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Multiple Heterogeneous Structural Breaks Models and Estimation Methods for Panel Data Analysis

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Abstract

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This paper introduces three panel data models and their estimation methods that take heterogeneous structural breaks in the slope coefficients into consideration. Despite the fact that many empirical studies in various domains suggest the effects of certain factors may be unstable and different across clusters, not many studies have considered the development of such techniques in panel data analysis. Thus, this paper seeks to fill this gap. We model the heterogeneities through a latent group structure and consider both the case of a static group pattern and the case when the group pattern changes after each break. For the static group case, we introduce the grouped adaptive group fused lasso (GAGFL) algorithm with a penalized least squares (PLS) method to estimate the exogenous model, and propose the incorporation with a penalized GMM (PGMM) method to estimate the endogenous model. To deal with the dynamic group pattern, we propose the dynamically grouped heterogeneous structural breaks (DGHB) estimation method. Through two sets of Monte Carlo simulations, we demonstrate that our methods give high accuracy in classifications, breaks detections and coefficients estimations. We further apply our GAGFL with PGMM method to investigate the effect of foreign direct investment (FDI) inflow on economic growth. The new evidence we obtained in this application confirms the usefulness of our methods in empirical studies.

Keywords: Panel data, heterogeneous slope coefficients, structural breaks, grouped fixed-effects, dynamic group pattern, penalized least squares, penalized GMM

Contents

1 Introduction

Panel data analysis is a crucial technique as it enables the analysis on both cross-sectional and time dimensions. When analyze panel data, it is important to consider heterogeneous structural breaks in the slope coefficients through a group pattern. Abundant evidence from various domains has shown that certain factors may cause different effects among individuals in different clusters, and the effects are usually unstable over time due to certain political events, financial crisis, technology breakthroughs or preference changes. For example, Owyang and Wall (2005) conduct a study by regions and find that the effect of monetary policy experienced a dramatic break at the beginning of the Volcker-Greenspan era and this effect differs significantly across different regions in the U.S. Okui and Wang (2017) find that the effects of several typical determinants on savings rate are different for countries with different economic status and these effects contain structural breaks. Nonetheless, traditional panel data models tend to ignore the existence of such grouped heterogeneous structural breaks, which can lead to inconsistent estimates and erroneous conclusions. To address this problem, this paper focuses on the modelings and estimations of heterogeneous structural breaks in panel data under different situations.

Specifically, we consider three panel data models that consider the heterogeneous structural breaks in the slope coefficients through an unobserved group pattern. In these models, individuals in the same group share the same slope coefficients every time while the coefficients may differ across different groups. Allowing the occurrence of structural breaks means the slope coefficients can change in disjoint time intervals. We use an exogenous linear model introduced by Okui and Wang (2017) as our benchmark model and adopt their grouped adaptive group fused lasso (GAGFL) algorithm for estimation. Although Okui and Wang (2017)'s GAGFL algorithm works well for exogenous case, they do not consider the endogenous case in their study. However, endogeneity issues widely occur in empirical studies, and treat them the same way as the exogenous case results in inconsistent estimates. Thus, taking this endogeneity into consideration, we construct our second model, the fixed effects endogenous model. To estimate this model, we extend the GAGFL algorithm by employing a penalized GMM estimation (PGMM) introduced by Qian and Su (2016). Another issue previous studies do not consider is a changing group pattern. As also mentioned by Okui and Wang (2017), allowing the group pattern to change is advisable because significant events such as financial crisis may severely change the relationships between variables and shift the group pattern. In this research we also attempt to address this problem. Thus, we construct a dynamic group pattern model which allows part of the group memberships (i.e. the groups each individuals belong to) to change after each break. To estimate this model, we propose the dynamically grouped heterogeneous structural breaks (DGHB) estimation method.

In this paper, we aim to investigate if our estimation methods for the three models can correctly cluster individuals in to different groups, detect the true structural break dates and give consistent slope coefficient estimates. Therefore, we first conduct several Monte Carlo simulations to test the finite sample performances of these estimation methods. We start with a replication of Okui and Wang (2017)'s simulation tests to check the ability of GAGFL with

PLS method. After that, we apply the same tests to check our GAGFL with PGMM for the endogenous model and DGHB for the dynamic group pattern model. Through simulation studies, we find that when the the data are not very noisy, both GAGFL algorithm with PLS and PGMM methods give accurate classifications, break date detections and consistent slope coefficient estimates. As for the dynamic group pattern case, we find our DGHB method gives perfect estimation of the break date even when the noise in the data is substantial. Moreover, DGHB also gives accurate clusterings and good slope coefficient estimates.

Apart from the simulation studies, we also examine the performance of our GAGFL with PGMM method in empirical study. In particular, we apply our fixed effects endogenous model and GAGFL with PGMM estimation method to investigate the effect of foreign direct investment (FDI) inflow on economic growth, which researches have been studying for decades but fail to give clear conclusions with the use of traditional panel data method (e.g. GMM). With our approach, we find that there are indeed heterogeneous structural breaks in the effect of FDI on growth. Given the optimal number of groups, countries are mainly categorized through the status of their developments, and the estimation method detects break points that correspond to policy changes and crisis. This new evidence we find takes us one step closer towards solving this puzzle and proves the usefulness of our model and methods in reality.

Our main findings in this research support two main contributions of this research. Firstly, the GAGFL with PGMM method gives accurate estimates of heterogeneous structural breaks and has the potential to become an influential tool in various empirical studies. Secondly, the DGHB estimation method proves to be a good alternative to GAGFL with PLS method when the group patten is dynamic.

The rest of this paper is as follows. Section 2 gives a review of the related literature. Section 3 presents the benchmark model and GAGFL with PLS method. Section 4 introduces the fixed effects endogenous model and GAGFL with PGMM method. Section 5 gives the dynamic group pattern model and proposes the DGHB method. In section 6, we conduct the Monte Carlo simulation. Section 7 illustrates the GAGFL with PGMM method through an application. Finally, section 8 concludes.

2 Literature review

The study of modeling heterogeneous slope coefficients in panel data through a latent group pattern dates back to Sun (2005) who constructs a finite mixture model with unknown group memberships and proposes a maximum likelihood estimator. Later, Lin and Ng (2012) introduce the use of k-means clustering and Su, Shi, and Phillips (2016) introduce the classifier-lasso clustering. A recent research by Bonhomme and Manresa (2015) elaborates previous works by modeling time-varying grouped fixed-effects (GFE) and introducing the GFE estimator. Their study provides us a good foundation to develop estimation methods for estimating multiple models with heterogeneous structural breaks in the coefficients. Specifically, the GAGFL algorithms we use in this research incorporates a GFE-type of estimator to give preliminary group membership estimates. In addition, our DGHB method uses the GFE estimator in the final step to give slope coefficient estimates.

Apart from the development of modeling heterogeneous slope coefficients, recent studies also focus on modeling structural breaks in the slope coefficients in panel data regressions. One distinctive study by Qian and Su (2016) extends Qian and Su (2014)'s Lasso-type shrinkage approach for time series regressions and develops the use of adaptive group fussed lasso (AGFL) penalty to detect an unknown number of homogeneous structural breaks in slope coefficients for panel data models. The proposed penalized least squares (PLS) estimation for exogenous regressions and the penalized GMM (PGMM) estimation for endogenous regressions give consistent estimates of both the break dates and slope coefficients. In our study, we employ their PLS and PGMM methods to detect the structural breaks and coefficients in each group for static group pattern models.

Although literatures on heterogeneous coefficients and homogeneous structural breaks are abundant, not many studies have considered heterogeneous structural breaks in coefficients. In the latest study, Okui and Wang (2017) construct a model concerning both of these two issues. To estimate their model, they propose a grouped AGFL (GAGFL) method which is a hybrid procedure that uses a GFE-type of estimation to give the group pattern and PLS estimation to explicitly detect structural break regimes and slope coefficients in each group. Their model improves the estimation efficiency through the parsimonious group pattern and give consistent estimates. Thus, it provides us an important cornerstone. Based on their model, we extend it to endogenous case and propose a similar estimation method, the GAGFL with PGMM estimation.

Another study that models the heterogeneous structural breaks in slope coefficients is done by Baltagi, Feng, and Kao (2016). This study is more restrictive than Okui and Wang (2017) as it allows the coefficients to differ among individuals but assumes all the individuals have the same break dates. The fact that this method permits individual-specific coefficients provides as an ideal tool to develop our DGHB estimation method which is a hybrid algorithm that uses Baltagi et al. (2016)'s method to detect the common break date and Bonhomme and Manresa (2015)'s GFE method to estimate the group pattern and slope coefficients between each consecutive breaks.

3 Benchmark model

In the first part of this section, we present our benchmark model. To estimate this model, we introduce the GAGFL with PLS estimation method proposed by Okui and Wang (2017) in the second part and assume the total number of groups is given. In the third part, we discuss the choice of the tunning parameter for this method.

3.1 Model setup

The benchmark model capture the heterogeneous structural break through a static group pattern. Suppose $\{\{y_{it}, x_{it}\}_{t=1}^{T}\}_{i=1}^{N}$ is a panel data set with $t = 1, ..., T$ time periods and $n = 1, ..., N$ cross-sectional individuals. x_{it} denotes a vector containing k explanatory variables and y_{it} denotes the dependent variable. To model the group pattern, let $\mathbb{G} = \{1, ..., G\}$ be the set containing all the group numbers and let $g_i \in \mathbb{G}$ be the group number of the i_{th} individual. Furthermore, let $\beta_{g,t}$ denote the slope coefficient of group g at time period t. Then, our benchmark model takes the folowing form:

$$y_{it} = x'_{it}\beta_{g_i,t} + \epsilon_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T, \tag{1}$$

where ϵ_{it} is the error term. To model the structural breaks through the time-varying slope coefficients $\{\beta_{g,1}, ..., \beta_{g,T}\}$ with $g \in \mathbb{G}$, we introduce the following notations. Let m_g be the number of breaks in group g and let $\mathcal{T}_{m_g,g} = \{T_{g,1}, ..., T_{g,m_g}\}$ be the set of break dates. We allow structural breaks to occur at any disjoint time intervals and let the coefficients $\beta_{g,t}$ be a constant between two consecutive break dates and only change after each break. We also introduce $\alpha_{g,j}$ with $j = 1, ..., m_g$ to be the value of the coefficients between the $j - 1_{th}$ and j_{th} break point. In addition, let $\alpha_{g,m_{g+1}}$ be the value of the coefficient after the last break of group g and define $T_{g,0} = 1$ and $T_{g,m_{g+1}} = T + 1$. Therefore, we have the following relation:

$$\beta_{g,t} = \alpha_{g,j}, \quad if \ T_{g,j-1} \leq t < T_{g,j}. \tag{2}$$

According to Okui and Wang (2017), this benchmark model is, on the one hand, parsimonious, and on the other hand, general and flexible as it does not post any restrictions on the number of breaks and group pattern. Moreover, it permits the identification of the number of breaks. For this reason, we set this model as our benchmark and elaborate based on it.

3.2 GAGFL with penalized least squares (PLS) estimation

Before presenting the method, we first clarify some extra notations to use hereafter. Suppose the total number of groups is G. Let k be the number of explanatory variables in the regression, and let $\mathcal{B} \subset \mathbb{R}^k$ be the parameter space for each β_{gt}. We introduce $\beta = (\beta'_{1,1}, ..., \beta'_{1,T}, \beta'_{2,1}, ..., \beta'_{G,T})$ to be a vector that stacks all β_{gt}. Thus, the parameter space for β is $\mathcal{B}^{GT}$. In addition, we denote the group memberships for all the individuals by an N-dimensional vector γ, with$\gamma = \{g_1, ..., g_N\}$. Then, $\mathbb{G}^N$ is the parameter space for γ. Furthermore, define each period t for which $\hat{\beta}_{g,t} - \hat{\beta}_{g,t-1} \neq 0$ as a break date, and let $\hat{\mathcal{T}}_g = \{t \in \{2, ..., T\} | \hat{\beta}_{g,t} - \hat{\beta}_{g,t-1} \neq 0\}$. Thus, the estimated number of breaks for group g is equal to the cardinality of $\hat{\mathcal{T}}_g$.

Given the total number of groups G, we need to estimate the group memberships for each individual and the slope coefficients for each group. To do this, we follow Okui and Wang (2017) and use the GAGFL algorithm with PLS method. The GAGFL algorithm is an iterative procedure to update the optimal coefficient parameters and group memberships.

As a starting point, GAGFL uses the GFE-type of estimation introduced by Bonhomme and Manresa (2015). To give a consistent result, we adjust the standard GFE estimation method of Bonhomme and Manresa (2015) by allowing the slope coefficients to be time variant. Starting from this initial group assignment, we then conduct the iterative updates.

In each iteration, we first update the coefficient estimates β using the group assignment of the previous iteration. We do this by applying the AGFL method with PLS estimation introduced by Qian and Su (2016) in each group, and minimize the following penalized least squares objective function:

$$(\hat{\beta}, \hat{\gamma}) = \underset{(\beta,\gamma)\in\mathcal{B}^{GT}\times\mathbb{G}^N}{\arg\min} \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - x_{it}'\beta_{g_i,t})^2 + \lambda\sum_{g\in\mathbb{G}}\sum_{t=2}^{T}\dot{w}_{g,t}\left\|\beta_{g,t} - \beta_{g,t-1}\right\|, \tag{3}$$

where

$$\dot{w}_{g,t} = \left\|\dot{\beta}_{g,t} - \dot{\beta}_{g,t-1}\right\|^{-\kappa} \tag{4}$$

with κ a user-specified constant and $\dot{\beta}$ the preliminary GFE-type of estimates obtained by

$$(\dot{\beta}, \dot{\gamma}) = \underset{(\beta,\gamma)\in\mathcal{B}^{GT}\times\mathbb{G}^N}{\arg\min}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{i,t} - x_{it}'\beta_{g_i,t})^2. \tag{5}$$

The second term of (3) is a lasso-type of penalty term that has a sparse property. The λ in the expression is a tunning parameter and $\dot{w}_{g,t}$ is an adaptive weight. This penalization heavily penalize the minimization problem when $\dot{\beta}_{g,t} - \dot{\beta}_{g,t-1}$ closes to zero and $\beta_{g,t} - \beta_{g,t-1} = 0$. As noticed by Qian and Su (2016) and Okui and Wang (2017), this penalty method can give consistent estimations of the break dates.

After updating the slope coefficients, we next update the group memberships by minimizing the sum of squared errors for each individual. To do this, we use the estimated slope coefficients in the current iteration and assign individual i to the group whose coefficients give individual i the smallest sum of squared residuals. We repeat this iterative procedure until it meets some numerical convergence criterion specified by the user. Algorithm 1 shows the pseudo code of this GAGFL with PLS method.

Algorithm 1: GAGFL algorithm with PLS method

Initialization: initial GFE group assignment $\hat{\gamma}^{(0)}$ given by 5, $s = 0$
while *not numerical convergence* **do**
 Step 1: For the given $\hat{\gamma}^{(s)}$, compute

$$\hat{\beta}^{(s)} = \underset{\beta\in\mathcal{B}^{GT}}{\arg\min}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - x_{it}'\beta_{g_i,t}^{(s)})^2 + \lambda\sum_{g\in\mathbb{G}}\sum_{t=2}^{T}\dot{w}_{g,t}\left\|\beta_{g,t} - \beta_{g,t-1}\right\| \tag{6}$$

 where we obtain $\dot{w}_{g,t}$ by (4) and (5).
 Step 2: For all $i \in \{1, ..., N\}$, compute:

$$\hat{g_i}^{(s+1)} = \underset{g\in\mathbb{G}}{\arg\min}\sum_{t=1}^{T}(y_{i,t}x_{i,t}' - \hat{\beta}_{g,t}^{(s)})^2 \tag{7}$$

 Step 3: Set $s = s + 1$.
end

3.3 Choice of the tunning parameter

To choose the optimal tunning parameter λ for the lasso penalty in (3) and (6) in each iteration, we follow Qian and Su (2016) and Okui and Wang (2017) and minimize the following information criterion:

$$IC(\lambda) = \frac{1}{NT}\sum_{j=1}^{m+1}\sum_{t=T_{j-1}+1}^{T_j}\sum_{i=1}^{N}(y_{it} - x_{it}'\hat{\alpha}_{g_i,j})^2 + \rho_{NT}k(\hat{m}_\lambda + 1), \tag{8}$$

where $\hat{\alpha}_{g_i,j}$ is the post-lasso estimate for group g in the period between the j_{th} and $j-1_{th}$ break, $\hat{m}_\lambda$ is the estimated number of breaks corresponding to the tuning parameter λ, and $\rho_{NT} = cln(NT)/\sqrt{NT}$ with $c = 0.05$, which decides the penalization level on the number of breaks.

4 Extension 1: Fixed effects endogenous model

In this section, we present our first extension. In the first part, we introduce our fixed effects endogenous model. In the second part of this section, we propose an extension of Okui and Wang (2017)'s GAGFL algorithm and employ the PGMM estimation introduced by Qian and Su (2016) to tackle the endogeneity issues.

4.1 Model setup

The fixed effects endogenous model differs to the benchmark model in two aspects. First, this model contains an additional time-invariant individual fixed effect. Second, this model allows endogeneity issues which may be caused by for example, measurement errors, omitted variables or simultaneity problems. Let μ_i denote the fixed effect of individual i. Then, our model takes the form of the following expression:

$$y_{it} = \mu_i + x_{it}'\beta_{g_i,t} + \epsilon_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T. \tag{9}$$

Here we do not post any restrictions on μ_i and allow μ_i to be correlated with the regressors x_{it}.

Compared to the benchmark model, this model is even less restrictive and can be of significance use in practice. Firstly, as mentioned by Bonhomme and Manresa (2015), fixed effects are desirable because they allow the correlations between unobserved effects and covariates. Secondly, consider endogeneity issues is important since it can be hard to construct strictly exogenous regressions in reality.

4.2 GAGFL with penalized GMM (PGMM) estimation

To estimate our fixed effects endogenous model, we need to consider two problems. Firstly, the individual fixed effect can be correlated with the regressors. Thus, we cannot directly apply the standard GAGFL algorithm to estimate this model. To address this problem, we use the first-differencing technique to eliminate the individual fixed effects following Okui and Wang (2017)'s extension. After the first-differencing, we get the following equation:

$$\Delta y_{it} = x_{it}'\beta_{g_i,t} - x_{i,t-1}'\beta_{g_i,t-1} + \Delta\epsilon_{it}. \tag{10}$$

Secondly, we assume there are endogeneity issues in this model which cannot be completely removed by first-differencing. In such situation, applying the GAGFL with PLS estimation is not consistent. Therefore, we propose to replace the PLS estimation by the PGMM estimation. We introduce an additional notation z_{it} to be a vector including q instrumental variables and assume $q \geq k$.

After the first-differencing and the introduction of the instrumental variables, the procedure of this method is similar to the standard GAGFL algorithm given by Algorithm 1. But one main difference is that instead of minimizing the objective function given by equation (6), we use the following equation corresponding to the PGMM estimation with the first-differenced data:

$$\hat{\beta}^{(s)} = \underset{\beta \in \mathcal{B}^{GT}}{\arg\min} \frac{1}{T} \sum_{t=2}^{T} \left\{ \frac{1}{N} \sum_{t=1}^{N} \rho_{g_i,t} \right\}' W_{g,t} \left\{ \frac{1}{N} \sum_{t=1}^{N} \rho_{g_i,t} \right\} + \lambda_2 \sum_{t=2}^{T} \ddot{w}_{g,t} \left\| \beta_{g,t} - \beta_{g,t-1} \right\| \tag{11}$$

where $\rho_{g_i,t} = z_{it}(\Delta y_{it} - \beta'_{g,t} x_{it} + \beta'_{g,t-1} x_{i,t-1})$, and $W_{g,t}$ is a $q \times q$ symmetric weight matrix which is positive-definite when the sample size is large. To employ this PGMM estimation, we need to choose the weight matrix for each group in each time period. For the choice of this matrix, we follow the study of Qian and Su (2016) and use their two-step strategy.

Similar as the GAGFL with PLS estimation, the second term of equation (11) is also a lasso-type of penalty, with λ_2 a tunning parameter. In this method, we also impose an adaptive weight given by $\ddot{w}_{g,t} = \left\| \ddot{\beta}_{g,t} - \ddot{\beta}_{g,t-1} \right\|^{-\kappa_2}$, where κ_2 is a user-specified constant and $\ddot{\beta}$ is the preliminary GFE estimate obtained by applying the extended GFE method to the first-differenced data. That is, we obtain the preliminary GFE estimates by

$$(\ddot{\beta}, \ddot{\gamma}) = \underset{(\beta,\gamma) \in \mathcal{B}^{GT} \times \mathbb{G}^N}{\arg\min} \sum_{i=1}^{N} \sum_{t=1}^{T} (\Delta y_{it} - \beta'_{g,t} x_{it} + \beta'_{g,t-1} x_{i,t-1})^2. \tag{12}$$

Here, we rely on the ability of the GFE estimation to give consistent group assignments on the first-differenced data. In addition, as noticed by Bonhomme and Manresa (2015), the GFE-type of group assignment estimator is consistent even when the regressors are not strictly exogenous. Thus, our approach here is adequate.

4.3 Choice of the tunning parameter

Following Qian and Su (2016), we use a similar criterion as the one in GAGFL with PLS estimation to decide the tunning parameter λ_2 for the GAGFL with PGMM estimation. But one difference is that we calculate the information criterion use the first-differenced data and consider the use of the instrumental variables. Thus, we minimize the following expression:

$$\begin{aligned} IC_2(\lambda) = \frac{1}{T-1} \sum_{j=1}^{m+1} \left[\frac{1}{N} \sum_{t=T_{j-1}+1}^{T_j - 1} \sum_{i=1}^{N} \hat{\rho}_{g_i,t} \right]' W_{g,j} \left[\frac{1}{N} \sum_{t=T_{j-1}+1}^{T_j - 1} \sum_{i=1}^{N} \hat{\rho}_{g_i,t} \right] + \\ \sum_{j=1}^{m} \left[\frac{1}{N} \sum_{i=1}^{N} \hat{\rho}_{1 g_i, T_j} \right]' W_{g,T_j} \left[\frac{1}{N} \sum_{i=1}^{N} \hat{\rho}_{1 g_i, T_j} \right] + \rho_{NT} k (\hat{m}_{\lambda_2} + 1) \end{aligned} \tag{13}$$

where $\hat{\rho}_{g_i,t} = z_{it}(\Delta y_{it} - \hat{\alpha}'_{g_i,j} \Delta x_{it})$ and $\hat{\rho}_{1 g_i, T_j} = z_{i T_j}(\Delta y_{i T_j} - \hat{\alpha}'_{g_i,j+1} x_{i T_j} + \hat{\alpha}'_{g_i,j} x_{i,T_j - 1})$ with $\hat{\alpha}_{g_i,j}$ the post-lasso estimates. Same as $W_{g,t}$, $W_{g,j}$ is also a $q \times q$ symmetric weight matrix that we need to determine. For the choice of this weight matrix, we also adopt the two-step strategy by Qian and Su (2016). The third term of equation (13) is the same as equation (8).

5 Extension 2: Dynamic group pattern model

Our second extension to the benchmark model is a dynamic group pattern model which allows the group membership of each individual to change after every structural breaks. Same as the benchmark model and fixed effects endogenous model, we assume the total number of groups is known for now. In addition, as few previous study has considered such dynamic group pattern, in this research we only provide a preliminary methodology to take the changing group pattern into consideration. Thus, we consider a basic dynamic group model where a part of the individuals have one structural break in the slope coefficients during the estimation period and some of the individuals change their group memberships after the break. In this section, we first present the dynamic group pattern model. After that, we propose a new algorithm to estimate this model.

5.1 Model setup

In this model, we consider the case of linear regression with strictly exogenous regressors and adopt the same notations used in section 3.2. To model this dynamic group pattern, we add a time subscription to the group number and let g_{it} denotes the group membership of individual i in time period t, with $g_{it} \in \mathbb{G}$ and $\mathbb{G} = 1, ..., G$. Thus, our model takes the following form

$$y_{it} = x_{it}' \beta_{g_{it},t} + \epsilon_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T. \tag{14}$$

Modeling dynamic group pattern directly using this model instead of using the benchmark model and increasing the total number of groups has two main advantages. First, allowing group memberships to change enables us to use more observations to estimate the parameters in each stable time periods, which can give higher estimation accuracy. Second, tracing the changes of the group memberships can give us more insight to explain phenomenon in empirical studies.

5.2 Dynamically grouped heterogeneous structural breaks (DGHB) estimation

To estimate our dynamic group pattern model with one common break date, we propose the DGHB algorithm. In this algorithm, the first step is to detect the single break date. Baltagi et al. (2016) introduce an estimation method in their study to detect a common structural break in the slope coefficients for part of the explanatory variables. In our case, we use the same method for break date detection but consider a simplified case when the slope coefficients for all the explanatory variables change at the same time.

Let $Y_i = (y_{i1}, y_{i2}, ..., y_{iT})'$, $X_i = (x_{i1}, x_{i2}, ..., x_{iT})'$ and $\epsilon_i = (\epsilon_{i1}, \epsilon_{i2}, ..., \epsilon_{iT})'$ be the vectors that include all the observations and errors for an individual i. In addition, suppose $b^0 + 1$ is the true common break date, then we need to estimate the following model for each individual i, for $i = 1, ..., N$, :

$$y_{it} = \begin{cases} x_{it}'\beta_{1i} + \epsilon_{it} & t = 1, ..., b^0 \\ x_{it}'\beta_{2i} + \epsilon_{it} & t = b^0 + 1, ..., T, \end{cases} \tag{15}$$

where β_{1i} is the slope coefficient before the break and β_{2i} is the slope coefficient after the break for individual i.

To estimate this model, we first define some auxiliary variables. For all the possible break point $b = 1, ..., T-1$, we define auxiliary matrices $X_{1i}(b) = (x_{i1}, ..., x_{ib}, ..., 0, ..., 0)'$ and $X_{2i}(b) = (0, ..., 0, ..., x_{i,b+1}, ..., x_{iT})'$. Thus, when b is the true break point b^0, we have $X_{1i}^0 = X_{1i}(b^0)$ and $X_{2i}^0 = X_{2i}(b^0)$. Let $\mathbb{X}_i(b) = (X_{1i}(b), X_{2i}(b))$, then we can rewrite the piecewise model (15) as

$$Y_i = X_{1i}(b^0)\beta_{1i} + X_{2i}(b^0)\beta_{2i} + \epsilon_i = \mathbb{X}_i^0 B_i + \epsilon_i, \quad i = 1, ..., N, \tag{16}$$

where $B_i = [\beta_{1i}, \beta_{2i}]'$. Therefore, given any possible break point $b = 1, ..., T-1$, we can estimate B_i use the ordinary least squares estimation,

$$\hat{B}_i(b) = \begin{bmatrix} \hat{\beta}_{1i} \\ \hat{\beta}_{2i} \end{bmatrix} = [\mathbb{X}_i(b)'\mathbb{X}_i(b)]^{-1}\mathbb{X}_i(b)'Y_i, \quad i = 1, ..., N. \tag{17}$$

After obtaining the slope coefficient estimates for all the possible break dates for each individual, we can determine the common break date based on the total model fitness. Specifically, we set the estimate $\hat{b}$ to the one that gives the smallest sum of squared residuals. That is

$$\hat{b} = \underset{1 \le b \le T-1}{\arg\min} \sum_{i=1}^{N} SSR_i(b), \tag{18}$$

where SSR_i is the sum-of-squared residuals for individual i if the break date is b. Given there is only one common break date, according to Baltagi et al. (2016), this kind of estimation method is consistent.

Given the consistent common break date estimate $\hat{b} + 1$, our next step is to determine the group memberships before and after the break as well as the slope coefficients. To do this, we first separate the observations before and after this break date. If the break date estimate is consistently estimated, then there should not be any structural breaks in the pre-break and post-break datasets after the separation. Thus, we use the standard GFE estimation introduced by Bonhomme and Manresa (2015) to estimate the stable slope coefficients and group memberships for the pre and post-break periods separately. The pseudo code shown at the beginning of the next page summarizes the whole process of this algorithm.

6 Monte Carlo simulation

In this section, we conduct two sets of Monte Carlo simulation experiments to investigate the performances of our three methods in estimating the proposed models in finite samples. The first set of experiments consider the case of static group pattern. In this set of the experiments, we examine the abilities of the methods to correctly classify units, detect break dates and give accurate slope coefficients. The second set of experiments is for the case of dynamic group pattern which aims to examine if our method can give correct estimates of the common break date, changing group memberships and the slope coefficients.

Algorithm 2: DGHB algorithm

1 **Step 1:** Estimate the break date $\hat{b}$ use equation 17 and equation 18.
2 **Step 2:** Separate the observations from the pre-break period and the post-break period.
3 **Step 3:** Estimate the slope coefficients and group memberships:

$$(\hat{\beta}^{pre}, \hat{\gamma}^{pre}) = \underset{(\beta,\gamma)\in\mathcal{B}^{GT}\times\mathbb{G}^N}{\arg\min} \sum_{i=1}^{N}\sum_{t=1}^{\hat{b}} (y_{i,t} - x'_{it}\beta^{pre}_{g_i})^2. \tag{19}$$

where $\hat{\beta}^{pre}$ and $\hat{\gamma}^{pre}$ denote the stacked coefficients and group memberships for the pre-break period.

$$(\hat{\beta}^{post}, \hat{\gamma}^{post}) = \underset{(\beta,\gamma)\in\mathcal{B}^{GT}\times\mathbb{G}^N}{\arg\min} \sum_{i=1}^{N}\sum_{t=\hat{b}}^{T} (y_{i,t} - x'_{it}\beta^{post}_{g_i})^2. \tag{20}$$

where $\hat{\beta}^{post}$ and $\hat{\gamma}^{post}$ denote the stacked coefficients and group memberships for the post-break period.

Under each case, we present the data generation process, evaluation criteria and the results. For the static group pattern tests, we have an additional section for the tunning parameter selections. To test how our methods perform in different situations, we design two data generation processes for each method and consider different levels of the noise in the data and different number of available observations. In addition, we set the total number of groups $G = 3$ for all the generation processes. We report the results based on 200 replications to get a fairly reliable conclusion. All the experiments are conducted in MATLAB. For the GFE part of each algorithm, we base our code on the code by Bonhomme and Manresa (2015). For the parts of PLS and PGMM estimations, we borrow the code from Qian and Su (2016).

6.1 The case of static group pattern

We first present the Monte Carlo experiments for our models and methods that deal with the case of a static group pattern.

6.1.1 Data generation process

In this section, we design DGP 1 and DGP 2 to generate the data for the tests using the benchmark model and design DGP3 and DGP 4 for the tests using the fixed effects endogenous model.

Benchmark model

DGP 1: In DGP 1, we generate the data follow the benchmark model given by:

$$y_{it} = x'_{it}\beta_{g_i,t} + \epsilon_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T,$$

where $x_{it}\sim$ i.i.d. $N(0, 1)$, $\epsilon_{it}\sim$ i.i.d. $N(0, \sigma^2_\epsilon)$ and $\sigma_\epsilon = \{0.5, 0.75\}$. For the sample size, we consider $N = \{50, 100\}$ and $T = \{10, 20, 40\}$. Let N_g with $g = 1, 2, 3$, denote the number of individuals in group g, and we set $N_1 : N_2 : N_3 = 0.3 : 0.3 : 0.4$. The slope coefficients $\beta_{g,t}$ for the three groups are given by:

$$\beta_{1,t} = \begin{cases} 1 & if 1 \leq t < \lfloor T/2 \rfloor \\ 2 & if \lfloor T/2 \rfloor \leq t < \lfloor 5T/6 \rfloor \\ 3 & if \lfloor 5T/6 \rfloor \leq t \leq T \end{cases}, \quad \beta_{2,t} = \begin{cases} 3 & if 1 \leq t < \lfloor T/3 \rfloor \\ 4 & if \lfloor T/3 \rfloor \leq t < \lfloor 5T/6 \rfloor \\ 5 & if \lfloor 5T/6 \rfloor \leq t \leq T \end{cases},$$

and

$$\beta_{3,t} = 1.5, \quad for \ \ 1 \leq t \leq T.$$

where $\lfloor \cdot \rfloor$ means taking the integer part of the number inside this sign.

DGP 2: DGP 2 is the same as DGP 1 except that we let the error term ϵ_{it} follows an AR(1) process. For each individual i we set $\epsilon_{it} = 0.5\epsilon_{i,t-1} + u_{it}$, where $u_{it}\sim$ i.i.d. $N(0, \sigma_u^2)$ and $\sigma_u = \{0.5, 0.75\}$.

Fixed effects endogenous model

DGP 3: We generate DGP 3 use the fixed effects endogenous model and set the individual fixed effect to be the average of the explanatory variable over the estimation period:

$$y_{it} = \mu_i + x_{it}'\beta_{g_i,t} + \sigma_\epsilon \epsilon_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T,$$

where $\mu_i = T^{-1} \sum_{t=1}^{T} x_{it}$ and $\epsilon_{it}\sim$ i.i.d. $N(0, 1)$. We generate the endogenous explanatory variable x_{it} through $x_{it} = \sqrt{2/3}\delta_{it} + \sqrt{1/3}\epsilon_{it}$ where $\delta_{it}\sim$ i.i.d. $N(0, 1)$ and independent of ϵ_{it}. To generate the instrumental variable, we use $z_{it} = \sqrt{2/3}\delta_{it} + \sqrt{1/3}u_{it}$, where $u_{it}\sim$ i.i.d. $N(0, 1)$ and is independent of ϵ_{it}. Considering the complexity of the fixed effects endogenous model, we first consider a lower level of noise in the data for the simulation experiments. Thus, we use $\sigma_\epsilon = (0.2, 0.4)$. For the same reason, we choose larger sample sizes and set $N = \{200, 400\}$. We use the same time period lengths as the benchmark case, that is $T = \{10, 20, 40\}$. For the coefficients $\beta_{g_i,t}$ and the break dates in each group, we use the same settings as DGP 1.

DGP 4: DGP 4 is the same as DGP 3 except that δ_{it} follows an AR(1) process. For each individual i, we set $\delta_{it} = 0.5\delta_{i,t-1} + u_{it}$ with $u_{it}\sim$ i.i.d. $N(0, 0.75)$.

6.1.2 Evaluation criteria

To evaluate the performances of GAGFL with PLS and GAGFL with PGMM algorithms, we propose four evaluation criteria following Okui and Wang (2017).

Firstly, to test the abilities of our algorithms to classify individuals into the right groups, we look at the misclassification frequency (MF). Let g_i^0 be the true group membership for individual i, and let $I(\cdot)$ be the indicator function. Then, we define

$$MF = \frac{1}{N} \sum_{i=1}^{N} I(\hat{g}_i \neq g_i^0). \tag{21}$$

Secondly, to examine whether our algorithms can give reliable perditions of the number of breaks, we count the percentage of times GAGFL algorithms correctly estimate the true number of breaks in each group. We also calculate the average of the estimated number of breaks in each group over the 200 replications and compare it with the true number of breaks. The fourth criterion tests the accuracy of the break dates estimates. To do this, we follow Qian and Su (2016) and Okui and Wang (2017) and calculate the Hausdorff distance (HD) defined by

$$HD(\hat{T}^0_{g,\hat{m}}, T^0_{g,m^0}) \equiv max\{\mathcal{D}(\hat{T}^0_{g,\hat{m}}, T^0_{g,m^0}), \mathcal{D}(T^0_{g,m^0}, \hat{T}^0_{g,\hat{m}})\} \tag{22}$$

where $\mathcal{D}(A, B) \equiv sup_{b \in B} inf_{a \in A} |a - b|$ for any set A and B. We also report $100 \times HD(\hat{T}^0_{g,\hat{m}}, T^0_{g,m^0})/T$.

Finally, to evaluate the accuracy of the coefficient estimates, we use the root mean squared error (RMSE) and the coverage probability (CP) of the two-sided nominal 95% confidence interval. We compute RMSE use the following formula

$$RMSE(\hat{\beta}_{it}) = \sqrt{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(\hat{\beta}_{it} - \beta_{it})^2}, \tag{23}$$

and we use the following expression to calculate the coverage probability

$$CP(\hat{\beta}_{it}) = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} I(\hat{\beta}_{it} - 1.96\hat{\sigma}_{\beta,it} \leq \beta_{it} \leq \hat{\beta}_{it} + 1.96\hat{\sigma}_{\beta,it}). \tag{24}$$

6.1.3 Tunning parameter selections

Before discussing the results, we first present some practical choices for the experiments. Firstly, both GAGFL with PLS and GAGFL with PGMM methods require a preliminary group membership estimates through the GFE type of estimation. To obtain a good preliminary group assignment as the starting point, we choose to perform the GFE estimation 10 times and set the preliminary result as the one that gives the smallest sum of squared errors. Secondly, we need to select the value for the tunning parameters λ and λ_2 in the PLS step and PGMM step. For both PLS and PGMM, we follow Okui and Wang (2017) and Qian and Su (2016) to search the optimal tunning parameter that gives the lowest information criterion from a 40-evenly-distributed logarithmic grids in the interval [0.01, 100]. The lower bound leads to frequent breaks whereas the upper bound results in no break. Moreover, we set the user-specified constants κ and κ_2 equal to 2. Lastly, we choose to stop the algorithm when the norm difference between the estimated slope coefficients in the consecutive iterations is zero.

6.1.4 Results

In this section, we discuss the results for the static group pattern models. The result statistics are shown in A.1 and A.2.

Classification accuracy

Table 1 reports the average misclassification frequencies for the benchmark model. In general, the results confirm that GAGFL with PLS method can accurately detect the group pattern given sufficient number of observations or when there are little noise in the data. For both DGP 1 and DGP 2, the misclassification frequencies are within 0.1 except for the case of $N = 50$, $T = 10$ and $\sigma_\epsilon = 0.75$. In addition, the clustering accuracy increases significantly when T increases but does not necessarily increase with N, although on average the accuracy for $N = 100$ is higher than

$N = 50$. The above findings are consistent with Okui and Wang (2017). However, compared to their study, our results are more sensitive to the changes of σ_ϵ. One possible explanation is that we only conduct 200 replications while Okui and Wang (2017) conduct 1000 replications. It is possible that the method shows a higher level of robustness if we run more replications.

Table 5 under A.2 shows the average misclassification frequencies for the fixed effects endogenous model use our GAGFL with PGMM method. All the misclassification frequencies in this case are within 5%, which confirms that our proposal to deal with the fixed effects and endogenous regressors with first difference are very reliable for detecting the group pattern. Same as the exogenous case, the clustering accuracy increases with T but not N.

Break detection ability

Next, we talk about the abilities of the proposed methods to detect the break dates. Table 2 shows the average statistics of the number of breaks estimates for the benchmark model use GAGFL with PLS method. According to the average frequencies of correct estimation of the number of breaks, our results are consistent with Okui and Wang (2017) for the case when the errors are not very noisy ($\sigma_\epsilon = 0.5$), and the method can give 100% accuracies in some cases. Besides, our results also show that the break detection accuracy increases with the increase of both N and T. However, for both DGP 1 and DGP 2, the average frequencies for the correct estimation of number of breaks are low when $\sigma_\epsilon = 0.75$. In the worst case of DGP 1, when $N = 50$ and $T = 10$, the correct frequency is only around 25% for the first Group. But a good point is that, the results become better quickly when N and T become larger. For example, when $N = 100$ and $T = 40$, the accuracies for the third group reach 98% and 100% for DGP 1 and DGP 2 respectively. Table 2 also presents the average estimated number of breaks. From these statistics, we find that the PLS step of the algorithm has a tendency to overestimate the break numbers for all the three groups. To investigate the accuracy of the break date estimates, table 3 reports the Hausdorff errors between the estimated break dates and the true break dates. However, we find that even in the case of $N = 50$, $T = 10$ and $\sigma_\epsilon = 0.75$ for DGP 1 when the number of breaks is the most severely overestimated, the Hausdorff error is still in an acceptable region (0.1355). Thus, we conclude that this method has a issue of overestimating the break numbers but does not ignore the true break dates.

Table 6 presents the statistics of the break numbers estimates for the fixed effects endogenous model use GAGFL with PGMM method. Overall, we find this method gives perfect estimation of the number of breaks since most of the frequencies of correct estimation of breaks for all the cases are above 90%. When $T = 20$ or $T = 40$, the correct frequencies are most of the time 100% for both DGP 3 and DGP 4, which means the GAGFL with PGMM method almost estimates the true number of breaks every time. From the average estimated number of breaks in the same table, we find GAGFL with PGMM also tends to slightly overestimate the number of breaks. However, the overestimation level is very low and the estimated number of breaks is very closed to the true break numbers of each group. Table 7 reports the Hausdorff errors of the break date estimates for this model. Most of the Hausdorff errors shown are within 0.01, which means GAGFL with PGMM method gives very accurate break date estimates. From the above discussed results, we conclude that incorporating the PGMM method with GAGFL algorithm keeps PGMM's superior ability of structural breaks detection. Moreover, our results also prove that our method with the first-differencing step gives highly accurate break estimates even with the presence of individual fixed effects. This is in contrast with the result of Okui and Wang (2017) who add the individual fixed effects to the exogenous model and cannot get good break estimates use GAGFL with PLS and the first-differenced data.

Slope coefficient estimation accuracy

Table 4 reports the average RMSEs and the coverage probabilities of the coefficient estimates for the benchmark model. The results are again consistent with Okui and Wang (2017), and we conclude that GAGFL with PLS gives good coefficient estimates in general. On average, the values of RMSE are small, which verifies Okui and Wang (2017)'s conclusion that the slope coefficients are accurate even the break numbers are overestimated because of the consistency of the reliable group pattern estimates. The coverage probabilities for this model are closed to 95% but tends to be slightly bigger than 95% especially when N is small, which means the 95% nominal confidence interval is conservative. This suggests the estimator is not very efficient for small N but we also expect the results can become better if we run more replications.

Shown in table 8 are the average RMSEs and the coverage probabilities for the fixed effects endogenous model. The results show that given a good instrument variable, our GAGFL with PGMM method gives very high accuracy of the slope coefficient estimates even with the presence of individual fixed effects and endogenous issues. For both DGP 3 and DGP 4, the RMSEs are within 0.1 for different levels of noise in the data, and the RMSE decreases with both T and N. The coverage probabilities are slightly lower than 95% especially for DGP 4. However, the coverage probability increases when N and T become larger. In the best case of DGP 4 when $N = 400$, $T = 40$ and $\sigma_\epsilon = 0.2$, the coverage probability reaches 94.50%. Because the classification and break estimates also have very high accuracy for this model, we conclude the high coefficient estimates accuracy for this model relies on both the GAGFL with PGMM's high classification ability and its high break dates detection ability.

6.2 The case of dynamic group pattern

Finally in this section, we present the Monte Carlo experiments for the dynamic group pattern model with the use of our DGHB estimation algorithm.

6.2.1 Data generation process

DGP 5: DGP 5 generates the data with a changing group pattern follows the dynamic group pattern model:

$$y_{it} = x_{it}'\beta_{g_{it},t} + \epsilon_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T,$$

where $x_{it}\sim$ i.i.d. $N(0, 1)$, $\epsilon_{it}\sim$ i.i.d. $N(0, \sigma_\epsilon^2)$ and $\sigma_\epsilon = (0.5, 0.75)$. We choose $N = \{50, 100\}$ and $T = \{10, 20, 40\}$. For the single structural break, we set the break date at $\lfloor T/2 \rfloor$. Let N_{g_1,g_2} for $g_1 = 1, 2, 3$ and $g_2 = 1, 2, 3$ denote the number of individuals who are in group g_1 before the break and in group g_2 after the break. We fix the proportions of the number of individuals with different group memberships to be $N_{1,1} : N_{1,2} : N_{2,2} : N_{2,3} : N_{3,3} : N_{3,1} = 0.2 : 0.1 : 0.2 : 0.2 :$

0.1 : 0.3 : 0.1. For the slope coefficients of each group, we use the following equations:

$$\beta_{1,t} = \begin{cases} 5 & if 1 \leq t < \lfloor T/2 \rfloor \\ 3 & if \lfloor T/2 \rfloor \leq t \leq T \end{cases}, \quad \beta_{2,t} = \begin{cases} 3 & if 1 \leq t < \lfloor T/2 \rfloor \\ 8 & if \lfloor T/2 \rfloor \leq t \leq T \end{cases}.$$

and

$$\beta_{3,t} = 1.5, \quad for \quad 1 \leq t \leq T.$$

By doing this, we design a data generation process that considers a complex scenario in the sense that there are individuals who change their group memberships and also individuals whose memberships do not change. Moreover, we have groups that have structural breaks but also group who has no break.

DGP 6: DGP 6 is the same as DGP 5 except that the error term ϵ_{it} follows an AR(1) process, same as the error term in DGP 2.

6.2.2 Evaluation Criteria

To examine the performance of our DGHB algorithm, we investigate whether DGHB can give the correct estimate of the one single break date, identify the correct group memberships for each individuals in all periods and give consistent estimation of the slope coefficients.

To check the break date detection ability, we count the frequencies that DGHB does not estimate correctly the one single break date over the 200 replications. Suppose the true break date is $b^0 + 1$ and the estimated break date is $\hat{b} + 1$, then the mis-estimation frequency of the break date is given by

$$MB = \frac{1}{200} \sum_{j=1}^{200} I(\hat{b}_j \neq b^0) \tag{25}$$

where $I(\cdot)$ again denotes the indicator function.

For testing the ability of DGHB to give accurate individual classifications, we calculate the misclassification frequency use the same formula as the static group pattern models given by equation 21. However, here we calculate and report the misclassification frequencies both before and after the estimated break date. Lastly, to evaluate the accuracy of the coefficient estimates of DGHB, we calculate the RMSE and the coverage probability use the same formulas given by equation 23 and equation 24.

6.2.3 Results

In this section, we discuss the simulation results for the dynamic group pattern model use our DGHB method. Firstly, table 9 shows the mis-estimation frequencies of the single break date over 200 replications. The results verify that our DGHB method can perfectly detects the correct break date because DGHB almost always gives the correct break date even when the noise in the data is large ($\sigma_\epsilon = 0.75$).

Secondly, from the results shown in table 10, the misclassification frequencies for both the pre and post-break periods are within 10% for all cases and reaches to zero when N and T are large. This suggests our DGHB estimation gives highly accurate clusterings. On average, the classification accuracies are higher for the post-break period than the pre-break period. A potential explanation for this is that in our DGP 5 and DGP 6, the pre-break coefficients are set to 5, 3, and 1.5 for the three groups while the post-break coefficients are set to 3, 8, and 1.5. Thus, the post-break period has more distinctive slope coefficients across different groups which makes the GFE estimation easier to detect the correct group pattern.

Lastly, table 11 gives the average RMSEs and the coverage probabilities of the coefficient estimates for the dynamic group pattern model. The RMSEs are in an acceptable small level but slightly larger than the results for the benchmark model. This may stem from the fact that this method estimates the pre and post-break coefficients separately which reduces the number of observations considered in each estimation. The coverage probabilities tend to be larger than 95% but the gaps are not very big.

In general, these results imply that our DGHB method can be a good alternative to the GAGFL with PLS method in the presence of a changing group pattern. However, this method requires a sufficient number of time observations in each estimation periods. But from our case, we conclude five time observations is already a sufficient length.

7 Application: FDI and growth – new evidence from fixed effects endogenous model

In this section, we illustrate our GAGFL with PGMM estimation method through an empirical application. Specifically, we revisit Qian and Su (2016)'s application about the effect of foreign direct investment (FDI) inflow on economic growth.

7.1 Data and setup

In this application, we borrow the panel dataset from Qian and Su (2016), which includes the data of 88 countries over the time period from 1972 to 2012. For the economic growth measurement, we use the five-year average growth of logarithmic GDP per capita and denote this variable by $growth_{i,t}$ for country i in time period t. We use the ratio of net FDI inflow to total GDP as the proxy of FDI level, denoted by $FDI_{i,t}$. Besides, we include the lagged term $growth_{i,t-1}$ in the regression. Finally, to control the initial income level, we include the logarithmic GDP per capita $Y^0_{i,t}$. For the instrumental variables, we use $z_{i,t} = (growth_{i,t-1}, FDI_{i,t}, FDIi, t-1, Y^0_{i,t}, Y^0_{i,t-1})'$. In our study, we include the country-specific fixed effects μ_i in the model. Moreover, we consider the case that the slope coefficients have the same structural breaks dates for all variables and the breaks are heterogeneous across groups. Thus, our model is given by:

$$growth_{i,t} = \mu_i + \beta_{1,g_i,t} growth_{i,t-1} + \beta_{2,g_i,t} FDI_{i,t} + \beta_{3,g_i,t} Y^0_{i,t} + \epsilon_{i,t}.$$

To implement GAGFL with PGMM, we specify the parameter $\kappa_2 = 2$ and select the optimal tunning parameter λ_2 from 200 evenly-distributed logarithmic girds on the interval [0.5, 100], where 0.5 results in many breaks and 100 leads to almost zero break. For the selection of this tunning parameter, we use the information criterion introduced in section 5 given by equation 13. To get the best initial group assignment, we run the GFE-type of estimation 1000 times.

Furthermore, in this application, we need to select the optimal total number of groups. To do this, we adopt the method of Okui and Wang (2017) and Bonhomme and Manresa (2015) to select the total number of groups G by minimizing the following Bayesian information criterion (BIC):

$$BIC(G) = \frac{1}{NT}\sum_{j=1}^{m+1}\sum_{t=T_{j-1}+1}^{T_j}\sum_{i=1}^{N}(y_{it} - x'_{it}\hat{\alpha}_{g_i,j})^2 + \hat{\sigma}^2\frac{n_p(G)+N}{NT}lnNT \tag{26}$$

where $\hat{\alpha}_{g_i,j}$ is the post lasso estimate, σ^2 is the variance of $\hat{\epsilon}^2_{i,t}$ when $G = 1$, and $n_p(G)$ is the number of estimated coefficients corresponds to G. In this study, we set the candidate number of groups to be $\{1, 2, 3, 4, 5\}$.

7.2 Results

According to our results, the BIC criterion selects the total number of groups $G = 3$ as the optimal group number. Thus, we first analyze the result given $G = 3$. However, as noted by Okui and Wang (2017), the BIC may not correctly select the true number of groups when the dataset has a short time dimension. For this reason, we also discuss the results given $G = 5$ after discussion of $G = 3$ to investigate how our results change given a larger number of groups.

Analysis under the optimal number of groups, $G = 3$

With $G = 3$, our GAGFL with PGMM method gives one group with two structural breaks in the early periods, one group with only one structural break occurs at the end of the sample period, and the last group with six structural breaks occur at any disjoint time periods. Figure 1 shows the estimated group pattern given three groups through a world map, and table 12 lists the country names with their income levels according to their estimated group memberships. Table 13 presents the estimated coefficients and their stand errors as well as the break regimes of each group. Figure 2 illustrates the trends of the effects of FDI inflow on economic growth.

The first group contains two structural breaks in the early stage of the whole sample period (1983-1987 and 1988-1992), and we refer this group to the "early transitions" group. From table 12, most of the countries in this group are middle income countries. After a further examination, we find that a lot of these countries are classified as the top-performing emerging economies by the International Monetary Found (IMF). Shown in the figure 1, this includes several typical emerging countries in Asia, South America and Africa. For example, China, Mexico and South Africa. From table 13, the three explanatory variables are all insignificant before the second structural break at 1988-1992. Starting from the period of 1988-1992, both the AR(1) coefficient of the growth and the FDI inflow coefficient becomes significant at 1% significance level. Specifically, the effect of FDI inflow becomes positive with a moderate magnitude (0.7358) and remains stable till the end of the sample period. This result provides some interesting insights. Firstly, the break point reveals the influences of certain policy changes on the relation between FDI and growth. Several countries in this group experienced an unprecedented economic boom in the early 1990s followed by the liberalizations of economic policies and social structures. For instance, this includes China and India's increase of trade openness and South Africa's ending of Apartheid. These liberalizations help attract substantial foreign investment and enhance the local financial market thus explains why the effect of FDI on economic growth turns significantly positive from 1988 onwards. Secondly, we note that several countries in this group are labor-intensive countries (e.g. China, India and Brazil) and their education levels increase dramatically since the 1990s which may also be the reason that turns effect of FDI relevant and positive to economic growth. This finding is consistent with Borensztein, De Gregorio, and Lee (1998)'s study, which states FDI has positive effect on developing countries where there is a sufficient stock of human capital.

In the second group, the effect of FDI on growth is not significant over the estimation periods and its estimated coefficients are closed to zero. Thus, we name this group the "no significant effect" group. Presented in table 12 and figure 1, this group includes mostly the high-income countries located in Europe, North America, Australia and a small part of south-east Asia. This result confirms the intuitive statement that the positive spillover effect of FDI inflow on economic growth does not apply to the developed countries since these counties are already at the leading positions in various industries. However, one surprising point is that several middle-income countries and two low-income countries Togo and Uganda are also classified into this group. A possible explanation for this is that the economic growths of these countries do not count on FDI inflows or they are lack of the ability to attract sufficient foreign investments, which causes the effect of FDI irrelevant to growth. However, the initial income level of this group has significant negative coefficients. In addition, the relevance becomes stronger and the magnitude becomes larger after the break. This finding is in line with the famous idea of convergence in economic growth. Moreover, the structural break point for this coincides with the global financial crisis of 2007-2008 which severely harmed the economies of the developed countries in this group.

Finally, the third group has six structural breaks in the coefficient estimates, and we refer it to the "highly unstable" group. According to table 12 and figure 1, the members of this group seems to be counter-intuitive in the sense that it contains almost the even numbers of countries from the three different income classes, and the countries are located across different continents. From table 13, the effect of FDI inflow in this group is significantly positive at 1% confidence level for all the estimation periods except the period of 1983-1987. Figure 2 shows that this effect starts with a large positive value then experienced a dramatic drop and finally fluctuates till the end of the estimation period. As the impact of FDI on economic growth may be correlated with a large amount of different factors, we cannot explicitly explain why the effect of FDI on growth for the countries in this group is that volatile. But one important fact we notice is that some of the countries in this group have highly unrest political status in recent decades, for instance, Iran and Zimbabwe, which may be the cause for an unstable effect of FDI inflow on growth. Apart from that, we also notice that countries such as South Korea and Japan in this group are among the highly-innovative countries. The fast change in technology and new ideas in these countries may have similar impacts as the changes of political situations which can lead to an unstable effect of FDI. Thus, we conclude the classification of this group is mainly characterized by the fluctuations of the effect of FDI. Apart from FDI, the effect of initial income in this group is significantly negative, which again is in line with the economic convergence theory.

Analysis under $G = 5$

We now present the results given $G = 5$. Similar as the case of $G = 3$, figure 3 and table 14 shows the estimated group memberships while table 15 and figure 4 summarize the coefficient and regime estimates. According to table 14 and figure 3, increasing the total number of groups to five basically further separates the countries to different groups according to their income levels. Besides, after the separation, the effect of FDI on growth becomes more significant in each group and from figure 4, the effects become more unrest for some groups.

In particular, the new Group 1 includes most of the middle income countries from the "early transitions" group under $G = 3$ but five of the low income countries are removed. Same as before, the effects of FDI becomes significantly positive at 1% significance level since the period of 1988-1992. However, under $G = 5$, period 2003-2007 and 2008-2012 also exhibit structural breaks, and the effects of FDI becomes even larger after each break. These two breaks may correspond to the recent technology breakthroughs diffused globally which enhances the positive FDI influence on emerging markets.

The second group now mainly contains the highly-developed western countries that previously in the "no significant effect" group. Contrast to before, the effect of FDI is significantly positive from the period of 1988-1992 although the magnitude is much smaller than the emerging market group (Group 1). Again, there is a structural break in the period of 2008-2012, which corresponds to the post global financial-crisis period. This suggests FDI can also have positive effects for the big developed countries (e.g. U.S and U.K).

The third group consists of some lower middle-income countries that are previously from the "no significant effect" group, and the fourth group contains the majority of the countries in the "highly unstable" group. Both of these groups have significant and high volatile effects of FDI on growth, and they experience structural breaks in every period which may reflect the unstable situations of these countries in various aspects.

Lastly, the fifth group mainly contains the mainland European countries previously from the "no significant effect" group. Apart from that, Japan and South Korea are also categorized into this group. This group has four continuous structural breaks occur at the second half of the estimation period. Although the FDI effect is significantly positive in all periods, the estimated effect reaches a peak at 1993-1997 and has decreasing trend till 2008-2012.

7.3 Discussion

Both our results under the optimal number of groups $G = 3$ and under the case of more groups $G = 5$ verify that there are heterogeneous structural breaks in the effect of FDI inflow on economic growth.

Compared to other studies in this area, our method provides some new and interesting evidence. Qian and Su (2016) conduct the same study and assume the effect of FDI on economic growth is time-variant but homogeneous. Under their homogeneity assumption, they conclude the effect of FDI is significantly positive at 5% significance level in all periods, provided the number of breaks is set to the optimum. However, according to our study, their result only holds true for part of the countries and may not give impeccable conclusions. Carkovic and Levine

(2005) conduct an influential panel data analysis use the GMM method but fail to find any cross-country evidence that supports a significant effect of FDI inflow on economic development, which may be an incomplete conclusion. In general, the new and insightful evidence we find in this application verifies the importance of modeling heterogeneous structural breaks and confirms the usefulness of the GAGFL with PGMM method in empirical panel data studies.

8 Conclusion

In this paper, we introduce three heterogeneous structural breaks models and their corresponding estimation methods for panel data analysis. We set Okui and Wang (2017)'s exogenous model with a static group pattern as our benchmark model. We first replicate their simulation work to investigate the performance of their GAGFL with PLS method. Our finding is in line with Okui and Wang (2017), which confirms that the GAGFL with PLS method can consistently estimate the unobserved group structure, the dates of structural breaks and the slope coefficients in each group. The estimation accuracy of GAGFL with PLS is especially high when the data contain a low level of noise. After that, we extend the benchmark model by adding individual fixed effects and release the exogeneity assumption. To estimate this model, we work on the first-differenced data and propose the GAGFL with PGMM estimation method. Our simulation results for this model show that despite the complexity of this model, GAGFL with PGMM gives ideal estimation of the group pattern, break dates and the regression slope coefficients. We further demonstrates the usefulness of this fixed effects endogenous model and the GAGFL with PGMM method through an application. Taking heterogeneous structural breaks into consideration through a group pattern, we find new evidence about the effect of FDI on economic growth that traditional panel data studies on this topic has not discovered. Finally, in this paper we also consider a model that allows the group pattern to change after each structural break which has not been considered by previous literature. We propose the DGHB algorithm for estimation which, from our simulation results, gives high-accuracy estimations of the break date and the changing group memberships. The slope coefficient estimates are slightly less accurate compared to the results of other two methods, but are still sufficiently good. In general, we conclude that the three models and methods introduced in this paper perform well in finite samples and can be of significant use in empirical studies.

Future research can extend our methodologies to several interesting topics. First, we assume the slope coefficients of all the regressors share the same break dates for each individual. In reality, it is possible that certain events change the effects of some regressors but not all of them or their structural breaks occur at different points. Thus, it is desirable to model partial structural breaks in slope coefficients. Second, given the fact that many literature has stressed the importance of modeling cross-sectional dependence in panel data, future research can consider the existence of such dependence within each group. Third, in the dynamic group pattern model, we only allow the level of the structural breaks to be heterogeneous but assume all the units share one common break date. Thus, our methodology can still be elaborated to allow the occurrence of multiple structural breaks in different dates.

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A Monte Carlo Simulation

A.1 Benchmark model

Table 1: Misclassification frequency

		N = 50			N = 100		
	σ_ϵ	T = 10	T = 20	T = 40	T = 10	T = 20	T = 40
DGP1	0.5	0.0442	0.0041	0.0023	0.0102	0.0015	0.0000
	0.75	0.1341	0.0496	0.0017	0.0460	0.0155	0.0015
DGP2	0.5	0.0173	0.0029	0.0002	0.0188	0.0030	0.0001
	0.75	0.1261	0.0543	0.0088	0.0841	0.0178	0.0033

Table 1: This table presents the average misclassification frequency for the benchmark model over 200 replications. Shown in this table are the results for DGP 1 and DGP 2 with total number of individuals $N \in \{50, 100\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.5, 0.75\}$.

Table 2: Average frequency of correct estimation of the number of breaks and average estimated number of breaks

		N = 50						N = 100					
		T = 10		T = 20		T = 40		T = 10		T = 20		T = 40	
	Group	correct	$\hat{m}_g$	correct	$\hat{m}_g$	% correct	$\hat{m}_g$	correct	$\hat{m}_g$	correct	$\hat{m}_g$	correct	$\hat{m}_g$
DGP1													
$\sigma_\epsilon = 0.5$	G1 (2)	0.7000	2.3600	0.8500	2.1850	0.9500	2.0550	0.9100	2.1050	0.9800	2.0200	1.0000	2.0000
	G2 (2)	0.7700	2.2850	0.8850	2.1650	0.9250	2.1000	0.9150	2.0850	0.9800	2.0200	1.0000	2.0000
	G3 (0)	0.8550	0.2100	0.9800	0.0250	0.9900	0.0150	0.9500	0.0500	1.0000	0.0000	1.0000	0.0000
$\sigma_\epsilon = 0.75$	G1 (2)	0.2450	3.4000	0.2800	3.7050	0.5200	2.9550	0.5450	2.6750	0.6350	2.5250	0.8500	2.1900
	G2 (2)	0.3400	3.1650	0.3050	3.5650	0.5100	2.8950	0.5650	2.5800	0.7450	2.3550	0.8850	2.1550
	G3 (0)	0.3450	1.5100	0.5450	0.7950	0.8700	0.1950	0.6250	0.5500	0.8350	0.2200	0.9800	0.0200
DGP2													
$\sigma_\epsilon = 0.5$	G1 (2)	0.9350	2.0650	0.9900	2.0100	1.0000	2.0000	1.0000	5.0000	1.0000	2.0000	1.0000	2.0000
	G2 (2)	0.9350	2.0650	0.9850	2.0150	1.0000	2.0000	0.9900	2.0100	1.0000	2.0000	1.0000	2.0000
	G3 (0)	0.9650	0.0350	0.9950	0.0050	1.0000	0.0000	0.9950	0.0000	1.0000	0.0000	1.0000	0.0000
$\sigma_\epsilon = 0.75$	G1 (2)	0.2610	2.7600	0.7600	2.3450	0.8800	2.1750	0.7950	2.2400	0.9750	2.0300	0.9900	2.0100
	G2 (2)	0.5550	2.6350	0.7500	2.3400	0.9200	2.0800	0.8550	2.1900	0.9450	2.0550	0.9900	2.0100
	G3 (0)	0.6000	0.5500	0.8350	0.2200	0.9800	0.0300	0.8550	0.2000	0.9850	0.0150	1.0000	0.0000

This table presents the average frequency of the correct estimation of the number of breaks and the average estimated number of breaks over 200 replications for the benchmark model. Shown in this table are the results for DGP 1 and DGP 2 with total number of individuals $N \in \{50, 100\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.5, 0.75\}$. The second column presents the group name from G1 to G3 with the numbers in the parentheses show the true number of breaks. % correct denotes the average frequency of the correct estimation of the number of breaks and $\hat{m}_g$ represents the average estimated number of breaks for each group.

Table 3: Hausdorff error of break date estimates

	Group	N = 50			N = 100		
		T = 10	T = 20	T = 40	T = 10	T = 20	T = 40
DGP1							
$\sigma_\epsilon = 0.5$	G1 (2)	0.0525	0.0195	0.0063	0.0120	0.0015	0.0000
	G2 (2)	0.0390	0.0168	0.0085	0.0110	0.0025	0.0000
$\sigma_\epsilon = 0.75$	G1 (2)	0.1355	0.1675	0.0954	0.0905	0.0588	0.0241
	G2 (2)	0.1130	0.1220	0.0728	0.0730	0.0410	0.0124
DGP2							
$\sigma_\epsilon = 0.5$	G1 (2)	0.0120	0.0018	0.0000	0.0000	0.0000	0.0000
	G2 (2)	0.0090	0.0020	0.0000	0.0015	0.0000	0.0000
$\sigma_\epsilon = 0.75$	G1 (2)	0.1055	0.0515	0.0148	0.0425	0.0055	0.0021
	G2 (2)	0.0770	0.0368	0.0133	0.0250	0.0058	0.0019

This table presents the average Hausdorff error of break date estimates for the benchmark model over 200 replications. Shown in this table are the results for DGP 1 and DGP 2 with total number of individuals $N \in \{50, 100\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.5, 0.75\}$.

Table 4: Root mean squared error and coverage probability of coefficient estimates

			RMSE		CP	
σ_ϵ	N	T	DGP 1	DGP 2	DGP 1	DGP 2
0.5	50	10	0.1345	0.0049	0.9638	0.9547
	50	20	0.0095	0.0018	0.9766	0.9551
	50	40	0.0071	0.0008	0.9792	0.9583
	100	10	0.0038	0.0019	0.9534	0.9423
	100	20	0.0014	0.0007	0.9564	0.9461
	100	40	0.0006	0.0003	0.9620	0.9509
0.75	50	10	0.5453	0.2962	0.8903	0.9345
	50	20	0.2171	0.0712	0.9363	0.9650
	50	40	0.0992	0.0239	0.9488	0.9700
	100	10	0.3504	0.1275	0.9232	0.9556
	100	20	0.1467	0.0022	0.9606	0.9587
	100	40	0.0024	0.0009	0.9627	0.9670

This table presents the root mean squared error and coverage probability of the coefficient estimates for the benchmark model over 200 replications. Shown in this table are the results for DGP 1 and DGP 2 with total number of individuals $N \in \{50, 100\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.5, 0.75\}$. The column names RMSE represents root mean squared error and CP represents coverage probability.

A.2 Fixed effects endogenous model

Table 5: Misclassification frequency

		N = 200			N = 400		
	σ_ϵ	T = 10	T = 20	T = 40	T = 10	T = 20	T = 40
DGP3	0.2	0.0043	0.0000	0.0000	0.0041	0.0001	0.0000
	0.4	0.0184	0.0016	0.0000	0.0179	0.0001	0.0000
DGP4	0.2	0.0120	0.0007	0.0000	0.0112	0.0001	0.0000
	0.4	0.0349	0.0042	0.0000	0.0328	0.0032	0.0003

This table presents the average misclassification frequency for the fixed effects endogenous model over 200 replications. Shown in this table are the results for DGP 2 and DGP 3 with total number of individuals $N \in \{200, 500\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.2, 0.4\}$.

Table 6: Average frequency of correct estimation of the number of breaks and average estimated number of breaks

		N = 200						N = 4000					
		T = 10		T = 20		T = 40		T = 10		T = 20		T = 40	
	Group	correct	$\hat{m}_g$	correct	$\hat{m}_g$	correct	$\hat{m}_g$	correct	$\hat{m}_g$	correct	$\hat{m}_g$	correct	$\hat{m}_g$
DGP3													
$\sigma_\epsilon = 0.2$	G1 (2)	0.9200	2.0800	0.9900	2.0100	1.0000	2.0000	0.9900	2.0100	1.0000	2.0000	1.0000	2.0000
	G2 (2)	0.9000	2.1100	0.9900	2.0100	1.0000	2.0000	0.9800	2.0200	1.0000	2.0000	1.0000	2.0000
	G3 (0)	0.9400	0.0600	1.0000	0.0000	0.9900	0.0100	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
$\sigma_\epsilon = 0.4$	G1 (2)	0.9200	2.1400	0.9300	2.0900	0.9100	2.1000	0.9900	2.0200	1.0000	2.0000	0.9900	2.0100
	G2 (2)	0.8900	2.1300	0.9400	2.0600	0.8100	2.2000	0.9700	2.0300	1.0000	2.0000	0.9900	2.0100
	G3 (0)	0.9700	0.0400	0.9800	0.0200	1.0000	0.0000	0.9900	0.0100	1.0000	0.0000	1.0000	0.0000
DGP4													
$\sigma_\epsilon = 0.2$	G1 (2)	0.9200	2.0900	0.9400	2.0600	1.0000	2.0000	0.9900	2.0100	1.0000	2.0000	1.0000	2.0000
	G2 (2)	0.9400	2.0600	0.9700	2.0300	1.0000	2.0000	0.9800	2.0200	1.0000	2.0000	1.0000	2.0000
	G3 (0)	0.9100	0.1100	0.9900	0.0100	0.9800	0.0200	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
$\sigma_\epsilon = 0.4$	G1 (2)	0.9300	2.0700	0.9300	2.0700	0.9500	2.0500	0.9900	2.0100	1.0000	2.0000	1.0000	2.0000
	G2 (2)	0.9000	2.1300	0.9600	2.0400	0.9600	2.0400	0.9800	2.0200	1.0000	2.0000	1.0000	2.0000
	G3 (0)	0.9100	0.1000	0.9800	0.0200	1.0000	0.0000	0.9800	0.0200	1.0000	0.0000	1.0000	0.0000

This table presents the average frequency of the correct estimation of the number of breaks and the average estimated number of breaks over 200 replications for the fixed effects endogenous model. Shown in this table are the results for DGP 3 and DGP 4 with total number of individuals $N \in \{50, 100\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.5, 0.75\}$. The second column presents the group name from G1 to G3 with the numbers in the parentheses show the true number of breaks. % correct denotes the average frequency of the correct estimation of the number of breaks and $\hat{m}_g$ represents the average estimated number of breaks for each group.

Table 7: Hausdorff error of break date estimates

	Group	N = 200			N = 400		
		T = 10	T = 20	T = 40	T = 10	T = 20	T = 40
DGP3							
$\sigma_\epsilon = 0.2$	G1 (2)	0.0090	0.0035	0.0000	0.0010	0.0000	0.0000
	G2 (2)	0.0130	0.0005	0.0000	0.0040	0.0000	0.0000
$\sigma_\epsilon = 0.4$	G1 (2)	0.0170	0.0150	0.0320	0.0010	0.0000	0.0023
	G2 (2)	0.0160	0.0095	0.0422	0.0060	0.0000	0.0023
DGP4							
$\sigma_\epsilon = 0.2$	G1 (2)	0.0120	0.0140	0.0000	0.0010	0.0000	0.0000
	G2 (2)	0.0080	0.0050	0.0000	0.0030	0.0000	0.0000
$\sigma_\epsilon = 0.4$	G1 (2)	0.0100	0.0160	0.0185	0.0020	0.0000	0.0000
	G2 (2)	0.0150	0.0055	0.0105	0.0030	0.0000	0.0000

This table presents the average Hausdorff error of break date estimates for the fixed effects endogenous model over 200 replications. Shown in this table are the results for DGP 3 and DGP 4 with total number of individuals $N \in \{50, 100\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.5, 0.75\}$.

Table 8: Root mean squared error and coverage probability of coefficient estimates

			RMSE		CP	
σ_ϵ	N	T	DGP 3	DGP 4	DGP 3	DGP 4
0.2	200	10	0.0258	0.0365	0.8703	0.8744
	200	20	0.0162	0.0273	0.9109	0.9411
	200	40	0.0129	0.0240	0.9447	0.9408
	400	10	0.0163	0.0252	0.9233	0.8810
	400	20	0.0109	0.0173	0.9483	0.9099
	400	40	0.0083	0.0142	0.9528	0.9450
0.4	200	10	0.0548	0.0772	0.8758	0.8267
	200	20	0.0372	0.0565	0.9208	0.8916
	200	40	0.0271	0.0468	0.9390	0.9030
	400	10	0.0334	0.0532	0.9237	0.8850
	400	20	0.0221	0.0350	0.9400	0.9083
	400	40	0.0166	0.0272	0.9409	0.9132

This table presents the root mean squared error and coverage probability of the coefficient estimates for the fixed effects endogenous model over 200 replications. Shown in this table are the results for DGP 3 and DGP 4 with total number of individuals $N \in \{200, 400\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.2, 0.4\}$. The column names RMSE represents root mean squared error and CP represents coverage probability.

A.3 Dynamic group pattern model

Table 9: Mis-estimation frequency of the break date

		N = 50			N = 100		
	σ_ϵ	T = 10	T = 20	T = 40	T = 10	T = 20	T = 40
DGP5	0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.75	0.0000	0.0050	0.0000	0.0000	0.0000	0.0000
DGP6	0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.75	0.0150	0.0000	0.0000	0.0000	0.0000	0.0000

This table presents the mis-estimation frequency of the break date for the dynamic group pattern model over 200 replications. Shown in this table are the results for DGP 5 and DGP 6 with total number of individuals $N \in \{50, 100\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.5, 0.75\}$.

Table 10: Average misclassification frequency before and after the estimated break

		N = 50			N = 100		
	Period	T = 10	T = 20	T = 40	T = 10	T = 20	T = 40
DGP5							
$\sigma_\epsilon = 0.5$	pre	0.0188	0.0002	0.0000	0.0194	0.0007	0.0000
	post	0.0037	0.0030	0.0000	0.0033	0.0003	0.0002
$\sigma_\epsilon = 0.75$	pre	0.0611	0.0067	0.0001	0.0599	0.0057	0.0000
	post	0.0199	0.0022	0.0015	0.0196	0.0021	0.0001
DGP6							
$\sigma_\epsilon = 0.5$	pre	0.0285	0.0008	0.0000	0.0277	0.0018	0.0000
	post	0.0068	0.0004	0.0000	0.0080	0.0008	0.0001
$\sigma_\epsilon = 0.75$	pre	0.0770	0.0129	0.0007	0.0780	0.0120	0.0004
	post	0.0291	0.0049	0.0003	0.0284	0.0054	0.0003

This table presents the average misclassification frequency before and after the estimated break date for the dynamic group pattern model over 200 replications. Shown in this table are the results for DGP 5 and DGP 6 with total number of individuals $N \in \{50, 100\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.5, 0.75\}$. The second columns show the period. pre denotes the pre-break period and post denotes the post-break period.

Table 11: Root mean squared error and coverage probability of coefficient estimates

			RMSE		CP	
σ_ϵ	N	T	DGP 5	DGP 6	DGP 5	DGP 6
0.5	50	10	0.1485	0.2046	0.9706	0.9714
	50	20	0.0434	0.0492	0.9664	0.9605
	50	40	0.0257	0.0303	0.9490	0.9523
	100	10	0.1560	0.1991	0.9707	0.9723
	100	20	0.0327	0.0493	0.9503	0.9583
	100	40	0.0231	0.0233	0.9664	0.9647
0.75	50	10	0.3164	0.3736	0.9549	0.9459
	50	20	0.0993	0.1410	0.9704	0.9737
	50	40	0.0437	0.0510	0.9708	0.9535
	100	10	0.3136	0.3736	0.9576	0.9467
	100	20	0.0887	0.1387	0.9680	0.9718
	100	40	0.0295	0.0376	0.9494	0.9473

This table presents the root mean squared error and coverage probability of the coefficient estimates for the dynamic group pattern model over 200 replications. Shown in this table are the results for DGP 5 and DGP 6 with total number of individuals $N \in \{50, 100\}$, time periods $T \in \{10, 20, 40\}$ and the standard error of the error term $\sigma_\epsilon \in \{0.5, 0.75\}$. The column names RMSE represents root mean squared error and CP represents coverage probability.

B Application

B.1 Analysis under three groups

Figure 1: Estimated group pattern under $G = 3$

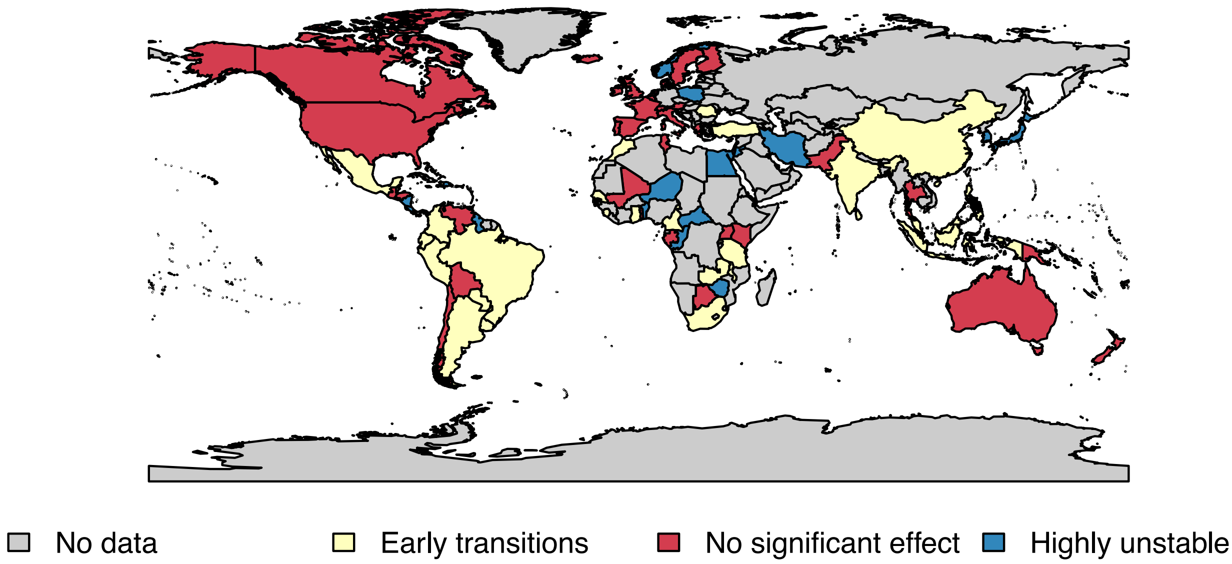

Figure 2: Effects of FDI inflow on economic growth under $G = 3$

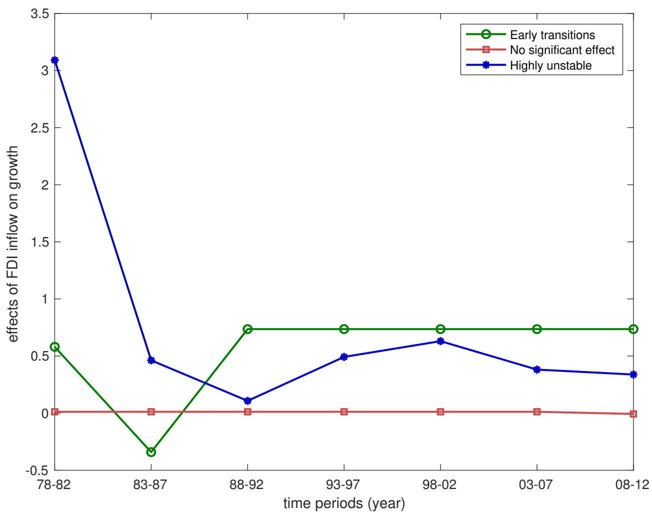

Table 12: List of countries by group, $G = 3$

Group 1 (Early transition)	Income level	Group 2 (No significant effect)	Income level	Group 3 (Highly unstable)	Income level
Argentina	H	Australia	H	Benin	L
Brazil	UM	Austria	H	Central African Republic	L
Cameroon	LM	Belgium	H	Congo	L
China	UM	Bolivia	LM	Costa Rica	UM
Colombia	UM	Botswana	UM	Dominican Republic	UM
Ecuador	UM	Canada	H	Egypt	LM
Ghana	LM	Chile	H	Guyana	UM
Haiti	L	China, Hong Kong	H	Iran	UM
India	LM	Côte d'Ivoire	LM	Japan	H
Indonesia	LM	Cyprus	H	Jordan	UM
Malawi	L	Denmark	H	South Korea	H
Malaysia	UM	El Salvador	LM	Mauritius	UM
Mexico	UM	Fiji	UM	Nicaragua	LM
Morocco	LM	Finland	H	Niger	L
Paraguay	UM	France	H	Norway	H
Peru	UM	Gabon	UM	Poland	H
Philippines	LM	Greece	H	Trinidad and Tobago	H
Romania	UM	Guatemala	LM	Zimbabwe	L
Rwanda	L	Honduras	LM		
Senegal	L	Iceland	H		
Sierra Leone	L	Ireland	H		
South Africa	UM	Israel	H		
Sri Lanka	LM	Italy	H		
Turkey	UM	Jamaica	UM		
Tanzania	L	Kenya	LM		
Uruguay	H	Mali	L		
Zambia	LM	Malta	H		
		Netherlands	H		
		New Zealand	H		
		Pakistan	LM		
		Papua New Guinea	LM		
		Portugal	H		
		Singapore	H		
		Spain	H		
		Sweden	H		
		Switzerland	H		
		Thailand	UM		
		Togo	L		
		Tunisia	LM		
		Uganda	L		
		United Kingdom	H		
		United States	H		
		Venezuela	UM		

This table lists the name of the countries according to their group memberships given the total number of groups is three. The second, fourth and sixth columns give the income levels of each country classified by the world bank. H represents the high income level, UM and LM represent the upper and lower middle income levels respectively and L indicates the low income level.

Table 13: FDI and economic growth: Coefficient and regime estimates of $G = 3$

		1 (78-82)	2 (83-87)	3 (88-92)	4 (93-97)	5 (98-02)	6 (03-07)	7 (08-12)
Group 1	$growth_{i,t-1}$	0.2096	-0.2640	-0.5008 ***				
(Early transition)		(-0.181)	(0.1790)	(0.0689)				
	$FDI_{i,t}$	0.5806	-0.3451	0.7358 ***				
		(0.6712)	(0.2276)	(0.1571)				
	$Y^0_{i,t}$	0.5168	0.5292	0.4006				
		(0.4877)	(0.4628)	(-0.4882)				
Group 2	$growth_{i,t-1}$	-0.0752						-0.1461
(No significant effect)		(0.0530)						(-0.2732)
	$FDI_{i,t}$	0.0120						-0.0071
		(0.0341)						(0.0532)
	$Y^0_{i,t}$	-0.5862 *						-0.7937 **
		(0.3062)						(0.3165)
Group 3	$growth_{i,t-1}$	-0.8267 ***	-0.1207	-0.0347	-0.0784	-0.1908	1.3902 ***	0.0856
(Highly unstable)		(0.1109)	(0.1108)	(0.1058)	(0.1025)	(0.1410)	(0.3271)	(0.2608)
	$FDI_{i,t}$	3.0912 ***	0.4623	0.1076 ***	0.4923 ***	0.6303 ***	0.3814 ***	0.3380 ***
		(0.8065)	(-0.4304)	(0.5252)	(-0.0371)	(0.1156)	(0.1041)	(0.0775)
	$Y^0_{i,t}$	-3.1731 **	-3.1128 **	-3.0434 ***	-2.8682 **	-2.9338 ***	-3.1084 ***	-2.781 **
		(1.0310)	(0.9857)	(0.9064)	(0.9219)	(0.8909)	(0.9471)	(0.9281)

This table presents the results of the application given the total number of groups is three. Shown in the table are the estimated slope coefficients and their standard errors in the parentheses for each group. Only the period after each structural breaks are associated with numbers, the blank space means this period has the same slope coefficients as the previous period.
* indicates statistical significance at 10% level.
** indicates statistical significance at 5% level.
*** indicates statistical significance at 1% level.

B.2 Analysis under five groups

Figure 3: Estimated group pattern under $G = 5$

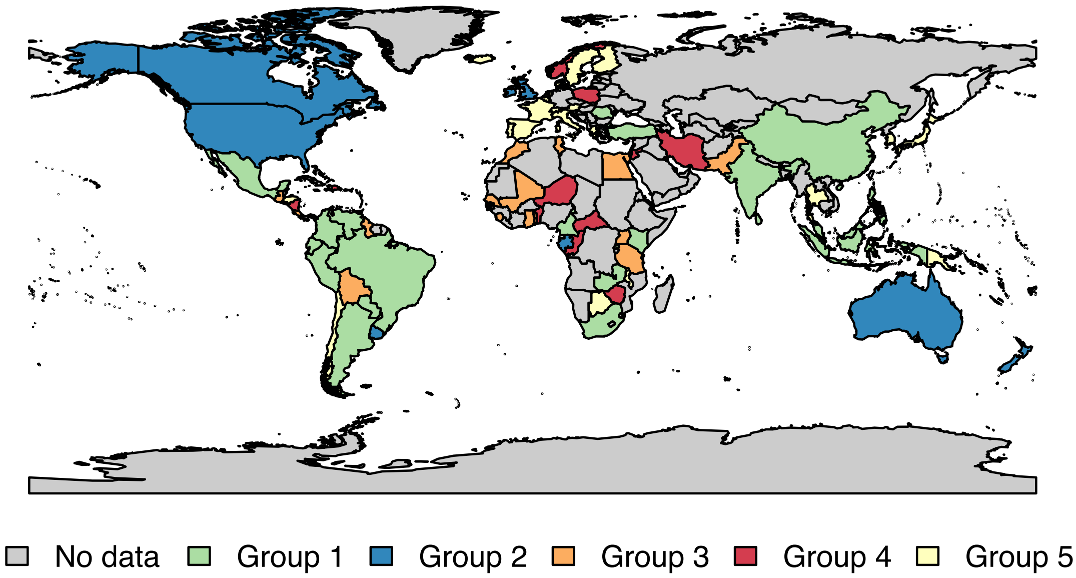

Figure 4: Effects of FDI inflow on economic growth under $G = 5$

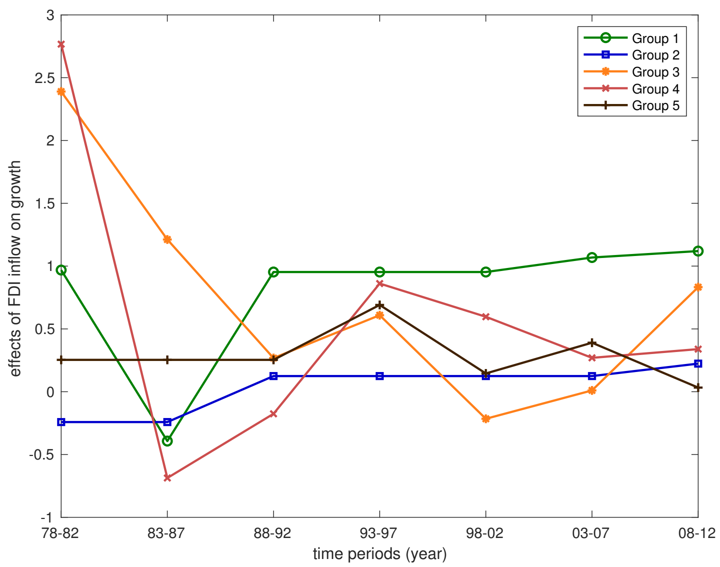

Table 14: List of countries by group, $G = 5$

Group 1	Income level	Group 2	Income level	Group 3	Income level	Group 4	Income level	Group 5	Income level
Argentina	H	Australia	H	Bolivia	LM	Benin	L	Austria	H
Brazil	UM	Canada	H	Costa Rica	UM	Central African Republic	L	Belgium	H
Cameroon	LM	Fiji	UM	Côte d'Ivoire	LM	Congo	L	Botswana	UM
China	UM	Gabon	UM	Egypt	LM	Dominican Republic	UM	Chile	H
Colombia	UM	Ireland	H	El Salvador	LM	Iran (Islamic Republic of)	UM	Hong Kong	H
Ecuador	UM	Israel	H	Ghana	LM	Jordan	UM	Cyprus	H
Haiti	L	Malta	H	Guatemala	LM	Mauritius	UM	Denmark	H
India	LM	New Zealand	H	Guyana	UM	Nicaragua	LM	Finland	H
Indonesia	LM	Singapore	H	Mali	L	Niger	L	France	H
Kenya	LM	United Kingdom	H	Morocco	LM	Norway	H	Greece	H
Malaysia	UM	United States	H	Pakistan	LM	Poland	H	Honduras	LM
Mexico	UM	Uruguay	H	Senegal	L	Trinidad and Tobago	H	Iceland	H
Paraguay	UM			Sierra Leone	L	Zimbabwe	L	Italy	H
Peru	UM			Togo	L			Jamaica	UM
Philippines	LM			Tunisia	LM			Japan	H
Romania	UM			Uganda	L			South Korea	H
Rwanda	L			Tanzania	L			Malawi	L
South Africa	UM							Netherlands	H
Sri Lanka	LM							Papua New Guinea	LM
Turkey	UM							Portugal	H
Venezuela	UM							Spain	H
Zambia	LM							Sweden	H
								Switzerland	H
								Thailand	UM

This table lists the name of the countries according to their group memberships given the total number of groups is five. The second, fourth and sixth columns give the income levels of each country classified by the world bank. H represents the high income level, UM and LM represent the upper and lower middle income levels respectively and L indicates the low income level.

Table 15: FDI and economic growth: Coefficient and regime estimates of $G = 6$

		1 (17-82)	2 (83-87)	3 (88-92)	4 (93-97)	5 (98-02)	6 (03-07)	7 (08-12)
Group 1	$growth_{i,t-1}$	-0.1854	-0.3343	-0.2125 **			-0.3468	0.1387
		(0.2574)	(0.2495)	(0.0919)			(0.2156)	(0.3126)
	$FDI_{i,t}$	0.9685	-0.3930	0.9529 ***			1.0684 ***	1.1196 ***
		(1.1278)	(0.9428)	(0.2214)			(0.1913)	(0.1765)
	$Y^0_{i,t}$	-3.2615 ***	-3.1643 ***	-3.4144 ***			-3.0992 ***	-3.1204 ***
		(0.7964)	(0.8283)	(0.8670)			(0.8431)	(0.8392)
Group 2	$growth_{i,t-1}$	-0.1377 *		-0.3989 ***				-0.6415
		(0.0833)		(0.0770)				(0.5046)
	$FDI_{i,t}$	-0.2416 *		0.1246 **				0.2236 ***
		(0.1359)		(0.0419)				(0.0516)
	$Y^0_{i,t}$	-0.6444		-0.5672				-0.7473
		(0.6065)		(0.5531)				(0.5342)
Group 3	$growth_{i,t-1}$	0.4716	0.5133 ***	0.4701 **	1.3727 ***	-0.3814 ***	0.5175 **	0.3473
		(0.2924)	(0.1417)	(0.1514)	(0.2795)	(0.0750)	(0.1988)	(0.3571)
	$FDI_{i,t}$	2.3897 ***	1.2117 ***	0.2679	0.6091 ***	-0.2155 **	0.0103	0.8325 ***
		(0.4791)	(0.2753)	(0.3807)	(0.0635)	(0.0771)	(0.1927)	(0.1634)
	$Y^0_{i,t}$	0.9618	1.5353	1.7337	1.2114	1.9965	1.7693	1.0246
		(0.4749)	(0.5050)	(0.4515)	(0.5019)	(0.4550)	(0.4554)	(0.4788)
Group 4	$growth_{i,t-1}$	-1.0547 ***	-0.2464 **	-0.2863	0.0812	-0.0616	1.5483 ***	0.0409
		(0.1453)	(0.0987)	(0.1991)	(0.2336)	(0.1464)	(0.2826)	(0.2236)
	$FDI_{i,t}$	2.7669 ***	-0.6868	-0.1758	0.8612***	0.5967 ***	0.2697 ***	0.3391 ***
		(0.8119)	(1.2250)	(0.4485)	(0.3291)	(0.1340)	(0.0939)	(0.0553)
	$Y^0_{i,t}$	-3.1848 ***	-3.1412 ***	-3.2294 ***	-2.988 ***	-3.0923 ***	-3.2917 ***	-2.9778 ***
		(1.2087)	(1.1740)	(1.0701)	(1.1086)	(1.0586)	(1.0409)	(1.0201)
Group 5	$growth_{i,t-1}$	0.3091 ***			-0.272 ***	-1.0093 ***	-0.1185	0.666 *
		(0.0798)			(0.0841)	(0.1980)	(0.1661)	(0.3834)
	$FDI_{i,t}$	0.2536 ***			0.6908 ***	0.1454 ***	0.3901 ***	0.0328 *
		(0.0947)			(0.1465)	(0.0408)	(0.1020)	(0.0727)
	$Y^0_{i,t}$	-0.2738			-0.3299	-0.2129	-0.3878	-0.6117 *
		(0.4211)			(0.4001)	(0.3903)	(0.3688)	(0.3680)

This table presents the results of the application given the total number of groups is five. Shown in the table are the estimated slope coefficients and their standard errors in the parentheses for each group. Only the period after each structural breaks are associated with numbers, the blank space means this period has the same slope coefficients as the previous period.
* indicates statistical significance at 10% level.
** indicates statistical significance at 5% level.
*** indicates statistical significance at 1% level.