The Problems with Growth Accounting:

_Unrealistic Assumptions and Biased Results_

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Abstract

The traditional methodology of growth accounting is dependent on the unrealistic assumptions of perfect competition and simultaneous expenditures and revenues for labor and intermediaries. When these assumptions do not hold, the economic growth attributed to labor and intermediaries is underestimated, the economic growth attributed to physical capital is overestimated and the economic growth attributed to productivity can be biased in both directions. The empirical relevance of this issue is assessed with a regression analysis, using panel data on private industries from the United States. The results indicate that the economic growth due to intermediaries is indeed significantly underestimated and growth due to physical capital significantly overestimated. The economic growth attributed to labor is substantially overestimated, but not significantly so, and whether growth due to productivity is significantly biased depends on the circumstances.

Keywords: Growth accounting, Methodology, Bias, Regression

JEL Classification: E25, E22, O47
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1. Introduction

The explanation of economic growth is one of the primary goals of economics. The most popular tool for this purpose is growth accounting (Renelt, 1991; Crafts, 2009), which attributes economic growth to four causes: growth in the quantity of labor, intermediaries, and physical capital used in production and growth in the productivity of the production process. Since its inception by Solow (1957), growth accounting has been used to explain the economic growth of virtually every country in the world, and many industries within those countries.

Growth accounting has been used at the national level for the East Asian Tigers (Young, 1995), Ireland (De Freitas, 2000), Portugal (Lains, 2002), the United States (Denison, 1962; Jones, 2002), Egypt (Ahmed, 2006), India (Bosworth & Collins, 2008), China (ECB, 2013), South Africa (Inglesi-Lotz et al., 2014), Poland (Gradzewicz et al., 2014), Japan (Broadberry et al., 2015), Spain (Franjo & Díaz, 2016), and almost all other countries (Senhadji, 2000; Conference Board, 2017). Growth accounting has also been used extensively at the industry level, such as in Brazil (Bonelli, 1992), the Netherlands (Van der Wiel, 1999), South Korea (Pyo et al., 2006), Germany (Eicher & Roehn, 2007), the European Union (Koszerek et al., 2007), the United States (Bartelsman & Beaulieu, 2004; Rosenthal et al., 2014), China (Wu, 2015), Zambia (Mulungu & Ng’ombe, 2017) and the United Arab Emirates (Alshehhi & Oláh, 2017).

Many conclusions about economic phenomena have been based on the results of growth accounting. Krugman (1994), for example, found that both the Soviet Union and the East Asian Tigers had little increased productivity during their respective industrialization periods, and predicted a slowdown in East Asian growth rates based on this. Further conclusions have been made about the effects of trade policies (Nishimizu & Robinson, 1986, Bonelli, 1992), the rise of information technology (Basu et al., 2003; Van Ark et al., 2002), the German reunification (Burda & Severgnini, 2017) and the sustainability of economic growth in Latin America (Sosa et al., 2013).

The methodology of growth accounting has also been criticized for various reasons (Hulten, 1978; Felipe & Fisher, 2003; Zuleta & Sturgill, 2015). This thesis consists of an additional criticism, which has remained unaddressed so far. Growth accounting is only correct when the expenditures on labor and intermediaries are equal to their contributions to the revenue. As will be shown, this condition is dependent on explicit and implicit assumptions which do not hold in reality. With a regression, using empirical data on private industries from the United States, this is shown to result in significant and substantial bias to the results of traditional growth accounting. The economic growth attributed to growth intermediaries is understated, the economic growth attributed to growth in physical capital is vastly overstated and the economic growth attributed to productivity growth can be biased in both directions.
2. Theory

In this chapter, the flaws in the traditional methodology of growth accounting and their consequences are theoretically explained. Before this is done however, a brief introduction will be given to the functioning of firms and the methodology of growth accounting itself.

2.1 The Functioning of Firms

Growth accounting concerns itself with aggregates of firms, such as all firms within an industry or country. To understand the purpose and methodology of growth accounting, the basic functioning of firms must be understood.

The activity of firms consists of three steps:\footnote{These steps are in a causal order from the point of view of the firm, not necessarily in chronological order. Expenditures can be paid after the factors of production have been received, and revenue from products can be obtained before they are delivered.}

1. Expenditures are made to obtain factors of production.
2. These factors of production are used to create products.
3. The products are sold for a revenue.

*Figure 1: A schematic representation of the activity of a firm. The orange boxes depict financial units, the blue boxes real units.*

The factors of production can be divided into three categories (OECD, 2005; O’Mahony & Timmer, 2009; Deardorff, 2017):

1. Labor, defined as work performed by humans.
2. Intermediaries, defined as any factor of production other than labor which can only be used a single time. This consists of materials, energy and services (BEA, 2016; OECD, 2005).
3. Physical capital is the remaining category, and as such contains factors of production other than labor, which can be used more than a single time. These are structures and equipment (OECD, 2001).
The expenditures, products and revenues can also be divided by the factor of production associated with them (Solow, 1956; OECD, 2005; O’Mahony & Timmer, 2009). The expenditures can be divided into expenditures on labor, expenditures on intermediaries and expenditures on physical capital. The product can be divided in the proportions produced by labor, intermediaries and physical capital and the revenue can be divided into the parts obtained by selling these respective products of labor, intermediaries and physical capital.

In mathematical notation, the factors of production labor, intermediaries and physical capital will be respectively denoted as \( L, I, \) and \( K \), the expenditures made on them as \( EL, EI \) and \( EK \), the products produced by them as \( PL, PI \) and \( PK \), and the revenues generated by them as \( RL, RI \) and \( RK \).

**Figure 2:** The expenditures, products and revenues are all associated with a specific factor of production.

Growth accounting considers revenues during periods \( t \), for aggregations of firms \( s \), such as all firms within an industry or country. These revenues \( R \) can be split into the parts generated by each factor of production:

\[
R_{ts} = RL_{ts} + RI_{ts} + RK_{ts}
\]

The contribution each factor of production makes to the revenue, is a function of the proportion of the product produced by this factor of production, which is a function of the quantity of the factor of production used, which is in turn a function of the expenditures made on this factor of production.

\[
\begin{align*}
RL_{ts} &= F(PL_{Rts}) = F(L_{Rts}) = F(EL_{Rts}) \\
RI_{ts} &= F(PI_{Rts}) = F(I_{Rts}) = F(EI_{Rts}) \\
RK_{ts} &= F(PK_{Rts}) = F(K_{Rts}) = F(EK_{Rts})
\end{align*}
\]
The products, factors of production and expenditures don’t have the subscript \( t \), because they do not necessarily happen during the period \( t \). A revenue obtained at a specific time, can be obtained from the sale of a product that was produced at a different time with factors of production obtained at yet another time, on which expenditures were made at a still other time. The revenues obtained during a period can therefore be a function of products, factors of production and expenditures which fall entirely or partly outside of this period. What matters is not the period in which these products, factors of production and expenditures occur, but that they are associated with the revenue obtained during period \( t \), denoted with the subscript \( R_t \).

With this basic knowledge about the functioning of firms, the methodology of growth accounting, and the criticism thereof, can be understood.

2.2 Traditional Growth Accounting

Growth accounting is a method to attribute growth in either revenue\(^2\) or gross value added to its causes. In this thesis, the attribution of growth in revenue is considered, because it is the most general and easiest to explain. Growth accounting for gross value added is briefly discussed in the appendix.

It has already been explained that revenues are functions of the quantities of labor, intermediaries and physical capital used to generate them:

\[
R_{ts} = R_L + R_I + R_K
\]

\[
R_{ts} = F(L_{Rt}) + F(I_{Rt}) + F(K_{Rt})
\]

Changes in revenues are caused by changes in the quantities of these factors of production, and changes in the productivity \( A \) with which they are used. These changes can be expressed in growth rates. The growth rate of the revenue is denoted by \( gR \), of labor by \( gL \), of intermediaries by \( gI \), of physical capital by \( gK \) and of productivity by \( gA \). The total change in revenue is the sum of the changes in revenue resulting from each cause:

\[
gR_{ts} = gR_{ts} \text{ due to } gL_{Rts} + gR_{ts} \text{ due to } gI_{Rts} + gR_{ts} \text{ due to } gK_{Rts} + gR_{ts} \text{ due to } gA_{Rts}
\]

---

\(^2\) In growth accounting papers, revenue is usually referred to as gross output. The term revenue is used in this thesis because it is more well-known.
Table 1: The causes of changes in revenues specified by growth accounting.

<table>
<thead>
<tr>
<th>In Words</th>
<th>In Symbols</th>
</tr>
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<tbody>
<tr>
<td>Change in revenue due to change in the quantity of labor</td>
<td>$g_{R_{ts}}$ due to $g_{L_{Rts}}$</td>
</tr>
<tr>
<td>Change in revenue due to change in the quantity of intermediaries</td>
<td>$g_{R_{ts}}$ due to $g_{I_{Rts}}$</td>
</tr>
<tr>
<td>Change in revenue due to change in the quantity of physical capital</td>
<td>$g_{R_{ts}}$ due to $g_{K_{Rts}}$</td>
</tr>
<tr>
<td>Change in revenue due to change in productivity</td>
<td>$g_{R_{ts}}$ due to $g_{A_{Rts}}$</td>
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</table>

Traditional growth accounting starts with the growth rate of the revenue, and uses a set of simple formulas to attribute this growth to the specified causes. The derivation of these formulas falls outside the scope of this thesis, and can be obtained from other papers (Solow, 1957; OECD, 2005). The growth in revenue due to the growth of each factor of production, is equal to the contribution of this factor of production to the revenue, divided by the total revenue, and multiplied by the change in the quantity of this factor of production:

\[
g_{R_{ts}} \text{ due to } g_{L_{Rts}} = \frac{R_{L_{ts}}}{R_{ts}} \cdot g_{L_{Rts}}
\]

\[
g_{R_{ts}} \text{ due to } g_{I_{Rts}} = \frac{RI_{ts}}{R_{ts}} \cdot g_{I_{Rts}}
\]

\[
g_{R_{ts}} \text{ due to } g_{K_{Rts}} = \frac{RK_{ts}}{R_{ts}} \cdot g_{K_{Rts}}
\]

The remainder of the growth in revenue is due to changes in productivity:

\[
g_{R_{ts}} \text{ due to } g_{A_{Rts}} = g_{R_{ts}} - g_{R_{ts}} \text{ due to } g_{L_{Rts}} - g_{R_{ts}} \text{ due to } g_{I_{Rts}} - g_{R_{ts}} \text{ due to } g_{K_{Rts}}
\]

\[
g_{R_{ts}} \text{ due to } g_{A_{Rts}} = g_{R_{ts}} - \frac{R_{L_{ts}}}{R_{ts}} \cdot g_{L_{Rts}} - \frac{RI_{ts}}{R_{ts}} \cdot g_{I_{Rts}} - \frac{RK_{ts}}{R_{pt}} \cdot g_{K_{Rps}}
\]

2.3 Existing Criticism of Growth Accounting

While this basic methodology continues to be widely used in empirical research, it has also been criticized in a variety of ways, which will be shortly discussed.
Growth accounting requires data for the growth rates of labor, intermediaries and physical capital. This is problematic, because these factors of production consist of heterogeneous objects, that can’t be accurately compared to each other in a quantitative manner (Felipe & Fischer, 2003). It’s hard if not impossible to combine kilograms of material, kilowatt hours of electricity and numbers of services into a single quantity of intermediaries. The same problem exists for the varying types and qualities of labor and physical capital. Without a common unit with which different kinds of labor, intermediaries and physical capital can be compared, it’s impossible to obtain a growth rate of these factors of production.

Another problem occurs when growth in labor, intermediaries or physical capital is itself caused by growth in other factors of production or growth in productivity (Hulten, 1978; Rodrik, 2008). When this is the case, it is unclear whether the increase in revenue should be attributed to the factor of production which directly generated this revenue, or to the ultimate cause which changed the quantity of this factor of production. An increase in productivity might lead to an increased production of physical capital, which might lead to an increased production of intermediaries, which would finally lead to an increase in revenue. This final increase in revenue is the direct result of an increase in the quantity of intermediaries, but it wouldn’t have occurred without an increase in the physical capital stock, or the original increase in productivity. This makes it impossible to properly attribute this growth in revenue to its causes.

Finally, the contributions to the revenue of each factor or production must remain constant proportions of the total revenue, for the results of growth accounting to be unbiased (Zuleta & Sturgill, 2015). As an example, if the growth in revenue from 2015 to 2016 is attributed to its causes, and the contribution to the revenue of intermediaries is 50% in 2015, then this contribution to the revenue must remain 50% in 2016. If the proportional contribution to the revenue increases or decreases, the attribution of the growth in revenue is no longer correct.

Despite all of this criticism, the aforementioned methodology is accepted as valid for the purposes of this thesis. This thesis solely concerns itself with the subsequent step in the traditional methodology of growth accounting: the equation of the contributions to the revenue of labor and intermediaries to the expenditures made on these factors of production.

2.4 New Criticism of Traditional Growth Accounting

The contributions to the revenue of labor, intermediaries and physical capital are critical inputs into the formulas of growth accounting. Because data for these variables is not available, their values must be estimated. This thesis criticizes the method by which these estimates are obtained.
Traditional growth accounting makes the explicit assumption of perfect competition, and claims that this makes the contributions of labor and intermediaries to the revenue equal to the expenditures made on them (Solow, 1957; OECD, 2005; O’Mahony & Timmer, 2009). The contribution of labor to the revenue of a period should be equal to the expenditures made on this labor, and the contribution of intermediaries to the revenue should be equal to the expenditures made on these intermediaries.

Stated mathematically:

\[
\text{Claimed } RL_{ts} = EL_{Rts}
\]

\[
\text{Claimed } RI_{ts} = EI_{Rts}
\]

Because the total revenue is the sum of the contributions of each factor of production to the revenue, the contribution of physical capital is simply the remainder of the revenue:

\[
\text{Claimed } RK_{Rts} = R_{ts} - EL_{Rts} - EI_{Rts}
\]

Therefore, traditional growth accounting explains changes in revenues with the following formulas in practice:

\[
\text{Claimed } gR_{ts} \text{ due to } gL_{Rts} = \frac{EL_{Rts}}{R_{ts}} \times gL_{Rts}
\]

\[
\text{Claimed } gR_{ts} \text{ due to } gI_{tps} = \frac{EI_{Rts}}{R_{ts}} \times gI_{Rts}
\]

\[
\text{Claimed } gR_{ts} \text{ due to } gK_{Rts} = \left(1 - \frac{EL_{Rts}}{R_{ts}} - \frac{EI_{Rts}}{R_{ts}}\right) \times gK_{Rts}
\]

\[
\text{Claimed } gR_{ts} \text{ due to } gA_{Rts} = gR_{ts} - \frac{EL_{Rts}}{R_{ts}} \times gL_{Rts} - \frac{EI_{Rts}}{R_{ts}} \times gI_{Rts} - \left(1 - \frac{EL_{Rts}}{R_{ts}} - \frac{EI_{Rts}}{R_{ts}}\right) \times gK_{Rts}
\]

The traditional methodology of growth accounting critically depends on the equivalence of the contributions to the revenue of labor and intermediaries and the expenditures made on these factors of production. If these contributions and expenditures differ in value, the results of traditional growth accounting are incorrect. Despite its importance, this condition has never been properly theoretically explored or empirically tested. Both is done in this thesis.

The remainder of this chapter consists of a theoretical criticism of this condition, consisting of two parts:

1. The assumption of perfect competition is insufficient.
2. The assumption of perfect competition is unrealistic.

Each is discussed in turn.
Traditional growth accounting claims that perfect competition makes the expenditures on labor and intermediaries equal to their contributions to the revenue. This is not necessarily true. As will be explained in this section, perfect competition creates a different, more general condition. The assumption of perfect competition is therefore insufficient to justify the traditional methodology of growth accounting.

Perfect competition makes the present value of the expenditures on factors of production equal to the present value of their respective contributions to the revenue (Fischer, 1930). The values of the contributions to the revenue of labor and intermediaries in perfect competition $PC RL$ and $PC RI$, are therefore equal to the present value of the respective expenditures made on this labor and these intermediaries at the time of this revenue $PV(EL)$ and $PV(El)$. The present value of the expenditures on labor and intermediaries at the time of the revenue, can be obtained by multiplying these expenditures by the sum of one and the relevant interest rate $i$ (Investopedia, 2017a):

$$PC RL_{ts} = PV(EL_{ts}) = (1 + i_{Lts}) * EL_{Rts}$$
$$PC RI_{ts} = PV(El_{ts}) = (1 + i_{Its}) * EI_{Rts}$$

The sign of these interest rates is determined by the difference in time between expenditures and revenues, and their size by investment risk (Kealhofer, 2003). Because this time difference and investment risk differs between factors of production, labor and intermediaries each have a separate interest rate. These interest rates can be positive, negative or zero, depending whether the expenditures occur earlier than, later than or at the same time as the revenues.

When the expenditures on labor and intermediaries precede the revenue to which this labor and these intermediaries make a contribution, the relevant interest rate is positive. In this case, the contributions to the revenue of labor and intermediaries exceed the expenditures made on them:

$$i_{Lts} > 0 \quad i_{Its} > 0$$

$$PC RL_{ts} = (1 + i_{Lts}) * EL_{Rts} > EL_{Rts} \quad PC RI_{ts} = (1 + i_{Its}) * EI_{Rts} > EI_{Rts}$$

Example 1: A construction company purchases intermediaries for 50.000 euro, and uses them to build a house. One year after the purchase of these intermediaries, the house is sold for a revenue. By this time, the value of the expenditure on the intermediaries is no longer equal to 50.000 euro. The expenditures have to be compounded by the yearly interest rate to reach the present value of the expenditures at the time of the revenue. If the yearly interest rate is equal to 10\% for example, this present value is equal to 55.000 euro. Therefore, the contribution of these intermediaries to the revenue
is 55.000 euro in perfect competition, instead of the 50.000 euro claimed by traditional growth accounting.

When the revenues on labor intermediaries precede their associated expenditures, the situation is reversed. In this case, the relevant interest rate is negative, and the expenditures are larger than the contributions to the revenue in perfect competition:

\[ i_{LtS} < 0 \quad i_{ItS} < 0 \]

\[ PC \, RL_{ts} < EL_{Rts} \quad PC \, RI_{ts} < EI_{Rts} \]

**Example 2:** A bartender works in a bar at the first day of a month and contributes to the revenue that same day. However, he is only paid at the end of the month. The contribution the bartender makes to the revenue is not equal to the wage he will receive for the work performed that day, which is for example 100 euro. This 100 euro in expenditures on labor has to be discounted by the monthly interest rate to reach the present value of these expenditures. When this monthly interest is 1%, the contribution of the labor to the revenue is 99 euro, slightly less than the 100 euro claimed by traditional growth accounting.

When the expenditures on labor and intermediaries are made at the same time as the revenue generated by these factors of production is obtained, the interest rate is equal to zero, and the contributions to the revenue are equal to the expenditures:

\[ i_{LtS} = 0 \quad i_{ItS} = 0 \]

\[ PC \, RL_{ts} = EL_{Rts} \quad PC \, RI_{ts} = EI_{Rts} \]

**Example 3:** A self-employed farmer is rewarded for his labor at the same moment as he receives the revenue from the sale of his crops. Because there is no time difference between expenditures and revenues, the relevant interest rate is zero, and no compounding or discounting of the expenditures is necessary. The contribution of this labor to the revenue in perfect competition, is the same as the proportion of the revenue which is a reward for labor.

As has been shown, the assumption of perfect competition is insufficient to make the contributions to the revenue of labor and intermediaries equal to the expenditures made on these factors of production. Even in perfect competition, the contributions to the revenue differ from the expenditures when the revenues and expenditures occur at distinct times. In addition to the explicit assumption of perfect competition, the traditional methodology of growth accounting therefore relies on a second, implicit assumption. Namely, that the expenditures on labor and intermediaries are made at the same time as the revenues to which they make a contribution are received.
The assumption of simultaneous expenditures and revenues of labor and intermediaries is unrealistic. In reality, these expenditures tend to precede the revenues generated by them. Expenditures on labor are usually made shortly after this labor has been supplied. In the United States, the country considered in this thesis, the most common payroll-period is bi-weekly (Cheek, 2008; US Department of Labor, 2012), so that expenditures on labor are made a just few days after this labor has been obtained. Expenditures on intermediaries happen mostly at the moment of the delivery of these intermediaries, and the revenues are mostly received at the moment when the product produced by this labor and these intermediaries is delivered to the buyer (Petersen & Rajan, 1997). Because the labor and intermediaries must be obtained before the product can be delivered, the expenditures on labor and intermediaries generally precede the revenues to which they contribute.

Because the expenditures on labor and intermediaries precede the revenues they generate, their associated interest rates are positive and the contributions to the revenue are larger than the expenditures. Therefore, traditional growth accounting underestimates the contributions of labor and intermediaries to the revenue, even if perfect competition holds:

\[ i_{lt} > 0 \]
\[ (1 + i_{lt}) \times EL_{rst} > EL_{rts} \]
\[ PC \, RL_{ts} > Claimed \, RL_{ts} \]

So far, labor and intermediaries have been discussed, but perfect competition also creates specific conditions for the contribution to the revenue of physical capital. Again, the initial expenditures must be increased by the interest rate, to reach the contribution to the revenue. The only difference is that physical capital contributes to more than a single revenue over multiple periods, so that only a proportion of the initial expenditure, known as the depreciation rate \( \delta \), needs to be covered by the revenue of a single period. The contribution of physical capital to the revenue in perfect competition is equal to the expenditures on this factor of production, multiplied by the sum of the relevant interest rate and the depreciation rate:

\[ PC \, RK_{ts} = (\delta_{ts} + i_{Kts}) \times EK_{rts} \]

Traditional growth accounting ignores this equation, and calculates the contribution of physical capital to the revenue by subtracting the claimed contributions of labor and intermediaries from the total revenue.

3 A notable exception is the labor of the self-employed, which receives its reward at the moment when the revenue is obtained. However, this is only a small category in the United States (BEA, 2017)
\[
\text{Claimed } RK_{ts} = R_{ts} - \text{Claimed } RL_{ts} - \text{Claimed } RI_{ts} = R_{ts} - EL_{RTS} - EI_{RTS}
\]

Likewise, the contribution of physical capital to the revenue in perfect competition can be calculated by subtracting the contributions of labor and intermediaries in perfect competition from the revenue:

\[
PC \text{ } RK_{ts} = R_{ts} - PC \text{ } RL_{ts} - PC \text{ } RI_{ts} = R_{ts} - (1 + i_{Lts}) \times EL_{RTS} - (1 + i_{Its})
\]

When the interest rates on labor and intermediaries are positive, and the contributions of labor and intermediaries to the revenue are underestimated, the contribution of physical capital to the revenue is overestimated in perfect competition:

\[
i_{Lts} > 0 \quad i_{Its} > 0
\]

\[
R_{ts} - (1 + i_{Lts}) \times EL_{RTS} - (1 + i_{Its}) \times EI_{RTS} < R_{ts} - EL_{RTS} - EI_{RTS}
\]

\[
PC \text{ } RK_{ts} < \text{Claimed } RK_{ts}
\]

The assumption of perfect competition is insufficient to justify the methodology of growth accounting. In addition, it has to be assumed that expenditures on labor and intermediaries are made at the exact same moment as the revenues generated by them are obtained, so that the interest rates on labor and intermediaries are equal to zero. In reality, expenditures tend to precede revenues. This results in an underestimation of the contributions of labor and intermediaries to the revenue, and an overestimation of the contribution to the revenue of physical capital.

2.6 Perfect Competition is Unrealistic

In addition to the implicit assumption of coinciding expenditures and revenues of labor and intermediaries, the methodology of growth accounting depends on the explicit assumption of perfect competition. This assumption does not hold in reality either, and this leads to an even larger underestimation of the contributions to the revenue of labor and intermediaries, and an even larger overestimation of the contribution to the revenue of physical capital.

When there are deviations from perfect competition, the contributions to the revenue of factors of production are no longer equal to the present value of the expenditures made on them. The amount by which the former exceeds the latter, is known as the economic profit (Investopedia, 2017b). This economic profit can be expressed as a proportion of the expenditures \(\pi\), which will be referred to as the rate of economic profit:

\[
\pi_{Lts} = \frac{RL_{ts} - (1 + i_{Lts}) \times EL_{RTS}}{EL_{RTS}}
\]

\[
\pi_{Its} = \frac{RI_{ts} - (1 + i_{Its}) \times EI_{RTS}}{EI_{RTS}}
\]
\[ \pi_{Kts} = \frac{RI_{ts} - (\delta_{ts} + i_{Kts}) \cdot EK_{RTS}}{EK_{RTS}} \]

The true contributions to the revenue are equal to the product of the expenditures and the sum of the rate of depreciation, interest and economic profit:

\[ RL_{ts} = (1 + i_{Lts} + \pi_{Lts}) \cdot EL_{RTS} \]
\[ RI_{ts} = (1 + i_{Its} + \pi_{Its}) \cdot EI_{RTS} \]
\[ RK_{ts} = (\delta_{ts} + i_{Kts} + \pi_{Kts}) \cdot EK_{RTS} \]

These formulas are true by definition. The interest rate accounts for any difference between the expenditures and the present value of the expenditures, and the rate of economic profit covers any difference between this present value and the contribution to the revenue.

Deviations from perfect competition occur when there are a limited number of competing firms within an industry. This can be caused by increasing returns to scale, limited access to natural resources, or legislation (Krugman & Wells, 2009). When there is limited competition, firms can charge a higher price for their products than the present value of their expenditures, without being undercut by their competitors (Görzig, 2014). If firms have market power, the contributions to the revenue can therefore exceed present value of the expenditures, and the rate of economic profit is positive. Because it is common for firms to have some degree of market power (US Census Bureau, 2017a), it is common for rate of economic profit to be positive.

**Example 4: Again consider the construction company of example 1, which made expenditures of 50.000 euro on intermediaries, one year before these intermediaries generated a revenue. Because the yearly interest rate was 10%, the contribution of these intermediaries to the revenue in perfect competition would be 55.000 euro. If this construction company has market power, the contribution to the revenue of the intermediaries can exceed the expenditures by a positive rate of economic profit, in addition to the yearly interest rate. If the rate of economic profit is 10%, the contribution to the revenue of intermediaries exceeds the expenditures made on them by 20%, which makes it equal to 60.000 euro, instead of the 50.000 euro ascribed to it by traditional growth accounting.**

Deviations from perfect competition do not lead to negative rates of economic profit. If the present value of the expenditures exceeds the present value of the contribution to the revenue for a factor of production, rational firms simply don’t purchase this factor of production. This doesn’t depend on the assumption of perfect competition.
With positive rates of economic profit in addition to positive interest rates, the contributions of labor and intermediaries are even further underestimated by the traditional methodology of growth accounting. The true contributions to the revenue not only exceed the claimed contributions by the interest rate, but also by the rate of economic profit:

\[ i_{Ltss} + \pi_{Ltss} > 0 \]
\[ (1 + i_{Ltss} + \pi_{Ltss}) * EL_{Rs} > EL_{Rs} \]
\[ RL_{ts} > Claimed RL_{ts} \]

\[ i_{Its} + \pi_{Its} \]
\[ (1 + i_{Its} + \pi_{Its}) * EI_{Rs} > EI_{Rs} \]
\[ RI_{ts} > Claimed RI_{ts} \]

These even greater underestimations of the contributions to the revenue of labor and intermediaries lead to an even greater overestimation of the contribution of physical capital to the revenue:

\[ R_{ts} - (1 + i_{Ltss}) * EL_{Rs} - (1 + i_{Its}) * EI_{Rs} < R_{ts} - EL_{Rs} - EI_{Rs} \]
\[ PC RK_{ts} < Claimed RK_{ts} \]

To summarize, in reality the revenue is the sum of the real contributions of each factor of production to the revenue:

\[ R_{ts} = (1 + i_{Ltss} + \pi_{Ltss}) * EL_{Rs} + (1 + i_{Its} + \pi_{Its}) * EI_{Rs} + (\delta_{ts} + i_{Kts} + \pi_{Kts}) * EK_{Rs} \]

The traditional methodology of growth accounting is only correct, when the interest rates and rates of economic profit are equal to zero for labor and intermediaries, which generates the following restricted equation:

\[ R_{ts} = EL_{Rs} + EI_{Rs} + (\delta_{ts} + i_{Kts} + \pi_{Kts}) * EK_{Rs} \]

These interest rates and rates of economic profit are only equal to zero when the assumptions of simultaneous expenditures and revenues, and perfect competition hold. Because neither of these assumptions holds in reality, the traditional methodology of growth accounting generates incorrect values for the contributions to the revenue of each factor of production.

---

4 While perfect competition also requires a rate of economic profit equal to zero for physical capital, this is actually not necessary for the methodology of growth accounting to be correct. Perfect competition is both insufficient and superfluous.
2.7 The Hypothesis

As has been theoretically explained, the traditional methodology of growth accounting should underestimate the contributions of labor and intermediaries to the revenue, and overestimate the contribution of physical capital to the revenue. In the remainder of this thesis, it is empirically tested whether this can also be observed in reality, for private industries within the United States.

The main hypothesis is the following:

*Traditional growth accounting attributes the correct values to the contributions of each factor of production to the revenue.*

This main hypothesis can be divided into two subhypotheses:

**The contribution of labor to the revenue is equal to the expenditures made on this labor.**

**The contribution of intermediaries to the revenue is equal to the expenditures made on these intermediaries.**

\[(1 + i_{LTS} + \pi_{LTS}) \times EL_{R_{TS}} = EL_{TS}\] & \[(1 + i_{ITS} + \pi_{ITS}) \times EI_{R_{TS}} = EI_{R_{TS}}\]

The two subhypotheses are mathematically equivalent to the following:

**The sum of the interest rate and rate of economic profit for labor is equal to zero.**

**The sum of the interest rate and rate of economic profit for intermediaries is equal to zero.**

\[i_{LTS} + \pi_{LTS} = 0\] & \[i_{ITS} + \pi_{ITS} = 0\]

This final specification is the way in which the main hypothesis will be tested. If the sum of the rates of interest and economic profit for either labor or intermediaries differs from zero, the contributions of labor or intermediaries are not equal to the expenditures made on this labor and these intermediaries, and the traditional methodology of growth accounting doesn’t generate correct estimates for the contributions to the revenue of each factor of production.

Because the main hypothesis is based on the unrealistic assumptions of perfect competition and simultaneous expenditures and revenues, it is expected that it will be rejected.
3. Methodology

First, the hypothesis is tested for the average industry with an ordinary least squares regression. Subsequently it is assessed whether this regression suffers from omitted variable bias. Because growth accounting is commonly done for countries as well as industries, a second, weighted, regression is performed to test the hypothesis at the national level.

3.1 Main Regression

The hypothesis that the contributions to the revenue claimed by traditional growth accounting are correct, is tested with an ordinary least squares regression, using panel data from the United States which covers 62 industries over 18 years. This makes the subscript t refers to the year, Rt to the expenditures associated with the revenue in year t, and s to the industry. The methodology follows from the theory and is explained in this section.

Total revenues are equal to the sum of the contributions to the revenue of each factor of production. These contributions are the product of the expenditures made on these factors of production and the sum of their associated rates of depreciation, interest and economic profits:

$$R_{ts} = (1 + i_{Lts} + \pi_{Lts}) \times EL_{Rts} + (1 + i_{Its} + \pi_{Its}) \times EI_{Rts} + (\delta_{ts} + i_{Kts} + \pi_{Kts}) \times EK_{Rts}$$

Traditional growth accounting assumes a specific restricted version of this equation, where the sums of the rate of interest and economic profit are zero for labor and intermediaries.

$$i_{Lts} + \pi_{Lts} = 0 \quad i_{Its} + \pi_{Its} = 0$$

$$R_{ts} = EL_{Rts} + EI_{Rts} + (\delta_{ts} + i_{Kts} + \pi_{Kts}) \times EK_{Rts}$$

Whether this is true or not, can be tested with an ordinary least squares multiple regression analysis, with the revenue as the explained, and the expenditures on labor, intermediaries and physical capital as the explanatory variables:

$$R_{ts} = \beta_0 + \beta_L \times EL_{Rts} + \beta_I \times EI_{Rts} + \beta_K \times EK_{Rts}$$

Because the revenue is the sum of the contributions each factor of production, there are no omitted variables which could alter the value of the revenue independently of that of the expenditures on these factors of production. All remaining variables which influence the revenue, do so through a change in the rates of depreciation, interest or economic profit. Their influence is therefore already captured by the coefficients in the current specification, and adding additional variables to the regression would lead to overcontrol. Time and industry fixed effects are omitted for the same reason. To prevent issues with serial correlation and heteroskedasticity, the standard errors are clustered by industry (Cameron & Miller, 2015).
Theoretically, the constant should be zero, and the coefficients obtained from the regression should represent the average sums of the rates of depreciation, interest and economic profit for each factor of production:

\[ \beta_0 = 0 \]
\[ \beta_L = 1 + \bar{r}_{LTS} + \bar{\pi}_{LTS} \]
\[ \beta_I = 1 + \bar{r}_{ITS} + \bar{\pi}_{ITS} \]
\[ \beta_K = \bar{\delta}_{TS} + \bar{r}_{KTS} + \bar{\pi}_{KTS} \]

If traditional growth accounting is correct, and the rates of interest and economic profits are zero for both labor and intermediaries, the coefficients on the expenditures on labor and intermediaries are equal to 1:

\[ \beta_L = 1 \]
\[ \beta_I = 1 \]

This also implies a specific value for the coefficient on the expenditures on physical capital. In an ordinary least squares regression, the average value of the dependent variable is equal to the outcome of the regression equation, when the average values of the independent variables are used as input (Field, 2013). This means:

\[ \bar{R}_{ts} = \beta_0 + \beta_L * \bar{E}_{L_{TS}} + \beta_I * \bar{E}_{I_{TS}} + \beta_K * \bar{E}_{K_{TS}} \]

When the constant is equal to zero, and the coefficients on expenditures on both labor and intermediaries are equal to 1, as is required by the traditional methodology of growth accounting, the coefficient on the expenditures on physical capital must be equal to a specific value:

\[ \bar{R}_{ts} = 0 + 1 * \bar{E}_{L_{TS}} + 1 * \bar{E}_{I_{TS}} + \beta_K * \bar{E}_{K_{TS}} \]

\[ \bar{R}_{ts} = \bar{E}_{L_{TS}} + \bar{E}_{I_{TS}} + \beta_K * \bar{E}_{K_{TS}} \]

\[ \beta_K = \frac{\bar{R}_{ts} - \bar{E}_{L_{TS}} - \bar{E}_{I_{TS}}}{\bar{E}_{K_{TS}}} \]

The dataset used for the regression has average values for revenues, expenditures on labor, expenditures on intermediaries and expenditures on physical capital of 342, 104, 153 and 478 billion dollar respectively. This would make the coefficient on expenditures on physical capital equal to 17.8%:

\[ \beta_K = \frac{\bar{R}_{ts} - \bar{E}_{L_{TS}} - \bar{E}_{I_{TS}}}{\bar{E}_{K_{TS}}} = \frac{342 - 104 - 153}{478} = 17.8\% \]
If traditional growth accounting distributes the factors of production correctly, the coefficients on expenditures on labor, intermediaries and physical capital, should generate values of one, one, and 17.8% respectively. If the coefficients differ significantly from these values, the traditional methodology of growth accounting incorrectly measures the contributions to the revenue of each factor of production. This is the way in which the hypothesis will be tested. As explained, it is expected that the rates of interest and economic profits on both labor and intermediaries are positive. This would make the coefficients on their expenditures larger than one, and the coefficient on physical capital smaller than 17.8%.

To test whether the coefficients differ significantly from the values ascribed to them by traditional growth accounting, t-tests are performed. The required values are subtracted from the obtained coefficients and the outcome is divided by the standard errors of the coefficients:

\[
\begin{align*}
    t_L &= \frac{\hat{\beta}_L - 1}{\text{S.E. of } \hat{\beta}_L} \\
    t_I &= \frac{\hat{\beta}_I - 1}{\text{S.E. of } \hat{\beta}_I} \\
    t_K &= \frac{\hat{\beta}_K - 17.8\%}{\text{S.E. of } \hat{\beta}_K}
\end{align*}
\]

The obtained t-values are subsequently tested for significance. If the coefficients differ significantly from their required values, the interest and profits generated by expenditures on labor and intermediaries differ significantly from zero, and the real contributions to the revenue differ significantly from the contributions claimed by traditional growth accounting. Although it is expected that the t-statistics on labor and intermediaries are positive, and the t-value on physical capital is negative, theoretically the reverse is also possible. Because of this, the t-tests are two-sided. A significance level of 5% will be maintained.

To summarize, a least squares regression is performed on panel data at the industry-level from the United States, with revenue as the explained and expenditures on labor, intermediaries and physical capital as the explanatory variables. If the coefficients differ significantly from the values they would have if the traditional methodology of growth accounting is correct, the hypothesis that traditional growth accounting is correct will be rejected.
Correlations

While there are no variables which could alter the revenue independently of the expenditures on labor, intermediaries and physical capital, the existence of omitted variables could still bias the results. To prevent omitted variable bias from resulting in an unjust rejection of the hypothesis, several correlations are performed.

Even if the hypothesis is correct, and the sums of rates of interest and economic profits are zero for both labor and intermediaries, the coefficients in the regression can differ significantly from their required values due to omitted variable bias. To see this, reconsider the equation which is assumed by the traditional methodology of growth accounting and tested with the regression:

$$R_{ts} = EL_{Rts} + EI_{Rts} + (\delta_{ts} + i_{Kts} + \pi_{Kts}) \times EK_{Rts}$$

There are six variables determining the amount of revenue, but the regression only uses three of them as independent variables:

$$R_{ts} = \beta_0 + \beta_L \times EL_{Rts} + \beta_I \times EI_{Rts} + \beta_K \times EK_{Rts}$$

The three remaining variables, the depreciation rate, interest rate and rate of economic profit on physical capital, are not included in the regression. When these omitted variables are correlated with the expenditures on labor or intermediaries, their coefficients become biased.

Positive correlations between the expenditures on labor and intermediaries and the rates of depreciation, interest and economic profit on physical capital lead to an overestimation of the coefficients on the expenditures on labor and intermediaries, and negative correlations lead to an underestimation of these coefficients. The results of the regression are only unbiased, if there is no correlation between the expenditures on labor and intermediaries on the one hand, and the rate of depreciation, interest and economic profit on physical capital on the other hand:

$$\text{Corr}(EL_{Rts}, \delta_{ts}) = 0 \quad \text{Corr}(EI_{Rts}, \delta_{ts}) = 0$$
$$\text{Corr}(EL_{Rts}, i_{Kts}) = 0 \quad \text{Corr}(EI_{Rts}, i_{Kts}) = 0$$
$$\text{Corr}(EL_{Rts}, \pi_{Kts}) = 0 \quad \text{Corr}(EI_{Rts}, \pi_{Kts}) = 0$$

Fortunately, there is no theoretical reason for these correlations to be either positive or negative. However, to ensure the initial regression results are unbiased, this is empirically tested. A number of Pearson correlations are performed and their significance is assessed.
Because data for depreciation rates is available, the first set of correlations can be performed directly. The average depreciation rate of each industry is correlated with the average expenditures on labor and intermediaries of these industries. Unfortunately, no data is available for the interest rate and rate of economic profit on physical capital. However, their determinants are known, and can be used as proxies.

The rate of interest is determined by investment risk, for which the unlevered beta is used as an indication. The unlevered beta captures the correlation between industry returns and national returns on investment, and is widely recognized as a determinant of investment risk (Sharpe, 1964; Lintner, 1965; Fama & McBeth, 1973; Pettengill et al., 1995; Görzig et al.; 2014). Positive correlations of the expenditures on labor and intermediaries with the unlevered beta, indicate positive correlations of these expenditures with the interest rate, and an overestimation of the coefficients on the expenditures on labor and intermediaries. A negative correlation indicates an underestimation of these coefficients.

The rate of economic profit is determined by market power, for which the concentration ratio provides an indication. The concentration ratio is the ratio of the revenue of the largest firms to the total industry revenue (Mahajan, 2006).

\[
Concentration Ratio_{ts} = \frac{Revenue \ of \ Firms \ with \ Highest \ Revenue_{ts}}{Industry \ Revenue_{ts}}
\]

The concentration ratio doesn’t take the distribution of revenue within the groups of largest and smallest firms into account. It also doesn’t consider the amount of competition between industries. Despite this, it should be informative to some extent. The concentration ratios of the four and twenty largest firms are correlated with the expenditures on labor and intermediaries. Positive correlations indicate a positive correlation with the rate of economic profit, and an overestimation of the coefficients on the expenditures on labor and intermediaries. Again, negative correlations indicate underestimated coefficients.

Both the interest rate and rate of economic profit are also influenced by the amount of time between the expenditures and revenues. While this exact time is unknown, the investment rate can be used as an indication. The investment rate is defined as ratio of the investment on physical capital during the current year \(t\) to the value of the expenditures on physical capital made in previous years:

\[
Investment \ Rate_{ts} = \frac{Investment \ in \ Physical \ Capital_{ts}}{Value \ of \ Physical \ Capital \ Stock_{ts}}
\]
The higher the investment rate, the more recent the expenditures on physical capital are made, and the less time between the expenditures and the revenues of this physical capital. When there is less time between the expenditures and revenues, the rates of interest and economic profit on physical capital should be lower. A positive correlation between the expenditures on labor and intermediaries and the investment rate could therefore lead to an underestimation of their coefficients, and a negative correlation to an overestimation.

In total, eight correlations are performed and assessed for significance.

\[
\begin{align*}
\text{Corr}(EL_{RTs}, \delta_{ts}) & \quad \text{Corr}(EI_{RTs}, \delta_{ts}) \\
\text{Corr}(EL_{RTs}, \text{Unlevered Beta}_{ts}) & \quad \text{Corr}(EL_{RTs}, \text{Unlevered Beta}_{ts}) \\
\text{Corr}(EL_{RTs}, \text{Concentration Ratio}(4)_{ts}) & \quad \text{Corr}(EL_{RTs}, \text{Concentration Ratio}(4)_{ts}) \\
\text{Corr}(EL_{RTs}, \text{Concentration Ratio}(20)_{ts}) & \quad \text{Corr}(EL_{RTs}, \text{Concentration Ratio}(20)_{ts}) \\
\text{Corr}(EL_{RTs}, \text{Investment Rate}_{ts}) & \quad \text{Corr}(EI_{RTs}, \text{Investment Rate}_{ts})
\end{align*}
\]

If the correlations with the depreciation rate, the unlevered beta or the concentration ratios is significantly positive, or the correlations with the investment rate are significantly negative, the coefficients on labor and intermediaries are overestimated. In this case, these coefficients could significantly exceed the value of one, even if the assumptions of traditional growth accounting hold. Therefore, if such correlation are found, the hypothesis shouldn’t be rejected, even if the regression coefficients significantly differ from their required values.

3.3 Weighted Regression

With the initial regression it is established whether the assumptions of traditional growth accounting hold for the average industry. The conclusion is not necessarily valid for growth accounting at the national level, because industries differ in size. Whether traditional growth accounting is correct at the national level is assessed with a second, weighted, regression.

Industries can differ dramatically in size. The real estate industry for example, is a hundred times as large as the pipeline transportation industry, covering 10% and 0.1% of the national revenue respectively, but in the initial regression, both are equally weighted as a single observation. If the results of the initial regression were solely caused by industries of negligible size, these results are not applicable to the national level. To test whether this is the case or not, a second regression is performed where each observation is weighted by revenue. The standard errors remain clustered by industry, and the specification is the same as the first regression:
\[ R_{ts} = \beta_0 + \beta_L \cdot EL_{Rts} + \beta_I \cdot EI_{Rts} + \beta_K \cdot EK_{Rts} \]

Again, the required values for the coefficients on expenditures on labor and intermediaries should be equal to one, if traditional growth accounting is correct. The required coefficient on expenditures on physical capital can be calculated with the weighted averages of variables, and should be equal to 13.6%.

\[ \beta_K = \frac{R_{ts} - EL_{Rts} - EI_{Rts}}{EK_{Rts}} = \frac{769 - 197 - 302}{1990} = 13.6\% \]

Again, the coefficients on the expenditures on labor, intermediaries and physical capital are compared to their required values with two-sided t-tests, maintaining a significance level of 5%.
4. Data

Data for the regressions consists of revenues and expenditures on labor, intermediaries and physical capital. This data is obtained from the Bureau of Economic Analysis (BEA hereafter), for a panel of 62 private industries from the United States, with yearly data from 1998 to 2015, making a total of 1116 observations.

Data for revenues is directly available. The expenditures on the factors of production should theoretically capture the expenditures on factors of production used to generate the revenue of the period. Unfortunately, this data is unavailable. However, close proxies can be found.

For labor and intermediaries, the expenditures made on them during the year itself are used. This misses the expenditures happening in other years on labor and intermediaries contributing to the revenue of the current year. It also includes expenditures made on labor and intermediaries in the current year, that only contribute to the revenues of other years. While this may introduce a small bias to the value of these expenditures, growth accounting faces the same problem and takes the same approach. Therefore, this doesn’t detract from the ability to test the hypothesis.

For expenditures on labor, there is an additional complication. The expenditures on labor partly consist of the compensation of employees, and partly of the rewards the self-employed receive for their labor. The former is accounted for by the BEA, but the latter is not. To estimate a value, it is assumed that self-employed persons receive the same compensation for their labor as full-time employees of the same industry and year. With this assumption, the expenditures on labor can be calculated in the following way:

\[
EL_{Res} = \frac{\#Self\text{ }Employed\text{ }Persons_{ts} + \#Full\text{ }Time\text{ }Equivalent\text{ }Employees_{ts}}{\#Full\text{ }Time\text{ }Equivalent\text{ }Employees_{ts}} \times \text{Compensation of Employees}_{ts}
\]

While this assumption is undoubtedly wrong, the size of this self-employed labor is relatively small, so even a substantial bias only affects the value of the expenditures on labor to a limited extent.

This leaves the expenditures on physical capital. Expenditures on physical capital during the current year are captured by the BEA as investment in structures and equipment. However, for physical capital, the expenditures made during any specific year are far smaller than the expenditures made on all physical capital that contributed to the revenues of this specific year. This is the case because a unit of physical capital makes contributions to the revenue in many different years. To resolve this issue, the value of the stock physical capital at the beginning of the year, measured as the stock of structures and equipment against current costs by the BEA, is used. This value is equivalent to the expenditures which
would have been made if the physical capital stock was purchased at the beginning of the year. The contribution of physical to the revenue during a year doesn’t depend at all on the time when it was purchased, so this shouldn’t bias the regression. The total expenditures on physical capital are calculated in the following way:

\[ EK_{ts} = \text{Stock of Structures against Current Costs}_{ts} + \text{Investment in Structures}_{ts} \]
\[ \quad + \text{Stock of Equipment against Current Costs}_{ts} + \text{Investment in Equipment}_{ts} \]

The data for all variables which are used in the regression comes from a single official source, so it should be consistent and relatively accurate. While the exact expenditures on the labor of the self-employed remain uncertain, this is unlikely to affect the regression in a substantial way.

Table 2: Summary statistics for the variables in the regressions. Values are in billions of US dollar.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>1116</td>
<td>342</td>
<td>382</td>
<td>16</td>
<td>2965</td>
</tr>
<tr>
<td>Expenditures on Labor</td>
<td>1116</td>
<td>104</td>
<td>123</td>
<td>1</td>
<td>645</td>
</tr>
<tr>
<td>Expenditures on Intermediaries</td>
<td>1116</td>
<td>153</td>
<td>148</td>
<td>3</td>
<td>807</td>
</tr>
<tr>
<td>Expenditures on Physical Capital</td>
<td>1116</td>
<td>478</td>
<td>2012</td>
<td>15</td>
<td>20738</td>
</tr>
</tbody>
</table>

For the correlations, additional data is required for the depreciation rates, unlevered betas, concentration ratios and investment rates.

The BEA has data for the absolute value of the depreciations. These values are divided by the previously calculated expenditures on physical capital to obtain the depreciation rate:

\[ \delta_{ts} = \frac{\text{Depreciation}_{ts}}{EK_{Rts}} \]

Because both the investment in physical capital and the value of the physical capital stock are known, the investment rate can also be calculated:

\[ \text{Investment Rate}_ts = \frac{\text{Investment in Structures}_{ts} + \text{Investment in Equipment}_{ts}}{\text{Stock of Equipment against Current Costs}_{ts} + \text{Stock of Structures against Current Costs}_{ts}} \]

---

5 Again, this is consistent with the practice of growth accounting, which uses the physical capital stock as the measurement for the quantity of physical capital (Solow, 1957).
Unfortunately, the BEA has no data for the unlevered betas and concentration ratios, so these variables must be obtained from other sources.

The unlevered industry betas are obtained from Palacios (2013), who used data from the Center for Research in Security Prices and Compustat to calculate their values for the period 1947-2010. This period differs from that of the other data, but betas can only be estimated over long periods of time. Besides this issue with the time-dimension, data is only available for 41 of the 62 industries used in the regression.

The concentration ratio of the four and twenty largest firms per industry is obtained from the US Census Bureau (2017b). Again the dataset is much smaller than in the original regression. Data is only available for the single year of 2007, again for just 41 of the 62 industries.

The data for depreciation and investment rate has the same source as the data for the regressions and has no omitted values. The variables for the unlevered beta and concentration ratio are from different sources and cover only 41 of the 62 private US industries. While this latter data is far from perfect it should be informative to some extent.

Table 3: Summary statistics for the variables which are exclusively used in the correlations.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation Rate</td>
<td>62</td>
<td>5.1%</td>
<td>2.4%</td>
<td>0.1%</td>
<td>13.1%</td>
</tr>
<tr>
<td>Unlevered Beta</td>
<td>41</td>
<td>1.04</td>
<td>0.21</td>
<td>0.25</td>
<td>1.44</td>
</tr>
<tr>
<td>Concentration Ratio (4)</td>
<td>41</td>
<td>16.7%</td>
<td>10.4%</td>
<td>2.5%</td>
<td>45.4%</td>
</tr>
<tr>
<td>Concentration Ratio (20)</td>
<td>41</td>
<td>35.8%</td>
<td>18.9%</td>
<td>8.5%</td>
<td>88.1%</td>
</tr>
<tr>
<td>Investment Rate</td>
<td>62</td>
<td>10.6%</td>
<td>4.7%</td>
<td>3.4%</td>
<td>27.4%</td>
</tr>
</tbody>
</table>
5. Results

As explained in the methodology, first an unweighted regression is performed to test the hypothesis at the industry level, subsequently omitted variable bias is assessed with Pearson correlations and finally a revenue-weighted regression is performed to test the hypothesis at the national level.

5.1 Main Regression

A regression is performed with revenue as the dependent variable, and expenditures on labor, intermediaries and physical capital as the independent variables. The results are presented in the table below:

*Table 4: The regression results. The dependent variable is the revenue and the standard errors are clustered by industry. The observations cover 62 private US industries over the 18 years from 1998 to 2015, making a total of 1116 observations.*

<table>
<thead>
<tr>
<th>Coefficient (Standard Error)</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-5.529 (6.953)</td>
</tr>
<tr>
<td>Labor</td>
<td>1.137 (0.107)***</td>
</tr>
<tr>
<td>Intermediaries</td>
<td>1.231 (0.052)***</td>
</tr>
<tr>
<td>Physical Capital</td>
<td>0.086 (0.002)***</td>
</tr>
</tbody>
</table>

* denotes significance at the 10% level, ** at the 5% level, and *** at the 1% level.

As indicated by the high R-squared, almost all variation in the revenue has been explained. The constant is small and insignificant, consistent with expectations. All coefficients are significantly different from zero, which means that all factors of production make a significant contribution to the revenue. As expected, the coefficients on expenditures on labor and intermediaries substantially exceed the value of one ascribed to them by traditional growth accounting, by respectively 13.7% and 23.1%. Conversely, the coefficient on physical capital is much smaller than predicted by traditional growth accounting. Instead of 0.178, the obtained coefficient is only 0.086, just 48.1% of its required value.

The differences between the coefficients and the values ascribed to them by traditional growth accounting are tested for significance with t-tests, of which the results are presented in the underlying table:

*Table 5: The t-tests on the coefficients of the regression.*

<table>
<thead>
<tr>
<th>Factor of Production</th>
<th>Coefficient</th>
<th>Required Value</th>
<th>Difference</th>
<th>Standard Error</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>1.137</td>
<td>1.000</td>
<td>0.137</td>
<td>0.107</td>
<td>1.28</td>
<td>0.205</td>
</tr>
<tr>
<td>Intermediaries</td>
<td>1.231</td>
<td>1.000</td>
<td>0.231</td>
<td>0.052</td>
<td>4.46</td>
<td>0.000***</td>
</tr>
<tr>
<td>Physical Capital</td>
<td>0.086</td>
<td>0.178</td>
<td>-0.092</td>
<td>0.002</td>
<td>-53.49</td>
<td>0.000***</td>
</tr>
</tbody>
</table>

* denotes significance at the 10% level, ** at the 5% level, and *** at the 1% level.
The coefficient on the expenditures on labor is not significantly different from one, which indicates that the average contribution of labor to the revenue is not significantly different from the average expenditures on labor. However, this lack of significance is likely caused by large differences in the interest rates and rates of economic profit on labor between industries, rather than these rates being always equal to zero. If traditional growth accounting was correct, the point estimate for the coefficient on the expenditures on labor should be close to one, with a small standard error. Instead, the point estimate is substantially different from one, and the standard error is high. The difference is insignificant due to a large uncertainty about the true value of the coefficient, not because it is certainly close to one.

The coefficient on expenditures on intermediaries is significantly larger than one at the 1% level of significance. This makes the average contribution of intermediaries to the revenue significantly larger than the average expenditures on intermediaries, and indicates that the traditional methodology of growth accounting significantly underestimates the contribution of intermediaries to the revenue.

The coefficient on the expenditures on physical capital differs even more significantly from the value of 17.8%. Because this coefficient is significantly smaller than the value it would obtain if the traditional methodology of growth accounting was correct, the contribution of physical capital to the revenue is significantly overestimated by this methodology.

As the contribution to the revenue of intermediaries is significantly underestimated, and the contribution to the revenue of physical capital is significantly overestimated by the traditional methodology of growth accounting, the main hypothesis must be rejected. Traditional growth accounting does not attribute the correct values to the contributions of each factor of production to the revenue, for private industries within the United States.

5.2 Correlations

To assess whether the deviations of the coefficients on labor and intermediaries from one were caused by omitted variable bias, several correlations are performed. If the coefficients on the expenditures on intermediaries and labor were overestimated, due to positive correlations with the depreciation rate, unlevered beta or the concentration ratios, or negative correlations with the investment rate, the hypothesis would have been wrongly rejected. The results of the correlations are presented in the following table:
Table 6: The Pearson correlations of the expenditures on labor and intermediaries with the depreciation rate, the unlevered beta, the concentration ratios for the 4 and 20 largest firms, and the investment rate. The observations cover all 62 private US industries for the depreciation rate and investment rate, but just 41 of these industries for the unlevered beta and the concentration ratios.

<table>
<thead>
<tr>
<th></th>
<th>Depreciation Rate</th>
<th>Unlevered Beta</th>
<th>Concentration Ratio (4)</th>
<th>Concentration Ratio (20)</th>
<th>Investment Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditures on Labor</td>
<td>0.21</td>
<td>-0.05</td>
<td>-0.37***</td>
<td>-0.40***</td>
<td>0.24*</td>
</tr>
<tr>
<td>Expenditures on Intermediaries</td>
<td>0.07</td>
<td>-0.18</td>
<td>-0.12</td>
<td>0.14</td>
<td>0.10</td>
</tr>
</tbody>
</table>

* denotes significance at the 10% level, ** at the 5% level, and *** at the 1% level.

The expenditures on intermediaries don’t have any significant correlation, so there is no indication that the coefficient on the expenditures on intermediaries was overestimated by omitted variable bias.

The expenditures on labor have three significant correlations. They are negatively correlated with the concentration ratios of both the four and twenty largest firms at the 1% level of significance, and positively correlated with the investment rate at the 10% level of significance. These positive correlations with the concentration ratios and negative correlation with the investment rate all indicate an underestimation of the coefficient on the expenditures on labor. While far from certain, it is possible that the coefficient on the expenditures on labor would have been significantly different from one in the absence of this omitted variable bias.

While omitted variable bias is detected, this leads to an underestimation instead of an overestimation of the coefficient on labor. Therefore, the rejection of the hypothesis was fully justified.

5.3 Weighted Regression

Finally, the regression is weighted by revenue to assess whether the results are just caused by industries of negligible size, making them invalid at the national level.

Table 7: The results of the revenue-weighted regression. Again, the dependent variable is the revenue, the standard errors are clustered by industry, and the observations cover 62 private US industries over the 18 years from 1998 to 2015, making a total of 1116.

<table>
<thead>
<tr>
<th>Coefficient (Standard Error)</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-4.404 (17.612)</td>
</tr>
<tr>
<td>Labor</td>
<td>1.218 (0.127)***</td>
</tr>
<tr>
<td>Intermediaries</td>
<td>1.198 (0.074)***</td>
</tr>
<tr>
<td>Physical Capital</td>
<td>0.086 (0.002)***</td>
</tr>
</tbody>
</table>

* denotes significance at the 10% level, ** at the 5% level, and *** at the 1% level.
The weighted and unweighted regressions provide similar results. Again, the R squared is high and the constant negligible. The coefficient on expenditures on physical capital is also the same. In this case however, the coefficient on expenditures on labor is slightly higher and the coefficient on expenditures on intermediaries slightly lower, although the difference is within the standard error.

The traditional methodology of growth accounting requires the values of the coefficients on expenditures on labor and intermediaries to be equal to one, and on the expenditures on physical capital to be equal to 13.6%. The estimated coefficients differ from these required values by respectively 21.8%, 19.8% and -36.8%. To see if these differences are significant, new t-tests are performed, as can be seen in the following table:

<table>
<thead>
<tr>
<th>Factor of Production</th>
<th>Coefficient</th>
<th>Required Value</th>
<th>Difference</th>
<th>Standard Error</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>1.218</td>
<td>1.000</td>
<td>0.218</td>
<td>0.127</td>
<td>1.72</td>
<td>0.091*</td>
</tr>
<tr>
<td>Intermediaries</td>
<td>1.198</td>
<td>1.000</td>
<td>0.198</td>
<td>0.074</td>
<td>2.69</td>
<td>0.009***</td>
</tr>
<tr>
<td>Physical Capital</td>
<td>0.086</td>
<td>0.136</td>
<td>-0.050</td>
<td>0.002</td>
<td>-24.63</td>
<td>0.000***</td>
</tr>
</tbody>
</table>

* denotes significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Both the coefficients on intermediaries and physical capital retain their significance at the 1% level. The coefficient on expenditures on labor becomes significant at the 10% level. This indicates that the traditional methodology of growth accounting is not just defective for the average industry within the United States, but also for the United States as a whole.
6. Interpretation

Given the unrealistic assumptions on which the traditional methodology of growth accounting is based, it isn’t surprising that its distribution of the contributions to the revenue is mismeasured. More interesting is the degree to which these contributions differ from their ascribed values, and the consequences this has for the results of traditional growth accounting. Both will be discussed in this chapter.

6.1 Biased Contributions to the Revenue

With the obtained coefficients, the real contributions to the revenue of each factor of production can be estimated. The estimated betas are simply multiplied by the average expenditures to obtain the estimated average real contribution to the revenue.

\[
\hat{\beta}_L \cdot \bar{E}_L = \bar{R}_L
\]

\[
\hat{\beta}_I \cdot \bar{E}_I = \bar{R}_I
\]

\[
\hat{\beta}_K \cdot \bar{E}_K = \bar{R}_K
\]

These values can be compared to the values for the average contributions to the revenue claimed by traditional growth accounting. The latter can be obtained either by multiplying the required coefficients to the average expenditures or using the original methodology. Both methods generate the same results:

\[
1 \cdot \bar{E}_L = \bar{E}_L = Claimed \bar{R}_L
\]

\[
1 \cdot \bar{E}_I = \bar{E}_I = Claimed \bar{R}_I
\]

\[
17.8\% \cdot \bar{E}_K = \bar{R}_K - \bar{E}_L - \bar{E}_I = Claimed \bar{R}_K
\]

The estimated real contributions and the contributions claimed by traditional growth accounting are compared in the figure on the next page.
Figure 3: The contributions to the revenue for the average private industry in the United States, according to the regression results and traditional growth accounting respectively. 95% confidence intervals are depicted.

The average contribution of physical capital to the revenue is 41 billion dollar. The contribution of physical capital to the revenue claimed by traditional growth accounting, which has a value of 85 billion dollar, is overestimated by 44 billion dollar. This implies that the contributions of labor and intermediaries to the revenue are jointly underestimated by the same amount. The distribution of this underestimation between labor and intermediaries remains uncertain. The contribution of intermediaries is significantly underestimated by traditional growth accounting, but to what degree is less clear. For labor the situation is even more unclear. The confidence interval is extremely wide, and includes overestimations of the contribution of labor to the revenue as well as severe underestimations. As point estimates, the average contribution of intermediaries to the revenue is underestimated by 36 billion dollar, and the average contribution of labor by 14 billion dollar.  

\[ \text{The sum of the over- and underestimations differs from zero by 6 billion, the value of the constant in the regression.} \]
6.2 Biased Attributions of Economic Growth

The traditional methodology of growth accounting has been shown to be unsound. The remaining question is how the defects in this methodology affect the results of traditional growth accounting. First, the attribution of economic growth to changes in labor, intermediaries and physical capital will be discussed, subsequently the attribution to changes in productivity.

The regression coefficients can be used to make point estimates about the true causes of growth in revenue:

\[
g_{R_{ts}} \text{ due to } g_{L_{Rts}} = \frac{R_{L_{Rts}}}{R_{ts}} \star g_{L_{Rts}} = \frac{\beta_{L} \star E_{L_{Rts}}}{R_{ts}} \star g_{L_{Rts}} = 113.7\% \star E_{L_{Rts}} \approx 113.7\% \star g_{L_{Rts}}
\]

\[
g_{R_{ts}} \text{ due to } g_{I_{Rts}} = \frac{R_{I_{Rts}}}{R_{ts}} \star g_{I_{Rts}} = \frac{\beta_{I} \star E_{I_{Rts}}}{R_{ts}} \star g_{I_{Rts}} = 123.1\% \star E_{I_{Rts}} \approx 123.1\% \star g_{I_{Rts}}
\]

\[
g_{R_{ts}} \text{ due to } g_{K_{Rts}} = \frac{R_{K_{Rts}}}{R_{ts}} \star g_{K_{Rts}} = \frac{\beta_{K} \star E_{K_{Rts}}}{R_{ts}} \star g_{K_{Rts}} = 8.6\% \star E_{K_{Rts}} \approx 8.6\% \star g_{K_{Rts}}
\]

These point estimates differ from the values claimed by traditional growth accounting:

\[
\text{Claimed } g_{R_{ts}} \text{ due to } g_{L_{Rts}} = \frac{E_{L_{Rts}}}{R_{ts}} \star g_{L_{Rts}}
\]

\[
\text{Claimed } g_{R_{ts}} \text{ due to } g_{I_{Rts}} = \frac{E_{I_{Rts}}}{R_{ts}} \star g_{I_{Rts}}
\]

\[
\text{Claimed } g_{R_{ts}} \text{ due to } g_{K_{Rts}} = \left(1 - \frac{E_{L_{Rts}}}{R_{ts}} - \frac{E_{I_{Rts}}}{R_{ts}}\right) \star g_{K_{Rts}} = \frac{17.8\% \star E_{K_{Rts}}}{R_{ts}} \approx 17.8\% \star g_{K_{Rts}}
\]

The estimated real growth in revenue caused by each factor or production can be expressed as a proportion of the growth in revenue claimed by traditional growth accounting:

\[
g_{R_{ts}} \text{ due to } g_{L_{Rts}} \approx 113.7\% \star \text{Claimed } g_{R_{ts}} \text{ due to } g_{L_{Rts}}
\]

\[
g_{R_{ts}} \text{ due to } g_{I_{Rts}} \approx 123.1\% \star \text{Claimed } g_{R_{ts}} \text{ due to } g_{I_{Rts}}
\]

\[
g_{R_{ts}} \text{ due to } g_{K_{Rts}} \approx 48.1\% \star \text{Claimed } g_{R_{ts}} \text{ due to } g_{K_{Rts}}
\]

These point estimates differ in their certainty. In figure on the following page, the associated confidence intervals can be found:
Figure 4: The growth in revenue due to growth in each factor of production for the average industry as estimated by the regression, as a proportion of the growth in revenue due to this factor of production claimed by traditional growth accounting.

Changes in the revenue caused by changes in the quantity of the factor of production have the same proportional bias as the coefficients and the contributions the revenue. In the case of physical capital, the point estimate for the real growth in revenue due to growth in physical capital is only 48% of the claimed growth, with a very small confidence interval. At the same time, the revenue growth caused by intermediaries growth is substantially and significantly underestimated. The amount of bias for the revenue growth due to labor growth remains uncertain.

This leaves the growth in the revenue caused by a growth in productivity. In this case, the results of traditional growth accounting are also biased but in a more complicated way. The point estimate for the real value of revenue growth due to changes in productivity is the following:

\[
gR_{ts} \text{ due to } gA_{RTS} = gR_{ts} - \frac{RL_{ts}}{R_{ts}} * gL_{RTS} - \frac{RI_{ts}}{R_{ts}} * gI_{RTS} - \left(1 - \frac{RL_{ts}}{R_{ts}} - \frac{RI_{ts}}{R_{ts}} \right) * gK_{RTS} = \\
gR_{ts} - \frac{\beta_L * EL_{RTS}}{R_{ts}} * gL_{RTS} - \frac{\beta_I * EI_{RTS}}{R_{ts}} * gI_{RTS} - \left(1 - \frac{\beta_L * EL_{RTS}}{R_{ts}} - \frac{\beta_I * EI_{RTS}}{R_{ts}} \right) * gK_{RTS} \approx \\
gR_{ts} - \frac{113.7\% * EL_{RTS}}{R_{ts}} * gL_{RTS} - \frac{123.1\% * EI_{RTS}}{R_{ts}} * gI_{RTS} \\
- \left(1 - \frac{113.7\% * EL_{RTS}}{R_{ts}} - \frac{123.1\% * EI_{RTS}}{R_{ts}} \right) * gK_{RTS}
\]
Again, this is clearly different from the growth in revenue due to a growth in productivity claimed by traditional growth accounting:

\[
\text{Claimed } gR_{ts} \text{ due to } gA_{Rts} = R_{ts} - \frac{EL_{Rts}}{R_{ts}} * gL_{Rts} - \frac{EI_{Rts}}{R_{ts}} * gI_{Rts} - (1 - \frac{EL_{Rts} + EI_{Rts}}{R_{ts}}) * gK_{Rts}
\]

In this case, it is easier to express the bias as an absolute difference:

\[
gR_{ts} \text{ due to } gA_{Rts} - \text{Claimed } gR_{ts} \text{ due to } gA_{Rts} \approx \frac{13.7\% * EL_{Rts}}{R_{ts}} * (gL_{Rts} - gK_{Rts}) + \frac{23.1\% * EI_{Rts}}{R_{ts}} * (gI_{Rts} - gK_{Rts})
\]

When the growth rates of labor and intermediaries exceed the growth rate of physical capital, the growth in revenue due to productivity growth is overestimated, and when the growth rate of physical capital exceeds those of labor and intermediaries, the growth in revenue due to productivity growth is underestimated. This finding casts doubts on the findings of little productivity growth in the Soviet Union and the East Asian Tigers during their industrialization periods (Young, 1995; Krugman, 1994), mentioned in the introduction. These rapidly industrializing countries had high growth rates of physical capital, which could lead to an underestimation of productivity growth.

Most results of traditional growth accounting turn out to be substantially biased. Only for the growth in revenue due to growth in labor does the bias remain uncertain. The growth in revenue due to growth in intermediaries is substantially underestimated, and the growth in revenue due to growth in physical capital is only half as large as claimed by traditional growth accounting. The growth in revenue due to growth in productivity is also biased, but the direction and size of this bias depends on the specifics of the situation.

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7 In the first half of the 1990s, South Korea had an average growth rate of physical capital of 14.7%, while its growth rate of labor (including quality) averaged just 2.4%, a difference of 12.3%. For Taiwan these figures were respectively 10.9% and 2.7%, a difference of 8.2% (The Conference Board, 2017). These massive differences could lead to a substantial underestimation of productivity growth.
7. Conclusion

Traditional growth accounting makes the unrealistic assumptions of instantaneous expenditures and revenues and perfect competition for labor and intermediaries. Because these assumptions don’t hold in reality, theoretically the contributions to the revenue of labor and intermediaries should be underestimated and the contribution to the revenue of physical capital overestimated by traditional growth accounting.

The regression analysis shows with data from the United States, that the contribution to the revenue of intermediaries is indeed significantly underestimated and the contribution to the revenue of physical capital significantly overestimated. The contribution to the revenue of labor is substantially underestimated but not significantly so, although this lack of significance could be the result of omitted variable bias.

The mismeasurement of the contributions to the revenue of the factors of production, leads to a misattribution of economic growth. Too little growth is attributed to intermediaries and too much growth is attributed to physical capital. The growth attributed to productivity is also biased, but the direction of this bias depends on the specifics of the situation.

The confidence in these findings is relatively high. They conform to the theoretical expectations. All of the data used in the regression comes from the same trustworthy source, and the regression specification is well defined by the theory. Although omitted variable bias reduced the power of the empirical tests, the results were highly significant.

If growth accounting is continued, it should be recognized that its results could suffer from substantial bias. Conclusions based on the historical use of growth accounting should be reassessed. While it is possible that these conclusions are valid, they could also be based on the defects in the methodology.
References


Appendix: Growth Accounting for Gross Value Added

In this appendix, it will be shown that growth accounting for gross value added suffers from the same problems as growth accounting for revenues. First the general and correct methodology of growth accounting for gross value added is explained, subsequently the methodology which is actually used in practice, and which is dependent on the unrealistic assumption that the rates of interest and economic profit for labor and intermediaries are equal to zero.

Gross value added $Y$ is defined as the revenue minus the expenditures on intermediaries which were required to produce this revenues (BEA, 2006):

$$Y_{ts} = R_{ts} - EI_{Rts}$$

Given the determinants of the revenue, gross value added is determined in the following way:

$$R_{ts} = (1 + i_{Lts} + \pi_{Lts}) \times EL_{Rts} + (1 + i_{Its} + \pi_{Its}) \times EI_{Rts} + (\delta_{ts} + i_{Kts} + \pi_{Kts}) \times EK_{Rts}$$

$$Y_{ts} = (1 + i_{Lts} + \pi_{Lts}) \times EL_{Rts} + (i_{Its} + \pi_{Its}) \times EI_{Rts} + (\delta_{ts} + i_{Kts} + \pi_{Kts}) \times EK_{Rts}$$

The contributions of labor and physical capital to gross value added are equal to their contributions to the revenue but the contribution of intermediaries to the gross value added only consists of the interest and profits generated by intermediaries:

$$YL_{ts} = (1 + i_{Lts} + \pi_{Lts}) \times EL_{Rts}$$

$$YI_{ts} = (i_{Its} + \pi_{Its}) \times EI_{Rts}$$

$$YK_{ts} = (\delta_{ts} + i_{Kts} + \pi_{Kts}) \times EK_{Rts}$$

Changes in gross value added can be attributed to changes in labor, changes in intermediaries, changes in physical capital and changes in productivity in the same way as growth accounting for revenue. The contributions to the revenue are just replaced by the contributions to the gross value added, and these contributions are divided by the gross value added instead of the revenue:

$$gY_{ts} = gY_{ts} \text{ due to } gL_{Rts} + gY_{ts} \text{ due to } gI_{Rts} + gY_{ts} \text{ due to } gK_{Rts} + gY_{ts} \text{ due to } gA_{Rts}$$

$$gY_{ts} \text{ due to } gL_{Rts} = \frac{YL_{ts}}{Y_{ts}} \times gL_{Rts} = \frac{(1 + i_{Lts} + \pi_{Lts}) \times EL_{Rts}}{Y_{ts}} \times gL_{Rts}$$

$$gY_{ts} \text{ due to } gI_{Rts} = \frac{YI_{ts}}{Y_{ts}} \times gI_{Rts} = \frac{(i_{Its} + \pi_{Its}) \times EI_{Rts}}{Y_{ts}} \times gI_{Rts}$$
\[ gY_{ts} \text{ due to } gK_{ts} = \frac{YK_{ts}}{Y_{ts}} * gK_{ts} = \frac{(\delta_{ts} + i_{Kts} + \pi_{Kts}) * EK_{Rts}}{Y_{ts}} * gK_{Rts} = \frac{Y_{ts} - (1 + i_{Lts} + \pi_{Lts}) * EL_{Rts} - (i_{Its} + \pi_{Its}) * EI_{Rts}}{Y_{ts}} * gK_{Rts} \]

\[ gY_{ts} \text{ due to } gA_{Rts} = \frac{(1 + i_{Lts} + \pi_{Lts}) * EL_{Rts}}{Y_{ts}} * gL_{Rts} - \frac{(i_{Its} + \pi_{Its}) * EI_{Rts}}{Y_{ts}} * gI_{Rts} = \frac{Y_{ts} - (1 + i_{Lts} + \pi_{Lts}) * EL_{Rts} - (i_{Its} + \pi_{Its}) * EI_{Rts}}{Y_{ts}} * gK_{Rts} \]

This correct version of growth accounting for gross value added is never used in practice. Traditional growth accounting for gross value added, like traditional growth accounting for revenue, assumes that the sum of the interest rate and rate of economic profit are zero for labor and intermediaries:

\[ i_{Lts} + \pi_{Lts} = 0 \]
\[ i_{Its} + \pi_{Its} = 0 \]

This results in the following contributions to the gross value added of each factor of production:

\[ \text{Claimed } YL_{ts} = 1 * EL_{Rts} = EL_{Rts} \]
\[ \text{Claimed } YI_{ts} = 0 * EI_{Rts} = 0 \]
\[ \text{Claimed } YK_{ts} = Y_{ts} - EL_{Rts} \]

And the following claimed growth rates caused by changes in the factors of production and productivity:

\[ \text{Claimed } gY_{ts} \text{ due to } gL_{Rts} = \frac{EL_{Rts}}{Y_{ts}} * gL_{Rts} \]
\[ \text{Claimed } gY_{ts} \text{ due to } gI_{Rts} = 0 * gI_{Rts} = 0 \]
\[ \text{Claimed } gY_{ts} \text{ due to } gK_{Rts} = \frac{Y_{ts} - EL_{Rts}}{Y_{ts}} * gK_{Rts} \]

\[ \text{Claimed } gY_{ts} \text{ due to } gA_{Rts} = \left( gY_{ts} - \frac{EL_{Rts}}{Y_{ts}} * gL_{Rts} - \frac{Y_{ts} - EL_{Rts}}{Y_{ts}} \right) * gK_{Rts} \]

Like with growth accounting for revenue, the contributions intermediaries and the economic growth attributed to them are understated, the contribution of physical capital and the economic growth attributed to it is overstated, and the direction of the bias on the attribution of economic growth to productivity depends on the growth rates of the factors of production.
\[
g_{Y_{ts}} \text{ due to } g_{L_{Rts}} \approx 113.7\% \\
\text{Claimed } g_{Y_{ts}} \text{ due to } g_{L_{Rts}} \\
g_{Y_{ts}} \text{ due to } g_{I_{Rts}} \approx 123.1\% \\
\text{Claimed } g_{Y_{ts}} \text{ due to } g_{I_{Rts}} \\
g_{Y_{ts}} \text{ due to } g_{K_{Rts}} \approx 49.1\% \\
\text{Claimed } g_{Y_{ts}} \text{ due to } g_{K_{Rts}}
\]

\[
\frac{\text{Claimed } g_{Y_{ts}} \text{ due to } g_{A_{Rts}} - g_{Y_{ts}} \text{ due to } g_{A_{Rts}}}{Y_{ts}} \\
\approx \frac{13.7\% \times E_{L_{Rts}}}{Y_{ts}} \times (g_{L_{Rts}} - g_{K_{Rts}}) + \frac{23.1\% \times E_{I_{Rts}}}{Y_{ts}} \times (g_{I_{Rts}} - g_{K_{Rts}})
\]

The results of the thesis are just as relevant for growth accounting for growth in gross value added as they are for growth accounting for revenue. Again, the growth attributed to intermediaries is understated and the growth attributed to physical capital vastly overstated by traditional growth accounting.