ERASMUS UNIVERSITY ROTTERDAM

Erasmus School of Economics

Master Thesis

Econometrics and Management Science
Econometrics

Bond Market Dependence during Expansions and Recessions in the Euro-area

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August 16, 2018

*The author gratefully acknowledges the advice and supervision of many individuals. Foremost, Anna Kiriliouk and Chen Zhou, for your insightful comments, enthusiasm and clear explanations. Many thanks as well to all others at DNB for their on- and off-topic support.

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Abstract

Multivariate extremes exhibit either tail dependence or tail independence. Instead of assuming tail (in)dependence upfront, we use a flexible copula model by Huser and Wadsworth (2017) to model extreme government bond losses in European countries. We find that bond losses across countries in the north exhibit tail dependence, in contrast to tail independence across countries in the south. Between northern and southern country pairs, we also observe tail independence, but the level of remaining tail dependence is higher than for pairs of southern countries. We show that tail (in)dependence coefficients across European countries significantly vary from expansions to recessions. Tail dependence of bond losses paired with Finland or Spain significantly decreases during recessions, whereas it increases for pairs involving France or the Netherlands, but to a lesser extent.

Keywords: Tail dependence, tail independence, Copula, Bond market linkages, Expansions and Recessions

1 Introduction

Economies experience booms and recessions on a national and international level. Asset markets are more volatile during crises and are prone to large losses. For example, during the financial turmoil of the 2008 crisis and the sovereign debt crisis in 2010, European asset markets often crashed together, which shows signs of dependence across these countries. The more dependent the Euro-area is, the more vulnerable it will be to a shock in a single country. These shocks occur in different asset markets, where each market crash has different consequences for individual European governments. Sovereign bonds in particular are required for a stable funding mechanism to finance all government operations. Bond prices can fluctuate drastically, which makes borrowing on the open market increasingly difficult or even impossible, such that in some cases, intervention by the central bank is required (Andrade, Breckenfelder, De Fiore, Karadi, & Tristani, 2016). In this paper, we analyse the dependence of heavy losses in government bond returns in Europe during periods of recession and expansion. Tail dependence of sovereign bond markets gives an indication of the relations between European countries that are more exposed to systemic risk. Consequently, the effectiveness of policy shaped to reduce national risks is related to the dependence on other countries. In addition, timing of policy could play an important role in its effectiveness, as dependence across countries could be affected by the state of the economy.

The challenge in modelling extremes of two or more variables is to determine the appropriate type of dependence, since most models are either tail dependent or tail independent. Tail dependence implies that there is a positive probability of a crash in one market conditional on a crash in another market as the crashes become more extreme. When this probability goes to zero, joint crashes are tail independent. However, we can still analyse the speed at which this probability of joint extreme crashes approaches zero as an indication of the remaining dependence. Using a new and flexible model, we do not specify the type of dependence prior to the analysis, which allows us to estimate the degree of tail (in)dependence from the data. This leads to the following research question:

How does tail dependence in the bond market in Europe differ during recessions and ex-

pansions?

In the latter part of this thesis, we analyse different regions in the Euro-area which have different dependence characteristics. Differences in dependence structures between and within for instance the northern and southern regions of Europe could give useful insights for policy makers. The overall picture of tail dependence helps to shape policy in both recessions and expansions and to minimise the consequences of contagion by identifying the strongest links. Therefore, we pose a second research question:

To what extent is tail dependence of the European bond market related to certain regions?

Copulas are a natural choice for this extreme value analysis, because we are interested in the dependence structure in the tail of the loss distribution. A copula is the link between the marginal distributions and a joint distribution. This is a useful tool to identify comovements between markets, because it contains all the information regarding the dependence. It is particularly useful for financial applications as it can incorporate non-normal marginals. Modelling dependence with a copula framework is favoured over the conventional Pearson correlation measure, because copulas allow for non-linear dependence and are invariant to marginal transformations (Embrechts, McNeil, & Straumann, 2002). Copulas can be used to summarise tail dependence into a tail (in)dependence coefficient, which goes beyond the linear properties of correlation and allows for a convenient comparison between groups of countries.

The copula model for the extreme observations used in this thesis is based on the model by Huser and Wadsworth (2017) (from now on HW). This model allows for a flexible tail dependence structure, since one does not need to assume tail dependence or tail independence, in contrast to previous models for copulas. Assuming an incorrect tail dependence structure can lead to a bias and erroneous applications in for instance risk management due to either over- or underestimation of joint tail risks. The HW model encompasses both structures in a single parametric model, much like the generalised extreme-value distribution.

In addition, the assumption of a static dependence structure can be misleading. For instance, Poon, Rockinger, and Tawn (2003b) shows that tail dependence tends to increase over time. This suggests that an analysis regarding different periods with a flexible model can be insightful.

The drawback of the HW model compared to a non- or semi-parametric model is that one has to assume a parametric model for the tail distribution. However, this is partly offset by the goal of this paper, which requires that there is no ex ante assumption about tail dependence. Besides, we use visualisations of the parametric fit in comparison with the non-parametric estimate to show that the HW model is able to capture the tail dependence structure.

We apply the HW model to bivariate pairs of bond losses in two regimes, expansions and recessions. This characterises the parametric differences between the two regimes and the HW model allows for statistical comparison between the dependence parameters across these regimes. Both regimes have their own parameters for the dependence measure and the copula model. We use different copula models, including the asymmetric logistic model, which is an extension of the HW symmetric logistic model. A test statistic is developed to examine changes in tail (in)dependence, using the asymptotic normality of the estimators. We test whether the bilateral tail dependence structure significantly changes between expansions and recessions. Afterwards, we use a network analysis to identify groups of countries that can be identified as tail dependent or tail independent during either expansions or recessions. The aim is to identify regions in Europe that are most likely to experience joint collapses. To analyse the extent to which this tail dependence poses a risk within certain regions of Europe, we test whether the tail (in)dependence coefficients differ for each group of tail dependent and independent countries across expansions and recessions.

We identify two regions in Europe with different tail dependence structures. Northern Europe, including the Netherlands, Finland, Belgium, Germany, France, and Austria exhibit tail dependence in expansions as well as recessions. The only exception is Finland, which exhibits significantly lower tail dependence during recessions. The countries that experience the strongest tail dependence increases with other countries resulting from a recession, are France and the Netherlands. Other than Italy and Spain, southern Europe, including Greece, Ireland, Italy, Portugal and Spain are tail independent. This is also observed for north-south relations, although this tail dependence is stronger than merely the south. Recessions only significantly affect the tail dependence of Spain with other countries, which decreases. These findings are robust to model variations.

The outline of the remainder of this paper is as follows. First, the relevant literature is discussed on the augmented methodology, as well as the linkages between bond markets. Section 3 describes the methodology of Huser and Wadsworth (2017) for bivariate returns and the implementation of the regime switching method. A simulation is performed in section 4. This is followed by an application into the European bond market in Section 5 and 6. The final section concludes.

2 Related literature

The framework in this paper builds on the work of Huser and Wadsworth (2017) to model tail dependence of sovereign debt between European countries. Thereby it relates to three strands of literature. First of all, it is connected to the modelling of tail dependence. The motivation for the application of the HW model originates from the issue whether data is tail dependent or tail independent, which is far from clear in practice. The HW model is one of the first to implement a flexible tail dependence structure for a spatial process. The model has similarities to Wadsworth, Tawn, Davison, and Elton (2017) and Huser, Opitz, and Thibaud (2017), who also try to encompass tail (in)dependence. However, these models are more complex, less flexible in the parameter space of the dependence measure and less flexible in the choice of the parametric distribution. Huser et al. (2017) in particular, develop the building blocks for the HW model, namely the combination of a random variable with a Gaussian model to achieve a smooth transition between tail dependence and tail independence. However, the contribution of HW allows for a more diverse selection of distributions, which are not necessarily Gaussian. Parametric models beyond the Gaussian are the broad range of max-stable models (Coles & Tawn, 1991). We evaluate multiple parametric models and test tail dependence in different states of the world.

Secondly, this thesis relates to the literature on dependence changes. Poon et al. (2003b) were the first to indicate the dynamics of tail dependence. In their paper, they analyse five global indices and study their dependence properties. They split the sample into subperiods, keeping the percentage of threshold exceedances relatively stable, and find evidence for complex non-stationary tail dependence between CAC (France), DAX (Germany) and FTSE (UK), which tends to increase over time. However, significance of the tail dependence coefficients is computed separately for each time period and no significance test is performed to test whether this increase in dependence is significant. We will use a parametric model that is able to test this dependence relation. In a related paper, they found evidence of tail independence in certain financial markets, in particular for stock returns (Poon, Rockinger, & Tawn, 2003a). Heteroskedasticity is found to be a major source of tail dependence and when filtered out, the resulting tail dependence structure often changes to independence. A GARCH filter was also used in Poon et al. (2003b). This motivates the use of a GARCH filter for the return series before the analysis of tail dependence.

A more recent study by Castro-Camilo, de Carvalho, and Wadsworth (2018) indicates a similar trend of increasing tail dependence over time for financial markets. Using the angular density of a number of extreme value distributions, they introduce a regression model that enables inference of tail dependence given a covariate. Again, no specific test of significance is performed, but now all available data is used to show that dependence takes a different form over time. They conclude that tail dependence increased over time for the European stock markets by comparing the dependence structure at a particular point in time, specifically after significant European financial events. In contrast, our method uses a more simple setup to compare dependence and focusses on the bond market, which might behave differently across countries than the stock market.

As a result of this apparent instability of dependence, recent literature explores structural breaks using copulas, where the copula parameter is allowed to vary over time, the functional form is time-varying, or both (Manner & Reznikova, 2012). As a result of their survey, they recommend a regime switching model to capture time dynamics for copulas, due to its performance and relative simplicity. An example is the use of Markov switching copulas by Rodriguez (2007), who allows a specific regime to determine the functional form of the copula. He shows evidence for changing dependence structures during periods of crises, due to changes in the regime and the related copula form. Wang, Wu, and Lai (2013) also stress the findings in the existing literature that a copula framework without temporal changes may not be appropriate to model financial returns. They conclude this based on a copula switching model that examines the dependence between stock and currency markets. Our approach differs in two ways. Firstly, we do not assume tail (in)dependence in any regime. This differs from Rodriguez (2007), because in that case, regimes have a particular copula structure with fixed tail (in)dependence. Secondly, our approach can be used to test for the significance of tail dependence changes across periods of expansion and recession.

Finally, this thesis adds to the bond market literature, with a specific focus on dependence. Until the paper of Hartmann, Straetmans, and Vries (2004), not much was known about bond market linkages. Using tail dependence coefficients from a non-parametric model on weekly bond returns, they show that there are relations between simultaneous crashes in international bond markets, which are less frequent than stock market cocrashes. Using a non-parametric test, most pairs of countries seem to be tail dependent and from this, they calculate expected number of market crashes. They do not find a general geographical pattern between extreme linkages, but show that occurrences of domestic and international joint collapses of asset markets are surprisingly similar due to financial integration. The consequences of deeper financial integration also increases the spillover of interest spreads after bad news (Beetsma, Giuliodori, De Jong, & Widijanto, 2013; Mink & De Haan, 2013). This seems to affect the entire European area, but to a lesser extent for northern countries affected by the south.

Reboredo and Ugolini (2015) use a systemic risk measure, CoVaR, in combination with copulas to estimate the tail dependence of the European bond market before and after the onset of the European sovereign debt crisis. The analysis is performed with six copula models, ranging from tail dependent to independent, where the model selection is based on the AIC criterion. They find that systemic risk before this crisis was stronger in the EU as a whole. After the onset of the debt crisis, markets decoupled and showed lower overall dependence. The systemic risk of Greek debt appears to have less detrimental effect for northern European countries but it did particularly influence Portuguese bonds. Results from Philippas and Siriopoulos (2013), using a semi-parametric model, confirm that probabilities of joint collapses with Greece, because of an overall type of contagion, appear to be low during recessions.

In summary, the contribution of this paper to the literature is a sound analysis into the tail dependence structure of the European bond market without ex ante assumptions on dependence, normality or linearity. On top of that, we develop a test for changes in tail dependence across multiple regimes.

3 Methodology

This section presents the method applied in this paper, developed by Huser and Wadsworth (2017). Instead of a spatial application, we use it to model bivariate financial returns in two regimes to test for temporal dependence changes.

We are interested in the distribution of a crash in a specific bond market, given the collapse in another country. To analyse these properties of extremal dependence, let (Y_1, Y_2) be a random vector denoting bond returns of two countries. For the ease of notation, we take the negative of the returns, such that extremes in the right tail correspond to large losses.

The information contained in (Y_1, Y_2) not only reflects the dependence, but also the behaviour of the marginal distributions. Marginal transformations are not related to the dependence structure. Therefore, we consider models for the copula of the joint distribution, which is unaffected by monotone increasing transformations of the marginals. Let $(U_1, U_2) = \big(F_1(Y_1), F_2(Y_2)\big)$, where F_1 and F_2 are the marginal distribution functions of Y_1 and Y_2 respectively. The random vector (U_1, U_2) can be represented on a unit square. The copula links these uniform marginals to the joint distribution F of (Y_1, Y_2) (Sklar, 1959)

$$C_Y(u_1, u_2) = F\big(F_1^{-1}(u_1), F_2^{-1}(u_2)\big), \qquad u_1, u_2 \in [0, 1]. \tag{1}$$

To model the extreme observations of the bond market losses, we use a parametric approach following the HW model. Let $X = (X_1, X_2)$ be a random vector, for which the copula of X only applies to the extreme observations in Y, such that

$$C_Y(u_1, u_2) = C_X(u_1, u_2) \qquad \text{for } u_1 > u_1^* \text{ or } u_2 > u_2^*, \tag{2}$$

where u_1^*, u_2^* are extremal thresholds. Note that this is a model for the tail of Y, described by the copula of X. A copula model exhibits a certain form of tail dependence, characterised by two metrics: the tail dependence coefficient and the tail independence coefficient, for respectively tail dependent and tail independent models. These two metrics summarise the tail (in)dependence structure in a two numbers.

3.1 Tail (in)dependence coefficients

Dependence for multivariate extremes occurs at the top-right corner of the copula distribution. When extremes tend to occur simultaneously, X is called tail dependent. A well known metric for the strength of tail dependence is the tail dependence coefficient

$$\chi_X = 2 - \lim_{p \to 0} \frac{1 - C_X(1-p, 1-p)}{p}. \tag{3}$$

Equation (3) can be written as $\lim_{u \to 1} P(F_1(X_1) > u \,|F_2(X_2) > u)$ where F_1 and F_2 are the marginals of X. This is the probability of a crash in a particular country given a crash in another country. Since it is a probability, it lies between 0 and 1.

If $\chi_X = 0$, this probability of a conditional co-crash is zero. In this case, the countries are tail independent. Nevertheless, there is still a certain level of residual dependence in this case. Therefore, we consider the tail independence coefficient, which measures this remaining dependence. We introduce the tail independence coefficient for random vectors with standard Pareto marginal distribution functions. Suppose $W = (W_1, W_2)$ is a random vector with standard Pareto margins and assume

$$P(W_1 > w, W_2 > w) = L_W(w) w^{-1/\eta_W}, \qquad w \geq 1, \tag{4}$$

where $0 < \eta_W < 1$ and L_W is slowly varying at infinity, which means that $\lim_{w \to \infty} \frac{L_W(aw)}{L_W(w)} \to 1 \; \forall \; a > 0$. Then the coefficient of tail independence η_W summarises the degree to which dependence vanishes in the limit (Ledford & Tawn, 1996).

Note that standard Pareto margins are not essential to define this model of tail independence, because η_W is related to the copula instead of the marginal distributions. We can derive this using the standard Pareto distribution function $P(W \leq w) = 1 - \frac{1}{w}$ and

$$\begin{aligned}
C_W(1-p, 1-p) &= P\left(1 - \frac{1}{W_1} \leq 1-p, 1 - \frac{1}{W_2} \leq 1-p\right) \\
&= P\left(W_1 \leq \frac{1}{p}, W_2 \leq \frac{1}{p}\right) \\
&= P\left(W_1 \leq \frac{1}{p}\right) + P\left(W_2 \leq \frac{1}{p}\right) - P\left(W_1 \leq \frac{1}{p} \ or \ W_2 \leq \frac{1}{p}\right) \\
&= (1-p) + (1-p) - \left(1 - P\left(W_1 > \frac{1}{p}, W_2 > \frac{1}{p}\right)\right) \\
&= 1 - 2p + L_W\left(\frac{1}{p}\right)\left(\frac{1}{p}\right)^{-1/\eta_W}.
\end{aligned} \tag{5}$$

So, η_W is a characteristic of the copula C_W, irrespective of the form of the marginal distributions. This can be linked to the tail dependence coefficient as follows

$$\chi_W = \lim_{p \to 0} 2 - \frac{1 - C_W(1-p, 1-p)}{p} = \lim_{p \to 0} L_W\left(\frac{1}{p}\right) p^{1/\eta_W - 1}. \tag{6}$$

Since $\eta_W < 1$, when $p \to 0$, $\chi_W \to 0$, which indicates the tail independence of W. The exponent $\frac{1}{\eta_W} - 1$ indicates how fast the conditional probability tends to zero. Consider for instance the case of independence, where $C_W(1-p, 1-p) = (1-p)^2$ and $\chi_W = p$. This implies that $\eta_W = \frac{1}{2}$.

Another example of the tail independence coefficient lies in its relation with the correlation coefficient for a bivariate normal random vector. A normally distributed random vector is tail independent when its correlation coefficient $\rho < 1$. It can be derived that the tail independence coefficient in that case equals $\eta = \frac{1+\rho}{2}$.

Tail dependence can thus be summarised depending on the tail dependence structure. For tail independent variables, $\chi_X = 0$ and $0 \leq \eta_X < 1$. For tail dependent variables, $0 < \chi_X \leq 1$ and $\eta_X = 1$. Higher values of χ_X or η_X imply stronger dependence at extreme levels.

3.2 The model

Recall that the tail dependence structure in Y is modelled via that of X as in Equation (1). The HW model for $X = (X_1, X_2)$ is given by

$$X_j = R^{\delta} W_j^{1-\delta}, \qquad \delta = [0, 1]\ , j = 1, 2, \tag{7}$$

where R is a univariate standard Pareto random variable, δ is the dependence parameter and $W = (W_1, W_2)$ is a random vector with standard Pareto margins. W is tail independent with a coefficient of tail independence η_W as defined in Equation (4). The construction of Model (7) uses the parameter δ to determine the type of extremal dependence, where the two extremes occur at the boundary points of the parameter space. If $\delta > 0.5$, X exhibits tail dependence with tail dependence coefficient

$$\chi_X = \frac{2\delta - 1}{\delta} \,\mathrm{E}\left(\min(W_1, W_2)^{\frac{1-\delta}{\delta}} \right), \tag{8}$$

as derived in Huser and Wadsworth (2017). If $\delta < 0.5$, (X_1, X_2) is tail independent and the coefficient of tail independence is characterised by

$$\eta_X = \begin{cases} 1, & \text{if } \delta \geq \frac{1}{2} \\ \frac{\delta}{1-\delta}, & \text{if } \frac{\eta_W}{1+\eta_W} < \delta < \frac{1}{2} \\ \eta_W, & \text{if } \delta \leq \frac{\eta_W}{1+\eta_W} \end{cases} . \tag{9}$$

The model shows appealing dependence properties by allowing for both types of tail dependences structures in a single model. The flexibility of this model stems from W, for which we can consider multiple distributions. We examine a Gaussian model and three inverted max-stable models: logistic, asymmetric logistic and Dirichlet. The main difference across these models is the coefficient of tail independence of W and the different shapes of the copula, which directly affect the aforementioned coefficients of tail (in)dependence of X.

3.2.1 Gaussian model

Let $Z = (Z_1, Z_2)$ be a standard normally distributed random variable with correlation coefficient ρ. Let

$$W_j = \frac{1}{1 - \Phi(Z_j)}, \tag{10}$$

where Φ is the standard normal distribution. Then W has a Gaussian copula with standard Pareto margins and it has a coefficient of tail independence $\eta_W = \frac{1+\rho}{2}$.

The Gaussian model is symmetric in its dependence structure. If $\delta > 0.5$, there is no explicit form for χ_X, but it can be estimated through numerical integration. When $\delta < 0.5$, X is tail independent with a tail independence coefficient

$$\eta_X = \begin{cases} 1, & \text{if } \delta \geq \frac{1}{2} \\ \frac{\delta}{1-\delta}, & \text{if } \frac{1+\rho}{3+\rho} < \delta < \frac{1}{2} \\ \frac{1+\rho}{2}, & \text{if } \delta \leq \frac{1+\rho}{3+\rho} \end{cases}$$

3.2.2 Inverted max-stable models

For this class of models, we make use of a max-stable random vector, which is constructed as follows. Consider the i.i.d. copies $\big((Y_{11}, Y_{12}), \ldots, (Y_{n1}, Y_{n2})\big)$ of a random vector $Y = (Y_1, Y_2)$. Let $M_n = (M_{n1}, M_{n2})$ be a random vector with componentwise sample maxima $M_{nj} = \max(Y_{1j}, \ldots, Y_{nj})$ for $j = 1, 2$. If there exist normalising constants $a_{nj} > 0$ and $b_{nj} \in \mathbb{R}$ for $j = 1, 2$ such that

$$P\left(\frac{M_{n1} - b_{n1}}{a_{n1}} < z_1, \frac{M_{n2} - b_{n2}}{a_{n2}} < z_2\right) \to G(z_1, z_2), \qquad n \to \infty,$$

for each $z_1, z_2 \in \mathbb{R}$, then G is called a multivariate generalised extreme value distribution or max-stable. If G has standard Fréchet marginals, we can apply a logarithmic transformation to derive the exponent function

$$V(z_1, z_2) = -\log G(z_1, z_2). \tag{11}$$

For necessary and sufficient conditions on V, see de Haan and Resnick (1977). In particular, V is homogeneous of degree -1, such that $V(a,a) = a^{-1}V(1,1)$ for $a \in \mathbb{R}$.

To obtain an inverted max-stable copula, let

$$W_j = \frac{1}{G_j(Z_j)} = \exp\left(\frac{1}{Z_j}\right), \tag{12}$$

where G_j are the standard Fréchet marginal distributions of Z_j. As a result, W has standard Pareto distributed marginals and it has an inverted max-stable copula with a coefficient of tail independence that depends on the exponent function. To derive this, we first consider the copula of W

$$\begin{aligned} P(W_1 > w, W_2 > w) &= P\left(\exp\left(\frac{1}{Z_1}\right) > w, \exp\left(\frac{1}{Z_2}\right) > w\right) \\ &= P\left(Z_1 < \frac{1}{\log(w)}, Z_2 < \frac{1}{\log(w)}\right) \\ &= \exp\left(-V\left(\frac{1}{\log(w)}, \frac{1}{\log(w)}\right)\right) = \exp\big(-\log(w)\ V(1,1)\big) \\ &= w^{-V(1,1)}, \end{aligned} \tag{13}$$

where $w > 1$ and we use the fact that V is an homogeneous function. By comparing this result with Equation (4), we get $\eta_W = \frac{1}{V(1,1)}$. We analyse different models with parametric exponent functions $V(z_1, z_2; \boldsymbol{\theta})$.

For the logistic model (Gumbel, 1960) the exponent function is

$$V(z_1, z_2; \theta) = (z_1^{-\frac{1}{\theta}} + z_2^{-\frac{1}{\theta}})^{\theta}, \qquad \theta \in (0,1]. \tag{14}$$

In this case, $\eta_W = 2^{-\theta}$.

An extension compared to HW is the asymmetric logistic distribution, which extends the symmetric model with two additional parameters. This model was introduced by Tawn (1990) and has exponent function

$$V(z_1, z_2; \psi_1, \psi_2, \theta) = \frac{1-\psi_1}{z_1} + \frac{1-\psi_2}{z_2} + \left(\left(\frac{\psi_1}{z_1}\right)^{\frac{1}{\theta}} + \left(\frac{\psi_2}{z_2}\right)^{\frac{1}{\theta}}\right)^{\theta}, \qquad \theta \in (0,1];\ \psi_1, \psi_2 \in [0,1]. \tag{15}$$

It simplifies to the symmetric model if $\psi_1 = \psi_2 = 1$. Its coefficient of tail independence equals

$$\eta_W = \frac{1}{2 - \psi_1 - \psi_2 + (\psi_1^{\frac{1}{\theta}} + \psi_2^{\frac{1}{\theta}})^\theta}.$$

The Dirichlet model (Coles & Tawn, 1991) is asymmetric and the corresponding exponent function is

$$V(z_1, z_2; \alpha, \beta) = \frac{1}{z_1}\left(1 - \text{Beta}\left(\frac{\alpha z_1}{\alpha z_1 + \beta z_2}; \alpha + 1, \beta\right)\right) + \frac{1}{z_2}\text{Beta}\left(\frac{\alpha z_1}{\alpha z_1 + \beta z_2}; \alpha, \beta + 1\right), \quad \alpha, \beta > 0. \tag{16}$$

The coefficient of tail independence then becomes

$$\eta_W = \left(1 - \text{Beta}\left(\frac{\alpha}{\alpha + \beta}, \alpha + 1, \beta\right) + \text{Beta}\left(\frac{\alpha}{\alpha + \beta}, \alpha, \beta + 1\right)\right)^{-1}.$$

For all inverted max-stable models $\eta_W \in [0.5, 1)$, which implies that these models cannot exhibit negative tail dependence. If $\delta > 0.5$ the bivariate extremes are tail dependent with tail dependence coefficient

$$\chi_X = \frac{2\delta - 1}{1 - (1 - \delta)(1 + \eta_W)}. \tag{17}$$

Consequently, for inverted max-stable models, χ_X is explicit and depends on both δ and η_W. If $\delta < 0.5$, X exhibits tail independence, $\chi_X = 0$ and the coefficient of tail independence is determined through Equation (9) combined the corresponding values of η_W and δ.

Like the Gaussian model, the logistic model is symmetric in its tail dependence structure. The asymmetric logistic and the Dirichlet model may have a better fit for pairs with more asymmetric tail dependence structure.

3.3 Estimation

Let (Y_{1t}, Y_{2t}) be observations of pairs of daily bond losses over time with $t = 1 \ldots T$. This sample is then transformed to uniform marginals as follows. Since the marginal distribution functions are unknown, we will use the empirical cumulative distribution function independently for both vectors of observations, which transforms each margin through their ranks. This process is modified such that the uniform variables never reach 1:

$$(U_{1t}, U_{2t}) = \left(\frac{\text{rank}(X_{1t})}{n+1}, \frac{\text{rank}(X_{2t})}{n+1} \right). \tag{18}$$

We fit the extreme observations (U_{1t}, U_{2t}) which lie above a certain threshold (u_1^*, u_2^*) to the copula of X, where X follows the HW model. This means that certain observations have to be censored to avoid a bias caused by the influence of non-extreme data points. All observations enter the likelihood, but in different forms depending on the location of the observation. Figure 1 displays the censoring in two dimensions. Since we use a copula, the marginals are uniform and the bivariate joint distribution can be displayed on a unit square. Region 1 contains the extreme observations above the thresholds u_1^* and u_2^*. These observations contribute fully to the model and require no censoring. Regions 2 and 3 contain observations where only one variable is above the threshold, and thus considered extreme. The censoring pushes the observation in region 3 (4) to the boundary u_2^* (u_1^*), which then represents the distribution of the entire area. Hence the censored observations are of the form $\big(\max(U_{1t}, u_1^*), \max(U_{2t}, u_2^*)\ \big)$. Finally, observations in region 4 do not exceed the thresholds in any dimension. They enter the likelihood at the boundary values (u_1^*, u_2^*), which represents the cumulative copula distribution over region 4.

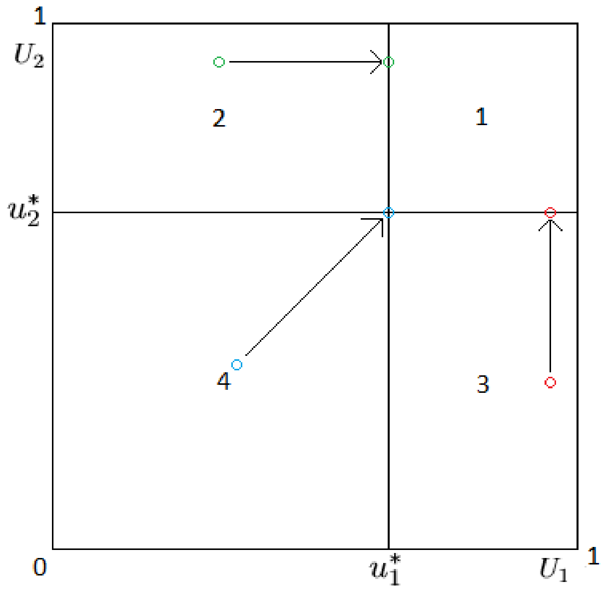

Figure 1: Regions of the censored likelihood

We use the log likelihood as defined in HW, with T, the number of observations. We can then write

$$L(\boldsymbol{\psi}) = \sum_{t=1}^{T} \log\big(L_t(\boldsymbol{\psi})\big), \tag{19}$$

where $\boldsymbol{\psi} = (\delta, \boldsymbol{\theta})$ and $\boldsymbol{\theta}$ are the parameters of the exponent function. So, according to Figure 1, the likelihood can be written for three categories in different ways:

$$L_t(\boldsymbol{\psi}) = \begin{cases} c(U_{1t}, U_{2t}; \boldsymbol{\psi}), & \text{region 1} \\ C_{\mathcal{J}_t}\big(\max(U_{1t}, u_1^*), \max(U_{2t}, u_2^*); \boldsymbol{\psi}\big), & \text{region 2 or 3} \\ C(u_1^*, u_2^*; \boldsymbol{\psi}), & \text{region 4} \end{cases}$$

where $\mathcal{J}_t$ denotes the set of observations with at least one threshold exceedance. The copula densities are described in Section 3.1 of Huser and Wadsworth (2017). The maximum likelihood estimate $\hat{\boldsymbol{\psi}}$ converges to a normal distribution with a speed of convergence based on the number of observations k that exceed the thresholds (region 1, 2 and 3) as $n \to \infty$,

$$\sqrt{k}(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}) \quad \to \quad \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}_{\psi}) \tag{20}$$

The parametric model for W is chosen based on the AIC criterion, such that the copula of X has the best fit with respect to the amount of parameters:

$$AIC = 2p - 2L(\boldsymbol{\psi})$$

where p is the number of parameters and $L(\boldsymbol{\psi})$ is the censored log likelihood. A lower AIC implies a better fit.

3.4 Testing dependence and regime switches

To test for the tail dependence structure, we use the asymptotic normality of the maximum likelihood estimator for the parameter δ. We test $H_0 : \delta = \frac{1}{2}$ versus $H_a : \delta < \frac{1}{2}$ or $H_a : \delta > \frac{1}{2}$ for respectively tail independent and tail dependent countries. We conduct this test in two subsamples, namely expansions and recessions. This means that we can create four categories of either similar dependence structures (TD-TD, TI-TI) or changing dependence structures (TD-TI, TI-TD) from expansions to recessions. We then proceed to test whether the change in the value of δ is significant.

The parameter δ is particularly useful in the context of tail dependence versus tail independence, and not so much the degree of dependence. This is because tail dependence is not solely determined by the value of δ, but also by the copula parameters. However, this test gives a first indication into the behaviour of the tail dependence across the regimes. We will incorporate the change in the copula parameters when we test for changes in the coefficient of tail (in)dependence.

Let δ_r be the dependence parameter in the two regimes $r = 1, 2$. The hypothesis of similar tail dependence structures is $H_0 : \delta_1 = \delta_2$. Let k_r be the amount of observations above the thresholds in each regime. Under the null hypothesis, it holds that

$$\hat{\delta}_1 - \hat{\delta}_2 \quad \overset{d}{\approx} \quad \mathrm{N}\left(0, \frac{\sigma_1^2}{k_1} + \frac{\sigma_2^2}{k_2}\right), \tag{21}$$

where σ_r^2 is the corresponding variance of $\hat{\delta}_r$. If the parameter δ is significantly different during recessions in contrast to expansions, there are again four possible outcomes. One possibility is a significant change in the value of δ with a different or similar tail dependence structure. The value of δ may also change without altering the tail dependence structure.

To test for tail dependence changes, we examine changes in χ_X and η_X. They are both functionals of the parameters δ and $\boldsymbol{\theta}$. Theoretically, based on the Cramer's delta method, the asymptotic distributions of $\hat{\chi}_X$ and $\hat{\eta}_X$ are both normal. Specifically, when we apply a function h

$$\sqrt{k_r}\big(h(\hat{\boldsymbol{\psi}}_r) - h(\boldsymbol{\psi}_r)\big) \xrightarrow{p} \mathrm{N}\big(\mathbf{0}, \nabla h(\boldsymbol{\psi}_r)' \boldsymbol{\Sigma}_{\boldsymbol{\psi}_r} \nabla h(\boldsymbol{\psi}_r)\big). \tag{22}$$

Calculating the gradient can become arduous for the Dirichlet model, so we use a parametric bootstrap instead of direct inference on the coefficient of tail (in)dependence.

The hypothesis of equal dependence is $H_0 : \chi_{X,1} = \chi_{X,2}$ versus $H_a : \chi_{X,1} \neq \chi_{X,2}$ for tail dependent countries and $H_0 : \eta_{X,1} = \eta_{X,2}$ versus $H_a : \eta_{X,1} \neq \eta_{X,2}$ for countries that are tail independent. We obtain 1000 samples of $\boldsymbol{\psi}_r$ by simulating from a normal distribution with mean $\hat{\boldsymbol{\psi}}_r$ and variance $\frac{1}{k_r}\hat{\boldsymbol{\Sigma}}_r$. With these parameters, we calculate the coefficients of tail (in)dependence and test the difference between an expansion and a recession per country pair, using an asymptotically normal test

$$T_d = \frac{\hat{\chi}_{X,1} - \hat{\chi}_{X,2}}{\sigma[\hat{\chi}_{X,1} - \hat{\chi}_{X,2}]}, \qquad T_i = \frac{\hat{\eta}_{X,1} - \hat{\eta}_{X,2}}{\sigma[\hat{\eta}_{X,1} - \hat{\eta}_{X,2}]}, \tag{23}$$

where the denominator denotes the standard deviation of the difference in the bootstrap sample $\hat{\chi}_{X,1} - \hat{\chi}_{X,2}$ and the same applies to the statistic for η_X.

4 Simulation

To examine whether the proposed models can be estimated correctly we perform simulations. Given that HW performed simulations for the Gaussian model, we do simulations for the inverted max-stable models, including the novel asymmetric logistic model.

We first simulate from the HW copula with the asymmetric logistic model for W. Figure 2 shows the results for the parameter δ with shape parameters $\phi_1 = 0.25$, $\phi_2 = 0.75$ and dependence parameter $\theta = 0.4$. The estimation of the parameter δ in this model performs well, especially in the tail dependent case ($\delta > \frac{1}{2}$). For lower values of δ, there is a higher variance and a downward bias. This is comparable to simulation results in Huser and Wadsworth (2017) of the Gaussian model. Since $\eta_W > 0.5$ for inverted max-stable models, Equation (9) implies that η_X will be the same for $\delta < \frac{1}{3}$. Different values for the dependence parameter θ were considered. As a result, lower values of θ (stronger dependence) result in a better fit for the parameter δ. Especially when $\theta > 0.6$, estimates of tail independent values for the parameter δ become more volatile and the downward bias increases due to the low dependence.

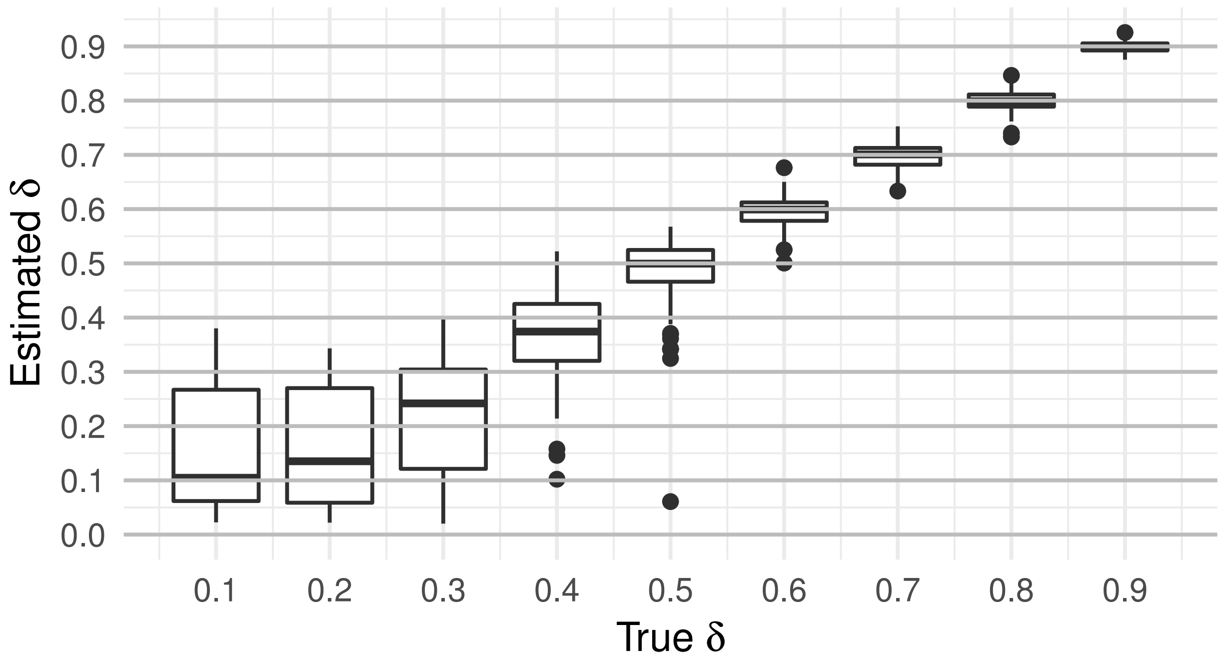

Figure 2: Boxplots for $\delta = 0.1 \ldots 0.9$ simulated from the asymmetric logistic model with parameters $\phi_1 = 0.25$, $\phi_2 = 0.75$, $\theta = 0.4$. Per value of δ, 100 simulations are performed, with $n = 1000$ observations. The solid grey lines display the true values of δ. When $\delta = 0.1 \ldots 0.4$, $\eta_X = 0.57, 0.57, 0.57, 0.67$. For $\delta = 0.6 \ldots 0.9$, $\chi_X = 0.53, 0.75, 0.87, 0.94$.

The estimation of the copula parameters is more variable. Figure 3 shows the boxplots for the three parameters. The estimation of all parameters is better for lower values of θ (stronger dependence), but the bias of the parameters ϕ_1 and ϕ_2 is larger than the bias of the parameter δ for any value of θ. Also, the bias of the parameter θ seems to be affected by the value of δ. This effect is persistent for multiple values of θ in the simulated distribution.

We interpret this observed effect as follows. Both parameters δ and θ contribute to the degree of dependence and they appear to counteract, i.e. when the value of δ is low (high), the estimate of θ is high (low). This should be taken into account in the empirical analysis, since this could also lead to a bias of the tail (in)dependence coefficients.

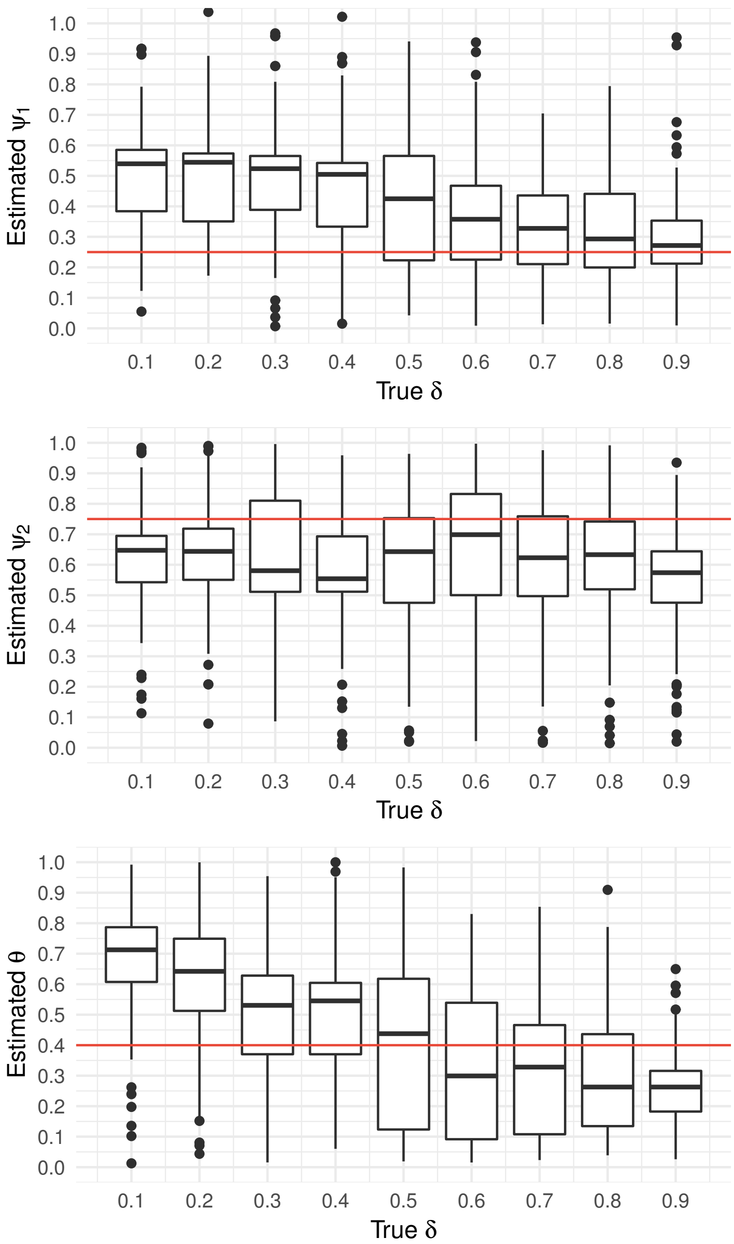

Figure 3: Boxplots for $\psi_1 = 0.25$, $\psi_2 = 0.75$ and $\theta = 0.4$ simulated from the asymmetric logistic model with $\delta = 0.1, \ldots, 0.9$. Per value of δ, 100 simulations are performed, with $n = 1000$ observations. The solid red lines display the true values of the parameters.

Figure 4 presents the simulation for the (symmetric) logistic model. Estimation for this model with a single parameter does not show a bias over the range of the parameter δ. The only effect of a larger value of δ is a wider distribution around the true value of θ, due to the strong dependence. This is also observed for the Gaussian parameters in Huser and Wadsworth (2017). The volatility increases for simulations with larger values of the parameter θ.

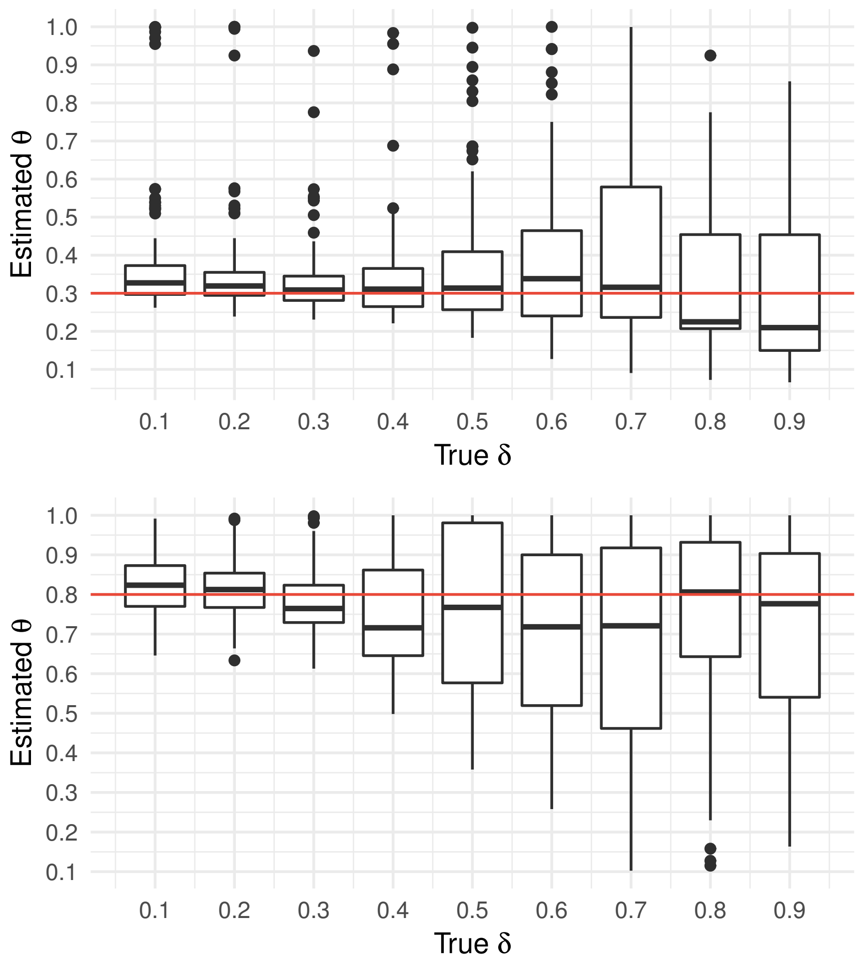

Figure 4: Boxplots for 2 simulations of the logistic model with $\theta = 0.3$ (top) and $\theta = 0.8$ (bottom). It is simulated from the symmetric logistic model with $\delta = 0.1, \ldots, 0.9$. Per value of δ, 100 simulations are performed, with $n = 1000$ observations. The solid red lines display the true values of the parameters.

Finally, we analyse the Dirichlet model. Figure 5 shows the results for the parameter δ and Figure 6 gives the results for the copula parameters. We simulate with shape parameters $\alpha = 0.15$ and $\beta = 0.75$, such that we obtain similar tail (in)dependence coefficients as in the asymmetric logistic model. The estimates of the parameter δ are slightly more variable than in the asymmetric logistic model, but are estimated relatively well. Estimates of lower values of δ are the most volatile.

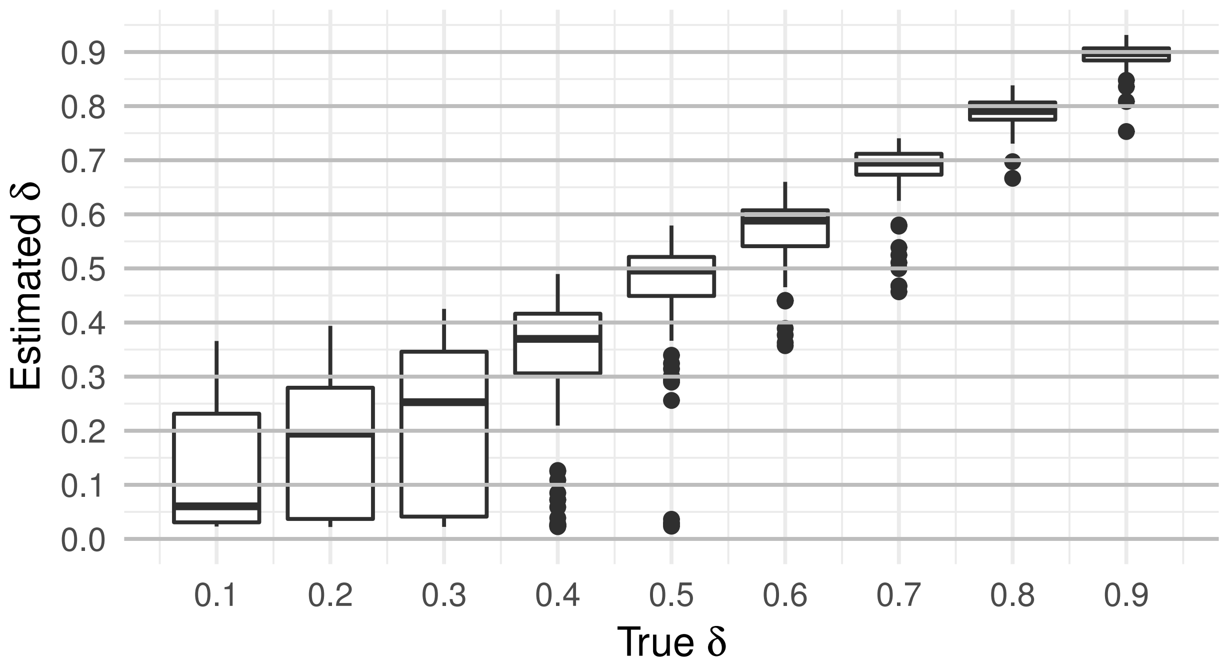

Figure 5: Boxplots for $\delta = 0.1 \ldots 0.9$ simulated from the Dirichlet model with parameters $\alpha = 0.15$ and $\beta = 0.75$. Per value of δ, 100 simulations are performed, with $n = 1000$ observations. The solid grey lines display the true values of δ. When $\delta = 0.1 \ldots 0.4$, $\eta_X = 0.57, 0.57, 0.57, 0.67$. For $\delta = 0.6 \ldots 0.9$, $\chi_X = 0.54, 0.76, 0.88, 0.95$.

Estimation of the parameters in the Dirichlet model is less accurate than that for the parameter δ. Although estimation of the a parameter α is good for the tail independent case, increasing the value of δ increases its bias and variance. Estimates of the parameter β are biased downwards and volatile. This pattern is unrelated to the value of δ. Finally, we observe that many outliers are present for both parameters.

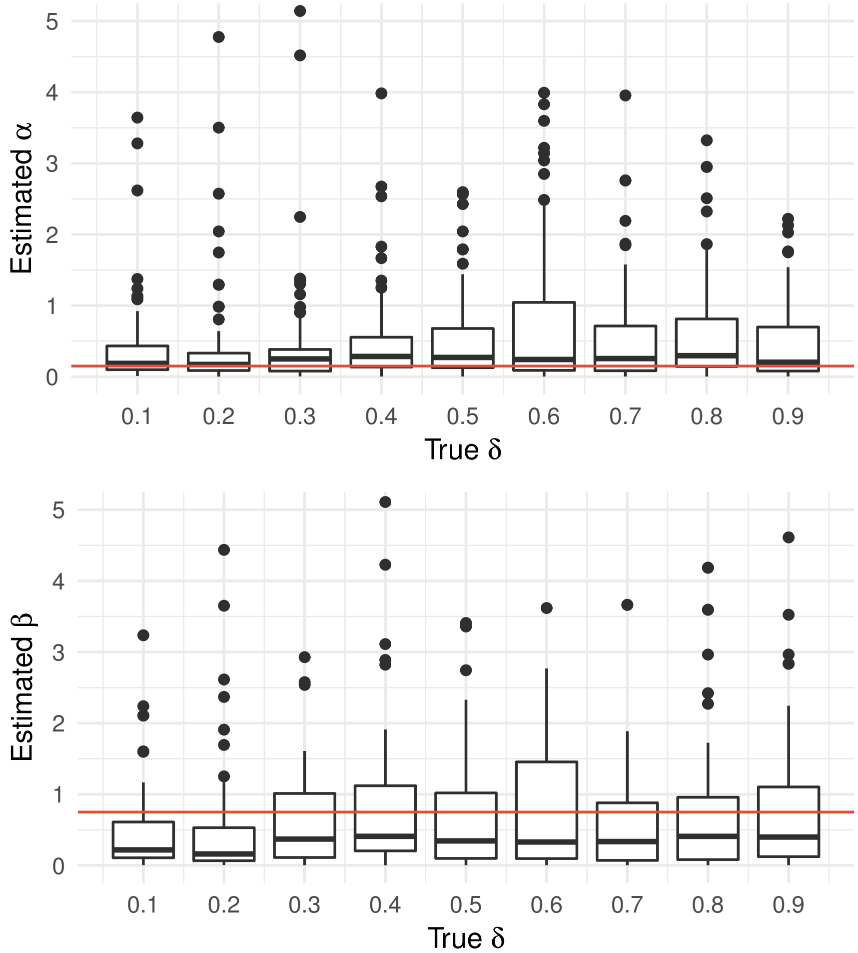

Figure 6: Boxplots for $\alpha = 0.15$ and $\beta = 0.75$ simulated from the Dirichlet model with $\delta = 0.1, \ldots, 0.9$. Per value of δ, 100 simulations are performed, with $n = 1000$ observations. The solid red lines display the true values of the parameters.

These simulations indicate that estimation of the copula parameters is prone to a bias, especially in asymmetric models. Therefore, a robustness analysis of an asymmetric model should be performed with a symmetric model which is less biased. This is especially relevant for the calculation of χ_X and η_X, which depend on the copula parameters.

5 Data

We use daily bond price data from Datastream. These are 10-year all traded benchmark government bond price indices from the following countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain. Losses are calculated as

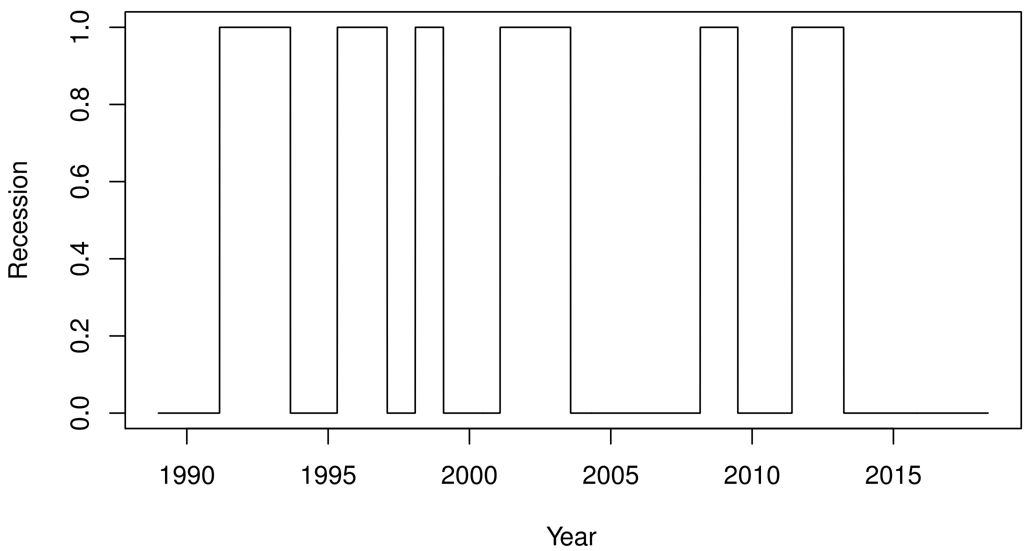

Figure 7: Daily European recession indicator based on OECD data. The value 0 is an expansionary period, while 1 indicates a recession.

the negative of the daily percentage index returns. The sample starts at December 30, 1988 and ends on May 8, 2018. Not all countries have data available from the start of the sample, but the number of observations is large enough for this extreme value analysis. Data on recessions is available at the FRED database[1]. It is an OECD based Recession Indicator on a daily basis, which is not seasonally adjusted. The sample contains six periods of recessions, which in total consist of 2849 observations. Figure 7 presents an overview.

Table 1 provides the summary statistics for the daily returns on government bonds. Overall, returns in Spain, Portugal and Italy are the highest, while the Netherlands, Germany and Austria have the lowest. The higher returns possibly reflect the higher risk profiles of these bonds. Standard deviations are also higher in the former three countries.

The phenomenon of a flight to quality towards bonds during recessions appears to be present for most countries. Average returns are higher during recessions, indicating a rising demand in all countries, except Greece. At the same time, volatility increases during recessions. Expansions are characterised by low and relatively similar average bond returns.

[1]https://fred.stlouisfed.org/series/EURORECD

Finally, the largest losses during either expansions and recessions are fairly similar per country. Also, most of these minima occurred during recessions. The most notable daily losses happened in Greece. This reflected the turmoil of two disturbing events for the Greek economy. During the recession in 2012, worries rose that Greece would possibly default. Then in 2015, while Europe was in an expansion, the EU rejected a bid to extend the Greek bailout, creating a severe loss on the bond market. Minima during expansions are more scattered across the sample than during recessions, which might indicate stronger tail dependence in turbulent periods.

Table 1: Summary Statistics: 10-year government bond price index returns in percentages and standard deviations in parentheses.

	n	Mean	Expansion Mean	Recession mean	Expansion minimum	Recession minimum
Austria	7657	0.008	0.001	0.020	-2.323	-2.303
		(0.310)	(0.304)	(0.320)		
Belgium	7527	0.010	0.002	0.024	-2.323	-2.634
		(0.345)	(0.333)	(0.363)		
Finland	6962	0.012	0.003	0.026	-4.398	-3.400
		(0.372)	(0.343)	(0.413)		
France	7657	0.010	0.002	0.024	-1.991	-1.996
		(0.357)	(0.351)	(0.368)		
Germany	7657	0.008	0.000	0.020	-2.037	-2.494
		(0.335)	(0.327)	(0.347)		
Greece	4984	0.011	0.018	-0.003	-25.315	-19.482
		(1.606)	(1.543)	(1.746)		
Ireland	7657	0.009	-0.004	0.031	-4.960	-4.934
		(0.457)	(0.426)	(0.504)		
Italy	7072	0.014	0.004	0.028	-3.134	-3.621
		(0.460)	(0.405)	(0.532)		
Netherlands	7657	0.008	0.000	0.022	-1.712	-1.794
		(0.317)	(0.311)	(0.327)		
Portugal	6462	0.015	0.000	0.044	-6.496	-10.977
		(0.643)	(0.565)	(0.769)		
Spain	7157	0.018	0.006	0.035	-2.701	-2.605
		(0.441)	(0.386)	(0.512)		

To reduce the effect of heteroskedasticity on tail dependence, we fit a Garch(1,1) model and use the residuals of this series for the transformation to uniform margins (Poon et al., 2003b). Two examples of these transformations are shown in Figures 8 and 9. Each plot shows a different type of tail (in)dependence in expansions or recessions. In Figure 8, note the apparent dependence of the Netherlands and Germany. The largest losses seem to cluster together in the upper right corner, which tends to increase during a recession.

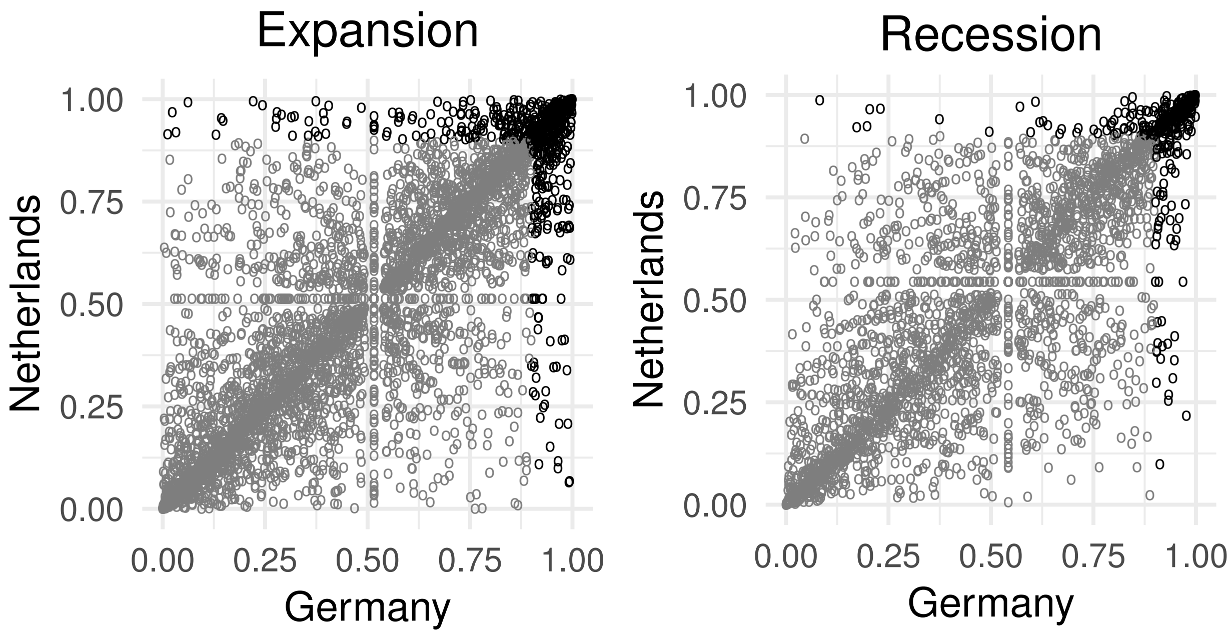

Figure 8: Scatterplot of the marginally transformed residuals of daily losses for the Netherlands and Germany after a Garch fit. The dark area indicates the losses above the 90% threshold. There is an indication of tail dependence.

Figure 9 shows stronger signs of tail independence. A lot of points are scattered across the grid instead of being clustered in the upper right corner. Also, the relation looks more asymmetric. This motivates the use of a flexible model to evaluate the tail dependence and to incorporate asymmetric models like the Asymmetric Logistic and Dirichlet.

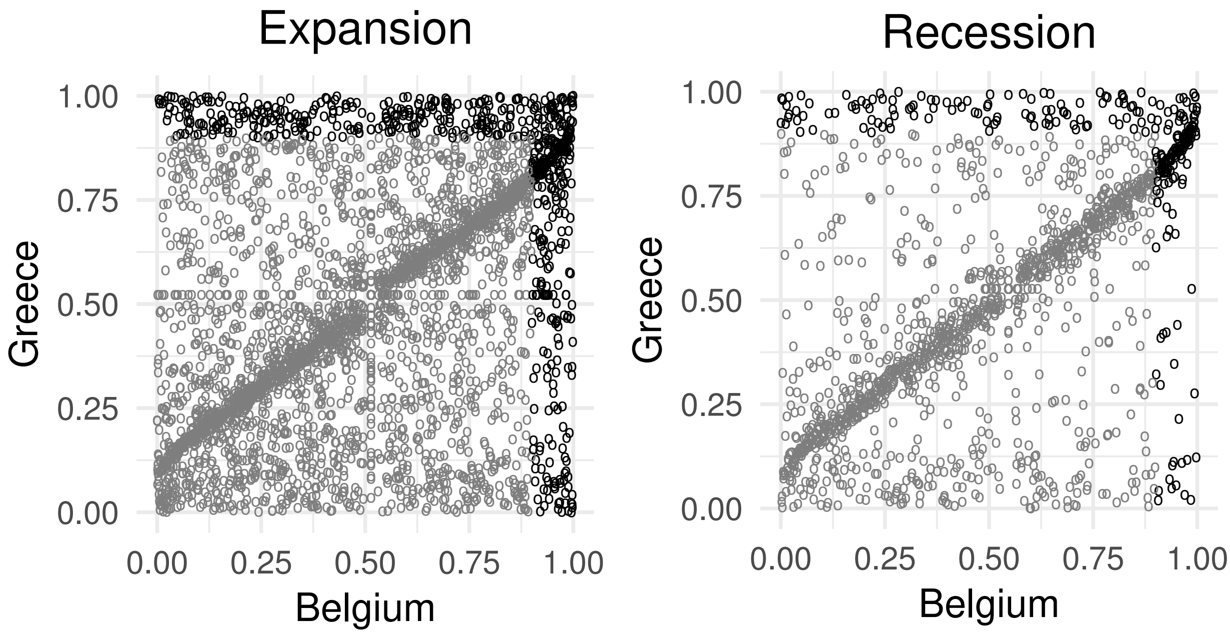

Figure 9: Scatterplot of the marginally transformed residuals of daily losses for Belgium and Greece after a Garch fit. The dark area indicates the losses above the 90% threshold. There is an indication of tail independence

6 Results

In this section, we describe the results of the HW model applied to the European bond market losses. We start with comparing the goodness-of-fit of the models. Then we group countries using a network analysis and compare dependence changes based from expansions to recessions. We end with a robustness analysis.

6.1 Model selection and evaluation

We fit each pair of countries to the four models and compare their relative fit based on the AIC criterion. Table 2 indicates how many times a specific model has the best parametric fit out of the 55 country combinations. We use 90% as the threshold for all country pairs. The asymmetric models outperform the symmetric models, and the Gaussian model fits poorly. One drawback of the asymmetric logistic model however is its variability regarding the initial values in the optimisation. This affects the outcome δ to an extent that the optimisation is able to find solutions with both $\delta > 0.5$ and $\delta < 0.5$. When we consider the relative performance without the asymmetric logistic model, the Dirichlet model has the best fit in 35 and 38 times in respectively expansions and recessions.

Table 2: Model selection using AIC criterion: Sum of cases where a model has the best fit

	Model			
Regime	Gaussian	Logistic	Dirichlet	Asymmetric Logistic
Expansion	9	11	19	16
Recession	5	12	22	16

To evaluate the performance of the HW model we to compare the non-parametric estimates of χ_X with the parametric estimates. Figure 10 gives an example of the fit compared to the non-parametric estimates for France and Italy. All models indicate tail independence in this figure. The asymmetric models follow the empirical estimate best over the range of u. For most country pairs, the largest deviations from the empirical estimate occur at the endpoints, where the asymmetric logistic model has the steepest descent in χ_X and therefore it indicates tail independence more often than the other three models. Most variation in the non-parametric estimates occur close to $u = 1$, but most models are able to capture this variation.

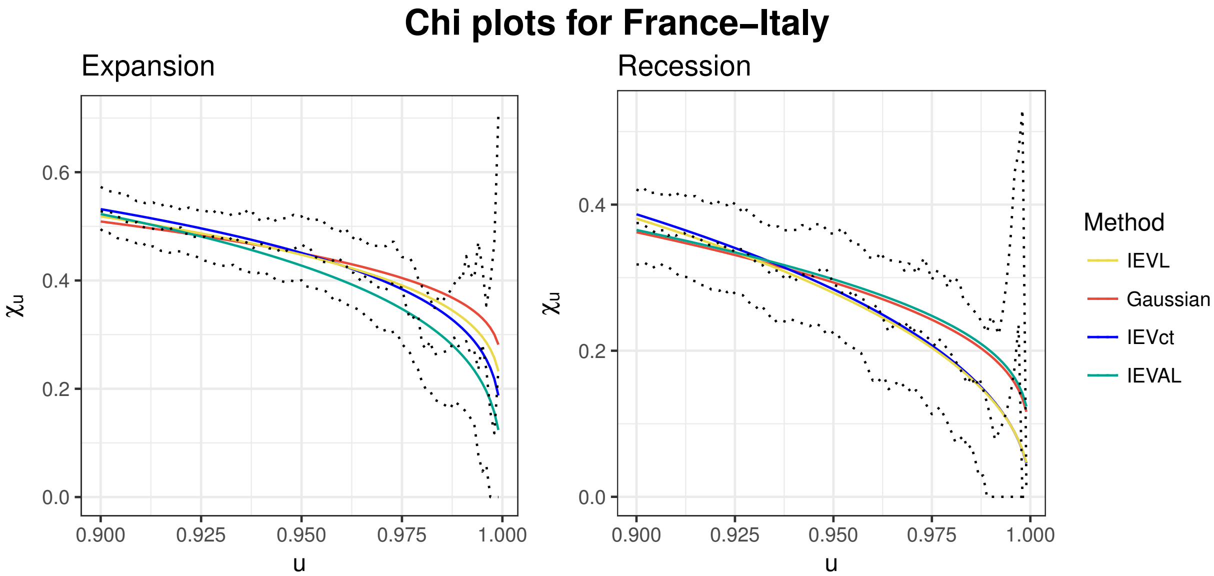

Figure 10: Estimates for $\chi_{X,u}$ for $u = 0.9, \ldots, 1$. The black dotted lines are the empirical estimates with a 95% confidence interval. The models are logistic (IEVL), Gaussian, Dirichlet (IEVct) and asymmetric logistic (IEVAL).

An example of a more debatable result is presented in Figure 11. In both recessions and expansions, all models except the asymmetric logistic model indicate tail dependence. In the asymmetric logistic model however, the parameter estimate of δ is sensitive to the initial parameters (varying the starting value of θ changes the point estimate for the parameter δ from 0.47 to 0.61), which makes the model hard to interpret even though the goodness-of-fit is good for both values of θ. The algorithm converges in both cases, and the likelihood appears relatively flat, which could explain these varying parameter estimates.

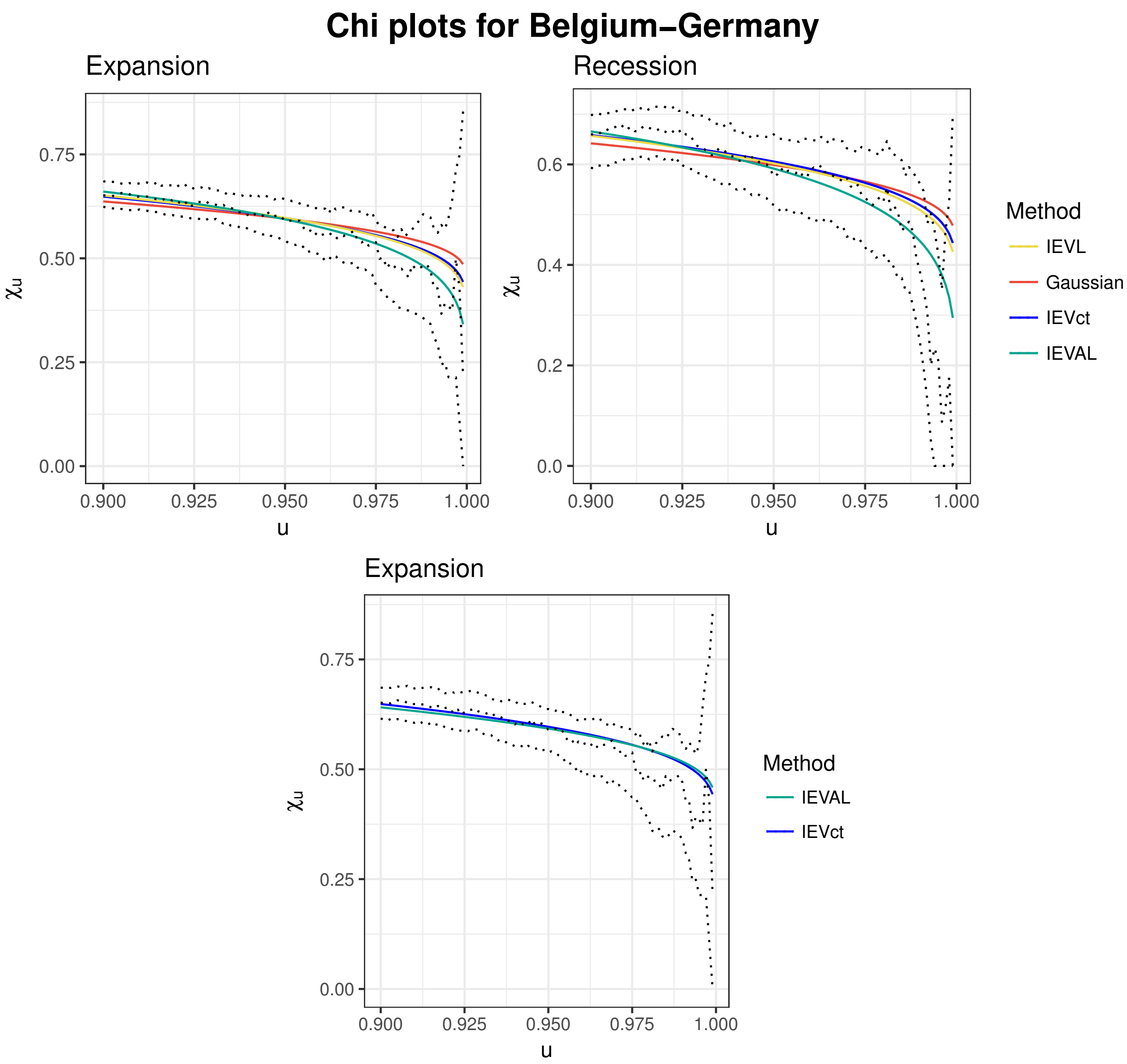

Figure 11: Estimates for $\chi_{X,u}$ for $u = 0.9, \ldots, 1$. The black dotted lines are the empirical estimates with a 95% confidence interval. The models are logistic (IEVL), Gaussian, Dirichlet (IEVct) and asymmetric logistic (IEVAL). The bottom figure shows the same figure as the expansion with different starting values for the asymmetric logistic model, showing tail dependence.

Based on the model selection criteria in combination with the parametric fit and the variability of the asymmetric logistic results, the Dirichlet model is the best candidate. We continue the analysis with this model.

6.2 Network analysis

We now categorise the bilateral dependence relations into the four categories depending on the value of δ, as in Table 3 and Figures 12 and 13. The missing values in Table 3 are due to numerical issues in calculating the standard errors. These will be disregarded from the analysis.

The cluster of countries that results from the division in tail (in)dependence is visualised in Figure 12. The inter-linkages in the bond markets are clearly divided between the north and the south of Europe, where the north consists here of Germany, France, Finland, Belgium, Austria and the Netherlands. Among the southern countries, consisting of Greece, Ireland, Italy, Spain and Portugal, there is hardly any tail dependence. The single exception is Italy - Spain, which also exhibits tail dependence.

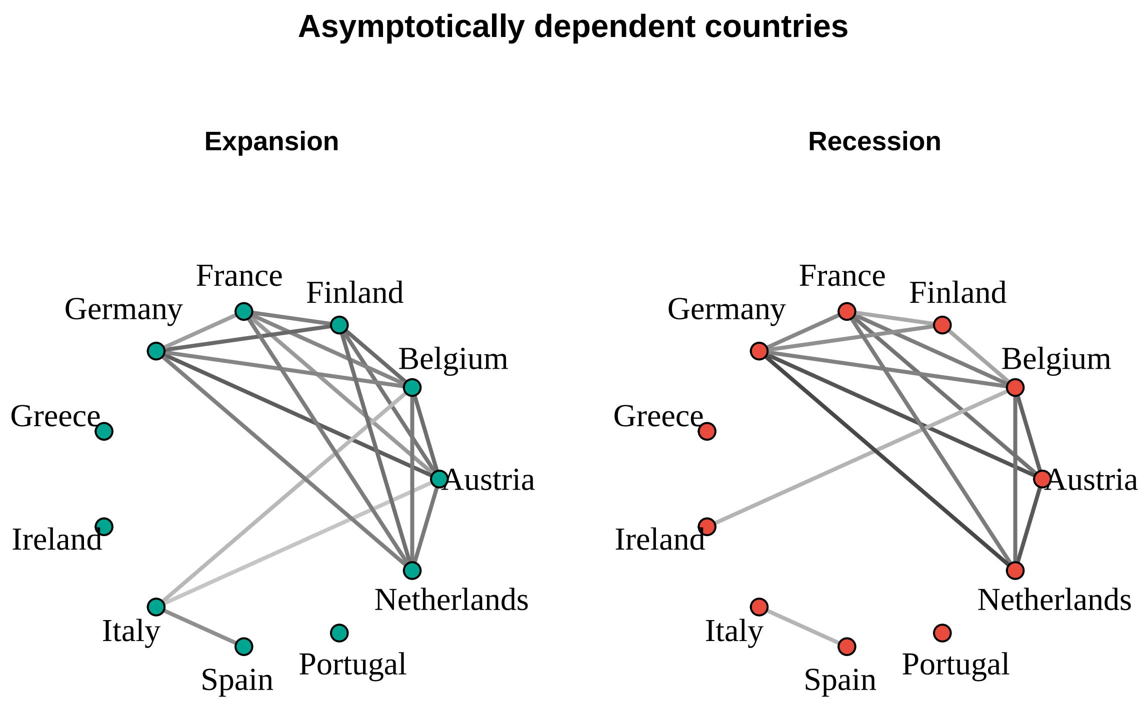

Figure 12: All connected countries with $\delta > 0.5$. The colour intensity of the edges represents the value of χ_X.

Tail dependence is found to be strongly significant in the northern countries of Europe during expansions. Italy is also connected to some northern countries. However, this linkage with Belgium and Austria is weak, insignificant and it vanishes in recessions. The other connections continue to exist during recessions, except for the pairs Finland - Netherlands and Finland - Austria, which become (insignificantly) tail independent. The pair Ireland - Belgium becomes tail dependent during recessions. Nevertheless, this tail dependence is weak and insignificant. During recessions, the parameter δ becomes less significant or insignificant for the pairs involving Belgium and for the pair Italy - Spain.

Five out of fourteen tail dependent country pairs experience a significant change in the value of δ across regimes. The change in the value of δ can be both significantly positive or negative.

From these results, it is evident that the northern connections during expansions are more often significantly dependent than during recessions, which is not in contrast with the literature. It is shown that during European recessions, spillover effects in the north are limited and that during the onset of a crises, dependence is strongest (Reboredo & Ugolini, 2015).

Table 3: Estimates of δ and asymptotically normal P-values of each country pair (Dirichlet model).

	Expansion		Recession		Comparison
Tail Dependence	δ	$P(\delta > 0.5)$	δ	$P(\delta > 0.5)$	$P(\delta_{rec} = \delta_{exp})$
Austria-Belgium	0.57	0.00	0.56	0.22	0.38
Austria-France	0.56	0.00	0.59	0.00	0.02
Austria-Germany	0.63	0.00	0.57	0.00	0.00
Austria-Netherlands	0.56	0.00	0.58	0.00	0.28
Belgium-Finland	0.56	0.00	0.50	0.88	0.02
Belgium-France	0.55	0.00	0.56	0.00	0.40
Belgium-Germany	0.53	0.01	0.53	0.25	0.44
Belgium-Netherlands	0.54	0.00	0.55	0.00	0.29
Finland-France	0.55	0.00	0.51	0.45	0.02
Finland-Germany	0.56	0.00	0.56	0.00	0.40
France-Germany	0.54	0.00	0.54	0.00	0.35
France-Netherlands	0.56	0.00	0.60	0.00	0.02
Germany-Netherlands	0.56	0.00	0.53	0.40	0.30
Italy-Spain	0.54	0.00	0.54	0.22	0.40
Tail Independence	δ	$P(\delta < 0.5)$	δ	$P(\delta < 0.5)$	$P(\delta_{rec} = \delta_{exp})$
Austria-Greece	0.03		0.12	0.08	
Austria-Ireland	0.43		0.46	0.00	
Austria-Portugal	0.24	0.00	0.32	0.00	0.10
Austria-Spain	0.49	0.70	0.25	0.00	0.00
Belgium-Greece	0.06	0.20	0.02	0.00	0.46
Belgium-Portugal	0.38	0.15	0.35	0.00	0.36
Belgium-Spain	0.48	0.53	0.37	0.06	0.08
Finland-Greece	0.06	0.30	0.17	0.00	0.40
Finland-Ireland	0.46	0.06	0.48	0.34	0.27
Finland-Italy	0.50	0.87	0.44		
Finland-Portugal	0.40	0.00	0.39	0.00	0.39
Finland-Spain	0.48	0.37	0.39	0.00	0.01
France-Greece	0.10	0.47	0.13	0.05	0.48
France-Ireland	0.47	0.20	0.50	0.91	0.17
France-Italy	0.45	0.06	0.31	0.06	0.08
France-Portugal	0.14	0.10	0.33	0.00	0.19
France-Spain	0.41	0.01	0.39	0.03	0.38
Germany-Greece	0.17	0.00	0.13		
Germany-Ireland	0.43	0.00	0.47	0.27	0.10
Germany-Italy	0.46	0.00	0.07	0.06	0.05
Germany-Portugal	0.27	0.00	0.34	0.00	0.14
Germany-Spain	0.49	0.46	0.29	0.00	0.00
Greece-Ireland	0.09	0.11	0.21	0.01	0.34
Greece-Italy	0.04	0.04	0.28	0.01	0.16
Greece-Netherlands	0.04	0.15	0.13		
Greece-Portugal	0.05	0.00	0.35	0.01	0.02
Greece-Spain	0.10	0.00	0.37	0.02	0.02
Ireland-Italy	0.44	0.01	0.47		
Ireland-Netherlands	0.40	0.00	0.46	0.29	0.11
Ireland-Portugal	0.43	0.00	0.48	0.49	0.04
Ireland-Spain	0.42	0.00	0.42	0.03	0.49
Italy-Netherlands	0.47	0.22	0.21	0.01	0.01
Italy-Portugal	0.47	0.15	0.45	0.07	0.29
Netherlands-Portugal	0.28	0.00	0.30	0.00	0.34
Netherlands-Spain	0.02		0.31	0.00	
Portugal-Spain	0.49	0.74	0.45	0.01	0.14
TI → TD	δ	$P(\delta < 0.5)$	δ	$P(\delta > 0.5)$	$P(\delta_{rec} = \delta_{exp})$
Belgium-Ireland	0.45	0.60	0.52	0.33	0.24
TD → TI	δ	$P(\delta > 0.5)$	δ	$P(\delta < 0.5)$	$P(\delta_{rec} = \delta_{exp})$
Austria-Finland	0.59	0.00	0.50	0.91	0.00
Austria-Italy	0.51	0.72	0.03		
Belgium-Italy	0.52	0.11	0.04	0.20	0.09
Finland-Netherlands	0.53	0.01	0.49	0.57	0.03

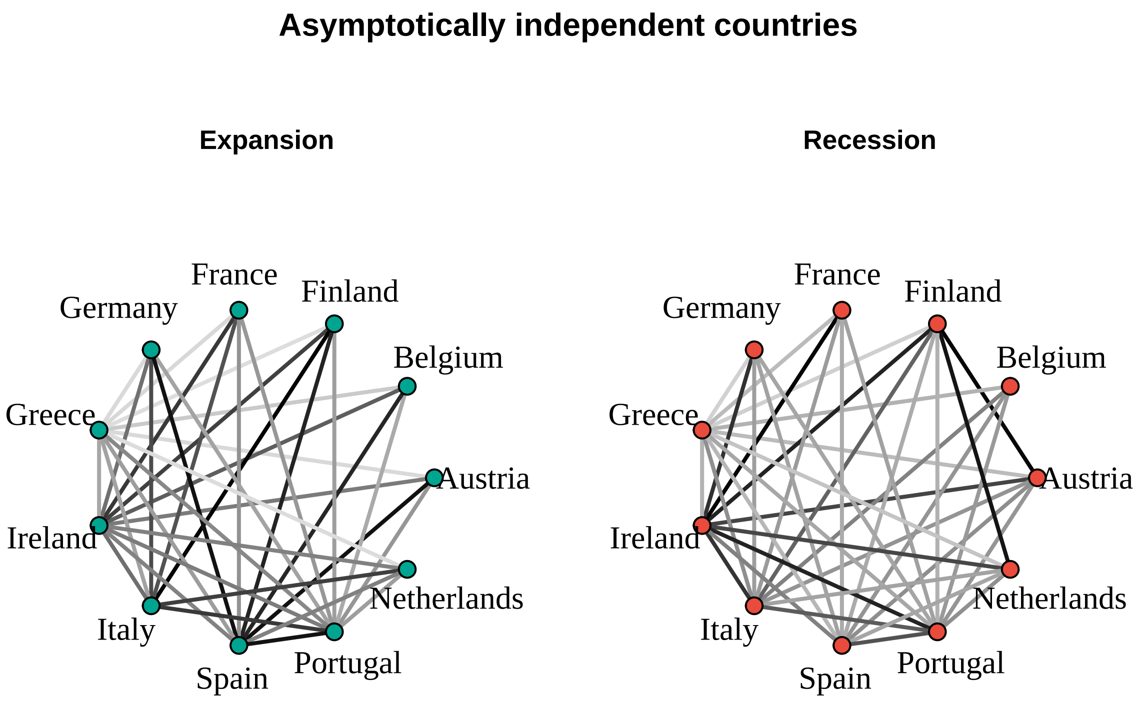

Figure 13: All connected countries with $\delta < 0.5$. The colour intensity of the edges represents the value of η_X.

Turning now to the second group of countries, Figure 13 shows that the majority of country pairs in the euro-area are tail independent. The relations are not so clear cut as before, but there are some noteworthy results.

First of all, tail dependence in the south of Europe during expansions is the highest for the pairs Portugal - Spain and Italy - Portugal. These pairs are also insignificantly tail independent. The tail independence coefficient decreases during recessions, which is reflected by the pair Portugal - Spain, which becomes significantly tail independent.

Secondly, Greece has low tail dependence coefficients in combination with the northern countries during expansions, which slightly increases during recessions. The difference in the value of δ from expansions to recessions is also insignificant in at least half of the pairs. The strongest links between the north and the south in expansions are via pairs with either Spain or Italy. This can be seen from the dark colours representing η_X and the fact that most estimates for the parameter δ are not significantly different from 0.5 for pairs involving Italy or Spain. During recessions, these relations weaken and instead, Ireland has the strongest links with all northern countries.

Tail independent estimates of the parameter δ more often have a higher variance than tail dependent estimates. This could also be expected based on the simulation results, which showed greater variability for $\delta < 0.5$. There is no clear pattern in the change of value in δ from expansions to recessions. In many cases, the hypothesis of equal δ can not be rejected. To gain more insight into dependence movements, we will analyse the tail (in)dependence coefficients.

6.3 Dependence comparison

Because the parameter δ does not provide all information on the tail dependence structure, we consider the tail (in)dependence coefficients χ_X and η_X. Denote $\tilde{\chi} = \hat{\chi}_{X,rec} - \hat{\chi}_{X,exp}$ as the difference between the bootstrapped values of χ_X in recessions and expansions. Similarly $\tilde{\eta}_X = \hat{\eta}_{X,rec} - \hat{\eta}_{X,exp}$.

Figure 14 shows that dependence can either significantly increase or decrease for tail dependent countries. In all cases where the parameter δ is significantly different between expansions and recessions, so is the corresponding tail dependence coefficient. For example, France has higher tail dependence coefficients with northern European countries during recessions and this increase is significant in combination with Austria and Germany. Finland is the exception to this. However, Finland experiences a significant decrease in its tail dependence coefficient during recessions in combination with Belgium, France and Germany. On average, this decrease is higher than 10%. Finland also becomes tail independent in combination with Austria and the Netherlands. So, Finland deviates from the northern European countries during recessions.

The countries that have exclusively higher tail dependence coefficients with tail dependent countries in recessions are Austria and the Netherlands. The Netherlands and Germany specifically appear strongly affected by recessions, which increases the probability of a collapse in a bond market by 10% conditional on a crash in the other bond market.

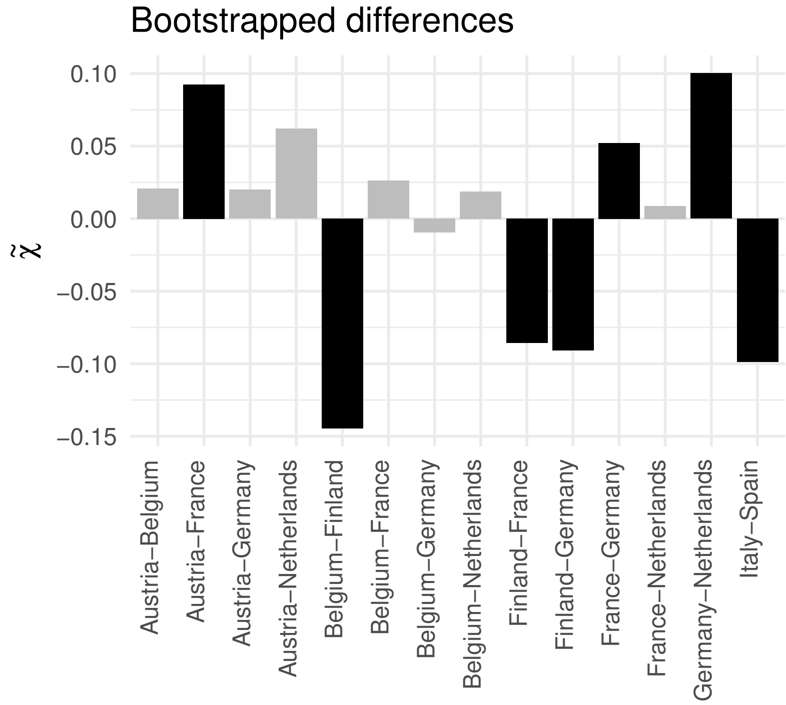

Figure 14: Differences in χ_X from expansions to recessions. The black bars are significant at 5% based on 1000 bootstraps.

The comparison of the tail independence coefficients show a different picture in Figure 15. The significant changes in the coefficient of tail independence are negative and occur for Spain in combination with Austria, Finland and Germany. Similar to Finland in the north, Spain segregates from countries in the north and south during recessions. Other southern countries do not seem to be significantly affected by changes in tail dependence due to crises. Also, a significant difference in the value of δ from expansions to recessions for tail independent countries does not automatically lead to a significant change in the coefficient of tail independence. To summarise, collapses in government debt that occur for southern countries appear to be more idiosyncratic. This aspect also holds for the relationships between the north and the south, in particular for Spain.

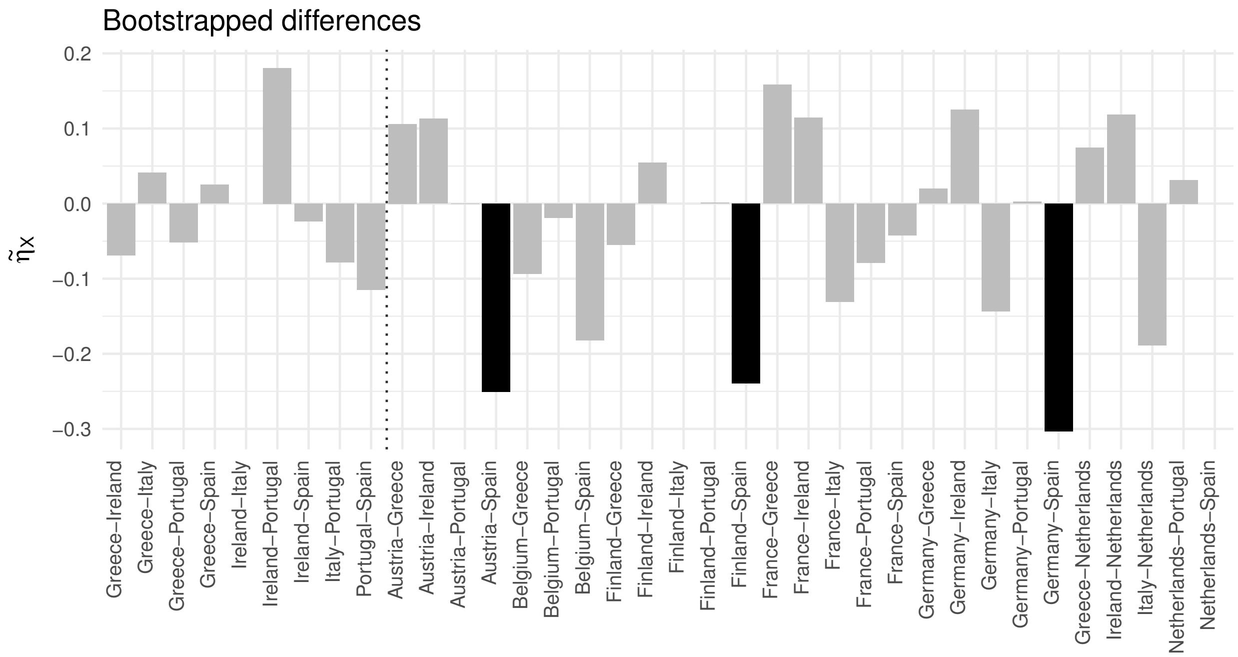

Figure 15: Differences in η_X from expansions to recessions. Countries on the left are southern countries, on the right are the interactions between north and south. The black bars are significant at 5% based on 1000 bootstraps.

6.4 Robustness analysis

Since estimation of the copula parameters can be biased using an asymmetric model as noted in Section 4, we perform a robustness check using the best symmetric alternative: the logistic model. The network plots are expected to be similar, because estimation of δ is unbiased in all models. However, the coefficients of tail (in)dependence could be affected by the bias in the copula parameters.

For the network of tail dependent countries in figure 16, we find a similar structure. Finland differs from the previous model since it loses its tail dependence with the Netherlands in expansions and with Germany and Austria in recessions. However, the relation with the Netherlands is not significantly tail independent. Finland's decrease in dependence with northern countries during recessions was noted in the previous section as well, which results in these tail independent estimates in the logistic model.

During expansions, Spain shows weak tail dependence with Germany, Belgium and Austria, which vanishes during recessions. These decreases in δ are significant. This is in line with the previous model, which indicated a general drop in dependence for Spain. In the logistic model the point estimates of δ are similar, but have a larger variance.

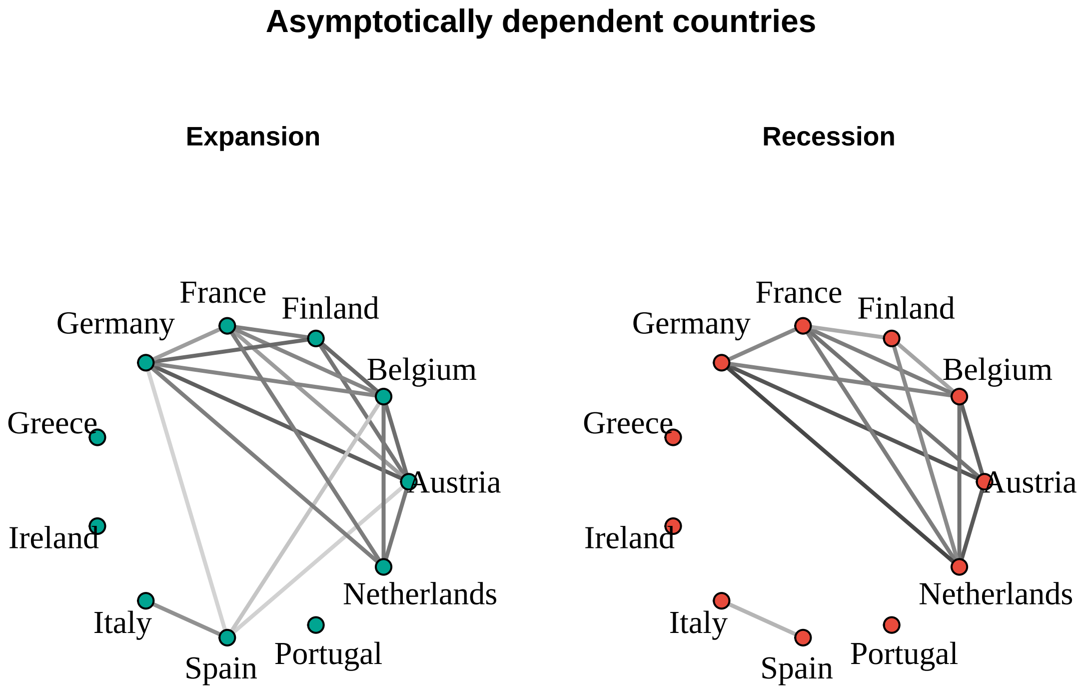

Figure 16: All connected countries with $\delta > 0.5$. The colour intensity of the edges represents the value of χ_X. Estimates of the logistic model.

For tail independent countries in Figure 17, we find the expected higher tail independence coefficients for Finland in combination with other northern countries, which are on the border of tail dependence. One notable difference from the previous model is the stronger connection between Greece and Portugal during expansions, a phenomenon that was also seen in Reboredo and Ugolini (2015). Greece also had a relatively strong connection with Portugal in the Dirichlet model, but it was not the strongest southern link.

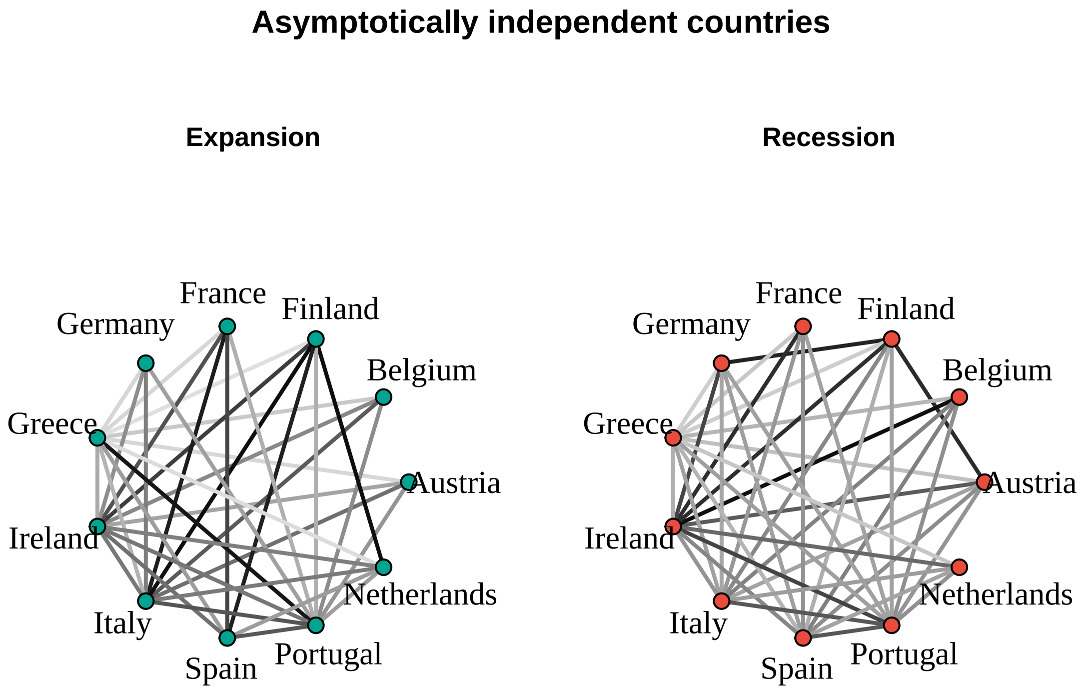

Figure 17: All connected countries with $\delta < 0.5$. The colour intensity of the edges represents the value of η_X. Estimates of the logistic model.

Figure 18 displays the differences in χ_X and shows that the logistic model matches the Dirichlet model to a great extent. Finland experiences dependence decreases of at least 10%, which generates significantly less vulnerability to northern European shocks during recessions. In contrast to the previous model, France has significant tail dependence coefficient increases with only Austria and instead, tail dependence for the Netherlands increases significantly with Austria and Germany. More so, the change in dependence between the Netherlands and Germany grows to 13%, compared to 10% in the previous model.

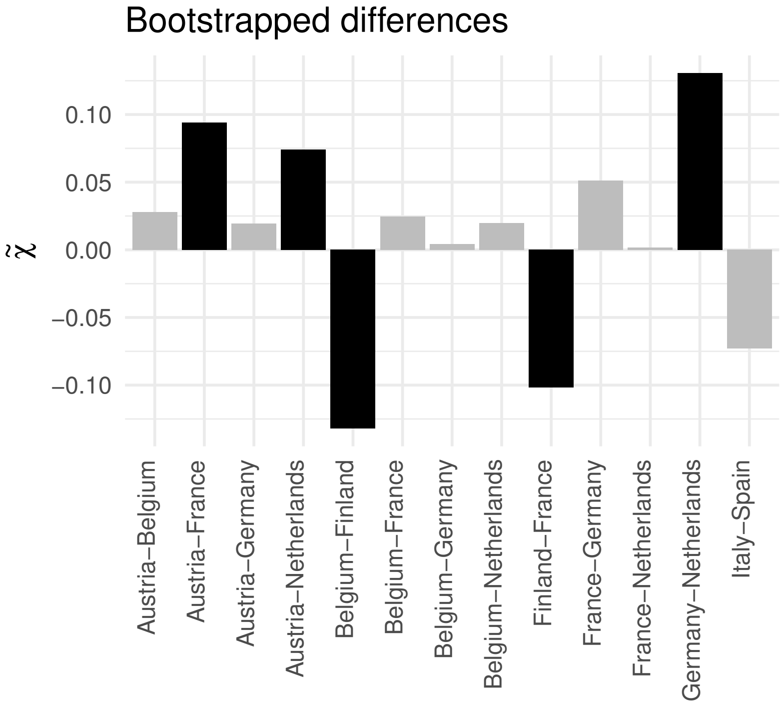

Figure 18: Differences in χ_X from expansions to recessions based on the symmetric logistic model. The black bars are significant at 5% based on 1000 bootstraps.

For tail independence in Figure 19, we see a different picture compared to the Dirichlet model for two reasons. First, Spain exhibits tail dependence in combination with Germany, Belgium and Austria, so these pairs do not show up in this figure. However, the decrease in the value of δ is significant for the combinations with Belgium and Germany, which indicates that Spain becomes less tail dependent during recessions. Secondly, the coefficient of tail independence of Greece and Portugal during expansions is high and decreases significantly during recessions. This is similar to the pair France and Italy, where the dependence decrease is now significant.

Again, no significant increases in the coefficient of tail independence during recessions occur for the southern or northern-southern relations.

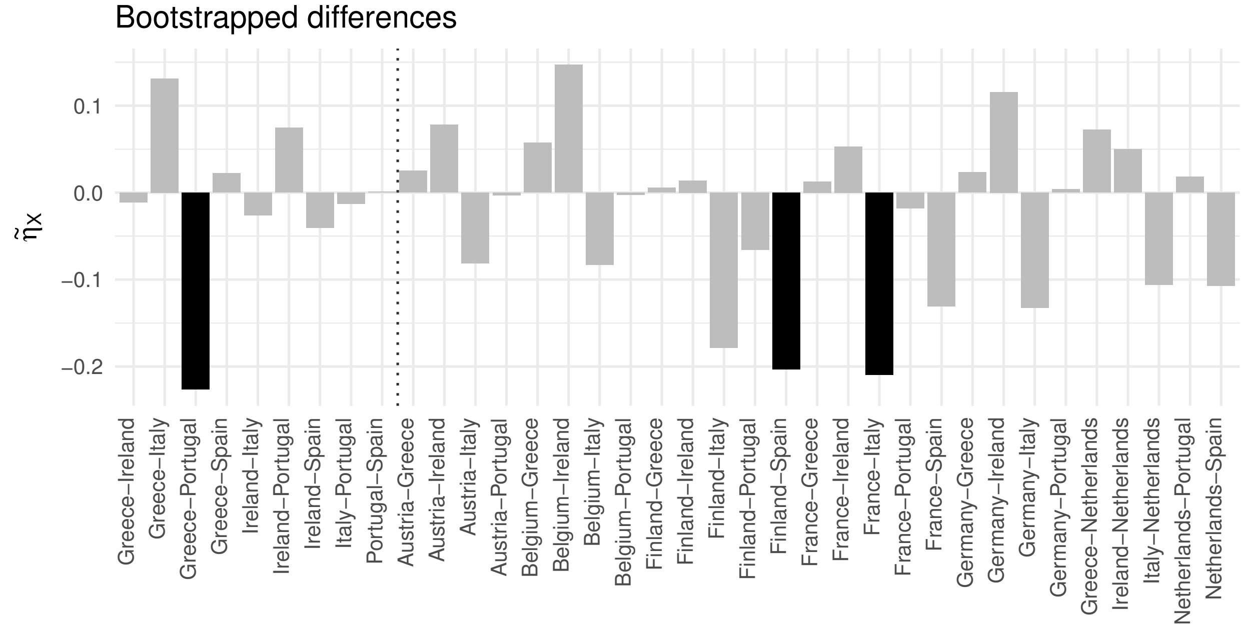

Figure 19: Differences in η_X from expansions to recessions based on the symmetric logistic model. Countries on the left are southern countries, on the right are the interactions between north and south. The black bars are significant at 5% based on 1000 bootstraps.

Overall, results from the Dirichlet model are in line with the symmetric logistic model. Significance of changes in the parameter δ are less indicative for tail (in)dependence changes in the logistic model, but tail (in)dependence coefficients show a similar picture as the asymmetric Dirichlet model. This holds specifically for the most deviating countries, Spain and Finland. Pairs involving France or the Netherlands experience most significant tail dependence increases in both models.

7 Conclusion & Discussion

In this paper, we analysed the dependence of extreme losses in the European bond markets using a novel copula model by Huser and Wadsworth (2017). This model bridges tail dependence and tail independence in a single model using a dependence parameter. Both tail dependence and tail independence occur for different groups of countries, which strengthens the need for a flexible tail dependence structure instead of assuming tail (in)dependence. Using this framework, we estimated tail (in)dependence in recessions and expansions and developed a test statistic to evaluate changes in tail (in)dependence coefficients across regimes.

Linkages of heavy losses in the European bond markets during recessions and expansions are characterised by tail dependence in the north of Europe and tail independence for southern and north-south relations. We find that tail dependence is persistent for most bilateral relations in the north except for Finland, which experiences significantly reduced tail dependence during recessions. On average, the probability of a co-crash with Finland decreases by more than 10%. The strongest increases in tail dependence occur for France and the Netherlands in combination with Austria and Germany.

By contrast, southern Europe is characterised by tail independence, except for Italy and Spain. In recessions, the coefficient of tail independence does not significantly increase, but it decreases significantly for Spain, similar to Finland in the tail dependent case. So, bond market crises in southern Europe appear less systemic than in the north. This also holds for relations between the north and the south, which are mostly tail independent. However, specifically during expansions, north-south linkages are stronger than the southern links. In short, tail dependence across the different regions in Europe can thus be summarised by $north > north - south > south$.

From a policy perspective, this implies that regulation with a national focus will be more effective in the south than in the north. Based on the results, the importance of the timing of policies is less important for southern countries, except Spain. There, the best time to adopt regulations that could be affected by its dependence on other countries will be during recessions. The effectiveness of regulations in northern Europe may be more strongly affected by the dependence on other countries in the north, because they exhibit tail dependence. During recessions, this is specifically applies to France, the Netherlands, Germany and Austria.

The model by Huser and Wadsworth (2017) has useful financial applications using tail (in)dependence coefficients and it is an effective tool to differentiate between tail dependent and tail independent series. However, there are certain limitations to this model as well, apart from the ones mentioned in Huser and Wadsworth (2017). First of all, simulation showed that estimation of the copula parameters can be biased and subject to a large variance. The symmetric logistic model with a single copula parameter showed the lowest bias, but increasing variance for larger values of δ. Estimation of δ is unbiased for all models, but is more variable for lower values of δ. Inverted max-stable models in particular, exhibit a constant tail independence coefficient for $\delta < \frac{1}{3}$. The bias can be reduced by increasing the number of dimensions, but this requires the assumption of d-wise tail (in)dependence.

The second drawback of this model is the computation time for datasets with many pairs or high dimensions. This especially applies to a bootstrap procedure for inverted max-stable models to calculate standard errors. Therefore, we use the Hessian as the representation of the asymptotic variance, which may affect the power of the significance tests.

Finally, when the dependence structure changes from recessions to expansions, we can not simply compare the tail (in)dependence coefficients. As a result, we can only assess the statistical significance of the differences in the model parameters, but not for the tail (in)dependence coefficients for these country pairs.

This copula model is one of the first to specifically model the tail dependence structure and has many possible extensions. One can easily implement new models, such as the asymmetric logistic model. In any case, a careful simulation analysis should be considered to assess a possible bias or instability of the parameters with respect to the initial values. This particularly holds for analyses with few dimensions and with asymmetric models. A possible improvement is to consider a new model for W that is not inverted max-stable, but which is asymmetric, unlike the Gaussian model. On the one hand, this overcomes the issue of a constant tail independence coefficient for $\delta < \frac{1}{3}$. At the same time, an asymmetric structure is preferred based on the results from the AIC criterion and the parametric fit.

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Appendix

Table 4: Estimates of δ and asymptotically normal P-values for each country pair (logistic model).

	Expansion		Recession		Comparison
Tail Dependence	δ	$P(\delta > 0.5)$	δ	$P(\delta > 0.5)$	$P(\delta_{rec} = \delta_{exp})$
Austria-Belgium	0.56	0.01	0.53	0.32	0.23
Austria-France	0.52	0.33	0.60	0.00	0.02
Austria-Germany	0.61	0.00	0.56	0.04	0.14
Austria-Netherlands	0.55	0.00	0.57	0.01	0.30
Belgium-Finland	0.55	0.03	0.51	0.65	0.15
Belgium-France	0.54	0.01	0.56	0.04	0.30
Belgium-Germany	0.52	0.32	0.52	0.43	0.45
Belgium-Netherlands	0.53	0.22	0.54	0.09	0.30
Finland-France	0.53	0.10	0.51	0.81	0.25
France-Germany	0.52	0.40	0.54	0.14	0.28
France-Netherlands	0.52	0.56	0.58	0.01	0.08
Germany-Netherlands	0.55	0.00	0.51	0.89	0.13
Italy-Spain	0.53	0.20	0.54	0.25	0.44
Tail Independence	δ	$P(\delta < 0.5)$	δ	$P(\delta < 0.5)$	$P(\delta_{rec} = \delta_{exp})$
Austria-Greece	0.07	0.22	0.10		
Austria-Ireland	0.40	0.08	0.45	0.21	0.23
Austria-Italy	0.29	0.00	0.31	0.05	0.45
Austria-Portugal	0.20	0.03	0.31	0.00	0.24
Belgium-Greece	0.11	0.02	0.14	0.25	0.47
Belgium-Ireland	0.42	0.01	0.49	0.79	0.04
Belgium-Italy	0.45	0.38	0.14	0.02	0.03
Belgium-Portugal	0.16		0.31	0.00	
Finland-Greece	0.10	0.29	0.12		
Finland-Ireland	0.47	0.12	0.47	0.24	0.39
Finland-Italy	0.49	0.77	0.42	0.06	0.07
Finland-Portugal	0.35	0.50	0.32	0.00	0.45
Finland-Spain	0.48	0.19	0.38	0.11	0.09
France-Greece	0.06	0.12	0.20		
France-Ireland	0.45	0.07	0.48	0.35	0.28
France-Italy	0.48	0.58	0.27		
France-Portugal	0.34	0.10	0.34	0.00	0.47
France-Spain	0.46	0.29	0.30	0.00	0.01
Germany-Greece	0.06	0.17	0.15	0.14	0.41
Germany-Ireland	0.41	0.00	0.46	0.17	0.12
Germany-Italy	0.39	0.42	0.25	0.03	0.22
Germany-Portugal	0.22	0.00	0.33	0.00	0.13
Greece-Ireland	0.11	0.33	0.17	0.19	0.45
Greece-Italy	0.31		0.35	0.42	
Greece-Netherlands	0.11	0.04	0.15	0.58	0.48
Greece-Portugal	0.49	0.54	0.32	0.02	0.01
Greece-Spain	0.09	0.03	0.36	0.09	0.09
Ireland-Italy	0.43	0.01	0.41	0.15	0.38
Ireland-Netherlands	0.38	0.01	0.44	0.06	0.16
Ireland-Portugal	0.29	0.01	0.46	0.30	0.02
Ireland-Spain	0.43	0.01	0.42	0.02	0.36
Italy-Netherlands	0.43	0.15	0.14	0.00	0.01
Italy-Portugal	0.45	0.16	0.45	0.10	0.48
Netherlands-Portugal	0.25	0.01	0.28	0.00	0.37
Netherlands-Spain	0.40	0.50	0.33	0.00	0.32
Portugal-Spain	0.45		0.45	0.03	
TI → TD	δ	$P(\delta < 0.5)$	δ	$P(\delta > 0.5)$	$P(\delta_{rec} = \delta_{exp})$
Finland-Netherlands	0.49	0.79	0.51		
TD → TI	δ	$P(\delta > 0.5)$	δ	$P(\delta < 0.5)$	$P(\delta_{rec} = \delta_{exp})$
Austria-Finland	0.54	0.11	0.48	0.51	0.07
Austria-Spain	0.52		0.24	0.00	
Belgium-Spain	0.51	0.79	0.25	0.00	0.00
Finland-Germany	0.54	0.14	0.48	0.65	0.13
Germany-Spain	0.52	0.28	0.28	0.00	0.00