ERASMUS UNIVERSITY ROTTERDAM

ERASMUS SCHOOL OF ECONOMICS

MSC. INTERNATIONAL ECONOMICS

Bargaining with incomplete information under loss-aversion and reference points for the seller

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This paper examines the bargaining situation with a seller who is assumed loss-averse and has an exogenously determined reference point with buyers who are assumed to maximize their individual payoffs. By introducing reference points the seller has certain expectations about the value of the object being sold. These expectations in turn are responsible for the pricing scheme the seller introduces. Since the seller is also assumed loss-averse, the pricing scheme of the seller also changes as soon as the realized gains fall below the reference point. The consumer who purchases the object will always always be worse off in this new situation, whereas the seller with loss aversion and a relatively low reference point might see its profits increasing.

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November 7, 2017

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1 Introduction

Conventional economics makes a clear distinction between consumers and sellers. Sellers are often depicted as firms or profit maximizing agents, but of course also a consumer can eventually become a seller. Especially more recently this distinction is becoming less clear as it has become easier than ever to sell objects, via for instance online-platforms such as E-bay, Marktplaats.nl and Facebook. In these instances the consumer actually becomes the seller. The assumptions in economics, and particularly in behavioral economics are often of a consumer which maximizes utility and not necessarily payoff[1] and a profit maximizing seller. However, taking the previous into account it becomes clear that this utility maximization can also be transferred to the seller's side of the bargaining process, for a seller who receives utility by not purely maximizing profits.

The objects we tend to sell have a certain value to us, which gives us a certain expectation with regard to the price we would like to receive when selling the object. Consider a homeowner who is contemplating selling his property. This property was bought a while ago. Therefore the initial purchasing value of the house does not necessarily resemble the current expected value of the house. The seller must therefore set a price below, equal to, or above his reference point, in which the reference point is the price the seller expects/wants to receive for the house. If no buyer is found for the price set by the seller of the house, the seller should lower its price in order to find a potential buyer. It becomes clear that the seller has a certain reference point to which he compares the possible gains or losses it acquires in the selling process. This paper aims to model and describe the behavior of the seller, in which expectations for the seller remain the same during the bargaining process. The buyers therefore will perhaps face different prices for the same object as the seller, of course, aims to minimize the feeling of a loss and at the same time maximize the feeling of a gain. The main question which needs to be answered in this paper will be who benefits most in this new bargaining situation and how is the efficiency of the bargaining process affected. The model formulated in this paper adds to the literature by introducing sellers which have reference points and are assumed to be loss averse. This has not been investigated previously as the seller is mostly thought of a profit maximizing agent. However in everyday life, sellers are not necessarily profit maximizing but are lead on by their expectations. Departing from perspective will therefore provide new insights in the bargaining process.

The expectation is that the reference point in combination with the effects of loss aversion will create higher prices. This is due to the fact that the seller would want to minimize the felt losses, which is best done by increasing the prices in order to not make a loss on that transaction. However this could have a reverse impact as increased prices would imply that a smaller quantity will be sold, consequently out weighing the decision to ask higher prices. In-line with the expectations, it is in fact found in this paper that prices do increase as sellers are assumed to have certain reference points in com-

[1]To clarify: the utility maximization for consumers can lead to a different situation than a pay-off maximizing consumer (such as valuation minus the price)

bination with loss aversion. The consumers which have relatively low valuations for the object don't purchase anymore - implying that less items will be sold at each stage of the bargaining process.

This paper's structure is as follows: Section 2 provides a brief overview of the related literature, and a more in depth analysis of the benchmark to which this paper's extensions will be examined. Section 3 will show this paper's model and the methodology involved. Section 4 will solve the model under the different cases: The bargaining process. The results are discussed in Section 5, in which comparisons are made between the separate Cases and the benchmark Case. Section 6 will be dedicated to discuss the model's limitations and extension possibilities. Finally section 7 will give a summarized conclusion of the findings in this paper.

2 Literature Review

This paper aims to create a sequential bargaining model with incomplete information on the seller's side. Here the seller is assumed to have reference points in combination with loss aversion. In order to give the reader more information on the creation of bargaining models and loss aversion, this section will be dedicated to discuss the origin and progression of literature which has emerged over the past years with regard to these subjects. The section can be divided in two parts, the first paragraph discusses overall ideas and contributions of various authors to the literature. The second paragraph will be an in-depth analyses of one of the models which is used as underlying bargaining model for this paper.

2.1 Bargaining models and loss aversion

One of the first bargaining models which incorporated one-sided incomplete information was formulated by Fudenberg & Tirole (1983). They were the founders of this string of literature by creating the simple framework in which the bargaining system consisted of two people bargaining in a two-period framework. The seller makes the offers, whereas the consumer can either accept or reject the offer. Once the offer is rejected the seller has one more opportunity to sell to the consumer. After this second round the game ends. This will be examined further in the next paragraph.

A different approach had been introduced by Kahneman & Tversky (1979). They argue that utility should be examined in a different manner. The reasoning is that the expected utility is a function of two parts: the weighting function and the value function. The first part resembles the regular utility which is achieved when purchasing an object, this part also captures the phenomena that people normally overestimate a small probability outcome of occurring and underestimate the chance of a high probability outcome occurring. The second part takes into account the reference point. If the realized outcome is higher than the reference point, it is considered a gain. If it is lower than the reference point it is considered a loss. Since people are loss averse, one of the implications will therefore be that losses are felt more severely than gains of equal magnitude. Further research on Kahneman & Tversky's (1979) work has been done by Köszegi & Rabin (2006). They create a model which combines choice and also the determination of the reference point. They state that the "gain-loss utility" is derived from the standard "consumption utility", whereas the reference point is determined endogenously by economic environment (Köszegi & Rabin, 2006). They show that consumers are actually willing to pay more for an object as the probability of purchasing the object increases. Also they show that as the consumer expects to purchase the object, an increase in price will also increase the consumer's willingness to pay. This boils down to the fact that an increase in expectation/probability of purchasing leads to an attachment effect, and therefore the consumer is willing to pay higher prices.

As has been shown by various authors, purely the fact of having expectations on the good in a bargaining situation alters the purchasing decision of the consumers. More research has been done on the field of sequential bargaining models with reference dependent preference by Rosato (2013), and by Thiel (2015). Rosato's model can be seen as a combination of Fudenberg & Tirole (1983) and Köszegi & Rabin's model (2006). He creates a two period framework with consumers who are uniformly distributed in [0,1], with reference points which are fixed for the consumers during the bargaining situation. His findings are that relatively high type consumers are worse-off and relatively low type consumers benefit in this model as the first period price increases whereas the second period price decreases. Thiel (2015) adjusted the reference points to be endogenously determined instead of static, which eventually lead to the results which were in-line with Rosato's (2013) findings.

The models discussed create interesting insights into how loss aversion and reference points can alter the bargaining process. This work mostly focused on changing the assumptions on the consumers side, while leaving the seller as profit maximizing agents. Little theoretical work has been done on these assumptions for the seller. However it has been shown empirically that restaurant owners can behave in a loss-aversive manner (Kapoor, 2017). This bridges the gap for the assumption that loss aversion can indeed also apply for the seller's side.

This paper aims to use the insights gained from Fudenberg & Tirole (1983) and add the reference points in combination with loss aversion as discussed by Rosato (2013). Therefore the coming paragraph will thoroughly examine the benchmark case of Fudenberg & Tirole (1983), after which in the next section loss aversion and exogenous reference points are introduced.

2.2 Solving the basic model a la Fudenberg and Tirole

This paper will show the effect of sellers with different preferences compared to a model where consumers and sellers behave in a neutral manner[2]. The bargaining model as formulated by Fudenberg and Tirole (1983) is used as as benchmark in order to see how the seller and consumer's behavior changes. These authors created the first

[2]Neutral refers to the situation where the agents are not assumed loss averse, nor have any reference points.

string of literature for a simple two period, two player model with incomplete information. Their model shows how a seller, who makes all the offers, and a consumer who can either accept or reject these offers behave. The notion of neutrality in this benchmark is derived from the fact that neither the consumers nor the seller are loss averse and do not have reference points.

Fudenberg and Tirole (1983) set up their model in such a way that the seller sets the prices and the consumers have the option to either accept or reject the offers. If the first period offer is accepted, the game ends. If the first period offer is rejected, the game moves on to the second period. Regardless of whether the offer is accepted or rejected, the game ends after the second period. However, bargaining is costly (discount factor δ_b for the consumers and δ_s for the seller). The seller and consumer want to come to an agreement sooner rather than later because of these discount factors. The surprising results yielded from the comparative statistics show that a decrease in the buyers discount factor might in fact make him better off (a decrease in the discount factor implies $\delta < 1$). The reasoning for this, as explained by Fudenberg and Tirole, is the fact that a consumer cannot credibly commit to a sub-optimal strategy, which would compromise its optimal strategy, in advance. Therefore, if the consumer's discount factor is decreased compared to the discount factor of the seller, it will give the consumer a different optimal strategy. This different optimal strategy is then incorporated in the seller's optimal strategy. Which will eventually make the consumer better off. A decrease in the discount factor would imply the consumer is more impatient and would rather consume today than in the future. The seller understands this process and therefore asks a higher first period price, harming the relatively high valued consumers, while asking a lower second period price, which is beneficial to the relatively low valued consumers. The net effect will be such that more consumers will be able to purchase the object. Vice versa, this also applies that an increase in the consumers' discount factor actually makes him the relatively low valued consumers worse-off and the relatively high valued consumers better off. This as the first period price will decrease, whereas the second period price will increase in this situation.[3]

3 The Model: Exogenous reference points with loss aversion

In this section I depart from the Fudenberg and Tirole (1983) setting which was discussed in the previous section. This framework adopts their one-sided bargaining model under incomplete information. The model however will include a loss averse seller, where the feeling of loss originates from the fact that the seller sells the object below its reference point (p_x) [4]. If the item is sold above the reference point the feeling of a small gain is included. This model's setup assumes one loss averse seller which makes all the offers and a unit mass of neutral consumers. These players bargain over

[3]To illustrate this for the interested reader the model as proposed by Fudenberg & Tirole (1983), which is used as benchmark for this paper, is solved and evaluated in Appendix A.

[4]This can also be seen as an exogenously given expectation

an indivisible object. The seller's fixed and marginal costs are assumed to be zero. The game lasts for a maximum of two periods. After the first period the buyer can either accept or reject the first period offer. If the first period offer is rejected the game moves on to the second period, if accepted the game ends. After the rejection of the first period offer the buyer can accept or reject the second period offer. Either way, the game ends after the second period.

The consumer makes the decision to purchase the object if the price asked by the seller is equal or lower than the valuation of the consumer. The first stage utility for the consumer which purchases the object in the first period is therefore:

$$U_{1,buyer} = v - p_{1,i}$$

If the consumer decides to purchase the object in the second period, $t = 2$, the discount factor should be taken into account. Therefore the utility of the consumer which purchases the object in the second period changes to:

$$\delta U_{2,buyer} = \delta(v - p_{2,i})$$

If the consumer does not buy in either stage he will not receive any utility, nor incur any costs. The consumer in this model therefore behaves identical to the consumer as in Fudenberg & Tirole (1983). This paper adjusts the behavior of the seller, compared to the benchmark setting. Therefore the seller's first period utility function becomes:

$$U_{1,Seller} = \begin{cases} (1 - v_{x,i})((1-\alpha)p_{1,i} + \alpha(p_{1,i} - p_x)), & \text{if } p_{1,i} - p_x \geq 0 \\ (1 - v_{x,i})((1-\alpha)p_{1,i} + \alpha\theta(p_{1,i} - p_x)), & \text{if } p_{1,i} - p_x < 0 \end{cases}$$

The second period utility function in $t = 2$ for the seller is conditional on the first period price being rejected. This function takes the discount factor, δ, into account and therefore becomes:

$$\delta U_{2,Seller}(p_{2,i}|p_{1,i}\text{rejected}) = \begin{cases} \delta\frac{(v_{x,i}-v_{y,i})}{v_{x,i}}((1-\alpha)p_{2,i} + \alpha(p_{2,i} - p_x)), & \text{if } p_{2,i} - p_x \geq 0 \\ \delta\frac{(v_{x,i}-v_{y,i})}{v_{x,i}}((1-\alpha)p_{2,i} + \alpha\theta(p_{2,i} - p_x)), & \text{if } p_{2,i} - p_x < 0 \end{cases}$$

If the seller does not sell the object, zero utility will be received as the seller does not incur any fixed or marginal costs. The utility functions of the seller states that the seller does not only derive utility from selling the object, but rather that the seller receives utility from selling the object relative to a reference point (p_x). Comparing the selling price to the reference point can either yield a gain or loss. In the first and second period the seller receives utility by means of these two parts in the utility function.[5] The first part this paper calls the *selling utility*, which can be found at the $(1 - \alpha)$ part,

[5]This can be compared to the literature on consumer utility with reference points as formulated by Kahneman & Tversky (1979), with appended work from Köszegi and Rabin (2006). Here the two parts are called intrinsic consumption utility and gain-loss utility. Where the intrinsic consumption utility describes the utility the consumer derives from consuming the object, whereas gain-loss utility is the utility the consumer receives from comparing the consumption utility with the expectation. This phenomena is converted to the seller's utility in this paper, partly the utility is received from profits: selling utility, and partly from the gain-loss utility: reference utility.

which is the utility received by selling the object against the first and second period price. This is depicted by the quantity sold in that period, multiplied by the price in that period. This boils down to the profits being multiplied by $(1-\alpha)$. The second part of the utility function can be found at the (α) part. In this paper we call this the *reference utility*. If the object is sold for a price in the first or second stage which is above the reference point p_x the seller will experience the feeling of a gain when selling the object. However if the price received for the object is lower than the reference point p_x the seller will experience the sensation of a loss in the *reference utility*. Combining both parts of the utility function will accumulate to the overall utility of selling an object at a certain price. As introduced by Kahneman & Tversky (1979), losses are felt more severely than gains of equal magnitude. This finding is also implemented in this paper. Therefore it will be assumed that the seller is loss averse, losses will be felt more severely than gains of equal magnitude. This is captured by the coefficient of loss aversion: $\theta > 1$. In this model the parameter $\alpha \in [0,1]$ measures the relative importance of the *reference utility* compared to the *selling utility*. The higher α , the more emphasis is on the *reference utility*. p_t Depicts the price the seller asks in period t, $v \in [0,1]$ denotes the valuation the consumer has for the good, $v_{x,i}$ denotes the valuation of the indifferent consumer which still buys at price $p_{1,i}$, whereas $v_{y,i}$ denotes the valuation of the consumer which still marginally purchases the object at price $p_{2,i}$. The reference point , $p_x \in [0,1]$, denotes the reference point the seller has. This reference point will be compared to the price the seller receives for the object which determines if selling the object yields a gain or a loss. This can also be seen as an exogenous expectation the seller has with regard to selling the object. The difference between p_t and p_x can be seen as the surplus the seller receives from selling the object for a higher price than expected, or as a loss when this difference is negative. In this framework the index $_i$ denotes the different cases which result from evaluating threshold values of p_x for which the seller is located at different cases. This model differs compared to Fudenberg and Tirole (1983) in the fact that two very distinct features are added. The first one is the *reference utility*: here the seller evaluates the received price to a reference point. The second feature is loss aversion. As the seller experiences the feeling of a loss, this feeling is felt more severely than a gain of equal magnitude. In other words, sellers receive a premia from selling for a price above the reference point, but a more than proportional punishment by selling for a price under the reference point.

The game lasts for a maximum of two stages. If the first period price is accepted by the consumer, the game ends. If the consumer rejects the first period price, the seller has the opportunity to set a new price: $p_{2,i}$. Irrespective if the consumer accepts or rejects $p_{2,i}$ the game ends after the second period.

As the above section clearly illustrate, there will be multiple cases to take into account. Namely, the pricing in either period will be above or below the reference point. Therefore the following cases arise:

Case 1 (Gain): $p_1 \geq p_2 \geq p_x$
Case 2 (Gain/Loss): $p_1 \geq p_x \geq p_2$
Case 3 Loss: $p_x \geq p_1 \geq p_2$

Case 4: $p_2 \geq p_1$ [6]

The first case will not receive the sensation of a loss. However in the second case, the second period price will be below the reference point and the loss which is felt will be multiplied by θ (the coefficient of loss aversion) in the *reference utility* section. The third case prices below the reference point in both the first period and second period, implying that the *reference utility* section is multiplied by θ in both periods. The coming sections will analyze the different cases.

4 Solving the bargaining process

The coming section will analyze the previously described setting of the three different cases. First the consumer's behavior is analyzed, after which the cases will be discussed.

The indifferent consumer is indifferent between buying in $t = 1$ and $t = 2$ at price $p_{1,i}$, this consumer's valuation is denoted by $v_{x,i}$ and there is a consumer which is indifferent between buying in $t = 2$ and not buying at all at price $p_{2,i}$, this consumer's valuation is denoted by $v_{y,i}$. The discount rate for the seller and consumers are identical and equal to $\delta \in [0, 1]$.

The first step in order to solve the model with backward induction is to find out what the value ($v_{y,i}$) for the indifferent consumer is at $t = 2$. The consumer with this valuation is indifferent between buying in the second period and not buying at all. This indifferent consumer can be found by equalizing the second period indifferent consumer's valuation with the second period price. The reasoning behind this, is that there is no credible strategy for the consumer not to buy at $t = 2$ for $p_{2,i}$ if the valuation ($v_{y,i}$) of this consumer is at least equal to $p_{2,i}$. Resulting in:

$$v_{y,i} - p_{2,i} = 0 \tag{1}$$

This yields the consumer who is indifferent between buying in the second period and not buying at all, with the valuation thus as shown in Equation 1 being equal to $v_{y,i} = p_{2,i}$. Therefore implying consumers with a valuation for the object which is below $v_{y,i}$ do not purchase at any stage in the bargaining process.

The consumer with the valuation which is indifferent at buying in the first or second period ($v_{x,i}$) will only be indifferent if the utility is at least equal to the utility this valued consumer would receive when postponing the purchasing decision to the second period, whilst taking into account the discount factor δ. Resulting with the indifferent

[6] Taking a look at the above cases it becomes evident that Case 4 should not be considered, as the buyers will all anticipate that the seller will, in this case, sell for a higher price in the second period. Therefore they will all consume in the first period, and less profits are extracted for the seller. In this setting p_1 would equalize to p_2, however this is already considered in case one, two and three. Therefore case 4 is discarded.

consumer's valuation in the first period being equal to $U_{1,buyer} = \delta U_{2,buyer}$.

$$v_{x,i} - p_{1,i} = \delta(v_{x,i} - p_{2,i}) \tag{2}$$

However, $p_{2,i}$ in Equation 2 is still a function of $v_{x,i}$. So far the findings replicate the initial model as shown in Fudenberg and Tirole (1983). In order to find the optimal value of $p_{2,i}$ the seller's behavior must now be optimized, which is where this paper departs from the initial model. Therefore the coming section will start with the analysis of the different cases.

4.1 Case 1: Gain

This section will analyze the gain case implying that the first period profit and the second period profits are greater or equal compared to the reference point

$$p_{1,1} \geq p_{2,1} \geq p_x$$

Therefore both prices are above the reference point (p_x). As Fudenberg and Tirole (1983) showed in their paper, the seller updates its information on the buyers conditional on the acceptance (or rejection) of the first period price. The seller knows that the consumer's valuation is uniformly distributed. With means of backwards induction the seller can optimize its overall profits, by first looking at the optimization of the second period profits conditional on the first period price being rejected:

$$E(\pi_{2,1}|p_{1,1 \text{is rejected}}) = \frac{(v_{x,1} - v_{y,1})}{v_{x,1}}((1-\alpha)p_{2,1} + \alpha(p_{2,1} - p_x)) \tag{3}$$

As the seller knows the consumers' valuation is uniformly distributed, a new distribution is considered in which $p_{1,1}$ is rejected, for which the consumers with a higher valuation than $v_{x,1}$ have already purchased the object. Taking into account $v_{y,1} = p_{2,1}$ and maximizing Equation 3 with regard to $p_{2,1}$ yields:

$$\frac{\partial \pi_{2,1}}{\partial p_{2,1}} = 0 \tag{4}$$

Solving Equation 4 for the second period price yields:

$$p_{2,1} = \frac{v_{x,1} + \alpha p_x}{2} \tag{5}$$

In order for the consumer, which is indifferent to buy in the first stage and the second stage, to actually make the decision to buy in the first stage instead of in the second stage, the utility the consumer derives from the decision to buy in the first stage $(v_{x,1} - p_{1,1})$ must be equal or larger compared to the utility gained by buying in the second period $\delta(v_{x,1} - p_{2,1})$. Therefore the consumer needs to be found which is indifferent between buying in either period. This indifferent consumer can be found as shown in Equation 2, which leads to the following equality:

$$v_{x,1} - p_{1,1} = \delta(v_{x,1} - p_{2,1})$$

Filling in $p_{2,1}$ yields:

$$v_{x,1} - p_{1,1} = \delta(v_{x,1} - \frac{v_{x,1} + \alpha p_x}{2}) \tag{6}$$

Solving Equation 6 for ($v_{x,1}$) the indifferent consumer's valuation yields:

$$v_{x,1} = \frac{2p_{1,1} - p_x \alpha \delta}{2 - \delta} \tag{7}$$

Then in the first stage the seller behaves in a manner which maximizes overall expected profits:

$$E(\pi_{overall}) = (1 - v_{x,1})((1-\alpha)p_{1,1} + \alpha(p_{1,1} - p_x)) + \delta(v_{x,1} - v_{y,1})((1-\alpha)p_{2,1} + \alpha(p_{2,1} - p_x)) \tag{8}$$

Substituting Equation 5, for both $p_{2,1}$ and $v_{y,1}$, and Equation 7 into Equation 8 yields:

$$\begin{aligned} E(\pi_{overall}) &= (1 - (\frac{2p_{1,1} - p_x\alpha\delta}{2-\delta}))((1-\alpha)p_{1,1} + \alpha(p_{1,1} - p_x)) \\ &+ \delta((\frac{2p_{1,1} - p_x\alpha\delta}{2-\delta}) - (\frac{\frac{2p_{1,1}-p_x\alpha\delta}{2-\delta} + \alpha p_x}{2}))((1-\alpha)(\frac{(\frac{2p_{1,1}-p_x\alpha\delta}{2-\delta}) + \alpha p_x}{2}) + \alpha((\frac{(\frac{2p_{1,1}-p_x\alpha\delta}{2-\delta}) + \alpha p_x}{2}) - p_x)) \end{aligned} \tag{9}$$

Taking the partial derivative of Equation 9 with regards to $p_{1,1}$, and solving for $p_{1,1}$ yields:

$$\frac{\partial \pi_{overall}}{\partial p_{1,1}} = 0 \tag{10}$$

I have shown that the optimal price in the first period is equal to:

$$p^*_{1,1} = \frac{\alpha(\delta^2 + 2\delta - 4)p_x - (\delta - 2)^2}{6\delta - 8} \tag{11}$$

rewritten to:

$$p^*_{1,1} = \frac{\alpha(4 - \delta^2 - 2\delta)p_x + (\delta - 2)^2}{8 - 6\delta} \tag{12}$$

The optimal first period price, as shown in Equation 12 can now be substituted in Equation 7 in order to calculate the value of the indifferent consumer in the first period.

$$v^*_{x,1} = \frac{\delta + 2\alpha(\delta - 1)p_x - 2}{3\delta - 4} \tag{13}$$

The indifferent consumer's valuation in the first period, which is shown in Equation 13, can now be substituted in Equation 5 in order to calculate the optimal second period price, which is equal to the indifferent valued consumer in the second period, as shown in Equation 1:

$$p^*_{2,1} = v^*_{y,1} == \frac{\delta + \alpha(5\delta - 6)p_x - 2}{6\delta - 8} \tag{14}$$

4.2 Case 2: Gain/Loss

This section will analyze the second case where the seller only experiences the feeling of a loss in the *reference utility* in the second period. Therefore $p_1 \geq p_x \geq p_2$. The steps to calculate the optimal behavior are exactly the same as in Case 1. The only difference can be found in the second period profit function for the seller, this changes to:

$$E(\pi_{2,2}|\text{rejected } p^*_{1,2}) = \frac{(v_{x,2} - v_{y,2})}{v_{x,2}}((1-\alpha)p_{2,2} + \alpha\theta(p_{2,2} - p_x)) \tag{15}$$

The overall profit function therefore changes to:

$$E(\pi_{overall}) = (1 - v_{x,2})((1-\alpha)p_{1,2} + \alpha(p_{1,2} - p_x)) + \delta(v_{x,2} - v_{y,2})((1-\alpha)p_{2,2} + \alpha\theta(p_{2,2} - p_x)) \tag{16}$$

As can be seen, the only parameter which is added is: θ. This parameter depicts loss aversion when $p_x > p_t$, which in Case 2 implies $p_x > p_2$. Therefore θ is added in front of the *reference utility* part of the seller's utility function. This models for the phenomena of loss aversion for the seller. The calculations are executed in the same manner as before, therefore only the optimal results will be discussed .[7]

$$p^*_{1,2} = \frac{-(\delta-2)^2 + 2\alpha^2(\theta-1)p_x(\delta\theta+\delta-2) + \alpha(p_x(\delta^2\theta + 2\delta - 4) - (\delta-2)^2(\theta-1))}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{17}$$

$$v^*_{x,2} = \frac{\delta + \alpha^2(\theta-1)x(\delta\theta-2) + \alpha((\delta-2)(\theta-1) + 2p_x(\delta\theta-1)) - 2}{(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{18}$$

$$p^*_{2,2} = v^*_{y,2} == \frac{\delta + 2\alpha^2(\theta-1)p_x(\delta\theta-1) + \alpha((\delta-2)(\theta-1) + p_x(5\delta\theta - 4\theta - 2)) - 2}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{19}$$

4.3 Case 3: Loss

This section analyses the third case of exogenous reference points where the seller experiences the feeling of a loss in the *reference utility* in both periods, therefore $p_x > p_1 \geq p_2$. The steps to calculate Case 3 are identical to the ones in Case 1 and 2, however in this case the seller experiences the feeling of a loss in both periods. Therefore, in comparison to Case 2, the only adjustments in the calculations are found in the first period utility function for the seller, which is part of the overall utility function of the seller:

$$\pi_{overall} = (1 - v_{x,3})((1-\alpha)p_{1,3} + \alpha\theta(p_{1,3} - p_x)) + \delta(v_{x,3} - v_{y,3})((1-\alpha)p_{2,3} + \alpha\theta(p_{2,3} - p_x)) \tag{20}$$

Since in this case $p_x > p_{1,3}$, θ is introduced in the part of the *reference utility* for the seller in the overall utility function. The calculations are executed in the same manner as before, therefore only the optimal results will be discussed.[8]

$$p^*_{1,3} = \frac{\alpha(\delta^2(\theta(p_x-1)+1) + 2\delta(\theta(p_x+2)-2) - 4(\theta+\theta p_x - 1)) - (\delta-2)^2}{2(3\delta-4)(\alpha(\theta-1)+1)} \tag{21}$$

[7] The interested reader is referred to Appendix B in order to find the calculations of Case 2 Gain/Loss step by step.

[8] The interested reader is referred to the Appendix B in order to find the calculations of Case 3 Loss step by step.

$$v_{x,3}^{*} = \frac{\delta + \alpha(\delta(\theta + 2\theta p_x - 1) - 2(\theta + \theta p_x - 1)) - 2}{(3\delta - 4)(\alpha(\theta - 1) + 1)} \tag{22}$$

$$p_{2,3}^{*} = v_{y,3}^{*} == \frac{\delta + \alpha(\delta(\theta + 5\theta p_x - 1) - 2(\theta + 3\theta p_x - 1)) - 2}{2(3\delta - 4)(\alpha(\theta - 1) + 1)} \tag{23}$$

4.4 Analysis interval

All optimal values for the cases have been calculated. Before the comparison is made between these cases and the benchmark, it should first be considered if for all the following values the model is solvable: $\theta \in [1, 4]$[9], $\alpha \in [0, 1], \delta \in [0, 1]$ and $p_x \in [0, 1]$. As can be seen in Figure 2 the model is not continuous in p_x. Therefore this section will analyse p_x in-depth.

The seller cannot sell in Case 1 for a price $(p_{2,1})$ which is below the reference point (p_x)[10]. This poses a problem as the seller only wants to experience the feeling of a loss in the *reference utility* at a threshold of the reference point (p_x) which is substantially higher than the ending of Case 1. The same holds for Case 3 where the seller cannot sell for a price $(p_{1,3})$ which is above the reference point. Case 2 ends at a lower threshold of the reference point p_x compared to the relatively high reference point for which the seller is willing to receive a feeling of a loss in both periods (Case 3). These intervals are shown in both Figure 1 and Figure 2.

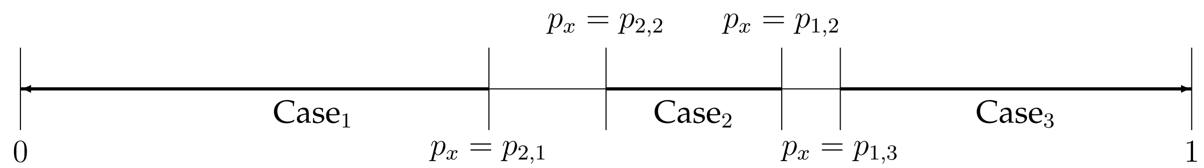

Figure 1: The interval is shown for which the seller is not analyzable: the thin black lines. Here the behavior of the seller is currently not clear.

11

[9]Emperical studies on loss aversion found θ to be as low as 1.4 (Schmidt and Traub (2002)) and up to 4.5 (Fishburn and Kochenberger (1979)) therefore the analyzable interval of $[1, 4]$ is used

[10]Keep in mind that selling for a price below the reference point implies the jump to the next case, as selling for a price below the reference point implies experiencing the sensation of a loss in the *reference utility*. Conditional on $\alpha > 0$

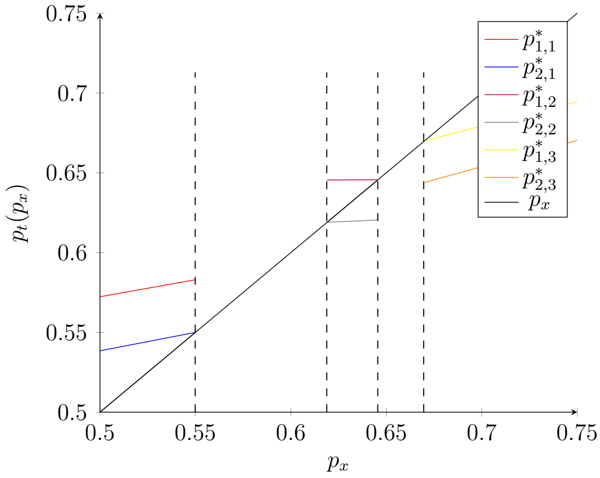

Figure 2: The effect of the reference point on the first and second period prices in the three cases. Using the parameters: $\alpha = 0.4$, $\delta = 0.9$ and $\theta = 2$.

As the seller's expectation about the object increases, there is a certain threshold for which the seller switches from Case 1 to the next Case. However, the next Case is not Case 2 (Gain/Loss), but rather an in-between Case which this paper refers to as Case A. As the reference point surpasses the threshold for the seller to be in Case 1, the best response from the seller will be to alter its pricing scheme in order to postpone the entering of Case 2: Gain/Loss. This is done by keeping $p_{2,A}$ equal to p_x. This is because the seller optimally chooses to follow his expectations with regard to the object in the second period, instead of asking a price which is below p_x in the second period (which would result in the sensation of a loss). Therefore the seller sets a price in the second period of $p_{2,A}$=p_x, in order to postpone the decision to receive the feeling of a loss in the second period. This will however not go-on forever, as following p_x comes at a price, the increased price diminishes the quantity sold. At a certain threshold it becomes more interesting to receive the feeling of a loss in the second period than stubbornly asking the reference price in the second period indefinitely. Asking a relatively low second period price has the downside of receiving the feeling of a loss in that period. However, the benefit this creates is that the quantity sold is relatively larger for a relatively lower $p_{2,2}$, than sticking to a relatively high p_x: Thus entering Case 2: Gain/Loss.

As the seller's reference point increases even further the threshold is met for which the seller switches from Case 2 to the next Case. However, the next Case is not Case 3 (Loss), but rather another in-between Case which this paper refers to as Case B. The same reasoning applies here, as it did to the gap between Case 1 and Case 2. As the reference point increases, the best response of the seller will be to postpone the decision to ask a first period price which yields the sensation of a loss in the *reference utility*, as it already does so in the second period. This will yield a first period price which is equal to the reference point: $p_{1,B} = p_x$. Therefore the first period price will now be equal to the reference point. This will remain the optimal strategy for the seller until the threshold is met for which the reference point becomes large enough to permit the seller to sell for the sensation of a loss in both periods, implying $p_{1,3} < p_x$, and thus entering Case 3: Loss.

It is now clear how the seller behaves for the second period price in Case A, and the first period price in Case B. However, this second period price in Case A yields a corresponding optimal first period price in Case A. The same applies to changing the first period price in Case B will yield a corresponding optimal second period price. Therefore the two coming paragraphs will analyze the optimal prices in Case A & B.

Case [A] This section will analyze the Case between Case 1 and Case 2 of exogenous reference points where the seller sets $p_{2,A}(v_{x,A}(p_{1,A})) = p_x$. Taking into account the optimal second period price in Case A, the optimal first period price will be calculated. In order to calculate the first period price in Case A, the optimal response of the seller and consumer can be used as was the done in Case 1. Case A takes the optimal second period price and imposes the restriction that this is equal to p_x. Plugging in the optimal response functions for $v_{x,A}$ in $p_{2,A}$, conditional on $p_{2,A}$ being equal to p_x and solving for $p_{1,A}$ yields the optimal first period price for which the seller optimally demands a second period price which is exactly equal to p_x.

$$p_{1,A} = p_x(2 - \alpha - \delta + \alpha\delta)$$

$$v_{x,A} = p_x(2 - \alpha)$$

$$p_{2,A} = v_{v,A} == p_x$$

For proof the reader is directed to Appendix C

Case [B] This section will analyze the Case between Case 2 and Case 3 of exogenous reference points where the seller sets $p_{1,B} = p_x$. Taking into account the optimal first period price in Case B which is equal to p_x, the indifferent valued consumers and the optimal second period price can be calculated. In order to calculate the optimal second period price in Case B, the optimal response of the seller and consumer can be used as was done in Case 2[12]. Case B takes the optimal first period price and imposes the restriction that this is equal to p_x. Plugging in the optimal response functions for $v_{x,B}$ in $p_{2,B}$, conditional on $p_{1,B}$ being equal to p_x and solving for $p_{2,B}$ and $v_{x,B}$ yields the optimal second period price and indifferent valued consumers, for which the seller optimally demands a first period price which is exactly equal to p_x.

$$p_{1,B} = p_x$$

$$v_{x,B} = \frac{p_x(\alpha((\delta - 2)\theta + 2) - 2)}{(\delta - 2)(\alpha(\theta - 1) + 1)}$$

$$p_{2,B} = v_{y,B} == \frac{p_x(\alpha((\delta - 2)\theta + 1) - 1)}{(\delta - 2)(\alpha(\theta - 1) + 1)}$$

For proof the reader is directed to Appendix D

[12]See Appendix B for the optimal response functions of $v_{x,2}$ and $p_{2,2}$

4.5 Final model

Combining Cases $[1, 2, 3]$ and Cases $[A, B]$ the graphs depicted in Figure Figure 3 can be constructed with prices on the y axis and the reference point on the x axis.

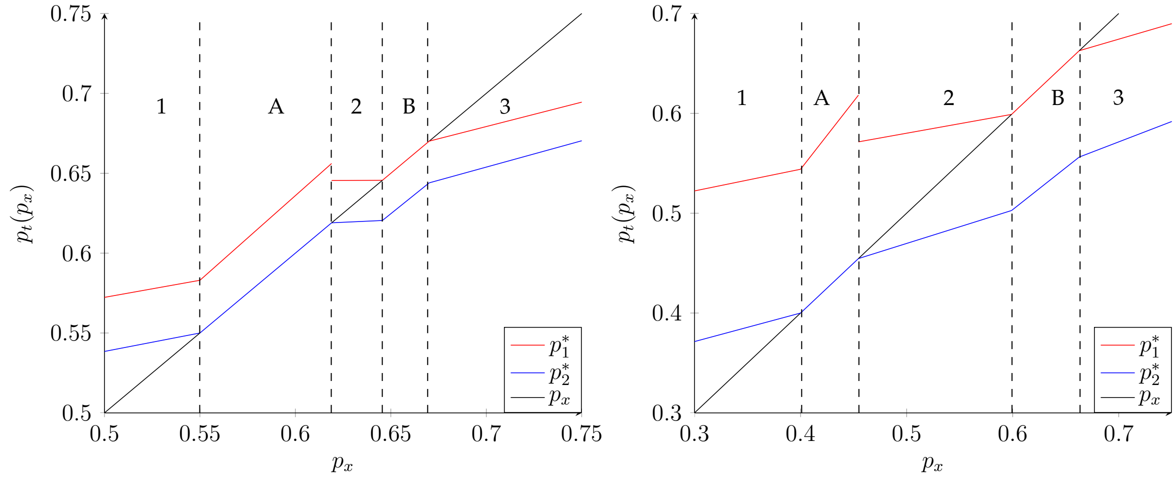

Figure 3: The effects of p_x on p_t, while keeping the following parameters constant:$\alpha = 0.4$, $\theta = 2$ and $\delta = 0.9$ for the initial graph with an $\delta = 0.4$ for the second graph

In Figure 3 the effects of the reference point on the prices becomes clear and can quickly be analyzed. As the reference point increases the price in either period increases as well. The following paragraph will discuss the prices as the reference point increases

As the reference point of the seller increases (ceteris paribus), the prices the seller asks increases. This is true for both the first and second period prices. As the reference point increases even further, the seller finds it beneficial to follow the reference point with its second period pricing, the seller leaves Case 1 and enters Case A. As the seller is located in Case A, there will be no feeling of a loss in the *reference utility*. In order for the seller to keep the feeling of a loss away, as the reference point increases, his best response is to ask an even larger first and second period price. At a certain level the seller's reference point reaches a height at which committing to this rapidly increasing price becomes inefficient. It is true that committing to this high price has the benefit of not yielding the feeling of a loss in the *reference utility*, however the quantity sold suffers greatly from this. After a certain point it becomes more interesting for the seller to deviate from the strategy of pricing equal to its reference point in the second period. The seller enters Case 2. The benefits of selling a relatively larger quantity outweighs the decision to ask high prices, and thus now the seller will experience the feeling of a loss in the second stage. As is illustrated in Figure 3 there is a drop in first period prices as the seller enters Case 2. The reason for this drop is that as the seller priced relatively high with reference points which kept the seller in Case A, this was only the best strategy for the seller as the reference point was not high enough yet to permit asking a price in the second period which would optimally yield a loss in the *reference* *utility*. Furthermore, the seller could not credibly ask a lower price than $p_{1,A}$, as asking a lower price would optimally force the seller to ask a price which is below p_x in the second period. As the seller does not yet want to experience the feeling of a loss, asking a lower first period price is not a proper strategy for him. However, as became clear, the reference point is now high enough for the seller to be willing to experience the sensation of a loss in the *reference utility*. Once the seller decides to price lower than p_x in the second period its first period price can drop slightly. This will increase the quantity sold to consumers in the first period. As the reference point of the seller increases even further and reaches its limit of Case 2 the best strategy for the seller will be to postpone experiencing the sensation of loss again. Just as the seller did in Case A, the seller now does not want the first period price to drop below p_x as this would imply the seller sells the object in both periods below its reference point and therefore experiences a loss in both periods. The seller sets its first period price equal to p_x, and adjusts $p_{2,B}$ accordingly. This strategy increases the price the seller asks relatively quickly as the reference point increases. This strategy will also not go on indefinitely. At a certain point it is not optimal for the seller to follow the reference point with the seller's first period price. As the seller decides to optimally set its first period price lower than the reference point the seller consequentially enters Case 3. Now a loss is felt in both periods. In this instance the seller does not 'drop' the prices in order to sell a larger quantity, as was the situation between Case A and Case 2. The reason why there is no drop is because optimally the seller would ask a lower first period price in Case A, in order to sell a larger quantity in the first period. However, as discussed previously this is not possible as the seller has a commitment in the second period to p_x, and setting a first period price below $p_{1,A}$ would subsequently compromise this commitment to p_x. However, this commitment is different in Case B. The seller sets the first period price equal to p_x for which it adjusts the second period price optimally. Therefore the quantity sold is followed optimally in the first period and as soon at it becomes more interesting to abandon the increasing first period price, which follows p_x, the seller decides to enter Case 3 and therefore sells a relatively larger amount for a relatively lower price.[13]

5 Results

In the coming section comparative statistics will be examined in order to analyze the previous cases. These cases will not only be compared to the benchmark setting of Fudenberg & Tirole (1983), but also to the other cases in order to examine the effects of the seller's reference points in combination with loss aversion on prices and quantities.

This model yields the results from Fudenberg and Tirole (1983) when $\alpha = 0$ or when $p_x = 0$.

$$p_1^*|_{\alpha=0} = \frac{(2-\delta)^2}{(8-6\delta)} \tag{24}$$

$$v_x^*|_{\alpha=0} = \frac{(2-\delta)}{(4-3\delta)} \tag{25}$$

[13] Appendix E covers the the parameter values which determine what Case the seller is in.

$$p_2^*|_{\alpha=0} = \frac{(2-\delta)}{(8-6\delta)} \tag{26}$$

$$v_y^*|_{\alpha=0} = \frac{(2-\delta)}{(8-6\delta)} \tag{27}$$

This can be compared to the model in which the seller has *reference utility* and is loss averse. The conclusion which can be drawn is that for all values of $\alpha > 0$ and $p_x > 0$, $p_1^* \geq p_1^*|_{\alpha=0}$ and $p_2^* \geq p_2^*|_{\alpha=0}$. The reference point determines which Case is currently effective. The interval for when a certain case becomes active for a certain reference point is determined by three other factors: δ, α and θ. Manipulating these parameters will influence the interval location of the Cases for a changing p_x. In order to get a better idea about how the parameters influence which Case should be analyzed the reader is redirected to Appendix E, here the formal definition of each Case's boundaries are shown with also the a summary of the optimal prices and indifferent valued consumer at each instance. The following graph will illustrate how the optimal prices and indifferent consumers are located compared to Fudenberg and Tirole (1983). The next intervals are calculated using fixed values for the following parameters: $\delta = 0.9$, $\alpha = 0.4$ and $\theta = 2$ while p_x changes. As can be seen in Figure 4 a change in the reference

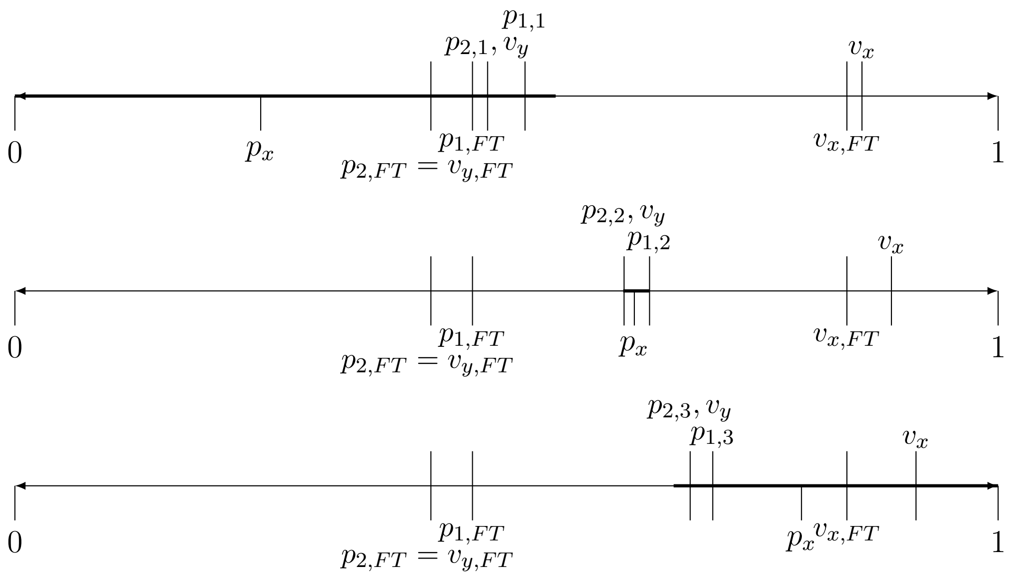

Figure 4: The influence of p_x on the prices and indifferent consumers. Where the thick black line represents the parameter values p_x can take which coincides with Case 1, 2 and 3 respectively.

point for the seller has great implications on the prices and the indifferent consumers, keeping all other parameters constant. This is due to the fact that the seller experiences *reference utility* and also has loss aversion. Therefore less consumers will purchase the object at higher price in both periods, when the reference point increases. In the coming section the influence of δ, α, θ & p_x on the optimal prices and indifferent valued consumers will be discussed.

5.1 First stage price: $p_{1,t}$

The model as formulated by Fudenberg & Tirole (1983) predicts that for a low δ, p_1 is relatively high, but decreases as δ increases. A low δ indicates that the future is severely discounted, for both the consumer and seller. As the future is heavily discounted, not coming to an agreement in the first period is costly. Consumers are therefore willing to accept relatively high prices in the first period, compared to a situation where δ is higher. The reason for this is that as δ becomes higher, waiting becomes more attractive for the consumers. This since consumers expect p_2 to be lower than p_1. More consumers will be interested in purchasing in $t = 2$ for p_2 which gives an incentive for the seller to lower p_1 in order to sell a larger quantity in the first period. Furthermore the seller also experiences benefits from a higher discount factor as selling in the second period becomes more attractive (waiting is less costly). However as δ becomes even larger (at around $\delta = \frac{2}{3}$), p_1 increases again. After this point it becomes more attractive to sell to the high valued consumers in t_1 at a higher price, and to the lower valued consumers in t_2. Plotting p_1 against δ leads to the described U-shaped curve, as can be seen in Figure 6.

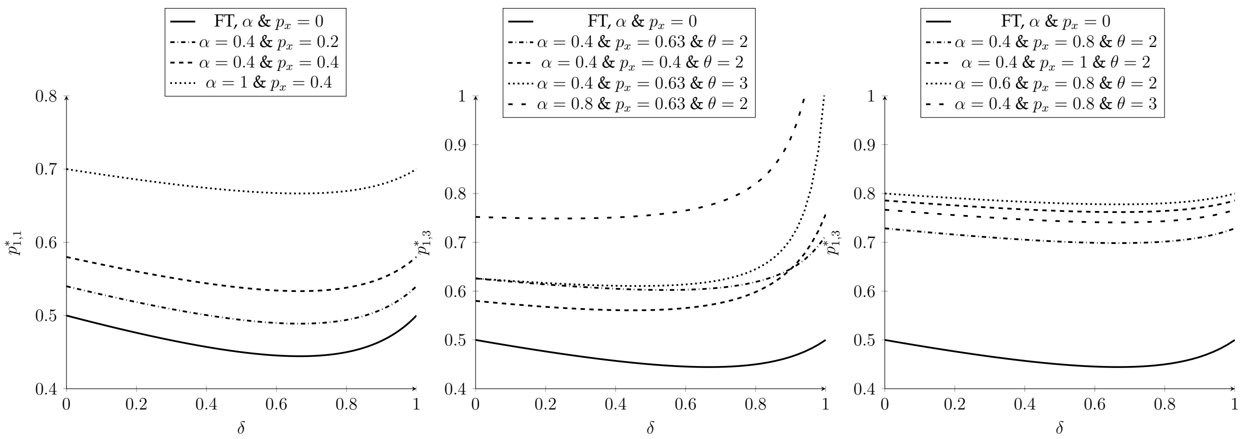

Figure 5: The effects of δ on the first period price in Case 1, 2 and 3 respectively.

In the three Cases in Figure 5, taking α as a given, an increase in p_x increases the first period price for all instances of δ. This by cause of the reference utility becoming increasingly important. Not increasing the first period price would decrease the feeling of a gain in the first period ($p_1 - p_x$). Therefore in order to increase the reference utility the price in the first period should increase. The same reasoning applies when p_x is a given and α is adjusted. The more the emphasis is on the reference utility, the more the seller is inclined to adjust prices in order to reconcile its lost gains. The seller will sell a smaller quantity but at a higher price and feel less lost gains. Furthermore looking at Case 3 the same reasoning applies as before for a given α and p_x, but also for θ this applies. The higher is the loss aversion of the seller - the more will the losses be felt - the more incentive the seller has to make as little loss per transaction as possible. Just as with α and p_x this is done by increasing the pricing scheme in order to diminish the experienced reference utility losses.

However as can be seen in Figure 5 the intuition for Case 2 is a bit different than for Cases 1 and 3. The reason is because the feeling of a loss only occurs in the second period. When the seller becomes more loss averse θ increases, holding other parameters as given, the pricing grows even more exponentially as δ increases. The reasoning for this is that normally the exponential growth is dedicated to the fact that the second period profits become more important as δ increases. However as θ is the feeling of loss, which only occurs in period 2 (in the second case), as δ becomes higher, more emphasis will be given to the second period reference utility in order to decrease the feeling of a loss in this second period. This is best done by increasing its second period prices as δ increases. Which the seller includes in its first stage price. As this strategy, despite selling a smaller quantity, will maximize the seller's utility.

It is good to keep in mind that Case 2 is only a small interval, dependent on whether p_1 is larger than p_x, and p_x is larger than p_2. Which I have already showed, depend on the underlying parameters. Figure 5 is designed to illustrate the behavior of the optimal first period pricing with respect to δ as the other parameter are assumed not to change. In order to calculate in which Case the seller is currently present, the bounds as shown in Appendix E should be consulted.

The overall take-away from this section is that the first period price increases as any parameter increases compared to the benchmark of Fudenberg & Tirole (1983). In other words, the more important reference utility becomes for the seller (higher α), the higher the expectations (higher p_x), or the more loss averse the seller is (higher θ), prices in the first period will increase.

5.2 Second stage price: $p_{2,t}$

In the model as formulated by Fudenberg & Tirole (1983) the second stage price (p_2) increases exponentially in δ. As δ reaches its limit of 1, p_2 becomes equal to p_1. When $\delta < 1$ all values of p_2 will below that of the first stage price, $p_1 > p_2$. For high values of δ, which implies the second period to be relatively important compared to the first period, p_2 will also be relatively high. The lower δ becomes, the relatively less important the second stage becomes, leading to a relative low p_2. As mentioned before, a low δ implies that consumers (and sellers) find waiting to be relatively unattractive and therefore are less inclined to postpone the purchasing decision from the first period to the second period. The consumer will therefore want to be compensated in order to postpone the purchasing decision to the second stage by having to pay a lower second stage price. The seller on the other hand has the incentive to obey to these preferences by the consumer in order to sell to consumers who would otherwise have a too low valuation to make the purchasing decision in the first period, therefore creating profits in the second period.

Looking at Figure 6 for the separate cases, it can be seen that α and p_x each have a positive influence on the second period prices. The same applies for θ and when δ increases as well. As α increases the seller's preferences change to be more dependent on the reference utility part. Therefore the seller experiences higher utility as the second period price increases relative to the reference point. The same reasoning applies to the reference point. As p_x increases the optimal strategy for the seller is also

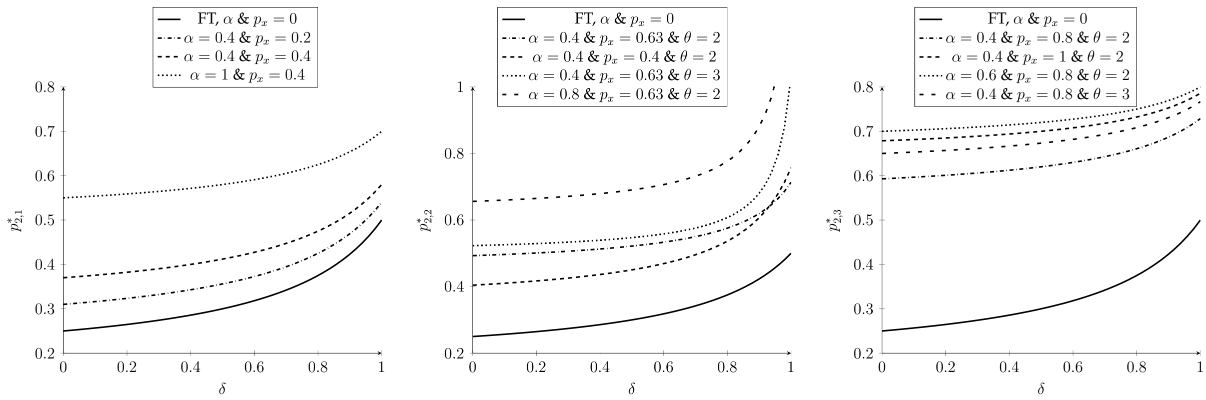

Figure 6: The effects of δ on the second period price in Case 1, 2 and 3 respectively.

to increase p_2. This is due to the fact that the reference utility will feel less like a gain as p_x increases, or even as a loss if p_x becomes large enough. In order to increase the 'feeling' of selling for an acceptable price, the seller wishes to sell for a higher second period price as the reference point increases. The parameter θ has the property that it magnifies the feeling of a loss as the seller's losses are felt much more severely than gains of equal magnitude, it is beneficial to diminish this feeling as much as possible. This is done by increasing p_2 more (exponentially) when the relative importance of the second period also increases (δ increases). As the losses will be felt more severely the seller's best strategy will be to diminish the feeling of a loss, which is thus done by increasing the second period price. This can best be seen in Case 2. Here the feeling of a loss is only felt in the second period, therefore increasing the relative importance of the second period it is intuitive that the seller would argue for a higher second period price when θ is also high, as δ increases.

So far it can be seen that this model compared to the benchmark case of Fudenberg & Tirole (1983) will always have higher prices in both stages for any positive value for the parameters (α, p_x and θ). Now it becomes interesting to see where the indifferent consumers are located, compared to the benchmark case.

5.3 Indifferent consumer first period, $v_{x,t}$

As shown in Fudenberg and Tirole's (1983) model, the consumer which is indifferent between purchasing in the first period and in the second period, has a valuation which increases in δ. The relatively more important the second period becomes for the buyer, the more the buyer is willing to wait in order to decide to purchase the object. This results in the indifferent consumer in the first period to have a valuation which increases in δ. Furthermore, as δ increases the first period price also increases, making the purchasing decision less attractive in the first period. However the discount factor argument and increasing price in the first period are related to each other: The higher the price in the first period, the more attractive it becomes to postpone the purchasing decision.

Now looking at this paper's model, the indifferent consumer in the first period increases in either of the following parameters (ceteris paribus): α, p_x and θ, as can be

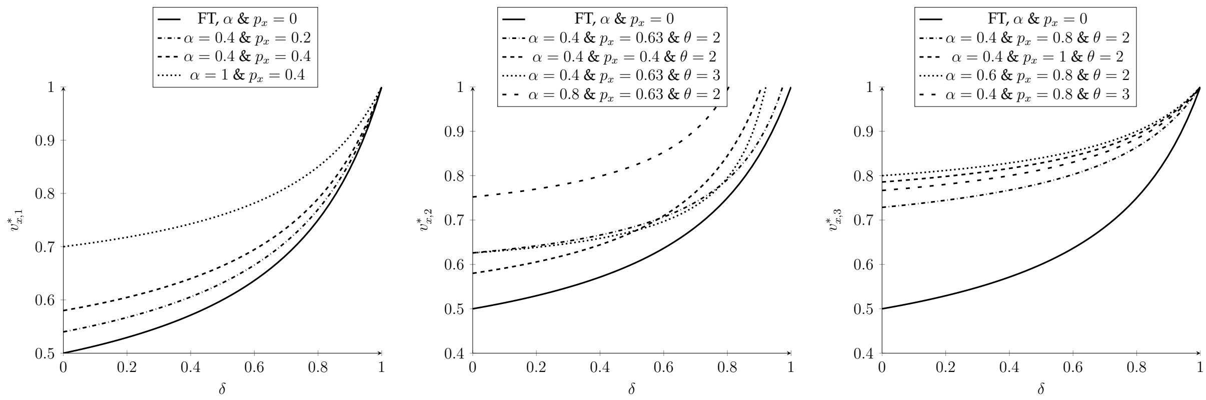

Figure 7: The effects of δ on the first period indifferent consumer in Case 1, 2 and 3 respectively.

seen in Figure 7. The higher α becomes, the more interesting it becomes for the seller to increase the prices. Therefore the indifferent consumer in the first period will be inclined to postpone the purchasing decision to the second stage, where the prices are expected to be lower. The same reason applies to both p_x and θ. The higher p_x becomes, the smaller will be the gains (or the larger the feeling of a loss) for the seller. This gives the incentive to increase the first period price, which will as mentioned before give the incentive for the indifferent consumer in the first period to postpone the purchasing decision to the second period, where the prices are expected to be lower. The higher θ becomes, the larger will be the feeling of a loss, in the reference utility part of the seller. This gives the incentive for the seller to increase the prices in order to diminish the feeling of a loss. Which again, gives the incentive for the indifferent consumer to postpone the purchasing decision to the second period. Also as the discount factor δ increases, implying the consumer becomes more patient, the value of the indifferent consumer in the first period increases as it becomes more attractive to postpone the purchasing decision. The model formulated in this paper therefore has the property that the indifferent consumer in the first period and the first period price move along the same direction as the underlying parameters change.

5.4 indifferent consumer second period: $v_{y,t}$

The indifferent consumer in the second period, as formulated by Fudenberg & Tirole's (1983) model increases in δ. As δ increases the second period becomes relatively more important thus more consumers would be interested in purchasing in the second period. Also the second period indifferent consumer follows the second period prices formulated by the seller. As the indifferent consumer in the second period is indifferent between buying and not buying when $v = p_2$. If the valuation of the buyer is larger than p_2 he will always purchase, either in the first or in the second period. Therefore, as p_2 is formed, it also forms the benchmark for the formation of v_y.

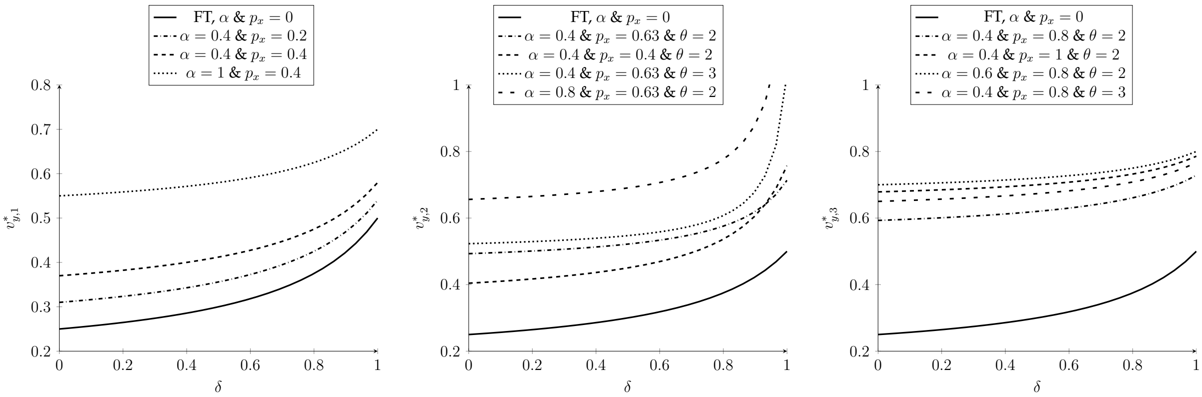

Figure 8: The effects of δ on the second period indifferent consumer in Case 1, 2 and 3 respectively.

In this paper's model for any positive value of the parameters α, p_x, θ and δ the second period indifferent consumer will lie above the indifferent consumer as formulated by Fudenberg & Tirole (1983). This is clearly depicted in Figure 8. The reason for this is that these parameters have a positive influence on the second period price (as mentioned before). And the decision to purchase in the second period (or not) depends on the fact if the consumer's valuation for the object is in fact larger than the price which has to be pay. The consumer will in fact deem it worthy to purchase the object until the point is reached where the valuation for the good is equal to the second period price. For a detailed overview of the comparative statistics the reader is directed to Appendix F.

To summarize the previous findings Table 1 is constructed with the parameter's effects on the optimal first and second period prices and the indifferent consumers in each period:

	p_1	p_2 (& v_y)	v_x
α	$\Uparrow$	$\Uparrow$	$\Uparrow$
p_x	$\Uparrow$	$\Uparrow$	$\Uparrow$
θ	$\Uparrow$	$\Uparrow$	$\Uparrow$
δ	First $\Downarrow$, then $\Uparrow$	$\Uparrow$	$\Uparrow$

Table 1: The summarized effects of the parameters α, δ, θ and p_x on the prices and indifferent consumers.

The results thus far show a clear trend of the prices and valuation of the indifferent consumer which moves move up as any parameter increases, except for the first period price with regards to a low value of δ, which has a convex form. What can easily be seen from this is that the consumers are definitely worse of in this situation if they decide to purchase the object compared to the situation without reference points and loss aversion for the seller. If the consumer does not decide to purchase the object in this model, and conditional on the corresponding valuation of the consumer for the object would also not high enough to purchase purchase in any stage in Fudenberg and Tirole's (1983) model, then the conclusion can be drawn that the consumer would not be worse-off, but would rather enjoy zero utility in both situations.

However, what is still inconclusive is if the seller is better or worse off in this new situation. Of course the model in this paper examines utility for the seller, whereas Fudenberg and Tirole (1983) examine profits of the seller, making it ambiguous if the seller is necessarily better or worse-off in this new situation. Comparing utility with profits will therefore not be a fair comparison. However, this model can compare the seller's profits compared to the seller's profits as proposed in Fudenberg and Tirole's (1983) setting. So far it has been shown that the prices and indifferent valued consumer in each stage increases in this model. However, these prices and indifferent valued consumers do not necessarily change at the same rate. This as the prices might increase at a higher rate than the indifferent consumer's valuation. If this is true, it would lead to the fact that a few less consumers purchase the object, however at a relatively higher price. This could in fact increase total revenue of the seller. This will be examined in the next section.

5.5 Profits

The profits of the seller are defined by the quantity sold times the price in each period, whilst taking the discount factor into account. According to the model of Fudenberg & Tirole (1983) the profits decrease as δ increases until about $2/3$, after which the profits start to increase again as δ increases. This is due to the fact that the prices in the first period decrease until δ is large enough to ask higher first period prices as also relatively high second period prices can be asked, which maximizes profits, since the indifferent valued consumer puts more weight into buying in the second period as well after δ becomes high enough.

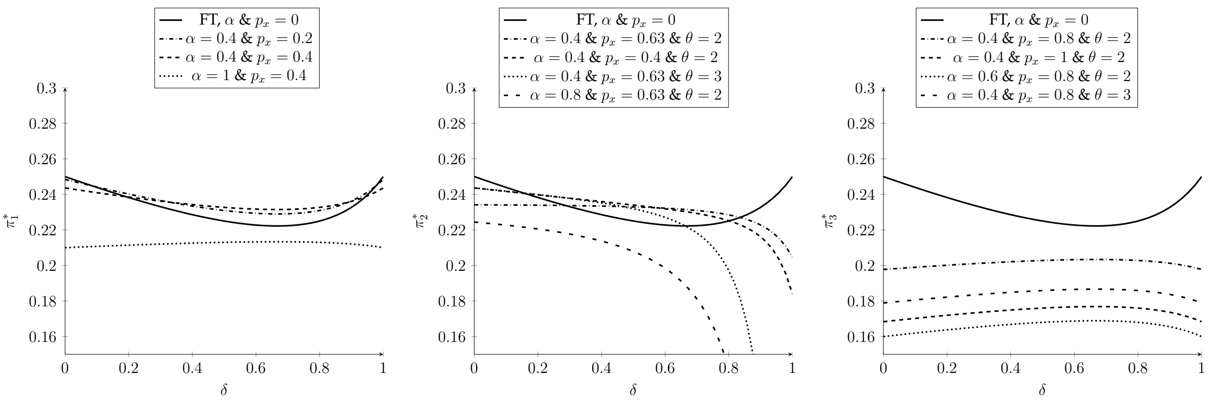

Figure 9: The effects of δ on the on overall profits. Case 1, 2 & 3 respectively.

This mechanism is still in place in this paper's model. Very interestingly the finding is that as δ is not too low, and not too high, it can actually increase the seller's profits in the first and second case, dependent on the other parameter values as can be seen in Figure 9. The reason for this is that as the importance of the reference point increases or if the degree of loss aversion increases, the seller will ask higher prices in the first and second period, which will also increase the value of the indifferent consumer in the first period. However, the indifferent consumer's valuation in the first period increases at a lower rate than the first period price in Case 1 and 2, under certain parameter values. This leads to the conclusion that profits can in fact increase in the model formulated in this paper. In Case 3 the profits are always lower as the seller charges prices which increase the indifferent valued consumer's in each period to such an extent that the the income lost by selling to a smaller quantity are not regained by the increased price.

5.6 Case A & Case B

The Cases A and B are only a small interval of the total analyzable region in this model and will be discussed in this section. Their interval regions and existence depend on certain parameters of δ, α and θ. To see how the pricing and indifferent consumers change as certain parameters change for these Cases the reader is redirected to Appendix E.

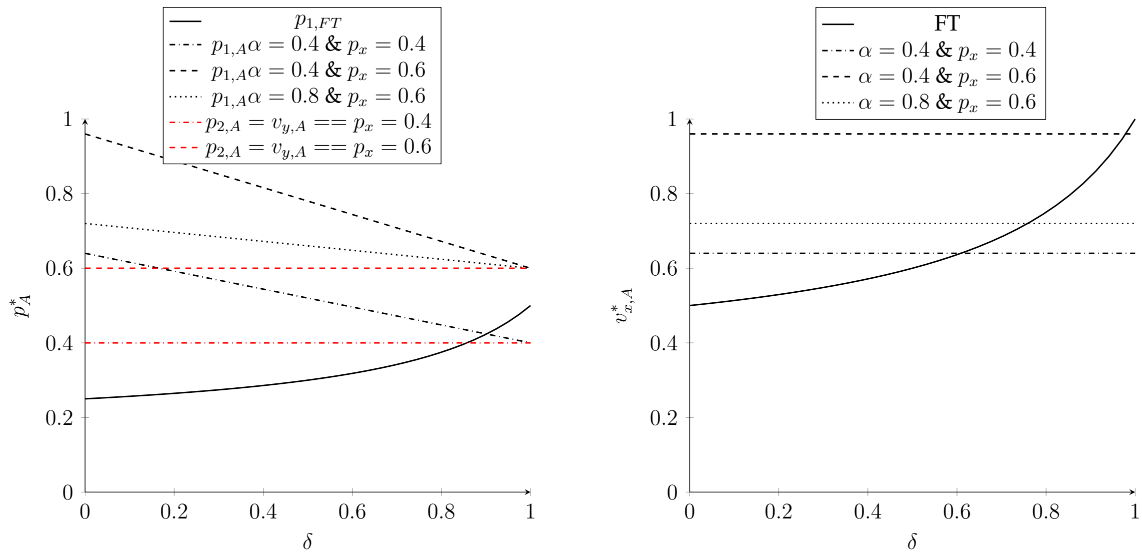

Figure 10: The effects of δ on the optimal prices and indifferent valued consumer in Case A

Figure 10 illustrates the pricing of the seller for Case A & B. For Case A the first period price increases in p_x. This is due to the fact that when p_x increases, the second period price increases[14]. As the reference point of the seller increases, the optimal first period price the seller asks in Case A will also be higher. This in order to keep the optimal second period price constant at p_x. Asking a lower price would force the seller to optimally deviate from this strategy, which the seller does not want as he does not

[14]As p_2 is fixed at p_x in order to not experience the feeling of a loss in the reference utility

want to experience the feeling of a loss yet[15]. Furthermore, $p_{1,A}$ decreases in α, this is due to the fact that as α increases more emphasis is on the *reference utility* of the first period[16]. As the seller is still selling for a price which is above its reference point, the incentive is still there to lower the first period price in order to keep the second period price equal to p_x in order to not receive the sensation of a loss yet in the *reference utility*. Also $p_{1,A}$ decreases in δ, implying that as the second period becomes more important, the seller asks a lower price in the first period in order to make the decision to purchase in the first period more attractive. The reason for this is that as δ becomes higher it becomes more attractive for the consumer to postpone the purchasing decision to the second period, where $p_2 = p_x$. In order to sell to a larger amount of consumers in the first period, the first period price decreases as the discount factor δ increases. These two effects offset the change in the valuation of the first period indifferent consumer, as he would be inclined to postpone the purchasing decision as he becomes more patient, however the lower price in the first period is now a large enough incentive to purchase in the first period. As can be seen in Figure 14, the valuation of the indifferent consumer remains constant. This is important for the seller, as the seller wants the second period price to be exactly equal to the reference point in Case A. Therefore it is crucial that the seller adjusts the first period price in order to optimally set the second period price exactly equal to p_x, and therefore not receiving the sensation of a loss in the second period. This is the reason $p_{2,A}$ and $v_{y,A}$ remain constant. The optimal first period price is adjusted in order to keep $p_{2,A}$ optimally equal to p_x, which in turn keeps the indifferent consumer in the first period at a constant valuation.

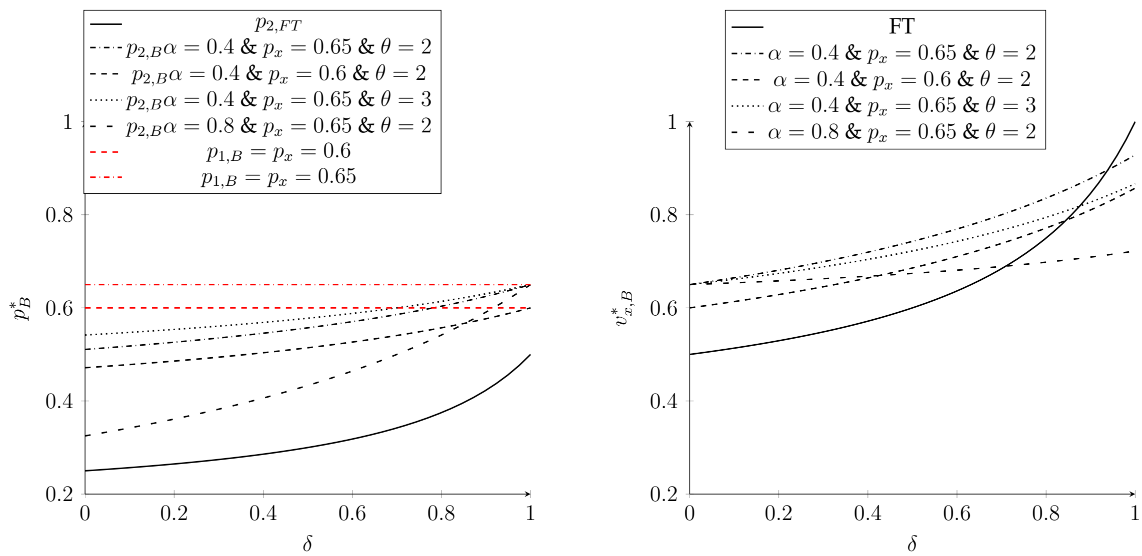

Figure 11: The effects of δ on the optimal prices and indifferent valued consumer in Case B

For Case B the reasoning is almost identical identical, but the seller commits itself

[15]His reference point is not high enough yet to justify entering Case 2, and thus feeling the sensation of a loss

[16]As $p_{2,A}$ is equal to the reference point there will be no feeling of a gain nor loss in the second period

to a first period price which is equal to p_x. By imposing this constraint on the first period price the seller postpones entering the next case, and consequently therefore does not experience the sensation of a loss in the first period. If the seller is located in Case B the reference point is not high enough to strategically ask a lower first period price. committing to $p_{1,B} = p_x$ is therefore the optimal strategy. As δ increases $p_{2,B}$ tends to p_x at $\delta = 1$ at that point the second period price becomes as large as possible: $p_{2,B} = p_x$. When α increases, the second period price decreases. This as an increase in α shows that the emphasis on the *reference utility* increases. In Case B the *reference utility* is only experienced in the second period, in order to diminish the feeling of a loss in this part, the second period price would be expected to increase in α. However, the reason it does not increase in α is because currently the reference point for the seller is still low enough to have the incentive to price according to $p_{1,B} = p_x$. Since the seller's best strategy is to price according a fixed p_x in the first period, the seller must price according to its best response to this first period price, in the second period. Therefore, as the seller keeps p_1 constant, the best response in the second period will be to decrease p_2 as the α increases. The commitment for the first period price to p_x also applies to an increase in θ, as the seller becomes more loss averse, the optimal strategy will become to increase the second periods price for low values of δ, whereas the second period price tends to p_x as δ increases. Here the reasoning is that the optimal strategy in the second period, with a higher θ, will be to ask a relatively higher price as the feeling of a loss is multiplied by a fixed feeling of a loss per transaction. Nevertheless it becomes clear that the optimal prices in Case B increase compared to the benchmark Case.

The indifferent valued consumer's valuation in Case B increases in α, p_x and θ. This as these parameters increase the optimal second period price in Case B. The seller must increase the optimal second period price in such a manner that the seller does not want to price lower in the first period, this as the seller is committed to $p_{2,A} = p_x$, as was discussed in Section 4.5. It is shown in Appendix F how in Case A & B the prices and the indifferent valued consumer change under varying parameter values.

The profits in the benchmark model of Fudenberg & Tirole (1983) are assumed to be convex in δ. Which was already discussed in Section 5.5.

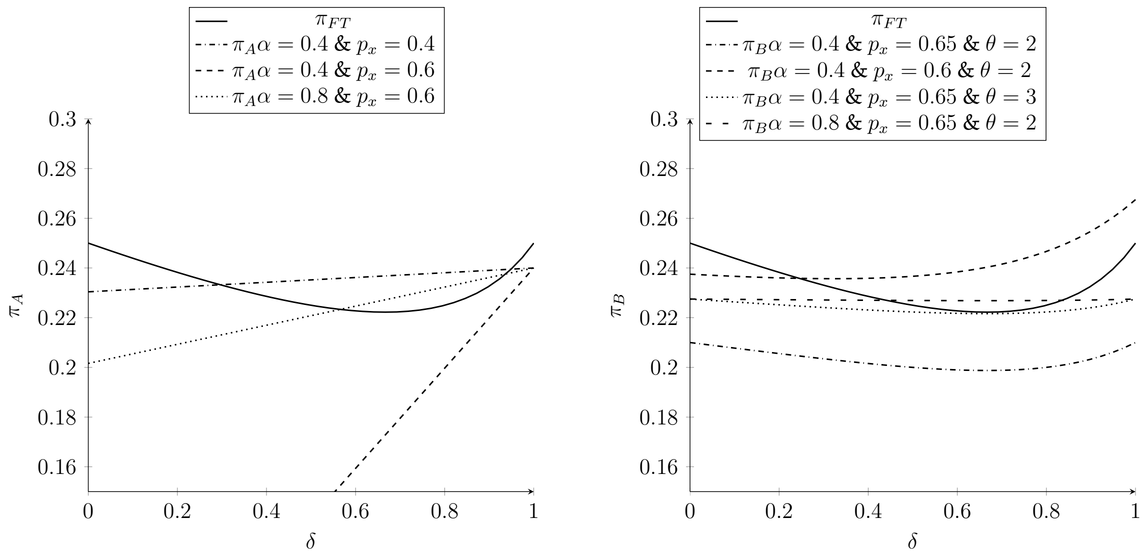

Figure 12: The effects of δ on the optimal prices and indifferent valued consumer in Case A

Figure 12 shows the profits for Cases A and B. Profits increase when δ is not too high and not too low. Furthermore the profits increase as both α and p_x are relatively low. This is since when these parameters are relatively low, the seller sets the prices in the first and second period in such a manner that the indifferent consumer in the first period is relatively high compared to the first period price. This in turn creates are relatively large profit, even larger than the benchmark case of Fudenber & Tirole (1983). As discussed earlier, the increase in profits is due to the fact that the increase in first period prices increases at a higher rate than the indifferent consumer, consequently leading to greater profits. The only reason the seller can credible ask these higher prices is due to the fact that the seller also cares about not experiencing the sensation of a loss in the *reference utility*, therefore the consumers acknowledge that the seller will optimally maximize its utility, which results in higher first and second period prices. This in turn can yield an indifferent consumer which still purchases the object in the first at a relatively lower valuation[17]. This in turn can create a larger overall profit for the seller.

5.7 Appended reference point

So far it is assumed that the reference point is included in the overall utility of the seller. This is done by multiplying the selling utility with $(1 - \alpha)$ and the reference utility with (α). However it could be argued that it would make sense to append instead of incorporate the selling utility with the *reference utility*. This since the reference part could be seen as an additional gain, or as an additional loss. In order to realize this the model can be adjusted by excluding the $(1 - \alpha)$ in front of the selling utility. Then the *reference utility* will be an additional feeling with regard to the *selling utility*. These results are shown in Appendix D. The results show the same findings as

[17] The indifferent consumer in the first period purchases at relatively lower level than the increased first period price.

the previous sections: The prices and the location of the indifferent valued consumers increase as the seller experiences reference points and is assumed to be loss averse. This robustness check shows that the previous findings are reliable under changing circumstances.

6 Discussion

In order to analyze the behavior of the seller and consumers several assumptions had to be implemented to the model. It is important to stress that these assumptions create a more elegant model, with having the downside of not necessarily fulfilling the most realistic setting. This stylized version of reality however does contribute to the understanding of the complex dynamics which are inherent to everyday life. One assumption in this paper which could be examined more in future research is the fact that the consumers are drawn up as loss-neutral, which in reality is not the case which became clear in the literature review. Therefore future research could incorporate a model of a loss averse seller which has a reference point (perhaps even endogenously determined) with also consumers which are loss-averse. This would create a model which is a step closer to reality. Another note-worthy addition future research could look into is the form of loss aversion itself. In this model the seller is assumed to have linear loss aversion. However Kahneman and Tversky (1979) argue that loss aversion is in fact not linear, but the feeling of loss changes at certain thresholds, for instance small losses are felt more intensely than larger ones. This model only uses two-time periods, which is a good first step to analyze a theoretical model, but in reality bargaining processes can include many more rounds which could influence the decision to price a certain way, or it could affect the consumer's decision to purchase. Another aspect future research can look at is the discount factor: δ. In this paper it is assumed equal for the consumer and the seller, however it could very well be that these two agents have a very different discount factor. Also, this paper makes the assumption that the consumers valuation is uniformly distributed. Perhaps a different distribution would give different results. All these suggestions for future research could be a meaningful contribution to the field of economics and push the understanding of behavioral aspects of sellers (and consumers) to a new level.

7 Conclusion

This paper's main focus is to examine the bargaining effects of a seller who is assumed to be loss-averse and has an exogenously determined reference point with a consumer who is assumed to maximize its pay-off. By introducing reference points the seller has certain expectations about the value of the object being sold. These expectations in turn are responsible for the pricing scheme the seller introduces. Since the seller is also assumed loss-averse, losses are felt more severely than gains of equal magnitude, the pricing scheme of the seller also changes as soon as the realized gains fall below the reference point. The tendency of the seller will be such to increase the prices in order to meet his own expectations compared to the situation where there are no reference points for the seller. This will be disadvantageous for the consumer who decides to purchase the object. As the prices increase, less consumers will decide to purchase the object in either stage, as their value for the object will become lower than the price requested by the seller for the object. Not only the position of the consumers who purchase the object worsens, but also the position of consumers who now do not purchase the object anymore, but would purchase the object if the seller priced as if he was not assumed loss-averse with reference points. Therefore the most crucial finding in this paper is that loss aversion in combination with the reference point creates a credible strategy for the seller to demand higher first and second period prices. Even though the prices will be higher which leads to less consumers who purchase the object, in some instances these prices can increase more rapidly than the marginal consumer's value for the object. This in turns implies that profits for the seller may actually increase under the right circumstances.

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Appendices

A Solving the benchmark: Fudenberg & Tirole

In the coming section the simple model as formulated by Fudenberg and Tirole (1983) will be solved. The underlying principle will coincide as the benchmark for the appended work later on in this paper.

The utility for the consumers is denoted as follows

$$U_{1,buyer} = v_x - p_1$$

$$U_{2,buyer} = v_y - p_2$$

The indifferent consumer in the first period is indifferent between buying in $t = 1$ and $t = 2$ at price p_1, this consumer's valuation is denoted by v_x and there is a consumer which is indifferent at buying in $t = 2$ and not buying at all at price p_2, this consumer's valuation is denoted by v_y. The discount rate for the seller is denoted as δ_s, and for the consumer as δ_b, with $\delta_s, \delta_b \in [0, 1]$.

The first step in order to solve the model with backward induction is to find out what the value (v_y) for the indifferent consumer is at $t = 2$. The consumer with this valuation is indifferent between buying in the second period and not buying at all. This indifferent consumer can be found by equalizing the second period indifferent consumer's valuation with the second period price. The explanation for this is that there is no credible strategy for the consumer not to buy at $t = 2$ for p_2 if the valuation (v_y) of this consumer is at least equal to p_2. Resulting in:

$$v_y - p_2 = 0 \tag{28}$$

This yields the consumer who is indifferent between buying in the second period and not buying at all, with the valuation thus as shown in Equation28 being equal to $v_y = p_2$.

Thus resulting with the indifferent consumer's valuation in the first period being equal to $U_{1,buyer} = \delta_b(U_{2,buyer})$.

$$v_x - p_1 = \delta_b(v_x - p_2) \tag{29}$$

However, p_2 in Equation29 is still a function of v_x. In order to find the optimal value of p_2 the seller's behavior must now be optimized.

As Fudenberg and Tirole (1983) showed in their paper, the seller updates its information on the buyers conditional on the acceptance (or rejection) of the first period price. The seller knows that the consumer's valuation is uniformly distributed. With means of backwards induction the seller can optimize its overall profits, by first looking at the optimization of the second period profits conditional on the first period price being rejected:

$$(\pi_2 | p_{1 \text{ is rejected}}) = \frac{(v_x - v_y)}{v_x} p_2 \tag{30}$$

Taking into account $v_y = p_2$ and maximizing Equation 30 with regards to p_2 yields:

$$\frac{\partial \pi_2}{\partial p_2} = 1 - \frac{2p_2}{v_x} \tag{31}$$

Solving Equation 31 for the second period price yields:

$$p_2 = \frac{v_x}{2} \tag{32}$$

In order for the consumers to make the decision to buy in the first period instead of in the second period, the utility the consumer derives from the decision to buy in the first stage must be equal or larger compared to the utility gained by buying in the second period. Therefore the consumer needs to be found which is indifferent between buying in either period.This consumer can be found as shown in 29 this leads to the following equality:

$$v_x - p_1 = \delta_b(v_x - p_2)$$

Filling in p_2 yields:

$$v_x - p_1 = \delta_b(v_x - \frac{v_x}{2}) \tag{33}$$

Solving Equation 33 for v_x the indifferent consumer's valuation yields:

$$v_x = \frac{2p_1}{2 - \delta_b} \tag{34}$$

Then in the first stage the seller behaves in order to maximize overall profits:

$$\pi_{overall} = (1 - v_x)p_1 + \delta_s(v_x - v_y)p_2 \tag{35}$$

Substituting Equation 32, for both p_2 and v_y, and Equation 34 in to Equation 35 yields:

$$\pi_{overall} = (1 - \frac{2p_1}{2 - \delta_b})p_1 + \delta_s(\frac{2p_1}{2 - \delta_b} - \frac{\frac{2p_1}{2-\delta_b}}{2})\frac{\frac{2p_1}{2-\delta_b}}{2}) \tag{36}$$

Taking the partial derivative of Equation 36 with regards to p_1, and equalizing for p_1 yields:

$$\pi_{overall} = (1 - \frac{2p_1}{2 - \delta_b})p_1 + \delta_s(\frac{2p_1}{2 - \delta_b} - \frac{\frac{2p_1}{2-\delta_b}}{2})\frac{\frac{2p_1}{2-\delta_b}}{2}) \tag{37}$$

$$\frac{\partial \pi_{overall}}{\partial p_1} = 0 \tag{38}$$

$$p_1 = \frac{(2 - \delta_b)^2}{2(4 - 2\delta_b - \delta_s)} \tag{39}$$

I have shown that the optimal price in the first period is equal to:
$p_1^* = \frac{(2-\delta_b)^2}{2(4-2\delta_b-\delta_s)}$.

$$p_1^* = \frac{(2 - \delta_b)^2}{(8 - 4\delta_b - 2\delta_s)} \tag{40}$$

The optimal first period price can now be substituted in Equation 34 to calculate the value of the indifferent consumer in the first period:

$v_x^* = \frac{2\frac{(2-\delta_b)^2}{(8-4\delta_b-2\delta_s)}}{2-\delta_b}$

$$v_x^* = \frac{(2-\delta_b)}{(4-2\delta_b-\delta_s)} \tag{41}$$

The indifferent consumer's valuation in the first period, which is shown in Equation 41, can now be substituted in Equation 32 in order to calculate the optimal second period price, which is equal to the indifferent consumer in the second period, as shown in Equation 28:

$p_2^* = \frac{\frac{(2-\delta_b)}{(4-2\delta_b-\delta_s)}}{2}$

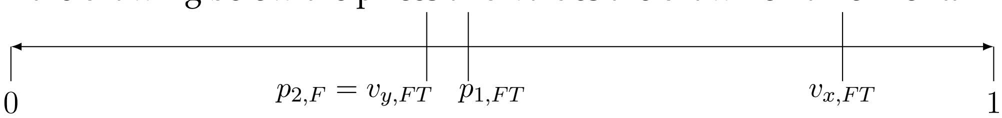
$$p_2^* = \frac{(2-\delta_b)}{(8-4\delta_b-2\delta_s)} \tag{42}$$

$v_y^* = \frac{\frac{(2-\delta_b)}{(4-2\delta_b-\delta_s)}}{2}$

$$v_y^* = \frac{(2-\delta_b)}{(8-4\delta_b-2\delta_s)} \tag{43}$$

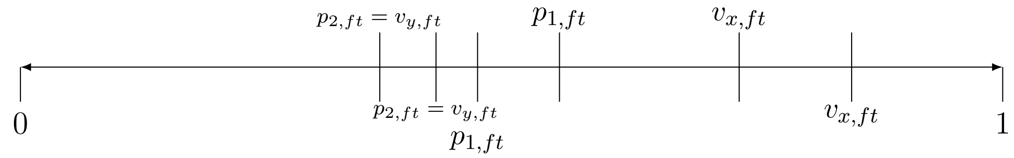
Comparative Statistics: Fudeberg & Tirole

In this paragraph some comparative statistics will be examined inorder to show the implications of the model. Filling in $\delta_b = \delta_s == 0.9$ yield the following results:
$p_1^* = 0.4654$
$v_x^* = 0.8462$
$v_y^* = p_2^* = 0.4231$
In the drawing below the prices and values are drawn on a horizontal line.



Now it will be interesting to compare the previous situation where delta's are the same to the one where $\delta_s > \delta_b$. Assume $\delta_s = 0.9$ and $\delta_b = 0.5$, this yields the following values:
$p_1^* = 0.54878$
$v_x^* = 0.7317$
$v_y^* = p_2^* = 0.36585$
Filling in the interval depicted below, where the bottom values are equivalent to the case with equal discounting, and the values depicted above make use of the following

parameters: $\delta_s > \delta_b$ [$\delta_s = 0.9; \delta_b = 0.5$].



Proposition 1: *The optimal first period price depends negatively on the relative discount factor $\frac{\delta_b}{\delta_s}$, where as the optimal second period prices depend positively on this parameter.*

Proof of 1: *It can easily be seen if the buyer discounts the good more, thus δ_b becomes smaller, that the optimal first period price does in fact increase, however eventually the second period price decreases more compared to the optimal prices with equal δ's. Therefore the conclusion can be drawn that more consumers will buy the good, as a lower δ_b will result in a larger area of customers, namely: $1 - v_y$. Eventually this will be beneficial for the consumers as some of them would not have bought the good for the higher second period price, and now they would.Taking the partial of p_1 and p_2 with regards to δ_b yields a positive value. Resulting in the proof that if δ_b increases, thus the discounting becomes closer to 1 (which is equal to not discounting at all) in which the prices in p_1 and p_2 increase. Analogously, decreasing the discount factor for buyers is beneficial for the buyers. Therefore when the buyer becomes discounts the future heavily, the seller acknowledges this and reacts appropriately. The seller wants to maximize profits and does this by increasing the first period price, but decreasing the second period price in order to sell to the heavily discounting consumers, which would otherwise not purchase the object.*

B Analyzing Cases 2 & 3

Case 2: Gain period 1, loss period 2

This section will analyze Case 2, Gain/Loss, therefore $p_1 \geq p_x \geq p_2$. The indifferent consumer is indifferent between buying in $t = 1$ and $t = 2$ at price p_1, this consumer's valuation is denoted by v_x and there is a consumer which is indifferent at buying in $t = 2$ and not buying at all at price p_2, this consumer's valuation is denoted by v_y. The discount rate for the seller and consumers are identical and equal to $\delta \in [0, 1]$.

The first step in order to solve the model with backward induction is to find out what the value (v_y) of the indifferent consumer is at $t = 2$, which is indifferent between buying in the second period and not buying at all. This can be found by equalizing the second period indifferent consumer's valuation with the second period price. Resulting in: $v_y - p_2 = 0$. This yields the consumer who is indifferent between buying in the second period and not buying at all, with the valuation being equal to $v_y = p_2$. Therefore the indifferent consumer's valuation in the first period is when $U_{1,buyer} = \delta U_{2,buyer}$.

$$v_x - p_1 = \delta_b(v_x - p_2) \tag{44}$$

However, p_2 is still a function of v_x. In order to find the optimal value of p_2 the seller's behaviour must now be optimized.

In this paragraph we analyze the situation where the seller sets the first period price above its exogenous reference point (p_x), but the second period price below its exogenous reference point. The seller updates its information about the buyer after the acceptance (or rejection) of the first period price. With means of backwards induction the seller can optimize its overall profits, looking first at the optimization of the second period profits, taking into account that the first period price has been rejected.

$$\pi_2 = \frac{(v_x - v_y)}{v_x}((1-\alpha)p_2 + \alpha\theta(p_2 - p_x)) \tag{45}$$

Taking into account $v_y = p_2$ and maximizing the second period profit function with respect to p_2 yields:

$$\frac{\partial \pi_2}{\partial p_2} = 0 \tag{46}$$

Solving this for the second period price yields:

$$p_2 = \frac{v_x(\alpha(\theta-1)+1) + \alpha\theta p_x}{2\alpha(\theta-1)+2} \tag{47}$$

In order for the consumer to make the decision to buy in the first stage instead of in the second stage, the utility the consumer derives from the decision to buy in the first stage must be larger or equal compared to the utility gained by buying in the second period. Therefore the indifferent valued consumer needs to be found: $U_{1,buyer} = U_{2,buyer}$ this leads to the following equality:

$$v_x - p_1 = \delta(v_x - p_2) \tag{48}$$

Filling in p_2 yields:

$$v_x - p_1 = \delta(v_x - \frac{v_x(\alpha(\theta-1)+1) + \alpha\theta p_x}{2\alpha(\theta-1)+2}) \tag{49}$$

Solving this inequality for (v_x) the indifferent consumer yields between buying in $t = 1, t = 2$ becomes:

$$v_x = \frac{\alpha\delta\theta p_x - 2p_1(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)} \tag{50}$$

Then in the first stage the seller behaves in order to maximize overall profits:

$$\pi_{overall} = (1-v_x)((1-\alpha)p_1 + \alpha(p_1 - p_x)) + \delta(v_x - v_y)((1-\alpha)p_2 + \alpha\theta(p_2 - p_x)) \tag{51}$$

Taking the partial derivative of $\pi_{overall}$ with regards to p_1, and equalizing for p_1 yields:

$$\frac{\partial \pi_{overall}}{\partial p_1} = 0 \tag{52}$$

$$p_1^* = \frac{-(\delta-2)^2 + 2\alpha^2(\theta-1)p_x(\delta\theta+\delta-2) + \alpha\left(p_x\left(\delta^2\theta+2\delta-4\right) - (\delta-2)^2(\theta-1)\right)}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{53}$$

With the optimal first period price the optimal other prices and values can be calculated.

$$v_x^* = \frac{\delta + \alpha^2(\theta-1)p_x(\delta\theta-2) + \alpha((\delta-2)(\theta-1)+2p_x(\delta\theta-1)) - 2}{(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{54}$$

$$p_2^* = v_y^* == \frac{\frac{\delta+\alpha^2(\theta-1)p_x(\delta\theta-2)+\alpha((\delta-2)(\theta-1)+2p_x(\delta\theta-1))-2}{\delta(\alpha(\theta-1)+3)-4} + \alpha\theta p_x}{2\alpha(\theta-1)+2} \tag{55}$$

Now values for the parameters need to be filled in. In order to model "exogenous expected value" (p_x). Just as before the interval for p_x needs to be calculated. The appropriate parameter values are as before: $\alpha = 0.4$, $\delta = 0.9$. The missing parameter's value is θ. As shown in the literature review the bound we apply to θ is $\in [1.5, 4]$. With these bounds the decision is made to choose $\theta = 2$.

The interval which p_x can take and still fulfill the requirement of $p_1 \geq p_x \geq p_2$ should be shown in order to determine an appropriate value. The optimal functions have been calculated for p_1^* and p_2^*.

- For p_1 to be larger than p_x it follows that p_x cannot become larger than 0.6456, thus $p_x \leq 0.6456$.

  This is a necessary, but not sufficient condition, furthermore the second period price can also not exceed p_x.

- For p_2 to be smaller than p_x it follows that p_x should not become smaller than 0.61897, thus $p_x \geq 0.61897$

This yields the following interval in where p_x can be located: $p_x \in [0.61897; 0.6456]$

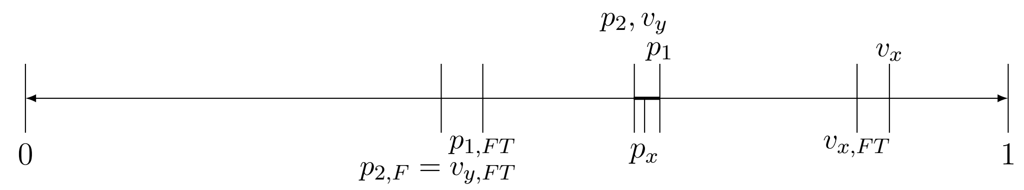
Taking the value $p_x = 0.63$, δ takes the value of 0.9, implying limited discounting and α with value of 0.4, showing that sellers take the reference point into account, but only partially. And $\theta = 2$, this leads to the following optimal values:

$$p_1^* = 0.645532$$

$$v_x^* = 0.879149$$

$$p_2^* = v_y^* == 0.619574$$

This can be compared to the benchmark case where the seller has a different preference. This is depicted in the illustration below. The bottom values depict the benchmark case, the upper values depict Case 2 (gain / loss). The (small) thick line represents all the values p_x could take and for which this case would still be relevant..



Case 3: Loss

This section will analyze the Loss case in both periods, therefore $p_x \geq p_1 \geq p_2$. The indifferent consumer is indifferent between buying in $t = 1$ and $t = 2$ at price p_1, this consumer's valuation is denoted by v_x and there is a consumer which is indifferent at buying in $t = 2$ and not buying at all at price p_2, this consumer's valuation is denoted by v_y. The discount rate for the seller and consumers are identical and equal to $\delta \in [0, 1]$.

The first step in order to solve the model with backward induction is to find out what the value (v_y) of the indifferent consumer is at $t = 2$, which is indifferent between buying in the second period and not buying at all. This can be found by equalizing the second period indifferent consumer's valuation with the second period price. Resulting in: $v_y - p_2 = 0$. This yields the consumer who is indifferent between buying in the second period and not buying at all, with the valuation being equal to $v_y = p_2$. Therefore the indifferent consumer's valuation in the first period is when $U_{1,buyer} = \delta U_{2,buyer}$.

$$v_x - p_1 = \delta(v_x - p_2) \tag{56}$$

However, p_2 is still a function of v_x. In order to find the optimal value of p_2 the seller's behavior must now be optimized.

In this paragraph we analyze the situation where the seller sets the first period price above its exogenous reference point (p_x), but the second period price below its exogenous reference point. The seller updates its information about the buyer after the acceptance (or rejection) of the first period price. With means of backwards induction the seller can optimize its overall profits, looking first at the optimization of the second period profits, taking into account that the first period price has been rejected.

$$\pi_2 = \frac{(v_x - v_y)}{v_x}((1-\alpha)p_2 + \alpha\theta(p_2 - p_x)) \tag{57}$$

Taking into account $v_y = p_2$ and maximizing the second period profit function with respect to p_2 yields:

$$\frac{\partial \pi_2}{\partial p_2} = 0 \tag{58}$$

Solving this for the second period price yields:

$$p_2 = \frac{v_x(\alpha(\theta - 1) + 1) + \alpha\theta p_x}{2\alpha(\theta - 1) + 2} \tag{59}$$

In order for the consumer to make the decision to buy in the first stage instead of in the second stage, the utility the consumer derives from the decision to buy in the first stage must be larger or equal compared to the utility gained by buying in the second period. Therefore the indifferent valued consumer needs to be found: $U_{1,buyer} = U_{2,buyer}$ this leads to the following equality:

$$v_x - p_1 = \delta(v_x - p_2) \tag{60}$$

Filling in p_2 yields:

$$v_x - p_1 = \delta(v_x - \frac{v_x(\alpha(\theta-1)+1) + \alpha\theta p_x}{2\alpha(\theta-1)+2}) \tag{61}$$

Solving this inequality for (v_x) the indifferent consumer yields between buying in $t = 1, t = 2$ becomes:

$$v_x = \frac{\alpha\delta\theta p_x - 2p_1(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)} \tag{62}$$

Then in the first stage the seller behaves in order to maximize overall profits:

$$\pi_{overall} = (1-v_x)((1-\alpha)p_1 + \alpha\theta(p_1 - p_x)) + \delta(v_x - v_y)((1-\alpha)p_2 + \alpha\theta(p_2 - p_x)) \tag{63}$$

$$\begin{aligned}
\pi_{overall} = {} & (1 - \frac{\alpha\delta\theta p_x - 2p_1(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)})((1-\alpha)p_1 + \alpha\theta(p_1 - p_x)) \\
& + \delta(\frac{\alpha\delta\theta p_x - 2p_1(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)} - \frac{(\frac{\alpha\delta\theta p_x - 2p_1(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)})(\alpha(\theta-1)+1) + \alpha\theta p_x}{2\alpha(\theta-1)+2}) \\
& ((1-\alpha)(\frac{(\frac{\alpha\delta\theta p_x - 2p_1(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)})(\alpha(\theta-1)+1) + \alpha\theta p_x}{2\alpha(\theta-1)+2}) + \alpha\theta(\frac{(\frac{\alpha\delta\theta p_x - 2p_1(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)})(\alpha(\theta-1)+1) + \alpha\theta p_x}{2\alpha(\theta-1)+2} - p_x))
\end{aligned} \tag{64}$$

Taking the partial derivative of $\pi_{overall}$ with regards to p_1, and equalizing for p_1 yields:

$$\frac{\partial \pi_{overall}}{\partial p_1} = 0 \tag{65}$$

$$p_1^* = \frac{\alpha\left(\delta^2(\theta(p_x-1)+1) + 2\delta(\theta(p_x+2)-2) - 4(\theta+\theta p_x - 1)\right) - (\delta-2)^2}{2(3\delta-4)(\alpha(\theta-1)+1)} \tag{66}$$

With the optimal first period price the optimal other prices and values can be calculated.

$$v_x^* = \frac{\delta + \alpha(\delta(\theta + 2\theta p_x - 1) - 2(\theta + \theta p_x - 1)) - 2}{(3\delta-4)(\alpha(\theta-1)+1)} \tag{67}$$

$$p_2^* = v_y^* == \frac{\delta + \alpha(\delta(\theta + 5\theta p_x - 1) - 2(\theta + 3\theta p_x - 1)) - 2}{2(3\delta-4)(\alpha(\theta-1)+1)} \tag{68}$$

C Case A

In Case A the restriction is made that the optimal second period price is exactly equal to p_x. This section gives the proof how the optimal first period price is calculated for Case A.
The first step will be to see how the seller optimally chooses $p_{2,A}$, conditional on $p_{1,A}$ being rejected:

$$E(\pi_{2,A}|p_{1,A\text{is rejected}}) = \frac{(v_{x,A} - v_{y,A})}{v_{x,A}}((1-\alpha)p_{2,A} + \alpha(p_{2,A} - p_x))$$

$$\frac{\partial \pi_{2,A}}{\partial p_{2,A}} = 0$$

Solving for $p_{2,A}$ yields:

$$p_{2,A} = \frac{v_{x,A} + \alpha p_x}{2}$$

The consumer who is indifferent at buying at $p_{1,A}$ and $p_{2,A}$ can be found by equalizing the payoff the indifferent valued consumer would receive in either period (taking into account the discount factor):

$$v_{x,A} - p_{1,A} = \delta(v_{x,A} - p_{2,A})$$

Filling in the optimal second period price and solving for the indifferent consumer yields:

$$v_{x,A} = \frac{2p_{1,A} - p_x\alpha\delta}{2-\delta}$$

As illustrated above, it is shown that the optimal second period price is a function of $v_{x,A}$ which is a function of $p_{1,A}$. Case A takes the optimal second period price and imposes the restriction that this is equal to p_x. Plugging in the optimal response functions for $v_{x,A}$ in $p_{2,A}$, conditional on $p_{2,A}$ being equal to p_x and solving for $p_{1,A}$ yields the optimal first period price for which the seller optimally demands a second period price which is exactly equal to p_x.

$$(p_{2,A}|_{(p_{2,A}=p_x)}) : \frac{\frac{2p_{1,A}-p_x\alpha\delta}{2-\delta} + \alpha p_x}{2} = p_x$$

Solving the above equation for $p_{1,A}$ yields:

$$p_{1,A} = p_x(2 - \alpha - \delta + \alpha\delta)$$

filling in the optimal $p_{1,A}$ in $v_{x,A}$ yields:

$$v_{x,A} = \frac{2 * (p_x(2 - \alpha - \delta + \alpha\delta)) - p_x\alpha\delta}{2-\delta}$$

Simplifying yields:

$$v_{x,A} = p_x(2-\alpha)$$

$$p_{2,A} = v_{v,y} == p_x$$

This optimal first period price will result in the second period price being exactly equal to p_x. However, of course as the seller sets this optimal first period price, there must be no incentive to deviate from this strategy. It must therefore be considered for what first period price the seller would still optimally choose p_x in the second period in Case 2. If the optimal strategy for the seller would be to charge a lower price than p_x with the fixed $p_{1,A}$, this will result in the feeling of a loss in the *reference utility* in the second period. If this is true, than it can be concluded that $p_{1,A}$ is a sub-optimal strategy and the seller will not be able to commit to this strategy. Therefore now the seller's optimal first period price will be examined for which the seller's commits to p_x in the second period, in Case 2: Gain / Loss.

The optimal first period price will be found by constraining the second period price to be equal to p_x:

$$p_{2,2}(v_{x,2}(p_{1,2})) = p_x$$

The optimal second period price is a function of the value of the indifferent valued consumer in the first period, which subsequently is a function of optimal first period price. In Appendix B it is shown that the optimal second period price and optimal valuation of the indifferent valued consumer in Case 2 are as follows:

$$p_{2,2} = \frac{v_{x,2}(\alpha(\theta-1)+1)+\alpha\theta p_x}{2\alpha(\theta-1)+2}$$

$$v_{x,2} = \frac{\alpha\delta\theta p_x - 2p_{1,2}(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)}$$

Plugging $v_{x,2}$ into $p_{2,2}$ yields:

$$p_{2,2} = \frac{\frac{\alpha\delta\theta p_x - 2p_{1,2}(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)}(\alpha(\theta-1)+1)+\alpha\theta p_x}{2\alpha(\theta-1)+2}$$

Imposing $p_{2,2} = p_x$ yields:

$$\frac{\frac{\alpha\delta\theta p_x - 2p_{1,2}(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)}(\alpha(\theta-1)+1)+\alpha\theta p_x}{2\alpha(\theta-1)+2} = p_x$$

Solving the above equation for $p_{1,2}|_{(p_{2,2}=p_x)}$ yields:

$$p_{1,2}|_{(p_{2,2}=p_x)} = \frac{p_x(\alpha(\delta+\theta-2)-\delta+2)}{\alpha(\theta-1)+1}$$

It must be true that the first period price in Case A abides to the constraint of being at least equal or larger than the optimal first period price in Case 2, with an imposed second period price of p_x, in order to be incentive compatible for the seller to credibly charge $p_{1,A}$:

$$p_{1,A}|_{(p_{2,A}=p_x)} \geq p_{1,2}|_{(p_{2,2}=p_x)}$$

$$p_x(2-\alpha-\delta+\alpha\delta) \geq \frac{p_x(\alpha(\delta+\theta-2)-\delta+2)}{\alpha(\theta-1)+1}$$

$$p_x(2-\alpha-\delta+\alpha\delta)-\frac{p_x(\alpha(\delta+\theta-2)-\delta+2)}{\alpha(\theta-1)+1}\geq 0$$

Simplifying:

$$\frac{(1-\alpha)\alpha(1-\delta)(\theta-1)p_x}{\alpha(\theta-1)+1}\geq 0$$

As α and δ are larger than 0, and $\theta > 1$ the numerator will always be positive. As $\theta > 1$, and $\alpha > 0$ the denominator will always be positive. Implying that for all values of the parameters the requirements of $p_{1,A}|_{(p_{2,A}=p_x)} \geq p_{1,2}|_{(p_{2,2}=p_x)}$ are fulfilled. Therefore the optimal first period price in Case A fulfills the requirement of incentive compatibility. Therefore $p_{1,A}$ is the optimal pricing strategy in Case A.

D Case B

In Case B the restriction is made that the optimal first period price is exactly equal to p_x. This section gives the proof how the optimal second period price and indifferent valued consumers are calculated for Case B.
The first steps will be to see how the seller optimally chooses $p_{2,B}$, conditional on $p_{1,B}$ being rejected. After which the value of the indifferent valued consumer in the first period can be calculated. This will be exactly the same as in Case 2. The reason for this is that by means of backwards induction, altering the last step of the optimization does not alter the optimization process of the previous steps. Nevertheless, the calculations are shown next:

$$E(\pi_{2,B}|p_{1,B}\text{is rejected})=\frac{(v_{x,B}-v_{y,B})}{v_{x,B}}((1-\alpha)p_{2,B}+\alpha\theta(p_{2,B}-p_x))$$

$$\frac{\partial\pi_{2,B}}{\partial p_{2,B}}=0$$

Solving for $p_{2,B}$ yields:

$$p_{2,B}=\frac{v_{x,B}(\alpha(\theta-1)+1)+\alpha\theta p_x}{2\alpha(\theta-1)+2}$$

The consumer who is indifferent at buying at $p_{1,B}$ and $p_{2,B}$ can be found by equalizing the payoff the indifferent valued consumer would receive in either period (taking into account the discount factor):

$$v_{x,B}-p_{1,B}=\delta(v_{x,B}-p_{2,B})$$

Filling in the optimal second period price and solving for the indifferent consumer yields:

$$v_{x,B}=\frac{\alpha\delta\theta p_x-2p_{1,B}(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)}$$

Plugging $v_{x,B}$ into $p_{2,B}$ yields:

$$p_{2,B}=\frac{\frac{\alpha\delta\theta p_x-2p_{1,B}(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)}(\alpha(\theta-1)+1)+\alpha\theta p_x}{2\alpha(\theta-1)+2}$$

Imposing $p_{1,B} = p_x$ yields:

$$p_{1,B} = p_x$$
$$v_{x,B} = \frac{p_x(\alpha((\delta-2)\theta+2)-2)}{(\delta-2)(\alpha(\theta-1)+1)}$$
$$p_{2,B} = v_{y,B} == \frac{p_x(\alpha((\delta-2)\theta+1)-1)}{(\delta-2)(\alpha(\theta-1)+1)}$$

Here the seller commits $p_{1,B}$ to p_x. However, setting this first period price results in a fixed second period price, there must be no incentive to deviate from this strategy by the seller. It must therefore be considered for what first period price the seller would still optimally choose p_x in the first period in Case 3, instead of a lower p_x. If the seller would prefer setting a lower $p_{1,3}$, with the optimal second period price derived from Case B: $p_{2,B}$, then the seller is not behaving incentive compatible. If the optimal strategy for the seller would be to charge a lower price than p_x for $p_{1,3}$, with a fixed $p_{2,3} = p_{2,B}$. This will result in the feeling of a loss in the *reference utility* in the first period. If this is true, than it can be concluded that $p_{1,A}$ is a sub-optimal strategy and the seller will not be able to commit to this strategy. Therefore now the seller's optimal second period price will be examined for which the seller's commits to p_x in the first period, in Case 3: Loss.

As the optimal first period price is calculated in the exact same manner as in Case 2 it becomes clear that indeed the optimal strategy for the seller who commits to $p_{2,3} = \frac{p_x(\alpha((\delta-2)\theta+1)-1)}{(\delta-2)(\alpha(\theta-1)+1)}$ leads to an optimal first period price which is identical to the first price in Case B: $p_{1,B} = p_x$.

The formal proof:
As $p_{1,B} = p_x$ it must be such that: $p_{2,3}(v_{x,3}(p_{1,3})) = p_{2,B}|_{(p_{1,B}=p_x)}$. Solving the above equation for $p_{1,3}$ should result in a first period price being no less than p_x.

$$p_{2,3}(v_{x,3}(p_{1,3})) = p_{2,B}|_{(p_{1,B}=p_x)}$$
$$p_{2,3} = \frac{\frac{\alpha\delta\theta p_x - 2p_{1,3}(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)}(\alpha(\theta-1)+1)+\alpha\theta p_x}{2\alpha(\theta-1)+2}$$
$$v_{x,3} = \frac{\alpha\delta\theta p_x - 2p_{1,3}(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)}$$

Filling in $v_{x,3}$ in $p_{2,3}$ yields:

$$\frac{\frac{\alpha\delta\theta p_x - 2p_{1,3}(\alpha(\theta-1)+1)}{(\delta-2)(\alpha(\theta-1)+1)}(\alpha(\theta-1)+1)+\alpha\theta p_x}{2\alpha(\theta-1)+2} = \frac{p_x(\alpha((\delta-2)\theta+1)-1)}{(\delta-2)(\alpha(\theta-1)+1)}$$

solving for $p_{1,3}$ yields:

$$p_{1,3}|_{(p_{2,3}=p_{2,B})} = p_x$$

Therefore the seller is behaving optimally by pricing $p_{1,B} = p_x$ and $p_{2,B} = \frac{p_x(\alpha((\delta-2)\theta+1)-1)}{(\delta-2)(\alpha(\theta-1)+1)}$.
The seller has no incentive to price any lower than p_x in Case B.

E Bounds on cases and optimal values

Assuming loss aversion in combination with reference utility for the seller, the optimal pricing and the indifferent consumers' location are calculated using the following optimal functions, depending on the location of the reference point of the seller.

For a Case to end the criteria must be met which ends that Case, which subsequently triggers the beginning of the new Case. This section shows the bounds of a case imposed by its restrictions.

Case 1 requirement is $p^*_{2,1} \geq p_x$

$$\frac{\delta + \alpha(5\delta - 6)p_x - 2}{6\delta - 8} \geq p_x$$

Simplified to:

$$\frac{2 - \delta}{5\alpha\delta - 6\alpha - 6\delta + 8} \geq p_x$$

A seller with reference point $p_x \in [0, \frac{2-\delta}{5\alpha\delta - 6\alpha - 6\delta + 8}]$ will yield the following scheme **(Case 1)**:

$$p^*_{1,1} = \frac{\alpha(4 - \delta^2 - 2\delta)p_x + (\delta - 2)^2}{8 - 6\delta} \tag{69}$$

$$v^*_{x,1} = \frac{\delta + 2\alpha(\delta - 1)p_x - 2}{3\delta - 4} \tag{70}$$

$$p^*_{2,1} = v^*_{y,1} == \frac{\delta + \alpha(5\delta - 6)p_x - 2}{6\delta - 8} \tag{71}$$

Case A requirements are $p^*_{2,2} \geq p_x \geq p^*_{2,1}$

$$\frac{\delta + 2\alpha^2(\theta - 1)p_x(\delta\theta - 1) + \alpha((\delta - 2)(\theta - 1) + p_x(5\delta\theta - 4\theta - 2)) - 2}{2(\alpha(\theta - 1) + 1)(\delta(\alpha(\theta - 1) + 3) - 4)} \geq p_x \geq \frac{\delta + \alpha(5\delta - 6)p_x - 2}{6\delta - 8} \tag{72}$$

Simplified to:

$$\frac{\delta + 2\alpha^2(\theta - 1)p_x(\delta\theta - 1) + \alpha((\delta - 2)(\theta - 1) + p_x(5\delta\theta - 4\theta - 2)) - 2}{2(\alpha(\theta - 1) + 1)(\delta(\alpha(\theta - 1) + 3) - 4)} \geq p_x \geq \frac{2 - \delta}{5\alpha\delta - 6\alpha - 6\delta + 8} \tag{73}$$

A seller with reference points $p_x \in [p^*_{2,1}(p_x), p^*_{2,2}]$ will yield the following scheme **(Case A)**:

$$p_{1,A} = p_x(2 - \alpha - \delta + \alpha\delta) \tag{74}$$

$$v_{x,A} = p_x(2-\alpha) \tag{75}$$

$$p_{2,A} = v_{v,A} == p_x \tag{76}$$

Case 2 requirements are $p^*_{1,2} \geq p_x \geq p^*_{2,2}$

$$\frac{-(\delta-2)^2 + 2\alpha^2(\theta-1)p_x(\delta\theta+\delta-2) + \alpha(p_x(\delta^2\theta+2\delta-4) - (\delta-2)^2(\theta-1))}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \geq p_x \geq \frac{\delta + 2\alpha^2(\theta-1)p_x(\delta\theta-1) + \alpha((\delta-2)(\theta-1) + p_x(5\delta\theta-4\theta-2)) - 2}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{77}$$

Simplified to:

$$\frac{(\delta-2)^2(\alpha(\theta-1)+1)}{4\alpha^2(\delta-1)(\theta-1) + \alpha\left(\delta^2\theta + \delta(10-8\theta) + 8\theta - 12\right) - 6\delta + 8} \geq p_x \geq -\frac{(\delta-2)(\alpha(\theta-1)+1)}{2\alpha^2(\delta-1)(\theta-1) + \alpha(\delta(8-3\theta) + 4\theta - 10) - 6\delta + 8} \tag{78}$$

A seller with a reference point $p_x \in [p^*_{2,2}, p^*_{2,1}]$ will yield the following scheme **(Case 2)**:

$$p^*_{1,2} = \frac{-(\delta-2)^2 + 2\alpha^2(\theta-1)p_x(\delta\theta+\delta-2) + \alpha(p_x(\delta^2\theta+2\delta-4) - (\delta-2)^2(\theta-1))}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{79}$$

$$v^*_{x,2} = \frac{\delta + \alpha^2(\theta-1)x(\delta\theta-2) + \alpha((\delta-2)(\theta-1) + 2p_x(\delta\theta-1)) - 2}{(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{80}$$

$$p^*_{2,2} = v^*_{y,2} == \frac{\delta + 2\alpha^2(\theta-1)p_x(\delta\theta-1) + \alpha((\delta-2)(\theta-1) + p_x(5\delta\theta-4\theta-2)) - 2}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{81}$$

Case B requirements are $p^*_{1,3} \geq p_x \geq p^*_{1,2}$

$$\frac{\alpha\left(\delta^2(\theta(p_x-1)+1) + 2\delta(\theta(p_x+2)-2) - 4(\theta+\theta p_x-1)\right) - (\delta-2)^2}{2(3\delta-4)(\alpha(\theta-1)+1)} \geq p_x \geq \frac{-(\delta-2)^2 + 2\alpha^2(\theta-1)p_x(\delta\theta+\delta-2) + \alpha(p_x(\delta^2\theta+2\delta-4) - (\delta-2)^2(\theta-1))}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} \tag{82}$$

Simplified to:

$$\frac{(\delta-2)^2(\alpha(\theta-1)+1)}{\alpha\left(\delta^2\theta - 4\delta\theta + 6\delta + 4\theta - 8\right) - 6\delta + 8} \geq p_x \geq \frac{(\delta-2)^2(\alpha(\theta-1)+1)}{4\alpha^2(\delta-1)(\theta-1) + \alpha\left(\delta^2\theta + \delta(10-8\theta) + 8\theta - 12\right) - 6\delta + 8} \tag{83}$$

A seller with reference point $p_x \in [p^*_{1,2}(p_x), p_{1,3}]$ will yield the following scheme **(Case B)**:

$$p^*_{1,B} = p_x \tag{84}$$

$$v^*_{x,B} = \frac{p_x(\alpha((\delta-2)\theta+2)-2)}{(\delta-2)(\alpha(\theta-1)+1)} \tag{85}$$

$$p^*_{2,B} = v^*_{y,B} == \frac{p_x(\alpha((\delta-2)\theta+1)-1)}{(\delta-2)(\alpha(\theta-1)+1)} \tag{86}$$

Case 3 requirement is $p_x \geq p^*_{1,3}$

$$p_x \geq \frac{\alpha\left(\delta^2(\theta(p_x-1)+1)+2\delta(\theta(p_x+2)-2)-4(\theta+\theta p_x-1)\right)-(\delta-2)^2}{2(3\delta-4)(\alpha(\theta-1)+1)}$$

Simplified to:

$$p_x \geq \frac{(\delta-2)^2(\alpha(\theta-1)+1)}{\alpha\left(\delta^2\theta-4\delta\theta+6\delta+4\theta-8\right)-6\delta+8}$$

A seller with reference point $p_x \in (p^*_{1,3}(p_x), 1]$ will yield the following scheme **(Case 3)**:

$$p^*_{1,3} = \frac{\alpha(\delta^2(\theta(p_x-1)+1)+2\delta(\theta(p_x+2)-2)-4(\theta+\theta p_x-1))-(\delta-2)^2}{2(3\delta-4)(\alpha(\theta-1)+1)} \tag{87}$$

$$v^*_{x,3} = \frac{\delta+\alpha(\delta(\theta+2\theta p_x-1)-2(\theta+\theta p_x-1))-2}{(3\delta-4)(\alpha(\theta-1)+1)} \tag{88}$$

$$p^*_{2,3} = v^*_{y,3} == \frac{\delta+\alpha(\delta(\theta+5\theta p_x-1)-2(\theta+3\theta p_x-1))-2}{2(3\delta-4)(\alpha(\theta-1)+1)} \tag{89}$$

F Comparative statistics

The following shows that $p^*_{1,1}$ is increasing in p_x and α. However $p^*_{1,1}$ decreases in δ until $\delta = (2/3)$, after which $p^*_{1,1}$ increases.

$$\frac{\partial p^*_{1,1}}{\partial \alpha} = \frac{(4-\delta^2-2\delta)\,p_x}{8-6\delta} > 0$$

$$\frac{\partial p^*_{1,1}}{\partial p_x} = \frac{\alpha\,(4-\delta^2-2\delta)}{8-6\delta} > 0$$

$$\frac{\partial p^*_{1,1}}{\partial \delta} = \frac{(3\delta^2-8\delta+4)\,(\alpha p_x-1)}{2(4-3\delta)^2} \text{ before } \delta = 2/3 \;\&\; < 0 \text{ after } > 0^{18}$$

$$\frac{\partial^2 p^*_{1,1}}{\partial \delta} = \frac{4(\alpha p_x-1)}{(3\delta-4)^3} > 0$$

[18] $p^*_{1,1}$ increases in δ until $\delta = (2/3)$, after which $\frac{\partial p^*_{1,1}}{\partial \delta} < 0$

The following shows that $v^*_{x,1}$ is increasing in p_x, α and δ.

$$\frac{\partial v^*_{x,1}}{\partial p_x} = \frac{2\alpha(1-\delta)}{4-3\delta} > 0$$

$$\frac{\partial v^*_{x,1}}{\partial \delta} = \frac{2-p_x\alpha}{(4-3\delta)^2} > 0$$

$$\frac{\partial v^*_{x,1}}{\partial \alpha} = \frac{2p_x(1-\delta)}{4-3\delta} > 0$$

As is shown below, $p^*_{2,1}$ is increasing in p_x, α and δ.

$$\frac{\partial p^*_{2,1}}{\partial p_x} = \frac{\alpha(6-5\delta)}{8-6\delta} > 0$$

$$\frac{\partial p^*_{2,1}}{\partial \delta} = \frac{1-p_x\alpha}{(4-3\delta)^2} > 0$$

$$\frac{\partial p^*_{2,1}}{\partial \alpha} = \frac{p_x(6-5\delta)}{8-6\delta} > 0$$

The following shows that $p^*_{1,2}$ is increasing in p_x and α. However $p^*_{1,2}$ first decreases in δ then increases in δ.

$$\frac{\partial p^*_{1,2}}{\partial \alpha} = \frac{(\delta-2)^2\delta(\theta-1)(\alpha(\theta-1)+1)^2}{2(\alpha(\theta-1)+1)^2(\delta(\alpha(\theta-1)+3)-4)^2} - \frac{p_x\left(\alpha^2(\theta-1)^2\left(\delta^3\theta-2\delta^2(4\theta+3)+4\delta(2\theta+5)-16\right)-4\alpha(3\delta-4)(\theta-1)(\delta\theta+\delta-2)-(3\delta-4)\left(\delta^2\theta+2\delta-4\right)\right)}{2(\alpha(\theta-1)+1)^2(\delta(\alpha(\theta-1)+3)-4)^2} > 0 \quad (90)$$

$$\frac{\partial p^*_{1,2}}{\partial p_x} = \frac{2\alpha^2(\theta-1)(\delta\theta+\delta-2)+\alpha\left(\delta^2\theta+2\delta-4\right)}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} > 0$$

$$\frac{\partial p^*_{1,2}}{\partial \delta} = \frac{\alpha p_x\left(4\alpha^2(\theta-1)^2+\alpha(\theta-1)\left(\left(\delta^2-8\right)\theta+8\right)+3\delta^2\theta-8\delta\theta+4\right)-(\delta-2)(\alpha(\theta-1)+1)(\alpha(\delta+2)(\theta-1)+3\delta-2)}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)^2} \text{ first } < 0 \text{ then } > 0$$

$$\frac{\partial^2 p^*_{1,2}}{\partial \delta} = \frac{4\left(\alpha^3(\theta-1)^2p_x-\alpha^2(\theta-1)(-\theta+2(\theta-2)p_x+1)+\alpha(2\theta-4\theta p_x+3x-2)+1\right)}{(\delta(\alpha(\theta-1)+3)-4)^3} > 0$$

The following shows that $v^*_{x,2}$ is increasing in p_x, α and δ.

$$\frac{\partial v^*_{x,2}}{\partial p_x} = \frac{2\alpha(1-\delta)}{4-3\delta} > 0$$

$$\frac{\partial v^*_{x,2}}{\partial \delta} = \frac{2-p_x\alpha}{(4-3\delta)^2} > 0$$

$$\frac{\partial v^*_{x,2}}{\partial \alpha} = \frac{2p_x(1-\delta)}{4-3\delta} > 0$$

As is shown below, $p^*_{2,2}$ is increasing in p_x, α and δ.

$$\frac{\partial p^*_{2,2}}{\partial p_x} = \frac{\alpha(6-5\delta)}{8-6\delta} > 0$$

$$\frac{\partial p^*_{2,2}}{\partial \delta} = \frac{1 - p_x\alpha}{(4-3\delta)^2} > 0$$

$$\frac{\partial p^*_{2,2}}{\partial \alpha} = \frac{p_x(6-5\delta)}{8-6\delta} > 0$$

The following shows that $p^*_{1,1}$ is increasing in p_x and α. However $p^*_{1,1}$ decreases in δ until $\delta = (2/3)$, after which $p^*_{1,1}$ increases.

$$\frac{\partial p^*_{1,1}}{\partial \alpha} = \frac{(4-\delta^2-2\delta)\, p_x}{8-6\delta} > 0$$

$$\frac{\partial p^*_{1,1}}{\partial p_x} = \frac{\alpha\,(4-\delta^2-2\delta)}{8-6\delta} > 0$$

$$\frac{\partial p^*_{1,1}}{\partial \delta} = \frac{(3\delta^2-8\delta+4)\,(\alpha p_x - 1)}{2(4-3\delta)^2} < 0 \text{ before } \delta = 2/3 \ \& > 0 \text{ after}$$ [19]

$$\frac{\partial^2 p^*_{1,1}}{\partial \delta} = \frac{4(\alpha p_x - 1)}{(3\delta-4)^3} > 0$$

The following shows that $v^*_{x,1}$ is increasing in p_x, α and δ.

$$\frac{\partial v^*_{x,1}}{\partial p_x} = \frac{2\alpha(1-\delta)}{4-3\delta} > 0$$

$$\frac{\partial v^*_{x,1}}{\partial \delta} = \frac{2 - p_x\alpha}{(4-3\delta)^2} > 0$$

$$\frac{\partial v^*_{x,1}}{\partial \alpha} = \frac{2p_x(1-\delta)}{4-3\delta} > 0$$

As is shown below, $p^*_{2,1}$ is increasing in p_x, α and δ.

$$\frac{\partial p^*_{2,1}}{\partial p_x} = \frac{\alpha(6-5\delta)}{8-6\delta} > 0$$

$$\frac{\partial p^*_{2,1}}{\partial \delta} = \frac{1 - p_x\alpha}{(4-3\delta)^2} > 0$$

$$\frac{\partial p^*_{2,1}}{\partial \alpha} = \frac{p_x(6-5\delta)}{8-6\delta} > 0$$

The following shows that $p^*_{1,2}$ is increasing in p_x and α. However $p^*_{1,2}$ first decreases in δ then increases in δ.

$$\frac{\partial p^*_{1,2}}{\partial \alpha} = \frac{(\delta-2)^2\delta(\theta-1)(\alpha(\theta-1)+1)^2}{2(\alpha(\theta-1)+1)^2(\delta(\alpha(\theta-1)+3)-4)^2} - \frac{p_x\left(\alpha^2(\theta-1)^2\left(\delta^3\theta-2\delta^2(4\theta+3)+4\delta(2\theta+5)-16\right)-4\alpha(3\delta-4)(\theta-1)(\delta\theta+\delta-2)-(3\delta-4)\left(\delta^2\theta+2\delta-4\right)\right)}{2(\alpha(\theta-1)+1)^2(\delta(\alpha(\theta-1)+3)-4)^2} > 0 \tag{91}$$

$$\frac{\partial p^*_{1,2}}{\partial \theta} = \frac{\alpha\delta\Big(-4\alpha^3(\delta-1)(\theta-1)^2p_x-\alpha^2(\theta-1)\Big(p_x\Big(\delta^2(\theta+1)+\delta(12-8\theta)+8(\theta-2)\Big)-(\delta-2)^2(\theta-1)\Big)-2\alpha\Big(2p_x\Big(\delta^2-3\delta\theta+\delta+4\theta-3\Big)-(\delta-2)^2(\theta-1)\Big)+\delta^2(3p_x+1)-4\delta(p_x+1)+4\Big)}{2(\alpha(\theta-1)+1)^2(\delta(\alpha(\theta-1)+3)-4)^2} > 0$$

[19] $p^*_{1,1}$ increases in δ until $\delta = (2/3)$, after which $\frac{\partial p^*_{1,1}}{\partial \delta} < 0$

$$\frac{\partial p_{1,2}^*}{\partial p_x} = \frac{2\alpha^2(\theta-1)(\delta\theta+\delta-2)+\alpha\left(\delta^2\theta+2\delta-4\right)}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} > 0$$

$$\frac{\partial p_{1,2}^*}{\partial \delta} = \frac{\alpha p_x\left(4\alpha^2(\theta-1)^2+\alpha(\theta-1)\left(\left(\delta^2-8\right)\theta+8\right)+3\delta^2\theta-8\delta\theta+4\right)-(\delta-2)(\alpha(\theta-1)+1)(\alpha(\delta+2)(\theta-1)+3\delta-2)}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)^2} < \text{ first 0 then } > 0$$

$$\frac{\partial^2 p_{1,2}^*}{\partial \delta} = \frac{4\left(\alpha^3(\theta-1)^2p_x-\alpha^2(\theta-1)(-\theta+2(\theta-2)p_x+1)+\alpha(2\theta-4\theta p_x+3x-2)+1\right)}{(\delta(\alpha(\theta-1)+3)-4)^3} > 0$$

The following shows that $v_{x,2}^*$ is increasing in p_x, α and δ.

$$\frac{\partial v_{x,2}^*}{\partial \alpha} = \frac{2x\left(\alpha^2(\theta-1)^2\left(\delta^2\theta-\delta(2\theta+3)+4\right)+\alpha(3\delta-4)(\theta-1)(\delta\theta-2)+(3\delta-4)(\delta\theta-1)\right)-(\delta-2)\delta(\theta-1)(\alpha(\theta-1)+1)^2}{(\alpha(\theta-1)+1)^2(\delta(\alpha(\theta-1)+3)-4)^2} > 0$$

$$\frac{\partial v_{x,2}^*}{\partial p_x} = \frac{\alpha(\alpha(\theta-1)(\delta\theta-2)+2\delta\theta-2)}{(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} > 0$$

$$\frac{\partial v_{x,2}^*}{\partial \theta} = -\frac{\alpha\delta\left((\delta-2)(\alpha(\theta-1)+1)^2+x\left(\alpha^3(\delta-2)(\theta-1)^2-2\alpha^2(\theta-1)(\delta(\theta-3)-2\theta+4)+\alpha(\delta(11-6\theta)+8\theta-14)-6\delta+8\right)\right)}{(\alpha(\theta-1)+1)^2(\delta(\alpha(\theta-1)+3)-4)^2} > 0$$

$$\frac{\partial v_{x,2}^*}{\partial \delta} = \frac{2\left(\alpha^3(\theta-1)^2p_x-\alpha^2(\theta-1)(-\theta+2(\theta-2)p_x+1)+\alpha(2\theta-4\theta p_x+3x-2)+1\right)}{(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)^2} > 0$$

As is shown below, $p_{2,2}^*$ is increasing in p_x, α and δ.

$$\frac{\partial p_{2,2}^*}{\partial \alpha} = \frac{(3\delta-4)p_x\left(\alpha^2(\theta-1)^2(\delta\theta-2)+4\alpha(\theta-1)(\delta\theta-1)+(5\delta-4)\theta-2\right)-(\delta-2)\delta(\theta-1)(\alpha(\theta-1)+1)^2}{2(\alpha(\theta-1)+1)^2(\delta(\alpha(\theta-1)+3)-4)^2} > 0$$

$$\frac{\partial p_{2,2}^*}{\partial p_x} = \frac{\alpha(2\alpha(\theta-1)(\delta\theta-1)+(5\delta-4)\theta-2)}{2(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)} > 0$$

$$\frac{\partial p_{2,2}^*}{\partial \theta} = -\frac{\alpha\left((\delta-2)\delta(\alpha(\theta-1)+1)^2+p_x\left(2\alpha^3(\delta-1)\delta(\theta-1)^2-\alpha^2\delta(\theta-1)(3\delta\theta-13\delta-4\theta+16)-2\alpha\left(\delta^2(6\theta-13)+\delta(23-8\theta)-8\right)-15\delta^2+32\delta-16\right)\right)}{2(\alpha(\theta-1)+1)^2(\delta(\alpha(\theta-1)+3)-4)^2} > 0$$

$$\frac{\partial p_{2,2}^*}{\partial \delta} = \frac{\alpha^3(\theta-1)^2p_x-\alpha^2(\theta-1)(-\theta+2(\theta-2)p_x+1)+\alpha(2\theta-4\theta p_x+3p_x-2)+1}{(\alpha(\theta-1)+1)(\delta(\alpha(\theta-1)+3)-4)^2} > 0$$

The following shows that $p_{1,3}^*$ is increasing in p_x and α. However $p_{1,3}^*$ first decreases in δ then increases as δ becomes higher.

$$\frac{\partial p_{1,3}^*}{\partial \alpha} = \frac{\left(\delta^2+2\delta-4\right)\theta p_x}{2(3\delta-4)(\alpha(\theta-1)+1)^2} > 0$$

$$\frac{\partial p_{1,3}^*}{\partial \theta} = -\frac{(\alpha-1)\alpha\left(\delta^2+2\delta-4\right)p_x}{2(3\delta-4)(\alpha(\theta-1)+1)^2} > 0$$

$$\frac{\partial p_{1,3}^*}{\partial p_x} = \frac{\alpha\left(\delta^2\theta+2\delta\theta-4\theta\right)}{2(3\delta-4)(\alpha(\theta-1)+1)} > 0$$

$$\frac{\partial p_{1,3}^*}{\partial \delta} = \frac{\left(3\delta^2-8\delta+4\right)(\alpha(\theta(p_x-1)+1)-1)}{2(4-3\delta)^2(\alpha(\theta-1)+1)} < \text{ first 0 then } > 0$$

$$\frac{\partial^2 p^*_{1,3}}{\partial \delta} = \frac{4(\alpha(\theta p_x - 1) + 1) - 1)}{(3\delta - 4)^3(\alpha(\theta - 1) + 1)} > 0$$

The following shows that $v^*_{x,3}$ is increasing in p_x, α and δ.

$$\frac{\partial v^*_{x,3}}{\partial \alpha} = \frac{2(\delta - 1)\theta p_x}{(3\delta - 4)(\alpha(\theta - 1) + 1)^2} > 0$$

$$\frac{\partial v^*_{x,3}}{\partial p_x} = \frac{\alpha(2\delta\theta - 2\theta)}{(3\delta - 4)(\alpha(\theta - 1) + 1)} > 0$$

$$\frac{\partial v^*_{x,3}}{\partial \theta} = \frac{2(\alpha - 1)\alpha(\delta - 1)p_x}{(3\delta - 4)(\alpha(\theta - 1) + 1)^2} > 0$$

$$\frac{\partial v^*_{x,3}}{\partial \delta} = -\frac{2(\alpha(\theta(p_x - 1) + 1) - 1)}{(4 - 3\delta)^2(\alpha(\theta - 1) + 1)} > 0$$

As is shown below, $p^*_{2,3}$ is increasing in p_x, α, θ and δ.

$$\frac{\partial p^*_{2,3}}{\partial \alpha} = \frac{(5\delta - 6)\theta p_x}{2(3\delta - 4)(\alpha(\theta - 1) + 1)^2} > 0$$

$$\frac{\partial p^*_{2,3}}{\partial p_x} = \frac{\alpha(5\delta\theta - 6\theta)}{2(3\delta - 4)(\alpha(\theta - 1) + 1)} > 0$$

$$\frac{\partial p^*_{2,3}}{\partial \theta} = -\frac{(\alpha - 1)\alpha(5\delta - 6)p_x}{2(3\delta - 4)(\alpha(\theta - 1) + 1)^2} > 0$$

$$\frac{\partial p^*_{2,3}}{\partial \delta} = \frac{\alpha(\theta + \theta(-p_x) - 1) + 1}{(4 - 3\delta)^2(\alpha(\theta - 1) + 1)} > 0$$

As is shown below, $p^*_{1,A}$ is increasing in p_x but decreasing in α and δ.

$$\frac{\partial p^*_{1,A}}{\partial \alpha} = p_x\delta - p_x < 0$$

$$\frac{\partial p^*_{1,A}}{\partial p_x} = 2 - \alpha - \delta + \alpha\delta > 0$$

$$\frac{\partial p^*_{1,A}}{\partial \delta} = p_x\alpha - p_x < 0$$

As is shown below, $p^*_{v,A}$ is decreasing in α, increasing in p_x, and constant in δ:

$$\frac{\partial p^*_{v,A}}{\partial \alpha} = -p_x < 0$$

$$\frac{\partial p^*_{v,A}}{\partial p_x} = 2 - \alpha > 0$$

$$\frac{\partial p^*_{v,A}}{\partial \delta} = 0$$

As is shown below, $p^*_{2,A} = v_{y,A}$ is increasing in p_x and constant in α and δ.

$$\frac{\partial p^*_{2,A} = v_{y,A}}{\partial \alpha} = 0$$

$$\frac{\partial p_{2,A}^* = v_{y,A}}{\partial p_x} = 1 > 0$$

$$\frac{\partial p_{2,A}^* = v_{y,A}}{\partial \delta} = 0$$

As is shown below, $p_{1,B}^*$ is increasing in p_x, but constant in α, δ and θ.

$$\frac{\partial p_{1,B}^*}{\partial \alpha} = 0$$

$$\frac{\partial p_{1,B}^*}{\partial p_x} = 1 > 0$$

$$\frac{\partial p_{1,B}^*}{\partial \theta} = 0$$

$$\frac{\partial p_{1,B}^*}{\partial \delta} = 0$$

As is shown below, $v_{x,B}^*$ is increasing in p_x, but constant in α, δ and θ.

$$\frac{\partial v_{x,B}^*}{\partial \alpha} = \frac{\delta\theta p_x}{(\delta - 2)(\alpha(\theta - 1) + 1)^2} < 0$$

$$\frac{\partial v_{x,B}^*}{\partial p_x} = \frac{\alpha((\delta - 2)\theta + 2) - 2}{(\delta - 2)(\alpha(\theta - 1) + 1)} > 0$$

$$\frac{\partial v_{x,B}^*}{\partial \theta} = -\frac{(\alpha - 1)\alpha\delta p_x}{(\delta - 2)(\alpha(\theta - 1) + 1)^2} < 0$$

$$\frac{\partial v_{x,B}^*}{\partial \delta} = -\frac{2(\alpha - 1)p_x}{(\delta - 2)^2(\alpha(\theta - 1) + 1)} > 0$$

As is shown below, $p_{2,B}^*$ is increasing in p_x, but constant in α, δ and θ.

$$\frac{\partial p_{2,B}^*}{\partial \alpha} = \frac{(\delta - 1)\theta p_x}{(\delta - 2)(\alpha(\theta - 1) + 1)^2} > 0$$

$$\frac{\partial p_{2,B}^*}{\partial p_x} = \frac{\alpha(\delta - 2)\theta + \alpha - 1}{(\delta - 2)(\alpha(\theta - 1) + 1)} > 0$$

$$\frac{\partial p_{2,B}^*}{\partial \theta} = -\frac{(\alpha - 1)\alpha(\delta - 1)p_x}{(\delta - 2)(\alpha(\theta - 1) + 1)^2} > 0$$

$$\frac{\partial p_{2,B}^*}{\partial \delta} = \frac{p_x - \alpha p_x}{(\delta - 2)^2(\alpha(\theta - 1) + 1)} > 0$$

G Model with appended reference utility

In this section I depart from this paper's assumption that the reference utility is non-separable, implying that the reference utility is appended to the profits the seller makes. Compared to the initial model this paper describes the main implication is that $(1-\alpha)$ is completely eliminated. This separates the selling utility from the reference utility. The other assumptions about the model are still in place. The coming section will solve this new model, and show that the result reinforce the initial model as previously formulated in this paper. For every price the seller sets, if it is below the sellers "expected" price p_x the seller experiences a feeling of a loss when selling the item. This model therefore uses exogenous reference points, independent from each period in order to determine the "feeling" of the seller. The seller's profit/utility function therefore changes to:

$$\pi_1 = \begin{cases} (1-v_x)p_1 + \alpha(1-v_x)(p_1-p_x), \text{if } p_1 - p_x \geq 0 \\ (1-v_x)p_1 + \alpha\theta(1-v_x)(p_1-p_x), \text{if } p_1 < p_x \end{cases}$$

Simplified to:

$$\pi_1 = \begin{cases} (1-v_x)(p_1 + \alpha(p_1-p_x)), \text{if } p_1 - p_x \geq 0 \\ (1-v_x)(p_1 + \alpha\theta(p_1-p_x)), \text{if } p_1 < p_x \end{cases}$$

For the second period profits, it becomes:

$$(\pi_2|p_1 beingrejected) = \begin{cases} \frac{(v_x-v_y)}{v_x}p_2 + \alpha\frac{(v_x-v_y)}{v_x}(p_2-p_x), \text{if } p_2 - p_x \geq 0 \\ \frac{(v_x-v_y)}{v_x}p_2 + \alpha\theta\frac{(v_x-v_y)}{v_x}(p_2-p_x), \text{if } p_2 - p_x < 0 \end{cases}$$

Simplified to:

$$(\pi_2|p_1 beingrejected) = \begin{cases} \frac{(v_x-v_y)}{v_x}(p_2 + \alpha(p_2-p_x)), \text{if } p_2 - p_x \geq 0 \\ \frac{(v_x-v_y)}{v_x}(p_2 + \alpha\theta(p_2-p_x)), \text{if } p_2 - p_x < 0 \end{cases}$$

The utility for the consumer remains:

$$U_{1,buyer} = v_x - p_1 \tag{92}$$

$$U_{2,buyer} = v_y - p_2 \tag{93}$$

As the above setting illustrates, there are multiple cases which should be considered. These cases depend on the exogenous reference points of the seller. If $p_1 \geq p_x$ and/or $p_2 \geq p_x$ regardless if the item is sold, it will feel like a gain for the seller. If $p_1 < p_x$ and/or $p_2 < p_x$ regardless if the item is sold, it will feel like a loss to the seller. This since the price asked will be below the expected exogenous price.
Therefore this paper considers 3 cases:
* Case 1 Gain: $p_1 \geq p_x$ and $p_1 \geq p_x$
* Case 2 Gain / loss: $p_1 \geq p_x$ and $p_1 < p_x$
* Case 3 Loss: $p_1 < p_x$ and $p_1 < p_x$

Case 1: gain

This section will analyse the gain case, therefore $p_1 \geq p_2 \geq p_x$. The indifferent consumer is indifferent between buying in $t = 1$ and $t = 2$ at price p_1, this consumer's valuation is denoted by v_x and there is a consumer which is indifferent at buying in $t = 2$ and not buying at all at price p_2, this consumer's valuation is denoted by v_y. The discount rate for the seller and consumers are identical and equal to $\delta \in [0, 1]$.

The first step in order to solve the model with backward induction is to find out what the value (v_y) of the indifferent consumer is at $t = 2$, which is indifferent between buying in the second period and not buying at all. This can be found by equalizing the second period indifferent consumer's valuation with the second period price. Resulting in: $v_y - p_2 = 0$. This yields the consumer who is indifferent between buying in the second period and not buying at all, with the valuation being equal to $v_y = p_2$. Therefore the indifferent consumer's valuation in the first period is when $U_{1,buyer} = \delta U_{2,buyer}$.

$$v_x - p_1 = \delta(v_x - p_2) \tag{94}$$

However, p_2 is still a function of v_x. In order to find the optimal value of p_2 the seller's behaviour must now be optimized.

In this paragraph we analyze the situation where the seller sets both prices above its exogenous reference point (p_x).Just like in Fudenberg and Tirole (1983) the seller updates its information about the buyer after the acceptance (or rejection) of the first period price. With means of backwards induction the seller can optimize its overall profits, looking first at the optimization of the second period profits, taking into account that the first period price has been rejected.

$$\pi_2 = \frac{(v_x - v_y)}{v_x} p_2 + \alpha \frac{(v_x - v_y)}{v_x} (p_2 - p_x) \tag{95}$$

Taking into account $v_y = p_2$ and maximizing the second period profit function with respect to p_2 yields:

$$\frac{\partial \pi_2}{\partial p_2} = 0 \tag{96}$$

Solving this for the second period price yields:

$$p_2 = \frac{v_x}{2} + \frac{p_x \alpha}{2(1 + \alpha)} \tag{97}$$

In order for the consumer to make the decision to buy in the first stage instead of in the second stage, the utility the consumer derives from the decision to buy in the first stage must be larger or equal compared to the utility gained by buying in the second period. Therefore the indifferent valued consumer needs to be found: $U_{1,buyer} = U_{2,buyer}$ this leads to the following equality:

$$v_x - p_1 = \delta(v_x - p_2) \tag{98}$$

Filling in p_2 yields:

$$v_x - p_1 = \delta(v_x - (\frac{v_x}{2} + \frac{p_x \alpha}{2(1 + \alpha)})) \tag{99}$$

Solving this inequality for (v_x) the indifferent consumer yields:

$$v_x = \frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta}{(1+\alpha)(2-\delta)} \tag{100}$$

Then in the first stage the seller behaves in order to maximize overall profits:

$$\pi_{overall} = (1-v_x)p_1 + \delta(v_x - v_y)p_2 + \alpha((1-v_x)(p_1-p_x) + \delta(v_x-v_y)(p_2-p_x)) \tag{101}$$

filling in the above equation leads to:

$$\begin{aligned}\pi_{overall} = {} & (1-(\frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta}{(1+\alpha)(2-\delta)}))p_1 + \delta((\frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta}{(1+\alpha)(2-\delta)}) \\ & -(\frac{((\frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta}{(1+\alpha)(2-\delta)}))}{2} + \frac{p_x\alpha}{2(1+\alpha)}))(\frac{((\frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta}{(1+\alpha)(2-\delta)}))}{2} + \frac{p_x\alpha}{2(1+\alpha)}) \\ & +\alpha((1-((\frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta}{(1+\alpha)(2-\delta)})))(p_1-p_x) + \delta(((\frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta}{(1+\alpha)(2-\delta)})) \\ & -(\frac{((\frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta}{(1+\alpha)(2-\delta)}))}{2} + \frac{p_x\alpha}{2(1+\alpha)}))((\frac{((\frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta}{(1+\alpha)(2-\delta)}))}{2} + \frac{p_x\alpha}{2(1+\alpha)}) - p_x))\end{aligned} \tag{102}$$

Taking the partial derivative of $\pi_{overall}$ with regards to p_1, and equalizing for p_1 yields:

$$\frac{\partial\pi_{overall}}{\partial p_1} = 0 \tag{103}$$

$$p_1^* = \frac{(1+\alpha+p_x\alpha + \frac{p_x\alpha\delta}{2-\delta} - \frac{p_x\alpha\delta^2}{(2-\delta)^2})}{\frac{4(1+\alpha)}{2-\delta} - \frac{2\delta(1+\alpha)}{(2-\delta)^2}} \tag{104}$$

Simplified to:

$$p_1^* = \frac{\alpha\left(\left(\delta^2+2\delta-4\right)p_x - (\delta-2)^2\right) - (\delta-2)^2}{2(\alpha+1)(3\delta-4)} \tag{105}$$

With the optimal first period price the optimal other prices and values can be calculated.

$$v_x^* = \frac{\delta + \alpha(\delta + 2(\delta-1)p_x - 2) - 2}{(\alpha+1)(3\delta-4)} \tag{106}$$

$$p_2^* = v_y^* == \frac{\delta + \alpha(\delta + (5\delta-6)p_x - 2) - 2}{2(\alpha+1)(3\delta-4)} \tag{107}$$

Now values for the parameters need to be filled in. In order to model "exogenous expected value" (p_x), the expected price for the seller needs to be calculated in order to show the interval between where p_x is allowed to be picked from. Therefore the other values should be discussed first. Just as the benchmark case, this case will use a discount factor of $\delta = 0.9$, to show limited discounting. Furthermore a value for $\alpha = 0.4$ is chosen to show that the reference utility is only a part of the overall utility. With these two parameters an interval for p_x can be calculated which suffices the following condition for the gain case: $p_1 \geq p_2 \geq p_x$.

The following conditions need to be fulfilled: $p_1 \geq p_x$ and $p_2 \geq p_x$.

Filling in both optimal p_1^* and p_2^* equations and filling in all appropriate parameters the following conditions are obtained:

- For p_1 to be larger than p_x it follows that p_x cannot become larger than 0.5493, thus $p_x \leq 0.5493$.

  This is a necessary, but not sufficient condition, furthermore p_x can also not exceed the second period price.

- For p_2 to be larger than p_x it follows that p_x should not become larger than 0.5066, thus $p_x \leq 0.5066$

Therefore the interval for which this case requirements with regards to p_x are fulfilled is when $p_x \in [0; 0.5066]$.

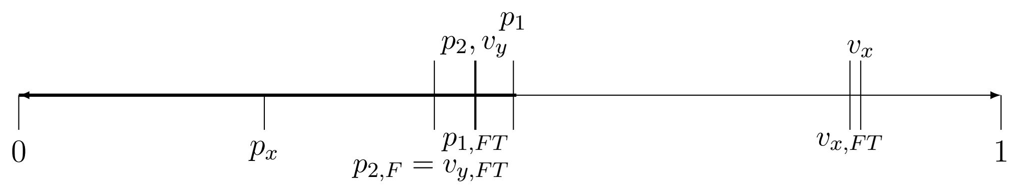
Any value for p_x can be taken which fulfills the above condition. This case will take the middle value: $p_x = 0.25$. The other paramaters remain the same: $\alpha = 0.4$ and $\delta = 0.9$. This yields the following results:

$$p_1^* = 0.503571$$

$$v_x^* = 0.85714$$

$$p_2^* = v_y^* == 0.46429$$

This can be compared to the benchmark case where the seller has a different preference. This is depicted in the illustation below. The bottom values depict the benchmark case, the upper values depict Case 1 (gain). The thick line represents the values p_x could be located in.



Case 2: Gain period 1, loss period 2

This section will analyse the gain with loss case, therefore $p_1 \geq p_x \geq p_2$. The indifferent consumer is indifferent between buying in $t = 1$ and $t = 2$ at price p_1, this consumer's valuation is denoted by v_x and there is a consumer which is indifferent at buying in $t = 2$ and not buying at all at price p_2, this consumer's valuation is denoted by v_y. The discount rate for the seller and consumers are identical and equal to $\delta \in [0,1]$.

The first step in order to solve the model with backward induction is to find out what the value (v_y) of the indifferent consumer is at $t = 2$, which is indifferent between buying in the second period and not buying at all. This can be found by equalizing the second period indifferent consumer's valuation with the second period price. Resulting in: $v_y - p_2 = 0$. This yields the consumer who is indifferent between buying in the second period and not buying at all, with the valuation being equal to $v_y = p_2$. Therefore the indifferent consumer's valuation in the first period is when $U_{1,buyer} = \delta U_{2,buyer}$.

$$v_x - p_1 = \delta(v_x - p_2) \tag{108}$$

However, p_2 is still a function of v_x. In order to find the optimal value of p_2 the seller's behaviour must now be optimized.

In this paragraph we analyze the situation where the seller sets the first period price above its exogenous reference point (p_x), but the second period price below its exogenous reference point. We now depart from the Fudenberg and Tirole (1983) framework, as we introduce loss aversion for the seller. The seller updates its information about the buyer after the acceptance (or rejection) of the first period price. With means of backwards induction the seller can optimize its overall profits, looking first at the optimization of the second period profits, taking into account that the first period price has been rejected.

$$\pi_2 = \frac{(v_x - v_y)}{v_x} p_2 + \alpha\theta \frac{(v_x - v_y)}{v_x}(p_2 - p_x) \tag{109}$$

Simplified to:

$$\pi_2 = \frac{(v_x - v_y)}{v_x}(p_2 + \alpha\theta(p_2 - p_x)) \tag{110}$$

Taking into account $v_y = p_2$ and maximizing the second period profit function with respect to p_2 yields:

$$\frac{\partial \pi_2}{\partial p_2} = 0 \tag{111}$$

Solving this for the second period price yields:

$$p_2 = \frac{v_x}{2} + \frac{p_x \alpha\theta}{2(1 + \alpha\theta)} \tag{112}$$

In order for the consumer to make the decision to buy in the first stage instead of in the second stage, the utility the consumer derives from the decision to buy in the first stage must be larger or equal compared to the utility gained by buying in the second period. Therefore the indifferent valued consumer needs to be found: $U_{1,buyer} = U_{2,buyer}$ this leads to the following equality:

$$v_x - p_1 = \delta(v_x - p_2) \tag{113}$$

Filling in p_2 yields:

$$v_x - p_1 = \delta(v_x - (\frac{v_x}{2} + \frac{p_x \alpha\theta}{2(1 + \alpha\theta)})) \tag{114}$$

Solving this inequality for (v_x) the indifferent consumer yields:

$$v_x = \frac{2p_1}{2-\delta} - \frac{p_x\alpha\delta\theta}{(1+\alpha\theta)(2-\delta)} \tag{115}$$

Then in the first stage the seller behaves in order to maximize overall profits:

$$\pi_{overall} = (1-v_x)(p_1+\alpha(p_1-p_x)+\delta(v_x-v_y)(p_2+\alpha\theta(p_2-p_x)) \tag{116}$$

$$\begin{aligned}\pi_{overall} = \delta(\alpha\theta(&\frac{\frac{\alpha\theta(\alpha\delta\theta p_x-2p(\alpha\theta+1))}{(\delta-2)(\alpha\theta+1)}+\frac{\alpha\delta\theta p_x-2p(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)}+\alpha\theta p_x}{2(\alpha\theta+1)}-p_x)\\ &+\frac{\frac{\alpha\theta(\alpha\delta\theta p_x-2p(\alpha\theta+1))}{(\delta-2)(\alpha\theta+1)}+\frac{\alpha\delta\theta p_x-2p_1(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)}+\alpha\theta x}{2(\alpha\theta+1)})(\frac{\alpha\delta\theta p_x-2p_1(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)}\\ -\frac{\frac{\alpha\theta(\alpha\delta\theta p_x-2p_1(\alpha\theta+1))}{(\delta-2)(\alpha\theta+1)}+\frac{\alpha\delta\theta p_x-2p_1(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)}+\alpha\theta p_x}{2(\alpha\theta+1)}&)+(\alpha(p_1-p_x)+p_1)(1-\frac{\alpha\delta\theta p_x-2p_1(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)})\end{aligned} \tag{117}$$

Taking the partial derivative of $\pi_{overall}$ with regards to p_1, and equalizing for p_1 yields:

$$\frac{\partial\pi_{overall}}{\partial p_1} = 0 \tag{118}$$

$$p_1^* = \frac{-(\delta-2)^2+\alpha^2\theta\left(p_x\left(\delta^2+2\delta\theta-4\right)-(\delta-2)^2\right)+\alpha\left(p_x\left(\delta^2\theta+2\delta-4\right)-(\delta-2)^2(\theta+1)\right)}{2(\alpha\theta+1)(\alpha(\delta(\theta+2)-4)+3\delta-4)} \tag{119}$$

With the optimal first period price the optimal other prices and values can be calculated.

$$v_x^* = \frac{\delta+\alpha^2\theta(\delta+p_x(\delta\theta+\delta-2)-2)+\alpha((\delta-2)(\theta+1)+2p_x(\delta\theta-1))-2}{(\alpha\theta+1)(\alpha(\delta(\theta+2)-4)+3\delta-4)} \tag{120}$$

$$p_2^* = v_y^* == \frac{\delta+\alpha^2\theta(\delta+p_x(2\delta\theta+3\delta-6)-2)+\alpha((\delta-2)(\theta+1)+p_x(5\delta\theta-4\theta-2))-2}{2(\alpha\theta+1)(\alpha(\delta(\theta+2)-4)+3\delta-4)} \tag{121}$$

Now values for the parameters need to be filled in. In order to model "exogenous expected value" (p_x). Just as before the interval for p_x needs to be calculated. The appropriate parameter values are as before: $\alpha = 0.4$, $\delta = 0.9$. The missing parameter's value is θ. As mentioned in the literature review, empirical work establishes θ to be bound $\in [1.5, 4]$. Using these bounds, the decision is made to choose a value which is located somewhere in between $\theta = 2$.

The interval which p_x can take and still fulfill the requirement of $p_1 \geq p_x \geq p_2$ should be shown in order to determine an appropriate value. The optimal functions have been calculated for p_1^* and p_2^*.

- For p_1 to be larger than p_x it follows that p_x cannot become larger than 0.59148, thus $p_x \leq 0.59148$.

  This is a necessary, but not sufficient condition, furthermore the second period price can also not exceed p_x.

- For p_2 to be smaller than p_x it follows that p_x should not become smaller than 0.559774, thus $p_x \geq 0.559774$

This yields the following interval in where p_x can be located: $p_x \in [0.559774; 0.59148]$

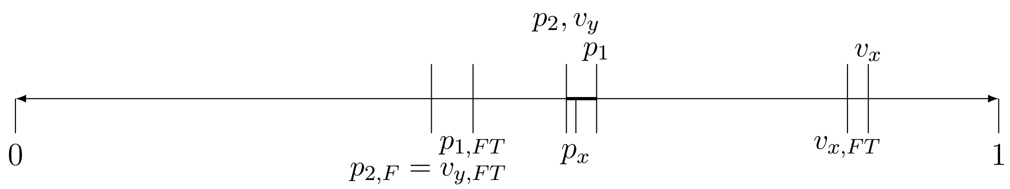
Taking the value $p_x = 0.57$, δ takes the value of 0.9, implying limited discounting and α with value of 0.4, showing that sellers take the reference point into account, but only partially. And $\theta = 2$, this leads to the following optimal values:

$$p_1^* = 0.59107$$

$$v_x^* = 0.867397$$

$$p_2^* = v_y^* == 0.56037$$

This can be compared to the benchmark case where the seller has a different preference. This is depicted in the illustration below. The bottom values depict the benchmark case, the upper values depict Case 2 (gain / loss). The (small) thick line represents all the values p_x could take and for which this case would still be relevant.[20].



Case 3: Loss

This section will analyse the loss (in both periods), therefore $p_x \geq p_1 \geq p_2$. The indifferent consumer is indifferent between buying in $t = 1$ and $t = 2$ at price p_1, this consumer's valuation is denoted by v_x and there is a consumer which is indifferent at buying in $t = 2$ and not buying at all at price p_2, this consumer's valuation is denoted by v_y. The discount rate for the seller and consumers are identical and equal to $\delta \in [0, 1]$.

The first step in order to solve the model with backward induction is to find out what

[20]The location of p_1 and p_2 is actually closer to each other, however for visual purposes they are moved a bit more away from each other on the horizontal line. The interpretation stays the same.

the value (v_y) of the indifferent consumer is at $t = 2$, which is indifferent between buying in the second period and not buying at all. This can be found by equalizing the second period indifferent consumer's valuation with the second period price. Resulting in: $v_y - p_2 = 0$. This yields the consumer who is indifferent between buying in the second period and not buying at all, with the valuation being equal to $v_y = p_2$. Therefore the indifferent consumer's valuation in the first period is when $U_{1,buyer} = \delta U_{2,buyer}$.

$$v_x - p_1 = \delta(v_x - p_2) \tag{122}$$

However, p_2 is still a function of v_x. In order to find the optimal value of p_2 the seller's behavior must now be optimized.

In this paragraph we analyze the situation where the seller sets the first and second period prices below its exogenous reference point (p_x). This is also where I depart from the Fudenberg and Tirole (1983) framework, as I introduce loss aversion for the seller. The seller updates its information about the buyer after the acceptance (or rejection) of the first period price. With means of backwards induction the seller can optimize its overall profits, looking first at the optimization of the second period profits, taking into account that the first period price has been rejected.

$$\pi_2 = \frac{(v_x - v_y)}{v_x}(p_2 + \alpha\theta(p_2 - p_x)) \tag{123}$$

Taking into account $v_y = p_2$ and maximizing the second period profit function with respect to p_2 yields:

$$\frac{\partial \pi_2}{\partial p_2} = 0 \tag{124}$$

Solving this for the second period price yields:

$$p_2 = \frac{v_x}{2} + \frac{p_x\alpha\theta}{2(1 + \alpha\theta)} \tag{125}$$

In order for the consumer to make the decision to buy in the first stage instead of in the second stage, the utility the consumer derives from the decision to buy in the first stage must be larger or equal compared to the utility gained by buying in the second period. Therefore the indifferent valued consumer needs to be found: $U_{1,buyer} = U_{2,buyer}$ this leads to the following equality:

$$v_x - p_1 = \delta(v_x - p_2) \tag{126}$$

Filling in p_2 yields:

$$v_x - p_1 = \delta(v_x - (\frac{v_x}{2} + \frac{p_x\alpha\theta}{2(1 + \alpha\theta)})) \tag{127}$$

Solving this inequality for (v_x) the indifferent consumer yields:

$$v_x = \frac{2p_1}{2 - \delta} - \frac{p_x\alpha\delta\theta}{(1 + \alpha\theta)(2 - \delta)} \tag{128}$$

Then in the first stage the seller behaves in order to maximize overall profits:

$$\pi_{overall} = (1 - v_x)(p_1 + \alpha\theta(p_1 - p_x)) + \delta(v_x - v_y)(p_2 + \alpha\theta(p_2 - p_x)) \tag{129}$$

$$\begin{aligned}
\pi_{overall} = \delta(\alpha\theta(&\frac{\frac{\alpha\theta(\alpha\delta\theta p_x - 2p(\alpha\theta+1))}{(\delta-2)(\alpha\theta+1)} + \frac{\alpha\delta\theta p_x - 2p_1(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)} + \alpha\theta p_x}{2(\alpha\theta+1)} - p_x) \\
&+ \frac{\frac{\alpha\theta(\alpha\delta\theta p_x - 2p_1(\alpha\theta+1))}{(\delta-2)(\alpha\theta+1)} + \frac{\alpha\delta\theta p_x - 2p_1(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)} + \alpha\theta p_x}{2(\alpha\theta+1)}) \\
&(\frac{\alpha\delta\theta p_x - 2p_1(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)} - \\
&\frac{\frac{\alpha\theta(\alpha\delta\theta p_x - 2p_1(\alpha\theta+1))}{(\delta-2)(\alpha\theta+1)} + \frac{\alpha\delta\theta p_x - 2p_1(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)} + \alpha\theta p_x}{2(\alpha\theta+1)}) + (\alpha\theta(p_1 - p_x) + p_1)(1 - \frac{\alpha\delta\theta p_x - 2p_1(\alpha\theta+1)}{(\delta-2)(\alpha\theta+1)})
\end{aligned} \tag{130}$$

Taking the partial derivative of $\pi_{overall}$ with regards to p_1, and equalizing p_1 to 0 yields:

$$\frac{\partial \pi_{overall}}{\partial p_1} = 0 \tag{131}$$

$$p_1^* = \frac{\delta^2(\alpha\theta(p_x - 1) - 1) + 2\delta(\alpha\theta(p_x + 2) + 2) - 4(\alpha\theta(p_x + 1) + 1)}{2(3\delta - 4)(\alpha\theta + 1)} \tag{132}$$

With the optimal first period price the other price and values can be calculated.

$$v_x^* = \frac{\delta(\alpha(\theta + 2\theta p_x) + 1) - 2(\alpha\theta(p_x + 1) + 1)}{(3\delta - 4)(\alpha\theta + 1)} \tag{133}$$

$$p_2^* = v_y^* == \frac{\alpha\delta\theta - 2\alpha\theta + \delta + 5\alpha\delta\theta p_x - 6\alpha\theta p_x - 2}{2(3\delta - 4)(\alpha\theta + 1)} \tag{134}$$

Now values for the parameters need to be filled in. In order to model "exogenous expected value" (p_x). Just as before the interval for p_x needs to be calculated. The appropriate parameter values are as before: $\alpha = 0.4$, $\delta = 0.9$. The missing parameter's value is θ. Taking a look at the literature usual loss aversion parameters range anywhere between $\in [1.5, 4]$. Using other papers as a useful benchmark in assessing a reliable value, the decision is made to choose $\theta = 2$.

The interval which p_x can take and still fulfill the requirement of $p_x \geq p_1 \geq p_2$ should be shown in order to determine an appropriate value. The optimal functions have been calculated for p_1^* and p_2^*.

- For p_1 to be smaller than p_x it follows that p_x should always be larger than 0.610426, thus $p_x \geq 0.610426$.

  This is a necessary, but not sufficient condition, furthermore the second period price can also not exceed p_x.

- For p_2 to be smaller than p_x it follows that p_x should always be larger than 0.568966, thus $p_x \geq 0.568966$

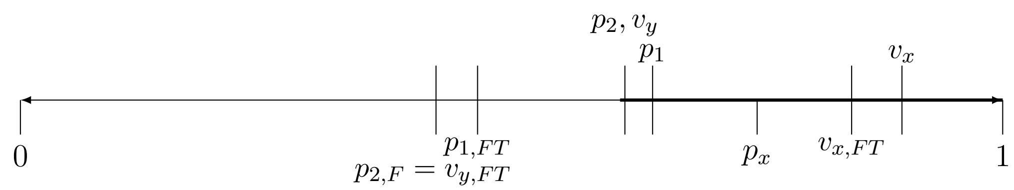
In order to fulfill the above requirements, the largest value should be taken to ensure the feeling of a loss is felt in both periods. This yields the following interval in where p_x can be located: $p_x \in [0.610426; 1]$

Taking the value $p_x = 0.75$, δ takes the value of 0.9, implying limited discounting and α with value of 0.4, showing that sellers take the reference point into account, but only partially. And $\theta = 2$, this leads to the following optimal values:

$$p_1^* = 0.64359$$

$$v_x^* = 0.897436$$

$$p_2^* = v_y^* == 0.615385$$



Overall conclusion: appending the reference point

As the previous cases have shown, the overall conclusion from including the reference utility as an appended utility create the same trend as having reference utility as part of the inclusive utility function. The prices and indifferent consumers behave according to the findings in the initial model. Therefore this section reinforces our previous findings