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# Global yield curve macroeconomic dynamics and interactions: A dynamic Nelson Siegel approach

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**Abstract**

The Dynamic Nelson Siegel (DNS) model is a latent factor model frequently fitted to the government bond yield curves of individual countries. In this paper, I apply the methodology of Diebold et al. (2008) who extend it to a global setting. Of central interest is the existence of global factors that drive country specific latent factors. I extend the work of Diebold et al. (2008) by including local macroeconomic factors, considering the effects of persistent oversimplification of the DNS and generating one step ahead forecasts. My results reaffirm the existence of global latent factors. In addition, I show that macroeconomic factors have a strong effect only for the UK and Japan. Finally, I propose a model that ameliorates the effects of oversimplification and improves forecast performance.

# Contents

# 1 Introduction

Academics and practitioners alike have been interested in modeling government bond yield curves because of their power as a predictor of future economic activity, inflation levels and borrowing costs, as well as their central role in the management of bond market risk. This interest has originated a vast literature on yield curve modeling over the past decades. Since the seminal work of Nelson and Siegel (1987), much of that literature has been focused on explaining the term structure of interest rates with a set of factors. In many cases, these factors can be interpreted as the level, slope and curvature in the manner of Diebold and Li (2006). It is important to note that although these factors are given names that relate to the shape of the yield curve, their relationship with macroeconomic indicators is not explicit. Since they are not directly observed, they are referred to as latent factors. Other models as Diebold et al. (2006) have also incorporated observable macroeconomic factors, such as manufacturing capacity utilization, inflation and the federal funds rate in its capacity as a monetary policy instrument.

Most of this literature studies the yield curve of a single country in isolation. While the resulting country-specific factors are informative, especially for large and closed economies like the US, a natural extension would be to consider the existence of global factors that drive the yield curves of multiple countries. This conjecture is plausible because global factors are often present in asset markets. Hence, the understanding of these factors would be central to understanding global interest rates.

There are numerous questions to pursue regarding such global factors: whether they indeed exist, what their dynamic properties are, how much do local factors depend on global and how much must be explained by idiosyncratic factors, how country factors load on global factors, whether they are still relevant after controlling for macroeconomic factors and whether the resulting framework uses all available information when generating one step ahead forecasts. I address these research questions in the current paper. Much of my methodology is built on a somewhat simplified version of Diebold et al. (2008), in which the authors investigate the existence and properties of global yield curve factors for five countries - Japan, Germany, UK and the US. They do so through a nested approach. First, they estimate the local factors of each country through the Nelson-Siegel three factor model as made time variant by Diebold and Li (2006). Next, they apply principal component analysis (PCA) on the resulting local factors to extract global factors and use them to estimate the loadings of local on global factors. In principle, various dynamics can be imposed on both the global factors and the loading equations. The authors use autoregressive dynamics for global factors because experience shows the shape of the yield curve to be often persistent. I apply the methodology of Diebold et al. (2008) to a contemporary dataset (April 1995 to December 2017) to see how well their results hold up in the recent low interest rate environment.

I extend the work of Diebold et al. (2008) by following their suggestions for future research in three directions. First, I control for macroeconomic factors when modeling local latent factors. This is done by adding inflation, manufacturing capacity utilization (an indicator of real activity), unemployment and an interest rate (the primary monetary instrument of central banks) as explanatory variables to the latent factor model of each country. In the spirit of Diebold et al. (2006) and similar to global latent factors, one can estimate the loadings of local on macroeconomic factors. The purpose is to improve fit by explaining some of the idiosyncratic factors of each country by its macroeconomic indicators. Second, I incorporate a moving average dynamic into the DNS model by using a two step method with a panel data regression. I do so because it is likely that DNS persistently oversimplifies the yield curve at certain maturities, which results in residual autocorrelation. Finally, I use a 10 year moving window to generate one step ahead forecasts of country factors in the manner of Diebold and Li (2006) and evaluate the adequacy of the various model specifications using these forecasts. Forecast evaluation is done by a comparison of mean squared errors (MSE) and a Diebold-Marianno test (Diebold and Mariano, 1995).

The rest of this paper is organized as follows. In Section 2 I describe my global yield curve modeling methods based on a simplified version of Diebold et al. (2008). In section 3 I specify my extensions to their framework. In section 4 I explain the construction of my dataset and discuss descriptives. Results are provided in section 5 in which I discus latent factor estimation (5.1), factor dynamics estimation (5.2), the results of my extensions to the framework (5.3 and 5.4) and forecast evaluation (5.5). Finally, I conclude in section 6.

# 2 Methodology

The single country Nelson-Siegel model has been shown to accurately approximate the yield curve and provide good forecasts (Diebold et al., 2005). Here, I present the methodology of Diebold et al. (2008) who extends the single country model to an environment with multiple countries.

## 2.1 Single country model

Nelson and Siegel's famous factorization of the of the yield curve in three factors can be stated in the form of Diebold-Li at any particular point in time as:

$$y_i(\tau) = l_i + s_i\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau}\right) + c_i\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau} - e^{-\lambda_i\tau}\right) + v_i(\tau) \quad (1)$$

where $y_i(\tau)$ is the continuously compounded zero coupon nominal yield of a government bond with $\tau$ remaining months to maturity in country $i$. Since each yield curve is constructed through a selection of maturities, some authors employ a notation such as $\tau_j$ where $j = 1\dots M$ denotes the exact maturity. I omit this for simplicity but note that $\tau$ varies across the same $M$ maturity levels for all countries. The three factors are present in the unknown parameters $l_i$, $s_i$ and $c_i$ and can be interpreted as level, slope and curvature. The interpretation of the latent factors comes from their loadings - a constant, a decreasing function of maturity and a concave function of maturity. In addition, there is the parameter $\lambda_i$ that is more difficult to interpret but determines the maturity at which the curvature factor $c_i$ is maximized. The random disturbance $\nu_i(\tau)$ has a standard deviation $\sigma_i(\tau)$.

This specification has been dynamized by Diebold and Li (2006) to allow for time varying latent factors:

$$y_{it}(\tau) = l_{it} + s_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau}\right) + c_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau} - e^{-\lambda_{it}\tau}\right) + \nu_{it}(\tau) \tag{2}$$

This is known as the Dynamic Nelson-Siegel (DNS) model. Time varying factors are introduced, because they allow for the exploration of the dynamic properties of factors and thus enable forecasting. This model is simplified by imposing that the parameter $\lambda_{it}$ be constant (i.e. $\lambda_{it} = \lambda = 0.0609$ for $\forall i,t$) following Diebold and Li (2006), because doing so results in little loss of generality and because accurate estimation of the parameter is often not possible. A constant $\lambda$ also simplifies the estimation procedure by reducing the model to a linear form. The series of latent factors can then be estimated using OLS from the cross section of maturity levels $\tau$ for each time moment $t = 1\ldots T$ and country $i = 1\ldots N$.

## 2.2 Multi-country model

The multi-country correlation matrix for each factor at any point in time is calculated from the series obtained in (2). The global factor is taken to be the first principal component of correlation matrix of each local factor. In this manner, global factors $L_t$, $S_t$ and $C_t$ are derived for level, slope and curvature. Simple autoregressive dynamics can be endowed on them:

$$\begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} = \Phi \begin{pmatrix} L_{t-1} \\ S_{t-1} \\ C_{t-1} \end{pmatrix} + \begin{pmatrix} U_t^l \\ U_t^s \\ U_t^c \end{pmatrix} \tag{3}$$

where $\Phi$ is a $3\times 3$ autoregressive coefficient matrix and $U_t^n$ are disturbances with $E[U_t^n U_{t'}^{n'}] = (\sigma^n)^2$ when $n = n'$ and $t = t'$ and 0 otherwise, $n = l,s,c$. I restrict $\Phi$ to be diagonal, following Diebold et al. (2008), to allow for more tractable estimation and forecasting.

The yield curve for each country is still determined by (2) but I now allow the factors common between countries - $l_{it}$, $s_{it}$ and $c_{it}$, to load on their corresponding global factors $L_t, S_t$ and $C_t$, as well as country idiosyncratic factors:

$$l_{it} = \alpha_i^l + \beta_i^l L_t + \varepsilon_{it}^l \tag{4a}$$

$$s_{it} = \alpha_i^s + \beta_i^s S_t + \varepsilon_{it}^s \tag{4b}$$

$$c_{it} = \alpha_i^c + \beta_i^c C_t + \varepsilon_{it}^c \tag{4c}$$

where $\{\alpha_i^l, \alpha_i^s, \alpha_i^c\}$ are constant terms, $\{\beta_i^l, \beta_i^s, \beta_i^c\}$ are local factor loadings on global factors and $\{\varepsilon_{it}^l, \varepsilon_{it}^s, \varepsilon_{it}^c\}$ are the country idiosyncratic factors. Naturally, the coefficients are estimated for each available country $i = 1 \ldots N$.

As with global factors, first order autoregressive dynamics are also be endowed on the country idiosyncratic factors:

$$\begin{pmatrix} \varepsilon_{it}^l \\ \varepsilon_{it}^s \\ \varepsilon_{it}^c \end{pmatrix} = \Theta_i \begin{pmatrix} \varepsilon_{i,t-1}^l \\ \varepsilon_{i,t-1}^s \\ \varepsilon_{i,t-1}^c \end{pmatrix} + \begin{pmatrix} u_{it}^l \\ u_{it}^s \\ u_{it}^c \end{pmatrix} \tag{5}$$

where $\Theta_i$ is a $3 \times 3$ autoregressive coefficient matrix of country $i$ and $u_{it}^n$ are disturbances such that $E[u_{it}^n u_{i't'}^{n'}] = (\sigma_i^n)^2$ if $i = i'$, $t = t'$ and $n = n'$ and 0 otherwise, $n = l, s, c$. Similar to $\Phi$ in (3), each $\Theta_i$ is restricted to be diagonal. Finally, an assumption is made that the shocks to global factors $U_t^n$ and the shocks to the country-specific factors $u_{it}^n$ are orthogonal $E[U_t^n u_{i,t-s}^{n'}] = 0$ for all $n, n', t, i, s$.

# 3 Extensions

I extend the framework of Diebold et al. (2008) with the aim of improving explanatory power and forecast accuracy. I do so by introducing country macroeconomic factors to the model for country latent factors and a moving average specification for yield curve forecasting. Additionally, I provide a detailed description of my forecasting procedure.

## 3.1 Macroeconomic factors

Diebold et al. (2006) found strong evidence of the effects of US macro variables on future movements in the US yield curve. They added three macroeconomic factors to the DNS model - manufacturing capacity utilization (an indicator of real activity), the inflation rate and the federal funds rate (the primary monetary policy instrument of the Federal Reserve). I extend this model to a global setting:

$$l_{it} = \alpha_i^l + \beta_i^l L_t + \Gamma_i^{l\prime} F_{i,t-1} + \varepsilon_{it}^l \tag{6a}$$

$$s_{it} = \alpha_i^s + \beta_i^s S_t + \Gamma_i^{s\prime} F_{i,t-1} + \varepsilon_{it}^s \tag{6b}$$

$$c_{it} = \alpha_i^c + \beta_i^c C_t + \Gamma_i^{c\prime} F_{i,t-1} + \varepsilon_{it}^c \tag{6c}$$

where $F_{it} = \{mcu_{it}, i_{it}, r_{it}, u_{it}\}'$ is a vector of macroeconomic factors (manufacturing capacity utilization, inflation rate, interest rate and unemployment) with a parameter vector $\Gamma_i = \{\gamma_{i,mcu}, \gamma_{i,i}, \gamma_{i,r}, \gamma_{i,u}\}'$ for country $i$ and time $t$. The residuals are, as before, endowed with the autoregressive dynamics defined in

(5). The interest rate is the primary monetary policy instrument of the country's central bank - the Bank of England's base rate, the Bank of Japan's official discount rate, the overnight money market financing rate of the Bank of Canada, the ECB's interest rate on main refinancing operations (MRO) and the aforementioned federal funds rate of the Federal Reserve.

### 3.2 Moving average extension in the Dynamic Nelson-Siegel model

It is possible that the error terms in of the DNS are autocorrelated because the latent factors in (2) persistently oversimplify the yield curve for certain maturities and therefore do not capture all the information relevant for forecasting. I address this, by using the lagged Nelson-Siegel residual as an additional regressor in what I call the DNS-MA specification:

$$y_{it}(\tau) = \alpha_{i\tau} + \beta_i \bar{y}_{it}(\tau) + \theta_i \hat{v}_{i,t-1}(\tau) + \omega_{it} \tag{7}$$

where $\bar{y}_{it}(\tau)$ is the fitted yield, $\hat{v}_{i,t-1}$ is the estimated lagged residual for maturity $\tau$ from the DNS (2) and $\omega_{it}$ is the error term for country $i$ and time $t$. In contrast to the DNS, the parameter vector $\{\alpha_i, \beta_i, \theta_i\}$ is estimated per country using all the observations for all maturities $\tau$ and time periods $t$. This model is effectively a panel data model with fixed effects, in which $\alpha_{i\tau}$ is the individual intercept of maturity $\tau$.

I estimate this specification with a two step method for each country. I first estimate the DNS (2) fitted yield $\bar{y}_{it}(\tau)$ and error term $\hat{v}_{it}(\tau)$ for all time periods $t = 1 \dots T$. Then, I use them as regressors in the panel data model (7) and obtain parameter estimates with the least square dummy variable (LSDV) method.

### 3.3 Forecasting

Forecasting local latent factors can easily be done on a one step ahead horizon, by using their loadings on global latent and idiosyncratic factors (4):

$$\hat{n}_{i,t+1} = \hat{\alpha}_i^n + \hat{\beta}_i^n \hat{N}_{t+1} + \hat{\Gamma}_i^{n\prime} F_{i,t-1} + \hat{\varepsilon}_{i,t+1}^n \tag{8}$$

where $\hat{n}_{i,t+1}$ is the forecast of an arbitrary local latent factor, $\hat{N}_{t+1}$ the forecast of its global counterpart and $F_{i,t-1}$ the lagged macroeconomic factor (if present). Deriving the one step ahead forecasts of $\hat{N}_{t+1}$ and $\hat{\varepsilon}_{i,t+1}^n$ is also trivial due to the structure of the VAR(1) models (3) and (5), by which they are specified:

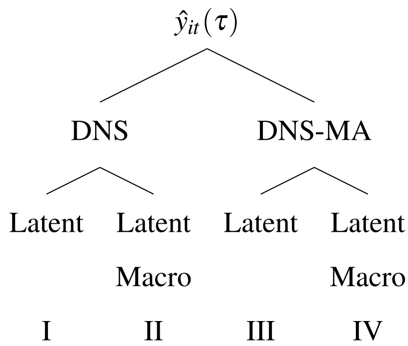
$$\begin{pmatrix} \hat{L}_{t+1} \\ \hat{S}_{t+1} \\ \hat{C}_{t+1} \end{pmatrix} = \hat{\Phi} \begin{pmatrix} \hat{L}_t \\ \hat{S}_t \\ \hat{C}_t \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} \hat{\varepsilon}_{i,t+1}^l \\ \hat{\varepsilon}_{i,t+1}^s \\ \hat{\varepsilon}_{i,t+1}^c \end{pmatrix} = \hat{\Theta}_i \begin{pmatrix} \hat{\varepsilon}_{i,t}^l \\ \hat{\varepsilon}_{i,t}^s \\ \hat{\varepsilon}_{i,t}^c \end{pmatrix} \tag{10}$$

Forecasts for the yields themselves can finally be obtained through the DNS model or the DNS-MA specification:

$$\bar{y}_{it}(\tau) = \Lambda(\tau)'\hat{f}_{it} \qquad (11) \qquad\qquad \hat{y}_{it}(\tau) = \hat{\alpha}_{i\tau} + \hat{\beta}_i \bar{y}_{it}(\tau) + \hat{\theta}_i \hat{v}_{i,t-1}(\tau) \qquad (12)$$

where $\Lambda(\tau) = \{1, \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right), \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)\}'$ is a vector of the Nelson-Siegel factorization and $f_{it} = \{\hat{l}_{it}, \hat{s}_{it}, \hat{c}_{it}\}'$ is a vector of the latent factor estimates for country $i$ and time $t$. This framework results in four possible forecasting specifications, depending on whether or not macroeconomic variables are used to forecast local latent factors and whether the DNS or the DNS-MA model is used to forecast yields.




I derive forecasts for all models from a 10 year rolling window and compare their performance through MSE and a Diebold-Marianno test (Diebold and Mariano, 1995).

# 4 Data

In this section, I specify my data sources and yield curve construction procedure. I also provide a discussion of yield curve descriptives focusing on the commonalities between countries.

## 4.1 Sources and construction

I use continuously compounded zero coupon constant maturity yields of government bonds for the United States, United Kingdom, Japan, Germany and Canada for the period April 1995 to December 2017. The yield curves are obtained primarily from each country's central bank and, whenever necessary, supplemented by secondary market data. Each yield curve contains maturities for at least 6, 12, 24, 36, 48, 60, 84 and 120 months (although some countries contain more maturity levels). Following Diebold et al. (2008), [1] I linearly interpolate the available maturities to obtain yield curves with consistent maturities

[1] The authors find that their yields are highly correlated with results by Brennan and Xia (2006) who use a cubic spline

of 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months for each country.

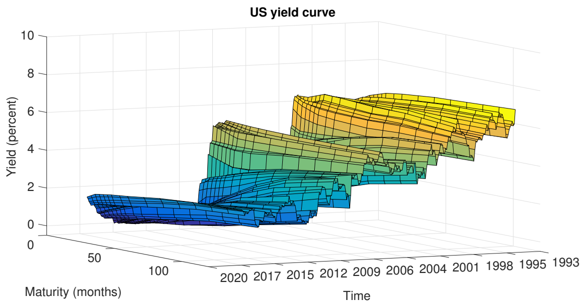
The macroeconomic data consists of manufacturing capacity utilization, annualized inflation rate, annual interest rate and unemployment rate on a monthly basis for the same period (April 1995 to December 2017) as the yields curves. The data was obtained from each country's central banking authority or national statistical bureau. All data sets were reported on a monthly basis except the manufacturing capacity utilization rates of Canada, Germany and Japan, which were reported quarterly. Those series were converted to a monthly frequency through linear interpolation.

### 4.2 Description

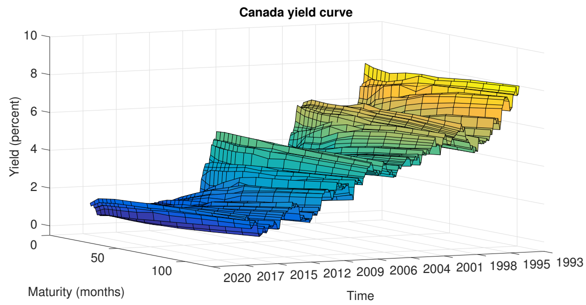
The yield curves of all five countries are plotted across time in Figure 1. Each yield curve undergoes substantial level changes, sharply decreasing after the financial crisis and mostly remaining low until present day. Further common movements across countries appear to occur with business cycles, dropping after global slowdowns (such as the Asian crisis of 1998 at the Dot-com bubble) and increasing thereafter. Each country has its own idiosyncrasies as well - persistently low interest rates in Japan, negative yields at low maturities in Germany and recent hikes in the US and Canada.

Slopes seem to vary less as can be seen from the general tendency towards flattening before a drop and tilting towards longer maturities thereafter for most countries. They do, off course, vary which is evident for instance from the flatter slope of recent Canadian yields as compared to the US, a difference that likely reflects growth expectations for the US. Curvature variance is more difficult to recognize but if one interprets the factor as the rapidity of the rate of change of yield with respect to maturity, it is easy to see that for some countries such as the UK and Canada yields climb faster for lower maturities than they do for higher ones. Here, too, a commonality appears to be present.

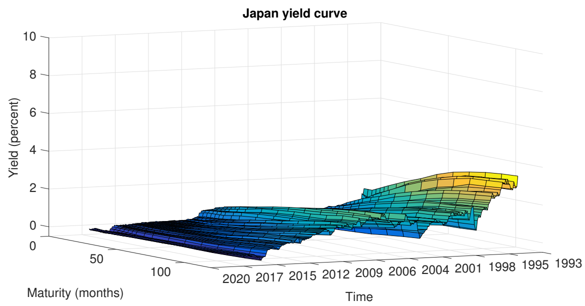
Table 1 reports descriptive statistics for the bond yields at selected maturities. Japanese yields are the lowest over the period due to the prolonged stagnation of the Japanese economy. All curves slope upwards on average with a maturity spread between 94 and 156 basis points. Yield standard deviation decreases for longer maturities in the US and UK but remains constant or increases for Canada, Japan and Germany. These findings disagree with those of Diebold et al. (2008), who found standard deviations to be uniformly decreasing with respect to maturities in their sample period (September 1985 to August 2005) for all countries. This is likely because my sample period includes the Great Recession, the European sovereign debt crisis of 2011-12 and their aftermaths. Both economic events cast uncertainties about the mid to long term inflation and growth prospects of advanced economies and at times even raised the possibility of defaults. Finally, yield curves are highly persistent with first order autocorrelations close to unity for all countries and maturities except Japan where they still measure above 0.90. This suggests that simple autoregressive dynamics might be sufficient to capture the behavior of yield curves.



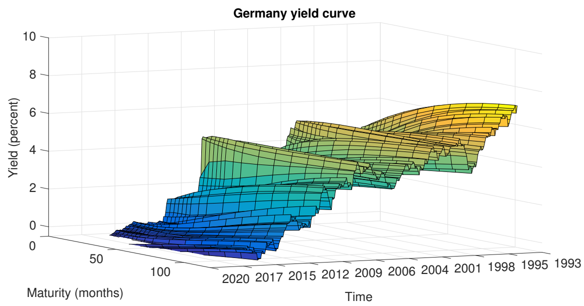

(a) United States



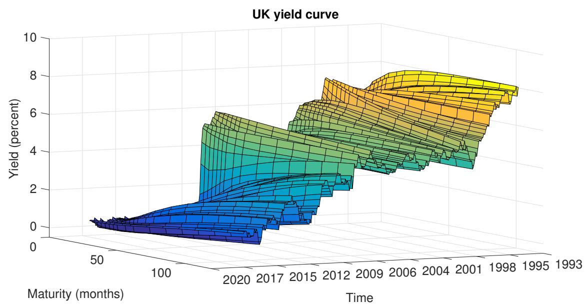

(b) Canada




(c) Japan




(d) Germany




(e) United Kingdom

Figure 1: Yield curves across time in monthly frequency for five countries from oldest to most recent.

| $\tau$ (Months) | Mean | Std Dev | Min | Max | $\hat{\rho}(1)$ | $\hat{\rho}(12)$ | $\hat{\rho}(30)$ |
|---|---|---|---|---|---|---|---|
| US | | | | | | | |
| 6 | 2.46 | 2.24 | 0.03 | 6.42 | 0.99 | 0.80 | 0.38 |
| 12 | 2.57 | 2.21 | 0.09 | 6.37 | 0.99 | 0.82 | 0.42 |
| 60 | 3.43 | 1.84 | 0.59 | 6.88 | 0.98 | 0.83 | 0.56 |
| 120 | 4.02 | 1.53 | 1.46 | 7.07 | 0.98 | 0.81 | 0.58 |
| Canada | | | | | | | |
| 6 | 2.69 | 1.84 | 0.28 | 7.93 | 0.98 | 0.70 | 0.46 |
| 12 | 2.83 | 1.87 | 0.41 | 7.78 | 0.98 | 0.73 | 0.49 |
| 60 | 3.55 | 1.87 | 0.59 | 7.93 | 0.98 | 0.81 | 0.59 |
| 120 | 4.06 | 1.79 | 0.98 | 8.31 | 0.98 | 0.81 | 0.57 |
| Japan | | | | | | | |
| 6 | 0.16 | 0.24 | -0.35 | 1.36 | 0.91 | 0.54 | 0.07 |
| 12 | 0.20 | 0.27 | -0.33 | 1.35 | 0.91 | 0.60 | 0.11 |
| 60 | 0.72 | 0.63 | -0.32 | 2.85 | 0.95 | 0.69 | 0.30 |
| 120 | 1.33 | 0.78 | -0.24 | 3.60 | 0.96 | 0.70 | 0.33 |
| Germany | | | | | | | |
| 6 | 2.00 | 1.76 | -0.92 | 5.11 | 0.99 | 0.76 | 0.48 |
| 12 | 2.07 | 1.78 | -0.92 | 5.17 | 0.99 | 0.77 | 0.51 |
| 60 | 2.86 | 1.93 | -0.63 | 6.49 | 0.98 | 0.81 | 0.58 |
| 120 | 3.59 | 1.90 | -0.21 | 7.35 | 0.98 | 0.80 | 0.54 |
| UK | | | | | | | |
| 6 | 3.26 | 2.46 | -0.02 | 7.37 | 0.99 | 0.83 | 0.60 |
| 12 | 3.28 | 2.47 | -0.03 | 7.21 | 0.99 | 0.84 | 0.61 |
| 60 | 3.81 | 2.14 | 0.22 | 8.30 | 0.98 | 0.81 | 0.58 |
| 120 | 4.20 | 1.81 | 0.66 | 8.40 | 0.98 | 0.76 | 0.45 |

Table 1: Descriptive statistics of government bond yields for selected maturities across countries for the period from April 1995 to December 2017.

# 5 Results

In this section I present the results of my study. I begin by estimating the local and global latent factors in subsection 5.1 and proceed to fit them with autoregressive dynamics in subsection 5.2. I estimate my macroeconomic extension in subsection 5.3 and moving average extension in subsection 5.4. I close the section by evaluating 10 year moving window forecasts provided by the various models with a MSE comparison and a Diebold-Marianno test in subsection 5.5. Latent factors have been shown to be linked to macroeconomic variables by Diebold et al. (2006) and Ang and Piazzesi (2003). Specifically, level is linked to inflation and slope to real activity. It is natural to evaluate whether this relationship bears out in a global setting. Thus, in my discussion I focus on links to macroeconomic variables.

## 5.1 Latent factor estimation

Table 2 summarizes descriptives for local latent factors estimated from the DNS model for all five countries. The average level is lowest for Japan due to prevalently low interest rates, the average absolute slope is largest for the US and Germany, reflecting optimistic growth prospects for these countries on average throughout the period. Mean absolute curvature is highest for Germany and Japan, indicating a larger maturity spread in short to mid term bonds than there is on mid to long term bonds. A possible interpretation could be short term to mid term uncertainty about growth prospects in these countries throughout the sample period. This explanation seems plausible, because the period contains the rise of "Abenomics" and the European debt crisis. The high autocorrelations indicate persistence of all latent factors throughout the period, with curvature less persistent than level and slope.

| Factor | Mean | Std Dev | Min | Max | $\hat{\rho}(1)$ | $\hat{\rho}(12)$ | $\hat{\rho}(30)$ |
|---|---|---|---|---|---|---|---|
| US | | | | | | | |
| $\hat{l}_{US,t}$ | 4.60 | 1.33 | 1.84 | 7.16 | 0.96 | 0.72 | 0.50 |
| $\hat{s}_{US,t}$ | -2.11 | 1.54 | -5.15 | 1.01 | 0.97 | 0.47 | -0.22 |
| $\hat{c}_{US,t}$ | -2.37 | 2.34 | -7.12 | 2.91 | 0.95 | 0.65 | 0.25 |
| Canada | | | | | | | |
| $\hat{l}_{CA,t}$ | 4.49 | 1.83 | 1.15 | 8.70 | 0.98 | 0.78 | 0.50 |
| $\hat{s}_{CA,t}$ | -1.79 | 1.37 | -5.17 | 0.41 | 0.97 | 0.29 | -0.31 |
| $\hat{c}_{CA,t}$ | -1.79 | 1.44 | -5.33 | 1.78 | 0.83 | 0.38 | -0.04 |
| Japan | | | | | | | |
| $\hat{l}_{JP,t}$ | 1.80 | 1.09 | -0.26 | 4.78 | 0.97 | 0.68 | 0.33 |
| $\hat{s}_{JP,t}$ | -1.44 | 0.95 | -4.20 | 0.02 | 0.97 | 0.65 | 0.33 |
| $\hat{c}_{JP,t}$ | -2.61 | 1.29 | -7.34 | -0.16 | 0.94 | 0.51 | 0.19 |
| Germany | | | | | | | |
| $\hat{l}_{DE,t}$ | 4.21 | 2.04 | -0.02 | 8.42 | 0.98 | 0.78 | 0.46 |
| $\hat{s}_{DE,t}$ | -2.05 | 1.28 | -5.07 | 0.37 | 0.97 | 0.42 | -0.17 |
| $\hat{c}_{DE,t}$ | -3.07 | 1.71 | -6.71 | 0.57 | 0.92 | 0.38 | -0.09 |
| UK | | | | | | | |
| $\hat{l}_{UK,t}$ | 4.59 | 1.65 | 0.89 | 8.82 | 0.97 | 0.67 | 0.25 |
| $\hat{s}_{UK,t}$ | -1.26 | 1.77 | -5.26 | 2.63 | 0.98 | 0.55 | 0.02 |
| $\hat{c}_{UK,t}$ | -1.77 | 2.76 | -7.72 | 5.10 | 0.95 | 0.70 | 0.56 |

Table 2: Descriptive statistics of the estimated latent DNS factors across countries for the period from April 1995 to December 2017.

The central interest of Diebold et al. (2008) is commonality in local latent factors. Figure 2 explores this further by superimposing the estimated DNS level, slope and curvature factors for all countries. It is clear that level factors move closely together, decreasing shortly after economic slowdowns (such as Asian debt crisis of 1998, Dot-com burst in 2001 and the Great Recession) and increasing consequently.

Slope factors vary less closely together with Japan standing somewhat apart from other countries and exhibiting lower variability. Common tendencies towards flattening out before slowdowns are visible for all countries, although movements in Japan are less pronounced. Finally, curvature factors vary less closely than either level and slope, although a common pattern is still visible.

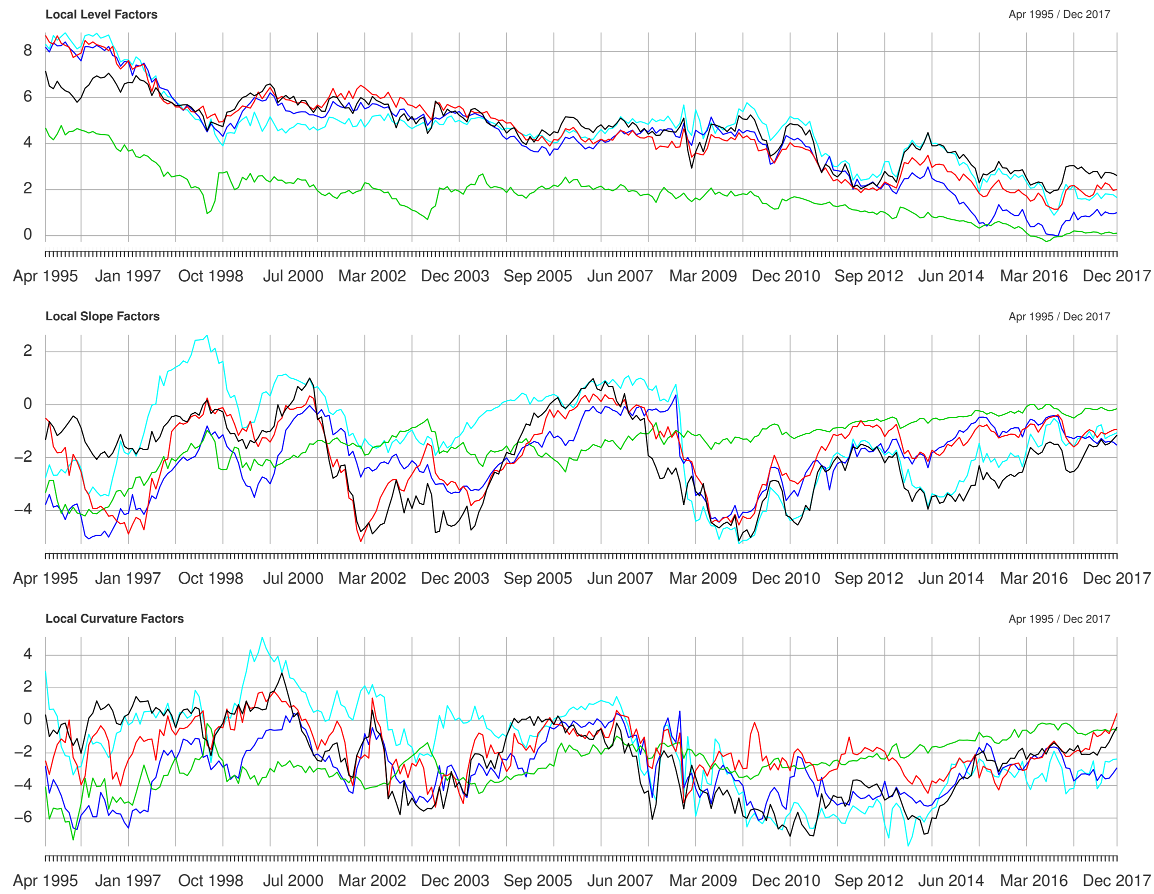



Figure 2: Estimated values of the latent DNS factors (from top to bottom - level, slope and curvature) for the US (black), Canada (red), Japan (green), Germany (blue) and the UK (cyan) in monthly frequency for the period from April 1995 to December 2017.

I estimate global factors through principal component analysis of the correlation matrix of all five countries for each latent factor. The first principal component of each correlation matrix is taken to be the global factor. The results are presented in Table 3, which reports the five eigenvalues of each correlation matrix and the portion of variance attributable to its corresponding principal component. These findings agree with the work of Diebold et al. (2008), by suggesting the existence of global factors that explain a large portion of the variability of local factors. The first principal component of the level factor explains two thirds of level variation. A natural interpretation is the existence of a global level factor that drives country level factors although this global factor is not as dominant in my sample period as it is for Diebold et al. (2008). The prominence of the global level factor is likely due to the global business cycle simultaneously pushing inflation and affecting central bank decisions in highly interconnected economies. The first principal components of slope and curvature explain close to forty percent of local factor variation. These factors are likely less prominent than the global level factor because the growth and inflation expectations of each country depend more strongly on the economic policy of that country's government than do inflation levels.

| | | PC1 | PC2 | PC3 | PC4 | PC5 |
|---|---|---|---|---|---|---|
| Level | Eigenvalue | 2.16 | 0.43 | 0.27 | 0.21 | 0.16 |
| | Prop. of variance | 0.67 | 0.13 | 0.08 | 0.06 | 0.05 |
| | Cumulative prop. | 0.67 | 0.80 | 0.89 | 0.95 | 1.00 |
| Slope | Eigenvalue | 1.71 | 1.22 | 0.60 | 0.34 | 0.30 |
| | Prop. of variance | 0.41 | 0.29 | 0.14 | 0.08 | 0.07 |
| | Cumulative prop. | 0.41 | 0.70 | 0.85 | 0.93 | 1.00 |
| Curvature | Eigenvalue | 1.68 | 1.13 | 0.66 | 0.57 | 0.36 |
| | Prop. of variance | 0.38 | 0.26 | 0.15 | 0.13 | 0.08 |
| | Cumulative prop. | 0.38 | 0.64 | 0.79 | 0.92 | 1.00 |

Table 3: Results of principal component analysis on the latent DNS factor correlation matrix of the US, Canada, Germany, Japan and the UK.

Global level, slope and curvature factors are plotted in Figure 3. The global level factor trends downwards, coinciding with the decrease in inflation across developed countries throughout the sample period and the especially low (and sometimes negative) inflation in the post-crisis period. The global slope factor moves with the business cycle and peaks before recessions (1998, 2001 and 2008). The global level factor also decreases after recessions but it does not seem to react as strongly in an all cases. For instance, the decrease in 1998 is sharp, but the one in 2001 is less pronounced, perhaps because the latter did not have a severe effect on inflation. Curvature lacks clear links to macroeconomic fundamentals as shown in Diebold et al. (2006). It rises some time after drops in slope, which could indicate lower demand for mid term maturity bonds during economic expansions, perhaps due to competing investment opportunities. This could be an interesting topic for future research.

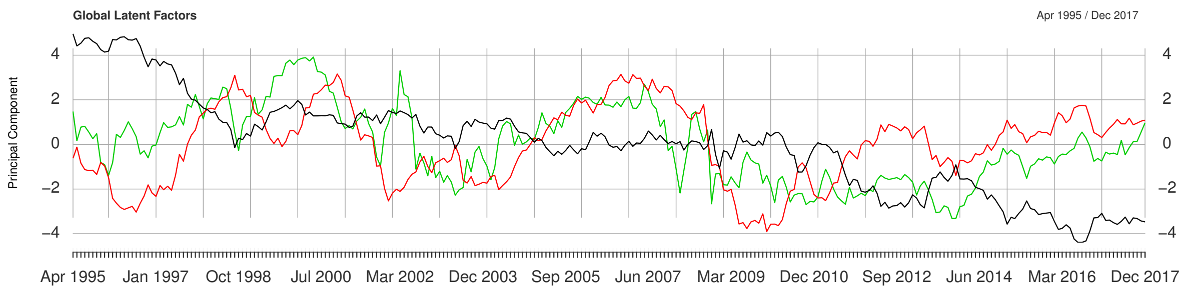



Figure 3: Estimated global level (black), slope (red) and curvature (green) factors for the period from April 1995 to December 2017. The global factors are the first principal components of their corresponding local latent factors

## 5.2 Factor dynamics

I estimate the models for global factor dynamics (3), local factor loadings on global factors (4) and idiosyncratic factor dynamics (5). Results for global factor dynamics are presented in Table 4. As suggested by the descriptives of the local factors, all global factors are highly persistent although slope and curvature are slightly less so. Global curvature is more volatile than slope, which in turn is more volatile than global level.

| Global level | Global slope | Global curvature |
|---|---|---|
| $L_t = \underset{(0.01)}{0.99} L_{t-1} + 0.08\mathscr{V}_t^l$ | $S_t = \underset{(0.01)}{0.97} S_{t-1} + 0.15\mathscr{V}_t^s$ | $C_t = \underset{(0.02)}{0.94} C_{t-1} + 0.32\mathscr{V}_t^c$ |

Table 4: Parameter estimation results for equation (3) specifying the dynamics of global latent factors using the definition $\mathscr{V}_t^n \equiv U_t^n/\sigma^n$ such that $U_t^n = \sigma^n \mathscr{V}_t^n$ for factor $n \in \{l, s, c\}$. Standard errors are in parenthesis.

Table 5 summarizes the estimation results for country factor loadings on global and country specific factors. Consider first local level factors, which load positively on the highly serially correlated global level factor for all countries. The loadings are all significant with parameter estimates much larger than their standard errors. Idiosyncratic factors are also persistent to the same extent for all countries, although less so than the findings of Diebold et al. (2008) suggest. Germany is not as heavily dependent on global factors as it is during the authors' sample period, indicating that the country's local level dynamics no longer match those of global level as closely. This could be because the German economy has integrated more closely with other European economies and has become more dependent for growth on exports to China in the last fifteen years. It is clear, nevertheless, that Germany, Canada and the UK load heavily on the global factors, implying that their bond yield levels react relatively more to changes in global yield levels. This is sensible because their comparatively smaller and more open economies would be more sensitive to changes in global inflation than the US and Japan. Japan is most disconnected from the rest, which is natural because of its unusually low interest rates throughout the period.

Consider next the results for the local slope factors. All countries again exhibit positive loadings on the global slope factor, which is also highly autocorrelated although somewhat less than global level. The estimates are all significant with slightly higher standard deviations than the load estimates. The loading for Japan is the smallest and the only one markedly different than the rest. Its movements appear almost completely idiosyncratic. This is similar to the results for the level factor, suggesting that Japan is most removed from the rest of the countries in the sample. An important reason besides the prominence of idiosyncratic factors might be the omission of Japan's biggest trading partner China. The slope loadings of the US and the UK are slightly higher and the idiosyncratic factors more persistent than they are for the level factor. Thus, although local slopes are more reactive to changes in global slope factors, idiosyncratic factors explain more of their variation. Contrary to level loadings, German slope loadings are the second lowest after Japan. This finding indicates that the German business cycle is more independent from changes in global economic conditions than the business cycles of other western economies. The Canadian slope loading is again one of the highest underlying the country's dependence on global factors. This is likely due to the importance of US trade to Canada and the size of the American economy.

| | |
|---|---|
| Local level factors | |
| $l_{US,t} = \underset{(0.03)}{4.60} + \underset{(0.01)}{0.59} L_t + \varepsilon^l_{US,t}$ | $\varepsilon^l_{US,t} = \underset{(0.03)}{0.79}\varepsilon^l_{US,t-1} + 0.25 v^l_{US,t}$ |
| $l_{CA,t} = \underset{(0.03)}{4.49} + \underset{(0.01)}{0.83} L_t + \varepsilon^l_{CA,t}$ | $\varepsilon^l_{CA,t} = \underset{(0.03)}{0.70}\varepsilon^l_{CA,t-1} + 0.25 v^l_{CA,t}$ |
| $l_{JP,t} = \underset{(0.02)}{1.80} + \underset{(0.01)}{0.48} L_t + \varepsilon^l_{JP,t}$ | $\varepsilon^l_{JP,t} = \underset{(0.03)}{0.71}\varepsilon^l_{JP,t-1} + 0.23 v^l_{JP,t}$ |
| $l_{DE,t} = \underset{(0.02)}{4.21} + \underset{(0.01)}{0.93} L_t + \varepsilon^l_{DE,t}$ | $\varepsilon^l_{DE,t} = \underset{(0.03)}{0.75}\varepsilon^l_{DE,t-1} + 0.21 v^l_{DE,t}$ |
| $l_{UK,t} = \underset{(0.03)}{4.59} + \underset{(0.01)}{0.74} L_t + \varepsilon^l_{UK,t}$ | $\varepsilon^l_{UK,t} = \underset{(0.03)}{0.75}\varepsilon^l_{UK,t-1} + 0.28 v^l_{UK,t}$ |
| Local slope factors | |
| $s_{US,t} = \underset{(0.06)}{-2.11} + \underset{(0.03)}{0.71} S_t + \varepsilon^s_{US,t}$ | $\varepsilon^s_{US,t} = \underset{(0.02)}{0.86}\varepsilon^s_{US,t-1} + 0.55 v^s_{US,t}$ |
| $s_{CA,t} = \underset{(0.04)}{-1.79} + \underset{(0.02)}{0.74} S_t + \varepsilon^s_{CA,t}$ | $\varepsilon^s_{CA,t} = \underset{(0.03)}{0.79}\varepsilon^s_{CA,t-1} + 0.33 v^s_{CA,t}$ |
| $s_{JP,t} = \underset{(0.06)}{-1.44} + \underset{(0.03)}{0.15} S_t + \varepsilon^s_{JP,t}$ | $\varepsilon^s_{JP,t} = \underset{(0.02)}{0.85}\varepsilon^s_{JP,t-1} + 0.54 v^s_{JP,t}$ |
| $s_{DE,t} = \underset{(0.04)}{-2.05} + \underset{(0.02)}{0.65} S_t + \varepsilon^s_{DE,t}$ | $\varepsilon^s_{DE,t} = \underset{(0.03)}{0.83}\varepsilon^s_{DE,t-1} + 0.40 v^s_{DE,t}$ |
| $s_{UK,t} = \underset{(0.07)}{-1.27} + \underset{(0.04)}{0.82} S_t + \varepsilon^s_{UK,t}$ | $\varepsilon^s_{UK,t} = \underset{(0.02)}{0.84}\varepsilon^s_{UK,t-1} + 0.61 v^s_{UK,t}$ |
| Local curvature factors | |
| $c_{US,t} = \underset{(0.08)}{-2.37} + \underset{(0.05)}{1.14} C_t + \varepsilon^c_{US,t}$ | $\varepsilon^c_{US,t} = \underset{(0.03)}{0.71}\varepsilon^c_{US,t-1} + 0.77 v^c_{US,t}$ |
| $c_{CA,t} = \underset{(0.06)}{-1.79} + \underset{(0.03)}{0.78} C_t + \varepsilon^c_{CA,t}$ | $\varepsilon^c_{CA,t} = \underset{(0.04)}{0.62}\varepsilon^c_{CA,t-1} + 0.60 v^c_{CA,t}$ |
| $c_{JP,t} = \underset{(0.09)}{-2.61} - \underset{(0.05)}{0.10} C_t + \varepsilon^c_{JP,t}$ | $\varepsilon^c_{JP,t} = \underset{(0.03)}{0.78}\varepsilon^c_{JP,t-1} + 0.81 v^c_{JP,t}$ |
| $c_{DE,t} = \underset{(0.08)}{-3.07} + \underset{(0.04)}{0.77} C_t + \varepsilon^c_{DE,t}$ | $\varepsilon^c_{DE,t} = \underset{(0.03)}{0.76}\varepsilon^c_{DE,t-1} + 0.76 v^c_{DE,t}$ |
| $c_{UK,t} = \underset{(0.09)}{-1.77} + \underset{(0.05)}{1.35} C_t + \varepsilon^c_{UK,t}$ | $\varepsilon^c_{UK,t} = \underset{(0.04)}{0.77}\varepsilon^c_{UK,t-1} + 0.84 v^c_{UK,t}$ |

Table 5: Parameter estimation results of equation (4) specifying the loading of local on global latent factors and equation (5) specifying country idiosyncratic factor dynamics using the definition $v^n_{i,t} \equiv u^n_{i,t}/\sigma^n_i$ such that $u^n_{i,t} = \sigma^n v^n_{i,t}$ for factor $n \in \{l, s, c\}$. Standard errors are in parenthesis.

Finally, consider the curvature factor. All significant loadings are positive and idiosyncratic autocorrelations are mostly the same. Japan is again divorced from the rest of the countries with a small (and insignificant) global curvature loading. Local curvature factors load heavily on global curvature for the US and the UK but less so for Canada and Germany. Interpretation for this factor, however, is more difficult. The unexplained portion of variance $\sigma^c_i$ for this factor is also higher than it is for level and slope. The local on global loadings are higher for all factors compared to Diebold et al. (2008), indicating a higher sensitivity of local economies to changes in global economic conditions brought about an increasing degree of globalisation.

### 5.3 Macroeconomic factors extension

I attempt to elucidate country idiosyncratic factors by adding lagged macroeconomic variables for each country as regressors in the local factor model (6). All macroeconomic factors are in percentages and their plots can be found in figure 7 in the Appendix. The estimated parameters are reported in Table 6.

Country factor loadings on the global level remain unchanged for the US and Japan but decrease slightly for Canada and Germany while increasing slightly for the UK. Overall, the differences are not big enough to alter the conclusions of the previous section. Lagged manufacturing capacity utilization (MCU) is a significant determinant of yield levels for all countries except Japan. The effect is positive for Canada and the UK, and negative for the US and Germany. On average, one percentage increase in MCU changes the level factor by 0.05 in countries where the effect is significant. Lagged inflation rates and lagged interest rates are only important for Germany and the UK. Lagged inflation rates contribute positively to local levels in both countries but the effect of interest rates in the UK is negative. This result mostly agrees with Ang and Piazzesi (2003), who argue that yield levels reflect inflation levels, but it is not significant for all countries. The interest rate effect makes sense for the UK, which has sovereign monetary policy - an interest rate hike would be a reaction to high actual or expected future inflation. This relationship is not as straightforward for Germany because its monetary policy is controlled by the ECB, which likely considers Eurozone-wide inflation for policy decisions. Unemployment is the only macroeconomic variable that is significant for all countries, with a negative relationship in the US and Japan and positive elsewhere. This result is particularly interesting, because it implies that low employment actually increases interest rate levels for some countries in spite of the Philips curve. It is possible that increased global competition prevents domestic price levels from increasing with employment in the short run. The inclusion of macroeconomic factors explains a portion of the idiosyncratic factors for all countries. This is evident from the decrease of the moving average parameter and unexplained volatility $\sigma_i^l$ for each country. It is, nevertheless, difficult to make a general conclusion on the effect of a particular macroeconomic factor on local levels. They seem to be most successful for Canada, Japan and the UK and less so for others.

The global slope loadings decrease slightly upon the introduction of macroeconomic factors. Japan maintains the smallest loading and Canada's loading becomes the largest. MCU is is significant for the US, Canada and Japan with a positive parameter for the US and negative for Canada and Japan. Unemployment is significant for the US, Germany and the UK with effects again higher than those of the level factor. It seems that monthly slope variation can indeed be explained by the business cycle.

Inflation holds no significance except for Japan. The interest rate is significant for all countries except Canada with an effect much higher than that for the level. This is plausible because the objective of interest rate changes is, at least partially, to dampen the cyclical nature of demand.

| Local level factors | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\alpha_i^l$ | $L_t$ | $mcu_{i,t-1}$ | $i_{i,t-1}$ | $r_{i,t-1}$ | $u_{i,t-1}$ | $\varepsilon_{i,t-1}^l$ | $\sigma_i^l$ |
| $l_{US,t} =$ | 9.94 (0.81) | 0.60 (0.02) | −0.06 (0.01) | −0.01 (0.02) | 0.02 (0.02) | −0.14 (0.02) | 0.74 (0.03) | 0.22 |
| $l_{CA,t} =$ | −0.76 (1.02) | 0.73 (0.02) | 0.05 (0.01) | 0.06 (0.03) | 0.05 (0.02) | 0.12 (0.04) | 0.69 (0.03) | 0.23 |
| $l_{JP,t} =$ | 2.60 (0.36) | 0.48 (0.01) | 0.00 (0.01) | −0.05 (0.02) | 0.13 (0.12) | −0.21 (0.04) | 0.67 (0.03) | 0.22 |
| $l_{DE,t} =$ | 7.92 (0.56) | 0.78 (0.02) | −0.05 (0.01) | 0.14 (0.03) | 0.15 (0.02) | 0.04 (0.01) | 0.71 (0.03) | 0.18 |
| $l_{UK,t} =$ | 1.41 (0.76) | 0.81 (0.03) | 0.04 (0.01) | 0.08 (0.03) | −0.10 (0.02) | 0.09 (0.03) | 0.70 (0.03) | 0.23 |
| Local slope factors | | | | | | | | |
| | $\alpha_i^s$ | $S_t$ | $mcu_{i,t-1}$ | $i_{i,t-1}$ | $r_{i,t-1}$ | $u_{i,t-1}$ | $\varepsilon_{i,t-1}^s$ | $\sigma_i^s$ |
| $s_{US,t} =$ | −9.45 (1.33) | 0.61 (0.03) | 0.07 (0.02) | −0.07 (0.04) | 0.34 (0.03) | 0.21 (0.04) | 0.75 (0.03) | 0.37 |
| $s_{CA,t} =$ | 5.00 (1.37) | 0.80 (0.03) | −0.08 (0.02) | 0.02 (0.04) | 0.01 (0.03) | −0.01 (0.05) | 0.75 (0.03) | 0.30 |
| $s_{JP,t} =$ | 4.31 (0.66) | 0.27 (0.03) | −0.05 (0.01) | 0.16 (0.05) | −1.34 (0.22) | 0.06 (0.07) | 0.76 (0.03) | 0.43 |
| $s_{DE,t} =$ | 0.13 (1.44) | 0.67 (0.03) | −0.02 (0.02) | 0.10 (0.05) | −0.12 (0.03) | −0.10 (0.02) | 0.78 (0.03) | 0.34 |
| $s_{UK,t} =$ | −2.26 (1.29) | 0.59 (0.03) | 0.02 (0.02) | −0.03 (0.04) | 0.32 (0.02) | −0.28 (0.04) | 0.71 (0.03) | 0.37 |
| Local curvature factors | | | | | | | | |
| | $\alpha_i^c$ | $C_t$ | $mcu_{i,t-1}$ | $i_{i,t-1}$ | $r_{i,t-1}$ | $u_{i,t-1}$ | $\varepsilon_{i,t-1}^c$ | $\sigma_i^c$ |
| $c_{US,t} =$ | −13.43 (2.35) | 0.84 (0.06) | 0.16 (0.03) | −0.25 (0.06) | 0.04 (0.05) | −0.22 (0.06) | 0.71 (0.04) | 0.65 |
| $c_{CA,t} =$ | 5.94 (2.50) | 0.94 (0.05) | −0.09 (0.03) | 0.11 (0.07) | −0.10 (0.05) | 0.01 (0.07) | 0.64 (0.04) | 0.57 |
| $c_{JP,t} =$ | 3.28 (1.16) | −0.02 (0.05) | −0.03 (0.01) | 0.22 (0.08) | −1.60 (0.38) | −0.52 (0.13) | 0.78 (0.03) | 0.73 |
| $c_{DE,t} =$ | −10.09 (2.30) | 0.84 (0.05) | 0.09 (0.03) | 0.03 (0.11) | −0.35 (0.06) | 0.03 (0.05) | 0.73 (0.03) | 0.70 |
| $c_{UK,t} =$ | 9.51 (2.14) | 0.91 (0.05) | −0.13 (0.03) | −0.35 (0.07) | 0.41 (0.04) | −0.22 (0.06) | 0.76 (0.04) | 0.65 |

Table 6: Parameter estimation results of equation (6) specifying the loading of local on global latent and macroeconomic factors and equation (5) specifying country idiosyncratic factor dynamics.Standard errors are in parenthesis.

Curvature loadings decrease for the US and the UK but remain mostly unchanged for other countries. MCU is significant for all countries and unemployment is significant with a strong negative effect for the US, Japan and the UK. These findings provide further evidence for the hypothesis that the short term movements in curvature coincide with those of real activity. It is reasonable that higher employment would drive yields of early to mid maturity bonds by increasing returns on alternative investments. Macroeconomic effects are the strongest for the UK followed by the US and Japan.

### 5.4 MA extension

It is likely that the DNS persistently oversimplifies the yield curve for certain maturities. To investigate this, I estimate the DNS-MA model as specified in (7) and present the results in Table 7. The results are obtained through a LSDV regression, where the different maturities represent a cross section for each country. The $\alpha_{i\tau}$ parameter is the individual effect of maturity $\tau$ while $\theta$ is the fixed effect of the lagged error term. If the error terms of the DNS (2) are not "forecastable", one would expect both the maturity specific intercepts and the MA parameter to be zero.

The fitted DNS yield $\bar{y}_{it}(\tau)$ is a good approximation to the yield curve, as indicated by its parameter, which is not significantly different than unity for any country. The MA effect that indicates oversimplification across all maturities is significant for all countries with particularly large estimates for Japan, Germany and the UK. The model will likely be able to decrease the forecast errors of all maturities for these countries.

| Moving average model |
|---|
| $y_{US,t}(\tau) = \alpha_{US,\tau} + \underset{(0.01)}{1.00}\bar{y}_{US,t}(\tau) + \underset{(0.01)}{0.85}\nu_{US,t-1}(\tau) + 0.02\eta_{US,t}$ |
| $y_{CA,t}(\tau) = \alpha_{CA,\tau} + \underset{(0.01)}{1.00}\bar{y}_{CA,t}(\tau) + \underset{(0.01)}{0.60}\nu_{CA,t-1}(\tau) + 0.04\eta_{CA,t}$ |
| $y_{JP,t}(\tau) = \alpha_{JP,\tau} + \underset{(0.01)}{1.00}\bar{y}_{JP,t}(\tau) + \underset{(0.01)}{0.89}\nu_{JP,t-1}(\tau) + 0.01\eta_{JP,t}$ |
| $y_{DE,t}(\tau) = \alpha_{DE,\tau} + \underset{(0.00)}{1.00}\bar{y}_{DE,t}(\tau) + \underset{(0.01)}{0.86}\nu_{DE,t-1}(\tau) + 0.01\eta_{DE,t}$ |
| $y_{UK,t}(\tau) = \alpha_{UK,\tau} + \underset{(0.00)}{1.00}\bar{y}_{UK,t}(\tau) + \underset{(0.01)}{0.90}\nu_{UK,t-1}(\tau) + 0.02\eta_{UK,t}$ |

Table 7: Parameter estimation results of equation (7) specifying moving average dynamics for a DNS fitted yield curve using the definition $\eta_{i,t} \equiv \omega_{i,t}/\sigma^i$ such that $\omega_{i,t} = \sigma^i\eta_{i,t}$. Standard errors are in parenthesis.

The intercepts $\alpha_{i\tau}$ are plotted in Figure 4. The effect is present for most countries but is less than a basis point for all but the largest maturities. The only exception is Canada where oversimplification appears to be the strongest and can be as much as two basis points.

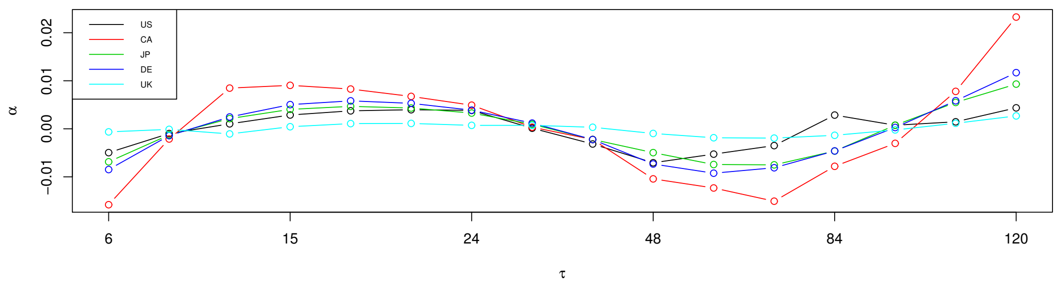



Figure 4: Estimated values of the $\alpha_{i\tau}$ parameter in equation (7) for the US, Canada, Germany, Japan and the UK across $\tau$ (maturity in months).

The pattern is sinusoidal and appears to be common for most countries, underestimating short to mid level maturity yields and reversing for mid to long maturities. The strongest oversimplifications are visible in the shortest and longest end of the curve. The effect is practically not present in the UK. One would expect this procedure to flatten forecasting errors between maturities mostly for Canada, followed by Germany, Japan and the US.

### 5.5 Forecasting

I evaluate the forecast accuracy for the four specifications defined in subsection 5.5 by a comparison of mean squared errors (MSE) in Figure 5 and a Diebold-Marianno (DM) test statistic in Figure 6. The forecasts are produced with estimates from a 10 year moving window.

Consider first the MSE comparison. The addition of macroeconomic factors improves the forecast of countries in which they have a strong overall effect - Canada (red), Japan (green) and especially the UK (cyan) although they do not smooth out the MSE for different maturity levels. The large drop in MSE for the UK indicates that the spanning hypothesis[2] might not be true for that country. The MA model smooths out the MSE for countries with large swings in the maturity specific intercept $\alpha_{i\tau}$ - Canada (red), Japan (green) and Germany (blue). It is evident from these findings that there is indeed information in the yield curves of these countries that is not taken into account by the DNS. Overall, the combination of both macroeconomic factors and the MA specification provides the best improvement overall by smoothing out MSE across maturities and decreasing the MSE level for countries in which macroeconomic factors are important. The US is the only country for which the DNS extensions provide almost no improvement.

Finally consider the DM test statistics. According to this criterion, the MA specification alone provides the best forecasts for the US (black), Canada (red) and Germany (blue). The DNS already provides mostly accurate forecasts for the US but the MA extension smooths out the test statistic for all maturities. This is done through its incorporation of the country's fluctuating intercept $\alpha_{i\tau}$. The improvement is noticeable in short to mid term and the longest term maturities for which DNS forecasts errors are significant. The MA specification also improves performance for the shorter term halves of the yield curves of Canada and Germany. There seems to be persistent overestimation for the highest maturities of these countries that is minimized but not eliminated by the MA model. Forecasts for Japan and the UK produced by all models are mostly inaccurate although they approach critical values for specification IV. Forecasts of the longer maturity half of the Japanese curve produced by that model are not significantly inaccurate. These results are to be expected because both Japan's macroeconomic factors and maturity specific intercept have strong effects on the yield curve. Note that the MSE for Japan is very

[2] The spanning hypothesis holds that all information relevant to the yield curve is already incorporated within it and therefore level, slope and curvature should be sufficient to forecast bond yields.

low compared to other countries despite its high DM due to Japan's low interest rates. Models including macroeconomic factors are again very important for the UK and somewhat important for Canada.

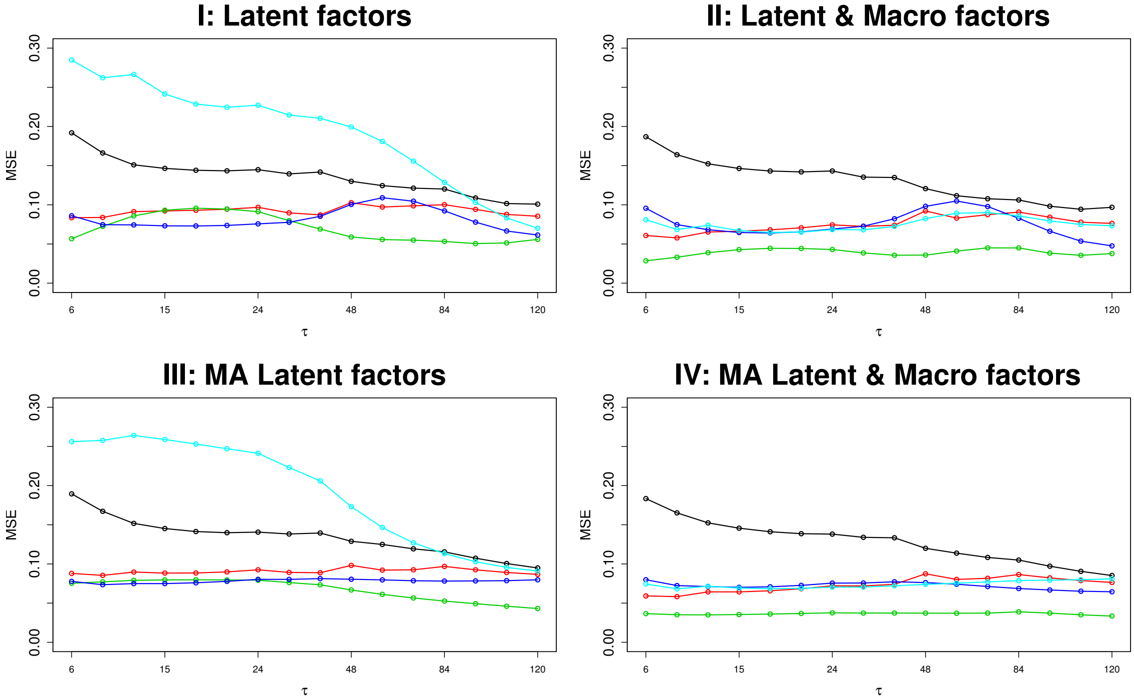



Figure 5: Mean Squared Error of 10 year rolling window forecasts from each of the four model specifications across five countries - the US (black), Canada (red), Japan (green), Germany (blue) and the UK (cyan). The values are plotted against $\tau$ (maturity in months).

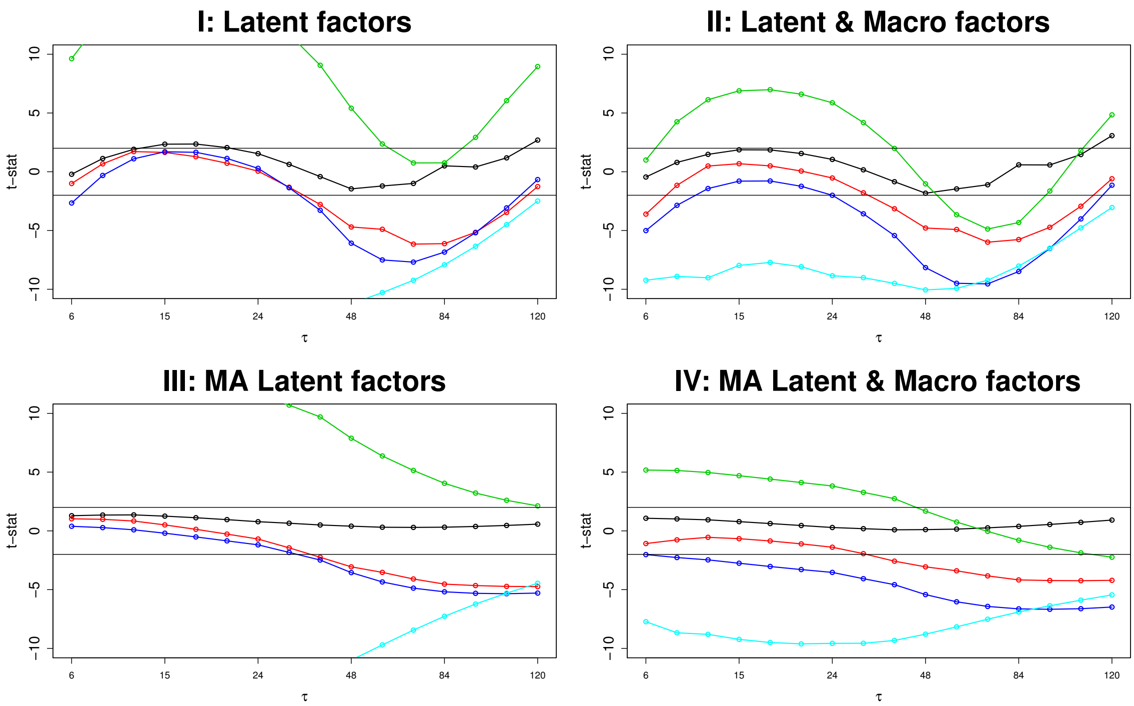



Figure 6: Diebold-Marianno test statistics of 10 year rolling window forecasts from each of the four model specifications across five countries - the US (black), Canada (red), Japan (green), Germany (blue) and the UK (cyan). The statistics are plotted against $\tau$ (maturity in months) with critical values $(-2, 2)$ displayed as solid lines.

# 6 Conclusion

I replicate the methodology of Diebold et al. (2008), who extend the yield curve model of Nelson and Siegel (1987) as dynamized by Diebold and Li (2006) to a global setting. I use a monthly frequency sample for the US, Canada, Japan, Germany and the UK from April 1995 to December 2017, which contains more recent data compared to the authors' period between September 1985 to August 2005. My work confirms the central results of the authors - there exist global level, slope and curvature factors that explain a large portion of local latent factor variance. The global level factor, explaining two thirds of local variability, is most prominent, followed by slope and curvature which explain close to forty percent. The global factors are not as dominant as in the authors' results. This is likely due to the omission of China, which has risen to global economic prominence during the time period of my sample. Thus, a possible direction for future research would be to evaluate how the prominence of global factors is changed by the inclusion of Chinese latent factors. Another challenge in my sample period has been the prevalence of extremely low interest rates in the post-crisis economic environment. Near-zero interest rates make PCA less effective and forecasting more difficult especially for short term maturities. Improvements have been made by Christensen and Ruderbusch (2015) and Lemke and Vladu (2017) through the definition of an unobserved rate for short term maturities that can turn negative. Such a model could have implications for estimating global factors and has the potential to improve forecasts in low yield periods.

Diebold et al. (2008) further propose a hierarchical specification that models local factors through the highly autocorrelated global and idiosyncratic factors. I extend their specification by including the curvature latent factor and apply it to my sample period. The results mostly agree with those of the authors, although local factor loadings on global factors are slightly higher for most countries indicating a higher degree of globalisation present in my sample. I extend the model of Diebold et al. (2008) by introducing local macroeconomic variables as regressors to the local latent factor equations. The effects of a particular macroeconomic factor differ between countries but are strongest for the UK, Japan and Canada overall. Unemployment is related to level and manufacturing capacity utilization to curvature for all countries. Interest rates have a strong effect on slope for all countries except Canada. The results further provide some evidence that curvature varies with short term changes in real activity although the relationship should be explored further.

I produce one step forecasts using the estimated parameters from 10 year rolling window. The results show that forecasts can be improved by introducing a lagged error term to the fitted Dynamic Nelson Siegel estimates due to the models' persistent oversimplification of the curve at certain maturities. The forecasting results are best for the US, Canada and Germany although the addition macroeconomic factors results in further performance improvements for the UK and especially for long term maturities in Japan.

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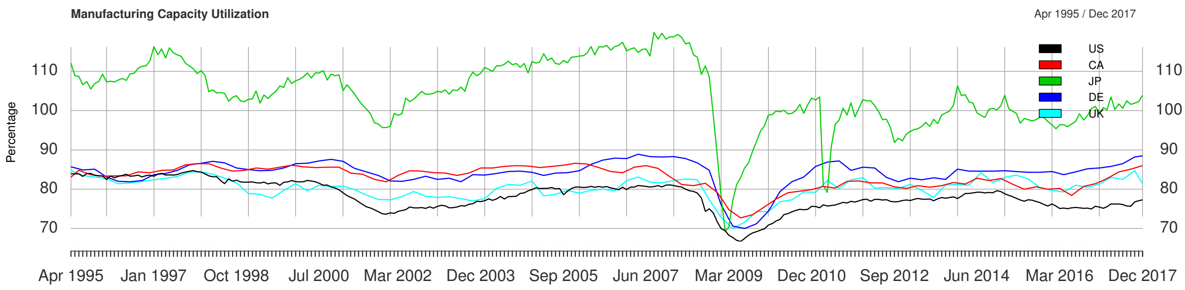
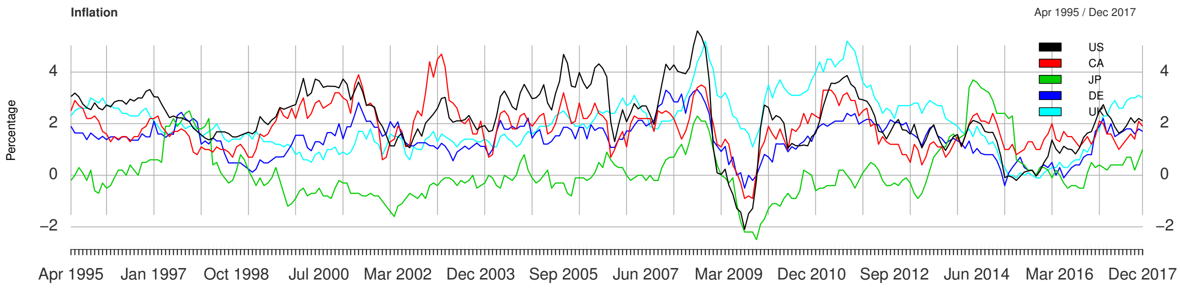
# Appendix

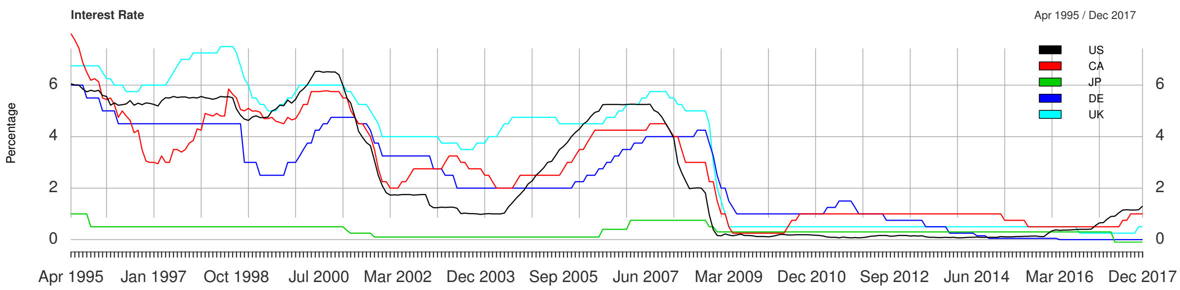
Macroeconomic factors in percentages for the five countries are plotted in Figure 7.



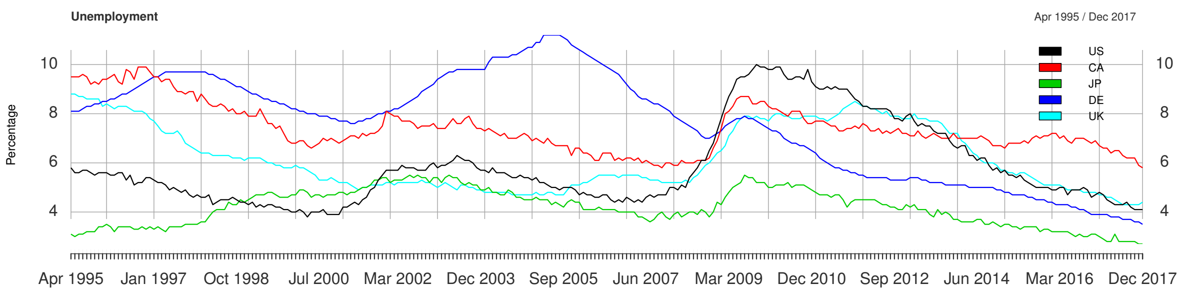

(a) Manufacturing Capacity Utilization




(b) Inflation




(c) Interest Rate




(d) Unemployment

Figure 7: Macroeconomic variables (from top to bottom - manufacturing capacity utilization, inflation, interest rate and unemployment) in percentages for five countries - the US, Canada, Japan, Germany and the UK. Sample period from April 1995 to December 2017.