ERASMUS UNIVERSITY ROTTERDAM

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Bachelor Thesis (IBEB)

Cryptocurrencies: the digital gold of the 21st century? Statistical and optimizational approaches to the hedging properties of cryptocurrencies

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This paper addresses the hedging (negatively correlated with stocks) and safe haven (negatively correlated with stocks in extreme stock market declines) properties of cryptocurrencies through three methods: GARCH regressions, pairwise dynamic conditional correlation (DCC) parameters and mean-variance portfolio optimization. While Ethereum and Ripple had mixed performances, the results advocate a significant hedging ability of Bitcoin against movements in European or Japanese stocks, but also through an important 9.02% average portfolio weight. Additionally, the inclusion of cryptocurrencies in a stock indices portfolio generates a substantial 105% quarterly average improvement in the optimal Sharpe ratio of the portfolio, from an average of 0.17 to 0.35.

1. Introduction

In the last decade since the financial crisis in 2008, numerous financial markets all over the globe have been passing through periods of high volatility and abnormal negative performances. While trade volume and value of stock shares have been steadily increasing since then, the expanding interdependence between assets and stock markets have created difficulties in discovering an efficient solution for reducing financial damages in times of downturns (Baur & Lucey, 2010).

This paper aims to scrutinize the relationship between the most traded cryptocurrencies and the most important geographic stock indices. The final purposes are to determine whether cryptocurrencies can be considered hedges or safe havens against movements in stock markets and explore the validity of the findings by integrating into the analysis of optimal portfolios stock indices from a diversified array of markets (US, China, Japan, Europe). Therefore, the following research question is formulated:

Can cryptocurrencies be considered hedges and/or safe havens against movements in stock indices?

The decision to invest in cryptocurrencies as hedges or as safe havens against stock markets in periods of turbulences could represent an innovative solution in terms of asset allocation. For the pension funds industry, Ibbotson & Kaplan (2000) have argued that about 90% of the variability of a pension fund's return over time is influenced significantly by the asset allocation policy chosen by the fund. Thereby, the decision to allocate a proportion of the assets in cryptocurrencies might have a potential positive benefit to the pension funds' expected returns and risks, and implicitly for the mass of fund clients.

From a scientific perspective, cryptocurrencies' potential in the investing process is rather unexplored, mostly because investors are still predisposed to speculation and rumors in managing virtual currencies. In this sense, Wu & Pandey (2014) have examined the consequences of including Bitcoin in an investment portfolio and have advocated that the portfolio effectiveness has experienced a positive effect. By taking in consideration these facts, this paper establishes its scientific value by placing in the "spotlight" of investment opportunities other cryptocurrencies besides Bitcoin and observing how different virtual coins react in a similar research design.

Divided in two directions, the paper first analyzes the statistical relationship between stock indices and cryptocurrencies through GARCH model regressions and pairwise dynamic conditional correlations (DCC) to determine any hedge or safe haven properties in the daily or weekly returns of these assets. By combining these two techniques which focus on how the returns of stock indices affect the returns and dynamic correlations of the cryptocurrencies, the paper aims to generate more valid results, verified through the two different techniques and through two different frequencies of returns (daily and weekly).

In the second section of this paper, the effect on the Sharpe ratio of an optimal portfolio of stock indices is analyzed after introducing cryptocurrencies in the portfolio. For improved insights, the analysis is performed in a static case, characterized by the assumption that an investor does not change the asset allocation of the portfolio, and a dynamic case, where the investor is assumed to modify the portfolio's asset allocation in each observed quarter.

The results of the research phase suggested relatively favorable conclusions. For the statistical part, while Bitcoin and Ripple exhibited significant hedging effects against US, European, Japanese or Chinese stocks, no significant results had been discovered regarding the safe haven properties of cryptocurrencies. In addition, the change in frequency of returns from daily to weekly presented significant influence over the results, as most daily hedging abilities seemed to lose their significance in the weekly analysis.

Surprisingly, the influence of adding cryptocurrencies to a stock index portfolio was extremely consistent both over the entire studied period and over each quarter comprising this period. For a better metric evaluation, in the dynamic case, an average cryptocurrency allocation weight of 19.40% contributed to an increase in optimal Sharpe ratio from a quarterly average of 0.17 to 0.35, or approximately 105% on average.

The structure of this paper is as follows. In the next section, a literature review on the most relevant topics used in this research is provided, and the theoretical framework is discussed, along the main stated hypotheses. The following section contains the data and methodology used for the statistical analysis. Lastly, the results of the study are presented and in the final sections, they are interpreted and discussed.

2. Literature review

Past academic literature concentrated around the topics of cryptocurrencies and their potential benefits for finance professionals is extremely narrow, and it is mostly biased towards one of the most elderly and prominent virtual coins, Bitcoin. Several reasons have been previously analyzed in relation to the scarcity of research regarding Bitcoin. Firstly, Bitcoin is relatively obscure for any professional or amateur investors who does not present an affinity to the fields of computing and cryptography, two fields which are at the base of cryptocurrencies (Lee, 2013). Secondly, Velde (2013) has argued that the market value of Bitcoin might still be categorized as not significant, compared to the dimension of the global economy. This aspect might pose less importance in the present, as nowadays numerous cryptocurrencies are exchanged at huge volumes (€11B daily volume for the five most traded cryptocurrencies, compared to S&P 500's daily volume of approximatively €3B, at the start of 2018).

In the last years, there has been a modest sample of studies focused on the investment potential of Bitcoin. Significant attention was placed on the aspect of correlation, as it represents an important element of optimal portfolio formation. An example is the research performed by Burniske & White (2016), which have measured the correlation between Bitcoin and several acknowledged financial references, such as S&P 500, US Real Estate, oil, gold and bonds. Their conclusion is that Bitcoin has the potential to transform into a differentiator among other assets, and even to transform the entire financial community. In another example, Baur, Hong & Lee (2016) have compared Bitcoin to a selection of 16 different assets in terms of returns and correlations. Among the compared assets, the authors analyzed a variety of instruments, such as precious metals, energy, currencies and the typical stocks and bonds. Their results advocate that

Bitcoin presents the highest average return and standard deviation relative to the other analyzed assets. In addition, no significant correlation between Bitcoin and the other instruments has been found, fact which is in accordance with past literature on the topic.

In terms of considering Bitcoin into the process of optimal asset allocation, Eisl, Gasser & Weinmayer (2015) have dived into the theme by analyzing historical data of portfolios containing a wide array of assets, including Bitcoin. The results depicted that while the highest asset allocation of the virtual currency is only approximately 7.69%, the allocation of Bitcoin in the average portfolio contributes positively to the specific portfolios by improving the Sharpe ratio. In a more complex research framework, Briere et al. (2015) have included Bitcoin into a diversified portfolio of US assets and concluded that this allocation decision improved the Sharpe ratio of the portfolio. In addition, Bouri et al. (2016) have advanced the idea that an optimal combination of Bitcoin and US equities can significantly reduce the variance, implicitly the risk of a portfolio.

However, even if the selected research papers supported a positive effect of adding Bitcoin to portfolios, other academics warn about the intrinsic nature of Bitcoin and how it influences the effectiveness of the currency as a safe haven. In this sense, Bouri et al. (2016) and Kristoufek (2015) have concluded that Bitcoin does not represent an effective safe haven, because its hedging capabilities vary significantly between time horizons. The bubble in 2013 and the crash following it illustrate perfect examples of how Bitcoin had previously lost its potential abilities of safe haven. Moreover, there is a debate surrounding whether or not Bitcoin has reached the investment grade status, which represents a necessary condition for including a financial instrument into a portfolio of diversified assets. On one side, Cheah & Fry (2015) and Urquhart (2016) have supported the perception that Bitcoin has failed yet to achieve this status, but on the other side, more recent research performed by Nadarajah & Chu (2017) have suggested the opposite. Therefore, whether or not Bitcoin or other cryptocurrencies should be included in an optimal portfolio persists to remain an open question.

Another aspect far from being exhaustively documented, it appears that past literature on cryptocurrencies as investment instruments or as hedges/safe havens is concentrated predominantly on the perspective of an US investor. A main problem from this perspective is that the US assets analyzed in these papers have limited exposure to financial markets outside of US, such as China. Considering that China dominates Bitcoin exchanges, with up to 99% of the total global cryptocurrency transactions, the decision to not include research on Chinese optimal portfolios might greatly affect the validity of past results. In fact, movements in the Chinese economy directly influence the CNY Bitcoin market, which in turn presents the ability to substantially affect the US market (Kristoufek 2015).

3. Theoretical framework

As emphasized in the previous section, prior academic literature has predominantly neglected the potential capacity of Bitcoin and of other major cryptocurrencies as a safe haven and has missed to differentiate among the concepts of hedging and safe haven properties, as described by Baur & Lucey (2010). In addition, past papers have focused on a limited geographical sample, where attention is placed on US assets and financial instruments, which may not represent the most suitable portfolio components to illustrate the world-scale potential of cryptocurrencies. Consequently, this paper addresses the identified literature gaps by assessing whether or not five of the currently most important cryptocurrencies by popularity, market value and trading volume (*Bitcoin*, *Ripple*, *Ethereum*, *Litecoin* and *Stellar*) have the potential to act as a hedge or as a safe haven against fluctuations in prices of various stock indices, representing important geographical financial markets (*S&P 500* and *DIJA* for the US, *Nikkei 225* for Japan, *Shanghai Stock Exchange* for China, and *Stoxx Europe 600* for Europe).

Hypothesis 1: Cryptocurrency assets do not represent a strong (weak) hedge against movements in stock indices.

A strong (weak) hedge is characterized as an asset that is negatively correlated (uncorrelated) with another asset on average. As an addition to this definition, it must be stated that a hedge does not necessarily present the ability to reduce losses in periods of market declines, as the respective asset could exhibit a positive correlation in the poor performing periods and a negative correlation in good performing periods. This situation might result in a negative correlation on average.

Hypothesis 2: Cryptocurrency assets do not represent a strong (weak) safe haven against movements in stock indices.

A strong (weak) safe haven is characterized as an asset that is negatively correlated (uncorrelated) with a specific stock market in time intervals of extreme stock market declines. The specific property that distinguishes between hedges and safe havens is that in the case of the latter, the asset presents non-positive correlation with a specific stock market in severe market conditions. However, it must be added that this property does not impose the correlation to be either positive or negative on average, but only to be equal to zero or negative in specific periods when stock markets are declining.

Hypothesis 3: Cryptocurrency assets do not represent a practical hedge in terms of mean-variance optimization of investment portfolios.

A practical hedge is considered to present a substantial impact on the risk of the portfolio (standard deviation) without a significant effect on the portfolio return (mean) in the moment when it is introduced in a portfolio. In the mean-variance framework developed by Markowitz (1952), the main technique used to judge the effect of cryptocurrencies included in portfolios in this paper is the Sharpe ratio (1966), which focuses on the mean and standard deviation of the portfolio and the risk-free rate.

4. Data & Methodology

4.1. Time series

Daily and weekly observations of the period from January 1st, 2016 to May 1st, 2018 regarding the returns of stock indices and cryptocurrencies were collected from the databases of Bloomberg, in the case of stock indices, and from the data published by CoinMarketCap, a website that stores historical data on all existing cryptocurrencies, in the case of the virtual coins.

In order to avoid the risk of currency fluctuations, inflation and natural differences between the virtual coins, all studied variables were measured in percentages, representing the price performance of the specific variable at time (t) relative to its price at time (t-1). *Table 4.1* depicts a visualization of the relevant daily data (Panel A) and weekly data (Panel B), and of the transformations performed.

Table 4.1. Descriptive statistics summary

Variable	Observations	Mean	Std. dev.	Min	Max	Skewness	Kurtosis
Panel A: Daily data							
Cryptocurrencies							
R_BITC	852	0.004	0.042	-0.187	0.252	0.253	7.821
R_ETHE	852	0.010	0.070	-0.270	0.353	0.802	6.766
R_RIPP	852	0.009	0.103	-0.460	1.793	7.649	116.543
R_LITE	852	0.006	0.062	-0.326	0.476	1.948	16.067
R_STEL	852	0.011	0.108	-0.306	1.060	3.773	31.446
Stock indices							
R_SP50	852	0.001	0.006	-0.040	0.271	-0.931	10.935
R_DIJA	852	0.001	0.006	-0.046	0.028	-1.099	12.337
R_NIKK	852	0.001	0.010	-0.079	0.071	-0.240	14.649
R_SHAN	852	-0.001	0.008	-0.070	0.042	-2.272	21.207
R_STOX	852	0.001	0.007	-0.070	0.036	-0.965	14.792
Panel B: Weekly data							
Cryptocurrencies							
R_BITC	122	0.031	0.115	-0.224	0.509	0.584	5.062
R_ETHE	122	0.077	0.247	-0.268	1.422	2.460	12.012
R_RIPP	122	0.071	0.310	-0.331	1.999	3.242	16.615
R_LITE	122	0.046	0.205	-0.293	1.401	3.309	19.567
R_STEL	122	0.082	0.359	-0.360	2.742	4.192	28.015
Stock indices							
R_SP50	122	0.002	0.014	-0.059	0.031	-1.151	6.779
R_DIJA	122	0.002	0.016	-0.061	0.053	-0.815	6.217
R_NIKK	122	0.001	0.026	-0.111	0.067	-1.001	6.226
R_SHAN	122	-0.001	0.022	-0.099	0.051	-1.699	8.603
R_STOX	122	0.001	0.019	-0.066	0.047	-0.555	4.129

Figure 4.2 (see *Appendix*) for the daily data and *Figure 4.3* (see *Appendix*) for the weekly data illustrate the price evolution of Bitcoin (R_BITC), Ethereum (R_ETHE), Ripple (R_RIPP), Litecoin (R_LITE), Stellar (R_STEL), S&P 500 (R_SP50), DIJA (R_DIJA), Nikkei 225 (R_NIKK), Shanghai Stock Exchange (R_SHAN), and Stoxx Europe 600 (R_STOX) between 1st January 2016 (t=1) and 1st May 2018 (t=852 for daily data and t=122 for weekly data). As of important remarks, none of the variables exhibited any clear trend. On the side of cryptocurrencies, several outliers were existent, fact which might have had slight implications in the performed regressions.

To assess the stationarity of the studied variables, the paper utilized the Dickey-Fuller test under the null hypothesis of a unit root. If this hypothesis cannot be rejected, then the tested variables pose a non-stationary trend, which can bias relevant coefficients for the analysis. A potential problem would be that some independent variables would have a higher likelihood to appear related even when they were not, a phenomenon called spurious regression. In both Panel A and Panel B, the above-mentioned variables comprised neither a trend nor an intercept in their equations, fact which is of great relevance in the Dickey-Fuller test equation and its results' validity. In the end, all variables, proved stationary (see *Appendix*, *Table 4.4*) implying that there was no degree of risk that could skew the results.

In addition, to better represent the effect of cryptocurrencies on stock indices in periods of markets' relative downturn, three dummy variables were introduced for each stock index variable. For example, in the case of stock indices, the paper proposed the dummy variables D_{10}, D_{05}, and D_{01}, which had value of 1 when the respective stock index's share price was among the worst 10%, 5%, respectively 1% performers during the observed time interval, and 0 otherwise.

4.2. Methodology

4.2.A GARCH regression & Dynamic Conditional Correlation (DCC)

This paper's first subject of interest was principally interested in the reaction of cryptocurrencies to fluctuations in the stock indices' share prices in order to scrutinize whether the selected cryptocurrencies represent a hedge or a safe haven for the specific stock indices. This section is focused on two main regression perspectives: GARCH regressions and dynamic conditional correlation models (DCC) from the multivariate GARCH family.

First of all, constant correlation matrixes were computed for the entire period and worst 10%, 5% and 1% market days to observe any indication of possible hedge and safe haven attributes of cryptocurrencies. In case the correlation between a cryptocurrency and a stock index is negative or relatively insignificant (lower than 0.1), then the specific cryptocurrency exhibits a potential indication of its hedging capacities.

After studying the specific constant correlations, the relation between cryptocurrencies and stock indices was first tested using OLS regression models, which focused both on the daily or weekly return of stock indices and on the interaction effects encountered in the worst 10%, 5% and 1% periods of the financial markets. Judging after the skewness values of the return variables exhibited in both the daily and the weekly panel data, the probability that volatility clustering was present in the data points was significant, fact that motivated the usage of a system of equations that would also consider the variance of the variables. Therefore, the basic regression models were exemplified through the following mean and variance equations, incorporated in a GARCH (1,1) model:

$$\text{R\_Crypto}_{i,t} = \beta_0 + \beta_1 * \text{R\_Index}_{i,t} + \alpha_1 * D_{10} * \text{R\_Index}_{i,t} + \alpha_2 * D_{05} * \text{R\_Index}_{i,t} + \alpha_3 * D_{01} * \text{R\_Index}_{i,t} + \varepsilon_t \quad (1)$$

$$h_t = \omega + \Omega * \varepsilon_{t-1}^2 + \pi * h_{t-1} \quad (2)$$

In this regression form, $\text{R\_Crypto}_{i,t}$ represented the return of one of the cryptocurrencies' price at time t, while $\text{R\_Index}_{i,t}$ expressed the return of the specific stock index at time t. The parameters that had to be estimated were β_1 for hedging effects and α_1, α_2, and α_3 for safe haven abilities, while the error term at time t was illustrated by ε_t. The dummy variables D_{10}, D_{05}, and D_{01} were included in the regression to account for the interaction terms in order to represent asymmetries of positive or negative shocks and to focus on downturn market return periods. For example, if the stock index's return was higher than the 10^{th} quartile, the value of the dummy variable D_{10} would be zero, implicitly the value of the 10% quartile interaction effect would be also zero, otherwise it would be the initial value. In addition, the variance equation from the GARCH (1, 1) model was incorporated in the calculation (equation 2) for the purpose of accounting for the heteroskedasticity of the time series data. The parameters h_t, ω, Ω, and π represented the conditional variance, the constant, the short-run consistency of the ARCH effect, respectively the long-run consistency of the GARCH effect.

For the second part of the regression section, a deeper econometric perspective was adopted through the use of a bivariate DCC model developed by Engle (2002), a model which is particularly relevant for estimating dynamic correlations between the returns of the studied variables. In contrast to multivariate GARCH models such as constant conditional correlation

(CCC) models, the DCC model possesses the potential to capture the time-varying connections among the return of cryptocurrencies and stock indices with relatively few computational complications (Parhizgari & Cho, 2008), fact that is of great importance in this situation where the present number of return variables can exhibit computational difficulties.

The computation of the bivariate DCC model was conducted in two separate steps. Firstly, an univariate GARCH (1,1) model was calculated as following:

$r_t = \mu_t + \omega * r_{t-1} + \varepsilon_t$ (3)

$h_t = c + a * \varepsilon_{t-1}^2 + b * h_{t-1}$ (4)

In equation 3, r_t represented the vector of the daily or weekly price return of a cryptocurrency and that of a stock index, while μ_t portrayed the conditional mean vector of r_t and ε_t was the vector of residuals. The parameters in equation 4 basically represented the same type of calculations such as in the case of equation 2, therefore the description of the parameters was already offered.

In the next step, the DCC model is described as following:

$H_t = D_t * R_t * D_t$

$R_t = \text{diag}\ (Q_t)^{-1} * Q_t * \text{diag}\ (Q_t)^{-1}$

$Q_t = (1 - \alpha - \beta) * \overline{Q} + \alpha * (\varepsilon_{t-1}, \varepsilon_{t-1}) + \beta * Q_{t-1}$

$\varepsilon_t = D_t^{-1} * r_t$

Where H_t represents a time varying covariance matrix, R_t a time varying correlation matrix, Q_t a time varying covariance matrix of the standardized residuals, $\overline{Q}$ the unconditional covariance matrix of the returns/residuals, D_t a diagonal matrix of time varying standard deviations resulted from the previous univariate GARCH (1,1) model, r_t the vector of the daily or weekly price return of a cryptocurrency and that of a stock index and ε_t the standardized returns.

The D_t, R_t and H_t matrixes were computed and constructed as in the following forms:

$$D_t = \begin{pmatrix} \sqrt{h_{1,t}} & 0 \\ 0 & \sqrt{h_{2,t}} \end{pmatrix}; \; R_t = \begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix};$$

$$H_t = D_t{*}R_t{*}D_t = \begin{pmatrix} \sqrt{h_{1,t}} & 0 \\ 0 & \sqrt{h_{2,t}} \end{pmatrix}\begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix}\begin{pmatrix} \sqrt{h_{1,t}} & 0 \\ 0 & \sqrt{h_{2,t}} \end{pmatrix} = \begin{pmatrix} \sigma_{1,t}^2 & \sigma_{12,t} \\ \sigma_{21,t} & \sigma_{2,t}^2 \end{pmatrix}$$

Where ρ_{ij} is the dynamic correlation between assets i and j, and it was calculated as follows:

$$\rho_{ij,t} = \frac{E_{t-1}(r_{1,t},r_{2,t})}{\sqrt{E_{t-1}(r_{1,t}^2)E_{t-1}(r_{2,t}^2)}} = \frac{E_{t-1}(\varepsilon_{1,t},\varepsilon_{2,t})}{\sqrt{E_{t-1}(\varepsilon_{1,t}^2)E_{t-1}(\varepsilon_{2,t}^2)}} = E_{t-1}(\varepsilon_{1,t},\varepsilon_{2,t})$$

Which represents the conditional covariance of the disturbances. In continuation, the standardized residuals matrix was constructed:

$$\varepsilon_t = D_t^{-1}{*}r_t = \begin{pmatrix} 1/\sigma_{1,t} & 0 \\ 0 & 1/\sigma_{1,t} \end{pmatrix}\begin{pmatrix} r_{1,t} \\ r_{2,t} \end{pmatrix} = \begin{pmatrix} \frac{r_{1,t}}{\sigma_{1,t}} \\ \frac{r_{2,t}}{\sigma_{2,t}} \end{pmatrix}$$

$$E_{t-1}(\varepsilon_t, \varepsilon_t) = R_t = \begin{pmatrix} 1 & \frac{q_{12,t}}{\sqrt{q_{1,t},q_{2,t}}} \\ \frac{q_{12,t}}{\sqrt{q_{1,t},q_{2,t}}} & 1 \end{pmatrix}$$

Where $q_{ij,t}$ is defined as the dynamic conditional correlation between assets i and j at time of observation t. In addition, each $q_{ij,t}$ variable was visualized in a simple GARCH (1,1) type structure:

$$q_{ij,t} = \bar{\rho}_{ij}{*}(1 - \alpha - \beta) + \alpha{*}(\varepsilon_{i,t-1}, \varepsilon_{j,t-1}) + \beta{*}(q_{ij,t-1})$$

Or expressed in a multivariate form, where α and β were calibrated during the estimation of R_t :

$$Q_t = \bar{Q}{*}(1 - \alpha - \beta) + \alpha{*}(\varepsilon_{i,t-1}, \varepsilon_{j,t-1}) + \beta{*}Q_{t-1}$$

Ultimately, for the purpose of identifying the hedge and safe haven properties of the cryptocurrencies assets, the dynamic conditional correlation parameters were separated from the previous DCC model in distinct time series and then regressed in a simple form:

$$DCC_t = m_0 + m_1{*}D_{10}{*}r_{\text{other asset}} + m_2{*}D_{05}{*}r_{\text{other asset}} + m_3{*}D_{01}{*}r_{\text{other asset}} + v_t \quad (5)$$

Where DCC_t illustrated the pairwise conditional correlation between a cryptocurrency and a stock index at time t, $r_{other\ asset}$ represented the return of the stock index variable, and v_t was interpreted as the error term at time t.

In connection with the first two hypotheses, equation (1) was utilized to explain the statistical relationship between the returns of cryptocurrencies and stock indices. Thereby, for hypothesis one, the parameter β_1 which represented the parameter attached to the independent variable of the stock index's return, must be lower than zero in order for the specific cryptocurrency to present hedge properties against the price movements of the respective stock index. For the second hypothesis, the sum of parameter β_1 with each of the α_1, α_2, or α_3 coefficients must be lower than zero for the specific cryptocurrency to act as a safe haven against the respective stock index.

Lastly, for the same two hypotheses, the focus was removed from mean equations and directed to the dynamic correlations of the studied assets in the equation (5). In this case, the cryptocurrency asset represents a weak hedge against volatility in a stock index if the parameter m_0 is approximately zero, or a strong hedge if it is negative. In respect to the safe haven abilities, the cryptocurrency asset represents a weak safe haven against volatility in a stock index if the coefficients m_1, m_2, and m_3 are close to zero, or a strong safe haven if all those parameters are significantly negative.

4.2.B Portfolio optimization

Regarding the portfolio asset allocation aspect, the paper's focus was predominantly on a two-side comparison of portfolio allocations. On one side, an index portfolio comprising of the five stock indices was analyzed in different time intervals, while on the other side a mixed portfolio involving both stock indices and cryptocurrencies was observed in parallel.

The two portfolios were compared in two distinct settings, depending on the typical investor's or portfolio manager's tendency in adjusting his/her portfolio components' weights. In the first setting (static case), the investor does not adjust his portfolio, choosing a constant weight distribution through the entire observed time interval (28 months). In the second instance (dynamic case), the assumption was that the investor adjusts the weight distribution inside the portfolio at the beginning of every quarter (9 quarters over 27 months, excluding April 2018).

In terms of performed calculations, the performance of a portfolio was mathematically visualized through the mean-variance framework developed by Markowitz (1952), and it was objectively analyzed relative to its optimal performance through the ratio promoted by Sharpe (1966). Therefore, the first calculation performed was the expected return of the portfolio:

$$E(R_P) = W^T\mu;$$

$$W = \begin{pmatrix} W_1 \\ W_2 \\ . \\ . \\ W_K \end{pmatrix}; \quad W^T = (W_1 \quad W_2 \quad . \quad . \quad W_K); \quad \mu = \begin{pmatrix} E(R_1) \\ E(R_2) \\ . \\ . \\ E(R_K) \end{pmatrix};$$

Where W^T represents the transpose of the portfolio components' weights vector, and μ serves as the vector of expected returns of the portfolio assets. Secondly, the other component of calculation performed was the standard deviation of the portfolio:

$$\sigma_P = (W^T\Sigma_W)^{1/2}$$

$$\Sigma_W = \begin{pmatrix} \sigma_{11} & \sigma_{12} & . & . & \sigma_{1K} \\ \sigma_{21} & \sigma_{22} & . & . & \sigma_{2K} \\ . & . & . & . & . \\ . & . & . & . & . \\ \sigma_{K1} & \sigma_{K2} & . & . & \sigma_{KK} \end{pmatrix};$$

Where $\sigma_{XY} = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})$ illustrates the covariance between asset x and asset y, within n daily observations. In this case, Σ_W represents the variance-covariance matrix of the portfolio components' weights. To calculate the variance-covariance matrix, the paper followed two steps. Firstly, an excess return matrix was computed (matrix X) through calculating the average daily return for each of the k assets and subtracting this average return from each daily return of the respective asset. Secondly, the excess return matrix was used in the variance-covariance matrix formula:

$$\Sigma_W = \frac{1}{n}(X^TX)$$

Finally, the performance of the portfolio was objectively assessed through the above-mentioned Sharpe's ratio, whose formula is:

Sharpe ratio = $\frac{R_P - R_F}{\sigma_P}$

Where R_P is the expected return of the portfolio, σ_P the standard deviation of the portfolio, and R_F the risk-free rate. However, in reality risk-free rates have been approximately zero in most developed economies (or at least in the economies surrounding the chosen stock indices), thereby the Sharpe ratio utilized in this paper had ignored the effect of the risk-free rate, as it appeared insignificant in the calculations of portfolio performance. Considering this, the updated formula is:

Sharpe ratio (updated) = $\frac{R_P}{\sigma_P}$

After the calculation of the Sharpe ratio was performed, the Solver tool offered in Excel was used to determine the maximum ratio that could be obtained by modifying the asset weights in the respective portfolio. As of constraints, short-selling, leveraging, or leftover cash were considered not possible, as illustrated below:

1. $\sum_{i=1}^{10} W_i = 1$ (sum of the asset weights must be equal with 100%, thus no leverage or leftover cash allowed)
2. $\forall W_i \geq 0$ (the weight of any asset must be positive, thus not allowing short-selling)

In the special case no allocation of assets can provide a positive expected portfolio return, an investor would prefer to not invest his/her funds at all (keep money in cash).

Following these calculations, the paper assessed the correctness of the last hypothesis by observing whether the mixed portfolio containing both stock indices and cryptocurrencies significantly outperformed the index portfolio containing only stock indices in all selected time settings.

5. Results

In this section, the study demonstrated and described the results of the statistical analysis performed on the observed variables. For a clear distinction between the interests of this research, the section was divided in two subsections, one focused on the regressions studying the relationship between cryptocurrencies and stock indices, and the other one concentrated on the effect of introducing cryptocurrencies into an investment portfolio.

A. Regressions

In advance of regressions, a number of four static correlation matrixes were constructed, as seen in *Table 5.1*. From the correlation matrix that covers the correlation coefficients over the entire period, in all 25 relevant situations analyzed the correlation value between a stock index and a cryptocurrency was either negative or extremely small (lower than 0.1), fact which offered strong indications of the possible hedging properties of cryptocurrencies.

However, when the matrixes of the correlation coefficients during the 10%, 5%, respectively 1% worst market days for each stock index were analyzed, the situation was different in terms of safe haven abilities of cryptocurrencies. While in the 10% setting, in 17 out of 25 cases (68%) the correlation coefficients were negative or lower than 0.1, that number was equal to 11 (44%) in the 5% setting, and 13 (52%) in the 1% setting. The most consistent safe haven abilities seemed concentrated against European stocks, represented by the R_STOX variable, which exhibited negative correlation coefficients with cryptocurrencies in 14 out of 15 cases (93.33%).

Table 5.1. Constant correlation matrixes of returns of cryptocurrencies and of stock indices

	Correlation over entire period						Correlation during the worst 10% market days				
	R_BITC	**R_ETHE**	**R_RIPP**	**R_LITE**	**R_STEL**		**R_BITC**	**R_ETHE**	**R_RIPP**	**R_LITE**	**R_STEL**
R_SP50	0.0141	0.0034	-0.0425	0.0296	-0.0086	**R_SP50**	0.0940	0.1731	0.0409	0.1546	0.1196
R_DIJA	0.0529	0.0295	0.0371	0.0639	0.0459	**R_DIJA**	0.1045	0.0510	0.0840	-0.0508	-0.0021
R_NIKK	0.0497	-0.0442	0.0082	0.0498	0.0282	**R_NIKK**	0.2337	-0.1724	0.2040	0.1550	0.1352
R_SHAN	-0.0168	0.0398	-0.0282	-0.0350	-0.0300	**R_SHAN**	0.0209	0.0279	0.0131	0.0420	0.0591
R_STOX	-0.0353	-0.0469	0.0374	0.0103	0.0129	**R_STOX**	-0.1736	-0.2368	-0.0878	-0.3587	-0.2910

	Correlation during the worst 5% market days						Correlation during the worst 1% market days				
	R_BITC	**R_ETHE**	**R_RIPP**	**R_LITE**	**R_STEL**		**R_BITC**	**R_ETHE**	**R_RIPP**	**R_LITE**	**R_STEL**
R_SP50	0.1136	0.2785	0.0746	0.1682	0.0643	**R_SP50**	0.3397	0.3641	-0.1302	0.5199	0.4498
R_DIJA	0.1804	0.2149	0.1485	0.1618	0.1094	**R_DIJA**	0.0739	0.1383	0.0234	0.0843	0.0579
R_NIKK	0.2998	-0.1876	0.2253	0.1497	0.2716	**R_NIKK**	0.4242	-0.3945	0.3677	0.2493	0.3802
R_SHAN	-0.0356	0.1235	0.0506	0.1004	0.0775	**R_SHAN**	-0.0923	0.6470	0.2338	0.6212	-0.0960
R_STOX	-0.2225	-0.2907	-0.0082	-0.3973	-0.3673	**R_STOX**	-0.2035	-0.0561	0.0992	-0.3197	-0.2815

Correlation matrixes of cryptocurrencies with stock indices in four different settings, concerning the returns of cryptocurrencies and stock indexes over the entire period, and during the worst 10%, 5%, 1% market periods. The correlation parameters are verified against the following stock indices: R_SP50 (S&P 500), R_DIJA (Dow Jones Industrial Average), R_NIKK (Nikkei 225), R_SHAN (Shanghai Stock Exchange), R_STOX (Stoxx Europe 600).

After the computation of the correlation matrixes, the focus was shifted towards the first regressions performed on the return variables. Through a GARCH (1,1) model, parameters of both mean and variance equations were calculated and analyzed, but only the results of the mean equation are illustrated in *Table 5.2* for both the daily and weekly returns of the cryptocurrency assets and of the stock indices, due to space constraints. However, it was worth noting that in the variance equation, the coefficients associated with the ARCH and GARCH effects was approximately close to the value of one, fact that advocated an increased degree of persistence in the variance process.

Table 5.2. Estimation results GARCH (1, 1) model on the hedge and safe haven abilities of cryptocurrencies against stock indices (Equation 1)								
	Panel A: daily data				Panel B: weekly data			
	10% quantile (α_1)	5% quantile (α_2)	1% quantile (α_3)	Hedge (β_1)	10% quantile (α_1)	5% quantile (α_2)	1% quantile (α_3)	Hedge (β_1)
Bitcoin								
R_SP50	0.714	0.370	0.112	-0.211	0.991	2.103	-2.486	0.545
R_DIJA	0.273	-0.440	1.750***	-0.246	0.003	1.959	-2.032	-0.273
R_NIKK	-0.117	-0.219	0.244	0.215**	1.280	-0.895	0.150	-0.212
R_SHAN	0.657	-0.665	0.331*	0.030	1.767	0.261	-1.574	-0.477
R_STOX	0.670*	-0.198	0.298	-0.373***	2.733***	-1.994	-0.416	-0.403
Ethereum								
R_SP50	0.336	0.733	0.178	-0.233	-9.075*	8.172	1.912	-0.008
R_DIJA	0.338	-1.904	1.406	1.030***	-1.777	2.123	1.602	1.045
R_NIKK	0.266	-0.062	0.133	-0.182	1.624	0.432	-7.286	-0.613
R_SHAN	-0.019	-0.191	-0.309	0.469	-1.859	0.619	0.896	-1.528
R_STOX	-1.476	1.338	-0.382	0.749***	-2.317	-1.437	4.259	0.266
Ripple								
R_SP50	-0.903**	3.034***	-0.309	-1.660***	-3.590	3.552	0.671	-0.577
R_DIJA	3.159***	-7.896***	4.610***	0.785***	-0.356	1.791	-0.979	0.306
R_NIKK	1.429***	-2.683***	1.272**	0.198	4.411	-0.916	-2.029	-1.761
R_SHAN	0.420	-3.328***	2.787***	0.212	3.982	-0.957	0.396	-3.070
R_STOX	2.273***	-4.419***	1.511***	0.670***	4.388	-3.838	-0.145	-0.138
Litecoin								
R_SP50	-0.361	0.154	-0.033	0.432	-6.314***	7.469***	-1.758	0.485
R_DIJA	0.886	-1.334	0.206	0.060	-2.017	-1.385*	3.224	-2.601
R_NIKK	-0.805	0.480	0.002	0.190	0.252	-1.947	-0.391	2.143
R_SHAN	-3.910***	4.166***	-0.062	-0.202	-2.542*	1.694	-0.800	-1.048
R_STOX	0.434	-0.483	-0.598	-0.095	-1.577	-0.510***	1.929	1.581
Stellar								
R_SP50	0.060	0.836	-0.815	-0.307	-0.499	-1.489	2.857	-0.451
R_DIJA	0.225	-2.198	0.480	1.518***	-0.016	-1.029	0.789	0.729
R_NIKK	-0.022	0.511	0.579	-0.552	1.959	0.523	-2.803	-2.132***
R_SHAN	-1.743	0.396	0.498	0.562	-0.072	-0.940	1.103	0.144
R_STOX	-0.343	-0.495	-1.182	0.961**	1.741	-0.793	2.111	-2.634***

*Notes: *** P<0.01, **P<0.05, *P<0.10.*

The parameters estimated in Table 5.2 were extracted from Equation 1, where the time series of one of the cryptocurrencies' price was regressed on the price of one of the stock indices (β_1), and on three dummy variables (α_1, α_2, α_3), which represented extreme volatility in the selected stock indices in the worst 10^{th}, 5^{th} and 1^{th} quantiles of daily and weekly return distributions. In addition, the parameters presented in the table were distributed into two panels focused on either regressions of daily data (Panel A) or regressions of weekly data (Panel B).

Altogether, the results of the GARCH model applied on data in Panel A illustrated an important number of significant coefficients, most of which were concentrated in the regressions performed on Ripple. In terms of hedging properties, Bitcoin seemed to act as a hedge against movements in European stocks, represented by the R_STOX variable, as it presented a significant negative coefficient (-0.373). Moreover, Ripple presented a strong hedge ability against US stock through the R_SP50 variable, as it also had a significant negative coefficient (-1.660). Nonetheless, in terms of safe haven abilities the results were lacking any meaningful finding, as the situation of obtaining significant coefficients for all variables was relatively difficult (2 situations out of 25) and even then, at least one sum of the parameters was highly positive, fact which affected the probability of discovering safe haven properties.

On the other side of the table in Panel B, the weekly data captured less significant coefficients than the daily data. Hedging properties were found only on the side of Stellar against Japanese and European stocks, represented by the R_NIKK (-2.132), respectively the R_STOX (-2.634) variables. No regression model was close to capture any safe haven abilities, as most dummy coefficients were not significant.

As the paper analyzed constant correlation matrixes and GARCH model regressions in the pursuit of discovering meaningful significant implications, the last technique used in this section was computing a DCC model and simply regressing the estimated pairwise dynamic conditional correlation parameters on the return of stock indices and dummy variables in order to investigate dynamic movements in correlation between cryptocurrencies' and stock indices' return variables. The results of this technique are summarized in *Table 5.3*.

Table 5.3. Estimation results Dynamic Conditional Correlation (DCC) regression on the hedge and safe haven abilities of cryptocurrencies against stock indices (Equation 5)								
	Panel A: daily data				Panel B: weekly data			
	10% quantile (m_1)	5% quantile (m_2)	1% quantile (m_3)	Hedge (m_0)	10% quantile (m_1)	5% quantile (m_2)	1% quantile (m_3)	Hedge (m_0)
Bitcoin								
R_SP50	0.065	0.057	0.041	0.027	0.043	0.109	0.110	0.041
R_DIJA	-0.001	-0.001	0.024	-0.001	0.093**	0.041	-0.256	0.099
R_NIKK	0.022	0.010***	0.060	-0.068**	-0.114	0.009	0.323	-0.007
R_SHAN	-0.016	-0.023	-0.033	0.002	-0.020	0.068*	0.080	-0.068**
R_STOX	-0.071	-0.008	0.045	-0.001*	-0.006	0.017	0.025	0.229
Ethereum								
R_SP50	0.056	0.062	0.045	0.027	-0.027	-0.053	0.200	0.003
R_DIJA	0.011	0.001	0.037	0.054	0.084	0.033	0.236	0.126
R_NIKK	-0.044	-0.028	0.033	-0.037	-0.003	-0.058	0.003	0.042**
R_SHAN	-0.026	-0.042	-0.032	0.018	0.007	0.029	0.161	0.082
R_STOX	0.069	-0.035	0.025	0.064*	0.035	0.003	0.174	0.053
Ripple								
R_SP50	-0.087**	-0.074*	0.030	-0.109***	-0.122	-0.071	-0.046*	-0.022
R_DIJA	0.019	0.017	0.032	0.046	-0.012	0.079	0.301	0.054
R_NIKK	-0.011*	-0.099*	-0.006	-0.058***	-0.096**	0.048	-0.021	0.131
R_SHAN	-0.131***	-0.105***	-0.012	-0.067	0.090*	0.185	0.153*	0.073
R_STOX	-0.054	0.025*	0.049	-0.017**	-0.035	-0.019	-0.015	0.006
Litecoin								
R_SP50	0.040	0.036	0.023	0.037	-0.040	0.001	0.357	0.069
R_DIJA	0.008	-0.017	0.011	0.017	0.042	0.126	-0.036	0.129
R_NIKK	0.136	0.032*	-0.027	-0.026**	0.084	-0.040	-0.154	0.075
R_SHAN	-0.084**	-0.037	-0.042	-0.048	0.003	-0.023	-0.039	0.090
R_STOX	-0.034**	-0.054	-0.045	0.050	-0.124	0.144*	0.140	0.007
Stellar								
R_SP50	0.019	0.022	0.029	-0.002	0.034	0.054	0.008	-0.041
R_DIJA	-0.001	-0.015	0.005	0.047	0.075	0.080	0.081	0.120
R_NIKK	-0.070	0.050	0.029	0.005	-0.015	0.010	-0.208	-0.049
R_SHAN	-0.081*	-0.076*	-0.020	-0.054	0.013	0.007	0.067*	0.042
R_STOX	-0.003	-0.017	-0.001	0.069*	-0.031	-0.100	-0.275	0.064

*Notes. *** P<0.01, **P<0.05, *P<0.10.*

The coefficients calculated in Table 5.3 were resulted after regressing the pairwise dynamic conditional correlation parameters on the return of stock indices and dummy variables, with the constant parameter (m_0) indicating the hedge properties, and the coefficients attached to the dummy variables (m_1, m_2, m_3) indicating the movements of the dynamic correlation during the worst 10th, 5th, and 1th quantiles of daily and weekly return distributions, implicitly the safe haven properties. Similarly, to the last table the results were distributed in two sections focusing on daily data (Panel A) and weekly data (Panel B).

At a first glance at Panel A, hedging relationships were present in relatively numerous occasions. For the start, Bitcoin displayed hedging properties against movements in Japanese and European stock, through the negative significant coefficients associated with the R_NIKK (-0.068) and the

R_STOX (-0.001) variables. Litecoin also joined the scene as a hedge for Japanese stocks, through the R_NIKK (-0.026) variable. On a larger scale, Ripple exhibited hedging abilities against a wide array of stocks, including US, Japanese and European stocks, represented by the R_SP50 (-0.109), R_NIKK (-0.058), respectively R_STOX (-0.017) variables. However, significant parameters of the dummy variables were present in few numbers, suggesting that the safe haven abilities of cryptocurrencies were limited.

In respect to Panel B, the decrease in significant parameters was similar to the phenomenon experienced in the previous table, as few parameters were useful for the analysis. While no safe haven effect could have been identified, Bitcoin was the only cryptocurrency which presented hedging abilities, in this case against Chinese stocks, represented by the R_SHAN (-0.068) variable.

B. Portfolio optimization

For the initial phase of portfolio optimization section, the mean, the standard deviation and the Sharpe ratio of each daily return variable was calculated. As mentioned in the methodology section, the focus of this paper is on two cases, in which a typical investor does not adjusts his/her portfolio during the observed time interval (static case), or the investor adjusts the asset allocation inside the portfolio quarterly (dynamic). The relevant information such as mean, standard deviation, and Sharpe ratio of the stock indices' and cryptocurrencies' daily returns are summarized in *Table 5.4* (see *Appendix*) for the static case, and in *Table 5.5* (see *Appendix*) for the dynamic case.

Succeeding the computation of means and standard deviations, the stock index portfolio (IP) and the mixed stock index and cryptocurrency portfolio (MP) were constructed in order to compare the efficiency of adding the selected cryptocurrencies to the initial portfolio composed only of stock indices. Through Solver, the computational instrument offered in Excel, the optimal Sharpe ratio of both portfolios was calculated, by adjusting the weights of the portfolios' assets. Of great relevance, *Table 5.6* for the static case and *Table 5.7* for the dynamic case advocate an important contribution of including cryptocurrencies into the portfolio, as they positively influence the optimal Sharpe ratio of the portfolio.

Table 5.6. Comparison between stock Index only Portfolio (IP) and Mixed Portfolio (MP) in terms of asset weight allocation over the entire observed period		
Assets	**IP weights**	**MP weights**
Cryptocurrencies		
R_BITC	0.00%	5.36%
R_ETHE	0.00%	9.24%
R_RIPP	0.00%	2.40%
R_LITE	0.00%	1.57%
R_STEL	0.00%	2.64%
Total weight	**0.00%**	**21.21%**
Stock indices		
R_SP50	40.44%	37.71%
R_DIJA	57.05%	37.42%
R_NIKK	2.51%	3.66%
R_SHAN	0.00%	0.00%
R_STOX	0.00%	0.00%
Total weight	**100%**	**78.79%**
Return	0.04%	0.21%
St. dev.	0.50%	1.11%
Sharpe	**0.07**	**0.19**

Notes: comparison of IP and MP in terms of asset weights of stock indices or cryptocurrencies included in the portfolios and the daily return, standard deviation, and the Sharpe ratio of the respective portfolios in the static case.

Judging after *Table 5.6*, the mixed portfolio clearly outperformed the index portfolio with an impressive increase of 171.42% in the optimal Sharpe ratio of the portfolio. In addition, an important highlight is the substantial 21.21% weight contribution of cryptocurrencies in the mixed portfolio, with Bitcoin and Ethereum, the two most popular cryptocurrencies, as important contributors. On the side of stock indices, US stocks lost ground, as both S&P 500 and DIJA presented a reduced contribution in the mixed portfolio, while Japanese stock represented by Nikkei 225 actually had a higher presence.

Table 5.7. Comparison between Index only Portfolio (IP) and Mixed Portfolio (MP) in terms of asset weight allocation over each of the first eight quarters analyzed

Assets	Quarter 1 (Jan 16 – Mar 16)		Quarter 2 (Apr 16 – Jun 16)		Quarter 3 (Jul 16 – Sep 16)		Quarter 4 (Oct 16 – Dec 16)	
	IP	**MP**	**IP**	**MP**	**IP**	**MP**	**IP**	**MP**
Cryptocurrencies								
R_BITC	0.00%	0.00%	0.00%	42.45%	0.00%	0.00%	0.00%	20.65%
R_ETHE	0.00%	24.89%	0.00%	5.13%	0.00%	1.38%	0.00%	0.77%
R_RIPP	0.00%	9.08%	0.00%	0.00%	0.00%	3.91%	0.00%	0.00%
R_LITE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.27%
R_STEL	0.00%	12.02%	0.00%	2.47%	0.00%	2.21%	0.00%	0.50%
Total weight	**0.00%**	**45.99%**	**0.00%**	**50.05%**	**0.00%**	**7.50%**	**0.00%**	**23.19%**
Stock indices								
R_SP50	0.00%	0.00%	29.74%	0.00%	47.15%	58.37%	0.00%	0.00%
R_DIJA	100%	54.02%	70.26%	36.05%	22.51%	7.35%	68.51%	63.29%
R_NIKK	0.00%	0.00%	0.00%	0.00%	9.20%	9.85%	10.51%	8.65%
R_SHAN	0.00%	0.00%	0.00%	0.00%	11.94%	12.92%	20.98%	4.87%
R_STOX	0.00%	0.00%	0.00%	13.89%	9.19%	4.02%	0.00%	0.00%
Total weight	**100%**	**54.01%**	**100%**	**49.95%**	**100%**	**92.50%**	**100%**	**76.81%**
Return	0.02	0.88	0.01	0.27	0.04	0.07	0.08	0.17
St. dev.	0.92	2.38	0.50	1.40	0.32	0.47	0.37	0.47
Sharpe	**0.02**	**0.37**	**0.03**	**0.19**	**0.14**	**0.18**	**0.22**	**0.37**
		(+1750%)		(+533%)		(+28%)		(+68%)

Assets	Quarter 5 (Jan 17 – Mar 17)		Quarter 6 (Apr 17 – Jun 17)		Quarter 7 (Jul 17 – Sep 17)		Quarter 8 (Oct 17– Dec 17)	
	IP	**MP**	**IP**	**MP**	**IP**	**MP**	**IP**	**MP**
Cryptocurrencies								
R_BITC	0.00%	0.42%	0.00%	4.83%	0.00%	2.42%	0.00%	1.42%
R_ETHE	0.00%	3.32%	0.00%	6.21%	0.00%	0.00%	0.00%	1.93%
R_RIPP	0.00%	2.43%	0.00%	1.37%	0.00%	0.00%	0.00%	0.46%
R_LITE	0.00%	0.00%	0.00%	2.54%	0.00%	0.08%	0.00%	0.00%
R_STEL	0.00%	0.00%	0.00%	0.33%	0.00%	0.00%	0.00%	0.69%
Total weight	**0.00%**	**6.17%**	**0.00%**	**15.28%**	**0.00%**	**2.50%**	**0.00%**	**4.50%**
Stock indices								
R_SP50	46.53%	50.11%	31.90%	23.07%	29.63%	32.81%	0.00%	46.04%
R_DIJA	18.22%	33.44%	41.43%	32.50%	52.68%	50.03%	100%	40.79%
R_NIKK	0.00%	0.43%	26.67%	29.14%	0.00%	0.00%	0.00%	8.66%
R_SHAN	16.49%	9.85%	0.00%	0.00%	17.69%	14.66%	0.00%	0.00%
R_STOX	18.76%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Total weight	**100%**	**93.83%**	**100%**	**84.72%**	**100%**	**97.50%**	**100%**	**95.50%**
Return	0.06	0.17	0.04	0.36	0.05	0.07	0.11	0.18
St. dev.	0.23	0.38	0.24	0.78	0.23	0.26	0.34	0.32
Sharpe	**0.24**	**0.44**	**0.17**	**0.46**	**0.21**	**0.25**	**0.32**	**0.56**
		(+83%)		(+170%)		(+19%)		(+75%)

Notes: comparison of IP and MP in terms of asset weights in stock indices or cryptocurrencies included in the portfolios and the daily return, standard deviation, and the Sharpe ratio of the respective portfolios in the dynamic case over the first eight quarters. The parameters in the brackets represent the improvement in the optimal Sharpe ratio of a portfolio after including cryptocurrencies in a portfolio of stock indices.

In respect to *Table 5.7*, the mixed portfolio categorically outperforms the index portfolio in the dynamic case over every quarter analyzed, except for Quarter 9, the last one included in the observed time interval. As observed in *Table 5.5* (see *Appendix*), during Quarter 9 all variables had a negative mean return, therefore no optimal portfolio could had been constructed in this case. For this reason, Quarter 9 was dropped from the portfolio comparison analysis. As of a metric overview of the two portfolio's performance, the index portfolio had a quarterly average Sharpe ratio of 0.17, while the mixed portfolio presented a superior quarterly average Sharpe ratio of 0.35, with an average cryptocurrency asset weight of 19.40%, distributed on average as 9.02% in Bitcoin, 5.45% in Ethereum, 2.16% in Ripple, 0.49% in Litecoin, and 2.28% in Stellar. Therefore, if in the static case, the optimal Sharpe ratio of the index portfolio was improved by 171% after adding cryptocurrencies, in the dynamic case the improvement was worth an average 105%, while the average cryptocurrency weight was relatively close at around 20% of the portfolio's assets.

Table 5.8. Cryptocurrency weights included in the mixed portfolio over the first eight quarters in the dynamic case

Time interval	R_BITC	R_ETHE	R_RIPP	R_LITE	R_STEL
Quarter 1	0.00%	24.89%	9.08%	0.00%	12.02%
Quarter 2	42.45%	5.13%	0.00%	0.00%	2.47%
Quarter 3	0.00%	1.38%	3.91%	0.00%	2.21%
Quarter 4	20.65%	0.77%	0.00%	1.27%	0.50%
Quarter 5	0.42%	3.32%	2.43%	0.00%	0.00%
Quarter 6	4.83%	6.21%	1.37%	2.54%	0.33%
Quarter 7	2.42%	0.00%	0.00%	0.08%	0.00%
Quarter 8	1.42%	1.93%	0.46%	0.00%	0.69%
Average	**9.02%**	**5.45%**	**2.16%**	**0.49%**	**2.28%**
St. dev.	15.16%	8.14%	3.13%	0.94%	4.05%
Minimum	0.00%	0.00%	0.00%	0.00%	0.00%
Maximum	42.45%	24.89%	9.08%	2.54%	12.02%

Additional information about the average, standard deviation, minimum and maximum of the weights is provided.

For an improved overview of the comparison between portfolios, *Table 5.8* exhibits the weight contribution of the cryptocurrency assets over the eight quarters. As of interesting implications emphasized in the figure above, Bitcoin had the highest variance, ranging from a maximum weight of 42.45% in Quarter 2 to a minimum of 0% in Quarters 1 and 3. Ethereum could be considered the "champion" of consistency, as it has positive weight in seven out of eight quarters, the only exception being represented by Quarter 7, when the allocation of cryptocurrencies in total was also the lowest among the eight quarters. Besides these facts, Ripple and Stellar had more modest contributions, most significant ones being isolated in Quarter 1, when Ripple and Stellar had weights of 9.08%, respectively 12.02%. If there would have been a "loser" in this equation, then Litecoin could had occupy that spot without difficulty, as the cryptocurrency had a positive weight in only three quarters, the most limited quarterly representation among cryptocurrencies, and a meager maximum weight of just 2.49% in Quarter 6.

6. Discussion

In the previous sections, the performed research analyzed data supporting the research question of the hedge and safe haven effects of cryptocurrencies on specific stock indices. It examined a total of three hypotheses, which presupposed beneficial effects of allocating cryptocurrencies into investment portfolios including global stock indices.

For the first section focused on regressions, both GARCH model regressions and pairwise dynamic conditional correlations were utilized to validate the first two hypotheses, focused on the existence of hedging properties (H1) and on the existence of safe haven abilities (H2). For a more comprehensive overview over the relationships between cryptocurrencies and stock indices, two different panels of data focused on either daily or weekly returns of assets to observe whether the frequency of returns is relevant in assessing the validity of the two hypotheses.

The results of the GARCH regression model with daily data suggested that Bitcoin and Ripple had significant hedging abilities against volatility in US stocks (for both cryptocurrencies) and in European stocks (Ripple). Yet, the weekly data GARCH regression models generated totally different results in the aspect of hedging, as only Stellar had been found to display a significant ability against Japanese and European Stocks. Though, both daily and weekly data presented no significant safe haven ability of any cryptocurrency against stock indices.

In the second part of the regression section, the use of pairwise dynamic conditional correlation parameters proved to be more insightful, as several more hedging properties have been discovered. In respect to daily data, Bitcoin, Ripple, and Litecoin had implied hedging abilities against Japanese stocks (all three), European stocks (Bitcoin and Ripple), respectively US stocks (Ripple). The results from the weekly data were less significant, as Bitcoin had been the only cryptocurrency to possess hedging properties, in this case against Chinese stocks. However, the safe haven related estimates were identically disappointing with the GARCH approach, as no cryptocurrency had any significant relationship in either daily or weekly return data.

As observed in both the GARCH and DCC approaches, the results of regression models based on daily data were significantly different from those based on weekly data. This fact highlights that the aspect of frequency is extremely relevant to investors, at least in the cryptocurrency market, as the hedging and safe haven abilities of the cryptocurrencies analyzed in this paper differed across time horizons. For example, in the GARCH section, while Bitcoin and Ripple have lost their hedging properties when the data was shifted from daily to weekly, Stellar actually presented new hedging abilities. Similarly, in the DCC section, Bitcoin, Ripple, and Litecoin have all lost most of their daily hedging ability after the move to weekly frequency of returns. Consequently, investors appeared to react differently to daily and weekly fluctuations in the relevant markets. This circumstance has had been previously explained by Ciaian et al. (2016), which in the case of Bitcoin they have advocated that hedge and safe haven properties at different time horizons are significantly affected by extremely distinctive factors.

In reference to the implications resulted above, an additional explanation must be offered related to how the existence of hedge properties does not naturally imply the existence of safe haven properties. Theoretically, it is conceivable that cryptocurrencies could be negatively correlated with stock indices on average (hedging abilities), but positively correlated with stock indices in extreme market conditions. A justification for this possibility could be associated with cross-asset contagion, which implies that either cryptocurrencies or stock indices “infect” the price evolution of the other asset. Furthermore, it is also feasible that cryptocurrencies do not lose value in extreme stock market conditions (safe haven abilities) but co-move with stock indices on average (absence of hedging abilities).

For the portfolio optimization part, in both static and dynamic cases, comprehensive analyses composed of return matrixes, excess return matrixes, variance-covariance matrixes and mean-variance optimization had been performed in order to evaluate the development of a portfolio’s optimal Sharpe ratio attributed to the inclusion of cryptocurrencies. While regression analyses were used to validate the first two hypotheses, the portfolio optimization section’s final purpose was to investigate the third hypothesis (H3), regarding the practical benefits of the cryptocurrencies’ hedging properties.

While in the static case, a straight-forward inclusion of 21.21% cryptocurrency weight in the portfolio was needed to generate an optimal Sharpe ratio improvement of 171%, the dynamic case offered invaluable implications through the quarterly consistency of the cryptocurrencies' addition in portfolios. Except for Quarter 9, where investors would have preferred to invest rather in cash than in any other asset class, cryptocurrencies improved the optimal Sharpe ratio of portfolios in all observed quarters, with weights of cryptocurrencies included in the mixed portfolio ranging from 2.5% in Quarter 7 to an impressive 50.05% in Quarter 2.

In addition, an interesting contrast can be observed in the dynamic case between the downward trend of cryptocurrency weight included in the portfolio, and the upward trend of improvement in the Sharpe ratio of the mixed portfolio relative to the index portfolio (*Figure 6.1*). While in the first two quarters, cryptocurrency allocation is close to 50% and the Sharpe ratio improvement is equal with 1750%, respectively 533% per quarter, the allocation takes a drastic negative movement towards an average of 9.86% cryptocurrency weight for the rest of six quarters. However, this reduced level of cryptocurrency allocation had still generated a mixed portfolio that outperformed the index portfolio by a quarterly average of 73% in terms of Sharpe ratio in the last six quarters.

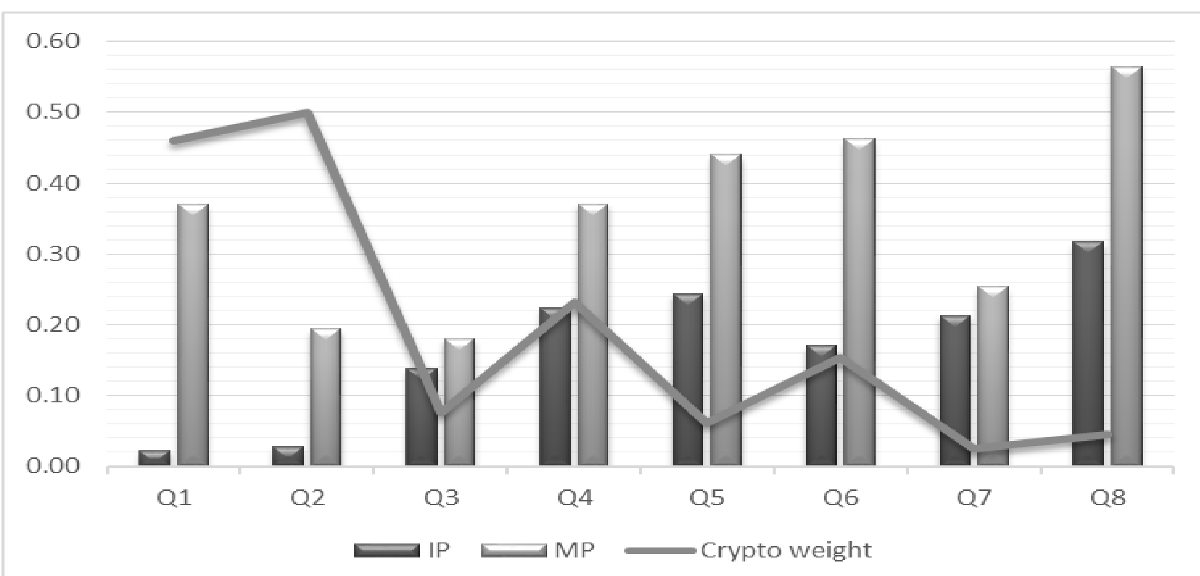

Figure 6.1. Comparison between the optimal Sharpe ratios of the Index Portfolio (IP) and Mixed Portfolio (MP), considering the allocated portfolio weight in cryptocurrencies over the eight studied quarters.

Another aspect worth more attention was the similarity in the generated results of the two sections analyzed in this paper. Both research methods (regressions and mean-variance optimization) advocated extremely positive results in the case of Bitcoin, mixed outcomes for Ethereum and Ripple, and mostly negative conclusions for Litecoin and Stellar. In a more comprehensive overview, Bitcoin had both a significant weight allocation in the portfolio optimization part and significant hedging properties in the statistical part. While Ripple presented important results in regressions but weak inferences in optimal portfolios, the situation was reversed for Ethereum. Lastly, for Litecoin and Stellar, the results were mostly sluggish in both the regression and portfolio sections.

7. Conclusions

The focus of this paper has been concentrated around the statistical relationship between the daily and weekly returns of specific cryptocurrencies and stock indices by including a more econometric approach through GARCH and DCC models, and on the optimizational consequences of including cryptocurrencies into an optimal investment portfolio containing stock indices. This study aimed to bring more clarity on the relatively unknown financial nature of cryptocurrencies and expose to investors and to the scientific community the still hidden properties of these virtual coins, which have the potential power to revolutionize the financial markets. In order to coherently outline the valuable insights generated during the research phase, this paper has aimed to answer the following central question:

Can cryptocurrencies be considered hedges and/or safe havens against movements in stock indices?

The first hypothesis stated that cryptocurrency assets do not represent a strong (weak) hedge against movements in stock indices. In both the GARCH and the DCC approaches, results have suggested hedging properties for several of the studied cryptocurrencies. While Bitcoin, Ripple, and Litecoin exhibited numerous significant effects in the daily data, Ripple and Litecoin had to cede their place for Stellar in the weekly data. Therefore, the hypothesis clearly can be rejected in the case of Bitcoin, as its effect is persistent in both daily and weekly frequencies (at least in the GARCH approach). For Ripple, Litecoin, and Stellar results are mixed, thus there is not enough consistency in results to reject the hypothesis, while for Ethereum absolutely no significant result has been discovered to reject the hypothesis.

The second hypothesis assumed that cryptocurrency assets do not represent a strong (weak) safe haven against movements in stock indices. Although in the DCC approach no cryptocurrency has been even close to exhibit safe haven properties in the daily or weekly analyses, Ripple presented the nearest significant parameters for the analysis in the daily data. Considering this, the hypothesis certainly cannot be rejected for all the studied cryptocurrencies, except for Ripple, whose results might partially reduce the certainty of the hypothesis.

Lastly, the third hypothesis presupposed that cryptocurrencies do not represent a practical hedge in terms of mean-variance optimization of investment portfolios. Fortunately, the portfolio optimization part of this paper has generated numerous valuable insights into the quarterly behavior of optimal portfolios. Bitcoin and Ethereum presented impressive performances in terms of weight of cryptocurrencies allocated in the optimal mixed portfolio, while Ripple had mixed effectiveness, and Litecoin and Stellar failed to have a substantial contribution to the portfolio. Taking all of these in consideration, the third hypothesis is rejected in the case of Bitcoin and Ethereum, but it cannot be rejected for Ripple, Litecoin and Stellar.

Altogether, the paper could not expose compelling statistical relationships between cryptocurrencies and stock indices in terms of safe haven properties, but it achieved several insights regarding cryptocurrencies' hedging properties and overwhelming understandings of cryptocurrencies' behavior inside optimal stock index portfolios. The most important and popular virtual coin, Bitcoin, and in less regard, Ethereum and Ripple, presented significant benefits for the average investor which seeks to diversify portfolios more efficiently, considering the actual assets interdependence environment. In this sense, pension funds could gradually allocate over a long-term horizon a minimal weight of their portfolios into investments in the three mentioned cryptocurrencies. In addition, the scientific community can utilize the generated information to explore more in-depth the quarterly behavior of portfolios containing cryptocurrencies. As the current research has proved, academicians cannot ignore the absolute superiority of mixed optimal portfolios against the simple optimal stock index portfolios.

In terms of limitations that might have had affected the results of this paper, an important notice must be given to the limited number of daily and weekly observations used in the research methodology. As numerous cryptocurrencies have gained popularity or just have began being traded two or three years ago, the available number of relevant daily and weekly returns will remain relatively low in the next 5-10 years. In addition, a substantial proportion of scientific literature studying financial hedges and safe havens is focused on gold or on VIX, therefore the methodology of this paper could be substantially improved in the future, by more efficiently adapting the statistical techniques used in previous papers for the unique nature of cryptocurrencies.

For further research, regression models should be adapted to focus more on sub-samples of data to observe seasonal effects or how prices fluctuated around important financial news. For the portfolio optimization side, an integration of the cryptocurrency return variables into the CAPM model would be essential for a larger perspective on their role in an optimal portfolio, in case the assumptions of the model could be realistically met. In addition, a broader range of performance measures could be implemented besides the Sharpe ratio to improve the validity of the results.

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9. Appendix

Figure 4.2. Distribution of daily data points during the observed time interval of each return variable.

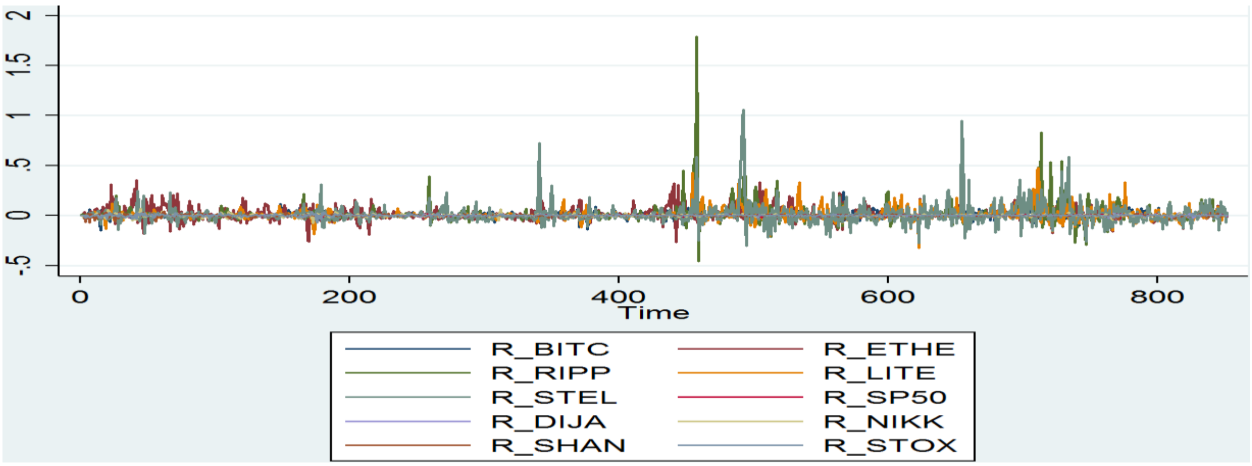

Figure 4.3. Distribution of weekly data points during the observed time interval of each return variable.

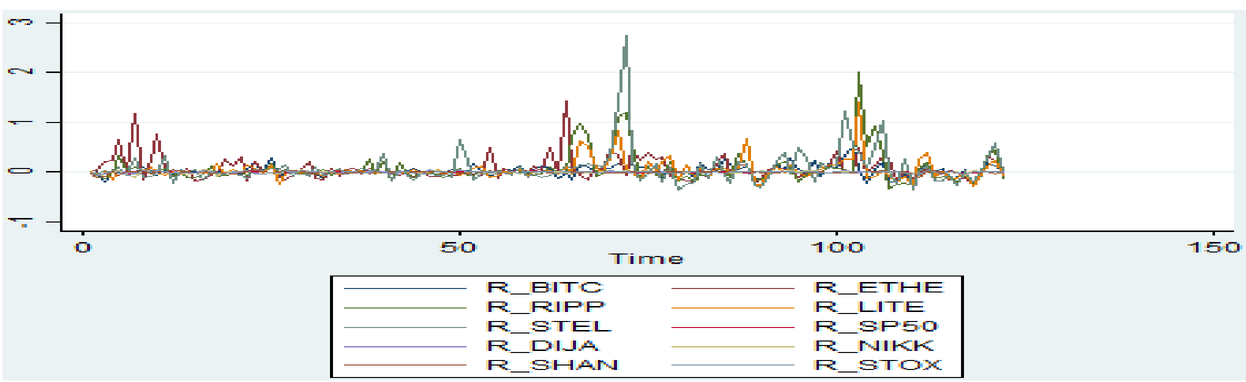

Table 4.4. Results of Dickey-Fuller test on the stock indices' and cryptocurrencies' daily and weekly return variables.

Variable	Test statistic	1% Critical value	P-value
Panel A: Daily data			
Cryptocurrencies			
R_BITC	-28.894	-3.430	0.001
R_ETHE	-28.058	-3.430	0.001
R_RIPP	-29.426	-3.430	0.001
R_LITE	-26.633	-3.430	0.001
R_STEL	-24.395	-3.430	0.001
Stock indices			
R_SP50	-31.854	-3.430	0.001
R_DIJA	-31.675	-3.430	0.001
R_NIKK	-29.769	-3.430	0.001
R_SHAN	-31.926	-3.430	0.001
R_STOX	-29.829	-3.430	0.001
Panel B: Weekly data			
Cryptocurrencies			
R_BITC	-9.634	-3.503	0.001
R_ETHE	-10.293	-3.503	0.001
R_RIPP	-6.881	-3.503	0.001
R_LITE	-9.605	-3.503	0.001
R_STEL	-7.435	-3.503	0.001
Stock indices			
R_SP50	-11.279	-3.503	0.001
R_DIJA	-11.350	-3.503	0.001
R_NIKK	-11.561	-3.503	0.001
R_SHAN	-10.450	-3.503	0.001
R_STOX	-10.933	-3.503	0.001

Table 5.4. Daily mean, standard deviation and Sharpe ratio of the stock indices' and cryptocurrencies daily returns over the entire observed period.

Variable	Mean	St. dev.	Sharpe ratio
Cryptocurrencies			
R_BITC	0.45%	4.25%	0.11
R_ETHE	1.01%	7.04%	0.14
R_RIPP	0.98%	10.30%	0.09
R_LITE	0.62%	6.25%	0.10
R_STEL	1.14%	10.83%	0.10
Stock indices			
R_SP50	0.03%	0.63%	0.05
R_DIJA	0.04%	0.63%	0.06
R_NIKK	0.03%	1.08%	0.02
R_SHAN	-0.01%	0.89%	-0.01
R_STOX	0.01%	0.78%	0.01

Table 5.5. Daily mean, standard deviation and Sharpe ratio of the stock indices' and cryptocurrencies' daily returns for each quarter over the observed period.

Variables	Measures	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
R_BITC	**Mean**	-0.01%	0.58%	-0.09%	0.51%	0.21%	0.99%	0.78%	1.46%	-0.60%
	St. Dev.	2.82%	3.17%	1.99%	1.80%	4.25%	3.55%	5.69%	6.03%	6.00%
	Sharpe	**-0.01**	**0.18**	**-0.04**	**0.29**	**0.05**	**0.28**	**0.14**	**0.24**	**-0.10**
R_ETHE	**Mean**	3.21%	0.36%	0.20%	-0.44%	2.32%	2.25%	0.30%	1.16%	-0.49%
	St. Dev.	9.71%	7.16%	5.24%	3.68%	8.00%	7.82%	7.52%	5.54%	6.66%
	Sharpe	**0.33**	**0.05**	**0.04**	**-0.12**	**0.29**	**0.29**	**0.04**	**0.21**	**-0.07**
R_RIPP	**Mean**	0.29%	-0.06%	0.41%	-0.30%	1.63%	4.42%	-0.03%	3.42%	-1.25%
	St. Dev.	3.45%	3.46%	4.90%	2.83%	8.92%	22.41%	6.83%	13.99%	8.88%
	Sharpe	**0.09**	**-0.02**	**0.08**	**-0.11**	**0.18**	**0.20**	**-0.01**	**0.24**	**-0.14**
R_LITE	**Mean**	-0.03%	0.33%	-0.07%	0.18%	0.60%	2.36%	0.63%	1.98%	-0.47%
	St. Dev.	2.62%	3.61%	2.18%	2.40%	5.43%	8.99%	7.77%	9.41%	7.83%
	Sharpe	**-0.01**	**0.09**	**-0.03**	**0.07**	**0.11**	**0.26**	**0.08**	**0.21**	**-0.06**
R_STEL	**Mean**	0.35%	0.17%	0.44%	0.18%	0.07%	4.16%	-0.30%	4.70%	-0.07%
	St. Dev.	6.32%	4.96%	5.26%	2.40%	5.04%	19.77%	9.14%	16.00%	11.02%
	Sharpe	**0.06**	**0.04**	**0.08**	**0.07**	**0.01**	**0.21**	**-0.03**	**0.29**	**-0.01**
R_SP50	**Mean**	0.02%	0.01%	0.05%	0.03%	0.06%	0.03%	0.04%	0.07%	-0.01%
	St. Dev.	0.95%	0.71%	0.52%	0.47%	0.35%	0.36%	0.39%	0.29%	1.03%
	Sharpe	**0.02**	**0.01**	**0.10**	**0.07**	**0.17**	**0.08**	**-0.10**	**0.26**	**-0.01**
R_DIJA	**Mean**	0.02%	0.02%	0.02%	0.08%	0.05%	0.04%	0.05%	0.11%	-0.02%
	St. Dev.	0.92%	0.69%	0.50%	0.43%	0.35%	0.38%	0.33%	0.33%	1.11%
	Sharpe	**0.02**	**0.03**	**0.05**	**0.20**	**0.14**	**0.10**	**0.16**	**0.33**	**-0.02**
R_NIKK	**Mean**	-0.16%	-0.02%	0.07%	0.16%	-0.01%	0.07%	0.02%	0.12%	-0.06%
	St. Dev.	1.84%	1.54%	0.98%	1.06%	0.78%	0.57%	0.49%	0.61%	1.19%
	Sharpe	**-0.09**	**-0.01**	**0.07**	**0.15**	**-0.01**	**0.11**	**0.04**	**0.20**	**-0.05**
R_SHAN	**Mean**	-0.16%	-0.02%	0.03%	0.04%	0.04%	-0.01%	0.05%	-0.01%	-0.04%
	St. Dev.	2.03%	0.90%	0.64%	0.56%	0.41%	0.47%	0.44%	0.47%	0.92%
	Sharpe	**-0.08**	**-0.03**	**0.04**	**0.07**	**0.10**	**-0.02**	**0.12**	**-0.03**	**-0.05**
R_STOX	**Mean**	-0.08%	-0.02%	0.04%	0.06%	0.06%	0.00%	0.03%	0.00%	-0.05%
	St. Dev.	1.34%	1.33%	0.69%	0.54%	0.43%	0.44%	0.49%	0.39%	0.72%
	Sharpe	**-0.06**	**-0.01**	**0.06**	**0.11**	**0.14**	**-0.01**	**0.05**	**0.01**	**-0.07**