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Downstream Environmental Taxes and Upstream Innovation

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Abstract

In this article a model is studied that analyses a downstream environmental tax in the setting of two vertically related monopolists. The upstream firm supplies intermediate inputs to the downstream firm and can innovate in order to make these inputs cleaner in use, which decreases the tax downstream. This upstream innovation is effective in a decreasingly decreasing manner. This paper shows that for all relevant levels of taxes on emissions or inputs/production, the upstream firm will always choose to innovate. This result is not found for an environmental tax on value added. The innovation increases with the tax, up to a certain level of this tax. When a downstream firm has higher initial emissions per unit of production the upstream firm innovates more. Besides the innovation that the tax induces, it also decreases the production in the vertical chain more when the initial pollution downstream is higher. Competition downstream increases the amount of upstream innovation whereas the inclusion of an upstream tax besides the downstream tax decreases the innovation. The result of this paper provides insights for policymaking as the analysed tax captures both the static and the dynamic goal of environmental policy. Furthermore, it shows that taxes do not only have influence on the taxed firm itself, but also on a related firm.

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1. Introduction

Global warming is an important aspect of climate change, a global problem that currently receives a lot of attention from world leaders. Several conferences have been held debating on how to reduce the impact human activity has on the environment. Progressively, it is understood that climate change and air pollution are connected to each other (UNECE, 2018). Therefore, economic policies to reduce air pollution can be beneficial in the act to reduce global warming and climate change. Policymakers can use the effect that air pollution has on climate change to diminish both problems at the same time.

Environmental pollution is a negative externality of the production of certain goods and services (Jaffe, Newell, & Stavins, 2005). This means that the private optimum of production is not the same as the optimal production from a social welfare point of view, because the costs of production to the environment are not taken into account. In essence, taxes make a firm pay for their pollution and thereby encourages them to abate more or produce less, which is the static environmental benefit of the use of taxes (Requate, 2005). In order to deal with the current environmental problems these two effects are not enough. In order to solve the problem, firms have to make lasting changes. For example, cleaner production processes have to be developed. This can be achieved by the dynamic effect of the use of taxes, as they give incentives to firms to keep innovating in order to achieve a cleaner production process (Jaffe et al., 2005). Requate (2005) even argues that, out of all the instruments, taxes result in one of the highest incentives to firms to adopt a cleaner technology. Besides these effects, it is argued that taxes do not only influence the taxed firm itself. The effects of taxes, for example, can "leak" into other industries or even to other countries. Leakage means that a policy in a certain industry or sector can result in more emissions in other sectors that are not regulated, which is mainly seen across borders (Mansur, 2011). Likewise, the influence of environmental effects are not always restricted to only one firm in the supply chain (Yang, Lin, Chan, & Shue, 2010). Therefore, it might also be true that an environmental tax has an influence on other firms in the vertical production chain. Specifically, a downstream environmental tax could result in more innovation in the environment upstream. Franco and Marin (2017) argue in an empirical study among eight European countries, that a tax on value added downstream can increase the amount of innovation of an upstream supplier. However, it is never studied theoretically why this would happen.

Therefore, in this paper a model is studied in which there is an upstream monopolist that supplies intermediate products to a downstream monopolist. The downstream monopolist has to pay an environmental tax. The upstream firm can innovate in order to supply cleaner products or products that are cleaner in use to the downstream firm, which can reduce the amount of tax the downstream firm pays. This setting makes the model different from previous models that study taxes in vertical relationships, as they either consider a tax upstream and downstream or that both firms have the possibility to innovate (e.g. Iida, Mizuno, & Yoshida, 2014; Sugeta & Matsumoto, 2007). A reason for the decrease in tax when there is more innovation upstream is, for example, that the downstream firm is able to process the inputs in a more environmentally friendly manner or, in the case of energy as input, have less polluting emissions. An example of this are taxes on the usage of "unclean" energy, which are the second most common taxes used in the European Union (European commision, International labour organisation, 2010). The demand of the downstream firm does not depend on the degree of environmental awareness of the production chain. This sets the model apart from similar models that assume that the market demand increases due to the environmental actions that the firms take, because they assume that the consumer values the environment (e.g. Shi, Qian & Dong, 2017; Yalabik & Fairchild , 2011). This assumption makes it more likely to find a positive result, regardless of whether a firm in itself or a vertical chain is considered. The reason being that more innovation will always lead to more demand independent of a tax. Therefore, the effect of the tax in itself is less clear. Furthermore, it is difficult to estimate the degree of awareness of consumers to be able to use the results in practice. Therefore, the model studied in this paper assumes that consumers do not value the environmental actions that a firm takes, in order to detach the effect of the tax form the possible effect consumer preferences can have. If it is true that a downstream tax can lead to more innovation upstream, as argued empirically by Franco and Marin (2017), this would provide insights for policymakers. The downstream environmental tax can then be used to make the downstream firm produce less and to make the upstream firm innovate. Thus capturing both the static and the dynamic effect of environmental policy. As mentioned before, this dynamic effect is of specific importance to achieve the long term environmental goals.

The article will continue as follows. First, literature will be presented, on both the effects of taxes on innovation and the effects that taxes have in vertical relations. Then, the model will be presented, which will be analysed for different kinds of taxes, after which the discussion and conclusion will follow.

2. Theoretical framework

2.1 Taxes as environmental policy instrument and their influence on innovation

Besides taxes, there are several instruments to make firms internalise the externality imposed on the environment in order to produce the amount that is socially optimal (Fullerton, Leicester, & Smith, 2008). The environmental instruments can broadly be divided into two categories: command and control, and market based instruments (Requate, 2005). Market based instruments work through incentives that induces firms to reduce pollution themselves through a combination of a decrease in production and investments that reduce the emissions. A specific level of innovation can also be achieved by forcing firms to adopt a new technology, which is an example of a command and control instrument. Environmental taxes are an example of a market based instrument (Fullerton et al., 2008). Polluting firms can either be indirectly taxed on their production, or directly on emissions.

Employment of taxes has several advantages. A first advantage of the use of taxes, and of market based instruments over command and control in general, is that it is efficient when all firms have different marginal costs of abatement, making it easy for policymakers to use when acquiring specific information is difficult or costly (Requate, 2005). The choice between an environmental tax on output or on emissions also depends on possible information problems (Schmutzler & Goulder, 1997). If monitoring is very costly an output tax is optimal. A tax on emissions is optimal when it is less costly to monitor the pollution. Another advantage is that taxes provide incentives to innovate (Requate, 2005). One way in which taxes seem to increase the incentive to innovate is when tax benefits are related to the innovation costs (Owens & Ash, 2010). A tax can increase the costs for a firm for using a more polluting production technology, and therefore make the firm invest in a less polluting technology (Carrora & Siniscalco, 1994). However, innovation itself is also costly for a firm. In practice, innovation has a diminishing negative influence on the reduction of the degree of pollution (Popp, 2005). As firms innovate more the marginal impact will decrease. However, the costs will increase when there is more innovation. This means that it becomes increasingly costly to innovate, as the costs increase and at the same time the impact decreases. The effect of upstream innovation on the downstream pollution and the costs of innovation in this paper are based on this result. Krass, Nedorezov and Ovchinnikov (2013) find that the effect of an environmental tax on innovation is non-monotone, and very high taxes could lead to less innovation and even dirtier technologies. The last reason that taxes are often used is because they generate income for governments (European commision, International labour organisation, 2010).

As already mentioned, environmental protection is an additional cost for firms, which is generally seen as a burden to the firm. However, the innovation that it possibly induces, might be able to offset the negative influence the environmental policy has (Porter & van der Linde, 1995). This is because innovation does not only lead to less pollution but also to an improvement of the production process. This idea is known as the Porter hypotheses. The weak version of the Porter hypothesis reduces this idea to the fact that stricter environmental policies lead to more innovation with regard to the environment (Ambec, Cohen, Elgie, & Lanoie, 2013). This would imply that innovation increases with the stringency of the policies. If this analysis shows that this is in fact the case in vertical relationships, it can be used as support for the Porter hypothesis.

2.2 Environmental taxes and innovation in vertical relationships

Environmental taxes induce the taxed firm to innovate with regard to making their production more environmentally friendly. However, as mentioned before the effects of taxes could also influence other firms that are vertically related to the taxed firm, as argued by Franco and Marin (2017). The approximation of the tax they used in their empirical study is the average percentage of value added that firms spend on environmental taxes. This approximation is a tax on value added, which is a form of an ad valerum tax. They find that a downstream environmental tax has a positive effect on innovation, measured as environmental patents. When an upstream tax is added next to the downstream tax, the amount of innovation decreases again. Furthermore, they find that a downstream tax alone can result in upstream innovation. When a tax on value added is levied on a downstream firm in a vertical chain it is more efficient than when it is imposed on the upstream firm (Peitz & Reisinger, 2014). This result holds for an oligopoly up- and downstream, and several demand functions. Therefore, a downstream tax is considered in this paper. However, the effect of the inclusion of an upstream tax on the result is analysed as well. When a tax is levied on production it does not matter for the efficiency whether it is up- or downstream, because both kinds of taxes result in the same amount of after tax production.

Few models exist in which (environmental) taxes and their effects on innovation are analysed in a vertical relationship. The first line of literature analyses the effects of an environmental tax downstream on the upstream market for abatement technology. In a model in which there is a polluting downstream sector and an upstream sector that produces abatement technology, higher environmental taxes lead to more innovation upstream (Greaker, 2006). The reason for this is that the higher tax increases the demand for the upstream products, which leads to the fact that lower mark-ups are needed to cover development costs, resulting in more entrance and thus innovation in new technologies in that market. In contrast to this line of literature, this paper studies an upstream industry for intermediate products, rather than a market for abatement technology. Therefore, due to the environmental policy the demand for the upstream firm will decrease instead of increase.

In the second line of literature, the upstream and downstream firms are both producer of an intermediate or final good not related to abatement itself, which is more similar to the model studied in the current paper. However, one or both of the firms are able to innovate to reduce their own emissions instead of the emissions of the other firm. Iida, Mizuno and Yoshida (2014) study a model in which both the upstream and the downstream monopolist in a vertically related chain face an environmental tax on their emissions. The downstream firm can innovate in order to decrease both the up- and downstream tax. They find that a change in the division of the taxes up and downstream does not increase the innovation downstream. This changes when there is competition downstream. The reason for this is that when a downstream firm innovates, the upstream tax will decrease as well as the downstream tax, resulting in a lower price upstream and less costs for both downstream firms, leading to less incentive to innovate.

Sugeta and Matsumoto (2007) consider a model in which an upstream monopolist supplies a downstream duopoly. The downstream firms face an emissions tax and can abate. The upstream firm faces an input tax. Independent of the pricing of the upstream firm and the amount of competition or heterogeneity of costs downstream, an increase in the tax will result in more abatement. They also find that in this setting, a switch from an input tax to an emissions tax results in less tax revenue, but it would increase social welfare. This model does not directly consider costly innovation in order to reduce the emissions. On the contrary, Shi, Qian and Dong (2017) do consider costly innovation as a means to reduce pollution in relation to the power structure. In this analysis the downstream firm is assumed to be a retailer. The model assumes environmentally aware consumers and an upstream environmental tax, of which their negative effect can be reduced with the costly innovation. This means that the results capture the positive effect innovation has on both the tax and consumer demand. The sustainable effort increases under all power structures with the environmental awareness of the consumer. When the innovation is downstream, the more power the firm has, the more effort it will exert to become more environmentally friendly. If the innovation is upstream there is the least innovation when the upstream firm is leader in the chain. Shi et al. do not study the effect of the tax on its own, which is done in the model studied in this paper. Furthermore, they consider an upstream tax and upstream innovation.

Yalabik and Fairchild (2011) consider a model in which pollution results in less consumer demand and more regulation for a monopolist firm. This firm can then decide to innovate in order to reduce their emissions. This model does not consider a vertical relationship. However, the way consumer demand is modelled makes it very similar to a situation in which the production of a downstream firm is negatively affected by an environmental tax, which is considered in the model in the current paper. If the firm innovates, the tax will decrease and the downstream firm will produce more, similar to consumers that are environmentally aware. However, the difference is that the firm that innovates also faces an environmental tax itself. Yalabik and Fairchild find that the monopolist will innovate more when the environmental pressure increases. This result only holds when the initial emissions are below a certain threshold. Above this threshold, the more initial emissions, the less a firm will innovate.

3. The model

Consider an upstream (u) and downstream monopolist (d). The demand of the downstream monopolist is given by a linear inverse demand function: $P(Q) = \alpha - Q$, in which the Q represents the quantity of the downstream firm and α is a constant. The demand of the upstream firm is the amount that the downstream firm chooses to produce (Q). For simplicity, the marginal costs of the downstream firm are equal to the price upstream (p_u) and the production of one unit needs one unit of the input made by the upstream firm. Furthermore, the downstream firm pollutes a certain amount (β) per unit of production, which is defined as $E(Q) = \beta Q$. An environmental tax (t) is levied on the downstream firm that depends on the characteristics with regard to the harm to the environment due to the inputs used by the downstream firm (γ). This harm decreases when the impact of the emissions per unit of production decreases because the input is cleaner in use, or because of a cleaner input itself.[1] The harm to the environment is modelled separately from the initial emissions implying that the model can also be used to analyse a tax levied on the use of certain inputs itself ($\beta = 1$)[2]. In this case a lower γ would decrease the tax directly, due to the policy design, instead of it decreasing the emissions downstream.

The upstream firm has constant marginal costs (c). It can innovate (x) which reduces the harm to the environment γ. This γ is a factor between zero and one and is defined as $\gamma(x)$ with $\gamma'(x) < 0$ and $\gamma''(x) > 0$. Essentially, innovation decreases γ, which means it decreases the impact that the production has on the environment, which results in a lower tax for the downstream firm. If the upstream firm chooses to innovate, it has costs of innovation, which are defined as $C(x) = x^2$.

The basic profit functions of the upstream and the downstream firms without any tax are defined in (1). The result of this model can be used as comparison for different taxes. In any case the downstream firm will first decide their production, followed by the upstream firm choosing their price and level of innovation.

$$\begin{aligned} \pi_d &= (\alpha - Q - p_u)Q \\ \pi_u &= (p_u - c)Q - x^2 \end{aligned} \tag{1}$$

The result of this model is $Q = \frac{\alpha - p_u}{2}$, $p_u = \frac{\alpha + c}{2}$, $x = 0$.[3] Obviously, this shows that in absence of any tax the optimal innovation is zero, because there is no benefit for the upstream firm. The profit functions defined in (1) are modified to include different taxes. The taxable bases considered are value

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[1] Note that β itself does not change as a result of the innovation upstream because beta is a fixed initial level of emissions of a downstream firm.

[2] Because of the fact that the production uses one input, the tax can also be seen as a tax on production that decreases with the level of innovation.

[3] Proof in Appendix A.

added and emissions or inputs/production. The tax on emissions is levied on $\beta\gamma(x)Q$. When the tax is levied on inputs/production it is levied on $\gamma(x)Q$. This is essentially a simplified version of the tax on emissions with $\beta = 1$. It is then analysed whether or not this tax gives the upstream firm, which does not directly pay the tax, more incentive to innovate in order to produce inputs that are more environmentally friendly in use for the downstream firm. Lastly, a tax upstream is added.

4. Analysis

The following section consists of the analysis of different taxes in the model described above. The first one is a tax on value added like the finding of the empirical study of Franco and Marin (2017). After that a general model for taxes on emissions or inputs/production is studied and illustrated for a specific function of γ(x). Lastly, an upstream tax is included besides the downstream tax on emissions, production or inputs. All proofs to the theorems and results in 4.1 can be found in Appendix B. The proofs to the results in 4.2 and 4.3 are presented in Appendix C and D, respectively.

4.1 Tax on value added

The tax on value added is levied on the difference between the price and the costs of the firm downstream. The profit functions are defined in (2) below.

$$\begin{gathered} \pi_d = (1-t)(\alpha - Q - p_u)Q \\ \pi_u = (p_u - c)Q - x^2 \end{gathered} \tag{2}$$

The first order condition for the downstream firm $\left(Q = \frac{(\alpha - p_u)}{2}\right)$, shows that the quantity choice of the downstream firm does not depend on the tax level. This makes sense because a tax on value added is essentially a percentage reduction in the total profits, which implies that the quantity decision does not change as a result of a change in the tax and thus the innovation decision of the upstream firm cannot change. Furthermore, it is not surprising that in this setting there is no innovation as there are no benefits of the innovation for the upstream firm, so even when the quantity does depend on the tax, innovation would not occur. Therefore, the result does not change when it is added that the tax decreases in the innovation, which for example could happen because a cleaner firm reaches a lower tax level. The profit functions are in that case defined in (3).

$$\begin{gathered} \pi_d = (1-\gamma(x)t)(\alpha - Q - p_u)Q \\ \pi_u = (p_u - c)Q - x^2 \end{gathered} \tag{3}$$

From this follows the same level of downstream production as before. The result that the quantity does not depend on the tax and that because of that no innovation occurs, does not change when n identical downstream firms in quantity competition are considered. The downstream firm chooses $Q = n(\alpha - Q - p_u)$. Again, this does not depend on the level of the environmental tax. From this analysis follows theorem 1.

Theorem 1: An environmental tax on value added does not lead to innovation upstream.

Franco and Marin (2017) argued empirically that a tax on the value added of a downstream firm might increase the incentives to innovate for an upstream supplier of that firm. However, the result of the model studied in this section shows that this is not straightforward.

4.2 Tax on emissions or input/production

4.2.1 General analysis

Beside a tax on value added it is also possible to levy a tax on the emissions of the downstream firm or on the inputs that the downstream firm uses. If the taxable base is the downstream emissions, β represents the amount of emissions per unit of production, as defined above. If the tax is levied on the production or the used inputs, β is equal to one. γ represents the impact that the downstream production or emissions have on the environment and thus reduces the effective tax paid when the upstream firm innovates. The resulting model is, therefore, general for a tax on production, emissions and inputs and can be used for several functions for γ. This is true as long as γ is decreasing in the upstream innovation and between zero and one as defined above. The profit functions as defined in (1) can be modified to include this tax (4).

$$(4) \qquad \begin{aligned} \pi_d &= (\alpha - Q - p_u)Q - \beta\gamma(x)Qt \\ \pi_u &= (p_u - c)Q - x^2 \end{aligned}$$

First, this model is solved for the downstream firm resulting in its quantity choice $Q = \frac{a - p_u - t\beta\gamma(x)}{2}$. This quantity depends negatively on the environmental tax. When the tax increases, the optimal quantity decreases. However, when the impact on the environment due to the inputs the upstream firm supplies the downstream firm decreases (lower values of γ), the optimal production decreases less severe as a result of a higher tax. This implies that the upstream firm could reduce the impact of the environmental tax on their own demand by innovating, in order to reduce γ(x).

Choice of innovation

As the downstream quantity is equal to the demand of the upstream firm, the quantity can be substituted into the profit function of the upstream firm. Solving this for the optimal upstream price and the amount of innovation gives $p_u = \frac{c+\alpha-\beta\gamma(x)t}{2}$ and $x = \frac{-(p_u-c)\beta\gamma\prime(x)t}{4}$, which together result in the definition for the optimal amount of innovation (5).

$$(5) \quad x = \frac{-(\beta\gamma'(x)t)(\alpha - c - \beta\gamma(x)t)}{8}$$

There are two conditions that have to be satisfied in order for this relationship to be valid. First, there has to be production downstream, which is the case only for $t < \frac{\alpha-c}{\beta\gamma(x)}$. Furthermore, it is important that the upstream firm has positive profits, because otherwise the firm will go out of business. The condition for which this is satisfied is: $t < \frac{\alpha-c-\sqrt{8}x}{\beta\gamma(x)}$. The former is always true when the latter is. This means that for any level of tax for which there is upstream profit, there is downstream production.

As expected, in absence of a tax, the innovation upstream is zero. However, for any positive tax that satisfies the conditions above, the optimal level of innovation is positive. This implies that an environmental tax levied on a downstream firm can, in this model, lead to upstream innovation that reduces the overall impact the production has on the environment. Furthermore, for all values of t that satisfy $< \frac{\alpha-c}{2\beta\gamma(x)}$, the amount of innovation increases when the tax increases. After this point the level of innovation decreases until the point where the profit becomes negative. This means that the tax can be too high. This is because at a certain point the marginal cost of innovation becomes higher than the marginal benefit this innovation has on the profits of the upstream firm. Therefore, a lower level of innovation will be chosen, when the tax increases. This result is summarized in theorem 2.

Theorem 2: If $\gamma(x)$ is a function with $\gamma'(x) < 0$ and $\gamma''(x) > 0$, then the optimal x is a positive value for the combinations of t and x for which the upstream profit and downstream production is positive. The optimal x is an increasing function of t if and only if $t < \frac{\alpha-c}{2\beta\gamma(x)}$.

The specific level of innovation for a certain level of the environmental tax, depends on the specific function of γ that is applicable and the values for the several constants in the model (α, c, β). However, in general, the conclusion depends in the same way on these constants. The maximum level of innovation only depends on α-c. The tax at which this maximum level of innovation is reached depends negatively on the amount of initial emissions per unit of production (β). As the maximum level of innovation does not depend on β, and a higher β means that the tax at which the maximum level of innovation is reached, more initial emissions per unit of production lead to a steeper relationship between the tax and the innovation. Furthermore, the higher α-c, the higher the maximum level of tax that can be levied. These results are summarised in the following corollaries.

Corollary 1: If α-c is larger, the maximum amount of innovation is larger and the maximum tax is higher.

Corollary 2: The larger β, the more responsive the innovation is on the tax.

The results above imply that the lower the costs per unit of the upstream firm, the more it will innovate. This makes sense because when the costs of the upstream firm decrease, the optimal price that the upstream firm asks will decrease as well. As the optimal quantity choice downstream depends negatively on the upstream price, this means that the lower the upstream costs, the higher the upstream demand. This implies that for each level of the environmental tax, the income of the upstream firm is higher, which means that the firm can afford to innovate more. The tax necessary to achieve the maximum level of innovation increases with the level of emissions per unit of production downstream, which means that the amount of innovation increases more quickly. Intuitively, this makes sense because higher emissions mean a higher effective tax, and therefore more marginal benefits from innovation that reduces the effective tax while at the same time the costs remain the same.

The function for the impact on the environmental that the downstream production has (γ(x)), follows directly from (5) and is by definition a decreasing function of the total amount of innovation. The optimal γ as a function of the tax and the optimal innovation is defined in (6).

$$(6) \quad \gamma(t) = \frac{8x}{\beta^2\gamma'(x)t^2} + \frac{\alpha - c}{\beta t}$$

When the tax increases to a level lower than the tax that results in maximum innovation, γ(x) will decrease. The reason for this is that when the tax increases up to that level the optimal level of innovation also increases. Thus, up to a certain level, the degree of impact on the environment decreases with the tax and after that it increases again, which again shows that the tax can be too high. This result is summarized in corollary 3.

Corollary 3: If the tax increases, $\gamma(x)$ decreases if and only if the tax fulfils the condition $t < \frac{\alpha-c}{2\beta\gamma(x)}$.

The results found in this section show that up to a certain level of tax the upstream firm indeed uses its possibility to diminish the impact an increase in tax has on their demand. For any higher tax it is not beneficial anymore to innovate more as costs of innovation become too high compared to the benefits.

Competition

It is interesting to analyse whether the result above depends on competition in the industry. The result found for the tax on value added did not depend on competition as the optimal quantity in the downstream market is not affected by the environmental tax. To analyse the possible impact competition has on the chosen innovation described in (5), both upstream and downstream competition is considered. This is done by considering n identical firms.

Upstream competition does not change the result found above.[4] However, downstream competition does slightly change the result for the tax on emissions, production or inputs. When n identical downstream firms are considered, the resulting total optimal quantity downstream is $Q = \frac{n(\alpha - P_u - \beta\gamma(x)t)}{(1+n)}$. From this follows that the optimal level of innovation is defined as in (7).

$$(7) \qquad x = \frac{-(n\beta\gamma'(x)t)(\alpha - c - \beta\gamma(x)t)}{4(1+n)}$$

The optimal level of innovation still depends in the same way on all variables. However, the factor $\frac{n}{4(1+n)}$ shows that when there is one firm the monopoly result arises again. As this factor is increasing in the number of downstream firms (n), overall the amount of innovation will increase when the number of downstream firms increases. This makes sense as, independent of the tax, the total optimal output in the downstream industry is larger when there are more downstream firms. From this follows theorem 3.

Theorem 3: If n increases the total innovation in the industry increases.

A combination of both upstream and downstream competition does not change the result compared to when there is only downstream competition. All upstream firms will innovate an equal part of the amount of innovation an upstream firm would choose to do. As competition does not result in changes in the general effects that the tax has on the innovation, the remainder of the paper will consider the monopoly model again.

[4] The only difference is that each firm produces $1/n^{th}$ of the total downstream production. Therefore, each firm will innovate $1/n^{th}$ of the innovation a monopolist would do. This is based on the assumption that the part of the total demand for a single upstream firm does not depend on their own innovation. If the amount of the total production that one upstream firm gets depends on the amount it innovates, innovation would increase the demand of an upstream firm more than only the increase in total demand. Therefore, it is possible that there is more innovation overall because each upstream firm has more incentives to innovate. A formal analysis of this is beyond the scope of this paper. However, as all upstream firms are assumed to be identical, it is likely that the division of total demand in equilibrium will be equal again over all firms.

Influence on profits and output

Returning to the model with monopolists, it is interesting to see what happens to the equilibrium profit when the tax increases, as the upstream firm is able to increase its demand again when it chooses a higher level of innovation. This could mean that the tax is beneficial for the upstream firm, which would be support for the Porter hypothesis. The profit function for the upstream firm for the optimal values of the price and the innovation for the different values of the environmental tax, is defined in (8).

$$(8) \qquad \pi_u(t) = \frac{(\alpha - c - \beta\gamma(x)t)^2}{8} - x^2$$

For a given value of t with its corresponding level of innovation, this function gives the profit in the optimal point. So, for a particular tax, the optimum of this function corresponds to the level of innovation that is optimal. The equilibrium profit depends negatively on the tax.

Theorem 4: For the optimal values of x for values of t, the profit is smaller when the tax is higher. However, the profit is higher than when innovation would not have been possible.

The result in theorem four makes sense because when the amount of innovation increases, the costs of innovation increase more than the profits gained from the reduction of γ. This means that a higher environmental tax decreases the upstream profits. The upstream profits overall are also lower than in a model without any taxation. However, they are higher than when there would not have been the possibility to decrease the effective tax with innovation. This means that the results found are evidence for the weak Porter hypothesis, which claims that environmental awareness of a firm can diminish the negative effects of a policy. However, the tax is not beneficial for the upstream firm, because the profits do decrease.

It is also important to look at what happens to the total production in the market as a function of the tax and how that is related to the degree of pollution that still remains. The reason for this is that the tax should still capture the static effect that environmental policy is supposed to have. The production is a function of α-c and β. However, it also depends on the tax; the amount of innovation and the corresponding γ. The optimal quantity in the vertical production chain as a function of the tax is defined as in (9).

$$(9) \qquad Q(t) = \frac{1}{4}\left((1-\beta)(\alpha - c) - \frac{8x}{\gamma'(x)t}\right)$$

The production depends negatively on both the costs of the upstream firm and the amount of emissions. This implies that the higher the initial emissions, the higher the decrease in production, which means that the static effect of environmental policy is still present. As expected, the quantity also depends negatively on the tax. However, it is clear that the negative effect of an environmental tax on the total quantity produced is diminished because of the innovation. This is not surprising as the innovation leads to cleaner production downstream and, therefore, to a reduction in the effective tax paid by the downstream firm.

Government income

One of the reasons for the government to levy taxes is to gain income. The tax income for the government might change as a result of the innovation, as this decreases the effective tax that is paid. The total income for the government is defined as in (10)

$$(10) \quad G(t) = \frac{(1-2\beta)(\alpha-c)8x}{\gamma'(x)t} - \frac{(8x)^2}{(\gamma(x)t)^2} + (\beta-\beta^2)(\alpha-c)^2$$

This function depends positively on the tax. However, the optimal innovation for a certain tax increases for most levels of taxes. Therefore, the government income will increase less with the tax than without innovation.

4.2.2 Illustration of the innovation choice for a specific function for γ

The general result described above can be illustrated by considering a specific function for γ. An example of a function that satisfies the assumptions made is $\gamma(x) = \frac{1}{1+x}$. This results in the definition for the optimal amount of innovation in (11), which depends positively, following theorem 2, on the tax for all values of x and t for which the constraints hold.[5]

$$(11) \quad x = \frac{\left(\frac{\beta t}{(1+x)^2}\right)\left(\alpha - c - \frac{\beta t}{1+x}\right)}{8}$$

Figure 1 shows that the amount of innovation depends, for the most part, positively on the tax and how this result depends on the several constants in the model. The right side of the figure illustrates that the maximum amount of innovation depends positively on α-c. So, when the upstream costs decrease the maximum level of innovation increases as well. It also shows that the maximum possible tax is higher, the lower the costs. The left-hand side shows that the tax necessary to achieve the maximum amount of innovation is lower the higher the emissions (β) per unit of production are initially. Furthermore, it can be seen that after some point the amount of innovation decreases with the tax. This implies that, for this particular function for γ(x), it is possible to have too high taxes.

[5] $t < \frac{\alpha-c}{2\beta\left(\frac{1}{1+x}\right)}$, $t < \frac{\alpha-c-\sqrt{8x}}{\beta\left(\frac{1}{1+x}\right)}$ and $t < \frac{\alpha-c}{\beta\left(\frac{1}{1+x}\right)}$.

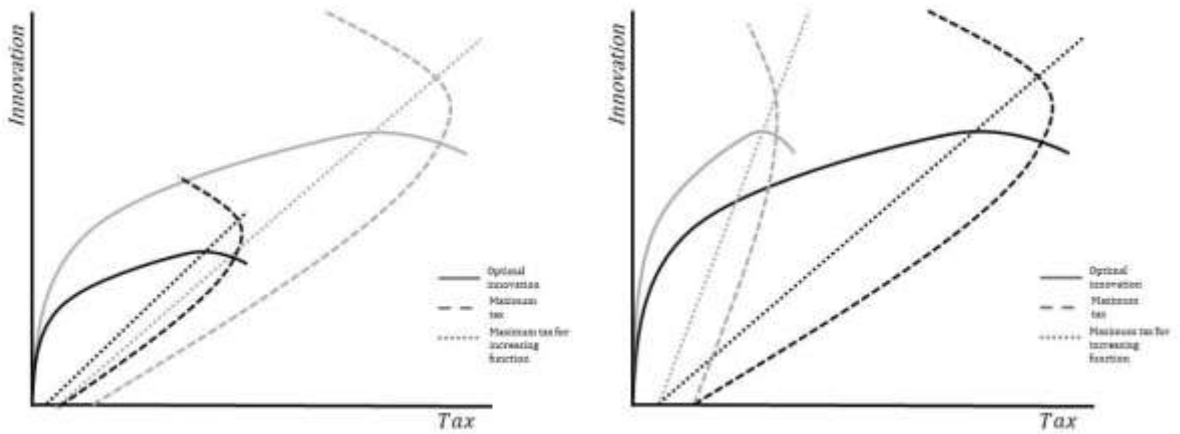

Figure 1. Optimal innovation as a function of the environmental tax, keeping α-c constant on the left and keeping β constant on the right. The lighter figure represents the higher values of α-c or β.

4.3 Both upstream and downstream firms are taxed on emissions or inputs/production

4.3.1 General analysis

The former sections studied the model in which only the downstream firm is taxed. However, it is interesting to analyse whether the result changes when the upstream firm itself is taxed as well. This could for example be because that firm itself is also polluting. The innovation is assumed to decrease both the impact to the environment downstream and upstream. For simplicity, the impact on both is the same. In this section the emissions downstream are defined as β_d and the emissions upstream are defined as β_u. This results in the profit functions as in (12).

$$(12) \qquad \begin{aligned} \pi_d &= (\alpha - Q - p_u)Q - \beta_d \gamma(x) Q t \\ \pi_u &= (p_u - c - \beta_u \gamma(x) t) Q - x^2 \end{aligned}$$

From this follows the same downstream quantity as in the last section $Q = \frac{\alpha - p_u - t\beta\gamma(x)}{2}$. Again, this can be substituted into the upstream profit function. The optimal upstream price that results from this is $p_u = \frac{c + \alpha + \beta_u \gamma(x) t - \beta_d \gamma(x) t}{2}$. The optimal amount of innovation $x = \frac{-(p_u - c - \beta_u \gamma(x) t) \beta_d \gamma\prime(x) t}{4}$. This results in the definition for the optimal innovation as in (13).

$$(13) \qquad x = \frac{-(\beta_d \gamma'(x) t)(\alpha - c - (\beta_u + \beta_d)\gamma(x) t)}{8}$$

Again, in order for this function to be valid the upstream profit and the downstream production should be positive. The condition for a positive downstream production is $t < \frac{\alpha - c}{(\beta_u + \beta_d)\gamma(x)}$. The condition that has to be satisfied in order for the upstream firm to have positive profits is $t < \frac{\alpha - c - \sqrt{8}x}{(\beta_u + \beta_d)\gamma(x)}$. Again, for all values of the environmental tax for which the upstream profits are positive, there is downstream production. If there are positive upstream emissions, these conditions are more restrictive than when the upstream firm is not taxed. This makes sense because the upstream tax decreases the upstream profits and increases the price upstream, which increases the downstream costs and decreases the downstream quantity.

The level of innovation is positive for all positive values of the environmental tax and is increasing in the tax if and only if $t < \frac{\alpha - c}{2(\beta_u + \beta_d)\gamma(x)}$. Up until this point the only difference with the analysis before is that β_u also has an influence on the optimal innovation and the several restrictions and definitions. The maximum amount of innovation does still depend positively on α-c. However, it now also depends on the emission of both the upstream and the downstream firm. This is a change compared to the section before, as there the downstream emissions per unit of production only changed the responsiveness of the innovation on the tax.

The maximum innovation depends negatively on the upstream emissions per unit of production. On the one hand this is surprising as more upstream emissions mean that the tax is higher and innovation could decrease the tax more, leading to more benefits of innovation. However, higher emissions per unit of production increase the effective tax, and therefore the "costs" of the upstream firm are also higher. This decreases the innovation again. Therefore, the overall innovation is lower when an upstream tax is added. What is even more surprising is that keeping the upstream amount of emissions constant, the maximum amount of innovation increases when there are more downstream emissions. So, when there are upstream emissions, downstream emissions increase the innovation. This is in contradiction to the result when there were no upstream emissions, because then the downstream emissions did not have an influence on the maximum amount of innovation. However, it makes sense that more downstream emissions make the innovation increase more quickly with the tax, because downstream emissions give more incentives to innovate because the upstream firm can diminish the negative effect the tax has on its own demand. The fact that the total innovation increases also makes sense because the increase in innovation due to the higher downstream emissions also decreases the tax that the upstream firm pays (when the upstream emissions are kept constant), which results in lower costs upstream and therefore higher maximum innovation as before. From this follows theorem 5.

Theorem 5: An upstream tax together with a downstream tax, decreases the maximum amount of innovation. However, keeping the upstream emissions per unit constant, more downstream emissions increase the maximum amount of innovation.

4.3.2 Illustration of the innovation choice for a specific function for γ

To illustrate the above findings, again, the specific function $\gamma(x) = \frac{1}{1+x}$ is used. This results in the definition of the optimal innovation as in (14).

$$(14) \qquad x = \frac{-\left(\frac{\beta_d t}{(1+x)^2}\right)\left(\alpha - c - \frac{(\beta_u + \beta_d)t}{1+x}\right)}{8}$$

Figure 2 shows the effects of changes in upstream emissions and downstream emissions. Firstly, the overall shape of the relation is the same as when the upstream firm was not taxed itself. It can be seen that a higher level of upstream emissions per unit of production results in a similar increase in innovation as a result of a higher tax. However, it increases to a lower level. On the right-hand side of figure 2 can be seen that higher downstream emissions result in a faster increase of the innovation, which increases to a higher maximum level.

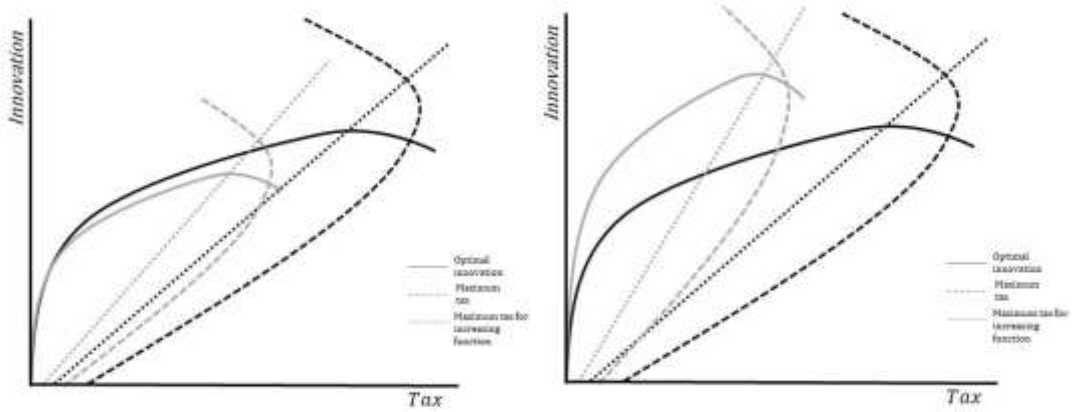

Figure 2. Innovation as a function of the environmental tax. On the left side for a changing β_u keeping β_d constant and on the left for a changing β_d keeping β_u constant. The lighter figure represents the higher value of both variables.

5. Discussion

In this paper a model for environmental taxation in vertical relationships is studied, in which an upstream and a downstream monopolist are considered. The model is used to analyse the effects that a downstream environmental tax has on the upstream innovation choice. The first environmental tax was a tax on value added. The analysis shows that, theoretically, a tax on value added cannot result in a change in innovation upstream through their demand, which follows from the fact that the quantity choice downstream does not depend on the tax. This is in contradiction to the empirical result of Franco and Marin (2017), which seems surprising. However, it could be that the result Franco and Marin found, is driven by another mechanism, as empirical data has its limitations with respect to the accurate measuring of certain variables. One possible explanation for the differences in results, can be that the tax was in fact not levied on value added. For example, on emissions or production, which are often the taxable bases of environmental taxes in practice. In fact, the approximation for the variable tax used by Franco and Marin is the percentage of value added that is spent on environmental taxes. This means that it is possible that the taxes were in fact levied on something else, which makes it sensible to assume that their result captures a mechanism that will not be found when analysing a tax on value added theoretically. Another reason could be that the studied period was subject to more environmental awareness overall, leading to firms claiming more patents for environmentally friendly production and/or products. This because firms assume they might be able to gain competitive advantage over other firms in case environmental awareness keeps increasing in the future.

In the model with the tax on value added, it is possible to find a positive level of upstream innovation when some factors are added to the model. The first factor is when consumer demand depends negatively on the impact that the production has on the environment. This would mean that the consumer demand depends negatively on $\gamma(x)$, meaning that if the upstream firm innovates to decrease $\gamma(x)$, the consumer demand increases. In this case the downstream firm will have more demand when the upstream firm innovates. Therefore, the upstream firm has an incentive to innovate independent of the tax. It is also true that the tax on value added still does not have an influence on the quantity choice of the downstream firm. Therefore, the only reason that the upstream firm, in this case, will innovate is because of the increased customer demand. So, this effect would also exist when there was no tax included in the model. Of course, for this effect to exist the consumer has to be able to observe the innovation and its effects.

However, even without environmentally aware consumers, the model can show positive upstream innovation as a result of a tax on the value added of the downstream firm. In the model the downstream firm does not have other costs besides the inputs it buys from the upstream firm. If there are additional costs and these costs are taxed in the same way as the inputs, nothing changes compared to the situation in which there are only input costs. However, when there are additional cost components that are excluded from the taxable profit margin, the result will change. Additional non-deductible costs imply that the profit per unit of production for the downstream firm is lower than its taxable profit. If this is the case the downstream quantity choice will depend on the tax and the level of upstream innovation, which it previously (when all costs are deductible) did not. Non-deductible cost components result in a lower downstream production level, which is lower the higher the tax is. However, the downstream production decreases less when there is upstream innovation, because upstream innovation decreases the effective tax the downstream firm pays. Therefore, the upstream firm has an incentive to innovate as it can increase its demand by innovating. This means that the fact that a tax on value added has no influence on the quantity downstream and therefore cannot have influence on the upstream innovation, is restricted to the case in which the tax on value added is levied over the entire value added, meaning that it is levied on the selling price minus all the costs per unit of production. The result Franco and Marin (2017) found might also be the result of the fact that in reality some costs are excluded from the taxable value added of a firm. For example, personnel costs which cannot be deducted from the profit when determining the taxable profit.

It is also possible that there are environmentally aware consumers and some of the costs are non-deductible. If this is the case, the positive effect that upstream innovation has on the downstream production choice through fact that consumers are environmentally aware is weakened by the effect the tax has in case some costs are non-deductible. This results from the fact that innovation upstream increases the consumer demand and therefore the downstream production. However, this also implies that the payable tax downstream increases because the taxable base is larger, which decreases the production downstream again. Therefore, the incentives to innovate are smaller with the tax than without it. Thus, if consumers are environmentally aware and there are non-deductible costs, a tax on value added decreases the incentives to innovate that were already there due to consumer demand. If it is in reality true that both factors are present, positive innovation upstream can be found, because the positive effect of environmentally aware consumers still exists. However, positive innovation would exist because of this and not as a result of the tax, because the effect of the tax decreases the incentive to innovate of the upstream firm, which is inconsistent with the findings of Franco and Marin (2017).

The second environmental tax which is considered in this paper is a tax on emissions or inputs/production. In contradiction to the result found for the tax on value added, the optimal quantity of the downstream firm is influenced by the level of environmental tax. The analysis shows that for cases in which the innovation has a decreasingly decreasing influence on the degree of pollution downstream, at first the optimal innovation is an increasing function of the environmental tax. However, after some amount of tax (depending on the function for $\gamma(x)$ and the other constants in the model) the optimal level of innovation could go down. This is similar to the result of Krass et al. (2017), who found that the optimal innovation as a result of an environmental tax is a non-monotone function in a setting without a vertical relationship. Furthermore, the fact that the optimal amount of innovation can decrease with the tax, means that the analysis does not find hard evidence for the weak Porter hypothesis. However, there exist functions for γ for which the model does show an increasing function for the relation between the environmental tax and the innovation for all relevant taxes.[6]

The results also show that the higher the initial emissions, the more responsive the innovation is on the tax. Thus, for each tax level, higher initial pollution per unit of production results in more

[6] This is $\gamma(x) = \frac{1}{e^x}$

innovation, which makes sense as higher initial emissions lead to a higher effective tax downstream. Therefore, the upstream firm has more benefits from innovation. The fact that the responsiveness of the innovation to the tax depends positively on the initial amount of emission is partly contradictory to the result of Yalabik and Fairchild (2011). They find that the initial emissions increase innovation up to a certain threshold and then the innovation decreases again. This could be because in their model the innovating firm itself is also subject to an environmental tax. If the upstream firm then has more initial emission it will also pay more taxes, possibly resulting in too high costs of innovation in relation to the possible benefits.

Besides the effect on innovation upstream, the downstream tax also has an influence on the upstream profits. The profit of the upstream firm decreases when the tax becomes higher, which shows that the environmental tax does not benefit the upstream firm. Furthermore, the profits are lower than when there is no tax at all. This is evidence against the strong Porter hypothesis that claims that more environmental protection could lead to benefits for firms. The profits with innovation are higher than the profits without any innovation for the same level of tax. Thus, the negative effect that the tax has on the upstream firm can partially be offset by the innovation of the upstream firm, which supports the weak Porter hypothesis.

One of the goals of environmental policy, and taxation in general, is a reduction in the amount firms produce. In this paper the quantity produced does decline with the tax, even though a higher tax results in cleaner production through the effect of upstream innovation. So, besides the dynamic effect of an environmental tax the static effect is also still present. If the initial production is cleaner, the optimal quantity decreases at a slower rate than when the initial emissions are higher. This is an expected result as the effective tax is higher when the initial emissions per unit of production are higher. Therefore, more innovation would be needed in order to reach the same level of impact on the environment as a firm that initially was cleaner already.

Competition can often change results. Therefore, upstream and downstream competition was considered in this paper. For the tax with value added as taxable base downstream competition did not change the result. The total production in the downstream market was still independent of the tax. However, competition downstream did change the result. The more downstream firms there are, the more upstream innovation results, because more downstream firms lead, for any tax, to more production and therefore more upstream demand and higher incentives to innovate. Upstream competition does not change the result. It should be noted that the competition in this paper is based on identical firms and could therefore be the reason that there are only minor differences. However, it is unlikely that as a result of differences in firms, a positive level of innovation is not chosen anymore.

Lastly, it is studied what the effects are of the inclusion of the same tax upstream next to the tax downstream that already existed. The inclusion of an upstream tax decreases the overall amount of innovation, which is the same result as Franco and Marin (2017) found in their empirical study. Iida et al. (2014) found that when there are upstream and downstream taxes, and the downstream firm can innovate to decrease both taxes, the division of the taxes upstream and downstream does not change the result. The results in the current study show that when the innovating firm is upstream, this division does matter. When there are more upstream emissions compared to downstream emissions, thus more effective tax upstream compared to downstream, the innovation is lower than when the opposite holds.

6. Conclusion

This paper found evidence that in a vertical relationship with an upstream and a downstream monopolist, a tax on emissions or inputs/production levied on the downstream firm, can result in innovation upstream. This is not the case for a tax on value added. Based on the assumptions made in this study, this innovation is, for most possible values of the environmental tax, a positive function of the environmental tax. Downstream competition increases the total amount of innovation of the upstream monopolist. Furthermore, the upstream firm is able to diminish the effects that the tax has on its profits. When an upstream tax is included next to the downstream tax the upstream firm still chooses to innovate. However, to a smaller extent than without the upstream tax.

These findings have important implications for policymakers in the field of environmental policy. As mentioned before, only the static effects of policy are not enough to solve the environmental problems the world faces nowadays. It is the dynamic part that it can have that is important, which means that the policy can result in innovation in for example a production process that results in less pollution. Besides being effective for the static goal of environmental policy, the tax studied in this paper results in this dynamic effect, as the tax induces innovation upstream. Moreover, this means that a tax can have a positive influence on related firms that are not directly taxed, which implies that the positive effects of a tax are possibly larger than one would expect. A possible downside is the necessity of information on the amount of emissions of the firm, in order to be able to reduce the tax when a firm is cleaner. However, the effect is still found when the tax is levied on production or inputs. This requires only information on the degree of pollution due to the inputs produced upstream. Another important finding for policymaking is that taxation of both the upstream and the downstream firm is less effective than a tax on the downstream firm alone. This means that it is better to only tax downstream.

Future research could clarify if the result also holds when the innovation does not have a decreasingly decreasing effect on the level of pollution, but a linear or an increasingly decreasing effect.

Even though the relationship assumed in this paper is a common one, including more relations would improve the generality of the findings. Furthermore, it would be interesting to see whether product differentiation upstream, leading to several input options for the downstream firm, changes the conclusion. The competition analysed in this article was based on the existence of identical firms. This might be the reason that upstream competition does not have an influence on the total amount of innovation. It could be that if firms differ in their costs, some firms innovate more and others less, possibly resulting in a larger overall amount of innovation. This because a monopolist with lower costs innovates more.

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Appendix A

The first order condition of the downstream profit function to Q is $\alpha - 2Q - p_u = 0 \Leftrightarrow Q = \frac{(\alpha - p_u)}{2}$

The upstream profit function is then $\pi_u = (p_u - c)\left(\frac{(\alpha - p_u)}{2}\right) - x^2$. The first order condition of this profit function to the price is $\frac{-2p_u + \alpha + c}{2} = 0 \Leftrightarrow p_u = \frac{\alpha + c}{2}$. The optimal investment by the upstream firm is zero, as $2x = 0 \Leftrightarrow x = 0$.

Appendix B

Proof Theorem 1

The derivation of this result is the following. The downstream firm pays a tax on the value added. First, a value added tax that does not decrease in the innovation is considered. This gives the following profit function: $\pi_d = (1 - \gamma(x)t)(\alpha - Q - p_u)Q$. The first order condition to the quantity Q is $2Q + 2tQ = \alpha - p_u - t\alpha + tp_u \Leftrightarrow Q = \frac{(1-t)(\alpha - p_u)}{2(1-t)} \Leftrightarrow Q = \frac{(\alpha - p_u)}{2}$. This means that the upstream demand does not depend on the tax.

When the downstream tax is influenced by the innovation the profit function is: $\pi_d = (1 - \gamma(x)t)(\alpha - Q - p_u)Q$. The resulting first order condition is: $2Q + 2\gamma(x)tQ = \alpha - p_u - \gamma(x)t\alpha + \gamma(x)tp_u \Leftrightarrow Q = \frac{(1-\gamma(x)t)(\alpha - p_u)}{2(1-\gamma(x)t)} \Leftrightarrow Q = \frac{(\alpha - p_u)}{2}$.

When considering n firms the profit function for firm i out of n firms is: $\pi_{di} = (1 - \gamma(x)t)(\alpha - q_i - Q_{-1} - p_u)$. The, from this function, resulting first order condition for firm i is: $q_i = \frac{(1-\gamma(x)t)(\alpha - Q - c)}{(1-\gamma(x)t)}$. This quantity can be added up over all n firms, which gives: $Q = n(\alpha - Q - c) \Leftrightarrow Q = \frac{n(\alpha - c)}{1+n}$.

Appendix C

Proof Theorem 2

The quantity choice and thus the demand for the upstream monopolist is derived in the following way. The downstream profit function is: $\pi_d = (\alpha - Q - p_u)Q - \beta\gamma(x)tQ$. Taking the first order derivative to Q gives: $\alpha - p_u - \beta\gamma(x)t = 2Q \Leftrightarrow Q = \frac{\alpha - p_u - \beta\gamma(x)t}{2}$. The upstream profit function that follows from this is: $\pi_u = (p_u - c)\left(\frac{\alpha - p_u - \beta\gamma(x)t}{2}\right) - x^2$. Taking the first order condition to the amount of innovation gives the optimal innovation $\frac{-(p_u - c)\beta\gamma'(x)t}{2} - 2x = 0 \Leftrightarrow x = \frac{-(p_u - c)\beta\gamma'(x)t}{4}$. The first order condition of the upstream price is: $-\left(\frac{(p_u-c)}{2}\right)+\left(\frac{\alpha-p_u-\beta\gamma(x)t}{2}\right)=0 \Leftrightarrow p_u=\frac{c+\alpha-\beta\gamma(x)t}{2}$. These can be combined to find the optimal level of innovation: $4x=-\left(\frac{-\left(\beta\gamma'(x)t\right)(\alpha-c-\beta\gamma(x)t)}{2}\right) \Leftrightarrow x=\frac{-\left(\beta\gamma'(x)t\right)(\alpha-c-\beta\gamma(x)t)}{8}$.

There are certain conditions under which the above condition is economically valid. This first criteria for it to be possible is that there must be downstream production. This is when the optimal quantity is positive: $Q>0 \Leftrightarrow \frac{\alpha-c-\beta\gamma(x)t}{4}>0 \Leftrightarrow \alpha-c-\beta\gamma(x)t>0 \Leftrightarrow t<\frac{\alpha-c}{\beta\gamma(x)}$. The second criteria is that the upstream profit has to be positive, otherwise this firm will not exist: $\pi_u>0 \Leftrightarrow \left(\frac{\alpha-c-\beta\gamma(x)t}{2}\right)\left(\frac{\alpha-c-\beta\gamma(x)t}{4}\right)-x^2>0 \Leftrightarrow \frac{(\alpha-c-\beta\gamma(x)t)^2}{8}>x^2 \Leftrightarrow \alpha-c-\sqrt{8}x>\beta\gamma(x)t \Leftrightarrow t<\frac{\alpha-c-\sqrt{8}x}{\beta\gamma(x)}$.

The optimal amount of innovation is an increasing function of the tax only for certain values of the tax. In order to show this $x=\frac{-(\beta\gamma'(x)t)(\alpha-c-\beta\gamma(x)t)}{8}$ is divided into two functions, which can be done because they are equal. These are shown in (1).

$$f(x,t)=x$$
$$(1) \quad f(x,t)=\frac{-(\beta\gamma'(x)t)(\alpha-c-\beta\gamma(x)t)}{8}$$

If the second function is increasing in t, then x is also increasing when t increases. Taking the first order derivative to t of the second function gives: $f'(t)=\frac{-(\alpha-c)\beta\gamma'(x)+2\beta^2\gamma'(x)\gamma(x)t}{8}=\frac{\beta\gamma\prime(x)(2\beta\gamma(x)t-(\alpha-c))}{8}$.
As the first derivative of γ(x) is negative this above function is positive when $2\beta\gamma(x)t-(\alpha-c)<0$, which is the case for all values that satisfy the following condition: $t<\frac{\alpha-c}{2\beta\gamma(x)}$. The point for which $t=\frac{\alpha-c}{2\beta\gamma(x)}$ is satisfied is the maximum level of innovation that is attainable, which also proves that the higher α-c the higher the maximum level of innovation, which can be shown by substituting the condition in the optimal innovation: $x=\frac{-(\beta\gamma'(x)t)(\alpha-c-\beta\gamma(x)t)}{8}=\frac{-\left(\beta\gamma'(x)\left(\frac{\alpha-c}{2\beta\gamma(x)}\right)\right)\left(\alpha-c-\beta\gamma(x)\left(\frac{\alpha-c}{2\beta\gamma(x)}\right)\right)}{8}=\frac{-\gamma\prime(x)(\alpha-c)^2}{4\gamma(x)}$.

Writing the above definition for x as a function of y(x) gives the following function for $\gamma(x)$ as a function of t. $\gamma(t)=\frac{8x}{\beta^2\gamma\prime(x)t^2}+\frac{\alpha-c}{\beta t}$. Because it is the same definition it decreases with t for the values of t that satisfy the condition $t<\frac{\alpha-c}{2\beta\gamma(x)}$

Proof Theorem 3:

There are n identical downstream firms considered. For each i out of these n firms the profit function is: $\pi_{di}=(\alpha-q_i-Q_{-1}-p_u)q_i-\beta\gamma(x)tq_i$. The first order condition to q_i gives: $\alpha-Q_{-1}-2q_i-p_u-$

$\beta\gamma(x)t \Leftrightarrow q_i = \alpha - Q - p_u - \beta\gamma(x)t$. This can be added up over all n firms, which results in: $Q = \frac{n(\alpha - p_u - \beta\gamma(x)t)}{(1+n)}$.

The upstream profit function is $\pi_u = (p_u - c)\left(\frac{n(\alpha - p_u - \beta\gamma(x)t)}{(1+n)}\right) - x^2$. The first order condition to the upstream price p_u gives $-\frac{n}{1+n} * (p_u - c) + \frac{n(\alpha - p_u - \beta\gamma(x)t)}{1+n} = 0 \Leftrightarrow p_u = \frac{c+\alpha-\beta\gamma(x)t}{2}$. The first order condition to the amount of innovation filling in the function of p_u gives: $\frac{-n\beta\gamma'(x)t(p_u - c)}{1+n} - 2x = 0 \Leftrightarrow \frac{-n\beta\gamma(x)t(\alpha-c-\beta\gamma(x)t)}{2} = (1+n)2x \Leftrightarrow x = \frac{-n\beta\gamma'(x)t(\alpha-c-\beta\gamma(x)t)}{4(1+n)}$.

Proof theorem 4:

The upstream profit function is $\pi_u(x,t) = \frac{(\alpha-c-\beta\gamma(x)t)^2}{8} - x^2$. To see how this depends on the level of t, the derivative to t can be taken: $\frac{d\pi_u}{dt} = \frac{2(\alpha-c-\beta\gamma(x)t)}{8} * -\beta\gamma(x) = \frac{-\beta\gamma(x)(\alpha-c-\beta\gamma(x)t)}{4}$. As $\alpha - c - \beta\gamma(x)t$ is always positive for existing values of x and t and γ(x) are positive, this function is negative. This means that the profits depend negatively on the tax. If the upstream firm would not be able to innovate and decrease the tax, the upstream profit would be: $\pi_u(t) = \frac{(\alpha-c-\beta t)^2}{8}$. This follows from a downstream quantity of $Q = \frac{\alpha-p_u-\beta t}{2}$, which leads to the upstream profit function $\pi_u = (p_u - c)\left(\frac{\alpha-p_u-\beta t}{2}\right)$. The upstream price is: $p_u = \frac{c+\alpha-\beta t}{2}$. This profit function implies that for every $t > \frac{\alpha-c}{\beta}$, the profit is zero or negative.

Production:

The optimal quantity in the vertical chain is $Q = \frac{\alpha-c-\beta\gamma(x)t}{4} = \frac{\alpha-c-\beta t\left(\frac{8x}{\beta\gamma(x)t}+\frac{\alpha-c}{t}\right)}{4} = \frac{1}{4}\left((1+\beta)(\alpha-c) - \frac{8x}{\gamma'(x)t}\right)$. To show how this depends on t the first order derivative can be taken: $\frac{dQ}{dt} = \frac{8x}{\gamma'(x)*t^2}$.

This shows that Q depends negatively on t as the derivative is negative because $\gamma'(x)<0$. The quantity is a decreasing function of t when the negative effect t has on Q is compensated by the positive effect that x has on Q.

Government income:

The income for the government is G(t)= βγ(x)tQ. Therefore, the following definition results $G(t) = \frac{(1-2\beta)(\alpha-c)8x}{\gamma'(x)t} - \frac{(8x)^2}{(\gamma(x)t)^2} + (\beta - \beta^2)(\alpha-c)^2$. Taking the first order derivative to t gives: $\frac{dG}{dt} =$

$-\frac{(1-2\beta)(\alpha-c)8x}{\gamma'(x)*t^2}+\frac{16X^2}{\gamma\prime(x)^2t^3}$. This is positive as γ'(x) is negative, which means that the government income depends positively on the tax.

Appendix D

Proof Theorem 4:

The downstream quantity decision is still the same as in the proof of theorem 2. This means that the upstream profit function is $\pi_u=(p_u-c-\beta_u\gamma(x)t)\left(\frac{\alpha-p_u-\beta_d\gamma(x)t}{2}\right)-x^2$. The first order derivative to P_u gives: $-\left(\frac{(p_u-c-\beta_u\gamma(x)t)}{2}\right)+\left(\frac{\alpha-p_u-\beta_d\gamma(x)t}{2}\right)=0 \Leftrightarrow p_u=\frac{c+\alpha+\beta_u\gamma(x)t-\beta_d\gamma(x)t}{2}$. The first order condition for the level of innovation gives: $\frac{-(p_u-c-\beta_u\gamma(x)t)\beta_d\gamma\prime(x)t}{2}-2x=0 \Leftrightarrow x=\frac{-(p_u-c-\beta_u\gamma(x)t)\beta_d\gamma\prime(x)t}{4}$. Substituting the optimal upstream price into the definition for the optimal innovation, gives the following definition for the optimal innovation for the upstream firm $x=\frac{-(\beta_d\gamma'(x)t)(\alpha-c-(\beta_u+\beta_d)\gamma(x)t)}{8}$.

The conditions for the definition for x to be valid are derived as follows. $Q>0 \Leftrightarrow \frac{\alpha-c-(\beta_u+\beta_d)\gamma(x)t}{4}>0 \Leftrightarrow \alpha-c-(\beta_u+\beta_d)\gamma(x)t>0 \Leftrightarrow t<\frac{\alpha-c}{(\beta_u+\beta_d)\gamma(x)}$ and $\pi_u>0 \Leftrightarrow \left(\frac{\alpha-c-(\beta_u+\beta_d)\gamma(x)t}{2}\right)\left(\frac{\alpha-c-(\beta_u+\beta_d)\gamma(x)t}{4}\right)-x^2>0 \Leftrightarrow \frac{(\alpha-c-(\beta_u+\beta_d)\gamma(x)t)^2}{8}>x^2 \Leftrightarrow \alpha-c-\sqrt{8}x>(\beta_u+\beta_d)\gamma(x)t \Leftrightarrow t<\frac{\alpha-c-\sqrt{8}x}{(\beta_u+\beta_d)\gamma(x)}$.

The optimal amount of innovation is an increasing function of the tax only for certain values of the tax. In order to show this $x=\frac{-(\beta_d\gamma'(x)t)(\alpha-c-(\beta_u+\beta_d)\gamma(x)t)}{8}$ is divided into two functions, which can be done because they are equal. These are shown in (2).

$$f(x,t)=x$$
$$(2) \quad f(x,t)=\frac{-(\beta_d\gamma'(x)t)(\alpha-c-(\beta_u+\beta_d)\gamma(x)t)}{8}$$

If the second function is increasing in t, then x is also increasing when t increases. Taking the first order derivative to t of the second function gives: $f'(t)=\frac{-(\alpha-c-(\beta_u+\beta_d)\gamma(x)t)\beta_d\gamma'(x)+2\beta_d(\beta_u+\beta_d)\gamma'(x)\gamma(x)t}{8}=\frac{\beta_d\gamma\prime(x)(2(\beta_u+\beta_d)\gamma(x)t-(\alpha-c))}{8}$. This depends positively on the tax when $2(\beta_u+\beta_d)\gamma(x)t-(\alpha-c)<0 \Leftrightarrow$: $t<\frac{\alpha-c}{2(\beta_u+\beta_d)\gamma(x)}$. The point for which the maximum innovation is reached is defined as: $x*=\frac{-(\beta_d\gamma'(x)t)(\alpha-c-(\beta_u+\beta_d)\gamma(x)t)}{8}=\frac{-\left(\beta_d\gamma'(x)\left(\frac{\alpha-c}{2(\beta_u+\beta_d)\gamma(x)}\right)\right)\left(\alpha-c-(\beta_u+\beta_d)\gamma(x)\left(\frac{\alpha-c}{2(\beta_u+\beta_d)\gamma(x)}\right)\right)}{8}=\frac{-\beta_d\gamma\prime(x)(\alpha-c)^2}{4(\beta_u+\beta_d)\gamma(x)}$. It is straightforward to see that this depends positively on α-c. In order to show how it depends on β_u, the first order derivative of the definition for the maximum innovation can be taken $\frac{dx*}{d\beta_u} = \frac{\beta_d \gamma\prime(x)(\alpha - c)^2}{4\gamma(x)(\beta_u + \beta_d)}$. As $\gamma'(x)$ is negative, this function is negative, which means that the maximum innovation depends negatively on β_u. To see how it depends on β_d the derivative can be taken as well: $\frac{dx*}{d\beta_d} = \frac{\gamma\prime(x)(\alpha - c)^2(\beta_d - (\beta_d + \beta_u))}{4\gamma(x)(\beta_u + \beta_d)^2}$. This is positive when $\beta_d - (\beta_d + \beta_u) < 0 \iff -\beta_u < 0$. This means that the maximum innovation depends positively on β_d when β_u is positive.