Title of thesis: DTRP - An extension of the Delay Time model to include Replacement Possibility.

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Abstract

Forecasting demand for upcoming periods is an issue that has been thoroughly discussed and researched. It is of great importance as company strategies and schedules are made based on these forecasts every day. This thesis investigates and compares four types of existing forecasting methods and proposes a new one that is consistent with a failure mode in which items can also be replaced with newer models instead of invariably fixed. Specifically, there are two major contributions made to the existing literature.

Firstly, it provides a detailed explanation of a comparison of four existing methods used in forecasting of demand. It evaluates the strength and weaknesses of these methods, and states the most efficient method given the setup of the sample. The findings indicate that the best forecasting method depends on the error function and the sample size of the items.

Secondly, this paper defines two modes of failure and contributes a new forecasting method tailored to it. The first failure mode is set up in such a way that a failure of an item is always cause for demand. In the second failure mode, an item is either fixed or replaced. Only the items that are fixed are cause for demand. In accordance with the second failure mode, a new method is proposed. This method is under certain circumstances an improvement to existing methods.
## Contents

1 Problem Statement  ........................................... 1

2 Literature  .................................................. 2

3 Simulation study  ........................................... 4

4 Methodology  .................................................. 6
   4.1 Time Series Methods  .................................... 7
   4.2 Delay Time model: DT  .................................. 8
   4.3 Delay Time model with replacement possibility: DTRP  ........ 11

5 Results  ......................................................... 12
   5.1 Reproduction  ............................................ 12
   5.2 The effect of variance coefficient and sample size on DT  ........ 14
   5.3 Delay Time Replacement Possibility: DTRP  .................... 15

6 Conclusion  ..................................................... 18
List of Figures

1  mba Insight graph illustrating the average value of an aircraft spare part over time.
   source: www.mba.aero/ ................................................................. 1
2  Schematic demand pattern .............................................................. 5
3  Block scheme, age based maintenance $o^*$ is set to infinity if replacement is disregarded. 6
4  Demand simulation ........................................................................ 13
5  A sum of the absolute forecasting error for different types of forecasting: SBA, TSB, LES and DT ........................................................................ 14
7  Demand simulation Replacement ....................................................... 16
8  Errors of five forecasting methods .................................................... 17
9  Advice on which method to apply ...................................................... 19
10 Appendix C: A snippit of the mathematics used in Wang et al. [11]. ............... 25

List of Tables

1  Attributes of three time series methods for demand forecast .................... 7
2  Overview of the performances of four forecasting methods, one failure mode. ......................................................... 13
3  Distribution characteristics and errors per demand pattern and sample size. ......................................................... 15
4  Overview of the performances of five forecasting methods, two failure modes. ......................................................... 16
5  Appendix A: Overview of values for the distribution of the initial/replacement time used in the paper. ......................... 21
6  Appendix B: Gridsearch .................................................................... 24
1 Problem Statement

Malfunctioning of appliances can have significant economic consequences. The problem must be resolved as swiftly as possible. At a private level, it causes discomfort. At a corporate level it causes loss in revenue, delays, lower quality and, again, discomfort.

Researchers have been investigating and are able to make reasonable forecasts of spare part demand, occurring whenever an item fails. Fixing a failed item is called corrective maintenance. The other cause for demand is regular preventive maintenance. At preventive maintenance, the parts of an appliance are checked to catch a faulty item before it causes a failure. Preventive maintenance causes a spike in demand at the size of faulty appliances at the time of inspection, because there is a delay time between the appearance of an error and the failure time. This causes, along with the randomness of error and failure time, an intermittent demand pattern [11]. The types of maintenance differ in their nature: preventive maintenance is deterministic in occurrence and stochastic in demand size, whereas corrective maintenance is stochastic in occurrence and demand size is always one.

The variation in demand occurrence is not all. Additionally the sample size can also to predict for can also vary over time. As an appliance gets outdated, or is discarded, a user can choose to update to a newer model, rather than have it fixed again. A practical application of this is the maintenance of wind turbines. Proper lubrication and other preventive actions can avoid expensive and unnecessary repairs. At the same time, rapid development ensures that companies are producing larger windmills and maintenance equipment providers, such as lubrication producers, need to produce materials that keep up with the industry [5]. Moreover as illustration, Figure 1, shows that spare parts for outdated aircraft model no longer sell well over time.

![Spare Parts Value Over Time](http://www.mba.aero/)

**Figure 1.** mba Insight graph illustrating the average value of an aircraft spare part over time. source: www.mba.aero/

Intermittent demand patterns and forecasting for such has been researched thoroughly. The literature in section 2 shows four existing methods to forecast demand, three time series model, and the Delay Time model (DT) [11]. The DT model is the cornerstone for this thesis. In addition to existing literature, the DT method is compared to more methods than just SBA, as it has been in the paper by Wang et al. [11]. It is now also compared to TSB and LES. Also, this thesis provides an insight
into a new model in which there are two failure modes. Originally there was only one failure mode in which a failed item was always fixed and cause for demand of a spare part, as applied in the paper by Wang et al. [11]. The second (new) failure mode, a failed/faulty item is either fixed OR replaced with a new item, based on an additional variable that states at which point in time an item becomes outdated. If an item is replaced, there is no demand for a spare part. These research goals can be summarized in the research questions (RQ) stated below.

RQ1: How can one incorporate the two failure modes in a DT model to forecast spare parts demand for items that might be replaced?

RQ2: What is the best method for forecasting spare part demand for one and for two failure modes?

The build-up of the paper is as follows: section 2 discusses past literature, section 3 provides insight in how the data used in this paper is generated and applied, in section 4 the methodology is discussed, section 5 discusses tests and their results for the methodology and lastly section 6 states a summary and some concluding remarks.

2 Literature

There have been many models to forecast demand over the years. One of the first and a basis for future models was put forward by Croston, 1972 [2]. He proposed a method that deals with demand patterns by forecasting the interval between the occurrence of demand and the demand size when this happens uncorrelated to each other. Croston time series method outperformed the Single Exponential Smoothing (SES). Croston’s method estimated the mean demand per period. In time period \( t \), \( z_t \) is the actual demand size, \( z'_t \) is the smoothed estimate of the next period for \( z_t \), \( p_t \) is the actual demand interval, \( p'_t \) is the smoothed estimate of the next period for \( p_t \), \( \alpha \) is a smoothing parameter where \( 0 \leq \alpha \leq 1 \), \( F_t \) is the forecast for the next period. In the method, the variables are only updated in periods with non-zero demand. The forecast is given in equation 1.

\[
F_t = \frac{z'_t}{p_t}
\]  

where if \( z_t > 0 \): \( p'_t = p'_{t-1} + \alpha(p_t - p'_{t-1}), z'_t = z'_{t-1} + \alpha(z_t - z'_{t-1}) \)

The method by Croston was proven to be biased in 2001 by Syntetos and Boylan [8], with a bias equal to \( Bias \approx \frac{\sigma^2}{2\alpha} \cdot E[z_t] \cdot \frac{p-1}{p^2} \). Subsequently, Syntetos and Boylan, 2005 [9] put forward a solution that combats the bias, transforming the forecast from equation 1 slightly. They use the same definition for the variables. The estimator is called the Syntetos Boylan Approximator (SBA). The forecast of the SBA method is given in equation 2.

\[
F_t = (1 - \frac{\alpha}{2}) \cdot \frac{z'_t}{p_t}
\]  

\( 2 \)
The paper by Teunter et al., 2011 [10] extended the method with an unbiased estimator using demand occurrence probability and instead of demand interval, named after Teunter, Syntetos and Babai (TSB). This allegedly incorporates a faster reaction to a decrease in demand and had a smaller variance than the SBA and Croston methods given that the right smoothing constants are used. When an item becomes obsolescent its forecasts degenerate exponentially to zero. The variable are defined with $\rho_t$ as the occurrence of demand in period $t$ and $\rho_t'$ as the probability of a demand occurrence at the end of period $t$. Furthermore, they introduce two different smoothing variables $\alpha$ and $\beta$ for demand size and demand occurrence respectively. The forecast is given in equation 3:

$$F_t = \rho_t' \cdot z_t'$$

where if $\rho_t = 0$: $\rho_t' = \rho_t' + \beta(0 - \rho_t')$, $z_t' = z_{t-1}'$

else $\rho_t = 1$: $\rho_t' = \rho_t' + \beta(1 - \rho_t')$, $z_t' = z_{t-1}' + \alpha(z_t - z_{t-1}')$

Another forecasting method showing promising results is Linear-Exponential Smoothing (LES) put forward by Prestwich et al., 2014 [6]. It is similar to the method of Hyperbolic-Exponential Smoothing (HES) [7] put forward by Prestwich et al, 2014. HES estimates demand size, $z_t'$, and inter-demand interval, $p_t'$, and increases the interval in periods where demand is zero, $z_t = 0$ which decreases the forecast for the next period: $F_t > F_{t-1}$. This is comparable to a combination of TSB, updating when nonzero, and SBA working with demand interval. The forecast is given in equation 4.

$$F_t = \begin{cases} 
\frac{z_t'}{p_t'} & \text{if } z_t > 0 \\
\frac{z_t'}{p_t' + \frac{\beta p_t}{2}} & \text{if } z_t = 0
\end{cases}$$

where if $z_t > 0$: $p_t' = p_{t-1}' + \beta(p_t - p_{t-1}')$, $z_t' = z_{t-1}' + \alpha(z_t - z_{t-1}')$

The method LES [6] differs from HES [7] in the way it updates $F_t$, and enables LES to forecast zero demand, which was not possible yet with HES or TSB. They update $z_t'$ and $p_t'$ the same as with HES. The forecasts for LES are given in equation 5.

$$F_t = \begin{cases} 
\frac{z_t'}{p_t'} & \text{if } z_t > 0 \\
\frac{z_t'}{p_t'} \cdot \max\{0, 1 - \frac{\beta p_t}{2p_t'}\} & \text{if } z_t = 0
\end{cases}$$

In variance to all of the methods mentioned (Croston, SBA, TSB, HES, LES) that react to data, there is also a method proposed by Wang et al., 2011 [11] that incorporates the sources of demand patterns to develop a pro-active mannerism to forecast the demand for spare parts. The forecast equals the addition of the probabilities of failure within the time period and the probability of a part being faulty at moment of inspection at the beginning of the time period. The method is called
Delay Time method (DT), as it builds upon the intermittent nature of demand sizes provided by the delay time between the point an error arose and the point it caused a failure. Allegedly, given that the distributions for occurrence of error and failure, it outperformed SBA. A fuller description of the mathematics to support their method is given in section 4.2.

To summarize the literature, when the paper by Wang et al. [11] was published in 2011, SBA was best shown to outperform other Croston-based methods by a paper from Eaves et al. [3], Gutierrez et al. [4] and Syntetos et al. [9], but DT outperformed SBA still. Since then, improvements different time series based methods have been published: TSB, HES and LES. HES is not included in the analysis, because HES and LES are very similar, but LES has been proven superior in Prestwich et al, [6]. The new model proposed in this paper: DTRP is an extension of DT that also includes the probability of items being replaced. DTRP will be compared to DT, SBA, TSB and LES in order to provide a thorough understanding of the possibilities or limitations of DTRP contrasting ingrained methods in the field of forecasting spare parts demand. In addition, the findings from [11] are reproduced when comparing the performances of DT and SBA.

3 Simulation study

The data used to test the performances is simulated. Two types of data are considered, distinguished as two different failure modes. The first one is in accommodation with the DT model where all failures/faulty items are always fixed and cause demand. The second failure mode is in accommodation with the DTRP model, where failures/faulty items can either be fixed and cause demand or replaced with a new model, without causing demand for a spare part. The maintenance schedule is age based, meaning that an item is only checked if it was checked longer than a certain time ago. This is a practical situation, which saves a repair man some time.

For the data, the failure pattern is implemented described in the paper by Wang et al. [11]. A total of \( N \) items are in the sample size. The time point item \( i \) presents an error is given at \( U_i \), if an error has not occurred already. This error causes a failure at time point \( S_i \). The time it takes between \( U_i \) and \( S_i \) is called the delay time, \( H_i \). Items are inspected at a regular interval of length \( t \). If the item is inspected and there is an error, it is fixed immediately. Also, if a failure occurs, the item is fixed/replaced immediately. The error time is Weibull distributed and the delay time is exponentially distributed. The last time an item was placed is given as: \( t_{ip} \). The variable that indicates from what usage time on an item is to be inspected, is given with \( \tau \). The last checkup time is \( t_{ip} \). Using age based maintenance, if an item is very freshly placed it does not have to be inspected; if \( t_{now} - t_r \leq \tau \).

When the data includes replacement possibility, an item gets outdated at time point \( O_i \). If an item is inspected and faulty or fails after \( O_i \), it is replaced with a new model. If this is the case, the item doesn’t cause demand for a spare part. The occurrence of a part needing replacement is
A representation of the failure pattern is given in figure 2.

To implement the simulation with a solid foundation, five assumptions have to be made in the age based maintenance scheme.

1. If the item is inspected, any present error is fixed, causing demand equal to 1 per item, or the item is replaced, causing no demand.

2. Inspections are undertaken at a regular interval $t$.

3. When the technicians performed an inspection or fixes a failure, they put a label on the item showing the time of the inspection or replacement so that the next inspection is taking place after $t$ time from that particular day.

4. The distribution for the time elapsed between the introduction of a new item and the point $U_i$ is Weibull; the delay time distribution is exponential (fitted result).

5. The technicians goes around every period $t$ but only those associated with a time elapsed since the last inspection or replacement of more than $\tau$ are inspected again according to the label attached to the appliance.

The current time period looked at is time period $m$, ranged between $1 \leq m \leq int(T/t)$. The moment of replacement is given as $O_i$. If an item is never replaced, $O_i$ is not generated but set to infinity, meaning that the blue part is always true. If items are replaced, $O_i$ is generated as the expiration date of an item, and as soon as the item is faulty at inspection or failed, it is replaced. With these variables, the data is simulated in a manner as represented in figure 3. The gray boxes indicate an update mechanism of planned maintenances that are happening in periods without failures/errors, in order to keep track of the last time of inspection: $t_{ip}$. These blocks are an addition to the block scheme in the paper by Wang et al. [11].
4 Methodology

The performance of three types of models is to be compared: time series models based on Croston’s method [2], the DT model [11] and an extension of the DT model: DTRP. First the Croston based methods and their implementations are given in section 4.1. Secondly, the math for the DT method is described as it is implemented in this paper in section 4.2. This is different, but roughly equivalent, to how it was proposed in the original paper by Wang et al. [11], which is stated in Appendix C. Lastly, the DTRP method is proposed in section 4.3.
4.1 Time Series Methods

Croston based time series methods use previously observed demand and the time passed between previous non-zero demand periods to forecast the next period’s average demand. As discussed in section 2, three methods are considered: SBA [9], TSB [10] and LES [6]. These models are one step ahead forecasts, relying on the actual values from last period and using these to adjust to previous forecasts. An overview of the methods is given in table 1.

<table>
<thead>
<tr>
<th>Method</th>
<th>Variables</th>
<th>Forecast</th>
<th>Starting Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>SBA [9]</td>
<td>smoothing var $\alpha$</td>
<td>$F_t = (1 - \frac{z}{2}) \cdot \frac{z_t}{p_t}$</td>
<td>$z_0 = 0$; $p_0 = 0$; $\rho_0 = 0$;</td>
</tr>
<tr>
<td>TSB [10]</td>
<td>smoothing var $\alpha$, $\beta$</td>
<td>$F_t = z_t \cdot p_t$</td>
<td>$z_0 = 0$; $p_0 = 0$; $\rho_0 = 0$;</td>
</tr>
<tr>
<td>LES [6]</td>
<td>smoothing var $\alpha$, $\beta$</td>
<td>$F_t = \begin{cases} \frac{z_t}{p_t} &amp; \text{if } z_t &gt; 0 \ \left(\frac{z_t}{p_t}\right) \cdot (1 - \beta p_t / 2p_t)^+ &amp; \text{if } z_t = 0 \end{cases}$</td>
<td>$z_0 = 0$; $p_0 = 0$; $\rho_0 = 0$;</td>
</tr>
</tbody>
</table>

Table 1. Attributes of three time series methods for demand forecast

Note that these three methods only rely on past demand. This means two things to keep in mind for the testing. Firstly, they need some adjustment time to cope with the demand pattern: in each test the first 20/40 time periods are disregarded as an in-sample period for the time-series based methods. Secondly, because these methods do not rely on the number of items that are in place in the model, they are practically equal for the model with one failure mode as for the model with two failure modes.

As the smoothing variable is very important for the implications of these methods, when the distributive values for the initial and delay time are known. As the distribution is known for the DT and DTRP prediction, this can also be applied for the grid-search without making any new assumptions. The best values for $\alpha$ and $\beta$ for each of the methods is calculated using grid search with step sizes equal to 0.05 along the values ranging in $[0.05, 1]$. An example of grid-search is stated in the values displayed in Appendix B, used for a graph in section 5.1.
4.2 Delay Time model: DT

The DT model uses the distributive characteristics of the initial time and delay time function of items to pro-actively forecast demand. It is not based on past demand occurrence like the time series method. The initial time distribution function is distributed Weibull(λ, κ) by assumption. The delay time distribution function is distributed Exponential(µ). These distributions are used to forecast the probability of an item needing to be fixed and therefore demand for spare parts, in a time period of length t.

The pdf. and the cdf. of the initial time of an error is given by \( f_U(u) \) and \( F_U(u) \). The conditional distribution uses that the initial time \( U_i \) is zero at the time if placement of item \( i \), \( t_{ir} \), given that \( i \) was last inspected and non-faulty at \( t_{ip} \). This is gives the conditional pdf. for general \( U \) is given in equation 6.

\[
 f_{U}(u|U > t_{ip} - t_{ir}) = \begin{cases} 
 \frac{f_U(u+t_{ip}-t_{ir})}{\int_{t_{ip}-t_{ir}}^{\infty} f_U(u) du} & t_{ir} < t_{ip}, \ 0 < u < \infty \\
 f_U(u) & t_{ir} = t_{ip}, \ 0 < u < \infty 
\end{cases} 
\] (6)

For the delay time, the pdf. and cdf. are given as \( f_H(h) \) and \( F_H(h) \) respectively. It is known that after an error occurs, if \( H \) has not already happened at the current time, \( t_{now} \). The conditional pdf. for general \( H \) is given in equation 7.

\[
 f_{H}(h|H > t_{now}) = \begin{cases} 
 \frac{f_H(h+t_{now})}{\int_{t_{now}}^{\infty} f_H(h) dh} & u < t_{now}, \ 0 < h < \infty \\
 f_H(h) & u \geq t_{now}, \ 0 < h < \infty 
\end{cases} 
\] (7)

For simplicity we make Assumption 1: In the calculations of the DT model, an item has never a failure after replacement within the time span of \( t: U_i + H_i > t \). E.q. it is not possible to fail or be faulty at inspection at \( t_{now} \), be replaced and have another error occurring between \( (t_{now}, t_{now} + t) \). This calculation error made because of this assumption, \( \epsilon \), is expected to be zero.

These definitions of the distributions for \( f_U(u) \) and \( f_H(h) \) and Assumption 1 are taken from the paper by Wang et al. [11]. But as there were some typing mistakes and inconsistencies, the following demand forecasts are different than used in that paper. The main difference is that in this way every time period is forecasted for, whereas the paper by Wang et al. [11] simulates for two time periods. Though, the formulations are equivalent as they both still rely on the initial time and delay time functions, \( f_U(u) \) and \( f_H(h) \) respectively. Their mathematics of the DT model is given in Appendix C.

The expected demand for a period with age based maintenance data is calculated. For this type of calculation, there are two scenarios for all items \( i \in N \):

**Scenario 1:** the item is inspected at period \( t_{now} \).

**Scenario 2:** the item does not have to be inspected at period \( t_{now} \).
First, consider Scenario 1, in which the item is inspected at this moment \((t_{\text{now}})\), and has not been inspected since \(t_{ip}\). The probability of an item \(i\) needing fixing at moment \(t_{\text{now}}\) is the probability of an item an error having occurred between the last inspection/failure and this one: between \((t_{ip}, t_{\text{now}})\), plus the probability of an error occurring and failure between \((t_{\text{now}}, t_{now} + t)\). Assumption 1 states that there is no possibility of a second failure within \((t_{now} + t)\). Therefore, the only other possibility is that an error presents after \(t_{\text{now}}\) meaning \(t_{\text{now}} + t - t_{ip} > U_i > t_{\text{now}} - t_{ip}\). For notation, \(Dem_1(i, s_1, s_2)\) is the event of there being demand for item \(i \in N\) with maintenance in both \(s_1\) and period \(s_2\) for time period \((s_1, s_2)\) for \(\forall s_1, s_2 \in \{1, T\}\). The demand due to items in scenario 2 is given in equation 8.

\[
Pr(Dem_1(t_{\text{now}}, t_{now} + t)) = Pr(\text{error in } (t_{ip}, t_{\text{now}})) + Pr(\text{error} \wedge \text{failure in } (t_{\text{now}}, t_{\text{now}} + t)) \\
= Pr(U < t_{\text{now}} - t_{ip}|U > t_{ip} - t_{ir}) \\
+ Pr(t_{\text{now}} - t_{ip} < U < t_{\text{now}} + t|U > t_{ip} - t_{ir}, H < t_{\text{now}} + t) + \epsilon \\
= \int_0^{t_{\text{now}} - t_{ip}} f_U(u|U > t_{ip} - t_{ir}) du + \\
+ \int_{t_{\text{now}} - t_{ip} + t}^{t_{\text{now}} - t_{ip}} f_U(u|U > t_{ip} - t_{ir}) \int_0^{u - t_{ip}} H(h) dh du + \epsilon \\
\approx \int_0^{t_{\text{now}} - t_{ip}} f_U(u|U > t_{ip} - t_{ir}) du \\
+ \int_0^{t_{\text{now}} - t_{ip}} f_U(u|U > t_{ip} - t_{ir}) (H(t - u)) du
\]  

(8)

Now consider an item \(i\) in Scenario 2. The last time it was inspected was time \(t_{ip}\) and at this time it was non-faulty and still working. At the current time \(t_{\text{now}}\), it has not yet failed, and it was checked last for errors/ fixed at \(t_{ip}\). Therefore, three things can happen to \(i\). The first possibility is that it is found to be non-faulty at the next inspection at \(t_{\text{now}} + t\) and remains non-faulty until \(t_{\text{now}} + 2t\), two inspection periods from now. This would not cause any demand in general. Secondly, it can be found faulty at the next inspection, at \((t_{\text{now}} + t)\) is replaced and hadn’t failed. This would only cause demand in the following period, as is also evident from equation 8. Thirdly, a failure occurs before \(t_{\text{now}} + t\), before the next inspection. This is the only scenario that causes demand for the current period. In this case either an error has occurred between \((t_{ip}, t_{\text{now}})\) and the failure occurs between \(t_{\text{now}}, t_{\text{now}} + t\) or an error and failure combination occurs as error in \((t_{\text{now}}, u)\) and failure in \((u, t_{\text{now}} + t)\), for \(t_{\text{now}} < u < t_{\text{now}} + t\). Again, due to Assumption 1, it is not possible to have a second failure before \(t_{\text{now}} + t\). For notations, \(Dem_2(i, s_1, s_2)\) is the event of there being demand for item \(i \in N\) with maintenance is not held at \(s_1\) and is done at \(s_2\) for time period \((s_1, s_2)\) for \(\forall s_1, s_2 \in \{1, T\}\). The
demand due to items in scenario 2 is given in equation 9.

\[ Pr(Dem2_i(t_{now}, t_{now} + t)) = Pr(error \text{ in} (t_{ip}, t_{now}) \land failure \text{ in} t_{now}, t_{now} + t) \]
\[ + Pr(error \land failure \text{ in} (t_{now}, t_{now} + t)) \]
\[ = Pr(U < t_{now} | U > t_{ip} - t_{ir}, H < t_{now} + t | H > t_{now})du \]
\[ + Pr(t_{now} - t_{ip} < U < t_{now} + t | U > t_{ip} - t_{ir}, H < t_{now} + t | H > t_{now} + t - u) + \epsilon \]
\[ = \int_{0}^{t_{now} - t_{ip}} f_U(u) f_H(h) dh du \]
\[ + \int_{t_{now} - t_{ip} + u}^{t_{now} - t_{ip}} f_U(u) f_H(h) dh du + \epsilon \]
\[ \approx \int_{0}^{t_{now} - t_{ip}} f_U(u) f_H(h) dh du \]
\[ + \int_{0}^{t} f_U(u) f_H(h) dh du \]
\[ = \int_{0}^{t_{now} - t_{ip}} f_U(u) f_H(h) dh du \]
\[ + \int_{t_{now} - t_{ip} + u}^{t_{now} - t_{ip}} f_U(u) f_H(h) dh du \]
\[ \approx \int_{0}^{t} f_U(u) f_H(h) dh du \]
\[ (9) \]

The total expected number of replacements, \( F(t_{now}, t_{now} + t) \), and therefore the forecasted demand in the coming period \( (t_{now}, t_{now} + t) \) is given in equation 10.

\[ E[F(t_{now}, t_{now} + t)] = \sum_{i=1}^{N} Dem1_i(t_{now}, t_{now} + t) \cdot \delta_i + \sum_{i=1}^{N} Dem2_i(t_{now}, t_{now} + t) \cdot (1 - \delta_i) \]
\[ = \sum_{i=1}^{N} \left( \int_{0}^{t_{now} - t_{ip}} f_U(u) f_H(h) dh du \right) \cdot \delta_i \]
\[ + \int_{0}^{t} f_U(u) f_H(h) dh du \]
\[ \approx \int_{0}^{t} f_U(u) f_H(h) dh du \]
\[ \approx \int_{0}^{t} f_U(u) f_H(h) dh du \]
\[ (10) \]

where \( \delta_i = \begin{cases} 
1 & \text{if the item is in Scenario 1} \\
0 & \text{if the item is in Scenario 2} 
\end{cases} \)

If an item goes obsolete in the period in between inspections, this will be noted at the next inspection and the item will be taken out of inspection rotation and demand is noted to be zero for all coming periods.
4.3 Delay Time model with replacement possibility: DTRP

For the DTRP method, there are two failure modes. An item is either fixed or replaced after inspected faulty/failure. The moment $O_i$, is distributed by the Weibull distribution with pdf. $f_{O}(o)$ and cdf. $F_{O}(o)$. After an item is outdated, it gets replaced at the following failure or when it is inspected and faulty.

At every point in time of prediction, that $O_i$ is larger than the moment of last fixing the item, $t_{ir}$. Then there are two cases. Either the last moment of fixing an item has happened in the previous period, or it happened before. The conditional pdf. for general $O$ is given in equation 11.

$$f_{O}(o|O > t_{ir}) = \begin{cases} \frac{f_{O}(o+t_{ip})}{f_{O}(o)} & t_{ir} < t_{ip}, \ 0 < d < \infty \\ f_{O}(o) & t_{ir} = t_{ip}, \ 0 < o < \infty \end{cases}$$

Subsequently, the probability of replacement, $q_{i(s1,s2)}'$, is given in equation 12.

$$q_{i(s1,s2)}' = Pr(i \text{ replaced in period } (s1,s2)) = Pr(O < s_2 | O > t_{ir}) = \int_{s_2}^{\infty} f_{O}(o | O > t_{ir}) do$$

In the forecast, items are dismissed if they have been replaced. In this case, variable $D_i$ is set to 0, else it is 1. The DT method is thusly transformed to predict the possibility of error/failure occurrences that are not replaced in that time period. Multiplying equation 10 with the probability that they are not replaced, gives the forecasts of the DTRP model. The forecasted demand in the coming period $(t_{now}, t_{now}+t)$ is given in equation 13.

$$E[F(t_{now}, t_{now}+t)] = \sum_{i=1}^{N} D_i (1 - q_{i(t_{now},t_{now}+t)}) \cdot \left( \int_{0}^{t} f_{U}(u | U > t_{ip} - t_{ir}) F_{H}(t_{now} + t - u) du + \int_{t_{now} - t_{ip}}^{t_{now} - t_{ir}} f_{U}(u | U > t_{ir} - t_{ip}) du \cdot \delta_i + \int_{0}^{t_{now} + t - u} f_{H}(h | H > t_{now}) dh \cdot (1 - \delta_i) \right)$$

Using this, it is possible to adjust the forecasts to the replacement of old parts, using the original DT model. Thus far, RQ1 is theoretically answered. Although, to evaluate the performance of the DTRP method, tests should be run and analyzed. Only then it can be concluded if this model is a satisfactory improvement.
5 Results

To analyze the performance of the methods considered, the total absolute errors of the forecasts are compared. The total absolute errors of $\text{Meth} \in \{\text{SBA, DT, TSB, LES}\}$ is given in equation 14.

$$\text{Error}_{\text{Meth}} = \frac{\sum_{s=1}^{\text{Number of simulations}} \sum_{t=1}^{T} |z_t - \text{Meth}(F_t)|}{\text{Number of simulations}}$$  \hspace{1cm} (14)

For the distributions following, the initial time is distributed by the Weibull distribution with scale ($\lambda_u$) and shape ($\kappa_u$) parameters, given in equation 15. The delay time is exponentially distributed with parameter ($\mu$), given in equation 16.

$$f(u; \lambda, \kappa) = \frac{\kappa}{\lambda} \left(\frac{u}{\lambda}\right)^{\kappa-1} e^{-\left(\frac{u}{\lambda}\right)^\kappa}$$  \hspace{1cm} (15)

$$f(h; \mu) = \frac{1}{\mu} e^{-\frac{h}{\mu}}$$  \hspace{1cm} (16)

In section 5.1 the results found in the paper by Wang et al. [11] are reproduced and the findings are validated: DT outperforms SBA. Additionally, DT is compared to TSB and HES. Section 5.2 shows a theoretical experiment that investigates different situations and to find in which situation DT is preferred over LES and vice versa. In section 5.3, the performance and additional value of the DTRP method is discussed and compared to other methods.

5.1 Reproduction

In the paper by Wang et al. [11], an aggregated demand graph for a block based scheme was published. In a block based scheme, the maintenance check-ups are consistently scheduled and performed every time period $t$, regardless of an items’ age. Additionally, the paper calculates the errors for two types of pumps with age based inspection. This is reproduced using solely age based data (see section 3) and the definition of DT that predicts for every $t$ and not every $2t$ (see section 4.2. Proceeding, mostly the same variables are used as in the original paper.

First a demand simulation is produced. The set-up of the experiment is 20 simulation runs with 20 items, the initial time is distributed with $\lambda$ and $\kappa$ parameters equal to 1052.6 and 8.907 respectively and the delay time with $\mu$ equal to 574.71. The time periods are adjusted for age based data: using shorter periods of $t = 12$, for a maximum of 140 periods, so $T = 1680$, where the first 40 are out of sample. Lastly, $\tau$ is taken at 21. To find the the optimal values for the smoothing coefficients, $\alpha$ and $\beta$, grid search is used. These are equal for SBA to $\alpha = 0.05$, for TSB to $\alpha = 0.05$ and $\beta = 1$ and for LES to $\alpha = 1$ and $\beta = 0.05$ (for total table, see Appendix B). The demand aggregation and forecasts are given in figure 4. The main difference between this graph and the one in Wang et al. [11] is that 4 presents spikes. This is due to the age based data and the fact that there is a prediction for each
period. If there is no inspection in a period, the forecasted demand is far lower than if there is an inspection period. The total errors are calculated as in equation 14 given in table 2.

![Figure 4. Demand simulation](image)

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<th>LES</th>
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**Table 2.** Overview of the performances of four forecasting methods, one failure mode.

Next in the paper by Wang et al. [11], specific values are used in age based maintenance scheme to test DT again, using values fitted for pumps in a paper by Baker et al. [1]. This case uses two types of pumps: volumetric and peristaltic. For both types, there are 2000 time units, $t$ lasts 28 time units, meaning that there are 71 forecasting periods of which 20 are used to initialize the Croston based methods. The minimum age of inspection is 42 time units.

For the volumetric pump, it is used that $\lambda = 588.24$, $\kappa = 1.42$, $\mu = 57.4712$, and $N = 105$. Moreover, SBA uses $\alpha = 1$, TSB uses $\alpha = 0.05$ and $\beta = 1$, LES uses $\alpha = 1$ and $\beta = 0.05$. The total absolute errors are shown in figure 5a.

For the peristaltic pumps, it is used that $\lambda$ equals 1369,863, $\kappa$ equals 2.41, $\mu$ equals 1111.11, and $N$ equals 35. Moreover, SBA uses $\alpha = 0.05$, TSB uses $\alpha = 0.05$ and $\beta = 0.1$, LES uses $\alpha = 0.7$ and $\beta = 0.05$. The results are shown in figure 5b.

Reproducing these findings and comparing the errors produced shows similar results to those in the paper by Wang et al. [11]: DT outperforms SBA. Additionally to the reproduction, LES and TSB are considered and compared as forecasters. It shows that TSB performs worst for the pumps. LES performs best in figure 4 because the systematic low forecasts and fit ultimately well with the actual values of single run demand, even though the average of DT fits closer. The difference in relative performances between DT and LES is noteworthy for the pumps in figure 5. LES performs best in figure 5b, though DT is close. DT performs best in figure 5a. Assessing the properties of these different situations (Appendix A) shows that the difference in performance between LES and DT could
be because the variance is higher for the case in figure 5b, standard deviation equals 537.02, than in figure 5a, standard deviation equals 379.24. The use of the DT model is more accurate if the variance is low, because in that case the intermittent demand is better matched by the probability distribution. Contrarily, the Croston based methods profit from a larger variance, since larger variance in their case means a more flattened out demand pattern. These methods are unable to predict demand spikes and rely on the past observations, as is indicated in figure 4. Another influence can be the sample size. DT does not predict demand equal to zero, so in periods in advance to actual demand, predictions are already off, if there is no demand. As the sample size increases the chance of demand occurring in the tail periods leading up to the mean of the initial time distribution. A larger sample size leads to a better fit of predictions if the variance is relatively high. To conclude, a relatively high variance, which is present in all figures 4, and 5 is beneficial for LES in general, unless the sample size is also large. In that case, DT might be preferred.

The limitation was not touched upon in [11], because the largest variance considered was 26000. This restraint means that demand is never spread out so far that DT loses its accuracy significantly compared to SBA. It is merely hinted at in their findings, as they find that the difference in errors of DT and SBA is larger when variance is low and mean is high. They state: “...as the variance decreases and mean increases the advantage of the DT approach becomes even more obvious”, but the limitations are not further considered.

5.2 The effect of variance coefficient and sample size on DT

This section backs up the theory that variance and sample sizes affect which method performs best. Consider the following theoretical experiment exploiting the two extreme cases of failure distribution: a single spike because of a small variance coefficient ($CV = 0.0028$), and a flattened out demand because of high variance coefficient ($CV = 2.1$). The coefficient of the variance is given as $CV = \frac{\sqrt{Var}}{\text{mean}}$. 
For both cases, \( t = 12, T = 1680, \tau = 21, \mu = 57.4713 \) and the number of repetitions is equal to 20. For the low variance coefficient, the initial time is simulated with \( \lambda = 427.7515 \) and \( \kappa = 0.5192 \). The distribution of LES is used with \( \alpha = \beta = 0.05 \), optimized using grid search. The figures are given in 6a and 6c. For the low variance coefficient, the initial time is simulated with \( \lambda = 801.0059 \) and \( \kappa = 457.9269 \). The distribution of LES is used with \( \alpha = \beta = 0.05 \), optimized using grid search. The figures are given in 6b and 6d. These figures are run for \( N = \{20, 100\} \), to evaluate the difference in demand as a result of variance and of number of items. The corresponding errors are given in table 3.

![Graphs showing demand patterns for different scenarios](image)

(a) \( N=20 \), high variance coefficient  
(b) \( N=20 \), low variance coefficient  
(c) \( N=100 \), high variance coefficient  
(d) \( N=100 \), low variance coefficient

<table>
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<td>94</td>
<td>1.9231</td>
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</table>

**Table 3.** Distribution characteristics and errors per demand pattern and sample size.

As is evident from table 3, DT performs always best for a distribution with a small variance coefficient. For the high variance coefficient, LES performs better if the sample size is small. The larger sample size makes that in general the demand simulated fits the initial distribution closer and therefore DT is still closer. This can be seen in the figures 6a and 6c. These results help formulate an answer to RQ2 for one failure mode.

5.3 Delay Time Replacement Possibility: DTRP

Next, simulations with two failure modes are considered to examine the performance of DTRP.

First a demand graph is evaluated. Using the same variables as used for graph 4, the variables are given by 20 simulation runs with 20 items, the error time is distributed with \( \lambda_u \) and \( \kappa_u \) parameters.
equal to 1052.6 and 8.907 respectively, \( \mu = 574.71, \ t = 12, \ \tau = 21, \ T = 1680, \) of which the first 20 are out of sample for time series methods to catch up. Lastly, The optimal smoothing coefficients are found using grid search and are equal for SBA to \( \alpha = 0.05, \) for TSB to \( \alpha = 0.05 \) and \( \beta = 1 \) and for LES to \( \alpha = 1 \) and \( \beta = 0.05. \) Starting of, the replacement function is assumed to be distributed according to the Weibull distribution, with scale parameter \( \lambda_u = \lambda_o \) and shape parameter \( \kappa_u = \kappa_o. \) The demand graph for two failure modes is given in figure 7.

Comparing to figure 4, the small demand spikes from periods 120+ have disappeared. This is because time point \( O_i \) is smaller than 120*t for \( \forall i \in \{1, \ldots, N\}. \) These items are either replaced earlier, or are otherwise replaced when they fail in these later periods.

Furthermore, it is seen that maximum demand is lower than before. This was to be expected, as there is no demand for the items that are replaced.

Lastly, it can be seen that the demand decline is more abrupt. The probability that the simulated replacement time has passed is negatively correlated with \( U_i. \) Starting at period 80, this is displayed more obvious in the graph. Mathematically this is stated for arbitrary time period \( \forall s \in \{1, \ldots, N\} \) as:

\[
Pr(O_i < U_i | U_i > s) > Pr(O_i < U_i | U_i < s).
\]

Table 4 shows the errors. LES has the lowest errors. The ranking is similar to 2. Again LES is the lowest, due to small sample sizes in combination with a relatively large variance coefficient.

![Figure 7. Demand simulation Replacement](image)

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Table 4. Overview of the performances of five forecasting methods, two failure modes.

Next, the shortcomings of DTRP are evaluated and the performance of DTRP compared to other methods. The variables are similar to the case in figure 7, except for the distribution of the replacement function, \( f_O(o) \) and the sample size. These are varied to assess what specifically affects performance. Two means are considered: one low mean equal to 795, in the middle of the total time period, and
a high mean equal to 1095, at $\frac{3}{4}$ of the total time period. Two sample sizes are considered: a small one; $N = 20$, and a large one; $N = 100$. A total of 20 different values for coefficient of variation are considered, for $CV \in \{0.05, ..., 1\}$ with step size equal to 0.05. The errors are given in figure 8.

Firstly, the effect of the sample sizes is examined. For SBA and LES the sample size has no effect on their relative performances. TSB performs relatively better (better than DT) with a small sample size than a large sample size. This is due to the fact that the forecasts rely on demand probability, set very low, creating always a low forecast as seen in figure 7. If the sample size is low, and the demand stays lower, the TSB prediction are less far off. Furthermore, it is evident that the preference of DTRP over DT switches as the sample size is increased. This is due to the nature of DTRP. DTRP has as a benefit that it can predict non-demand if replacement occurs. But a disadvantage is that if there is no replacement, the forecasts of DTRP are systematically too low. Apparently, as the sample size increases, the relative benefits of predicting non-demand in one period is too small to compensate the lower predictions. Only for a low mean and a very low variance ($CV < 0.2$), DTRP performs better in a large sample.

Next, the effect of the distribution of the replacement function is examined. It is evident that the errors for all methods are lower when the mean is small in general. The items are replaced at an earlier stage. DT and DTRP don't predict after replacement from the next point on (noticed at maintenance) and the LES, and TSB adjust to consistent zero demand, aggregating low errors. SBA’s predictions remain low, along with the demand. This means the errors are not so far off.

There is a difference in the effect of the variance depending on the mean. If the mean is low, the errors are positively correlated with the variance. The reason for this is that some replacements are
happening later, causing larger errors in general that are not compensated by the benefit of earlier replacements. If the mean is high, errors are negatively correlated with the variance. This is due to the fact that the decline of demand is less abrupt, benefiting the time series methods as well as DT and DTRP, through DT’s characteristics as the parent method. It is shown that for a small sample size and a larger variance, the benefit of DTRP over DT grows, because the items that are not replaced benefit from the consistent adjustments made by incorporating replacement possibility. For a large mean, the difference between errors is inconsistent.

Considering all of these influences seen so far, the goal is to answer RQ2 for the two failure modes. SBA is the worst predictor in every case considered. This is also because SBA does not adjust to obsolescence, and is therefore a poor predictor after all items are replaced. LES is the best predictor in any case, because of the high variance of the initial time distribution (see section 5.2), the quick adjustment to zero demand and consistently conservative predictions harmonizing with the replacements. Even in large sample sizes, DT is still not better than LES. Both DT and DTRP are stable in their predictions. If the sample size is low, DTRP is better than DT. If the sample size is high, DT is preferred. TSB is not such a good predictor, except when the mean and sample size are very low, but for suspicious reasons: TSB always has low predictions. Therefore, DTRP can be preferred in this case to TSB.

To answer RQ2 to the best of our abilities:

**Apparent from table 3:** If there is no replacement (one failure mode), and the sample size is very large, DT is most efficient.

If there is no replacement (one failure mode), the sample size is small and the variance of the initial function is small, DT is most efficient.

If there is no replacement (one failure mode), the sample size is small and the variance of the initial function is large, LES is most efficient.

**Apparent from figures 6a and 8:** If there are is replacement (two failure modes) and the variance of the initial function is large, LES is most efficient.

**Apparent from figures 6b, 8a and 8b:** If there are is replacement (two failure modes), the variance is very small and the sample size is small, DTRP is most efficient.

**Apparent from figures 6d, 8c and 8d:** If there are is replacement (two failure modes), the variance is very small and the sample size is large, DT is most efficient.

6 Conclusion

Following the paper by Wang et al. (2011) [11], the first goal of this thesis was to confirm their findings and investigate the grounds of these findings. In the paper, DT is said to outperform SBA [9], but
is only applicable if the initial time and delay time distributions are known. Otherwise, SBA should be applied instead. It is now found that there are three things wrong with this assumption. Firstly, apparent after this research and others’, SBA is not the best alternative out there. The inclusion of obsolescence and quick response to intermittent demand from LES [6] has been proven to outperform SBA in each of the experiments performed. This is merely and update to their theory, as LES was not discovered yet in 2011. Secondly, it is found that not only knowledge of the distribution is needed for proper forecasting in the DT model, but also the variance should be small, otherwise time-series methods might be preferred (see table 3). Thirdly, if the function is unknown or the variance is large and the sample size is very large, DT is still preferred to other time series methods 3.

Next, a new model was proposed to incorporate the probability of replacement at a certain point in time was added to the DT model to create the DTRP model. Under multiple conditions the characteristics of DT and DTRP were compared, along the time series based methods. DTRP is still not yet exclusively superior to DT, which just disregards replacement possibility in each period. The abilities of DTRP that strengthen it to predict replacements, are undone in a period no replacement is made: the forecasts to low. Also, LES performs better in all experiments performed regarding two failure modes, so DTRP is not yet as well as idealized. At this point, further research should be done to correct the DTRP model such that it takes into account these problems. A conclusion of which method is best applied in which situation is given in figure 9.

For further research it might be useful to define the exact points which classify variance and sample size as ‘large’, such to make a clear distinction when to use LES or DT/DTRP. Ultimately, a goal could be to redefine the DTRP method in such a way that it is no longer crippling itself in periods with zero replacement, and would for real outperform DT in any scenario.

![Figure 9. Advice on which method to apply.](image_url)

Further in the paper: Bibliography, Appendix A in which a table gives an overview of distribution values used in this paper, Appendix B in which an overview of values is found using grid-search as an example for the reader and Appendix C which gives a description of the DT-math used in the paper by Wang et al. [11].
References


Appendix A

A clear overview of different values for the distribution of the Weibull function in table 5.

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<th>Description use</th>
<th>figure items</th>
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Table 5. Appendix A: Overview of values for the distribution of the initial/replacement time used in the paper.

Where the mean and variance of the Weibull distribution with scale λ and shape κ are given in equation 17 and equation 18 respectively.

\[
mean_{Wb} = \lambda \Gamma(1 + \frac{1}{\kappa}) \tag{17}
\]

\[
var_{Wb} = \lambda^2 \Gamma(1 + \frac{2}{\kappa}) - \mean_{Wb}^2 \tag{18}
\]
Appendix B

The table 6 of values found for grid-search with step size=0.05, derived from an average run of demand with $t = 12$, $N = 20$, $scale = 1/0.00095$, $shape = 8.907$, $T = 140 + t$, $\tau = 21$, $\mu = 1/0.00174$. Errors are calculated as in equation 14. Used as an example, similar for other cases.

Best values found are for SBA; $\alpha = 0.05$, the error is equal to 29.5612.

Best values found are for TSB; $\alpha = 0.05$, $\beta = 1$, the error is equal to 28.9251.

Best values found are for LES; $\alpha = 1$, $\beta = 0.05$, the error is equals to 18.5201. LES performs best overall.
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Table 6. Appendix B: Gridsearch
Appendix C

The mathematics for the DT model is given here as it was given in the paper by Wang et al. [11]. The conditional function for $f_U(u)$ is the same, and stated again in 11

$$f_U(u|U > t_p - t_r) = \begin{cases} \frac{f_U(u+t+p-t_r)}{f_{t_p-t_r} f_U(u)} & t_r < t_p, \ 0 < u < \infty \\ f_U(u) & t_r = t_p, \ 0 < u < \infty \end{cases}$$

(19)

There are two scenarios defined an item can be in, reversed from how it is stated in section 4.2.

Scenario 1: the item does not have to be inspected at the current check-up point.

Scenario 2: the item is inspected at the current check-up point.

The concluding equation is given in figure 10. Where it says $E[N_r(t_p + t, t_p + 2t)]$, it should say $E[N_r(t_p + t, t_p + 2t)]$, as forecasts for failure is still made for items in scenario 1 until $t_p + 3t$, and it says that they predict for 28 time units, and that $t$ is set to 14. This is strange, as the prediction by the way data is simulated is updated and the nature of demand vector $x$ is divided for every time unit $t$.

The period in which they are forecasting is given at $t_p + t$ if an item is in scenario 1.

The period in which they are forecasting seems given at $t_p$ if an item is in scenario 2, the probability of failure is included from $(t_p, t_p + 2t)$. There are two problems with this statement. Though, starting at $t_p$, seems to be off as the prediction starts a period later at $t_p + 1$: “$E[N_r(t_p + t, t_p + 2t)]$”. Secondly, this is not in accommodation with the way data is simulated. This predicts that the maintenance in an period counts as demand for the beginning of that period. This is stated in figure 3 and taken from the paper by Wang et al. [11]. There is no prediction for the current immediate inspection and it is not clear if there is a fix at the beginning in this period, which should update $t_p$.

$$E(N_r(t_p + t, t_p + 2t)) \approx \sum_{i=1}^{N} \left( \frac{\Pr(\text{the } i\text{th item identified at } t_p + 2t \text{ to be faulty})}{\Pr(\text{the } i\text{th item having a failure in } (t_p + t, t_p + 3t))} \delta(i) + \sum_{i=1}^{N} \left( \frac{\Pr(\text{the } i\text{th item identified at } t_p + 2t \text{ to be faulty})}{\Pr(\text{the } i\text{th item having a failure in } (t_p, t_p + 2t))} (1 - \delta(i)) \right) \right)$$

$$= \sum_{i=1}^{N} \left( \int_{0}^{3t} f_U(u|U > t_p - t_u) du + \int_{3t}^{2t} f_U(u|U > t_p - t_u) F_U(3t - u) du \right) \delta(i)$$

$$+ \sum_{i=1}^{N} \int_{0}^{2t} f_U(u|U > t_p - t_u) du (1 - \delta(i)),$$

where

$$\delta(i) = \begin{cases} 1 & \text{if the } i\text{th item is in scenario (1)} \\ 0 & \text{otherwise} \end{cases}.$$