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Forecasting spare parts demand for aperiodic inspection intervals

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Abstract

Advanced technical systems are complex machines that consist of several components. For these systems, it is important to forecasts defects and failure, which generate the demand for spare parts. Recently, Wang and Syntetos (2011) introduced a maintenance-based model to forecast the intermittent and lumpy demand for spare parts. In this paper, we tested how well this model performs with aperiodic inspection intervals. We introduce a simple policy to create those aperiodic intervals. It turns out that the maintenance-based model with aperiodic intervals performs as well as with periodic intervals. In most of the cases it outperforms the time series-based benchmark.

Contents

1 Introduction

Advanced technical systems are complex machines that consist of several components. This requires that companies have a good maintenance strategy. Maintenance strategies can be broadly classified into Corrective Maintenance (CM) and Preventive Maintenance (PM) strategies (Duffuaa et al., 2001). Corrective maintenance is a strategy that only let maintenance take place after a component has failed. This results in a high downtime of the machine and large costs for the company (Tsang, 1995). On the other hand, we have preventive maintenance, which is a strategy that uses inspections to check if there are any defect components. Usher et al. (1998) show that preventive maintenance results in a lower downtime and less costs compared to corrective maintenance. In this paper, we focus on the use of block-based inspections for the preventive maintenance part. During such an inspection, all the items are inspected regardless their age and condition and are replaced if needed.

In case of intermittent and lumpy demand, e.g. in the aircraft industry (Ghobbar and Friend, 2002), it is hard to have a good preventive maintenance strategy because it is very difficult to forecast the intermittent and lumpy demand for spare parts. This is one of the reasons why there is an increasing demand for research in this area. Wang and Syntetos (2011) introduce a new way of solving this problem by looking at the sources of the demand. The authors created a maintenance driven forecast model which outperforms their benchmark forecast model under certain assumptions and conditions. One of these assumptions is stationary intervals.

There are not many models who are able to forecast spare part demand for non-stationary intervals. That is reason why we focus in this research on forecasting spare parts demand for aperiodic intervals. We use the maintenance driven model introduced by Wang and Syntetos (2011) as our base model and the time series-based model from Syntetos and Boylan (2005) as the benchmark model. With these models we create one period ahead forecasts. The research questions of this paper are formulated as followed:

- How well does the base model perform with aperiodic intervals in comparison with periodic intervals in terms of prediction the spare part demand?
- What is the effect of aperiodic intervals in combination with different distribution parameters on the predictions?

In order to answer the first research question, we will replicate the work of Wang and Syntetos (2011) by using the base model and the benchmark model for the periodic intervals. Next, we will extent their work by describing and using a policy to create aperiodic intervals and looking at the forecasts models for the aperiodic intervals. Finally, we will compare the obtained forecasts and try to answer the research question. For the second research question, we take a look at different scale and shape parameters for the distribution that is used to model the arrival of a defect.

In the next section we explain the delay time concept and review the literature that is relevant to our research. This is followed by the description of the used data in Section 3. Next, Section 4 explains the implementation of the models. The results are described in Section 5 and in Section 6 the overall conclusions of this paper are given and we will also mention possible future research possibilities.

2 Research Background

2.1 Delay time concept

The delay time concept has been introduced and discussed by many authors, e.g. Christer and Waller (1984a), Christer and Waller (1984b), and Christer (1987). This concept defines two stages in the failure process. The defect will arise in the first stage and in the second stage, the defect will result in a failure. In Fig. 1, a visual representation is given to make this delay time concept more clear. We denote (0,u) as the initial time (first stage) and (u,s) as the delay time (second stage).

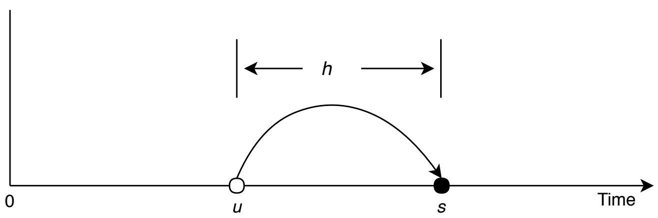

Figure 1: Illustration of the delay time concept, in which u is the time of defect, h is the delay time, and s is time of failure. ∘ defect arrives, • failure of the item.

Fig. 2 shows the combination of the delayed time concept and the block-based inspections. We define the interval ((k-1)t,kt] as inspection interval k. One can see that in inspection interval 1, there is a failure that needs corrective maintenance and a defect that needs preventive maintenance. The failure will be self-announcing and the defect will be detected during the inspection.

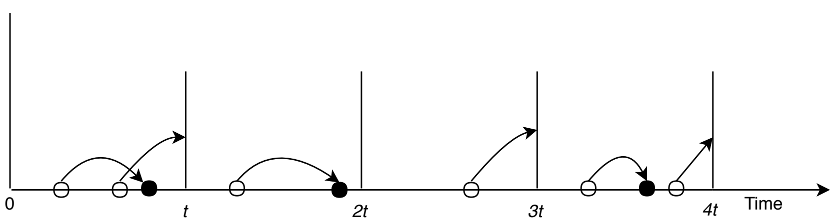

Figure 2: Illustration of the delay time concept in combination with the block-based inspection, in which t is the length of an inspection interval. ∘ defect arrives, • failure of the item.

2.2 Literature review

Wang and Syntetos (2011) introduced a delay time-based model to forecast the next time-period demand. The authors simulated the data by using the two-stage failure process described above. They compared their model with a time series-based model, the benchmark model, and concluded that their maintenance driven model outperforms the benchmark model under certain conditions. For the time series-based model they use the Syntetos-Boylan Approximation (SBA), which is introduced by Syntetos and Boylan (2005). The authors show in their paper that SBA outperforms other time series-based models that are used to forecast intermittent demand. This has also been shown in Eaves and Kingsman (2004) and Gutierrez et al. (2008). SBA has proven to be a good benchmark, which we can use to answer our research question. In Section 4, we will discuss the maintenance-based model and the SBA in more detail.

Taghipour and Banjevic (2011) and Wang (2009) introduced an inspection optimization model to optimize the inspection intervals to reduce the total costs of having inspections and failures. Both showed with an numerical example that their inspection optimization model cost reducing is. This gives us a reason to use aperiodic inspection intervals.

3 Data

In order to get data that is suitable to answer our research questions, we make use of a simulation study. Before we take a closer look at our simulation, we take a look at some assumptions that we have to make on forehand. The assumptions that have been made for the block-based inspections are the following:

- We start with an inspection and replace all the items to ensure that they are all unused at the start of the simulation.
- During an inspection all defect items will be found, if there are any.
- If a defect item is found during inspection, it will be replaced immediately.
- If a failure occurs, the item will be replaced immediately.
- There are always enough items in stock to perform a replacement.
- An inspection interval will start right after an inspection and include the next inspection.
- There will be no defects or failures during the inspection.
- An item that is replaced during the inspection will not be defect during that same inspection.

In Fig. 3, the scheme for the simulation is given. We distinguish two cases: a failure occurs between two inspections and a defect is found during an inspection. Table 1 contains all the variables that are used in the simulation. We assume that the delay-time (h) and the initial time (u) follow an exponential distribution and a Weibull distribution, respectively (Baker and

Wang, 1991). Furthermore, we make use of an integer function (Int), which returns the largest integer less or equal to the argument. In case of the periodic intervals $tnew$ is equal to 14, 28 or 42 days and in case of aperiodic intervals $tnew$ will be defined by our policy, which we will introduce in the next section. We aggregated the total demand within each inspection interval to get the demand per inspection interval.

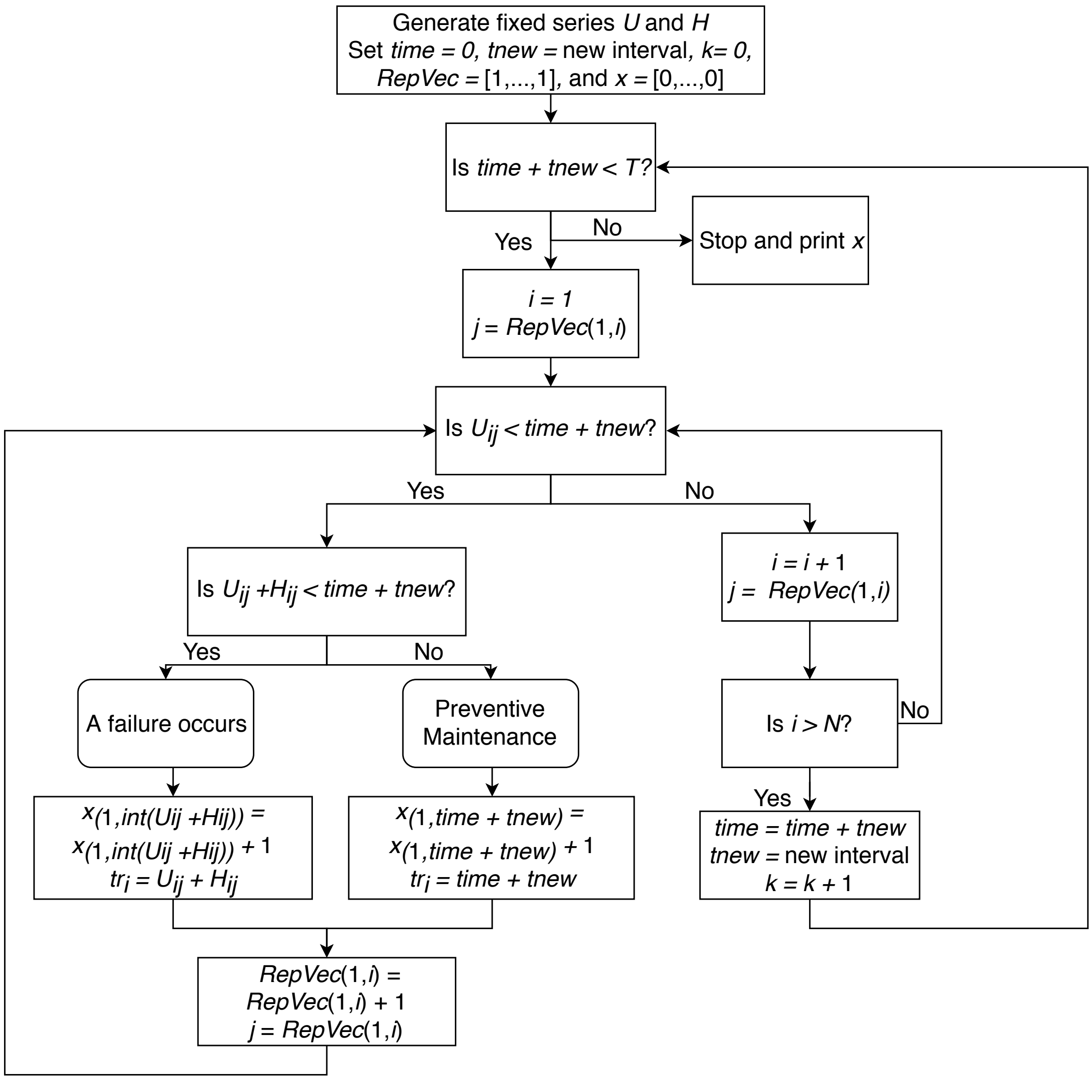

Figure 3: Simulation scheme for the block-based inspections

Table 1: Description of all variables used in the simulation

Variable	Definition
x_k	Number of replacements during the kth day
T	Simulation length in days
k	Inspection interval counter, starts with $k = 1$
$tnew$	Size of the new inspection interval in days
$time$	Days counter
tr_i	Previous replacement point of item i
u_{ij}	Random initial time of item i that is replaced j times
h_{ij}	Random delay time of item i that is replaced j times
N	Number of identical items to be inspected

One has to beware that this is another simulation scheme than described in Wang and Syntetos (2011). The difference is that our simulation is per inspection interval and their simulation is per item. The reason why we chose to adjust the simulation is that we need the data per inspection interval in order to use our policy for creating the aperiodic intervals. We will tell more about this in Section 4.2. Another reason to adjust the simulation is that we want to ensure more fairness when we compare different inspection interval sizes. Specific details about this can be found in Section 5.1. The simulation of Wang and Syntetos (2011) can be found in Appendix A together with some explanation about the simulation.

4 Methodology

In this section, we will first take a look at the models that we will use to make forecasts for the periodic intervals. We start with the maintenance-based model introduced by Wang and Syntetos (2011). Thereafter, we will discuss the time series-based model from Syntetos and Boylan (2005). Next, we will have a look at the models that we will use for the aperiodic intervals. We will discuss the maintenance-based model, the time series-based model and an adjusted time-series model. We have to keep in mind that all the information that is available at inspection interval k can be used to forecast the one period ahead forecast for inspection interval $k + 1$.

4.1 Periodic intervals

4.1.1 Maintenance driven model

As mentioned before, we use the maintenance driven model that is introduced by Wang and Syntetos (2011). Let U be the random initial time and H the random delay time. Next, let $f_U(u),\ F_U(u)\ and\ f_H(h),\ F_H(h)$ be the probability density function (PDF) and the cumulative density function (CDF) of u and h, respectively. Now, we can formulate the following:

$$f_U(u|U > (k-1)t - tr_i) = \begin{cases} \frac{f_U((k-1)t - tr_i + u)}{\int_{(k-1)t-tr_i}^{\infty} f_U(u)du} & tr_i < (k-1)t,\ 0 < u \leq \infty \\ f_U(u) & tr_i \geq (k-1)t,\ 0 < u < \infty\ . \end{cases} \tag{1}$$

As we have seen in Figure 2, we have two types of replacement. The first one is when an item fails during an inspection interval and the second one is when an item is defect during an inspection. Consequently, the expected number of replacements in period $((k-1)t,\ kt]$, denoted by $E(N_r((k-1)t,\ kt])$, is given by:

$$\begin{aligned} E(N_r((k-1)t,\ kt]) &= \sum_{i=1}^{N}(\text{Pr(the } i\text{th item is replaced at } kt) \\ &\quad + \text{Pr(the } i\text{th item fails in } ((k\text{-}1)t,\ kt))) \\ &\approx \sum_{i=1}^{N} F_U(t|U > (k-1)t - tr_i), \end{aligned} \tag{2}$$

where t is equal to the size of the inspection interval. The proof of this model can be found in Appendix B.1.

4.1.2 Time series-based model

As the benchmark model, we will use the time series-based model that is described in Syntetos and Boylan (2005). The demand is assumed to occur according to a Bernoulli process and the inter-demand intervals are geometrically distributed. We also assume that the demand sizes and intervals are stationary. This has some consequences for the usability of this model. We will come back to this in the next section.

The forecast, F_{k+1} for the next inspection interval is given by:

$$F_{k+1} = (1 - \frac{\alpha}{2})\frac{z'_k}{p'_k}, \tag{3}$$

where $p'_k = p'_{k-1} + \alpha(p_k - p'_{k-1})$ and $z'_k = z'_{k-1} + \alpha(z_k - z'_{k-1})$. The vectors z' and p' will be updated after demand occurs, if no demand occurs the estimates will be the same.

Syntetos and Boylan (2005) suggest a smoothing parameter α close to zero. The authors made some comparison tests with $\alpha = 0.05$, 0.10, 0.15, and 0.20. They got the best results for $\alpha = 0.10$, 0.15, and 0.20. We will discuss in Section 5, which smoothing parameter we will use for the time series-based model.

In Table 2, one can find the definition of all the variables that are used in the time series-based model.

Table 2: Description of all variables used in the time series-based model

Variable	Definition
t	Size of the inspection interval
k	Inspection interval counter, starts with $k = 1$
z_k	Actual demand size at inspection interval k
z'_k	Exponential smoothed estimate of the demand size at inspection interval k
p_k	Actual demand interval at inspection interval k
p'_k	Exponential smoothed estimate of the demand interval at inspection interval k
α	Smoothing parameter $(0 \leq \alpha \leq 1)$
F_{k+1}	Demand forecast for inspection interval $k + 1$
γ	Mean of the geometrically distributed inter-demand intervals
μ	Mean of the Bernoulli distributed demand size

To make this time series-based model more clear, we provide a fictional running example in Appendix B.2.

4.2 Aperiodic intervals

As we have mentioned before, it is cost reducing if the inspections intervals are as optimal as possible. This gives us a reason to use aperiodic intervals to approximate the optimal intervals. In order to create the aperiodic intervals, we will first take a look at the CDF of the initial time. The initial time follows a Weibull distribution. When the up-time of an item increase, the probability of failure also increase because of the increasing failure rate. We used this fact and the average up-time of all the items to create multiple thresholds for the CDF-value of initial time. The reason why we use the average up-time and not the up-time of a single item is that we focus on block based inspection, where all items are inspected regardless their age and condition. When the CDF-value of the average up-time comes above a certain threshold, the interval length will become shorter. The reason for this is that the probability of a failure increases and we try to prevent failures and the corresponding Corrective Maintenance by using a shorter inspection interval. We can create a simple basic policy, like the following: 'If the CDF-value of the average up-time $> \zeta$, then reduce the current inspection interval size'.

4.2.1 Maintenance driven model

The maintenance-based model will almost stay the same for the aperiodic intervals. The only difference is that we do not have $(k-1)t$ and kt but we have the beginning (*start*) and the end (*end*) of an interval. This is because the intervals will be aperiodic and that is why we cannot use the old notation anymore. Consequently, the model will be as follows:

$$f_U(u|U > start - tr_i) = \begin{cases} \frac{f_U(start - tr_i + u)}{\int_{start-tr_i}^{\infty} f_U(u)du} & tr_i < start,\ 0 < u \leq \infty \\ f_U(u) & tr_i \geq start,\ 0 < u < \infty\ . \end{cases} \quad (4)$$

$$E(N_r(start,\ end]) = \sum_{i=1}^{N}(\Pr(\text{the } i\text{th item is replaced at } end) + \Pr(\text{the } i\text{th item fails in } (start,\ end))) \approx \sum_{i=1}^{N} F_U(t|U > start - tr_i), \quad (5)$$

4.2.2 Time series-based model

The time series-based model stays the same but there is something important to keep in mind. One of the main assumptions is that the intervals are stationary. In case of the aperiodic intervals, we will violate that assumption, so we have to be careful when we interpret the forecasts and results of this model.

4.2.3 Adjusted time series-based model

A solution, for the problem that is described above, is adjusting the time series-based model such that we have stationary intervals. Since we will be using 14, 28 and 42 days inspection intervals, we will temporary rescale all the intervals back to multiple intervals of 14 days and equally divide the demand. So, for example an interval of size 28 days with a spare part demand of two, will become two inspections intervals of 14 days with a demand of one each. For the forecasts we will use the same formula:

$$F_{k+1} = (1 - \frac{\alpha}{2})\frac{z'_k}{p'_k}, \quad (6)$$

where $p'_k = p'_{k-1} + \alpha(p_k - p'_{k-1})$ and $z'_k = z'_{k-1} + \alpha(z_k - z'_{k-1})$. If the original inspection interval consists of multiple small inspection intervals of 14 days, we will first make a forecasts for the first interval of 14 days and use this forecast to create the next forecast. Eventually, we will sum those forecasts up to one forecast for the original inspection interval. To make this adjusted time series-based model more clear, we provide a fictional running example in Appendix C.1.

5 Results

5.1 Set-up and implementation

Before we take a look at the results and answer our research questions, we want to stress that in the following comparisons the number of defect items does not match perfectly across the different inspection intervals sizes. This is due the random draws and different inspection intervals. We took multiple measures to have as fair as possible comparisons. The first measure is the usage of a fixed seed for all the draws. Next, we changed the simulation model from Wang and Syntetos (2011) to simulate data per inspection interval and not per item. Finally, we created a fixed series of random draws for each item to ensure the fairness.

For the implementation of the simulation, we need to point out some small details. We will use the following means for the initial time distribution: 200, 400, 600, 800, and 1000. We combine those means with the following variances: 6000, 10000, 14000, 18000, 22000, and 24000. The distribution parameters for the initial time are the same as the distribution parameters that are used in Wang and Syntetos (2011). These can be found in Table 9, which is stated in Appendix A. For the distribution parameter of the delay time, α_h, we use the value 0.0174 from the case study performed by Baker and Wang (1991). Moreover, we use 20 items, 20 simulations and a simulation length of 2000 days. These settings are the same as in Wang and Syntetos (2011), so that we can compare our results with the results that are mentioned in their paper.

Next, due the intermittent and lumpy demand, we use an initialization period of 20 inspection intervals to ensure that the time series driven model is initialized. The smoothing parameter that we use is equal to 0.20. We chose for this smoothing parameter because this parameter gives the best results for the time series-based model.

Table 3 contains the threshold values and corresponding inspection interval sizes that we will use in this paper to create the aperiodic intervals.

Table 3: Threshold values and corresponding inspection interval sizes

Inspection interval size	Threshold values
42 days	$0.00 \leq x < 0.05$
28 days	$0.05 \leq x < 0.10$
14 days	$0.10 \leq x \leq 1.00$

Where x is the average up-time of all the items.

More specific details about the implementation can be found in Appendix D.

5.2 Periodic intervals

As has been mentioned before, we will use the maintenance driven model created by Wang and Syntetos (2011) as our base model and the time series-based model from Syntetos and Boylan (2005) as the benchmark model. We start with replicating the models with an inspection interval size of 28 days. In Fig. 4, one can see a simulation run with the forecasted demand using both models.

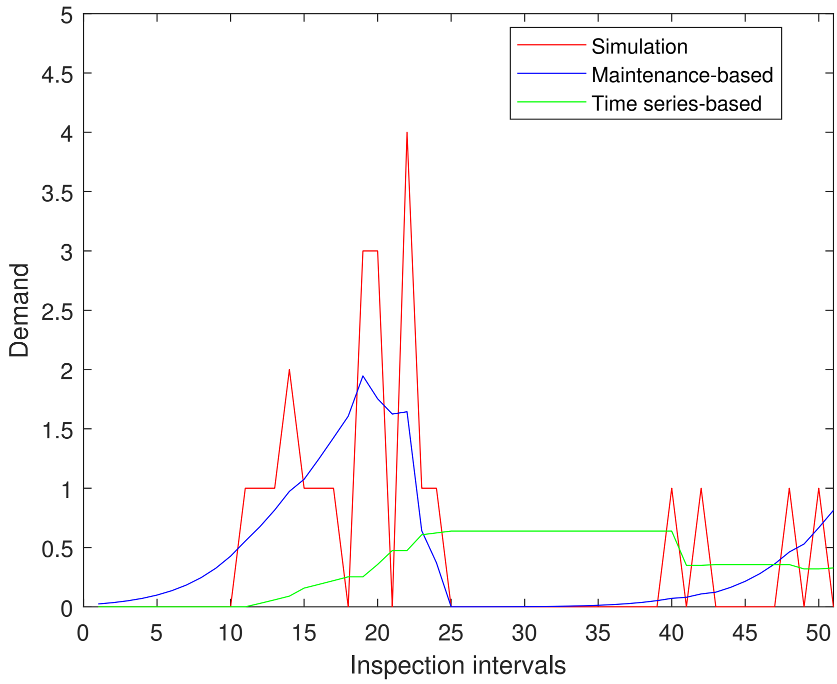

Figure 4: A simulation run with forecasted demand using the maintenance-based model and the time series-based model. The inspection interval length is 28 days.

Here, the mean and variance of the initial time are 1000 and 18,000 respectively. Fig. 4 is very similar to Fig. 8 from Wang and Syntetos (2011). The difference is caused by the fact that we use a different seed. The authors do not mention their seed, thus it is almost impossible to recreate the same figure. Similar figures, like Fig. 4, can be made with an inspection interval length of 14 or 42 days.

We compare some statistics to see the effect of different inspection interval sizes more clearly. We use the following statistics: average number of failures, average number of preventive replacements, average number of inspections and the average ratio between failures and preventive replacements. If we take the same mean and variance, that we used for Fig. 4, we get the results that are displayed in Table 4.

Table 4: Comparison of different inspection interval sizes with multiple statistics

Interval size	Failures	Preventive rep.	Inspections	F/P ratio
14 days	2.60	21.80	142	0.13
28 days	4.20	19.95	71	0.22
42 days	6.20	17.70	47	0.37

Where the mean and variance of the initial time are 1000 and 18,000 respectively. All statistics are the average of 20 simulations.

One can see that the simulations with inspection interval size of 14 days, have the lowest average number of failures and the highest average number of preventive replacements. This results in a low failures/replacements ratio, which can be explained by the high number of inspections. The high number of inspections ensures that more defects will be noticed earlier, before they can result in failures. On the one hand a short inspection interval is desired because of the low failures/replacements ratio but on the other hand it is time-consuming and expensive to have a high number of inspections.

In order to compare the maintenance-based model with the time series-based model, we use the following formulas:

$$\texttt{Total absolute error} = \sum_{k=1}^{K} |\texttt{forecasted demand}_k - \texttt{actual demand}_k|, \tag{7}$$

$$\texttt{Difference} = \frac{\sum \texttt{Total absolute errors}_{TB}}{\texttt{Number of simulations}} - \frac{\sum \texttt{Total absolute errors}_{MB}}{\texttt{Number of simulations}}, \tag{8}$$

where K is the total number of inspections, TB stands for time series-based model and MB for maintenance-based model. So, if the difference is positive, it indicates that the maintenance-based model gives a better prediction than the time series-based model. If we use the same mean and variance that we used for Fig. 4, we get the following differences:

Table 5: Differences with different inspection intervals

Inspection interval size	14 days	28 days	42 days
Difference	8.81	12.06	14.59

Where the mean and variance of the initial time are 1000 and 18,000 respectively.

One can see that in this case, the maintenance-based model gives a better prediction regardless the inspection interval size.

The example above is one of the thirty cases of Table 9. In the following figures, the differences of all the thirty cases are displayed. Starting with 28 days, followed by 14 and 42 days.

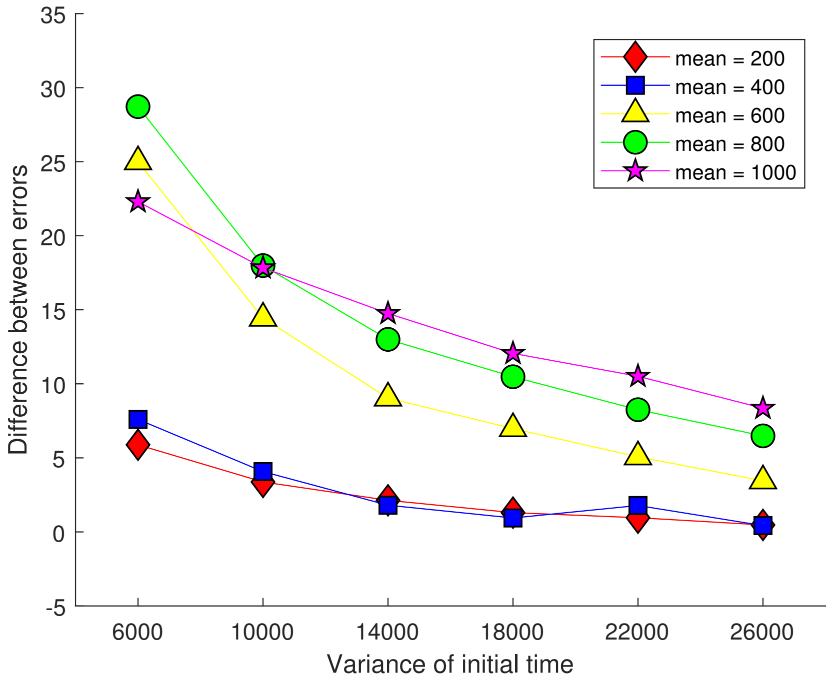

Figure 5: The inspection interval length is 28 days.

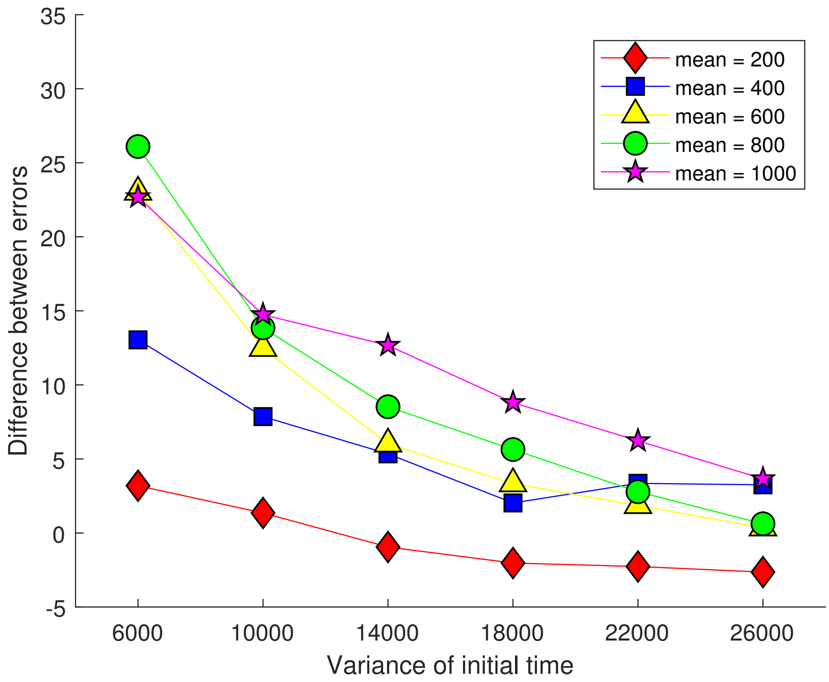

Figure 6: The inspection interval length is 14 days.

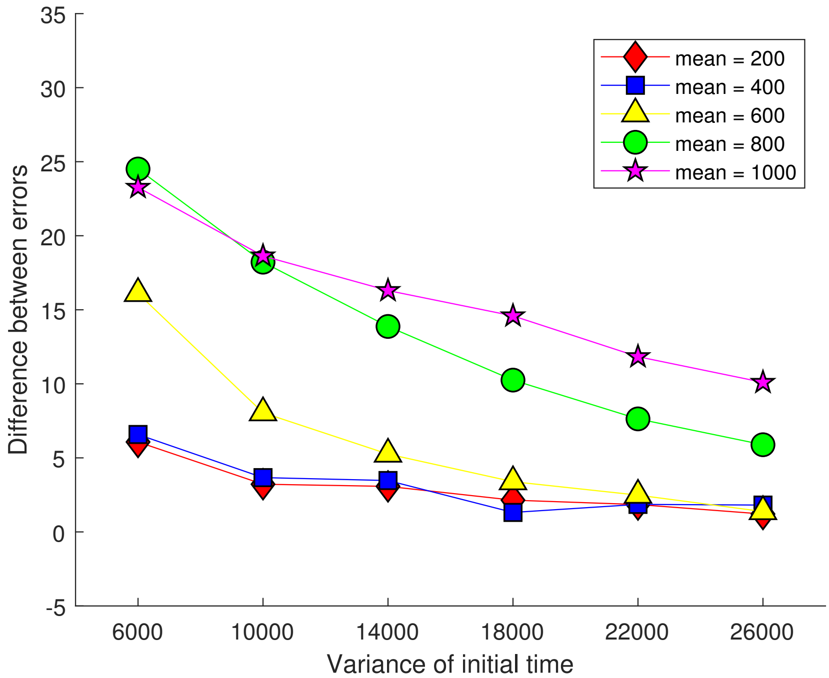

Figure 7: The inspection interval length is 42 days.

Fig. 5 is similar to Fig. 9 from Wang and Syntetos (2011). The differences can be explained by the fact that we use a different seed and a different simulation scheme. Let us take a look at figures 5, 6, and 7. In all the three figures, we see that the differences are larger when the mean of the initial time is large compared to differences with a low mean. The general structure is in all the three the figures the same but we notice some small differences. One of those differences is the fact that there are negative values in Fig. 6. This indicates that the time series-based model forecasts the spare parts better than the maintenance-based model. Overall, the differences are the smallest for inspection intervals with the length of 14 days and the largest for inspection intervals with the length of 28 days. Another difference is the shape of the Fig. 7, which looks more flattened compared to the other figures. However, the conclusion here is the same as the conclusion that has been made in Wang and Syntetos (2011): The maintenance-based model outperforms the time-series model in almost all the cases.

5.3 Aperiodic intervals

For the aperiodic intervals, we will use the maintenance-based model created by Wang and Syntetos (2011), the time series-based model from Syntetos and Boylan (2005), and the adjusted time-series model that is introduced in this paper. First, we have to repeat the remark that the time series-based model from Syntetos and Boylan (2005) is based on stationary intervals, so we have to be careful when we interpreter the results from that model. In Fig. 8, one can see a simulation run with the forecasted demand using the three models.

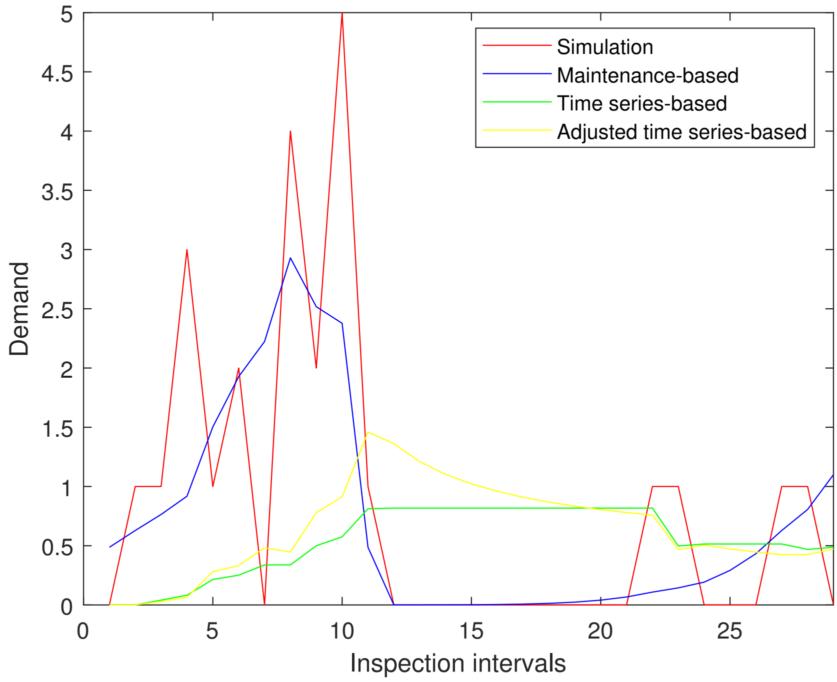

Figure 8: A simulation run with forecasted demand using the maintenance-based model, the time series-based model and the adjusted time series-based model. Aperiodic inspection intervals are used in this case.

Here, the mean and variance of the initial time are 1000 and 18,000 respectively. We see that maintenance-based forecasts fit better than the time series-based forecasts. If we take a look at the time series-based forecasts and the adjusted time series-based forecasts, we see that there is not a model superior to the other model because sometimes the adjusted time series-based forecasts fit better and sometimes the time series-based forecasts.

We make use of the same statistics that we used in Table 4. If we take the same mean and variance, that we used for Fig. 4, we get the results that are displayed in Table 6.

Table 6: Comparison of different inspection interval sizes with multiple measurements

Interval size	Failures	Preventive rep.	Inspections	F/P ratio
Aperiodic	5.50	18.5	49.5	0.31

Where the mean and variance of the initial time are 1000 and 18,000 respectively. All statistics are the average of 20 simulations.

If we compare Table 4 and Table 6, we see that the values of the aperiodic intervals lie between the interval sizes 28 and 42. This can be explained by the fact that the policy, in combination with the mean and variance of the initial time, uses many inspection intervals of 42 days and sometimes inspection intervals 14 and 28 days.

In order to compare the base model with the time series-based model, we use equations 7 and 8 but if we want to compare with the adjusted time series-based model, we will use the following equation:

$$\texttt{Difference} = \frac{\sum \texttt{Total absolute errors}_{TBA}}{\texttt{Number of simulations}} - \frac{\sum \texttt{Total absolute errors}_{MB}}{\texttt{Number of simulations}}, \quad (9)$$

where K is the total number of inspections, TBA stands for adjusted time series-based model and MB for maintenance-based model. If we use the same mean and variance, that we used for Fig. 4, we get the following differences:

Table 7: Differences with different inspection intervals

Inspection interval size	Aperiodic_{TB}	Aperiodic_{TBA}
Difference	12.06	12.88

Where the mean and variance of the initial time are 1000 and 18,000 respectively. TB stands for time series-based model and TBA for adjusted time series-based model.

In Table 7, we see that the adjusted time series-based model has a higher difference than the time series-based model. This indicates, given this mean and variance of the initial time, that the adjusted time series-based model performs a little worse than the other time series-based model but we have to keep in mind that we are dealing with non-stationary inspection intervals.

In the following figures, we displayed the differences for all the thirty cases. The first figure is with the time series-based model and the second figure is with the adjusted time series-based model.

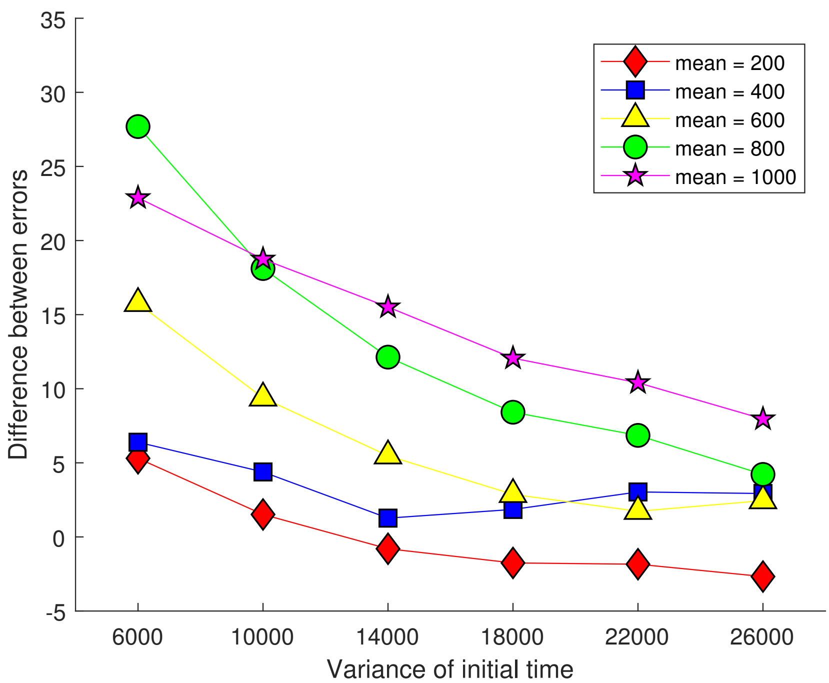

Figure 9: Compared with the time series-based model.

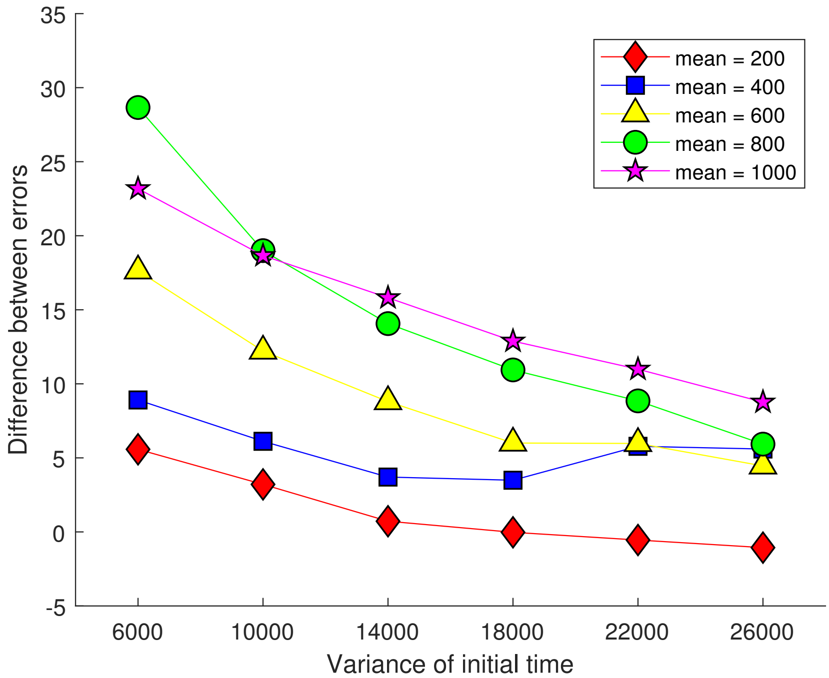

Figure 10: Compared with the adjusted time series-based model.

Figures 9 and 10 are very similar. We take a closer look at differences between the time series-based models by looking at the average per variance in Fig. 11.

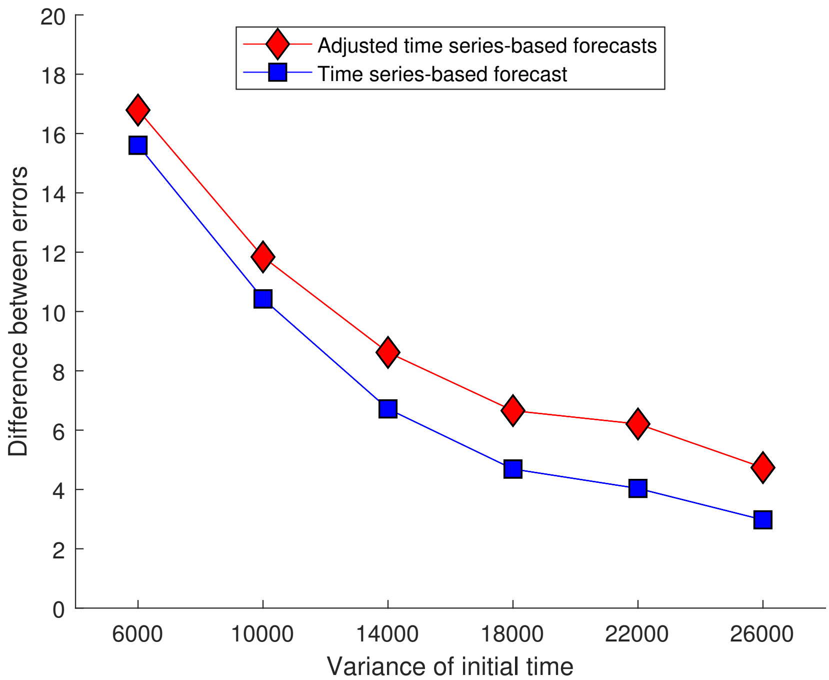

Figure 11: Average per variance of the initial time.

Here, we see again that the shapes are similar. This gives us more reason to use the adjusted time series-based model as the benchmark model for the aperiodic intervals. One of the assumptions for the time series-based model was the assumption of stationary intervals. Because of the violation of this assumption, it is not fair to compare the maintenance-based model and the time series-based model. So, from now on we will only consider Fig. 10. In Fig. 10, we see that in 28 of the 30 cases the maintenance-based model gives a better prediction for the spare parts demand than the adjusted time series-based model. Only in cases with a mean of 200 and a variances of 22,000 and 24,000 the maintenance-based model predicts worse.

If we compare Fig. 10 with the three figures from the periodic intervals, we see some specific similarities. In case of a low mean (200 and 400), Fig. 10 looks similar to Fig. 6. On the other hand, in case of a high mean (600, 800, and 1000), we see that Fig. 10 looks similar to figures 5 and 7. This can be explained by the fact that our policy uses short intervals of 14 days when the average probability of a failure is high. In case of a low mean, the threshold will be easily met and thus results in the use of an interval of 14 days. The reasoning can also be used in case of the high mean. In case of a high mean for the initial time, the thresholds ensures that most of the times an interval with a length 28 or 42 days will be used.

6 Conclusion

Forecasting spare part demand is a difficult task, especially when the demand is intermittent and lumpy. Recently, Wang and Syntetos (2011) introduced a maintenance-based model to forecast the intermittent and lumpy demand for spare parts. In this paper, we tested how well this model performs, with aperiodic inspections, compared to the time series-based model of Syntetos and Boylan (2005). We introduced a simple and effective policy to create those aperiodic intervals. Furthermore, we came to the conclusion that it is not fair to compare the maintenance-based model with the time series-based model when we deal with aperiodic intervals. This is the reason why we introduced the adjusted time series-based model.

In Section 5, we saw that the maintenance-based model with periodic intervals outperforms the time series-based model in almost all cases. The maintenance-based model did not outperform the time series-based model in the cases where the initial time had a small mean and a high variance. This conclusion corresponds with the conclusion that Wang and Syntetos (2011) made in their paper. In Fig. 10, we saw that in 28 of the 30 cases, the maintenance-based model with aperiodic intervals gives a better prediction for the spare parts demand than the adjusted time series-based model. Only in cases with a mean of 200 and a variances of 22,000 and 24,000 the maintenance-based model predicts worse. Overall, the maintenance-based model of Wang and Syntetos (2011) gives in most of the cases a better prediction than the benchmark models.

For future research, one could use another policy for creating aperiodic inspection intervals. This can be done by testing with more or different threshold values and changing the inspection interval sizes. Another option is to use an optimizing algorithm to create more optimal inspection intervals instead of using a policy. Further, one can improve the benchmark model or replace it with model that is focused on aperiodic intervals. Another idea for future research is to use aperiodic inspection intervals in combination with age-based or condition-based inspections instead of block-based inspections.

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Appendices

A Simulation

The scheme for the simulation that is used in Wang and Syntetos (2011) can be found in Fig. 12. We distinguish the same two cases: a failure occurs between two inspections and a defect is found during an inspection. Table 8 contains all the variables that are used in the simulation. We also assume that the delay-time (h) and the initial time (u) follow an exponential distribution and a Weibull distribution, respectively (Baker and Wang, 1991). Furthermore, we make use of the integer function (Int), which returns the largest integer less or equal to the argument.

In Figure 9, one can see the distribution parameters that are used for the different means and variances of the initial time.

Table 8: Description of all variables used in the simulation

Variable	Definition
x_j	Number of replacements during the jth day
T	Simulation length in days
t	Size of the inspection interval in days
k	Inspection interval counter, starts with $k = 1$
tr_i	Previous replacement point of item i
u	Accumulated random initial time of the item
u^*	Actual initial time
h	Random delay time
h^*	Random delay time
m	Number of inspections before u, where $mt < u < (m+1)t$
N	Number of identical items to be inspected

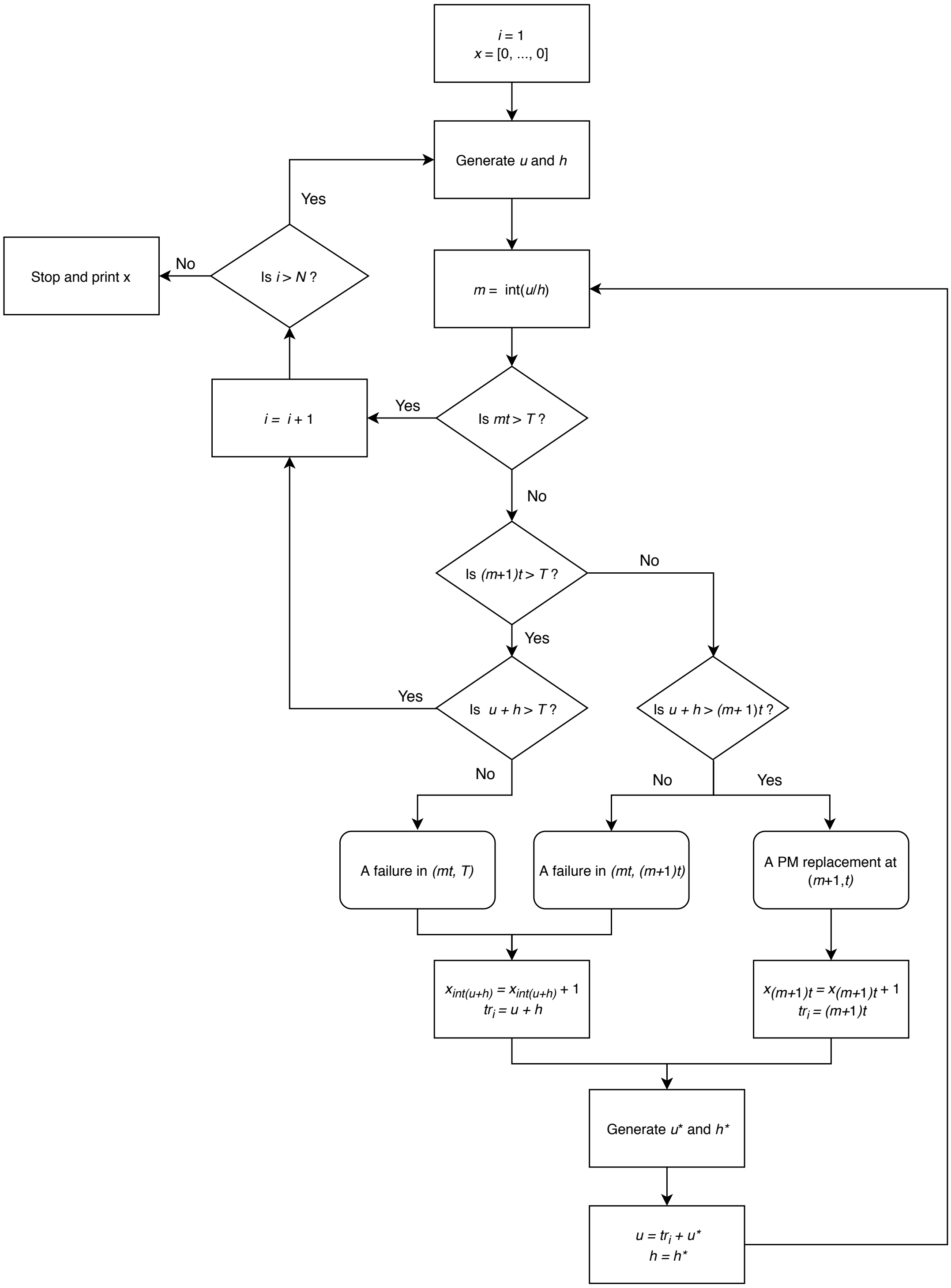

Figure 12: Simulation scheme used in Wang and Syntetos (2011)

Table 9: Distribution parameters of the initial time

Variance	Mean 200	400	600	800	1000
6000	0.0045	0.0023	0.0016	0.0012	0.0009
	2.7945	6.0042	9.2792	12.5721	15.8771
10000	0.0044	0.0023	0.0016	0.0011	0.0009
	2.1013	4.5422	7.0613	9.6031	12.1532
14000	0.0044	0.0022	0.0015	0.0011	0.0009
	1.7439	3.7722	5.8868	8.0273	10.1782
18000	0.0045	0.0022	0.0015	0.0011	0.0009
	1.5198	2.2807	5.1334	7.0412	8.9072
22000	0.0045	0.0022	0.0015	0.0011	0.0009
	1.3638	2.9336	4.5986	6.2941	8.0025
26000	0.0046	0.0022	0.0015	0.0011	0.0009
	1.2481	2.6723	4.1945	5.7487	7.3171

Top: scale parameter, bottom: shape parameter.

B Periodic intervals

B.1 Maintenance driven model

B.1.1 Proof maintenance driven model

We made some assumptions in Section 3 about the distribution and arrivals of failures. For example, the assumption that there will be no defects or failures during the inspection. The probability that this occurs will be almost zero, so we will not take this in consideration and leave it out of the equation.

$$\begin{aligned}
\Pr(\text{the } i\text{th item is replaced at } kt) &= \Pr(U < kt - tr_i | U > (k-1)t - tr_i,\ H > kt - u) \\
&= \int_0^t f_U(u | U > (k-1)t - tr_i) \int_{kt-u}^{\infty} f_H(h) dh du \\
&= \int_0^t f_U(u | U > (k-1)t - tr_i)(1 - F_H(kt-u)) du,
\end{aligned} \tag{10}$$

$$\begin{aligned}
\Pr(\text{the } i\text{th item is replaced at } kt) &= \Pr(U < kt - tr_i | U > (k-1)t - tr_i,\ H > kt - u) \\
&= \int_0^t f_U(u | U > (k-1)t - tr_i) \int_0^{kt-u} f_H(h) dh du \\
&= \int_0^t f_U(u | U > (k-1)t - tr_i) F_H(kt-u) du,
\end{aligned} \tag{11}$$

$$
\begin{aligned}
E(N_r((k-1)t,\ kt]) = &\sum_{i=1}^{N}(\Pr(\text{the } i\text{th item is replaced at } kt) \\
&+ \Pr(\text{the } i\text{th item fails in } ((k\text{-}1)t,\ kt))) \\
&= \sum_{i=1}^{N}(\int_0^t f_U(u|U > (k-1)t - tr_i)(1 - F_H(kt-u))du \\
&+ \int_0^t f_U(u|U > (k-1)t - tr_i)F_H(kt-u)du) \\
&= \sum_{i=1}^{N}\int_0^t f_U(u|U > (k-1)t - tr_i)du \\
&\approx \sum_{i=1}^{N} F_U(t|U > (k-1)t - tr_i).
\end{aligned}
\tag{12}
$$

B.2 Time series-based model

B.2.1 Running example time series-based model

To make the time series-based model more clear, we provide a fictional running example below. Let us take the following intermittent and lumpy demand, z:

$$z = [0\ 8\ 2\ 0\ 1\ 0\ 0\ 4]$$

This results in the following inter-demand vector p:

$$p = [0\ 2\ 1\ 0\ 2\ 0\ 0\ 3]$$

Subsequently, z' and p' will be as follow:

$$z' = [0.00\ 0.80\ 0.92\ 0.92\ 0.93\ 0.93\ 0.93\ 1.24]$$

$$p' = [0.00\ 0.20\ 0.28\ 0.28\ 0.45\ 0.45\ 0.45\ 0.71]$$

Finally, this leads to the following forecast vector F:

$$F = [0.00\ 0.00\ 3.80\ 3.12\ 3.12\ 1.95\ 1.95\ 1.95]$$

C Aperiodic intervals

C.1 Adjusted time series-based model

C.1.1 Running example adjusted time series-based model

To make the adjusted time series-based model more clear, we provide a fictional running example below. Let us take the following intermittent and lumpy demand, z, and the corresponding interval size vector m:

$$z = [0\ 8\ 2\ 0\ 1\ 0\ 0\ 4]$$

$$m = [42\ 28\ 28\ 28\ 14\ 14\ 28\ 28]$$

If we divide all the demand equally and split the intervals, then this results in the following vector w :

$$w = [0\ 0\ 0\ 4\ 4\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 2\ 2]$$

Subsequently, the forecasts per 14 day interval will be as follows:

$$F' = [0.00\ 0.00\ 0.00\ 0.00\ 0.00\ 1.57\ 1.55\ 1.36\ 1.35\ 1.36\ 0.97\ 0.97\ 0.86\ 0.97\ 0.71]$$

Finally, this leads to the following forecast vector F:

$$F = [0.00\ 0.00\ 3.12\ 2.71\ 1.36\ 0.97\ 1.83\ 1.68]$$

D Implementation

First of all, we used MATLAB for our implementation. For the delay (h) and initial time (u) we used the following probability density functions:

$$f_h = \alpha_h^{-(\alpha_h h)}, \tag{13}$$

$$f_u = \alpha_u \beta_u (\alpha_u u)^{\beta_u - 1} e^{-(\alpha_u u)^{\beta_u}}, \tag{14}$$

where α_h is 0.0174 (Baker and Wang, 1991) and α_u, and β_u are given in Table 9. Next, the seed that has been used in this research is '12345'.