Effect of market liquidity on distressed debt hedge funds

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Abstract

In this paper I investigate the returns of distressed debt hedge funds. Historically, these types of investors managed to outperform the market by a significant margin. I examine if traditional risk factors combined with a liquidity risk factor are able to capture the common variation in the returns of the distressed debt hedge funds. A liquidity proxy is constructed by a Nelson-Siegel model with time-varying loading parameter and volatility. The time-varying volatility is modeled by a GARCH process and is used as a proxy for the liquidity in the market. Using the Fama-Macbeth procedure, I then construct a liquidity factor which is being used in the risk factor model. I find that there is significant evidence that the liquidity factor is able to explain part of the variation in the returns of distressed debt hedge funds. Moreover, the factors altogether explain a significant part of the common variation in the returns of distressed debt hedge funds.

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1 Introduction

The hedge fund industry has suffered since the financial crisis. Many of these funds use sophisticated strategies that are lowly correlated with overall financial market dynamics, but still encountered huge losses during the financial crisis. However, some hedge funds still managed to generate satisfying returns. An important subset of these hedge funds are distressed debt funds who, as their name implies, invest in companies in distress, often in the form of (high-yield) debt and loans. Altman (2014) estimates that there are, today, more than 200 financial institutions investing between $400-450 billion in the distressed debt market in the U.S. and a substantial number and amount operating in Europe and in other markets. Empirical return data shows that these particular funds have consistently outperformed the general stock market. Jiang, Li & Wang (2012) studied the effect of hedge funds that get involved in the Chapter 11 bankruptcy process. They find that hedge funds balances the power between the debtor and secured creditors. This effect manifests in higher probabilities of bankruptcy emergence and payoff to junior claims.

The objective of my research is to identify potential risk factors that are able to explain the returns accomplished by these specific hedge funds. The investment strategy of distressed debt hedge funds is by definition value-oriented. It depends on assessing the true value of a company's assets vs. liabilities and invest if there exists an opportunity where the market price does not reflect the underlying value. Therefore I suspect that distressed debt hedge fund returns are subject to value and market factors. Fama & French (1993) constructed a 3-factor model with a market, size and value factor. Carhart (1997) extended the model with a momentum factor. I will use this 4-factor model to see if it can explain the returns of distressed debt hedge funds.

The aforementioned 4-factor model did especially well in explaining the returns of mutual funds. However, hedge funds use more unconventional investment styles such as long-short positions (hedging out market risk), merger arbitrage (not size-or value based) et cetera. Hence, it seems likely that the returns of hedge funds are also prone to other risk factors. Fung & Hsieh (2004) already conducted research to explaining returns of hedge funds. They came up with a 7-factor model (or 8-factor if an emerging market risk factor is included) that performed well in explaining hedge fund returns. Moreover, they investigated if traditional common risk factors were able to explain hedge fund returns but concluded that they did a far worse job than their 7-factor model. However, it must be noted that the 7-factor model did well in explaining returns of multi-strategy hedge funds, so not distressed debt hedge funds in particular.

Distressed debt funds are subject to strategy-specific risks that impact their returns. In his book, David Moyer (2005) names three key factors driving the excess returns of distressed debt investing:

(i) Information asymmetry: Hedge funds often have extensive specialised research units that are likely to have more experience / knowledge / information in situations when a company defaults than other parties. When the general public is unaware of the value hidden in some of the distressed companies, hedge funds who were able to locate this value can make an investment at a highly discounted price.

(ii) Forced selling: Many traditional asset managers are contractually obliged to exit their positions in a company when the company becomes distressed. Especially large institutional managers are constrained from owning distressed securities. Hence, in distressed situations there often occurs a forced sell-off, driving down the security prices and the potential returns for the distressed investors up.

(iii) Liquidity: Distressed debt is highly illiquid, especially during the bankruptcy process. There are not many buyers available who wish to buy bonds of a company that is in default or is likely to default. Hence, if an investor in a distressed asset wishes to sell its stake, it faces a severe liquidity risk. It is often the case that the investor himself is involved in the reorganisation process. He then can only sell his stake if he lets the counterparty know that he is involved in the process and has access to non-public information. This makes it even more difficult to sell for the investor.

The third factor can be seen as a risk factor; liquidity risk. Liquidity risk forms an integral part of asset pricing literature in both the bond as the equity space. For equity markets, empirical evidence on priced liquidity risk is provided by, e.g., Brennan and Subrahmanyam (1996), Haugen and Baker (1996), Brennan et al. (1998), Chordia et al. (2001) and Amihud and Mendelson (1986). Pástor & Stambaugh (2003) found that expected stock returns are related cross-sectionally to the sensitivities of returns to fluctuations in aggregate liquidity. For the bond markets, most research has been done on U.S. treasury bonds as price data is easily available and bonds are issued on a regular basis. Hu, Pan & Wang (2013) constructed the 'Noise' measure to proxy liquidity risk in U.S. treasury bonds. They found that their proxy captures liquidity crises and using it as a priced risk factor, it helps explaining cross-sectional returns of hedge funds. Hence, there clearly exists sufficient evidence to believe that a liquidity risk factor is present in the equity and bond markets. It seems very reasonable to believe that there exists a liquidity factor in the distressed debt space as well, as it is an extremely illiquid environment.

Therefore I want to extend the Fung & Hsieh factors with a liquidity risk factor. As aforementioned, Hu, Pan & Wang constructed a liquidity proxy and factor. Their Noise measure is a simple but effective way of measuring market liquidity in the U.S. treasury bond market. In this paper, I will use a more sophisticated approach to construct a liquidity proxy and apply it to the high-yield bond market as this is the investment space of many distressed debt hedge funds. There is no research as of yet that investigates which risk factors distressed debt hedge funds are exposed to, so this research contributes to the existing literature by examining these risk factors.

First, I use a Nelson-Siegel model with time-varying factor loading and volatility as proposed by Koopman, Mallee & van der Wel (2010) to construct a proxy for liquidity. This model extends the traditional Nelson-Siegel model (1987) by having a time-varying loading parameter and a GARCH process. Hu, Pan & Wang constructed a liquidity measure by squaring the difference between the theoretical yield and the observed yield. This approach might be too rigorous as other noise that could influence the deviation is also measured as liquidity. Therefore I implement a more subtle method by assuming that all yields are prone to a common liquidity component with a time-varying volatility (modeled by a GARCH process). This GARCH volatility can then be viewed as the magnitude of the liquidity in the market at each specific time-point. In this way, this practice enhances a more subtle way in approximating market liquidity as the common liquidity component filters out most of the noise which is being captured by the error term.

Distressed debt is non investment-grade (their rating is below BBB- as by Standard's & Poors and Fitch or below Ba1 as by Moody's). To assemble a liquidity proxy that is representative for the instruments that distressed debt firms invest in I will look at the yields of non investment-grade bonds, also known as high-yield bonds.

To then transform the liquidity proxy into a priced risk factor I will use the well-known Fama-Macbeth procedure (1973). By following a portfolio sorting approach based on the sensitivity of the bonds to the liquidity proxy, it is possible to construct a priced liquidity risk factor. This risk factor can then be used in my 8-factor model (7-factor model of Fung & Hsieh plus the liquidity risk factor).

Secondly, I investigate if the returns of the distressed debt hedge funds can be explained by this 8-factor model. To do this, I use two different return benchmarks. The first benchmark is the Event Driven Distressed hedge fund index from Credit Suisse and the other is a well-weighted distressed debt hedge fund index which is constructed by EurekaHedge.

I find that a liquidity proxy can be created from the high-yield bond data, and that its shape makes economically sense as well. For instance, the liquidity proxy tends to increase during financial crises such as the dot-com bubble or the more recent financial crisis. Moreover, when I form a liquidity factor using the Fama-Macbeth procedure I find significant statistical evidence that this liquidity factor can explain common variation in distressed debt hedge fund returns. Given that the coefficient sign for the liquidity factor is positive implies that when the high-yield bond market become less liquid, this enhances the returns of distressed debt hedge funds.

The rest of the paper is build up as follows; section 2 describes the data, section 3 the method to extract a liquidity premium and section 4 shows how to explain the distressed debt hedge fund returns. In section 5 I show my findings and the rationale behind them and the conclusion follows in section 6.

2 Data

For my research I use three different types of data. To extract a liquidity premium I will use a high yield bond index from Bank of America Merrill Lynch. Furthermore, I will use the common risk factors from Fama & French (market, value, size and momentum) and Fung & Hsieh. Finally, individual fund returns and a distressed debt fund index will be used as a proxy for the returns generated by distressed debt hedge funds. Hedge funds' popularity increased critically after the dot-com bubble as investors wanted to reduce their exposure anymore to the market. Therefore, the period I will consider for my research spans from January 2000 to July 2016 on a monthly basis. Below I will describe each dataset in more detail.

2.1 Bank of America Merrill Lynch High Yield Bond Index

The Bank of America Merrill Lynch U.S. High Yield Bond Index (BoAML HYBI) consists of a large amount of non-investment grade bonds from U.S. companies. To qualify for inclusion in the index, securities must have a below investment grade rating (based on an average of Moody's, S&P, and Fitch) and an investment grade rated country of risk (based on an average of Moody's, S&P, and Fitch foreign currency long term sovereign debt ratings). Each security must have greater than 1 year of remaining maturity, a fixed coupon schedule, and a minimum amount outstanding of $100 million. Original issue zero coupon bonds, "global" securities (debt issued simultaneously in the eurobond and US domestic bond markets), 144a securities and pay-in-kind securities, including toggle notes, qualify for inclusion in the Index. Callable perpetual securities qualify provided they are at least one year from the first call date. Fixed-to-floating rate securities also qualify provided they are callable within the fixed rate period and are at least one year from the last call prior to the date the bond transitions from a fixed to a floating rate security. DRD-eligible and defaulted securities are excluded from the Index. The following table contains descriptive statistics of the BoAML data from January 2000 to July 2016:

Table 1: BoAML HYBI descriptive stats (1/2000 - 7/2016)

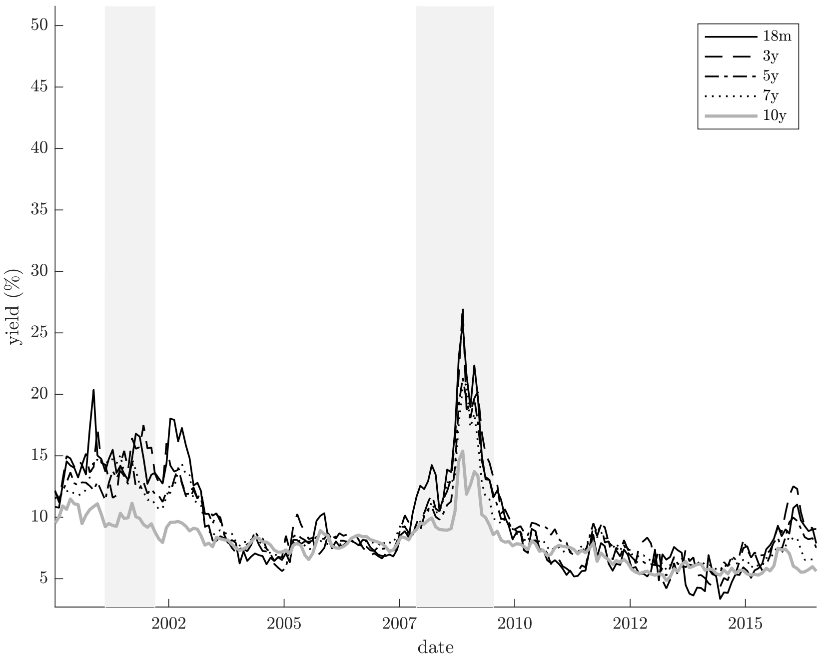
	Mean	Median	Min.	Max.
Amount of issuers	1229	1240	1138	1301
Amount of bonds	2200	2232	2003	2335
Yield (%)	7.88	6.07	-9.21	1854
Price (base: 100)	100.8	103.8	0.01	157.0
Coupon (%)	7.72	7.63	0.00	17.0
Maturity (years)	6.61	5.93	1.00	86.3

Binning procedure

The composition of the bonds in the BoAML HYBI varies each month. Bonds are added to the index and bonds that do not meet the criteria (i.e. their rating changed to IG or maturity < 1 year) anymore are removed from the index. However, this gives rise to a complication as the Nelson-Siegel model only allows for a fixed number of maturities over the sample period. A solution for this is to 'group' all bonds in a maturity bin. There are 18 bins in total, ranging from 18 to 120 months to maturity, divided in half-yearly steps (6 months each). For every month, each bond in the index is put in the corresponding maturity bin. The weighted mean (based on market cap) over all the bond yields in each bin is then the approximation. The yields are plotted in the graph below:

Figure 1: BoAML HYBI: Selection of maturity bins

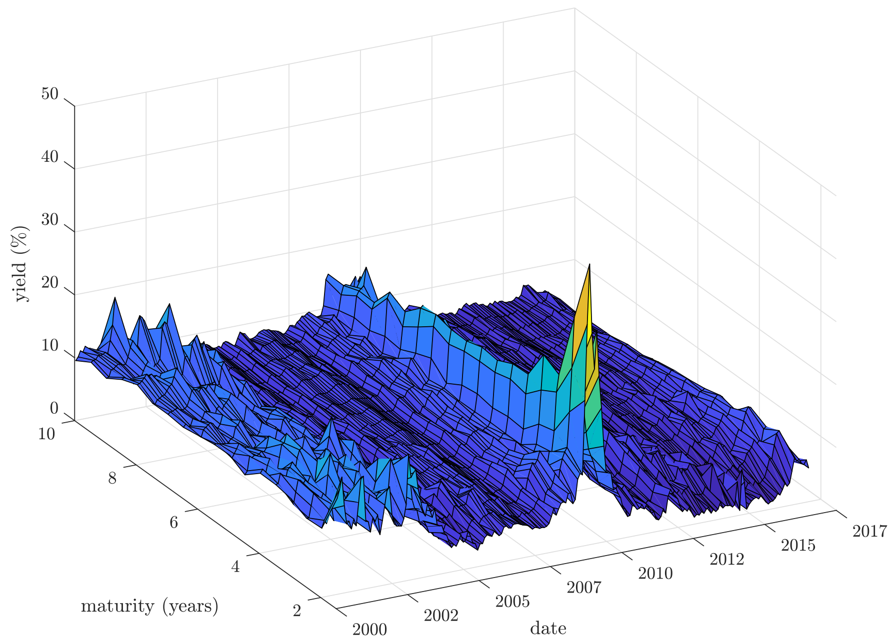
This figure shows the yield curves of a selection of maturities. The gray shaded areas correspond to recessions as identified by the NBER, respectively the dot-com bubble and the financial crisis.



In some months, some bins may be empty as no feasible bonds are available at that time. Although this problem has minimal impact on the estimation uncertainty (missing bins consist of $< 0.1\%$ of all bins) I have accounted for it by using interpolation. The estimated yield for each empty bin is the average of the first available upper and lower bin. An overview of the sample statistics of each maturity can be found in Appendix A.

Figure 2: BoAML HYBI: 3-D plot

This 3-D plot displays the cross-section of yield and maturities over the sample period.



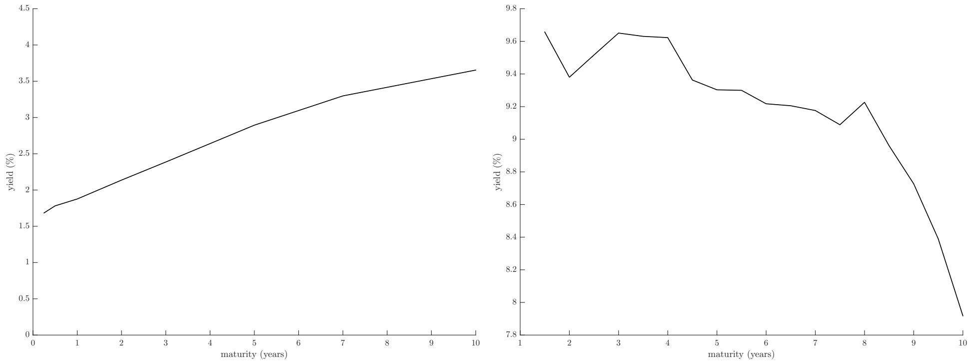
Principal component analysis (PCA)

The term structure of treasury yields can be well explained by three common factors: level, slope and curvature. The factor loadings of these factors correspond to the first three principal components of the treasury yields. These can be interpreted as level, slope and curvature factor loadings. The level factor loading is flat for all maturities, the slope factor loading is high for short maturities and low for long maturities and the curvature factor loading is low for both short and long maturities and high for medium maturities. The three factors normally explain around 95-99% of the variance of the yield curve. The shape of the principal components form the basis of the factor loading of the Nelson-Siegel model, which is more extensively described in the next section. I will apply PCA as described by Abdi & Williams (2010) to the BoAML High-Yield bond index to see if the same factor loadings are present in the dataset.

When comparing the average yield curve of the BoAML High Yield Bond index with the treasury yield curve as in Figure 3, one can easily observe the differences in shape. It is therefore necessary to check if the same factors are still shaping the high yield curve, and see if the Nelson-Siegel model can still be applied. I did a principal component analysis on the BoAML yields from which the first three components are displayed in Figure 4. These first

Figure 3: Average yield curves (1/2000 - 12/2016)

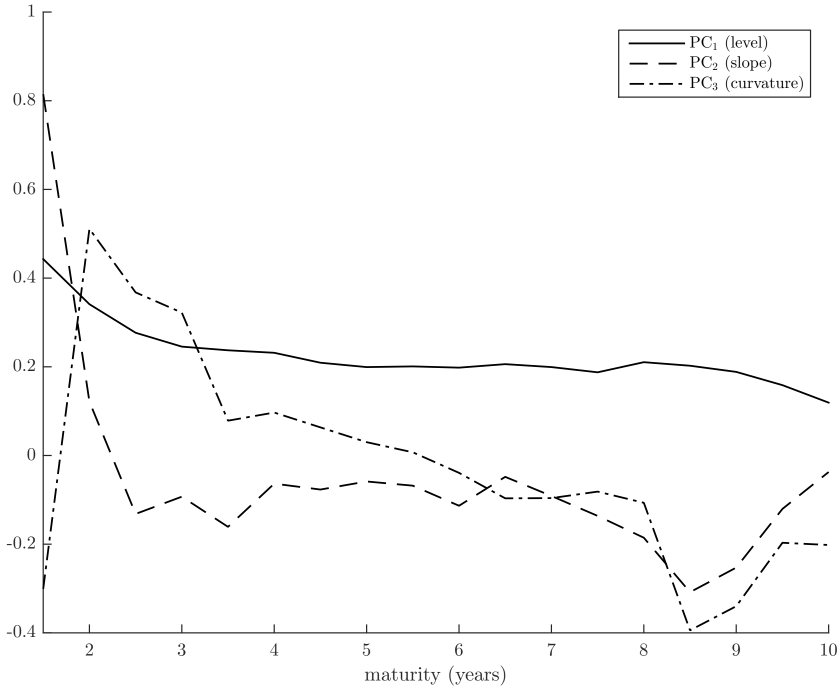
(a) U.S. Treasury yield curve **(b) BoAML high-yield curve**



three components explain 94.3% of the total variance. Moreover, the shape of the components is similar to the common level, slope and curvature factor loadings. Hence, there is sufficient evidence that the level, slope and curvature factors explain most of the variation in the yield curve and therefore the Nelson-Siegel model can be used for estimating the BoAML high yield bond yields.

Figure 4: Principal components

The figure displays the first three principal components of the BoAML high-yield bond index. The components altogether explain 94.3% of the total variance. The first component (capturing 88.5% of the variance) is a relatively flat line and can be interpreted as the level factor loading. The second component (capturing 4.2%) is high for short maturities and low for long maturities and corresponds to the slope factor loading. The third component (capturing 1.6%) reaches its peak at medium-term maturity and therefore can be seen as the curvature factor loading.



2.2 Risk factors

In this part I will describe the risk factors I will use to explain the returns of the distressed debt funds. Fama & French constructed a set of common risk factors that have significant explanatory power in the equity space. However, as the funds I am targeting differ substantially from the common equity market, I will also look at the hedge fund risk factors that Fung & Hsieh created. Below I will elaborate deeper on the different risk factors.

Fama & French common risk factors

Fama & French (1993) first identified additional common risk factors[1] that helped explain asset returns next to the market return factor alone as assumed in the simple CAPM model. These factors include the size and value which could be measured by firm size (market cap) and book-to-market ratio, respectively. The market factor is defined as a well-weighted portfolio of stocks that represents the stock market.

Carhart (1997) extended the 3-factor model of Fama & French with a new factor, momentum[1]. Momentum basically implies that today's winners are also today's winners or in mathematical notation: $\mathrm{E}[r_t|r_{t-1} > 0] > 0, \mathrm{E}[r_t|r_{t-1} < 0] < 0$ for a return time-series r_t.

I will apply this 4-factor model on the distressed debt hedge funds returns to see if they can help explain returns. However, it must be noted that these funds implement strategies that are unconventional compared to traditional mutual funds. Therefore, other factors might be more helpful such as the hedge fund risk factors designed by Fung & Hsieh:

Fung & Hsieh hedge fund risk factors

Fung & Hsieh have dedicated much research on hedge funds. In their paper "Hedge Fund Benchmarks: A Risk-Based Approach" they find that the traditional value, size and market risk factors do a poor job in explaining hedge fund returns. This is because evidence shows that hedge fund returns have different characteristics than those of traditional funds. For example, Fung & Hsieh (1997) found that hedge fund returns have a much lower correlation to standard asset returns than mutual fund returns. An explanation could be that hedge funds are exposed to risks that are different from mutual fund risks. Therefore, Fung & Hsieh came

[1] The Fama & French risk factors including the momentum factor can be downloaded via `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data%5Flibrary.html`

up with a set benchmarks that capture the common risk factors in hedge funds.[2] They used three hedge-fund return indices as industry benchmarks and found that the factors can explain a significant part of the common variation of the hedge fund returns. The R^2s from their factor regression ranged from 0.48 to 0.84.

Seven risk factors have been identified. Equity long / short hedge funds are exposed to two equity risk factors. Fixed income hedge funds are exposed to two interest rate-related risk factors. Trend-following hedge funds are exposed to three portfolios of options. Empirical evidence shows that these seven risk factors jointly capture the majority of the variation in hedge fund returns. Each risk factor is defined as follows:

Trend-Following Factors

Fung & Hsieh construct three trend-following risk factors using lookback straddle models. These factors tend to be uncorrelated with the standard equity, bond, commodity and currency indices. The trend-following risk factors are *Bond Trend-Following Factor (BFF)*, *Commodity Trend-Following Factor (CoFF)* and *Currency Trend-Following Factor (CuFF).*

Equity Risk Factors

Equity Market Factor (SPTRI): The Standard & Poors 500 index monthly total return˙ *Size Spread Factor (RMS)*: Russell 2000 index monthly total return - Standard & Poors 500 monthly total return.

Interest-Related Risk Factors

Bond Market Factor (BMF): The monthly change in the 10-year treasury constant maturity yield (month end-to-month end).

Credit Spread Factor (CSF): The monthly change in the Moody's Baa yield less 10-year treasury constant maturity yield (month end-to-month end).

2.3 Hedge fund returns

For the returns generated by distressed debt hedge funds, I will look at two datasets: (i) the EurekaHedge Distressed Debt index[3] and (2) Credit Suisse Event Driven distressed debt hedge fund index[4]. The reason for using two sets of data is to see if the explanatory power of the

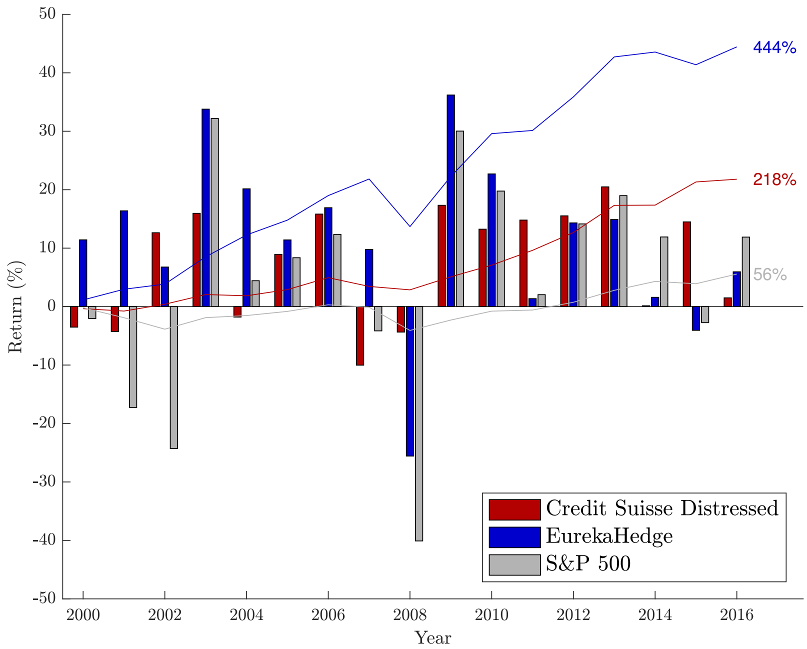
[2] The hedge fund factors constructed by Fung & Hsieh can be downloaded via `https://faculty.fuqua.duke.edu/~dah7/HFRFData`

[3] The EurekaHedge index returns are available via `http://www.eurekahedge.com/Indices/IndexView/Eurekahedge/477/Eurekahedge_Distressed_Debt_Hedge_Fund_Index`

[4] The Credit Suisse index returns can be downloaded via `https://lab.credit-suisse.com/#/en/index/HEDG/HEDG_EVDRV/HEDG_DISTR/overview`

liquidity factor is consistent for different distressed debt hedge funds return sets.

Figure 5: Return summary of Credit Suisse, EurekaHedge and S&P500



The EurekaHedge Distressed Debt Hedge Fund index is an equally weighted index of 28 funds specialised in distressed debt and is rebalanced each month. To remove survivorship-and backfill bias, if a new fund with historical performance is added to the index, all returns are rebalanced accordingly on a going-forward basis. This means that if a new fund satisfies the criteria of the index, from that moment on its returns are being added to the index. If a fund collapses or stops reporting, its track record will remain permanently in the index. Furthermore, since the rationale behind the index is relative benchmarking (rather than making them investible), funds that are closed for further capital inflows are also included in the index.

The event driven distressed debt hedge fund database from Credit Suisse, constitutes of 19 hedge funds in total during the period 01/2000 - 07/2016. A coincise summary of all the reporting hedge funds over the sample period are displayed in Appendix B. To reduce survivorship bias as much as possible, a hedge fund is added to the index on a going-forward basis. Hence, past returns of the index are unaffected. Moreover, if a fund stops reporting or is liquidated, its past returns remain in the index.

In Table 2 and Figure 5 a summary of the returns of each of the hedge fund data sources

Table 2: Distressed debt hedge funds descriptive stats (1/2000 - 7/2016)

This table shows the monthly return statistics of the two distressed debt hedge fund indices and the S&P 500 index. The annualized return, the minimum and maximum monthly loss, the standard deviation and the maximum drawdown over the sample period are shown for each index. The maximum drawdown is computed as the largest consecutive loss incurred until a positive return occurs.

	Return Statistics(%)					# Reporting
	Annualized	Min	Max	St.dev.	Max d.d.	Entire period
Credit Suisse	7.22	-5.66	4.15	1.58	-20.80	19
EurekaHedge	10.76	-9.53	6.62	1.91	-23.93	33
S&P 500	2.71	-16.9	10.78	4.32	-30.14	-

are displayed and benchmarked against the S&P 500 index. The data indeed shows that distressed debt hedge funds outperform the market by a significant margin. Whereas the S&P 500 generates an annualized return of mere 2.7% over the entire period, the EurekaHedge index gains an impressive 10.8% and the Credit Suisse Index a respectable average of 7.2%. This results in a total return of 444% for EurekaHedge, 218% for the Credit Suisse Event Driven Distressed Hedge Fund index and only 56% for the S&P 500. Moreover, the volatility and the maximum drawdown of the distressed debt hedge fund returns are much lower than of the S&P 500, implying that these funds indeed are able to achieve superior investment performance over the market average.

3 Extracting a liquidity proxy

To extract a liquidity factor, I will build further on the latent factor model developed for the yield curve as proposed by Nelson & Siegel (1987). Following the approach of Koopman et al. (2010), I will extend the model by implementing time-varying components that can serve as a proxy for liquidity. First, I will introduce the simple model, after I will elaborate on the time-varying model in state-space form. Moreover, I introduce a simple measure for liquidity from the paper of Hu, Pan & Wang (2013) for comparison.

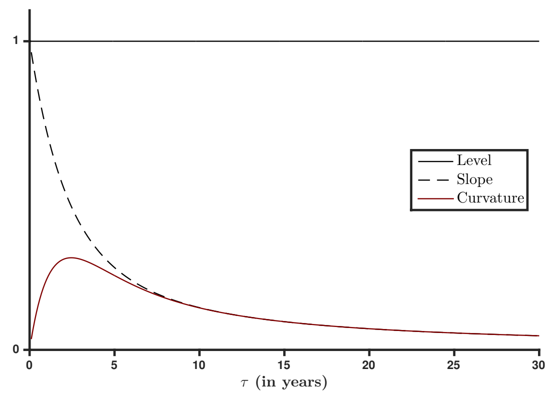
3.1 Nelson-Siegel model

Nelson and Siegel (1987) designed a parametrically parsimonious model for yield curves that is able to represent the curves generally associated with yield curves. The Nelson-Siegel model is based on 3 components: level, slope and curvature. These 3 components together are able to estimate the shape of the yield curve in a dynamic way. When denoting the yield as $y_t(\tau)$ with maturity τ, and defining a smoothing function $\theta_t(\tau)$ representing the yields as a function of τ, the Nelson-Siegel model is given by:

$$\theta_t(\tau) = \beta_{1t} \cdot \underbrace{1}_{\text{Level}} + \beta_{2t} \underbrace{\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right)}_{\text{Slope}} + \beta_{3t} \underbrace{\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)}_{\text{Curvature}} \tag{1}$$

where $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})'$ are the factors for given t, λ and maturity τ that determines the exponential decay of the slope and curvature factor loadings as displayed in the figure below.

Figure 6: Factor loadings of Nelson-Siegel model ($\lambda = 0.7308$)



As can be seen from Figure 6, the first factor loading is fixed at 1 and can therefore be seen as a level factor that equally affects the short-term as well as the long-term yields. The second factor, commonly referred to as the slope factor, converges to 0 as $\tau \uparrow \infty$ and to 1 as $\tau \downarrow 0$. Hence, the slope factor has the most influence on short-term yields. The third factor also goes to 0 as $\tau \uparrow \infty$ but is concave and therefore mainly affects medium-term yields.

When observing a set of yields with n different maturities, we can estimate the yield curve by the following regression model:

$$
\begin{aligned}
y_t(\tau_i) &= \theta_t(\tau_i) + \epsilon_{it} && (2)\\
&= \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i}\right) + \epsilon_{it} && (3)
\end{aligned}
$$

for $i = 1, \ldots, n$ and $t = 1, \ldots, T$. As λ is fixed and the error terms are independent with mean zero and constant variance by assumption, the least squares method can be applied to find estimates for the factors β_{jt}, $j = 1, 2, 3$.

Diebold, Rudebusch and Aruoba (2006) went a step further by treating the factors β_{jt} as latent. They put the Nelson-Siegel model in a state-space framework. This is being done by assuming that the factors β_{jt} follow a first-order vector autoregressive process. To illustrate this, the transition equation, which governs the dynamics of the state vector, is

$$
\begin{pmatrix} \beta_{1t} - \bar{\beta}_1 \\ \beta_{2t} - \bar{\beta}_2 \\ \beta_{3t} - \bar{\beta}_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \beta_{1,t-1} - \bar{\beta}_1 \\ \beta_{2,t-1} - \bar{\beta}_2 \\ \beta_{3,t-1} - \bar{\beta}_3 \end{pmatrix} + \begin{pmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \end{pmatrix} \tag{4}
$$

where $\bar{\beta}j$ denotes the sample average of β_{jt} for $j = 1, 2, 3$ and η_t are disturbances with mean 0 and covariance matrix Σ_η. The transition equation is the same as Equation (1) but the factors now follow the VAR(1) process as in Equation (4). In vector notation, I may now define the state space model as follows:

$$
\begin{aligned}
y_t &= \Lambda(\lambda)\beta_t + \epsilon_t, && \epsilon_t \sim \mathrm{WN}(0, \Sigma_\epsilon) && (5)\\
\beta_t &= \mu + \Phi(\beta_{t-1} - \mu) + \eta_t, && \eta_t \sim \mathrm{WN}(0, \Sigma_\eta) && (6)
\end{aligned}
$$

where the (i, j)th element of the loading matrix Λ is given by

$$\Lambda_{ij}(\lambda) = \begin{cases} 1, & j = 1 \\ 1 - e^{-\lambda\tau_i}/\lambda\tau_i, & j = 2 \\ 1 - e^{-\lambda\tau_i}/\lambda\tau - e^{-\lambda\tau_i}, & j = 3 \end{cases} \tag{7}$$

and where y_t is a $n \times T$ matrix and β_t is a $3 \times T$ matrix. This model is estimated by maximizing the likelihood function from the above set of equations using an EM algorithm. The step-by-step estimation procedure is described later in this section and in Appendix C. As it is a state space model, I will apply the Kalman filter where I use the same initialisations as Diebold, Rudebusch and Aruoba (2006),where Σ_ϵ is assumed to be diagonal and Σ_η to be non-diagonal.

3.2 Time-varying Nelson-Siegel model

Time-varying loading parameter

In the standard NS model, a constant loading parameter λ is assumed. This loading parameter determines the shape (slope and curvature) of the yield curve. Diebold and Li (2006) use a fixed value of 0.0609 for λ and Diebold, Rudebusch and Aruoba (2006) fix λ at 0.077. These values maximize the curvature loading in the medium term at 30 and 23.3 months, respectively.

Most of the research regarding the loading parameter has been done to government paper or investment-grade corporate bonds. However, the yield curves of the high-yield bonds differ substantially in terms of shape as Figure 3 in the previous section shows.
As the yield curves of high-yield bonds follow a fundamentally different shape, the aforementioned fixed loading parameters ($\lambda = 0.0609$ and $\lambda = 0.077$) are most likely not optimal for my NS model. Conveniently, now the NS model is put in a state space framework, it is relatively easy to estimate λ by including it in the parameter vector Θ.

Moreover, high-yield bonds are very prone to overall market conditions. Yields are relatively stable under normal conditions but they explode during market downturns. If I fix λ at a specific value, the speed of the decay of the slope factor β_{2t} and the maturity at which the curvature factor β_{3t} is maximized are fixed as well over the entire sample period. This fixation of λ could be too restrictive, as it is plausible that the value changes over time, especially if you consider the major events during the sample period with for instance the dot-com bubble and the financial crisis. Therefore I propose to treat the loading parameter as the fourth latent factor β_{4t} in the NS model and thereby allowing it to vary over time. The new 4×1 vector β_t is modeled by a VAR(4) process, allowing for dynamic interactions between the four factors. The model dimensions remain the same but note that the new observation equation is now non-linear:

$$y_t = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right) + \epsilon_t \tag{8}$$

$$y_t = \Lambda(\lambda_t) \cdot (\beta_{1t}, \beta_{2t}, \beta_{3t})' + \epsilon_t \tag{9}$$

Time-varying volatility

Now that I have introduced the Nelson-Siegel model in state space form, I will extend the model in order to estimate a liquidity measure. This can be done by implementing a time-varying volatility component in the NS model which is able to capture the amount of liquidity in the market.

The volatility of each yield $y_t(\tau)$ at time point t implies the deviation of the observed yield from the 'true' theoretical yield given by the NS model. Hu, Pan & Wang (2013) conducted research to the liquidity of US government bonds and found that during periods of liquidity shortage, the volatility of the yields tends to rise. They captured a liquidity measure by taking the squared differences between the observed and theoretical NS yields and called it the "Noise measure".

In this case, the central idea is that changes in the volatilities of the yields are assumed to be driven by a single time-varying liquidity component. This liquidity measure increases during periods of liquidity shortage and falls when there is liquidity abundance. Following the thoughts of Hu, Pan & Wang, I treat the theoretical yields from the NS model as the 'true' yields. However, I do not assume that any deviations from these theoretical yields are only caused by liquidity as they do. Instead, I view the volatility of the deviations of the observed yields from the theoretical yields as a result of a common liquidity factor and idiosyncratic disturbances. Koopman et al. (2010) came up with a NS model that has these kind of time-varying variance matrices via a common volatility component that is modeled by a GARCH$(1, 1)$ process, see Bollerslev (1986):

$$\epsilon_t = \Pi_\epsilon \epsilon_t^* + \epsilon_t^+ \tag{10}$$

where Π_ϵ is a $n \times 1$ loading vector, ϵ_t^* is the common liquidity component and ϵ_t^+ is a $n \times T$ disturbance vector. The distribution of the liquidity component and disturbance vector is as following:

$$\epsilon_t^* \sim \mathcal{N}(0, h_t), \quad \epsilon_t^+ \sim \mathcal{N}(0, \Sigma_\epsilon^+), \quad t = 1, \dots, T \tag{11}$$

where h_t follows a symmetric GARCH$(1,1)$ process:

$$h_{t+1} = \delta_0 + \delta_1 \epsilon_t^{*2} + \delta_2 h_t, \quad t = 1, \dots, T \tag{12}$$

with $\delta_0 > 0$, $0 < \delta_1 + \delta_2 < 1$ and $h_1 = \delta_0(1 - \delta_1 - \delta_2)^{-1}$. This specification results in a time-varying variance matrix for ϵ which is given by

$$\Sigma_\epsilon(h_t) = h_t \Pi_\epsilon \Pi_\epsilon' + \Sigma_\epsilon^+ \tag{13}$$

which is a function of the GARCH$(1,1)$ process as given by (12). The loading vector Π_ϵ is not identified however. To do so, I will implement the normalization constraint $\Pi_\epsilon' \Pi_\epsilon = 1$. I will refer to this model as DNS-TVLGARCH from now on. The liquidity proxy is assumed to be equal to the square root of time-varying variance of the common error component, which is h_t, where h_t is obtained from the GARCH process (12). From now on, I refer to this liquidity proxy by $\alpha_t (= \sqrt{h_t})$.

3.3 Estimation procedure

Nelson-Siegel model

The basic Nelson-Siegel model as given by (6) is estimated by an EM procedure. The factors are treated as latent and are modeled via a VAR process in a state-space set-up. The latent factors are first estimated by the Kalman filter, then they undergo a smoothing procedure to obtain smoothed estimates $\xi_{t|T}$. Having obtained these smoothed estimates, the maximization step gives the parameters that maximize the prediction error decomposition (14), given the latent factor estimates from the Kalman filter and smoother:

$$\begin{aligned} l(\Theta) = \frac{T}{2}\log|\Sigma_\epsilon^{-1}| - \frac{1}{2}\sum_{t=1}^{T}\big(y_t - \Lambda(\lambda)'\xi_t\big)'\Sigma_\epsilon^{-1}\big(y_t - \Lambda(\lambda)'\xi_t\big) \\ + \frac{T}{2}\log|\Sigma_\eta^{-1}| - \frac{1}{2}\sum_{t=1}^{T}\big(\xi_t - \Phi\xi_{t-1}\big)'\Sigma_\eta^{-1}\big(\xi_t - \Phi\xi_{t-1}\big) \end{aligned} \tag{14}$$

Fortunately, for this linear state-space model I find algebraic expressions for the parameters. These expressions and their derivation are given in Appendix C. After the M-step, one iteration of the EM-algorithm is completed. In total, I iterate this algorithm $10,000$ times.

DNS-TVLGARCH model

The DNS-TVLGARCH model now consists of an autoregressive process λ_t and a GARCH$(1,1)$ process h_t. As h_t depends on past values of the latent disturbance term ϵ_t^* and factor loading λ_t depends on the unobserved Nelson-Siegel factors (level, slope and curvature), it is not possible to determine them a-priori. Following Koopman et al. (2010), in order to make a feasible estimation, I view these two variables as latent and put them in the state vector ξ_t. The new state space model is defined as follows:

$$y_t = \Gamma(\xi_t) + \epsilon_t^+, \quad \epsilon_t^+ \sim \mathcal{N}(0, \Sigma_\epsilon^+), \quad t = 1, \dots, T \tag{15}$$

$$\xi_{t+1} = \mu + A(\xi_t - \mu) + \omega_{t+1}, \qquad \omega_t \sim \mathcal{N}(0, \Sigma_\omega) \tag{16}$$

where

$$\Gamma(\xi_t) = \Lambda(\lambda_t)(\beta_{1t}, \beta_{2t}, \beta_{3t})' + \Pi_\epsilon \epsilon_t^* \tag{17}$$

or in extended matrix form:

$$y_t = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right) + \underbrace{\Pi_\epsilon \epsilon_t^* + \epsilon_t^+}_{\epsilon_t} \tag{18}$$

$$\begin{pmatrix} \beta_{1,t+1} \\ \beta_{2,t+1} \\ \beta_{3,t+1} \\ \lambda_{t+1} \\ \epsilon_{t+1}^* \end{pmatrix} = c + \begin{pmatrix} \Phi & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \\ \lambda_t \\ \epsilon_t^* \end{pmatrix} + \begin{pmatrix} \eta_t \\ \epsilon_{t+1}^* \end{pmatrix}, \quad \begin{pmatrix} \eta_t \\ \epsilon_{t+1}^* \end{pmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} \Sigma_\eta & 0 \\ 0 & h_{t+1} \end{bmatrix}\right) \tag{19}$$

for $t = 1, \dots, T$ and where $c = [\mu'(I - \Phi)', 0]'$. However, h_{t+1} is a function of $\{\epsilon_i^*\}_{i=1}^t$ which are unknown so I cannot compute h_{t+1}. To solve this, I replace $h_t + 1$ by an estimate based on observations $y_1, \dots, y_t$

$$\hat{h}_{t+1|t} = \delta_0 + \delta_1 e_t^2 + \delta_2 \hat{h}_{t|t-1}, \quad t = 1, \dots, T, \tag{20}$$

where e_t is an estimate of ϵ_t^* which is obtained by applying the filtering step of the Kalman filter to (19).

The Kalman filter only works with linear observation equations. With the addition of the time-varying loading parameter λ_t and the decomposition of the disturbance vector the new observation is clearly non-linear in xi_t as can be seen from (18). In order to apply the Kalman filter, I locally linearize the observation equation in $\xi_t = \xi_{t|t-1}$:

$$y_t = \Gamma_t(\xi_{t|t-1}) + \mathrm{d}\Gamma_t \cdot (\xi_t - \xi_{t|t-1}) + \epsilon_t^+, \quad t = 1, \ldots, T \tag{21}$$

where $\mathrm{d}\Gamma_t = \delta\Gamma_t(\xi_t)/\delta\xi_t'|_{\xi_t=\xi_{t|t-1}} = [\mathrm{d}\gamma_{1t}', \mathrm{d}\gamma_{2t}', \ldots, \mathrm{d}\gamma_{Nt}']'$ with

$$\mathrm{d}\gamma_{it} = [1, \Lambda_{i2}(\lambda_{t|t-1}), \Lambda_{i3}(\lambda_{t|t-1}), \xi_{2,t|t-1}\mathrm{d}\Lambda_{i2}(\lambda_{t|t-1}) + \xi_{3,t|t-1}\mathrm{d}\Lambda_{i3}(\lambda_{t|t-1}), \Pi_{\epsilon,i}] \tag{22}$$

in which $\mathrm{d}\Lambda_{ij}$ denotes the first-order derivative of Λ_{ij} with respect to the loading parameter λ_t for $i = 1, \ldots, N$ and $j = 2, 3$. The prediction step for $\xi_{t|t-1}$ and its covariance matrix $P_{t|t-1}$ is as follows:

$$\xi_{t|t-1} = c + \xi_{t-1|t-1}, \quad P_{t|t-1} = P_{t-1|t-1} \tag{23}$$

with a diffuse prior for $\xi_{0|0}$ and $P_{0|0}$. Given an estimate for $\xi_{t|t-1}$ and $P_{t|t-1}$, the filtering step is given by

$$\xi_{t|t} = \xi_{t|t-1} + P_{t|t-1}\mathrm{d}\Gamma_t' H_t^{-1} u_t, \quad P_{t|t} = P_{t|t-1} - P_{t|t-1}\mathrm{d}\Gamma_t' H^{-1}\mathrm{d}\Gamma_t P_{t|t-1} \tag{24}$$

where $H_t = \mathrm{d}\Gamma_t' P_{t|t-1} + \Sigma_\epsilon^+$ and $u_t = y_t - \Gamma(\xi_t)$.

Due to the linearization of the observation equation (21) and the replacement of h_{t+1} by $\hat{h}_{t+1}$ in (19) the estimates $\xi_{t|t-1}$ and $\xi_{t|t}$ are sub-optimal. Consequently, $P_{t|t-1}$ and $P_{t|t}$ are labeled as approximate MSE matrices. The recursive algorithm of the subsequent prediction and filtering steps is known as the extended Kalman filter, originally proposed by Smith, et al. (1962) when they conducted research for NASA Ames. After each filtering and updating step, the updated parameters Θ are obtained by maximizing the following loglikelihood function:

$$l(\Theta) = -\frac{NT}{2}\log(2\pi) - 1/2\sum_{t=1}^{T}\log|H_t| - 1/2\sum_{t=1}^{T} u_t' H_t^{-1} u_t \tag{25}$$

3.4 Benchmark proxy: Noise measure

Hu, Pan & Wang (2013) came up with a convenient and simple measure for liquidity in the U.S. treasury bond market. The idea behind the measure is that during normal times, the treasury yield curve is smoothed out by abundant arbitrage capital and keeps the deviations from the theoretical yields small. During market crises, the shortage of arbitrage capital leaves the yield move more freely relative to the theoretical yields and thus causes noise in the yields. Hence, they argue that this noise could be informative about the liquidity conditions within the market. The standard Nelson-Siegel model is used to estimate theoretical yields. The noise measure is calculated by calculating the root mean squared distance between the market yields and the model-implied yields:

$$\text{Noise}_t = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}[y_{i,t} - y_{i,t}^{NS}]^2} \tag{26}$$

where $y_{i,t}^{NS}$ is the yield for maturity i obtained from the Nelson-Siegel model at time t. I use this simple Noise measure as a relative benchmark to the liquidity proxy from the DNS-TVLGARCH model for explaining the returns of distressed debt hedge funds.

4 Explaining distressed debt hedge funds returns

4.1 Fama-Macbeth procedure

To implement the liquidity proxies in investment portfolios, we follow the methodology of Hu et al. (2013) by employing the popular Fama-MacBeth portfolio sorting approach (Fama & MacBeth, 1973). That means that I want to sort assets into portfolios, using the sensitivity of the assets to the various liquidity proxies. The assets being considered are high-yield bonds weighted on market capitalisation. An important detail about the data is that it is based on the BoAML HYBI selection methodology. This means that individual bonds can enter and exit the index at varying times. Some bonds are therefore only present during a short interval in the sample period, whereas other bonds are present for almost the entire sample. The amount of bonds per final portfolio therefore varies throughout the sample. This is not harmful for the replicability of my results because, even though the total amount of assets changes throughout time, the selection criteria for choosing these assets is freely available. There is no need to worry about survivorship bias because the constituents of the index are selected continuously throughout time and the index is updated on the last business day of each month.

Portfolio sorting approach

The sorting approach has four parts: (1) first I sort the assets between 4 categories according to their risk profile (measured by their credit rating), (2) then I regress the returns of the assets on a liquidity proxy, (3) then I use the estimated sensitivities of the assets to the liquidity proxy to sort the assets into two categories. This yields 8 (2×4) final portfolios. Hence, I employ a double sort approach where I take the assets' exposure to liquidity risk and the corresponding credit risk profile into account.

The first thing I do is to sort the assets into four risk categories: basket (1) contains all bonds rated BB1, BB2 and BB3, basket (2) all bonds rated B1, B2 and B3, basket (3) all bonds rated CCC1, CC2 and CC3, and basket (4) all bonds with rating CC, C and D. Important to note is that a *high* rating is supposed to represent a *low* risk profile for the respective asset. For example, the highest credit rating is 'AAA' (S&P) or 'Aaa' (Moody's) and corresponds with the lowest credit risk. In my case, the highest rating possible is Ba1 or BB+ due to the nature of the dataset. The lowest rating possible is C or D.

The second part of the approach is to regress the returns of the individual assets on the liquidity proxy and the returns on the market portfolio:

$$R_{t,i} = \beta_0 + \beta_i^L \Delta\text{Liq}_t + \beta_i^M R_t^M + \varepsilon_t^i \tag{27}$$

Here $R_{t,i}$ is the one-period return of asset i at time t and R_t^M is the return on the market, which is simply the weighted-average of all bonds in the BoAML HYBI. The returns are the first differences of the prices of the assets. The prices themselves are taken as the middle of the bid and ask of the prevailing asset prices, expressed at a percent of the asset's original face value. This is regressed on ΔLiq_t, the change in liquidity proxy.

This is being done for all bonds i and hence I acquire the pre-ranking β_i^L sensitivities from Equation (27). Important to note here is that a negative β_L implies that when the liquidity proxy increases, the return of the corresponding asset decreases. The liquidity proxy is thought to increase during crises, such that assets with a low liquidity beta will have lower returns during those periods. This also means that assets with a high liquidity beta have a high exposure to liquidity risk. Likewise, assets with a high market beta have a high exposure to overall market conditions.

Now that the 8 portfolios are formed, I can estimate their risk premia. Before I do this, I first need to calculate the post-ranking portfolio betas as follows:

$$R_{t,p} = \beta_0 + \beta_p^L \Delta\text{Liq}_t + \beta_p^M R_t^M + \varepsilon_t^p \tag{28}$$

For all portfolios $p = 1, \ldots, 8$. Here $R_{t,p}$ is the simple average return of portfolio p. The coefficients β_p^L and β_p^M are the portfolio's liquidity and market sensitivities respectively. These are also referred to as the 'post-ranking' betas. These are estimated over the full sample. The liquidity risk factor is the difference between the portfolio with the highest and lowest post-ranking liquidity beta. The portfolios are still also controlled for credit risk. As distressed debt hedge funds tend to invest in companies in default or approaching default, the two highest risk portfolios are most suitable for the liquidity risk factor in this space. Therefore, the liquidity risk factor I will use in my analysis is defined as follows:

$$\text{IML}_t = R_{t,(1,3)} - R_{t,(2,3)} \tag{29}$$

Now that I have the portfolio betas, I can also estimate the risk premia. I will estimate the risk premia in the following set-up. Assuming that common variation in returns can be explained by liquidity and a market risk factor, the estimation for the two risk factors goes as follows:

$$
\begin{aligned}
R_{t,1} &= \gamma_t^I + \gamma_t^L \beta_1^L + \gamma_t^M \beta_1^M + \varepsilon_t^1 \\
R_{t,2} &= \gamma_t^I + \gamma_t^L \beta_2^L + \gamma_t^M \beta_2^M + \varepsilon_t^2 \\
&\vdots \\
R_{t,8} &= \gamma_t^I + \gamma_t^L \beta_8^L + \gamma_t^M \beta_8^M + \varepsilon_t^8
\end{aligned} \tag{30}
$$

These risk premia are calculated cross-sectionally for each period $t = 1, \ldots, T$ such that we end up with a liquidity and market premium for each period.

4.2 Risk factors to explain hedge fund returns

With these liquidity-sorted portfolios I may now construct a liquidity risk factor in a similar way as Fama & French (1994) did. That is, I create mimicking returns for the liquidity factor by subtracting the portfolio with the lowest sensitivity to liquidity from the portfolio with the highest sensitivity to obtain the liquidity risk factor IML (Illiquid Minus Liquid). By means of a simple regression I can assess the explanatory power of the liquidity risk factor in combination with the Fama & French and Fung & Hsieh risk factors:

$$
r_{i,t} = \Theta_{j,t} + \gamma \mathrm{IML}_t + \zeta_{i,t} \tag{31}
$$

where $\Theta_{j,t}$ denotes the set of risk factors from either Fama & French or Fung & Hsieh and i denotes the set of hedge fund returns. The individual significance of the factors can be determined by a t-test and the combined significance by an F-test.

5 Empirical findings

5.1 Liquidity proxy

To extract a liquidity premium from the raw bond data, I employ the Dynamic Nelson-Siegel model with a GARCH component as described in section 3. This model includes a time-varying loading parameter λ_t and time-varying volatility modeled by a common volatility component that follows a GARCH(1,1) process. The square root of the GARCH variance α_t is used as a proxy that is negatively related to the liquidity in the high-yield bond market. In other words, if this liquidity proxy increases, the amount of liquidity in the market decreases. Moreover, I include the simple Noise measure as developed by Hu, Pan & Wang (2013) as a benchmark liquidity measure. The DNS-TVLGARCH liquidity proxy and factor are displayed together with the Noise measure and liquidity factor in Figure 7a and Figure 7b, respectively.

Table 3: Performance of DNS models

This table displays the log-likelihood value from the prediction error decomposition in (25), the Akaike Information Criterion (AIC), the number of parameters (#Θ) and the likelihood-ratio statistic.

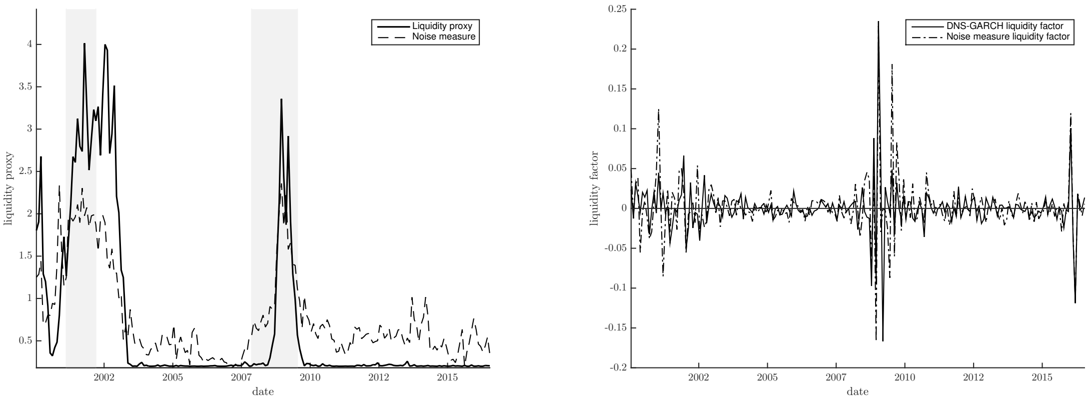
	Log-likelihood	AIC	#Θ	LR-statistic
DNS - base	−5907.5	11881.1	33	-
DNS - TVL	−5541.2	11214.4	66	732.6
DNS - GARCH	−5504.0	11122.1	57	807.0
DNS - TVLGARCH	−5486.0	11110.0	69	843.1

The parameter estimates of the latent factors are displayed in Table 5 together with their standard errors. To see how much the model fit improves by implementing the two time-varying components, I also estimated the basic Dynamic Nelson-Siegel model which is given by (6) and the Dynamic Nelson-Siegel models with only time-varying λ (DNS-TVL) or time-varying volatility (DNS-GARCH). In Table 3 the two models are compared based on their likelihood and the Akaike Information Criterion. From the table it is clear that the DNS-TVLGARCH model with the time-varying factor loading and volatility components provide a better fit than the other dynamic Nelson-Siegel model. The time-varying volatility seems to be the primary driver of the improved model fit, based on the log-likelihood and AIC values. The (log)-likelihood of the DNS-TVLGARCH model is higher, and the AIC of the DNS-TVLGARCH, which corrects for the number of parameters, is also lower, which indicates a better model fit. If you look at the VAR coefficients of λ_t and its (co-)variance estimates, they are all highly significant. From Figure 2 it can be seen that the spread between the yields of different maturities varies hugely

Figure 7: Liquidity proxy and factor

Figure (a) shows the liquidity proxy α_t as obtained from (12) in the DNS-TVLGARCH model and the Noise measrue, (b) shows the estimated liquidity factor of the two proxies from the the Fama-Macbeth procedure

(a) Liquidity proxy **(b) Liquidity factor**

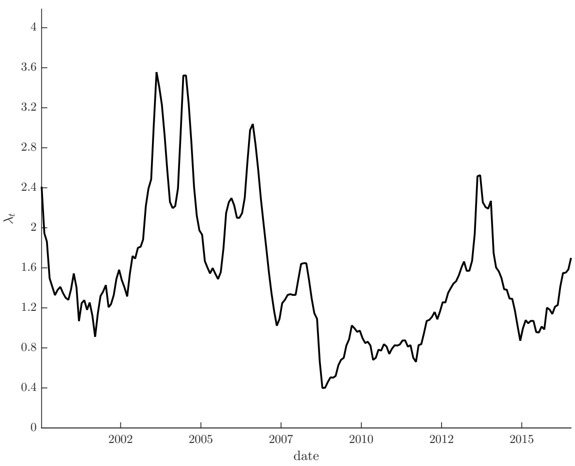


over the sample period. It is not surprising therefore that the estimated loading paramater λ_t also fluctuates substantially over the sample period as Figure 5a shows. For instance, when the yield curve is more volatile, as was the case during the beginning of this millennium, a low loading parameter flattens the slope and curvature factor loadings. Vice versa, a higher loading parameter occurs in lower volatility periods as was the case between the two recessions and after the financial crisis. The impact of the loading parameter on the slope and curvature factor loadings are displayed in Figure 8b and Figure 8c, respectively. Hence, it may be concluded that by including a time-varying loading parameter the performance of the Nelson-Siegel model significantly improves.

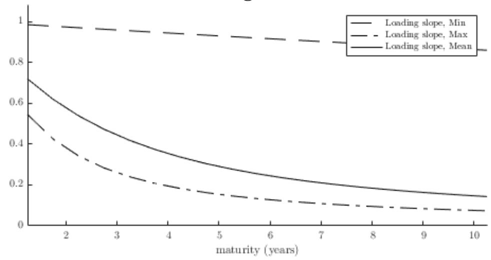
Moreover, each of the yields were subject to a single liquidity factor which was modeled by process (11). As discussed in section 3, this process models the average liquidity in the high-yield bond market where each yield i has a certain sensitivity $\Pi_{\epsilon,i}$ to this liquidity factor. The idiosyncratic sensitivities to the liquidity factor $\Pi_{\epsilon,i}$ are given in Table 4. The shortest maturities have the highest sensitivity loadings whereas the longest maturities have the lowest. This can be explained by thinking of the liquidity proxy as the average amount of liquidity in the market. The sensitivity loadings therefore mainly serve to make a relative distinction between the liquidity of the different maturities as a high loading means that it is highly dependent on market liqudiity and a low coefficient means the opposite. Hence, what this tells us, is that the ongest maturities are the most liquid securities whereas the maturities with the highest sensitivity loadings belong to the most illiquid class. The idiosyncratic disturbance terms $(\Pi_{\epsilon,i}\epsilon_t^*)$

Figure 8: Loading parameter

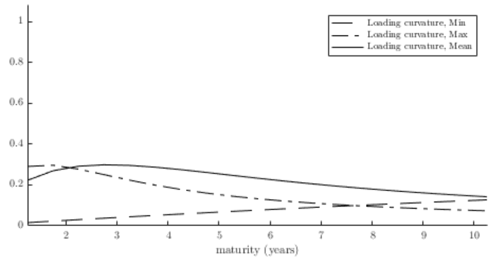
These three figures summarise the loading parameter λ_t. Figure (a) shows the values of the loading parameter over the sample period. Figure (b) shows the slope factor loading for the minimum, maximum and the average of λ_t during the sample period. Figure (c) shows the same, only then for the curvature factor loading.



(a)



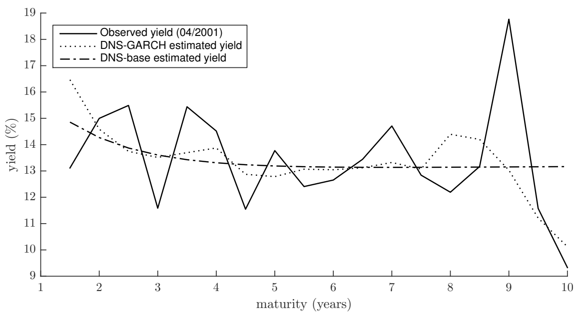
(b)



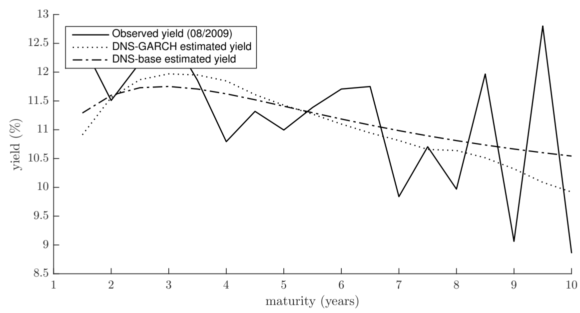
(c)

Figure 9: Estimated yields for some maturities

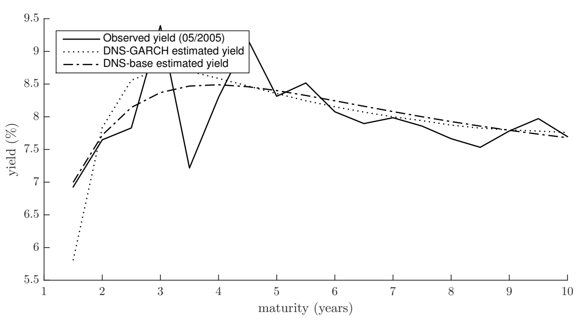
Below are the yield curves (solid line) of four different dates plotted together with the estimated filtered yields from the DNS-base model (stripes) and the DNS-TVLGARCH model (dotted line).



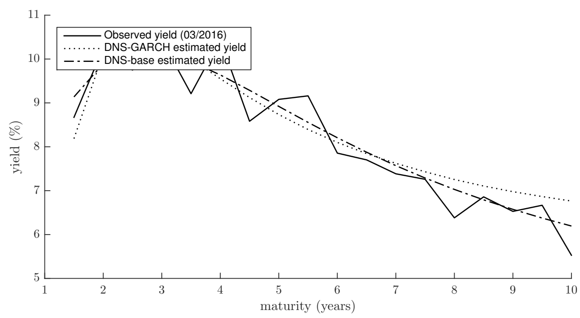
(a) April 2001



(c) August 2009



(b) May 2005



(d) March 2016

for each time-point t also allow for more flexibility in estimating the yields. Instead of having a smooth exponential curve as in the standard Nelson-Siegel model, the DNS-TVLGARCH model allows for all types of shapes via this liquidity factor, resulting in a superior model fit. Figure 9 shows estimated yields for the DNS-TVLGARCH model and the basic DNS model. The DNS-TVLGARCH fitted yields are clearly closer to the true yields and takes on a more rigid shape. Table 4 displays the means and errors of the full model specification and the base model. It shows that the DNS-TVLGARCH model provides slightly better yield estimates, as the average standard deviation of the errors is lower and provides the best yield estimates for most of the maturities.

Table 4: Filtered errors - model comparison

This table shows the filtered error statistics for the baseline model and the DNS-TVLGARCH model. The filtered errors are defined as the difference between the observed yields, and the filtered estimates obtained from the Kalman filter. For each maturity, the mean and standard deviation of the filtered errors are provided as summary statistics. Also in the table is the liquidity loading Π_{ϵ_i} for each maturity i, which shows the dependence of each maturity's yield on the liquidity in the market.

Maturity	DNS - base		DNS-TVLGARCH				DNS - base		DNS-TVLGARCH		
(months)	Mean	St.dev.	Mean	St.dev.	$\Pi_{\epsilon,i}$		Mean	St.dev.	Mean	St.dev.	$\Pi_{\epsilon,i}$
18	0.501	2.576	0.682	2.240	0.769**	72	0.021	0.443	0.027	0.476	0.098**
24	−0.014	0.979	−0.059	0.994	0.345**	78	0.086	0.648	0.065	0.659	0.118**
30	−0.112	1.178	−0.178	1.169	0.154**	84	0.103	0.590	0.029	0.619	0.146**
36	−0.132	1.139	−0.200	1.108	0.088**	90	0.103	0.598	0.065	0.625	0.135**
42	0.051	1.073	−0.056	1.061	0.093*	96	0.299	1.132	−0.012	0.883	0.259**
48	0.099	0.968	−0.038	0.983	0.114**	102	0.146	1.234	−0.139	1.085	0.250**
54	−0.078	0.597	−0.018	0.630	0.040	108	−0.032	1.049	−0.100	1.097	0.155**
60	−0.079	0.527	−0.005	0.517	0.047	114	−0.334	1.164	−0.058	1.007	0.002
66	0.009	0.424	0.021	0.452	0.086**	120	−0.781	1.151	−0.302	0.621	−0.088
Mean							−0.010	0.964	−0.015	0.901	
Median							0.015	1.014	−0.028	0.933	
# Lower							8		10		

The standard errors are reported below each estimate. An asterisk (*) denotes significance at a 5% level or less and two asterisks (**) a significance at a 1% level or less.

The liquidity proxy α_t obtained from the DNS-TVLGARCH model and the liquidity factor are plotted in Figure 7. The gray shaded areas refer to the recession periods as identified by the NBER. During the dot-com bubble in the early 00s, the liquidity proxy spikes which indicates a severe liquidity shortage in the high-yield bond market. This can be attributed to the fact that during this period many tech and internet companies were experiencing life-threatening financial problems when the bubble burst and valuations plummeted towards zero. Before the burst, the world was in a long bull market which started in the early 90s. When the tech frenzy collapsed, it took all the financial markets with them. Yields spiked and throughout the entire bubble, everyone wanted to sell, resulting in a large liquidity shortage as also shown by the

Table 5: Estimates of latent factors from VAR model and GARCH process

This table displays the estimated parameters of the VAR model for the latent factors (Panel A), its covariance matrix (Panel B) and the estimated common coefficients δ_1 and δ_2 for the GARCH process (Panel C).

Panel (A) : Autoregressive coefficients of VAR				
	$Level_{t-1}$	$Slope_{t-1}$	$Curvature_{t-1}$	$Loading_{t-1}$
$Level_t$	0.9921^{**} (0.0105)	0.0110^{**} (0.0006)	0.0064^{**} (0.0008)	0.2446 (0.0483)
$Slope_t$	0.1812^{**} (0.0504)	0.7329^{**} (0.0060)	-0.2078^{**} (0.0093)	-1.5542^{**} (0.1010)
$Curvature_t$	-0.3436^{**} (0.0614)	0.1603^{**} (0.0180)	1.1342^{**} (0.0128)	-1.2334^{**} (0.1447)
$Loading_t$	-0.0105^{**} (0.0001)	0.0056^{**} (0.0000)	0.0044^{**} (0.0000)	1.0285^{**} (0.0062)

The standard errors are reported below each estimate. An asterisk (*) denotes significance at a 5% level or less and two asterisks (**) a significance at a 1% level or less.

Panel (B) : Covariance matrix of VAR				
	$Level_t$	$Slope_t$	$Curvature_t$	$Loading_t$
$Level_t$	0.0363 (0.0345)	-0.7195^{**} (0.1105)	0.5188^{**} (0.1419)	0.0313^{**} (0.0013)
$Slope_t$		15.9086^{**} (0.2177)	-9.9855^{**} (0.1652)	-0.5575^{**} (0.0028)
$Curvature_t$			12.1823^{**} (0.0956)	0.3179^{**} (0.0084)
$Loading_t$				0.0247^{**} (0.0000)

The standard errors are reported below each estimate. An asterisk (*) denotes significance at a 5% level or less and two asterisks (**) a significance at a 1% level or less.

Panel (C) : Parameters of GARCH(1, 1) process			
	δ_0	δ_1	δ_2
Estimate	0.7060^{**} (0.0107)	0.9677^{**} (0.0704)	0.0182 (0.0699)

The standard errors are reported below each estimate. An asterisk (*) denotes significance at a 5% level or less and two asterisks (**) a significance at a 1% level or less.

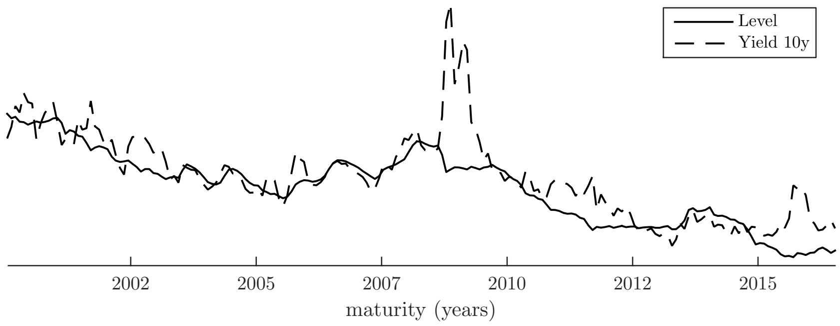
shape of the liquidity factor during this period. When things returned to normal after and investor's confidence grew, so did market liquidity.

What is apparent however, is that during the financial crisis, market liquidity seems to be slightly less impacted as shown by the liquidity proxy in Figure 7a, but the liquidity factor is bigger. What was different here, is the nature of the recession and how market liquidity was affected. The dot-com bubble mainly affected tech companies and the financial markets but did not harm the global economy so much. The financial crisis however, did not only hurt the financial markets, but it also had a devastating effect on the global financial system and

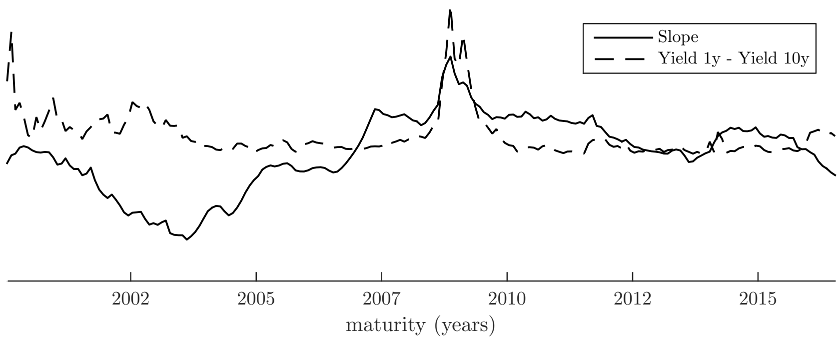
Figure 10: Level, slope and curvature

In the figures below, the estimated level, slope and curvature are plotted together with its sample estimate. (a) shows the level together with the yield of the longest maturity (10 year), (b) the slope with the shortest maturity (18 months) minus the longest maturity, and (c) displays the curvature with two times a medium-term maturity (2.5 year) minus the shortest and longest maturity.

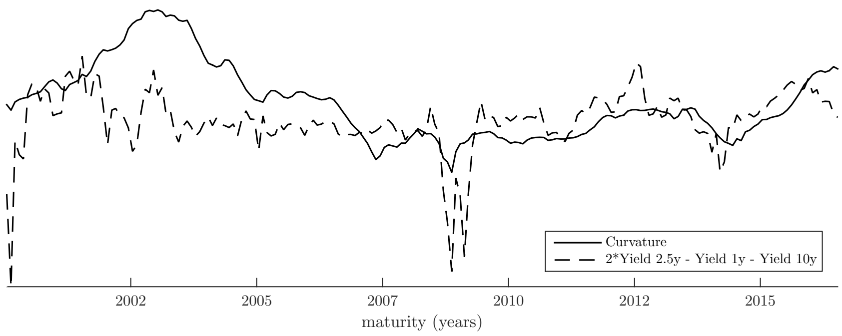
(a) Level



(b) Slope



(c) Curvature



the world economy. Therefore, governments decided to actively intervene to try to soften the impact of this recession. Unconventional monetary policies such as quantitative easing were designed to provide liquidity to the market. Moreover, the US government and Europe decided to continue with QE after the recession ended which on their part helps to understand why the liquidity proxy wasn't as large as during the dot-com bubble and quickly returned to its normal state. However, during the financial crisis bond returns were impacted much more by the overall market liquidity which explains why the liquidity factor is bigger than during the dot-com bubble.

In the next part, I will use the liquidity factor obtained from the DNS-TVLGARCH model in a explanatory factor model to see if it can explain common variation in the returns of distressed debt hedge funds.

5.2 Return analysis

In this part I will analyse if there is evidence that the constructed liquidity factor can capture common variation in the returns of distressed debt hedge funds. As aforementioned in section 2, I will use the EurekaHedge distressed debt hedge fund index and the Credit Suisse Distressed Debt Hedge Fund index as benchmarks for industry-wide returns. I use the liquidity factor together with the four Fama & French factors (value, growth, market and momentum) and with the 7 hedge fund risk factors from Fung & Hsieh. By means of an ordinary least squares regression, I assess the explanatory power of the factors. The results are summarised in Table 10.

The table shows some interesting information. First I look at the Fama & French factors. Using these risk factors and the liquidity factor as explanatory variables, I obtain an R-squared of 0.638 and 0.559 which implies that there is significant explanatory power in these risk factors. Moreover, four out of five factors are significant under a 1% level. Not surprisingly, Rm has a significant positive coefficient for both return benchmarks. Most hedge funds are exposed to the market and have non-zero beta, although by their nature their beta is overall relatively low. Moreover, the SMB and HML coefficients are both positive for both return benchmarks. This provides evidence that distressed debt hedge funds are exposed to value-and size related risk factors. This makes sense when thinking of periods when the value factor generally outperforms the market. This is during periods of economic distress when overvalued companies plummet and value-based strategies outperform. Distressed debt hedge funds investing strategy can also be seen as a value-based strategy as they make rigid analyses of distressed companies and invest when the underlying value does not match the price of the security. Also not surprisingly is that the momentum factor is not significant. Hedge funds and especially distressed debt hedge funds tend not to follow momentum-related strategies. Lastly, there is highly significant evidence that the liquidity factor captures common variation in the returns of distressed debt hedge funds. The positive coefficient implies that when the markets experience liquidity shortage (liquidity factor increases), the returns of distressed debt hedge funds increase. In other words, they get rewarded by facing liquidity risk.

The second set of risk factors I consider are the 7 hedge fund risk factors from Fung & Hsieh plus the liquidity factor. Here there seems to be evidence as well that the factors hold significant explanatory power when looking at the large R-squared values and coefficient statistics. From the trend-following risk factors, only the bond factor coefficient is significant. The other two trend-following factors, the currency and the commodity factors are insignificant which seems logical as distressed debt hedge funds do not operate in this space. For the EurekaHedge benchmark, the equity risk factors RMS & SP500 are positive and significant which can be explained by the same rationale as for the Rm factor in the previous paragraph. Also here, the liquidity risk factor is positive and highly significant.

Table 6: Results of factor regressions

This table reports the performance of the Fama & French and the Fung & Hsieh factors in explaining the returns of distressed debt hedge funds represented by the EurekaHedge index and the Credit Suisse index. In the left panel, the Fama & French value factor (HML), size factor (SMB), market return factor (Rm) and the momentum factor (MOM) are used together in the explanatory model with the constructed liquidity factor. In the right panel, the Fung & Hsieh trend-following factors (BFF, CuFF & CoFF), equity factors (SP500 & RMS), and the interest-rate related factors (BMF & CSF) are used together with the liquidity factor (IML). For each factor on each index return, I report the coefficient, the t-statistic of the coefficient and its p-value.

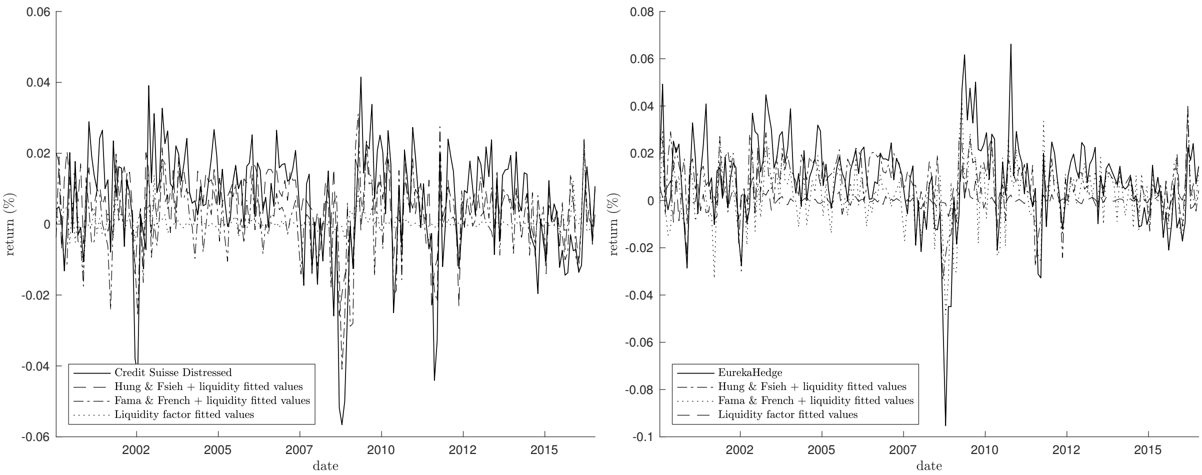
Fama & French factors						
	EurekaHedge			Credit Suisse		
	b	$t(b)$	p-value	b	$t(b)$	p-value
C	0.007	7.060	0.000**	0.005	5.383	0.000**
SMB	0.133	4.192	0.000**	0.091	3.439	0.000**
HML	0.113	3.429	0.001**	0.113	4.108	0.000**
Rm	0.253	10.116	0.000**	0.217	10.342	0.000**
MOM	−0.001	−0.049	0.481	0.513	0.736	0.304
IML	0.139	4.203	0.009**	0.112	4.015	0.000**
R^2		0.484			0.473	
AIC		−2837.6			−2978.3	

Fung & Hsieh factors						
	EurekaHedge			Credit Suisse		
	b	$t(b)$	p-value	b	$t(b)$	p-value
C	0.017	2.513	0.006**	0.017	2.954	0.002**
BFF	−0.039	−4.677	0.000**	−0.031	−4.548	0.000**
CuFF	−0.000	−0.101	0.456	−0.002	−0.324	0.373
CoFF	−0.014	−1.681	0.047*	−0.011	−1.529	0.064
BMF	0.000	0.716	0.238**	0.000	0.329	0.371
SP500	0.148	5.420	0.000**	0.125	5.508	0.000**
RMS	0.148	5.397	0.000**	0.125	5.496	0.000**
CSF	−0.005	−2.971	0.002**	−0.005	−3.665	0.000**
IML	0.122	3.211	0.000**	0.089	2.843	0.003**
R^2		0.336			0.334	
AIC		−2731.0			−2878.9	

The standard errors are reported below each estimate. An asterisk (*) denotes significance at a 5% level or less and two asterisks (**) a significance at a 1% level or less.

Hence, from these results it can reasonably derive that the liquidity factor has strong explanatory power in explaining distressed debt hedge fund returns. In Figure 11 below, I plotted the fitted values for both set of risk factors and return benchmarks. From here it can be observed that the factors capture the common variation in the returns quite well. Moreover, it is interesting to see that the liquidity risk factor captures most of the variation during the two recessions, when liquidity in the market tends to decrease a lot.

Figure 11: Index returns and fitted values



The regression results from the Noise measure liquidity factor are summarized in Table 7. An extensive overview of the regression results can be found in Appendix D. As can be seen in Figure 7a, the Noise measure has some similarities in shape compared to the DNS-TVLGARCH liquidity proxy. Both are relatively low during normal times and peak during recession periods. The simple Noise measure is useful for explaining the returns of distressed debt hedge funds as its coefficient estimates are significant and positive.

Table 7: Noise measure results

This table shows the R-squared of the same factor regressions as in Table 10, but now with the liquidity factor from the Noise measure. An asterisk (∗) means that the factors plus the DNS-TVLGARCH liquidity factor (IML) has a higher R^2 or lower AIC.

	Fama & French factors		Fung & Hsieh factors	
	EurekaHedge	Credit Suisse	EurekaHedge	Credit Suisse
R^2	0.461*	0.437*	0.333*	0.315*
AIC	−2802.3*	−2952.0*	−2729.1*	−2867.6*
ΔAIC	35.3	26.3	1.9	11.3

However, the Noise measure has its limitations because it is somewhat simplistic. All deviations from the Nelson-Siegel curve are assumed to be caused by the amount of liquidity in the market. Therefore, its explanatory power it not as high as the more subtle liquidity proxy α_t from the DNS-TVLGARCH model when looking at the regression R^2 & AIC and p-values of the liquidity factor coefficients. As a rule of thumb, a difference in the AIC smaller than 2 can be seen as two indistinguishable models. For any difference larger than 2, this indicates an improved model with the lower AIC. The table above gives strong evidence that although the Noise measure is a convenient and straight-forward way to approximate the liquidity in the market, the DNS-TVLGARCH model provides a better factor for explaining the returns of distressed debt hedge funds.

5.3 Robustness

In this section I do a robustness check of the results by comparing different samples. I split the sample into two equal parts; (i) January 2000 - December 2008 and (ii) January 2009 - July 2016. Table 8 shows the regression results for each of the two time period with period (i) having bold-faced numbers. The effect of most of the factors remain relatively similar over the two periods. It is noteworthy that the factors SMB and HML seem to have less explanatory power after 2009 while momentum seems to become a more important factor. This is in line with how the markets have behaved recent years. Market levels seem to have diverged from their fundamental values and momentum strategies have prospered. This partly explains why active investors, such as hedge funds who often chase value-oriented strategies have underperformed passive index funds who follow momentum-based strategies. This could also be derived when looking at the coefficient of the coefficient. In the first subsample, it was positive and highly significant. In other words, despite the different risk factors explaining a large part of the common variation, the distressed debt hedge funds were still able to generate positive alpha which is included in the constant. This positive alpha can be seen as a proxy for the skills of the hedge fund managers which can't be explained by all the risk factors. However, in the second subsample, after accounting for the Fung & Hsieh risk factors, the constant is not significant anymore. This implies, that distressed debt hedge fund managers had difficulty to leverage their investing skills in the market environment prevailing from 2009 on. Moreover, the IML factor appears to perform slightly worse in the second subsample. This is most likely due to the fact that the first subsample had two major recessions and the second did not. These recessions are typically the moments when liquidity tends to become an important factor as common market liquidity is absent during these periods.

Overall, the results appear to be fairly robust. The explanatory power of the factors taken altogether seems to be slightly better in the first subsample when considering R^2 but the results are relatively similar between the two subsamples. The differences can be mainly attributed to the changed market conditions and its impact on some of the risk factors as briefly discussed in the previous paragraph.

Table 8: Robustness results

This table reports a comparison of the regression results in the two subsamples. The estimates from the first subsample (01/2000 - 12/2008) have been boldfaced such that the reported values should be read as: **Subsample 1 estimate**/Subsample 2 estimate.

	Fama & French factors					Fung & Hsieh factors			
	EurekaHedge		Credit Suisse			EurekaHedge		Credit Suisse	
	b	p-value	b	p-value		b	p-value	b	p-value
C	**0.007**/0.007	**0.000**/0.000	**0.006**/0.002	**0.000**/0.016	C	**0.024**/−0.012	**0.042**/0.122	**0.034**/−0.000	**0.003**/0.487
SMB	**0.158**/0.010	**0.000**/0.437	**0.112**/−0.003	**0.001**/0.477	BFF	**−0.029** /−0.035	**0.011**/0.002	**−0.021**/−0.035	**0.003**/0.000
HML	**0.142**/0.048	**0.000**/0.223	**0.124**/0.085	**0.001**/0.042	CuFF	**0.001** /0.000	**0.452**/0.498	**−0.003**/0.002	**0.366**/0.429
Rm	**0.278**/0.267	**0.000**/0.000	**0.225**/0.242	**0.000**/0.000	CoFF	**−0.011** /−0.001	**0.184**/0.186	**−0.004**/−0.015	**0.368**/0.046
MOM	**0.025**/−0.054	**0.167**/0.035	**0.007**/0.000	**0.384**/0.495	BMF	**0.001** /0.007	**0.328**/0.003	**−0.002**/0.004	**0.191**/0.020
IML	**0.148**/0.132	**0.008**/0.001	**0.114**/0.115	**0.017**/0.000	SP500	**0.133**/0.155	**0.000**/0.000	**0.120**/0.115	**0.000**/0.000
					RMS	**0.133**/0.156	**0.000**/0.000	**0.120**/0.115	**0.000**/0.000
					CSF	**−0.009**/0.000	**0.000**/0.445	**−0.008**/−0.002	**0.000**/0.140
					IML	**0.164**/0.064	**0.013**/0.086	**0.136**/0.049	**0.016**/0.095
R^2	**0.518**/0.495		**0.464**/0.539			**0.421**/0.349		**0.377**/0.364	

6 Conclusion

The main objective of this research is to see if a set of risk factors can explain variation in the returns of distressed debt hedge funds. Distressed debt funds invest in securities of companies in distress, which means they are already in financial trouble and are likely to default. Often, investors stay far away from these kinds of securities for different reasons, making this area of investing relatively more illiquid than conventional investing fields. These hedge funds however, manage to generate large excess returns by investing in this unconventional types of securities. There has not been any literature on how these returns could be explained and which risk factors these funds face. As these hedge funds are able to invest in a broad set of asset classes, ranging from bonds to bank loans to convertibles to common equity, my idea was to use a large set of common risk factors. Moreover, these funds operate in a space which tends to be very illiquid by its nature, so including a liquidity risk factor could provide significant explanatory power.

However, such a liquidity factor does not readily exist. Some research has already been done to liquidity in financial markets but this was mainly limited to common equity markets. To quantify a measure for liquidity in the investing space of distressed debt hedge funds, I first came up with a Dynamic Nelson-Siegel (DNS) state-space model to estimate yields of high-yield bonds. Under the assumption that any deviations of the estimated DNS yields from the true yields were caused by liquidity in the market, l implemented a latent liquidity factor in the DNS model using a GARCH process. This latent liquidty proxy affected all bonds, and could thus be seen as a measure for liquidity in the high-yield bond market. Using the liquidity proxy from the DNS-TVLGARCH model, I then created a liquidity factor which could be used in an explanatory factor model for distressed debt hedge fund returns.

Using the EurekaHedge and Credit Suisse distressed debt hedge fund index as industry return benchmarks, I used two sets of risk common factors plus the liquidity factor to see if they could explain the common variation in the returns of these indices. The results showed that both sets of risk factors had significant explanatory power for the distressed debt hedge fund returns. Moreover, there was highly significant evidence that the constructed liquidity factor captured common variation in both return benchmarks. This in turn proves that liquidity plays an important role as a risk factor for distressed debt hedge funds.

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A Sample statistics maturity bins

Table 9: BoAML HYBI - yield statistics

This table displays the sample statistics of the binned maturities. There are 18 bins in total, ranging from 18 months to 10 years to maturity, divided in half-yearly steps (0.5 year). Every month, each bond in the index is put in the corresponding maturity bin corresponding to its market cap weight. The weighted average over all the bond yields in each bin is then used in the Nelson-Siegel model.

Maturity	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
Mean	9.69	9.52	9.57	9.60	9.77	9.76	9.51	9.43	9.43	9.36	9.34	9.28	9.21	9.34	9.13	8.89	8.54	8.05
St.dev	6.55	4.86	3.98	3.56	3.43	3.32	2.96	2.79	2.79	2.77	2.89	2.80	2.67	3.12	3.14	2.90	2.49	1.81
Minimum	2.99	2.97	3.36	4.21	4.24	4.47	4.74	5.25	5.53	5.22	5.23	5.69	5.38	5.24	5.15	4.90	4.98	4.86
Maximum	46.88	36.61	25.79	27.20	22.34	22.21	21.03	21.35	20.62	20.65	21.30	20.87	18.19	19.56	22.33	19.77	20.59	15.38
ρ_1	0.897	0.936	0.939	0.935	0.942	0.954	0.953	0.957	0.961	0.958	0.965	0.965	0.966	0.966	0.962	0.953	0.936	0.938
ρ_{12}	0.244	0.316	0.428	0.394	0.450	0.420	0.438	0.455	0.485	0.523	0.428	0.497	0.566	0.538	0.578	0.570	0.477	0.601
Maturity (mean)	17.88	24.54	29.73	36.08	41.61	48.30	53.79	60.06	65.93	72.01	77.70	84.24	89.95	95.87	101.33	108.07	114.59	119.95

B Constituents - Credit Suisse Event Driven Distressed Hedge Fund Index

The Credit Suisse Index is created by the combined average weighted returns of the following 19 distressed debt funds:

- AG Corporate Credit Opportunities
- AG Diversified Income Fund Plus
- AG Super Fund
- Archer Capital Master Fund LP
- Candlewood Special Situations Fund
- Castlerigg International Limited
- Cerberus
- DW Catalyst Fund
- Highland Crusader Offshore Partners
- JLP Credit Opportunity Fund
- King Street Capital
- Knighthead Master Fund
- Marathon Special Opportunity Fund
- Merced Partners
- Monarch Debt Recovery Fund
- Owl Creek
- Paulson Special Situations Fund
- Styx Partners
- York Credit Opportunities Fund LP

C EM-algorithm for basic DNS, expressions and derivations

Below I show one loop of the EM-algorithm I use to estimate the basic DNS model. The derivations for the M-step are also given. First, I conduct the expectation step were I use the Kalman Filter and Smoother to estimate the latent factors:

Prediction equations:

$$\xi_{t|t-1} = \Phi\xi_{t-1|t-1}, \quad P_{t|t-1} = \Phi P_{t-1|t-1}\Phi' + \Sigma_\eta. \tag{32}$$

Filtering equations:

$$\xi_{t|t} = \xi_{t|t-1} + (P_{t|t-1}\Lambda(P_{t|t-1}\Lambda)^{-1}(y_t - \Lambda'\xi_{t|t-1}), \tag{33}$$

$$P_{t|t} = P_{t|t-1} - (P_{t|t-1}\Lambda(\Lambda' P_{t|t-1}\Lambda(\lambda)' + \Sigma_\epsilon)^{-1}(\Lambda' P_{t|t-1}). \tag{34}$$

Smoothing equations:

$$\xi_{t|T} = \xi_{t|t} + P_{t|t}\Phi' P_{t+1|t}(\xi_{t+1|T} - \xi_{t+1|t}), \tag{35}$$

$$P_{t|T} = P_{t|t} - P_{t|t}\Phi' P^{-1}_{t+1|t}(P_{t+1|T} - P_{t+1|t})P^{-1}_{t+1|t}\Phi P_{t|t}, \tag{36}$$

$$P_{t+1|t} = P_{t+1|T}P^{-1}_{t+1|t}\Phi P_{t|t}. \tag{37}$$

where $\xi_{0|0} = 10^6 \cdot [1, 1, 1]'$, $P_{0|0} = 10^6 \cdot I_3$ and $\xi_{T|T}$ is the filtered estimate for $t = T$. As to find a derivation for the maximization of the parameters of model (6), I take first-order conditions with respect to the prediction error decomposition:

$$l(y_t|\Theta) = \frac{T}{2}\log|\Sigma_\epsilon^{-1}| - \frac{1}{2}\sum_{t=1}^{T}(y_t - \Lambda(\lambda)'\xi_t)'\Sigma_\epsilon^{-1}(y_t - \Lambda(\lambda)'\xi_t)$$
$$+ \frac{T}{2}\log|\Sigma_\eta^{-1}| - \frac{1}{2}\sum_{t=1}^{T}(\xi_t - \Phi\xi_{t-1})'\Sigma_\eta^{-1}(\xi_t - \Phi\xi_{t-1})$$

The following matrix derivative rules are useful for determining the FOC:

$$\frac{\mathrm{d}}{\mathrm{d}\mathbf{A}}\log|\mathbf{A}| = (\mathbf{A}')^{-\mathbf{1}},$$
$$\frac{\mathrm{d}}{\mathrm{d}\mathbf{A}}\mathbf{x}'\mathbf{A}\mathbf{x} = \mathbf{x}\mathbf{x}',$$
$$\frac{\mathrm{d}}{\mathrm{d}\mathbf{A}}\mathrm{Tr}(\mathbf{A}\mathbf{B}) = \frac{\mathrm{d}}{\mathrm{d}\mathbf{A}}\mathrm{Tr}(\mathbf{B}\mathbf{A}) = \mathbf{B}'$$
$$\frac{\mathrm{d}}{\mathrm{d}\mathbf{A}}\mathrm{Tr}(\mathbf{A}'\mathbf{B}') = \frac{\mathrm{d}}{\mathrm{d}\mathbf{A}}\mathrm{Tr}(\mathbf{B}'\mathbf{A}') = \mathbf{B}'$$

Now, taking the partial derivatives to each parameter I obtain the following expressions:

$$\begin{aligned}
0 = \frac{\mathrm{d}l}{\mathrm{d}\Phi} &= -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\Phi}\mathrm{Tr}\left[\Sigma_\eta^{-1}\sum_{t=1}^{T}(\xi_t - \Phi\xi_{t-1})(\xi_t - \Phi\xi_{t-1})'\right] \\
&= -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\Phi}\mathrm{Tr}\left[\Sigma_\eta^{-1}\sum_{t=1}^{T}(\xi_t\xi_t' - \xi_t\xi_{t-1}'\Phi' - \Phi\xi_{t-1}\xi_t' + \Phi\xi_{t-1}\xi_{t-1}'\Phi')\right] \\
&= -\frac{1}{2}\sum_{t=1}^{T}[\Sigma_\eta^{-1}\xi_t\xi_{t-1}' - (\xi_{t-1}\xi_t'\Sigma_\eta^{-1}) + (\xi_{t-1}\xi_{t-1}'\Phi'\Sigma_\eta^{-1})' + \Sigma_\eta^{-1}\Phi\xi_{t-1}\xi_{t-1}'] \\
0 = \frac{\mathrm{d}l}{\mathrm{d}\Sigma_\eta^{-1}} &= \frac{1}{2}\sum_{t=1}^{T}(\xi_t - \Phi'\xi_{t-1})(\xi_t - \Phi'\xi_{t-1})'
\end{aligned}$$

Now, please note that in order to retain comparibility between the DNS-TVLGARCH and the base model, that we need to have a diagonal Σ_ϵ matrix, so we get for the diagonal elements $\sigma_{\epsilon,i}$

$$0 = \frac{\mathrm{d}l}{\mathrm{d}\sigma_{\epsilon,\mathrm{i}}^2} = -\frac{T}{2}\sigma_{\epsilon,i} + \frac{1}{2\sigma_{\epsilon,i}}\sum_{t=1}^{T}a_t(j)'a_t(j) \quad \text{where } a_t(j) = (y_t - \Lambda(\lambda)'\xi_t)_j$$

which solutions are given by:

$$\begin{aligned}
&\iff \Phi = \left(\sum_{t=1}^{T}(\xi_t\xi_{t-1}'\right)\left(\sum_{t=1}^{T}\xi_{t-1}\xi_{t-1}'\right), \\
&\iff \Sigma_\eta = \frac{1}{T}\sum_{t=1}^{T}(\xi_t - \Phi'\xi_{t-1})(\xi_t - \Phi'\xi_{t-1})', \\
&\iff \Sigma_\epsilon = \frac{1}{T}\sum_{t=1}^{T}(y_t - \Lambda(\lambda)'\xi_t)(y_t - \Lambda(\lambda)'\xi_t)'.
\end{aligned}$$

As we work with conditional expectations, I replace ξ with the following expressions, using the fact that $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y) + \mathrm{Cov}(X, Y)$

$$\begin{aligned}
\mathbb{E}[\xi_t] &= \xi_{t|T}, \\
\mathbb{E}[\xi_t \xi_t'] &= \xi_{t|T}\xi_{t|T}' + P_{t|T}, \\
\mathbb{E}[\xi_t \xi_{t-1}'] &= \xi_{t|T}\xi_{t-1|T}' + P_{t,t-1|T}.
\end{aligned}$$

where

$$P_{t,t-1|T} = P_{t+1|T}P_{t+1|t}^{-1}\Phi P_{t|t}.$$

now, using these conditional expectations and the FOCs, I can derive an analytical expression for the ML estimates for each of the parameters, given the state variable obtained from the Kalman filter and smoother ($\xi_{t|t}$ and $\xi_{t|T}$ respectively):

$$\begin{aligned}
\Phi &= \left(\sum_{t=1}^{T}(\xi_{t|T}\xi_{t-1|T}' + P_{t,t-1|T}\right)\left(\sum_{t=0}^{T-1}\xi_{t|T}\xi_{t|T}' + P_{t|T}\right)^{-1} \\
\Sigma_\eta &= \frac{1}{T}\sum_{t=1}^{T}(\xi_t - \Phi'\xi_{t-1})(\xi_t - \Phi'\xi_{t-1})' \\
\sigma_{\epsilon,i} &= \frac{1}{T}\sum_{t=1}^{T}\left[\left(y_t y_t' - \Lambda(\lambda)'\xi_{t|T}y_t' - y_t\xi_{t|T}'\Lambda(\lambda) + \Lambda(\lambda)'[\xi_{t|T}\xi_{t|T}' + P_{t|T}]\Lambda(\lambda)\right)'\right]_j \\
\Sigma_\epsilon &= diag(\sigma_{\epsilon_i=1:19})
\end{aligned}$$

D Results factor regressions with Noise measure

Table 10: Results of factor regressions

In this table I report the performance of the Fama & French and the Fung & Hsieh factors in explaining the returns of distressed debt hedge funds represented by the EurekaHedge index and the Credit Suisse index. In the left panel, the Fama & French size factor (SMB), value factor (HML), market return factor (Rm) and the momentum factor (MOM) are used together in the explanatory model with the constructed liquidity factor. In the right panel, the Fung & Hsieh trend-following factors (BFF, CuFF & CoFF), equity factors (SP500 & RMS), and the interest-rate related factors (BMF & CSF) are used together with the liquidity factor from the Noise measure. For each factor on each index return, I report the coefficient, the t-statistic of the coefficient and its p-value.

	Fama & French factors							Fung & Hsieh factors					
	EurekaHedge			Credit Suisse				EurekaHedge			Credit Suisse		
	b	$t(b)$	p-value	b	$t(b)$	p-value		b	$t(b)$	p-value	b	$t(b)$	p-value
C	0.007	6.967	0.000**	0.005	5.323	0.000**	C	0.016	2.387	0.009**	0.016	2.823	0.003**
SMB	0.127	3.918	0.000**	0.089	3.223	0.001**	BFF	−0.036	−4.357	0.000**	−0.029	−4.173	0.000**
HML	0.094	2.844	0.003**	0.096	3.426	0.000**	CuFF	−0.004	−0.531	0.298	−0.004	−0.765	0.223
Rm	0.240	9.465	0.000**	0.206	9.585	0.000**	CoFF	−0.012	−1.434	0.077	−0.010	−1.385	0.084
MOM	0.002	0.098	0.461	0.008	0.471	0.319	BMF	0.001	−0.215	0.197	0.000	0.437	0.331**
IML	0.083	2.920	0.002**	0.038	1.590	0.057	SP500	0.114	4.789	0.000**	0.130	4.988	0.000**
							RMS	0.115	4.763	0.000**	0.130	4.975	0.000**
							CSF	−0.005	−2.893	0.002**	−0.005	−3.550	0.000**
							IML	0.098	3.052	0.001**	0.041	1.532	0.064
R^2		0.461			0.437				0.333			0.315	

The standard errors are reported below each estimate. An asterisk (*) denotes significance at a 5% level or less and two asterisks (**) a significance at a 1% level or less.