Bachelor Thesis Financial Economics

GARCH Modeling of Bitcoin, S&P-500 and the Dollar

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Abstract

Bitcoin and cryptocurrencies are relatively new financial asset and have gotten more attention in the worldwide media. This paper performs a volatility analysis of Bitcoin and compares the volatility process of Bitcoin with the S&P-500 and the EUR/USD exchange rate. The goodness-of-fit and forecast ability are examined for the GARCH(p,q), TGARCH(1,1), EGARCH(1,1) and APARCH(1,1) models. The goodness-of-fit and forecast ability are evaluated with the use of Akaike/Bayesian information criteria and the Mincer-Zarnowitz regression, respectively. The EGARCH(1,1) model fitted the volatility best for Bitcoin and the S&P-500, the APARCH(1,1) fitted the volatility best for the EUR/USD exchange rate. Only the EGARCH(1,1) model with a student's t distribution with 5 degrees of freedom seemed to have fairly accurate forecast power for Bitcoin. The APARCH(1,1) model predicts the conditional variance for the EUR/USD exchange rate relative well. The EGARCH(1,1) did not succeed in performing unbiased predictions for the S&P-500. From a volatility point of view, the analysis performed in this paper shows that Bitcoin is closer to a security than a true currency.

Keywords— Bitcoin, Volatility, GARCH, Maximum likelihood

Contents

1 Introduction

The first introduction of Bitcoin was in late 2008 in the white paper: "Bitcoin: A Peer-to-Peer Electronic Cash System" of a pseudo anonymous writer called Satoshi Nakamoto (Nakamoto, 2008). Recently Bitcoin and other cryptocurrencies have gotten more and more attention in the worldwide media because of their enormous market capitalization growth (see Figure 1). Different government agencies are researching the possibilities for fitted regulation. This makes the long run outlook for Bitcoin and other cryptocurrencies uncertain, which is an important reason why just a few institution investors are holding Bitcoin or other cryptocurrencies in their investment portfolio.

At this stage, the amount of research that has been done on Bitcoin and other cryptocurrencies is still in short supply. Brire, Oosterlinck and Szafarz investigated Bitcoin as a method to further diversify a investment portfolio with traditional assets. There research showed that Bitcoin has a remarkably low correlation with traditional assets and that Bitcoin offers significant diversification methods (Brière, Oosterlinck, and Szafarz, 2015). Dyhrberg explored the financial asset capabilities of Bitcoin with reference to the dollar and gold using GARCH models. The conclusion of his analysis points out that Bitcoin deserves a place on the financial markets and in portfolio management as it can be classified as a asset class between gold and the dollar (Dyhrberg, 2016). Pitch and Kaizoij modeled the volatility of BTC using the day-to-day distribution of logarithmic return and realized volatility (sum of the squared logarithmic returns on 5-min basis). They found that the Heterogeneous Autoregressive model for realized volatility fits reasonably well for there BTCUSD dataset (Pichl and Kaizoji, 2017). Another study analyzed 12 different GARCH-type models not just for Bitcoin, but for the seven most popular cryptocurrencies. The results of their work shows that the IGARCH and GJRGARCH models provide the best fits, in terms of modeling the volatility (Chan, Chu, Nadarajah, and Osterrieder, 2017). Katsiampa found that the CGARCH model with a AR transformation fitted the data best, which emphasis the significance of including both the short and long run component of the conditional variance (Katsiampa, 2017).

The question for regulators remains: 'Are Bitcoin and other cryptocurrencies, currencies or a securities?'. According to The European Central Bank (ECB) the main motivation why Bitcoin is not a true currency is the volatility aspect of the new asset class (ECB, 2015). In this paper the volatility aspect of Bitcoin will be researched and compared with the volatility of currencies (EUR/USD exchange rate) and of securities (S&P-500).

The structure of this research is divided into five sections. First, the data will be described in Section 2. Second, the method and the different GARCH-type models that are used for the volatility predictions of Bitcoin, S&P-500 and the EUR/USD exchange rate are described in Section 3. Subsection 3.2 includes the selection criteria and measurements of forecast ability. The results will be discussed in Section 4 of this paper. Section 5 concludes with a brief overview, discussion and suggestions for future research.

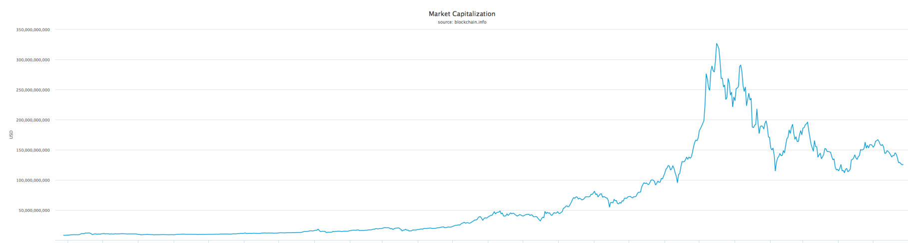

Figure 1: Market capitalization Bitcoin

2 Data

The data that is going to be applied in this paper are the historical daily closing prices for Bitcoin, S&P500-index and the EUR/USD exchange rate. The one-day returns for the Bitcoin, S&P500-index and the EUR/USD exchange rate are calculated from the daily closing prices (P_t) where the return for day t is calculated as:

$$Return_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

To use the most accurate Bitcoin prices, the average closing prices on the basis of volume are used in the paper, as performed in (Chan, Chu, Nadarajah, and Osterrieder, 2017). The average closing prices are obtained from Coinmarketcap (Coinmarketcap, 2018). Daily closing prices of the S&P500-index are sourced from the Datastream database and the closing prices of the EUR/USD exchange rate are obtained from Investing (Investing, 2018). The dataset contains prices starting at 01-05-2013 up to 01-03-2018, with 1804 observations for Bitcoin, 1217 for the S&P-500 and 1252 for the EUR/USD exchange rate. From Table 1 we see that the average return for Bitcoin is 0.32%, 0.04% for the S&P-500 and almost 0% for the EUR/USD exchange rate. Bitcoin highest one-day return is 42.97% and the lowest one-day return is -23.37% The high kurtosis value (12.90) suggest that Bitcoin is a relative risky assets compared to the S&P-500 (6.36) and the EUR/USD exchange rate (5.88). This can also be seen in the relatively high standard deviation of the returns of Bitcoin compared to the other two assets. From the skewness values we can conclude that Bitcoin is moderately right skewed, the S&P-500 moderately left skewed and the USD/EUR exchange rate is approximately symmetric (Bulmer, 1979).

Table 1: Data descriptives

	Bitcoin	S&P-500	EUR/USD
Observations	1804	1217	1252
Mean	0.318	0.045	-0.004
Standard dev.	4.524	0.773	0.526
Minimum	-23.371	-4.098	-2.293
Maximum	42.968	3.903	3.161
Skeweness	0.511	-0.524	0.264
Kurtosis	12.897	6.357	5.881

Mean, Standard dev., Minimum, and the Maximum are in percentage points

3 Method

In this paper several Generalized Autoregressive Conditionally Heteroskedasticity (GARCH)-type models are used to predict the one-day ahead volatility (conditional variance) for the three asset classes. But first, a short introduction is provided for the four GARCH-type models: GARCH(p,q), Exponential(E)-GARCH(1,1), Threshold (T)ARCH(1,1) and Asymmetric(A)-Power(P)ARCH(1,1). After the introduction, the criteria will be described that will be used to choose and evaluate the forecasting models.

3.1 Models

The main reason why different GARCH-type models are investigated in this paper is because of a assumption from the Classical Linear Regression Model (CRLM). One of the assumptions of CLRM states that that the variance of the errors is constant, also known as *homoscedasticity* ($var(r_t) = \sigma^2$). If the variance of the errors are not constant, this would be know as *heteroscedasticity*. If the errors are heteroscedastic, but assumed homoscedastic, an implication would be that the standard errors estimates could be wrong. It is unlikely in the context of financial time series that the variance of the errors will be constant over time (Brooks, 2014). See Appendix A for the variance of the returns for the three different assets. Another important feature of many series of financial assets returns that provides a motivation of for the ARCH/GARCH class of models, is known as 'volatility clustering'. Volatility clustering describes the tendency of large changes in asset prices to follow large changes and small changes to follow small changes. In other words the level of volatility in period t_1 tends to be positively correlated with the level of volatility in period t_{-1} (Brooks, 2014).

All the GARCH-type models that are used in this analysis have the same structure. Let r_t represent the daily returns of the three different asset classes. Then the basic structure of a GARCH model can be denoted by:

$$r_t = \mu_t + \sigma_t z_t = \mu_t + \epsilon_t \tag{2}$$

Where μ_t represents the conditional mean, σ_t represents the volatility process and z_t the residuals of the volatility. In this paper the Gaussian (normal) distribution, student's t distribution with 5 degrees of freedom and student's t distribution with 7 degrees of freedom will be fitted as the distribution of the z_t. The average return of

Bitcoin is relative far from zero compared to the other two classes, as visible in Table 1. For this reason the GARCH-structure with a autoregressive (AR) process for the return is also tested for Bitcoin. The AR process is denoted by the y_{t-1} term and represents the return for the previous day. The representation of the AR process can be denoted as:

$$r_t = y_t + \sigma_t z_t, \quad where\ y_t = \mu + \phi y_{t-1} \tag{3}$$

GARCH(p,q)

The ARCH model was introduced by (Engle, 1982) in a study of inflation rates and there has since been a barrage of proposed parametric and nonparametric specifications of autoregressive conditional heteroscedasticity (Stata, 2018). The ARCH model has a specification of the conditional mean and variance, where the variance is a function of the σ_t^2 in (4). Bollerslev added a lagged term of the conditional variance to the ARCH model of Engle, what is now known as a GARCH model (Bollerslev, 1986). The basic GARCH(p,q) model as described in Bollerslev, 1986 can be specified as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_1 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta \sigma_{t-j}^2 \tag{4}$$

For $\alpha_0 > 0$, $\alpha_1 > 0$ and $\beta > 0$. In his study of inflation rates, Engle, 1982 assumed that the error term followed a Gaussian distribution ($\epsilon_t \sim N(0, \sigma_t^2)$. Nevertheless, like Mandelbrot, 1997 and many others pointed out, is that the extreme stock returns are more frequent than would be expected by the Gaussian distribution. In other words, the tails of the distribution of the error terms are much fatter than the normal distribution would expect. In a wide range of papers the use of other distributions are suggested (e.g. student's t distribution). The student's t distribution has fatter tails than the normal distribution. Where the degrees of freedom of the student's t distribution approaches infinity, the student's t distribution converges to a normal distribution (Stata, 2018).

A range of GARCH(p,q) models with different p and q values and different distributions of the error terms will be evaluated on the basis of the evaluation criterion that will be discussed in the end of this section.

E-GARCH

The Exponential GARCH (EGARCH) model is proposed in 1991 by (Nelson, 1991). There are various ways to express the EGARCH equation, the specification as in (Brooks, 2014) is:

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}] \tag{5}$$

The model has several advantages over the GARCH model specified in (4). First of all, the $\ln(\sigma_t^2)$ is modeled, so even if the parameters are negative, σ_t^2 will be positive. So there is no need for artificially impose non-negative constraints on the model parameters. Second, asymmetries are allowed in the EGARCH specification, since if the relationship between volatility and returns is negatively, γ will be negative. The EGARCH model will be tested with the following distributions: Gaussian (normal), student's t with 5 and student's t with 7 degrees of freedom.

TGARCH

A major registrations of the ARCH and GARCH models is that they both have a symmetric specification. By this we mean that the variance estimation is based on the absolute value of the innovation because of the squared residuals. In other words large positive shocks have the same effect as large negative effects in the ARCH/GARCH models. However, for financial time-series and especially equities it has been observed that negative shocks have more impact than negative shocks with the same attitude. This problem is addressed by work of (Zakoian, 1994) with the Threshold GARCH model. The main purpose of the TGARCH model is the asymmetric specification in the sign of the returns (Asteriou and Hall, 2015). The specification of a TGARCH(1,1) model:

$$\sigma_t = \alpha_0 + \alpha\varepsilon_{t-1}^2 - \gamma\varepsilon_{t-1}^2 n_{t-1} + \beta\sigma_{t-1} \tag{6}$$

Where n_t takes the value 1 for a negative value of ε, and 0 otherwise. Which means that the impact of negative shocks is higher $(\alpha + \gamma)$ than the impact of positive shocks (α), with the same magnitude. If $\gamma > 0$ we observe asymmetry, if $\gamma = 0$ than we can conclude symmetry in shocks (Asteriou and Hall, 2015)

A-PARCH

(Ding, Granger, and Engle, 1993) investigated the so called 'long memory' property of stock returns. Their findings shows that not only the absolute returns have a higher correlation compared to the normal returns, but the power transformation of the absolute returns ($|r_t|^d$) results in high autocorrelation for long lags as well. The ARCH type specifications are based upon squared returns, however Ding et al.(1993) concluded that the autocorrelation function for a fixed lag has a unique maximal point for $d = 1$. What makes the linear relationship among absolute returns neither a necessary nor efficient property of the ARCH specification. For this reason Ding et al.(1993) proposed a new class ARCH models named: Asymmetric(A)-Power(P)GARCH model.

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j\sigma_{t-i}^\delta, \quad where \tag{7}$$

$$\begin{gathered} \alpha_0 > 0, \delta \geq 0, \\ \alpha_i \geq 0, i = 1, ..., p, \\ -1 < \gamma_i < 1, i = 1, ..., p, \\ \beta_j \geq 0, j = 1, ..., q, \end{gathered}$$

Ding et al.(1993) estimated the power δ in (7) for the S&P-500 returns and found a value of 1.43. The 1.43 is significantly different from the 1 and 2 that is used in the TGARCH (6) and GARCH models (4), respectively.

3.2 Selection & Evaluation

All the GARCH-type models are estimated based on the Maximum Likelihood. To compare the models on goodness of fit the following two criteria have been used:

- Akaike information criterion, as described by (Akaike, 1974)
- Bayesian information criterion, as described by (Schwarz et al., 1978)

The model with the lowest AIC/BIC score is assumed to be the model with the best fit.

The models with the lowest AIC and BIC scores within there distributions will be used as forecasting model. The main purpose of these models is to forecast the one-day ahead conditional variance (volatility). The coefficients of the GARCH-type models will be estimated using a rolling-window. Which means that the parameters estimates are updated for every one-day ahead forecast. In this paper the rolling window length will be equal to 500 observations. In other words the last 500 observations will be predicted with the use of a rolling window where the estimated parameters are updated for every prediction. For example take Bitcoin, $[t_1; t_{1304}]$ will be used to estimate the parameters that will be used to predict the conditional variance for t_{1305}. $[t_2; t_{1305}]$ will be used to calculate the parameters for t_{1305}, so on so fourth. If the parameters are constant, this will also result in constant parameters over the sample. When the parameters are constant over the sample, the parameters for the rolling window with length h should not be very different compared to a fixed window with the same length h. But if the parameters change over the sample period, then the rolling window estimates of the parameters would capture this instability (Zivot and Wang, 2006).

The Mincer-Zarnowitz regression is used to evaluate the accuracy of the predictions of the selected model. The first introduction of the Mincer-Zarnowitz regression was in 1969 in the *The evaluation of economic forecasts* (Mincer and Zarnowitz, 1969). The Mincer-Zarnowitz regression can be denoted by:

$$\tilde{\sigma}_t^2 = \alpha_0 + \alpha_1 \hat{\sigma_t^2} + e_t \tag{8}$$

Where $\tilde{\sigma_t^2}$ is a proxy for the ex-post volatility which is not observable and suppose we can calculate one day out-of-sample predictions of the variance ($\hat{\sigma_t^2}$) using model (3) for the periods $t = 1, ...n$. A forecast is deemed "efficient" if $\alpha_0 = 0$ and $\alpha_1 = 1$. Efficiency is tested by the application of ordinary least squares to the available sample (Granger and Newbold, 2014). The following hypothesis are formulated and tested with the use of a Wald test:

$$H_0: \ \alpha_0 = 0 \cap \alpha_1 = 1$$
$$H_1: \ \alpha_0 \neq 0 \cup \alpha_1 \neq 1$$

In earlier work researchers used daily squared returns (r_t^2) as proxy for the ex-post volatility in. As visible in Table 1 the conditional mean of the daily returns of Bitcoin is practically not zero which gives rise to a alternative proxy. It was not possible to retrieve and compute realized variance on a basis of 15-min returns, which seems to be a relative good proxy (Patton, 2011). For this reason, the squared residuals of the returns will be used as proxy for the ex-post volatility for Bitcoin, i.e. $\hat{\sigma_t^2} = e_t^2$ in (8). For the S$P-500 and the EUR/USD exchange rate the squared returns will be used as ex-post volatility proxy, i.e. $\hat{\sigma_t^2} = r_t^2$ in (8).

In practice the coefficients for $\hat{\sigma_t^2}$ are subject to estimation error, which results in a standard errors-in-variables problem and a downward bias in the regression coefficient (α_1 in (8)) Andersen and Bollerslev, 1998. Christensen and Prabhala, 1998 concluded this problem with evaluating volatility forecasts with the use of implied volatility, established from option prices. However a combination of coefficients from (8), in other words R^2, could provide a relationship between the $\hat{\sigma_t^2}$ and the used ex-post volatility proxy e_t^2 for Bitcoin and r_t^2 for S&P-500 and the EUR/USD exchange rate). The R^2 is therefore often interpreted as a measurement of the degree of predictability in the volatility process (Andersen and Bollerslev, 1998).

4 Results

In this section, first, the AIC and BIC scores for the different GARCH-type models for the three asset classes will be discussed. Secondly, the rolling window forecasts for the models with the lowest BIC and AIC values will be evaluated based on the Mincer-Zarnowitz regression introduced in the method section.

Table 2 shows the AIC and BIC values for the different GARCH models with the Gaussian (normal) distribution and the student's t distribution with 5 and 7 degrees of freedom for Bitcoin. The values in **bold** are the lowest within there respective distribution. The EGARCH(1,1) in (5) without the AR(1) process results in the lowest AIC and BIC scores. The EGARCH(1,1) without the AR(1) process is also the model with the lowest AIC and BIC scores for the student's t distributions. The lowest average AIC and BIC scores are visible in the student's t distribution column with 5 degrees of freedom. In other words, the student's t distribution with 5 degrees of freedom fitted the distribution of the error terms for Bitcoin most accurately. For the EGARCH with AR process and APARCH without AR process, Stata (statistical software program) runs in 'flat likelihood' errors. Stata represents this output if it struggles with finding optimal parameters using Maximum Likelihood Estimation (MLE). Ordinary Least Squares (OLS) can always be computed, when taking covariance of variance. For MLE it is has to find the maximum values, which is not always possible. However, a larger dataset could resolve this problem (Quaedvlieg, 2018).

Table 2: AIC and BIC scores Bitcoin

	Gaussian		**t (7df)**		**t (5df)**	
Models	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**
$GARCH(1,1)$	9885.24	9885.24	9517.36	9517.36	9464.68	9486.67
$GARCH(1,1)^*$	9886.17	9913.66	9519.36	9546.84	9466.64	9494.13
$GARCH(1,2)$	9872.59	9900.07	9514.67	9542.16	9462.34	9489.83
$GARCH(1,2)^*$	9879.60	9912.59	9516.08	9549.07	9463.65	9496.64
$GARCH(2,1)$	9878.20	9905.69	9514.09	9541.58	9461.75	9489.24
$GARCH(2,1)^*$	9873.85	9906.84	9516.67	9549.65	9464.25	9497.24
$GARCH(2,2)$	9874.48	9907.46	9516.53	9549.51	9464.32	9497.30
$GARCH(2,2)^*$	9875.69	9914.17	9518.53	9557.01	9466.24	9504.72
$TGARCH(1,1)$	9887.21	9914.70	9519.24	9546.72	9466.49	9493.98
$TGARCH(1,1)^*$	9888.15	9921.14	9521.23	9554.22	9468.43	9501.42
$EGARCH(1,1)$	**9856.80**	**9884.29**	**9504.00**	**9531.49**	**9452.42**	**9479.91**
$EGARCH(1,1)^*$	-	-	9505.90	9538.89	9454.07	9487.06
$APARCH(1,1)$	9854.15	9892.64	9505.17	9543.66	-	-
$APARCH(1,1)^*$	9855.38	9899.36	9507.16	9551.14	9455.86	9499.85

* indicates the GARCH-type model with a AR process for the return. The Gaussian, t(7df) and the t(5df) columns represents the Gaussian distribution, student's t distribution with 7 degrees of freedom and the student's t distribution with 5 degrees of freedom, respectively. The GARCH model can be found in (4), TGARCH in (5), EGARCH in (6) and APARCH in (7).

The AIC and BIC scores for the GARCH-type models regarding the S&P-500 are visible in Table 3. The EGARCH(1,1) model has the best volatility fit for S&P-500 with all the different distributions tested. The student's t distribution with 7 degrees of freedom yields the lowest average AIC and BIC scores. Analyzing this, we can conclude that the distribution of the error terms for S&P-500 are closer to a normal distribution compared to the distribution of the error terms for Bitcoin. As the degrees of freedom approaches infinity, the distribution converges to normal.

Table 3: AIC and BIC scores S&P-500

	Gaussian		**t (7df)**		**t (5df)**	
Models	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**
$GARCH(1,1)$	2549.79	2570.21	2473.96	2494.38	2470.10	2490.51
$GARCH(1,2)$	2551.04	2576.56	2475.96	2501.48	2472.09	2497.61
$GARCH(2,1)$	2551.32	2576.84	2475.96	2501.48	2472.09	2497.61
$GARCH(2,2)$	2553.54	2584.17	2468.57	2499.20	2473.80	2504.42
$TGARCH(1,1)$	2491.79	2517.31	2417.93	2443.45	2418.13	2443.65
$EGARCH(1,1)$	**2462.01**	**2487.53**	**2404.90**	**2430.42**	**2406.89**	**2432.41**
$APARCH(1,1)$	2495.39	2531.12	2421.21	2456.94	2421.11	2456.84

The Gaussian, t(7df) and the t(5df) columns represents the Gaussian distribution, student's t distribution with 7 degrees of freedom and the student's t distribution with 5 degrees of freedom, respectively. The GARCH model can be found in (4), TGARCH in (5), EGARCH in (6) and APARCH in (7).

Table 4 shows the AIC and BIC values for the EUR/USD exchange rate. Looking at Table 4 we can see that the Threshold(T) GARCH (1,1) model gives the lowest AIC and BIC values for all three different distributions for the EUR/USD exchange rate. The student's t distribution with 7 degrees of freedom yields the lowest AIC and BIC values, similar to S&P-500. The AIC and BIC values for the EGARCH(1,1) model and the APGARCH (1,1) with a normal distribution could not be calculated because of the problem with the MLE described earlier in the interpretation of the AIC and BIC values for Bitcoin.

Table 4: AIC and BIC scores EUR/USD exchange rate

	Gaussian		**t (7df)**		**t (5df)**	
Models	AIC	BIC	AIC	BIC	AIC	BIC
$GARCH(1,1)$	1801.91	1822.44	1732.74	1753.27	1735.42	1755.95
$GARCH(1,2)$	1803.70	1829.36	1734.46	1760.12	1737.14	1762.80
$GARCH(2,1)$	1803.80	1829.46	1734.42	1760.08	1737.06	1762.72
$GARCH(2,2)$	1802.18	1832.97	1736.69	1767.48	1739.03	1769.83
$TGARCH(1,1)$	**1785.53**	**1811.20**	**1728.98**	**1754.64**	**1732.81**	**1758.47**
$EGARCH(1,1)$	-	-	-	-	-	-
$APARCH(1,1)$	-	-	1732.25	1768.17	1734.22	1765.02

The Gaussian, t(7df) and the t(5df) columns represents the Gaussian distribution, student's t distribution with 7 degrees of freedom and the student's t distribution with 5 degrees of freedom, respectively. The GARCH model can be found in (4), TGARCH in (5), EGARCH in (6) and APARCH in (7).

The one-day ahead rolling window forecasts for Bitcoin, S&P-500 and the EUR/USD exchange rate are visible in Figure 2, 3 and 4, respectively. Figure 2 reflects the conditional variance predications for Bitcoin based on the EGARCH(1,1) model with a 5 degrees of freedom student's t distribution. In Figure 3 the conditional variance predictions are shown for the S&P-500, with a EGARCH(1,1) model and a 7 degrees of freedom student's t distribution. Last, the variance predictions for the EUR/USD exchange rate are based on the TGARCH(1,1) model with a student's t distribution including 7 degrees of freedom and are visible in Figure 4. The conditional variances predictions for the EUR/USD exchange rate is relatively constant compared to the predictions for Bitcoin and the S&P-500. Looking at the the figures it shows that the spikes are better captured for the S&P-500 when comparing the predictions to Bitcoin. In order to make any conclusions about the forecast ability, we need to interpret the Mincer-Zarnowitz regression first.

The Mincer-Zarnowitz regressions for Bitcoin are shown in Table 5. At the significance level of 5%, the H_0 hypothesis will be rejected for the EGARCH model including the Gaussian and t distribution with 7 degrees of freedom. This means that the predictions are suffering from a systematic bias. However, the H_0 hypothesis for the EGARCH model with a t distribution with 5 degrees of freedom is not rejected. In other words, the EGARCH model with a t distribution of 5 degrees of freedom provides unbiased predictions for Bitcoin. The EGARCH(1,1) model with student's t distributions give higher R^2 values than when a normal distribution is assumed. From this we can conclude that the ex-post volatility proxy (e_t^2) is explained more accurately when the distribution of the errors are assumed to have a student's t distribution. These results are parallel to the AIC and BIC values showed earlier.

Table 5: Mincer-Zarnowitz regression Bitcoin

Distribution	α_0	t	α_1	t	R^2	F
Gaussian (normal)	-20.560	-2.92	1.884	5.33	0.336	6.52*
Student's t (5df)	-14.954	-2.34	1.828	5.10	0.340	2.75
Student's t (7df)	-15.834	-2.44	1.969	5.13	0.341	3.21*

The α_0, α_1 coefficients, the relevant t-score and R^2 are obtained from the Mincer-Zarnowitz regression (8). The F column represents the F-score for the performed Wald test, where * indicates a rejection of the H_0 at a 5% significance level.

Table 6 captures the The Mincer-Zarnowitz regressions for the S&P-500. The constant values in the regressions are remarkable lower compared to the values that are obtained in the regressions for Bitcoin, due to the difference in size of the conditional variances. All the null hypothesis for the three EGARCH models are rejected at the 5% significance level, so none of the tested models provide unbiased conditional variance predictions. The R^2 values are higher than reported for Bitcoin. In other words, the ex-post volatility proxy (r_t^2) is better explained for the S&P-500, with the use of EGARCH(1,1) models.
Last, we see that the highest R^2 is obtained when a normal distribution is assumed for the errors, what is not analogue with the AIC and BIC values in Table 3.

The Mincer-Zarnowitz regressions for the EUR/USD exchange rate are visible in Table 7. The constant values in the regressions are again remarkable lower compared

Table 6: Mincer-Zarnowitz regression S&P-500

Distribution	α_0	t	α_1	t	R^2	F
Gaussian (normal)	-0.340	-2.44	1.655	4.50	0.518	8.20*
Student's t (5df)	-0.256	-2.01	1.297	4.23	0.497	15.21*
Student's t (7df)	-0.267	-2.09	1.434	4.31	0.509	9.48*

The α_0, α_1 coefficients, the relevant t-score and R^2 are obtained from the Mincer-Zarnowitz regression (8). The F column represents the F-score for the performed Wald test, where * indicates a rejection of the H_0 at a 5% significance level.

to the values observed for Bitcoin and the S&P-500. None of the null hypothesis for the three TGARCH models are rejected at the 5% significance level, so all the TGARCH models provide unbiased conditional variance predictions. The R^2 values are substantially lower than reported for Bitcoin and the S&P-500. One explanation could be that the EUR/USD predictions are constant around the mean, but do not capture the high and low squared returns well. So the TGARCH models provide unbiased predictions, it is still questionable if these predictions are also efficient. The R^2 values are higher for the t distributions, what is similar to the interpretation of the AIC and BIC values in Table 4.

Table 7: Mincer-Zarnowitz regression EUR/USD exchange rate

Distribution	α_0	t	α_1	t	R^2	F
Gaussian (normal)	0.031	0.32	0.832	1.74	0.017	0.08
Student's t (5df)	-0.097	-0.82	1.333	2.46	0.035	1.72
Student's t (7df)	-0.068	-0.60	1.296	2.31	0.031	0.30

The α_0, α_1 coefficients, the relevant t-score and R^2 are obtained from the Mincer-Zarnowitz regression (8). The F column represents the F-score for the performed Wald test, where * indicates a rejection of the H_0 at a 5% significance level.

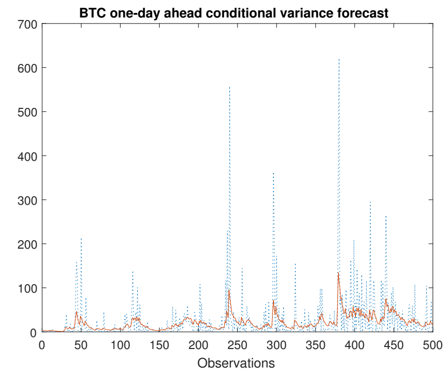

Figure 2: One-day ahead rolling window forecast Bitcoin, with a rolling window length of 500 observations. Orange line represents the forecast, dotted blue line the squared residuals.

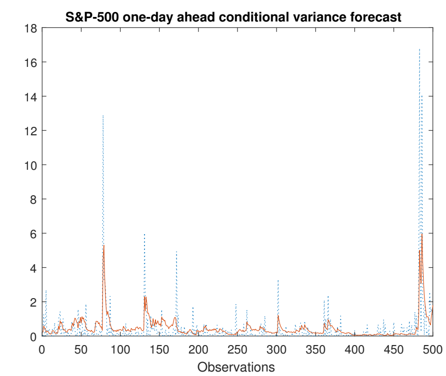

Figure 3: One-day ahead rolling window forecast S&P-500, with a rolling window length of 500 observations. Orange line represents the forecast, dotted blue line the squared returns.

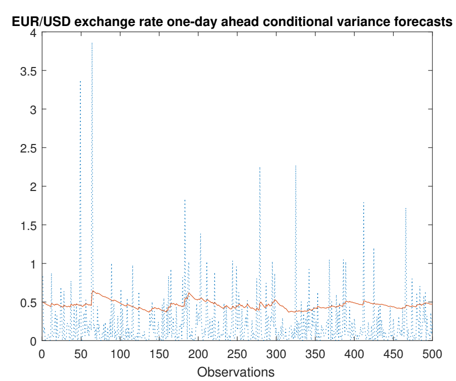

Figure 4: One-day ahead rolling window forecast EUR/USD exchange rate, with a rolling window length of 500 observations. Orange line represents the forecast, dotted blue line the squared returns.

With the results and interpretations of the Mincer-Zarnowitz regression it is now possible to make statements about the forecasting ability of the volatility. Comparing the forecasting abilities, the conclusion can be made that the EUR/USD exchange rate can be predicted without biasness. However, the TGARCH models do not explain the volatility very well for the EUR/USD exchange rate. The EGARCH model can be used to yield unbiased predictions for Bitcoin when a student's t distribution is assumed with 5 degrees of freedom. The volatility process is also explained relatively well by the EGARCH model. The EGARCH model did not succeed in generating unbiased predictions for the S&P-500 in the setting of this paper. Nevertheless, the EGARCH model explains the volatility process for the S&P-500 well. The highest R^2 is obtained for the S&P-500 (0.518), (0.035) for the EUR/USD exchange rate and (0.341) for Bitcoin.

One important remark is the influence of the ex-post volatility proxy. The use of less accurate measurements of the ex-post volatility proxy ($\tilde{\sigma}_t^2$) influences the accuracy of α_0 and α_1. What makes the Mincer-Zarnowitz regression less precise in the way it detects deviations from forecast optimality (Patton and Sheppard, 2009). This emphasis the need for a more detailed analysis on whether the squared residuals (e_t^2) is a suited ex-post volatility proxy for Bitcoin.

5 Conclusion

The analysis shows that there are similarities between Bitcoin, S&P-500 and the EUR/USD rate from a volatility process point of view. Where the EGARCH models show the best fit for both Bitcoin and the S&P-500. The high volatility aspect is distinctive for Bitcoin. Ex-post volatility proxies are observed for Bitcoin of over 600, which is 35 times as high as the maximum values observed for the S&P-500 and almost 200 times as high as observed for the EUR/USD exchange rate. The European Central Bank has a valid motivation for not labeling Bitcoin as a true currency due to the high volatility (ECB, 2015). When looking at the volatility aspect, the analysis performed in this paper shows that Bitcoin is closer to a security than a currency.

It is still questionable why all three EGARCH models are rejected for the S&P-500 and only two EGARCH models for Bitcoin. One explanation could be that the asymmetries of the returns are higher for Bitcoin than for the S&P-500 and therefore capture the tails relatively better. To see if this explanation holds, the process of the parameters needs to be investigated and compared to each other. The fact that the student's t distribution with 5 degrees of freedom is the only error distribution that is not rejected for Bitcoin shows that relatively high and low returns are observed more often than a normal distribution assumes.

Lastly, more analysis around choosing the ex-post volatility proxy is needed in order to make hard conclusions regarding the volatility prediction ability for Bitcoin. One suggestion for future research is measuring the performance of one-day ahead conditional variance predictions with the use of high frequency data.

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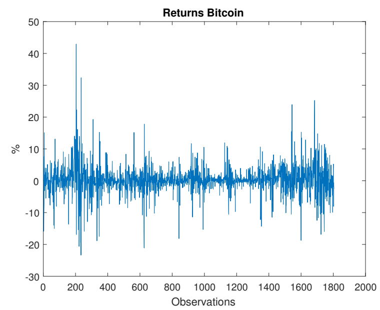
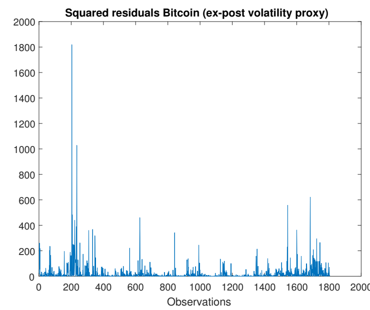
Appendix A: Returns and ex-post volatility proxies for Bitcoin, S&P-500 and the EUR/USD exchange rate

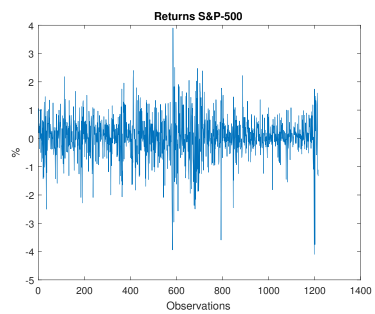


(a) Returns

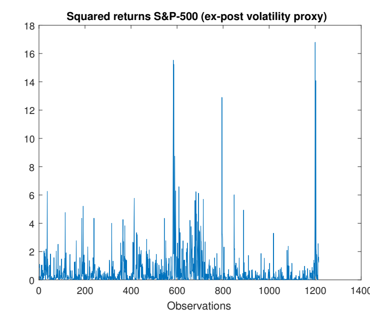


(b) Squared residuals

Figure 5: Visualization of a) the returns and b) the squared residuals (ex-post volatility proxy for Bitcoin)

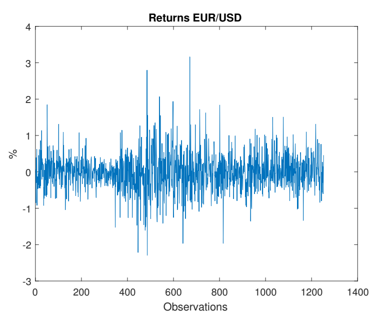


(a) Returns

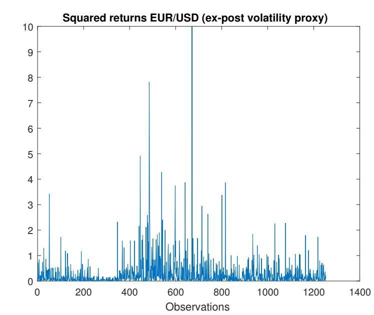


(b) Squared Returns

Figure 6: Visualization of a) the returns and b) the squared returns (ex-post volatility proxy) for S&P-500



(a) Returns



(b) Squared Returns

Figure 7: Visualization of a) the returns and b) the squared returns (ex-post volatility proxy) for EUR/USD exchange rate