Erasmus School of Economics

Bachelor Thesis Econometrics & Operations Research

Regional and global yield curve dynamics and interactions

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July 6, 2018

Abstract

In this paper I examine whether regional and/or global yield factors exist, what their dynamics and interactions are, and how they perform in forecasting the country yield factors and curves. For this purpose I extend the single-country dynamized Nelson-Siegel model to a multi-country context; regionally and globally. I use monthly government bond yield data for different maturities of countries in the regions America, Europe and Asia/Pacific. I find important regional and global yield factors, where the regional factor is more important than the global factor for at least 67% of the countries. This is also confirmed by an analysis of the prediction performance, where the regional factor outperforms the global factor.

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Contents

1 Introduction

An important concept in the bond market is the yield curve. This curve shows the relationship between the yield or interest rate and the time to maturity ('term') of a certain bond in a specific currency. The shape of the yield curve can give useful information about the expected economic activity and inflation. Therefore the yield curve is of great interest both to economists and academics, but the curve is also closely watched by traders and several financial institutions, because their decisions are partly based on the yield curve. Due to the fact that the yield curve is important for so many people, it is interesting to understand which latent factors are driving this yield curve and how they vary over time. This will be the main goal of my research, in particular whether regional and/or global yield factors exist, how they vary over time and how they interact with each other. Furthermore, I evaluate the prediction performance of the regional and global yield factors to see whether one outperforms the other.

In the past much literature is published about this topic, such as Diebold & Li (2006) in which the authors propose a dynamized version of the Nelson-Siegel model to model the yield curve, and in which they interpret the latent time-varying yield factors as level, slope and curvature. Furthermore, they propose and estimate autoregressive models for the factors to produce term-structure forecasts at both short and long horizons.

However, most of the literature, including Diebold & Li (2006), only focus on the yield curve of a single country and relate domestic yields to domestic yield factors, while nowadays the global yield curve becomes more important due to a more integrated global bond market. Therefore, the importance of domestic yield factors will gradually diminish and it is interesting to examine whether common global or regional yield factors exist, how they vary over time and what the implications are for cross-country yield curve interactions. In the literature of Diebold et al. (2008) the authors already address these research issues by modeling a large set of country yield curves, in which country yields may depend on country factors, and country factors may depend on global factors. They also do an empirical analysis by estimating the dynamic factor model and extract the global yield curve factors using government bond yields for the US, Japan, Germany and the UK over the sample period 1985 until 2005. The authors show that the global yield factors explain significant fractions of country yield curve dynamics.

However, an important shortcoming in the research of Diebold et al. (2008) is that their results for the global yield curve are based on four countries. The global bond market consists of many countries and their results can easily change by including more countries. In particular the addition of less developed countries, because in general the authors examine four developed countries. Moreover, it should be better to do an intermediate step by first looking to the presence of regional yield factors instead of global factors. In this way I expect to obtain richer dynamic cross-country bond yield interactions in a specific region. Subsequently, it is possible to examine to what extent regional factors depend on global factors.

In my research I am going to address this shortcoming in the literature of Diebold et al. (2008). Firstly, I replicate a part of the work of Diebold et al. (2008). Secondly, I extend their work by including regional yield factors, in which case country factors may depend on regional factors. This is a more appropriate and structured way to analyze the yield curve. I do not only examine the presence of common regional yield factors and their dynamics, I also use more countries in my research to obtain more reliable results. Furthermore, I investigate the prediction performance between the regional and global yield factors to see which factor is preferred in general.

In order to pursue the goal of my research as well as possible, I formulate the central research question as follows:

"Do regional and/or global yield factors exist, what are their dynamics and interactions, and how do they perform in forecasting the country yield factors and curves?"

In order to answer the research question I will use several techniques and methods which are already developed and applied in previous literature, such as Diebold & Li (2006) and Diebold et al. (2008). For example, I will apply the Nelson-Siegel model to model a set of country yield curves in a framework that takes regional, global and country yield factors into account. Besides, I use three metrics, namely the mean squared error, mean absolute error and the mean correct prediction to evaluate the prediction performance.

In the literature of Sopov & Seidler (2011) the authors already apply Diebold et al. (2008) to a regional context, but they only consider the Central European region, in particular the currencies' yield curves of Czech Republic, Hungary, Poland and Slovakia. They conclude that the yield curve of Czech Republic possesses its own dynamics corresponding to country yield factors, whereas the yield curves of the other countries are strongly influenced by the regional yield factors.

However, the findings of Sopov & Seidler (2011) are not appropriate for use in a follow-up situation in which research can be done to what extent regional yield factors may depend on global yield factors. Therefore, I consider multiple regions in my research where I define regions as continents, such as America, Europe and Asia/Pacific. I apply the methodology to these regions separately to examine the presence of regional common factors. For this purpose I use monthly government bond yield data for different maturities of countries which are part of these regions, namely the US, Canada, Brazil, the UK, Germany, France, Japan, Singapore and Hong Kong. Subsequently, as a topic for further research my findings can be used to investigate the relationship between regional yield factors and global yield factors.

One of my main results is that important regional and global yield factors exist, especially Europe and Asia/Pacific exhibit strong and dominant regional yield factors. For Europe this is due to the existence of the Eurozone and the European Union (EU). For Asia/Pacific the country yield factors are also positively correlated with the regional yield factor, which indicates the similarity of Asian countries in terms of inflation and real economic activity. Besides, for the region America there are big differences between developed countries, such as the US and Canada, and less developed countries, such as Brazil. Due to fundamental economic and political problems, such as high governments debts and high inflation rates, Brazil performs the worst in comparison with the other countries which results in high yields. Finally, the analysis of the prediction performance between the regional and global yield factors in forecasting the country's yield curve shows that the regional factor outperforms the global factor for at least 56% of the countries. In forecasting the country yield factors this is even the case for at least 67% of the countries.

As already mentioned, the literature of Diebold et al. (2008) only considers global yield factors. The authors do not take regional yield factors into account and they do not conduct an analysis of the prediction performance. Therefore, my research contributes in an important way. Next to the global yield factors it shows the existence of strong and dominant regional yield factors, which have a relatively large impact on the country yield factors. Moreover, for at least 67% of the countries the regional yield factor is more important than the global yield factor. Furthermore, the regional yield factor performs better than the global yield factor in forecasting the country's yield curve for at least 56% of the countries and in forecasting the country yield factors for at least 67% of the countries.

In the following section I will describe the data in more detail and provide some descriptive statistics. In the third section I will explain the econometric methods and techniques in much more detail which I am going to use to investigate the research problem. Subsequently, the fourth section reports my results and I explain and discuss my main findings. Finally, in the fifth section I draw my conclusions to give answer on my research question and I discuss some limitations of my research and provide some topics for further research.

2 Data

For my research I source Thomson-Reuters data from the financial database Datastream. Specifically, I use monthly zero-coupon government bond yields over the sample period January 2006 until May 2018 which results in 149 observations per series. For all bond yields I consider seventeen monthly maturities τ for τ = 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, 120. However, the yield data is only available for a couple of maturities, so I obtain the yield data of the remaining maturities by using cubic spline interpolation which is also used in the literature of Brennan & Xia (2006). Furthermore, due to the fact that I examine regional yield factors, I consider three regions in my research, namely America, Europe and Asia/Pacific. For this purpose I use bond yield data of three countries per region, so in total nine countries, namely Brazil, Canada, the US, France, Germany, the UK, Hong Kong, Japan and Singapore. Table 1 shows the descriptive statistics for the bond yields for representative maturities of all countries, which are part of the regions America, Europe and Asia/Pacific.

In general, I see that the yield curves for all countries are upward sloping and the yields behave less volatile when maturity increases. Furthermore, the autocorrelations show that the yields are very persistent over time. This means that the yield data structure remains roughly unchanged over time, which I also see from a small variation relative to the mean.

Besides, I see that the yields for Brazil are the highest in comparison with the yields of the other countries. This big difference between Brazil and the other countries in my research may have impact on the results of the global yield factor, so by taking regional yield factors into account I can examine the difference in impact. Literature of Favero & Giavazzi (2002) and Balliester Reis (2018) argue that Brazil suffers from big fundamental economic and political problems[1].

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[1] This is also emphasized by a Bloomberg article: https://www.bloomberg.com/gadfly/articles/2016-02-25/does-anyone-love-brazil-anymore-yes-bond-buyers

Table 1: Descriptive statistics government bond yields

Maturity (months)	Mean	St. dev.	Min.	Max.	$\hat{\rho}(1)$	$\hat{\rho}(6)$	$\hat{\rho}(12)$
Panel A: Region America							
Brazil							
3	11.50	1.95	6.19	14.18	0.95	0.54	0.02
12	11.36	2.42	6.30	16.33	0.95	0.57	0.09
60	11.26	2.59	5.09	17.21	0.93	0.61	0.19
120	12.22	1.72	9.18	17.33	0.89	0.44	-0.01
Canada							
3	1.41	1.31	0.16	4.55	0.98	0.86	0.62
12	1.60	1.31	0.41	4.75	0.98	0.84	0.63
60	2.14	1.13	0.57	4.58	0.97	0.82	0.69
120	2.67	1.01	1.01	4.60	0.98	0.84	0.71
US							
3	1.06	1.67	-0.01	5.10	0.98	0.84	0.60
12	1.26	1.65	0.09	5.26	0.98	0.84	0.63
60	2.17	1.23	0.59	5.11	0.97	0.78	0.64
120	2.92	1.01	1.46	5.15	0.96	0.75	0.62
Panel B: Region Europe							
France							
3	0.88	1.58	-0.85	4.33	0.99	0.87	0.69
12	1.02	1.63	-0.72	4.64	0.99	0.87	0.71
60	1.73	1.54	-0.43	4.76	0.98	0.89	0.79
120	2.51	1.36	0.13	4.79	0.98	0.89	0.80
Germany							
3	0.81	1.62	-0.98	4.33	0.99	0.88	0.70
12	0.96	1.67	-0.86	4.66	0.99	0.87	0.71
60	1.48	1.60	-0.56	4.64	0.99	0.88	0.78
120	2.13	1.44	-0.13	4.61	0.98	0.89	0.80
UK							
3	1.55	1.99	0.15	5.95	0.99	0.86	0.68
12	1.49	1.92	0.06	5.93	0.99	0.85	0.69
60	2.22	1.57	0.21	5.66	0.98	0.86	0.74
120	2.88	1.31	0.67	5.44	0.98	0.85	0.73
Panel C: Region Asia/Pacific							
Hong Kong							
3	0.82	1.26	-0.38	4.08	0.95	0.76	0.55
12	0.99	1.32	0.07	4.47	0.96	0.78	0.60
60	1.81	1.16	0.27	4.67	0.96	0.75	0.64
120	2.33	1.07	0.64	4.99	0.96	0.70	0.56
Japan							
3	0.12	0.24	-0.38	0.67	0.97	0.84	0.64
12	0.17	0.29	-0.32	0.83	0.98	0.86	0.71
60	0.44	0.49	-0.35	1.53	0.98	0.85	0.75
120	0.89	0.60	-0.25	1.97	0.98	0.87	0.77
Singapore							
3	0.93	0.87	0.19	3.37	0.97	0.79	0.56
12	1.01	0.87	0.20	3.25	0.97	0.80	0.58
60	1.58	0.78	0.31	3.40	0.96	0.76	0.60
120	2.38	0.54	1.30	3.62	0.91	0.50	0.37

Note: The table shows the descriptive statistics for monthly government bond yields for representative maturities of all countries, which are part of the regions America (Panel A), Europe (Panel B) and Asia/Pacific (Panel C), over the sample period 2006.01 - 2018.05. $\hat{\rho}(\tau)$ represents the sample autocorrelation at lag τ.

Brazil faces high government debts and the interest payments are a big burden on the government's budget. Moreover, Brazil experienced several economic recessions and the country belongs to the countries with the highest inflation rate worldwide. All these problems and risks are incorporated in the Brazilian yields which explain the high values.

For Japan the yields are the lowest in comparison with the yields of the other countries. Literature of Akram & Das (2013) argue that the Japanese yields are low due to the fact that Japan has monetary sovereignty and the inflation rates are low over time.

For the region America the yields of Canada and the US are relatively similar, also in terms of volatility. As already noted, there is a big difference between Brazil and countries, such as Canada and the US, due to the high yields for Brazil. In general, Canada and the US are more developed than Brazil.

The yields of the European countries are also comparatively similar, this holds in particular for France and Germany. The UK has in comparison with France and Germany higher yields for all maturities. This small difference can be explained due to the fact that the UK is not part of the Eurozone and has its own monetary policy.

For the region Asia/Pacific the yields of Hong Kong and Singapore are relatively similar, although the yields of Singapore behave less volatile. They are also comparable with the yields of France and Germany. As already noted, there is a clear difference between the yields of Japan and those for Hong Kong and Singapore, because of the unique characteristics of the Japanese monetary policy. The Japanese yields are the lowest and they behave the least volatile in comparison with other countries.

3 Methodology

3.1 Modeling and estimating the regional and global yield curves

In the beginning of my research I consider the single-country context which is a framework that models the yield curve of each country i at time t and only allows for country-specific yield factors. For this purpose I use the dynamized version of the Nelson-Siegel model as empathized by Diebold & Li (2006) which is defined as:

$$y_{it}(\tau) = l_{it} + s_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau}\right) + c_{it}\left(\frac{1-e^{-\lambda_{it}\tau}}{\lambda_{it}\tau} - e^{-\lambda_{it}\tau}\right) + v_{it}(\tau), \tag{1}$$

where $y_{it}(\tau)$ is the continuously-compounded zero-coupon nominal yield on a τ-month government bond, l_{it} is the level factor, s_{it} is the slope factor, c_{it} is the curvature factor and $v_{it}(\tau)$ is a disturbance term with standard deviation $\sigma_i(\tau)$ for country $i = 1, \ldots, N$ at time $t = 1, \ldots, T$. I estimate the parameters l_{it}, s_{it}, c_{it} and λ_{it} via non-linear least squares (NLS) regressions for each country and month separately, where the parameters λ_{it} are optimized in such a way that they determine the maturity at which loadings on the curvature reach their maximum.

Firstly, to obtain the results of the global yield factors I replicate the work of Diebold et al. (2008), in which the global yield model is defined as:

$$Y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + C_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \tag{2}$$

where $Y_t(\tau)$ are unobservable global yields, and L_t, S_t and C_t are global yield factors at time $t = 1, \ldots, T$.

Next to the replication part of my research, I also consider for my extension the multi-country context which is a framework that models the yield curve of each region j at time t and allows for regional yield factors. Building on the methodology of Diebold et al. (2008) I define the regional yield model as:

$$Y_{jt}(\tau) = L_{jt} + S_{jt}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + C_{jt}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right), \tag{3}$$

where $Y_{jt}(\tau)$ are unobservable regional yields, and L_{jt}, S_{jt} and C_{jt} are regional yield factors for region $j = 1, \ldots, M$ at time $t = 1, \ldots, T$. The dataset in my research only consists of country yields, so I need to compute the global and regional yields. By applying principal component analyses I extract the global and regional yield factors. Subsequently, by substituting these values in Eq. (2) and (3) I compute the global and regional yields, respectively.

To formulate the way in which the country factors depend on the regional factors, I need to take into account that not all countries $i = 1, \ldots, N$ are part of a specific region j where $j = 1, \ldots, M$. So building on the methodology of Diebold et al. (2008), $\forall i \in j$ I define the relationship as:

$$l_{it} = \alpha_i^l + \beta_i^l L_{jt} + \epsilon_{it}^l \tag{4a}$$

$$s_{it} = \alpha_i^s + \beta_i^s S_{jt} + \epsilon_{it}^s \tag{4b}$$

$$c_{it} = \alpha_i^c + \beta_i^c C_{jt} + \epsilon_{it}^c, \tag{4c}$$

where $\{\alpha_i^l, \alpha_i^s, \alpha_i^c\}$ are constants, $\{\beta_i^l, \beta_i^s, \beta_i^c\}$ are loadings on regional factors and $\{\epsilon_{it}^l, \epsilon_{it}^s, \epsilon_{it}^c\}$ are country idiosyncratic factors. I assume that $E\left[\epsilon_{it}^n\right] = 0$ and the innovations to regional factors have standard deviation equal to 1, so $\sigma^n = 1$, for $n = l, s, c$. For the global context, I also use Eq. (4a) - (4c), but the only difference is that the regional factors are replaced by the global factors.

As an extension I also examine the impact on the country yield factors by including the global yield factor next to the regional yield factor in Eq. (4a) - (4c). In this way I compare the impact of the regional and global yield factor on the country yield factors. So building on the methodology of Diebold et al. (2008), $\forall i \in j$ I define this combined yield factor model as:

$$l_{it} = \alpha_i^l + \beta_{1i}^l L_{jt} + \beta_{2i}^l L_t + \epsilon_{it}^l \tag{5a}$$

$$s_{it} = \alpha_i^s + \beta_{1i}^s S_{jt} + \beta_{2i}^s S_t + \epsilon_{it}^s \tag{5b}$$

$$c_{it} = \alpha_i^c + \beta_{1i}^c C_{jt} + \beta_{2i}^c C_t + \epsilon_{it}^c, \tag{5c}$$

where in addition to the explanation of Eq. (4a) - (4c), $\{\beta_{2i}^l, \beta_{2i}^s, \beta_{2i}^c\}$ are loadings on the global factors.

To model the dynamics, I specify the regional yield factors in the following way:

$$\begin{pmatrix} L_{jt} \\ S_{jt} \\ C_{jt} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} L_{jt-1} \\ S_{jt-1} \\ C_{jt-1} \end{pmatrix} + \begin{pmatrix} U^l_{jt} \\ U^s_{jt} \\ U^c_{jt} \end{pmatrix}, \tag{6}$$

where U^n_{jt} are disturbance terms with $E\left[U^n_{jt} U^{n'}_{j't'}\right] = (\sigma^n)^2$ if $t = t'$ and $n = n'$ for $n = l, s, c$, and 0 otherwise. For the global context, I also use Eq. (6), but the only difference is that the regional yield factors are replaced by the global yield factors.

Besides, I specify the country idiosyncratic factors in the following way:

$$\begin{pmatrix} \epsilon^l_{it} \\ \epsilon^s_{it} \\ \epsilon^c_{it} \end{pmatrix} = \begin{pmatrix} \phi_{i,11} & \phi_{i,12} & \phi_{i,13} \\ \phi_{i,21} & \phi_{i,22} & \phi_{i,23} \\ \phi_{i,31} & \phi_{i,32} & \phi_{i,33} \end{pmatrix} \begin{pmatrix} \epsilon^l_{i,t-1} \\ \epsilon^s_{i,t-1} \\ \epsilon^c_{i,t-1} \end{pmatrix} + \begin{pmatrix} u^l_{it} \\ u^s_{it} \\ u^c_{it} \end{pmatrix}, \tag{7}$$

where u^n_{it} are disturbance terms with $E\left[u^n_{it} u^{n'}_{i't'}\right] = (\sigma^n_i)^2$ if $i = i'$, $t = t'$ and $n = n'$ for $n = l, s, c$, and 0 otherwise. Furthermore, $E\left[U^n_{jt} u^{n'}_{i,t-s}\right] = 0$ and $E\left[U^n_t u^{n'}_{i,t-s}\right] = 0$, for all n, n', i, s.

To examine the presence of an underlying common factor in the country yield curves of a specific region, I apply principal component analysis (PCA) on the estimated country level, slope and curvature factors. I also do this in a global context. Subsequently, I use the PCA results to extract the regional and global yield factors. For simplicity and to replicate the research of Diebold et al. (2008) as well as possible, I assume the dynamic matrices in (6) and (7) to be diagonal. Finally, I obtain the estimates of Eq. (2) - (7) by using ordinary least squares (OLS).

3.2 Prediction performance of the regional and global yield factors

Next to the impact of the regional and global yield factors on the country yield factors, I also investigate the prediction performance between the regional and global yield factor. I use separate equations in forecasting the country yield factors, namely Eq. (4a) - (4c) for the regional yield factor and for the global context I replace the regional yield factor by the global yield factor in these equations. Besides, I evaluate the prediction performance of the regional and global yield factor in forecasting the yield curve of each country. For this purpose I substitute the predicted country yield factors from Eq. (4a) - (4c) in Eq. (1) to compute the predicted country's yield curve. To evaluate the prediction performance I use three metrics, namely the mean squared error (MSE), mean absolute error (MAE) and the mean correct prediction (MCP).

As stated in Heij et al. (2004), the mean squared error (MSE) is defined as:

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2, \tag{8}$$

which is the average squared deviations of the observed values (y_i) and the predicted values ($\hat{y}_i$), where N denotes the number of observations in the prediction sample.

Another metric that I use in my research is the mean absolute error (MAE). An advantage of the MAE is that it is more robust to outliers than the MSE, because it makes use of absolute deviations instead of squared deviations. As shown in Heij et al. (2004), the mean absolute error (MAE) is defined as:

$$MAE = \frac{1}{N}\sum_{i=1}^{N} |y_i - \hat{y}_i|, \tag{9}$$

which is the average absolute deviations of the observed values (y_i) and the predicted values ($\hat{y}_i$), where N denotes the number of observations in the prediction sample. The model with the lowest MSE and MAE value is preferred.

Finally, the last metric that I use is the mean correct prediction (MCP). The MCP counts the number of times when the sign of changes is correctly predicted, divided by the number of observations in the prediction sample. An advantage of the MCP is that it evaluates the directional movement of a variable of interest, while the MSE and MAE consider quantitative errors. Based on the work of Chalamandaris & Tsekrekos (2010), the mean correct prediction (MCP) is defined as:

$$MCP = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{sign(y_i - y_{i-1}) == sign(\hat{y}_i - \hat{y}_{i-1})}, \tag{10}$$

where $\mathbf{1}$ is the indicator function when the sign of change of the observed values (y_i) is equal to the sign of change of the predicted values ($\hat{y}_i$), and N denotes the number of observations in the prediction sample. The model with the highest MCP value is preferred.

4 Results

4.1 Modeling and estimating the regional and global yield curves

As a first step in the process to obtain the estimates of the regional and global yield factors, I estimate the level ($\hat{l}_{it}$), slope ($\hat{s}_{it}$) and curvature ($\hat{c}_{it}$) factors, $i = 1, \ldots, N$ and $t = 1, \ldots, T$, in Eq. (1) for each country separately by using non-linear least squares (NLS). Table 2 shows the descriptive statistics for estimated country level ($\hat{l}_{it}$), slope ($\hat{s}_{it}$) and curvature ($\hat{c}_{it}$) factors of all countries, which are part of the regions America, Europe and Asia/Pacific.

Not only for the region America, but also globally, I see that the mean level factor is greatest for Brazil. Furthermore, the mean slope and curvature factor for Brazil have a positive sign while these factors for all other countries have a negative sign. As already noted in Section 2, it is clear that Brazil faces some serious fundamental economic and political problems. In general, due to these problems bond investors consider Brazilian government bonds as a risky investment and they are pessimistic about the Brazilian growth expectations which is reflected in the means of the Brazilian yield factors.

For the region Europe I see that the mean level and curvature factors of the countries are comparatively equal to each other. Both regionally and globally the mean absolute slope factor is greatest for France and Singapore, which reflects the relatively optimistic average growth expectations for those countries.

Finally, I see that the mean level factor is relatively low for Japan and Hong Kong in a global context, where it is lowest for Japan both regionally and globally. In general, the autocorrelations show that the yield factors of all countries are persistent over time.

Table 2: Descriptive statistics estimated country level, slope and curvature factors

Country factor	Mean	St. dev.	Min.	Max.	$\hat{\rho}(1)$	$\hat{\rho}(6)$	$\hat{\rho}(12)$
Panel A: Region America							
Brazil							
$\hat{l}_{it}$	10.79	4.08	0.002	17.21	0.78	0.48	0.045
$\hat{s}_{it}$	1.11	4.19	-6.06	15.73	0.71	0.46	0.12
$\hat{c}_{it}$	0.71	18.24	-49.00	68.98	0.61	0.40	0.071
Canada							
$\hat{l}_{it}$	4.06	0.84	1.87	6.06	0.65	0.39	0.11
$\hat{s}_{it}$	-2.67	1.26	-5.28	0.11	0.82	0.62	0.36
$\hat{c}_{it}$	-2.06	1.77	-6.99	2.61	0.69	0.29	0.14
US							
$\hat{l}_{it}$	4.52	1.23	1.76	7.15	0.89	0.76	0.63
$\hat{s}_{it}$	-3.42	1.79	-6.78	0.22	0.94	0.78	0.58
$\hat{c}_{it}$	-3.19	2.67	-7.27	4.37	0.82	0.59	0.53
Panel B: Region Europe							
France							
$\hat{l}_{it}$	4.90	1.09	2.31	7.98	0.71	0.18	0.16
$\hat{s}_{it}$	-4.03	1.97	-7.95	-0.37	0.90	0.69	0.57
$\hat{c}_{it}$	-3.93	2.49	-8.42	3.10	0.88	0.63	0.53
Germany							
$\hat{l}_{it}$	4.29	1.10	1.41	7.28	0.84	0.36	0.27
$\hat{s}_{it}$	-3.49	1.77	-7.21	0.03	0.93	0.71	0.57
$\hat{c}_{it}$	-3.42	2.50	-8.57	3.51	0.88	0.54	0.44
UK							
$\hat{l}_{it}$	4.02	1.14	1.35	6.18	0.89	0.68	0.61
$\hat{s}_{it}$	-2.32	1.97	-5.74	1.77	0.94	0.78	0.61
$\hat{c}_{it}$	-3.77	2.69	-9.02	5.33	0.88	0.64	0.52
Panel C: Region Asia/Pacific							
Hong Kong							
$\hat{l}_{it}$	3.20	1.11	0.003	5.32	0.86	0.54	0.43
$\hat{s}_{it}$	-2.34	1.14	-4.59	0.54	0.82	0.55	0.41
$\hat{c}_{it}$	-2.35	2.11	-5.82	5.35	0.73	0.55	0.42
Japan							
$\hat{l}_{it}$	2.85	1.50	0.09	5.82	0.96	0.86	0.73
$\hat{s}_{it}$	-2.72	1.41	-5.66	-0.23	0.95	0.85	0.72
$\hat{c}_{it}$	-2.66	1.99	-6.38	2.83	0.94	0.74	0.62
Singapore							
$\hat{l}_{it}$	5.06	2.05	2.24	11.22	0.79	0.47	0.40
$\hat{s}_{it}$	-4.01	2.57	-10.77	-0.04	0.86	0.61	0.52
$\hat{c}_{it}$	-5.04	3.15	-13.18	0.004	0.87	0.65	0.51

Note: The table shows the descriptive statistics for estimated country level ($\hat{l}_{it}$), slope ($\hat{s}_{it}$) and curvature ($\hat{c}_{it}$) factors of all countries, which are part of the regions America (Panel A), Europe (Panel B) and Asia/Pacific (Panel C), over the sample period 2006.01 - 2018.05. $\hat{\rho}(\tau)$ represents the sample autocorrelation at lag τ.

As a next step I apply principal component analysis (PCA) on the estimated country level, slope and curvature factors of countries for each region separately and of all countries together to examine the existence of regional and global yield factors. Table 3 shows the results of these four principal component analyses.

The first principal component for level factors explains 49% of level variation on a global base, 53% for the region America, 70% for Asia/Pacific and 71% for Europe. I interpret this as the existence of one dominant level factor for the regions Asia/Pacific and Europe, and one important level factor for the region America and on a global base. However, in the paper of Diebold et al. (2008) the authors find the existence of one dominant global level factor which explains 91% of level variation. In comparison with my results it seems that if I examine more countries on a global base, the importance of the global level factor decreases. Another reason can be the choice of the sample period, because I use more recent years in my research.

For the slope factors the first principal component explains 56% of slope variation for the region America, 59% on a global base, 81% for Asia/Pacific and 89% for Europe. I interpret this as the existence of one dominant regional and global slope factor, in particular the regional slope factor for Asia/Pacific and Europe is very important. In comparison with the results of Diebold et al. (2008) the importance of the global slope factor has increased in my research. The authors find in their paper that the first principal component for slope factors explains 50% of slope variation, while it explains 59% in my research.

Finally, the first principal component for curvature factors explains 45% of curvature variation for the region America, 53% on a global base, 80% for Europe and 82% for Asia/Pacific. For the region America and on a global base I interpret the PCA results as the existence of one important curvature factor and for Europe and Asia/Pacific one dominant curvature factor.

In general, I see that the regions Europe and Asia/Pacific exhibit strong and dominant regional yield factors. For Europe this can be explained by the presence of the Eurozone and the European Union (EU). For example, France and Germany are part of the Eurozone and therefore share the same currency and are under supervision of one central bank, namely the European Central Bank (ECB). Furthermore, the European Union has created an internal single market and develops policies and laws which apply in all EU countries, including France, Germany and the UK which are part of my research. For the region Asia/Pacific I only consider Japan, Hong Kong and Singapore in my research which are all developed countries and where much wealth is concentrated. This can be a reason why those countries share important common factors. For example, for the region America I consider Brazil which suffers from severe fundamental problems and is a less developed country than Canada and the US. This difference can be an explanation why the regional yield factors for America are less important than those for Europe and Asia/Pacific.

To extract the regional and global yield factors I use the results of the principal component analyses. In Appendix A Fig. 1, 2 and 3 show how the extracted regional and global factors behave over time for the level, slope and curvature, respectively.

For the extracted regional and global level factors I see a pattern of commonality in the dynamics. In particular the European and Asian level factor behave relatively similar. However, the American level factor is comparatively divorced from the other level factors which reflects the unique situation of Brazil. In general, I see that the regional and global level factors reach their highest values between 2010 and the end of 2013, except for America which attains its maximum values around 2007. From the end of 2013 onwards the level factors decline with short peaks in the meantime, except for the American level factor which already declines from the end of 2010 onwards.

For the extracted regional and global slope factors there is also a clear pattern of commonality over time. In general, from 2008 onwards the slope factors decline with the lowest values between 2010 and 2014, so when the level factors reach their maximum values, the slope factors attain their minimum values. Afterwards the slope factors show an increasing trend.

Finally, the extracted regional and global curvature factors also behave relatively similar over time. Just as the level factor, the American curvature factor behaves uniquely during certain periods, especially at the beginning of the sample period there is a large deviation in comparison with the other curvature factors. Like the slope factors, the curvature factors are negative during most of the time.

Table 3: PCA for estimated country level, slope and curvature factors

	Level factors ($\hat{l}_{it}$)			Slope factors ($\hat{s}_{it}$)			Curvature factors ($\hat{c}_{it}$)		
	PC 1	PC 2	PC 3	PC 1	PC 2	PC 3	PC 1	PC 2	PC 3
Panel A: Region America									
St. dev.	1.26	0.94	0.72	1.30	1.02	0.53	1.16	0.94	0.88
Variance prop.	0.53	0.30	0.17	0.56	0.34	0.10	0.45	0.29	0.26
Cumulative prop.	0.53	0.83	1.00	0.56	0.90	1.00	0.45	0.74	1.00
Panel B: Region Europe									
St. dev.	1.46	0.84	0.41	1.63	0.53	0.25	1.55	0.71	0.31
Variance prop.	0.71	0.24	0.05	0.89	0.09	0.02	0.80	0.17	0.03
Cumulative prop.	0.71	0.95	1.00	0.89	0.98	1.00	0.80	0.97	1.00
Panel C: Region Asia/Pacific									
St. dev.	1.45	0.79	0.53	1.56	0.58	0.49	1.57	0.57	0.45
Variance prop.	0.70	0.21	0.09	0.81	0.11	0.08	0.82	0.11	0.07
Cumulative prop.	0.70	0.91	1.00	0.81	0.92	1.00	0.82	0.93	1.00
Panel D: Global									
St. dev.	2.11	1.29	0.93	2.30	1.17	1.07	2.18	1.34	0.94
Variance prop.	0.49	0.18	0.10	0.59	0.15	0.13	0.53	0.20	0.10
Cumulative prop.	0.49	0.67	0.77	0.59	0.74	0.87	0.53	0.73	0.83

Note: The table shows the results of the principal component analysis (PCA) for estimated country level ($\hat{l}_{it}$), slope ($\hat{s}_{it}$) and curvature ($\hat{c}_{it}$) factors of the countries in the regions America (Panel A), Europe (Panel B), Asia/Pacific (Panel C) and all together (global setting; Panel D), over the sample period 2006.01 - 2018.05. For the global setting only PC 1 until PC 3 are reported, because the other PC's explain 9% or less of total variance and are therefore irrelevant to mention.

Using the extracted regional and global yield factors, I estimate Eq. (2) - (7) by using OLS. For each region Table 4 shows the parameter estimates of the regional yield curve model. Table 5 shows the parameter estimates of the global yield curve model.

For the country level factors I see that for the region Europe and Asia/Pacific they all load positively on the regional level factor. In general, the regional level factor loadings are estimated with high precision. Furthermore, the results for Europe and Asia/Pacific are highly serially correlated. As already mentioned, the regional European level factor is very important. For example, if the European level factor increases with one unit the German level factor increases on average with a value of 0.64. An already mentioned explanation may be the existence of the European Union and the Eurozone. In comparison with France and Germany the level loading of the UK on the European level factor is smaller and the UK-specific level factor is more persistent, which means that the European level factor is less important for the UK than for France and Germany. As explained in the literature of Diebold et al. (2008) and Sopov & Seidler (2011) a main determinant of the level factor is inflation. Due to the fact that the UK is not part of the Eurozone, it does not have the same central bank as France and Germany. Therefore, the UK has another monetary policy which can be the reason why the European level factor is less important for the level factor of the UK. For the region Asia/Pacific the country level factors are also highly positively correlated with the regional level factor. For example, if the Asian level factor increases with one unit the level factor of Singapore increases on average with a value of 0.80. Like the European level factor, these positively correlated results reflect an important common factor driving the monetary policies of the three Asian countries. However, the level loading of Hong Kong on the Asian level factor is smaller than Japan and Singapore. Furthermore, the country-specific level factor of Hong Kong is more persistent, so the Asian level factor is less important for Hong Kong than for Japan and Singapore. For the region America the country level factors also load positively on the regional level factor, except Brazil which loads negatively on the American level factor. This difference in sign shows once again the problematic situation in Brazil. For example, the mean inflation rate of Brazil over my sample period is 5.58% which is comparatively quite high than the inflation rates of Canada and the US, namely 1.70% and 1.96%, respectively[2].

For the country slope factors I see that for all regions they all load positively on the regional slope factor, where the factor loadings are also estimated with high precision. In the literature of Diebold et al. (2008) and Sopov & Seidler (2011) the authors already explained that a main determinant of the level factor is inflation, but they also argue that a main determinant of the slope factor is real economic activity. Hence, these results reflect the common characteristics of the economies of the countries in these regions. For Europe this is in line with the results of the regional level factor, where I see the importance of the European Union and the Eurozone in terms of monetary policy and real economic activity. This is also confirmed by the average GDP annual growth of the European countries over my sample which are relatively similar, namely 1.08% for France, 1.67% for Germany and 1.36% for the UK[2].

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[2] I source inflation and real GDP growth data from the International Monetary Fund; World Economic Outlook database: http://www.imf.org/external/datamapper/datasets/WEO

Table 4: Parameter estimates of the regional yield curve models

America	Europe	Asia/Pacific
$L_{AM,t} = 0.83L_{AM,t-1} + U^l_{AM,t}$ (0.05)	$L_{EU,t} = 0.86L_{EU,t-1} + U^l_{EU,t}$ (0.04)	$L_{AS,t} = 0.92L_{AS,t-1} + U^l_{AS,t}$ (0.03)
$S_{AM,t} = 0.90S_{AM,t-1} + U^s_{AM,t}$ (0.04)	$S_{EU,t} = 0.95S_{EU,t-1} + U^s_{EU,t}$ (0.02)	$S_{AS,t} = 0.93S_{AS,t-1} + U^s_{AS,t}$ (0.03)
$C_{AM,t} = 0.65C_{AM,t-1} + U^c_{AM,t}$ (0.06)	$C_{EU,t} = 0.92C_{EU,t-1} + U^c_{EU,t}$ (0.03)	$C_{AS,t} = 0.92C_{AS,t-1} + U^c_{AS,t}$ (0.03)
Country	Country	Country
$l_{BR,t} = 12.27 - 1.56L_{AM,t} + \epsilon^l_{BR,t}$ (0.18) (0.07)	$l_{FR,t} = 0.41 + 0.59L_{EU,t} + \epsilon^l_{FR,t}$ (0.22) (0.03)	$l_{HK,t} = 1.09 + 0.33L_{AS,t} + \epsilon^l_{HK,t}$ (0.20) (0.03)
$l_{CA,t} = 3.88 + 0.19L_{AM,t} + \epsilon^l_{CA,t}$ (0.06) (0.03)	$l_{GM,t} = -0.57 + 0.64L_{EU,t} + \epsilon^l_{GM,t}$ (0.15) (0.02)	$l_{JP,t} = -0.90 + 0.59L_{AS,t} + \epsilon^l_{JP,t}$ (0.16) (0.02)
$l_{US,t} = 4.19 + 0.35L_{AM,t} + \epsilon^l_{US,t}$ (0.08) (0.03)	$l_{UK,t} = 0.27 + 0.49L_{EU,t} + \epsilon^l_{UK,t}$ (0.33) (0.04)	$l_{SP,t} = -0.02 + 0.80L_{AS,t} + \epsilon^l_{SP,t}$ (0.22) (0.03)
$s_{BR,t} = 3.68 + 0.62S_{AM,t} + \epsilon^s_{BR,t}$ (0.74) (0.16)	$s_{FR,t} = -0.55 + 0.61S_{EU,t} + \epsilon^s_{FR,t}$ (0.10) (0.02)	$s_{HK,t} = -0.40 + 0.37S_{AS,t} + \epsilon^s_{HK,t}$ (0.10) (0.02)
$s_{CA,t} = -0.41 + 0.55S_{AM,t} + \epsilon^s_{CA,t}$ (0.10) (0.02)	$s_{GM,t} = -0.34 + 0.55S_{EU,t} + \epsilon^s_{GM,t}$ (0.08) (0.01)	$s_{JP,t} = -0.38 + 0.45S_{AS,t} + \epsilon^s_{JP,t}$ (0.13) (0.02)
$s_{US,t} = -0.26 + 0.77S_{AM,t} + \epsilon^s_{US,t}$ (0.16) (0.03)	$s_{UK,t} = 0.95 + 0.57S_{EU,t} + \epsilon^s_{UK,t}$ (0.15) (0.02)	$s_{SP,t} = 0.76 + 0.91S_{AS,t} + \epsilon^s_{SP,t}$ (0.14) (0.02)
$c_{BR,t} = 4.92 + 1.59C_{AM,t} + \epsilon^c_{BR,t}$ (0.27) (0.02)	$c_{FR,t} = -0.16 + 0.59C_{EU,t} + \epsilon^c_{FR,t}$ (0.14) (0.02)	$c_{HK,t} = 0.44 + 0.48C_{AS,t} + \epsilon^c_{HK,t}$ (0.16) (0.02)
$c_{CA,t} = -1.96 + 0.04C_{AM,t} + \epsilon^c_{CA,t}$ (0.15) (0.01)	$c_{GM,t} = 0.39 + 0.60C_{EU,t} + \epsilon^c_{GM,t}$ (0.13) (0.02)	$c_{JP,t} = 0.12 + 0.48C_{AS,t} + \epsilon^c_{JP,t}$ (0.12) (0.02)
$c_{US,t} = -2.96 + 0.09C_{AM,t} + \epsilon^c_{US,t}$ (0.21) (0.02)	$c_{UK,t} = -0.27 + 0.55C_{EU,t} + \epsilon^c_{UK,t}$ (0.25) (0.03)	$c_{SP,t} = -0.55 + 0.77C_{AS,t} + \epsilon^c_{SP,t}$ (0.17) (0.02)
$\epsilon^l_{BR,t} = 0.78\epsilon^l_{BR,t-1} + u^l_{BR,t}$ (0.05)	$\epsilon^l_{FR,t} = 0.60\epsilon^l_{FR,t-1} + u^l_{FR,t}$ (0.07)	$\epsilon^l_{HK,t} = 0.84\epsilon^l_{HK,t-1} + u^l_{HK,t}$ (0.05)
$\epsilon^l_{CA,t} = 0.62\epsilon^l_{CA,t-1} + u^l_{CA,t}$ (0.06)	$\epsilon^l_{GM,t} = 0.51\epsilon^l_{GM,t-1} + u^l_{GM,t}$ (0.07)	$\epsilon^l_{JP,t} = 0.54\epsilon^l_{JP,t-1} + u^l_{JP,t}$ (0.07)
$\epsilon^l_{US,t} = 0.79\epsilon^l_{US,t-1} + u^l_{US,t}$ (0.07)	$\epsilon^l_{UK,t} = 0.78\epsilon^l_{UK,t-1} + u^l_{UK,t}$ (0.05)	$\epsilon^l_{SP,t} = 0.65\epsilon^l_{SP,t-1} + u^l_{SP,t}$ (0.06)
$\epsilon^s_{BR,t} = 0.74\epsilon^s_{BR,t-1} + u^s_{BR,t}$ (0.06)	$\epsilon^s_{FR,t} = 0.60\epsilon^s_{FR,t-1} + u^s_{FR,t}$ (0.07)	$\epsilon^s_{HK,t} = 0.57\epsilon^s_{HK,t-1} + u^s_{HK,t}$ (0.07)
$\epsilon^s_{CA,t} = 0.59\epsilon^s_{CA,t-1} + u^s_{CA,t}$ (0.07)	$\epsilon^s_{GM,t} = 0.57\epsilon^s_{GM,t-1} + u^s_{GM,t}$ (0.07)	$\epsilon^s_{JP,t} = 0.70\epsilon^s_{JP,t-1} + u^s_{JP,t}$ (0.06)
$\epsilon^s_{US,t} = 0.77\epsilon^s_{US,t-1} + u^s_{US,t}$ (0.05)	$\epsilon^s_{UK,t} = 0.75\epsilon^s_{UK,t-1} + u^s_{UK,t}$ (0.05)	$\epsilon^s_{SP,t} = 0.58\epsilon^s_{SP,t-1} + u^s_{SP,t}$ (0.07)
$\epsilon^c_{BR,t} = 0.72\epsilon^c_{BR,t-1} + u^c_{BR,t}$ (0.06)	$\epsilon^c_{FR,t} = 0.63\epsilon^c_{FR,t-1} + u^c_{FR,t}$ (0.06)	$\epsilon^c_{HK,t} = 0.46\epsilon^c_{HK,t-1} + u^c_{HK,t}$ (0.07)
$\epsilon^c_{CA,t} = 0.66\epsilon^c_{CA,t-1} + u^c_{CA,t}$ (0.06)	$\epsilon^c_{GM,t} = 0.60\epsilon^c_{GM,t-1} + u^c_{GM,t}$ (0.07)	$\epsilon^c_{JP,t} = 0.55\epsilon^c_{JP,t-1} + u^c_{JP,t}$ (0.07)
$\epsilon^c_{US,t} = 0.76\epsilon^c_{US,t-1} + u^c_{US,t}$ (0.05)	$\epsilon^c_{UK,t} = 0.78\epsilon^c_{UK,t-1} + u^c_{UK,t}$ (0.05)	$\epsilon^c_{SP,t} = 0.48\epsilon^c_{SP,t-1} + u^c_{SP,t}$ (0.07)

Note: The table shows the parameter estimates of the regional yield curve models (3)-(7) with standard errors presented in brackets. The estimates are based on the sample period 2006.01 - 2018.05.

For the region Asia/Pacific it is also clear that Hong Kong, Japan and Singapore share an important common factor in terms of monetary policy and that they are comparatively homogeneous regarding their business cycle. An already mentioned explanation is that these Asian countries are all developed countries where much wealth is concentrated.

For the region America the country slope factors also load positively on the regional slope factor, where the estimates are highly serially correlated. For example, if the American slope factor increases with one unit the slope factor of the US increases on average with a value of 0.77. These results reflect that Brazil, Canada and the US are comparatively homogeneous regarding their business cycle. I also see this in terms of their average GDP annual growth which is equal to 2.18% for Brazil, 1.82% for Canada and 1.66% for the US.

All country curvature factors load positively on their corresponding regional curvature factor which shows the similarity of each country regarding their curvature. Most of the countries are highly positively correlated with their corresponding regional curvature factor, except Canada and the US. Their factor loading is relatively small and the persistence of the country-specific curvature factor large, which means that the curvature factor of those countries is relatively divorced from the regional curvature factor.

Table 5 shows the parameter estimates of the global yield curve model, in which I see that all country level factors load positively on the global level factor, except Brazil. Furthermore, in general all factor loadings are estimated with high precision. In a global context I see that every country shares a common factor regarding monetary policy and inflation. However, the Brazilian level factor is negatively correlated with the global level factor. For example, if the global level factor increases with one unit the Brazilian level factor decreases on average with a value of 0.37. This difference in sign in comparison with the other countries in my research clearly reflects that Brazil suffers from big problems concerning high government debts and high inflation, Brazil even corresponds to the countries in the world with the highest inflation rate. It is clear that Brazil is the least developed country, while all other countries in my research are quite developed from a global perspective. The authors in Diebold et al. (2008) also find that their country level factors load positively on the global level factor.

For the country slope factors I see similar results in comparison with the country level factors, because in this case they also load positively on the global slope factor, except Brazil which has a negative loading. Furthermore, the estimates are highly serially correlated. Thus, in general I see a strong positive impact of the global slope factor on the country slope factors. I interpret this as when the global economy performs well, the economy of most of the countries - especially developed countries - also performs well. A less developed country, such as Brazil, is an exception in this case regarding the sign of the factor loading. As already explained this is due to the big economic problems in Brazil. In this case my results are in line with the findings of Diebold et al. (2008), in which the authors also find a positive relationship between the country slope factor and the global slope factor. However, in general the global slope factor has become more important in my research. Besides, I still see some similarities, such as the fact that the global slope factor is relatively less important for Japan and very important for the UK. Only Singapore has a stronger relationship with the global slope factor than the UK.

Finally, I see that all country curvature factors also load positively on the global curvature factor which reflects the homogeneity of the curvature for each country.

I also examine the impact on the country yield factors by including both the regional and global yield factor in a combined model, which is defined in Eq. (5a) - (5c). I estimate these Eq. (5a) - (5c) by using OLS. Table 6 shows the corresponding results of the estimated yield curve model with the regional and global factor combined. For the interpretation of these results I use the *ceteris paribus* assumption and a significance level of 5%.

For the country level factors of the region America I see that they all load positively on both the regional and global level factor, except Brazil for which it only loads negatively on the regional level factor. Furthermore, for Brazil and Canada the regional level factor is more important than the global level factor, while just the opposite holds for the US. The negative correlation between the Brazilian level factor and the regional level factor reflects the unique Brazilian monetary policy and inflation rate regionally. However, there exists a positive correlation between the Brazilian level factor and the global level factor. For Europe the regional level factor has a relative strong impact on the level factor of France and Germany, which reflects the importance of the Eurozone. The UK is not part of the Eurozone and this is also clearly visible in the results, because the regional level factor for the UK is insignificant. Thus, the UK level factor only loads positively on the global level factor. For the region Asia/Pacific the regional level factor for Hong Kong and the global level factor for Japan is not significant. Hence, the level factor of Hong Kong only loads positively on the global level factor and the level factor of Japan only loads positively on the regional level factor. As already noted, Japan even has the lowest inflation rate of all countries in my research which may explain the independence of the Japanese monetary policy globally. Lastly, the regional level factor has a relative strong positive impact on the level factor of Singapore, while the global level factor has a negative impact. Thus, Singapore is unique globally, but it shows common features within the region. Finally, I see that for 67% of the country level factors the regional level factor is more important than the global level factor.

Concerning the country slope factors, for the region America the global slope factor for Canada is insignificant. All country slope factors load positively on both the regional and global slope factor, except Brazil which loads negatively on the global slope factor. Globally, Brazil has unique characteristics regarding the economy due to the already mentioned economic problems. For Europe all country slope factors load positively on both the regional and global slope factor, except France and Germany which only loads negatively on the global slope factor. It shows once again the relative strong impact of the European slope factor on the country slope factors, especially for France and Germany. For Hong Kong the global slope factor is not significant, so in contrast with the level factor of Hong Kong which only loads on the global level factor, the slope factor only loads on the regional slope factor. The slope factor of Japan loads negatively on the global slope factor which reflects the unique developments in the Japanese real economy from a global perspective. Finally, I see that for all country slope factors the regional slope factor is more important than the global slope factor.

Table 5: Parameter estimates of the global yield curve model

Level factor	Slope factor	Curvature factor
$L_t = 0.93L_{t-1} + U^l_t$	$S_t = 0.96S_{t-1} + U^s_t$	$C_t = 0.86C_{t-1} + U^c_t$
(0.03)	(0.02)	(0.04)
Country level factors	Country slope factors	Country curvature factors
$l_{BR,t} = 14.58 - 0.37L_t + \epsilon^l_{BR,t}$	$s_{BR,t} = -0.51 - 0.18S_t + \epsilon^s_{BR,t}$	$c_{BR,t} = 17.32 + 1.85C_t + \epsilon^c_{BR,t}$
(1.24) (0.12)	(0.81) (0.08)	(1.30) (0.11)
$l_{CA,t} = 2.68 + 0.13L_t + \epsilon^l_{CA,t}$	$s_{CA,t} = -0.75 + 0.22S_t + \epsilon^s_{CA,t}$	$c_{CA,t} = -1.41 + 0.07C_t + \epsilon^c_{CA,t}$
(0.24) (0.02)	(0.17) (0.02)	(0.21) (0.02)
$l_{US,t} = 0.60 + 0.38L_t + \epsilon^l_{US,t}$	$s_{US,t} = 0.00 + 0.39S_t + \epsilon^s_{US,t}$	$c_{US,t} = -1.30 + 0.21C_t + \epsilon^c_{US,t}$
(0.19) (0.02)	(0.16) (0.02)	(0.25) (0.02)
$l_{FR,t} = 2.99 + 0.19L_t + \epsilon^l_{FR,t}$	$s_{FR,t} = -0.77 + 0.37S_t + \epsilon^s_{FR,t}$	$c_{FR,t} = -2.06 + 0.21C_t + \epsilon^c_{FR,t}$
(0.30) (0.03)	(0.25) (0.03)	(0.23) (0.02)
$l_{GM,t} = 1.55 + 0.27L_t + \epsilon^l_{GM,t}$	$s_{GM,t} = -0.42 + 0.35S_t + \epsilon^s_{GM,t}$	$c_{GM,t} = -1.43 + 0.22C_t + \epsilon^c_{GM,t}$
(0.25) (0.02)	(0.20) (0.02)	(0.22) (0.02)
$l_{UK,t} = 0.28 + 0.36L_t + \epsilon^l_{UK,t}$	$s_{UK,t} = 1.57 + 0.44S_t + \epsilon^s_{UK,t}$	$c_{UK,t} = -1.32 + 0.27C_t + \epsilon^c_{UK,t}$
(0.16) (0.02)	(0.15) (0.02)	(0.19) (0.02)
$l_{HK,t} = 0.07 + 0.30L_t + \epsilon^l_{HK,t}$	$s_{HK,t} = -0.37 + 0.22S_t + \epsilon^s_{HK,t}$	$c_{HK,t} = -0.90 + 0.16C_t + \epsilon^c_{HK,t}$
(0.23) (0.02)	(0.13) (0.01)	(0.21) (0.02)
$l_{JP,t} = -1.88 + 0.46L_t + \epsilon^l_{JP,t}$	$s_{JP,t} = -0.71 + 0.23S_t + \epsilon^s_{JP,t}$	$c_{JP,t} = -1.13 + 0.17C_t + \epsilon^c_{JP,t}$
(0.24) (0.02)	(0.21) (0.02)	(0.18) (0.01)
$l_{SP,t} = -0.69 + 0.56L_t + \epsilon^l_{SP,t}$	$s_{SP,t} = 0.87 + 0.55S_t + \epsilon^s_{SP,t}$	$c_{SP,t} = -2.65 + 0.27C_t + \epsilon^c_{SP,t}$
(0.42) (0.04)	(0.24) (0.02)	(0.28) (0.02)
$\epsilon^l_{BR,t} = 0.78\epsilon^l_{BR,t-1} + u^l_{BR,t}$	$\epsilon^s_{BR,t} = 0.71\epsilon^s_{BR,t-1} + u^s_{BR,t}$	$\epsilon^c_{BR,t} = 0.60\epsilon^c_{BR,t-1} + u^c_{BR,t}$
(0.05)	(0.06)	(0.004)
$\epsilon^l_{CA,t} = 0.60\epsilon^l_{CA,t-1} + u^l_{CA,t}$	$\epsilon^s_{CA,t} = 0.68\epsilon^s_{CA,t-1} + u^s_{CA,t}$	$\epsilon^c_{CA,t} = 0.65\epsilon^c_{CA,t-1} + u^c_{CA,t}$
(0.07)	(0.06)	(0.06)
$\epsilon^l_{US,t} = 0.72\epsilon^l_{US,t-1} + u^l_{US,t}$	$\epsilon^s_{US,t} = 0.82\epsilon^s_{US,t-1} + u^s_{US,t}$	$\epsilon^c_{US,t} = 0.69\epsilon^c_{US,t-1} + u^c_{US,t}$
(0.06)	(0.05)	(0.06)
$\epsilon^l_{FR,t} = 0.68\epsilon^l_{FR,t-1} + u^l_{FR,t}$	$\epsilon^s_{FR,t} = 0.83\epsilon^s_{FR,t-1} + u^s_{FR,t}$	$\epsilon^c_{FR,t} = 0.75\epsilon^c_{FR,t-1} + u^c_{FR,t}$
(0.06)	(0.05)	(0.05)
$\epsilon^l_{GM,t} = 0.77\epsilon^l_{GM,t-1} + u^l_{GM,t}$	$\epsilon^s_{GM,t} = 0.85\epsilon^s_{GM,t-1} + u^s_{GM,t}$	$\epsilon^c_{GM,t} = 0.70\epsilon^c_{GM,t-1} + u^c_{GM,t}$
(0.05)	(0.04)	(0.06)
$\epsilon^l_{UK,t} = 0.54\epsilon^l_{UK,t-1} + u^l_{UK,t}$	$\epsilon^s_{UK,t} = 0.68\epsilon^s_{UK,t-1} + u^s_{UK,t}$	$\epsilon^c_{UK,t} = 0.56\epsilon^c_{UK,t-1} + u^c_{UK,t}$
(0.07)	(0.06)	(0.07)
$\epsilon^l_{HK,t} = 0.79\epsilon^l_{HK,t-1} + u^l_{HK,t}$	$\epsilon^s_{HK,t} = 0.65\epsilon^s_{HK,t-1} + u^s_{HK,t}$	$\epsilon^c_{HK,t} = 0.55\epsilon^c_{HK,t-1} + u^c_{HK,t}$
(0.05)	(0.06)	(0.07)
$\epsilon^l_{JP,t} = 0.73\epsilon^l_{JP,t-1} + u^l_{JP,t}$	$\epsilon^s_{JP,t} = 0.90\epsilon^s_{JP,t-1} + u^s_{JP,t}$	$\epsilon^c_{JP,t} = 0.80\epsilon^c_{JP,t-1} + u^c_{JP,t}$
(0.06)	(0.04)	(0.05)
$\epsilon^l_{SP,t} = 0.70\epsilon^l_{SP,t-1} + u^l_{SP,t}$	$\epsilon^s_{SP,t} = 0.64\epsilon^s_{SP,t-1} + u^s_{SP,t}$	$\epsilon^c_{SP,t} = 0.69\epsilon^c_{SP,t-1} + u^c_{SP,t}$
(0.06)	(0.06)	(0.06)

Note: The table shows the parameter estimates of the global yield curve model with standard errors presented in brackets. The estimates are based on the sample period 2006.01 - 2018.05.

Table 6: Parameter estimates of the yield curve model with regional and global factor combined

Level factors	Slope factors	Curvature factors
$l_{BR,t} = 6.36 - 2.01L_{AM,t} + 0.62L_t + \epsilon^l_{BR,t}$	$s_{BR,t} = 1.00 + 2.51S_{AM,t} - 1.18S_t + \epsilon^s_{BR,t}$	$c_{BR,t} = 0.26 + 1.99C_{AM,t} - 0.64C_t + \epsilon^c_{BR,t}$
(0.53) (0.07) (0.05)	(0.54) (0.18) (0.09)	(0.40) (0.03) (0.05)
$l_{CA,t} = 3.24 + 0.14L_{AM,t} + 0.07L_t + \epsilon^l_{CA,t}$	$s_{CA,t} = -0.42 + 0.56S_{AM,t} - 0.004S_t + \epsilon^s_{CA,t}$	$c_{CA,t} = -1.01 - 0.05C_{AM,t} + 0.13C_t + \epsilon^c_{CA,t}$
(0.26) (0.03) (0.03)	(0.11) (0.04) (0.02)	(0.31) (0.03) (0.04)
$l_{US,t} = 1.03 + 0.10L_{AM,t} + 0.33L_t + \epsilon^l_{US,t}$	$s_{US,t} = 0.25 + 0.41S_{AM,t} + 0.22S_t + \epsilon^s_{US,t}$	$c_{US,t} = 0.61 - 0.22C_{AM,t} + 0.49C_t + \epsilon^c_{US,t}$
(0.21) (0.03) (0.02)	(0.13) (0.04) (0.02)	(0.32) (0.03) (0.04)
$l_{FR,t} = 0.36 + 0.90L_{EU,t} - 0.22L_t + \epsilon^l_{FR,t}$	$s_{FR,t} = -0.90 + 0.80S_{EU,t} - 0.16S_t + \epsilon^s_{FR,t}$	$c_{FR,t} = -0.02 + 0.73C_{EU,t} - 0.09C_t + \epsilon^c_{FR,t}$
(0.16) (0.03) (0.02)	(0.09) (0.03) (0.02)	(0.13) (0.03) (0.01)
$l_{GM,t} = -0.59 + 0.73L_{EU,t} - 0.07L_t + \epsilon^l_{GM,t}$	$s_{GM,t} = -0.52 + 0.65S_{EU,t} - 0.08S_t + \epsilon^s_{GM,t}$	$c_{GM,t} = 0.47 + 0.68C_{EU,t} - 0.05C_t + \epsilon^c_{GM,t}$
(0.14) (0.03) (0.02)	(0.09) (0.02) (0.02)	(0.13) (0.03) (0.01)
$l_{UK,t} = 0.35 - 0.02L_{EU,t} + 0.38L_t + \epsilon^l_{UK,t}$	$s_{UK,t} = 1.52 + 0.27S_{EU,t} + 0.26S_t + \epsilon^s_{UK,t}$	$c_{UK,t} = -0.53 + 0.28C_{EU,t} + 0.16C_t + \epsilon^c_{UK,t}$
(0.20) (0.04) (0.02)	(0.13) (0.04) (0.03)	(0.23) (0.05) (0.03)
$l_{HK,t} = -0.15 - 0.14L_{AS,t} + 0.42L_t + \epsilon^l_{HK,t}$	$s_{HK,t} = -0.34 + 0.32S_{AS,t} + 0.04S_t + \epsilon^s_{HK,t}$	$c_{HK,t} = 0.44 + 0.50C_{AS,t} - 0.01C_t + \epsilon^c_{HK,t}$
(0.25) (0.07) (0.06)	(0.11) (0.04) (0.02)	(0.16) (0.03) (0.02)
$l_{JP,t} = -1.08 + 0.52L_{AS,t} + 0.06L_t + \epsilon^l_{JP,t}$	$s_{JP,t} = -0.64 + 0.65S_{AS,t} - 0.15S_t + \epsilon^s_{JP,t}$	$c_{JP,t} = 0.12 + 0.46C_{AS,t} + 0.01C_t + \epsilon^c_{JP,t}$
(0.23) (0.07) (0.06)	(0.13) (0.05) (0.03)	(0.12) (0.03) (0.01)
$l_{SP,t} = 1.31 + 1.30L_{AS,t} - 0.44L_t + \epsilon^l_{SP,t}$	$s_{SP,t} = 0.95 + 0.76S_{AS,t} + 0.11S_t + \epsilon^s_{SP,t}$	$c_{SP,t} = -0.55 + 0.78C_{AS,t} - 0.001C_t + \epsilon^c_{SP,t}$
(0.29) (0.08) (0.07)	(0.15) (0.05) (0.03)	(0.17) (0.04) (0.02)

Note: The table shows the parameter estimates of the yield curve model including both the regional and global factor with standard errors presented in brackets. The estimates are based on the sample period 2006.01 - 2018.05.

For the country curvature factors I see that the regional curvature factor for Canada is not significant. All other country curvature factors load positively on the corresponding regional curvature factor, except the US which is unique regarding its curvature regionally. Besides, I see that the global curvature factor for Hong Kong, Japan and Singapore is insignificant, so all country curvature factors in the region Asia/Pacific only load on the regional curvature factor. For Europe the country curvature factors of France and Germany load negatively on the global curvature factor, although the impact is quite weak. Finally, I see for 78% of the country curvature factors that the regional curvature factor is more important than the global curvature factor.

4.2 Prediction performance of the regional and global yield factors

In the previous subsection I examine the impact of the regional and global yield factors on the country yield factors. In general, Table 6 shows that the regional yield factor is more important than the global yield factor. Therefore, it is interesting to investigate whether the regional yield factor has a better prediction performance than the global yield factor. Subsequently, it is interesting to link the obtained results to macroeconomic determinants, such as inflation and real economic activity. Besides, I evaluate the prediction performance of the regional and global yield factor in forecasting the yield curve of each country. For this purpose I use the three metrics (8) - (10), namely the mean squared error (MSE), mean absolute error (MAE) and the mean correct prediction (MCP), respectively, to evaluate the prediction performance. Table 7 shows the results of each metric for both the regional and global yield factor and curve model of all countries. The model with the lowest MSE and MAE value, and the highest MCP value is preferred regarding its prediction performance.

For 67% of the country level factors it is clear that the regional level factor has a better prediction performance than the global level factor. Only for the US, the UK and Hong Kong the global level factor performs better in forecasting the country level factor, although this only holds for the UK according to the MCP value. These results are in line with my findings in the previous subsection, in which the regional level factor for the UK and Hong Kong is not significant and the impact of the global level factor on the US level factor is larger than the regional level factor.

For at least 67% of the country slope factors, the regional slope factor is also better in forecasting the country slope factors than the global slope factor. Only for the US all metrics indicate that the global slope factor has a better prediction performance than the regional slope factor, although the differences are comparatively small. Moreover, the MSE and MAE values show that the global slope factor is preferred over the regional slope factor for the UK. This is also the case for Brazil according to the MAE. However, in the previous subsection I find that the impact of the regional slope factor on the US slope factor is greater than the global slope factor. Nevertheless, the global slope factor loading is also positive, so it reflects a common factor globally. An explanation can be the fact that the economy of the US belongs to the most important economies in the world. Therefore, the US is an important participant in the 'international business cycle'.

For 67% of the country curvature factors the regional curvature factor has a better prediction performance than the global curvature factor. Like the slope factor, all metrics show that for the US the global curvature factor is preferred over the regional curvature factor. Furthermore, the MSE and MAE values indicate that the global curvature factor is better in forecasting the curvature factor of Canada and the UK. These results are also in line with the findings in the previous subsection, in which the regional curvature factor for Canada is insignificant and the impact of the global curvature factor on the US curvature factor is in absolute sense twice as large as the regional curvature factor.

Finally, for the country's yield curve the three metrics indicate that for at least 56% of the countries the regional yield factor has a better prediction performance than the global yield factor. In general, I see that only for the US, Canada and Hong Kong the global yield factor performs better than the regional yield factor in forecasting the yield curve, although this does not hold for Hong Kong according to the MCP. This result is stronger for the US than for Canada and Hong Kong, which can be explained by the fact that the US bond market is the largest in the world. Therefore, the US has a big share in the global bond market. Besides, the result that the regional yield factor has a better prediction performance than the global yield factor is relatively strong for France and Germany, which indicates that these countries are important participants in the European bond market.

5 Conclusion

In this paper I examine whether regional and/or global yield factors exist, what their dynamics and interactions are, and how they perform in forecasting the country yield factors and curves. For this purpose I extend the single-country dynamized Nelson-Siegel model to a multi-country context; regionally and globally.

Table 7: MSE, MAE and MCP values of regional and global yield factor and curve models

Panel A: MSE	Level factor		Slope factor		Curvature factor		Yield curve	
	Regional	Global	Regional	Global	Regional	Global	Regional	Global
Brazil	**3.97**	15.52	**15.82**	16.88	**10.13**	110.50	**10.24**	14.97
Canada	**0.53**	0.57	**0.32**	0.79	2.96	**2.79**	0.48	**0.38**
US	0.88	**0.37**	0.71	**0.66**	6.13	**4.23**	0.73	**0.44**
France	**0.29**	0.92	**0.33**	1.60	**0.78**	3.38	**0.10**	0.85
Germany	**0.14**	0.64	**0.22**	1.08	**0.71**	3.04	**0.04**	0.51
UK	0.67	**0.26**	0.72	**0.61**	2.53	**2.38**	**0.27**	0.30
Hong Kong	0.66	**0.51**	**0.30**	0.45	**1.12**	2.76	0.55	**0.45**
Japan	**0.43**	0.60	**0.54**	1.10	**0.64**	2.03	**0.16**	0.80
Singapore	**0.85**	1.74	**0.61**	1.45	**1.27**	5.26	**0.26**	1.68

Panel B: MAE	Level factor		Slope factor		Curvature factor		Yield curve	
	Regional	Global	Regional	Global	Regional	Global	Regional	Global
Brazil	**1.66**	2.78	3.29	**3.26**	**2.71**	7.91	**2.47**	2.80
Canada	**0.55**	0.60	**0.46**	0.73	1.43	**1.40**	0.57	**0.50**
US	0.84	**0.48**	0.65	**0.64**	2.13	**1.60**	0.69	**0.54**
France	**0.45**	0.78	**0.45**	1.04	**0.70**	1.42	**0.24**	0.78
Germany	**0.26**	0.61	**0.35**	0.85	**0.60**	1.33	**0.15**	0.58
UK	0.70	**0.41**	0.68	**0.57**	1.29	**1.05**	0.41	**0.40**
Hong Kong	0.63	**0.52**	**0.41**	0.50	**0.77**	1.27	0.56	**0.53**
Japan	**0.50**	0.64	**0.60**	0.92	**0.61**	1.17	**0.32**	0.74
Singapore	**0.76**	1.07	**0.61**	0.94	**0.84**	1.85	**0.40**	1.03

Panel C: MCP	Level factor		Slope factor		Curvature factor		Yield curve	
	Regional	Global	Regional	Global	Regional	Global	Regional	Global
Brazil	**0.78**	0.62	**0.63**	0.50	**0.84**	0.70	**0.58**	0.49
Canada	**0.67**	0.64	**0.86**	0.64	**0.59**	0.51	0.54	0.54
US	**0.70**	0.66	0.68	**0.70**	0.68	**0.70**	0.61	**0.69**
France	**0.81**	0.64	**0.79**	0.65	**0.78**	0.62	**0.77**	0.54
Germany	**0.84**	0.68	**0.78**	0.61	**0.75**	0.70	**0.71**	0.61
UK	0.68	**0.70**	**0.71**	0.69	**0.69**	0.66	**0.63**	0.61
Hong Kong	**0.76**	0.70	**0.74**	0.69	**0.76**	0.59	**0.66**	0.61
Japan	**0.55**	0.53	**0.53**	0.49	**0.61**	0.56	0.52	0.52
Singapore	**0.83**	0.72	**0.80**	0.72	**0.82**	0.59	**0.68**	0.61

Note: The table shows the mean squared error (MSE; Panel A), mean absolute error (MAE; Panel B) and the mean correct prediction (MCP; Panel C) values of the regional and global yield factor and curve models for each country. Regarding prediction performance, the model with the lowest MSE/MAE and the highest MCP is preferred (for each case this MSE/MAE/MCP value is represented in bold).

Firstly, I estimate the single-country dynamized Nelson-Siegel model for each country separately to obtain the estimated country level ($\hat{l}_{it}$), slope ($\hat{s}_{it}$) and curvature ($\hat{c}_{it}$) factors. Subsequently, by applying principal component analyses on these estimated country yield factors I extract regional and global yield factors. In general, the results show that all regional and global yield factors are important, especially Europe and Asia/Pacific exhibit strong and dominant regional yield factors. For Europe this can be linked to the existence of the Eurozone and the European Union (EU), and for the region Asia/Pacific all considered countries are comparatively developed and there is much wealth concentrated.

As a next step to examine the dynamics and interactions of the regional and global yield factors, I estimate the regional and global yield curve models. For Europe I conclude that the countries are homogeneous regarding their business cycle due to the positive correlation between the country and regional slope factor. Furthermore, the countries share an important common regional factor in their inflation rate due to a strong positive correlation between the country and regional level factor, especially for France and Germany. This relationship is less important for the UK, reflecting the importance of the Eurozone with respect to monetary policy and inflation. Besides, for Asia/Pacific all countries share an important common factor in terms of both inflation and developments in the real economic activity due to a strong positive correlation. For the region America all countries are homogeneous regarding their business cycle, but in terms of inflation there is a big difference between the developed countries Canada and the US and the less developed country Brazil, which suffers from high inflation rates.

Globally, I conclude that every country shares a common factor regarding monetary policy and inflation, except Brazil which is negatively correlated with the global level factor due to big fundamental problems in that country. Furthermore, the results concerning the slope factors indicate that when the global economy performs well, the economy of developed countries also performs well.

Finally, I examine the impact on the country yield factors by including both the regional and global yield factor in a combined model. In general, I conclude that for almost all country yield factors the regional yield factor is more important than the global yield factor. This is also confirmed by the prediction performance, because in general for almost all countries the regional yield factor outperforms the global yield factor in forecasting the country yield factors, except for the US and the UK. For the country's yield curve the regional yield factor also has a better prediction performance than the global yield factor for most of the countries, except for the US, Canada and Hong Kong.

An important limitation of my research is the limited number of countries that I use for the analysis of the regional yield factors. As a topic for further research, it would be interesting to include more than three countries for each region to obtain more reliable results. For example, I examine relative developed countries for the region Asia/Pacific, so an extension with less developed countries can be interesting. It is also possible to extend the analysis to more regions. Lastly, I investigate how country yield factors load on the regional and global yield factors, but it is also interesting to know how the regional yield factors load on the global yield factors.

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Appendix A Plots of extracted regional and global yield factors

Figure 1: Extracted regional and global level factors

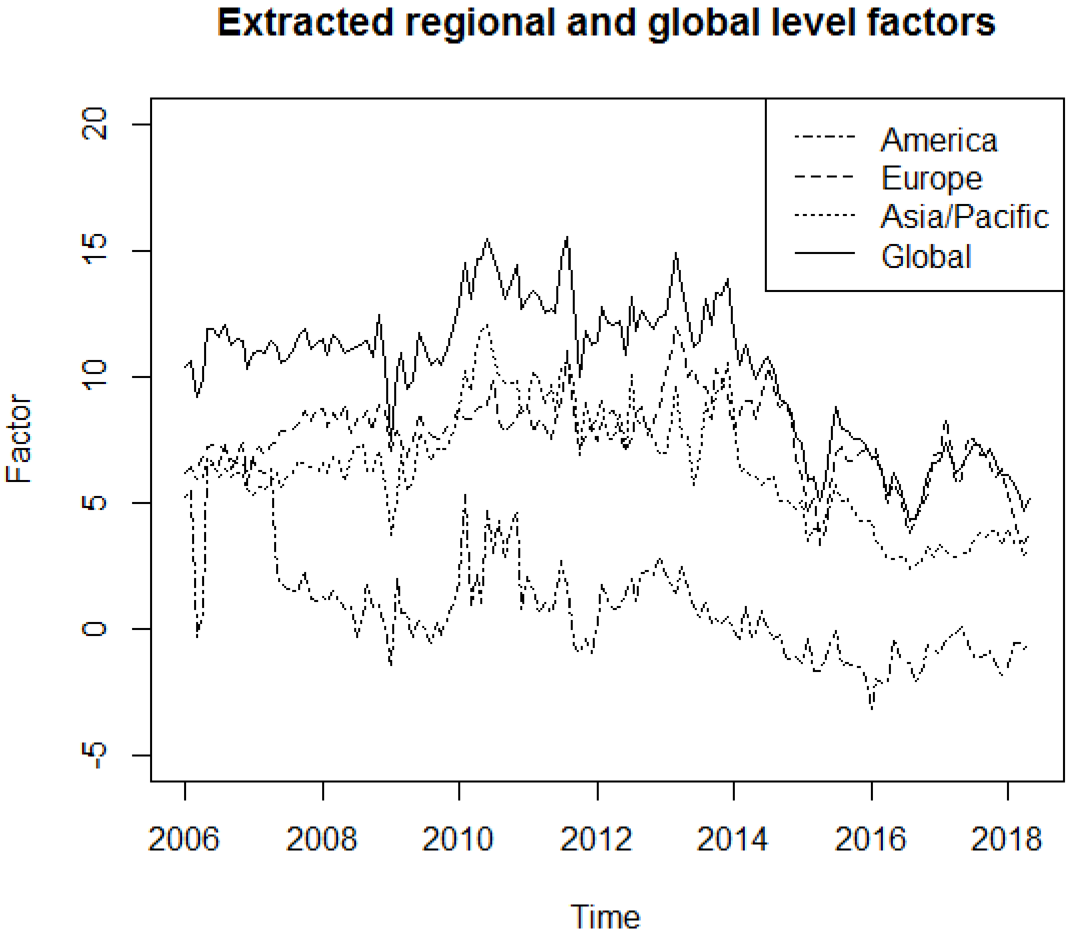

Figure 2: Extracted regional and global slope factors

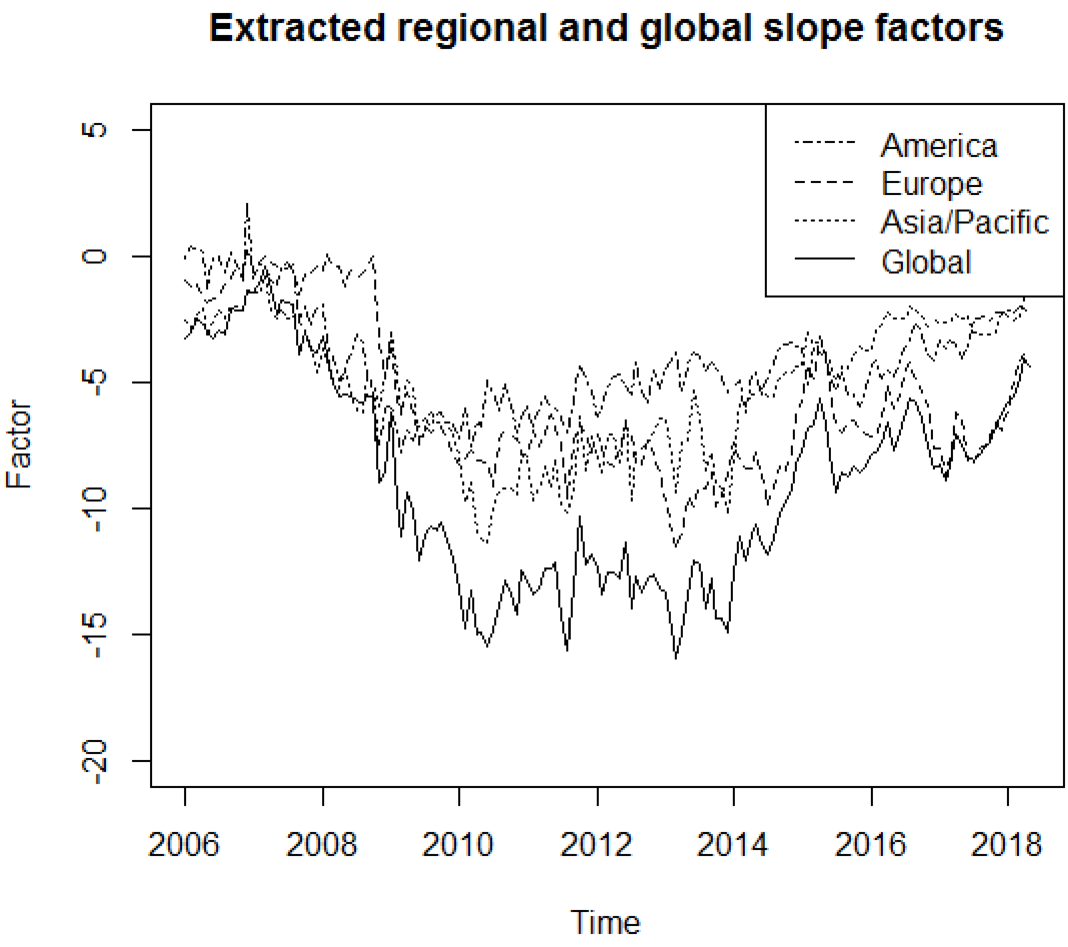

Figure 3: Extracted regional and global curvature factors