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Global Portfolio Diversification from a Eurozone Investor's Perspective

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Abstract

In this paper, we investigate global portfolio diversification from a Eurozone investor's perspective. We examine whether it adds value for Eurozone investors to have flexibility in the regional weights of a global equity portfolio instead of basing them on the regional allocation of global stock market indices. We do this by comparing portfolios based on the regional allocation in global stock market indices with various alternative portfolios. We furthermore investigate whether it adds value for Eurozone bond investors to diversify into U.S. bonds by analysing the source of the correlation between U.S. and Eurozone bonds, their hedging qualities and portfolios composed of these bonds. We find that following the regional allocation of global stock market indices is not always ideal. In particular, we find that it can add value to invest less in U.S. equity and more in Canadian, Swiss and Indian equity. We do not find that Eurozone bond investors should diversify into U.S. bonds: U.S. bonds are generally not found to have higher hedging qualities than Eurozone bonds and portfolio optimisation results large allocations to Eurozone bonds.

Keywords: Global equity diversification; global bond diversification; Eurozone investor;

JEL classification: G11; G15

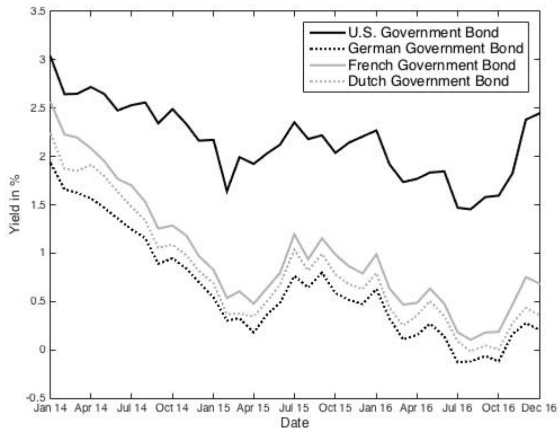
Contents

1 Introduction

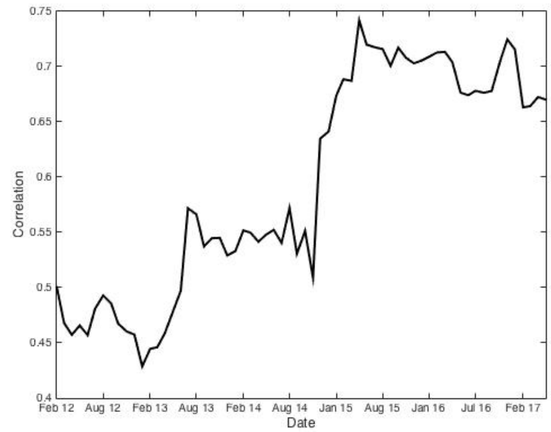
Portfolio diversification plays a prominent role in modern portfolio management for both equity investors and bond investors. It has been known for more than half a century that holding a well-diversified portfolio can significantly reduce an investor's exposure to risk (Markowitz, 1952). Global portfolio diversification in particular has become more popular in recent times due to e.g. globalisation (Campa & Fernandes, 2006; Stulz, 1999). Other possible explanations are the historical benefits of international portfolio diversification (Goetzmann, Li, & Rouwenhorst, 2005): Grubel (1968) and Levy and Sarnat (1970) e.g. find that investors in the 1950's and 1960's could have achieved a better risk-return trade-off by means of international diversification. Established knowledge holds that global diversification of equity adds value in terms of risk-adjusted returns (Litterman & Group, 2004; Madura & O'Brien, 1992; B. H. Solnik, 1995) and reduced risk in general (Heston & Rouwenhorst, 1994; Lessard, 1976; B. Solnik, 2000). This has led to the practice where investors often base the allocation to a particular region in their globally diversified portfolio on how much weight is assigned to that region in a global stock market index (Aon Hewitt Inc., 2015; Arslanalp & Tsuda, 2015; Kwa, 2010). The problem with this for Eurozone investors is that notable global stock market indices are often heavily concentrated in U.S. equity. For instance, the total allocation to American equity in the prominent MSCI World Index is equal to almost 60%. This implies that a Eurozone investor whose regional allocation in his globally diversified portfolio is based on the regional allocation of the MSCI World Index invests the majority of his wealth in U.S. equity. A large allocation to U.S. equity requires large amounts of U.S. dollar hedging for Eurozone investors. This often comes at a cost due to e.g. interest rate spreads (D'Antonio & Howard, 1994), bid-offer spreads (Bush, 2016) and political risks regulations (Bender, Kouzmenko, & Nagy, 2012) and therefore diminishes the benefits of diversifying into U.S. equity.

Where global diversification of equity portfolios is said to be superior to solely investing in domestic equity, the opposite is true for bond portfolios. Global diversification of bond portfolios is not believed to add value compared to only holding domestic bonds (Grauer & Hakansson, 1987; Kaplanis & Schaefer, 1991; Litterman & Group, 2004). Hence, most Eurozone investors only invest in euro-denominated bonds instead of holding a global bond portfolio (De Santis & Gerard, 2006; Schoenmaker & Bosch, 2008). The problem with this, however, is that due to the European Central Bank's quantitative easing programme, bonds denominated in euros currently suffer from historically low yields and compressed credit spreads (Kaya & Meyer, 2013). As Figure 1a illustrates, the yield to maturity of 7-10 years U.S. generic government bonds has been considerably higher than that of the largest Eurozone players of the bond market throughout recent years. Given that the U.S. is by far the largest player in the bond market and that U.S. dollar-denominated bonds nowadays have higher yields than euro-denominated bonds, diversifying into U.S. dollar-denominated bonds might be more profitable for a Eurozone investor than only considering domestic bonds. Bonds that are denominated in other currencies than the euro are furthermore not deemed suitable to hedge euro-denominated pension liabilities. Given the aforementioned recent developments in the Eurozone, however, this consensus may not be justified anymore. Due to their increased profitability relative to Eurozone bonds, U.S. bonds might currently be more suitable to hedge euro-denominated pension liabilities than they are currently believed to be.

In contrast to the large majority of the literature, we investigate global portfolio diversification from a Eurozone investor's perspective. We examine whether it could add value to have flexibility in the regional weights of an investor's global equity portfolio instead of simply basing the portfolio weights on the regional



(a) Yields of Eurozone and U.S. Bonds



(b) Correlation between Eurozone and U.S. Bonds

Figure 1: Yields (to maturity) of Eurozone and U.S. 7-10 years government bonds from January 2014 to April 2017 and correlations between Bloomberg Barclays Eurozone and U.S. bond indices January 2012 to April 2017.

allocation of a global stock market index. We also look for the theoretically optimal global equity allocation of a short-term, medium-term, and long-term Eurozone investor. As some investors only invest in developed markets, we first perform our research with only developed markets equity in the asset menu. Because diversifying into emerging markets can be beneficial for developed markets investors (Errunza, 1983; Meriç, Ding, & Meriç, 2016; Sappenfield & Speidell, 1992), we afterwards perform our analyses with both developed and emerging markets equity. Furthermore, we investigate whether it would add value for a euro-denominated investor to diversify into U.S. bonds. Because corporate bonds are considered to be more rewarding but also more risky than government bonds (Detzler, 1999), we incorporate and analyse both types of bonds in our research. We furthermore analyse the inflation and interest rate hedging qualities of bonds from a Eurozone point of view and also look for the theoretically optimal bond portfolio weights of both an asset-only investor and an asset-liability investor. As a lot of currency hedging is involved when investing globally, we simultaneously analyse to what extent the level of currency hedging affects the results while we investigate global equity and bond diversification.

This research is of value to the existing literature on global diversification and currency hedging because we approach our research from the perspective of a Eurozone investor, whereas the majority of the existing literature on international portfolio considers a U.S. based investor (e.g. Bekaert and Urias (1996); Glen and Jorion (1993); Huberman and Kandel (1987); Laopodis (2005)). Furthermore, because of the increasing popularity of investing internationally (Davis & Marquis, 2005; Poser, 2001; Schulz & Wolff, 2008), it is nowadays of great value to get a better understanding about (the benefits of) global diversification. The part of this research concerning bonds is particularly interesting now, as the European Central Bank's quantitative easing programme, that is responsible for the low profitability of Eurozone denominated bonds, was introduced quite recently and is expected to run until at least 2018 (Khan, 2017). The analysis of bond diversification is also of interest in other times when U.S. bonds are more profitable than Eurozone bonds. Examples of such times are periods of major economic or political events that affect interest rates and therefore indirectly affect bond returns. The inclusion of emerging markets in this research also makes it relevant in these times, their role in financial markets has grown rapidly over the last years and is expected to be even greater in the future (Hartmann & Khambata, 1993). Besides being of scientific relevance, this research is also of interest for practical application. As this research approaches global portfolio diversification from a Eurozone investor's perspective, it is particularly relevant for individual and institutional investors (e.g. pension funds and insurance companies) based in the Eurozone. The analysis of the interest rate hedging qualities of bonds in particular could be of interest for pension funds, as pension funds often want to hedge their liabilities against adverse changes in interest rates. The methodology of this research can furthermore be performed with another base currency than the euro to approach the research from the perspective of an investor based in another region.

We use monthly data throughout our entire research because monthly financial time series contain considerably more data points than their annual counterpart and are often more complete than daily time series. The sample period of our research begins in January 2002 and runs until December 2016. For the part of our research focused on global diversification of equity, we consider the main stock market indices of 10 developed markets and 5 emerging markets. For the part of our research focused on global diversification of bonds, we use data on U.S. bonds and the bonds of the aforementioned developed markets that are in the Eurozone. We consider 7-10 years government bonds and several government and corporate bond indices. Furthermore, data on several corresponding state variables (dividend yield, inflation rate, short-term interest rate and long-term interest rate) is required for both the analysis of equity and bonds. Because Dutch pension funds generally use interest rate swaps to hedge against adverse change in interest rates (De Horde, 2016; Duyvesteyn, 2012), we use euro interest rate swaps as a proxy for euro-denominated pension fund liabilities.

In order to find out whether it could add value to have flexibility in the regional weights of a globally diversified portfolio, we consider four types of portfolios that are relatively easy to construct and are frequently used in practice: the equal-weighted portfolio, a value-weighted portfolio, the global minimum variance portfolio and the mean-variance portfolio. Our value-weighed portfolio is based on the gross domestic product of each country in our sample. As we approach the research problem from a Eurozone investor's point of view, we also consider a portfolio that is heavily concentrated in the equity of Eurozone countries. We then compare the performances of each of these portfolios with that of a portfolio based on a global stock market index. The two global stock market indices we consider are the MSCI World Index, which only incorporates developed markets, and the MSCI (ACWI) Index, which also includes emerging markets. These global stock market indices are both often used by investors as benchmarks to decide their global equity allocations and are furthermore often considered in research (see e.g. Bekaert, Erb, Harvey, and Viskanta (1998); Gahlot and Datta (2011); Hau, Massa, and Peress (2010)). We evaluate portfolios by means of (in-sample) historical average returns, volatilities, Sharpe ratios, Value at Risk estimates, Expected Shortfall estimates, and the estimated fee a risk-averse investor would be willing to pay to switch from one portfolio to another. Furthermore, we find theoretically optimal dynamic portfolio weights by means of the recursive analytical solution of Jurek and Viceira (2011). Where the empirical part of their research considers a U.S. investor who invests in domestic assets, we compute the portfolio weights for a Eurozone equity investor who can invest internationally. Nevertheless, the methodology of Jurek and Viceira (2011) suits our research well, as it can be used for any kind of assets and can furthermore flexibly take a large number of assets into account where numerical methods often fail to.

We follow the framework of Viceira, Wang, and Zhou (2017) with Eurozone and U.S. bonds as investable assets to find out whether it adds value for a euro-denominated investor to diversify into American bonds. Cross-country correlations of global bond markets have increased significantly in recent times (Asness, Israelov,

& Liew, 2011; Quinn & Voth, 2008), which suggests that the benefits of global bond diversification have declined in recent times. Figure 1b illustrates that the correlation between U.S. dollar- and euro-denominated bonds have increased throughout recent years. As the source of the cross-country correlations determines to what extent the correlations between bonds are detrimental to a global bond investor, Viceira et al. (2017) explore the sources of the cross-country correlations of various global bond markets. While Viceira et al. (2017) approach their research from the perspective of an American investor who invests internationally, we consider a Eurozone investor who can diversify into U.S. bonds. As many institutional investors are pension funds with liabilities that are subject to interest rate risk (Hoevenaars, Molenaar, Schotman, & Steenkamp, 2008), we furthermore follow the framework of Hoevenaars et al. (2008) to analyse the hedging qualities of U.S. dollar- and euro-denominated bonds. More specifically, we investigate the interest rate hedging qualities by analysing the correlation between bond returns and euro pension liability returns, while we additionally investigate the inflation hedging qualities of bonds by analysing the correlation between bond returns and euro inflation. The framework of Hoevenaars et al. (2008) is also used to find the theoretically optimal global bond portfolio of both an asset-only investor and an asset-liability investor with euro-denominated pension liabilities. While Hoevenaars et al. (2008) compare various U.S. assets with each other, we use their methodology to compare Eurozone bonds with U.S. bonds.

In order to examine to what extent the level of currency hedging affects the results, the analysis of both global equity diversification and global bond diversification is done for a range of hedge ratios between 0% (unhedged) and 100% (fully hedged). The results are then compared across hedge ratios. We compute the currency hedged returns of foreign assets in the manner of Campbell, de Medeiros, and Viceira (2010), who consider an investor who hedges foreign assets by entering into forward contracts.

The mean-variance portfolio and especially the global minimum variance portfolio are found to generally outperform the portfolio based on the MSCI indices. This implies that having flexibility in the regional weights of a globally diversified portfolio can be superior to simply basing the portfolio weights on the regional allocation of a global stock market index. The high performances of these portfolios also imply that a Eurozone investor should primarily invest in Canadian, Swiss and Indian equity instead of U.S. equity. The weights of the theoretically optimal portfolio are comparable to those of the aforementioned global minimum variance and mean-variance portfolios. Most weight is allocated to Canadian, Hong Kong, Swiss and British equity. This makes sense given given the relatively high Sharpe ratios corresponding to these countries. On the other hand, French, German, Dutch, Brazilian and American stocks are generally short sold. The negative weights assigned to the Eurozone and Brazilian equity is due to their low Sharpe ratios, while U.S. equity is shorted due to the generally high correlation with other stocks. The effect of currency hedging to the euro on the composition and performance of the portfolios differs per portfolio. We do find some general patterns, however, such as that Sharpe ratios are highest and Value at Risk estimates lowest for the higher hedge ratios.

The benefits of global diversification between Eurozone and U.S. bonds in terms of portfolio risk have declined equally for short-term and long-term investors. This holds for all currency hedge ratios and for both government and corporate bonds. Currency hedged U.S. bonds are furthermore found to have a higher inflation hedging qualities than the Eurozone bonds, while unhedged U.S. bonds are a poor inflation hedging tool. Government bonds are furthermore found to be better inflation hedging instruments in the long run, while corporate bonds have higher inflation hedging qualities for short investment horizons. All Eurozone bonds considered are found to have higher euro interest rate hedging qualities than U.S. bonds. Also, government bonds are considerably better interest rate hedging instruments than corporate bonds in the short run, while the hedging qualities in the long run are similar. The theoretically optimal bond portfolio of the asset-only investor is primarily concentrated in German government bonds and Eurozone corporate bonds due to their high Sharpe ratios and low volatilities. The theoretically optimal asset-liability portfolio on the other hand is mostly concentrated in Dutch equity due to its relatively high hedging qualities. The effect of currency hedging on the theoretically optimal bond portfolios is furthermore found to be minimal due to the large allocation to Eurozone bonds.

All in all, while global diversification in general is found to be beneficial for Eurozone equity investors, following the regional allocation of global stock market indices is not always ideal. In particular, we find that it can add value to invest less in U.S. equity and more in other stocks. Portfolios concentrated in Canadian, Swiss and Indian equity in particular found to perform best. Diversifying into U.S. bonds is furthermore not found to clearly be more beneficial for a Eurozone investor than only holding domestic bonds: U.S. bonds are generally not found to have higher hedging qualities than the Eurozone bonds we consider and portfolio optimisation results in portfolios that are mostly concentrated in Eurozone bonds.

The remainder of this report is organised as follows: Section 2 presents and discusses the data used in our research, Section 3 describes our methodology, Section 4 presents the results of our research on the global diversification of equity, Section 5 presents the results corresponding to the global diversification of bonds, Section 6 concludes our research and Section 7 finally presents limitations of our research and avenues for further research.

2 Data

Section 2.1 gives an extensive description of all the data used in this research. Section 2.2 presents all the necessary data transformations applied before executing the methods described in the methodology section. Finally, Section 2.3 provides and discusses summary statistics of the equity and bond returns to give an impression of the assets considered in this research.

2.1 Data Description

For the part of our research focused on equity, we consider total return indices with net dividends of the stock market indices of various countries. We choose for total return indices instead of price indices because the latter only considers price movements while the former also incorporates interest, dividends, rights offerings and other distributions. Net dividends are chosen over gross dividends because the former incorporates taxes where the latter does not. For each country, we consider the most prominent stock market index of which we can find a sufficient number of observations as a proxy for the equity of that country. Because we compare several portfolios with a portfolio of which the regional allocation is based on that of the MSCI World and MSCI ACWI indices, we only consider countries that that are incorporated in these indices. In order to have time series long enough to produce reliable result and to avoid dimensionality issues when applying the methodologies of Jurek and Viceira (2011), Viceira et al. (2017) and Hoevenaars et al. (2008), we only consider a selection of the countries incorporated in the MSCI indices. The developed markets included in this research are the 10 markets to which the most weight is allocated to in the MSCI World Index. The emerging markets we consider are the 5 emerging markets to which the most weight is allocated to in the MSCI ACWI Index. Leaving the other countries out of our sample has no large consequences for our research, as the combined weights of the countries in our sample is equal to almost 95% for the MSCI World Index and 92% for the MSCI ACWI Index. Table 1 lists all countries considered in our research with the corresponding abbreviations used throughout this report.

Table 1: Market overview and the corresponding abbreviations

Developed Markets Equity		Emerging Markets Equity		Bonds	
Country	Abbreviation	Country	Abbreviation	Country	Abbreviation
Australia	AU	Brazil	BR	France	FR
Canada	CA	China	CN	Germany	DE
France	FR	India	IN	The Netherlands	NL
Germany	DE	South Korea	KR	The United States	US/US$_G$/US$_C$
Hong Kong	HK	Taiwan	TW	Eurozone	EU$_G$/EU$_C$
Japan	JP				
The Netherlands	NL				
Switzerland	CH				
The United Kingdom	UK				
The United States	US				

The subscripts G and C denote Bloomberg Barclays government and corporate indices, respectively.

As can be seen from Table 1, our data set contains a large variety of countries located all over the globe. The developed markets we consider are the largest stock markets in Europe, North-America and the Pacific Rim. The emerging markets in our research consist of Brazil and the TICK (Taiwan, India, China, (South) Korea) countries, which are said to be the current heavyweights among emerging markets (Johnson, 2016).

We consider several Bloomberg Barclays bond market return indices in our research: the US Government Index, the US Corporate Index, the Euro Aggregate Government-Related Index and the Euro Aggregate Corporate Index. Because the Euro Aggregate Government-Related Index does not take into account the heterogeneity among Eurozone countries, we also consider 7-10 years government bonds of the Eurozone countries in our data set (France, Germany and the Netherlands). We furthermore consider 7-10 years generic government bonds of the U.S. to match the maturity of the generic government bonds of the Eurozone countries. As there are no country-specific corporate bond indices available for our Eurozone countries with an extensive data history, we use the Eurozone aggregated corporate bond index for all analyses involving corporate bonds.

A set of state variables is needed when using the methodology of Jurek and Viceira (2011), Viceira et al. (2017) and Hoevenaars et al. (2008). For the case of global equity diversification, we use the dividend yield of the stock market index and the (month-over-month) inflation rate and short-term interest rate in the corresponding market as state variables. Economic time series in general (Franses, Dijk, & Opschoor, 2014) and inflation time series in particular (Bryan & Cecchetti, 1995) are widely known to contain some form of seasonality. We have therefore obtained seasonally adjusted time inflation series where possible and correct the inflation rates of the countries for which no seasonally adjusted series are available. As is common in practice and literature (see e.g. Moreni and Pallavicini (2014), Chevallier (2010) and Mandler (2002)), we use the 1-month Euro Interbank Offered Rate (Euribor) as a proxy for the short-term Eurozone interest rate. We consider the 1-month interbank deposit rate for all other countries except Brazil, China, India and Korea, for which no data on 1-month deposit rates over our entire sample period could be found. We therefore use the 3-month deposit rates divided by 3 for China, India and Korea and use the target interest rate set by the Brazilian central bank as the short-term interest rate of Brazil. The 1-month Euribor rate is furthermore used as risk-free rate because we approach our research from a Eurozone investor's point of view and the short-term Euribor rate is commonly used as a benchmark for the risk-free rate of a Eurozone investor. As yield spreads are required for the analysis of global bond diversification, we furthermore require the yields to maturity of all bonds we consider in our research.

As Dutch pension funds often use interest rate swaps to hedge against adverse change in interest rates (De Horde, 2016; Duyvesteyn, 2012), we consider euro interest rate swap (IRS) returns as a proxy for euro-denominated pension liability returns. The use of interest rate swap returns as a proxy for pension liabilities is also justified by the fact that pension fund liabilities are the present value of future obligations discounted at the real interest rate (Hoevenaars et al., 2008). To analyse to what extent the duration of the pension liabilities affects the results, we furthermore consider both 20 and 30 year IRS returns. The former corresponds to a duration of approximately 17 years as of June 2017, while the latter corresponds to a duration of approximately 25 years. We assume that the duration of pension liabilities matches that of the IRS returns.

As our value-weighted portfolio is based on each country's gross domestic product (GDP) relative to the total GDP of our sample, we also require historical data on the GDP of each country in our sample. To correctly compare the GDP's of countries with varying currencies, the GDP of each country must be measured in the same currency. An obvious choice for the currency to uniformly measure the GDP in is the U.S. dollar, as a large part of the obtained GDP data is measured in U.S. dollar.

Finally, we require foreign exchange data to analyse the effect of hedging for both all-equity and all-bond investors. The foreign exchange data consists of spot rates and forward points for all data points in our sample period. The forward points are solely used to compute forward rates.

We have obtained the seasonally adjusted consumer price index (CPI) time series of the U.S. from the publicly accessible Federal Reserve Economic Data (FRED) database. The GDP and (not seasonally adjusted) CPI data of India are obtained from the also publicly accessible database of the Organisation for Economic Co-operation and Development (OECD) and The World Bank, respectively. The Bloomberg Barclays bond indices used in our research are taken from Barclays Live. All other data is retrieved from the Bloomberg Professional Services database.

2.2 Data Transformations

We require the returns of all equity and bond indices in our data set. For time t, the return of a stock or bond index is computed as follows for country i:

$$R_{i,t} = \frac{P_{i,t}}{P_{i,t-1}} - 1, \tag{1}$$

where $P_{i,t}$ denotes the value of the total return index corresponding to country i at time t. Inflation rates are computed in the same manner, only with the consumer price index instead of the total return index.

As the inflation rates of France, Germany, Hong Kong, the Netherlands, Switzerland, Brazil, China, India and Taiwan are not seasonally adjusted, we adjust them for seasonality manually. We follow Brockwell and Davis (2016) and estimate and eliminate the seasonal component in the inflation series. To this end, we first estimate the trend component m_t in the inflation series with period $d = 2q = 12$ (months per year) for each month t as following:

$$\hat{m}_t = \frac{1}{d}(\frac{1}{2}\pi_{t-q}^{NSA} + \pi_{t-q+1}^{NSA} + \ldots + \pi_{t+q-1}^{NSA} + \frac{1}{2}\pi_{t+q}^{NSA}), \quad q < t \leq T - q, \tag{2}$$

where π_t^{NSA} denotes the not seasonally adjusted (NSA) inflation rate at time t and n is the number of observations. For each $k = 1 \ldots d$, we then compute the average ω_k of the deviations of the NSA inflation series from the trend, which are defined as follows:

$$(\pi_{k+jd}^{NSA} - \hat{m}_{k+jd}), \quad q < k + jd \leq n - q, \; j = 1, \ldots, \frac{T}{d}.$$

We can then estimate the seasonal component s_t for each month t:

$$\hat{s}_t = \begin{cases} \omega_t - \frac{1}{d}\sum_{i=1}^{d}\omega_i & \text{for } 1 \leq t \leq d \\ \hat{s}_{t-d} & \text{for } d < t \leq T. \end{cases} \tag{3}$$

The inflation data is then adjusted for seasonality by removing the seasonal component from the unadjusted inflation series:

$$\pi_t^{SA} = \pi_t^{NSA} - \hat{s}_t. \tag{4}$$

Because the frameworks of Viceira et al. (2017), Jurek and Viceira (2011) and Hoevenaars et al. (2008) we follow in this research are based on a logarithmic portfolio return framework and log returns are generally often preferred over regular returns in financial literature (Brooks, 2002), we transform all returns in our research into log returns as follows:

$$r_{i,t} = \log(1 + R_{i,t}), \tag{5}$$

where $R_{i,t}$ is the asset return of asset i at time t. We refer to the natural logarithm whenever logarithms (or logs) are mentioned in this research. We do not directly compute log returns from the return indices because non-logarithmic returns are required for the currency hedging method we use. The logarithms of bond yields, dividend yields and inflation rates are computed in the same manner. As the interest rates we have obtained are annualised while all other data is measured on a monthly basis, we follow Viceira et al. (2017) and compute monthly log interest rates as follows:

$$y_{i,t}^{s} = \frac{\log(1 + Y_{i,t}^{s,an})}{12}, \tag{6}$$

where $Y_{i,t}^{s,an}$ denotes the annualised interest rate of country i at time t. Furthermore, log yield spreads are required for the analysis of global diversification of bonds. In line with Viceira et al. (2017) among others, we compute log yield spreads by taking the difference of the log short-term yield and log long-term yield:

$$ys_{i,t} = y_{i,t}^{l} - y_{i,t}^{s}, \tag{7}$$

where $y_{i,t}^{s}$ denotes the log short-term yields (interest rates) at time t for country i, respectively. Furthermore, $y_{i,t}^{l}$ denotes the yield to maturity of a 10 year government bond index when we consider government bonds, while it denotes the yield to maturity of a corporate bond index when we consider corporate bonds.

All GDP time series and the CPI time series of Australia, France, Germany, the Netherlands and Switzerland are only available on a quarterly basis. As the rest of the data is observed on a monthly basis, we transform the quarterly data to monthly data by means of interpolation. The interpolation method we apply is cubic spline interpolation (De Boor, 1978), as this method is commonly used in practice.

We require both spot and forward rates with respect to the euro to compute currency hedged returns. While spot rates can be retrieved from the Bloomberg Professional Services database, forward rates must be computed manually as follows for each time t and currency i:

$$F_{i,t} = S_{i,t} + \frac{1}{x} FP_{i,t}, \tag{8}$$

where $S_{i,t}$ and $FP_{i,t}$ denote the spot rate and forward points, respectively, at time t for currency i. Furthermore, x is equal to 1 when the currency under consideration is the South Korean won or the New Taiwan dollar, 100 when we consider the Japanese Yen and 10,000 for every other currency.

2.3 Summary Statistics

Panel A of Table 2 shows the annualised historical average returns, volatilities, Sharpe ratios and first-order autocorrelations of unhedged log equity returns of the developed markets considered in this research. Panel B contains the same statistics for the unhedged log equity returns of the emerging markets we consider. Panels C and D present the summary statistics for fully hedged returns. The log equity returns of the Eurozone countries have relatively low average returns and high volatilities. Hence, the lowest Sharpe ratios correspond to the Eurozone countries. The stocks of all other countries besides the U.K., Brazil and China have considerably higher Sharpe ratios. The highest Sharpe ratios belong to Australian, Canadian, Hong Kong, Indian and Korean equity. Because the Australian and Canadian stocks have relatively high expected log returns and low volatilities, we expect that the mean-variance portfolio will be quite concentrated in the equity of these two countries. The global minimum variance portfolio can be expected to be quite concentrated in Swiss equity, as the stocks of Switzerland have the lowest volatility of all and furthermore have relatively low correlations with most non-Eurozone countries (see Section B.1). As could have been expected, the volatilities of the emerging markets stocks are considerably higher than those of developed markets stocks, which shows why some investors shun emerging markets equity. As all first-order autocorrelations presented in Table 2 are quite small, all log return series seem to be stationary.

To get an idea of how currency hedging will affect our results, we also have a look at the summary statistics of (fully) hedged returns. The currency hedging procedure in our research is that of Campbell et al. (2010) and is described in Section 3.3. We find that the average return of half of the non-Eurozone countries are lower for fully hedged returns than for unhedged returns while the average returns are higher for the other half. Slightly more countries see their volatility increase when increasing the hedge ratio from 0 to 100% than decrease. In terms of Sharpe ratios, half of the non-Eurozone countries presented in Panels C and D of Table 2 perform worse than when the stocks of these countries are unhedged. For the Sharpe ratios across all hedge ratios between 0 and 100%, we refer the reader to Figure A1 in Appendix A.

We also consider a number of corporate and government bonds in our research. Table 3 presents the annualised historical average returns, volatilities, Sharpe ratios and first-order autocorrelations of the log returns of all the bonds considered in this research. The summary statistics of the Eurozone bonds are very similar to

Table 2: Summary statistics of unhedged log equity returns

Panel A: Unhedged Developed Markets Equity										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
Average Return (%)	8.83	7.15	3.60	5.33	6.71	5.17	3.15	6.36	3.50	5.35
Volatility (%)	18.13	17.37	17.81	21.47	18.86	16.64	20.23	13.44	14.40	14.31
Sharpe Ratio	0.33	0.27	0.10	0.15	0.24	0.19	0.06	0.26	0.10	0.20
ACF(1)	0.15	0.08	0.10	0.07	0.08	0.01	0.09	0.15	0.14	0.16

Panel B: Unhedged Emerging Markets Equity					
	BR	CN	IN	KR	TW
Average Return (%)	6.52	6.17	11.87	8.36	6.21
Volatility (%)	35.20	28.17	26.90	24.42	20.76
Sharpe Ratio	0.12	0.12	0.35	0.25	0.21
ACF(1)	0.04	0.09	0.05	0.03	0.09

Panel C: Fully Hedged Developed Markets Equity										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
Average Return (%)	5.06	6.90	3.60	5.33	7.25	5.56	3.15	5.06	4.48	5.75
Volatility (%)	12.96	12.94	17.81	21.47	20.99	20.24	20.23	14.06	13.86	14.67
Sharpe Ratio	0.21	0.34	0.10	0.15	0.25	0.17	0.06	0.19	0.19	0.24
ACF(1)	0.11	0.22	0.10	0.07	0.13	0.16	0.09	0.20	0.02	0.15

Panel D: Fully Hedged Emerging Markets Equity					
	BR	CN	IN	KR	TW
Average Return (%)	1.49	4.95	15.36	8.07	7.61
Volatility (%)	24.14	29.06	23.76	19.62	19.55
Sharpe Ratio	-0.00	0.09	0.56	0.33	0.29
ACF(1)	0.10	0.09	0.06	0.02	0.10

Note: average returns, volatilities and Sharpe ratios are annualised. ACF(1) denotes the first-order autocorrelation of the log return series. Unhedged returns are converted to euros.

each other. Relatively high average returns and low volatilities result in Sharpe ratios around the value of 1. The U.S. bonds on the other hand have considerably lower Sharpe ratios than their Eurozone equivalent. This holds for both government and corporate bonds. As companies can take more risk than governments, it is not surprising that the Sharpe ratios of the U.S. corporate bonds are larger than those of U.S. government bonds. A possible explanation for the fact that this does not hold for Eurozone bonds is that the Eurozone government bonds are very rewarding due to the relatively low Eurozone interest rates. The effect of currency hedging the U.S. bonds is strong: fully hedged U.S. bonds have considerably higher average returns and far lower volatilities than their unhedged counterparts. The first order autocorrelations are once again quite low and therefore show no evidence of non-stationarity in the bond return series.

We furthermore compute the correlations between the return series of the assets we consider in our research. For the correlations between equity return series, we refer the reader to Section B.1 of the Appendix. Appendix B.2 presents the correlations between the returns of the bonds we consider in our research for both unhedged

Table 3: Summary statistics of log bond returns

	FR	DE	NL	US	US_H	EU_G	US_G	$\text{US}_{G,H}$	EU_C	US_C	$\text{US}_{C,H}$
Average Return (%)	6.00	5.80	5.93	4.00	4.89	4.91	2.91	3.83	4.70	4.34	5.12
Volatility (%)	4.98	4.85	4.97	11.57	6.62	3.77	10.55	4.18	3.60	10.01	5.85
Sharpe Ratio	1.02	0.96	0.99	0.21	0.58	0.92	0.13	0.58	0.69	0.26	0.58
ACF(1)	0.01	0.05	-0.02	0.05	0.02	0.10	0.03	0.04	0.14	-0.08	0.19

Note: average returns, volatilities and Sharpe ratios are annualised. ACF(1) denotes the first-order autocorrelation of the log return series. Unhedged returns are converted to euros and a subscript H denotes fully hedged returns.

and fully hedged returns. The correlations seem reasonable for both stock and bond returns and show expected patterns such as high correlations between European assets and higher correlations for fully hedged returns than unhedged returns.

3 Methodology

This section describes the complete methodology of our research. Section 3.1 describes the methodology behind our analyses on the global diversification of equity. We then turn our attention to the global diversification of bonds in Section 3.2. Section 3.3 describes how we compute currency hedged returns and how we analyse the effect of currency hedging on global portfolio diversification.

3.1 Global Diversification of Equity

Section 3.1.1 begins with the introduction of the portfolios with the regional allocation based on the MSCI World and MSCI ACWI indices before describing five alternative global equity portfolios and how we compute their weights. The computation of the theoretically optimal weights of an investor's global dynamic portfolio is described in Section 3.1.3. Section 3.1.4 presents the various methods of shrinkage we apply in our research to control for error maximisation when computing portfolios. Finally, Section 3.1.2 describes the statistical, economical and risk measures we use to compare portfolios.

3.1.1 Constructing the Portfolios

We consider two MSCI global stock market indices in this research. The portfolio based on the regional allocation of the MSCI World Index will be referred to as the MSCI World portfolio throughout the remainder of this report, while the portfolio based on the MSCI ACWI Index will be referred to as the MSCI ACWI portfolio. For each country, the corresponding weight in the MSCI World portfolio is then computed as follows:

$$w_{MW,i,t} = \frac{\sum_{k=1}^{M_i} w_{MSCI,i,k}}{\sum_{j=1}^{N} \sum_{k=1}^{M_j} w_{MSCI,j,k}}, \tag{9}$$

where $w_{MSCI,i,k}$ is the weight in the MSCI World Index assigned to asset k corresponding to country i, M_i the number of assets of country i incorporated in the MSCI World Index and N the number of countries we consider. Similarly, the weights of the MSCI ACWI portfolio are computed as follows:

$$w_{MA,i,t} = \frac{\sum_{k=1}^{M_i} w_{ACWI,i,k}}{\sum_{j=1}^{N}\sum_{k=1}^{M_j} w_{ACWI,j,k}}, \tag{10}$$

where $w_{ACWI,i,k}$ is the weight in the MSCI ACWI Index assigned to asset k corresponding to country i. Constructing the MSCI portfolios based on (9) and (10) ensures that the weights still add up to 100% while the interrelations between the country weights in the MSCI indices are preserved.

We compute the returns of five portfolios that are relatively easy to construct to compare them with the MSCI-based portfolios. The first four are often used in practice while the fifth is relevant for this research because we approach the problem from the perspective of a Eurozone investor. An obvious first choice is the notoriously hard to beat equal-weighted portfolio. The weights of the equal-weighted portfolio are constant across time and assets:

$$w_{EW,i,t} = \frac{1}{N}, \tag{11}$$

where N is the number of assets (countries in our case) in the portfolio and i denotes asset i. The equal-weighted portfolio is often used in practice (Huberman & Jiang, 2006; Thaler & Benartzi, 2001) and is empirically known to perform quite good (DeMiguel, Garlappi, & Uppal, 2009). As the equal-weighted portfolio makes no distinction between the assets in a portfolio, it is worth considering a value-weighted portfolio that assigns more weight to potentially more profitable assets. The value-weighted portfolio is frequently considered in literature (e.g. Asness (1997); Chen, Kan, and Miller (1993); French and Poterba (1991)) and its popularity among investors is increasing globally (Bhattacharya & Galpin, 2011). The value-weighted portfolio we consider in this research is based on the proportion of each country's GDP relative to the total GDP of our sample:

$$w_{VW,i,t} = \frac{GDP_{i,t}}{\sum_{j=1}^{N} GDP_{j,t}}, \tag{12}$$

where GDP_i denotes the GDP of country i.

We also consider the modern portfolio theory introduced by Markowitz (1952). In particular, we consider the global minimum variance (GMV) portfolio and the mean-variance (MV) portfolio. Even though the GMV and MV portfolios are theoretically optimal for one investment period, they can be easily compared with the other portfolios we consider because the other portfolio weights are all constant across investment periods. An investor's GMV portfolio has the lowest risk of all portfolios on the investor's efficient frontier. The GMV portfolio has consistently been advocated in literature (e.g. by Jagannathan and Ma (2003) and Ledoit and Wolf (2003)). Some strengths of the GMV portfolio are that it follows the Markowitz (1952) framework of searching an efficient portfolio by diversification and that it does not require the estimation of expected asset returns, which reduces the impact of estimation errors (Frahm, 2010). The portfolio weights of the global minimum variance portfolio at time t can be obtained by solving the following minimisation problem:

$$\begin{gathered}\min_{\boldsymbol{w}_t} \sigma^2_{p,t} = \boldsymbol{w}_t' \boldsymbol{\Sigma}_t \boldsymbol{w}_t, \\ \boldsymbol{w}_t' \boldsymbol{\iota} = 1,\end{gathered} \tag{13}$$

where $\boldsymbol{w}_t$, $\sigma_{p,t}$, and $\boldsymbol{\iota}$ denote the vector of portfolio weights, the portfolio volatility and a vector of ones, respectively. Furthermore, $\boldsymbol{\Sigma}_t$ denotes the covariance matrix of the asset returns computed with all information up to and including time t. The vector of GMV portfolio weights at time t are then defined as follows:

$$\boldsymbol{w}_{GMV,t} = \frac{\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\iota}}{\boldsymbol{\iota}'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\iota}}. \tag{14}$$

Appendix C contains the derivation of the GMV portfolio weights (14). We compute the GMV portfolio weights with the sample covariance matrix of the returns instead of the true covariance matrix $\boldsymbol{\Sigma}_t$. An investor with mean-variance preferences wants to make the optimal trade-off between the expected return of his portfolio and its risk. The following maximisation problem must be solved to obtain the optimal asset allocation in the MV portfolio at time t:

$$\begin{gathered}\max_{\boldsymbol{w}_t} \mu_{p,t} - \frac{\gamma}{2}\sigma_{p,t}^2 = \boldsymbol{w}_t'\boldsymbol{\mu}_t - \frac{\gamma}{2}\boldsymbol{w}_t'\boldsymbol{\Sigma}_t\boldsymbol{w}_t, \\ \boldsymbol{w}_t'\boldsymbol{\iota} = 1,\end{gathered} \tag{15}$$

where γ is the investor's coefficient of relative risk aversion, $\boldsymbol{\mu}_t$ denotes the expected asset returns of the asset returns computed with all information up to and including time t and all other variables are defined as before. The vector of MV portfolio weights at time t then has the following definition:

$$\boldsymbol{w}_{MV,t} = \frac{1}{\gamma}\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}_t + \frac{\gamma - \boldsymbol{\mu}_t'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\iota}}{\boldsymbol{\iota}'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\iota}}. \tag{16}$$

The derivation of the MV portfolio weights (16) can be found in Appendix D. We compute the MV portfolio weights by plugging in the sample mean vector and sample covariance matrix in (16). As we approach the research problem from a Eurozone investor's point of view, we also consider a portfolio heavily concentrated in equity of the Eurozone countries in our sample. This portfolio will be referred to as the euro portfolio throughout the remainder of this report and is constructed by spreading 50% of the portfolio weights evenly among the three Eurozone countries in our sample and spreading the remaining 50% over the other countries:

$$w_{EU,i,t} = \begin{cases} \frac{0.5}{N_1} & \text{if country } i \text{ is in the Eurozone,} \\ \frac{0.5}{N_2} & \text{if country } i \text{ is not in the Eurozone,} \end{cases} \tag{17}$$

where N_1 and N_2 are equal to the total number of euro countries in our sample and the total number of non-euro countries, respectively. As N_1 and N_2 add up to the total number of countries, the specification of (17) ensures that the weights of the euro portfolio add up to 100%.

3.1.2 Comparing the Portfolios

After computing the returns of the five alternative portfolios described in Section 3.1.1, we compare them with the portfolios of which the regional allocation is based on the MSCI World and MSCI ACWI indices by means of several (in-sample) of statistical, economic and risk measures. The first two measures we consider are the annualised average return and annualised volatility. The former is obtained by computing the monthly average of the log portfolio returns and multiplying by 12:

$$\mu_A = 12 * \frac{1}{T} \sum_{t=1}^{T} r_t, \tag{18}$$

where T denotes the number of time periods and r_t is the log portfolio return at time t. For each portfolio, the annualised volatility is obtained by computing the (sample) standard deviation of the monthly log returns and multiplying by $\sqrt{12}$:

$$\sigma_A = \sqrt{12} * \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (r_t - \bar{r})^2}, \tag{19}$$

where $\bar{r}$ denotes the (monthly) average log returns.

The next measure we consider is the Sharpe ratio (Sharpe, 1994). The Sharpe ratio is a measure of risk-adjusted return often considered in both literature and in practice and is defined as follows:

$$SR = \frac{\mu - R_f}{\sigma}, \tag{20}$$

where μ and σ respectively denote the (monthly) mean and standard deviation of the (log) portfolio returns and R_f is the risk-free rate. We compute monthly Sharpe ratios with the monthly average returns and sample standard deviation before converting them to annualised units in the manner of Lo (2002). In order to use this method, we need to assume that the return series are not independent and identically distributed (i.i.d.). This assumption is not unrealistic, as the assumption of i.i.d. returns in financial data has often been found to be violated (MacKinlay & Lo, 1999). Converting Sharpe ratios from one frequency to another is usually done by multiplying the higher-frequency Sharpe ratio by the square root of the number of time periods in the holding period of the lower-frequency Sharpe ratio (e.g. multiply monthly Sharpe ratios by $\sqrt{12}$ to obtain annual estimates). However, Lo (2002) shows that this method of time aggregation of Sharpe ratios is incorrect under the assumption of non-i.i.d. returns. The method of Lo (2002) on the other hand accounts for the autocorrelation in the return series and is therefore correct in the case of non-i.i.d. returns. According to Lo (2002), monthly Sharpe ratios are properly converted to annualised units as follows:

$$SR_A = \frac{12}{\sqrt{12 + 2\sum_{k=1}^{11}(12-k)\rho_k}} SR_M, \tag{21}$$

where SR_M denotes the monthly Sharpe ratio and ρ_k the kth order autocorrelation of the portfolio return series. To properly check whether a particular portfolio outperforms the portfolio based on the MSCI indices, we test for the significance of the difference between Sharpe ratios. The hypothesis test of Jobson and Korkie (1981) is often used to do this (see e.g. DeMiguel et al. (2009), DeMiguel and Nogales (2009) and Gasbarro, Wong, and Kenton Zumwalt (2007)). According to Ledoit and Wolf (2008), however, the Jobson-Korkie test is not valid when the returns under consideration are of time series nature, which is why we resort to their robust Sharpe ratio hypothesis testing method instead. As suggested by Ledoit and Wolf (2008), we consider their studentised time series bootstrap method with the prewhitened Quadratic-Spectral (QS) kernel of Andrews and Monahan

(1992). The p-value of the hypothesis test of equal Sharpe ratios is then computed as follows:[1]

$$PV = \frac{\{\tilde{d}^{*,m} \geq d\} + 1}{M + 1}, \tag{22}$$

where M denotes the number of bootstrap resamples, d the original studentised test statistic and $\tilde{d}^{*,m}$ the centred studentised statistic computed from the mth bootstrap sample. For the exact definitions of these statistics and the complete algorithm, we refer the reader to Ledoit and Wolf (2008).

We also consider the economic measure Fleming, Kirby, and Ostdiek (2001) use in their research to compare portfolios. Comparable measures are often used to give an economic value to portfolios (see e.g. Kole and Dijk (2017), Marquering and Verbeek (2004) and West, Edison, and Cho (1993)). Fleming et al. (2001) start by equating the average utility of an investor who invests in a portfolio A and the average utility of the same investor who invests in the alternative portfolio B. Portfolio B, however, is subject to daily expenses (Δ) expressed as a fraction of the invested wealth. Since the utilities are equated to each other, the investor is indifferent between portfolios A and B, which in turn implies that Δ can be interpreted as the maximum performance fee the investor would be willing to pay to switch from portfolio A to portfolio B. In line with Fleming et al. (2001), the estimates of Δ are obtained by solving the following equation for Δ:

$$\sum_{t=1}^{T} (R_{B,t} - \Delta) - \frac{\gamma}{2(1+\gamma)} (R_{B,t} - \Delta)^2 = \sum_{t=1}^{T} R_{A,t} - \frac{\gamma}{2(1+\gamma)} R_{A,t}^2, \tag{23}$$

where $R_{A,t}$ and $R_{B,t}$ denote the return at time t of respectively portfolio A and portfolio B while γ denotes the investor's coefficient of relative risk aversion.

Because some investors (e.g. pension funds) might be more interested in the risk of a portfolio than its return, we also consider two statistical risk measures to compare portfolios. The first is the Value at Risk (VaR), which is often used in practice to measure and quantify the level of financial risk (Jorion, 1997; Pritsker, 1997). The VaR of a portfolio at the confidence level α is equal to the smallest number X such that the probability that the loss of the portfolio exceeds X is not larger than (1-α)%. In this research, the VaRs of portfolios are computed by means of the historical simulation method. The advantage of this method is that it is a non-parametric method and therefore does not result in errors caused by false parametric assumptions. According to the historical simulation method, an estimate for the VaR can be acquired by first sorting the losses of the portfolio, which are equal to the returns multiplied by negative one, in ascending order. The estimated VaR corresponding to the confidence level α is then obtained as follows:

$$\widehat{\text{VaR}}_{\alpha} = L_{(\lceil \alpha T \rceil)}, \tag{24}$$

where $L_{\lceil \alpha T \rceil}$ is the (αT)th order statistic of the portfolio losses and T is the number of portfolio returns. The second risk measure we consider is the Expected Shortfall (ES). The ES is the average value of the losses exceeding the VaR and is defined as follows:

$$\text{ES}_{\alpha} = \mathbb{E}(L | L \geq \text{VaR}_{\alpha}) = \frac{1}{1-\alpha} \int_{\alpha}^{1} \text{VaR}_u \text{du}. \tag{25}$$

[1] We use the Matlab code of Ledoit and Wolf (2008) that is publicly available on the website of the University of Zurich.

The estimate for the ES corresponding to the VaR of (24) is then computed as follows:

$$\widehat{\mathrm{ES}}_\alpha = \frac{\lceil \alpha T \rceil - \alpha T}{(1-\alpha)T} L_{(\lceil \alpha T \rceil)} + \frac{1}{(1-\alpha)T} \sum_{i=\lceil \alpha T \rceil + 1}^{T} L_{(i)}, \tag{26}$$

where T, α and $L_{(i)}$ once again denote the number of portfolio returns, the significance level and the ith order statistic of the portfolio losses.

3.1.3 The Theoretically Optimal Portfolio of an Investor with a Defined Investment Horizon

We find an investor's theoretically optimal global dynamic portfolio weights of by means of the portfolio optimisation routine of Jurek and Viceira (2011). Even though the portfolio weights of Jurek and Viceira (2011) are obtained under short selling restrictions and may therefore not be of much practical use for some investors, they do give a clear idea of which countries should be invested in most and which countries are less profitable. The investor considered in the framework of Jurek and Viceira (2011) has power utility preferences over terminal wealth and the asset returns are assumed to be lognormally distributed. The lognormal power utility framework is often considered in finance due to the realisticness of the power utility function (it exhibits a decreasing absolute risk aversion and a constant relative risk aversion (Hakansson, 1971, 1972)) and the fact that security prices are well approximated by the lognormal distribution (Blattberg & Gonedes, 1974; Lintner, 1972; Rosenberg, 1973). In accordance to the lognormal power utility framework, the investor chooses a sequence of portfolio weights $\{\boldsymbol{w}^{(\tau)}_{opt,t+K-\tau}\}_{\tau=K}^{\tau=1}$ between time t and $t+K-1$ for an investment horizon K such that

$$\{\boldsymbol{w}^{(\tau)}_{opt,t+K-\tau}\}_{\tau=K}^{\tau=1} = argmax \ \mathrm{E}_t \left[\frac{W_{t+K}^{1-\gamma}}{1-\gamma} \right], \tag{27}$$

subject to his budget constraint

$$W_{t+1} = W_t(1 + R_{p,t+1}), \tag{28}$$

$$R_{p,t+1} = \sum_{j=1}^{N} w_{opt,i,t}(R_{j,t+1} - R_{1,t+1}) + R_{1,t+1}, \tag{29}$$

where W_t denotes the investors wealth at time t, γ the coefficient of relative risk aversion, $R_{j,t+1}$ the return on asset j and $R_{1,t+1}$ the return on a benchmark asset.

In order to acquire the investor's theoretically optimal intertemporal asset allocation, we first define the following first order Vector Autoregressive (VAR(1)) model in the manner of Jurek and Viceira (2011):

$$\boldsymbol{z}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{z}_t + \boldsymbol{v}_{t+1}, \tag{30}$$

where $\boldsymbol{z}_{t+1}$ is defined as follows:

$$\boldsymbol{z}_t = \begin{bmatrix} r_{1,t} \\ \boldsymbol{x}_t \\ \boldsymbol{s}_t \end{bmatrix}, \tag{31}$$

where $r_{1,t}$ denotes the log real return at time t of the asset we consider as the benchmark asset, $\boldsymbol{x}_t$ the vector containing the excess log returns of all other assets with respect to the benchmark and $\boldsymbol{s}_t$ the vector with the (log) state variables. The variables $\boldsymbol{x}_t$ and $\boldsymbol{s}_t$ contain the data of all countries stacked under each other in the order they are presented in Table 1. Furthermore, the error terms in the vector $\boldsymbol{v}_{t+1}$ in (30) are assumed to be homoskedastic and normally distributed. Jurek and Viceira (2011) note that even though these assumptions might not be true empirically, assuming them is relatively harmless in this particular framework. A VAR(1) model is more parsimonious than higher order VAR models because it contains fewer parameters to estimate, while it is not too restrictive because any VAR model can be rewritten as a VAR(1) model (Campbell, Chan, & Viceira, 2001). The VAR(1) formulation in the portfolio choice model of Jurek and Viceira (2011) assumes that one of the investable assets acts as a benchmark asset over which excess returns on the other assets are measured for convenience of their analysis. The portfolio weight corresponding to the benchmark asset is then obtained by subtracting the sum of all other weights from 100%. As the choice of benchmark asset does not affect the portfolio weights, we select the stock index of the U.S. as the benchmark asset $r_{1,t}$ for no other reason than the U.S. being the last developed market we consider in alphabetical order. This means that our $(N-1)$ vector of excess returns $\boldsymbol{x}_t$ looks as follows at time t:

$$\boldsymbol{x}_t = \begin{bmatrix} r_{AU,t} - r_{US,t} \\ r_{CA,t} - r_{US,t} \\ \vdots \\ r_{UK,t} - r_{US,t} \end{bmatrix}, \tag{32}$$

where $r_{i,t}$ denotes the log return of the stock index of country i at time t. Furthermore, we use the following set of state variables in our research for each country i:

$$\boldsymbol{s}_{i,t} = \begin{bmatrix} d_{i,t} \\ \pi_{i,t} \\ y^s_{i,t} \end{bmatrix}, \tag{33}$$

where $d_{i,t}, \pi_{i,t}$ and $y^s_{i,t}$ denote the log dividend yield, log inflation rate and log short-term interest rate, respectively, corresponding to country i. The log dividend yield is included as state variable because it is known to predict equity returns (Campbell & Shiller, 1988; Fama & French, 1989; Goetzmann & Jorion, 1993; Hodrick, 1992), while the log inflation rates and log short-term interest rates are included to capture the dynamics of inflation and interest rates (Jurek & Viceira, 2011; Viceira et al., 2017). Short-term interest rates have furthermore been found to also be effective predictors of stock returns (Campbell, 1985; Fama & Schwert, 1977; Glosten, Jagannathan, & Runkle, 1993).

We set the effect of the explanatory variable of one country on the dependent variables of other countries to zero, which helps avoid dimensionality problems. The only exception to this is the U.S., as the U.S. stock market index serves as our benchmark asset and is therefore present in every row of $\boldsymbol{x}_{i,t}$. In line with Jurek and Viceira (2011), we also impose the restriction that the lagged returns have no effect on the dependent variables because lagged returns forecast the returns of the next period at a monthly frequency positively. Jurek and Viceira (2011) state that this causes short-horizon effects in the asset allocations that are unrelated to the effects of persistent changes in investment opportunities, which are well captured by our state variables. Combined with the restriction that lagged variables do not predict variables of other countries, this has the additional desirable effect that the matrix $\mathbf{\Phi}_1$ in (30) contains a large number of zeros, which means working with such a large matrix (40×40 when we only consider developed markets and 60×60 when we also include emerging markets) does not result in dimensionality issues.

Jurek and Viceira (2011) then define the covariance matrix of the error term vector $\boldsymbol{v}_{t+1}$ as follows:

$$\mathbf{\Sigma}_v = \begin{bmatrix} \sigma_1^2 & \boldsymbol{\sigma}'_{1x} & \boldsymbol{\sigma}'_{1s} \\ \boldsymbol{\sigma}_{1x} & \mathbf{\Sigma}_{xx} & \mathbf{\Sigma}'_{xs} \\ \boldsymbol{\sigma}_{1s} & \mathbf{\Sigma}_{xs} & \mathbf{\Sigma}_s \end{bmatrix}, \tag{34}$$

where the diagonal elements are the variance of the return on the benchmark asset (σ_1^2), the covariance matrix of excess returns ($\mathbf{\Sigma}_{xx}$) and the covariance matrix of the shocks to the state variables ($\mathbf{\Sigma}_s$). The off-diagonal elements are the covariances of the return on the benchmark asset with the excess returns ($\boldsymbol{\sigma}_{1x}$), the covariance of the benchmark asset returns with the shocks to the state variables ($\boldsymbol{\sigma}_{1s}$) and the covariances of the excess returns with shocks to the state variables ($\mathbf{\Sigma}_{xs}$).

Jurek and Viceira (2011) then obtain the following recursive solution for the theoretically optimal intertemporal asset allocation of the power utility investor as an affine function of the state vector:

$$\boldsymbol{w}^{(\tau)}_{opt,t+K-\tau} = \boldsymbol{A}^{(\tau)} + \boldsymbol{B}^{(\tau)}\boldsymbol{z}_{t+K-\tau}, \tag{35}$$

where τ denotes the remaining investment horizon, t the current time period and K the total investment horizon. $A^{(\tau)}$ and $B^{(\tau)}$ are functions of the remaining investment horizon, the coefficient of relative risk aversion, the coefficients of the VAR system (30) and the covariance matrix (34). We compute the portfolio weights with the estimated VAR coefficients and the sample covariance matrix. The exact definition of these two parameters is given in Appendix E. The solution (35) can be interpreted as a discretised version of the exact continuous time solution obtained by Liu (2006). In order to properly compare the theoretically optimal portfolio with the MSCI portfolios of Section 3.1.1, we compute the theoretically optimal weights at every time point t of our sample period by only using data available up to and including time t. To ensure we use enough data to estimate the VAR model (30), we only compute the portfolio weights for the time period January 2003 to December 2016.[2] Furthermore, we compute the theoretically optimal portfolio for a short-term investor, a medium-term investor and long-term investor with investment horizons of 1, 10 and 20 years, respectively.

3.1.4 Shrinkage

Michaud (1989) states that portfolio optimisation methods that rely on estimated inputs (e.g. means and covariance matrices) tend to result in inaccurate results when the estimated inputs deviate too much from the true value. This is known as error maximisation and has a greater effect on large portfolios. As we use estimated values such as the sample covariance matrix and our research is furthermore conducted with a large number of assets, it is of importance to control for error maximisation. We therefore apply various methods of shrinkage in our research to reduce error maximisation.

Of all portfolios described in Section 3.1.1, only the GMV portfolio and the MV portfolio require the estimation of sample means and sample covariance matrices. We therefore solve both optimisation problems

[2] The portfolio weights are computed with the Matlab code of Jurek and Viceira (2011).

(13) and (15) with the restriction that the resulting portfolio weights should not be smaller than 0 or larger than 1. Imposing this short selling restriction does not only make it easier to compare the portfolio weights (14) and (16) with the weights of the other global equity portfolios described in Section 3.1.1, but is also a form of shrinkage that reduces error maximisation (Jagannathan & Ma, 2003).

Furthermore, we apply three methods of shrinkage when following the framework of Jurek and Viceira (2011). The first method of shrinkage is placing the restrictions on the matrix of coefficients $\mathbf{\Phi}_1$ of (30) mentioned earlier. As these restrictions set a lot of values of the matrix $\mathbf{\Phi}_1$ to zero, they do not only help avoid dimensionality issues, but also considerably shrink the matrix of coefficients. As all of the restrictions we place are of the sort that restrict the effect of an explanatory variable on the dependent variable to be zero, they are easily implemented by estimating VAR model (30) equation by equation (Zeng & Wu, 2013) and only including the variables of which we want the effect the be non-zero. We furthermore estimate each equation of the VAR model by means of a ridge regression (Hoerl & Kennard, 1970). The ridge regression is a method of shrinkage designed to improve the estimation of the coefficients of linear models (Efron & Hastie, 2016). As a ridge regression amounts to a Bayesian parameter estimation with an increased prior belief that the true value of the coefficient that has to be estimated lies near zero (Efron & Hastie, 2016), it can unproblematically be used to estimate the coefficients of our VAR model per equation. The ridge regression estimator is defined as follows for equation i in our VAR model:

$$\hat{\boldsymbol{\beta}}(\lambda) = (\boldsymbol{Z}'\boldsymbol{Z} + \lambda \boldsymbol{I})^{-1}\boldsymbol{Z}'\boldsymbol{y}, \tag{36}$$

where $\boldsymbol{I}$ is an identity matrix, $\boldsymbol{y}$ is the dependent variable of the equation under consideration and λ is the ridge parameter that determines the degree of shrinkage applied. Furthermore, $\boldsymbol{Z}$ is defined as following:

$$\boldsymbol{Z} = \begin{bmatrix} 1, & z_{1,1} & \dots & z_{K,1} \\ 1, & z_{1,2} & \dots & z_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ 1, & z_{1,T-1} & \dots & z_{K,T-1} \end{bmatrix}, \tag{37}$$

where K is the number of variables in $\boldsymbol{z}_t$. We use the method of generalised cross-validation of Golub, Heath, and Wahba (1979) to find the optimal value of the ridge parameter. Golub et al. (1979) recommend to estimate the ridge parameter with the minimiser of V(λ), which is definded as follows:

$$V(\lambda) = \frac{\frac{1}{T}||(\boldsymbol{I} - \boldsymbol{A}(\lambda))\boldsymbol{y}||^2}{\left[\frac{1}{T}\text{Trace}(I - \boldsymbol{A}(\lambda))\right]^2}, \tag{38}$$

where $\boldsymbol{A}(\lambda) = \boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z} + T\lambda\boldsymbol{I})^{-1}\boldsymbol{Z}'$, T denotes the number of observations in $\boldsymbol{y}$ and the other variables are defined as in (36) and (37). Furthermore, $||\cdot||$ indicates the Euclidean norm. We restrict the ridge parameter to be larger than 0, so that we ensure that some degree of shrinkage is applied to the parameter estimates (36). In line with Van Wieringen (2015), we consider values of the ridge parameter up to 10.

After applying the aforementioned restrictions on the coefficients of (30) and estimating the coefficients with a ridge regression, we can acquire the estimate $\boldsymbol{S}_v$ of the covariance matrix $\boldsymbol{\Sigma}_v$. Ledoit and Wolf (2004a) find that the sample covariance matrix $\boldsymbol{S}_v$ contains estimation error that is likely to perturb a mean-variance optimisation. Therefore, the final method of shrinkage we apply in this research is the shrinkage method of

Ledoit and Wolf (2004b) to reduce the estimation error of $\boldsymbol{S}_v$. Ledoit and Wolf (2004b) shrink the sample covariance matrix towards a one-parameter matrix, i.e. the covariance matrix $\mu \boldsymbol{I}$ with identical variances and covariances all equal to zero:

$$\boldsymbol{\Sigma}_{shrink} = \frac{\beta^2}{\delta^2}\mu \boldsymbol{I} + \frac{\alpha^2}{\delta^2}\boldsymbol{S}_v, \tag{39}$$

where μ is the average of the variances of the covariance matrix $\boldsymbol{\Sigma}_v$, $\boldsymbol{I}$ is an identity matrix and the following equation holds:

$$\alpha^2 + \beta^2 = \delta. \tag{40}$$

We refer the reader to Ledoit and Wolf (2004b) for the exact definition of the scalars α, β and δ and the methodology behind obtaining the estimates of these scalars.[3]

3.2 Global Diversification of Bonds

Our analysis of bonds starts with Section 3.2.1, which describes how we investigate whether it adds value for a Eurozone investor to diversify into U.S. bonds. Section 3.2.2 describes how we analyse the inflation and interest rate hedging qualities of bonds. Finally, Section 3.2.3 describes how the theoretically optimal portfolio weights of an asset-only and an asset-liability investor are computed by means of the portfolio optimisation framework of Hoevenaars et al. (2008).

3.2.1 Benefits of Diversifying into U.S. Bonds

We follow the framework of Viceira et al. (2017) with Eurozone and U.S. bonds as investable assets to find out whether it adds value for a euro-denominated investor to diversify into U.S. bonds. We first perform the analysis with Eurozone and U.S. government bonds before considering Eurozone and U.S. corporate bonds so that we are analysing the benefits of diversifying across regions and not across bond categories. Similarly, we treat (and refer to) the whole of the Eurozone as a single country instead of considering various Eurozone countries in order to ensure we are analysing the benefits of diversifying into American bonds and not the benefits of diversifying across Eurozone bonds.

Where it is traditionally assumed that discount rates are constant and all variation in asset returns is solely driven by changes in cash flows, recent research has found evidence of predictable variation in discount rates (Campbell, 1991; Cochrane, 2008, 2011; Vuolteenaho, 2002). This implies that realised asset returns vary over time due to both shocks to cash flows, which empirically appear to have a permanent effect, and shocks to discount rates, which appear to have a transitory effect (Campbell, 1991; Campbell & Shiller, 1988; Campbell & Vuolteenaho, 2004). Hence, both types of shocks drive return correlations between countries, but each with a different effect. Although the cross-country correlations of global bond markets have been increasing throughout recent times, it is the source of these correlations that determines how detrimental this increased correlation has been for global investors. Viceira et al. (2017) therefore explore the different types of sources of cross-country correlations and their respective contribution to the correlations. Viceira et al. (2017) begin their analysis by performing the decomposition of realised returns of Campbell (1991) to show that the unexpected log return on an asset consists of a term that represents changes in expected future cash flows and a term that represents

[3] We use the Matlab code of Ledoit and Wolf (2004b) that is publicly available on the website of the University of Zurich.

changes in expected future returns (or discount rates):

$$r_t - \mathbb{E}_t[r_t] \equiv N_{CF,t} - N_{DR,t}, \tag{41}$$

where r_t is the asset return at time t and $N_{CF,t}$ and $N_{DR,t}$ are the terms that represent the changes in respectively expected future cash flows and expected future discount rates, respectively. Viceira et al. (2017) refer to the former as cash flow news and to the latter as discount rate news, which is how we will refer to them as well throughout the remainder of this report. The discount rate news component is then decomposed further into news about excess returns (or risk premia news) and news about the return of the reference asset used to compute excess returns (or real rate news):

$$N_{DR,t} = N_{RR,t} + N_{RP,t}, \tag{42}$$

where $N_{RR,t}$ denotes the real rate news component and $N_{RP,t}$ the risk premia news component. Viceira et al. (2017) then find that discount rate shocks are transitory shocks and have a smaller impact on long-run portfolio returns than the permanent cash flow shocks. In particular, Viceira et al. (2017) state that when cross-country correlated discount rate news is the source of increased cross-country return correlations, the benefits of global portfolio diversification measured as a reduction of portfolio risk do not decline as much for long-term investors as for short-term investors. When cross-country cash flow news correlation is the source of increased cross-country return correlations, however, the benefits of global portfolio diversification decline equally for all investors.

The fact that cross-country correlations driven by discount rate news and cross-country correlations driven by cash flows news have different consequences for global portfolio diversification shows the added value of identifying which type of news component primarily drives the cross-country correlations of global bond markets. Because the news components Viceira et al. (2017) define are not directly observable, they must therefore be inferred from a return generating model. In line with Viceira et al. (2017), we follow Campbell (1991) and assume that the asset generating process follows the following VAR(1) model:

$$\boldsymbol{z}_{i,t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{z}_{i,t} + \boldsymbol{v}_{i,t+1}, \tag{43}$$

where index i denotes country i and the state vector $\boldsymbol{z}_{i,t}$ is specified as follows:

$$\boldsymbol{z}_{i,t} = \begin{bmatrix} xr_{i,t} \\ ys_{i,t} \\ \pi_{i,t} \\ y^s_{i,t} \end{bmatrix}, \tag{44}$$

where $xr_{i,t+1}$ denotes the excess log bond return, $ys_{i,t}$ the log yield spread, $\pi_{i,t+1}$ the log inflation rates and $y^s_{i,t}$ the log short-term interest rates. Inflation rates and the short-term interest rate are once again considered to capture the dynamics of inflation and interest rates, while yield spreads are known to be a good predictor of bond returns (Campbell et al., 2001; Campbell & Shiller, 1991; Campbell & Viceira, 2005; Fama & Bliss, 1987;

Fama & French, 1989). The excess log bond returns are computed as follows:

$$xr_{i,t} = r_{i,t} - y^1_{EU,t}, \tag{45}$$

where r_t denotes the log returns of country i at time t, while $y^1_{EU,t}$ denotes the log risk free rate, which is the 1-month Euribor rate in our case. Working with excess log returns ensures that the return decomposition of Campbell (1991) is currency independent (Campbell et al., 2010).

In line with Viceira et al. (2017), we assume that the estimates of $\boldsymbol{\Phi}_0$ and $\boldsymbol{\Phi}_1$ are constant across the countries in our sample and estimate these coefficients by means of pooled ordinary least squares to take as much cross-country correlation as possible into account. The pooled ordinary least squares estimates are obtained by means of the following regression:

$$\boldsymbol{Y} = \boldsymbol{Z}\boldsymbol{\Phi}' + \boldsymbol{V}, \tag{46}$$

where $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_0\ \boldsymbol{\Phi}_1]$ and the three variables are the stacked variants of the corresponding variables in (43):

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{z}_{1,t+1} \\ \boldsymbol{z}_{2,t+1} \end{bmatrix}, \boldsymbol{Z} = \begin{bmatrix} \boldsymbol{\iota} & \boldsymbol{z}_{1,t} \\ \boldsymbol{\iota} & \boldsymbol{z}_{2,t} \end{bmatrix}, \boldsymbol{V} = \begin{bmatrix} \boldsymbol{v}_{1,t+1} \\ \boldsymbol{v}_{2,t+1} \end{bmatrix}, \tag{47}$$

where ι_T is a vector of ones and $\boldsymbol{z}_{i,t}$ is defined as in (44). Furthermore, the subscripts 1 and 2 denote the Eurozone and the U.S., respectively. The estimate of $\boldsymbol{\Phi}$ is then obtained as follows:

$$\hat{\boldsymbol{\Phi}} = (\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{Z}'\boldsymbol{Y}. \tag{48}$$

The t-statistics corresponding to $\hat{\boldsymbol{\Phi}}_0$ and $\hat{\boldsymbol{\Phi}}_1$ are computed with the robust Driscoll-Kraay standard errors. These standard errors are computed in the manner of Hoechle (2007) and are capable of properly correcting for groupwise heteroscedasticity, autocorrelation and cross-sectional dependence (Driscoll & Kraay, 1998).

Viceira et al. (2017) then show that the three types of news components (cash flow news, risk premia news and real rate news) can be explicitly identified from the VAR(1) model (43) as follows:

$$xr_{i,t+1} - \mathbb{E}_t[xr_{i,t+1}] = \mathbf{e1}'\boldsymbol{v}_{i,t+1}, \tag{49}$$

$$N_{CF,i,t+1} = -\mathbf{e3}'(\sum_{j=1}^{n-1} \rho_b^j \boldsymbol{\Phi}_1^j)\boldsymbol{v}_{i,t+1}, \tag{50}$$

$$N_{RR,i,t+1} = \mathbf{e4}'(\sum_{j=1}^{n-1} \rho_b^j \boldsymbol{\Phi}_1^{j-1})\boldsymbol{v}_{i,t+1} + N_{CF,i,t+1}, \tag{51}$$

$$N_{RP,i,t+1} = N_{CF,i,t+1} - N_{RR,i,t+1} - (xr_{i,t+1} - \mathrm{E}_t[xr_{i,t+1}]), \tag{52}$$

$$N_{DR,i,t+1} = N_{RR,i,t+1} + N_{RP,i,t+1}, \tag{53}$$

where $\mathbf{eL}$ denotes a column vector with a 1 as the Lth element and 0's on all other positions. Furthermore, n is the maturity of the bonds and $\rho_b = 1/(1+\exp(-\bar{p}_n))$ with $\bar{p}_n$ the average bond price for maturity n. As we do not have bond prices at our disposal, we approximate them by setting the bond price of 1-month before our sample starts to 100 units of local currency and computing current prices at time t as $P_t = P_{t-1} * (1 + R_t)$. This follows from the fact that asset returns can be computed from price indices as follows: $R_{i,t} = \frac{P_{i,t}}{P_{i,t-1}} - 1$.

Small differences between the approximated and true bond prices hardly matter as the natural exponential of $-p_n$ is very close to zero for large values of p_n anyway.

Like Viceira et al. (2017) and Ammer and Mei (1996), we then use the news components of the unexpected excess bond returns computed with the estimate of the VAR(1) model to explore the sources of the cross-country correlations of the excess bond returns. In particular, we look at the contribution of the correlations between the news component to the total cross-country correlations between excess bond returns. This allows us to explore which type of news component is the greatest driver of the correlation between U.S. dollar-denominated bonds and euro-denominated bonds. Viceira et al. (2017) argue that the total excess bond return correlation across countries can be decomposed into pairs of covariances between news components and show that the contribution of the cross-country correlation of news component type k to the total cross-country correlation can be estimated as follows:

$$C_{NC_k} = \frac{\mathbb{C}[NC_{k,i}, NC_{k,j}]}{\sigma_{\tilde{xr}_i}\sigma_{\tilde{xr}_j}}, \tag{54}$$

where $\sigma_{\tilde{xr}_i}$ denotes the standard deviation of $\tilde{xr}_i = NC_{CF,i} - NC_{DR,i}$ and the subscript i denotes country i. The contributions should (approximately) add up the total cross-country correlation as follows:

$$\begin{aligned}
\mathrm{Corr}(\tilde{xr}_i, \tilde{xr}_j) &= \frac{\mathbb{C}[\tilde{xr}_i, \tilde{xr}_j]}{\sigma_{\tilde{xr}_i}\sigma_{\tilde{xr}_j}} \\
&= \frac{\mathbb{C}[NC_{CF,i}, NC_{CF,j}] - \mathbb{C}[NC_{CF,i}, NC_{DR,j}] - \mathbb{C}[NC_{DR,i}, NC_{CF,j}] + \mathbb{C}[NC_{DR,i}, NC_{DR,j}]}{\sigma_{\tilde{xr}_i}\sigma_{\tilde{xr}_j}} \\
&= C_{CF} - \frac{\mathbb{C}[NC_{CF,i}, NC_{DR,j}]}{\sigma_{\tilde{xr}_i}\sigma_{\tilde{xr}_j}} - \frac{\mathbb{C}[NC_{DR,i}, NC_{CF,j}]}{\sigma_{\tilde{xr}_i}\sigma_{\tilde{xr}_j}} + C_{DR},
\end{aligned} \tag{55}$$

where C_{CF} and C_{DR} denote the contributions of the cross-country correlation of cash flow news and discount rate news, respectively, obtained by plugging in the corresponding news component estimates in (54). Viceira et al. (2017) note that because the estimated news components come from innovations in excess returns and not actual excess returns, it is likely that the estimated news component contributions will not exactly sum up to the total cross-country correlation. Furthermore, (55) shows that it is possible that the contributions of the cross-country news components are larger than the total cross-country correlation in case of positive cross-country covariances between the cash flow and discount rate news components.

3.2.2 Inflation and Interest Rate Hedging Qualities of Bonds

To analyse the inflation risk and real interest rate risk hedging qualities of bonds, we follow Hoevenaars et al. (2008) and Spierdijk and Umar (2013) and look at the correlation between cumulative excess log bond returns and inflation for inflation hedging and the correlation between cumulative excess log bond returns and the cumulative excess log IRS returns for interest rate hedging. The idea behind the latter is that pension liabilities are the present value of future obligations discounted at a real interest rate (Hoevenaars et al., 2008). Furthermore, we consider the government bonds of the Eurozone countries in our data set instead of a single Eurozone government bond index, as the hedging qualities of bonds possibly differs between the Eurozone countries. The Bloomberg Barclays US Government Index is replaced by the 7-10 years U.S. government bond index to match the maturity of the bond indices of the Eurozone countries. In line with Hoevenaars et al.

(2008), we begin with specifying the following VAR(1) model:

$$\boldsymbol{z}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{z}_t + \boldsymbol{v}_{t+1}, \tag{56}$$

where the state vector $\boldsymbol{z}_{i,t}$ is specified as follows:

$$\boldsymbol{z}_t = \begin{bmatrix} r_{1,t} \\ \boldsymbol{s}_t \\ \boldsymbol{x}_t \end{bmatrix}, \tag{57}$$

where $r_{1,t}$ denotes the log real return of the asset we consider as the benchmark asset, $\boldsymbol{s}_t$ the vector with the (log) state variables and $\boldsymbol{x}_t$ defined as follows:

$$\boldsymbol{x}_t = \begin{bmatrix} \boldsymbol{x}_{A,t} \\ x_{L,t}, \end{bmatrix}, \tag{58}$$

where $\boldsymbol{x}_{A,t}$ denotes a vector of excess (with respect to the benchmark asset) log bond returns and $x_{L,t}$ the excess log returns of the liabilities, in our case the IRS returns. We replace the excess log IRS returns by the log inflation when we analyse the inflation hedging qualities of the assets. We use the risk-free rate (the 1-month Euribor rate) as the benchmark asset in (57) while we use the same set of state variables as in Section 3.2.1: the log yield spread, the log inflation rates and the log short-term interest rates.

The VAR(1) model (56) can then be used to obtain K-step-ahead forecasts of excess log bond returns, excess log IRS returns and log inflation rates. As Hoevenaars et al. (2008), who in turn follow Campbell and Viceira (2005), we assume homoskedastic errors. We forecast data for each path i by first drawing innovations $\boldsymbol{v}_{T+1}^{(i)}, \boldsymbol{v}_{T+2}^{(i)}, ..., \boldsymbol{v}_{T+K}^{(i)}$ from the multivariate normal distribution with a vector of zeros as mean and $\hat{\boldsymbol{\Sigma}}_v$ =Var($\hat{\boldsymbol{v}}_{t+1}$) as the covariance matrix of the residuals of the VAR(1) model (56), before combining them with the estimated coefficients of (56) as follows:

$$\begin{aligned} \boldsymbol{z}_{T+1}^{(i)} &= \hat{\boldsymbol{\Phi}}_0 + \hat{\boldsymbol{\Phi}}_1 \boldsymbol{z}_T + \boldsymbol{v}_{T+1}^{(i)} \\ \boldsymbol{z}_{T+2}^{(i)} &= \hat{\boldsymbol{\Phi}}_0 + \hat{\boldsymbol{\Phi}}_1 \boldsymbol{z}_{T+1}^{(i)} + \boldsymbol{v}_{T+2}^{(i)} \\ &\vdots \\ \boldsymbol{z}_{T+K}^{(i)} &= \hat{\boldsymbol{\Phi}}_0 + \hat{\boldsymbol{\Phi}}_1 \boldsymbol{z}_{T+K-1}^{(i)} + \boldsymbol{v}_{T+K}^{(i)}, \end{aligned} \tag{59}$$

where T denotes the last time point of our sample period. We simulate a total of 10,000 paths, which we find to be enough for convergence (see Appendix G). For each path, cumulative data at investment horizon K is then computed by taking the sum of the forecasted data over all investment horizons up to and including investment horizon K. This can be done because cumulative log returns at investment horizon K can be computed as follows:

$$\log(1 + R_{t+K}^{(K)}) = \log(1 + R_{t+1}) + ... + \log(1 + R_{t+K}), \tag{60}$$

where $\log(1 + R_{t+K})$ denotes the asset return at time $t + K$, respectively.

We can then compute the correlations between the forecasted cumulative excess log bond returns and forecasted cumulative log inflation rates and the correlations between the forecasted cumulative excess log bond returns and the forecasted cumulative excess log IRS returns. This is done for all investment horizons between 1 months and 30 years. As we use the euro as our base currency, we use the euro inflation as the inflation to hedge against. We perform the analysis separately with 30 and 20 year IRS returns to get an idea of how much the duration of the pension liabilities affects the results. In line with Bodie (1982) and Hoevenaars et al. (2008) among others, we interpret a higher coefficient of correlation as a higher hedging quality.

3.2.3 Theoretically Optimal Global Bond Portfolios

The framework of Hoevenaars et al. (2008) is also used to find the theoretically optimal global bond portfolio of both an asset-liability investor and an asset-only investor. For both investors, we first consider an asset menu consisting of only government bonds before we also include corporate bonds as investable assets.

The portfolio weights derived by Hoevenaars et al. (2008) are functions of the conditional mean and variance of the cumulative excess log returns, which are defined as follows:

$$\boldsymbol{\mu}_t^{(K)} = \frac{1}{K}\mathrm{E}_t[x_{t+K}^{(K)}] = \begin{bmatrix} \boldsymbol{\mu}_{A,t}^{(K)} \\ \mu_{L,t}^{(K)} \end{bmatrix}, \tag{61}$$

$$\boldsymbol{\Sigma}_t^{(K)} = \frac{1}{K}\mathrm{Var}_t[x_{t+K}^{(K)}] = \begin{bmatrix} \boldsymbol{\Sigma}_{AA}^{(K)} & \boldsymbol{\sigma}_{AL}^{(K)} \\ \boldsymbol{\sigma}_{AL}^{(K)\prime} & \sigma_L^{(K)2} \end{bmatrix}, \tag{62}$$

where $x_{t+K}^{(K)}$ denotes the cumulative excess log returns over K periods at time $t+K$ with x_t defined as (58), $\boldsymbol{\mu}_A^{(K)}$ the mean of the cumulative excess log bond returns, $\mu_L^{(K)}$ the mean of the cumulative excess log liability returns, $\boldsymbol{\Sigma}_{AA}^{(K)}$ the covariance matrix of the cumulative excess log bond returns, $\boldsymbol{\sigma}_{AL}^{(K)}$ the covariances between the cumulative excess log bond returns and the cumulative excess log liability returns and $\sigma_L^{(K)2}$ the variance of the cumulative excess log liability returns. In order to acquire $\mathrm{E}_t[x_{t+K}^{(K)}]$ and $\mathrm{Var}_t[x_{t+K}^{(K)}]$, we simulate 10,000 paths of data from the VAR(1) model (56) and compute cumulative returns as we did in Section 3.2.2, after which we take respectively the average and the variance of the cumulative simulated data. As a global portfolio that is heavily concentrated in the risk-free asset would complicate properly comparing the differences in allocation to the individual bonds we consider, we do not include the risk-free asset in the asset menu. Therefore, one of the bonds should be considered as the benchmark asset r_1 in (57). As in Section 3.1.3, we choose the U.S. asset as benchmark asset for no reason than the U.S. being the last of the countries in alphabetical order.

For the asset-liability investor, Hoevenaars et al. (2008) find approximate analytical portfolio weights by following Leibowitz, Kogelman, and Bader (1994) and approaching asset–liability management from a funding ratio (ratio of assets and liabilities) return perspective. In line with Van Binsbergen and Brandt (2007), Hoevenaars et al. (2008) find the portfolio weights of an investor with constant relative risk aversion preferences on the funding ratio at time $t+K$:

$$\boldsymbol{w}_{AL,t}^{(K)} = argmax\ \mathrm{E}_t\left[\frac{F_{t+K}^{1-\gamma}}{1-\gamma}\right], \tag{63}$$

where F and γ denote the funding ratio and coefficient of relative risk aversion, respectively. In order to stabilise the portfolio weights, Hoevenaars et al. (2008) furthermore assume that the investor keeps his portfolio weights fixed throughout his entire investment horizon. Moreover, Hoevenaars et al. (2008) note that fixed portfolio weights appear more closely connected to the industry practice than dynamic portfolio weights, as e.g. pension funds usually plan their portfolio on a constant mix basis and the investment plan of an institutional investor with a long horizon is typically only reviewed once every three to five years. Hoevenaars et al. (2008) aggregate the one-period portfolio returns under the assumption that the investor rebalances to the initial weights at the end of each period and subsequently find the following fixed, horizon specific portfolio weights for the asset-liability investor:

$$\boldsymbol{w}_{AL,t}^{(K)} = \frac{1}{\gamma}\left(\left(1-\frac{1}{\gamma}\right)\boldsymbol{\Sigma}_{AA}^{(K)} + \frac{1}{\gamma}\boldsymbol{\Sigma}_{AA}\right)^{-1}\left(\boldsymbol{\mu}_{A,t}^{(K)} + \frac{1}{2}\boldsymbol{\sigma}_{A}^{2} - (1-\gamma)\boldsymbol{\sigma}_{AL}^{(K)}\right), \tag{64}$$

where γ is the investor's coefficient of relative risk aversion, $\boldsymbol{\sigma}_A^2$ denotes a vector with the diagonal elements of $\boldsymbol{\Sigma}_{AA}$ and the other variables are defined as in (61) and (62). Once again, we consider 30 and 20 IRS returns to get an idea of how much the duration of the pension liabilities affects the results. We compute the portfolio weights with the estimated values of the variables in (64).

Hoevenaars et al. (2008) also compute the theoretically optimal portfolio of an asset-only investor. They do this by solving the following power utility problem:

$$\boldsymbol{w}_{AO,t}^{(K)} = argmax \ \mathrm{E}_t\left[\frac{W_{t+K}^{1-\gamma}}{1-\gamma}\right], \tag{65}$$

where W_t denotes the investors wealth at time t. For the asset-only investor, the liability returns are replaced by the benchmark asset in x_t and therefore also in (61) and (62). The theoretically optimal portfolio weights of the asset-only investor are then defined as follows:

$$\boldsymbol{w}_{AO,t}^{(K)} = \frac{1}{\gamma}\left(\left(1-\frac{1}{\gamma}\right)\boldsymbol{\Sigma}_{AA}^{(K)} + \frac{1}{\gamma}\boldsymbol{\Sigma}_{AA}\right)^{-1}\left(\boldsymbol{\mu}_{A,t}^{(K)} + \frac{1}{2}\boldsymbol{\sigma}_{A}^{2} + (1-\gamma)\boldsymbol{\sigma}_{Ar}^{(K)}\right), \tag{66}$$

where $\boldsymbol{\sigma}_{Ar}^{(K)}$ is a vector of covariances between the excess log bond returns and the log return on the benchmark asset over an investment horizon of K periods and the other variables are defined as in (61) and (62). We compute the portfolio weights by plugging in the corresponding estimates in (66).

As in the case of the theoretically optimal portfolio of equity investors, we compute the optimal portfolio weights for a short-term investor with an investment period of 1 year and a long-term investor with an investment period of 20 years. In contrast to in the part of our research focused on the global diversification of equity, however, we do not consider a middle-term investor with an investment period of 10 years. This is because including an extra investment horizon would lead to so much results (we already consider two types of investors, two different liabilities and three hedge ratios per investment horizon) that it would become too difficult to draw a clear conclusion from our analysis. Furthermore, we apply the same forms of shrinkage described in Section 3.1.4 to once again reduce estimation error.

3.3 The Effect of Currency Hedging on Globally Diversified Portfolios

There are various methods to compute currency hedged returns that are considered to be correct. One could for instance secure a currency hedge by investing in domestic and foreign money market instruments (Steiner,

2011) or futures (Hardie, 1983). In this research, we use the method of Campbell et al. (2010), who consider an investor who participates in currency hedging by entering into forward contracts. This method of currency hedging is both popular in literature (see e.g. Cantaluppi (1994); Glen and Jorion (1993); Kritzman (1993); Perold and Schulman (1988)) and in practice (Fung & Leung, 1991). Allow $F_{i,t}$ to denote the one month forward exchange rate in euros per foreign currency i at time t, $S_{i,t}$ the spot exchange rate, θ the hedge ratio and $R_{i,t}$ the return of the assets of country i denominated in the domestic currency. To obtain an expression for hedged returns, Campbell et al. (2010) assume that an investor exchanges $\theta/S_{i,t}$ units of the foreign currency denominated return $R_{i,t}/S_{i,t}$ back into the domestic currency at an exchange rate $F_{i,t}$ at time $t+1$. Afterwards, the investor exchanges the rest, $R_{i,t}/S_{i,t} - \theta/S_{i,t}$ units of foreign currency, at the spot exchange rate $S_{i,t+1}$. Collecting the asset returns for all countries implies the following expression for hedged asset returns at time $t+1$:

$$R^h_{i,t+1} = \left(\frac{S_{i,t+1}}{S_{i,t}}(1 + R_{i,t+1}) - \frac{S_{i,t+1}}{S_{i,t}}\theta + \frac{F_{i,t}}{S_{i,t}}\theta \right) - 1, \tag{67}$$

where $R_{i,t+1}$ denotes the return of country i at time $t+1$ denominated in the local currency, $S_{i,t}$ the spot exchange rate in euros per currency of country i, $F_{i,t}$ the 1-month forward exchange rate in euros per currency of country i and θ the hedge ratio. In contrast to Campbell et al. (2010), we keep the hedge ratio θ constant across time and currencies because of the large number of hedge ratios, time points and foreign assets we consider in our research. Time- and country-varying hedge ratios would distract from the main focus of this research.

The effect of currency hedging in the case of global equity diversification is analysed by constructing the portfolios described in Sections 3.1 and the statistical, economic and risk measures described in Section 3.1.2 for all hedge ratios between 0% (unhedged returns) and 100% (fully hedged returns) before comparing them across hedge ratios. In particular, we can examine how currency hedging affects the asset allocation of the GMV and MV portfolios of Section 3.1.1 and the theoretically optimal portfolio weights described in Section 3.1.3. Because performing the simulation methods described in Section 3.2.2 is quite time consuming, we only present and analyse portfolio weights for unhedged, half hedged and fully hedged returns. We furthermore investigate how currency hedging affects globally diversified bond portfolios. To this end, the news components analysis of Section 3.2.1 will be performed for the hedge ratios of 0, 50 and 100%. We also compute the theoretically optimal bond portfolios of the asset-only and asset-liability investors described in Section 3.2.3 for these three hedge ratios so that we can once again examine the effect of currency hedging on the investor's asset allocation. We do not consider all hedge ratios between 0 and 100%, as this would result in too much results which in turn would complicate giving a clear interpretation to the results. Because the only non-Eurozone country we consider in our analysis of global bond diversification, currency hedging furthermore plays a less significant role than when we investigate the global diversification of equity. This is also why we only consider unhedged and fully hedged U.S. bond returns for the hedging quality analysis of Section 3.2.2.

We exclude the transaction costs of currency hedging from our research, as transaction costs often depend heavily on factors that vary across investors. An example of such a factor is the investment fund or broker an investor chooses to consult. Due to the large variation in transaction costs across investors, choosing a single value would diminish the relevance of this research for investors with different transaction costs, while excluding transaction costs from our research ensures that it is equally relevant for all investors. We refrain from considering a range of transaction costs as this, combined with the large set of hedge ratios and investment horizons considered throughout this research, would complicate drawing relevant conclusions from our research.

Also, our research as it is now can easily be extended with any level of transaction costs the researcher or investor deems appropriate. Furthermore, as transaction costs of currency hedging are often negligibly small compared to the reward of risk reduction gained by diversification (Perold & Schulman, 1988), excluding transaction costs from our research will probably not have a large impact on our results. To to get some idea of how transaction costs of portfolio allocation might affect the results, we shortly discuss the turnovers of the portfolios discussed in Section 3.1.1, averaged over hedge ratios. We compute (monthly) turnovers in the same manner as Bianchi and Carvalho (2011), DeMiguel et al. (2009) and Plyakha, Uppal, and Vilkov (2012), among others:

$$\text{Turnover} = \frac{1}{T-1} \sum_{t=1}^{T-1} \sum_{i=1}^{N} \left(|w_{i,t+1} - w_{i,t^+}| \right), \tag{68}$$

where T and N are respectively the number of observations and assets, while $w_{i,t+1}$ and w_{i,t^+} denote the portfolio weight of asset i at time $t+1$ after and before rebalancing, respectively.

4 An Empirical Analysis of Global Equity Diversification

This section presents the results obtained by executing the methodology described in Section 3.1. As we do not research how an investor's risk aversion affects the portfolio choice, we follow Barnichon (2009), Bovenberg, Koijen, Nijman, and Teulings (2007) and Viceira et al. (2017), among others, and set the relative risk aversion coefficient of 5. A risk aversion coefficient of 5 corresponds to an average level of risk aversion and is therefore often considered in literature (Fugazza, Guidolin, & Nicodano, 2010). As we consider numerous portfolios and an even larger number of hedge ratios, considering a single level of risk aversion also ensures that the results of our research do not become too difficult to interpret. Section 4.1 presents the results of the analysis on whether it adds value for an investor to have flexibility in the regional weights of his globally diversified portfolio and to deviate from the regional allocation of the MSCI World Index. Section 4.2 presents the results on the computation of the theoretically optimal intertemporal portfolio of an investor with a defined investment horizon. The results presented in Section 4.1 are obtained over the full sample period (January 2002 to December 2016). The results presented in Section 4.2, on the other hand, are obtained over the period of January 2003 to December 2016 so that we use enough data to estimate the VAR model (30) accurately. In both Section 4.1 and Section 4.2, the portfolio weights we present are the weights computed for the last date in our sample period.

4.1 Deviating from Portfolio Weights Based on Global Stock Market Indices

Section 4.1.1 presents the results of our research for when the asset menu only consists of the equity of developed markets. The results in Section 4.1.2 correspond to an asset menu that consists of the equity of both developed and emerging markets. In both sections, we present GMV and MV portfolio weights computed without the shrinkage method of Ledoit and Wolf (2004b), as we have found that it has very little impact on the GMV and MV portfolio weights. This is because these portfolios are not as sensitive to estimation error as e.g. the theoretically optimal portfolios of Jurek and Viceira (2011) and Hoevenaars et al. (2008) and that restricting the GMV and MV portfolios weights to be between 0 and 100% has therefore already reduced the estimation error a lot.

4.1.1 Developed Markets Equity

Table 4 presents the asset allocation of the MSCI World, equal-weighted, value-weighted and euro portfolios described in Section 3.1.1. The weights of the value-weighted portfolio displayed in Table 4 are the weights computed at the latest date in our sample period (December 2016), while the weights of the other three portfolios are constant over time.

Table 4: The weights of the MSCI World (MW), equal-weighted (EW), value-weighted (VW) and euro portfolios (EU) with developed markets equity as investable assets

	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
MW	2.99%	3.82%	3.88%	3.62%	1.34%	9.17%	1.35%	3.40%	6.99%	63.43%
EW	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
VW	3.41%	4.20%	6.66%	9.38%	0.87%	13.38%	2.09%	1.79%	7.12%	51.11%
EU	7.14%	7.14%	16.67%	16.67%	7.14%	7.14%	16.67%	7.14%	7.14%	7.14%

With a weight of 63.43%, the U.S. is the country most weight is allocated to in the MSCI World portfolio. Japan is the country invested in most after the U.S. and the U.K. complements the top three. Hong Kong stocks and Dutch stocks are invested in least while the allocations to the other countries are comparable to each other. In the equal-weighted portfolio, the total investable wealth is distributed evenly among each asset in the portfolio. As the asset menu we consider consists of 10 countries, a weight of 10% is assigned to each country. As the U.S. is the world's largest economy, it is not surprising that our value-weighted portfolio based on GDP is heavily concentrated in U.S. equity with an allocation of 51.11%. Japan has the second largest GDP of all developed markets in our sample and is closely followed by Germany. The countries of which the GDP in proportion to the total GDP of our sample is smallest are by far the three smallest economies out of the developed markets we consider: Hong Kong, Switzerland and the Netherlands. The similarity of the value-weighted portfolio and the MSCI World portfolio was to be expected, as the allocations in the MSCI World Index are also based on an indicator of economic size (market capitalisation). As the euro portfolio is in fact nothing more than a combination of two equal-weighted portfolios, the similarity between the equal-weighted portfolio and the euro portfolio is no surprise. The weights of the euro portfolio corresponding to the three Eurozone countries in our sample (France, Germany and the Netherlands) are each equal to 16.67% while a weight of 7.14% is assigned to each of the other countries.

The portfolio weights of the GMV portfolio are obtained by plugging in the estimated sample mean and sample covariance matrix in (13) and solving for the weights. The portfolio weights of the MV portfolio are obtained similarly with (15) as the equation to be solved. Like the value-weighted portfolio, both the GMV portfolio and the MV portfolio vary over time. In contrast to the weights of the portfolios discussed so far, however, the weights of the GMV and MV portfolios depend on the asset returns and hence also differ per hedge ratio. Therefore, Table 5 presents the portfolio weights of the GMV and MV portfolios computed at the latest date in our sample period with unhedged, half hedged and fully hedged asset returns. The portfolio weights of the GMV portfolio vary considerably across hedge ratios. The GMV investor who does not currency hedge foreign stocks invests most in Swiss, U.S. and Japanese equity. This makes sense, as Swiss and American equity have the lowest volatilities of all unhedged stocks while unhedged Japanese equity has also has a relatively low volatility (see Table 2). Furthermore, all three of these assets have relatively high Sharpe ratios. The GMV portfolio

Table 5: The weights of the unhedged, half hedged and fully hedged global minimum variance (GMV) and mean variance (MV) portfolios with developed markets equity as investable assets

Panel A: Unhedged Returns										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
GMV	0.32%	7.97%	0.10%	0.04%	0.40%	16.37%	0.05%	48.65%	7.30%	18.81%
MV	29.48%	12.14%	0.01%	0.01%	0.08%	1.68%	0.01%	56.49%	0.01%	0.09%
Panel B: Half Hedged Returns										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
GMV	2.37%	25.86%	0.06%	0.03%	0.15%	6.05%	0.03%	45.61%	6.13%	13.70%
MV	17.98%	44.45%	0.01%	0.01%	0.08%	0.11%	0.01%	37.23%	0.02%	0.10%
Panel C: Fully Hedged Returns										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
GMV	30.59%	40.08%	0.06%	0.04%	0.06%	0.09%	0.04%	27.60%	1.26%	0.18%
MV	0.31%	83.59%	0.01%	0.01%	0.03%	0.03%	0.01%	15.93%	0.03%	0.05%

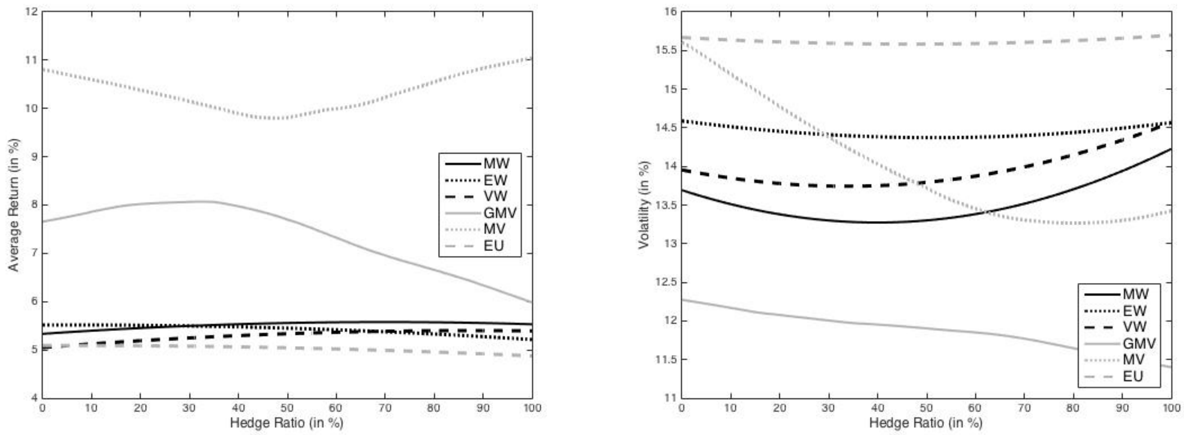
of the investor with a hedge ratio of 50% is almost completely concentrated in Swiss, Canadian and American equity. If the GMV investor opts for full hedging strategy, his portfolio is almost completely concentrated in Canadian, Swiss and Australian equity. This does not come as a surprise given the low volatilities and high Sharpe ratios in Table 2 corresponding to the fully hedged assets of Australia, Canada and Switzerland. For all hedge ratios considered in Table 5, the countries invested in least are Germany, the Netherlands and France. This is very reasonable, as Table 2 shows that the three Eurozone countries considered in our research have quite high volatilities and low Sharpe ratios compared to the other developed markets.

The portfolio of the MV investor is almost completely concentrated in the equity of Australia, Canada and Switzerland for all three hedge ratios considered in Table 5. This could have been expected given the high Sharpe ratios of Australian, Canadian and Swiss equity. The portfolio weights assigned to these three countries do vary across hedge ratios. The MV investor who chooses not to hedge foreign equity invests most in Swiss equity, then in Australian equity and after that in Canadian equity. The MV investor with a hedge ratio of 50% invests primarily in Canadian equity, then in Swiss equity and then in Australian equity. The MV portfolio of the investor who fully hedged foreign stocks is almost completely concentrated in Canadian equity with a weight of 83.59%. The rest of the portfolio is heavily concentrated in Swiss equity while the other stocks all receive weights considerably smaller than 1%. The extremely large allocation to Canadian equity makes perfect sense, as Canadian equity has the highest Sharpe ratio among all fully hedged developed markets. The fact that the allocation to Swiss equity in the fully hedged MV portfolio is far larger than the allocation to American equity, which has a higher Sharpe ratio, can be explained by the relatively low correlation between Canadian and Swiss equity and the far higher correlation between Canadian and American equity. As for the GMV portfolio, we find that the lowest weights of the MV portfolio for all hedge ratios correspond to French, German and Dutch equity. This is again due to the relatively high volatilities and low Sharpe ratios of these assets.

We have also performed a small robustness check concerning the portfolio weights of the GMV portfolio. The conclusion of this robustness check is that the GMV portfolio weights are more robust the more data is used. For the (monthly) data set we use, at least 5 years of data is required to produce relatively stable portfolio weights.

Given the similar method of construction, we assume the same results hold for the MV portfolio weights. The details of the robustness check can be found in Appendix Section F.

The weights of MSCI World, equal-weighted, value-weighted, GMV, MV and euro portfolios are used to compute the returns of each of these portfolios. Figure 2 presents the annualised historical average returns and volatilities of each of these portfolios across all hedge ratios.



(a) Average Returns

(b) Volatilities

Figure 2: The annualised historical average returns and volatilities of the MSCI World (MW), equal-weighted (EW), value-weighted (VW), GMV, MV and euro (EU) portfolios across hedge ratios. The asset menu only consists of developed markets equity

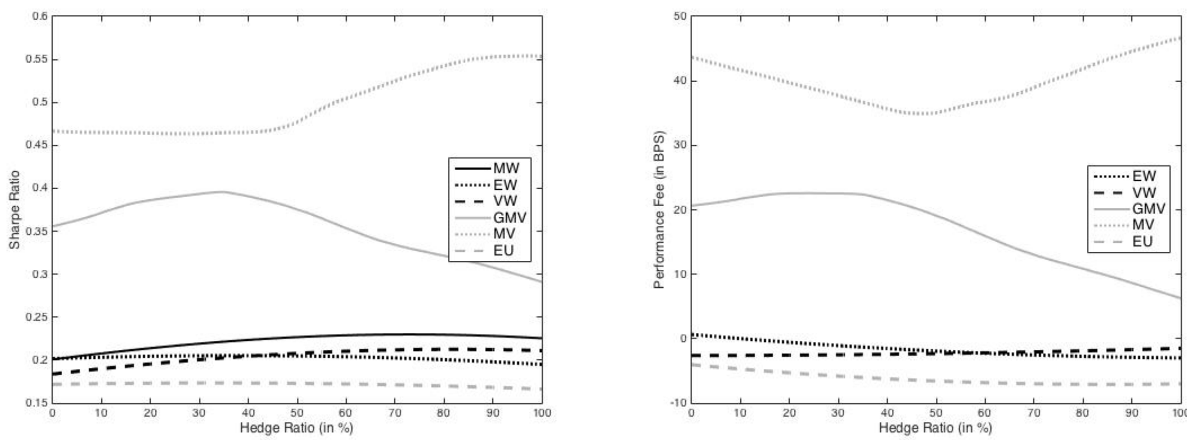
Figure 2a illustrates that the MV and GMV portfolios have considerably higher annualised historical average returns than the MSCI World portfolio for all hedge ratios between 0 and 100%. The equal- and value-weighted portfolios have comparable historical average returns to the MSCI World portfolio, while those of the euro portfolio are considerably lower.

The historical average return of the MV portfolio decreases as the hedge ratio increases until it reaches its minimum value at the hedge ratio of 48% before increasing to its maximum at the hedge ratio of 100%. For all other portfolios, the historical average return initially increases until it reaches its maximum, before decreasing again. This illustrates that each portfolio is affected differently by currency hedging. Figure 2a furthermore shows that the historical average returns of the GMV and MV portfolios are far more sensitive to changes in the hedge ratio than the other portfolios. This is because, unlike the weights of the other portfolios, the portfolio weights of the GMV and MV portfolios differ per hedge ratio.

The MSCI World portfolio has a lower annualised volatility than the equal-weighted, value-weighted, and euro portfolios for all hedge ratios. Figure 2b furthermore illustrates that the MV portfolio has a lower volatility than the MSCI World portfolio for hedge ratios of 63% or higher, while the GMV portfolio has the lowest annualised volatilities for all hedge ratios. The relatively low volatility of the GMV portfolio is because it is heavily concentrated in low-volatility assets.

The volatility of the GMV portfolio generally keeps decreasing as the hedge ratio increases. The volatilities of all other portfolio also initially decrease as the degree of currency hedging intensifies, but reach their minimum before the hedge ratio of 100% and increase again afterwards. As with the historical average returns, there does not seem to be a uniformly optimal hedge ratio.

We also consider the Sharpe ratios of the MSCI World, equal-weighted, value-weighted, GMV, MV and euro portfolios and the estimated maximum performance fees an investor would pay to switch from the MSCI World portfolio to the other portfolios. Figure 3 presents these two statistics across all hedge ratios.



(a) Sharpe Ratios

(b) Maximum Performance Fee

Figure 3: The annualised Sharpe ratios of the MSCI World (MW), equal-weighted (EW), value-weighted (VW), GMV, MV and euro (EU) portfolios and the maximum performance fee the an investor would pay to switch from the MSCI World Portfolio to the other portfolios across hedge ratios. The asset menu only consists of developed markets equity

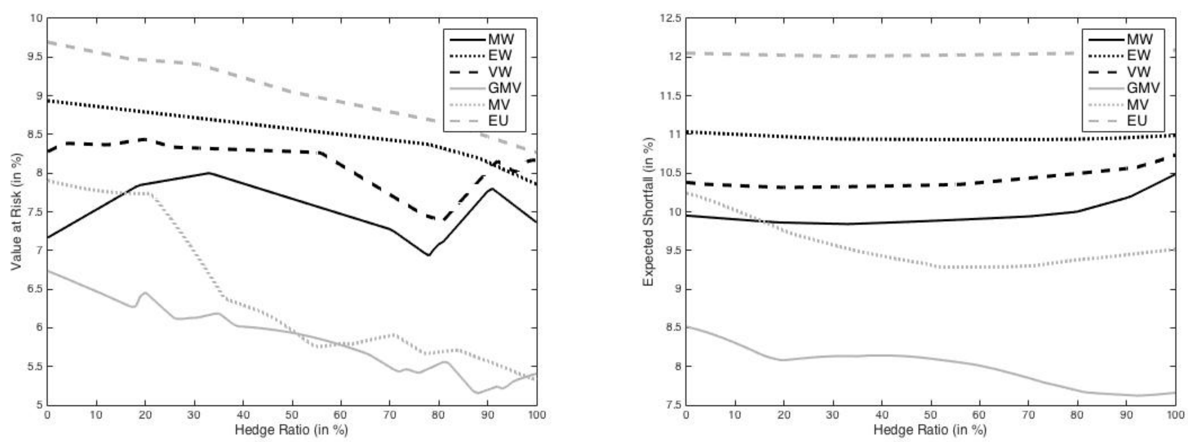
The performances of the portfolios in terms of Sharpe ratios are very comparable to their performances in terms of average returns and volatilities presented in Figure 2a. The MSCI World portfolio is not outperformed by the equal-weighted, value-weighted or euro portfolios for any hedge ratio. For all hedge ratios, the MV portfolio has a far higher annualised Sharpe ratio than each of the other portfolios. This makes perfect sense, as the computation of the MV portfolio is comparable to maximising the portfolio's Sharpe ratio. The MV portfolio has significantly higher Sharpe ratios than the MSCI World portfolio for all hedge ratios over 51% at the significance level of 0.10, while the difference is statistically significant for hedge ratios over 74% at a significance level of 0.05. Furthermore, the GMV portfolio also has higher annualised Sharpe ratios than the MSCI World portfolio. The difference in Sharpe ratios between the GMV portfolio and the MSCI World portfolio is statistically significant at the 0.10 significance level for hedge ratios up to 48%. At the significance level of 0.05, however, the difference is only statistically significant for (some) hedge ratios between 14 and 32%. The p-values of the hypothesis test of difference in Sharpe ratios can be found in Appendix H.1.

The effect of the hedge ratio on the Sharpe ratio is comparable to its effect on the average return for the MSCI World, equal-weighted, value-weighted, GMV and euro portfolios: the Sharpe ratio increases until its maximum value before decreasing again. The Sharpe ratio of the MV portfolio initially stays stable around the value of 0.46 until around the hedge ratio of 40%, before increasing to its maximum value.

Plotting the maximum performance fee an investor would pay to switch from the MSCI World portfolio to the alternative portfolios against the hedge ratio yields similar results as when we consider the Sharpe ratios of the portfolios. As the methodology behind computing the maximum performance fee takes into account the portfolio returns but not the volatilities of the portfolios, the resemblance is logically even greater when we compare it with Figure 2a. The MV portfolio is once again found to be the most favourable portfolio. Figure 3b furthermore shows that the investor would also pay a positive maximum performance fee to switch from the MSCI World portfolio to the GMV portfolio for all hedge ratios. The value-weighted portfolio only outperforms the MSCI World portfolio for hedge ratios up to and including 10%, while the investor would not want to switch from MSCI World portfolio to the equal-weighted and euro portfolios for any hedge ratio.

The effect of currency hedging on the performance fee measure is almost identical to the effect of hedging on the average return for the GMV and the MV portfolios. The performance fee the investor would be willing to pay to switch from the MSCI World portfolio to the MV portfolio initially decreases as the hedge ratio increases until its minimum, before increasing to its maximum value. A similar pattern is found for the euro portfolio, which is highest for unhedged returns and reaches its minimum at the hedge ratio of 85%. For the GMV portfolio, the performance fee initially increases until it reaches its maximum at the hedge ratio of 24%, before decreasing again. The performance fee estimate is furthermore strictly decreasing for the equal-weighted portfolio while it is strictly increasing for the value-weighted portfolio.

As some investors (e.g. pension funds) attach more importance to the risk of a portfolio than its expected profit, we now consider two risk measures to evaluate the portfolios. Figure 4 presents the monthly 95% VaRs and ES's of the MSCI World, equal-weighted, value-weighted, GMV, MV and euro portfolios across hedge ratios.



(a) 95% Value at Risk

(b) 95% Expected Shortfall

Figure 4: The monthly 95% Value at Risk and Expected Shortfall estimates of the MSCI World (MW), equal-weighted (EW), value-weighted (VW), GMV, MV and euro (EU) portfolios across hedge ratios. The asset menu only consists of developed markets equity

Figure 4a shows that the GMV portfolio has the lowest monthly 95% VaR for all but a few hedge ratios. This is not surprising given that the GMV portfolio is per definition a portfolio of relatively low risk. For all hedge ratios of 16% and higher, the MV portfolio also has a lower 95% VaR than the MSCI World portfolio. The portfolios for which the method of construction does not take the volatility of the assets into account (the equal-weighted, value-weighted and euro portfolios) have considerably higher VaR estimates than the GMV and MV portfolios. The equal-weighted, value-weighted and euro portfolios are furthermore also outperformed by the MSCI World portfolio in terms of monthly 95% VaR for all hedge ratios.

Although Figure 4a shows some spikes and dips for almost all portfolios, a general downward trend for the plots of the equal-weighted, GMV, MV and euro portfolio can be recognised clearly. Hence, increasing the hedge ratio generally results in a lower monthly 95% VaR for these portfolios. The downward trend of the VaR estimates of the value-weighted portfolio is more subtle while the VaRs of the MSCI World portfolio do not show a clear trend. In general, the lowest monthly 95% VaRs are found for the higher hedge ratios: the equal-weighted, MV and euro portfolios all have the lowest 95% VaR for the full hedging strategy while the 95% VaRs of the MSCI World, value-weighted and GMV portfolios are found for the hedge ratios of 78, 81 and 88%.

The performances of the portfolios in terms of ES are very similar to the performances in terms of VaR. Figure 4b illustrates that the GMV portfolio is by far the least risky portfolio. The monthly returns of GMV portfolio on average decrease with 7.62 to 8.51% when the corresponding 95% VaR is exceeded. In comparison, the monthly returns of the MSCI World portfolio on average decrease with 9.84 to 10.48%. For hedge ratios above 15%, the MV portfolio also has a lower 95% ES than the MSCI World portfolio. The equal-weighted, value-weighted and euro portfolios all have a higher ES than the MSCI World portfolio for all hedge ratios.

Figure 4 shows that the effect of the hedge ratio on the 95% ES is much smaller than its effect on the 95% VaR. While a clear downward trend is again visible for the ES estimates of the GMV and MV portfolios, the hedge ratio seems to have almost no effect on the equal-weighted and euro portfolios. The ES's of the MSCI World and value-weighted portfolios are also almost unaffected by the degree of currency hedging until a hedge ratio of around 80%, after which the monthly 95% ES slightly increases.

Table 6 presents the average returns, volatilities, Sharpe ratios, performance fee estimates, 95% VaRs and 95% ES's of the MSCI World, equal-weighted, value-weighted, GMV, MV and euro portfolios averaged over the hedge ratios. Furthermore, Table 6 also ranks each of the portfolios based on these measures.

Table 6: The average return, volatility, Sharpe ratio, performance fee, 95% Value at Risk and 95% Expected Shortfall estimates of the MSCI World (MW), equal-weighted (EW), value-weighted (VW), GMV, MV and euro (EU) portfolios averaged over the hedge ratios with developed markets equity as investable assets

	MW		EW		VW		GMV		MV		EU	
Average Return (in %)	5.51	(3)	5.42	(4)	5.30	(5)	7.37	(2)	10.34	(1)	5.02	(6)
Volatility (in %)	13.53	(2)	14.44	(5)	13.95	(3)	11.87	(1)	13.99	(4)	15.62	(6)
Sharpe Ratio	0.22	(3)	0.20	(5)	0.20	(4)	0.36	(2)	0.50	(1)	0.17	(6)
Performance Fee (in BPS)	-	(3)	-1.67	(4)	-2.23	(5)	16.93	(2)	39.74	(1)	-6.21	(6)
95% Value at Risk (in %)	7.57	(3)	8.54	(5)	8.12	(4)	5.89	(1)	6.42	(2)	9.05	(6)
95% Expected Shortfall (in %)	9.96	(3)	10.96	(5)	10.41	(4)	8.00	(1)	9.54	(2)	12.04	(6)

Note: the rank of the portfolio based on each measure is presented in parentheses. Average returns, volatilities and Sharpe ratios are annualised. The asset menu only consists of developed markets equity.

Averaged over the hedge ratios, the GMV portfolio and especially the MV portfolio outperform the MSCI World portfolio in terms of annualised average return while the equal-weighted, value-weighted and euro portfolios on average all have lower historical average returns. In terms of volatility, only the GMV portfolio outperforms the MSCI World portfolio. Averaged over hedge ratios, the Sharpe ratio and the performance fee estimates yield similar results as when we consider the average return: The MV portfolio is on average superior to all other portfolios and is followed by the GMV portfolio. Averaged over the hedge ratios, the GMV portfolio does not, however, statistically significantly outperform the MSCI World portfolio in terms of Sharpe ratios at the significance level of 0.10 (p-value of 0.11). With a p-value of 0.09, the MV portfolio does on average have a significantly higher Sharpe ratio than the MSCI World portfolio at the significance level of 0.10, but not at the significance level of 0.05. The low average VaR and ES estimates imply that the GMV is the least risky portfolio of all. The MV portfolio also (slightly) outperforms the MSCI World portfolio in terms of VaR and ES while the equal-weighted, value-weighted and euro portfolio are riskier than the MSCI World portfolio.

The equal-weighted, value-weighted and euro portfolio do not outperform the MSCI World portfolio for any measure considered after averaging over the hedge ratios. On the other hand, the GMV and MV portfolios are superior to the MSCI World portfolio in terms of almost all measures we consider in our research. Choosing the best portfolio based on the results in Table 6 therefore boils down to the choice between the GMV portfolio and the more rewarding but also slightly more risky MV portfolio. The fact that the GMV and MV portfolios outperform the MSCI World portfolio in terms of most measures we consider shows that it can certainly be of value to deviate from the regional allocation of the MSCI World Index.

The previous results are found in the absence of transaction costs. To get an idea of how transaction costs for the portfolio allocation could affect the results, we consider the portfolio turnovers averaged over the hedge ratios. Although the GMV and MV portfolios have higher average turnovers (7.78% and 16.08%, respectively) than the MSCI World portfolio (1.15%), the difference is not that large, which means that the GMV and MV portfolios will probably also outperform the MSCI World portfolio in the presence of (moderate) portfolio allocation transaction costs. To give an illustration, assume transaction costs of 50 BPS as done by Balduzzi and Lynch (1999) and DeMiguel et al. (2009). Averaged over the hedge ratios, the annualised historical average returns of the MSCI World, GMV and MV portfolios after transaction costs are then equal to 5.45, 6.90 and 9.37%, respectively. Although the values have decreased stronger for the GMV and MV portfolios than for the MSCI World portfolio, the average returns are still considerably higher. With average values of 1.91, 1.64 and 1.81%, respectively, the equal-weighted, value-weighted and euro portfolios furthermore have comparable turnovers to MSCI World portfolio.

4.1.2 Developed and Emerging Markets Equity

In contrast to Section 4.1.1, this section contains the results for an asset menu that consists of the equity of both developed and emerging markets. Table 7 presents the asset allocation of the MSCI World, equal-weighted, value-weighted and euro portfolios with an asset menu consisting of the equity of both developed and emerging markets. The presented weights of the value-weighted portfolio are the weights computed at the latest date in our sample period (December 2016), while the weights of the other three portfolios are constant over time. With a weight of 58.61%, the U.S. is the country most weight is allocated to in the MSCI ACWI portfolio. This was also the case when we considered the MSCI World portfolio. In fact, the list of countries that form the top 6 markets most invested in (the U.S., Japan, the U.K., Canada, France and Germany) remains unchanged from when the global stock market index was the MSCI World portfolio. The countries invested in most after these are Switzerland and China. The portfolio weights assigned to all emerging markets other than China are generally quite small, with the two countries least weight is allocated to being Brazil and India. As we now consider

Table 7: The weights of the MSCI ACWI (MA), equal-weighted (EW), value-weighted (VW) and euro portfolios (EU) with both developed and emerging markets equity as investable assets

Panel A: Developed Markets										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
MA	2.61%	3.59%	3.59%	3.27%	1.20%	8.50%	1.20%	3.05%	6.43%	58.61%
EW	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%	6.67%
VW	2.33%	2.87%	4.55%	6.41%	0.59%	9.14%	1.43%	1.22%	4.86%	34.93%
EU	4.17%	4.17%	16.67%	16.67%	4.17%	4.17%	16.67%	4.17%	4.17%	4.17%

Panel B: Emerging Markets					
	BR	CN	IN	KR	TW
MA	0.87%	3.05%	0.98%	1.63%	1.42%
EW	6.67%	6.67%	6.67%	6.67%	6.67%
VW	3.35%	20.63%	4.07%	2.61%	0.98%
EU	4.17%	4.17%	4.17%	4.17%	4.17%

a total of 15 countries, the portfolio weight allocated to each country in the equal-weighted portfolio is equal to 6.67%. The euro portfolio looks very similar as each Eurozone country has a weight of 16.67% assigned to while every non-Eurozone country receives a weight of 4.17%. The value-weighted portfolio differs substantially to when we only considered developed markets. For instance, China (20.63%) replaces Japan (9.14%) as the country with the second largest GDP while the weights assigned to Brazil and India are invested in more than a handful of developed markets. All in all, emerging markets pay a much larger role in the value-weighted portfolio than in all other portfolios.

Table 8 presents the portfolio weights of the GMV and MV portfolios computed at the latest date (December 2016) in our sample period with unhedged, half hedged and fully hedged asset returns. The results in Table 8 furthermore correspond to an asset menu that consists of both developed and emerging markets equity. Despite the introduction of emerging markets equity, the GMV portfolio weights presented in Table 8 do not differ much from the GMV portfolio weights of an investor who only considers developed markets equity. This is because the emerging markets stocks are considerably more volatile than most developed markets stocks and the GMV portfolio is per definition concentrated in assets of low volatility. The GMV investor who does not currency hedge foreign returns again invests most in Swiss, American and Japanese stocks. Once again, the GMV portfolio of the investor with a hedge ratio of 50% is heavily concentrated in Swiss, Canadian and U.S. equity while the GMV portfolio of the investor who fully hedges is almost entirely concentrated in Canadian, Australian and Swiss equity. For all hedge ratios, we find that German, Dutch and Brazilian stocks are invested in least. Of all emerging markets equity, we only find weights larger than 1% for Chinese stocks.

The MV portfolio differs considerably compared to before we introduced emerging markets equity to the asset menu. This is primarily due to the relatively large allocation to Indian equity. This could have been expected given the high Sharpe ratios of India presented in Table 2. The MV portfolio of the investor who does not currency hedge now is heavily concentrated in Swiss, Australian and Indian equity while the MV portfolio of the investor with a hedge ratio of 50% is heavily concentrated in the Swiss, Canadian and Indian equity. We find that the fully hedged MV portfolio is almost entirely concentrated in Canadian and Indian equity. As

Table 8: The weights of the unhedged, half hedged and fully hedged global minimum variance (GMV) and mean variance (MV) portfolios with both developed and emerging markets equity as investable assets

Panel A: Unhedged Returns, Developed Markets										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
GMV	0.26%	5.46%	0.11%	0.04%	0.14%	14.33%	0.05%	50.07%	4.01%	17.01%
MV	21.41%	4.34%	0.01%	0.01%	0.03%	0.08%	0.01%	54.41%	0.01%	0.07%

Panel B: Unhedged Returns, Emerging Markets					
	BR	CN	IN	KR	TW
GMV	0.03%	7.89%	0.07%	0.04%	0.48%
MV	0.01%	6.00%	13.56%	0.02%	0.03%

Panel C: Half Hedged Returns, Developed Markets										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
GMV	1.11%	22.14%	0.06%	0.03%	0.10%	4.37%	0.04%	45.85%	5.87%	13.02%
MV	2.87%	29.72%	0.01%	0.01%	0.02%	0.03%	0.01%	37.15%	0.02%	0.09%

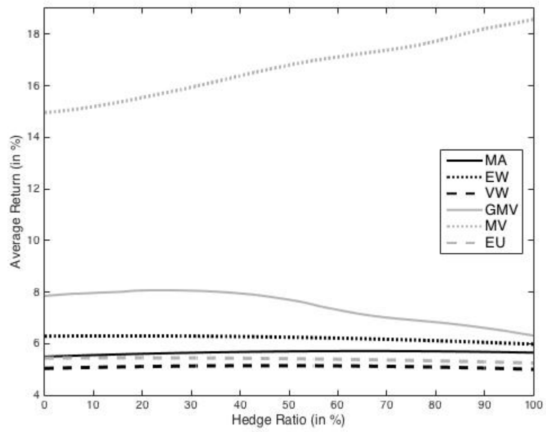
Panel D: Half Hedged Returns, Emerging Markets					
	BR	CN	IN	KR	TW
GMV	0.03%	6.39%	0.07%	0.14%	0.19%
MV	0.00%	1.36%	28.61%	0.03%	0.07%

Panel E: Fully Hedged Returns, Developed Markets										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
GMV	29.52%	38.88%	0.06%	0.04%	0.05%	0.09%	0.04%	27.61%	1.41%	0.17%
MV	0.06%	48.78%	0.01%	0.01%	0.01%	0.01%	0.01%	9.90%	0.02%	0.03%

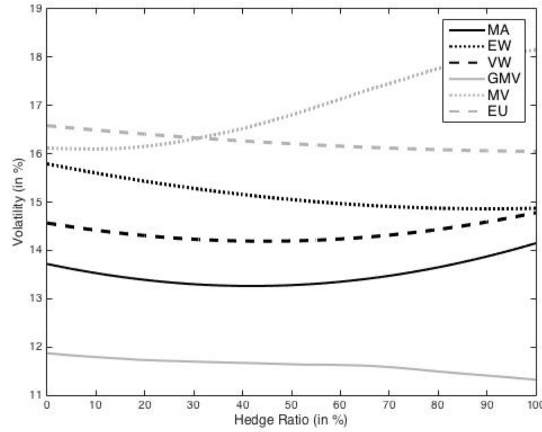
Panel F: Fully Hedged Returns, Emerging Markets					
	BR	CN	IN	KR	TW
GMV	0.05%	1.69%	0.06%	0.14%	0.19%
MV	0.00%	0.02%	41.05%	0.05%	0.04%

when we only considered developed markets equity, the allocation to French, German and Dutch equity is again very small (0.01%) for every hedge ratio. Brazilian equity is the only other asset to which the allocation is even smaller, as the portfolio weights corresponding to Brazilian equity are equal to zero to the second decimal place for the hedge ratios of 50 and 100%. The fact that hedged Brazilian stocks are virtually not invested in is very reasonable, given the negative Sharpe ratio of hedged Brazilian equity in Table 2.

We use the weights of the MSCI ACWI, equal-weighted, value-weighted, GMV, MV and euro portfolios to compute the returns of these portfolios. Figure 5 presents the annualised historical average returns and volatilities of these portfolios across all hedge ratios. The annualised historical average returns presented in Figure 5a differ slightly to when we only considered developed markets (Figure 2a). Although the MV portfolio and the GMV portfolio again have the highest annualised average returns, the equal-weighted portfolio now also has (slightly) higher average returns than the portfolio based on the MSCI benchmark. For all hedge ratios, the value-weighted and euro portfolios are inferior to the MSCI ACWI portfolio in terms of average return. We furthermore find that the average returns are generally slightly higher after the introduction of emerging



(a) Average Returns



(b) Volatilities

Figure 5: The annualised historical average returns and volatilities of the MSCI ACWI (MA), equal-weighted (EW), value-weighted (VW), GMV, MV and euro (EU) portfolios across hedge ratios. The asset menu consists of both developed and emerging markets equity.

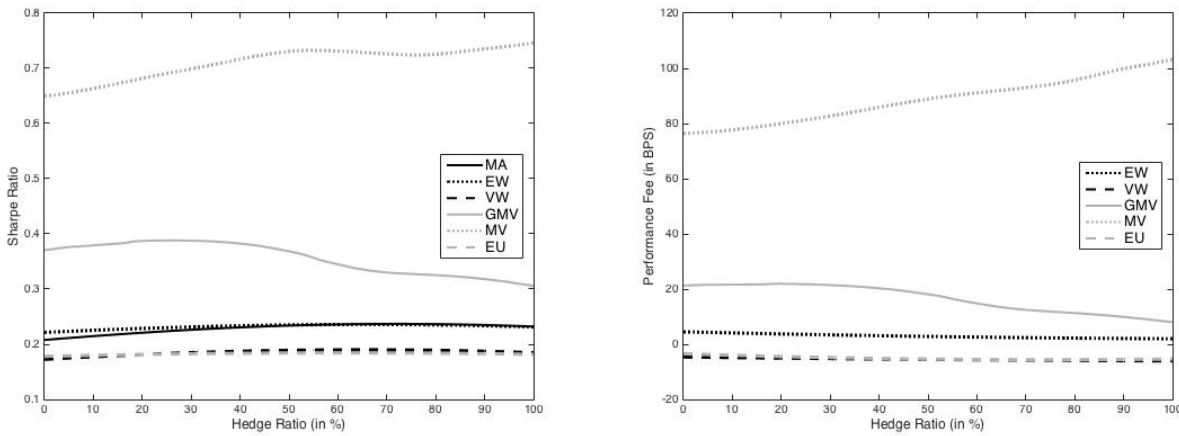
markets into the asset menu than before. For the MV portfolio in particular, we find that including emerging markets in the asset menu strongly increases the average return, which is caused by the high historical average return of Indian equity (see Table 2).

The effect of currency hedging on the historical average return of the MSCI ACWI portfolio is similar to its effect on the historical average return of the MSCI World portfolio: as the hedge ratio increases, the average return increases until reaching its maximum, before decreasing again. As before we introduced emerging markets equity to the asset menu, the same is true for the equal-weighted, value-weighted, GMV and euro portfolio. The effect of currency hedging on the average return of the MV portfolio differs considerably. Where it is initially decreasing and afterwards increasing when only developed markets are involved, the average return of the MV portfolio is strictly increasing as the hedge ratio increases for an asset menu consisting of both developed and emerging markets equity.

The volatilities of most portfolios have increased after including equity of emerging markets into the asset menu. For hedge ratios up to around 30%, the MSCI ACWI portfolio is slightly more volatile than the MSCI World portfolio, while it is less volatile for higher hedge ratios. The increase in volatility of most portfolios caused by introducing emerging markets equity to the asset menu is not surprising given the relatively high volatilities of the emerging markets presented in Table 2). The fact that there is no evident increase in volatility for the MSCI ACWI portfolio and the GMV portfolio could also have been expected, as the allocation to emerging markets equity in these portfolios is far smaller than for the other portfolios. Figure 5b furthermore illustrates that the GMV portfolio is again the least volatile portfolio. The GMV portfolio is furthermore the only portfolio that is less volatile than the MSCI ACWI portfolio.

The effect of the hedge ratio on the volatility remains unchanged to when the asset menu did not contain emerging markets equity for all portfolios except the MV and euro portfolios. Figure 5b illustrates that the volatility of the euro portfolio keeps decreasing as the hedge ratio increases while the volatility of the MV portfolio initially decreases, but starts increasing after the hedge ratio of 9%. The volatilities of MSCI ACWI, equal-weighted and value-weighted portfolios all follow the same pattern as those of the MV portfolio. Introducing emerging markets equity to the asset menu seems to have very little effect on the volatility of the GMV portfolio, as the volatility of the GMV portfolio is almost identical in size to when the asset menu only consists of developed markets equity and is furthermore once again a decreasing function of the hedge ratio.

We also consider the annualised Sharpe ratios of each of the portfolios and the estimated maximum performance fees an investor would pay to switch from the MSCI ACWI portfolio to the other portfolios. Figure 6 presents these two statistics across all hedge ratios for an investor with an average level of risk aversion.



(a) Sharpe Ratios

(b) Maximum Performance Fee

Figure 6: The annualised Sharpe ratios of the MSCI ACWI (MA), equal-weighted (EW), value-weighted (VW), GMV, MV and euro (EU) portfolios and the maximum performance fee the an investor would pay to switch from the MSCI World Portfolio to the other portfolios across hedge ratios. The asset menu consists of both developed and emerging markets equity.

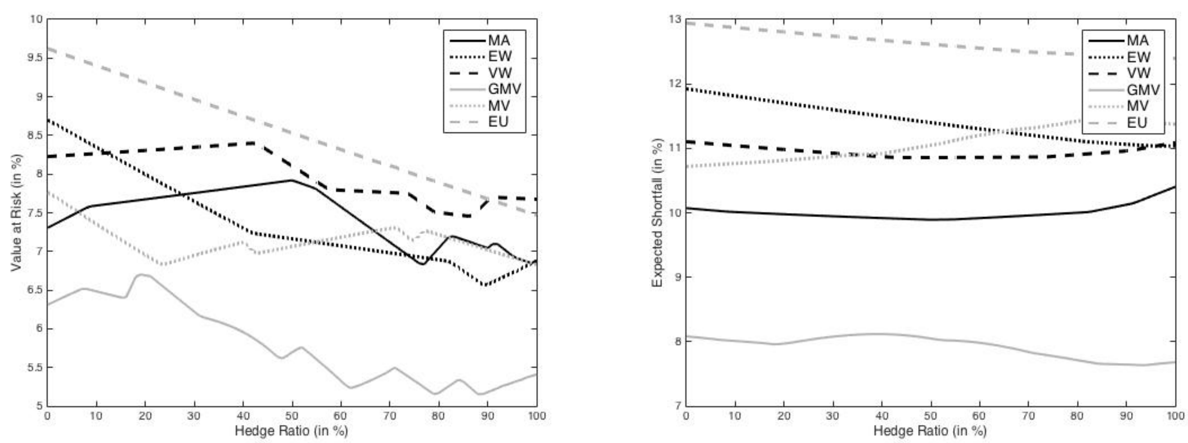
Although the annualised Sharpe ratios of the GMV portfolio are higher than those of the MSCI ACWI portfolio for all hedge ratios, the difference is not statistical at the 0.10 significance level. The MV portfolio has far higher annualised Sharpe ratios than the MSCI ACWI portfolio. The difference is furthermore statistically significant at the 0.05 significance level for all hedge ratios. For the p-values of the hypothesis test of difference in Sharpe ratios with both developed and emerging markets in the asset menu, we refer the reader to Figure A4b in Appendix H.1. Although the equal-weighted portfolio has slightly higher annualised Sharpe ratios than the MSCI ACWI portfolio for all hedge ratios up to 58%, the difference is negligibly small. As when we only compared developed markets equity, the value-weighted portfolio and the euro portfolio both have lower Sharpe ratios than the MSCI-based portfolio. We furthermore find that all Sharpe ratios are higher after including emerging markets in the asset menu. This is not surprising given the relatively high Sharpe ratios of Indian, South Korean and Taiwanese equity.

As before considering emerging markets, the Sharpe ratio of all portfolios other than the MV portfolio increases with the hedge ratio until reaching its maximum and decreasing again. Figure 6a furthermore illustrates that the Sharpe ratios of the MSCI ACWI, equal-weighted, value-weighted, GMV and euro portfolios lowest when the foreign stocks are unhedged. The only difference compared to when the asset menu only consists of the stocks of developed markets is that the Sharpe ratio of the MV portfolio now is in general an increasing function of the hedge ratio.

Plotting the maximum performance fee an investor would pay to switch from the MSCI ACWI portfolio to the other portfolios against the hedge ratio results in a similar image as when we plotted the average returns against the hedge ratios (Figure 5a). The MV portfolio once again outperforms all other portfolios in terms of this measure for all hedge ratios. As the values in Figure 6b corresponding to the equal-weighted portfolio are all slightly larger than 0, the investor is also willing to pay a (small) fraction of his wealth to switch from the MSCI ACWI portfolio to the equal-weighted portfolio. As the maximum performance fees the investor would be willing to pay to switch from the MSCI ACWI portfolio to the value-weighted and euro portfolios are (slightly) negative, these two portfolios are both (slightly) outperformed by the MSCI ACWI portfolio in terms of the maximum performance fee measure.

The effect of currency hedging on the performance fee measure is again almost comparable to the effect of hedging on the average return for the GMV and the MV portfolios. The performance fee the investor would be willing to pay to switch from the MSCI ACWI portfolio to the MV portfolio is a strictly increasing function of the hedge ratio. As the average return, the performance fee increases until reaching its maximum for the GMV and euro portfolios before decreasing again. For both the equal-weighted portfolio and the value-weighted portfolio, Figure 6b shows that the performance fee estimate is strictly decreasing.

Once again, we consider the VaR and ES risk measures. Figure 7 presents the monthly 95% VaRs and ES's of the MSCI ACWI, equal-weighted, value-weighted, GMV, MV and euro portfolios across hedge ratios.



(a) 95% Value at Risk

(b) 95% Expected Shortfall

Figure 7: The monthly 95% Value at Risk and Expected Shortfall estimates of the MSCI ACWI (MA), equal-weighted (EW), value-weighted (VW), GMV, MV and euro (EU) portfolios across hedge ratios. The asset menu consists of both developed and emerging markets equity.

The plots of the monthly 95% VaR estimates with an asset menu consisting of the equity of both developed and emerging markets look a lot like when we only considered developed markets. The largest differences are that the equal-weighted portfolio now has considerably lower VaR estimates than when we only considered developed markets and that the VaR estimates of the MV portfolio are now far higher than before. Figure 7a shows that the equal-weighted portfolio is less risky in terms of VaR than the MSCI ACWI portfolio for most hedge ratios over 28% while the MV portfolio has lower VaR estimates than the MSCI ACWI portfolio for hedge ratios between 7 and 66%. The GMV portfolio unsurprisingly turns out to be the least risky portfolio again and is furthermore far less risky than the MSCI ACWI portfolio.

We once again find a general downward trend when the monthly 95% VaR estimates are plotted against the hedge ratio. For the equal-weighted, MV and euro portfolios we even find that the VaR is highest for unhedged returns. Compared to when only developed markets are considered, the downward trend is more distinct now for the value-weighted and MSCI portfolio. For all portfolios, the highest monthly 95% VaR estimates are found for the lowest half of hedge ratios while the lowest 95% VaRs are found for the highest quarter of hedge ratios.

In terms of monthly 95% ES, only the GMV portfolio outperforms the MSCI ACWI portfolio. The monthly 95% ES estimates of the GMV portfolio are far lower than those of the other portfolios. Figure 7b shows that the estimated ES's of the equal-weighted, value-weighted, MV and euro portfolios are considerably higher than the ES estimates of the MSCI ACWI portfolio for every hedge ratio.

As when we only considered developed markets, the effect of currency hedging on the 95% ES is quite small. While a clear downward trend is visible for the 95% ES of the equal-weighted portfolio and the euro portfolio, we find an upward trend for the ES estimates of the MV portfolio. On the other hand, the hedge ratio seems to have little effect on the 95% ES of the MSCI ACWI, value-weighted and GMV portfolios. The ES's of the MSCI ACWI and value-weighted portfolios are almost completely unaffected by the hedge ratio until a hedge ratio of around 80%, after which they suddenly start increasing. The estimated monthly 95% ES of the GMV portfolio remains stable around a value of approximately 7.93% as the hedge ratio increases.

Table 9 presents the annualised average returns, annualised volatilities, annualised Sharpe ratios, monthly 95% VaRs and monthly 95% ES's averaged over the hedge ratios for the MSCI ACWI, equal-weighted, value-weighted, GMV, MV and euro portfolios. Table 9 also ranks each of the portfolios based on these measures.

Table 9: The average return, volatility, Sharpe ratio, performance fee, 95% Value at Risk and 95% Expected Shortfall estimates of the MSCI ACWI (MA), equal-weighted (EW), value-weighted (VW), GMV, MV and euro (EU) portfolios averaged over the hedge ratios with both developed and emerging markets equity as investable assets

	MA		EW		VW		GMV		MV		EU	
Average Return (in %)	5.65	(4)	6.21	(3)	5.10	(6)	7.46	(2)	16.71	(1)	5.39	(5)
Volatility (in %)	13.50	(2)	15.15	(4)	14.36	(3)	11.62	(1)	16.94	(6)	16.24	(5)
Sharpe Ratio	0.23	(4)	0.23	(3)	0.19	(5)	0.36	(2)	0.71	(1)	0.18	(6)
Performance Fee (in BPS)	-	(4)	2.97	(3)	-5.43	(6)	16.65	(2)	88.46	(1)	-5.03	(5)
95% Value at Risk (in %)	7.46	(4)	7.38	(3)	8.01	(5)	5.80	(1)	7.13	(2)	8.54	(6)
95% Expected Shortfall (in %)	10.00	(2)	11.42	(5)	10.94	(3)	7.93	(1)	11.08	(4)	12.63	(6)

Note: the rank of the portfolio based on each measure is presented in parentheses. Average returns, volatilities and Sharpe ratios are annualised. The asset menu consists of both developed and emerging markets equity.

As all annualised average returns presented in Table 9 are considerably higher than their counterparts in Table 6, we can conclude that adding the equity of emerging markets to the asset menu adds some value in terms of expected return. On the other hand, we find that the portfolios in general are unsurprisingly also more risky after the introduction of emerging markets equity. As the Sharpe ratios of most portfolios are slightly higher when the asset menu consists of both developed and emerging markets equity, however, we can conclude that in most cases the performances of the portfolios in terms of risk-adjusted returns are at least as good after including emerging markets equity into the asset menu as before.

Like in Section 4.1.1, the GMV portfolio and especially the MV portfolio outperform the MSCI portfolio in terms of historical average return. In contrast to then, however, we now also have that the equal-weighted portfolio has a higher historical average return. In terms of volatility, only the GMV outperforms the MSCI ACWI portfolio. Moreover, the MV portfolio is more risky after the introduction of emerging markets equity and is now furthermore the most volatile portfolio. This can be explained by the large allocation in the MV portfolio to the highly volatile Indian equity. Averaged over the hedge ratios, only the equal-weighted, GMV and MV portfolios have higher Sharpe ratios than the MSCI ACWI portfolio. The difference, however, is negligibly small for the equal weighted portfolio and not significant at the 0.10 significance level for the GMV portfolio (p-value of 0.21). With an average annualised Sharpe ratio that is almost twice as high as that of the MSCI ACWI portfolio, the MV portfolio significantly (p-value of 0.03) outperforms the MSCI ACWI portfolio when averaged over hedge ratios. The performance fee measure unsurprisingly yields similar results as the average return: The MV portfolio performs best, while the GMV portfolio and the equal-weighted portfolio also have higher values than the MSCI ACWI portfolio. The GMV, MV and equal-weighted portfolios also outperform the MSCI ACWI portfolio in terms of monthly 95% VaR. Only the GMV portfolio, however, also has a lower monthly 95% ES than the MSCI ACWI portfolio.

As when the asset menu only consists of developed markets equity, the value-weighted and euro portfolio do not outperform the MSCI-based portfolio for any measure we consider. The GMV and MV portfolios on the other hand outperform the MSCI ACWI portfolio in terms of almost all measures. We furthermore find that the equal-weighted portfolio is also slightly more rewarding than the MSCI ACWI portfolio. However, as the equal-weighted portfolio is also more risky than the MSCI ACWI portfolio and the difference in terms of annualised Sharpe ratio is negligibly, neither of the two portfolios can be said to be convincingly better than the other. Hence, the best portfolio is either the MV portfolio for investors who can bear the relatively high risk or the GMV portfolio for investors who prefer a safer option. As the GMV and MV portfolios outperform the MSCI ACWI portfolio in terms of various measures we consider, it can certainly be of value to deviate from the regional allocation of the MSCI ACWI Index.

Once again, we consider the portfolio turnovers averaged over the hedge ratios to get an idea of how transaction costs for the portfolio allocation could affect the results. The equal-weighted, value-weighted and euro portfolios have slightly higher turnovers (2.72, 2.48 and 2.31%, respectively) than the MSCI ACWI portfolio (1.49%). With average values of 8.02% and 18.87%, respectively, the difference in turnover is larger for the GMV and MV portfolios. Assuming transaction costs of 50 BPS again, however, the average returns of these portfolios (6.97 and 15.57%, respectively) are still considerably higher than that of the MSCI ACWI portfolio (5.56%) when we take the average over the hedge ratios. This implies that when the GMV and MV portfolios outperform the MSCI ACWI portfolio in terms of certain measures in the absence of transaction costs, they probably also outperform the MSCI ACWI portfolio in terms of these measures in the presence of (moderate) transaction costs.

4.2 The Theoretically Optimal Portfolio of an Investor with a Defined Investment Horizon

We now consider the theoretically optimal intertemporal portfolio of an investor with a defined investment horizon. The results presented in Section 4.2.1 correspond to an asset menu consisting of only the equity of developed markets while the asset menu consists of both developed and emerging markets equity in Section 4.2.2. We only report the results obtained by applying the forms of shrinkage described in Section 3.1.4, as the portfolio weights obtained without applying shrinkage are far too extreme for any practical use or inference.

4.2.1 Developed Markets Equity

We find similar results for the estimation of the matrix of coefficients $\mathbf{\Phi}_1$ of our VAR model (30) as Jurek and Viceira (2011) and Viceira et al. (2017) among others have found for the estimation of similar VAR models. We find for instance that the dividend yield on average is a positive predictor of stock returns while the short-term interest rate is a negative predictor. We also find that these variables are good predictors of themselves with estimated coefficients and R^2 statistics close to 1. Like e.g. Viceira et al. (2017), we find that the R^2 is quite low for the equations of the stock returns, which shows that stock returns are hard to predict. This is not a problem for us, as Campbell and Thompson (2005) show that a small R^2 can still be economically meaningful. The fact that our estimates produce well-known results show that our specification of the VAR model (30) is very reasonable.

Table 10 presents the theoretically optimal portfolio weights of an investor with an investment horizon of 1, 10 or 20 years who begins with investing in December 2016 and chooses for a hedge ratio of 0, 50 or 100%.

Table 10: The weights of the unhedged, half hedged and fully hedged theoretically optimal portfolio of an investor with an investment horizon (K) of 1, 10 or 20 years and developed markets equity as investable assets

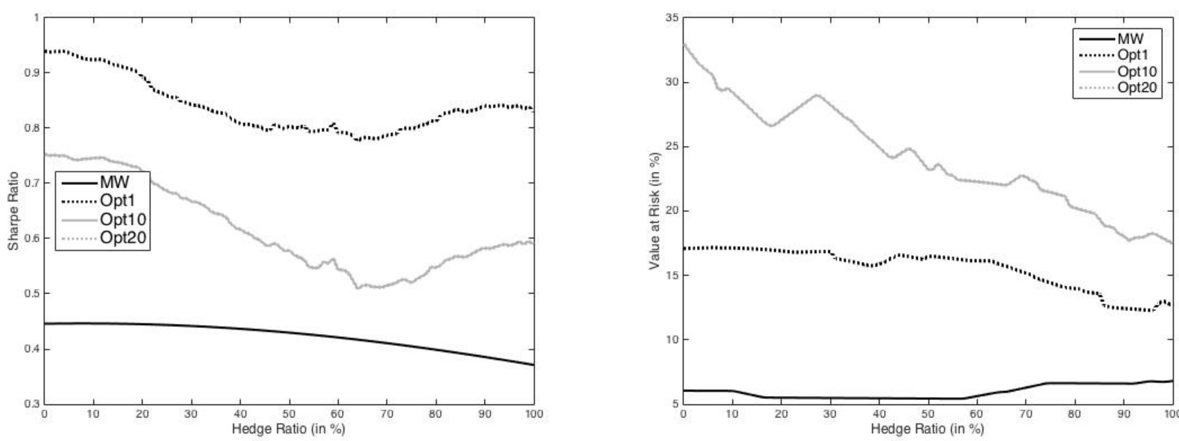
Panel A: Unhedged Returns										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
K=1	-53.2%	15.3%	-19.0%	-103.8%	254.2%	145.5%	-101.5%	162.8%	198.2%	-398.4%
K=10	-379.1%	65.0%	125.4%	-540.3%	399.7%	356.4%	-510.9%	590.4%	483.1%	-489.8%
K=20	-379.2%	65.0%	125.4%	-540.4%	399.8%	356.4%	-511.0%	590.5%	483.2%	-489.8%
Panel B: Half Hedged Returns										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
K=1	5.7%	113.5%	-34.2%	-31.9%	195.0%	34.5%	-116.2%	118.0%	119.7%	-304.1%
K=10	-128.7%	256.5%	-12.1%	-245.3%	196.7%	110.2%	-395.2%	472.1%	428.5%	-582.7%
K=20	-128.7%	256.6%	-12.1%	-245.4%	196.7%	110.2%	-395.3%	472.2%	428.6%	-582.7%
Panel C: Fully Hedged Returns										
	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
K=1	-10.3%	125.4%	-25.8%	10.6%	146.2%	-7.4%	-91.9%	68.2%	26.3%	-141.3%
K=10	48.0%	264.2%	-17.3%	-36.8%	137.7%	-11.8%	-193.5%	193.5%	126.7%	-410.7%
K=20	48.1%	264.3%	-17.3%	-36.8%	137.6%	-11.8%	-193.5%	193.6%	126.8%	-410.9%

Although they are more extreme due to the lack of short selling restrictions, the weights of the theoretically optimal portfolio are generally quite comparable to the GMV and MV portfolios in terms of which assets are invested in most. We find that the weights assigned to Canadian, Hong Kong, Swiss and British equity in the theoretically optimal portfolio are positive for all hedge ratios and investment horizons. The popularity of these assets can be explained by their relatively high Sharpe ratios or low volatilities. For French, German, Dutch and American equity on the other hand, we find that the portfolio weights are generally negative. This makes sense for the Eurozone assets given their low Sharpe ratios while the negative weights assigned to U.S. equity can be explained by the relatively high correlations with other countries.

The portfolio weights do not differ in sign across hedge ratios for all but a few cases. We do find some considerable differences in size that cause the set of assets invested in most to differ across hedge ratios. For unhedged returns, the assets invested in most are Swiss, British, Hong Kong and Japanese equity in that order. When the hedge ratio increases to 50%, the investor should invest more in Canadian equity than Hong Kong equity. As the Sharpe ratio of Canadian equity keeps increasing as the hedge ratio increases and ends up considerably higher than the Sharpe ratios of other stocks, it is no surprise that the most weight in the theoretically optimal portfolio is assigned to Canadian equity.

The portfolio weights of the middle-term investor and the long-term investor are almost identical. This is in line with the findings of Jurek and Viceira (2011) that the weights of the theoretically optimal portfolio are very similar for all investment horizons larger than around 10 years. The portfolio weights of the short-term investor only differ in sign with those of the middle- and long-term investors in a few cases. We do find considerable difference in terms of magnitude. In particular, we generally find that positive weights are a positive function of the investment horizon, while weights that are negative for the short-term investor are a negative function of the investment horizon. This is also found by Jurek and Viceira (2011) for all asset they consider.

To compare the theoretically optimal portfolios with the MSCI World portfolio, we compare the annualised Sharpe ratios and volatilities of these portfolios. Figure 8 presents these statistics across all hedge ratios.



(a) Sharpe Ratios

(b) 95% Value at Risk

Figure 8: The annualised Sharpe ratios and monthly 95% Value at Risk of the MSCI World (MW) portfolio and the theoretically optimal portfolio of an investor with an investment horizon of 1 year (Opt1), 10 years (Opt10) or 20 years (Opt20) across hedge ratios. The asset menu only consists of developed markets equity.

Figure 8a shows that the theoretically optimal portfolio has higher Sharpe ratios than the MSCI World portfolio for all investment horizons.[4] Even though the theoretically optimal portfolios have higher Sharpe ratios than the MSCI World portfolio, they are also far more risky. Figure 8b shows that the monthly 95% VaR estimates of the theoretically optimal portfolio ranges from 12.27 to 17.18% for the short-term investor and from around 17.47 to 32.97% for the middle- and long-term investor. The monthly 95% VaRs of the MSCI World portfolio range from 5.42 to 6.81% and are clearly far lower. Hence, an investor should carefully consider the high risk of the theoretically optimal portfolio before using the portfolio weights in practice.

The theoretically optimal portfolios are virtually identical in terms of Sharpe ratio and VaR between the investment horizons of 10 and 20 years. This is not surprising given the fact that the portfolios weights of the middle- and long-term investors presented in Table 10 are very similar to each other. Equally unsurprisingly, the statistics of the theoretically optimal short-term portfolio differ considerably. Figure 8 shows that the theoretically optimal portfolio has both higher Sharpe ratios and lower VaRs for the investment horizon of 1 year than for the investment horizons of 10 and 20 years. When we compare the Sharpe ratio of the theoretically optimal portfolio across hedge ratios, we find that it decreases to its minimum before increasing again. Figure 8a shows that the Sharpe ratio is highest for unhedged returns. The pattern is almost identical for all three investment horizons. The effect of the hedge ratio on the VaR is quite different: Figure 8b shows a clear decreasing trend for all investment horizons. This implies that the theoretically optimal portfolio is less risky for higher hedge ratios.

4.2.2 Developed and Emerging Markets Equity

In contrast to Section 4.2.1, the asset menu now consists of the equity of both developed and emerging markets. After including the equity of emerging markets in the asset menu, we find similar results for the estimation of the matrix of coefficients $\mathbf{\Phi}_1$ of our VAR model (30) as in Section 4.2.1. The finding are furthermore again in line with well-known results: the dividend yield and short-term interest rate are positive and negative predictors of stock returns, respectively, and are furthermore good predictors of themselves while stock returns are generally hard to predict. The specification of our VAR model (30) therefore seems to be reasonable.

Table 11 presents the theoretically optimal portfolio weights of an investor with an investment horizon of 1, 10 or 20 years who begins with investing in December 2016 and chooses for a hedge ratio of 0, 50 or 100%. The asset menu now consists of the equity of both developed and emerging markets.

We find that the weights of the theoretically optimal portfolio are again comparable to the GMV and MV portfolios in terms relative allocation. As before introducing emerging markets equity to the asset menu, the portfolio weights assigned to Canadian, Hong Kong, Swiss and British equity in the are again positive for all combinations of hedge ratio and investment horizon while the weights of French, German, Dutch and American equity are again mostly negative. The weights assigned to Chinese, Indian and Taiwanese equity are furthermore also always positive. This can be explained by the relatively low correlations of Chinese equity with other assets and the relatively high Sharpe ratios of Taiwanese and especially Indian equity. The investor should furthermore short sell Korean equity for most investment horizons and hedge ratios. This is probably because Korean equity is considerably more correlated with other assets than the other emerging markets stocks are. Brazilian equity should not bought for any hedge ratio or investment horizon, which is due to its relatively low Sharpe ratio.

[4] As the hypothesis testing method of Ledoit and Wolf (2008) requires long computations times and investigating the optimality of the portfolio optimisation routine of Jurek and Viceira (2011) is not the aim of our research, we refrain from testing the statistical significance of the differences in Sharpe ratios

Table 11: The weights of the unhedged, half hedged and fully hedged theoretically optimal portfolio for investment horizon (K) of 1, 10 or 20 years and both developed and emerging markets equity as investable assets

Panel A: Unhedged Returns, Developed Markets

	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
K=1	5.8%	24.0%	-26.9%	-50.2%	214.3%	128.9%	-55.1%	139.2%	108.0%	-377.6%
K=10	-199.9%	85.3%	41.5%	-360.0%	210.6%	315.1%	-325.0%	503.2%	284.4%	-518.7%
K=20	-199.9%	85.4%	41.5%	-360.0%	210.6%	315.1%	-325.0%	503.2%	284.4%	-518.7%

Panel B: Unhedged Returns, Emerging Markets

	BR	CN	IN	KR	TW
K=1	-13.7%	45.8%	12.6%	-110.5%	55.6%
K=10	-57.6%	143.3%	46.8%	-260.2%	191.1%
K=20	-57.6%	143.4%	46.8%	-260.3%	191.1%

Panel C: Half Hedged Returns, Developed Markets

	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
K=1	18.8%	105.1%	-35.9%	-19.2%	170.1%	22.6%	-87.6%	92.8%	93.9%	-296.1%
K=10	-65.5%	261.6%	-31.5%	-192.0%	97.8%	73.4%	-288.5%	393.9%	341.6%	-647.7%
K=20	-65.5%	261.7%	-31.5%	-192.1%	97.8%	73.4%	-288.6%	394.0%	341.6%	-647.8%

Panel D: Half Hedged Returns, Emerging Markets

	BR	CN	IN	KR	TW
K=1	-20.9%	23.4%	45.5%	-49.6%	37.1%
K=10	-82.4%	85.4%	96.8%	-68.5%	125.6%
K=20	-82.4%	85.5%	96.9%	-68.6%	125.6%

Panel E: Fully Hedged Returns, Developed Markets

	AU	CA	FR	DE	HK	JP	NL	CH	UK	US
K=1	-9.6%	122.0%	-22.0%	-5.9%	134.4%	-21.8%	-79.5%	64.4%	36.5%	-193.3%
K=10	47.3%	274.7%	-20.0%	-74.5%	93.3%	-33.5%	-183.7%	213.5%	163.3%	-594.9%
K=20	47.4%	274.9%	-20.0%	-74.6%	93.2%	-33.5%	-183.8%	213.7%	163.5%	-595.2%

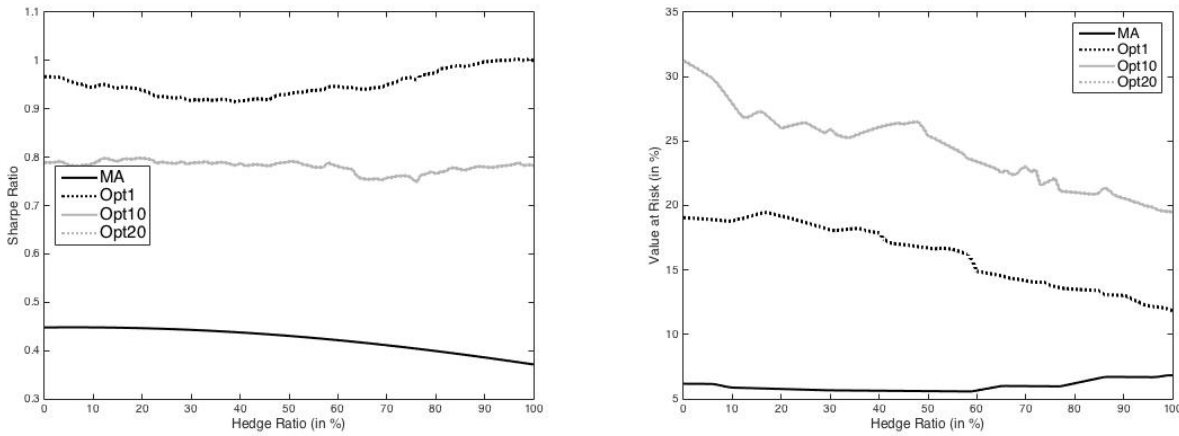
Panel F: Fully Hedged Returns, Emerging Markets

	BR	CN	IN	KR	TW
K=1	-18.3%	12.3%	67.2%	-8.6%	22.3%
K=10	-31.1%	41.7%	107.8%	45.6%	50.2%
K=20	-31.1%	41.7%	107.8%	45.7%	50.3%

Like when we only considered developed markets equity, the portfolio weights differ more in terms of magnitude than in terms of sign across hedge ratios. For unhedged returns, we find that Swiss, British, Hong Kong and Japanese stocks are again invested in most. The countries most weights is allocated to remain almost unchanged for the hedge ratios of 50 and 100% after introducing emerging markets equity: the only differences are that the allocation to Taiwanese equity is larger than the allocation to Hong Kong equity when the hedge ratio is 50% and that the fully hedging investor should buy more Indian equity than Hong Kong equity.

Table 11 furthermore shows that the allocation to Indian equity increases as the hedge ratio increases. As in the case of Canadian equity, this can be explained by the fact that the Sharpe ratio of Indian equity increases (strongly) as the degree of currency hedging intensifies (see Figure A1b in Appendix A). We furthermore find again that the portfolio weights of the middle-term investor are very similar to those of the long-term investor, while those of the short-term investor differ considerably.

We once again look at the annualised Sharpe ratio and monthly 95% VaR of the theoretically optimal portfolio. Figure 9 presents these statistics for the theoretically optimal portfolios of the short-term, middle-term and long-term investor and the MSCI ACWI portfolio across hedge ratios.



(a) Sharpe Ratios

(b) 95% Value at Risk

Figure 9: The annualised Sharpe ratios and monthly 95% Value at Risk of the MSCI World (MW) portfolio and the theoretically optimal portfolio of an investor with an investment horizon of 1 year (Opt1), 10 years (Opt10) or 20 years (Opt20) across hedge ratios. The asset menu consists of both developed and emerging markets equity.

Figure 9a illustrates the annualised Sharpe ratios of the theoretically optimal portfolio are considerably higher than those of the MSCI ACWI portfolio. As before we included emerging markets equity in the asset menu, the theoretically optimal portfolios are also far more risky than the MSCI-based portfolio.

Furthermore, Figure 9 shows that the Sharpe ratios and VaR estimates of the theoretically optimal portfolios are again almost identical between the investment horizons of 10 and 20 years while they differ considerably for the short-term investor. Also, the theoretically optimal portfolio once again has higher Sharpe ratios and lower VaRs for the investment horizon of 1 year. The effect of the hedge ratio on the Sharpe ratio of the theoretically optimal portfolio is slightly different compared to when the asset menu only consists of developed market equity, as it now remains fairly stable as the hedge ratio increases. Figure 9b shows that the effect of the hedge ratio on the VaR is similar to before we introduced emerging markets: a clearly decreasing trend is visible for the short-term, middle-term and long-term theoretically optimal portfolios alike.

5 An Empirical Analysis of Global Bond Diversification

This section presents and discusses the results of the part of our research on the global diversification of bonds. Section 5.1 presents the results of the analysis on the benefits of diversifying into U.S. dollar-denominated bonds for a Eurozone investor. As some investors want to hedge their liabilities against adverse changes in interest rates and inflation, we also analyse the hedging qualities of euro- and dollar-denominated bonds. Section 5.2

presents the results of this investigation. Finally, Section 5.3 presents the theoretically optimal portfolio weights of an asset-only and an asset-liability investor.

5.1 Benefits of Diversifying into U.S. Bonds

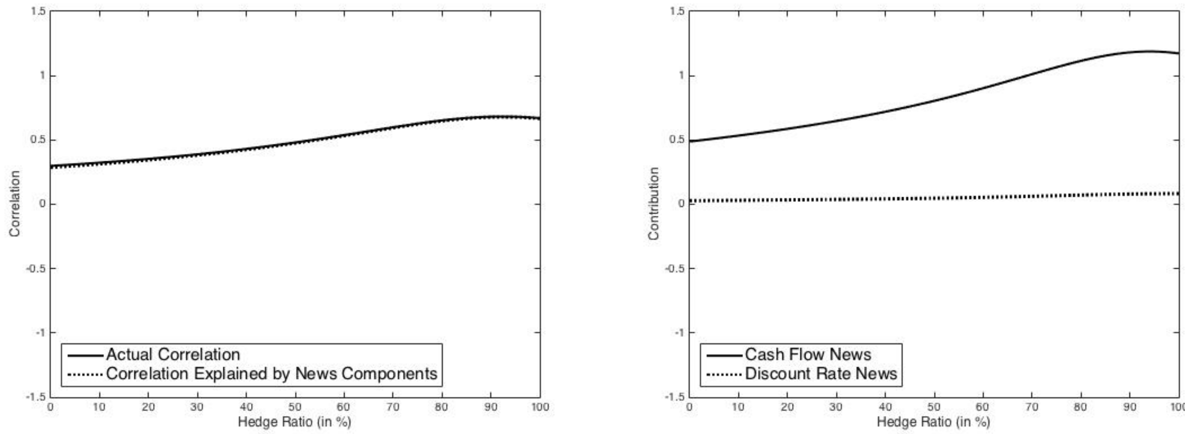
Government bonds and corporate bonds are considered separately in this section. Section 5.1.1 presents the results of our analysis on the benefits for a Eurozone investor of diversifying into U.S. dollar-denominated bonds with only government bonds as investable assets. Section 5.1.2 presents the results for an investor who can only invest in corporate bonds.

5.1.1 Government Bonds

We find similar results for the estimation of our pooled VAR model (43) as Viceira et al. (2017) find for theirs. As the results of the VAR model are not of much importance for our research, we only present a summary here and refer the reader to Table A8 in Appendix I.1 for a full overview of the parameter estimates. As each state variable is a significant predictor of itself, the variables we consider seem to be well-described by our VAR(1) specification. Like Viceira et al. (2017), we also find that the equations corresponding to bond returns have the lowest R^2's, which shows that returns are hard to predict accurately. The estimated coefficients do not differ much across hedge ratios. The only clear peculiarity is that the log yield spread is only a statistically significant predictor of excess log returns when the returns are fully hedged. As also found by Viceira et al. (2017), they are in this case positive predictors of log bond excess returns. As the R^2's are almost identical across hedge ratios, however, we can conclude that our pooled VAR(1) model is almost equally suitable for unhedged, half hedged and fully hedged bond returns.

We use the estimated coefficients of our pooled VAR(1) model to compute the cash flow news component (50) and the discount rate news component (53). We then compute the estimated contribution of the correlations of these news components to the total correlation between the Eurozone and U.S. excess bond returns by means of (54). Figure 10 presents the correlation between Eurozone and U.S. government bonds as of December 2016 and the estimated contribution of the correlations of the cash flow news and discount rate news components to the total correlation across hedge ratios. Figure 10a illustrates that the correlation between Eurozone and U.S. government bonds as of December 2016 slightly increases as the hedge ratio increases. The (small) increase in correlation as the hedge ratio increases implies that the global diversification between Eurozone and U.S. government bonds is (slightly) less attractive for the higher hedge ratios than for the lower hedge ratios. Figure 10a also shows that the correlation explained by the news components is almost exactly equal to the actual correlation for all hedge ratios, which means that our news components and their contributions to the actual correlation have been estimated accurately. The actual total correlation ranges from 0.299 to 0.682 while the correlation explained by the news components ranges from 0.287 to 0.680.

As Figure 10b clearly shows, the correlation between Eurozone and U.S. government bonds as of December 2016 is almost entirely driven by cross-country correlated cash flow news. Cross-country discount rate correlations barely contribute to the total cross country correlation. Therefore, the benefits of the global diversification between Eurozone and U.S. government bonds measured as a reduction of portfolio risk have declined equally for short-term investors and long-term investors. The intensity of currency hedging has virtually no effect on the contribution of cross-country discount rate news correlations. On the other hand, the cross-country discount



(a) Correlation Between Eurozone and U.S. Bonds

(b) Contribution of News Components

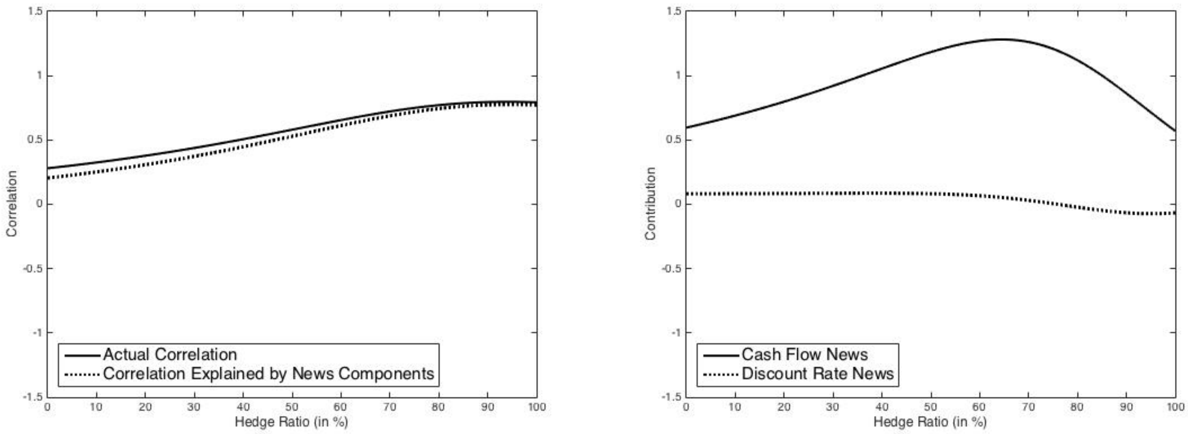
Figure 10: The correlation between Eurozone and U.S. government bonds as of December 2016 and the estimated contribution of the correlations of cash flow news and discount rate news to the total correlation across hedge ratios.

rate news correlations are clearly increasing as the hedge ratio increases. As the total correlation between Eurozone and U.S. government bonds is almost entirely driven by cross-country cash flow news correlations for each hedge ratio, the benefits of the global diversification between Eurozone and U.S. government bonds measured as a reduction of portfolio risk are not affected by to what extent a Eurozone investor chooses to currency hedge the American bonds.

5.1.2 Corporate Bonds

We now consider corporate bonds instead of government bonds. As when we considered government bonds, we only present a summary of the pooled VAR model estimates here and refer the reader to Table A9 in Appendix I.2 for the values of the parameter estimates. We find comparable results for the estimation of our pooled VAR model with corporate bonds as when we considered government bonds. Log yield spreads are once again statistically significant positive predictors of log bond excess returns while the state variables are again well-described by our VAR(1) specification. The R^2's are again almost identical across hedge ratios for the equations of the state variables. For the equations corresponding to the corporate bond returns, we find that the R^2's are still low for corporate bond returns, but slightly higher than for government bond returns.

We once again use the estimated coefficients of our pooled VAR(1) model to compute the cash flow news component (50) and the discount rate news component (53). We then compute the contribution of the correlations of these news components to the total correlation between Eurozone and U.S. corporate bonds by means of (54). Figure 11 presents the correlation between Eurozone and U.S. corporate bonds as of December 2016 and the estimated contribution of the correlations of the two news components to the total correlation across all hedge ratios. As when we considered government bonds, the correlation between Eurozone and U.S. corporate bonds as of December 2016 increases as the hedge ratio increases. For both government and corporate bonds, global diversification between Eurozone and U.S. bonds therefore becomes less attractive as the hedge ratio



(a) Correlation Between Eurozone and U.S. Bonds

(b) Contribution of News Components

Figure 11: The correlation between Eurozone and U.S. corporate bonds as of December 2016 and the estimated contribution of the correlations of cash flow news and discount rate news to the total correlation across hedge ratios.

increases. Furthermore, the correlations between the Eurozone and U.S. corporate bonds are very similar to the correlations between the Eurozone and U.S. government bonds. The correlations are almost identical for hedge ratios up to 9%, while we find that the correlations between the corporate bonds are clearly (slightly) higher than the correlations between the government bonds. Therefore, global diversification between Eurozone and U.S. corporate bonds seems to be slightly more attractive than global diversification between Eurozone and U.S. government bonds for most hedge ratios. Although the difference is slightly larger than in the case of government bonds, the correlation explained by the news components is again very similar to the actual correlation for each hedge ratio: the actual total correlation ranges from 0.280 to 0.797 while the correlation explained by the news components ranges from 0.205 to 0.776.

The estimated contributions of the news component correlations to the total correlations are very similar between when the investable bonds are corporate bonds and when they are government bonds: the correlations between Eurozone and U.S. corporate bonds are also almost entirely driven by cross-country correlated cash flow news while the contribution of cross-country discount rate correlations are close to 0. This holds for every hedge ratio between 0 and 100%. Therefore, the benefits of the global diversification between Eurozone and U.S. corporate bonds measured as a reduction of portfolio risk have declined equally for short-term investors and long-term investors. The hedge ratio once again has a very small effect on the contribution of cross-country discount rate news correlations. The cross-country cash flow news correlations are affected slightly differently by the hedging intensity when the investable assets are corporate bonds compared to when they are government bonds. The contribution of cross-country correlated cash flow news increases as the hedge ratio increases until 66%, after which it decreases again. As the total correlation between Eurozone and U.S. corporate bonds is nevertheless still almost entirely driven by cross-country cash flow news correlations for each hedge ratio, the benefits of the global diversification between Eurozone and U.S. corporate bonds measured as a reduction of portfolio risk are barely affected by the hedge ratio an investor uses.

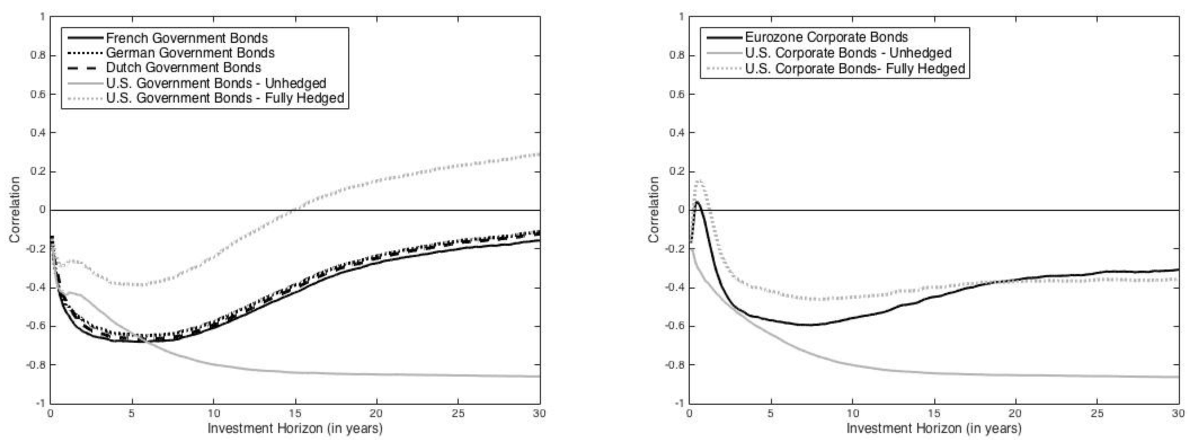
All in all, our results confirm the common consensus that even after taking into account the historically low yield of Eurozone bonds, it does not add value for a Eurozone investor to diversify into U.S. bonds. This holds for both short-term and long-term investors. We also find that the conclusion is the same for each hedge ratio and for both government and corporate bonds.

5.2 Hedging Qualities of Eurozone and U.S. Bonds

In this section, we analyse the hedging qualities of Eurozone and American bonds. We consider government and corporate bonds simultaneously now. Section 5.2.1 presents the results corresponding to the inflation hedging qualities of bonds. In Section 5.2.2, we analyse the interest rate hedging qualities of bonds. For both sections, we find similar results for the estimation of the VAR(1) model (56) as Hoevenaars et al. (2008). For instance, nominal interest rates are found to be quite persistent while yield spreads are generally found to be good predictors of bond returns. All in all, our VAR(1) model specification seems to be very reasonable.

5.2.1 Inflation Hedging Qualities

We first analyse the inflation hedging qualities of the Eurozone and U.S. bonds in our data set. Figure 12 presents the correlations between cumulative excess log bond returns and euro inflation rates across investment horizons.



(a) Government Bonds

(b) Corporate Bonds

Figure 12: The correlations between cumulative excess log bond returns and euro inflation rates across investment horizons. The U.S. bond returns returns are unhedged or fully hedged.

For most bonds and investment horizons, the hedging qualities of the bonds we consider are negative. This is caused by the inverse relation between bond prices and yield changes (Hoevenaars et al., 2008). We find that fully hedged U.S. bonds generally have higher inflation hedging qualities than the Eurozone bonds. For corporate bonds, this holds for for investment horizons shorter than 20 year while hedged U.S. government bonds have higher inflation hedging qualities than Eurozone government bonds for virtually all investment horizons. This implies that fully hedged U.S. bonds are more suitable to hedge against adverse changes in inflation rates than

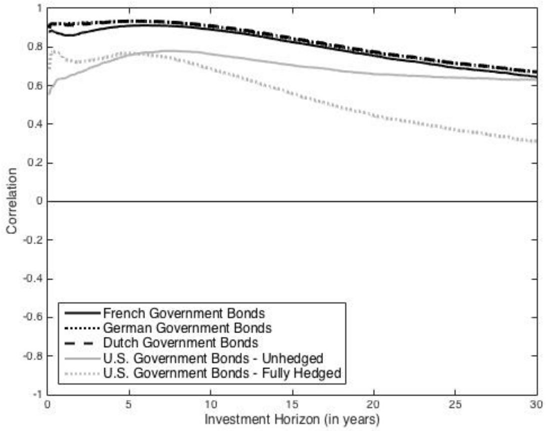
Eurozone bonds for the aforementioned investment horizons. The unhedged U.S. bonds on the other hand have the lowest inflation hedging qualities among the bonds we consider. Even though the hedged American bonds have the highest inflation hedging qualities, the hedging qualities are not positive for all investment horizons. The hedged U.S. government bonds are only positively correlated with the Eurozone inflation for investment horizons of over 15 years while the hedged U.S. corporate bonds only have positive inflation hedging qualities for a few short investment horizons. This implies that hedged U.S. government bonds in are a good hedge against inflation risk in the long-run while hedged U.S. corporate bonds are only good hedging instruments in the short run. The French, German and Dutch government bonds are not positively correlated with the euro inflation for any investment horizon. Moreover, the inflation hedging qualities of these bonds are unsurprisingly very similar to each other while they are also similar to the hedging qualities of Eurozone corporate bonds.

The correlations initially decrease for the Eurozone and hedged U.S. government bonds, but afterwards increase considerably as the investment horizon increases. This is in line with the findings of Hoevenaars et al. (2008). For the Eurozone and hedged U.S. corporate bonds, the correlations first increase sharply before showing a similar pattern. The correlations of the Eurozone inflation with the unhedged U.S. bond returns initially keep decreasing as the investment horizon increases, but remain stable for investment horizons over 15 years. For both government and corporate bonds, the fully hedged U.S. government bonds are superior over the unhedged government bonds. Although this holds for all investment horizons, the difference is largest for investment horizons higher than 10 years. When we compare the hedging qualities across bond types, we find that that the inflation hedging qualities of the French, German and Dutch government bonds are very similar to those of Eurozone corporate bonds. The same is true for unhedged U.S. bonds. On the other hand, the inflation hedging qualities of hedged U.S. corporate bonds are considerably higher than those of hedged U.S. government bonds for short-term investors, but also far lower for investment horizons of 10 years and higher. The correlations of the Eurozone bonds and the Eurozone inflation furthermore display a very similar pattern as the correlations of the Eurozone inflation with their hedged U.S. counterparts.

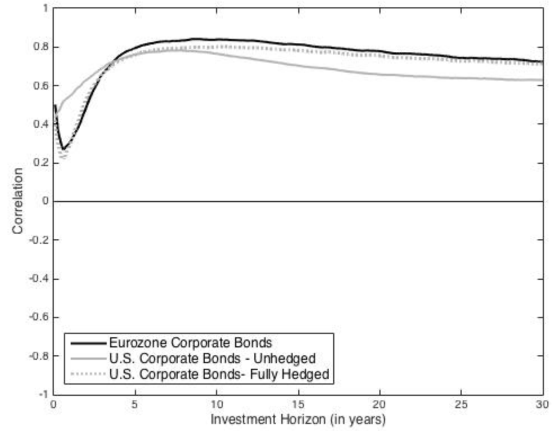
5.2.2 Interest Rate Hedging Qualities

Because pension fund liabilities are the present value of future obligations discounted at a real interest rate (Hoevenaars et al., 2008), also analyse the interest rate hedging qualities of bonds. Figure 13 presents the correlations between cumulative excess log returns of the U.S. and Eurozone bonds and the cumulative excess log euro IRS returns across investment horizons. We consider both 20 and 30 year IRS returns.

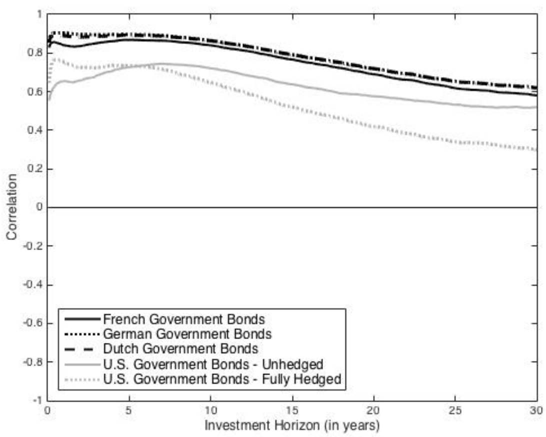
We find that all correlations with the IRS returns are positive. The interest rate hedging qualities of the Eurozone bonds are very similar to each other and are all higher than the hedging qualities of the U.S. bonds. This holds for all investment horizons when we consider government bonds and for investment horizons over 5 years when the bonds under consideration are corporate bonds. While the hedging qualities across bond types are comparable for longer investment horizons, the government bonds have considerably higher hedging qualities than the corporate bonds for investment horizons shorter than 5 years. We furthermore find that currency hedging the U.S. government bonds results in higher interest rate hedging qualities for investment horizons shorter than 5 years, but lower hedging qualities for investment horizons higher than 5 years. For U.S. corporate bonds, however, we find the opposite: currency hedging is in the long run better than not hedging.



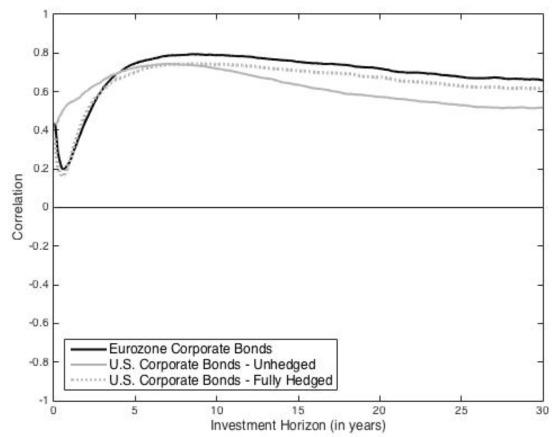
(a) IRS_{20}, Government Bonds



(b) IRS_{20}, Corporate Bonds



(c) IRS_{30}, Government Bonds



(d) IRS_{30}, Corporate Bonds

Figure 13: The correlations between cumulative excess log bond returns and interest rate swap returns across investment horizons. The U.S. bond returns returns are unhedged or fully hedged. Both 20 year (IRS_{20}) and 30 year (IRS_{30}) euro interest rate swaps are considered.

When we compare Figures 13a and 13b with Figures 13c and 13d, we find that the interest rate hedging qualities of the bonds are very similar between when pension liabilities have a duration of around 17 years and when they have a duration of around 25 years. Although the correlations between bond and IRS returns are slightly higher for the lower duration, the differences are quite small. This holds for both Eurozone and U.S. bonds and for both government and corporate bonds.

For the Eurozone and currency hedged U.S. government bonds, we find that the interest rate hedging qualities are lower in the long run than for low investment horizons. Hoevenaars et al. (2008) also find this for government bonds and state that the lower hedging qualities in the long run is due to cumulative inflation. The hedging qualities of the unhedged U.S. bonds initially increases as the investment horizon increases, but afterwards slowly decreases again and generally remains close to its initial value. The behaviour of the correlations over the investment horizons is almost identical for the Eurozone and hedged U.S. corporate bonds: an initial decrease is followed by a sudden increase, after which the correlations remain quite stable. The correlations of the unhedged U.S. corporate bonds with the IRS returns do not decrease for the first investment horizons, but further show the same pattern as the other corporate bonds. For both the Eurozone and the U.S., the interest rate hedging qualities of the corporate bonds are considerably higher in the long run than in the short run. All in all, the government bonds have higher hedging qualities than corporate bonds in the short-run, while the hedging qualities in the long run are similar between the government bonds and the corporate bonds.

5.3 Theoretically Optimal Global Bond Portfolios

This section presents the theoretically optimal global bond portfolio we have found by means of the framework of Hoevenaars et al. (2008) for both an asset-only investor and an asset-liability investor. In Section 5.3.1, we first consider an asset menu consisting of only government bonds. Afterwards, we include corporate bonds as investable assets in Section 5.3.2. As in the case of equity, we only present the portfolio weights computed for the last time point of our sample period. For both Sections 5.3.1 and 5.3.2, we furthermore find similar results for the estimation of the VAR(1) model (56) as we did in in Section 5.2 and as Hoevenaars et al. (2008), which implies that our VAR(1) model specification once again seems to be reasonable.

5.3.1 Government Bonds

Table 12 presents the theoretically optimal portfolio weights of an asset-only investor whose asset menu consists of French, German, Dutch and U.S. government bonds. We consider a short-term investor with an investment horizon of 1 year and a long-term investor with an investment horizon of 20 years.

Table 12: The weights of the unhedged, half hedged and fully hedged theoretically optimal portfolio of an asset-only investor with an investment horizon (K) of 1 or 20 years and government bonds as investable assets

	Panel A: Unhedged Returns			
	FR	DE	NL	US
K=1	17.69%	43.45%	15.38%	23.48%
K=20	23.25%	72.42%	12.59%	-8.26%
	Panel B: Half Hedged Returns			
	FR	DE	NL	US
K=1	18.80%	56.54%	11.25%	13.41%
K=20	27.22%	84.52%	6.07%	-17.81%
	Panel C: Fully Hedged Returns			
	FR	DE	NL	US
K=1	26.25%	47.86%	6.22%	19.67%
K=20	27.29%	73.25%	6.87%	-7.41%

For each hedge ratio and investment horizon, the most weight in the theoretically optimal portfolio is assigned to the German bond. In all cases except when the short-term investor does not currency hedge U.S. bonds, the French bond is the second most popular asset. The Dutch bonds are invested in least among the Eurozone bonds, which is probably because the French and German bonds are more correlated with the Dutch bonds than with each other. Tables A3 and A4 furthermore show that the correlations of the French and German bonds with the U.S. bonds are far lower than the correlations with the Dutch bonds, which explains why U.S. bonds receive larger weights than Dutch bonds in the short run. The overall dominance of the Eurozone assets does not come as a surprise given their high Sharpe ratios presented in Table 3.

When comparing the portfolios of the short-term investor and the long-term investor, we do not find any difference in sign for the Eurozone bonds. For U.S. bonds, however, we find that the investor should buy U.S. bonds in the short run, but not in the long-run. This could be because the correlations between the Eurozone bonds, which have considerably higher Sharpe ratios than the U.S. bonds, are less of a problem in the long run than in the short run. This also explains why the portfolio of the long-term investor is almost completely concentrated in French and especially German bonds. We furthermore do not find large differences across hedge ratios. An interesting result is that increasing the hedge ratio from 50% to 100% for the investment horizon of 20 years results in the investor short selling fewer U.S. bonds at the expense of buying fewer German bonds.

We now consider an asset-liability investor. Table 13 presents the theoretically optimal portfolio weights of the asset-liability investor for both liabilities with a duration of 17 years and liabilities with a duration of 25 years. The asset menu consists of the same bonds as for the asset-only investor while we once again consider investment horizons of 1 and 20 years.

Table 13: The weights of the unhedged, half hedged and fully hedged theoretically optimal portfolio of an asset-liability investor with an investment horizon (K) of 1 or 20 years and government bonds as investable assets

	Panel A: Unhedged Returns			
	FR	DE	NL	US
K=1, IRS_{20}	11.90%	6.26%	38.79%	43.05%
K=1, IRS_{30}	9.60%	-7.94%	41.92%	56.42%
K=20, IRS_{20}	12.88%	-5.25%	71.50%	20.87%
K=20, IRS_{30}	4.15%	-15.63%	68.78%	42.69%
	Panel B: Half Hedged Returns			
	FR	DE	NL	US
K=1, IRS_{20}	7.26%	-14.99%	59.94%	47.79%
K=1, IRS_{30}	-1.52%	-29.54%	57.57%	73.49%
K=20, IRS_{20}	1.31%	-24.46%	88.15%	35.00%
K=20, IRS_{30}	-13.50%	-39.69%	83.59%	69.60%
	Panel C: Fully Hedged Returns			
	FR	DE	NL	US
K=1, IRS_{20}	2.10%	1.48%	69.20%	27.21%
K=1, IRS_{30}	-13.35%	20.98%	59.35%	33.03%
K=20, IRS_{20}	0.43%	13.06%	80.67%	5.83%
K=20, IRS_{30}	-22.17%	26.97%	88.45%	6.76%

Note: IRS_{20} and IRS_{30} correspond to liability durations of around 17 and 25 years, respectively.

In contrast to the theoretically optimal portfolio of the asset-only investor, the optimal portfolio of the asset-liability investor is in general mostly concentrated in Dutch bonds. This can be explained by the fact that Dutch bonds have the highest interest rate hedging qualities of all bonds we consider and furthermore have relatively high Sharpe ratios. Moreover, the Dutch bonds also have relatively high inflation hedging qualities. U.S. bonds are the assets most weight is allocated to after Dutch bonds, which is probably because the Dutch bonds are far less correlated with the U.S. bonds than they are with the French and German bonds.

The theoretically optimal portfolio weights of the asset-liability investor do not differ much between when the duration of the liabilities is around 17 years and when it is around 25 years. The only differences in sign correspond to the weight of the German bond in the portfolio of the short-term investor who does not currency hedge and the weights of the French bond in the portfolio of the currency hedging investor. For all hedge ratios and investment horizons, the portfolio weights corresponding to U.S. bonds in Table 13 are higher for the duration of 25 years than for the duration of 17 years, while it is the exact opposite for the French bonds. This difference in weights is slightly larger for U.S. bonds when they are unhedged than when they are fully hedged. Figures 13a and 13c imply that this is because the interest rate hedging quality of fully (currency) hedged U.S. bonds decreases much more than that of unhedged U.S. bonds when the liability duration increases.

The differences in portfolio weights across investment horizons are also not large. In terms of sign, the only difference we find is for the weights assigned to German bonds in the portfolio of the investor who does not currency hedge and whose liabilities have a duration of around 17 years. Furthermore, we find for all hedge ratios that the allocation to U.S. bonds is considerably larger in the short run than in the long run, while the allocation to Dutch bonds is larger in the long run. The differences across hedge ratios are generally quite small. The most noteworthy difference is that fully hedged U.S. bonds are invested in far less than their unhedged and half hedged equivalent. A possible reason for this could be that the correlations between the U.S. bonds and the Eurozone bonds are far larger when the returns are fully hedged than when they are unhedged or half hedged. Tables A3 and A4 show that this is certainly the case between unhedged and fully hedged returns.

To compare the short-term and long-term portfolios of the asset-only and asset-liability investors, we look at the annualised average returns, volatilities and Sharpe ratios of these portfolios presented in Table 14. We find that the asset-liability portfolios have higher Sharpe ratios when the liabilities have a duration of around 17 years than when the duration is around 25 years. The difference is statistically significant at the 0.05 significance level for the hedge ratios of 0 and 50%, while it is not significant for fully hedged returns. This holds for both investment horizons we consider. For both the asset-only and the asset-liability investor, the Sharpe ratios are furthermore slightly higher for the investment horizon of 20 years than for the investment horizon of 1 year. This is because the long-term portfolios are more concentrated in the Eurozone bonds than the short-term portfolios. The Sharpe ratios of the asset-only portfolios are considerably higher than those of the asset-liability portfolios. This is because the asset-only portfolios favour bonds with the highest Sharpe ratios while the asset-liability portfolios also take into account the hedging quality of the bonds. For all corresponding p-values, we refer the reader to Table A5 in Appendix H.2. We find for the short-term investor that the differences in Sharpe ratios between the asset-only portfolio and the asset-liability portfolio are significant at the 0.05 significance level for the hedge ratios of 0 and 50%. This holds for both liability durations and for the the long-term investor with a liability duration of 25 years. For the long-term investor with the shorter liability duration, the difference is still significant at the significance level of 0.05 for half hedged returns, but only significant at the 0.10 significance level for unhedged returns. When the foreign bonds are unhedged, the difference in Sharpe ratios are not statistically significant for either investment horizon. We furthermore find that the asset-liability portfolios are

Table 14: The annualised average returns, volatilities and Sharpe ratios of the unhedged, half hedged and fully hedged theoretically optimal portfolio for the short-term (K=1) and long-term (K=20) asset-only (AO) and asset-liability (AL) investors with government bonds as investable assets.

	AO, K=1	AO, K=20	AL_{20}, K=1	AL_{20}, K=20	AL_{30}, K=1	AL_{30}, K=20
Panel A: Unhedged Returns						
Average Return (in %)	5.43	6.01	5.10	5.54	4.86	5.13
Volatility (in %)	5.72	4.74	6.95	5.63	7.92	6.94
Sharpe Ratio	0.73	1.04	0.53	0.78	0.43	0.54
Panel B: Half Hedged Returns						
	AO, K=1	AO, K=20	AL_{20}, K=1	AL_{20}, K=20	AL_{30}, K=1	AL_{30}, K=20
Average Return (in %)	5.69	6.08	5.31	5.49	4.98	5.03
Volatility (in %)	5.04	4.71	5.94	5.61	6.83	6.72
Sharpe Ratio	0.92	1.05	0.70	0.79	0.54	0.56
Panel C: Fully Hedged Returns						
	AO, K=1	AO, K=20	AL_{20}, K=1	AL_{20}, K=20	AL_{30}, K=1	AL_{30}, K=20
Average Return (in %)	5.68	5.93	5.65	5.85	5.55	5.81
Volatility (in %)	4.97	4.81	5.13	4.96	5.20	4.98
Sharpe Ratio	0.95	1.01	0.93	0.98	0.88	0.95

Note: AL_{20} and AL_{30} correspond to liability durations of around 17 and 25 years, respectively.

generally more volatile than the asset-only portfolio. A possible explanation for this is that an asset-liability investor needs to take more risk to meet his liabilities (Hoevenaars et al., 2008).

When we compare the portfolios across hedge ratios, we find for all portfolios other than the long-term asset-only portfolio that the Sharpe ratio increases as the hedge ratio increases. This overall increase in Sharpe ratios is probably caused by the overall positive effect of currency hedging on the Sharpe ratio of U.S. bonds. The p-values of the hypothesis test of differences in Sharpe ratios across hedge ratios are presented in A6 in Appendix H.2. The increases in Sharpe ratio when the hedge ratios increases from 0 to 50% are statistically significant at the significance level of 0.05 for the short-term asset-only investor while it also significant at the 0.10 significance level for the short-term asset-liability investor with a liability duration of 17 years. The increase in Sharpe ratio corresponding to increasing the hedge ratio from 50% to 100% is significant at the 0.10 significance level for the short-term asset-liability investor with the longer liability duration and at the 0.05 significance level for both long-term asset-liability investors. The difference in Sharpe ratios between an investor who does not currency hedge and an investor who fully hedges U.S. bonds is significant at the 0.10 significance level for both asset-liability investors and even at the 0.05 significance level for the long-term asset-liability investor with a liability duration of around 25 years.

5.3.2 Government and Corporate Bonds

We now include Eurozone and U.S. corporate bonds into the asset menu in addition to the government bonds considered in Section 5.3.1. Table 15 presents the theoretically optimal portfolio weights of the asset-only investor whose asset menu consists of both government bonds and corporate bonds. We once again consider investment horizons of 1 and 20 years.

Table 15: The weights of the unhedged, half hedged and fully hedged theoretically optimal portfolio of an asset-only investor with an investment horizon (K) of 1 or 20 years and both government and corporate bonds as investable assets

Panel A: Unhedged Returns						
	FR	DE	NL	US	EU_C	US_C
K=1	5.76%	43.70%	0.58%	18.56%	19.18%	12.22%
K=20	-9.76%	35.41%	-28.52%	99.31%	-21.60%	25.16%
Panel B: Half Hedged Returns						
	FR	DE	NL	US	EU_C	US_C
K=1	6.00%	40.14%	-10.53%	4.78%	23.17%	36.45%
K=20	-15.18%	43.77%	-37.79%	-26.64%	28.64%	107.19%
Panel C: Fully Hedged Returns						
	FR	DE	NL	US	EU_C	US_C
K=1	-2.03%	39.06%	-12.39%	29.21%	52.98%	-6.82%
K=20	-14.67%	39.83%	-30.14%	26.45%	116.17%	-37.64%

The portfolio weights differ considerably compared to when the asset menu only consists of government bonds. The only clear similarities we find is that German bonds consistently receive relatively large portfolio weights while Dutch bonds receive quite low weights. On the other hand, the allocation to U.S. bonds is considerably larger after the introduction of corporate bonds. The low correlations between U.S. government bonds and Eurozone corporate bonds presented in Tables A3 and A4 imply that the allocation to U.S. government bonds is mainly for diversification purposes. The allocation to corporate bonds is overall quite large as well and is probably due to their low correlation with the government bonds.

We find no difference in sign between the portfolio weights of the short-term investor and those of the long-term investor when the U.S. bonds are fully hedged. When the investor does not currency hedge, however, the allocation to French and Dutch government bonds and Eurozone corporate bonds is negative for an investment horizon of 20 years, but positive for an investment horizon of 1 year. In the case of half hedged returns, we find the same for French and U.S. government bonds. The differences of the portfolio weights across hedge ratios are less obvious. The most notable differences are that the allocation to U.S. corporate bonds is considerably larger for half hedged returns than for unhedged or fully hedged returns and that long-term investor invests far more in U.S. corporate bonds for a hedge ratio of 0% than for the other two hedge ratios. We furthermore find that the allocation to the Eurozone corporate bonds generally increases as the hedge ratio increases.

We now consider a Eurozone-based asset-liability investor who can invest in both government and corporate bonds. Table 16 presents the theoretically optimal portfolio weights of this investor for liabilities with a duration of 17 and 25 years. We once again consider investment horizons of 1 and 20 years while the asset menu consists of the same bonds as for the asset-only investor. As when the asset menu only consisted of government bonds, we find that most weight in the theoretically optimal asset-liability portfolio is assigned to Dutch bonds. In contrast to before we introduced corporate bonds, however, we now find that the allocation to French and German bonds is generally also quite large. To compensate for this, the portfolio weights of the Eurozone corporate bonds and

Table 16: The weights of the unhedged, half hedged and fully hedged theoretically optimal portfolio of an asset-liability investor with an investment horizon (K) of 1 or 20 years and both government and corporate bonds as investable assets

	FR	DE	NL	US	EU_C	US_C
Panel A: Unhedged Returns						
K=1, IRS_{20}	50.67%	41.07%	73.01%	-98.92%	39.12%	-4.95%
K=1, IRS_{30}	64.55%	53.93%	101.02%	-162.00%	61.33%	-18.83%
K=20, IRS_{20}	52.28%	46.21%	104.49%	-112.12%	38.74%	-29.61%
K=20, IRS_{30}	70.45%	55.34%	145.49%	-193.14%	52.76%	-30.90%
Panel B: Half Hedged Returns						
	FR	DE	NL	US	EU_C	US_C
K=1, IRS_{20}	45.73%	30.65%	81.20%	33.48%	2.51%	-93.58%
K=1, IRS_{30}	59.18%	55.21%	105.94%	68.06%	-15.77%	-172.63%
K=20, IRS_{20}	54.46%	31.32%	114.24%	47.89%	-27.49%	-120.42%
K=20, IRS_{30}	75.80%	48.66%	148.86%	65.85%	-28.92%	-210.24%
Panel C: Fully Hedged Returns						
	FR	DE	NL	US	EU_C	US_C
K=1, IRS_{20}	42.49%	29.46%	81.13%	4.89%	-86.00%	28.03%
K=1, IRS_{30}	61.54%	51.16%	113.09%	-13.16%	-165.48%	52.84%
K=20, IRS_{20}	52.31%	45.27%	117.52%	-33.87%	-128.35%	47.12%
K=20, IRS_{30}	69.76%	67.80%	168.23%	-45.76%	-219.21%	59.18%

Note: IRS_{20} and IRS_{30} correspond to liability durations of around 17 and 25 years, respectively.

the U.S. bonds are found to be negative more often than not. These are also the three asset with the lowest interest rate hedging qualities. Another reason for the overall low portfolio weights assigned to these bonds could be the high correlations between the unhedged U.S. corporate bonds and unhedged U.S. government bonds (0.87) and between fully hedged U.S. corporate bonds and Eurozone bonds (0.79).

The theoretically optimal portfolio weights of the asset-liability investor again do not differ much between when the liability duration is around 17 years and when it is 25 years. We do find for all hedge ratios and investment horizons that more weight is assigned to French, German and Dutch assets for the longer duration. For the U.S. bonds and the Eurozone corporate bonds on the other hand, the portfolio weights are usually smaller for the longer duration. A possible explanation for this could be that the fact that the interest rate hedging qualities of the corporate bonds and the U.S. government bonds decrease (slightly) more than those of the Eurozone government bonds when the duration of the liabilities increases from 17 to 25 years.

In general, we find for the Eurozone government bonds that the allocation is larger in the long run than in the short run, while it is the opposite for the U.S. government bonds and the corporate bonds. This could because it becomes safer to invest more in the Eurozone government assets, which have the highest Sharpe ratios and interest hedging qualities, in the long run than in the short run due to predictability of assets. A possible explanation for the lower allocation to government bonds in the long run is that they have considerably lower inflation hedging qualities in the long run than in the short run. The differences in portfolio weights across hedge ratios are minimal for the French, German and Dutch assets. For the U.S. government bonds and the corporate bonds, the effect is much larger. When the investor chooses not to currency hedge, he should always buy Eurozone corporate bonds, while the weights assigned to the U.S. bonds are always negative. For the hedge ratio of 50%, the portfolio weights are positive for the U.S. government bonds and usually negative for the corporate bonds. For an investor who chooses to fully hedge, the allocation to U.S. corporate bonds is positive while the portfolio weights corresponding to U.S. government bonds and Eurozone corporate bonds are negative in most cases. The fact that the weights assigned to fully hedged U.S. bonds are considerably larger than the weights corresponding to unhedged U.S. bonds is probably because the fully hedged bonds have both higher Sharpe ratios and higher hedging qualities.

We once again have a closer look at the portfolios of the asset-only and asset-liability investors. Table 17 presents the annualised average returns, volatilities and Sharpe ratios of the portfolios of these investors for both investment horizons and all three hedge ratios. As the statistics barely differ when we vary the duration of the liabilities, we once again only present the results for the liabilities with a duration of around 17 years.

Table 17: The annualised average returns, volatilities and Sharpe ratios of the unhedged, half hedged and fully hedged theoretically optimal portfolio for the short-term (K=1) and long-term (K=20) asset-only (AO) and asset-liability (AL) investors with both government and corporate bonds as investable assets.

Panel A: Unhedged Returns						
	AO, K=1	AO, K=20	AL_{20}, K=1	AL_{20}, K=20	AL_{30}, K=1	AL_{30}, K=20
Average Return (in %)	5.11	4.51	6.60	7.24	7.29	8.04
Volatility (in %)	5.18	3.17	8.51	7.86	10.58	10.98
Sharpe Ratio	0.70	0.89	0.64	0.83	0.58	0.66
Panel B: Half Hedged Returns						
	AO, K=1	AO, K=20	AL_{20}, K=1	AL_{20}, K=20	AL_{30}, K=1	AL_{30}, K=20
Average Return (in %)	5.07	4.44	6.68	7.27	7.51	8.19
Volatility (in %)	4.19	3.14	7.75	7.96	9.96	11.09
Sharpe Ratio	0.90	0.89	0.75	0.83	0.67	0.68
Panel C: Fully Hedged Returns						
	AO, K=1	AO, K=20	AL_{20}, K=1	AL_{20}, K=20	AL_{30}, K=1	AL_{30}, K=20
Average Return (in %)	4.98	4.47	6.70	7.45	7.65	8.58
Volatility (in %)	3.81	3.09	7.15	7.96	9.29	10.99
Sharpe Ratio	0.98	0.91	0.85	0.87	0.77	0.74

Note: AL_{20} and AL_{30} correspond to liability durations of around 17 and 25 years, respectively.

When we compare the statistics presented in Table 17 with their counterparts presented in Table 14, we find that the Sharpe ratios of the asset-only portfolios are in most cases higher before the introduction of corporate bonds. This seems reasonable, as e.g. the Eurozone corporate bonds have far lower Sharpe ratios than the Eurozone government bonds, but still receive considerable weights in the asset-only portfolios due to their lower volatility. For the asset-liability portfolios corresponding to the hedge ratios of 0 and 50%, we find that the Sharpe ratios are higher with corporate bonds in the asset menu than without. The differences, however, are not significant at the significance level of 0.10 for any combination of hedge ratio and liability duration (see Table A7 in Appendix H.2 for the p-values). We also find again that the Sharpe ratios are generally slightly higher for the long-term portfolios than the short-term portfolios. In contrast to when we only considered government bonds, however, this does not hold for every hedge ratio. The differences are also slightly smaller now. Furthermore, we find for all portfolios the Sharpe ratio increases as the hedge ratio increases. In contrast to when the asset menu only consists of government bonds, however, the difference is only significant at the significance level of 0.10 when the asset-only investor decided to half hedge U.S. bonds instead of not currency hedging at all.

The asset-liability portfolios once again have higher Sharpe ratios for the lower liability duration. For the long-term investor, the Sharpe ratio of the asset-liability portfolio for liabilities of which the duration is around 17 years is significantly higher than when the liability duration is around 25 years. This holds for the significance level of 0.05 and for all hedge ratios. For the short-term investor, the difference in Sharpe ratios is only statistically significant at the 0.05 significance level for unhedged returns. For half hedged returns, the difference is significant at the significance level of 0.10, while the difference is not statistically significant when the U.S. bonds are fully hedged. The Sharpe ratios of the asset-only portfolios are also again higher than those of the asset-liability portfolios for all hedge ratios and investment horizons. However, where the differences in Sharpe ratios between the asset-only and asset-liability portfolios are mostly found to be significant in Table 14, we now find that the differences are not significant at the 0.10 significance level for any combination of hedge ratio, investment horizon and liability duration. We once again refer the reader to Table A5 in Appendix H.2 for an overview of all corresponding p-values.

6 Conclusion

In this research, we have investigated whether it could add value for a Eurozone investor to deviate from the regional allocation of a global stock market index when choosing the allocation in a globally diversified portfolio. We did this by comparing various portfolios with portfolios based on MSCI global stock market indices. We have also looked for the theoretically optimal allocation of an investor with a defined investment horizon by means of the recursive analytical solution of Jurek and Viceira (2011). We have first only considered developed markets equity before introducing emerging markets equity to the asset menu as well. Furthermore, we have investigated whether it would add value for a euro-denominated investor to diversify into U.S. bonds by following the framework of Viceira et al. (2017). In particular, we have investigated the main driver behind the correlation between Eurozone and U.S. bonds to find whether the correlation is equally detrimental for short-term investors and long-term investors alike. The framework of Hoevenaars et al. (2008) was used to analyse the inflation and interest rate hedging qualities of Eurozone and U.S. bonds and to compute the theoretically optimal bond portfolio weights of both an asset-only and an asset-liability investor. Both government and corporate bonds are considered in our analysis. They were considered separately when we followed the framework of Viceira et al. (2017) and simultaneously when analysed the hedging qualities of bonds. The theoretically optimal bond portfolio was first computed with only government bonds in the asset menu and afterwards with both government and corporate bonds as investable assets. Furthermore, we have analysed to what extent currency hedging affects both global equity diversification and global bond diversification. This was done by performing our analyses for a diverse range of hedge ratios between 0% (unhedged returns) and 100% (fully hedged returns).

The mean-variance portfolio is found to be generally more rewarding, but also slightly more volatile than the portfolio based on the MSCI indices. The global minimum variance portfolio on the other hand is superior to the MSCI portfolios for all measures we have considered. The relatively high performances of the mean-variance and global minimum variance portfolios imply that it certainly adds value to have flexibility in the regional weights of a globally diversified portfolio compared to basing the portfolio weights on the regional allocation of a global stock market index. They furthermore indicate that Eurozone investor should primarily invest in Canadian, Swiss and Indian equity instead of U.S. equity. We furthermore unsurprisingly find that including emerging markets equity in the asset menu leads to more rewarding but also more risky portfolios. The weights of the theoretically optimal portfolio are comparable to those of the global minimum variance and mean-variance portfolios. The assets that that are bought most are Canadian, Hong Kong, Swiss and British equity while French, German, Dutch and American stocks are short sold for most combinations of hedge ratio and investment horizon. The effect of currency hedging foreign assets to the euro on the composition and performance of the portfolios furthermore differs considerably per portfolio and per asset menu. For instance, the Sharpe ratio of the mean-variance portfolio is only significantly higher than that of the portfolio based on the MSCI World Index for hedge ratios higher than 50%, while the mean-variance portfolio significantly outperforms the portfolio based on the MSCI ACWI Index for all hedge ratios. We do find some general patterns, however, such as that Sharpe ratios are highest and Value at Risk estimates lowest for high hedge ratios.

For both government and corporate bonds, the correlation between Eurozone bonds and U.S. bonds is found to be almost completely driven by changes in cash flows. This implies that the correlation between Eurozone and U.S. bonds is equally detrimental for short-term and long-term investors. This holds for all hedge ratios between 0 and 100%. Furthermore, we have found that fully (currency) hedged U.S. bonds have a higher inflation hedging qualities than the Eurozone bonds, while unhedged U.S. bonds are a poor inflation hedging tool. Moreover, government bonds are found to be better inflation hedging instruments in the long run, while corporate bonds have higher inflation hedging qualities for short investment horizons. All Eurozone bonds considered are found to have higher (euro) interest rate hedging qualities than the U.S. bonds. Furthermore, government bonds are considerably better interest rate hedging instruments than corporate bonds in the short run, while the hedging qualities are comparable in the long run. The theoretically optimal bond portfolio of the Eurozone asset-only investor is primarily concentrated in assets with high Sharpe ratios that have relatively low volatilities. German government bonds and Eurozone corporate bonds are therefore found to receive relatively large portfolio weights. The asset-liability investor on the other hand attaches more importance to the hedging qualities of bonds. The theoretically optimal asset-liability portfolio of a Eurozone investor is therefore mostly concentrated in Dutch equity. Since the theoretically optimal bond portfolios are mostly concentrated in Eurozone bonds, the effect of currency hedging is found to be much smaller than in the case of equity.

All in all, our research on global diversification from a Eurozone investor's perspective has produced three main findings. First, we have found that deviating from global stock market indices when choosing the regional allocation in a globally diversified portfolio is in most cases superior to basing it on the regional allocation of a globally stock market index. Although a portfolio that is concentrated in U.S. equity is by no means a poorly performing portfolio and superior to e.g. an equal-weighted portfolio, more rewarding and less risky portfolios can be obtained by allocating less weight to U.S. equity and more to for instance Canadian and Swiss equity. Secondly, our research does not provide strong evidence against the general consensus that only investing in Eurozone bonds is more rewarding for a Eurozone investor than diversifying into U.S. bonds. Eurozone bonds are overall found to have slightly higher hedging qualities than U.S. bonds while portfolio optimisation leads to large allocation to Eurozone bonds. Finally, currency hedging foreign equity to the euro is in most cases found to be preferable to not currency hedging. The optimal hedge ratio varies per set of assets most weight is allocated to. In the case of bonds, currency hedged U.S. bonds are generally found to have higher hedging qualities and lead to better portfolios than unhedged U.S. bonds.

7 Limitations and Further Research

A clear shortcoming of our research is that we did not put restrictions on short selling when we computed the theoretically optimal portfolio weights of an investor with a defined investment horizon in both the equity and the bond investment cases. This is because putting short selling restrictions on portfolios weights that are computed by means of simulation requires an impractical amount of computation time for a large asset menu and we did not find closed-form solutions that incorporate short selling restrictions. Further research on optimal portfolio allocation could be of greater practical relevance if dynamic portfolio weights could be restricted to be between 0 and 100% for a large asset menu. Due to the large number of portfolios we considered throughout out research, we furthermore refrained from testing the statistical significance of portfolio weights. For a more proper comparison of portfolios, one could test statistical of portfolio weights by means of e.g. the hypothesis tests of Britten-Jones (1999) and Frahm (2010).

When we investigated global bond diversification, we estimated a pooled Vector Autoregressive model in line with Viceira et al. (2017). The pooled estimation assumes identical asset return generating processes across countries. Since this is a fairly restrictive assumption, estimating the model with a less restrictive method might yield more accurate results. One could for instance consider models, such as the so-called mean group estimator of Pesaran and Smith (1995) or the principal components based estimator of Bai (2009), that take into account the heterogeneity across cross-sections. We have furthermore analysed the inflation and interest rate hedging qualities of Eurozone and U.S. bonds by examining their correlations with respectively inflation rates and pension liability returns. Further research could consider other measures of dependence than the correlation, such as the copula (Sklar, 1959) or Kendall's tau coefficient (Kendall, 1938).

Another shortcoming of our research is that we did not consider currency hedging transaction costs in our research due to the large variation in transaction costs across investors and the already large number of variables (e.g. investment horizons and hedge ratios) of our research. For a more realistic take on the effect of currency hedging on global portfolio diversification, one could perform our research with a range of transaction costs, while (institutional) investors could consider the level of currency hedging transaction costs they usually face.

Furthermore, we considered the cases of global diversification in equity and global diversification in bonds separately. An interesting extension to this research would be to investigate global diversification from a Eurozone point of view with an asset menu that includes both equity and bonds. It could also be of interest to consider alternative asset classes, such as commodities, credits and real estate.

In the empirical part of our research, we have only considered the investment horizons of 1, 10 and 20 years for our research on global equity diversification and the investment horizons of 1 and 20 years for our research on global bond diversification. It might be interesting to look at more, or perhaps even all, intermediate horizons. It might also be of interest to examine the effect of risk aversion on global portfolio allocation by performing this research with varying levels of risk aversion. Furthermore, we only focused on diversification benefits between the Eurozone and the U.S. bonds. With the growing popularity of emerging markets, it might be of practical interest to also consider emerging markets bonds.

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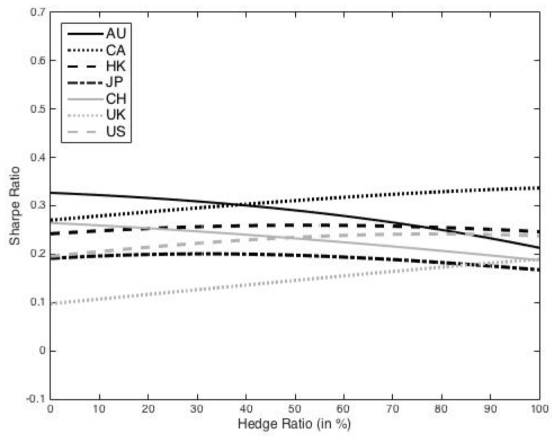
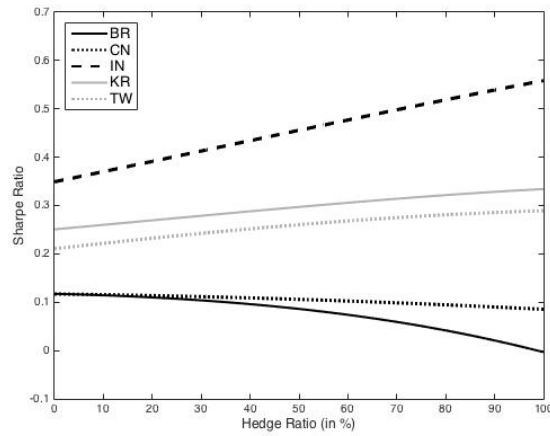
Appendix

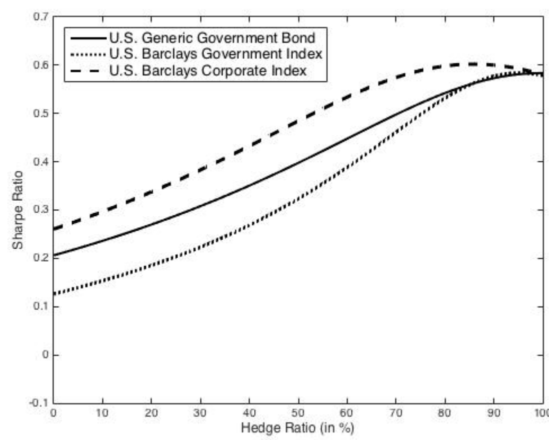
A Sharpe Ratios of Foreign Assets Across Hedge Ratios



(a) Developed Markets Equity



(b) Emerging Markets Equity



(c) Bonds

Figure A1: The annualised Sharpe ratios of all foreign assets in our data set across hedge ratios.

B Correlations of Asset Returns

B.1 Equity Returns

Table A1: Correlations of unhedged equity returns

	AU	CA	FR	DE	HK	JP	NL	CH	UK	US	BR	CN	IN	KR	TW
AU	1.00														
CA	0.75	1.00													
FR	0.68	0.65	1.00												
DE	0.65	0.62	0.92	1.00											
HK	0.69	0.67	0.59	0.61	1.00										
JP	0.51	0.52	0.47	0.50	0.54	1.00									
NL	0.69	0.71	0.92	0.88	0.64	0.51	1.00								
CH	0.61	0.53	0.76	0.75	0.56	0.57	0.74	1.00							
UK	0.74	0.75	0.84	0.79	0.72	0.56	0.86	0.76	1.00						
US	0.64	0.70	0.74	0.75	0.62	0.60	0.76	0.72	0.81	1.00					
BR	0.69	0.71	0.58	0.59	0.58	0.34	0.64	0.45	0.60	0.48	1.00				
CN	0.31	0.34	0.21	0.26	0.53	0.26	0.25	0.19	0.33	0.30	0.26	1.00			
IN	0.59	0.57	0.52	0.54	0.62	0.52	0.55	0.44	0.53	0.49	0.54	0.29	1.00		
KR	0.69	0.65	0.64	0.69	0.66	0.54	0.68	0.56	0.62	0.61	0.56	0.35	0.61	1.00	
TW	0.61	0.60	0.55	0.56	0.68	0.48	0.62	0.48	0.57	0.59	0.48	0.32	0.53	0.70	1.00

Table A2: Correlations of fully hedged equity returns

	AU	CA	FR	DE	HK	JP	NL	CH	UK	US	BR	CN	IN	KR	TW
AU	1.00														
CA	0.74	1.00													
FR	0.73	0.70	1.00												
DE	0.67	0.65	0.92	1.00											
HK	0.66	0.72	0.62	0.61	1.00										
JP	0.62	0.60	0.61	0.59	0.60	1.00									
NL	0.69	0.72	0.92	0.88	0.63	0.60	1.00								
CH	0.66	0.58	0.80	0.77	0.52	0.58	0.79	1.00							
UK	0.75	0.75	0.86	0.81	0.67	0.57	0.85	0.74	1.00						
US	0.77	0.78	0.83	0.81	0.71	0.65	0.81	0.75	0.84	1.00					
BR	0.60	0.69	0.55	0.52	0.66	0.43	0.56	0.43	0.61	0.63	1.00				
CN	0.36	0.36	0.27	0.30	0.56	0.35	0.28	0.24	0.26	0.34	0.40	1.00			
IN	0.57	0.61	0.54	0.54	0.66	0.57	0.55	0.45	0.56	0.58	0.57	0.35	1.00		
KR	0.59	0.62	0.64	0.67	0.65	0.56	0.66	0.52	0.62	0.64	0.56	0.38	0.62	1.00	
TW	0.58	0.66	0.60	0.58	0.71	0.52	0.63	0.44	0.59	0.63	0.53	0.37	0.58	0.68	1.00

B.2 Bond Returns

Table A3: Correlations of unhedged bond returns

	EU_G	US_G	FR	DE	NL	US	EU_C	US_C
EU_G	1.00							
US_G	0.29	1.00						
FR	0.92	0.40	1.00					
DE	0.85	0.47	0.94	1.00				
NL	0.89	0.43	0.96	0.97	1.00			
US	0.41	0.98	0.52	0.60	0.56	1.00		
EU_C	0.62	-0.06	0.55	0.46	0.54	0.02	1.00	
US_C	0.31	0.90	0.39	0.42	0.42	0.87	0.26	1.00

Table A4: Correlations of fully hedged bond returns

	EU_G	US_G	FR	DE	NL	US	EU_C	US_C
EU_G	1.00							
US_G	0.67	1.00						
FR	0.92	0.73	1.00					
DE	0.85	0.78	0.94	1.00				
NL	0.89	0.75	0.96	0.97	1.00			
US	0.68	0.98	0.74	0.80	0.77	1.00		
EU_C	0.62	0.33	0.55	0.46	0.54	0.33	1.00	
US_C	0.48	0.59	0.48	0.42	0.49	0.57	0.79	1.00

C Derivations of the Global Minimum Variance Portfolio

An investor's global minimum variance portfolio has the lowest risk of all portfolios on the investor's efficient frontier. The portfolio weights of the global minimum variance portfolio at time t can be obtained by solving the following minimisation problem:

$$\begin{aligned} \min_{\boldsymbol{w}_t} \sigma^2_{p,t} &= \boldsymbol{w}_t' \boldsymbol{\Sigma}_t \boldsymbol{w}_t, \\ \boldsymbol{w}_t' \boldsymbol{\iota} &= 1, \end{aligned} \tag{69}$$

where $\boldsymbol{w}_t$, $\sigma_{p,t}$, and $\boldsymbol{\iota}$ denote the vector of portfolio weights, the portfolio volatility and a vector of ones, respectively. Furthermore, $\boldsymbol{\Sigma}_t$ denotes the covariance matrix of the asset returns computed with all information up to and including time t. The corresponding Lagrangian and first order conditions (FOC's) then look as follows at time t:

$$\begin{aligned} L_t &= \frac{1}{2} \boldsymbol{w}_t' \boldsymbol{\Sigma}_t \boldsymbol{w}_t - \lambda_t (\boldsymbol{w}_t' \boldsymbol{\iota} - 1), \\ L'_{w,t} &= \boldsymbol{\Sigma}_t \boldsymbol{w}_t - \lambda_t \boldsymbol{\iota} = 0, \\ L'_{\lambda,t} &= \boldsymbol{w}_t' \boldsymbol{\iota} - 1 = 0, \end{aligned} \tag{70}$$

where λ_t is the Lagrange multiplier at time t and the $\frac{1}{2}$ term appears to facilitate computations. Rearranging the first of the two FOC's results in the following expression for the portfolio weights:

$$\mathbf{w}_t = \lambda_t \boldsymbol{\Sigma}_t^{-1} \boldsymbol{\iota}. \tag{71}$$

Filling in (71) in the second FOC results in the following expression for the Lagrange multiplier:

$$\lambda_t = \frac{1}{\boldsymbol{\iota}' \boldsymbol{\Sigma}_t^{-1} \boldsymbol{\iota}}. \tag{72}$$

Finally, filling in (72) in (71), results in the following expression for the global minimum variance portfolio weights at time t:

$$\boldsymbol{w}_{GMV,t} = \frac{\boldsymbol{\Sigma}_t^{-1} \boldsymbol{\iota}}{\boldsymbol{\iota}' \boldsymbol{\Sigma}_t^{-1} \boldsymbol{\iota}}. \tag{73}$$

D Derivations of the Mean-Variance Portfolio

An investor with mean-variance preferences wants to make the optimal trade-off between the expected return of his portfolio and its risk. The following maximisation problem must be solved to obtain the optimal asset allocation in the mean-variance investor's portfolio at time t:

$$\begin{aligned} \max_{\boldsymbol{w}_t} \mu_{p,t} - \frac{\gamma}{2}\sigma_{p,t}^2 = \boldsymbol{w}_t'\boldsymbol{\mu}_t - \frac{\gamma}{2}\boldsymbol{w}_t'\boldsymbol{\Sigma}_t\boldsymbol{w}_t, \\ \boldsymbol{w}_t'\boldsymbol{\iota} = 1, \end{aligned} \tag{74}$$

where $\boldsymbol{w}_t$, $\sigma_{p,t}$, $\boldsymbol{\iota}$ and γ denote the vector of portfolio weights, the portfolio volatility, a vector of ones and the investor's coefficient of relative risk aversion, respectively. Furthermore, $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$ denote respectively the expected asset returns and covariance matrix of the asset returns computed with all information up to and including time t. The Lagrangian and first order conditions (FOC's) now look as follows:

$$\begin{aligned} L_t &= \boldsymbol{w}_t'\boldsymbol{\mu}_t - \frac{\gamma}{2}\boldsymbol{w}_t'\boldsymbol{\Sigma}_t\boldsymbol{w}_t - \lambda_t(\boldsymbol{w}_t'\boldsymbol{\iota} - 1), \\ L_{w,t}' &= \boldsymbol{\mu}_t - \gamma\boldsymbol{\Sigma}_t\boldsymbol{w}_t - \lambda_t\boldsymbol{\iota} = 0, \\ L_{\lambda,t}' &= \boldsymbol{w}_t'\boldsymbol{\iota} - 1 = 0, \end{aligned} \tag{75}$$

where λ_t once again denotes the Lagrange multiplier at time t. Rearranging the first FOC gives the following equation:

$$\boldsymbol{w}_t = \frac{1}{\gamma}\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}_t - \frac{\lambda_t}{\gamma}\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\iota}. \tag{76}$$

Filling in (76) in the second FOC results in the following expression Lagrange multiplier:

$$\lambda_t = \frac{\boldsymbol{\mu}_t'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\iota} - \gamma}{\boldsymbol{\iota}'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\iota}}. \tag{77}$$

After filling in (77) in (76) and some rearrangements, we end up with the following weights for the MV portfolio at time t:

$$\boldsymbol{w}_{MV,t} = \frac{1}{\gamma}\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}_t + \frac{\gamma - \boldsymbol{\mu}_t'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\iota}}{\boldsymbol{\iota}'\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\iota}}. \tag{78}$$

E Coefficients of the Theoretically Optimal Intertemporal Portfolio Weights

The following expressions are all taken from the appendix of Jurek and Viceira (2011)

For $\tau = 1$, we have the following equations for $A_0^{(\tau)}$, $A_1^{(\tau)}$, $B_1^{(\tau)}$ and $B_2^{(\tau)}$:

$$A_0^{(\tau)} = \frac{1}{\gamma}\mathbf{\Sigma}_{xx}^{-1}(\mathbf{\Phi}_0^x + \frac{1}{2}\boldsymbol{\sigma}_x^2 + (1-\gamma)\boldsymbol{\sigma}_{1x}) \tag{79}$$

$$A_1^{(\tau)} = \frac{1}{\gamma}\mathbf{\Sigma}_{xx}^{-1}\mathbf{\Phi}_1^x \tag{80}$$

$$B_1^{(\tau)} = \mathbf{\Phi}_1^1 + A_0^{(\tau)'}(\mathbf{\Phi}_1^x - \gamma\mathbf{\Sigma}_{xx}A_1^{(\tau)}) + (\mathbf{\Phi}_0^x + \frac{1}{2}\sigma_x^2 + (1-\gamma)\boldsymbol{\sigma}_{1x})'A_1^{(\tau)} \tag{81}$$

$$B_2^{(\tau)} = A_1^{(\tau)'}(\mathbf{\Phi}_1^x - \frac{\gamma}{2}\mathbf{\Sigma}_{xx}A_1^{(\tau)}) \tag{82}$$

The coefficients $A_0^{(\tau)}$ and $A_1^{(\tau)}$ are defined as follows for $\tau \geq 2$:

$$A_0^{(\tau)} = \frac{1}{\gamma}\mathbf{\Sigma}_{xx}^{-1}(\mathbf{\Phi}_0^x + \frac{1}{2}\boldsymbol{\sigma}_x^2 + (1-\gamma)(\boldsymbol{\sigma}_{1x} + \mathbf{\Sigma}_x(B_1^{(\tau-1)'} + 2\tilde{B}_2^{(\tau-1)}\mathbf{\Phi}_0))) \tag{83}$$

$$A_1^{(\tau)} = \frac{1}{\gamma}\mathbf{\Sigma}_{xx}^{-1}(\mathbf{\Phi}_1^x + 2(1-\gamma)\mathbf{\Sigma}_x\tilde{B}_2^{(\tau-1)}\mathbf{\Phi}_1) \tag{84}$$

The coefficients $B_1^{(\tau)}$ and $B_2^{(\tau)}$ in (35) are defined as follows for $\tau \geq 2$ and a finite coefficient of risk aversion γ:

$$\begin{aligned} B_1^{(\tau)} =& \mathbf{\Phi}_1^1 + A_0^{(\tau-1)'}(\mathbf{\Phi}_1^x - \gamma\mathbf{\Sigma}_{xx}A_1^{(\tau-1)}) + (\mathbf{\Phi}_0^x + \frac{1}{2}\sigma_x^2 + (1-\gamma)\mathbf{\Sigma}_{1x})'A_1^{(\tau-1)} \\ &+ (B_1^{(\tau-1)} + 2\mathbf{\Phi}_0'\tilde{B}_2^{(\tau-1)})\mathbf{\Phi}_1 + (1-\gamma)(B_1^{(\tau-1)}\mathbf{\Sigma}_x' + \mathbf{\Phi}_0'\mathbf{\Xi}_x^{(\tau-1)'})A_1^{(\tau-1)} \\ &+ (1-\gamma)(4\mathbf{\Phi}_0'\tilde{\Lambda}^{(\tau-1)} + \mathbf{\Xi}_1^{(\tau-1)'} + A_0^{(\tau-1)'}\mathbf{\Xi}_x^{(\tau-1)'} + B_1^{(\tau-1)}\mathbf{\Xi}^{(\tau-1)'})\mathbf{\Phi}_1 \end{aligned} \tag{85}$$

$$B_2^{(\tau)} = A_1^{(\tau-1)'}(\mathbf{\Phi}_1^x - \frac{\gamma}{2}\mathbf{\Sigma}_{xx}A_1^{(\tau-1)}) + \mathbf{\Phi}_1'(B_2^{(\tau-1)}) + 2(1-\gamma)\mathbf{\Lambda}^{(\tau-1)})\mathbf{\Phi}_1 + (1-\gamma)\mathbf{\Phi}_1'\mathbf{\Xi}_x^{(\tau-1)'}A_1^{(\tau-1)} \tag{86}$$

where a matrix X with a tilde above it is defined as $\tilde{X}_j^{(i)} = \frac{1}{2}(X_j^{(i)} + X_j^{(i)'})$ and we introduce the auxiliary matrices $\mathbf{\Lambda}^{(\tau-1)} = \tilde{B}_2^{(\tau-1)}\mathbf{\Sigma}_v\tilde{B}_2^{(\tau-1)'}$ and $\mathbf{\Xi}^{(\tau-1)} = 2\mathbf{\Sigma}_v\tilde{B}_2^{(\tau-1)'}$ as in Jurek and Viceira (2011). Furthermore, a subscript 1 or x on a matrix that has no other subscripts or a superscript 1 or x on a matrix that does have other subscripts indicates that only the rows of the matrix that corresponds to the benchmark asset (1) or the excess returns (x) is selected.

F Robustness of the Global Minimum Variance Portfolio Weights

Figure A2 presents the weights of the global minimum variance portfolio computed with data up to either December 2013 or December 2014 with 3 years of data, 5 years of data or all data from January 2002 up to the final date.

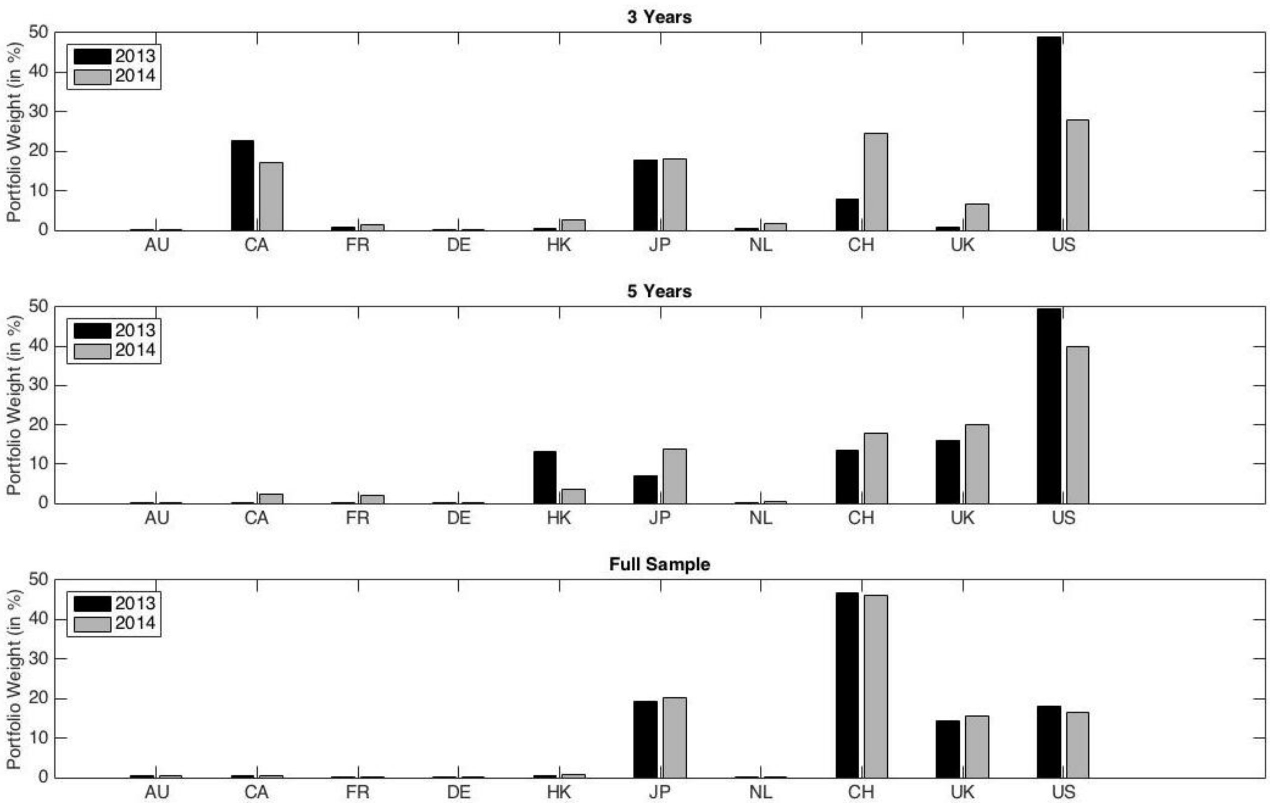
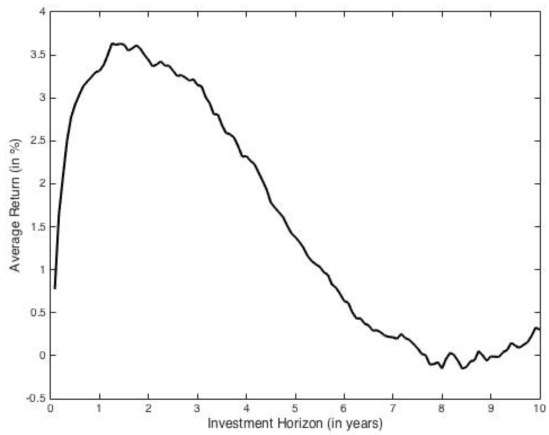


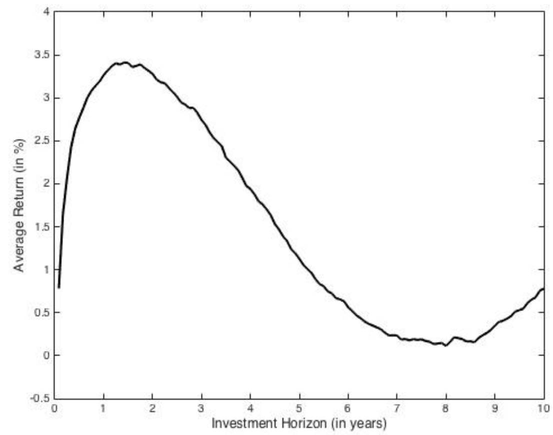
Figure A2: The weights of the global minimum variance portfolio on December 2013 or December 2014 with 3 years of data, 5 years of data or all data from January 2002 up to the final date.

Figure A2 shows that the portfolio weights of December 2013 and the weights of December 2014 differ quite a lot when only 3 years of prior data is used. The difference is considerably smaller when 5 years of data is used, while the portfolio weights of 2013 and 2014 are almost identical when 12 or 13 years of data is used. Hence, the more data is used to compute the global minimum variance portfolio weights, the more robust the weights are.

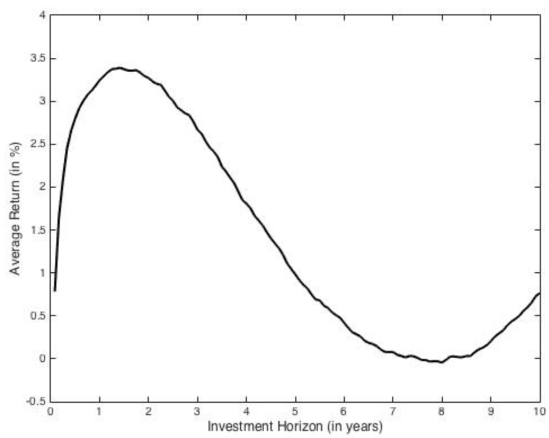
G Convergence of the Vector Autoregressive Model Simulation



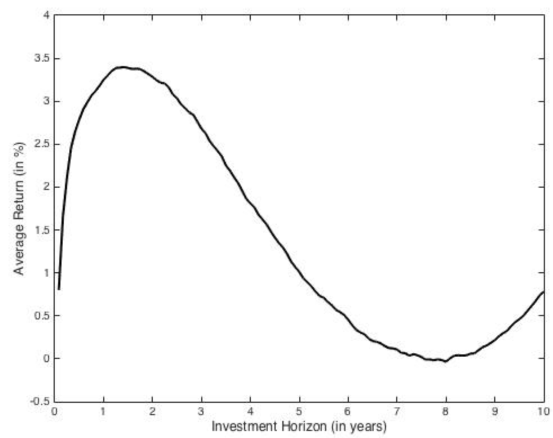
(a) 1000 Simulations



(b) 5000 Simulations



(c) 8000 Simulations



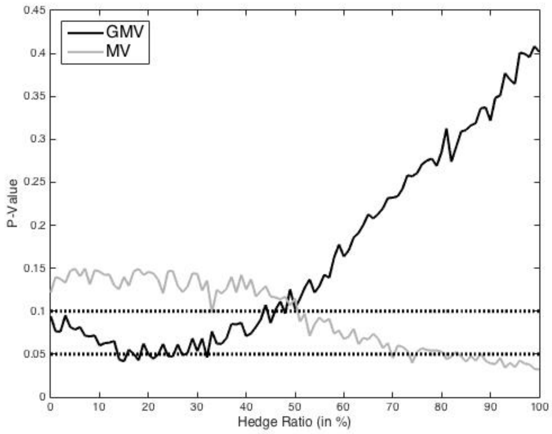
(d) 10,000 Simulations

Figure A3: Simulated returns of Dutch government bonds averaged over 1000, 5000, 8000 or 10,000 (all) simulated paths.

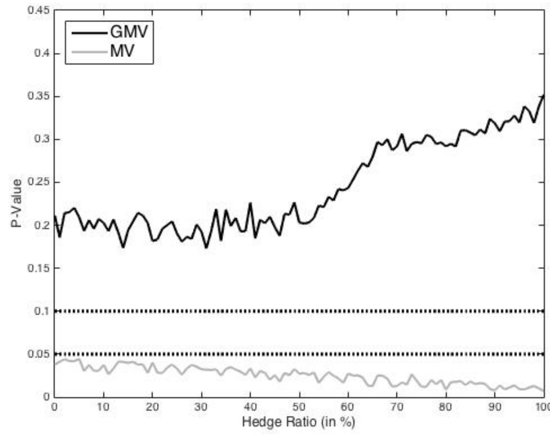
Figure A3 shows that the average simulated returns of Dutch government bonds are quite similar between when we consider 1000 and 5000 simulations. However, the difference is far smaller when we compare 5000 simulations with 8000 simulations, while the average simulated returns are virtually identical between when we use 8000 simulations and when we consider all 10,000 simulations. Hence, a total of 10,000 simulations seems to be (more than) enough for the convergence of simulated returns.

H Hypothesis Testing of Sharpe Ratios

H.1 Equity



(a) MSCI World



(b) MSCI ACWI

Figure A4: The p-values of the hypothesis test for the difference between the Sharpe ratio of the global minimum variance or mean-variance portfolio and the Sharpe ratio of the MSCI World (a) or MSCI ACWI portfolios (b)

H.2 Bonds

Table A5: The p-values of the hypothesis test of difference in Sharpe ratios for asset-only (AO) and asset-liability (AL) investors with investment horizons of 1 year (K=1) or 20 years (K=20) and asset menus consisting of either only government bonds or both government and corporate bonds

Panel A: Unhedged Returns						
	Government Bonds			Government and Corporate Bonds		
	AL_{20} vs. AL_{30}	AO vs. AL_{20}	AO vs. AL_{30}	AL_{20} vs. AL_{30}	AO vs. AL_{20}	AO vs. AL_{30}
K=1	0.001	0.016	0.004	0.029	0.427	0.294
K=20	0.009	0.059	0.033	0.026	0.436	0.304
Panel B: Half Hedged Returns						
	Government Bonds			Government and Corporate Bonds		
	AL_{20} vs. AL_{30}	AO vs. AL_{20}	AO vs. AL_{30}	AL_{20} vs. AL_{30}	AO vs. AL_{20}	AO vs. AL_{30}
K=1	0.011	0.022	0.009	0.076	0.208	0.156
K=20	0.017	0.044	0.022	0.032	0.482	0.300
Panel C : Fully Hedged Returns						
	Government Bonds			Government and Corporate Bonds		
	AL_{20} vs. AL_{30}	AO vs. AL_{20}	AO vs. AL_{30}	AL_{20} vs. AL_{30}	AO vs. AL_{20}	AO vs. AL_{30}
K=1	0.210	0.450	0.210	0.125	0.263	0.249
K=20	0.531	0.480	0.377	0.048	0.473	0.337

Note: AL_{20} and AL_{30} correspond to liability durations of around 17 and 25 years, respectively.

Table A6: The p-values of the hypothesis test of difference in Sharpe ratios across hedge ratios for asset-only (AO) and asset-liability (AL) investors with investment horizons of 1 year (K=1) or 20 years (K=20) and asset menus consisting of either only government bonds or both government and corporate bonds

Panel A: Half Hedged Returns vs. Unhedged Returns						
	Government Bonds			Government and Corporate Bonds		
	AO	AL_{20}	AL_{30}	AO	AL_{20}	AL_{30}
K=1	0.043	0.087	0.215	0.097	0.151	0.169
K=20	0.355	0.880	0.944	0.741	0.677	0.654
Panel B: Fully Hedged Returns vs. Half Hedged Returns						
	Government Bonds			Government and Corporate Bonds		
	AO	AL_{20}	AL_{30}	AO	AL_{20}	AL_{30}
K=1	0.808	0.104	0.060	0.524	0.310	0.346
K=20	0.268	0.034	0.024	0.614	0.373	0.233
Panel C: Fully Hedged Returns vs. Unhedged Returns						
	Government Bonds			Government and Corporate Bonds		
	AO	AL_{20}	AL_{30}	AO	AL_{20}	AL_{30}
K=1	0.151	0.090	0.065	0.237	0.231	0.255
K=20	0.454	0.078	0.033	0.614	0.580	0.295

Note: AL_{20} and AL_{30} correspond to liability durations of respectivy 17 and 25 years, while θ denotes the hedge ratio.

Table A7: The p-values of the hypothesis test of difference in Sharpe ratios between when the asset menu only consists of government bonds and when it consists of both government and corporate bonds for asset-liability (AL) investors with investment horizons of 1 year (K=1) or 20 years (K=20)

	K=1, $\theta = 0$	K=1, $\theta = 0.5$	K=20, $\theta = 0$	K=20, $\theta = 0.5$
AL_{20}	0.178	0.670	0.815	0.808
AL_{30}	0.146	0.308	0.464	0.457

Note: AL_{20} and AL_{30} correspond to liability durations of respectivy 17 and 25 years, while θ denotes the hedge ratio.

I Pooled VAR(1) Model Estimates

I.1 Government Bonds

Table A8: Pooled VAR(1) model parameter estimates with government bonds as investable assets

Panel A: Unhedged Returns					
	$xr_{i,t}$	$ys_{i,t}$	$\pi_{i,t}$	$y^s_{i,t}$	R^2
$xr_{i,t+1}$	0.03 (0.46)	-1.08 (-0.48)	0.05 (0.09)	-1.41 (-1.28)	0.01
$ys_{i,t+1}$	0.00 (-1.56)	0.91 (30.37)	-0.01 (-0.85)	-0.02 (-1.30)	0.87
$\pi_{i,t+1}$	-0.03 (-2.83)	0.24 (1.48)	0.33 (4.20)	0.19 (1.40)	0.23
$y^s_{i,t+1}$	0.00 (0.36)	0.05 (1.65)	0.02 (1.49)	1.00 (73.72)	0.98
Panel B: Half Hedged Returns					
	$xr_{i,t}$	$ys_{i,t}$	$\pi_{i,t}$	$y^s_{i,t}$	R^2
$xr_{i,t+1}$	0.06 (0.87)	1.01 (0.74)	-0.24 (-0.81)	-0.34 (-0.44)	0.01
$ys_{i,t+1}$	0.00 (-1.05)	0.91 (30.80)	-0.01 (-0.80)	-0.02 (-1.26)	0.87
$\pi_{i,t+1}$	-0.04 (-2.79)	0.24 (1.48)	0.32 (4.08)	0.19 (1.41)	0.22
$y^s_{i,t+1}$	0.00 (-0.35)	0.05 (1.62)	0.02 (1.46)	1.00 (73.99)	0.98
Panel C: Fully Hedged Returns					
	$xr_{i,t}$	$ys_{i,t}$	$\pi_{i,t}$	$y^s_{i,t}$	R^2
$xr_{i,t+1}$	0.05 (0.85)	3.05 (3.13)	-0.57 (-1.90)	0.70 (1.10)	0.04
$ys_{i,t+1}$	0.00 (0.91)	0.91 (32.16)	-0.01 (-0.58)	-0.02 (-1.15)	0.87
$\pi_{i,t+1}$	-0.01 (-0.93)	0.32 (1.65)	0.37 (3.68)	0.22 (1.64)	0.18
$y^s_{i,t+1}$	0.00 (-1.95)	0.05 (1.61)	0.02 (1.37)	1.00 (76.71)	0.98

Note: $xr_{i,t}$, $ys_{i,t}$, $\pi_{i,t}$ and $y^s_{i,t}$ denote excess log returns, log yield spreads, log inflation and log short-term interest rate, respectively. T-statistics are presented in parentheses.

I.2 Corporate Bonds

Table A9: Pooled VAR(1) model parameter estimates with corporate bonds as investable assets

Panel A: Unhedged Returns					
	$xr_{i,t}$	$ys_{i,t}$	$\pi_{i,t}$	$y^s_{i,t}$	R^2
$xr_{i,t+1}$	-0.08 (-1.12)	1.44 (1.09)	-0.31 (-0.58)	-2.57 (-3.08)	0.04
$ys_{i,t+1}$	0.00 (-1.00)	0.99 (39.56)	-0.01 (-1.04)	0.03 (1.19)	0.94
$\pi_{i,t+1}$	-0.01 (-1.61)	-0.09 (-0.38)	0.37 (3.94)	0.07 (0.29)	0.17
$y^s_{i,t+1}$	0.00 (1.17)	-0.03 (-2.06)	0.02 (1.88)	0.97 (49.38)	0.98
Panel B: Half Hedged Returns					
	$xr_{i,t}$	$ys_{i,t}$	$\pi_{i,t}$	$y^s_{i,t}$	R^2
$xr_{i,t+1}$	-0.03 (-0.54)	2.26 (2.36)	-0.43 (-1.14)	-1.70 (-2.10)	0.08
$ys_{i,t+1}$	0.00 (-1.24)	0.99 (42.39)	-0.01 (-1.16)	0.03 (1.20)	0.95
$\pi_{i,t+1}$	0.01 (0.62)	-0.09 (-0.39)	0.40 (3.72)	0.12 (0.60)	0.17
$y^s_{i,t+1}$	0.00 (1.15)	-0.03 (-2.16)	0.02 (1.86)	0.97 (53.95)	0.98
Panel C: Fully Hedged Returns					
	$xr_{i,t}$	$ys_{i,t}$	$\pi_{i,t+1}$	$y^s_{i,t}$	R^2
$xr_{i,t+1}$	0.14 (1.70)	2.93 (2.94)	-0.50 (-1.43)	-0.62 (-0.73)	0.11
$ys_{i,t+1}$	0.00 (-1.59)	0.99 (48.00)	-0.01 (-1.18)	0.02 (1.21)	0.95
$\pi_{i,t+1}$	0.05 (1.90)	-0.10 (-0.50)	0.42 (4.28)	0.19 (1.27)	0.23
$y^s_{i,t+1}$	0.00 (1.11)	-0.03 (-2.24)	0.02 (1.79)	0.97 (54.29)	0.98

Note: $xr_{i,t}$, $ys_{i,t}$, $\pi_{i,t}$ and $y^s_{i,t}$ denote excess log returns, log yield spreads, log inflation and log short-term interest rate, respectively. T-statistics are presented in parentheses.