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MSc Economics & Business
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The Search for the Optimal Debt Level

Determining the growth-maximising public debt-to-GDP ratio

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''A national debt if it is not excessive will be to us a national blessing''[1]

[1] Quote by Alexander Hamilton, one of the Founding Fathers of the United States, on the public debt debate.

Preface

During both my Bachelor- and Master studies, the 2008 financial crisis and its consequences for public debt played a role in a substantial number of courses. Notably, the *international financial markets* seminar sparked my interest on the implications of this increase in public debt. This enthusiasm ultimately resulted in a suitable topic for my *international economic* master thesis.

This thesis aims to estimate the optimal debt-to-GDP ratio in terms of economic growth. Thereby contributing to the discussion whether the current public debt levels have grown out of proportion.

In this preface, I would like to express my gratitude to my supervisor J.M.A Viaene for his supervision, and helpful advice during these past months. The constructive criticism stimulated me to work even harder, and this eventually resulted in the completion of my master thesis. I would also like to thank my family, friends and girlfriend for the continuous support, which played a crucial role in successfully conducting my research.

Abstract

This paper studies the impact of public debt-to-GDP ratios on the economic growth rate of 21 industrial countries during the 1990-2014 period. Thereby the aim is to determine the optimal debt-to-GDP ratio in terms of maximised economic growth. The debt-growth relation has been estimated by regression analyses, while taking alternative growth determinants, non-linearity and potential endogeneity issues into consideration. The findings confirm concavity of the debt-growth relation and the existence of upperbound values. The reported results indicate that moderate public debt levels stimulate economic growth, while diminished economic growth characterises debt ratios beyond the determined threshold. The most-encompassing instrumental variable specification indicates that the optimal debt ratio is about 129.55%, which is a higher public debt ratio than reported in previous studies. The conducted robustness checks show that the non-linear debt-growth relation prevails for alternative control variables, dependent variable, country sample and incorporated polynomial. Finally, the empirical analyses reported that the interaction with the real interest rate is a channel through which the public debt ratio reduces the economic growth rate. In the latter case, the inclusion of several interaction terms reduces the optimal debt-to-GDP ratio to about 100%.

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1. Introduction

Issuing national public debt is a practice of all times, as governments of ancient societies already resorted to public borrowing to finance expenditures. Historic records trace public borrowing back to the fourth century BC, the form of public borrowing did, however, significantly change over time (Salsman, 2017). In the medieval times, the borrowed funds mostly consisted of commodities, the duration of the loans was shorter, and loans were acquired from the wealthy local elites (Dyson, 2014). Nowadays, public debt is publically traded on financial markets, and the bonds mature in multiple years. Moreover, in ancient times public borrowing mostly served to finance a war, while in contemporary times public debt functions as a structural solution to budgetary deficits (Salsman, 2017).

In the past decade's public debt-to-GDP have been severely increasing, partly due to the frequent occurrence of these deficits (IMF, 1996);(Blanchard and Johnson, 2013). In 1999, the average Eurozone public debt-to-GDP ratio was 53.26%. In the year 2016, the ratio has substantially increased to the unprecedented level of 79.20%, which reflects a 48.70% increase[2]. In 2016, the debt ratio of Belgium, Cyprus, Greece, Italy, and Portugal were at levels higher than 100%. Whereas, the debt ratio of Austria, France, Germany Ireland and Spain lie within the range of 80% and 100% (Figure 1.1).

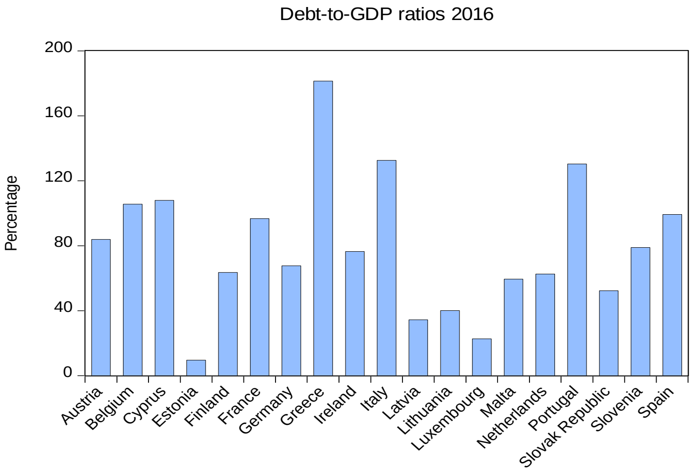

Figure 1.1: The debt-to-GDP ratios in 2016 for the Eurozone members

Notes: (1) based on data from IMF World Economic Outlook Database.

The current height and rapid increase of the debt-to-GDP ratios of the advanced economies have made politicians and academic to dispute the sustainability of these government debt levels. National governments introduced guidelines on acceptable debt-to-GDP ratios to prevent these situations of unsustainable debt. For example, the Eurozone countries are by the Maastricht treaty, and the Stability and Growth Pact (henceforth SGP) obliged to strive for a debt-to-GDP ratio of 60%. On the other hand, the United Kingdom aims at obtaining a ratio below 40% of GDP (European Union,

[2] Based on data from IMF World Economic Outlook Database.

1992);(Topalova and Nyberg, 2010). However, as the introduced debt ceilings are only adequate levels of debt, welfare is increased when the limit is equal to the optimal debt ratio (Topalova and Nyberg, 2010).

Based on this reasoning, several empirical studies, with different perspectives on how to quantify optimal debt, concentrated on determining which public debt ratio can be judged as optimal. One point of view is that public debt is at its optimum when a nations debt-to-GDP level is stationary, as it implies that when a shock to the debt-to-GDP level occurs, budgetary surpluses ensure that the debt level will revert to the initial debt level (Da Costa, 2010). In contrast, the paper of Aiyagari et al. (1998) quantifies the optimal debt ratio by examining for which debt ratio social welfare is maximized. As a result of this, the author assesses the welfare cost from being at a sub-optimal quantity of debt. In the search for the optimal debt ratio, this paper characterises the optimal debt-to-GDP ratio as the ratio for which economic growth is maximised, an approach applied by more recent academic papers. The existence of such an economic growth-maximising public debt level implies the existence of a non-linear debt-growth relationship, where issuing additional debt beyond the optimal debt threshold causes a reduction in GDP.

In the recent years, the number of publications concerned with the subject of determining the tipping point of public debt has increased substantially. The much-quoted paper of Reinhart and Rogoff (2010) investigated forty-five countries during the 1790-2009 period and found that passing the 90% debt-to-GDP ratio causes the average 3% growth to drop to 1.7%. The research of Cecchetti et al. (2011) concentrated on eighteen OECD countries from 1980 to 2010 and concluded that the optimum is found at 85%. Moreover, the related study of Checherita and Rother (2010) claim that the threshold is located in a range of 90% to 100%. However, the work of Egert (2015) contradicts these results, as even if optimal debt ratios do exist, the upperbounds are found around 20% to 60%.

The results of the outlined studies indicate that the found optima differ per study, as the used data, country sample and methodology varies per study. As a result, the heated debate concerning the potential unsustainability of the current debt levels and the disagreement on the level of the optimal debt ratio demands further investigation. Consequently, the results of this paper contribute to the discussion by determining the optimal growth-maximising debt ratio for a different set of countries, data and methodology. Therefore, the research question of the thesis is the following:

Which debt-to-GDP ratio could be labelled as optimal, and should, therefore be pursued?

The paper is subdivided into seven different chapters. The first section contains the introduction of the paper. In the second chapter, a numerical simulation of the debt dynamics of five case studies is performed. This chapter aims to gain further insight into the evolution of public debt levels and sheds light on the determinants of public debt. The third chapter describes the related literature on the effect of public debt on economic growth. Subsequently, the methodology used to determine the growth maximising debt-to-GDP ratio is presented in the fourth chapter. The fifth chapter discusses the data on which the empirical analysis is based. Next, the results of the analysis are reported in the sixth chapter. The final and seventh chapter contains the conclusion and discussion of the paper.

2. Public Debt Dynamics

This chapter aims to use numerical simulations to study the determinates of public debt and examine the development of national debt-to-GDP for future years. As a result, the often heard claim that the current debt levels are unsustainable is investigated, as sustainable debt should, at least, not continuously grow over time. The debt sustainability is assessed by numerically simulating the debt dynamics of the five most intensively involved countries in the 2009 Eurozone sovereign debt crisis, which are Portugal, Ireland, Italy, Greece and Spain (henceforth: PIIGS).

2.1 Background

The intuition behind the severe accumulation of debt over time, indicated in the introduction, is explained by basic economic principles, such as the requirement of a balanced public budget. The first step in determining a model that adequately describes the evolution of public debt is presenting the government budget constraint:

$$G_t + rD_{t-1} = \tau_t + (D_t - D_{t-1}) \tag{2.1}$$

where G_t reflects governmental expenditures, r indicates the real interest rate, D_t symbolises the absolute debt level of the relevant period and τ_t taxation income. Equation (2.1) is based on the argument that a government must run a balanced budget, as governments are required to finance all its expenditures. Therefore, the total of governmental costs, consisting of public expenditures and interest payments on existing public debt, must be equal to the total of governmental income, which originates from either tax revenue or a governmental debt issue.

Restructuring (2.1) in such a manner that current debt is a function of previous debt and the budgetary balance, results in the following equation:

$$D_t = (1 + r)D_{t-1} + (G_t - \tau_t) \tag{2.2}$$

The formula states that absolute debt at the end of period 1 is the sum of the debt of the previous period plus interest paid on the outstanding plus the budgetary deficit.

In general, the public debt level is measured relative to a countries national gross domestic product instead of absolute terms, as it indicates which percentage of the total national product needs to be devoted to repaying the public debt. Therefore, (2.2) is modified to reflect the development of the debt-to-GDP ratio:

$$d_t - d_{t-1} = \frac{r_t - g_t}{1 + g_t} * d_{t-1} + \frac{G_t - \tau_t}{Y_{t-1}} \tag{2.3}$$

where d_t reflects the debt-to-GDP ratio, Y_t symbolizes the current GDP, and the remaining variables are indicated by the previously named symbols. The derivation is included in Appendix A. Equation (2.3) states the recently increased debt ratios, and debt dynamics in general, are determined by either the occurrence of budgetary deficits as a percentage of GDP or by the positive difference between the real interest rate and the country's economic growth (Blanchard and Johnson, 2013). The first component of (2.3) implies that the debt-to-GDP ratio is reduced if economic growth is higher than the real interest rate by an amount of the $r_t - g_t$ multiplied by outstanding debt. The second component indicates the crucial role of the budgetary balance of a government. A fiscal surplus reduces the increase in public debt, as administrations can pay down outstanding debt. On the other hand, a budgetary deficit causes debt to increase, as additional public debt is required to finance all the governmental expenditures.

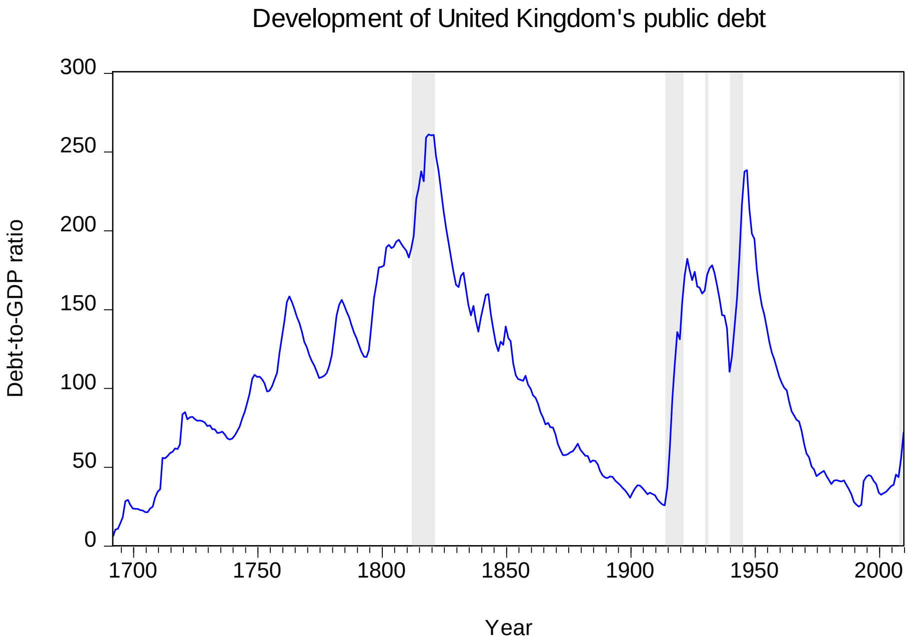

Figure 2.1: Development of United Kingdom's public debt

Notes: (1) Debt measured as total net central government debt to GDP. (2) Data obtained from Reinhart and Rogoff (2010).

The theoretical implications of (2.3) are confirmed by Figure 2.1, which reports the development of the public debt ratios of the United Kingdom from 1700 to 2010. The figure shows that each of the five shaded time periods, which are characterised by sharp rising public debt levels, occur during times of reduced economic growth, high real interest rates, public deficits or a combination of the factors. The shaded areas are explained by the following events in consecutive order; the post-Napoleonic depression; the post-World War I depression; the great depression; the occurrence of the Second World War and the 2008 financial crisis.

The figure and the included shaded areas indicate that public debt historically rises sharply during the times of war or financial crisis, as both events result in high public deficits and reduced economic growth. On the contrary, the time periods of diminishing public debt ratios are usually explained by increased economic growth, such as the decrease in the United Kingdoms public debt ratio during the nineteen-hundreds and around the 1970's. Furthermore, inflation affects the public debt development indirectly through the real interest rate (Chouraqui et al., 1986). Historically seen high inflation results in reductions in the debt ratios, as public debt is nominated in nominal terms; strong inflation thereby reduces the real value of debt (The Economist, 2011). Once more this is in line with (2.3), as inflation reduces *r*.

2.2 Numerical Simulations

The numerical simulations of the case studies' debt dynamics are based on parameter values of three different scenarios. Namely, a scenario reflecting the *actual* data, a scenario assuming *optimistic* development of the parameters and a scenario based on *pessimistic* values. The debt dynamics of the public debt-to-GDP ratios are assessed by simulating a deterministic model based on the previous work of De Luca (2012). As the rate of economic growth and interest rate fluctuate per scenario, the intuition behind (2.3) is further examined. According to the formula, economic growth should adverse affect public debt growth, while the real interest rate should enhance debt accumulation.

The deterministic model

The dynamics of public debt are commonly analysed by a difference equation, which implies that the future value of a variable is determined by its value of the previous periods. An approach justified by (2.2), as it shows that current debt is a function of outstanding debt of the prior period. Subsequently, (2.2) is forward substituted to be able to determine the debt level for each future period:

$$D_t = (1+r)^t [D_0 + \sum_{k=1}^{t} \frac{pb_k}{(1+r)^k}] \tag{2.4}$$

where pb_k indicates the public balance, $G_t - \tau_t$. Equation (2.4) implies that the current absolute debt level is a function of the initial debt level, the accumulated public deficits, time and the interest rate paid on the public debt.

Underlying identities

Equation (2.2) and (2.3) are equations commonly used to study the evolution of the debt level over time. The equations are, however, relatively simplistic. Therefore, the equations are further augmented with a more elaborate description of the evolvement of the budgetary deficit and by the introduction of economic growth. Resulting in a more accurate simulation of the debt dynamics. These factors are incorporated with the help of several underlying identities (Appendix B). By doing so, (2.5) is obtained, which describes the debt dynamics more comprehensively:

$$D_t = (1+r)^t D_0 + \frac{G_0}{r}[(1+r)^t - 1] - \tau_0\gamma_\alpha\left(\frac{1+g}{r-g}\right)[(1+r)^t - (1+g)^t] \qquad (2.5)$$

Equation (2.5) indicates that current debt is a function of the real interest rate, initial governmental expenditure, initial tax revenue, time and the economic growth rate.

Thereafter, both sides of (2.5) are divided by τ_0. Hereby, the absolute debt level is measured relative to the countries initial tax revenue:

$$\frac{D_t}{\tau_0} = (1+r)^t \frac{D_0}{\tau_0} + \frac{G_0}{\tau_0 * r}[(1+r)^t - 1] - \frac{\tau_{0*}\gamma_\alpha}{\tau_0}\left(\frac{1+g}{r-g}\right)[(1+r)^t - (1+g)^t] \qquad (2.6)$$

As a result, (2.6) indicates which percentage of the total national tax revenue needs to be devoted to repaying the public debt.

Parameterization

The subsequent step in simulating the debt dynamics is the parameterisation of (2.6). The initial values, indicated by a $t = 0$ subscript, differ per country, as the variables are country specific. The same holds for the economic growth and interest rate parameter values. However, these variables, additionally, differ per scenario, while the initial values remain constant throughout the simulation. The data of the *actual* scenario is based on the most recent available data and is obtained from the World Bank and the OECD. The *pessimistic* scenario builds on the average parameter value over the 2008-2015 period, while the *optimistic* scenario depends on the average over the 1999-2007 period. This approach can be substantiated by the fact that the first period is characterised by economic prosperity and low-interest rates, while the second period contains times of economic turmoil with lower economic growth and higher interest rates. Additionally, $\frac{b_0}{\tau_0}$ and $\frac{G_0}{\tau_0}$ are parameterised accordingly to each countries initial values. For convenience reasons, γ_α is assumed to be equal to unity. Consequently, the numerical simulations are based on the following data and scenarios:

Table 2.1: The parameter values per scenario and figure.

Scenarios	Actual[1]		Optimistic[2]		Pessimistic[3]		$\frac{d_0}{\tau_0}$[4]	$\frac{G_0}{\tau_0}$[5]
	(g)	(r)	(g)	(r)	(g)	(r)		
Portugal	1.39%	1.54%	2.09%	1.1%	-3.29%	4.54%	5.64	2.11
Italy	0.88%	0.74%	1.51%	2.12%	-0.78%	4.35%	5.58	2.13
Ireland	5.21%	1.94%	6.65%	0.39%	3.67%	4.35%	4.08	1.53
Greece	0.001%	7.32%	3.92%	-0.021%	-3.29%	9.88%	6.95	2.19
Spain	3.24%	1.08%	3.89%	0.86%	-0.003%	3.41%	6.89	3.02

Notes: (1) the economic growth rate (g) and interest rate (r) are based on 2015 data. (2) Parameter values builds on the average over 1999-2007. (3) Parameter values are based on the average over 2008-2015. (4) $\frac{D_0}{\tau_0}$ equals 2015 initial debt divided by initial tax revenue. (5) $\frac{G_0}{\tau_0}$ equals 2015 initial governmental expenditures divided by 2015 tax revenue.

2.3 Numerical Results

Simulating (2.6) results in the debt dynamics under the different parameter values of Table 2.1 for various periods. The most striking observation of Figure 2.2 is that, in both the *actual* and *pessimist* scenario, each examined country, besides Ireland, is confronted by an increasing debt ratio over time. The debt sustainability of Portugal, Italy and Spain is even more at risk, as the debt level is even increasing in the *optimistic* scenario and do not seem to be paid back over time. The only exceptions are found to be Ireland and Greece, as the *optimistic* debt curve eventually reaches $b_n = 0$, and public debt is entirely paid back. The Irish debt is sustainable to such a degree that debt is decreasing in all three scenarios.

The debt enhancing role of the real interest rate and the diminishing effect of the economic growth is clearly indicated by comparing the different outcomes of the simulated scenarios. For example, when an *optimistic* development of the economic growth and interest rate is assumed the Greek debt ratio vanishes over time, while the Greek debt increases under the economic circumstances of the alternative two scenarios. Moreover, as economic growth diminishes in the *pessimistic* scenario, public debt is present for a longer period, or even continuously increases over time, in all the Portuguese, Italian and Spanish scenarios. The deteriorating effect of economic growth and the stimulating effect of the interest rate on public debt is further emphasised by simulating scenarios based on artificial parameter values (Appendix C).

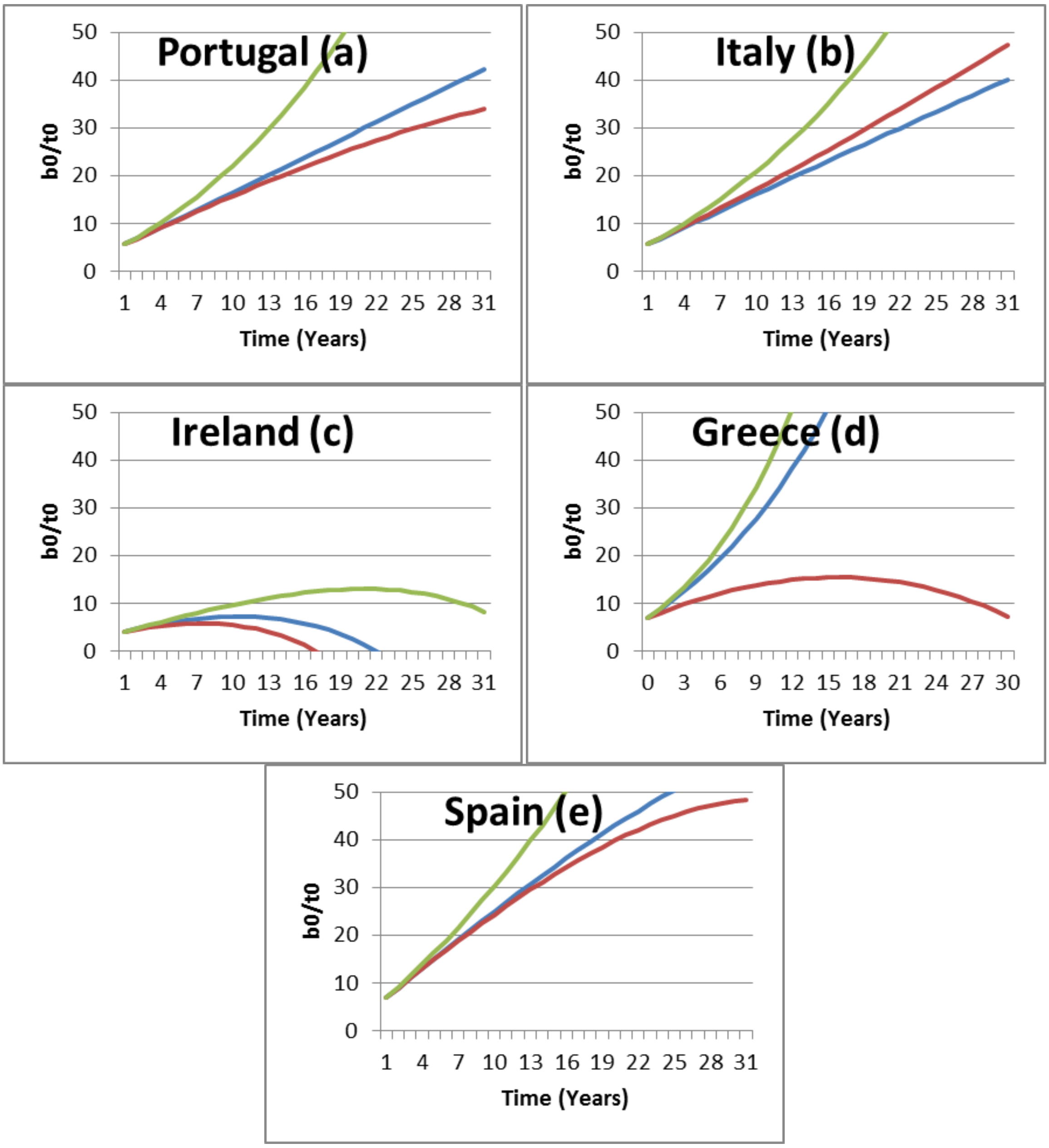

Figure 2.2: Debt Dynamics of the PIIGS countries.

Notes: (1) the blue line reflects the simulated debt dynamics of the *actual* scenario, the green line of the *pessimistic* scenario and the red line the *optimistic* scenario.

Sustainability results

The presented figures and interpretations are only preliminary evidence on debt sustainability, as debt developments take place beyond the timeframe included in the graph. Consequently, this paragraph aims to examine whether the considered scenarios truly represent continuously increasing or decreasing debt levels for all periods. Firstly, the debt sustainability of the highly indebted countries is further assessed by deriving the first derivative of (2.6) with respect to *t*:

$$\frac{1}{\tau_0}\frac{db_t}{dt} = (1+r)^t \ln(1+r)\,((\frac{d_0}{\tau_0}+\frac{G_0}{r\tau_0}-\gamma_\alpha(\frac{1+g}{r-g})(1-\frac{\ln(1+g)}{\ln(1+r)}\left(\frac{1+g}{1+r}\right)^t)) \qquad (2.7)$$

Subsequently, the found first derivative is set equal to zero and solved for the t. In case of the existence of a certain period, for which the first derivative is equal to zero, the debt curve is characterized by a turning point. Observations beyond such a turning point, the public debt will likely diminish as time passes and potentially reach $b_t = 0$. Secondly, the second derivative of (2.6), with respect to *t*, is derived to examine the shape of the debt curves. The second derivative is the following:

$$\frac{1}{\tau_0}\frac{db_t^2}{dt^2} = (1+r)^t \ln^2(1+r)\,(\frac{d_0}{\tau_0} + \frac{G_0}{r\tau_0} - \gamma_\alpha\left(\frac{1+g}{r-g}\right)\left(1 - \frac{\ln^2(1+g)}{\ln^2(1+r)}\left(\frac{1+g}{1+r}\right)^t\right)) \quad (2.8)$$

The second derivative reflects how the slope of the curves changes as time passes. In the light of this research, it is expected that countries and scenarios characterized by the elimination of debt have a negative second derivative for all periods. For instance, it is expected that all three Irish scenarios have a negative second derivative, as all three debt curves, in Figure 3.1C, potentially reach $b_n = 0$. In addition, a negative second derivative implies that the function is concave, which further strengthens the expectancy that the curve will eventually reach $b_t = 0$. On the other hand, scenarios with ever-increasing debt dynamics should have a positive second derivative, as this implies that the slope of the debt curve increases as time passes. Such a finding brings unsustainability to light, as public debt grows over time and, potentially, always be present.

The calculated first and second derivatives, the consequential turning point, and the shapes of the debt dynamics are presented in Table 2.2. Table 2.2 is restricted to the three scenarios of Ireland and Greece, where the complete results are found in Appendix D. These two countries have the most varying parameter values, which can serve to more clearly highlight the influence of the economic growth and the interest rate on a countries debt dynamics.

Table 2.2: Summary of Debt Sustainability results

Scenario	*Economic growth[1]: (1)*	*Interest rate[1]: (2)*	*Turning-point[2]: (3)*	*Debt repayment $b_n = 0$[3] (4)*	*Convex function[4]: (5)*	*Concave function[4]: (6)*
Ireland optimistic	6.65%	0.39%	*T=6.33*	*T=16*	-	*All time periods*
Ireland actual	5.21%	1.94%	*T=9.58*	*T=21*	-	*All time periods*
Ireland pessimistic	3.67%	4.35%	*T=20.1*	*T=36*	-	*All time periods*
Greece optimistic	3.92%	-0.021%	*T=15*	*T=35*	-	*All time periods*
Greece actual	0.001%	7.32%	∞	∞	*All time periods*	-
Greece pessimistic	-3.29%	9.88%	∞	∞	*All time periods*	-

Notes: (1) Parameter values based on Table 2.1. (2) Found by setting (2.7) equal to zero. (3) The period for which (2.6) is equal to zero. (4) The curvature of the scenarios for different periods found by equalising (2.8) to zero.

The results confirm the preliminary debt dynamics results presented in the previous figures. Firstly, the third column shows that the turning point, where the public debt starts diminishing, is negatively related to the to the economic growth and positively to the interest rate. The Irish scenarios show that the turning-point increase with lower economic growth and with higher interest rates. Therefore, the lower the economic growth, relative to the interest rate, the more time it takes the debt to diminish.

The number of years required to repay debt is included in the fourth column. The postponement of the debt turning point, the year in which the debt ratio is equal to zero also occurs at a further point in time. However, the Greek case reveals that under the wrong economic circumstances debt repayment is not taking place at all. Moreover, for the Greek *actual* and *pessimistic* case, no turning point is present, which results in public debt continuously growing over time. This result denotes that if these countries maintain to be confronted with similar growth and interest rates, the national debt is far from sustainable.

The results of the second derivative, once more, confirm the previously found results. The fifth and sixth column illustrates that the countries with turning points and debt curves reaching $b_n = 0$ over time are characterized by a concave function. Considering all case studies, it is shown that eight out of the fifteen scenarios show negative curvature for all periods, which implies concavity and $b_t = 0$ is reached over time. This ensures debt sustainability for these countries. This, additionally, holds for the two scenarios, which have a positive curvature for initial periods but are, eventually, characterized by a negative second derivative. Namely, the *actual* Portuguese scenario and the *optimistic* Italian scenario. However, five out of the fifteen scenarios are convex functions for all periods, which implies the debt curves are continuously increasing over time and therefore unsustainable. Specifically, the Portuguese, Italian and Spanish debt is unsustainable in the *pessimistic* scenario, as debt steadily grows over time and does not reach $b_n = 0$. Moreover, the simulation provides evidence that the Greek debt under, both, the *actual* and *pessimistic* scenario is unsustainable. The single PIIGS country with sustainable debt in all three scenarios is Ireland.

2.4 Preliminary Conclusion

In general, the comparison of the debt dynamics scenarios made clear that public debt is larger for lower economic growth rates relative to the real interest rate. Thereby confirming the intuition behind (2.3). The diminishing effect of the economic growth rate on public debt and the enhancing effect of the real interest rate manifests itself in either a continuously unsustainable debt level or a postponement of the period in which public debt is completely paid-down. The combination of the finding that economic growth adversely affects debt accumulation with, in the introduction, outlined empirical results that certain debt levels diminish economic growth indicates that debt sustainability is put further at risk.

3. Theoretical Review

The reported rising debt levels by both the introduction and the debt dynamics has raised concerns about its negative implications for the growth rates of the economy. The two main mechanisms through which public debt lowers economic growth are less budgetary space, as yearly-required interest payments increase with debt, and lower national investments, through an increased real interest rate (Chouraqui et al., 1986).

The occurrence of vicious cycles of ever-increasing public debt is an alternative adverse effect of high debt on economic growth. Where the debt dynamics simulation assumed a constant real interest rate, in practice the height of the debt, the growth rate and the real interest rate are related. The Eurozone sovereign debt crisis functions as a clear example that countries characterised by high debt ratios are confronted by higher interest rates and expenses, as investors demand more compensation due to increased risk of default. The increase in governmental expenses potentially forces the government into an additional public debt issue, due to the need for further financing, which not only increases the outstanding debt but also results in even higher demand interest rates and interest payments. Alternatively, countries could compensate the increased expenses by levying higher tax rates. However, the resulting tax distortions have several potential adverse side effects, such as a lower amount of investments and labour supply (Feldstein, 2008). Thereby, once more, diminishing the economic growth rate.

Despite the presented negative economic consequences of public debt, the current consensus is that, mainly, moderate debt levels, could be beneficial for economic growth and welfare. For instance, the public budget constraint of (2.1), shows that public borrowing enables the government to spend more funds than the immediate revenues. Running such an unbalanced budget could enhance the economic growth rate, as public borrowing permits stimulation of the economic growth and aggregated demand by influencing, for example, employment, domestic output and inflation with these additional funds. During the latest financial crisis, public borrowing enabled the government to break the vicious cycle of decreasing private aggregate demand, falling national income, reduced tax revenue and lowered employment by increasing its governmental expenditures. Thereby mitigating the negative effects of the economic crisis by stimulating aggregate demand and output with borrowed funds (International Monetary Fund, 2012). The fact that public debt could both be beneficial and harmful for economic growth results in the question for which exact public debt-to-GDP ratio economic growth is maximised and optimal regarding economic growth.

Existing Literature

Research on the relationship between economic growth and public debt gained momentum during the aftermath of the 2008 financial crisis, as the topic started to play a central role in both the political and academic arena. The famous study of Reinhart and Rogoff introduced the idea that the debt-growth relation could be non-linear to the general public. Such a non-linear relation indicates the existence of a growth maximising debt ratio, where debt accumulation beyond the optimal value results in diminished economic growth. The Reinhart and Rogoff (2010) study shows that countries with a 90% or higher debt ratio are faced with diminished economic growth for a sample of forty-five countries based on data from 1790 to 2009. Up to the derived 90% threshold, public debt does not severely affect growth rates. However, average growth drops from 3% to 1.7% once debt ratio is beyond the 90% threshold. The negative impact of debt on growth is even more severe when a shorter time sample is taken into account, as average growth rates fall from 3% to -0.1% (Reinhart and Rogoff, 2010). However, the findings of the Reinhart and Rogoff paper became controversial when a replication paper of Herndon et al. (2013) reported that database contained coding errors and when these errors were taken into account, the results turned out to be less dramatic. After correction of errors, the growth rate falls from 3% to 2.2% instead of -0.1%. Nevertheless, both studies show that debt-growth relation is negative and non-linear.

Comparable research of Baum et al. (2012) reports that debt accumulation of highly indebted countries, with debt ratios above 95%, are confronted with negative consequences for debt. Cecchetti et al. (2011) study on the debt-growth relationship based their conclusions on yearly data from 18 OECD countries during 1980 to 2010 and showed that the public debt stimulates growth up to 85%. Chechertia and Rother (2010) estimated a regression equation based on twelve euro area countries from 1970 to 2010 and confirms the existence of a non-linear relationship with a growth maximising debt ratio lies within the 90%-100% range. Further empirical support for a non-linear negative debt-growth relationship is provided by Caner et al. (2010), which is based on data from 101 countries during the 1980-2008 period. The results indicate a tipping point of 77%, a 1% increase beyond the tipping point diminishes economic growth by 0.017. Extending the analysis to emerging markets shows that the tipping point is found at a debt-to-GDP ratio of 64%.

However, several authors have opposing views on whether certain optimal debt-to-GDP ratios in terms of economic growth exist. For example, Pescatori et al. (2014) find no support for the claim that economic growth starts to diminishes when the debt ratio is beyond a general tipping point. For instance, the study shows that the compromised economic growth for highly indebted countries, with a debt ratio above 90%, is absent when the debt ratios are on a declining path. Based on this observation the authors conclude that the diminished economic growth is not due to high debt ratios, but the lower growth is caused by alternative variables related to debt accumulation. The existence of non-linearity in the debt-growth relation is put further into question by the work of Egert (2015). The results show that it is challenging to find such a relation and even when a threshold is found, then the upper bounds are located in the range of 20% to 60% of GDP.

The study of Panizza and Presbitero (2012) raises a more fundamental point of criticism by stating that no conducted study identified a true causal relationship between debt and growth due to incorrectly dealing with endogeneity. As an alternative explanation of the found negative impact of public debt on economic growth is the presence of reverse causality, which implies that weak economic growth causes high debt and not the other way around. For example, a country faced by with lower economic growth rate tends to issue additional public debt as governments pursue expansionary policies and public revenues are reduced due to the lower economic growth rate. Another endogeneity issue is potentially present, as it is plausible that periods characterised by both high debt and low economic growth are caused by an omitted variable. Failing to take these forms of endogeneity into account results in biased estimates and incorrect optimal debt-to-GDP values.

A widely applied solution for the presence of endogeneity problems is estimating the relation by instrumental variables. The approach implies that the endogenous debt variable is replaced by a variable that is related to the endogenous debt variable but only influences the dependent variable through its effect on debt (Bosker, 2017). For example, the paper of Cecchetti et al. (2011) introduces lagged values of the debt ratio as an instrument, which, however, could be inadequate as debt is persistent over time (Panizza and Presbitero, 2012). Checherita and Rother (2010) instrument debt by the average debt of the sample countries. Panizza and Presbitero (2012), who instrument debt by the interaction between foreign currency debt and the exchange rate, apply a more rigorous approach. The first two papers show the prevalence of a non-linear relationship and growth optimising debt ratio, while the paper of Panizza reports that the debt-growth relationship is insignificant when the link is estimated by the instrumental variable approach.

In general, the theoretical review indicates contradictions and differences in the results on the debt-growth relation. The optimal debt-to-GDP ratio in terms economic growth differs per study, which implies that the results are not robust and further empirical investigation on the topic of growth is required.

4. Empirical Methodology

The empirical methodology presented in this chapter aims at determining the causal effect of public debt on economic growth and determining the growth maximising debt-to-GDP ratio. The debt-growth relation is unrevealed by estimating a regression model with several specifications that differ in the incorporated control variables and estimation techniques.

The regression specification is, firstly, estimated with *ordinary least squares (*henceforth: OLS). However, as noted in previous research, the debt-growth relation is characterised by endogeneity issues, which results in biased estimates with the OLS estimator (Panizza and Presbitero, 2012). For example, it is likely that the baseline of (4.1) does not control for all relevant variables resulting in an omitted variable bias. To mitigate this bias, all the specifications are augmented with both forms of fixed effects (Brooks, 2014).

More problematic is bias introduced by the mentioned presence of reverse causality, as augmenting the specification with fixed effects is not sufficient to address the bias. To take the endogeneity problems into account, the debt-growth relation is subsequently estimated by an *instrumental variable* approach introduced by paper of Checherita and Rother (2012). Consequently, the public debt for country *i* at period *t* is instrumented by the average debt-to-GDP proportion of the remaining countries in the same period.

Model specification and identification strategy

The debt-growth relationship and the resulting optimal debt ratios are found by estimating the following baseline equation:

$$gdppc_{it} = \beta_0 + \beta_1 Debt_{it} + \beta_2 Debt_{it}^2 + \beta_3 X_{it} + \mu_i + \varphi_t + \varepsilon_{it} \quad (4.1)$$

where $gdppc_{it}$ is the equations dependent variable and reflects the growth of the GDP per capita of country i at time *t*, β_0 reflects the constant, $Debt_{it}$ is the relevant public debt variable. The discussed non-linearity of the relationship is captured by including the squared term of public debt. Moreover, X_{it} symbolises a vector of control variables. Equation (4.1) also incorporates country fixed effects, μ_i and time fixed effects, indicated by φ_t. The last symbol of the equation, ε_{it} is the error term of the equation, which contains non-included variables affecting economic growth. The coefficients of interest are β_1 and β_2 as these reflect the effect of public debt on economic growth. According to the literature, it is expected that debt's coefficient is positive, while debt^2 should have a negative sign. As these coefficients would result in the discussed concave non-linear shaped relation, where modest debt-to-GDP ratios stimulate economic growth, while the adverse effects dominate for higher levels of debt.

The control variables of (4.1) are included based on previous studies, where these variables are found to be growth affecting. The inclusion of these control variables enables to determine whether public debt affects economic growth while taking into account the effect of alternative growth affecting variables. The following control variables are included:

- National investments, population growth and technological advances, as they reflect the primary drivers of economic growth according to the prominent study of Solow (Romer, 2012);
- Initial GDP per capita to capture that less developed countries will develop more rapidly than economically more advanced countries (Rodrik, 2013);
- Human capital, as it is characterised as a catalyst for productivity (Kumar and Woo, 2010);
- government size to express that governments excessive in size could harm the economic growth through higher tax rates and required debt issues (Asimakopoulos and Kravias, 2016);
- The budgetary balance to control for the influence of governmental fiscal policies (Kumar and Woo, 2010);
- Trade openness and terms of trade reflect the beneficial character of trade and international competitiveness of a country (Frankel and Romer, 1999);(Checherita and Rother, 2010);(Borkin, 2006);
- Financial development reflects the external financial influence, as it, for instance, increases the available funds and costs of investment (Rajan and Zingales, 1998);
- Age composition to include the working population in the specification (Santacreu, 2016);
- The real interest rate to capture its influence on investments and governmental debt, as its role is reported by (2.3) and the debt dynamics section (Reserve Bank of India, 2013).

5. Data

The relationship between economic growth and public debt and the potential existence of growth-maximising debt ratios is examined for a set of 21 industrial countries for the period 1990-2014. The data is structured as panel data, which consist of observations for different countries for multiple years. Consequently, each observation is characterised by a cross-section, shown by the subscript i, and time series dimension, specified by the t subscript. The use of panel data enables the paper to fully benefit from the rich structure of the available data, with more accurate estimations of parameters as a result. The analysis is based on the following countries:

Australia	**France**	**Netherlands**	**Sweden**
Austria	**Germany**	**New Zealand**	**United Kingdom**
Belgium	**Greece**	**Norway**	**United States**
Canada	**Ireland**	**Portugal**	
Denmark	**Italy**	**Spain**	
Finland	**Japan**	**Switzerland**	

The examined 21 industrial countries are similar to the classification of Kose et al. (2006);(Appendix E). The sample is restricted to industrial countries, as these countries have relatively similar debt-to-GDP levels, economic structures, political systems and their debt ratios were most severely affected by the recent financial crisis. The relation is estimated for yearly data from the 1990-2014 period. The scope of the study contains the most recent available data, which enables the paper to consider the latest developments. Moreover, the limited availability of data restricts the thesis in analysing longer time periods. Especially, the data on specific control variables is only present for the recent periods. Table 5.1 reports the applied symbols, descriptions and data sources of all included variables of (4.1).

Table 5.1: Abbreviations, descriptions, expected sign and sources of the included variables

Symbol[1]	Variable name	Description	Expected sign[2]	Data source[3]
ggdpc	Economic growth	Yearly growth rate of GDP per capita. Base year 2010 US dollars.	Dependent variable	WWDI
debt	Public Debt-to-GDP ratio	General government gross debt as ratio of GDP	+	WEO
$debt^2$	(Public Debt-to-GDP ratio)^2	Squared term of the debt variable	-	Own calculations
inv	Investments	Gross capital formation as percentage of GDP.	+	WWDI
pop	Population growth	Yearly population growth rate	-	WWDI
tfp	Total factor productivity	Measured relative to the United States.	+	PWT
hc	Human Capital	Human capital index based on years and returns of education	+	PWT
gov_size	Government size	Government consumption as ratio of GDP at current PPP's	-	PWT
gov_bal	Government budget balance	Public revenue minus expenditures, both as percentage of GDP	+	WEO
tr_open	Trade Openness	Imports of goods and services plus exports of goods and services, both as ratio of GDP	+	WWDI
tot	Terms of Trade	Ratio of index export and import prices. 2010 base year.	+	OECD
fin_dev	Financial Development	Ratio of liquid liabilities to GDP	+	WWDI
age	Age Composition	Part of the population with an age below 15 or above 64,	-	WWDI
rir	Real interest rate	rir= [(1+ nom_ir)/(1+ inf)]–1	-	Own calculations
nom_ir	Nominal interest rate	The interest rate of public bonds with 10-year maturity.	-	OECD
inf	Inflation	Yearly growth rate of changes in the price level of the total economy	-	WWDI
igdppc	Initial Real GDP per capita	Reflected by the GDP per capita of the first year of every five years.	-	WWDI

Notes: (1) abbreviations used in the text. (2) expected influence on economic growth. (3) WWDI reflects the Worldbank World Development Indicators; WEO stands for IMF World Economic Outlook database; PWT symbolizes the Penn World Tables data; OECD indicates data obtained from the organisation's website.

5.1 Descriptive Statistics

Table 5.2 includes the descriptive statistics of the core variables over time 1990-2016 period. The complete table is included in Appendix F. The mean, median, maximum, minimum, standard deviation and the number of observations are reported for all variables. The variables are mainly measured as a percentage of GDP. The exceptions are the GDP per capita, human capital index, total factor productivity as an index for technological advances, age and the real interest rate. The discussion of the descriptive statistics is restricted to the primary variables of interest. Namely, the GDP per capita growth and the debt-to-GDP ratios.

Table 5.2: Descriptive statistics

	Mean (1)	Median (2)	Maximum (3)	Minimum (4)	St.dev (5)	Observations (6)
ggdpc	1.40	1.54	24.67	-8.99	2.52	567
debt	69.51	60.81	242.11	9.68	38.96	545
igdppc	41520.03	38969.32	89619.49	16687.26	13123.94	567
gov_bal	-2.31	-2.59	31.22	-32.13	4.85	526
hc	3.17	3.24	3.73	1.94	0.37	525
gov_size	16.38	16.52	25.18	6.96	3.69	525
tr_open	71.88	63.69	216.67	16.01	36.09	563
age	50.40	50.07	65.73	43.02	3.63	567
pop	0.61	0.51	2.89	-1.85	0.51	566
inv	22.63	22.61	34.22	9.83	3.39	563
tot	98.34	100.00	150.95	10.27	13.98	561
fin_dev	86.11	76.50	231.62	37.83	37.12	526
rir	2.79	2.54	22.95	-26.23	3.30	553
tfp	0.91	0.89	1.62	0.53	0.16	527

Notes: (1) values are rounded to two digits.

Firstly, it is seen that the lowest economic growth of the sample is rate minus 8.99%. This minimum value of GDP per capita is found in Greece in 2011. In 2011, financial markets demand higher interest rates, severe austerity policies are introduced and Greece receives a 109 billion bailout to maintain a sustainable debt and prevent default (Allen et al., 2011). Table 5.2 reports that the maximum GDP per capita growth is 24.67%. In light of the mean value, an economic growth of 24.67% is a seemingly odd finding. However, this observation is explained by the fact that numerous non-Irish firms mainly moved to Ireland in 2015 to benefit from a better tax environment (Inman, 2016). Secondly, the descriptive statistics of the debt-to-GDP make clear that the ratios differ substantially per period and country. The average debt ratio is 69.51% while the highest debt ratio is found for Japan in 2014 at

242.11%. The minimum debt ratio of 9.68% belongs to Australia in 2007. Further understanding of the data is obtained by reporting the graphical evolution of the debt ratio from 1990 to 2016:

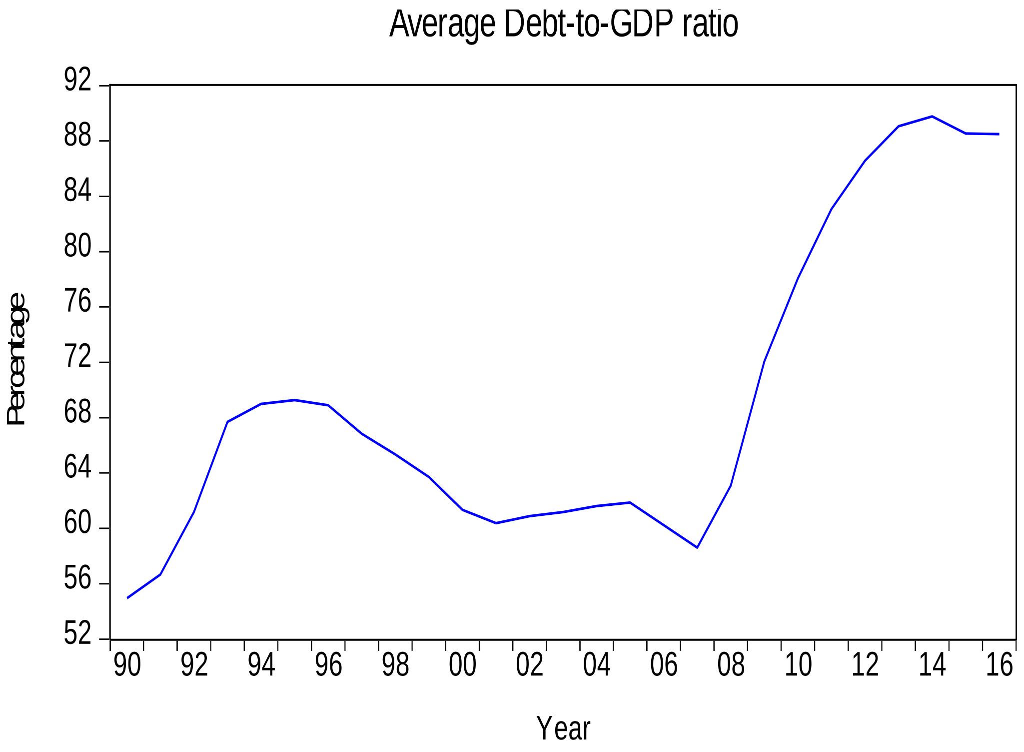

Figure 5.1: Development in the average debt-to-GDP ratio over time

Notes: (1) the average debt ratio is based on the 21 industrial countries (Appendix E)

The most striking observation of Figure 5.1 is the severe increase of the public debt ratios over time. In 1990, the average debt ratio was 54.95%, which increased to 88.50% in 2016. The figure indicates that debt ratios especially rose during the occurrence of the financial crisis of 2008. Before the crisis, the average debt ratio was in 2007 58.61%, while after the economic turmoil of the crisis the average debt ratio has increased to the current levels of 88.50%. Research shows that the issuance of public debt by the Eurozone countries approximately increased to twice its size relative to the pre-crisis period (De Broeck and Guscina, 2011).

The sample size of 1990-2016 includes a total number of 567 observations. However, the sixth column of Tablle 5.2 points out that most variables have missing observations. Most of the missing data can be accounted for by the fact that some data is extracted from the *Penn World Table*, as that database is limited to observation till 2014. Therefore, each series is characterised by 42 missing observations. In general, the missing observations are the consequence from either missing historical data on the earliest sample years or the fact that not all data is available for most recent years. These missing observations are random and non-systematic, as they are not related to certain countries, which prevents the omitted observations from biasing the results.

Correlation matrix

The correlation matrix reported in Appendix G shows that public debt is negatively correlated with GDP per capita growth. The found correlation of -0.16 implies that an increase in public debt diminishes economic growth, which is in line with the literature. Intuitively, debt is highly and positively correlated with debt^2, which is confirmed by the matrix. In general, the obtained correlation matrix indicates that when public debt increases, economic growth decreases. Mainly all the control variables have the expected correlations. Trade openness, technological innovations, population growth, investments and the government balance are positively correlated with economic growth. In line with the expectation, the real interest rate, government size, initial GDP per capita and the age composition are negatively correlated with economic growth. However, the finding that human capital, terms of trade and financial development are negatively correlated with economic growth is in contrast to the literature.

Scatterplot

The scatterplot of Figure 5.2 confirms the found negative correlation, as the figure shows a slightly negative linear relationship between the two variables. The OLS regression shows a slope of -0.008, which implies that a 10 % increase in public debt ratio results in a 0.08% reduction in economic growth. Once more, this suggests that a higher debt ratio is associated with a lower economic growth. The right-hand side panel of Figure 5.2 indicates that the relation is presumably more negative when only the countries that had a debt-to-GDP ratio of, at least, 100% during the 1990-2016 period are taken into account.

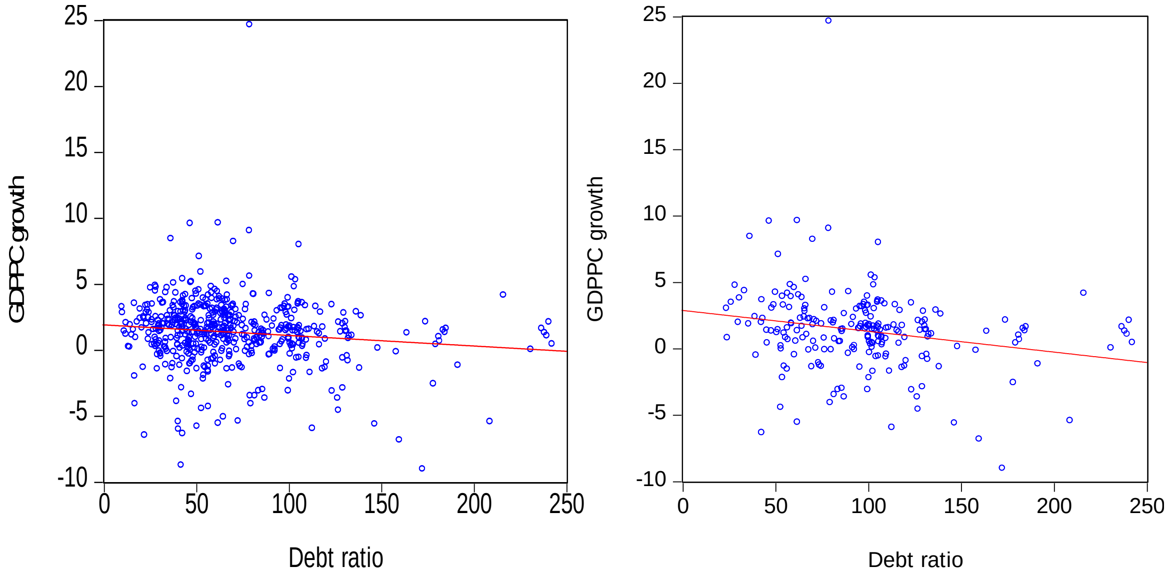

Figure 5.2: Scatterplot of the debt-to-GDP ratio and GDP per capita growth

Notes: (1) Left-hand side is based on the 21 industrial countries. (2) Right-hand side is based on the countries who had a ratio above 100% during 1990-2016.

The found negative relation serves only as preliminary evidence, as the presence of endogeneity issues biases the results. Thereby, causal claims on the exact debt-growth relation cannot be made. For instance, the scatterplot does not include control variables or does not take reverse causality or non-linearity into account. The identification strategy reported in chapter 4 should present evidence that is more conclusive on the matter.

5.2 Endogenous Variables

The suspicion that public debt is an endogenous variable is further examined by the *Hausman endogeneity test*, which is a sequence of regressions. First, the potentially endogenous variable is regressed on all exogenous variables, from which the residuals are obtained. Secondly, (4.1) is estimated, while the obtained residuals of the previous step are also included.

The *Hausman Test* provides evidence for the presence of endogeneity issues in the public debt variable, as the coefficient of the residuals is significant at the 10% significance level (Appendix H). Although the residuals are only significant at the 10% level, it can safely be assumed that debt is endogenous because the relevant literature further substantiates this assumption (Kumar and Woo, 2010);(Panizza and Presbitero, 2012). Therefore, it is correct to estimate the debt-growth relation by an IV approach.

Table 5.3: Results of the Hausman endogeneity test

Dependent variable is real GDP per capita growth	
Variable	(1)
debt	0.053*** (0.00)
Residuals	-0.041* (0.08)
Controls[1]	Yes
Time fixed effects:	Yes
Country Fixed effects:	Yes

Notes: (1) Included controls: tr_open, tot, tfp, rir, pop, inv, igdppc, hc, gov_size, gov_bal, fin_dev, age. (2) *,** and *** specify significance at 10%, 5% and 1% level. (3) The coefficients and probabilities, shown in parentheses, are round to three decimals.

5.3 Correctness of the Instrument

The outcomes of the Hausman endogeneity test provide evidence supporting the need for an instrument. However, the accuracy of the introduced instrument and the found results depend on the legitimacy of the instrument. Firstly, the instrument should be related to the endogenous variable. Secondly, the instrumental variable can only influence the dependent variable through its effect on the independent variable and is therefore uncorrelated with the error term. The second assumption cannot be verified, as the error term is unobservable. However, whether the first assumption holds is examined by regressing the endogenous variable on the instrument.

Table 5.4: First stage of the instrument

	Dependent variable is endogenous debt		
Variable	(1)	(2)	(3)
avg_debt_{it}[1]	0.373** (0.01)	-16.73*** (0.00)	-7.20*** (0.00)
Wald F-statistic	6.178** (0.01)	256.79*** (0.00)	535.92*** (0.00)
Controls	No	No	Yes
Time fixed effects:	No	Yes	Yes
Country Fixed effects:	No	Yes	Yes

Notes: (1) symbolizes the applied instrument for $debt_{it}$ (2) The controls are tot, tr_open, pop, hc, age, tfp, rir, inv, gov_size, gov_bal, fin_dev and igdppc. (3) *,** and *** specify significance at 10%, 5% and 1% level. (4) The coefficients and probabilities, shown in parentheses, are round to three decimals.

The result of Table 5.4 indicates that the introduced instrument is strongly correlated with the endogenous public debt variable in all specifications. Additionally, the best-specified equations reported in column (2) and (3) pass the rule of thumb of having an F-statistic higher than 10. Overall, the outcomes reported in Table 5.4 indicate that the average public debt ratio of the remaining countries is a valid instrument for the endogenous public debt variable.

5.4 Granger Causality

Subsequently, the Granger causality test is conducted, as the test helps in assessing whether the causality of the debt-growth relation runs in one or two ways. This result is obtained by estimating how the lagged values of the influencing variables are correlated with the future values of the influenced variable. The condition that both economic growth and public debt are stationary is met, as the Augmented Dickey-Fuller test, shown in Table 5.5, indicates that the unit root null hypothesis is rejected at a 5% significance level. Hereby, preventing the presence of non-stationary from biasing the results (Foresti, 2006).

The reported test results of Table 5.6 partly support the existence of bi-directional causality between economic growth and public debt. As the estimation results depend on the number of selected lags, the outcome of the test also differs per the number of selected lags. The test outcome reports that with 2,4 and 6 lags the causality runs in both directions. However, when more lags are selected, which is preferred as more information is incorporated, the results show that only $debt_{it}$ granger causes economic growth and not the other way around. Therefore, there is more conclusive evidence of uni-directional causality running from public debt to economic growth.

However, this finding should not be interpreted as proof of public debt causally influencing economic growth. The preformed Granger causality test only provides evidence that public debt precedence economic growth (Eviews, 2017). The true causal debt-growth relationship is found by the estimating the baseline equation presented in the methodological framework.

Table 5.5: Output results of the stationary test

Null hypotheses (1)	F-value (probability) (2)	Decision (3)
H0: ggdpc has a unit root	173.738*** (0.00)	Reject
H0: debt has a unit root	61.422*** (0.03)	Reject

Notes: (1) output based on ADF Fisher chi-square test. (2) Complete results in Appendix I. (3) *,** and *** specify significance at 10%, 5% and 1% level.

Table 5.6: Results of the Granger causality test on the direction of causation

Direction if causality (1)	Number of lags (2)	Probability (3)	Decision (4)
debt→ggdpc	2	0.06*	Reject
ggdpc → debt	2	0.00***	Reject
debt→ ggdpc	4	0.02**	Reject
ggdpc → debt	4	0.00***	Reject
debt→ ggdpc	6	0.00***	Reject
ggdpc → debt	6	0.26	Do not reject
debt→ ggdpc	8	0.00***	Reject
ggdpc → debt	8	0.52	Do not reject
debt→ ggdpc	10	0.00***	Reject
ggdpc → debt	10	0.01**	Do not reject

Notes: (1) debt→ggdpc symbolizes the H0: debt does not granger causes GDP per capita growth. (2) ggdpc→debt reflects the H0: GDP per capita growth does not granger cause debt. (3) *,** and *** specify significance at 10%, 5% and 1% level.

6. Results

This chapter reports the results of the estimated relationship between public debt and economic growth. The found results enable the paper to determine the optimal debt-to-GDP ratio in terms of economic growth. More specifically, the optimal ratio is found by determining for which debt-to-GDP ratio the highest GDP per capita growth is obtained, which is done by optimising the acquired debt and debt^2 coefficients. Where Figure 5.2 already hinted towards a negative debt-growth relationship, more conclusive and causal evidence is provided by the results of estimating (4.1). First, the, presumably biased, results of the OLS estimation are presented. Secondly, the results of the preferred *instrumental variable* estimation are reported.

Next, additional insight on the debt-growth relationship is obtained by estimating the influence of public debt on the real interest rate. Thereby, examining a potential channel through which public debt influences economic growth and further investigating the crucial role of the interest rate in a countries debt dynamics. Finally, numerous robustness checks are performed to determine whether the found results are robust to different specifications, such as interaction terms, and the used country sample.

6.1 Ordinary Least Squares estimates

The results of estimating (4.1) by OLS are shown in Table 6.1. The first column reports the OLS estimation of the specification with no included control variables. The specification of the second column includes variables that control for the beneficial character of trade and initial GDP per capita. In the third column, variables are included to control for the demographic and technological development of a country: population growth, human capital, age and technological advances. The fourth column contains estimates of the baseline (4.1) when including all control variables. The results of the *redundant fixed effects* test indicate that fixed effects should be preferred over random effects and that both forms of FE should be included in the specification (Appendix J).

The OLS estimates show that neither public debt nor public debt^2 significantly influences economic growth in three out of the four specifications. Although the OLS results report an insignificant relationship between economic growth and public debt, additional research is required as the OLS estimate could be insignificant due to endogeneity bias. The control variables, except terms of trade, human capital and financial development, do have the appropriate signs and significance according to previous studies. Further, the R^2 indicates that the estimated OLS models potentially lack explanatory power, as the models do not explain more than 30% of the variance of GDP per capita growth.

Table 6.1: Relation between economic growth and public debt estimated by OLS

	Dependent variable is real GDP per capita growth			
Variable	**(1)**	**(2)**	**(3)**	**(4)**
Constant	1.203** (0.02)	-0.447 (0.827)	19.147*** (0.00)	18.458** (0.01)
$debt_{it}$	**0.005** **(0.63)**	**-0.006** **(0.59)**	**-0.013** **(0.25)**	**0.026**** **(0.02)**
$debt^2_{it}$	**-0.00031** **(0.47)**	**0.000011** **(0.82)**	**-0.000011** **(0.82)**	**-0.000074** **(0.12)**
tot_{it}		0.012 (0.33)	-0.0216* (0.08)	-0.032*** (0.00)
tr_open_{it}		0.059*** (0.00)	0.034*** (0.00)	0.043*** (0.00)
$igdppc_{it}$		0.00008** (0.02)	-0.0004*** (0.00)	-0.0004*** (0.00)
pop_{it}			-0.104 (0.67)	-0.991*** (0.00)
hc_{it}			-2.86* (0.08)	-2.678 (0.11)
age_{it}			-0.038 (0.485)	-0.019 (0.724)
tfp_{it}			10.205*** (0.00)	6.157*** (0.00)
rir_{it}				-0.068*** (0.00)
inv_{it}				0.330*** (0.00)
gov_size_{it}				-0.278*** (0.00)
gov_bal_{it}				0.05** (0.04)
fin_dev_{it}				-0.009 (0.14)
Time fixed effects:	Yes	Yes	Yes	Yes
Country fixed effects:	Yes	Yes	Yes	Yes
R^2	0.495	0.527	0.640	0.731
F-statistic (probability)	10.147 (0.00)	10.62 (0.00)	15.07 (0.00)	19.20 (0.00)
Optimal debt-to-GDP ratio[3]	-	-	-	-

Notes: (1) *,** and *** specify significance at 10%, 5% and 1% level. (2) the coefficients and probabilities, shown in parentheses, are round to three decimals. (3) the optimum is in terms of economic growth.

6.2 Two-Stage Least Squares Estimates

The reported results of Table 6.1 are both counterintuitive and in contradiction to the outlined literature, as no conclusive evidence of public debt affecting economic growth is found. The contradictions are arguably caused by the simple strategy of estimating (4.1) by OLS, which results in biased and non-causal estimates. As indicated by the previous chapter, the bias is eliminated by estimating (4.1) by an instrumental variable approach. The results of the IV estimation are included in Table 6.2.

Table 6.2: Relation between economic growth and public debt estimated by instrumental variables

Dependent variable is real GDP per capita growth					
Variable	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Constant	0.072 (0.94)	-1.08** (0.63)	20.66*** (0.00)	21.78*** (0.00)	-2.349 (0.72)
$debt_{it}$	**0.03** **(0.21)**	**0.062** **(0.02)**	**0.039*** **(0.07)**	**0.097*** **(0.00)**	**0.111***** **(0.00)**
$debt^2_{it}$	**-0.00013** **(0.19)**	**-0.00028**** **(0.01)**	**-0.00023**** **(0.02)**	**-0.00037***** **(0.00)**	**-0.00044***** **(0.00)**
tot_{it}		-0.013 (0.42)	-0.035** (0.02)	-0.057*** (0.00)	-0.053*** (0.00)
tr_open_{it}		0.042*** (0.00)	0.023* (0.08)	0.028*** (0.00)	0.028** (0.00)
$igdpdpc_{it}$		0.00004 (0.24)	-0.0003*** (0.00)	-0.0003*** (0.00)	-0.0003*** (0.00)
pop_{it}			0.207 (0.45)	-0.767*** (0.00)	-0.820*** (0.00)
hc_{it}			-5.106*** (0.00)	-6.279*** (0.00)	
age_{it}			0.027 (0.68)	0.076 (0.26)	0.105 (0.160)
tfp_{it}			10.147*** (0.00)	5.609*** (0.00)	6.835*** (0.00)
rir_{it}				-0.055** (0.03)	-0.046* (0.09)
inv_{it}				0.363*** (0.00)	0.398*** (0.00)
gov_size_{it}				-0.258*** (0.00)	-0.221** (0.01)
gov_bal_{it}				0.109*** (0.00)	0.108*** (0.00)
fin_dev_{it}				-0.009 (0.19)	-0.01 (0.14)
Time fixed effects:	Yes	Yes	Yes	Yes	Yes
Country fixed effects:	Yes	Yes	Yes	Yes	Yes
R^2	0.495	0.489	0.623	0.703	0.676
F-statistic (probability)	10.147 (0.00)	10.87 (0.00)	15.00 (0.00)	19.93 (0.00)	20.30 (0.00)
Optimal debt-to-GDP ratio[6]	-	110.92[1]	88.48[2]	129.55[3]	125.91[4]

*Notes: (1) 3.44% is the found growth rate at the growth maximising debt-to-GDP ratio. (2) similarly, the maximum rate is 1.77%. (3) 6.28% is the maximum rate for the fourth specification. (4) the maximum obtainable rate is 7%. (5) *,*** and *** specify significance at 10%, 5% and 1% level. (5) the coefficients and probabilities, shown in parentheses, are round to three decimals. (6) the optimum is in terms of economic growth.

Debt-growth relationship

The reported results in Table 6.2 report distinctively different results in comparison to the estimation by OLS. Where debt and $debt^2$ were previously to be found insignificant, the IV results show that both debt and $debt^2$ are significant in all specifications, except in the first column. Moreover, the found coefficients confirm that the debt-growth relationship is a non-linear concave relationship, as the coefficient of debt is positive and the sign of debt squared is negative in all specifications. The estimated significant coefficients of public debt vary from 0.03 to 0.111, while the point estimate of $debt^2$ lies within a range from -0.00044 to -0.00013 %. The found debt point estimates confirm the concave debt-growth relationship. Such a finding implies that low debt-to-GDP ratios contribute to economic growth, up to the point that the debt ratios pass a certain threshold for which the adverse effects of public debt[2] are larger than the initial positive effects of debt.

Optimal debt-to-GDP ratio

The location of these thresholds, for which further debt accumulations causes economic growth to diminish, are found by maximising the retrieved debt-growth relationship. For example, the results indicate the following debt-growth relationship for the specification in column (4) of Table 6.2:

$$g_{it} = 0.0969 * DEBT_{it} - 0.000374 * DEBT_{it}^2 \quad (6.1)$$

Maximising (6.1) results in the location of the optimal public debt ratio in terms of economic growth. The same procedure is followed for the remaining specifications, the outcomes are included in the last row of Table 6.2. The reported results show that the optimal debt-to-GDP ratio differs per specification, as the thresholds lie within the range of 88% to 129% of GDP. The location of the growth-maximising debt ratio and the curvature of the debt-growth relation becomes more intuitive when the obtained relation is plotted for different debt-to-GDP ratios.

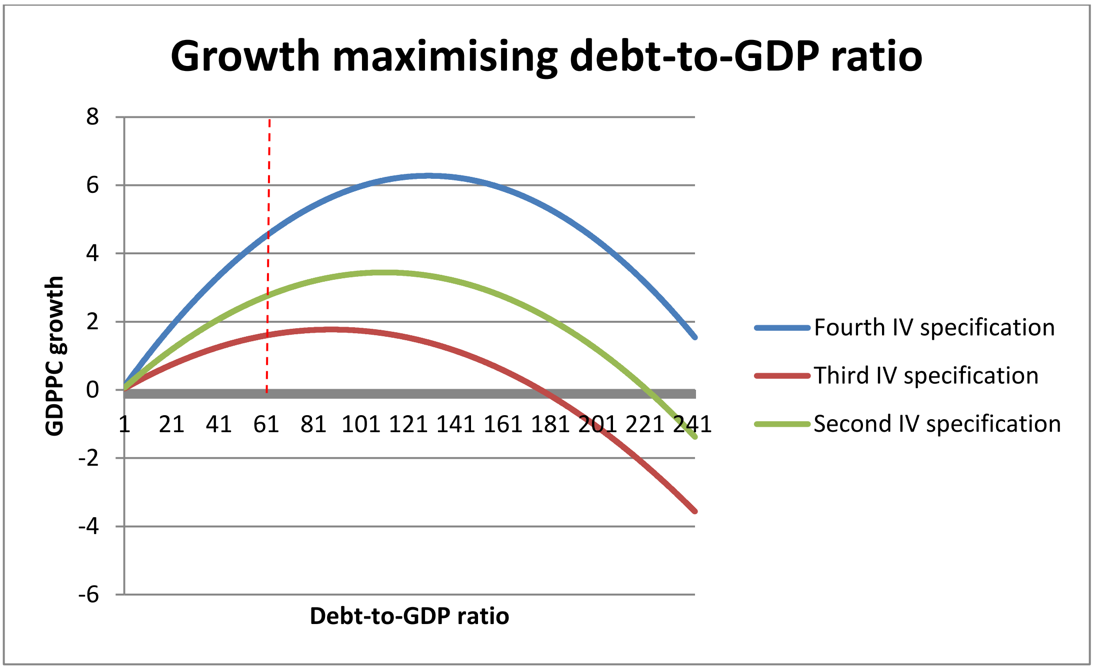

Figure 6.1: Optimal debt-to-GDP ratio in terms of GDPPC maximisation
Notes: (1) where the dotted line indicates the 60% SGP debt ceiling.

Figure 6.1 confirms that the debt-growth relationship is characterised by a concave curve and that issuing debt beyond the optimal debt values is harmful to growth. More specifically, the maximised economic growth rates are found to be 3.44% for the second specification, 1.77% for the third specification and 6.28% for the fourth specification. The maximum economic growth rate of the fourth IV specification, the most encompassing specification is relatively similar to the main finding of Checherita and Rother (2010), who find a maximum rate of 6%.

The estimated signs of the coefficients are similar to the previously discussed studies, as the characterising non-linearity and growth-maximising ratios are found. However, the determined optimal ratio does differ from the related literature. For example, the studies of Reinhart and Rogof (2010), Cecchetti et al. (2011) and Checherita and Rother (2010) find estimates within the range of 85% to 100%. The finding of different results is reasonable as the examined sample and timeframe are distinctively different, which influences the found regression estimates. Additionally, the findings of both the table and figure indicate that the 60% SGP debt ceiling is too strict in terms of economic growth for all specifications.

The results shown in Table 6.2 also further emphasises the necessity of the applied IV approach. First, the validity of the instrument is strengthened as the found debt coefficients do not differ substantially among the IV specifications. Secondly, the findings of the IV estimation also support the fact that public debt is endogenous, as the OLS and IV point estimates differ substantially. Thirdly, the table reports that the IV models explain more variation in the dependent variable, as the R^2 is substantially larger.

The found coefficients of the control variables are mostly consistent with the literature, both in sign as in significance. Terms of trade, human capital, age and financial development are, however, not in line with the expectations of Table 5.1. The point estimates of the first two variables have the wrong sign, while the latter two are insignificant. However, in the last decade, numerous studies are unsuccessful in finding a significant relationship between education and economic growth or even find a negative effect of education on growth (de la Fuente and Domenech, 2006). Therefore, column (5) of Table 6.2 excludes *human capital* and reports that the found non-linear debt-growth relation is robust to the exclusion of *human capital.* Additionally, the results show that population growth is only significant and has the correct negative sign in the fourth and fifth specification, while *igdppc* coefficient is insignificant in the second specification. Moreover, the relatively large coefficients of *tfp* and *human capital* are explained by the fact that both are measured as an index instead of in terms of GDP as most variables.

6.3 Further Results

The conducted analysis, so far, neglected the potential channels through which government debt affects economic growth. This section, especially, focused on the real interest rate as both a channel and as an important determinant of the debt dynamics. Specifically, the relationship between the real interest rate and public debt ratio is unravelled by estimating a baseline equation proposed by Checherita and Rother (2010):

$$rir_{it} = \beta_0 + \beta_1 Debt_{it} + \beta_2 X_{it} + \mu_i + \varphi_t + \varepsilon_{it} \quad (6.2)$$

Where the subscript *i* reflects the country and the *t* the period. The *rir* reflects the real interest rate and functions as the dependent variable of the equation. The main independent variable of interest is public debt, which is incorporated by $debt_{it}$. Additionally, numerous control variables are included by the symbol X. For instance, the paper of Alper and Forni advocates controlling for financial development (2011), while the work of Ardagna et al. (2004) includes a control variable of GDP growth rate. All variables included in (6.2) are described in Appendix K. The baseline equation (6.2) is augmented with both country fixed effects and time fixed effects, as the *redundant fixed effects test* states that including both effects is required (Appendix L).

Results

The result of estimating regression (6.2) are reported in Table 6.3. The table shows that public debt significantly and positively affects the real interest rate in all three specifications and is robust to the inclusion of several control variables. The found point estimates of debt vary within a range from 0.0255 to 0.031, which implicates that a 10% increase in the debt-to-GDP ratio causes the real interest rate to increase with at least 0.25%. The finding that debt-to-GDP ratios positively affects the real interest rate has several implications. First, it implicates that an increase in the public-debt-ratio further destabilises a country public debt sustainability, as an increase in the real interest rate is associated with increasing debt dynamics. Secondly, it suggests that the real interest rate is a channel through which public debt influences the economic growth.

Table 6.3: Estimated effect of public debt-to-GDP ratio on the real interest rate

Dependent variable is real GDP per capita growth			
Variable	**(1)**	**(2)**	**(3)**
constant	0.838*	0.196	-8.84*
	(0.08)	(0.88)	(0.07)
$debt_{it}$	**0.028*** **	**0.026*** **	**0.031*** **
	(0.00)	**(0.00)**	**(0.00)**
ca_{it}		0.011	0.036
		(0.82)	(0.52)
gov_bal_{it}		-0.113**	-0.026
		(0.01)	(0.59)
tr_open_{it}		0.008	0.047**
		(0.64)	(0.02)
$reer_{it}$			0.049**
			(0.02)
$output\_gap_{it}$			-0.0007
			(0.75)
$gdpg_{it}$			-0.315***
			(0.00)
fin_dev_{it}			0.023*
			(0.08)
Time fixed effects:	Yes	Yes	Yes
Country fixed effects:	Yes	Yes	Yes
R^2	0.318	0.328	0.377
F-statistic (probability)	4.802 (0.00)	4.641 (0.00)	4.79 (0.00)

Notes: (1) *,** and *** specify significance at 10%, 5% and 1% level. (2) the coefficients and probabilities, shown in parentheses, are round to three decimals.

6.4 Robustness of the Results

The robustness of the found results is determined by performing a variety of robustness tests. In the upcoming tables and figures, evidence is shown that the results are robust to the inclusion of: (1) additional controls; (2) alternative dependent variable; (3) different polynomials of debt; (4) alternative samples. Further, the sample of countries is extended to emerging markets to increase the external validity of the findings. The first column of both Table 6.4 and 6.5 reports the estimates of the most-encompassing fourth IV specification to make the comparison of the results more convenient. The description and sources of the variables are reported in Appendix F.

Alternative (control) variables

In the second and third column of Table 6.4 additional control variables are incorporated in the baseline specification. The pattern of public debt accumulation during times of financial crisis suggested by Figure 2.1 and further substantiated by the paper of Reinhart (2009) advocates for controlling for financial crises. Therefore, the second column reports estimates when a financial crises dummy variable is included in the specification. The dummy variable equals unity if there was a currency-, inflation-, stock market-, sovereign debt- or banking crisis in the relevant year and country and zero otherwise.

Additionally, study has shown that debt sustainability, and therefore economic growth, could be at risk in societies with high private debt level. For instance, the public revenues a government can generate over time are potentially lower due to high levels of private debt (Cecchetti et al., 2011). To consider this matter, the specification of the third column controls for private debt. Finally, a third-degree polynomial form of public debt is taken into account. The fourth column reports that the results are robust, as the non-linear and concave relation persists and the debt threshold is similar to the optimum of column (1). Moreover, incorporating $debt^3$ to the baseline equation (4.1) results in no significant outcomes.

Table 6.4 reveals that the findings are robust to the inclusion of additional control variables and alternative functional forms. The reported point estimates and significance levels of the alternative specifications indicate that the conclusions are not altered, as the found optimal debt-to-GDP ratios are only barely affected. Additionally, the validity of the instrument is strengthened, as public debt has a similar influence on economic growth when controlled for additional variables.

Alternative dependent variable

Next, an alternative dependent variable is taken into consideration to verify the robustness of the found debt-growth relationship. Column (5) reports the results of the debt-growth relationship when a rolling window of the 5-year average of real GDP per capita functions as an alternative dependent variable. The alternative dependent variable incorporates the cyclical fluctuations of the economy and ensures that long-term effects of public debt on economic growth are taken into consideration (Afonso, 2014). The estimations, once more, verify the finding of a non-linear debt-growth relationship, as $debt_{it}$ and $debt_{it}^2$ have the appropriate signs. However, the point estimates, and consequently the optimal debt ratio does differ substantially when the alternative dependent variable is used. In the first three columns, the optimal debt ratio lies between 127% and 130%, while estimations based on the alternative dependent variable finds an optimal ratio of 86.02%. The reduction of the optimum, when using the 5-year average GDP growth, is in line with the work of Afonso (2014). However, the substantial difference between the optima is remarkable.

Alternative country samples

In the sixth column, the debt-growth relationship is estimated by excluding all non-European countries from the analysed sample, as it could be doubted that these countries are entirely comparable to the European nations. The main conclusion is that the found non-linear debt-growth relation is robust to the alternative country sample, as the column indicates that when Australia, Canada, Japan, New-Zealand and the United States are excluded the debt coefficients remain significant and have the appropriate signs. However, the found point estimates do differ. The optimal debt ratio is substantially lower for the alternative sample, which implies that the negative effects of public debt prevail for lower debt-to-GDP ratios.

Finally, (4.1) is estimated for an alternative sample of emerging market economies (Appendix E). However, the estimation is due to lack of reliable data based on a restricted number of control variables and a shortened sample from 2001 to 2014. Firstly, the results of column seven confirm the non-linear debt-growth relation for emerging markets, as both debt variables are significant and have the appropriate sign. Secondly, the estimates reveal that the optimal debt-to-GDP ratio for emerging markets is considerably lower, which is in line with the findings of Caner et al. (2010). The latter study finds an optimal debt-ratio of 77% for the full sample and 64% for the emerging market sample, which implies that the negative effect of public debt on economic growth takes place for lower ratios.

Interaction terms

In Table 6.5, additional robustness checks are conducted by determining whether the results are robust to the inclusion of interaction terms. The inclusion of the interaction terms aims at obtaining a better approximation of the effect of public debt on economic growth, as it is expected that the variables interact. Based on the work of Calderon and Fuentes (2013) and Ahlborn and Schweickert (2015) the baseline specification is augmented with the interaction between public debt with initial GDP per capita and financial openness. The table, additionally, reports the interaction between debt and a countries fiscal balance and the real interest rate.

The results of column (6) report that the debt and financial development interaction term is positive and significant at the 10% level, which implies an increase in public debt, enhances financial development, which boosts higher economic growth. On the contrary, the interaction term of public debt and initial GDP per capita is negative significant, which indicates that a higher public debt, the lower initial GDP per capita, and the lower the GDP per capita growth of a country. The remaining interaction terms are insignificant and, therefore, do not influence the dependent variable in a significant manner. The finding of the significant interaction terms indicates that financial development and initial GDP per capita influence the effect of public debt on economic growth. Therefore, the found point estimates of the interaction terms should be taken into account when determining the optimal debt-to-GDP ratio.

Figure 6.2 contains the development of the GDP per capita growth for different values of public debt, while the mean values of 86.11 and 41520 are assumed for financial development and initial GDP per capita. For instance, the effect of linear debt on economic growth for the (6) column specification, while taking the found significant interaction terms into account, equals 0.144152-0.000002*41520+0.0004*86.11= 0.103. Subsequently, the obtained debt-growth relationship is maximised in a similar way as (6.1):

$$g_{it} = 0.103 * DEBT_{it} - 0.0000491 * DEBT_{it}^2 \tag{6.3}$$

Figure 6.2 indicates that when the interaction terms are included the optimal debt ratio is found for a lower threshold value, as the curve is shifted downwards relative to the original specification. Namely, 105.07%. Additionally, it is seen that the debt ceiling of the SGP is still too strict as economic growth can be stimulated by issuing debt beyond the limit set by the treaty.

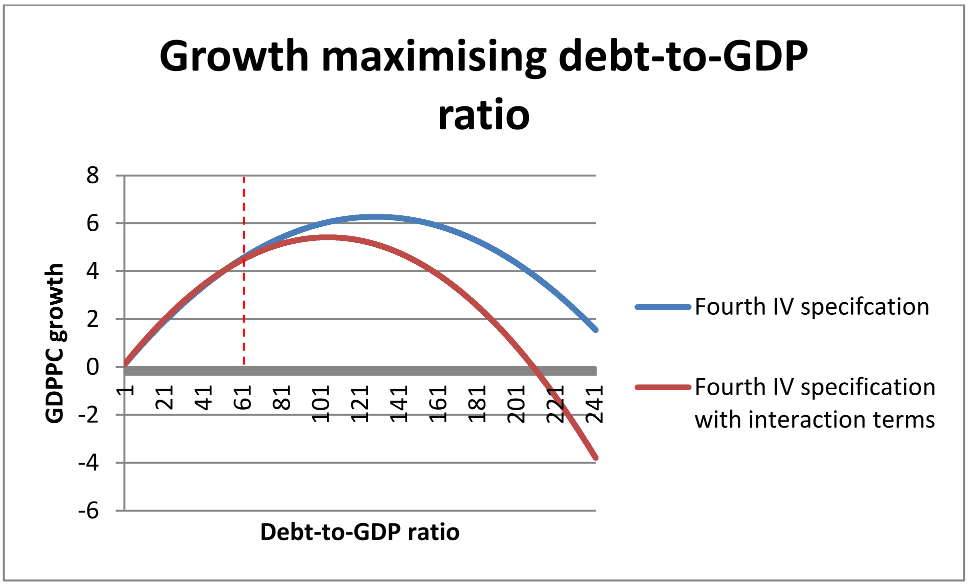

Figure 6.2: Optimal debt-to-GDP ratio in terms of economic growth

Notes: (1) where the dotted line indicates the 60% SGP debt ceiling.

Preliminary conclusion

In general, the preformed robustness checks reported in Table 6.4 and Table 6.5 confirm the concave non-linear relationship between public debt and economic growth, as the debt coefficient is positive and the squared public debt is associated with a negative point estimate. The relationship is robust to the inclusion of additional controls in the form of financial crisis dummy, and private debt, the exclusion of non-European countries, the alternative emerging market economies sample, a different dependent variable, the inclusion of interaction terms and the relation persists for a different debt polynomial. The results of the most extensive fourth IV specification indicates that the optimum is established at 129.55% while including interaction terms results in an optimum of 105.07%. Once more, it is found that the growth-maximising debt-to-GDP ratio lies beyond the 60% SGP debt ceiling of the Eurozone.

Table 6.4. Robustness checks

Variable	Dependent variable is real GDP per capita growth						
	(1)	(2)	(3)	(4)[4]	(5)	(6)	(7)
Constant	21.78*** (0.00)	21.06*** (0.00)	20.499** (0.01)	19.719** (0.01)	27.31*** (0.00)	5.55 (0.62)	19.73** (0.04)
$debti_t$	**0.097*** (0.00)**	**0.094*** (0.00)**	**0.094*** (0.00)**	**0.061*** (0.00)**	**0.035** (0.02)**	**0.123*** (0.00)**	**0.192** (0.03)**
$debt^2_{it}$	**-0.000374 (0.00)**	**-0.000369*** (0.00)**	**-0.0004*** (0.00)**	**-0.000001*** (0.00)**	**-0.00020*** (0.00)**	**-0.0008*** (0.00)**	**-0.0013** (0.01)**
tot_{it}	-0.057*** (0.00)	-0.055*** (0.00)	-0.056*** (0.00)	-0.057*** (0.00)	-0.059*** (0.00)	-0.046*** (0.00)	0.024*** (0.00)
tr_open_{it}	0.028** (0.04)	0.029** (0.03)	0.024* (0.08)	0.031** (0.02)	0.004 (0.58)	0.039*** (0.00)	0.005 (0.75)
$igdppci_t$	-0.0003*** (0.00)	-0.0003*** (0.00)	-0.0003*** (0.00)	-0.0003*** (0.00)	-0.00018*** (0.00)	-0.0003*** (0.00)	-0.0003 (0.25)
pop_{it}	-0.767*** (0.00)	-0.763*** (0.00)	-0.821*** (0.00)	-0.753*** (0.01)	0.061 (0.69)	-1.299*** (0.00)	-0.768* (0.09)
hc_{it}	-6.279*** (0.00)	-6.05*** (0.00)	-5.867** (0.01)	-5.93*** (0.00)	-7.91*** (0.00)	3.766 (0.18)	-3.21 (0.31)
age_{it}	0.076 (0.26)	0.075 (0.26)	0.082 (0.236)	0.098 (0.18)	0.019 (0.60)	-0.143** (0.03)	-0.157 (0.19)
tfp_{it}	5.609*** (0.00)	5.668*** (0.00)	5.357*** (0.00)	6.18*** (0.00)	6.691*** (0.00)	5.440*** (0.00)	-6.02** (0.04)
rir_{it}	-0.055** (0.03)	-0.054** (0.04)	-0.053** (0.04)	-0.058** (0.03)	-0.041*** (0.00)	-0.021 (0.52)	-
inv_{it}	0.363*** (0.00)	0.359*** (0.00)	0.363*** (0.00)	0.377*** (0.00)	0.219*** (0.00)	0.185*** (0.00)	0.255*** (0.00)
gov_size_{it}	-0.258*** (0.00)	-0.252*** (0.00)	-0.266*** (0.00)	-0.262*** (0.00)	0.024 (0.60)	-0.239** (0.01)	0.086 (0.24)
gov_bal_{it}	0.109*** (0.00)	0.106*** (0.00)	0.099*** (0.00)	0.109*** (0.00)	0.115*** (0.00)	0.160*** (0.00)	0.521*** (0.00)
fin_dev_{it}	-0.009 (0.198)	-0.007 (0.25)	-0.007 (0.319)	-0.006 (0.386)	0.002 (0.69)	-0.012 (0.10)	-0.07*** (0.00)
$crisis\_dum_{it}$	-	-0.220 (0.269)	-	-	-	-	-
$private\_debt_{it}$	-	-	-0.002 (0.645)	-	-	-	-
Time fixed effects:	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country fixed effects:	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.703	0.705	0.691	0.696	0.795	0.762	0.692
F-statistic (probability)	19.927 (0.00)	19.63 (0.00)	17.54 (0.00)	19.93 (0.00)	32.36 (0.00)	20.26 (0.00)	11.15 (0.00)
Optimal debt-to-GDP	129.55	127.83	128.83	139.71	86.02	76.08	71.11

Notes; (1) *,** and *** specify significance at 10%, 5% and 1% level. (2) the coefficients and probabilities, shown in parentheses, are round to three decimals. (3) the optimum is in terms of economic growth.(4) The $debt^2$ variable is replaced by $debt^3$ in the fourth specification.

Table 6.5: Robustness checks by augmenting the baseline equation with interaction terms

	Dependent variable is real GDP per capita growth					
Variable	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Constant	21.78*** (0.00)	27.326*** (0.00)	21.415** (0.00)	18.11** (0.02)	22.14*** (0.00)	23.26** (0.02)
$debt_{it}$	**0.097*** (0.00)**	**0.08*** (0.00)**	**0.098*** (0.00)**	**0.142*** (0.00)**	**0.099*** (0.00)**	**0.144*** (0.03)**
$debt^2_{it}$	**-0.000374 (0.00)**	**-0.0005*** (0.00)**	**-0.0004*** (0.00)**	**-0.0004*** (0.00)**	**-0.0004*** (0.00)**	**-0.0005*** (0.01)**
tot_{it}	-0.057*** (0.00)	-0.051*** (0.00)	-0.053*** (0.00)	-0.064*** (0.00)	-0.058*** (0.00)	-0.055*** (0.00)
tr_open_{it}	0.028** (0.04)	0.027** (0.05)	0.029** (0.03)	0.036** (0.01)	0.028** (0.04)	0.04*** (0.00)
$igdppc_{it}$	-0.0003*** (0.00)	-0.0003*** (0.00)	-0.0003*** (0.00)	-0.0002*** (0.00)	-0.0003*** (0.00)	-0.0003*** (0.00)
pop_{it}	-0.767*** (0.00)	-0.808*** (0.00)	-0.744*** (0.00)	-0.780*** (0.00)	-0.764*** (0.00)	-0.817* (0.09)
hc_{it}	-6.279*** (0.00)	-6.613*** (0.00)	-6.096** (0.01)	-6.373*** (0.00)	-6.482*** (0.00)	-6.618*** (0.00)
age_{it}	0.076 (0.26)	0.018 (0.77)	0.062 (0.37)	0.102 (0.14)	0.079 (0.25)	0.021 (0.73)
tfp_{it}	5.609*** (0.00)	6.542*** (0.00)	5.670*** (0.00)	5.512*** (0.00)	5.512*** (0.00)	6.824*** (0.00)
rir_{it}	-0.055** (0.03)	-0.05* (0.05)	0.002 (0.96)	-0.067** (0.01)	-0.055** (0.03)	0.0004 (0.99)
inv_{it}	0.363*** (0.00)	0.345*** (0.00)	0.350*** (0.00)	0.398*** (0.00)	0.363*** (0.00)	0.379*** (0.00)
gov_size_{it}	-0.258*** (0.00)	-0.239*** (0.00)	-0.243*** (0.00)	-0.287*** (0.00)	-0.253*** (0.00)	-0.262*** (0.00)
gov_bal_{it}	0.109*** (0.00)	0.109*** (0.00)	0.112*** (0.00)	0.099*** (0.00)	0.128** (0.02)	0.095* (0.07)
fin_dev_{it}	-0.009 (0.198)	-0.031* (0.05)	-0.009 (0.15)	0.005 (0.45)	-0.008 (0.25)	-0.036** (0.04)
$debt_t*fin\_dev_{it}$	-	0.0003 (0.12)	-	-	-	**0.0004* (0.06)**
$debt_t*rir_{it}$	-	-	-0.0007 (0.16)	-	-	-0.0008 (0.19)
$debt_t*igdppc_{it}$	-	-	-	-0.000001* (0.05)	-	**-0.000002*** (0.00)**
$debt_t*gov\_bal_{it}$	-	-	-	-	-0.0001 (0.65)	0.00003 (0.91)
Time fixed effects:	Yes	Yes	Yes	Yes	Yes	Yes
Country fixed effects:	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.703	0.704	0.705	0.704	0.702	0.709
F-statistic (probability)	19.927 (0.00)	19.546 (0.00)	19.83 (0.00)	19.54 (0.00)	19.55 (0.00)	18.75 (0.00)
Optimal debt-to-GDP ratio	129.55	81.23	132.94	130.52	128.63	105.07

Notes; (1) *,** and *** specify significance at 10%, 5% and 1% level; The coefficients and probabilities, shown in parentheses, are round to three decimals; the optimum is in terms of economic growth

7. Conclusion and Discussion

The substantial increase in the public debt-to-GDP ratios that took place in the recent years has resulted in an academic and political debate on the negative consequences on national economies. Especially of concern are the potentially negative effects on the economic growth rate when public debt ratios pass certain threshold values.

The paper has aimed to determine these threshold values. Thereby estimating the optimal debt-to-GDP ratios for which economic growth is maximised. The applied methodology is a regression framework based on several estimation methods for 21 industrial countries during the 1990-2014 period. The preferred technique is the IV estimation method, as it mitigates the endogeneity issues present in the debt-growth relation.

The obtained results of the numerically simulated debt dynamics indicated the determinates of public debt and it illustrates the crucial role of economic growth and the interest rate for the sustainability of a nation's debt.

The main empirical results provided evidence for the existence of a non-linear relationship between economic growth and public debt, as debt accumulation beyond the found optimal debt-to-GDP ratios negatively affects the growth rate. In the most encompassing equation specification, the growth-maximising- or optimal debt-to-GDP ratio is found at 129.55%. The policy implication of the obtained outcomes implies that while governments with national debt ratios beyond the found tipping point are sub-optimal and debt should therefore be reduced. On the contrary, governments with lower debt-to-GDP ratios have the opportunity to stimulate the economy by means of public borrowing up to the derived optimum of 129.55%. Conducted robustness checks result in relatively similar outcomes and optimal debt ratios, which strengthens the existence of the non-linear debt-growth relation. In each of the specifications the optimum is found for a higher ratio than the 60% SGP debt ceiling, which implies that this European Union policy is suboptimal in terms of economic growth.

The obtained optimal debt-to-GDP ratios are broadly consistent with the literature. However, in contrast to some previous findings, the growth-maximising ratio is found for a substantially higher percentage of GDP. A potential explanation for this result is the used database and analysed timeframe. The conducted research has some potential shortcomings and implications for future research. First, the paper only focused on the real interest rate as potential channel through which public debt affects the economic growth rate. The empirical estimation showed that the real interest rate negatively influences economic growth and is positively affected by the debt-to-GDP ratio. Future research should focus on alternative transmission channels that the current paper neglected, as it indicates through which channels economic growth is reduced. Secondly, augmenting the specification by interaction terms or by the alternative dependent variable results in an optimal debt ratio in the range of 86% to 105%. This interesting divergence between optima demands more thorough investigation to strengthen the presented outcomes. Thirdly, the conducted analyses potentially considers heterogeneity in a insufficient manner and neglects that the debt-growth relation could differ over time. Inadequately incorporating this heterogeneity between countries could result in biased estimates (Panizza and Presbitero, 2012). For instance, the robustness checks already indicated that the found optimal debt-to-GDP ratio differs per analyzed country set. This raises the question in how far the optimal debt ratio can be generalized for a sample of countries that differ in many aspects. For example, the current analyses disregards the composition of debt, such as the proportion of external debt, but also the trajectory of debt, whether debt is on an increasing or decreasing path. An alternative suggestion for future research is to estimate the debt-growth relationship with the use of different and potentially more complex constructed instrument variables. Despite the fact that the introduced instrumental variables is valid, it is relatively simplistic. In principle, the found point estimates and optima of Table 6.2 should not change. Finally, the current study has approached the issue of the optimal debt-to-GDP ratio in terms of maximised economic growth. However, the found optimum could differ when the matter is assessed from a different perspective, such as in terms of welfare. Follow-up research is needed to resolve these remaining questions and solidify the presented non-linear debt-growth relation and the resulting growth-maximising debt-to-GDP ratios.

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9. Appendix

Appendix A

Dividing (2.2) by the GDP of the relevant period gives:

$$\frac{D_t}{Y_t} = (1+r) * \frac{D_{t-1}}{Y_t} + \frac{G_t - \tau_t}{Y_t} \tag{A.1}$$

$\frac{D_{t-1}}{Y_t}$ is rewritten as, $\frac{D_t}{Y_{t-1}} * \frac{Y_{t-1}}{Y_t}$:

$$\frac{D_t}{Y_t} = (1+r) * \frac{D_{t-1}}{Y_{t-1}} * \frac{Y_{t-1}}{Y_t} + \frac{G_t - \tau_t}{Y_t} \tag{A.2}$$

Assuming a constant economic growth rate (Blanchard and Johnson, 2013) implies that $\frac{Y_{t-1}}{Y_t}$ can be replaced by $\frac{1}{(1+g)}$:

$$\frac{D_t}{Y_t} = \frac{(1+r)}{(1+g)} * \frac{D_{t-1}}{Y_{t-1}} + \frac{G_t - \tau_t}{Y_t} \tag{A.3}$$

Subsequently $\frac{(1+r)}{(1+g)}$ is rewritten as $1 + \frac{i-\gamma}{1+\gamma}$ (Escolano, 2010):

$$\frac{D_t}{Y_t} = (1 + \frac{i-\gamma}{1+\gamma}) * \frac{D_{t-1}}{Y_{t-1}} + \frac{G_t - \tau_t}{Y_t} \tag{A.4}$$

Equation (A.4) is equal to:

$$\frac{D_t}{Y_t} - \frac{D_{t-1}}{Y_{t-1}} = \frac{i-\gamma}{1+\gamma} * \frac{D_{t-1}}{Y_{t-1}} + \frac{G_t - \tau_t}{Y_t} \tag{A.5}$$

Equation (A.5) is similar to:

$$\frac{D_t}{Y_t} - \frac{D_{t-1}}{Y_{t-1}} = \frac{i}{1+\gamma} * \frac{D_{t-1}}{Y_{t-1}} - \frac{\gamma}{1+\gamma} * \frac{D_{t-1}}{Y_{t-1}} + \frac{G_t - \tau_t}{Y_t} \tag{A.6}$$

Subsequently, $\frac{\gamma}{1+\gamma}$ is replaced by $\frac{(1+\gamma)-1}{1+\gamma}$ which is equal to (Escolano, 2010):

$$\frac{(1+g)*(1+\pi)-1}{1+\gamma} = \frac{\pi+g+g\pi}{1+\gamma} = \frac{\pi+g(1+\pi)}{1+\gamma} = \frac{\pi}{1+\gamma} + \frac{g}{1+g} \tag{A.7}$$

Substituting (A.7) into (A.6) results in the following:

$$\frac{D_t}{Y_t} - \frac{D_{t-1}}{Y_{t-1}} = \frac{i_t}{1+\gamma_t} * \frac{D_{t-1}}{Y_{t-1}} - \frac{\pi_t}{1+\gamma_t} * \frac{D_{t-1}}{Y_{t-1}} - \frac{g_t}{1+g_t} * \frac{D_{t-1}}{Y_{t-1}} + \frac{G_t - \tau_t}{Y_t} \tag{A.8}$$

Subsequently, $\frac{i_t - \pi_t}{1+\gamma_t}$ is rewritten:

$$\frac{\{(1+r)(1+\pi)-1]-\pi}{1+\gamma} = \frac{r+\pi+r\pi-\pi}{1+\gamma} = \frac{r(1+\pi)}{(1+g)(1+\pi)} = \frac{r}{1+g} \tag{A.9}$$

Using (A.9) into (A.8), results in the following, and desired equation (2.3):

$$d_t - d_{t-1} = \frac{r_t - g_t}{1+g_t} * d_{t-1} + \frac{G_t - \tau_t}{Y_{t-1}} \tag{2.3}$$

where d_t is equal to $\frac{D_t}{Y_t}$

Appendix B

Initial governmental spending can be financed by initial tax revenue (τ_o) and initial debt financing (b_0), the two sources of governmental income. Therefore, total public spending must equal the sum of both financing sources:

$$G_0 = b_0 + \tau_o \tag{A.10}$$

Governmental spending for future time period *t* is a function of initial public spending plus a periodic increase in governmental expenditure, which is symbolized by A_t:

$$G_t = G_0 + A_t \tag{A.11}$$

The yearly public spending is the result of an increase in public means due to a potential increase in tax revenue due to economic growth. As a result, A_t depends on economic growth and, consequently, tax income:

$$A_t = \tau_t f(\frac{\tau_t}{Y_t}) \tag{A.12}$$

Which implies that A_t and, as a resultG_t, increases in tax revenue and the function$(\frac{\tau_t}{Y_t})$. According to the framework presented in De Luca(2012), $\alpha \frac{\tau_t}{Y_t}$ potentially represents $f\left(\frac{\tau_t}{Y_t}\right)$. Therefore, equation (A.3) becomes:

$$A_t = \alpha \frac{\tau_t^2}{Y_t} \tag{A.13}$$

This implies that A_t linearly depends on the taxation as a ratio of a countries economy. Consequently, an increase in total tax revenue causes governmental spending to increase through an increase in A_t.

Subsequently, (A.11) and (A.13) enables the paper to replace G_t in the budgetary balance (2.2). Consequently, pb_k can be described by the following equation:

$$pb_k = \alpha \frac{\tau_k^2}{Y_k} + G_0 - \tau_k \tag{A.14}$$

Instead of $pb_k = G_t - \tau_t$. Equation (2.4) now becomes the following:

$$D_t = (1+r)^t[D_0 + \sum_{k=1}^{t} \frac{\alpha\frac{\tau_k^2}{Y_k} + G_0 - \tau_k}{(1+r)^k}] \tag{A.15}$$

To adequately describe a countries debt dynamics, economic growth is included by the following equations:

$$Y_k = (1+g)^k Y_0 \tag{A.16}$$

$$\tau_k = (1+g)^k \tau_0 \tag{A.17}$$

With the help of (A.16) and (A.17), (A.15) can be rewritten into (2.5). First, the second component of A.15 can be rewritten in the following manner:

$$\sum_{k=1}^{t} \frac{\alpha\frac{\tau_k^2}{Y_k} + G_0 - \tau_k}{(1+r)^k} = G_0 * \sum_{k=1}^{t} \frac{1}{(1+r)^k} - \tau_0 * \left(\frac{1+r}{1+g}\right)^k + \frac{\alpha\tau_0^2}{Y_0} \sum_{k=1}^{n} \left(\frac{1+g}{1+r}\right)^k \tag{A.18}$$

Which can be simplified into the following:

$$G_0 * \sum_{k=1}^{t} \frac{1}{(1+r)^k} - \tau_0 * (1 - \frac{\alpha\tau_0}{Y_0}) \sum_{k=1}^{t} \left(\frac{1+g}{1+r}\right)^k \tag{A.19}$$

With the help of the mathematical rule presented in (A.20), (A.21) is obtained.

$$\sum_{k=1}^{t} x^k = x(\frac{1-x^t}{1-x}) \tag{A.20}$$

$$\sum_{k=1}^{t} \frac{\alpha\frac{\tau_k^2}{Y_k} + G_0 - \tau_k}{(1+r)^k} = \frac{G_0}{r} * \left(1 - \frac{1}{(1+r)^n}\right) - \tau_0 * \left(1 - \frac{\alpha\tau_0}{Y_0}\right) * \frac{1+g}{r-g} * [1 - \left(\frac{1+g}{1+r}\right)^t] \tag{A.21}$$

Insert (A.21) into (A.15) to obtain the following:

$$D_t = (1+r)^t * D_0 + (1+r)^t * [\frac{G_0}{r}\left(1 - \frac{1}{(1+r)^t}\right) - \tau_0 * \left(1 - \frac{\alpha\tau_0}{Y_0}\right) * \frac{1+g}{r-g} * (1 - \left(\frac{1+g}{1+r}\right)^n) \tag{A.22}$$

Which can be rewritten into:

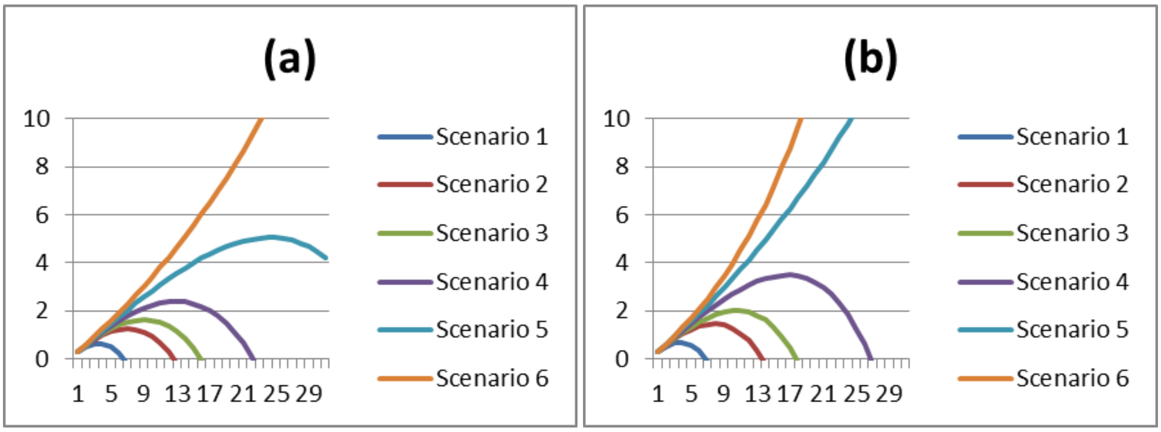
$$D_t = (1+r)^t * D_0 + (1+r)^t * \frac{G_0}{r} - (1+r)^t * \frac{G_0}{r} * \frac{1}{(1+r)^t} + \left[\tau_0\gamma_\alpha * \frac{1+g}{r-g} * (1+r)^t\right] - \tau_0\gamma_\alpha * \frac{1+g}{r-g} * \left(\frac{1+g}{1+r}\right)^t * (1+g)^t \tag{A.23}$$

Equation A.23 can be simplified into the desired equation (2.5):

$$D_t = (1+r)^t * D_0 + \frac{G_0}{r} * [(1+r)^t - 1] - \tau_0\gamma_\alpha * \left(\frac{1+g}{r-g}\right) * [(1+r)^t - (1+g)^t] \tag{2.5}$$

Where $\gamma_\alpha = (1 - \frac{\alpha\tau_0}{Y_0})$.

Appendix C



Notes: (1) panel A assumes a 6% real interest rate and panel B a 9% real interest rate. (2) the first scenario is based on 10% economic growth; the second on 5%; third on 4%; fourth on 3%; fifth on 2% and the sixth scenario is based on 1% economic growth.

Appendix D

Scenario	*Economic growth[1]: (1)*	*Interest rate[1]: (2)*	*Turning-point[2]: (3)*	*Debt repayment* $b_n = 0$[3] *(4)*	*Convex function[4]: (5)*	*Concave function[4]: (6)*
Portugal actual	1.39%	1.54%	*T=90*	*T=140*	*1 t/m 21*	*22 t/m ∞*
Portugal optimistic	2.09%	1.1%	*T=44*	*T=78*	*x*	*All time periods*
Portugal pessimistic	-3.29%	4.54%	∞	∞	*All time periods*	*x*
Italy actual	0.88%	0.74%	*T=122*	*T=197*	*x*	*All time periods*
Italy optimistic	1.51%	2.12%	*T=114*	*T=162*	*1 t/m 58*	*59 t/m ∞*
Italy pessimistic	-0.78%	4.35%	∞	∞	*All time periods*	*x*
Ireland actual	5.21%	1.94%	*T=9,58*	*T=21*	*x*	*All time periods*
Ireland optimistic	6.65%	0.39%	*T=6,33*	*T=16*	*x*	*All time periods*
Ireland pessimistic	3.67%	4.35%	*T=20,1*	*T=36*	*x*	*All time periods*
Greece actual	0.001%	7.32%	∞	∞	*All time periods*	*x*
Greece optimistic	3.92%	-0.021%	*T=15*	*T=35*	*x*	*All time periods*
Greece pessimistic	-3.29%	9.88%	∞	∞	*All time periods*	*x*
Spain actual	3.24%	1.08%	*T=40,64*	*T=68*	*x*	*All time periods*
Spain optimistic	3.89%	0.86%	*T=31,82*	*T=55*	*x*	*All time periods*
Spain pessimistic	-0.003%	3.41%	∞	∞	*All time periods*	*x*

Notes: (1) Parameter values based on Table 2.1. (2) Found by setting (2.7) equal to zero. (3) The period for which (2.6) is equal to zero. (4) The curvature of the scenarios for different periods found by equalising the second derivative to zero. (5) where a convex function implies a positive second derivative and concave function is found by a negative second derivative.

Appendix E

(1) Industrial countries (21)

Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and the United States.

(2) Emerging Market Economies (21)

Argentina, Brazil, Chile, China, Colombia, Egypt, India, Indonesia, Israel, Jordan, Korea, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, South Africa, Thailand, Turkey and Venezuela.

Appendix F

	ggdpc	**debt**	**$debt^2$**	**inv**	**fin_dev**	**gov_bal**	**avg_debt**	**avg_debt^2**
Mean	1.4005	69.51	63.46	22.63	86.11	-2.31	67.57	46.88
Median	1.54	60.81	36.97	22.61	76.50	-2.59	62.75	39.38
Maximum	24.67	242.11	586.19	34.22	231.62	31.22	92.55	85.65
Minimum	-8.99	9.68	0.94	9.83	37.83	-32.13	48.89	23.91
Std.Dev	2.52	38.96	82.73	3.39	37.12	4.85	11.03	16.02
Skewness	0.84	1.58	3.59	0.05	1.73	0.83	0.89	1.02
Kurtosis	17.15	6.69	19.28	4.06	6.09	11.29	2.39	2.56
Jarque-Bera (probability)	4799.72 (0.00)	538.04 (0.00)	7196.69 (0.00)	26.54 (0.00)	472.23 (0.00)	1638.82 (0.00)	84.26 (0.00)	103.40 (0.00)
Sum	794.10	37881.85	34588.12	12743.65	45294.61	-1269.11	38313.54	26578.38
Sum Sq.Dev	3600.033	825721.0	3723447.	6467.107	723360.6	12904.96	68901.07	145343.5
Observations	567	545	545	563	526	550	567	567

Appendix F continued

	age	tr_open	tot	tfp	rir	pop	hc	gov_size	igdppc
Mean	50.40	71.88	98.34	0.91	2.79	0.61	3.17	16.38	41520.03
Median	50.07	63.69	100.00	0.89	2.54	0.51	3.24	16.53	38969.32
Maximum	65.73	216.67	150.95	1.62	22.95	2.89	3.73	25.18	89619.49
Minimum	43.02	16.01	10.27	0.53	-26.23	-1.85	1.94	6.96	16687.26
Std.Dev	3.63	36.09	13.98	0.16	3.30	0.51	0.37	3.69	14123.94
Skewness	0.65	1.26	-1.04	1.13	-1.34	0.48	-0.86	-0.327	1.0876
Kurtosis	3.83	4.71	10.03	6.17	20.71	5.03	3.49	3.19	4.719
Jarque-Bera (probability)	56.13 (0.00)	216.85 (0.00)	1255.989 (0.00)	331.47 (0.00)	7391.452 (0.00)	118.93 (0.00)	70.85 (0.00)	10.15 (0.01)	181.599 (0.00)
Sum	28579.09	40467.23	55168.98	477.32	1544.33	343.95	1665.22	8598.862	23541856
Sum Sq.Dev	7450.19	732141.7	109410.3	13.48	6013.258	145.21	71.08	7123.99	1.13E+11
Observations	567	563	561	527	553	566	525	525	

Appendix G

	gddpc	debt	debt^2	tr_open	tot	tfp	rir	pop	hc	gov_size	inv	fin_dev	gov_bal	age	igdppc
ggdpc	**1.00**	-0.16	-0.16	0.11	-0.06	0.16	-0.13	0.005	-0.07	-0.23	0.23	-0.19	0.26	-0.05	-0.085
debt	-0.16	**1.00**	0.96	-0.06	0.32	-0.28	0.09	-0.49	-0.14	0.22	-0.28	0.48	-0.39	0.15	-0.215
debt^2	-0.16	0.95	**1.00**	-0.12	0.21	-0.28	0.08	-0.42	-0.06	0.22	-0.20	0.52	-0.26	0.25	-0.152
tr_open	0.11	-0.06	-0.12	**1.00**	0.02	0.37	-0.13	0.11	-0.07	0.03	-0.07	0.001	0.09	-0.02	0.263
tot	-0.06	0.32	0.21	0.02	**1.00**	-0.01	-0.07	-0.28	-0.14	-0.006	0.09	0.38	-0.16	-0.22	-0.122
tfp	0.16	-0.28	-0.28	0.37	-0.01	**1.00**	-0.24	0.26	0.18	-0.32	0.09	-0.25	0.49	-0.021	0.615
rir	-0.13	0.09	0.08	-0.13	-0.07	-0.24	**1.00**	-0.17	-0.18	0.06	-0.21	-0.05	-0.27	-0.07	-0.27
pop	0.005	-0.49	-0.42	0.11	-0.28	0.26	-0.17	**1.00**	0.18	-0.32	0.41	-0.07	0.24	-0.25	0.235
hc	-0.07	-0.14	-0.06	-0.07	-0.14	0.18	-0.18	0.18	**1.00**	-0.22	-0.09	0.17	0.28	0.14	0.601
gov_size	-0.23	0.22	0.22	0.03	-0.01	-0.32	0.06	-0.32	-0.22	**1.00**	-0.38	-0.12	-0.20	0.49	-0.324
inv	0.23	-0.28	-0.20	-0.07	0.09	0.09	-0.21	0.41	-0.09	-0.38	**1.00**	0.16	0.30	-0.41	0.1458
fin_dev	-0.19	0.48	0.52	0.001	0.38	-0.25	-0.05	-0.07	0.17	-0.12	0.16	**1.00**	-0.24	-0.16	0.044
gov_bal	0.26	-0.39	-0.26	0.09	-0.16	0.49	-0.27	0.24	0.28	-0.20	0.30	-0.24	**1.00**	0.13	0.517
age	-0.05	0.15	0.25	-0.02	-0.22	-0.02	-0.07	-0.25	0.14	0.49	-0.41	-0.16	0.13	**1.00**	0.108
igdppc	-0.085	-0.215	-0.152	0.263	-0.122	0.615	-0.27	0.235	0.60	-0.324	0.146	0.044	0.517	0.108	**1.00**

Appendix H

Variable	Dependent variable is GDP per capita growth (1)
Constant	19.009*** (0.00)
$debt_{it}$	0.053*** (0.00)
$debt^2_{it}$	-0.0002** (0.02)
tr_open_{it}	0.041*** (0.00)
tot_{it}	-0.038*** (0.00)
$tech_{it}$	5.859*** (0.00)
rir_{it}	-0.064*** (0.00)
pop_{it}	-0.905*** (0.00)
inv_{it}	0.355*** (0.00)
hc_{it}	-3.914** (0.03)
gov_size_{it}	-0.265*** (0.00)
gov_bal_{it}	0.072** (0.01)
fin_dev_{it}	-0.009 (0.156)
age_{it}	0.004 (0.94)
$igdppc_{it}$	-0.0003*** (0.00)
$residuals_{it}$	**-0.041*** **(0.08)**
R^2	0.732
F-statistic	19.017 (0.00)
Time fixed effects:	Yes
Country Fixed effects:	Yes

Notes: (1) *,** and *** specify significance at 10%, 5% and 1% level. (2) the coefficients and probabilities, shown in parentheses, are round to three decimals.

Appendix I

Unit root test			
Variable	**ADF- Fisher Chi^2**	**ADF- Choi Z-stat**	**Decision**
GDPPC Growth$_{it}$	173.738*** (0.00)	-9.235*** (0.00)	Reject h0: unit root
debt$_{it}$	61.429** (0.03)	-1.376 (0.08)	Reject h0: unit root

Notes: (1) *,** and *** specify significance at 10%, 5% and 1% level. (2) the coefficients and probabilities, shown in parentheses, are round to three decimals. (3) H0: the series contains a unit root and H1: the series is stationary.

Appendix J

Redundant fixed effects tests		
Effect test	**Statistic (Probability)**	**Decision**
Cross-section F	8.184*** (0.00)	Reject
Cross-section Chi^2	157.538***(0.00)	
Period F	13.994***(0.00)	Reject
Period Chi^2	280.639***(0.00)	
Cross-section/Period F	12.814***(0.00)	Reject
Cross-section/Period Chi^2	405.725***(0.00)	

Notes: (1) *,** and *** specify significance at 10%, 5% and 1% level. (2) the coefficients and probabilities, shown in parentheses, are round to three decimals. (3) H0: fixed effects are redundant.

Appendix K

Variable	Abbreviation	Description	Data source
Public Debt-to-GDP ratio	debt	General government gross debt as ratio of GDP	WEO
Government budget balance	gov_bal	Public revenue minus expenditures, both as percentage of GDP	WEO[4]
Trade Openness	tr_open	Imports of goods and services plus exports of goods and services, both as ratio of GDP	WWDI[1]
Financial Development	fin_dev	Ratio of liquid liabilities to GDP	WWDI
Real interest rate	rir	real_ir= [(1+ it)/(1+ π)]–1. Where *i*, is nominal interest rate and π inflation.	Own calculations
Nominal interest rate	nom_ir	The interest rate of public bonds with 10 year maturity.	OECD[3]
Inflation	inf	Yearly growth rate of changes in the price level of the total economy	WWDI
Output gap	output_gap	Output gap as ratio of potential GDP	WEO[4]
Current Account	ca	Current account balance as ratio of GDP	WEO
GDP growth	gdpc	Yearly growth rate of GDP. Based on constant 2010 U.S. dollars	WWDI
Real Effect Exchange Rate	reer	Real effective exchange rate index. Where 2010 is the base year.	WWDI
Crisis dummy variable	crisis_dum	Equals 1 when a currency-, inflation-, stock market-, sovereign debt- or banking crisis occurred and zero	Carmen Reinhart[5]
Private debt	private_debt	Domestic credit to private sector as ratio to GDP	WWDI
Instrument for debt	avg_debt	endogenous public debt variable for country *i* at time period *t* is instrumented by the average debt-to-GDP proportion	WEO

Notes: (1) WorldBank World Development Indicators. (2) Penn World Tables. (3) OECD database. (4) World Economic Outlook. (5) The authors website of Carmen Reinhart.

Appendix L

Redundant fixed effects tests		
Effect test	**Statistic (Probability)**	**Decision**
Cross-section F	3.130*** (0.00)	Reject
Cross-section Chi^2	62.705***(0.00)	
Period F	3.813***(0.00)	Reject
Period Chi^2	96.783***(0.00)	
Cross-section/Period F	3.999***(0.00)	Reject
Cross-section/Period Chi^2	165.343***(0.00)	

Notes: (1) *,** and *** specify significance at 10%, 5% and 1% level. (2) the coefficients and probabilities, shown in parentheses, are round to three decimals. (3) H0: fixed effects are redundant.