);
SalExp(1) = Salary(1);
Contribution = zeros(Years+1,1);
Contribution3 = zeros(Years+1,1);
Contribution(1) = (SalExp(1)-FranchiseM(1)) * (Conperc(StartAge)/100);
Contribution3(1) = (Salary2(1)-FranchiseM(1)) * (Conperc(StartAge)/100);
for i=2:Years
    % previous salary * growth factor * (average expected inflation +
    % excess salary growth)
    SalExp(i) = SalExp(i-1)* (1+0.035*0.926^(i-1)) * (1.0155 + 0.0026);
    Contribution(i) = (Conperc(StartAge-1+i)/100) * (SalExp(i)-FranchiseM(i*12))
    Contribution3(i) = (Conperc(StartAge-1+i)/100) * (Salary2(i)-FranchiseM(i*12
end

HumanCapital = zeros(Years+1,1);
%HumanCapital2 = zeros(Years,1);
Humantemp = zeros(Years,1);
Humantemp(Years) = Contribution(Years);
for i=1:Years-1
    %expected sum of nominal future incomes
    Humantemp(Years-i) = Humantemp(Years+1-i) + Contribution(Years-i);
end

for i=1:Years
    HumanCapital(i) = Humantemp(i)/((1+fY(shortrate(1+(12*(i-1)),1),1,Years*12))
end

ContributionT = zeros(T,1);
for i=2:T
    if mod((i-1),12) == 0
        ContributionT(i-12) = Contribution((i-1)/12);
    end
end

ContributionT3 = zeros(T,1);
```

```
ContributionT2 = zeros(T,1);
HumanCapitalT = zeros(T,1);
for i=1:T
    ContributionT2(i) = Contribution(floor(i/12)+1)/12;
    HumanCapitalT(i) = HumanCapital(floor(i/12)+1);

    ContributionT3(i) = Contribution3(floor(i/12)+1)/12;
end
```

8.3.12   Lifecycles

```
function [LC_St, LC_RE, LC_Bo, LC_Eq, LC_StRE, LC_StBo, LC_REBo, LC_Ex, ...
LC_M, LC_Mer, LC_Ac, LC_Am, LC_Mp] = Lifecycles(fY, T, ContributionT2, Y_new,...
 shortrate, Age, PensionAge, HumanCapitalT, dumrec)
%This function returns all the lifecycles paths
%%Lifecycles Return
%input:     fY, T, ContributionT2, Y_new, shortrate, Age, PensionAge,
%HumanCapitalT, dumrec
%output:    LC_St, LC_RE, LC_Bo, LC_Eq, LC_StRE, LC_StBo, LC_REBo, LC_Ex,
%LC_M, LC_Mer, LC_Ac, LC_Am, LC_Mp


%%Pure lifecycles

LC_St = zeros(T,1);                %Purely Stocks Lifecycle
LC_RE = zeros(T,1) ;               %Purely Real Estate Lifecycle
LC_Bo = zeros(T,1) ;               %Purely Bonds Lifecycle

LC_St(1) = ContributionT2(1);
LC_RE(1) = ContributionT2(1);
LC_Bo(1) = ContributionT2(1);
for i=2:T
    LC_St(i) = LC_St(i-1)*(1+Y_new(i-1,1)/100) + ContributionT2(i);
    LC_RE(i) = LC_RE(i-1)*(1+Y_new(i-1,2)/100) + ContributionT2(i);
    LC_Bo(i) = LC_Bo(i-1)*(1+fY(shortrate(i,1),i,i+240)) + ContributionT2(i);
end

%%Constant mixed lifecycles
%Equally weighted (1/3 in each Stocks, Real Estate and Bonds) Lifecycle
LC_Eq = zeros(T,1);
```

```
%Equally weighted (1/2 in each Stocks and Real Estate) Lifecycle
LC_StRE = zeros(T,1);
%Equally weighted (1/2 in each Stocks and Bonds) Lifecycle
LC_StBo = zeros(T,1);
%Equally weighted (1/2 in each Real Estate and Bonds) Lifecycle
LC_REBo = zeros(T,1);

LC_Eq(1) = ContributionT2(1);
LC_StRE(1) = ContributionT2(1);
LC_StBo(1) = ContributionT2(1);
LC_REBo(1) = ContributionT2(1);

for i=2:T
    LC_Eq(i) = LC_Eq(i-1)*((1/3)*((1+Y_new(i-1,1)/100) + ...
    (1+Y_new(i-1,2)/100) + (1+fY(shortrate(i,1),i,i+240)))) + ...
    ContributionT2(i);
    LC_StRE(i) = LC_StRE(i-1)*((1/2)*((1+Y_new(i-1,1)/100) + ...
    (1+Y_new(i-1,2)/100))) + ContributionT2(i);
    LC_StBo(i) = LC_StBo(i-1)*((1/2)*((1+Y_new(i-1,1)/100) + ...
    (1+fY(shortrate(i,1),i,i+240)))) + ContributionT2(i);
    LC_REBo(i) = LC_REBo(i-1)*((1/2)*((1+Y_new(i-1,2)/100) + ...
    (1+fY(shortrate(i,1),i,i+240)))) + ContributionT2(i);
end

%%Example lifecycle
%example only works for startage = 25 and pensionage = 67

LC_Ex  = zeros(T,1);                %example lifecycle

LC_Ex(1) = ContributionT2(1);

temp_Ex= zeros(T,3);
temp_Ex(:,2) = 0.15;
for i=1:T
    if i < 218
        temp_Ex(i,1) = 0.7;
    end
    if i > 217 && i < 278
        temp_Ex(i,1) = 0.7 - (i-217)*(0.02/12);
    end
    if i > 277 && i < 338
```

```
        temp_Ex(i,1) = 0.6 - (i-277)*(0.05/12);
    end
    if i > 337
        temp_Ex(i,1) = 0.35 - (i-337)*(0.025/12);
    end

    temp_Ex(i,3) = 1 - temp_Ex(i,1) - temp_Ex(i,2);
end

for i=2:T
    LC_Ex(i) = LC_Ex(i-1)*(temp_Ex(i,1)*(1+Y_new(i-1,1)/100) + ...
    temp_Ex(i,2)*(1+Y_new(i-1,2)/100) + ...
    temp_Ex(i,3)*(1+fY(shortrate(i,1),i,i+240))) + ContributionT2(i);
end

%%Lifecycles in practice
LC_Ac  = zeros(T,1);            %Achmea lifecycle
LC_Am  = zeros(T,1);            %Amersfoortse lifecycle

LC_Ac(1) = ContributionT2(1);
LC_Am(1) = ContributionT2(1);

temp_Am= zeros(T,3);
temp_Am(:,1) = 0.7;
temp_Am(:,2) = 0.15;
temp_Am(:,3) = 0.15;
for i=1:T
    if Age(i,1) > 52 && Age(i,1)<= 60
        temp_Am(i,1) = 0.4;
        temp_Am(i,2) = 0.05;
    end
    if Age(i,1)<= 52
        temp_Am(i,1) = 0.55;
        temp_Am(i,2) = 0.1;
    end
    if Age(i,1) > 60 && Age(i,1)< 65
        temp_Am(i,1) = 0.3;
        temp_Am(i,2) = 0.05;
    end
    if Age(i,1) >= 65 && Age(i,1)<= 66
        temp_Am(i,1) = 0.2;
```

```
            temp_Am(i,2) = 0.05;
        end
        if Age(i,1) > 66 && Age(i,1)<= 67
            temp_Am(i,1) = 0.1;
            temp_Am(i,2) = 0.05;
        end
        temp_Am(i,3) = 1 - temp_Am(i,2) - temp_Am(i,1);
end

temp_Ac= zeros(T,3);
for i=1:T
        if PensionAge - Age(i,1) >= 20
            temp_Ac(i,1) = 0.68;
            temp_Ac(i,2) = 0.15;
        end
        if PensionAge - Age(i,1) < 20
            temp_Ac(i,1) = temp_Ac(i-1,1)-(0.46/240);
            temp_Ac(i,2) = temp_Ac(i-1,2)-(0.095/240);
        end

        temp_Ac(i,3) = 1 - temp_Ac(i,2) - temp_Ac(i,1);
end

for i=2:T
        LC_Ac(i) = LC_Ac(i-1)*(temp_Ac(i,1)*(1+Y_new(i-1,1)/100) + ...
        temp_Ac(i,2)*(1+Y_new(i-1,2)/100) + ...
        temp_Ac(i,3)*(1+fY(shortrate(i,1),i,i+240))) + ContributionT2(i);
        LC_Am(i) = LC_Am(i-1)*(temp_Am(i,1)*(1+Y_new(i-1,1)/100) + ...
        temp_Am(i,2)*(1+Y_new(i-1,2)/100) + ...
        temp_Am(i,3)*(1+fY(shortrate(i,1),i,i+240))) + ContributionT2(i);
end


%%Merton's optimal lifecycle
%parameter values are from eviews
Covar_M = [17.4 18.7;18.7 33.1];
Mu_M = [0.32 0.19];
Gamma_M = 5;
w_M = zeros(T,2);

LC_M  = zeros(T,1);
```

```
LC_M(1) = ContributionT2(1);

temp_M = (Covar_M\(Mu_M'/Gamma_M * (1+HumanCapitalT(1)/LC_M(1))))';
temp_M = max(temp_M,0);
temp_M = min(temp_M,1);
w_M(1,:) = temp_M;

for i=2:T
    LC_M(i) = (w_M(i-1,1)*(1+Y_new(i-1,1)/100) + ...
    w_M(i-1,2) * (1+Y_new(i-1,2)/100) + ...
    (1-sum(w_M(i-1,:))) * (1+fY(shortrate(i,1),i,i+240))) * LC_M(i-1) + ...
    ContributionT2(i);
    temp_M = (Covar_M\(Mu_M'/Gamma_M * (1+HumanCapitalT(i,1)/LC_M(i))))';
    temp_M = max(temp_M,0);
    temp_M = min(temp_M,1);
    w_M(i,:) = temp_M;
end


%%Merton improved lifecycle
%parameter values are from eviews
Covar_Me = [12.1 13.5;13.5 26.5];
Mu_Me = [0.95 1.06];
Gamma_Me = 4.827;

Covar_Mr = [28.8 27.6;27.6 44.0];
Mu_Mr = [-1.64 -2.55];
Gamma_Mr = 5.546;

w_Mer = zeros(T,2);
w_Mer1 = zeros(T,2);

LC_Mer  = zeros(T,1);
LC_Mer(1) = ContributionT2(1);

temp_Mer = (Covar_Me\(Mu_Me'/Gamma_Me * (1+HumanCapitalT(1)/LC_Mer(1))))';
temp_Mer = max(temp_Mer,0);
temp_Mer = min(temp_Mer,1);
w_Mer1(1,:) = temp_Mer;
w_Mer(1,:) = w_Mer1(1,:)/sum(w_Mer1(1,:));
```

```
for i=2:T
    LC_Mer(i) = (w_Mer(i-1,1)*(1+Y_new(i-1,1)/100) + ...
    w_Mer(i-1,2) * (1+Y_new(i-1,2)/100) + ...
    (1-sum(w_Mer(i-1,:))) * (1+fY(shortrate(i,1),i,i+240))) * LC_Mer(i-1) ...
    + ContributionT2(i);
    if dumrec(i,4) == 1
        temp_Mer = (Covar_Mr\(Mu_Mr'/Gamma_Mr * ...
        (1+HumanCapitalT(i,1)/LC_Mer(i))))';
    else
        temp_Mer = (Covar_Me\(Mu_Me'/Gamma_Me * ...
        (1+HumanCapitalT(i,1)/LC_Mer(i))))';
    end
    temp_Mer = max(temp_Mer,0);
    temp_Mer = min(temp_Mer,1);
    w_Mer1(i,:) = temp_Mer;
    if sum(w_Mer1(i,:)) >1
        w_Mer(i,:) = w_Mer1(i,:)/sum(w_Mer1(i,:));
    else
        w_Mer(i,:) = w_Mer1(i,:);
    end
end

%merton based on Advies Commissie Parameters
%st dev stock return 20% per year, rra = 5, mean equity premium 4%
%=> 20% of wealth invested in risky stocks
w_Mp = zeros(T,1);

LC_Mp  = zeros(T,1);
LC_Mp(1) = ContributionT2(1);

temp_Mp = (1/Gamma_Me) * (1+HumanCapitalT(1)/LC_Mer(1))';
temp_Mp = max(temp_Mp,0);
temp_Mp = min(temp_Mp,1);
w_Mp(1) = temp_Mp;

for i=2:T
    LC_Mp(i) = (w_Mp(i-1)*(1+Y_new(i-1,1)/100) + (1-w_Mp(i-1) * ...
    (1+fY(shortrate(i,1),i,i+240)))) * LC_Mp(i-1) + ContributionT2(i);
    if dumrec(i,4) == 1
        temp_Mp = (1/Gamma_Mr) * (1+HumanCapitalT(i,1)/LC_Mp(i))';
    else
```

```
        temp_Mp = (1/Gamma_Me) * (1+HumanCapitalT(i,1)/LC_Mp(i))';
    end
    temp_Mp = max(temp_Mp,0);
    temp_Mp = min(temp_Mp,1);
    w_Mp(i) = temp_Mp;
end

end
```

8.3.13   Statistics

```
%%Simulation Statistics
%(S is matrix, V is vector, X is scalar, U is factor(scalar))
%Recession/Expansion
ALL_S_dumexp = abs(ALL_S_dumrec-1);              %determine expansion periods
%Vectors for each simulation run results
ALL_V_count_rec = sum(ALL_S_dumrec);             %number of recession periods
ALL_V_count_exp = sum(ALL_S_dumexp);             %number of expansion periods
ALL_V_avg_rec = mean(ALL_S_dumrec);              %fraction of reces periods
%Average over all simulation run results
ALL_U_avg_rec = mean(ALL_V_avg_rec);             %fraction of reces periods


%shortrate
ALL_V_avg_short = mean(ALL_S_shortrates);        %vector average per run
ALL_X_avg_short = mean(ALL_V_avg_short);         %overal average


ALL_V_std_short = std(ALL_S_shortrates);         %vector st dev per run
ALL_X_std_short = mean(ALL_V_std_short);         %average st dev


temp_RE = ALL_S_shortrates.*ALL_S_dumrec;        %recession returns
temp_RE2 = sum(temp_RE);
ALL_X_avg_short_rec = mean(temp_RE2./ALL_V_count_rec);     %in recession


temp_RE = ALL_S_shortrates.*ALL_S_dumexp;        %expansion returns
temp_RE2 = sum(temp_RE);
ALL_X_avg_short_exp = mean(temp_RE2./ALL_V_count_exp);     %in expansion


%Stock and Real Estate
%average Stock and Real Estate return per month (in percentages)
ALL_V_avg_S = mean(ALL_S_Stock);                 %vector average per run
ALL_X_avg_S = mean(ALL_V_avg_S);                 %overal average
```

```
ALL_V_std_S = std(ALL_S_Stock);        %vector st dev per run
ALL_X_std_S = mean(ALL_V_std_S);       %average st dev

ALL_V_avg_RE = mean(ALL_S_RE);                  %vector average per run
ALL_X_avg_RE = mean(ALL_V_avg_RE);              %overal average

ALL_V_std_RE = std(ALL_S_RE);          %vector st dev per run
ALL_X_std_RE = mean(ALL_V_std_RE);     %average st dev

%average Stock and Real Estate return in recessions (percentages)
temp_RE = ALL_S_Stock.*ALL_S_dumrec;
temp_RE2 = sum(temp_RE);
ALL_X_avg_S_rec = mean(temp_RE2./ALL_V_count_rec);    %in recession

temp_RE = ALL_S_Stock.*ALL_S_dumexp;
temp_RE2 = sum(temp_RE);
ALL_X_avg_S_exp = mean(temp_RE2./ALL_V_count_exp);    %in expansion

temp_RE = ALL_S_RE.*ALL_S_dumrec;
temp_RE2 = sum(temp_RE);
ALL_X_avg_RE_rec = mean(temp_RE2./ALL_V_count_rec);    %in recession

temp_RE = ALL_S_RE.*ALL_S_dumexp;
temp_RE2 = sum(temp_RE);
ALL_X_avg_RE_exp = mean(temp_RE2./ALL_V_count_exp);    %in expansion

%inflation
ALL_V_avg_infl = mean(ALL_S_Inflation);      %vector average per run
ALL_X_avg_infl = mean(ALL_V_avg_infl);       %overal average

ALL_V_std_infl = std(ALL_S_Inflation);       %vector st dev per run
ALL_X_std_infl = mean(ALL_V_std_infl);       %average st dev

temp_RE = ALL_S_Inflation.*ALL_S_dumrec;
temp_RE2 = sum(temp_RE);
ALL_X_avg_infl_rec = mean(temp_RE2./ALL_V_count_rec);    %in recession

temp_RE = ALL_S_Inflation.*ALL_S_dumexp;
temp_RE2 = sum(temp_RE);
ALL_X_avg_infl_exp = mean(temp_RE2./ALL_V_count_exp);    %in expansion
```

```
%percentilegraphs
ALL_Q_InflCPI = zeros(T-1,4);
temp_RE = sortrows(ALL_S_InflCPI(1:T-1,:)',T-1);
temp_RE = temp_RE';
ALL_Q_InflCPI(:,1) = temp_RE(:,floor(0.05*Nsim));
ALL_Q_InflCPI(:,2) = temp_RE(:,floor(0.5*Nsim));
ALL_Q_InflCPI(:,3) = temp_RE(:,floor(0.95*Nsim));
ALL_Q_InflCPI(:,4) = mean(temp_RE,2);

ALL_Q_RE = zeros(T-1,4);
temp_RE = sortrows(ALL_S_REZ(1:T-1,:)',T-1);
temp_RE = temp_RE';
ALL_Q_RE(:,1) = temp_RE(:,floor(0.05*Nsim));
ALL_Q_RE(:,2) = temp_RE(:,floor(0.5*Nsim));
ALL_Q_RE(:,3) = temp_RE(:,floor(0.95*Nsim));
ALL_Q_RE(:,4) = mean(temp_RE,2);

ALL_Q_Salary = zeros(T-1,4);
temp_RE = sortrows(ALL_S_Salary(1:T-1,:)',T-1);
temp_RE = temp_RE';
ALL_Q_Salary(:,1) = temp_RE(:,floor(0.05*Nsim));
ALL_Q_Salary(:,2) = temp_RE(:,floor(0.5*Nsim));
ALL_Q_Salary(:,3) = temp_RE(:,floor(0.95*Nsim));
ALL_Q_Salary(:,4) = mean(temp_RE,2);

% ALL_Q_shortrates = zeros(T-1,4);
% temp_RE = sortrows(ALL_S_shortrates(1:T-1,:)',T-1);
% temp_RE = temp_RE';
% ALL_Q_shortrates(:,1) = temp_RE(:,floor(0.05*Nsim));
% ALL_Q_shortrates(:,2) = temp_RE(:,floor(0.5*Nsim));
% ALL_Q_shortrates(:,3) = temp_RE(:,floor(0.95*Nsim));
% ALL_Q_shortrates(:,4) = mean(temp_RE,2);

ALL_Q_Stock = zeros(T-1,4);
temp_RE = sortrows(ALL_S_StockZ(1:T-1,:)',T-1);
temp_RE = temp_RE';
ALL_Q_Stock(:,1) = temp_RE(:,floor(0.05*Nsim));
ALL_Q_Stock(:,2) = temp_RE(:,floor(0.5*Nsim));
ALL_Q_Stock(:,3) = temp_RE(:,floor(0.95*Nsim));
```

```
ALL_Q_Stock(:,4) = mean(temp_RE,2);
```

8.3.14 StatisticsLC

```
%%Lifecycle Statistics
%expected utility, mean, variance, sharpe ratio,
%max, top 5%, top 25% ,median, bottom 25% ,bottom 5%, min,
%Expected shortfall 5%, chance income above 75% average, above 70% final

%initialize
A_LC_Ac = zeros(14,1);
A_LC_Am = zeros(14,1);
A_LC_Bo = zeros(14,1);
A_LC_Eq = zeros(14,1);
A_LC_Ex = zeros(14,1);
A_LC_M = zeros(14,1);
A_LC_Mer = zeros(14,1);
A_LC_RE = zeros(14,1);
A_LC_REBo = zeros(14,1);
A_LC_St = zeros(14,1);
A_LC_StBo = zeros(14,1);
A_LC_StRE = zeros(14,1);
A_LC_Mp = zeros(14,1);

%expected utility
A_LC_Ac(1) = mean(((ALL_LC_m_Ac/1000).^(1-5)-1)/(1-5));
A_LC_Am(1) = mean(((ALL_LC_m_Am/1000).^(1-5)-1)/(1-5));
A_LC_Bo(1) = mean(((ALL_LC_m_Bo/1000).^(1-5)-1)/(1-5));
A_LC_Eq(1) = mean(((ALL_LC_m_Eq/1000).^(1-5)-1)/(1-5));
A_LC_Ex(1) = mean(((ALL_LC_m_Ex/1000).^(1-5)-1)/(1-5));
A_LC_M(1) = mean(((ALL_LC_m_M/1000).^(1-5)-1)/(1-5));
A_LC_Mer(1) = mean(((ALL_LC_m_Mer/1000).^(1-5)-1)/(1-5));
A_LC_RE(1) = mean(((ALL_LC_m_RE/1000).^(1-5)-1)/(1-5));
A_LC_REBo(1) = mean(((ALL_LC_m_REBo/1000).^(1-5)-1)/(1-5));
A_LC_St(1) = mean(((ALL_LC_m_St/1000).^(1-5)-1)/(1-5));
A_LC_StBo(1) = mean(((ALL_LC_m_StBo/1000).^(1-5)-1)/(1-5));
A_LC_StRE(1) = mean(((ALL_LC_m_StRE/1000).^(1-5)-1)/(1-5));
A_LC_Mp(1) = mean(((ALL_LC_m_Mp/1000).^(1-5)-1)/(1-5));


%mean
```

```
A_LC_Ac(2) = mean(ALL_LC_m_Ac);
A_LC_Am(2) = mean(ALL_LC_m_Am);
A_LC_Bo(2) = mean(ALL_LC_m_Bo);
A_LC_Eq(2) = mean(ALL_LC_m_Eq);
A_LC_Ex(2) = mean(ALL_LC_m_Ex);
A_LC_M(2) = mean(ALL_LC_m_M);
A_LC_Mer(2) = mean(ALL_LC_m_Mer);
A_LC_RE(2) = mean(ALL_LC_m_RE);
A_LC_REBo(2) = mean(ALL_LC_m_REBo);
A_LC_St(2) = mean(ALL_LC_m_St);
A_LC_StBo(2) = mean(ALL_LC_m_StBo);
A_LC_StRE(2) = mean(ALL_LC_m_StRE);
A_LC_Mp(2) = mean(ALL_LC_m_Mp);

%st dev
A_LC_Ac(3) = std(ALL_LC_m_Ac);
A_LC_Am(3) = std(ALL_LC_m_Am);
A_LC_Bo(3) = std(ALL_LC_m_Bo);
A_LC_Eq(3) = std(ALL_LC_m_Eq);
A_LC_Ex(3) = std(ALL_LC_m_Ex);
A_LC_M(3) = std(ALL_LC_m_M);
A_LC_Mer(3) = std(ALL_LC_m_Mer);
A_LC_RE(3) = std(ALL_LC_m_RE);
A_LC_REBo(3) = std(ALL_LC_m_REBo);
A_LC_St(3) = std(ALL_LC_m_St);
A_LC_StBo(3) = std(ALL_LC_m_StBo);
A_LC_StRE(3) = std(ALL_LC_m_StRE);
A_LC_Mp(3) = std(ALL_LC_m_Mp);

%sharpe ratio
A_LC_Ac(4) = A_LC_Ac(2)/A_LC_Ac(3);
A_LC_Am(4) = A_LC_Am(2)/A_LC_Am(3);
A_LC_Bo(4) = A_LC_Bo(2)/A_LC_Bo(3);
A_LC_Eq(4) = A_LC_Eq(2)/A_LC_Eq(3);
A_LC_Ex(4) = A_LC_Ex(2)/A_LC_Ex(3);
A_LC_M(4) = A_LC_M(2)/A_LC_M(3);
A_LC_Mer(4) = A_LC_Mer(2)/A_LC_Mer(3);
A_LC_RE(4) = A_LC_RE(2)/A_LC_RE(3);
A_LC_REBo(4) = A_LC_REBo(2)/A_LC_REBo(3);
A_LC_St(4) = A_LC_St(2)/A_LC_St(3);
A_LC_StBo(4) = A_LC_StBo(2)/A_LC_StBo(3);
```

```
A_LC_StRE(4) = A_LC_StRE(2)/A_LC_StRE(3);
A_LC_Mp(4) = A_LC_Mp(2)/A_LC_Mp(3);

%max
A_LC_Ac(5) = max(ALL_LC_m_Ac);
A_LC_Am(5) = max(ALL_LC_m_Am);
A_LC_Bo(5) = max(ALL_LC_m_Bo);
A_LC_Eq(5) = max(ALL_LC_m_Eq);
A_LC_Ex(5) = max(ALL_LC_m_Ex);
A_LC_M(5) = max(ALL_LC_m_M);
A_LC_Mer(5) = max(ALL_LC_m_Mer);
A_LC_RE(5) = max(ALL_LC_m_RE);
A_LC_REBo(5) = max(ALL_LC_m_REBo);
A_LC_St(5) = max(ALL_LC_m_St);
A_LC_StBo(5) = max(ALL_LC_m_StBo);
A_LC_StRE(5) = max(ALL_LC_m_StRE);
A_LC_Mp(5) = max(ALL_LC_m_Mp);

%top 5%, %bottom 5%,
temp_RE = sort(ALL_LC_m_Ac);
A_LC_Ac(6) = temp_RE(0.95*Nsim);
A_LC_Ac(7) = temp_RE(0.75*Nsim);
A_LC_Ac(9) = temp_RE(0.25*Nsim);
A_LC_Ac(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = temp_RE';
temp_RE = sort(ALL_LC_m_Am);
A_LC_Am(6) = temp_RE(0.95*Nsim);
A_LC_Am(7) = temp_RE(0.75*Nsim);
A_LC_Am(9) = temp_RE(0.25*Nsim);
A_LC_Am(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_Bo);
A_LC_Bo(6) = temp_RE(0.95*Nsim);
A_LC_Bo(7) = temp_RE(0.75*Nsim);
A_LC_Bo(9) = temp_RE(0.25*Nsim);
A_LC_Bo(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_Eq);
A_LC_Eq(6) = temp_RE(0.95*Nsim);
A_LC_Eq(7) = temp_RE(0.75*Nsim);
A_LC_Eq(9) = temp_RE(0.25*Nsim);
```

```
A_LC_Eq(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_Ex);
A_LC_Ex(6) = temp_RE(0.95*Nsim);
A_LC_Ex(7) = temp_RE(0.75*Nsim);
A_LC_Ex(9) = temp_RE(0.25*Nsim);
A_LC_Ex(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_M);
A_LC_M(6) = temp_RE(0.95*Nsim);
A_LC_M(7) = temp_RE(0.75*Nsim);
A_LC_M(9) = temp_RE(0.25*Nsim);
A_LC_M(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_Mer);
A_LC_Mer(6) = temp_RE(0.95*Nsim);
A_LC_Mer(7) = temp_RE(0.75*Nsim);
A_LC_Mer(9) = temp_RE(0.25*Nsim);
A_LC_Mer(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_RE);
A_LC_RE(6) = temp_RE(0.95*Nsim);
A_LC_RE(7) = temp_RE(0.75*Nsim);
A_LC_RE(9) = temp_RE(0.25*Nsim);
A_LC_RE(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_REBo);
A_LC_REBo(6) = temp_RE(0.95*Nsim);
A_LC_REBo(7) = temp_RE(0.75*Nsim);
A_LC_REBo(9) = temp_RE(0.25*Nsim);
A_LC_REBo(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_St);
A_LC_St(6) = temp_RE(0.95*Nsim);
A_LC_St(7) = temp_RE(0.75*Nsim);
A_LC_St(9) = temp_RE(0.25*Nsim);
A_LC_St(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_StBo);
A_LC_StBo(6) = temp_RE(0.95*Nsim);
A_LC_StBo(7) = temp_RE(0.75*Nsim);
```

```
A_LC_StBo(9) = temp_RE(0.25*Nsim);
A_LC_StBo(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_StRE);
A_LC_StRE(6) = temp_RE(0.95*Nsim);
A_LC_StRE(7) = temp_RE(0.75*Nsim);
A_LC_StRE(9) = temp_RE(0.25*Nsim);
A_LC_StRE(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];
temp_RE = sort(ALL_LC_m_Mp);
A_LC_Mp(6) = temp_RE(0.95*Nsim);
A_LC_Mp(7) = temp_RE(0.75*Nsim);
A_LC_Mp(9) = temp_RE(0.25*Nsim);
A_LC_Mp(10) = temp_RE(0.05*Nsim);
A_LC__Sorted = [A_LC__Sorted temp_RE'];

%median,
A_LC_Ac(8) = median(ALL_LC_m_Ac);
A_LC_Am(8) = median(ALL_LC_m_Am);
A_LC_Bo(8) = median(ALL_LC_m_Bo);
A_LC_Eq(8) = median(ALL_LC_m_Eq);
A_LC_Ex(8) = median(ALL_LC_m_Ex);
A_LC_M(8) = median(ALL_LC_m_M);
A_LC_Mer(8) = median(ALL_LC_m_Mer);
A_LC_RE(8) = median(ALL_LC_m_RE);
A_LC_REBo(8) = median(ALL_LC_m_REBo);
A_LC_St(8) = median(ALL_LC_m_St);
A_LC_StBo(8) = median(ALL_LC_m_StBo);
A_LC_StRE(8) = median(ALL_LC_m_StRE);
A_LC_Mp(8) = median(ALL_LC_m_Mp);

%min,
A_LC_Ac(11) = min(ALL_LC_m_Ac);
A_LC_Am(11) = min(ALL_LC_m_Am);
A_LC_Bo(11) = min(ALL_LC_m_Bo);
A_LC_Eq(11) = min(ALL_LC_m_Eq);
A_LC_Ex(11) = min(ALL_LC_m_Ex);
A_LC_M(11) = min(ALL_LC_m_M);
A_LC_Mer(11) = min(ALL_LC_m_Mer);
A_LC_RE(11) = min(ALL_LC_m_RE);
A_LC_REBo(11) = min(ALL_LC_m_REBo);
```

```
A_LC_St(11) = min(ALL_LC_m_St);
A_LC_StBo(11) = min(ALL_LC_m_StBo);
A_LC_StRE(11) = min(ALL_LC_m_StRE);
A_LC_Mp(11) = min(ALL_LC_m_Mp);

%Expected shortfall 5%,
temp_RE = mean(A_LC__Sorted(1:(0.05*Nsim),:));
A_LC_Ac(12) = temp_RE(1);
A_LC_Am(12) = temp_RE(2);
A_LC_Bo(12) = temp_RE(3);
A_LC_Eq(12) = temp_RE(4);
A_LC_Ex(12) = temp_RE(5);
A_LC_M(12) = temp_RE(6);
A_LC_Mer(12) = temp_RE(7);
A_LC_RE(12) = temp_RE(8);
A_LC_REBo(12) = temp_RE(9);
A_LC_St(12) = temp_RE(10);
A_LC_StBo(12) = temp_RE(11);
A_LC_StRE(12) = temp_RE(12);
A_LC_Mp(12) = temp_RE(13);

%chance income above 75% average,
temp_RE = sum(ALL_LC_mn_Ac>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_Ac(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Am>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_Am(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Bo>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_Bo(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Eq>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_Eq(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Ex>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_Ex(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_M>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_M(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Mer>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_Mer(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_RE>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_RE(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_REBo>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_REBo(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_St>0.75*(mean(ALL_S_Salary(1:T-1,:))));
```

```
A_LC_St(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_StBo>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_StBo(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_StRE>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_StRE(13) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Mp>0.75*(mean(ALL_S_Salary(1:T-1,:))));
A_LC_Mp(13) = temp_RE*100/Nsim;

%above 70% final
temp_RE = sum(ALL_LC_mn_Ac>0.7*ALL_S_Salary(T-1,:));
A_LC_Ac(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Am>0.7*ALL_S_Salary(T-1,:));
A_LC_Am(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Bo>0.7*ALL_S_Salary(T-1,:));
A_LC_Bo(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Eq>0.7*ALL_S_Salary(T-1,:));
A_LC_Eq(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Ex>0.7*ALL_S_Salary(T-1,:));
A_LC_Ex(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_M>0.7*ALL_S_Salary(T-1,:));
A_LC_M(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Mer>0.7*ALL_S_Salary(T-1,:));
A_LC_Mer(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_RE>0.7*ALL_S_Salary(T-1,:));
A_LC_RE(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_REBo>0.7*ALL_S_Salary(T-1,:));
A_LC_REBo(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_St>0.7*ALL_S_Salary(T-1,:));
A_LC_St(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_StBo>0.7*ALL_S_Salary(T-1,:));
A_LC_StBo(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_StRE>0.7*ALL_S_Salary(T-1,:));
A_LC_StRE(14) = temp_RE*100/Nsim;
temp_RE = sum(ALL_LC_mn_Mp>0.7*ALL_S_Salary(T-1,:));
A_LC_Mp(14) = temp_RE*100/Nsim;

%summary
A_LC__Summary = [A_LC_Ac A_LC_Am A_LC_Bo A_LC_Eq A_LC_Ex A_LC_M ...
A_LC_Mer A_LC_RE A_LC_REBo A_LC_St A_LC_StBo A_LC_StRE A_LC_Mp];
A_LC__Sumnames = {'EU' ; 'mean'; 'std'; 'SR'; 'max'; '95'; '75'; ...
'median'; '25'; '5'; 'min'; 'ES5'; '0.75avg'; '0.7end'};
```

```
A_LC__Summary2 = dataset(A_LC__Sumnames, A_LC__Summary);
```

```
boxplot(A_LC__Sorted(1:950,:),'Whisker',5)
```