ERASMUS UNIVERSITY ROTTERDAM

Erasmus School of Economics

Bachelor Thesis Econometrics and Operational Research

Title of thesis: DTRP - An extension of the Delay Time model to include Replacement Possibility.

Name of Student: Veltman, S.

Student ID Number: 408927

Name of Supervisor: Eruguz Colak, A. S.

Name of Second assessor: Wang, W.

Final Version: 8-7-2018

Abstract

Forecasting demand for upcoming periods is an issue that has been thoroughly discussed and researched. It is of great importance as company strategies and schedules are made based on these forecasts every day. This thesis investigates and compares four types of existing forecasting methods and proposes a new one that is consistent with a failure mode in which items can also be replaced with newer models instead of invariably fixed. Specifically, there are two major contributions made to the existing literature.

Firstly, it provides a detailed explanation of a comparison of four existing methods used in forecasting of demand. It evaluates the strength and weaknesses of these methods, and states the most efficient method given the setup of the sample. The findings indicate that the best forecasting method depends on the error function and the sample size of the items.

Secondly, this paper defines two modes of failure and contributes a new forecasting method tailored to it. The first failure mode is set up in such a way that a failure of an item is always cause for demand. In the second failure mode, an item is either fixed or replaced. Only the items that are fixed are cause for demand. In accordance with the second failure mode, a new method is proposed. This method is under certain circumstances an improvement to existing methods.

Contents

List of Figures

List of Tables

1 Problem Statement

Malfunctioning of appliances can have significant economic consequences. The problem must be resolved as swiftly as possible. At a private level, it causes discomfort. At a corporate level it causes loss in revenue, delays, lower quality and, again, discomfort.

Researchers have been investigating and are able to make reasonable forecasts of spare part demand, occurring whenever an item fails. Fixing a failed item is called corrective maintenance. The other cause for demand is regular preventive maintenance. At preventive maintenances, the parts of an appliance are checked to catch a faulty item before it causes a failure. Preventive maintenance causes a spike in demand at the size of faulty appliances at the time of inspection, because there is a delay time between the appearance of an error and the failure time. This causes, along with the randomness of error and failure time, an intermittent demand pattern [11]. The types of maintenance differ in their nature: preventive maintenance is deterministic in occurrence and stochastic in demand size, whereas corrective maintenance is stochastic in occurrence and demand size is always one.

The variation in demand occurrence is not all. Additionally the sample size can also to predict for can also vary over time. As an appliance gets outdated, or is discarded, a user can choose to update to a newer model, rather than have it fixed again. A practical application of this is the maintenance of wind turbines. Proper lubrication and other preventive actions can avoid expensive and unnecessary repairs. At the same time, rapid development ensures that companies are producing larger windmills and maintenance equipment providers, such as lubrication producers, need to produce materials that keep up with the industry [5]. Moreover as illustration, Figure 1, shows that spare parts for outdated aircraft model no longer sell well over time.

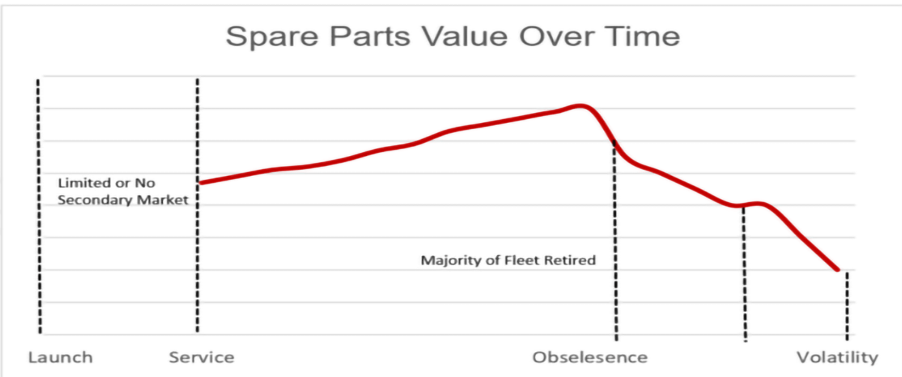


Figure 1. mba Insight graph illustrating the average value of an aircraft spare part over time. source: www.mba.aero/

Intermittent demand patterns and forecasting for such has been researched thoroughly. The literature in section 2 shows four existing methods to forecast demand, three time series model, and the Delay Time model (DT) [11]. The DT model is the cornerstone for this thesis. In addition to existing literature, the DT method is compared to more methods than just SBA, as it has been in the paper by Wang et al. [11]. It is now also compared to TSB and LES. Also, this thesis provides an insight into a new model in which there are two failure modes. Originally there was only one failure mode in which a failed item was always fixed and cause for demand of a spare part, as applied in the paper by Wang et al. [11]. The second (new) failure mode, a failed/faulty item is either fixed OR replaced with a new item, based on an additional variable that states at which point in time an item becomes outdated. If an item is replaced, there is no demand for a spare part. These research goals can be summarized in the research questions (RQ) stated below.

RQ1: How can one incorporate the two failure modes in a DT model to forecast spare parts demand for items that might be replaced?

RQ2: What is the best method for forecasting spare part demand for one and for two failure modes?

The build-up of the paper is as follows: section 2 discusses past literature, section 3 provides insight in how the data used in this paper is generated and applied, in section 4 the methodology is discussed, section 5 discusses tests and their results for the methodology and lastly section 6 states a summary and some concluding remarks.

2 Literature

There have been many models to forecast demand over the years. One of the first and a basis for future models was put forward by Croston, 1972 [2]. He proposed a method that deals with demand patterns by forecasting the interval between the occurrence of demand and the demand size when this happens uncorrelated to each other. Croston time series method outperformed the Single Exponential Smoothing (SES). Croston's method estimated the mean demand per period. In time period t, z_t is the actual demand size, z'_t is the smoothed estimate of the next period for z_t, p_t is the actual demand interval, p'_t is the smoothed estimate of the next period for p_t, α is a smoothing parameter where $0 \leq \alpha \leq 1$, F_t is the forecast for the next period. In the method, the variables are only updated in periods with non-zero demand. The forecast is given in equation 1.

$$F_t = \frac{z'_t}{p'_t} \tag{1}$$

where if $z_t > 0$**:** $p'_t = p'_{t-1} + \alpha(p_t - p'_{t-1})$, $z'_t = z'_{t-1} + \alpha(z_t - z'_{t-1})$

The method by Croston was proven to be biased in 2001 by Syntetos and Boylan [8], with a bias equal to $Bias \approx \frac{\alpha}{2-\alpha} \cdot E[z_t] \cdot \frac{p-1}{p^2}$. Subsequently, Syntetos and Boylan, 2005 [9] put forward a solution that combats the bias, transforming the forecast from equation 1 slightly. They use the same definition for the variables. The estimator is called the Syntetos Boylan Approximator (SBA). The forecast of the SBA method is given in equation 2.

$$F_t = (1 - \frac{\alpha}{2}) \cdot \frac{z'_t}{p'_t} \tag{2}$$

The paper by Teunter et al., 2011 [10] extended the method with an unbiased estimator using demand occurrence probability and instead of demand interval, named after Teunter, Syntetos and Babai (TSB). This allegedly incorporates a faster reaction to a decrease in demand and had a smaller variance than the SBA and Croston methods given that the right smoothing constants are used. When an item becomes obsolescent its forecasts degenerate exponentially to zero. The variable are defined with ρ_t as the occurrence of demand in period t and ρ'_t as the probability of a demand occurrence at the end of period t. Furthermore, they introduce two different smoothing variables α and β for demand size and demand occurrence respectively. The forecast is given in equation 3:

$$F_t = \rho'_t \cdot z'_t \tag{3}$$

$$\textbf{where if } \boldsymbol{\rho_t = 0}\textbf{:}\ \rho'_t = \rho'_t + \beta(0 - \rho'_t),\ z'_t = z'_{t-1}$$

$$\textbf{else } \boldsymbol{\rho_t = 1}\textbf{:}\ \rho'_t = \rho'_t + \beta(1 - \rho'_t),\ z'_t = z'_{t-1} + \alpha(z_t - z'_{t-1})$$

Another forecasting method showing promising results is Linear-Exponential Smoothing (LES) put forward by Prestwich et al., 2014 [6]. It is similar to the method of Hyperbolic-Exponential Smoothing (HES) [7] put forward by Prestwich et al, 2014. HES estimates demand size, z'_t, and inter-demand interval, p'_t, and increases the interval in periods where demand is zero, $z_t = 0$ which decreases the forecast for the next period: $F_t > F_{t-1}$. This is comparable to a combination of TSB, updating when nonzero, and SBA working with demand interval. The forecast is given in equation 4.

$$F_t = \begin{cases} \frac{z'_t}{p'_t} & \text{if } z_t > 0 \\ \frac{z'_t}{p'_t + \frac{\beta p_t}{2}} & \text{if } z_t = 0 \end{cases} \tag{4}$$

$$\textbf{where if } z_t > 0\textbf{:}\ p'_t = p'_{t-1} + \beta(p_t - p'_{t-1}),\ z'_t = z'_{t-1} + \alpha(z_t - z'_{t-1})$$

The method LES [6] differs from HES [7] in the way it updates F_t, and enables LES to forecast zero demand, which was not possible yet with HES or TSB. They update z'_t and p'_t the same as with HES. The forecasts for LES are given in equation 5.

$$F_t = \begin{cases} \frac{z'_t}{p'_t} & \text{if } z_t > 0 \\ \frac{z'_t}{p'_t} \cdot \max\{0, 1 - \frac{\beta p_t}{2p'_t}\} & \text{if } z_t = 0 \end{cases} \tag{5}$$

In variance to all of the methods mentioned (Croston, SBA, TSB, HES, LES) that react to data, there is also a method proposed by Wang et al., 2011 [11] that incorporates the sources of demand patterns to develop a pro-active mannerism to forecast the demand for spare parts. The forecast equals the addition of the probabilities of failure within the time period and the probability of a part being faulty at moment of inspection at the beginning of the time period. The method is called

Delay Time method (DT), as it builds upon the intermittent nature of demand sizes provided by the delay time between the point an error arose and the point it caused a failure. Allegedly, given that the distributions for occurrence of error and failure, it outperformed SBA. A fuller description of the mathematics to support their method is given in section 4.2.

To summarize the literature, when the paper by Wang et al. [11] was published in 2011, SBA was best shown to outperform other Croston-based methods by a paper from Eaves et al. [3], Gutierrez et al. [4] and Syntetos et al. [9], but DT outperformed SBA still. Since then, improvements different time series based methods have been published: TSB, HES and LES. HES is not included in the analysis, because HES and LES are very similar, but LES has been proven superior in Prestwich et al, [6]. The new model proposed in this paper: DTRP is an extension of DT that also includes the probability of items being replaced. DTRP will be compared to DT, SBA, TSB and LES in order to provide a thorough understanding of the possibilities or limitations of DTRP contrasting ingrained methods in the field of forecasting spare parts demand. In addition, the findings from [11] are reproduced when comparing the performances of DT and SBA.

3 Simulation study

The data used to test the performances is simulated. Two types of data are considered, distinguished as two different failure modes. The first one is in accommodation with the DT model where all failures/faulty items are always fixed and cause demand. The second failure mode is in accommodation with the DTRP model, where failures/faulty items can either be fixed and cause demand or replaced with a new model, without causing demand for a spare part. The maintenance schedule is age based, meaning that an item is only checked if it was checked longer than a certain time ago. This is a practical situation, which saves a repair man some time.

For the data, the failure pattern is implemented described in the paper by Wang et al. [11]. A total of N items are in the sample size. The time point item i presents an error is given at U_i, if an error has not occurred already. This error causes a failure at time point S_i. The time it takes between U_i and S_i is called the delay time, H_i. Items are inspected at a regular interval of length t. If the item is inspected and there is an error, it is fixed immediately. Also, if a failure occurs, the item is fixed/replaced immediately. The error time is Weibull distributed and the delay time is exponentially distributed. The last time an item was placed is given as: t_{ir}. The variable that indicates from what usage time on an item is to be inspected, is given with τ. The last checkup time is t_{ip}. Using age based maintenance, if an item is very freshly placed it does not have to be inspected; if $t_{now} - t_r \leq \tau$.

When the data includes replacement possibility, an item gets outdated at time point O_i. If an item is inspected and faulty or fails after O_i, it is replaced with a new model. If this is the case, the item doesn't cause demand for a spare part. The occurrence of a part needing replacement is distributed with the Weibull distribution starting from the moment item i was placed in the machine. A representation of the failure pattern is given in figure 2.

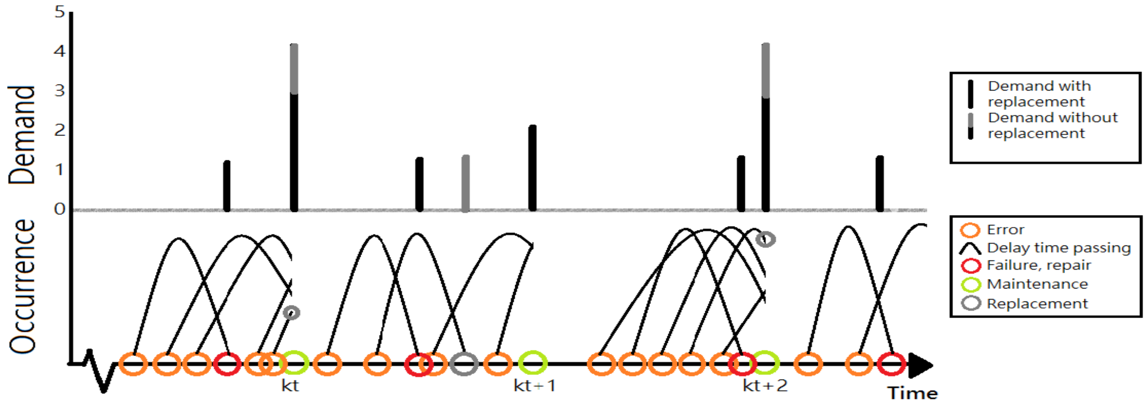


Figure 2. Schematic demand pattern.

To implement the simulation with a solid foundation, five assumptions have to be made in the age based maintenance scheme.

1. If the item is inspected, any present error is fixed, causing demand equal to 1 per item, or the item is replaced, causing no demand.

2. Inspections are undertaken at a regular interval t.

3. When the technicians performed an inspection or fixes a failure, they put a label on the item showing the time of the inspection or replacement so that the next inspection is taking place after t time from that particular day.

4. The distribution for the time elapsed between the introduction of a new item and the point U_i is Weibull; the delay time distribution is exponential (fitted result).

5. The technicians goes around every period t but only those associated with a time elapsed since the last inspection or replacement of more than τ are inspected again according to the label attached to the appliance.

The current time period looked at is time period m, ranged between $1 \leq m \leq int(T/t)$. The moment of replacement is given as O_i. If an item is never replaced, O_i is not generated but set to infinity, meaning that the blue part is always true. If items are replaced, O_i is generated as the expiration date of an item, and as soon as the item is faulty at inspection or failed, it is replaced. With these variables, the data is simulated in a manner as represented in figure 3. The gray boxes indicate an update mechanism of planned maintenances that are happening in periods without failures/errors, in order to keep track of the last time of inspection: t_{ip}. These blocks are an addition to the block scheme in the paper by Wang et al. [11].

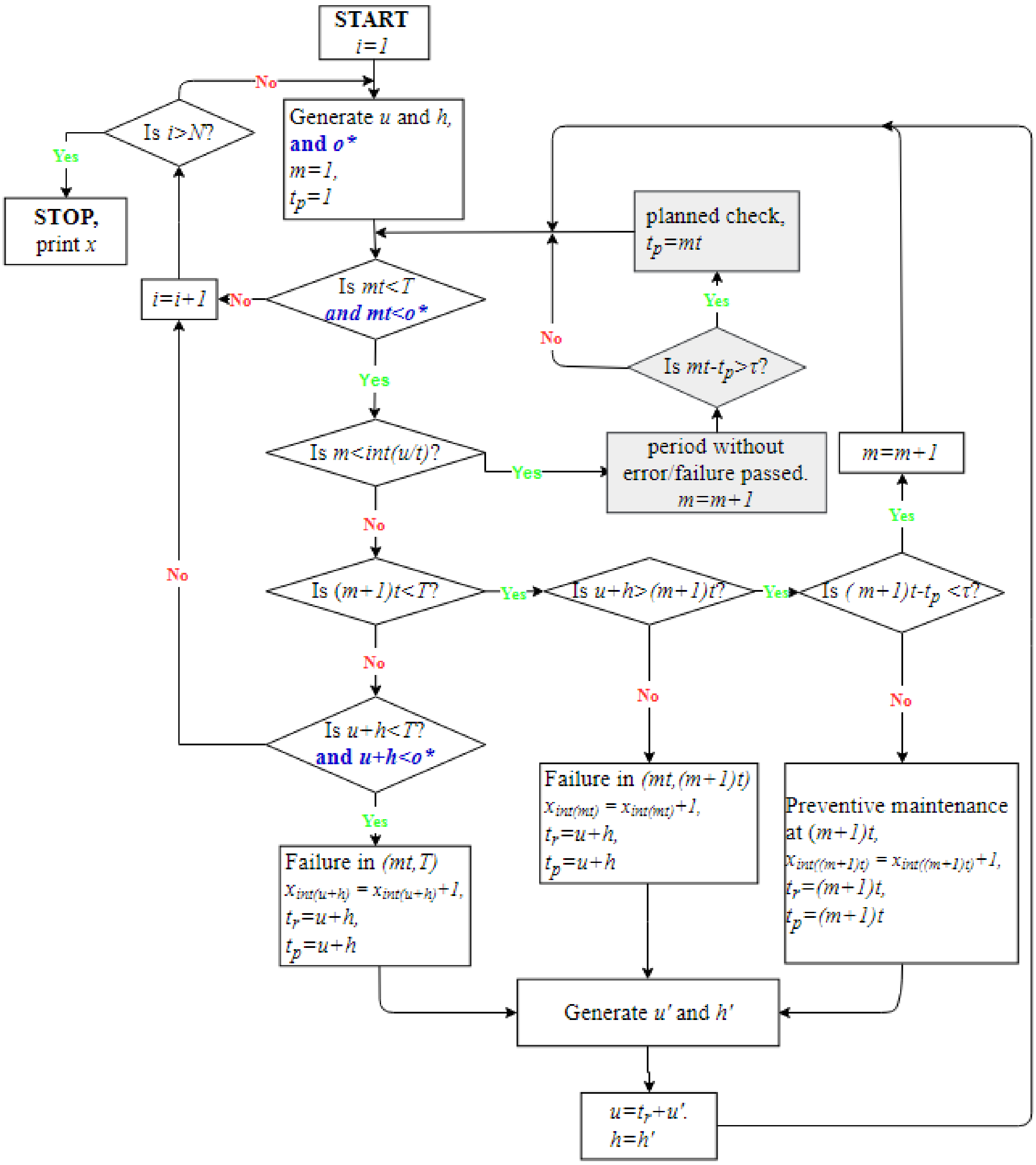


Figure 3. Block scheme, age based maintenance
o* is set to infinity if replacement is disregarded.

4 Methodology

The performance of three types of models is to be compared: time series models based on Croston's method [2], the DT model [11] and an extension of the DT model: DTRP. First the Croston based methods and their implementations are given in section 4.1. Secondly, the math for the DT method is described as it is implemented in this paper in section 4.2. This is different, but roughly equivalent, to how it was proposed in the original paper by Wang et al. [11], which is stated in Appendix C. Lastly, the DTRP method is proposed in section 4.3.

4.1 Time Series Methods

Croston based time series methods use previously observed demand and the time passed between previous non-zero demand periods to forecast the next period's average demand. As discussed in section 2, three methods are considered: SBA [9], TSB [10] and LES [6]. These models are one step ahead forecasts, relying on the actual values from last period and using these to adjust to previous forecasts. An overview of the methods is given in table 1.

	SBA [9]	TSB [10]	LES [6]
variables	smoothing var α demand in t: z_t forecast z_t: z'_t demand interval t: p_t forecast p_t: p'_t	smoothing var α, β demand in t: z_t forecast z_t: z'_t demand prob at t: ρ_t forecast ρ_t: ρ'_t	smoothing var α, β demand in t: z_t forecast z_t: z'_t demand interval t: p_t forecast p_t: p'_t
forecast	$F_t = (1 - \frac{\alpha}{2})\frac{z'_t}{p'_t}$	$F_t = z'_t \cdot \rho'_t$	$F_t = \begin{cases} z'_t/p'_t & \text{if } z_t > 0 \\ (z'_t/p'_t) \cdot (1 - \beta p_t/2p'_t)^+ & \text{if } z_t = 0 \end{cases}$
variable updates	$\mathbf{if\ z_t > 0}$ $z'_t = z'_{t-1} + \alpha(z_t - z'_{t-1})$ $p'_t = p'_{t-1} + \alpha(p_t - p'_{t-1})$ $\mathbf{else\ z_t = 0}$ $z'_t = z'_{t-1}$ $p'_t = p'_{t-1}$	$\mathbf{if\ \rho_t = 1}$ $z'_t = z'_{t-1} + \rho_t\alpha(z_t - z'_{t-1})$ $\rho'_t = \rho'_{t-1} + \beta(\rho_t - \rho'_{t-1})$ $\mathbf{else\ \rho_t = 0}$ $z'_t = z'_{t-1}$ $\rho'_t = \rho'_{t-1} + \beta(\rho_t - \rho'_{t-1})$	$\mathbf{if\ z_t > 0}$ $z'_t = \alpha z_t + (1 - \alpha)z'_t$ $p'_t = \beta p_t + (1 - \beta)p'_t$ $\mathbf{else\ z_t = 0}$ $z'_t = z'_{t-1}$ $p_t = p_t + 1$
staring values	$z'_0 = 0;$ $p'_0 = 0;$	$z'_0 = 0;$ $\rho'_0 = 0;$	$z'_0 = 0;$ $p'_0 = 0;$

Table 1. Attributes of three time series methods for demand forecast

Note that these three methods only rely on past demand. This means two things to keep in mind for the testing. Firstly, they need some adjustment time to cope with the demand pattern: in each test the first 20/40 time periods are disregarded as an in-sample period for the time-series based methods. Secondly, because these methods do not rely on the number of items that are in place in the model, they are practically equal for the model with one failure mode as for the model with two failure modes.

As the smoothing variable is very important for the implications of these methods, when the distributive values for the initial and delay time are known. As the distribution is known for the DT and DTRP prediction, this can also be applied for the grid-search without making any new assumptions. The best values for α and β for each of the methods is calculated using grid search with step sizes equal to 0.05 along the values ranging in [0.05,1]. An example of grid-search is stated in the values displayed in Appendix B, used for a graph in section 5.1.

4.2 Delay Time model: DT

The DT model uses the distributive characteristics of the initial time and delay time function of items to pro-actively forecast demand. It is not based on past demand occurrence like the time series method. The initial time distribution function is distributed Weibull(λ, κ) by assumption. The delay time distribution function is distributed Exponential(μ). These distributions are used to forecast the probability of an item needing to be fixed and therefore demand for spare parts, in a time period of length t.

The pdf. and the cdf. of the initial time of an error is given by $f_U(u)$ and $F_U(u)$. The conditional distribution uses that the initial time U_i is zero at the time if placement of item i, t_{ir}, given that i was last inspected and non-faulty at t_{ip}. This is gives the conditional pdf. for general U is given in equation 6.

$$f_U(u|U > t_{ip} - t_{ir}) = \begin{cases} \frac{f_U(u+t_{ip}-t_{ir})}{\int_{t_{ip}-t_{ir}}^{\infty} f_U(u)du} & t_{ir} < t_{ip},\ 0 < u < \infty \\ f_U(u) & t_{ir} = t_{ip},\ 0 < u < \infty \end{cases} \tag{6}$$

For the delay time, the pdf. and cdf. are given as $f_H(h)$ and $F_H(h)$ respectively. It is known that after an error occurs, if H has not already happened at the current time, t_{now}. The conditional pdf. for general H is given in equation 7.

$$f_H(h|H > t_{now}) = \begin{cases} \frac{f_H(h+t_{now})}{\int_{t_{now}}^{\infty} f_H(h)dh} & u < t_{now},\ 0 < h < \infty \\ f_H(h) & u \geq t_{now},\ 0 < h < \infty \end{cases} \tag{7}$$

For simplicity we make Assumption 1: In the calculations of the DT model, an item has never a failure after replacement within the time span of t: $U_i + H_i > t$. E.q. it is not possible to fail or be faulty at inspection at t_{now}, be replaced and have another error occurring between $(t_{now}, t_{now} + t)$. This calculation error made because of this assumption, ϵ, is expected to be zero.

These definitions of the distributions for $f_U(u)$ and $f_H(h)$ and Assumption 1 are taken from the paper by Wang et al. [11]. But as there were some typing mistakes and inconsistencies, the following demand forecasts are different than used in that paper. The main difference is that in this way every time period is forecasted for, whereas the paper by Wang et al. [11] simulates for two time periods. Though, the formulations are equivalent as they both still rely on the initial time and delay time functions, $f_U(u)$ and $f_H(h)$ respectively. Their mathematics of the DT model is given in Appendix C.

The expected demand for a period with age based maintenance data is calculated. For this type of calculation, there are two scenarios for all items $i \in N$:

Scenario 1: the item is inspected at period t_{now}.

Scenario 2: the item does not have to be inspected at period t_{now}.

First, consider Scenario 1, in which the item is inspected at this moment (t_{now}), and has not been inspected since t_{ip}. The probability of an item i needing fixing at moment t_{now} is the probability of an item an error having occurred between the last inspection/failure and this one: between (t_{ip}, t_{now}), plus the probability of an error occurring and failure between $(t_{now} + t)$. Assumption 1 states that there is no possibility of a second failure within $(t_{now} + t)$. Therefore, the only other possibility is that an error presents after t_{now} meaning $t_{now} + t - t_{ip} > U_i > t_{now} - t_{ip}$. For notation, $Dem1_i(s_1, s_2)$ is the event of there being demand for item $i \in N$ with maintenance in both s_1 and period s_2 for time period (s_1, s_2) for $\forall s_1, s_2 \in \{1, T\}$. The demand due to items in scenario 2 is given in equation 8.

$$
\begin{aligned}
Pr(Dem1_i(t_{now}, t_{now} + t)) &= Pr(error \text{ in } (t_{ip}, t_{now})) + Pr(error \wedge failure \text{ in } (t_{now}, t_{now} + t)) \\
&= Pr(U < t_{now} - t_{ip} | U > t_{ip} - t_{ir}) \\
&+ Pr(t_{now} - t_{ip} < U < t_{now} + t | U > t_{ip} - t_{ir}, H < t_{now} + t) + \epsilon \\
&= \int_0^{t_{now} - t_{ip}} f_U(u | U > t_{ip} - t_{ir}) du + \\
&+ \int_{t_{now} - t_{ip}}^{t_{now} - t_{ip} + t} f_U(u | U > t_{ip} - t_{ir}) \int_0^{t-u} f_H(h) dh du + \epsilon \\
&= \int_0^{t_{now} - t_{ip}} f_U(u | U > t_{ip} - t_{ir}) du \\
&+ \int_0^{t} f_U(u | U > t_{ip} - t_{ir}) F_H(t - u) du + \epsilon \\
&\approx \int_0^{t_{now} - t_{ip}} f_U(u | U > t_{ip} - t_{ir}) du \\
&+ \int_0^{t} f_U(u | U > t_{ip} - t_{ir}) (F_H(t - u)) du
\end{aligned}
\tag{8}
$$

Now consider an item i in Scenario 2. The last time it was inspected was time t_{ip} and at this time it was non-faulty and still working. At the current time t_{now}, it has not yet failed, and it was checked last for errors/fixed at t_{ip}. Therefore, three things can happen to i. The first possibility is that it is found to be non-faulty at the next inspection at $t_{now} + t$ and remains and remains non-faulty until $t_{now} + 2t$, two inspection periods from now. This would not cause any demand in general. Secondly, it can be found faulty at the next inspection, at $(t_{now} + t)$ is replaced and hadn't failed. This would only cause demand in the following period, as is also evident from equation 8. Thirdly, a failure occurs before $t_{now} + t$, before the next inspection. This is the only scenario that causes demand for the current period. In this case either an error has occurred between (t_{ip}, t_{now}) and the failure occurs between $t_{now}, t_{now} + t)$ or an error and failure combination occurs as error in (t_{now}, u) and failure in $(u, t_{now} + t)$, for $t_{now} < u < t_{now} + t$. Again, due to Assumption 1, it is not possible to have a second failure before $t_{now} + t$. For notation, $Dem2_i(s_1, s_2)$ is the event of there being demand for item $i \in N$ with maintenance is not held at s_1 and is done at s_2 for time period (s_1, s_2) for $\forall s_1, s_2 \in \{1, T\}$. The demand due to items in scenario 2 is given in equation 9.

$$\begin{aligned}
Pr(Dem2_i(t_{now}, t_{now}+t)) &= Pr(error \text{ in } (t_{ip}, t_{now}) \wedge failure \text{ in } t_{now}, t_{now}+t) \\
&+ Pr(error \wedge failure \text{ in } (t_{now}, t_{now}+t)) \\
&= Pr(U < t_{now} | U > t_{ip} - t_{ir}, H < t_{now}+t | H > t_{now}) du \\
&+ Pr(t_{now} - t_{ip} < U < t_{now}+t | U > t_p - t_{ir}, H < t_{now}+t | H > t_{now}+t-u) + \epsilon \\
&= \int_0^{t_{now}-t_{ip}} f_U(u|U > t_{ip} - t_{ir}) \cdot \int_0^{t-u} f_H(h|H > t_{now}) dh du \\
&+ \int_{t_{now}-t_{ip}}^{t_{now}-t_{ip}+t} f_U(u|U > t_{ip} - t_{ir}) \int_0^{t-u} f_H(h) dh du + \epsilon \\
&\approx \int_0^{t_{now}-t_{ip}} f_U(u|U > t_{ip} - t_{ir}) \cdot F_H(t-u|H > t_{now}) du \\
&+ \int_0^{t} f_U(u|U > t_{ip} - t_{ir}) F_H(t-u) du
\end{aligned} \tag{9}$$

The total expected number of replacements, $F(t_{now}, t_{now}+t)$, and therefore the forecasted demand in the coming period $(t_{now}, t_{now}+t)$ is given in equation 10.

$$\begin{aligned}
E[F(t_{now}, t_{now}+t)] &= \sum_{i=1}^{N} Dem1_i(t_{now}, t_{now}+t) \cdot \delta_i + \sum_{i=1}^{N} Dem2_i(t_{now}, t_{now}+t) \cdot (1-\delta_i) \\
&= \sum_{i=1}^{N} \Bigg(\int_0^{t_{now}-t_{ip}} f_U(u|U > t_{ip} - t_{ir}) du \\
&+ \int_0^{t} f_U(u|U > t_{ip} - t_{ir})(F_H(t-u)) du \Bigg) \cdot \delta_i \\
&+ \sum_{i=1}^{N} \Bigg(\int_0^{t_{now}-t_{ip}} f_U(u|U > t_{ip} - t_{ir}) \cdot F_H(t-u|H > t_{now}) du \\
&+ \int_0^{t} f_U(u|U > t_{ip} - t_{ir}) F_H(t-u) du \Bigg) \cdot (1-\delta_i) \\
&= \sum_{i=1}^{N} \int_0^{t} f_U(u|U > t_{ip} - t_{ir}) F_H(t-u) du \\
&+ \int_0^{t_{now}-t_{ip}} f_U(u|U > t_{ip} - t_{ir}) du \cdot \delta_i \\
&+ \int_0^{t_{now}-t_{ip}} f_U(u|U > t_{ip} - t_{ir}) \cdot F_H(t-u|H > t_{now}) du \cdot (1-\delta_i)
\end{aligned} \tag{10}$$

where $\delta_i = \begin{cases} 1 & \text{if the item is in Scenario 1} \\ 0 & \text{if the item is in Scenario 2} \end{cases}$

If an item goes obsolete in the period in between inspections, this will be noted at the next inspection and the item will be taken out of inspection rotation and demand is noted to be zero for all coming periods.

4.3 Delay Time model with replacement possibility: DTRP

For the DTRP method, there are two failure modes. An item is either fixed or replaced after inspected faulty/failure. The moment O_i, is distributed by the Weibull distribution with pdf. $f_O(o)$ and cdf. $F_O(o)$. After an item is outdated, it gets replaced at the following failure or when it is inspected and faulty.

At every point in time of prediction, that O_i is larger than the moment of last fixing the item, t_{ir}. Then there are two cases. Either the last moment of fixing an item has happened in the previous period, or it happened before. The conditional pdf. for general O is given in equation 11.

$$f_O(o|O > t_{ir}) = \begin{cases} \frac{f_O(o+t_{ir})}{\int_{t_{ir}}^{\infty} f_O(o)do} & t_{ir} < t_{ip},\ 0 < d < \infty \\ f_O(o) & t_{ir} = t_{ip},\ 0 < o < \infty \end{cases} \tag{11}$$

Subsequently, the probability of replacement, $q'_{i(s1,s2)}$, is given in equation 12.

$$\begin{aligned} q'_{i(s1,s2)} &= Pr(i \text{ replaced in period (s1,s2)}) \\ &= Pr(O < s_2 | O > t_{ir}) \\ &= \int_0^{s2} f_O(o|O > t_{ir})do \end{aligned} \tag{12}$$

In the forecast, items are dismissed if they have been replaced. In this case, variable D_i is set to 0, else it is 1. The DT method is thusly transformed to predict the possibility of error/failure occurrences that are not replaced in that time period. Multiplying equation 10 with the probability that they are not replaced, gives the forecasts of the DTRP model. The forecasted demand in the coming period $(t_{now}, t_{now} + t)$ is given in equation 13.

$$\begin{aligned} E[F(t_{now}, t_{now} + t)] = \sum_{i=1}^{N} D_i(1 - q'_{i(t_{now}, t_{now}+t)}) \cdot \Bigg(\int_0^t f_U(u|U > t_{ip} - t_{ir}) F_H(t_{now} + t - u) du \\ + \int_0^{t_{now} - t_{ip}} f_U(u|U > t_{ip} - t_{ir}) du \cdot \delta_i + \int_0^{t_{now}+t-u} f_H(h|H > t_{now}) dh \cdot (1 - \delta_i) \Bigg) \end{aligned} \tag{13}$$

Using this, it is possible to adjust the forecasts to the replacement of old parts, using the original DT model. Thus far, RQ1 is theoretically answered. Although, to evaluate the performance of the DTRP method, tests should be run and analyzed. Only then it can be concluded if this model is a satisfactory improvement.

5 Results

To analyze the performance of the methods considered, the total absolute errors of the forecasts are compared. The total absolute errors of $Meth \in \{SBA, DT, TSB, LES\}$ is given in equation 14.

$$Error_{Meth} = \sum_{s=i}^{\text{Number of simulations}} \frac{\sum_{t=1}^{T} |z_t - Meth(F_t)|}{\text{Number of simulations}} \tag{14}$$

For the distributions following, the initial time is distributed by the Weibull distribution with scale (λ_u) and shape (κ_u) parameters, given in equation 15. The delay time is exponentially distributed with parameter (μ), given in equation 16.

$$f(u; \lambda, \kappa) = \frac{\kappa}{\lambda}\left(\frac{u}{\lambda}\right)^{\kappa-1} e^{-(\frac{u}{\lambda})^{\kappa}} \tag{15}$$

$$f(h; \mu) = \frac{1}{\mu} e^{-\frac{h}{\mu}} \tag{16}$$

In section 5.1 the results found in the paper by Wang et al. [11] are reproduced and the findings are validated: DT outperforms SBA. Additionally, DT is compared to TSB and HES. Section 5.2 shows a theoretical experiment that investigates different situations and to find in which situation DT is preferred over LES and vice versa. In section 5.3, the performance and additional value of the DTRP method is discussed and compared to other methods.

5.1 Reproduction

In the paper by Wang et al. [11], an aggregated demand graph for a block based scheme was published. In a block based scheme, the maintenance check-ups are consistently scheduled and performed every time period t, regardless of an items' age. Additionally, the paper calculates the errors for two types of pumps with age based inspection. This is reproduced using solely age based data (see section 3) and the definition of DT that predicts for every t and not every $2t$ (see section 4.2. Proceeding, mostly the same variables are used as in the original paper.

First a demand simulation is produced. The set-up of the experiment is 20 simulation runs with 20 items, the initial time is distributed with λ and κ parameters equal to 1052.6 and 8.907 respectively and the delay time with μ equal to 574.71. The time periods are adjusted for age based data: using shorter periods of $t = 12$, for a maximum of 140 periods, so $T = 1680$, where the first 40 are out of sample. Lastly, τ is taken at 21. To find the the optimal values for the smoothing coefficients, α and β, grid search is used. These are equal for SBA to $\alpha = 0.05$, for TSB to $\alpha = 0.05$ and $\beta = 1$ and for LES to $\alpha = 1$ and $\beta = 0.05$ (for total table, see Appendix B). The demand aggregation and forecasts are given in figure 4. The main difference between this graph and the one in Wang et al. [11] is that 4 presents spikes. This is due to the age based data and the fact that there is a prediction for each period. If there is no inspection in a period, the forecasted demand is far lower than if there is an inspection period. The total errors are calculated as in equation 14 given in table 2.

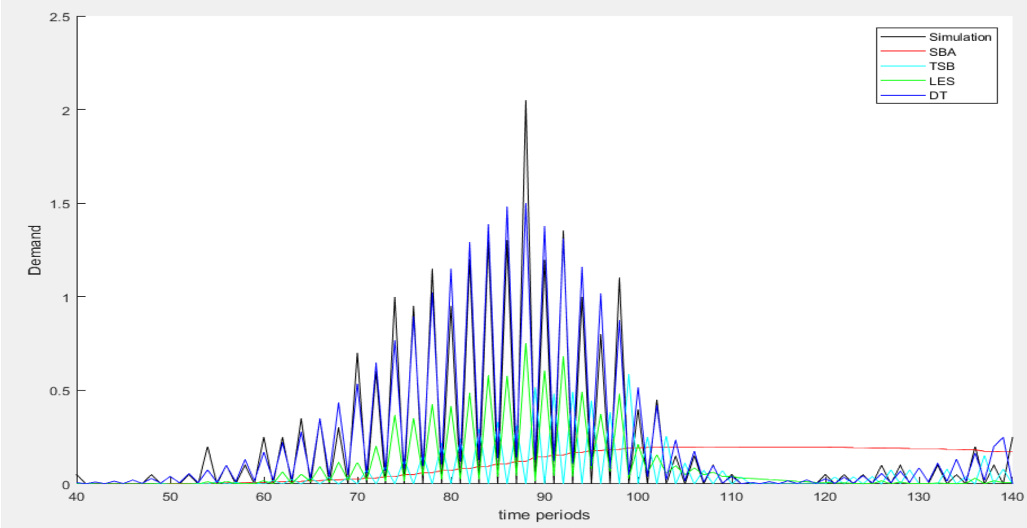


Figure 4. Demand simulation

	SBA	TSB	LES	DT
Average absolute errors	29.2750	26.9747	15.9310	19.2559

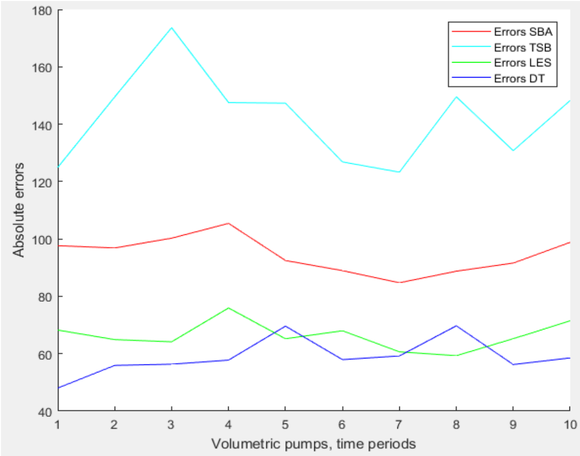
Table 2. Overview of the performances of four forecasting methods, one failure mode.

Next in the paper by Wang et al. [11], specific values are used in age based maintenance scheme to test DT again, using values fitted for pumps in a paper by Baker et al. [1]. This case uses two types of pumps: volumetric and peristaltic. For both types, there are 2000 time units, t lasts 28 time units, meaning that there are 71 forecasting periods of which 20 are used to initialize the Croston based methods. The minimum age of inspection is 42 time units.

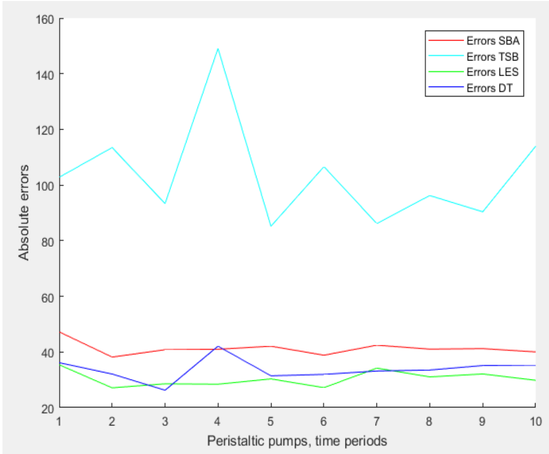
For the volumetric pump, it is used that $\lambda = 588.24$, $\kappa = 1.42$, $\mu = 57.4712$, and $N = 105$. Moreover, SBA uses $\alpha = 1$, TSB uses $\alpha = 0.05$ and $\beta = 1$, LES uses $\alpha = 1$ and $\beta = 0.05$. The total absolute errors are shown in figure 5a.

For the peristaltic pumps, it is used that λ equals 1369,863, κ equals 2.41, μ equals 1111.11, and N equals 35. Moreover, SBA uses $\alpha = 0.05$, TSB uses $\alpha = 0.05$ and $\beta = 0.1$, LES uses $\alpha = 0.7$ and $\beta = 0.05$. The results are shown in figure 5b.

Reproducing these findings and comparing the errors produced shows similar results to those in the paper by Wang et al. [11]: DT outperforms SBA. Additionally to the reproduction, LES and TSB are considered and compared as forecasters. It shows that TSB performs worst for the pumps. LES performs best in figure 4 because the systematic low forecasts and fit ultimately well with the actual values of single run demand, even though the average of DT fits closer. The difference in relative performances between DT and LES is noteworthy for the pumps in figure 5. LES performs best in figure 5b, though DT is close. DT performs best in figure 5a. Assessing the properties of these different situations (Appendix A) shows that the difference in performance between LES and DT could



(a) Errors: Volumetric pumps



(b) Errors: Peristaltic pumps

Figure 5. A sum of the absolute forecasting error for different types of forecasting: SBA, TSB, LES and DT

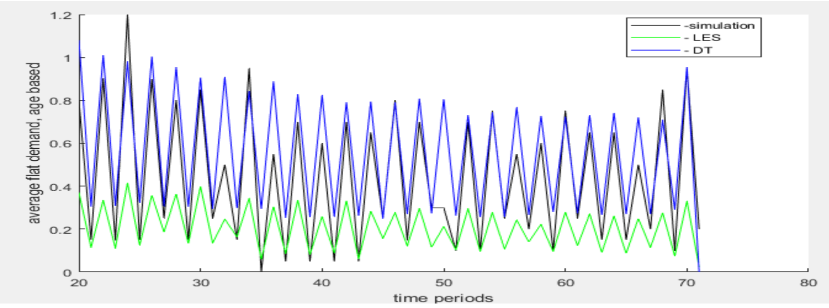
be because the variance is higher for the case in figure 5b, standard deviation equals 537.02, than in figure 5a, standard deviation equals 379.24. The use of the DT model is more accurate if the variance is low, because in that case the intermittent demand is better matched by the probability distribution. Contrarily, the Croston based methods profit from a larger variance, since larger variance in their case means a more flattened out demand pattern. These methods are unable to predict demand spikes and rely on the past observations, as is indicated in figure 4. Another influence can be the sample size. DT does not predict demand equal to zero, so in periods in advance to actual demand, predictions are already off, if there is no demand. As the sample size increases the chance of demand occurring in the tail periods leading up to the mean of the initial time distribution. A larger sample size leads to a better fit of predictions if the variance is relatively high. To conclude, a relatively high variance, which is present in all figures 4, and 5 is beneficial for LES in general, unless the sample size is also large. In that case, DT might be preferred.

The limitation was not touched upon in [11], because the largest variance considered was 26000. This restraint means that demand is never spread out so far that DT loses its accuracy significantly compared to SBA. It is merely hinted at in their findings, as they find that the difference in errors of DT and SBA is larger when variance is low and mean is high. They state: "...as the variance decreases and mean increases the advantage of the DT approach becomes even more obvious", but the limitations are not further considered.

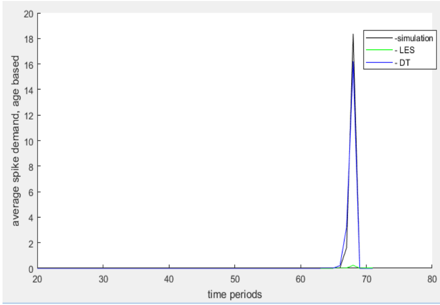
5.2 The effect of variance coefficient and sample size on DT

This section backs up the theory that variance and sample sizes affect which method performs best. Consider the following theoretical experiment exploiting the two extreme cases of failure distribution: a single spike because of a small variance coefficient ($CV = 0.0028$), and a flattened out demand because of high variance coefficient ($CV = 2.1$). The coefficient of the variance is given as $CV = \frac{\sqrt{var}}{mean}$.

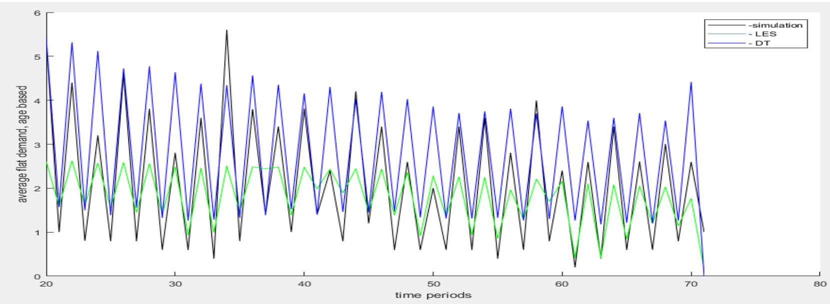
For both cases, $t = 12$, $T = 1680$, $\tau = 21$, $\mu = 57.4713$ and the number of repetitions is equal to 20. For the low variance coefficient, the initial time is simulated with $\lambda = 427.7515$ and $\kappa = 0.5192$. The distribution of LES is used with $\alpha = \beta = 0.05$, optimized using grid search. The figures are given in 6a and 6c. For the low variance coefficient, the initial time is simulated with $\lambda = 801.0059$ and $\kappa = 457.9269$. The distribution of LES is used with $\alpha = \beta = 0.05$, optimized using grid search. The figures are given in 6b and 6d. These figures are run for $N = \{20, 100\}$, to evaluate the difference in demand as a result of variance and of number of items. The corresponding errors are given in table 3.



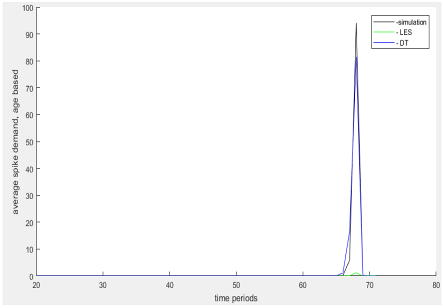
(a) N=20, high variance coefficient



(b) N=20, low variance coefficient



(c) N=100, high variance coefficient



(d) N=100, low variance coefficient

	High variance coefficient					**Low variance coefficient**				
N=	min. demand	max. demand	average demand	DT Error	LES Error	min. demand	max. demand	average demand	DT Error	LES Error
20	0	1.2	0.4385	41.4224	30.8922	0	18.35	0.3846	4.53282	19.7952
100	0.2	5.6	2.0923	104.4965	119.5737	0	94	1.9231	24.2315	98.9847

Table 3. Distribution characteristics and errors per demand pattern and sample size.

As is evident from table 3, DT performs always best for a distribution with a small variance coefficient. For the high variance coefficient, LES performs better if the sample size is small. The larger sample size makes that in general the demand simulated fits the initial distribution closer and therefore DT is still closer. This can be seen in the figures 6a and 6c. These results help formulate an answer to RQ2 for one failure mode.

5.3 Delay Time Replacement Possibility: DTRP

Next, simulations with two failure modes are considered to examine the performance of DTRP.

First a demand graph is evaluated. Using the same variables as used for graph 4, the variables are given by 20 simulation runs with 20 items, the error time is distributed with λ_u and κ_u parameters equal to 1052.6 and 8.907 respectively, $\mu = 574.71$, $t = 12$, $\tau = 21$, $T = 1680$, of which the first 20 are out of sample for time series methods to catch up. Lastly, The optimal smoothing coefficients are found using grid search and are equal for SBA to $\alpha = 0.05$, for TSB to $\alpha = 0.05$ and $\beta = 1$ and for LES to $\alpha = 1$ and $\beta = 0.05$. Starting of, the replacement function is assumed to be distributed according to the Weibull distribution, with scale parameter λ_u=λ_o and shape parameter κ_u=κ_o. The demand graph for two failure modes is given in figure 7.

Comparing to figure 4, the small demand spikes from periods 120+ have disappeared. This is because time point O_i is smaller than $120t$ for $\forall i \in \{1, .., N\}$. These items are either replaced earlier, or are otherwise replaced when they fail in these later periods.

Furthermore, it is seen that maximum demand is lower than before. This was to be expected, as there is no demand for the items that are replaced.

Lastly, it can be seen that the demand decline is more abrupt. The probability that the simulated replacement time has passed is negatively correlated with U_i. Starting at period 80, this is displayed more obvious in the graph. Mathematically this is stated for arbitrary time period $\forall s \in \{1, ..., N\}$ as: $Pr(O_i < U_i | U_i > s) > Pr(O_i < U_i | U_i < s)$.

Table 4 shows the errors. LES has the lowest errors. The ranking is similar to 2. Again LES is the lowest, due to small sample sizes in combination with a relatively large variance coefficient.

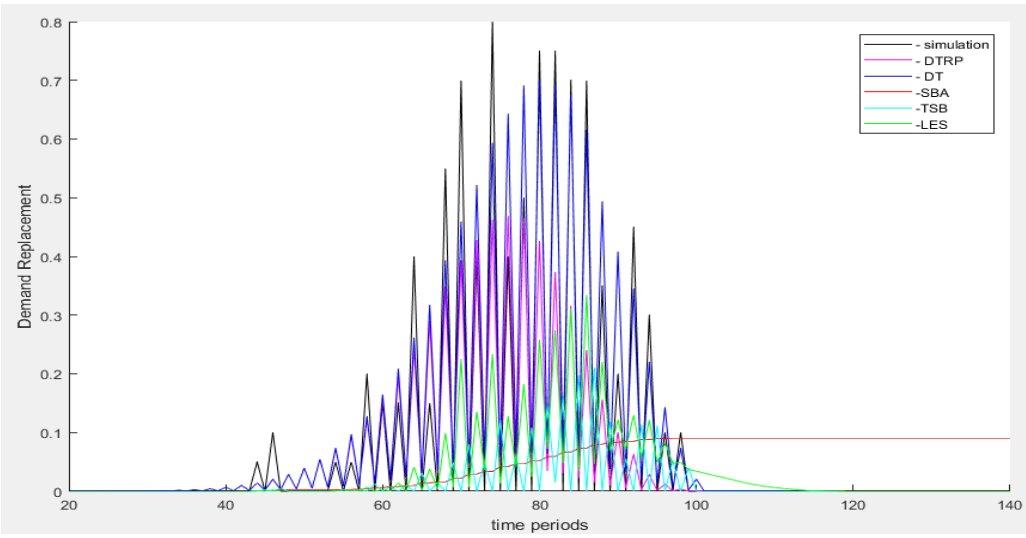
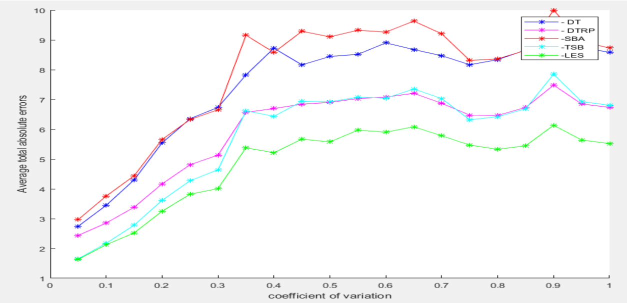


Figure 7. Demand simulation Replacement

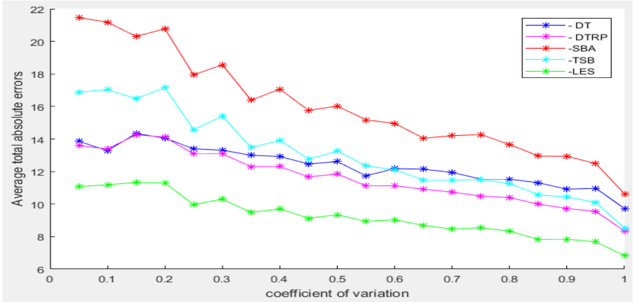
	SBA	TSB	LES	DT	DTRP
Average absolute errors	14.6799	10.8522	8.1755	11.3189	10.6939

Table 4. Overview of the performances of five forecasting methods, two failure modes.

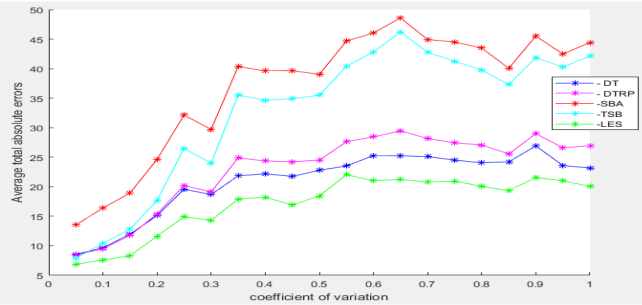
Next, the shortcomings of DTRP are evaluated and the performance of DTRP compared to other methods. The variables are similar to the case in figure 7, except for the distribution of the replacement function, $f_O(o)$ and the sample size. These are varied to assess what specifically affects performance. Two means are considered: one low mean equal to 795, in the middle of the total time period, and a high mean equal to 1095, at $\frac{3}{4}$ of the total time period. Two sample sizes are considered: a small one; $N = 20$, and a large one; $N = 100$. A total of 20 different values for coefficient of variation are considered, for $CV \in \{0.05, ..., 1\}$ with step size equal to 0.05. The errors are given in figure 8.



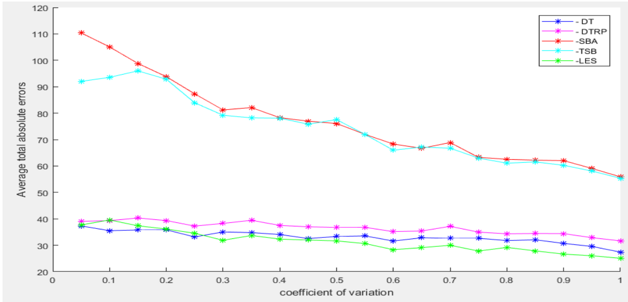
(a) Errors: N=20, Mean=795



(b) Errors: N=20, Mean=1095



(c) Errors: N=100, Mean=795



(d) Errors: N=100, Mean=1095

Figure 8. Errors of five forecasting methods

Firsly, the effect of the sample sizes is examined. For SBA and LES the sample size has no effect on their relative performances. TSB performs relatively better (better than DT) with a small sample size than a large sample size. This is due to the fact that the forecasts rely on demand probability, set very low, creating always a low forecast as seen in figure 7. If the sample size is low, and the demand stays lower, the TSB prediction are less far off. Furthermore, it is evident that the preference of DTRP over DT switches as the sample size is increased. This is due to the nature of DTRP. DTRP has as a benefit that it can predict non-demand if replacement occurs. But a disadvantage is that if there is no replacement, the forecasts of DTRP are systematically too low. Apparently, as the sample size increases, the relative benefits of predicting non-demand in one period is too small to compensate the lower predictions. Only for a low mean and a very low variance ($CV < 0.2$), DTRP performs better in a large sample.

Next, the effect of the distribution of the replacement function is examined. It is evident that the errors for all methods are lower when the mean is small in general. The items are replaced at an earlier stage. DT and DTRP don't predict after replacement from the next point on (noticed at maintenance) and the LES, and TSB adjust to consistent zero demand, aggregating low errors. SBA's predictions remain low, along with the demand. This means the errors are not so far off.

There is a difference in the effect of the variance depending on the mean. If the mean is low, the errors are positively correlated with the variance. The reason for this is that some replacements are happening later, causing larger errors in general that are not compensated by the benefit of earlier replacements. If the mean is high, errors are negatively correlated with the variance. This is due to the fact that the decline of demand is less abrupt, benefiting the time series methods as well as DT and DTRP, through DT's characteristics as the parent method. It is shown that for a small sample size and a larger variance, the benefit of DTRP over DT grows, because the items that are not replaced benefit from the consistent adjustments made by incorporating replacement possibility. For a large mean, the difference between errors is inconsistent.

Considering all of these influences seen so far, the goal is to answer RQ2 for the two failure modes. SBA is the worst predictor in every case considered. This is also because SBA does not adjust to obsolescence, and is therefore a poor predictor after all items are replaced. LES is the best predictor in any case, because of the high variance of the initial time distribution (see section 5.2), the quick adjustment to zero demand and consistently conservative predictions harmonizing with the replacements. Even in large sample sizes, DT is still not better than LES. Both DT and DTRP are stable in their predictions. If the sample size is low, DTRP is better than DT. If the sample size is high, DT is preferred. TSB is not such a good predictor, except when the mean and sample size are very low, but for suspicious reasons: TSB always has low predictions. Therefore, DTRP can be preferred in this case to TSB.

To answer RQ2 to the best of our abilities:

Apparent from table 3: If there is no replacement (one failure mode), and the sample size is very large, DT is most efficient.

If there is no replacement (one failure mode), the sample size is small and the variance of the initial function is small, DT is most efficient.

If there is no replacement (one failure mode), the sample size is small and the variance of the initial function is large, LES is most efficient.

Apparent from figures 6a and 8: If there are is replacement (two failure modes) and the variance of the initial function is large, LES is most efficient.

Apparent from figures 6b, 8a and 8b: If there are is replacement (two failure modes), the variance is very small and the sample size is small, DTRP is most efficient.

Apparent from figures 6d, 8c and 8d: If there are is replacement (two failure modes), the variance is very small and the sample size is large, DT is most efficient.

6 Conclusion

Following the paper by Wang et al. (2011) [11], the first goal of this thesis was to confirm their findings and investigate the grounds of these findings. In the paper, DT is said to outperform SBA [9], but is only applicable if the initial time and delay time distributions are known. Otherwise, SBA should be applied instead. It is now found that there are three things wrong with this assumption. Firstly, apparent after this research and others', SBA is not the best alternative out there. The inclusion of obsolescence and quick response to intermittent demand from LES [6] has been proven to outperform SBA in each of the experiments performed. This is merely and update to their theory, as LES was not discovered yet in 2011. Secondly, it is found that not only knowledge of the distribution is needed for proper forecasting in the DT model, but also the variance should be small, otherwise time-series methods might be preferred (see table 3). Thirdly, if the function is unknown or the variance is large and the sample size is very large, DT is still preferred to other time series methods 3.

Next, a new model was proposed to incorporate the probability of replacement at a certain point in time was added to the DT model to create the DTRP model. Under multiple conditions the characteristics of DT and DTRP were compared, along the time series based methods. DTRP is still not yet exclusively superior to DT, which just disregards replacement possibility in each period. The abilities of DTRP that strengthen it to predict replacements, are undone in a period no replacement is made: the forecasts to low. Also, LES performs better in all experiments performed regarding two failure modes, so DTRP is not yet as well as idealized. At this point, further research should be done to correct the DTRP model such that it takes into account these problems.A conclusion of which method is best applied in which situation is given in figure 9.

For further research it might be useful to define the exact points which classify variance and sample size as 'large', such to make a clear distinction when to use LES or DT/DTRP. Ultimately, a goal could be to redefine the DTRP method in such a way that it is no longer crippling itself in periods with zero replacement, and would for real outperform DT in any scenario.

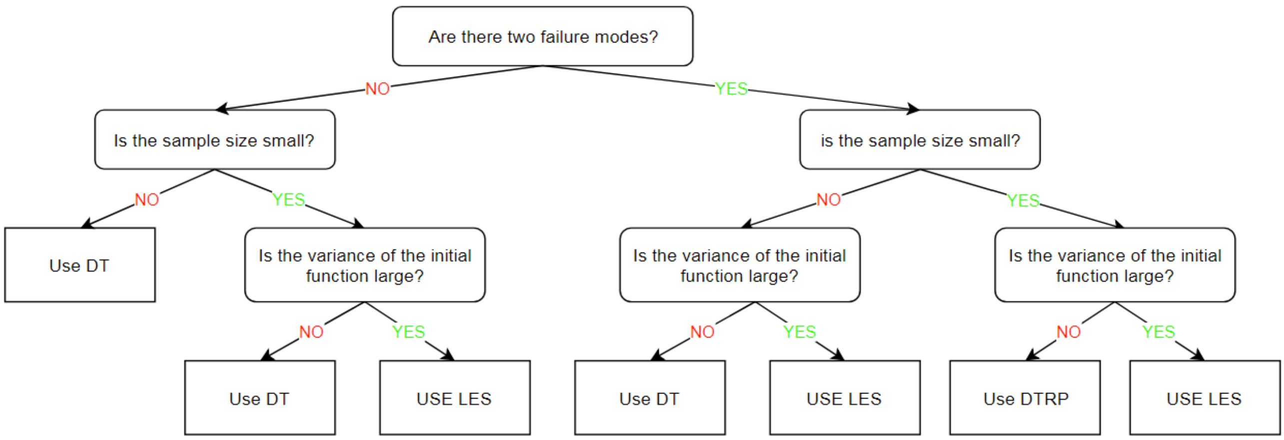


Figure 9. Advice on which method to apply.

Further in the paper: Bibilography, Appendix A in which a table gives an overview of distribution values used in this paper, Appendix B in which an overview of values is found using grid-search as an example for the reader and Appendix C which gives a description of the DT-math used in the paper by Wang et al. [11].

References

[1] R. Baker and W. Wang. Estimating the delay-time distribution of faults in repairable machinery from failure data. *IMA Journal of Management Mathematics*, 3(4):259–281, 1991.

[2] J. D. Croston. Forecasting and stock control for intermittent demands. *Journal of the Operational Research Society*, 23(3):289–303, 1972.

[3] A. H. Eaves and B. G. Kingsman. Forecasting for the ordering and stock-holding of spare parts. *Journal of the Operational Research Society*, 55(4):431–437, 2004.

[4] R. S. Gutierrez, A. O. Solis, and S. Mukhopadhyay. Lumpy demand forecasting using neural networks. *International Journal of Production Economics*, 111(2):409–420, 2008.

[5] J. Martino. Wind turbine lubrication and maintenance: Protecting investments in renewable energy. *Power Engineering*, 117(5), 2013.

[6] S. Prestwich, A. Tarim, R. Rossi, and B. Hnich. Intermittency and obsolescence: a croston method with linear decay. *arXiv preprint arXiv:1409.1609*, 2014.

[7] S. D. Prestwich, S. A. Tarim, R. Rossi, and B. Hnich. Forecasting intermittent demand by hyperbolic-exponential smoothing. *International Journal of Forecasting*, 30(4):928–933, 2014.

[8] A. A. Syntetos and J. E. Boylan. On the bias of intermittent demand estimates. *International journal of production economics*, 71(1-3):457–466, 2001.

[9] A. A. Syntetos and J. E. Boylan. The accuracy of intermittent demand estimates. *International Journal of forecasting*, 21(2):303–314, 2005.

[10] R. H. Teunter, A. A. Syntetos, and M. Z. Babai. Intermittent demand: Linking forecasting to inventory obsolescence. *European Journal of Operational Research*, 214(3):606–615, 2011.

[11] W. Wang and A. A. Syntetos. Spare parts demand: Linking forecasting to equipment maintenance. *Transportation Research Part E: Logistics and Transportation Review*, 47(6):1194–1209, 2011.

Appendix A

A clear overview of different values for the distribution of the Weibull function in table 5.

Descpription use	figure	items N	scale λ	shape κ	mean	variance	st dev
Demand simulation	fig 4 4	20	1052.6	8.907	996.31	17868	133.67
Vol pumps	fig 5a	105	588.24	1.43	534.42	143820	379.24
Per pumps	fig 5b	35	1369,863014	2.41	1214.5	288390	537.02
Flat Demand	fig 6a, 6c	20,100	427.7517	0.5192	800	288300	1697.9
Spike Demand	fig 6b, 6d	20,100	801.0059	457.9269	800	5.0044	2.2371
CV= 0.05 , low mean	fig 8a, 8c	20, 100	812.5529574	24.94977518	795	1580.0625	39.75
CV= 0.1 , low mean	fig 8a, 8c	20, 100	829.2149562	12.15343419	795	6320.25	79.5
CV= 0.15 , low mean	fig 8a, 8c	20, 100	844.6610479	7.906926805	795	14220.5625	119.25
CV= 0.2 , low mean	fig 8a, 8c	20, 100	858.5803726	5.797400066	795	25281	159
CV= 0.25 , low mean	fig 8a, 8c	20, 100	870.6907884	4.542213092	795	39501.5625	198.75
CV= 0.3 , low mean	fig 8a, 8c	20, 100	880.7517746	3.713772366	795	56882.25	238.5
CV= 0.35 , low mean	fig 8a, 8c	20, 100	888.5744549	3.128794406	795	77423.0625	278.25
CV= 0.4 , low mean	fig 8a, 8c	20, 100	894.027983	2.695621255	795	101124	318
CV= 0.45 , low mean	fig 8a, 8c	20, 100	897.0420067	2.36332506	795	127985.0625	357.75
CV= 0.5 , low mean	fig 8a, 8c	20, 100	897.6053947	2.101349095	795	158006.25	397.5
CV= 0.55 , low mean	fig 8a, 8c	20, 100	895.7617766	1.890252797	795	191187.5625	437.25
CV= 0.6 , low mean	fig 8a, 8c	20, 100	891.6026865	1.71708343	795	227529	477
CV= 0.65 , low mean	fig 8a, 8c	20, 100	885.2591896	1.572883123	795	267030.5625	516.75
CV= 0.7 , low mean	fig 8a, 8c	20, 100	876.892836	1.451263666	795	309692.25	556.5
CV= 0.75 , low mean	fig 8a, 8c	20, 100	866.6866656	1.347550851	795	355514.0625	596.25
CV= 0.8 , low mean	fig 8a, 8c	20, 100	854.8368175	1.258249263	795	404496	636
CV= 0.85 , low mean	fig 8a, 8c	20, 100	841.5451134	1.180695654	795	456638.0625	675.75
CV= 0.9 , low mean	fig 8a, 8c	20, 100	827.0128199	1.112827636	795	511940.25	715.5
CV= 0.95 , low mean	fig 8a, 8c	20, 100	811.4356549	1.053025256	795	570402.5625	755.25
CV= 1 , low mean	fig 8a, 8c	20, 100	795	1	795	632025	795
CV= 0.05 , high mean	fig 8b, 8d	20, 100	1119.176715	24.94977518	1095	2997.5625	54.75
CV= 0.1 , high mean	fig 8b, 8d	20, 100	1142.12626	12.15343419	1095	11990.25	109.5
CV= 0.15 , high mean	fig 8b, 8d	20, 100	1163.401066	7.906926805	1095	26978.0625	164.25
CV= 0.2 , high mean	fig 8b, 8d	20, 100	1182.572966	5.797400066	1095	47961	219
CV= 0.25 , high mean	fig 8b, 8d	20, 100	1199.25335	4.542213092	1095	74939.0625	273.75
CV= 0.3 , high mean	fig 8b, 8d	20, 100	1213.110935	3.713772366	1095	107912.25	328.5
CV= 0.35 , high mean	fig 8b, 8d	20, 100	1223.88557	3.128794406	1095	146880.5625	383.25
CV= 0.4 , high mean	fig 8b, 8d	20, 100	1231.397033	2.695621255	1095	191844	438
CV= 0.45 , high mean	fig 8b, 8d	20, 100	1235.548424	2.36332506	1095	242802.5625	492.75
CV= 0.5 , high mean	fig 8b, 8d	20, 100	1236.324412	2.101349095	1095	299756.25	547.5
CV= 0.55 , high mean	fig 8b, 8d	20, 100	1233.785088	1.890252797	1095	362705.0625	602.25
CV= 0.6 , high mean	fig 8b, 8d	20, 100	1228.056531	1.71708343	1095	431649	657
CV= 0.65 , high mean	fig 8b, 8d	20, 100	1219.319261	1.572883123	1095	506588.0625	711.75
CV= 0.7 , high mean	fig 8b, 8d	20, 100	1207.795793	1.451263666	1095	587522.25	766.5
CV= 0.75 , high mean	fig 8b, 8d	20, 100	1193.738237	1.347550851	1095	674451.5625	821.25
CV= 0.8 , high mean	fig 8b, 8d	20, 100	1177.416749	1.258249263	1095	767376	876
CV= 0.85 , high mean	fig 8b, 8d	20, 100	1159.109307	1.180695654	1095	866295.5625	930.75
CV= 0.9 , high mean	fig 8b, 8d	20, 100	1139.093129	1.112827636	1095	971210.25	985.5
CV= 0.95 , high mean	fig 8b, 8d	20, 100	1117.637789	1.053025256	1095	1082120.063	1040.25
CV= 1 , high mean	fig 8b, 8d	20, 100	1095	1	1095	1199025	1095

Table 5. Appendix A: Overview of values for the distribution of the initial/replacement time used in the paper.

Where the mean and variance of the Weibull distribution with scale λ and shape κ are given in equation 17 and equation 18 respectively.

$$mean_{Wb} = \lambda\Gamma(1 + \frac{1}{\kappa}) \tag{17}$$

$$var_{Wb} = \lambda^2\Gamma(1 + \frac{2}{\kappa}) - {mean_{Wb}}^2 \tag{18}$$

Appendix B

The table 6 of values found for grid-search with step size=0.05, derived from an average run of demand with $t = 12$, $N = 20$, $scale = 1/0.00095$, $shape = 8.907$, $T = 140 * t$, $\tau = 21$, $\mu = 1/0.00174$. Errors are calculated as in equation 14. Used as an example, similar for other cases.

Best values found are for SBA; $\alpha = 0.05$, the error is equal to 29.5612.

Best values found are for TSB; $\alpha = 0.05$, $\beta = 1$, the error is equal to 28.9251.

Best values found are for LES; $\alpha = 1$, $\beta = 0.05$, the error is equals to 18.5201. LES performs best overall.

λ	κ	error SBA	error TSB	error LES	λ	κ	error SBA	error TSB	error LES
0.05	0.05	29.56118044	267.750027	21.30395086	0.3	0.05	37.56065428	573.2758712	20.9653779
0.05	0.1	29.56118044	707.184759	21.30395086	0.3	0.1	37.56065428	1311.631048	20.9653779
0.05	0.15	29.56118044	3186.642553	21.30395086	0.3	0.15	37.56065428	5709.061578	20.9653779
0.05	0.2	29.56118044	19048.92788	21.30395086	0.3	0.2	37.56065428	33910.05291	20.9653779
0.05	0.25	29.56118044	139613.1373	21.30395086	0.3	0.25	37.56065428	248283.2195	20.9653779
0.05	0.3	29.56118044	1239883.671	21.30395086	0.3	0.3	37.56065428	2204659.414	20.9653779
0.05	0.35	29.56118044	13500329.55	21.30395086	0.3	0.35	37.56065428	24004781.91	20.9653779
0.05	0.4	29.56118044	184499215.9	21.30395086	0.3	0.4	37.56065428	328055412.8	20.9653779
0.05	0.45	29.56118044	3271060183	21.30395086	0.3	0.45	37.56065428	5816224481	20.9653779
0.05	0.5	29.56118044	78639524089	21.30395086	0.3	0.5	37.56065428	1.39828E+11	20.9653779
0.05	0.55	29.56118044	2.72036E+12	21.30395086	0.3	0.55	37.56065428	4.83704E+12	20.9653779
0.05	0.6	29.56118044	1.46872E+14	21.30395086	0.3	0.6	37.56065428	2.6115E+14	20.9653779
0.05	0.65	29.56118044	1.39E+16	21.30395086	0.3	0.65	37.56065428	2.47E+16	20.9653779
0.05	0.7	29.56118044	2.72E+18	21.30395086	0.3	0.7	37.56065428	4.83E+18	20.9653779
0.05	0.75	29.56118044	1.44E+21	21.30395086	0.3	0.75	37.56065428	2.55E+21	20.9653779
0.05	0.8	29.56118044	3.18E+24	21.30395086	0.3	0.8	37.56065428	5.65E+24	20.9653779
0.05	0.85	29.56118044	6.76E+28	21.30395086	0.3	0.85	37.56065428	1.20E+29	20.9653779
0.05	0.9	29.56118044	8.89E+34	21.30395086	0.3	0.9	37.56065428	1.58E+35	20.9653779
0.05	0.95	29.56118044	2.76E+45	21.30395086	0.3	0.95	37.56065428	4.91E+45	20.9653779
0.05	1	29.56118044	28.92507731	21.30395086	0.3	1	37.56065428	40.18822816	20.9653779
0.1	0.05	31.94659778	415.2233183	21.02609737	0.35	0.05	37.4352775	578.6015258	20.67419386
0.1	0.1	31.94659778	1061.01143	21.02609737	0.35	0.1	37.4352775	1294.750556	20.67419386
0.1	0.15	31.94659778	4755.031562	21.02609737	0.35	0.15	37.4352775	5597.946023	20.67419386
0.1	0.2	31.94659778	28396.35525	21.02609737	0.35	0.2	37.4352775	33206.33148	20.67419386
0.1	0.25	31.94659778	208088.921	21.02609737	0.35	0.25	37.4352775	243081.4389	20.67419386
0.1	0.3	31.94659778	1847964.446	21.02609737	0.35	0.3	37.4352775	2158412.662	20.67419386
0.1	0.35	31.94659778	20121288.76	21.02609737	0.35	0.35	37.4352775	23501164.95	20.67419386
0.1	0.4	31.94659778	274982982.5	21.02609737	0.35	0.4	37.4352775	321172758.3	20.67419386
0.1	0.45	31.94659778	4875282843	21.02609737	0.35	0.45	37.4352775	5694199032	20.67419386
0.1	0.5	31.94659778	1.17207E+11	21.02609737	0.35	0.5	37.4352775	1.36894E+11	20.67419386
0.1	0.55	31.94659778	4.05451E+12	21.02609737	0.35	0.55	37.4352775	4.73556E+12	20.67419386
0.1	0.6	31.94659778	2.18902E+14	21.02609737	0.35	0.6	37.4352775	2.55671E+14	20.67419386
0.1	0.65	31.94659778	2.07E+16	21.02609737	0.35	0.65	37.4352775	2.42E+16	20.67419386
0.1	0.7	31.94659778	4.05E+18	21.02609737	0.35	0.7	37.4352775	4.73E+18	20.67419386
0.1	0.75	31.94659778	2.14E+21	21.02609737	0.35	0.75	37.4352775	2.50E+21	20.67419386
0.1	0.8	31.94659778	4.74E+24	21.02609737	0.35	0.8	37.4352775	5.53E+24	20.67419386
0.1	0.85	31.94659778	1.01E+29	21.02609737	0.35	0.85	37.4352775	1.18E+29	20.67419386
0.1	0.9	31.94659778	1.32E+35	21.02609737	0.35	0.9	37.4352775	1.55E+35	20.67419386
0.1	0.95	31.94659778	4.12E+45	21.02609737	0.35	0.95	37.4352775	4.81E+45	20.67419386
0.1	1	31.94659778	33.32159026	21.02609737	0.35	1	37.4352775	40.82200364	20.67419386
0.15	0.05	34.09436334	496.3485788	21.00656148	0.4	0.05	36.95789482	580.5790266	20.29430637
0.15	0.1	34.09436334	1228.155523	21.00656148	0.4	0.1	36.95789482	1272.780341	20.29430637
0.15	0.15	34.09436334	5464.331403	21.00656148	0.4	0.15	36.95789482	5468.439344	20.29430637
0.15	0.2	34.09436334	32589.87731	21.00656148	0.4	0.2	36.95789482	32397.96777	20.29430637
0.15	0.25	34.09436334	238767.3853	21.00656148	0.4	0.25	36.95789482	237117.2746	20.29430637
0.15	0.3	34.09436334	2120344.472	21.00656148	0.4	0.3	36.95789482	2105400.571	20.29430637
0.15	0.35	34.09436334	23086975.14	21.00656148	0.4	0.35	36.95789482	22923894.26	20.29430637
0.15	0.4	34.09436334	315512743.8	21.00656148	0.4	0.4	36.95789482	313283545.1	20.29430637
0.15	0.45	34.09436334	5593850970	21.00656148	0.4	0.45	36.95789482	5554327920	20.29430637
0.15	0.5	34.09436334	1.34482E+11	21.00656148	0.4	0.5	36.95789482	1.33532E+11	20.29430637
0.15	0.55	34.09436334	4.6521E+12	21.00656148	0.4	0.55	36.95789482	4.61923E+12	20.29430637
0.15	0.6	34.09436334	2.51165E+14	21.00656148	0.4	0.6	36.95789482	2.49391E+14	20.29430637
0.15	0.65	34.09436334	2.37E+16	21.00656148	0.4	0.65	36.95789482	2.36E+16	20.29430637
0.15	0.7	34.09436334	4.65E+18	21.00656148	0.4	0.7	36.95789482	4.62E+18	20.29430637
0.15	0.75	34.09436334	2.46E+21	21.00656148	0.4	0.75	36.95789482	2.44E+21	20.29430637
0.15	0.8	34.09436334	5.44E+24	21.00656148	0.4	0.8	36.95789482	5.40E+24	20.29430637
0.15	0.85	34.09436334	1.16E+29	21.00656148	0.4	0.85	36.95789482	1.15E+29	20.29430637
0.15	0.9	34.09436334	1.52E+35	21.00656148	0.4	0.9	36.95789482	1.51E+35	20.29430637
0.15	0.95	34.09436334	4.72E+45	21.00656148	0.4	0.95	36.95789482	4.69E+45	20.29430637
0.15	1	34.09436334	36.1605286	21.00656148	0.4	1	36.95789482	41.28876725	20.29430637
0.2	0.05	35.90673027	539.4426718	21.07893068	0.45	0.05	36.33098668	580.6704553	19.90104786
0.2	0.1	35.90673027	1297.133766	21.07893068	0.45	0.1	36.33098668	1249.108193	19.90104786
0.2	0.15	35.90673027	5729.609534	21.07893068	0.45	0.15	36.33098668	5335.176075	19.90104786
0.2	0.2	35.90673027	34124.28565	21.07893068	0.45	0.2	36.33098668	31571.59486	19.90104786
0.2	0.25	35.90673027	249956.7646	21.07893068	0.45	0.25	36.33098668	231026.7608	19.90104786
0.2	0.3	35.90673027	2219647.522	21.07893068	0.45	0.3	36.33098668	2051271.008	19.90104786
0.2	0.35	35.90673027	24168136.56	21.07893068	0.45	0.35	36.33098668	22334461.68	19.90104786
0.2	0.4	35.90673027	330288073.6	21.07893068	0.45	0.4	36.33098668	305228135.7	19.90104786
0.2	0.45	35.90673027	5855808514	21.07893068	0.45	0.45	36.33098668	5411510267	19.90104786
0.2	0.5	35.90673027	1.40779E+11	21.07893068	0.45	0.5	36.33098668	1.30098E+11	19.90104786
0.2	0.55	35.90673027	4.86996E+12	21.07893068	0.45	0.55	36.33098668	4.50046E+12	19.90104786
0.2	0.6	35.90673027	2.62927E+14	21.07893068	0.45	0.6	36.33098668	2.42978E+14	19.90104786
0.2	0.65	35.90673027	2.48E+16	21.07893068	0.45	0.65	36.33098668	2.30E+16	19.90104786
0.2	0.7	35.90673027	4.87E+18	21.07893068	0.45	0.7	36.33098668	4.50E+18	19.90104786
0.2	0.75	35.90673027	2.57E+21	21.07893068	0.45	0.75	36.33098668	2.38E+21	19.90104786
0.2	0.8	35.90673027	5.69E+24	21.07893068	0.45	0.8	36.33098668	5.26E+24	19.90104786
0.2	0.85	35.90673027	1.21E+29	21.07893068	0.45	0.85	36.33098668	1.12E+29	19.90104786
0.2	0.9	35.90673027	1.59E+35	21.07893068	0.45	0.9	36.33098668	1.47E+35	19.90104786
0.2	0.95	35.90673027	4.94E+45	21.07893068	0.45	0.95	36.33098668	4.57E+45	19.90104786
0.2	1	35.90673027	38.03449312	21.07893068	0.45	1	36.33098668	41.64134671	19.90104786
0.25	0.05	37.09690039	561.9087681	21.09946038	0.5	0.05	35.6806229	579.6977477	19.54437738
0.25	0.1	37.09690039	1316.743381	21.09946038	0.5	0.1	35.6806229	1225.468024	19.54437738
0.25	0.15	37.09690039	5772.37937	21.09946038	0.5	0.15	35.6806229	5205.40302	19.54437738
0.25	0.2	37.09690039	34333.51513	21.09946038	0.5	0.2	35.6806229	30769.96292	19.54437738
0.25	0.25	37.09690039	251434.9614	21.09946038	0.5	0.25	35.6806229	225122.2495	19.54437738
0.25	0.3	37.09690039	2232708.554	21.09946038	0.5	0.3	35.6806229	1998799.313	19.54437738
0.25	0.35	37.09690039	24310267.25	21.09946038	0.5	0.35	35.6806229	21763086.46	19.54437738
0.25	0.4	37.09690039	332230360.9	21.09946038	0.5	0.4	35.6806229	297419508.2	19.54437738
0.25	0.45	37.09690039	5890243944	21.09946038	0.5	0.45	35.6806229	5273067921	19.54437738
0.25	0.5	37.09690039	1.41607E+11	21.09946038	0.5	0.5	35.6806229	1.2677E+11	19.54437738
0.25	0.55	37.09690039	4.8986E+12	21.09946038	0.5	0.55	35.6806229	4.38532E+12	19.54437738
0.25	0.6	37.09690039	2.64474E+14	21.09946038	0.5	0.6	35.6806229	2.36762E+14	19.54437738
0.25	0.65	37.09690039	2.50E+16	21.09946038	0.5	0.65	35.6806229	2.24E+16	19.54437738
0.25	0.7	37.09690039	4.90E+18	21.09946038	0.5	0.7	35.6806229	4.38E+18	19.54437738
0.25	0.75	37.09690039	2.59E+21	21.09946038	0.5	0.75	35.6806229	2.31E+21	19.54437738
0.25	0.8	37.09690039	5.72E+24	21.09946038	0.5	0.8	35.6806229	5.13E+24	19.54437738
0.25	0.85	37.09690039	1.22E+29	21.09946038	0.5	0.85	35.6806229	1.09E+29	19.54437738
0.25	0.9	37.09690039	1.60E+35	21.09946038	0.5	0.9	35.6806229	1.43E+35	19.54437738
0.25	0.95	37.09690039	4.97E+45	21.09946038	0.5	0.95	35.6806229	4.45E+45	19.54437738
0.25	1	37.09690039	39.30390458	21.09946038	0.5	1	35.6806229	41.91390991	19.54437738

α	β	error SBA	error TSB	error LES	α	β	error SBA	error TSB	error LES
0.55	0.05	35.07046032	578.1451313	19.24757277	0.8	0.05	33.10349799	569.6683259	18.64448302
0.55	0.1	35.07046032	1202.760164	19.24757277	0.8	0.1	33.10349799	1114.855603	18.64448302
0.55	0.15	35.07046032	5082.697196	19.24757277	0.8	0.15	33.10349799	4616.222836	18.64448302
0.55	0.2	35.07046032	30013.94917	19.24757277	0.8	0.2	33.10349799	27150.49356	18.64448302
0.55	0.25	35.07046032	219556.1299	19.24757277	0.8	0.25	33.10349799	198488.199	18.64448302
0.55	0.3	35.07046032	1949337.861	19.24757277	0.8	0.3	33.10349799	1762144.041	18.64448302
0.55	0.35	35.07046032	21224494.48	19.24757277	0.8	0.35	33.10349799	19186144.14	18.64448302
0.55	0.4	35.07046032	290058913.9	19.24757277	0.8	0.4	33.10349799	262202113.4	18.64448302
0.55	0.45	35.07046032	5142568941	19.24757277	0.8	0.45	33.10349799	4648684417	18.64448302
0.55	0.5	35.07046032	1.23632E+11	19.24757277	0.8	0.5	33.10349799	1.11759E+11	18.64448302
0.55	0.55	35.07046032	4.27679E+12	19.24757277	0.8	0.55	33.10349799	3.86606E+12	18.64448302
0.55	0.6	35.07046032	2.30903E+14	19.24757277	0.8	0.6	33.10349799	2.08727E+14	18.64448302
0.55	0.65	35.07046032	2.18E+16	19.24757277	0.8	0.65	33.10349799	1.97E+16	18.64448302
0.55	0.7	35.07046032	4.27E+18	19.24757277	0.8	0.7	33.10349799	3.86E+18	18.64448302
0.55	0.75	35.07046032	2.26E+21	19.24757277	0.8	0.75	33.10349799	2.04E+21	18.64448302
0.55	0.8	35.07046032	5.00E+24	19.24757277	0.8	0.8	33.10349799	4.52E+24	18.64448302
0.55	0.85	35.07046032	1.06E+29	19.24757277	0.8	0.85	33.10349799	9.61E+28	18.64448302
0.55	0.9	35.07046032	1.40E+35	19.24757277	0.8	0.9	33.10349799	1.26E+35	18.64448302
0.55	0.95	35.07046032	4.34E+45	19.24757277	0.8	0.95	33.10349799	3.92E+45	18.64448302
0.55	1	35.07046032	42.12910529	19.24757277	0.8	1	33.10349799	42.74833088	18.64448302
0.6	0.05	34.52689769	576.314533	19.01632516	0.85	0.05	32.94154969	568.85155	18.6628484
0.6	0.1	34.52689769	1181.462126	19.01632516	0.85	0.1	32.94154969	1103.85581	18.6628484
0.6	0.15	34.52689769	4968.81349	19.01632516	0.85	0.15	32.94154969	4557.495671	18.6628484
0.6	0.2	34.52689769	29313.6072	19.01632516	0.85	0.2	32.94154969	26790.23477	18.6628484
0.6	0.25	34.52689769	214401.5486	19.01632516	0.85	0.25	32.94154969	195838.3626	18.6628484
0.6	0.3	34.52689769	1903535.573	19.01632516	0.85	0.3	32.94154969	1738601.02	18.6628484
0.6	0.35	34.52689769	20725750.53	19.01632516	0.85	0.35	32.94154969	18929786.92	18.6628484
0.6	0.4	34.52689769	283242901.4	19.01632516	0.85	0.4	32.94154969	258698650.7	18.6628484
0.6	0.45	34.52689769	5021725081	19.01632516	0.85	0.45	32.94154969	4586570110	18.6628484
0.6	0.5	34.52689769	1.20727E+11	19.01632516	0.85	0.5	32.94154969	1.10266E+11	18.6628484
0.6	0.55	34.52689769	4.1763E+12	19.01632516	0.85	0.55	32.94154969	3.8144E+12	18.6628484
0.6	0.6	34.52689769	2.25477E+14	19.01632516	0.85	0.6	32.94154969	2.05938E+14	18.6628484
0.6	0.65	34.52689769	2.13E+16	19.01632516	0.85	0.65	32.94154969	1.95E+16	18.6628484
0.6	0.7	34.52689769	4.17E+18	19.01632516	0.85	0.7	32.94154969	3.81E+18	18.6628484
0.6	0.75	34.52689769	2.20E+21	19.01632516	0.85	0.75	32.94154969	2.01E+21	18.6628484
0.6	0.8	34.52689769	4.88E+24	19.01632516	0.85	0.8	32.94154969	4.46E+24	18.6628484
0.6	0.85	34.52689769	1.04E+29	19.01632516	0.85	0.85	32.94154969	9.48E+28	18.6628484
0.6	0.9	34.52689769	1.36E+35	19.01632516	0.85	0.9	32.94154969	1.25E+35	18.6628484
0.6	0.95	34.52689769	4.24E+45	19.01632516	0.85	0.95	32.94154969	3.87E+45	18.6628484
0.6	1	34.52689769	42.30231793	19.01632516	0.85	1	32.94154969	42.82301126	18.6628484
0.65	0.05	34.05795454	574.4087565	18.84726108	0.9	0.05	32.85704345	568.6599887	18.7258011
0.65	0.1	34.05795454	1161.842819	18.84726108	0.9	0.1	32.85704345	1095.69929	18.7258011
0.65	0.15	34.05795454	4864.640762	18.84726108	0.9	0.15	32.85704345	4513.013298	18.7258011
0.65	0.2	34.05795454	28673.87941	18.84726108	0.9	0.2	32.85704345	26516.77816	18.7258011
0.65	0.25	34.05795454	209694.2713	18.84726108	0.9	0.25	32.85704345	193826.6176	18.7258011
0.65	0.3	34.05795454	1861709.505	18.84726108	0.9	0.3	32.85704345	1720727.084	18.7258011
0.65	0.35	34.05795454	20270306.08	18.84726108	0.9	0.35	32.85704345	18735159.76	18.7258011
0.65	0.4	34.05795454	277018638.4	18.84726108	0.9	0.4	32.85704345	256038811.7	18.7258011
0.65	0.45	34.05795454	4911372592	18.84726108	0.9	0.45	32.85704345	4539412750	18.7258011
0.65	0.5	34.05795454	1.18074E+11	18.84726108	0.9	0.5	32.85704345	1.09132E+11	18.7258011
0.65	0.55	34.05795454	4.08452E+12	18.84726108	0.9	0.55	32.85704345	3.77518E+12	18.7258011
0.65	0.6	34.05795454	2.20522E+14	18.84726108	0.9	0.6	32.85704345	2.03821E+14	18.7258011
0.65	0.65	34.05795454	2.08E+16	18.84726108	0.9	0.65	32.85704345	1.93E+16	18.7258011
0.65	0.7	34.05795454	4.08E+18	18.84726108	0.9	0.7	32.85704345	3.77E+18	18.7258011
0.65	0.75	34.05795454	2.16E+21	18.84726108	0.9	0.75	32.85704345	1.99E+21	18.7258011
0.65	0.8	34.05795454	4.77E+24	18.84726108	0.9	0.8	32.85704345	4.41E+24	18.7258011
0.65	0.85	34.05795454	1.01E+29	18.84726108	0.9	0.85	32.85704345	9.38E+28	18.7258011
0.65	0.9	34.05795454	1.33E+35	18.84726108	0.9	0.9	32.85704345	1.23E+35	18.7258011
0.65	0.95	34.05795454	4.15E+45	18.84726108	0.9	0.95	32.85704345	3.83E+45	18.7258011
0.65	1	34.05795454	42.44424205	18.84726108	0.9	1	32.85704345	42.88878787	18.7258011
0.7	0.05	33.66436064	572.5794183	18.73346434	0.95	0.05	32.81797791	569.2677849	18.80994299
0.7	0.1	33.66436064	1144.083766	18.73346434	0.95	0.1	32.81797791	1090.818321	18.80994299
0.7	0.15	33.66436064	4770.740151	18.73346434	0.95	0.15	32.81797791	4484.735836	18.80994299
0.7	0.2	33.66436064	28097.81237	18.73346434	0.95	0.2	32.81797791	26341.84093	18.80994299
0.7	0.25	33.66436064	205456.2468	18.73346434	0.95	0.25	32.81797791	192538.8282	18.80994299
0.7	0.3	33.66436064	1824054.097	18.73346434	0.95	0.3	32.81797791	1709284.746	18.80994299
0.7	0.35	33.66436064	19860277.62	18.73346434	0.95	0.35	32.81797791	18610565.13	18.80994299
0.7	0.4	33.66436064	271415048.6	18.73346434	0.95	0.4	32.81797791	254336060.2	18.80994299
0.7	0.45	33.66436064	4812024275	18.73346434	0.95	0.45	32.81797791	4509223981	18.80994299
0.7	0.5	33.66436064	1.15686E+11	18.73346434	0.95	0.5	32.81797791	1.08406E+11	18.80994299
0.7	0.55	33.66436064	4.0019E+12	18.73346434	0.95	0.55	32.81797791	3.75008E+12	18.80994299
0.7	0.6	33.66436064	2.16061E+14	18.73346434	0.95	0.6	32.81797791	2.02465E+14	18.80994299
0.7	0.65	33.66436064	2.04E+16	18.73346434	0.95	0.65	32.81797791	1.91E+16	18.80994299
0.7	0.7	33.66436064	4.00E+18	18.73346434	0.95	0.7	32.81797791	3.75E+18	18.80994299
0.7	0.75	33.66436064	2.11E+21	18.73346434	0.95	0.75	32.81797791	1.98E+21	18.80994299
0.7	0.8	33.66436064	4.68E+24	18.73346434	0.95	0.8	32.81797791	4.38E+24	18.80994299
0.7	0.85	33.66436064	9.94E+28	18.73346434	0.95	0.85	32.81797791	9.32E+28	18.80994299
0.7	0.9	33.66436064	1.31E+35	18.73346434	0.95	0.9	32.81797791	1.23E+35	18.80994299
0.7	0.95	33.66436064	4.06E+45	18.73346434	0.95	0.95	32.81797791	3.81E+45	18.80994299
0.7	1	33.66436064	42.56246815	18.73346434	0.95	1	32.81797791	42.94736216	18.80994299
0.75	0.05	33.34565828	570.9566879	18.66766639	1	0.05	32.75	570.8730092	18.52005071
0.75	0.1	33.34565828	1128.353655	18.66766639	1	0.1	32.75	1089.739965	18.52005071
0.75	0.15	33.34565828	4687.675559	18.66766639	1	0.15	32.75	4475.096803	18.52005071
0.75	0.2	33.34565828	27588.53191	18.66766639	1	0.2	32.75	26280.02823	18.52005071
0.75	0.25	33.34565828	201710.0803	18.66766639	1	0.25	32.75	192082.092	18.52005071
0.75	0.3	33.34565828	1790769.714	18.66766639	1	0.3	32.75	1705225.168	18.52005071
0.75	0.35	33.34565828	19497846.3	18.66766639	1	0.35	32.75	18566359.68	18.52005071
0.75	0.4	33.34565828	266461939.4	18.66766639	1	0.4	32.75	253731933.3	18.52005071
0.75	0.45	33.34565828	4724208596	18.66766639	1	0.45	32.75	4498513169	18.52005071
0.75	0.5	33.34565828	1.13575E+11	18.66766639	1	0.5	32.75	1.08149E+11	18.52005071
0.75	0.55	33.34565828	3.92887E+12	18.66766639	1	0.55	32.75	3.74117E+12	18.52005071
0.75	0.6	33.34565828	2.12118E+14	18.66766639	1	0.6	32.75	2.01984E+14	18.52005071
0.75	0.65	33.34565828	2.00E+16	18.66766639	1	0.65	32.75	1.91E+16	18.52005071
0.75	0.7	33.34565828	3.93E+18	18.66766639	1	0.7	32.75	3.74E+18	18.52005071
0.75	0.75	33.34565828	2.07E+21	18.66766639	1	0.75	32.75	1.97E+21	18.52005071
0.75	0.8	33.34565828	4.59E+24	18.66766639	1	0.8	32.75	4.37E+24	18.52005071
0.75	0.85	33.34565828	9.76E+28	18.66766639	1	0.85	32.75	9.30E+28	18.52005071
0.75	0.9	33.34565828	1.28E+35	18.66766639	1	0.9	32.75	1.22E+35	18.52005071
0.75	0.95	33.34565828	3.99E+45	18.66766639	1	0.95	32.75	3.80E+45	18.52005071
0.75	1	33.34565828	42.66248574	18.66766639	1	1	32.75	43	18.52005071

Table 6. Appendix B: Gridsearch

Appendix C

The mathematics for the DT model is given here as it was given in the paper by Wang et al. [11]. The conditional function for $f_U(u)$ is the same, and stated again in 11

$$f_U(u|U > t_p - t_r) = \begin{cases} \frac{f_U(u+t+p-t_r)}{\int_{t_p-t_r}^{\infty} f_U(u)du} & t_r < t_p,\ 0 < u < \infty \\ f_U(u) & t_r = t_p,\ 0 < u < \infty \end{cases} \tag{19}$$

There are two scenarios defined an item can be in, reversed from how it is stated in section 4.2.

Scenario 1: the item does not have to be inspected at the current check-up point.

Scenario 2: the item is inspected at the current check-up point.

The concluding equation is given in figure 10. Where it says $E[N_r(t_p + t, t_p + 2t)]$, it should say $E[N_r(t_p + t, t_p + 3t)]$, as forecasts for failure is still made for items in scenario 1 until $t_p + 3t$, and it says that they predict for 28 time units, and that t is set to 14. This is strange, as the prediction by the way data is simulated is updated and the nature of demand vector x is divided for every time unit t.

The period in which they are forecasting is given at $t_p + t$ if an item is in scenario 1.

The period in which they are forecasting seems given at t_p if an item is in scenario 2, the probability of failure is included from $(t_p, t_p + 2t)$. There are two problems with this statement. Though, starting at t_p, seems to be off as the prediction starts a period later at $t_p + 1$: "$E[N_r(t_p + t, t_p + 2t)]$". Secondly, this is not in accommodation with the way data is simulated. This predicts that the maintenance in an period counts as demand for the beginning of that period. This is stated in figure 3 and taken from the paper by Wang et al. [11]. There is no prediction for the current immediate inspection and it is not clear if there is a fix at the beginning in this period, which should update t_p.

$$\begin{aligned} E(N_r(t_p+t,t_p+2t)) &\approx \sum_{i=1}^{N} \begin{pmatrix} \Pr(\text{the } i\text{th item identified at } t_p+2t \text{ to be faulty}) \\ +\Pr(\text{the } i\text{th item having a failure in}(t_p+t,t_p+3t)) \end{pmatrix} \delta(i) \\ &+ \sum_{i=1}^{N} \begin{pmatrix} \Pr(\text{the } i\text{th item identified at } t_p+2t \text{ to be faulty}) \\ \Pr(\text{the} i\text{th item having a failure in}(t_p,t_p+2t)) \end{pmatrix} (1-\delta(i)) \\ &= \sum_{i=1}^{N} \left(\int_0^{2t} f_U(u|U>t_p-t_{ir})du + \int_{2t}^{3t} f_U(u|U>t_p-t_{ir})F_H(3t-u)du \right) \delta(i) \\ &+ \sum_{i=1}^{N} \int_0^{2t} f_U(u|U>t_p-t_{ir})du(1-\delta(i)), \end{aligned}$$

where

$$\delta(i) = \begin{cases} 1 & \text{if the } i\text{th item is in scenario } (1) \\ 0 & \text{otherwise} \end{cases}.$$

Figure 10. Appendix C: A snippit of the mathematics used in Wang et al. [11].