

# Asymmetry in the Transmission of Return Shocks; The Skewed- $t$ Connectedness Framework

Master Thesis - Quantitative Finance

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## Abstract

In this report we study return spillovers among U.S. asset classes and global stock markets based on financial connectedness measures as introduced by Diebold and Yilmaz (2012). We show that the assumption of normally distributed residuals in the underlying VAR models is poorly substantiated. Therefore, we introduce new connectedness measures based on skewed- $t$  distributed innovations which account for excess kurtosis and skewness and enable us to make distinctions between the transmission of negative and positive extreme shocks. We find that the system's total connectedness is little affected by this new assumption, but that there are large differences in the transmission of extreme positive and negative shocks among individual markets. Since the original model of Diebold and Yilmaz (2012) is nested in our new model, we conclude that it is better to use our newly proposed skewed- $t$  connectedness measures.

*Keywords:* risk management, market risk, financial connectedness, vector autoregression, variance decomposition

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# 1 Introduction

The recent global financial and European sovereign debt crisis made it clear that unobserved cross-market connections can have a detrimental effect on the global economy once a shock hits one or more financial markets anywhere around the globe. In response to these major economic events, a large string of academic literature has emerged investigating the role of these unobserved market-to-market linkages. In more recent years, a wide variety of academics have adopted the financial connectedness methodology, as introduced by Diebold and Yilmaz (2009), to monitor and evaluate contagion and interdependence among markets and between countries.

In their connectedness framework, Diebold and Yilmaz (2009) use variance decompositions to measure how return and volatility shocks are being transferred across markets. Variance decompositions, as obtained from a vector autoregression, add information on the time-profile of returns and volatilities to the contemporaneous relations, as obtained from the covariance matrix. This results in asymmetric interdependence measures which are more informative regarding the system's connectedness than simple symmetric correlation measures.

Diebold and Yilmaz (2009) do not only use variance decomposition to reflect market-to-market return and volatility linkages. They also aggregate the information in these variance decompositions to connectedness measures related to the dependence of one specific market on all other markets in the system, and the dependence of the entire system on one specific market. Furthermore, they develop a so-called *Spillover Index* which represents the total level of connectedness in the system.

Newly proposed methods often bring valuable new insights to the table, however even more often, the further development of methods is necessary to improve accuracy and to indicate the opportunities and limitations of the methodology. Major methodological improvements already have been proposed, such as the adoption of the generalized VAR framework to make the connectedness measures invariant to the ordering of variables (Diebold and Yilmaz, 2012) and the LASSO (shrinkage and variable selection) technique, to improve the estimated dynamics of the VAR model (Demirer et al., 2015).

We argue that another major improvement can be made, which, to the best of our knowledge, has not yet been investigated. From an economic point of view, we expect markets to respond differently to negative versus positive and small versus large shocks. Therefore, we present a method which enables us to distinguish between these different origins of shocks.

In this report, we show that the assumption of Diebold and Yilmaz (2009) that the residuals

of the VAR model are normally distributed is not realistic (i.e. they are skewed and leptokurtic), and we propose a multivariate skewed- $t$  distribution to account for these characteristics. Since we move away from the elliptical shape of the normal distribution, the variance decompositions, and subsequently the related connectedness measures, are not invariant to shocks anymore but instead depend heavily on both the sign and the size of the shocks to the system. We choose to use the skewed- $t$  distribution, not only because it is able to account for skewness and kurtosis, but also because the normal distribution is nested in it. The skewed- $t$  distribution converges to the normal distribution as the skewness parameters tends to zero and the degrees of freedom parameter tends to infinity. Therefore, if the data is in fact normally distributed, our newly proposed model converges to the model of Diebold and Yilmaz (2012).

We investigate the opportunities and limitations of this new methodology in a study of spillovers among U.S. stock, bond, commodity and foreign exchange markets. This data is of particular interest because spillovers among U.S. asset classes may be representative for global financial market dynamics and may have been an important aspect of the global financial crisis which began in 2007 (Diebold and Yilmaz, 2012). Furthermore, we evaluate how the new distributional assumptions impact the estimated connectedness measures among a set of 10 global stock markets, of which six are located in industrial countries (United States, United Kingdom, France, Germany, Japan, and Australia) and four in emerging market economies (Brazil, China, India and Hong Kong). Since stock markets are an important barometer of current and expected future economic activity (Diebold and Yilmaz, 2015), it is interesting to apply the connectedness methodology to global stock market data because it may lead to insights on how shocks in economic activity and crises are transmitted around the globe.

Our results show that for the U.S. asset classes the alternative assumption and corresponding distribution yield different connectedness results when compared to the measures of Diebold and Yilmaz (2012). While the total system's connectedness is little affected, there are large differences between the transmission of extreme positive and extreme negative shocks among the individual U.S. asset classes. This is especially the case for the spillovers among stocks and bonds.

In the case of global stock markets we find little evidence of differences in positive and negative shock transmission and the estimated skewed- $t$  connectedness measures resemble those of Diebold and Yilmaz (2012). This may be explained by the fact that the skewed- $t$  distribution tends to normality when the dimension of the data becomes larger.

Since the model of Diebold and Yilmaz (2012) is nested in our new model, we conclude that, based on the assumption that the innovation terms of the VAR model may contain skewness and

kurtosis, it is best to move forward with our newly proposed skewed- $t$  connectedness measures. For further research it may be interesting to combine the skewed- $t$  assumption with the LASSO technique for the VAR estimation to further enhance the accuracy of the models.

This paper is organised as follows. Section 2 provides an overview of the literature regarding connectedness measures. Section 3 provides details on the proposed methodology. Section 4 reports the results regarding spillovers among U.S. asset classes, including a thorough evaluation of the methodologies. Section 5 reports the results regarding spillovers among ten global stock markets. Section 6 presents the conclusions of this research.

## 2 Literature Review

In early 2009, Diebold and Yilmaz (2009) introduced a simple and intuitive quantitative measure of interdependence of asset returns which they called the *Spillover Index*. This measure was derived from the forecast error variance decomposition of a covariance stationary VAR( $p$ ) model. They used a Cholesky factorization to identify the orthogonal innovations which are needed for the calculation of the variance decomposition. As a result of using this Cholesky factorization, these first constructions of spillover indices were dependent on the ordering of the variables in the VAR model. The results of this methodology are presented in so-called spillover tables (fixed window estimation), which indicate the interdependence structure in a specific time period, and spillover plots (rolling window estimation), which provide an overview of the changes in spillovers over time.

Using their spillover methodology, Diebold and Yilmaz (2009) found that return spillovers among a set of 19 global equity markets from the early 1990s to 2007 display a slightly increasing trend but no bursts. In their analysis on the same markets of volatility spillovers they did not find a trend but did observe clear bursts of volatility spillovers. Yilmaz (2010) and Diebold and Yilmaz (2011) extended this research by examining contagion and interdependence across the East Asian and Americas equity markets, respectively. They obtained similar results and observed that both return and volatility spillover indices reached their respective peaks during the global financial crisis of 2007-2008.

As stated before, the methods of Diebold and Yilmaz (2009) are dependent on the ordering of the variables in the VAR model. Diebold and Yilmaz (2012) circumvent this problem by exploiting the generalized VAR framework of Koop et al. (1996) and Pesaran and Shin (1998) which produces variance decompositions invariant to ordering. Furthermore they define so-called ‘directional spillovers’. One can think of the set of directional spillovers as providing a decomposition of total spillovers into those coming from (or to) a particular source.

Using these improved methods Diebold and Yilmaz (2012) show that despite significant volatility fluctuations in U.S. stock, bond, foreign exchange and commodities markets during the sample period (1999-2009), cross-market volatility spillovers were quite limited until the global financial crisis that began in 2007. As the crisis intensified so too did the volatility spillovers, with particularly important spillovers from the bond market to other markets taking place after the collapse of Lehman Brothers in September 2008.

Diebold and Yilmaz (2014) introduce a new way to interpret the spillover tables. They show

that variance decompositions define weighted, directed networks, so that their connectedness measures are intimately-related to key measures of connectedness used in the network literature. They visualize these networks with directed node-link diagrams which provide a clear illustration of the transmission of return and volatility shocks. Building on these insights, they track both average and daily time-varying connectedness of major U.S. financial institutions' stock return volatilities in recent years, including during the financial crisis of 2007-2008.

Diebold and Yilmaz (2016) employ this network connectedness approach and characterize equity return volatility connectedness in the network of major American and European financial institutions, 2004-2014. Their methods enable precise characterization of the timing and evolution of key aspects of the financial crisis. First, they found that during 2007- 2008 the direction of connectedness was clearly from the US to Europe, but that connectedness became bi-directional starting in late 2008. Second, they found an unprecedented surge in directional connectedness from European to US financial institutions in June 2011, consistent with massive deterioration in the health of EU financial institutions. Third, they identified particular institutions that played disproportionately important roles in generating connectedness during the US and the European crises.

Lanne and Nyberg (2016) note that with the standard choice of the order invariant generalized forecast error variance decomposition of Pesaran and Shin (1998) (used by Diebold and Yilmaz (2012)), the shares of the forecast error variation, do not add to unity, making it difficult to compare risk ratings and risk contributions at two different points in time. As a solution to this problem, they introduce a new generalized forecast error variance decomposition.

Finally, very recently, Barbaglia et al. (2017) employ a method which accounts for the possible fat-tailed distribution of the VAR innovation terms. They use a VAR model with errors following a multivariate Student  $t$ -distribution with unknown degrees of freedom and study volatility spillovers among energy, biofuel and agricultural commodities.

The connectedness literature provides researchers with great tools to evaluate interdependence and contagion among financial markets which are easy-to-interpret. However, to the best of our knowledge, in the existing literature no distinctions have been made between the transmission of positive versus negative and large versus small shocks. From an economic point of view, we expect that these shock characteristics should have an impact on how shocks are transferred among financial markets. Therefore, we expect that the introduction of the skewed- $t$  connectedness framework in this paper will add new insights to the existing literature on which can be build upon.

### 3 The Financial Connectedness Methodology

In this section we provide an overview of the methods applied in this research. We make use of the generalized financial connectedness methodology, introduced by Diebold and Yilmaz (2012), which is used to evaluate contagion in financial systems. The connectedness measures in this methodology are based on generalized impulse response functions and generalized forecast error variance decompositions as proposed by Koop et al. (1996) and Pesaran and Shin (1998).<sup>1</sup> Impulse response functions and variance decompositions are used to interpret the dynamics of estimated vector autoregressive (VAR) models and explain us how shocks move through financial systems.

In this report, we propose an adjustment of a fundamental assumption made by Diebold and Yilmaz (2012). This adjustment affects the impulse response functions and therefore causes changes in the connectedness measures. In order to explain how this new assumption affects the connectedness measures, we provide a detailed derivation of the connectedness measures.

#### 3.1 Generalized Impulse Response Functions

Let us consider a  $k$ -dimensional VAR( $p$ ) model for the vector  $\mathbf{y}_t = (y_{1,t}, y_{2,t}, \dots, y_{k,t})'$

$$\mathbf{y}_t = \sum_{i=1}^p \Phi_i \mathbf{y}_{t-i} + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}(0, \Sigma) \quad (1)$$

where  $\varepsilon_t$  is an independent and identically distributed error term with mean zero and covariance matrix  $\Sigma$ . Assuming weak stationarity,  $\mathbf{y}_t$  obtains the infinite-order moving-average VMA( $\infty$ ) representation

$$\mathbf{y}_t = \sum_{i=0}^{\infty} \mathbf{A}_i \varepsilon_{t-i}, \quad (2)$$

where the  $k \times k$  coefficient matrices  $\mathbf{A}_i$  can be obtained using the following recursive relations:

$$\mathbf{A}_i = \Phi_1 \mathbf{A}_{i-1} + \Phi_2 \mathbf{A}_{i-2} + \dots + \Phi_p \mathbf{A}_{i-p}, \quad i = 1, 2, \dots, \quad (3)$$

where  $\mathbf{A}_0$  is an identity matrix  $\mathbf{I}_k$  and  $\mathbf{A}_i = \mathbf{0}$  for  $i < 0$ .

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<sup>1</sup>Originally, Diebold and Yilmaz (2009) introduced orthogonalized connected measures which were obtained from *orthogonalized* impulse response functions based on an *orthogonalization* by using the Cholesky decomposition of the covariance matrix. As a result of this, the orthogonalized impulse response functions, and subsequently the orthogonalized connectedness measures, were dependent on the ordering of the variables in the VAR model. In Diebold and Yilmaz (2012), a solution to this problem was proposed, by using the *generalized* impulse response functions, introduced by Koop et al. (1996), which are in fact invariant to this ordering.



An impulse response function measures the time profile of the effect of arbitrary shocks at a given point in time on the (expected) future values of variables in a dynamic system. The best way to describe an impulse response is to view it as the outcome of a conceptual experiment in which the time profile of the effect of a hypothetical  $k \times 1$  vector of shocks  $\boldsymbol{\delta} = (\delta_1, \dots, \delta_k)'$  hitting the financial system at time  $t$  is compared with a base-line case in which there are no shocks (Pesaran and Shin, 1998).<sup>2</sup> The impulse response function is obtained by taking the difference between the expectation of the future values of the variables  $\mathbf{y}_{t+h}$  conditional on a shock  $\boldsymbol{\delta}$  occurring at time  $t$  and the expectation of the future values of the variables  $\mathbf{y}_{t+h}$  unconditional of such a shock. Koop et al. (1996) were the first to derive the generalized impulse response function (GIRF) for a VAR( $p$ ) model with an arbitrary current shock,  $\boldsymbol{\delta}$ , and history,  $\mathcal{F}_{t-1}$ .

$$\text{GIRF}(h, \boldsymbol{\varepsilon}_t = \boldsymbol{\delta}, \mathcal{F}_{t-1}) = \text{E}[\mathbf{y}_{t+h} | \boldsymbol{\varepsilon}_t = \boldsymbol{\delta}, \mathcal{F}_{t-1}] - \text{E}[\mathbf{y}_{t+h} | \mathcal{F}_{t-1}] \quad (4)$$

$$= \mathbf{A}_h \boldsymbol{\delta} \quad (5)$$

Clearly, the GIRF of the VAR( $p$ ) model does not depend on the history,  $\mathcal{F}_{t-1}$ , and thus we can choose an arbitrary  $\boldsymbol{\delta}$  and use estimated coefficient matrices  $\mathbf{A}_i$  to obtain the GIRF for  $h = 1, 2, \dots$

Since we are interested in the response to a shock in one particular variable  $j$  of size  $\delta_j$ , an obvious choice would be to set  $\boldsymbol{\varepsilon}_t = \delta_j \mathbf{e}_j$ , where  $\mathbf{e}_j$  is a  $k \times 1$  indicator vector with unity at entry  $j$  and zeros elsewhere. However, then we would not account for the contemporaneous correlation in the innovation terms as defined by the covariance matrix  $\boldsymbol{\Sigma}$  in Equation (1). Therefore, we take the expectation of the innovations  $\boldsymbol{\varepsilon}_t$  conditional on  $\varepsilon_{j,t} = \delta_j$  to account for this contemporaneous correlation and use this to obtain estimated GIRFs.

The use of the conditional expectation forces us to make a distributional assumption on the innovation terms. For the moment we follow Diebold and Yilmaz (2012) and assume normality.

$$\boldsymbol{\varepsilon}_t \sim \text{i.i.d. } N(0, \boldsymbol{\Sigma}) \quad (6)$$

$$\text{E}[\boldsymbol{\varepsilon}_t | \varepsilon_{j,t} = \delta_j] = \frac{\delta_j}{\sigma_j^2} \boldsymbol{\eta}_j, \quad (7)$$

where  $\sigma_j^2 = \text{E}[\varepsilon_{j,t}^2] = \mathbf{e}_j' \boldsymbol{\Sigma} \mathbf{e}_j$  and  $\boldsymbol{\eta}_j = \text{E}[\boldsymbol{\varepsilon}_t \varepsilon_{j,t}] = \boldsymbol{\Sigma} \mathbf{e}_j$ . The generalized impulse response function of the effect of a shock  $\delta_j$  to the  $j$ th disturbance term at time  $t$  on  $\mathbf{y}_{t+H}$  is now given

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<sup>2</sup>For notational purposes we denote the symbols of vectors in boldface and scalars in normal font style. Matrices are denoted in boldface capitals.

by

$$\text{GIRF}_j(h, \varepsilon_{j,t} = \delta_j, \mathcal{F}_{t-1}) = \text{E}[\mathbf{y}_{t+h} | \varepsilon_{j,t} = \delta_j, \mathcal{F}_{t-1}] - \text{E}[\mathbf{y}_{t+h} | \mathcal{F}_{t-1}] \quad (8)$$

$$= \mathbf{A}_h \text{E}[\boldsymbol{\varepsilon}_t | \varepsilon_{j,t} = \delta_j] \quad (9)$$

$$= \frac{\delta_j}{\sigma_j^2} \mathbf{A}_h \boldsymbol{\eta}_j \quad (10)$$

$$= \frac{\delta_j}{\sigma_j^2} \mathbf{A}_h \boldsymbol{\Sigma} \mathbf{e}_j. \quad (11)$$

Scaling the GIRF by setting  $\delta_j = \sigma_j$ , we obtain the GIRF for a 'unit' shock (i.e., a shock of one standard deviation in size) to the  $j$ th disturbance term, namely

$$\text{GIRF}_j(h, \varepsilon_{j,t} = \sigma_j, \mathcal{F}_{t-1}) = \left( \frac{1}{\sigma_j} \right) \mathbf{A}_h \boldsymbol{\Sigma} \mathbf{e}_j. \quad (12)$$

Next, we select the response in the  $i$ th variable  $\mathbf{y}_{i,t+h}$  on a unit shock to the  $j$ th disturbance term at time  $t$  by premultiplying by an indicator vector  $\mathbf{e}'_i$ .

$$\text{GIRF}_{ij}(h, \varepsilon_{j,t} = \sigma_j, \mathcal{F}_{t-1}) = \text{E}[\mathbf{y}_{i,t+h} | \varepsilon_{j,t} = \delta_j, \mathcal{F}_{t-1}] - \text{E}[\mathbf{y}_{i,t+h} | \mathcal{F}_{t-1}] \quad (13)$$

$$= \left( \frac{1}{\sigma_j} \right) \mathbf{e}'_i \mathbf{A}_h \boldsymbol{\Sigma} \mathbf{e}_j \quad (14)$$

The impulse responses from the GIRF are unique and are not affected by the ordering of the variables in  $\mathbf{y}_t$ .

### 3.2 Generalized Forecast Error Variance Decomposition

The generalized impulse response functions are used to derive the generalized forecast error variance decompositions, defined as the proportion of the  $H$ -step ahead forecast error variance of variable  $i$  which is accounted for by the innovations in the variable  $j$  (Pesaran and Shin, 1998). Since the innovation terms are assumed to have zero mean, the variance of the forecast error of variable  $i$  at time  $t+h$  caused by a shock in variable  $j$  at time  $t$  can be obtained by squaring the impulse response of variable  $i$  to the shock in  $j$ . By taking the sum of these squared impulse responses over the horizons  $h = 0, 1, 2, \dots, H$  and dividing it by the total  $H$ -step ahead forecast error variance we obtain the generalized forecast error variance decomposition. The elements of the variance decomposition  $\mathbf{D}_H = \{d_{ij}(H)\}$  are thus defined as

$$d_{ij}(H) = \frac{\sum_{h=0}^H (\text{GIRF}_{ij}(h, \varepsilon_{j,t} = \delta_j, \mathcal{F}_{t-1}))^2}{\sum_{j=1}^k \sum_{h=0}^H (\text{GIRF}_{ij}(h, \varepsilon_{j,t} = \delta_j, \mathcal{F}_{t-1}))^2} \quad (15)$$

$$= \frac{\sum_{h=0}^H (\text{GIRF}_{ij}(h, \varepsilon_{j,t} = \delta_j, \mathcal{F}_{t-1}))^2}{\sigma_i^2(H)}, \quad (16)$$

where  $\sigma_i^2(H)$  denotes the  $H$ -step ahead forecast error variance of the  $i$ th variable.

### 3.2.1 Special Case: Normally Distributed Innovations

By combining (14) and (15), we can show that for the special case of a VAR( $p$ ) model with the assumption of normal innovations and a shock to variable  $j$  of unit size ( $\delta_j = \sigma_j$ ), the generalized forecast error variance decomposition for variable  $i$  is defined as:

$$d_{ij}(H) = \frac{\sigma_j^{-2} \sum_{h=0}^H (\mathbf{e}'_i \mathbf{A}_h \boldsymbol{\Sigma} \mathbf{e}_j)^2}{\sum_{j=1}^k \sigma_j^{-2} \sum_{h=0}^H (\mathbf{e}'_i \mathbf{A}_h \boldsymbol{\Sigma} \mathbf{e}_j)^2} \quad (17)$$

By construction,  $\sum_{j=1}^k d_{ij}(H) = 1$  and  $\sum_{i=1}^k \sum_{j=1}^k d_{ij}(H) = k$ .

In this special case with normal innovations, we observe that the variance decomposition is independent of the size and sign of the shock  $\delta_j$  in variable  $j$  on which we condition the impulse responses since they cancel out in the numerator and denominator. Note that the impulse response functions are not independent of  $\delta_j$ .

### 3.3 Connectedness Measures

Following Diebold and Yilmaz (2012), we derive connectedness measures from the generalized forecast error variance decomposition. The *total spillover index* measures the contribution of spillovers of shocks across the  $k$  variables to the total forecast error variance. It is defined as the sum of all, except for the diagonal elements, of the forecast error variance decomposition  $\mathbf{D}_H$  as a fraction of the sum of all elements in  $\mathbf{D}_H$

$$S(H) = \frac{\sum_{i=1}^k \sum_{j=1|j \neq i}^k d_{ij}(H)}{\sum_{i=1}^k \sum_{j=1}^k d_{ij}(H)} \times 100 \quad (18)$$

$$= \frac{1}{k} \boldsymbol{\iota}' (\mathbf{D}_H - \text{diag}(\mathbf{D}_H)) \boldsymbol{\iota} \times 100, \quad (19)$$

where  $\boldsymbol{\iota}$  is a unit vector of dimension  $k \times 1$  and  $\text{diag}(\mathbf{D}_H)$  is a matrix consisting of the diagonal of  $\mathbf{D}_H$  and zeros elsewhere.

The idea of using generalized forecast error variance decomposition to measure financial connectedness not only helps us to understand how much of the shocks ‘spill over’ across the variables, but also enables us to learn about the direction of spillovers across variables. We obtain directional spillovers using the elements of the generalized variance decomposition matrix. We measure the directional return spillovers received by variable  $i$  from all other ‘source’ variables

$s$  as

$$S_{i \leftarrow s}(H) = \frac{\sum_{s=1|s \neq i}^k d_{is}(H)}{\sum_{s=1}^k d_{is}(H)} \times 100 = \sum_{s=1|s \neq i}^k d_{is}(H) \times 100 \quad (20)$$

$$= \mathbf{e}'_i \mathbf{D}_H (\mathbf{1} - \mathbf{e}_i) \times 100, \quad (21)$$

since  $\sum_{j=1}^k d_{ij}(H) = 1$ , as stated before. Note that this directional spillover from the *sources* to variable  $i$  is simply the fraction of the sum of row  $i$  in  $\mathbf{D}_H$  minus the value of diagonal element  $d_{ii}$  over the total sum of row  $i$ .

In similar fashion we measure directional spillovers transmitted from variable  $i$  to all other 'receiver' variables  $r$  as

$$S_{i \rightarrow r}(H) = \frac{\sum_{r=1|r \neq i}^k d_{ri}(H)}{\sum_{r=1}^k d_{ri}(H)} \times 100 \quad (22)$$

$$= \frac{(\mathbf{1}' - \mathbf{e}'_i) \mathbf{D}_H \mathbf{e}_i}{\mathbf{1}' \mathbf{D}_H \mathbf{e}_i} \times 100. \quad (23)$$

Note that this directional spillover from the variable  $i$  to the *receivers* is simply the sum of column  $i$  in  $\mathbf{D}_H$  minus the value of the diagonal element  $d_{ii}$  as a fraction of the total sum of column  $i$ .

One can think of the set of directional spillovers as providing a decomposition of total spillovers into those coming from (or to) a particular source. In order to identify which variables transfer or absorb most of the shocks in the system, we obtain the *net spillover* of variable  $i$  to or from all other variables  $j$  as

$$S_i(H) = S_{i \rightarrow r}(H) - S_{i \leftarrow s}(H) \quad (24)$$

The net spillover is simply the difference between gross shocks transmitted to and gross shocks received from all other variables.

### 3.4 Connectedness Tables

Diebold and Yilmaz (2012) construct connectedness tables which provide us with a natural interpretation of the connectedness measures. Table 1 illustrates the structure of connectedness tables. In the center of the table we find the forecast error variance decomposition matrix and in the bottom right corner we find the total spillover index. The right column and the bottom row provide the directional spillovers from and to a particular variable. Next to the spillover tables, Diebold and Yilmaz (2012) also introduce so-called spillover plots which provide an overview of

To	From				From others
	$y_1$	$y_2$	$\dots$	$y_k$	
$y_1$	$d_{11}(H)$	$d_{12}(H)$	$\dots$	$d_{1k}(H)$	$S_{1\leftarrow s}(H)$
$y_2$	$d_{21}(H)$	$d_{22}(H)$	$\dots$	$d_{2k}(H)$	$S_{2\leftarrow s}(H)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$y_k$	$d_{k1}(H)$	$d_{k2}(H)$	$\dots$	$d_{kk}(H)$	$S_{k\leftarrow s}(H)$
To others	$S_{1\rightarrow r}(H)$	$S_{2\rightarrow r}(H)$	$\dots$	$S_{k\rightarrow r}(H)$	$S(H)$

Table 1: Connectedness table for a  $H$ -step forecast horizon.

the connectedness dynamics over time. We obtain the dynamic total and directional spillovers by rolling window estimation and plot these over time.

### 3.5 Bootstrapped Standard Errors

Since connectedness measures are non-linear functions of the VAR parameter estimates, we cannot easily obtain exact standard errors. Therefore, we make use of a bootstrapping procedure to approximate the standard errors of the connectedness measures. From the total set of  $N \times k$  VAR innovation terms we randomly draw  $N$  sets of  $k$  corresponding innovations terms and calculate the connectedness measures for this random sample. We repeat this procedure  $n$  times (for  $n$  large) and obtain a set of  $n$  connectedness tables. Now we take the standard deviation for each separate connectedness measure over the  $n$  connectedness tables and use this as the standard error of the connectedness measures.

### 3.6 Alternative Assumptions

In the above, following Diebold and Yilmaz (2012), we assume the innovations in the VAR( $p$ ) model to be normally distributed. However, this assumption might be incorrect and alternative assumptions may improve the model.

It is commonly known that the empirical distributions of financial return series exhibit heavy-tails and asymmetry. Therefore, we expect the normality assumption to yield an incorrect

specification of the shock distribution. We let go of normality and propose two alternative distributions to overcome this problem. First, we impose a multivariate  $t$ -distribution on the innovation terms to account for heavy tails. Second, we propose using a skewed- $t$  distribution to also incorporate skewness in our approximating distribution.

### 3.6.1 $t$ -Distributed Innovations

Here, we assume the vector of random shocks,  $\boldsymbol{\varepsilon}_t$ , to follow a multivariate  $t$ -distribution, with location parameter  $\mathbf{0}$ , scale parameter  $\mathbf{V}$  and  $\nu$  degrees of freedom. We obtain impulse response functions that are only slightly different from those in the Diebold and Yilmaz (2012) connectedness framework. The covariance matrix  $\boldsymbol{\Psi}$  for a multivariate  $t$ -distribution with scale parameter  $\mathbf{V}$  is equal to  $\frac{\nu}{\nu-2}\mathbf{V}$ . Ding (2016) provides an extensive description of the conditional multivariate  $t$  distribution and some of its properties including the conditional expectation which we use to obtain a new generalized impulse response function.

$$\boldsymbol{\varepsilon}_t \sim \text{MVT}(\mathbf{0}, \boldsymbol{\Psi}, \nu) \quad (25)$$

$$\text{E}[\boldsymbol{\varepsilon}_t | \varepsilon_{j,t} = \delta_j] = \frac{\delta_j}{\psi_j^2} \boldsymbol{\zeta}_j, \quad (26)$$

$$\text{GIRF}_{ij}^t(h, \varepsilon_{j,t} = \psi_j, \mathcal{F}_{t-1}) = \left( \frac{1}{\psi_j} \right) \mathbf{e}'_i \mathbf{A}_h \boldsymbol{\Psi} \mathbf{e}_j \quad (27)$$

where  $\psi_j^2 = \text{E}[\varepsilon_{j,t}^2] = \mathbf{e}'_j \boldsymbol{\Psi} \mathbf{e}_j$  and  $\boldsymbol{\zeta}_j = \text{E}[\boldsymbol{\varepsilon}_t \varepsilon_{j,t}] = \boldsymbol{\Psi} \mathbf{e}_j$  are now obtained from the covariance matrix  $\boldsymbol{\Psi}$  and we used the scaling  $\delta_j = \psi_j$  to obtain the GIRF in 27. The generalized forecast error variance decomposition for a VAR( $p$ ) model with multivariate  $t$ -distributed innovations is defined as

$$d_{ij}^t(H) = \frac{\psi_j^{-2} \sum_{h=0}^H (\mathbf{e}'_i \mathbf{A}_h \boldsymbol{\Psi} \mathbf{e}_j)^2}{\sum_{j=1}^k \psi_j^{-2} \sum_{h=0}^H (\mathbf{e}'_i \mathbf{A}_h \boldsymbol{\Psi} \mathbf{e}_j)^2}, \quad (28)$$

which is different from the variance decomposition in the case of normality due to the difference in the covariance matrices  $\boldsymbol{\Sigma}$  for the multivariate normal distribution and  $\boldsymbol{\Psi}$  for the multivariate  $t$ -distribution. However, the differences between these covariance matrices are small and therefore there will be only small differences between using the multivariate  $t$ -distribution and the normal distribution.

### 3.6.2 Skewed- $t$ Distributed Innovations

Next, we assume the vector of random shocks,  $\boldsymbol{\varepsilon}_t$ , to follow a multivariate skewed- $t$  distribution. We make use of the multivariate skewed- $t$  distribution as defined by Azzalini and Capitanio (2003)

The probability density function of the multivariate skewed- $t$  distribution,  $\text{MST}_k(\boldsymbol{\xi}, \boldsymbol{\Omega}, \boldsymbol{\alpha}, \nu_s)$ , with location parameter  $\boldsymbol{\xi}$ , covariance matrix  $\boldsymbol{\Omega}$ , skewness parameter  $\boldsymbol{\alpha}$  and  $\nu_s$  degrees of freedom, is given by

$$f_{\text{MST}_k}(\mathbf{x}; \Theta) = 2t_k T \left\{ \boldsymbol{\alpha}' \mathbf{z} \left( \frac{\nu_s + k}{\nu_s + Q_{\mathbf{x}}^{\boldsymbol{\xi}, \boldsymbol{\Omega}}} \right)^{1/2}; \nu_s + k \right\}, \quad \mathbf{x} \in \mathbb{R}^k, \quad (29)$$

where  $\Theta = (\boldsymbol{\xi}, \boldsymbol{\Omega}, \boldsymbol{\alpha}, \nu_s)$ ,  $\mathbf{z} = \boldsymbol{\omega}^{-1}(\mathbf{x} - \boldsymbol{\xi})$ ,  $\boldsymbol{\omega}$  is the  $k \times k$  diagonal matrix containing the square roots of the diagonal elements of  $\boldsymbol{\Omega}$ ,  $Q_{\mathbf{x}}^{\boldsymbol{\xi}, \boldsymbol{\Omega}} = (\mathbf{x} - \boldsymbol{\xi})' \boldsymbol{\Omega}^{-1} (\mathbf{x} - \boldsymbol{\xi})$ ,  $t_k(\mathbf{x}; \boldsymbol{\xi}, \boldsymbol{\Omega}, \nu_s) = \Gamma\{(\nu_s + k)/2\} (1 + Q_{\mathbf{x}}^{\boldsymbol{\xi}, \boldsymbol{\Omega}}/\nu_s)^{-(\nu_s + k)/2} / \{|\boldsymbol{\Omega}|^{1/2} (\nu_s \pi)^{k/2} \Gamma(\nu_s/2)\}$  is the density of a  $k$ -variate Student's  $t$  distribution with  $\nu_s$  degrees of freedom, and  $T(\cdot; \nu_s + k)$  is the cumulative distribution function of a univariate Student's  $t$  distribution with  $\nu_s + k$  degrees of freedom.

Here, the skewness parameter  $\boldsymbol{\alpha}$  regulates the asymmetry of the distribution and the degrees-of-freedom parameter  $\nu_s > 0$  regulates the tails of the distribution. When an individual element of  $\boldsymbol{\alpha}$  is  $\alpha_i > 0$ , the distribution is skewed to the right, when  $\alpha_i < 0$  the distribution is skewed to the left. When all  $k$  components of  $\boldsymbol{\alpha}$  are zero, the multivariate skewed- $t$  density reduces to the multivariate Student's  $t$  density  $t_k(\cdot)$  and to the multivariate Normal density  $\phi_k(\cdot)$  when in addition  $\nu_s$  tends to infinity.

Since, there is no analytic solution for the conditional expectation of a skewed- $t$  distributed variable we have to approximate this conditional expectation numerically. For this approximation we take the following steps:

1. We set  $n$  to a large number and draw random values  $\boldsymbol{\varepsilon}_t^{(i)}$  from the multivariate skewed- $t$  distribution for  $i = 1, 2, \dots, n$ .

$$\boldsymbol{\varepsilon}_t^{(i)} \sim \text{MST}_k(\boldsymbol{\xi}, \boldsymbol{\Omega}, \boldsymbol{\alpha}, \nu_s) \quad (30)$$

2. We approximate the expectation of the shocks  $\boldsymbol{\varepsilon}_t$  conditional on a shock of at least size  $\varepsilon_{j,t}$  to the  $j^{\text{th}}$  variable by taking the conditional mean of  $\boldsymbol{\varepsilon}_t^{(i)}$ ,  $i = 1, 2, \dots, n$ . We obtain this conditional mean by imposing a certain restriction on  $\varepsilon_{j,t}$  and then taking the average of the subset left.

$$\mathbf{E}^*[\boldsymbol{\varepsilon}_t | \varepsilon_{j,t} \in \mathcal{C}] = \frac{\sum_{i=1}^n \boldsymbol{\varepsilon}_t^{(i)} \mathbf{I}[\varepsilon_{j,t}^{(i)} \in \mathcal{C}]}{\sum_{i=1}^n \mathbf{I}[\varepsilon_{j,t}^{(i)} \in \mathcal{C}]} \quad (31)$$

where  $\mathbf{I}[\varepsilon_{j,t}^{(i)} \in \mathcal{C}] = 1$  when the restriction holds and zero otherwise. Here  $\mathcal{C}$  defines a subset of the real numbers which defines the restriction on the conditioning shock  $\varepsilon_{j,t}$ . We can for example condition on shocks to the  $j^{\text{th}}$  variable that are larger than two standard deviations by setting the restriction to  $\varepsilon_{j,t}^{(i)} > 2\sigma_j$ .

Using the approximated conditional expectation  $E^*[\boldsymbol{\varepsilon}_t | \varepsilon_{j,t} \in \mathcal{C}]$  we can obtain the generalized impulse response functions

$$\text{GIRF}_j^{\text{skew-}t}(h, \varepsilon_{j,t} \in \mathcal{C}, \mathcal{F}_{t-1}) = E[\mathbf{y}_{t+h} | \varepsilon_{j,t} \in \mathcal{C}, \mathcal{F}_{t-1}] - E[\mathbf{y}_{t+h} | \mathcal{F}_{t-1}] \quad (32)$$

$$= \mathbf{A}_h E^*[\boldsymbol{\varepsilon}_t | \varepsilon_{j,t} \in \mathcal{C}]. \quad (33)$$

Again, we obtain  $\text{GIRF}_{ij}^{\text{skew-}t}(h, \varepsilon_{j,t} \in \mathcal{C}, \mathcal{F}_{t-1})$  by pre-multiplying with  $\mathbf{e}'_i$  and obtain the generalized forecast error variance decomposition

$$d_{ij}^{\text{skew-}t}(H) = \frac{\sum_{h=0}^H (\text{GIRF}_{ij}^{\text{skew-}t}(h, \varepsilon_{j,t} \in \mathcal{C}, \mathcal{F}_{t-1}))^2}{\sum_{j=1}^k \sum_{h=0}^H (\text{GIRF}_{ij}^{\text{skew-}t}(h, \varepsilon_{j,t} \in \mathcal{C}, \mathcal{F}_{t-1}))^2}. \quad (34)$$

Note that it is also possible to apply this simulation approach with the assumption of normal and  $t$ -distributed innovations. Furthermore, note that in our new approach we evaluate the responses to shocks coming from a specified interval of the shock distribution whereas the methods of Diebold and Yilmaz (2012) evaluate the responses to precisely defined shocks. However, since the connectedness measures of Diebold and Yilmaz (2012) are independent of the shock specification (size and sign), the use of this simulation approach in combination with the normal distribution will yield the same results as the original methodology of Diebold and Yilmaz (2012).



## 4 U.S. Asset Class Connectedness

In this section we perform a 'case-study' of financial connectedness among four U.S. asset classes in which we explore and evaluate the opportunities and limitations of the connectedness framework as introduced and developed by Diebold and Yilmaz (2012).

### 4.1 Data: Stocks, Bonds, Commodities, FX

We apply the connectedness methodology to measure return spillovers among four key U.S. asset classes: stocks, bonds, foreign exchange and commodities.<sup>3</sup> This data is of particular interest because spillovers among U.S. asset classes may be representative for global financial market dynamics and may have been an important aspect of the global financial crisis which began in 2007 (Diebold and Yilmaz, 2012).

We examine daily returns, calculated as the difference in log prices, on U.S. stock, bond, commodity, and foreign exchange markets. In particular, we examine the S&P 500 Index, the 10-year Treasury bond yield, the Dow-Jones/UBS Commodity Index, and the U.S. Dollar Index. The data is obtained from Bloomberg and spans the period of January 4, 2000 through October 16, 2017, for a total of 4,375 daily observations.

We derive the Treasury bond index from the 10-year Treasury bond yield. Assuming that the average coupon rate of the bonds that make up the index does not change from one day to the other, we can write the relationship between the yield  $y_t$  on day  $t$  and the price index  $P_t$  as  $P_t = 100 \times (1 + y_t)^{-10}$  (Diebold and Yilmaz, 2015). Having obtained the daily bond index, we measure the daily returns of the four different indices  $i$  as  $r_{i,t} = \ln(P_{i,t}/P_{i,t-1}) \times 100$ . Table 2 provides descriptive statistics of the annualized asset class returns.

Over the entire sample period, the investment yielding the highest average daily return was commodities. Second best was stocks, followed by bonds. The worst investment was the U.S. Dollar Index, which would even have resulted in an overall loss.

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<sup>3</sup>We opt for this dataset since it resembles the data used in Diebold and Yilmaz (2012). By taking this dataset, we enable ourselves to verify that we have correctly implemented the methods and are able to compare results. Our dataset differs from the data used by Diebold and Yilmaz (2012) in two ways. First, we evaluate a sample spanning the period 2000-2017 and Diebold and Yilmaz (2012) use a sample period of 1999 through 2010. Second, we are unable to derive the exact process used by Diebold and Yilmaz (2012) to cleanse the data. The choices we made during the cleaning of the data result in a slightly different dataset. We are unable to reproduce the summary statistics reported by Diebold and Yilmaz (2012) and therefore we also obtain slightly different values for the connectedness measures.

Table 2: Descriptive Statistics: U.S. Asset Classes

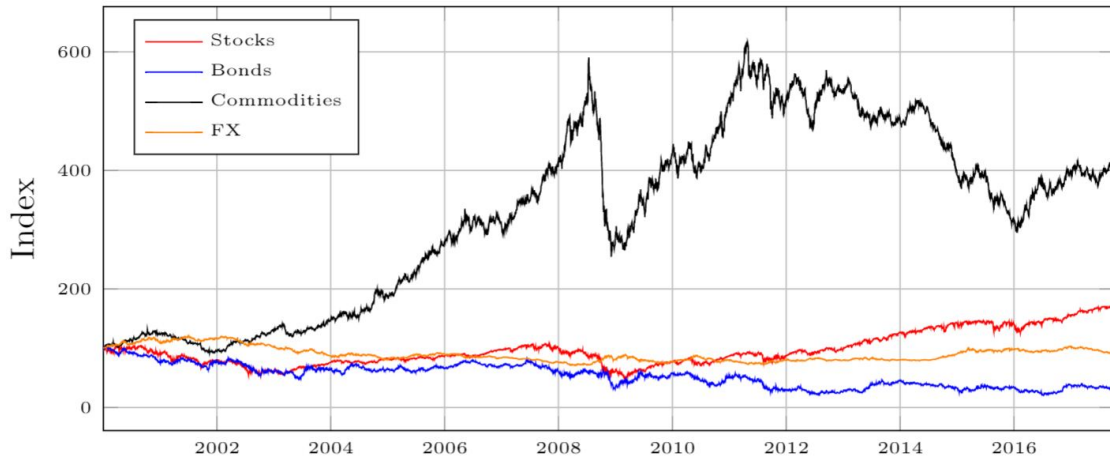
<b>(a) Annualized Log Returns</b>				
	Stocks	Bonds	Commodities	FX
Mean	3.02	2.40	7.76	-0.23
Median	12.41	9.64	8.96	-0.32
Minimum	-9.47	-2.57	-8.44	-2.73
Maximum	10.96	4.60	6.47	2.52
Standard Deviation	309.34	148.92	263.05	130.83
Skewness	-0.20	-0.03	-0.31	-0.02
Kurtosis	11.51	5.18	6.64	4.46
Jarque-Bera	13234.49	863.74	2490.09	391.13
P-Value	0.00	0.00	0.00	0.00
<b>(b) Correlation Matrix</b>				
	Stocks	Bonds	Commodities	FX
Stocks	1.000	-0.401	0.273	-0.075
Bonds	-0.401	1.000	-0.170	-0.129
Commodities	0.273	-0.170	1.000	-0.337
FX	-0.075	-0.129	-0.337	1.000

**Notes:** Panel (a) reports descriptive statistics for annualized daily returns on the four U.S. asset classes Stocks, Bonds, Commodities and FX for the period of January 4, 2000 through October 16, 2017. Panel (b) reports the correlation matrix for these four asset classes. The annualized returns are obtained by multiplying the daily returns by 252 (trading days). For the minimum and maximum returns we report the daily returns (de-annualized).

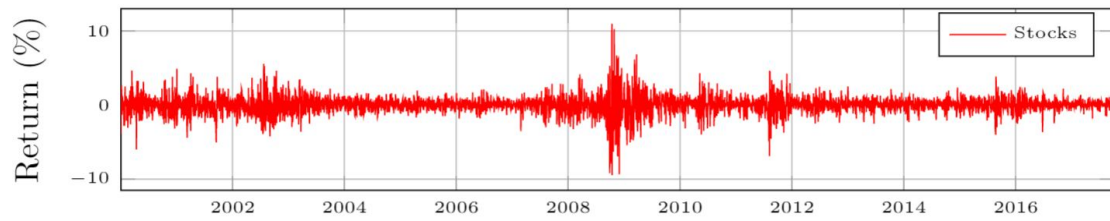
Figure 1 provides an overview of the indices and returns for the different asset classes over time. The plot in the top panel shows us the path which the indices followed over time. The plots in the lower panels illustrate the asset class returns. In these figures we can observe the volatility of the returns. As we would have expected, we observe a common feature of financial return series, namely that volatility is not constant over time. There exist alternating periods of predominantly high and low volatility (volatility clustering). Figure 1 shows that volatility was particularly high following the global financial crisis starting in 2007.

In order to get a general notion of how the asset classes are related to each other, we present the correlation matrix in Panel (b) of Table 2. The stocks asset class is strongest correlated to bonds (-0.401), followed by commodities (0.273). The correlation of stocks with FX is close to zero. The Treasury index shows, next to the strong correlation to stocks, weak correlations with the commodity and dollar indices (-0.170 and -0.129, respectively). Finally, the commodity and dollar indices exhibit strong negative correlation.

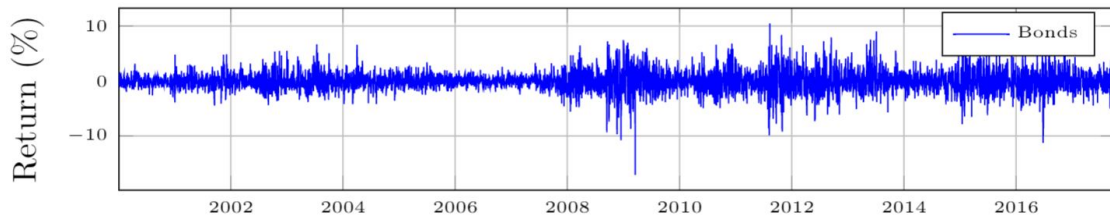
(a) Indices: U.S. Asset Classes



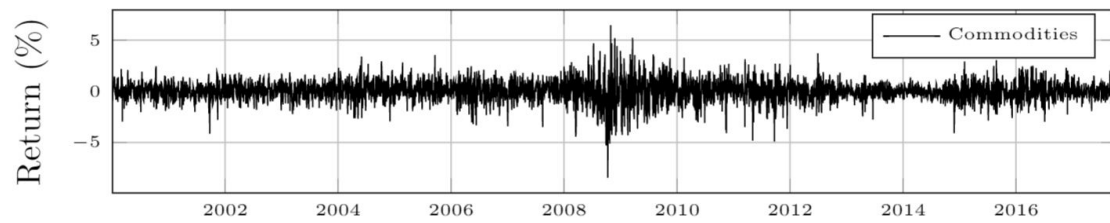
(b) Returns: Stocks



(c) Returns: Bonds



(d) Returns: Commodities



(e) Returns: FX

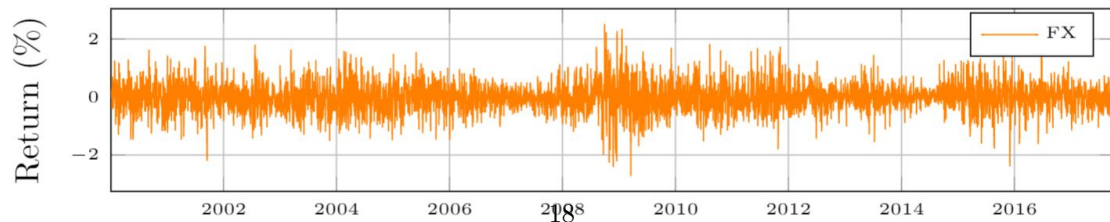


Figure 1: Indices and returns of four U.S. asset classes.

Table 3: VAR(5) Model

	Eq(-1)	Trs(-1)	Com(-1)	FX(-1)	Eq(-2)	Trs(-2)	Com(-2)	FX(-2)
Stocks	-0.075***	0.006	-0.031	-0.001	-0.077***	-0.062*	0.020	-0.052
Bonds	0.026***	-0.021	0.007	0.009	0.010	-0.043**	-0.013	0.001
Commodities	0.117***	0.038	-0.034**	-0.036	-0.024*	0.016	0.043***	-0.016
FX	-0.051***	-0.049***	-0.007	-0.031*	0.002	-0.009	-0.014	0.006
	Eq(-3)	Trs(-3)	Com(-3)	FX(-3)	Eq(-4)	Trs(-4)	Com(-4)	FX(-4)
Stocks	0.017	0.032	0.003	-0.035	-0.036**	-0.033	0.001	-0.020
Bonds	0.011	0.003	-0.022**	-0.005	0.008	0.023	0.005	-0.009
Commodities	-0.002	0.004	0.013	0.015	-0.008	-0.041	0.016	-0.025
FX	0.018**	-0.015	-0.006	-0.021	0.007	0.027*	0.002	0.026
	Eq(-5)	Trs(-5)	Com(-5)	FX(-5)	Const.	R <sup>2</sup>	F-stat	p-value
Stocks	-0.024	0.044	-0.023	-0.013	0.015	0.016	3.62***	0.000
Bonds	0.007	-0.014	0.019**	0.016	0.009	0.011	2.51***	0.000
Commodities	0.002	-0.008	0.000	0.071**	0.029*	0.021	4.70***	0.000
FX	0.001	0.003	-0.005	-0.006	0.000	0.017	3.86***	0.000

**Notes:** This table reports the regression coefficients, R-squared and F-statistics with corresponding p-values for the separate regression equations of the VAR(5) model in which constant terms are included. These statistics are estimated on the returns of four U.S. asset classes for the period of January 4, 2000 through October 16, 2017. In the notation of the explanatory variables Eq, Trs, Com and FX refer to the asset classes Stocks, Bonds, Commodities and FX respectively, and the lag order is denoted in parentheses. \*\*\*, \*\* and \* correspond to 1, 5 and 10 percent significance levels.

## 4.2 The Diebold-Yilmaz Connectedness Framework

### 4.2.1 Static (Full-Sample) Financial Connectedness

We estimate the dynamics in the system of asset classes with a VAR(5) model. The estimated model is presented in Table 3. Overall, we find the regression coefficients up to lag 5 to be jointly significant, but that the first lag variables account for most of the system dynamics. Especially the coefficients for the first lag of the stock returns are highly significant for all asset classes.<sup>4</sup>

Using the VAR(5) model parameters, we obtain the connectedness measures, as reported in Table 4, based on a 10-day ahead forecast error variance decomposition. Throughout this paper we use 10-day ahead forecast error variance decompositions unless specifically stated otherwise. Here, we focus on the full-sample connectedness as reported in Panel (a).

First, we focus on the forecast error variance decomposition given in the top-left part of the table. This variance decomposition provides estimated values of the asset-to-asset directional

<sup>4</sup>Note that these highly significant VAR coefficients suggest that there is some predictability in asset returns. However, this is in contradiction with the literature. Based on previous findings, we would only expect to find significant coefficients when evaluating the dynamics of the volatility of return series instead of the return series itself. Further research is needed to explain these highly significant lags.

spillover effects in the system. For example, based on the 'stocks-bonds' entry of this table, one can state that 12.7% of the forecast error variance of stocks comes from the spillover of return shocks in the bonds asset class.

It is easy to see that the largest return spillovers exist between stocks and bonds, in which the stocks asset class is the net sender and the bonds asset class is the net receiver of spillovers. We also observe strong connectedness among the commodity and FX market, in which the commodity market is the net sender and the FX market is the net receiver and has very low connectedness to other markets.

Second, the right part of the table reports the total directional connectedness measures, indicating to what extent the asset class returns are affected by return shocks in other asset classes. We observe that stocks and commodities receive relatively large spillover effects from the other asset classes and spillovers from the other asset classes have a relatively small impact on FX.

Third, the bottom part of the table gives the spillover of return shocks sent by a specific asset class to all other asset classes. Here we observe that stocks send relatively large spillovers to the other asset classes. Again, we see that FX is weakly connected to the other asset classes with a spillover to others of 11.0%.

Fourth, the Spillover Index for this financial system of asset classes is 17.5% (bottom right corner). This means that 17.5% of the total forecast error variance in the model comes from spillovers of return shocks across the four asset classes.

The variance decomposition estimates are related to the correlation matrix (as given in Table 2). Variance decompositions, which are key to the connectedness measures of Diebold and Yilmaz (2012), add information on the time-profile of returns, as obtained from the VAR( $p$ ) model, to the contemporaneous relations, as obtained from the covariance matrix. This results in asymmetric connectedness measures which are more informative regarding the system's connectedness than can be approximated by a simple symmetrical correlation matrix.

The connectedness measures can also be interpreted in a network setting. Diebold and Yilmaz (2014) show that variance decompositions define weighted, directed networks. Building on these insights, we visualize the financial network, as defined by Table 4 Panel (a) in Figure 2. The arrows follow the direction of the individual spillovers and large spillover effects are highlighted by the use of darker arrows. The size of the nodes are proportional to the spillovers each asset class sends to other asset classes. The color of the nodes indicates whether an asset class is a net sender or a net receiver of spillover effects. The color scheme for the nodes comes from a fading color

Table 4: Return Connectedness Tables, Multiple Time Periods

<b>(a) Full-Sample (2000-2017)</b>					
<i>To</i>	<i>From</i>				
	Stocks	Bonds	Commodities	FX	Others
Stocks	80.0 (1.5)	12.7 (1.1)	6.7 (0.9)	0.7 (0.3)	20.0 (1.5)
Bonds	13.4 (1.2)	82.3 (1.3)	2.9 (0.5)	1.4 (0.5)	17.7 (1.3)
Commodities	7.8 (0.9)	2.7 (0.5)	80.7 (1.4)	8.9 (0.8)	19.3 (1.4)
FX	1.8 (0.3)	1.6 (0.5)	9.5 (0.9)	87.2 (0.9)	12.8 (0.9)
Others	23.0 (1.7)	16.9 (1.3)	19.1 (1.6)	11.0 (0.9)	17.5 (1.0)

<b>(b) Crisis period (2012)</b>					
<i>To</i>	<i>From</i>				
	Stocks	Bonds	Commodities	FX	Others
Stocks	51.8 (2.7)	20.0 (2.1)	14.7 (2.2)	13.6 (1.9)	48.3 (2.7)
Bonds	22.9 (2.0)	61.6 (4.0)	8.8 (2.0)	6.7 (1.9)	38.4 (4.0)
Commodities	16.8 (2.1)	10.4 (1.7)	57.4 (3.7)	15.5 (2.4)	42.6 (3.7)
FX	14.7 (2.0)	10.5 (1.6)	16.5 (2.5)	58.3 (3.5)	41.7 (3.5)
Others	54.4 (3.9)	40.8 (4.3)	40.0 (5.1)	35.8 (4.6)	42.7 (3.0)

<b>(c) Recent period (2016-2017)</b>					
<i>To</i>	<i>From</i>				
	Stocks	Bonds	Commodities	FX	Others
Stocks	82.8 (4.2)	8.5 (2.9)	2.9 (1.7)	5.8 (2.4)	17.2 (4.2)
Bonds	10.5 (2.8)	69.6 (3.4)	1.5 (0.7)	18.4 (3.0)	30.4 (3.4)
Commodities	3.5 (1.7)	2.4 (0.9)	92.0 (1.9)	2.1 (0.9)	8.0 (1.9)
FX	8.1 (2.3)	20.3 (2.9)	1.6 (0.7)	70.0 (3.2)	30.0 (3.2)
Others	22.1 (4.7)	31.2 (4.5)	5.9 (1.8)	26.3 (3.9)	21.4 (2.5)

**Notes:** This table reports the return connectedness measures as developed by Diebold and Yilmaz (2012) with bootstrapped standard errors (10,000 iterations) in parentheses for the set of four U.S. asset classes. The variance decompositions in all panels are obtained via 10-day ahead impulse response functions based on VAR(5) models with constant terms included. Panel (a) reports the full sample connectedness measures, estimated for the period of January 4, 2000 through October 16, 2017 for a total of 4,375 observations. Panel (b) and (c) report connectedness measures for the sub samples January 3, 2012 through January 10, 2013 and October 3, 2016 through October 16, 2017, respectively.

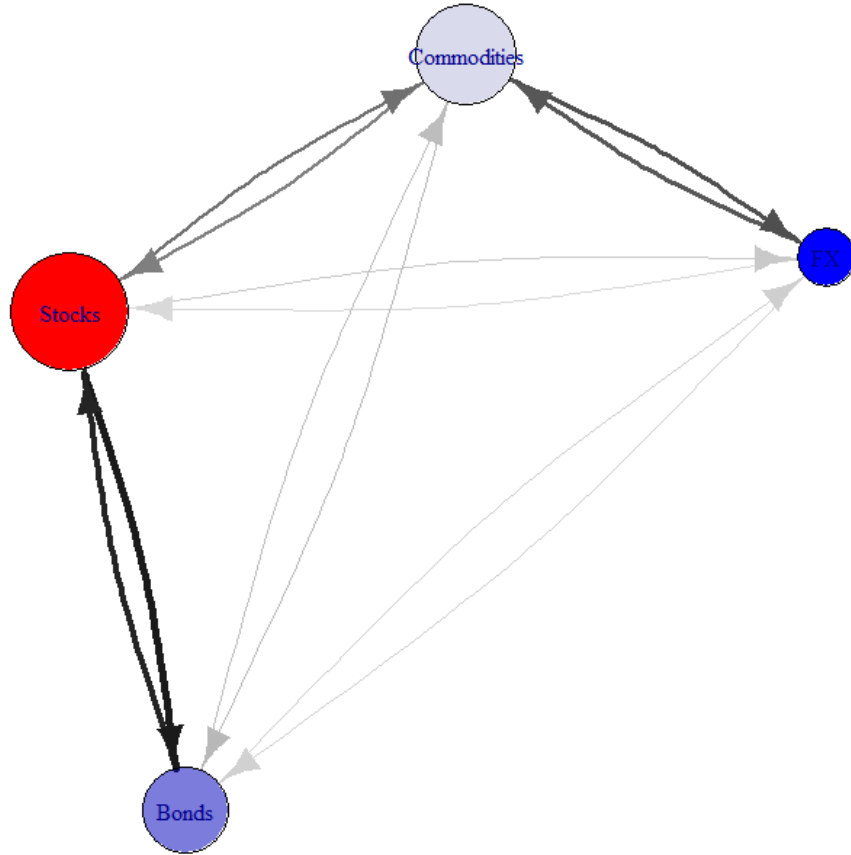


Figure 2: A network interpretation of financial connectedness.

scale from red to white to blue where red are net senders and blue are net receivers. Furthermore, the closer the nodes are positioned together, the stronger the connectedness between the two asset classes.

#### 4.2.2 Time-Variation in Financial Connectedness

In the above, we evaluate the full-sample connectedness spanning the period 2000-2017. As mentioned before, Figure 1 indicates that volatility varies over time which might lead to time variation in the dependence structure of the asset classes.

The Panels (b) and (c) in Table 4 illustrate that there is time-variation in connectedness measures. We observe large differences in connectedness between the crisis period and the more recent stable period. For example, we observe large changes in the directional spillover from commodities to stocks. In the more recent period, this connectedness measure is estimated to



Figure 3: Total return connectedness of stocks, bonds, commodities and FX.

be around 3%, while it is estimated to be close to 15% in 2012. The time-dynamics of the individual directional spillovers translate to a time-varying total Spillover Index which was very high in 2012 (43%), but has returned to a lower level (21%) in the more recent period.

Although point estimations, such as provided by Panel (b) and (c) in Table 4, give a notion of the time-profile of spillover effects, it is better to estimate the model in a rolling window setting in order to observe how the connectedness measures evolve over time. The resulting dynamic spillover index is presented in Figure 3.

Figure 3 illustrates that connectedness measures, and thus the underlying dependence structure of the financial system which we are trying to approximate, change heavily over time. We observe, for the VAR(5) model with a 1-year rolling window, that in the early 2000s the spillover index was relatively low at around 10-20% with a spike in 2003 to 30%. In 2006 the spillover index was at its lowest point (just below 10%) within the 2000-2017 sample period. In the following years, as the global financial and European debt crisis evolved, the system's connectedness increased and reached its maximum near the end of 2012 at 55%. In 2013, the spillover index decreased rapidly to a relatively low level of around 20%. In the following years the index remained in the 15-25% range with a brief spike in 2016 to 30%. The most recent observations are spillover indices around 20% in October 2017.



Note that, from 2006 onwards, the system's connectedness was increasing, which was probably caused by the state of distress the U.S. markets were in during the financial crisis. These markets stabilized and started recovering by the summer of 2009, and thus one might also expect the total spillover index to stabilize and eventually decrease. However, we observe that it kept increasing after 2009 and reached its maximum near the end of 2012 after which it quickly dropped back to a level of around 20%. For now, we cannot provide an easy explanation for this observation and further research is needed to evaluate this result.

Next, we observe that the level and time profile of the connectedness measure depends on the rolling window length. The timing ability of the model is better for the shorter 1-year rolling window. For the 3- and 5-year rolling window, the spillover dynamics are averaged out over a longer period and therefore the changes in connectedness indicated by these models are lagged. Although, the latter two models have less timing ability, they support the conclusion of the 1-year window model that connectedness was relatively low before the financial crisis, high during the crisis and has decreased to lower levels afterwards.

The timing ability of the model is important because in practice, risk managers and other financial agents need to have information about the current state of the markets to base decisions on. If they would use the 3- and 5-year window, they may be too late to respond effectively.

Therefore, one might argue that it is better to estimate the connectedness measures on an even shorter window than 1-year to increase the timing ability. However, the number of parameters in a VAR( $p$ ) model grows linearly in the order  $p$  and quadratically in the number of variables  $k$ . In this case-study with 4 financial variables and a VAR(5) model, we already need to fit 84 regression parameters (21 per regression equation). Therefore, we need as many observations as possible when estimating the model in order to keep the parameter uncertainty within an acceptable range (bias-variance tradeoff). We observe in Panel (b) and (c) of Table 4, which represent 1-year samples both in a crisis and more stable period, that the standard errors remain relatively low. Thus, we conclude that the increased parameter uncertainty does not seem to pose a worrying threat to the stability of our measures and we choose to move forward using a 1-year rolling window.

### 4.2.3 The Order $p$ of the VAR( $p$ ) Model

In all of the above, we interpret the connectedness of asset class returns estimated by a VAR(5) model. Here, we investigate the impact of the VAR-order  $p$  on the connectedness measures. Table 5 provides us with the connectedness measures derived from VAR( $p$ ) models, where  $p = 1, 3, 5$ ,

estimated on the most recent year of data.

The choice of the VAR-order has a weak effect on the dynamics in both the individual directional spillovers and the aggregated connectedness measures. There are some variations in the connectedness measures, but these variations lie within one standard deviation of each other and therefore there is no indication that they differ significantly.

Figure 3 gives the 1-year rolling spillover index for a VAR(1) model next to that of a VAR(5) model. The connectedness measures indicate slightly different levels of connectedness for the different model specifications but this does not impact the dynamic nature of the measures. The rolling spillover indices for VAR( $p$ ) models of order  $p = 2, 3, 4$  yield similar results.

The fact that the VAR(5) spillover index lies above the VAR(1) spillover index can be explained by the additional significant lags that are included in the VAR(5). This suggests that it might take a few days before the asset prices have fully adopted the new information that is related to the return shocks.

Going forward, we choose to continue with the VAR(5) model since the inclusion of more lags enables us to capture more of the time dynamics in the asset class returns. Again, in VAR( $p$ ) models, the choice for a high order  $p$  can introduce large parameter uncertainty. However, based on the low variation in the standard errors in Table 5, this is not the case here.

### 4.3 The Skewed- $t$ Connectedness Framework

#### 4.3.1 Conditional Expectations

In all of their works, Diebold and Yilmaz (2009); Yilmaz (2010); Diebold and Yilmaz (2011, 2012, 2014, 2016, 2015) impose a multivariate normal distribution on the innovation terms in the VAR model. However, Table 2 reports negative skewness and excess kurtosis in the returns of all markets. This suggests that the returns of the asset classes are not normally distributed and this is confirmed by significant Jarque-Bera normality test statistics.

From a risk perspective, one prefers a diversified portfolio in which assets show as little connectedness as possible. The results above show that the normality assumption used by Diebold and Yilmaz (2012) cannot be substantiated. By falsely assuming normality, we risk underestimating responses to shocks in the tails of the distribution and may be unable to identify differences in the responses to positive and negative shocks. Therefore, the Diebold-Yilmaz framework may suggest low portfolio connectedness, while there is in fact high connectedness for negative events and low connectedness for positive events.

Table 5: Return Connectedness Tables, Multiple VAR-orders

<b>(a) VAR(1)</b>					
<i>To</i>	<i>From</i>				
	Stocks	Bonds	Commodities	FX	Others
Stocks	87.8 (4.6)	6.5 (3.1)	2.0 (1.6)	3.6 (2.3)	12.2 (4.6)
Bonds	5.5 (2.7)	74.0 (3.9)	0.5 (0.6)	20.1 (3.3)	26.0 (3.9)
Commodities	2.4 (1.6)	0.8 (0.7)	95.2 (2.0)	1.5 (1.4)	4.8 (2.0)
FX	3.2 (2.1)	20.3 (3.4)	1.1 (1.1)	75.4 (3.4)	24.6 (3.4)
Others	11.1 (4.4)	27.7 (4.9)	3.6 (1.9)	25.2 (3.7)	16.9 (2.6)

<b>(b) VAR(3)</b>					
<i>To</i>	<i>From</i>				
	Stocks	Bonds	Commodities	FX	Others
Stocks	85.1 (4.5)	7.7 (3.1)	2.1 (1.6)	5.1 (2.5)	14.9 (4.5)
Bonds	8.7 (2.8)	71.9 (3.7)	1.0 (0.7)	18.5 (3.2)	28.1 (3.7)
Commodities	2.7 (1.7)	1.1 (0.9)	94.6 (2.0)	1.6 (1.3)	5.4 (2.0)
FX	7.4 (2.4)	19.9 (3.2)	1.1 (1.0)	71.6 (3.4)	28.4 (3.4)
Others	18.7 (4.7)	28.8 (4.7)	4.2 (1.8)	25.1 (4.0)	19.2 (2.6)

<b>(c) VAR(5)</b>					
<i>To</i>	<i>From</i>				
	Stocks	Bonds	Commodities	FX	Others
Stocks	82.8 (4.2)	8.5 (2.9)	2.9 (1.7)	5.8 (2.5)	17.2 (4.2)
Bonds	10.5 (2.7)	69.7 (3.5)	1.5 (0.7)	18.3 (3.0)	30.3 (3.5)
Commodities	3.5 (1.6)	2.3 (0.9)	92.0 (1.9)	2.1 (1.0)	8.0 (1.9)
FX	8.2 (2.3)	20.2 (3.0)	1.6 (0.8)	70.0 (3.3)	30.0 (3.3)
Others	22.2 (4.6)	31.1 (4.5)	5.9 (1.8)	26.3 (3.9)	21.4 (2.5)

**Notes:**

This table reports the return connectedness measures as developed by Diebold and Yilmaz (2012) with bootstrapped standard errors (10,000 iterations) in parentheses for the set of four U.S. asset classes. The variance decompositions in all panels are obtained via 10-day ahead impulse response functions based on VAR( $p$ ) models with constant terms included estimated on the period of October 3, 2016 through October 16, 2017 for a total of 246 observations. The connectedness tables in Panel (a), (b) and (c) are obtained via VAR(1), VAR(3) and VAR(5) models, respectively.

The rejection of normality and presence of skewness and kurtosis pave the way for the introduction of an alternative shock distribution which enables us to account this non-normality. In our newly proposed connectedness methodology, we impose a multivariate skewed- $t$  distribution on the innovation terms. One of the reasons why we have opted for the skewed- $t$  distribution is that the normal distribution is nested in it (i.e. we obtain the normal distribution from the skewed- $t$  distribution when we set the skewness parameters to zero and let the degrees of freedom run off to infinity). This enables the connectedness model to account for skewness and kurtosis.

Table 6: Skewed- $t$  Distribution Parameters for the VAR(5) innovation terms, U.S. asset classes.

	Stocks	Bonds	Commodities	FX
Location parameter ( $\xi$ )	0.267 (0.063)	0.017 (0.042)	0.032 (0.056)	-0.017 (0.029)
Skewness parameter ( $\alpha$ )	-0.408 (0.091)	-0.179 (0.101)	0.031 (0.084)	0.003 (0.082)
Degrees of Freedom ( $\nu$ )	4.947 (0.186)			
Covariance Matrix ( $\Omega$ )	Stocks	Bonds	Commodities	FX
<i>Stocks</i>	0.783 (0.036)	-0.145 (0.011)	0.160 (0.017)	-0.020 (0.009)
<i>Bonds</i>	-0.145 (0.011)	0.215 (0.006)	-0.051 (0.007)	-0.029 (0.004)
<i>Commodities</i>	0.160 (0.017)	-0.051 (0.007)	0.662 (0.020)	-0.102 (0.006)
<i>FX</i>	-0.020 (0.009)	-0.029 (0.004)	-0.102 (0.006)	0.171 (0.005)

**Notes:** This table reports the parameter estimates for the multivariate Skewed- $t$  distribution function as given in equation (29) fitted on the innovations terms. Bootstrapped standard errors are given in parentheses. The innovation terms are obtained from a VAR(5) model, estimated on the set of four U.S. asset classes Stocks, Bonds, Commodities and FX for the period of January 4, 2000 through October 16, 2017.

Table 6 reports the fitted parameters for the skewed- $t$  distribution of the U.S. asset classes. We observe that the estimated degrees of freedom parameter is close to five and thus the fitted distribution is fat-tailed. The skewness parameters also indicate non-normality as they are non-zero (especially for stocks and bonds). The four skewness parameters indicate that there is asymmetry in particular directions of the distribution. Based on these four parameters, we cannot make generalized statements on whether the full multivariate distribution is skewed to the left or to the right. For such statements measures of multivariate skewness are needed. In support of our choice to reject the assumption of normality, the full set of estimated parameters show that the skewed- $t$  distribution does not converge to the normal distribution.

Equation 9 shows that the impulse response functions, and subsequently the variance decompositions and connectedness measures, depend heavily on the conditional contemporaneous expectation of the innovation terms in the VAR( $p$ ) model. Therefore, we expect this alternative distributional assumption to have a large impact on the connectedness results.

In order to make a comparison between DY (Diebold and Yilmaz (2012)) and skewed- $t$  connectedness, we now evaluate the conditional contemporaneous expectations of the innovation terms obtained by these different methodologies. Because of the asymmetry in the skewed- $t$  distribution, we are unable to obtain analytic solutions for the conditional expectations, as is possible in the DY normality framework. Therefore, we apply a sampling algorithm to obtain an approximation of the conditional shock distribution. To make the results for the normal and skewed- $t$  distribution comparable, first, we also apply this sampling procedure to the normal distribution. Second, for each distribution, we obtain conditional expectations using the same conditioning shock size. We determine the shock size by taking quantiles of the empirical distribution. We condition on shocks that lie below the first and fifth and above the 95<sup>th</sup> and 99<sup>th</sup> percentiles of the empirical distribution (we refer to this as  $q(0.01)$ ,  $q(0.05)$ ,  $q(0.95)$ ,  $q(0.99)$ ). Finally, to evaluate how our simulated shocks relate to 'real' shocks, we compare the normal and skewed- $t$  conditional expectations with those of the empirical distribution.

The results are reported in Table 7. In Panel (a), (b), (c) and (d) we condition on extreme shocks to the asset classes stocks, bonds, commodities and FX, respectively. The first column gives the conditional expectations in the DY framework. The other columns report the conditional expectations based on the empirical distribution and the normal and skewed- $t$  distributions obtained with the sampling procedure. In each panel, the first values in each column give the conditioning shock size and the latter four values give the conditional shock vector which we use in Equation 9 later on.

Table 7 should be interpreted as follows. Given a negative return shock (impulse) to the asset class stocks of magnitude larger than 3.44, and given the assumption of skewed- $t$  distributed innovations, we expect a positive contemporaneous return shock (response) for the asset class bonds of size 0.82. Under the normality assumption, we conditionally expect this to be a positive contemporaneous return shock of size 0.73. From the empirical distribution of the innovations, we obtain that the average response in bonds to such a negative shock in stocks was 1.15 over the full sample. This is an indication that the skewed- $t$  distribution is able to model the transmission of such shocks between these two asset classes more accurately. However, we do note that we cannot make stronger statements than saying it is an indication since there are only 44 observations in

Table 7: Conditional Contemporaneous Return Shocks

<i>Distribution</i>	DY N	q(0.01)			q(0.05)			q(0.95)			q(0.99)		
		Emp.	N	Sk-t	Emp.	N	Sk-t	Emp.	N	Sk-t	Emp.	N	Sk-t
<b>(a) Response to Shock in Stocks</b>													
<i>Shock Size</i>	1.22	-3.44	-3.44	-3.44	-1.93	-1.93	-1.93	1.70	1.70	1.70	3.32	3.32	3.32
Stocks	1.22	-5.04	-3.80	-4.56	-3.01	-2.45	-2.79	2.72	2.25	2.38	4.47	3.69	4.27
Bonds	-0.23	1.15	0.73	0.82	0.67	0.47	0.52	-0.51	-0.43	-0.51	-0.76	-0.71	-0.96
Commodities	0.29	-1.75	-0.92	-0.92	-0.83	-0.59	-0.57	0.57	0.54	0.52	0.82	0.88	0.96
FX	-0.04	0.32	0.14	0.12	0.13	0.09	0.07	-0.05	-0.08	-0.05	-0.06	-0.13	-0.08
<b>(b) Response to Shock in Bonds</b>													
<i>Shock Size</i>	0.59	-1.57	-1.57	-1.57	-0.96	-0.96	-0.96	0.90	0.90	0.90	1.44	1.44	1.44
Stocks	-0.48	1.36	1.44	1.16	0.96	0.99	0.81	-1.47	-0.95	-1.02	-2.47	-1.35	-1.56
Bonds	0.59	-1.90	-1.75	-2.08	-1.34	-1.20	-1.37	1.25	1.16	1.30	1.88	1.63	1.93
Commodities	-0.18	0.58	0.55	0.47	0.29	0.38	0.32	-0.52	-0.36	-0.31	-1.22	-0.51	-0.48
FX	-0.07	0.32	0.19	0.29	0.19	0.14	0.19	-0.08	-0.13	-0.17	-0.22	-0.18	-0.25
<b>(c) Response to Shock in Commodities</b>													
<i>Shock Size</i>	1.03	-2.91	-2.91	-2.91	-1.64	-1.64	-1.64	1.62	1.62	1.62	2.60	2.60	2.60
Stocks	0.35	-2.11	-1.08	-1.18	-0.98	-0.70	-0.67	0.67	0.69	0.43	0.78	0.97	0.58
Bonds	-0.10	0.54	0.33	0.28	0.30	0.21	0.17	-0.25	-0.21	-0.19	-0.36	-0.29	-0.29
Commodities	1.03	-3.84	-3.22	-3.84	-2.43	-2.08	-2.36	2.24	2.06	2.33	3.31	2.93	3.46
FX	-0.17	0.63	0.52	0.60	0.38	0.34	0.37	-0.39	-0.34	-0.35	-0.62	-0.48	-0.52
<b>(d) Response to Shock in FX</b>													
<i>Shock Size</i>	0.51	-1.24	-1.24	-1.24	-0.85	-0.85	-0.85	0.83	0.83	0.83	1.34	1.34	1.34
Stocks	-0.10	0.34	0.29	-0.05	0.15	0.21	0.00	-0.41	-0.21	-0.24	-1.19	-0.30	-0.44
Bonds	-0.07	0.24	0.20	0.27	0.17	0.15	0.20	-0.16	-0.15	-0.21	0.08	-0.22	-0.32
Commodities	-0.34	1.21	0.93	0.98	0.71	0.70	0.71	-0.73	-0.69	-0.71	-1.50	-0.98	-1.07
FX	0.51	-1.62	-1.41	-1.67	-1.13	-1.06	-1.21	1.14	1.05	1.20	1.57	1.50	1.79

**Notes:** This table reports the expected contemporaneous responses in asset class returns to a return shock in another asset class. The first row of Panel (a) gives the minimum size of the shock in stocks on which the expected responses in the other asset classes are conditional. The first column in Panel (a) reports the expected responses of the asset classes, conditional on a one-standard deviation ( $\sigma$ ) shock in stocks, as obtained via the methods of Diebold and Yilmaz (2012). The next three columns report the expected return responses of the asset classes, conditional on a negative shock in stocks of size larger than the first percentile of the empirical distribution ( $q(0.01)$ ). The second column report the conditional responses as obtained via the empirical shock distribution, and the third and fourth column make use of 10,000,000 simulated values from the fitted normal and skewed- $t$  distributions. The remaining columns report similar values for negative return shocks smaller than the fifth percentile ( $q(0.05)$ ) and positive return shocks larger than the 95th and 99th percentile ( $q(0.95)$  and  $q(0.99)$ ). Panel (b), (c) and (d) are similar but give the expected conditional asset class responses to return shocks in bonds, commodities and FX, respectively. The results in this table are estimated on the full sample (January 4, 2000 through October 16, 2017) for a total of 4,375 observations.

the first percentile of the empirical distribution. For shocks coming from the top or bottom five percentiles we can make a more accurate comparison since these parts of the tail include 219 observations.

The first thing we notice in Table 7 is that, as expected, the different distributional assumptions result in different conditional expectations. While large differences can be observed, it is difficult to assess which distribution is more accurate (by comparing to the empirical conditional expectations). Overall, in 13 out of the 16 cases, the skewed- $t$  distribution seems to be more accurate than the normal distribution when it comes to approximating the conditional expectation of an extreme shock in the shocked asset class itself (i.e. the entries stocks-stocks, bonds-bonds, etc.).

In contrast to this, for the conditional expectations of the transmitted shocks (i.e. the entries stocks-bonds, stocks-commodities, stocks-FX, etc.), we observe that in 25 out of 48 cases, the normal distribution is better at approximating the empirical contemporaneous shock transmission than the skewed- $t$  distribution. Therefore, we conclude that the new distributional assumptions result in different conditional expectations, but that we are unable to assess which one performs better.

Furthermore, we observe that overall, there exist large differences between the empirical conditional shocks and both the normal and skewed- $t$  conditional shocks. This indicates that, while using the skewed- $t$  distribution may be an improvement to the model, this skewed- $t$  distribution might still be too restrictive to accurately model the contemporaneous relations between the return shocks.

Since the comparison to the empirical conditional shocks does not indicate which model performs better, we move forward with the assumption that the conditional expectations of the skewed- $t$  distribution are correct and compare how the conditional expectations of the normal distribution relate to it.

In Table 7 we observe for large negative shocks ( $q(0.01)$  and  $q(0.05)$ ), that the conditional responses are generally underestimated by the normal distribution when we assume that the skewed- $t$  distribution is the correct choice. In some cases, this underestimation of the conditional shocks by the normal distribution is large. For example, the shock to bonds conditional on a shock in stocks is underestimated by about 11% (0.82 versus 0.73). The only significant exception to this underestimation is the overestimation of the response in stocks to a shock in bonds.

For large positive shocks ( $q(0.95)$  and  $q(0.99)$ ), we observe a stronger underestimation by the normal distribution model. Now, the shock to bonds conditional on a shock in stocks is under-

estimated by about 26% (-0.96 versus -0.71). Here, the exception to this underestimation is the overestimation of the response in stocks conditional on an extreme shock in commodities. These large differences underwrite the importance of using a correctly specified shock distribution.

With the new distributional assumption, most conditional shocks change only in size. However, in the case of the expected response in stocks, conditional on an extreme negative return in FX ( $q(0.01)$ ), we see that not only the absolute magnitude changes, but also the sign of the conditional expectation changes (-0.03 versus 0.13). Although a shock response of -0.03 is a very small and therefore unlikely to incite asset-to-asset spillovers, it does show us that the system dynamics can fundamentally change as a result of changes in the underlying distributional assumptions on the innovation terms.

Note that the conditional expectations in Table 7 are obtained from VAR(5) residuals which have been estimated on the full data range. These conditional expectations will vary when evaluating subsets of data and when performing rolling window estimation on for the connectedness measures.

Furthermore, assuming the skewed- $t$  distribution to be the correct choice may be argued to be naive and other distributions might also be considered. However, the statement that the skewed- $t$  distribution is more suitable to fit the data than the normal distribution is a statement which we can make without hesitations. Since the normal distribution is nested inside the skewed- $t$  distribution, the maximum likelihood estimation procedure will automatically converge towards a parameter fit with zero skewness and very high degrees of freedom when the shock distribution is actually normal. However, as shown in Table 6 we do not observe such a tendency and observe that maximum likelihood estimation selects non-zero skewness parameters and degrees of freedom close to 5. Therefore, we conclude that the assumption of the skewed- $t$  distribution is the one to move forward with.

### 4.3.2 Full-Sample Skewed- $t$ Connectedness

Now that we have established that changes in the shock distribution have a large impact on the conditional expectations, we evaluate the subsequential impact it has on the connectedness measures. Table 8 provides three full-sample connectedness tables based on the transmission of large negative shocks ( $q(0.01)$ ). For ease of comparison, Panel (a) gives the connectedness measures based on the DY-methodology. Panel (b) and (c) report the connectedness measures based on the conditional shock vectors from Table 7 for the skewed- $t$  and empirical distribution, respectively. Table 9 provides similar connectedness measures, but now for the transmission of



large positive shocks ( $q(0.99)$ ).

Overall, in the Panels (b) of Table 8 and 9, we observe that the newly proposed skewed- $t$  methodology yields very different connectedness measures as compared to the DY-framework which lead to different economic interpretations. Both the individual directional spillovers and the aggregated connectedness measures of the skewed- $t$  framework show large deviations from the DY-connectedness measures.

For example, for the DY-framework we observe a directional return spillover from stocks to bonds and from bonds to stocks of 13.4% and 12.7%, respectively. Thus, in the DY-framework there is a net spillover effect from stocks to bonds of 0.7%. Therefore, one might argue that it is hard to make a sharp distinction between the sender and receiver of return spillovers. Note that these numbers in the DY-framework represent the full range of shocks and do not distinguish between positive and negative return shocks and do not account for the non-normal characteristics of the shocks.

Now, in Table 8, Panel (b), when we do distinguish between positive and negative shocks, and do account for skewness and kurtosis in the shocks distribution, we observe that for large negative shocks, the bonds-to-stocks and stocks-to-bonds directional return spillovers are 13.3% and 5.8%, respectively. Hence, now we do observe a large net spillover effect of return shocks (7.5%) from stocks to bonds and thus arrive at a new economic interpretation; large negative return shocks in stocks pose a larger threat to bond returns than the other way around.

Similarly, in Table 9, Panel (b), we observe that, this time for, large positive shocks, the bonds-to-stocks and stocks-to-bonds directional return spillovers are 19.3% and 11.6%, respectively. Again, we observe a large net spillover effect (7.7%) from stocks to bonds. And again, we arrive at a new economic interpretation; for large positive return shocks, shocks to stocks also have a larger impact on bond returns than the other way around. Additionally, the impact of shocks in the asset classes is larger for positive shocks.

Furthermore, we observe in Table 8 that for the DY-framework shocks to the long-term interest rate (bonds) were the main contributor to the forecast error variance in stocks, but that this has changed to the commodities asset class for the skewed- $t$  framework for large negative shocks. For large positive shocks it remains the same and the contribution of commodities is diminished to a very low level of 1.8%.

Next to changes in the individual directional spillovers, we also observe large changes in the aggregated connectedness measures. Where, in the DY-framework the net spillover effects to other asset classes were relatively low (2.9%, -0.8%, -0.3% and -1.9%, respectively), they are much

Table 8: Return Connectedness Table, Large Negative Return Shocks

(a) Diebold-Yilmaz Framework	<i>From</i>				<i>From</i>
	<i>To</i>	Stocks	Bonds	Commodities	FX
Stocks	80.0	12.7	6.7	0.6	20.0
Bonds	13.4	82.4	2.9	1.4	17.7
Commodities	7.8	2.7	80.7	8.9	19.3
FX	1.7	1.5	9.5	87.3	12.7
<i>To Others</i>	22.9	16.9	19.0	10.9	<i>Spillover</i>
<i>From Others</i>	20.0	17.7	19.3	12.7	<i>Index:</i>
<i>Net Spillover</i>	2.9	-0.8	-0.3	-1.9	17.4
(b) Skewed- <i>t</i> Lower-Tail Connectedness	<i>From</i>				<i>From</i>
<i>To</i>	Stocks	Bonds	Commodities	FX	<i>Others</i>
Stocks	88.1	5.8	6.1	0.1	11.9
Bonds	13.3	83.5	1.8	1.4	16.5
Commodities	6.4	1.3	86.6	5.7	13.4
FX	1.9	2.7	11.1	84.3	15.7
<i>To Others</i>	21.5	9.8	19.0	7.2	<i>Spillover</i>
<i>From Others</i>	11.9	16.5	13.4	15.7	<i>Index:</i>
<i>Net Spillover</i>	9.6	-6.7	5.6	-8.5	14.4
(c) Empirical Lower-Tail Connectedness	<i>From</i>				<i>From</i>
<i>To</i>	Stocks	Bonds	Commodities	FX	<i>Others</i>
Stocks	79.7	5.9	14.1	0.4	20.3
Bonds	25.4	67.7	5.8	1.1	32.3
Commodities	16.7	1.7	74.1	7.5	25.9
FX	4.7	3.1	12.4	79.8	20.2
<i>To Others</i>	46.7	10.7	32.3	9.0	<i>Spillover</i>
<i>From Others</i>	20.3	32.3	25.9	20.2	<i>Index:</i>
<i>Net Spillover</i>	26.4	-21.6	6.4	-11.2	24.7

**Notes:** Panel (a) reports the return connectedness measures as developed by Diebold and Yilmaz (2012) for the set of four U.S. asset classes. The connectedness measures in Panel (b) and (c) are obtained using the skewed-*t* and empirical distribution to calculate the expected contemporaneous asset class responses to positive return shocks of size larger than the 99th percentile of the empirical distribution ( $q(0.99)$ ). The variance decompositions in all panels are obtained via 10-day ahead impulse response functions based on VAR(*p*) models with constant terms included estimated on the period of January 7, 2000 through October 16, 2017 for a total of 4,375 observations.

Table 9: Return Connectedness Table, Large Positive Return Shocks

(a) Diebold-Yilmaz Framework		<i>From</i>				<i>From Others</i>
		Stocks	Bonds	Commodities	FX	
<i>To</i>						
Stocks	80.0	12.7	6.7	0.6		20.0
Bonds	13.4	82.4	2.9	1.4		17.7
Commodities	7.8	2.7	80.7	8.9		19.3
FX	1.7	1.5	9.5	87.3		12.7
<i>To Others</i>	22.9	16.9	19.0	10.9		<i>Spillover</i>
<i>From Others</i>	20.0	17.7	19.3	12.7		<i>Index:</i>
<i>Net Spillover</i>	2.9	-0.8	-0.3	-1.9		17.4
(b) Skewed- <i>t</i> Upper-Tail Connectedness		<i>From</i>				<i>From Others</i>
<i>To</i>	Stocks	Bonds	Commodities	FX		
Stocks	85.7	11.6	1.8	1.0		14.3
Bonds	19.3	76.7	1.9	2.1		23.3
Commodities	7.8	1.7	82.5	8.1		17.5
FX	1.3	1.9	7.6	89.3		10.7
<i>To Others</i>	28.4	15.1	11.3	11.1		<i>Spillover</i>
<i>From Others</i>	14.3	23.3	17.5	10.7		<i>Index:</i>
<i>Net Spillover</i>	14.1	-8.2	-6.3	0.4		16.5
(c) Empirical Upper-Tail Connectedness		<i>From</i>				<i>From Others</i>
<i>To</i>	Stocks	Bonds	Commodities	FX		
Stocks	71.1	21.6	2.3	5.0		28.9
Bonds	14.1	82.4	3.3	0.2		17.6
Commodities	5.8	9.7	69.9	14.6		30.1
FX	1.7	1.9	13.2	83.3		16.7
<i>To Others</i>	21.6	33.2	18.7	19.8		<i>Spillover</i>
<i>From Others</i>	28.9	17.6	30.1	16.7		<i>Index:</i>
<i>Net Spillover</i>	-7.4	15.6	-11.3	3.1		23.3

**Notes:** Panel (a) reports the return connectedness measures as developed by Diebold and Yilmaz (2012) for the set of four U.S. asset classes. The connectedness measures in Panel (b) and (c) are obtained using the skewed-*t* and empirical distribution to calculate the expected contemporaneous asset class responses to negative return shocks of size larger than the first percentile of the empirical distribution ( $q(0.01)$ ). The variance decompositions in all panels are obtained via 10-day ahead impulse response functions based on VAR(*p*) models with constant terms included estimated on the period of January 7, 2000 through October 16, 2017 for a total of 4,375 observations.

larger in the skewed- $t$  connectedness framework (14.1%, -8.2%, -6.3%, and 0.4%, respectively for large positive shocks) and are larger for large positive shocks than for large negative shocks. However, for large negative return shocks, all asset classes have a net spillover effect to others and for large positive return shocks, FX is neither a sender nor a receiver of return spillovers to other asset classes.

Finally, we observe a total spillover index of 17.4% for the DY-framework, 14.4% for large negative skewed- $t$  return shocks and 16.5% for large positive skewed- $t$  return shocks. Hence, we do not observe a large change in the system's total connectedness level but we do observe large changes in the asset-to-asset connectedness levels.

As a final note, the Panels (c) in Table 8 and 9 indicate that not only the DY, but also the skewed- $t$  framework may be too restrictive to accurately identify the system's 'real' connectedness.

### 4.3.3 Dynamic Skewed- $t$ Connectedness

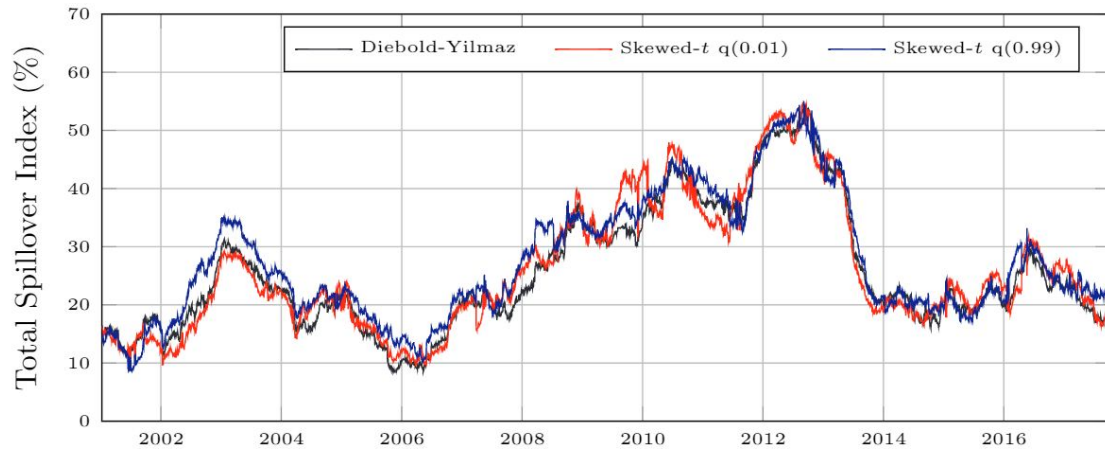
The differences between the normal and skewed- $t$  connectedness measures, which are discussed above, are based on the full sample period (2000-2017). However, as we have noted before, the level of connectedness varies over time. Therefore, in Figure 4, we compare the time profile of connectedness measures based on skewed- $t$  distributed innovations and compare it with those based on the normality assumption.

In Plot (a) of Figure 4, we observe that over time, there are some differences between the total spillover index of Diebold and Yilmaz (2012) based on normality and that of our skewed- $t$  framework. However, these differences are generally small and the overall dynamics of the spillover index are the same for both models. This was to be expected due to the small differences in the spillover indices in Table 8 and 9. We conclude that the new methodology has no significant impact on the total spillover index and thus the aggregated system's connectedness.

Differences in the financial connectedness measures become apparent when we evaluate individual directional spillovers, for example those from bonds to stocks. In Plot (b) for example, we observe that due to the new distributional assumptions, new economic insights arise for the period around 2003 and during and following the financial crisis (mid-2007 till end-2012).

Based on the DY-framework, one might argue that the directional spillover of bonds to stocks increased over 2003, with a peak at 30%. However, for the skewed- $t$  connectedness framework, we observe that large negative shocks ( $q(0.01)$ ) incited much higher spillovers, with a peak at 50%, and that for large positive shocks ( $q(0.99)$ ), there was an increase in spillovers during the

(a) Total Spillover Index: Stocks, Bonds, Commodities, FX



(b) Directional Spillover: From Bonds to Stocks

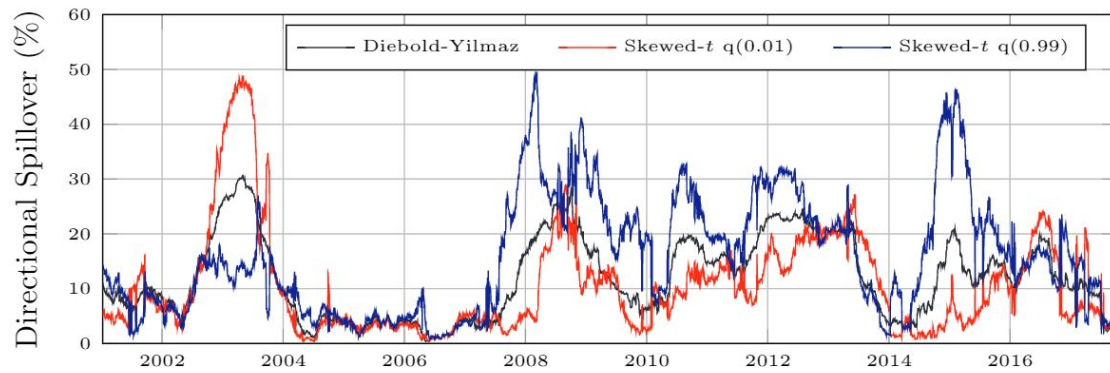


Figure 4: Dynamic Diebold-Yilmaz and Skewed- $t$  return connectedness measures, using a VAR(5) model and 1-year rolling window

second half of 2002, but the index halted around the 15% level and remained at this level for the following year before returning to lower levels in 2004.

Following the start of the global financial crisis in the summer of 2007, we observe that the DY directional spillover measure increased to a level of 20% and fluctuated around this level until the beginning of 2013, before returning to a lower level of 5% in the beginning of 2014. During the global financial and European sovereign debt crisis, we observe that the skewed- $t$  connectedness measures indicate much lower return spillovers resulting from large negative shocks and much higher values, with a peak at 50%, for spillovers resulting from positive return shocks. Also, for the period of mid-2014 until the start of 2016, we observe similar differences in the dynamic return spillover levels from bonds to stocks.

Based on all of the above, we conclude that the connectedness methodology of Diebold and Yilmaz (2012) is highly dependent on the assumption that the residuals of the VAR( $p$ ) model are normally distributed. However, as shown, this assumption is clearly unsubstantiated and it makes more sense to opt for a distribution which incorporates the high level of excess kurtosis and negative skewness exhibited by the residuals. Furthermore, by incorporating skewness, we are able to differentiate between the effects of positive and negative return shocks.

Note that we do not claim the skewed- $t$  assumption to be perfect, however, we think it is safe to say that it is better to go with skewed- $t$  distributed shocks than to keep the normal distribution for the estimation of the connectedness measures introduced by Diebold and Yilmaz (2012).

## 5 Global Stock Market Connectedness

In the previous section we have established that the newly proposed skewed- $t$  framework can have a large impact on financial connectedness measures. The spillovers related to the stocks asset class were affected the most. We expect that this was due to the fact that this asset class exhibited large skewness and the highest kurtosis. In this section we investigate financial connectedness among a broader set of stock markets around the world and assess the impact of the new methodology.

### 5.1 Data: 10 Global Stock Indices

In the analysis of global stock market connectedness we include 10 major stock markets. The list includes six industrial countries (the United States, the United Kingdom, France, Germany, Japan, and Australia) and four emerging market economies (Brazil, China, India and Hong Kong).<sup>5</sup>

Table 10: Summary Statistics: Global Equity Markets

	$\mu$	$\sigma$	<i>Median</i>	<i>Min*</i>	<i>Max*</i>	<i>Skew</i>	<i>Kurt</i>	<i>JB-stat</i>	<i>P-val</i>
USA	3.98	301.52	10.08	-9.47	10.96	-0.24	13.5	17366.55	0.00
UK	-0.70	298.05	5.04	-9.27	9.38	-0.14	10.54	8937.97	0.00
GER	2.87	373.34	17.64	-7.43	10.80	-0.01	8.02	3960.44	0.00
FRA	-2.24	367.91	5.04	-9.47	10.59	0.00	8.70	5101.18	0.00
JAP	2.38	371.42	0.00	-12.11	13.23	-0.49	10.94	10051.36	0.00
AUS	0.95	254.41	7.56	-8.70	5.63	-0.47	8.73	5292.08	0.00
HKG	1.64	356.60	2.52	-13.58	13.41	-0.02	13.46	17199.18	0.00
CHI	4.14	422.29	2.52	-9.70	8.97	-0.41	7.49	3274.32	0.00
IND	10.15	353.42	7.56	-11.81	15.99	-0.10	13.29	16628.71	0.00
BRA	6.79	435.78	0.00	-12.10	13.68	-0.07	7.77	3570.66	0.00

**Notes:**This table reports the summary statistics for annualized daily returns on 10 stock indices around the world for the period of January 7, 2003 through November 24, 2017 for a total of 3,771 observations. The annualized returns are obtained by multiplying the daily returns by 252 (trading days). \* Minimum and maximum daily returns (de-annualized).

For each country we use daily local-currency stock market indexes, taken from Bloomberg. The daily returns are calculated as the change in daily log prices. We provide descriptive statistics for the annualized returns in Table 10.

<sup>5</sup>Just as with the U.S. asset classes, we opt for this dataset since it resembles the data used by Diebold and Yilmaz (2015). By taking this dataset, we enable ourselves to compare results. Note that our sample period differs from that of Diebold and Yilmaz (2015). We evaluate a sample spanning the period 2002-2017 and Diebold and Yilmaz (2015) use a sample period of 1994 through 2013. Therefore we obtain slightly different values for the connectedness measures.

The developing economies India, Brazil and China have the best performing stock markets over the entire sample period, followed by the United States. France shows the worst performance over the sample period. The markets with the highest volatility are the emerging economies Brazil and China. Surprisingly, Germany has the highest volatility among the set of developed markets.

Most stock markets exhibit large negative skewness and all countries have excess kurtosis. The highly significant Jarque-Bera statistics verify the fact that the stock markets are not normally distributed and validate our choice for this dataset to illustrate the effect of dropping the normality condition of Diebold and Yilmaz (2012).

## 5.2 Full-Sample Return Connectedness

We estimate a VAR(5) model on the returns of the global stock markets and derive full-sample DY and skewed- $t$  connectedness measures from this. A graphical illustration of global stock market linkages based on DY connectedness is given in Figure 5 and DY and skewed- $t$  connectedness tables are given in Table 11.

In Figure 5, we observe that the largest directional spillover effects exist among the European countries and the United States. Both Australia, Brazil and Japan are related to, predominantly, the United States, but are mainly receivers of return spillovers. Among the Asian markets (Hong Kong, China and India), Hong Kong is the main sender of return spillovers to the other Asian markets. The United States, United Kingdom, Germany, France and Brazil are the net senders of returns spillovers and Japan, Australia, Hong Kong, China and India are the net receivers. Overall, the United States stock market has the strongest connectedness to other markets and the Chinese stock market the weakest. A more detailed representation of the system's connectedness can be found in the DY connectedness table in Panel (a) of Table 11.

In Panel (b) and (c) of Table 11, we report connectedness measures as obtained with the newly proposed skewed- $t$  connectedness methodology. For most individual directional spillovers we do not observe significant changes as a result of changing the distributional assumptions of the model. Especially the spillover effects among the United States and the European countries remain unchanged. For the emerging markets we observe some slight changes however these are not of significant size.

We obtain similar results for the aggregated connectedness measures. Both the total directional spillovers and the total spillover index show little deviation from the DY-connectedness measures. The only difference we observe is a lower spillover index of 54.5% for the transmission of large positive shocks compared to the spillover index of 62.6% in the original DY-connectedness



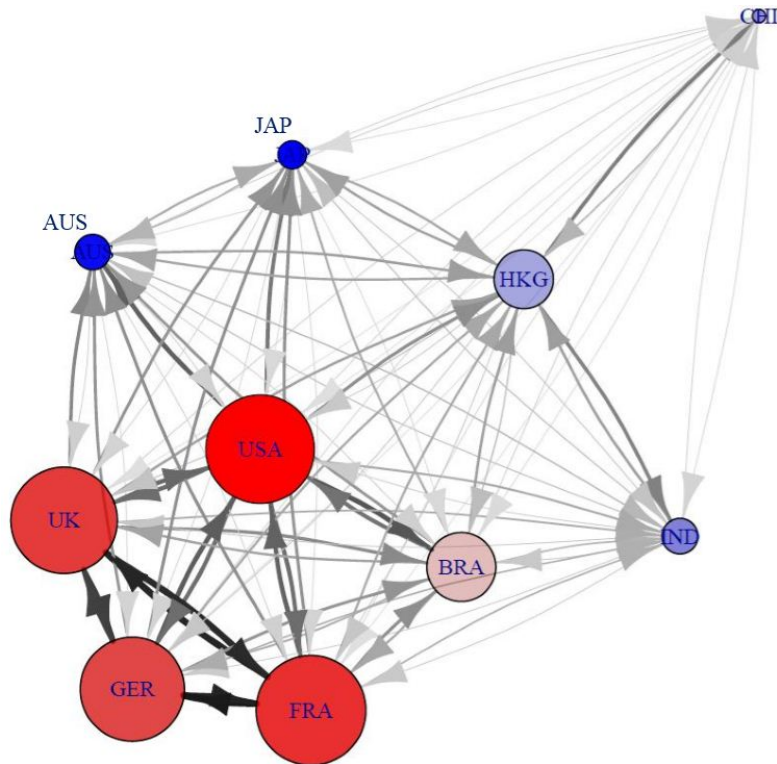


Figure 5: Return Connectedness among 10 global stock markets

framework.

We expected to find different upper- and lower tail connectedness, however the results do not comply with this expectation. There are multiple explanations. Perhaps, there simply is no difference in upper- and lower tail stock market behavior, however, this seems unlikely due to the skewness in the returns. Alternatively, there might be a problem with the estimation of the connectedness measures. Perhaps, by including 10 countries, the dimensions of the model gets too large to accurately fit the skewed- $t$  distribution. The algorithm we use in R to fit the skewed- $t$  distribution might be only able to accurately estimate the mean and variance parameters for such dimensions, and have difficulties to correctly identify the degrees of freedom and skewness parameters. This seems reasonable since the values in the skewed- $t$  connectedness tables resemble those in the DY-connectedness table.

To evaluate this hypothesis, we report the parameter estimates of the skewed- $t$  distribution for the ten stock markets in Table 12. We observe that the multivariate skewed- $t$  distribution has 5 degrees of freedom and non-zero skewness parameters. This parameter values show large

Table 11: Return Connectedness Table, Global Stock Markets

<b>(a) Diebold &amp; Yilmaz Connectedness</b>											<i>From</i>
<i>To</i>	USA	UK	GER	FRA	JPN	AUS	HKG	CHN	IND	BRA	<i>Others</i>
USA	34.1	13.9	15.1	14.6	1.4	2.1	3.1	0.4	2.6	12.7	65.9
UK	13.4	27.5	18.3	21.5	2.0	2.6	3.6	0.5	3.2	7.4	72.5
GER	13.5	18.7	28.2	23.1	1.9	1.9	3.2	0.4	2.9	6.3	71.9
FRA	13.3	20.9	21.8	26.6	2.0	2.1	3.2	0.4	2.9	6.7	73.3
JPN	12.2	9.1	9.6	9.5	33.7	6.6	7.6	1.0	4.1	6.5	66.2
AUS	14.9	10.6	9.1	9.7	5.4	29.0	7.1	1.2	4.0	8.9	70.9
HKG	9.8	7.6	6.9	6.8	7.0	7.9	32.6	5.5	8.0	8.0	67.5
CHN	1.9	2.1	1.5	1.7	2.1	2.8	11.6	71.2	2.6	2.5	28.8
IND	6.5	6.3	5.9	6.0	3.5	5.0	11.2	1.8	48.3	5.5	51.7
BRA	15.9	10.4	9.1	9.8	1.3	2.2	4.6	1.0	3.1	42.7	57.4
<i>To Others</i>	101.4	99.6	97.3	102.7	26.6	33.2	55.2	12.2	33.4	64.5	<i>Spillover</i>
<i>From Others</i>	65.9	72.5	71.9	73.3	66.2	70.9	67.5	28.8	51.7	57.4	<i>Index:</i>
<i>Net Spillover</i>	35.5	27.1	25.4	29.4	-39.6	-37.7	-12.3	-16.6	-18.3	7.1.0	62.6

<b>(b) Skewed-<i>t</i> Lower-Tail Connectedness</b>											<i>From</i>
<i>To</i>	USA	UK	GER	FRA	JPN	AUS	HKG	CHN	IND	BRA	<i>Others</i>
USA	34.3	14.3	14.9	15.1	1.7	2.3	2.6	0.8	2.3	11.7	65.7
UK	13.4	28.0	18.5	21.1	2.1	2.8	3.5	0.7	3.2	6.8	72.1
GER	13.2	18.8	28.3	23.7	2.0	2.0	3.0	0.5	2.9	5.5	71.6
FRA	13.2	20.5	22.6	27.1	2.1	2.1	3.0	0.6	2.9	5.9	72.9
JPN	12.1	9.1	9.6	9.7	36.7	6.0	6.2	0.9	3.6	6.1	63.3
AUS	14.1	10.5	8.8	9.4	5.4	31.8	6.7	1.4	3.6	8.4	68.3
HKG	9.7	7.8	7.0	6.9	6.1	7.3	34.9	5.6	6.9	7.9	65.2
CHN	2.0	2.1	1.5	1.7	1.7	2.6	10.7	73.3	2.0	2.4	26.7
IND	6.0	6.3	6.1	6.4	3.1	4.1	9.4	1.7	51.5	5.3	48.4
BRA	15.3	9.9	8.4	9.3	1.4	2.1	3.9	1.3	2.7	45.6	54.3
<i>To Others</i>	99.0	99.3	97.4	103.3	25.6	31.3	49.0	13.5	30.1	60.0	<i>Spillover</i>
<i>From Others</i>	65.7	72.1	71.6	72.9	63.3	68.3	65.2	26.7	48.4	54.3	<i>Index:</i>
<i>Net Spillover</i>	33.3	27.2	25.8	30.4	-37.7	-37.0	-16.2	-13.2	-18.3	5.7	60.9

<b>(c) Skewed-<i>t</i> Upper-Tail Connectedness</b>											<i>From</i>
<i>To</i>	USA	UK	GER	FRA	JPN	AUS	HKG	CHN	IND	BRA	<i>Others</i>
USA	40.9	14.2	15.1	15.1	0.5	0.9	1.1	0.1	0.9	11.2	59.1
UK	13.2	31.4	19.6	22.9	1.1	1.6	2.3	0.3	2.0	5.7	68.7
GER	12.9	19.7	31.5	25.7	1.1	1.0	1.8	0.2	1.8	4.3	68.5
FRA	13.0	21.7	24.3	29.8	1.1	1.1	1.9	0.2	1.9	4.9	70.1
JPN	11.3	7.9	8.5	8.4	43.8	5.9	6.0	0.7	2.5	5.0	56.2
AUS	14.8	9.8	7.9	8.5	4.8	37.9	5.8	0.9	1.9	7.8	62.2
HKG	9.5	6.8	5.8	5.8	5.6	6.6	41.9	5.2	5.7	7.1	58.1
CHN	1.2	1.4	0.8	1.0	1.0	1.7	9.6	80.6	1.1	1.6	19.4
IND	5.0	4.8	4.5	4.8	1.8	2.7	8.3	0.9	63.0	4.1	36.9
BRA	15.3	9.2	7.1	8.1	0.4	1.0	2.4	0.4	1.4	54.9	45.3
<i>To Others</i>	96.2	95.5	93.6	100.3	17.4	22.5	39.2	8.9	19.2	51.7	<i>Spillover</i>
<i>From Others</i>	59.1	68.7	68.5	70.1	56.2	62.2	58.1	19.4	36.9	45.3	<i>Index:</i>
<i>Net Spillover</i>	37.1	26.8	25.1	30.2	-38.8	-39.7	-18.9	-10.5	-17.7	6.4	54.5

**Notes:** Panel (a) reports the return connectedness measures as developed by Diebold and Yilmaz (2012) for a set of 10 stock markets. The connectedness measures in Panel (b) are obtained using the skewed-*t* distribution to calculate the expected contemporaneous asset class responses to negative return shocks of size larger than the first percentile of the distribution. The connectedness measures in Panel (c) are obtained using the skewed-*t* distribution to calculate the expected contemporaneous asset class responses to positive return shocks of size larger than the 99th percentile of the distribution. The variance decompositions in all panels are obtained via 10-day ahead impulse response functions based on VAR(*p*) models with constant terms included estimated on the period of January 7, 2003 through November 24, 2017 for a total of 3,771 observations.

Table 12: Skewed- $t$  Distribution, Global Stock Markets, VAR(5) innovation terms.

	USA	UK	GER	FRA	JAP
Location parameter ( $\xi$ )	0.262	0.209	0.269	0.240	0.117
Skewness parameter ( $\alpha$ )	-0.239	-0.079	-0.121	0.182	0.041
	AUG	HKG	CHI	IND	BRA
Location parameter ( $\xi$ )	0.161	0.216	0.177	0.240	0.323
Skewness parameter ( $\alpha$ )	-0.166	-0.059	-0.051	-0.124	-0.051
Degrees of Freedom ( $\nu$ )	5.047				

**Notes:** This table reports the parameter estimates for the multivariate Skewed- $t$  distribution function as given in equation (29) fitted on the innovations terms. The innovation terms are obtained from a VAR(5) model, estimated on the set of ten stock markets around the world, United States, United Kingdom, Germany, France, Japan, Australia, Hong Kong, China, Brazil for the period of January 7, 2003 through November 24, 2017.

resemblance with those of the four U.S. stock markets in Table 6. Therefore, we conclude that the parameter estimation does not cause the small impact of the new distributional assumptions on the connectedness measures.

Finally, when we evaluate the definition of the multivariate skewed- $t$  (MST) distribution as given in Equation (29), we find the reason which probably causes the fact that we do not find different levels of connectedness with the skewed- $t$  framework. In Equation (29), we see that the MST distribution function  $f_{MST_k}$  depends on the univariate Student's  $t$ -distribution  $T(\cdot; \nu_s + k)$ . As the dimension of the data  $k$  becomes larger, the degrees of freedom parameter increases and this  $t$ -distribution starts to resemble a normal distribution. In this example of global stock market connectedness we use a dataset of dimension  $k = 10$  where we used a dataset of dimension  $k = 4$  in the previous Section. Therefore, this univariate Student's  $t$ -distribution does not exhibit fat-tails anymore and shows more resemblance to the normal distribution. Therefore, the skewed- $t$  connectedness results resemble those of the DY connectedness framework.

This suggests that it is best to use the skewed- $t$  connectedness framework to evaluate linkages among a small set of markets in order to be able to identify asymmetries in shock transmission.

### 5.3 Dynamic Returns Connectedness

In the above, we performed full-sample estimation. However, as we have stated before, volatility changes over time, and therefore we expect the connectedness measures also to be time-varying. The dynamic connectedness measures are presented in Figure 6.

In Plot (a), we observe that the system's total connectedness fluctuated between 50% and

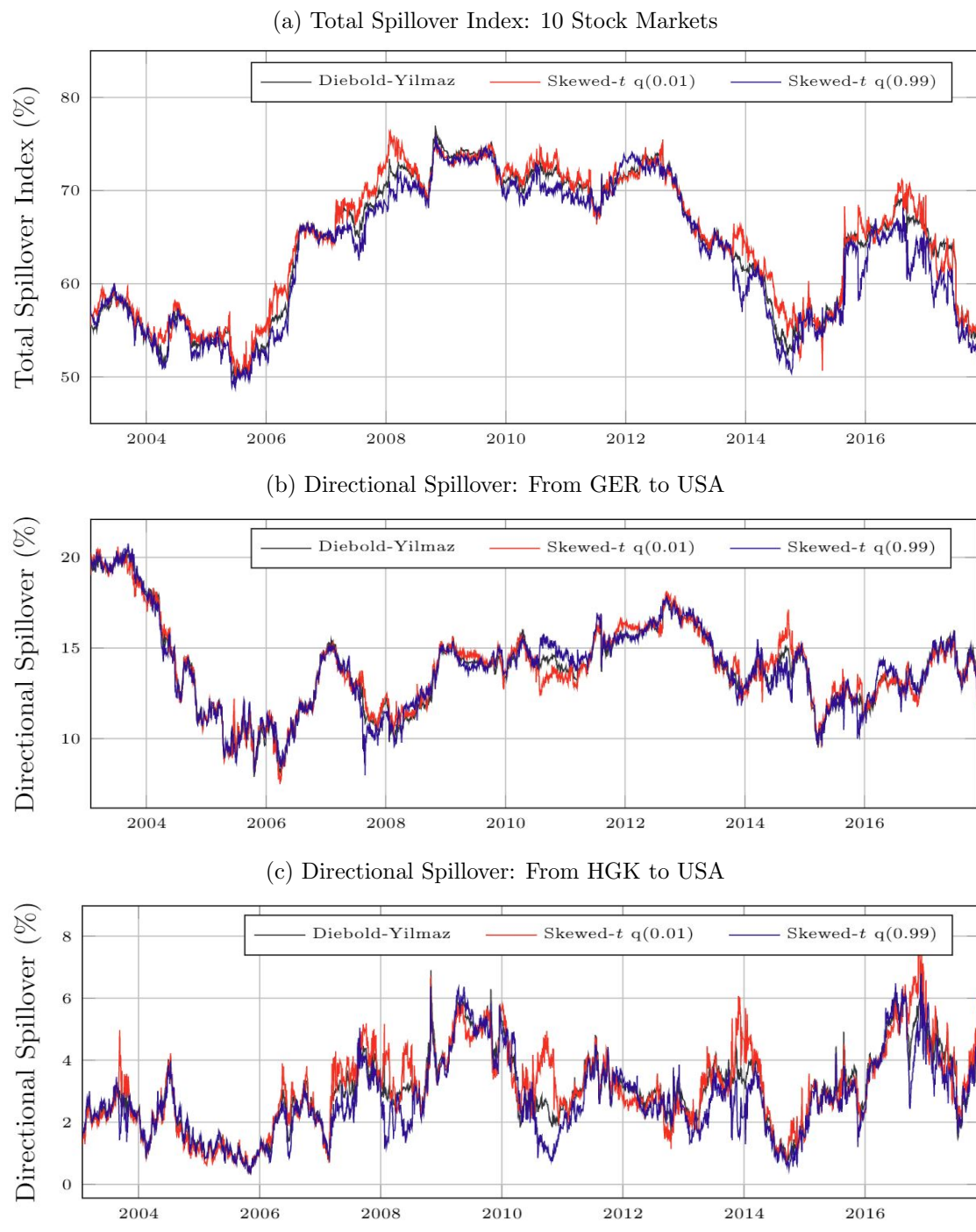


Figure 6: Dynamic Diebold-Yilmaz and Skewed- $t$  connectedness measures, using a VAR(5) model and 1-year rolling window.

60% in the early years of the sample. Mid 2005, before the global financial crisis, the system's connectedness reached its lowest recorded level at below 50%. From this moment onwards, the system's connectedness increased rapidly to a level of about 70% at the end of 2008. In the third quarter of 2012, the global stock market connectedness decreased over a two-year period to a level around 55%, before another surge of stock market connectedness mid 2015 to a level around 65%. Over the most recent year, the global stock market connectedness level is decreasing again towards a lower level of just above 50%.

From the above, one might derive that the total spillover index of global stock markets may function as an indicator of financial turmoil around the world. However, further research needs to be done before drawing conclusions on this subject.

In Plot (b) and (c) we present the dynamic connectedness measures for the spillover from Germany and Hong Kong to the United States, respectively. Interestingly, the directional spillover from Germany to the United States was relatively high in the first year of the sample and decreased afterwards to a level of 10% in 2006. During the global financial crisis, an increase in spillovers can be observed. However, the high spillover level of 2003 has not been reached since.

In some periods there are small but insignificant differences in total connectedness, but overall it remains unaffected by the new distributional assumptions. We make a similar observation for the directional spillovers for Germany and Hong Kong to the United States.

We conclude that the skewed- $t$  distribution has little to no effect on the full-sample and dynamic estimates of global stock market connectedness. Both for the directional spillover measures and the aggregated spillover indices.

## 6 Conclusion

We study the transmission of return shocks among U.S. asset classes and global stock markets using the financial connectedness methodology based on both normally and skewed- $t$  distributed shocks.

We find that return connectedness changes heavily over time. Furthermore, we find that the level of the estimated connectedness measures increases as a function of the amount of lags included in the VAR model, but that the dynamics and economic interpretation remain the same for the examined markets.

There are three main arguments to opt for the multivariate skewed- $t$  distribution to model return shocks in the connectedness methodology. First, return shocks are typically not normally distributed, i.e. they are skewed and leptokurtic. Both the U.S. asset class and the global stock market returns exhibit these non-normal characteristics.

Second, the asymmetry of the skewed- $t$  distribution enables us to distinguish between different market responses to negative versus positive shocks. In the case of U.S. asset returns we find that the total system's connectedness is little affected, but that there are large differences between the transmission of extreme positive and negative shocks among the individual asset classes. This is especially the case for the spillovers among stock and bonds.

In the case of global stock markets we find little evidence of differences in positive and negative shock transmission. This may be explained by the fact that the skewed- $t$  distribution tends to normality when the dimension of the data becomes larger.

Third, if the return shocks in fact turn out to be normally distributed, there is little harm in using the skewed- $t$  distribution since it then converges to the normal distribution and we obtain similar results as with the original methodology.

Based on our findings we prefer the use of the skewed- $t$  connectedness methodology over that of Diebold and Yilmaz (2012). Note that we do not claim the skewed- $t$  assumption to be perfect, however, there is little risk in using it and its underlying assumptions are more realistic.

Further research may be conducted to find more accurate methods to estimate the contemporaneous relations. For example the use of a less restrictive distribution function or a copula approach. Furthermore, it might be interesting to combine these more optimal methods to estimate the conditional contemporaneous expected shock transmissions with the more accurate LASSO technique to estimate the dynamics in the VAR model.

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