Effect of collateral floors on collateral setting in adverse selection credit markets

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Adverse selection in credit markets occupies a large volume within the economic literature. Most of this literature is dedicated to studying interest rate setting behaviour. This paper instead studies collateral setting behaviour by lenders and primarily how a collateral floor set by policy makers can influence the behaviour of lenders. Analysing the credit market using a model of asymmetric information yields interesting results. In a market free from intervention, no pure strategy equilibrium exists, instead a mixed strategy equilibrium arises due to the informational advantage of one lender. Lenders compete with each other in this equilibrium by demanding various amounts of collateral from their borrowers. If a collateral floor is imposed, assumptions on borrower behaviour crucially influence the equilibrium outcome of the credit market. When borrowers tend to always switch to competitor lenders, a pure strategy equilibrium arises. Contrary, when borrowers tend to always stay with their current lender, the mixed strategy equilibrium prevails. Ultimately this determines whether borrowers are made better off or worse off by the collateral floor.

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1 Introduction

Collateral is a much debated topic, especially since the 2008 financial crises. Each type of loan has its own type of collateral and the discussions in academic literature and policy circles draw very different conclusions for the different types of collateral. For example, the legal maximum loan-to-value ratio of Dutch residential mortgages was 110 percent during the crisis. Meaning that banks were allowed to make loans that exceed the value of the collateral, in this case the real estate property, by ten percent. Some banks found loopholes and made loans as large as 120 percent of the property value (Bijlo, 2012). Since 2011, the government has reduced the legal maximum gradually to 100% of property value in 2018 (Rijksoverheid, 2017) The Dutch central bank DNB (2015) argues that the loan to value ratio should be reduced further to 90%, a statement that is backed by the IMF (IMF, 2017). This would increase financial stability and prevent bubbles on the housing market by mostly making it harder for starters to purchase a house (DNB, 2015)

Contrary to the stricter rules for collateral on the Dutch mortgage market, the European Central Bank (ECB) has eased the requirements for collateral that it accepts. This policy was also born after the 2008 financial crisis, the ECB started accepting more different and riskier types of collateral for its loans to commercial banks (Belke, 2015). For example, the ECB initially accepted as collateral only those asset backed securities based on small and medium enterprise (SME) loans that had a 'triple A' rating. The minimum required rating was subsequently lowered to 'single A' and later to 'single B' (Aberg, 2013, Belke 2015). Consequently ,the use of asset backed securities as collateral ballooned from 182 billion euros in 2007 to 490 billion euros in 2010, in 2017 the total value of asset backed securities used as collateral for ECB loans still stands at 354 billion euros (ECB, 2017). Moreover, in the current zero interest rate era, collateral requirements might be the only effective policy tool that the ECB has to steer the money supply. One major aim of relaxing the minimum rating requirement was to increase bank lending to SME's and private households (Aberg, 2013).

To analyse all roles of collateral in credit markets would be beyond the scope of the paper. Instead, the focus will be on the use and effect of collateral floors in a credit market. Collateral floors are a legal requirement for lenders, that they should demand at least a certain amount of collateral for each loan they make. The lenders are left free to demand a higher amount of collateral, but they cannot demand less collateral than what is specified by the collateral floor.

One key problem in credit markets is information asymmetry, one party in the market has more information than other parties. Of course this influences the interactions between them, the side with more information is in a position of natural advantage. Moreover, information asymmetry can give rise to two fundamental problems, adverse selection and moral hazard. These two problems will be explained carefully in the related literature section. In a nutshell, adverse selection occurs ex ante to the market transaction. In the credit market it happens when the lender with less information receives a share of the borrowers that is of worse quality than the remaining share of borrowers. Thus this is a problem that occurs in the relationship between lenders.

A moral hazard on the other hand often occurs ex post and is a change of behaviour by one of the parties because of the superior information that he has. Usually this happens in a client and lender relationship, a client might not behave diligent after securing a loan contract for example. This paper will largely abstain from analysing the borrower side of the credit market and focuses on the behaviour of the lenders and the interactions between them.

The analysis in this paper will therefore focus on a credit market with adverse selection. Together this leads to the following research question:

What is the effect of a collateral floor on credit supply in adverse selection loan markets?

This paper will use a theoretical model to answer that research question. A mathematical framework is developed to depict the credit market and the economic agents involved. An objective function will be defined for each agent, lenders and borrowers. In simple terms, the objective function is the formula that an agent seeks to maximise, here it is the profit of the lenders. In this model, lenders will choose the amount of collateral they demand from borrowers and they will compete with each other on the amount of collateral only. The solution to this maximisation problem is dependent on the agent's interactions with the other economic agents in the model. As well as the variables of the model. In the end, if the model's design is sound, its solution should yield propositions that explain the role and effect of collateral floors in loan markets.

The next section will provide an overview of the literature on credit and collateral. Section three sets out the model explaining the roles of the different players, their payoff functions, and their strategies. The fourth section is dedicated to solving this model and derives interesting propositions from the model. In section five, collateral floors are introduced to the model. Section six is an extension to the model, the assumption on how ties are shared is changed. In the seventh section the results from analysing the model with and without collateral floor are compared. This section also shows the effect that a collateral floor has on lender profits, as well as borrower welfare. The final section summarises the findings to draw conclusions and make policy recommendations as well as recommendations for future research.

2 Related Literature

In this paper a model will be developed of a credit market with information asymmetry. Lenders have valuable information about the riskiness of borrowers in their portfolio, but they do not have that information about the borrowers in the portfolios of their competitors. As said before, information asymmetry can give rise to two problems, adverse selection and moral hazard. The adverse selection problem is the one being examined in this paper however, it might be helpful to provide a background into both problems.

Adverse selection in a credit market arises when the terms of a loan contract change the average quality of the borrowers that are willing to engage in the contract. For example, when the interest rate on a loan goes up, save borrowers might leave the market faster than high risk borrowers. High risk borrowers default more often, thus they pay the interest less often than low risk borrowers and are less tempted to leave the market if the interest increases. Of course, this would be bad for the lender, as he only attracts those clients that are the least profitable.

Moral hazard problems arise when the behaviour of borrowers is influenced by the terms of the contract after they agree to it. For example, an entrepeneur who has a loan with a high interest rate might invest this money in projects with more risk. Thus increasing the failure rate and decreasing the profit of the lender.

The literature on information asymmetries, and in particular adverse selection, has taken large steps since Stiglitz and Weiss (1981) published their theory of equilibrium rationing in credit markets. Asymmetric information theory has been applied to solve a wide variety of banking problems. Jain (1999) develops a model where some lenders have better information about the borrower pool than others to show why formal and informal credit markets coexist in developing countries.

More recent work uses a moral hazard framework to analyse interactions between the formal and informal markets (Madestam, 2014). Both moral hazard and adverse selection can clarify why firms favour trade credit in the short run and bank credit in the long run (Burkart & Ellingsen, 2004, Maksimovic & Frank, 2005).

Information asymmetry is also central to the academic literature on microfinance. The literature shows that joint liability lending can be a solution to information asymmetry for very small loans (Besley & Coate, 1995, Ghatak & Guinnane, 1999), competition in the micro-credit market increases asymmetric information (McIntosh & Wydick, 2005), micro-finance institutions can decrease borrower coverage in adverse selection markets (Demont et al., 2012).

Most of the above mentioned papers focus on the role of information asymmetry on interest rate setting behaviour by lenders. In microfinance obviously as the fundamental problem of micro credit is the lack of a credible collateral for the loan. More general, the interest rate is the most visible component in any loan contract.

This paper will instead focus on the amount of collateral that lenders demand when making loan contracts with borrowers. Wette (1983) was quick to analyse the role of collateral in the credit rationing framework. He found that increasing the collateral requirement can lead to the low risk borrowers leaving the market under both the assumptions of either risk neutral or risk averse borrowers (Wette, 1983). Bester (1985) established that collateral can be used as a screening device but when there is insufficient collateral, credit rationing might occur in imperfect markets. Competitive credit markets have an equilibrium where collateral is used as a sorting device (Besanko & Thakor, 1987).

The model deployed in this paper is similar to that of Varian (1980), which he used to explain sales. The equilibria for this model have been derived by Baye, Kovenock and de Vries (1996) although this paper is closely linked to the adaptation from Bijkerk and De Vries (2017). In their model, lenders compete with interest rates to attract borrowers from competitors and to defend their own borrower pool. In this paper, the interest rate will be an exogenous variable and firms will compete on collateral. Still this is a form of price competition, lenders do not compete on the quantity of loans they offer to borrowers. The collateral is the price that borrowers have to pay if they fail to meet their debt obligations.

Dell'Ariccia and Marquez (2006) examine a credit market with adverse selection, however they allow firms to compete on both interest and collateral. They show that banks profits, due to informational rents, are higher when there is more adverse selection (Dell'Ariccia & Marquez, 2006). If competition on both interest and collateral is allowed, a pooling equilibrium and a separating equilibrium exist. Since this paper allows only competition on collateral, no separating equilibrium exists.

Standard models of price competition predict a pure strategy equilibrium in which firms set price equal to the marginal cost. This way even in a two-firm market the firms make zero profits, equivalent to quantity competition in a market that is open to many firms. This result became know as the Bertrand result.

Baye and Morgan (1999) show that under price competition, many equilibria other than the zero profit Bertrand equilibrium exist. The necessary condition for their folk-theorem is that there exists no finite price for which demand is zero (Baye & Morgan, 1999). In such a market, a monopolist could have unbounded revenues. Kaplan and Wettstein (2000) confirm that unbounded revenues are a necessary condition for positive profit equilibria to exist. With unbounded monopolist revenues, competitors could device a mixed strategy equilibrium that secures a positive profit for all participants.

Hoernig (2007), adds that for these mixed strategy equilibria to exist, the tie-sharing rule must be "tie-decreasing ". Tie-decreasing means that, at a tie a player's expected payoff is lower than what he would receive at the same point were he not tied with his competitor (Hoernig, 2007). This paper will show that the chosen tie-sharing rule can crucially influence the equilibrium outcome after intervention in an asymmetric information credit market.

In 1990, Simon and Zame wrote that "endogenous " sharing rules could be used to summarise the consumer behaviour in Bertrand and Hotelling style models (Simon & Zame, 1990). In most Bertrand competition models, a simple sharing rule is used such as "both competitors split their clients evenly". By treating the sharing rule as an endogenous feature of the model, Simon and Zame allowed for a more general notion of the equilibrium without pinning the model to a certain tie-sharing rule.

For the credit market model as developed in this paper, the tie-sharing rule determines how many borrowers will switch between lenders when they are indifferent between the offers of two or more lenders. Consider for example the house-banker that offers a contract stipulating a certain amount of collateral C

to one of his clients, and it happens that a competing lender offers a similar contract so that the borrower is indifferent. It seems unrealistic to assume that half of the indifferent borrowers would switch from their current credit provider, to another lender. Which would be implied by a "fity-fifty" tie-sharing rule.

A little evidence can be drawn from the Dutch market of health insurance. In the Netherlands, clients must choose a health insurance policy every year and can switch freely at the end of each year. NRC, a Dutch newspaper, reported that clients can choose from more than 60.000 different health insurance policies (Steenbergen, 2017). More importantly, most of these health insurance policies are exactly the same and they boil down to about 55 varieties (Steenbergen, 2017). About seven percent of all clients switch to a different insurance company each year. Eventhough the consumer and market authority estimates that roughly three-quarters of all clients can save money by switching, for an average amount of 100 euros yearly (ACM, 2017). So in the health insurance market only seven percent of clients switch, although many people could save money yearly. It seems unlikely that indifferent consumers will have a habit of switching often.

This paper shows, that when the market is free from intervention, the tie-sharing rule between the inside lender and the outside lenders is unimportant. However, section five will show that when authorities impose a collateral floor on the market, the proportion of indifferent borrowers that switches from their current lender to an outside lender affects the equilibrium outcome.

3 Model

In this model there are three players, the lenders, that make loans to borrowers. Since this paper aims to analyse the collateral setting behaviour of the lenders, the borrowers are treated as more passive agents and not considered players. This portfolio considers the lenders i, j, and k, each of the lenders has a portfolio with a number α low risk and a number β of high risk borrowers. For brevity's sake, this paper focuses on the portfolio of lender i as the analysis is analogues for the other two lenders. Lender i knows the type of the individual borrowers within his own portfolio, and is therefore referred to as the inside lender. Lenders do not know the type of individual borrowers in the competitor's portfolios. Thus there is an information asymmetry, such that the outside lenders have less information than the inside lender i about borrowers in portfolio i.

The ratio of low risk borrowers over high risk borrowers $\frac{\alpha_i}{\beta_i}$ in a portfolio is common knowledge to all lenders and lenders can verify to which portfolio borrowers belong. Knowing the risk ratio implies that lenders know the probability with which a borrower from lender i's portfolio is either low or high risk.

Each lender offers borrowers in his portfolio a loan contract for a loan of size one, where the contract's conditions depend on the type of the borrower. A loan contract consists of an interest rate payable r and an amount of collateral C demanded in case the borrower defaults. Next to this, the lenders make contract offers to the borrowers in competitor's portfolios. Since the outside lenders cannot verify the type of individual borrowers in the portfolio of i, they cannot make separate contracts for low and high risk borrowers. In total, lender i has four decisions to make. The collateral he demands from low risk borrowers in his own portfolio, the demanded collateral from his high risk borrowers, and the contract that he offers to borrowers in portfolio's of lender j and k respectively.

The interest rate r is treated as exogenous, meaning that every lender sets the same interest rate for the same type of contract. Firstly, this is to simplify the analysis. Secondly, in the low interest rate world of today, interest rate margins are very small and collateral might become a more import instrument for lenders to compete with each other. As long as interest rates are close to zero, or even negative, there will not be much margin to compete over.

Lenders only compete on the collateral requirements C. Lenders act simultaneously by offering contracts to borrowers and try to maximise their profits, considering the risk types of borrowers and the possible actions of competitor lenders.

Each borrower has an inelastic loan demand of size one in the model. Funding of the loans by the lenders is taken to be exogenous, no assumptions are made about the cost of funds. Moreover, it is assumed that each lender has enough funds to serve all borrowers in the market. The borrowers then invest the loan in their projects and when successful they repay their loan and the interest rate. In the case of failure, the lender obtains the amount of collateral from the borrower that was demanded with the loan contract. Low risk borrowers have a probability p of success, whereas high risk borrowers succeed with probability q. Both p and q are distributed on the interval $(0, 1)$ and $p > q$ so that high risk borrowers default more often than low risk borrowers, default rates are assumed to be uncorrelated. For simplicity it is assumed that the borrower's outside option for a loan is equal to zero.

Borrowers aim to minimise the collateral requirement, thus they will choose the lender that offers the smallest collateral requirement. This is equivalent to maximising the loan that a borrower can obtain for a certain amount of collateral, but simplifying each loan size to one.

By chance it can happen that lenders tie and both offer a contract with the smallest amount of collateral demanded to a borrower. When there is an outiside-outside lender tie, and they demand a lower collateral than the inside lender, borrowers flip a coin to choose randomly between the two outside lenders. Hence each outside lender has a fifty percent chance to obtain the borrowers. Therefore, none of the outside lenders benefits from a tie with another outside lender, the tie-sharing rule is tie-decreasing.

This paper will consider the following tie sharing rule in case of an inside-outside lender tie, throughout the text it will assume that borrowers always switch to the outside lender upon an inside-outside lender tie.

It seems extreme to assume that all borrowers leave their current lender when they receive an equal offer from a competitor. However, this paper will show that the chosen inside-outside lender tie-sharing rule affects the results of the model, after a collateral floor is imposed on the credit market. This extreme tie-sharing rule is chosen to demonstrate its effect on the equilibrium outcome when a collateral floor is imposed.

Section six will provide an extension in which the opposite tie-sharing rule is chosen in case of an inside-outside lender tie. It shows that after a collateral floor is installed the result will be different under the opposite tie-sharing rule. Moreover, in a separate analysis in the appendix, it will be shown that changing the tie-sharing rule does not alter the results on a market without a collateral floor.

Finally, it is assumed that entry is free, so that expected profits of the outside lenders equal zero. A section four will show, the inside lenders will obtain a rent due to their informational advantage.

3.1 Collateral

The expected return that the informed lender i has on a low risk borrower is equal to

$$E[return|\alpha] = p(1+r) + (1-p)C_i^{\alpha} - 1$$

In p percent of the cases the borrower is successful, he will repay the loan plus interest. When the borrower defaults on his loan, with probability $1-p$, the lender obtains the collateral. To find the net return, the initial loan size of one needs to be deducted. Note that in the amount of collateral demanded C_i^{α} the subscript i determines which lender demands this collateral and the superscript α determines that it is demanded from low risk borrowers.

Similarly for high risk borrowers

$$E[return|\beta] = q(1+r) + (1-q)C_i^{\beta} - 1$$

To break even on low risk borrowers, which is to have zero expected return on a low risk borrower, lenders would have to demand a collateral of size

$$\underline{C}^{\alpha} = 1 - \frac{pr}{1-p}$$

from each low risk borrower. Where the underlining in $\underline{C}$ denotes the break even collateral.

For high risk borrowers the break even collateral is

$$\underline{C}^{\beta} = 1 - \frac{qr}{1-q}$$

Since $p > q$ and the interest rate r is exogenous, the collateral for high risk borrowers will always be larger than the collateral for low risk borrowers. The minimum collateral needed to break even decreases with both the success rate of the borrowers and the interest rate as

$$\frac{\partial \underline{C}^{\alpha}}{\partial p} = \frac{-r}{(1-p)^2} < 0 \ , \ \frac{\partial \underline{C}^{\alpha}}{\partial r} = \frac{-p}{1-p} < 0 \text{ (read } q \text{ in case of } \underline{C}^{\beta}\text{)}$$

The intuition behind this is straightforward, when less people default, the risk on the lender's side of making a loss on its portfolio decreases. He can therefore demand a smaller collateral to cover the losses from defaulting borrowers. When the successful borrowers pay a higher interest this also allows the bank to reduce the collateral in order to break even on average, ceteris paribus.

Since the informed lender knows the type of the borrowers in his portfolio, he can discriminate between them, making the contracts offered to each borrower independent of the ratio of low risk over high risk borrowers in his portfolio.

The competitors of lender i will also make contract offers to the borrowers that are currently in lender i's portfolio. The only information that the outside lenders $-i$ have about the portfolios of lender i is the ratio of low risk to high risk borrowers in each portfolio. From this, the outside lenders can deduce what the chances are that a randomly selected borrower from portfolio i is a low risk borrower or a high risk borrower. On a randomly selected borrower from portfolio i the outside lender thus has an expected return of

$$E[return| -i] = \begin{bmatrix} \left(\frac{\alpha_i}{\alpha_i+\beta_i i}\right)(p(1+r)+(1-p)C_{-i}) \\ +\left(\frac{\beta_i}{\alpha_i+\beta_i}\right)(q(1+r)+(1-q)C_{-i}) - 1 \end{bmatrix}$$

For C_{-i} the subscript $-i$ highlights that all outside lenders $\neq i$ face this break even collateral on the entire portfolio of lender i. In the remainder of this paper $-i$ will mean any of the outside lenders. If an individual outside lender, or his strategy, is mentioned j and k will be used.

From the expected return equation it follows that the break even collateral for lenders $-i$ on borrowers from i's portfolio is

$$\underline{C}_{-i} = 1 - \frac{(\alpha_i p + \beta_i q)r}{\alpha_i(1-p) + \beta_i(1-q)}$$

It is clear that in this case the break even collateral is a weighted average of the low risk borrowers and the high risk borrowers in the portfolio of lender i. The collateral decreases when the share of low risk borrowers goes up, since the average risk of default goes down. Similarly the collateral increases with the share of high risk borrowers as the average default risk goes up.

Naturally, the break even collateral for a low risk borrower is smaller than the break even collateral for a high risk borrower. And since the break even collateral for the entire portfolio j is a weighted average of both break even collaterals, it is in between the low risk and the high risk break even collaterals. When the share of high risk borrowers in a portfolio increases, the break even collateral on the entire portfolio is closer to the break even collateral on high risk borrowers but will never surpass it. The opposite holds when the share of high risk borrowers decreases, the break even collateral on the entire portfolio will be close to the low risk portfolio but never below this point.

Moreover, the break even collateral on an individual low risk or high risk borrower is static, since these collaterals only depend on exogenous variables. Figure one provides a view of the three break even collaterals and their position.

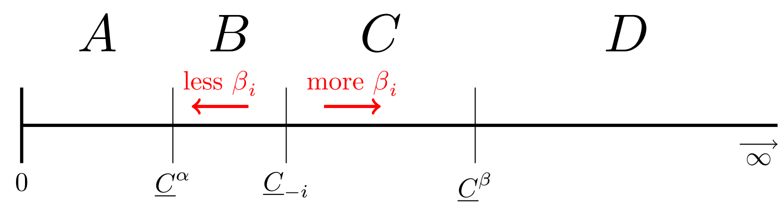

Figure 1: break even collaterals for portfolio of lender i.

Area A

Area A in figure one represents the amounts of collateral that are not sufficient to break even on the low risk borrowers nor the high risk borrowers. No lender would demand collateral in this area as any lender would make a loss on average.

Area B

Since the inside lender i, is able to verify the risk type of the borrowers in his portfolio, he can demand an amount of collateral from his low risk borrowers in the area B, and make a profit on average. The outside lender is unable to verify the risk type of the borrowers in the portfolio of lender i and would on average make a loss when the amount of collateral he demands falls in area B. This is because he will attract both the high risk and low risk borrowers from portfolio i.

Area C

Both lenders can make positive expected profits if they charge an amount of collateral that falls in area C. However, lender i will never charge an amount of collateral in area C to his high risk borrowers. Because the inside lender can verify the type of borrower and thus knows that this amount of collateral is not sufficient to break even on high risk types. Of course, the expected profits of the inside lender on amounts of collateral in area C are higher than the expected profits of the outside lenders.

Area D

Lastly, any amount of collateral in area D is larger than the high risk break even collateral. Demanding collateral of this amount would thus give both the inside and the outside lenders a positive expected profit on both types of borrowers.

As it is assumed that profits are zero for the outside lenders, it must be that the outside lenders never obtain any borrowers when they demand an amount of collateral in area D. Next to this, since the inside lender will never demand an amount of collateral from his high risk borrowers that is outside of area D, all high risk borrowers will switch to a competitor if this competitor demands collateral of an amount in area C. Lastly, the outside lenders will never demand an amount of collateral in area B as they will expect to make a loss. The inside lender can charge any amount of collateral within this area and be ensured that his borrowers do not receive a better offer from the competitors.

The dilemma for the outside lenders $-i$ is thus that if they want to obtain the profitable low risk borrowers, the outside lenders need to make competitive offers in area C to all borrowers in portfolio i. Surely the outside lenders will obtain all the high risk borrowers and make a loss on them, but it is not so certain that the outside lender will also obtain any low risk borrowers. Let alone that the profit on low risk borrowers offsets the losses on high risk borrowers. To deal with this dilemma, a clever strategy is needed.

3.2 Strategies

In the model with three lenders, $L_i(C)$ represents the strategy that lender i uses to set the collateral to his low risk borrowers. $H_i(C)$ is the strategy of lender i regarding the collateral for his high risk borrowers. Lastly, $U_{-i}(C)$ is the strategy that lenders $-i$ use to set the collateral for borrowers in the portfolio of lender i.

$L_i(C)$, $H_i(C)$, and $U_{-i}(C)$ are cumulative distribution functions (CDF) depending on the amount of collateral that lender i or $-i$ demands from the different types of borrowers. Take $L_i(C)$ as an example. Being a CDF implies that $L_i(C)$ takes values on the interval $[0,1]$. Consider that the inside lender i demands a collateral C_i^{α} then

$$\Pr\left\{C \leq C_i^{\alpha}\right\} = L_i(C)$$

hence the value of $L_i(C)$ gives the probability that the amount of collateral demanded C is equal or smaller than C_i^{α}. Similarly

$$\Pr\left\{C \leq C_i^{\beta}\right\} = H_i(C)$$

$H_i(C)$ gives the probability that the amount of collateral demanded C is equal or smaller than C_i^{β}, and

$$\Pr\left\{C \leq C_{-i}\right\} = U_{-i}(C)$$

$U_{-i}(C)$ gives the probability that the amount of collateral demanded C is equal or smaller than C. The range of values that C can take is called the support. Where $\underline{c}$ is the value of collateral at the bottom of the support and $\overline{c}$ is the upper limit of the support, taking again $L_i(C)$ as an example so that:

$$L_i(C) = \begin{cases} 0 & \text{if } C < \underline{c} \\ [0,1] & \text{if } C \in [\underline{c}, \overline{c}] \\ 1 & \text{if } C > \overline{c} \end{cases}$$

The inside lender never demands any amount of collateral strictly smaller than $\underline{c}$ from his low risk borrowers. And for any value of collateral above $\overline{c}$ there is no chance that the inside lender will demand this amount from his low risk borrowers. So there is a one-hundred percent chance that the demanded amount of collateral is equal to or smaller than $\overline{c}$. Since $L_i(C)$ is a CDF, it must be that it is a non-decreasing function.

A pure strategy is one in which the lender demands from all his low risk borrowers a single amount of collateral and never more or less than that amount. When the lender uses a pure strategy, the support consists of a single point $\underline{c} = \overline{c}$ and $L_i(C)$ either takes the values zero or one.

As mentioned before each lender has four simultaneous collateral setting strategies. For the low risk borrowers in his portfolio, strategy $L_i(C)$ sets a collateral that maximises the expected return

$$\max_C E\left[\underbrace{(1-U_j(C))(1-U_k(C))}_{\text{Probability of retaining low risk}}\ \underbrace{(p(1+r)+(1-p)C_i^{\alpha}-1)}_{\text{Expected return on low risk}}\alpha_i\right] = v_i^{\alpha}$$

As $U_{-i}(C)$ is the probability that an uninformed lender sets a collateral equal to or smaller than C, the probability that the uninformed lender sets a collateral above C is $1 - U_{-i}(C)$. Lender i retains his low risk lenders when both outside lenders demand a higher collateral than he does.

Next to this, strategy $H_i(C)$ maximises the returns of lender i with regard to his high risk borrowers

$$\max_C E\left[(1-U_j(C))(1-U_k(C))\left(q(1+r)+(1-q)C_i^{\beta}-1\right)\beta_i\right] = v_i^{\beta}$$

Finally, $U_j(C)$ is the collateral setting strategy that maximises the expected returns of lender j with regard to the borrowers of lenders i. The maximisation problem of lender j regarding the borrowers of lender i is

$$\max_C E\left[\begin{array}{c}(1-L_i(C))(1-U_k(C))(p(1+r)+(1-p)C_j-1)\alpha_i\\+(1-H_i(C))(1-U_k(C))\left(q(1+r)+(1-q)C_j-1\right)\beta_i\end{array}\right] = v_j$$

As mentioned before, it is assumed that uninformed lenders can verify to which portfolio a borrower belongs although the borrower's type is private information. Thus $U_j(C)$ maximises the expected return of lender j on the borrowers of lender i taking in account the behaviour of lender k who is also uninformed and competing for the portfolio of lender i. Whereas the fourth strategy $U_k(C)$ maximises the expected return of lender k on the borrowers of lender i.

The time line of this theoretical model is as follows. Firstly, all lenders observe the risk ratios in each portfolio, and in their own portfolios they observe who is a low risk and who is a high risk borrower. Next the lenders simultaneously make offers to all the borrowers trying to maximise their profit as explained above. The borrowers observe who has made the best offer, that is who demands the lowest amount of collateral, and choose that lender. In case two outside lenders make the lowest offer to a borrower, the borrower will flip a coin. In case the best offer is a tie between the inside and an outside lender, the borrower switches to the outside lender. The lenders evaluate their profits and after one period the model comes to an end.

Solving the model and its maximisation problems will lead to a Nash equilibrium, a powerful equilibrium concept named after its pioneer John Nash. This is a game of complete information, therefore the equilibrium concept is not a Bayesian Nash equilibrium. It may sound contradictionary to say that this game is a game of complete information, since information asymmetry is central to the model's problem. However, this information asymmetry is fully known to all lenders and more importantly there is no uncertainty about the characteristics of the lender's themselves nor their actions. The information asymmetry is in the type of the borrowers, which cannot be verified by outside lenders.

So the lenders have a complete information set about the other lenders, the risk ratios of their portfolios are known, and their maximisation problems are common knowledge. Each lender can therefore form a complete belief about the other lender's actions, by reasoning what the optimal behaviour of his competitors would be. Consequently, each lender can determine which of his actions are a best response to the beliefs he has formed about his competitors. Plainly, what is the best response of lender i given the believes that he has about the actions of the other lenders $-i$.

After reviewing his possible actions, each lender devises a strategy which is the best response given the strategies that the other lenders would play. The Nash equilibrium is thus a set of strategies for every lender i, in which every strategy is a best response to the strategies of the other lenders. The strategies are mutual best responses. In this case, no lender wants to change his strategy, as everyone is playing his best response strategy. Implying that at the Nash equilibrium no lender can make a higher profit by changing his strategy, given the strategies of his competitors.

Before continuing with solving this model, it might be useful to briefly summarise the model. On this credit market there are three lenders, each lender can verify whether his clients are low or high risk borrowers, but only knows the risk ratio of his competitors' portfolios. Each lender devices four strategies, two regarding the borrowers in his own portfolio, and one for each competitor's portfolio, from which he tries to capture borrowers.

4 Analysis without collateral floor

The strategies of all lenders are similar to the strategies of lender i, only the subscripts vary. Therefore the equilibrium consists of the strategies $L_i^*(C)$, $H_i^*(C)$, $U_j^*(C)$, $U_k^*(C)$. Which describes the strategies of all lenders regarding the borrowers of inside lender i.

In equilibrium, no lender must have an incentive to change his strategy, taking in account the strategies that his opponents use. As shown by Kaplan and Wettstein (2000) to ensure that no lender has an incentive to change his strategy, a strategy needs to meet the following two conditions.

1. *The expected profit of the lender when demanding an amount of collateral is the same for any point on the support*

2. *The expected profit of the lender when demanding an amount of collateral on the support is equal or larger than the expected profit for any point outside the support.*

4.1 Pure strategies

Standard models of price competition predict a pure strategy equilibrium in which profits are zero and firms set price equal to the marginal cost. Under marginal cost pricing, the support consists of a single point, thus condition one is satisfied by default. The second equilibrium condition is satisfied as pricing below the marginal cost results in losses on all clients. Next to this, pricing above the marginal cost will not attract any clients and result in zero profits. Thus any point outside the support gives equal or less profits. This pure strategy equilibrium is known as the Bertrand equilibrium and implies that price competition between two firms will result in pricing equal to the marginal cost. The Bertrand equilibrium of price competition is unique because under quantity competition, this result is not obtained when just two firms compete but only when many firms compete with each other.

The asymmetric information model of a lending market, as described in section three, is not a standard price competition model, and the Bertrand competition result is not obtained. When an outside lender captures borrowers from the inside lender at a profitable amount of collateral, the other outside lender will be quick to undercut him and take the profit. Moreover, the inside lender can allways undercut the outside lenders to defend his borrowers and he can go further than the outside lenders because he has an informational advantage. This makes that there is no pure strategy equilibrium.

Lemma 1 *There exists no pure strategy equilibrium in which lender i offers his low risk borrowers a contract with a fixed collateral $C_i^\alpha \in R^+$ and the outside lenders offer a contract with a fixed collateral $C_{-i} \in R^+$ to the borrowers of lender i.*

Proof. see appendix ■

Von Thadden (2004) provides a similar proof, although his model features two periods and is a model of incomplete information. He argues that the outside lenders suffer from a winner's curse (Von Thadden, 2004). If they manage to win over the borrowers from the inside lender it must be that they made an attractive offer to the borrowers and that the inside lender did not want to make an equally attractive bid, signalling that the bid is not profitable (Von Thadden, 2004).

Suppose that lender i demands with certainty a collateral $C_i^\alpha \in [\underline{C}_{-i}, \underline{C}^\beta)$ from the low risk borrowers in his portfolio. This is depicted by figure two, the vertical red dotted line denotes the mass point of mass one corresponding with a pure strategy. The outside lenders' best response is to demand an amount of collateral that matches the insides lender's demand C_i^α and gain all the low risk borrowers from portfolio i. In figure two, this is illustrated with a blue dotted line.

Now, the inside lender's best response is to undercut the outside lenders so his mass point moves following arrow number one. The movement is exaggerated in figure two for the purpose of clarity. In reality the inside lender would want to demand a collateral only marginally smaller than the lowest amount demanded by any of the two outside lenders.

When the inside lender changes his strategy, this also changes the best response of the outside lenders. Again, the outside lenders want to demand the same amount of collateral as the inside lender does, thus their mass point moves along arrow two.

The undercutting by the inside lender and matching by the outside lenders will go on until $C_i^\alpha < \underline{C}_{-i}$, this process of undercutting is illustrated by arrow three. Now the inside lender demands a collateral below the break even amount that the outside lenders need to demand. The outside lenders will no longer undercut the inside lender because they will expect to make a loss. Moreover, when the outside lenders demand exactly the portfolio break even amount of collateral $\underline{C}_{-i}$, they will only attract high risk borrowers because $C_i^\alpha < \underline{C}_{-i}$, and make a loss.

The best response of the outside lenders is to demand a collateral $C_{-i} = \underline{C}^\beta$ in this case they will only obtain high risk borrowers and make zero profit. Thus the mass point of the outside lenders shifts, following arrow four.

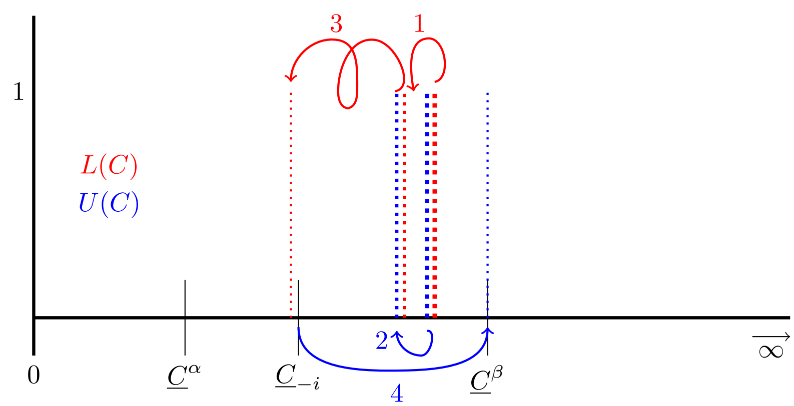

Figure 2: pure strategies and best responses.

This cannot be an equilibrium either as the inside lender now has an incentive to demand a collateral from his low risk borrowers marginally lower than $\underline{C}^\beta$. So that he maximises the expected profit on low risk borrowers. This will again prompt a change in strategies by the outside lenders and the reasoning starts over again.

Having proved that pure strategies are not an equilibrium, because they do not satisfy the equilibrium condition that no lender must have an incentive to change his strategy. The next section of this paper will deal with establishing an equilibrium in which lenders use mixed strategies. A mixed strategy is a strategy where the lender assigns a probability to the various amounts of collateral that he can demand from the borrowers. This way, lenders randomise the amount of collateral they demand over an interval

4.2 Mixed strategies

By assumption, the outside lenders make zero profit. The inside lender sets his strategy $L_i^*(C)$ so that in expectation the outside lenders always make zero profits. Thus the expected losses on capturing β_i high risk borrowers must be offset by the expected profits on capturing α_i low risk borrowers. Furthermore, since the inside lender i has an informational advantage, he will capture informational rents. These rents exist because the outside lenders will never demand a collateral $C_{-i} < \underline{C}_{-i}$. Given that the inside lender can verify the risk type of the borrowers in his portfolio, he will expect to make a profit on his low risk borrowers as the low risk break even collateral $\underline{C}^{\alpha} < \underline{C}_{-i}$.

Proposition 2 *In the model as described above, there exists a mixed strategy equilibrium. The collateral that the informed lender demands from his low risk borrowers is set according to the strategy $L_i^*(C)$ with*

$$L_i^*(C) = 1 - \tfrac{\beta_i}{\alpha_i}\left(\tfrac{1-q(1+r)-(1-q)C}{p(1+r)+(1-p)C-1}\right)$$
$$\textit{on the support } C \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$$

The collateral demanded from the borrowers in the portfolio of lender i by the outside lenders is set with strategy $U_{-i}^{i*}(C)$

$$U_{-i}^*(C) = 1 - \sqrt{\left(\tfrac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)\left(\tfrac{1}{p(1+r)+(1-p)C-1}\right)}$$
$$\textit{on the support } C \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$$

At the upper boundary there exists a masspoint with a size of

$$1 - U_{-i}^*(1 - \frac{qr}{1-q}) = \sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p) + \beta_i(1-q)}}$$

As $U_{-i}^*(1 - \frac{qr}{1-q}) = 1$ the inside lenders strategy demanding collateral from his high risk borrowers is

$$H_i^*(C) : C \geq \underline{C}^{\beta}$$

In the equilibrium, the informed lender earns informational rents on his low risk borrowers $\pi_i^{\alpha} = \frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\alpha_i$

Proof. see appendix ■

Figure three provides a plot of the CDF's $L_i^*(C)$ and $U_{-i}^*(C)$ corresponding with the equilibrium strategies.

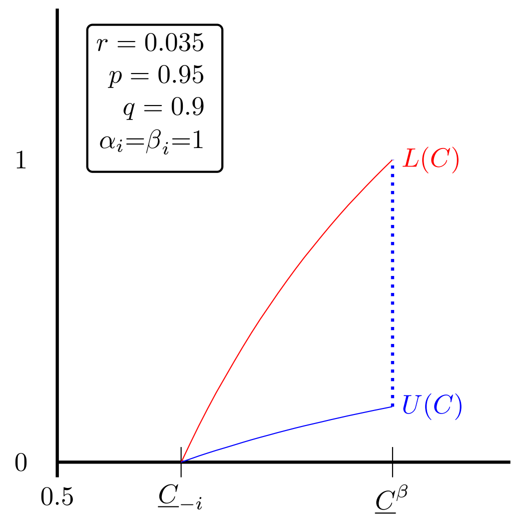

Figure 3: plot of strategies.

α_i, β_i, p, q, and r are arbitrary values. The blue dotted line indicates the size of the mass point at the upper bound of $U_{-i}^*(C)$. With the parameters set as in figure three, the masspoint has a size of 0.816, meaning that each outside lender demands the high risk break even collateral $\underline{C}^{\beta}$ in about 82 percent of the cases.

As explained in section three, $L_i^*(C)$ and $U_{-i}^*(C)$ give the chance that a certain collateral C or less is demanded. The red strategy $L_i^*(C)$ is always above the blue $U_{-i}^*(C)$. Thus for any amount of collateral C on the interval $\left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$, the chance that the inside lender demands this amount or less is larger than that the outside lender demands it. In other words, the inside lender demands more competitive amounts of collateral from his low risk borrowers. This is a consequence of the inside lender i's ability to verify the risk type of each borrower in his portfolio.

Before this paper continues with studying the effects of policy measures, the average collateral demanded from borrowers is calculated. This is important because it allows to compare the position of the borrowers before and after market intervention. To find out how much collateral is demanded on average, all possible collaterals C should be summed and weighted by the probability with which they occur. For the collateral that the informed lender i demands from the low risk borrowers this is:

$$\widetilde{C}_i = \int_{\underline{C}_{-i}}^{\underline{C}^{\beta}} l_i^*(C) C dC$$

where $l_i^*(C)$ is the probability density function that corresponds with $L_i^*(C)$, taking the first derivative of $L_i^*(C)$ gives:

$$l_i^*(C) = \frac{d}{dC} L_i^*(C) = \frac{\beta_i (p-q) r}{\alpha_i (p(1+r) + (1-p)C - 1)^2}$$

the average collateral $\widetilde{C}$ demanded by the informed bank from its low risk lenders is

$$\widetilde{C}_i = \frac{\beta_i}{\alpha_i} \frac{(p-q)r}{(1-p)^2} \ln \left(\frac{\alpha_i (1-p) + \beta_i (1-q)}{\beta_i (1-q)} \right) + 1 - \frac{pr}{1-p}$$

Both outside lenders bid for the borrowers in lender i's portfolio. However, the borrowers are only interested in the minimum amount of collateral demanded since they will choose the lender that offers them the contract with the lowest amount of collateral. Thus instead of calculating the average collateral for $U_{-i}^*(C)$, the average of $\min U_{-i}^*(C)$ should be calculated. The methodology for calculating the average minimum amount of collateral demanded by both outside lenders is based on Bijkerk and De Vries (2017) who calculate the minimum average interest rate. In this case $\left(1 - U_{-i}^*(C)\right)^2$ is the probability that both outside lenders demand a collateral larger than C. So that

$$\begin{aligned}
\min U_{-i}^*(C) &= 1 - \left(1 - U_{-i}^*(C)\right)^2 \\
\min U_{-i}^*(C) &= 1 - \frac{\beta_i (p-q) r}{\alpha_i (1-p) + \beta_i (1-q)} \frac{1}{p(1+r) + (1-p)C - 1}
\end{aligned}$$

is the probability that at least one of the borrowers demands a collateral smaller than C. To calculate the minimum average collateral demanded by the outside lenders:

$$\widetilde{C}_{-i} = \int_{\underline{C}_{-i}}^{\underline{C}^\beta} u_{-i}^{\min *}(C) C dC + \left(1 - U_i^*\left(\underline{C}^\beta\right)\right)^2 \underline{C}^\beta$$

Where u_{-i}^* is the PDF corresponding with $\min U_{-i}^*(C)$:

$$u_{-i}^{\min *} = \frac{d}{dC} \min U_{-i}^*(C) = \frac{\beta_i (p-q) r}{\alpha_i (1-p) + \beta_i (1-q)} \frac{1-p}{(p(1+r) + (1-p)C - 1)^2}$$

so that

$$\widetilde{C}_{-i} = \left[\begin{array}{c} \frac{\beta_i (p-q) r}{(1-p)(\alpha_i (1-p) + \beta_i (1-q))} \ln \left(\frac{\alpha_i (1-p) + \beta_i (1-q)}{\beta_i (1-q)} \right) \\ +1 - \frac{\beta_i q r + \alpha_i p r}{\alpha_i (1-p) + \beta_i (1-q)} \end{array} \right]$$

$\widetilde{C}_i < \widetilde{C}_{-i}$ for every value of the parameters p, q, r, α_i and β_i. The average collateral demanded by the outside lenders is higher because of the mass point at the upper bound of the support $\underline{C}^\beta$. It is unnecessary to account for the average collateral that the inside lender demands from his high risk borrowers, since high risk borrowers always switch to the outside lenders.

5 Analysis with a collateral floor

One policy that authorities can apply to credit markets in general, and the lenders in this model specifically, is to impose a collateral floor on loans. This means that the lender is compelled to demand at least a certain level of collateral for every loan it makes. Collateral floors on credit markets are similar to the minimum wage in the labour market. In the case of a collateral floor the lender can still demand any collateral larger than the collateral floor but he cannot demand a collateral below it.

A collateral floor can be an explicit rule: "banks need to demand at least an amount of collateral of size X for each loan of size Z ". But collateral floors can also arise more implicit from other regulations. The ECB requires commercial banks to provide collateral when they draw on ECB funds. Commercial banks often provide assets that are based on the loans in their portfolio. However, the ECB does not accept just any asset backed by a loan. Shortly after the crisis, it accepted only those assets that had a 'triple A' rating. Commercial banks channel these rating requirements to the borrowers in their portfolio. When the ECB imposes a rating requirement on the assets of banks, the banks will have to evaluate their portfolios and make sure that they are safe enough. This can have two effects, borrowers need to provide more collateral, an implicit collateral floor, so that the losses on loans are lower in case the project fails. On the other hand, banks can make less loans to risky borrowers, so that their portfolio is less risky and complies with the rating requirement. Since banks are not able, or willing, to screen all SME borrowers, the likely case is that the bank will install a collateral requirement for SME borrowers.

In 2011 the ECB lowered the rating requirements for many assets, one of the goals was to stimulate the markets for SME and private lending. The rating requirements for assets based on SME loans was first lowered to 'single A' and subsequently to 'single B' *(Aberg, 2013)*. Unlike the interest rate target and the quantitative easing program, these policy changes do not attract widespread attention in news media. Nonetheless, the channels that these policy changes work trough can have substantial impacts on credit markets.

The remainder of section five will start with analysing the effect of a collateral floor under the uninformed takes all tie-sharing rule that was also assumed in section four. Section six will then show what happens under the opposite tie-sharing rule, when no borrowers switch from inside to outside lenders upon a tie.

5.1 Uninformed takes all

This subsection will analyse the consequences of a collateral floor imposed on the model of the asymmetric information credit market. Under the tie-sharing rule that the uniformed lender obtains all borrowers in case of a tie between the inside and an outside lender. Recall that this is the extreme tie sharing rule that was assumed throughout sections three and four of this paper. Thus it is a natural starting point to start the analysis of a collateral floor imposed on the credit market.

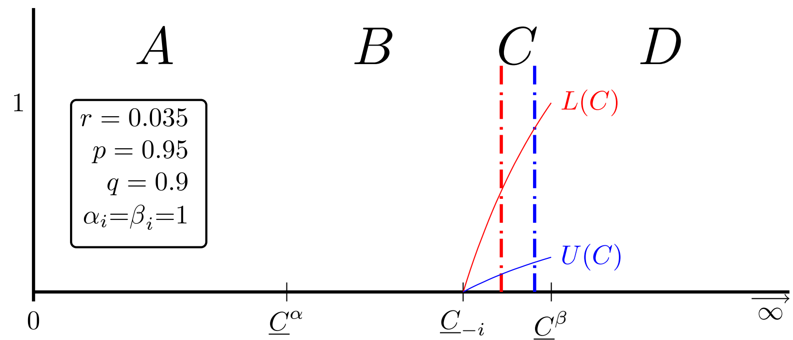

Figure 4: equilibrium strategies and average collaterals without collateral floor.

Figure four displays the equilibrium strategies in a market free from intervention. The dash-dotted lines depict the average collateral that the inside and outside lenders demand. As established above, the blue dash-dotted line gives the average lowest collateral that both outside lenders demand. The collateral floor, that is the minimum amount of collateral that a lender must demand, is denoted C_{floor}.

Section four showed that in a free market equilibrium, the collateral demanded C by any lender is always larger than $\underline{C}_{-i}$. Thus when $C_{floor} < \underline{C}_{-i}$ the collateral floor will have no effect, this corresponds with areas A and B in figure four. The collateral floor will not have an effect if it is smaller than $\underline{C}_{-i}$ because the inside lender can still use his informational advantage. The inside lender can always undercut the outside lenders at $\underline{C}_{-i}$ but the outside lenders can no longer retaliate. This advantages leads to the informational rent that the inside lender obtains in a free market.

If the collateral floor is placed at an amount of collateral within area C, the collateral floor will interfere with the strategies $L_i^*(C)$ and $U_i^*(C)$. In that case, the inside lender can no longer use his informational advantage because he can no longer undercut the outside lenders further than they can retaliate.

Proposition 3 *under the tie-sharing assumption that all indifferent borrowers switch upon a tie, there exists a pure strategy equilibrium after installing a sufficiently high collateral floor:*

$$\begin{array}{ll} L_i^*(C) & : C_i^{\alpha} \geq C_{floor} \\ U_{-i}^{i*}(C) & : C_{-i} = C_{floor} \\ H_i^*(C) & : C_i^{\beta} \geq C_{floor} \end{array}$$

when $C_{floor} \geq \underline{C}_{-i}$

Proof. see appendix ■

The lenders can no longer demand a collateral below C_{floor}, which implies that no lender can undercut his competitors at C_{floor}. If the outside lender demands a collateral equal to $C_{floor} \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right]$ from all lenders in portfolio i, all high risk borrowers will switch since the inside lender demands a collateral equal to or largen than C_{floor} from his high risk borrowers. When the inside lender i demands a collateral larger than C_{floor} he will be undercut by the outside lenders and his borrowers will leave his portfolio. Because of the tie-sharing rule, borrowers always switch to the outside lender upon an inside-outside lender tie, the outside lenders will also attract all the low risk borrowers when the inside lender i demands a collateral equal to C_{floor}.

At this pure strategy equilibrium, the inside lender has zero profit on both high and low risk borrowers in his portfolio since he always loses them. This implies that the inside lender i is indifferent between making an offer to his borrowers and not making an offer at all, both actions will give him zero profit. If the inside lender chooses to not make any offers to his borrowers, there will still be competition between the two outside lenders. Since there is no information asymmetry between the two outside lenders this competition will lead to the Bertrand outcome. Then the amount of collateral demanded by both outside lenders will equal the break even collateral or the collateral floor if $C_{floor} > \underline{C}_{-i}$. Thus the equilibrium outcome does not changes when the inside lender i chooses to participate or abstains from participating.

The outside lenders now have an expected profit on the borrowers in portfolio i

$$v_{-i} = \frac{(p(1+r)+(1-p)C_{floor}-1)\alpha_i + (q(1+r)+(1-q)C_{floor}-1)\,\beta_i}{2}$$

For $C_{floor} = \underline{C}_{-i}$ it will be that $v_{-i} = 0$ as this is the portfolio break even collateral.

$$\begin{aligned}
v_{-i} &= \frac{(p(1+r)+(1-p)\underline{C}_{-i}-1)\alpha_i + \left(q(1+r)+(1-q)\underline{C}_{-i}-1\right)\beta_i}{2} \\
&= \left[\begin{array}{c} (p(1+r)+(1-p)\left(1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)-1)\frac{\alpha_i}{2} \\ +\left(q(1+r)+(1-q)\left(1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)-1\right)\frac{\beta_i}{2} \end{array}\right] \\
&= 0
\end{aligned}$$

For any value $C_{floor} > \underline{C}_{-i}$ the collateral is higher than the portfolio break even collateral. Thus the profit of the outside lenders v_{-i} will be positive as well. Since free entry is assumed, this would trigger the inflow of many lenders, who will share the profit among a larger number so that profits become virtually zero.

The position of the collateral floor within area C determines wether borrowers are made better off or worse off. Given that the collateral floor is in area C, the following can be established about the borrowers in portfolio i. As long as $C_{floor} < \widetilde{C}_i$ both the low risk and high risk borrowers will be better off on average. When $\widetilde{C}_i < C_{floor} < \widetilde{C}_{-i}$, low risk borrowers will most likely be worse off on average. However the high risk borrowers will pay a smaller amount of collateral on average, thus they are better off. If $\widetilde{C}_{-i} < C_{floor}$ both types of borrowers will be worse off on average.

A collateral floor in section D, that is $C_{floor} > \underline{C}^{\beta}$ will unambiguously make borrowers worse off, not even on average. As borrowers now need to provide more collateral than they had to provide on the free market. Before a collateral floor is imposed, the strategies of all lenders are as defined in proposition two. The supports of $L_i^*(C)$ and $U_{-i}^*(C)$ are $C \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$ where $U_{-i}^*(C)$ has a mass point at the top of the support. This means that an amount of collateral larger than $\underline{C}^{\beta}$ is never demanded without a collateral floor. Therefore a collateral floor in section D always makes borrowers worse off.

In the case of $C_{floor} > \underline{C}^{\beta}$ the outside lenders will make positive profits on both the high and low risk borrowers. As the mandatory demanded amount of collateral is above the break even collateral for the high risk borrowers.

Note that under this particular tie-sharing rule, the best outcome for borrowers would occur when $C_{floor} = \underline{C}_{-i}$. Then all lenders make zero profit and borrowers provide a collateral equal to the lowest collateral demanded on the free market.

6 Extension: opposite tie-sharing rule

This section will analyse the impact of a collateral floor on an adverse selection credit market under the assumption that borrowers do not switch when they receive equal offers from the inside lender and an outside lender. Changing the tie sharing rule also changes the equilibrium when a collateral floor is installed on the market.

A free market, without a collateral floor, there is no effect of changing the tie-sharing rule. On a market free from collateral floors, the inside lender can fully use his informational advantage to keep his competitors down to zero profits. In essence, the inside lender would always be able to undercut the outside lenders and the tie-sharing rule does not affect his informational power. A deeper analysis of a free market under this alternative tie-sharing rule is referred to appendix two.

This extension will primarily analyse the consequences of a collateral floor with a different tie-sharing rule. Section five showed that when all borrowers switch upon a tie, the inside lender loses his informational advantage when the authorities impose a collateral floor $C_{floor} \geq \underline{C}_{-i}$. With a collateral floor the inside lender can no longer undercut the outside lenders below C_{floor} and since the tie-sharing rule does not work in his favour the inside lender has no market power.

In this section it is assumed that indifferent borrowers do not switch in case of a tie. This gives the inside lender greater market power because he will retain his borrowers upon a tie. Since the inside lender can verify the riskiness of the borrowers in his portfolio, a tie often implies that the low risk borrowers stay with the inside lender and the high risk borrowers leave his portfolio. Meaning that the outside lenders attract only high risk borrowers which is unprofitable as long as $C_{floor} < \underline{C}^{\beta}$. Hence the pure strategy equilibrium from the previous section breaks down partially, under the assumption that no borrowers switch when the inside lender ties with an outside lender.

Proposition 4 *A mixed strategy equilibrium arises when* $C_{floor} \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$ *under the inside lender retains all rule. The equilibrium strategies are*

$$L_i^*(C) = 1 - \tfrac{\beta_i}{\alpha_i}\left(\tfrac{1-q(1+r)-(1-q)C}{p(1+r)+(1-p)C-1}\right)$$
on the support $C \in \left(C_{floor}, \underline{C}^{\beta}\right]$

with a masspoint at the lower boundary when $C_{floor} > \underline{C}_{-i}$*. The size of the masspoint is directly related to the level of the collateral floor* C_{floor}*:*

$$L_i^*(C_{floor}) = 1 - \frac{\beta_i}{\alpha_i}\frac{\alpha_i\,(p-q)\,r - \varepsilon\,(1-q)\,(\alpha_i(1-p)+\beta_i(1-q))}{\beta_i\,(p-q)\,r + \varepsilon\,(1-p)\,(\alpha_i(1-p)+\beta_i(1-q))}$$

Let $C_{floor} = 1 - \frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)} + \varepsilon$*, with* $\varepsilon \in \left(0, \frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)} - \frac{qr}{1-q}\right)$*. Thus an increase in* ε *indicates a higher level of the collateral floor, resulting in an increase of the mass point at the bottom of the support.*

$$U_{-i}^*(C) = 1 - \left(\left(\tfrac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)} + (1-p)\,\varepsilon\right)\left(\tfrac{1}{p(1+r)+(1-p)C-1}\right)\right)^{0.5}$$
on the support $C \in \left[C_{floor}, \underline{C}^{\beta}\right)$

with a mass point at the upper bound of the support. The size of the masspoint at the upper bound is dependent on the level of ε *and thus directly related to* C_{floor}*:*

$$1 - U_{-i}^*\left(\underline{C}^{\beta}\right) = \sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)} + \frac{(1-p)\,(1-q)\,\varepsilon}{(p-q)\,r}}$$

Furthermore, the inside lender demands from his high risk borrowers any collateral equal to or larger than $\underline{C}^{\beta}$

$$H_i^*(C) : C_i^{\beta} \geq \underline{C}^{\beta}$$

Proof. see appendix ■

Under the informed lender keeps all tie-sharing rule, the equilibrium that arises for $C_{floor} \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$ is strikingly similar to the equilibrium in a credit market without collateral floor. Still, the outside lenders have zero expected profit. However, the informational rent of the inside lender increases with the collateral floor:

$$\pi_i^{\alpha} = \left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)} + (1-p)\,\varepsilon\right)\alpha_i$$

This implies that borrowers have to provide more collateral when the collateral floor increases. The borrowers pay for the inside lender's increased profit.

Corollary 5 *Under the informed keeps all tie-sharing rule, a collateral floor larger than $\underline{C}_{-i}$ makes all borrowers worse off on average.*

Proof. see appendix ■

All borrowers have to provide more collateral on average. Under the tie-sharing rule that no borrowers switch from inside to outside lenders, the inside lender uses his market power and informational advantage to keep the outside lenders at zero profits.

At the same time, the average collaterals demanded by all lenders increases. In effect, when indifferent borrowers never switch, a collateral floor $C_{floor} \in \left(\underline{C}_{-i}, \underline{C}^{\beta}\right)$ softens the competition between inside and outside lenders.

7 Results

Corollary six gives rise to an interesting thought. If borrowers could collude with each other and agree to always switch when they face two similar offers, the pure strategy equilibrium would exist for a collateral floor equal to the portfolio break even collateral $\underline{C}_{-i}$. By colluding they could influence the equilibrium outcome of the credit market. A collateral floor equal to or even slightly above $\underline{C}_{-i}$ could then significantly lower the average collateral demanded and make all borrowers better off on average.

It seems unlikely that borrowers would unite with each other and come to the agreement to always switch if they are indifferent. Nonetheless, there would be a business opportunity for a middleman, a credit broker of some sort. Who would represent a pool of borrowers at the bank and whenever he receives two similar offers for one of his clients, he switches. This thought gives rise to a whole different analysis of adverse selection, that goes beyond the scope of this paper, nonetheless an interesting thought.

Besides that proposition four and the earlier propositions are interesting from a theoretical viewpoint, they also have a large applicable relevance. The tie-sharing rules are the model's counterpart of client switching between lenders in credit markets. When an authority such as the ECB imposes a collateral floor on an asymmetric credit market, the equilibrium outcome is dependent on the fraction of clients that switches between lenders even when they receive similar offers. In turn, the type of equilibrium together with the level of the collateral floor determines which lender will benefit from a collateral floor policy. And maybe even more importantly, whether the policy improves the welfare of the borrowers in the market.

Table one shows the different types of equilibria that arise after a collateral floor is installed. The type of equilibrium differs with both the level of the collateral floor and the tie-sharing rule.

Note that with a collateral floor below $\underline{C}_{-i}$ the equilibrium is the same as on a market without a collateral floor. This is because the inside lender can fully use his informational advantage, a too low collateral floor will not affect the market outcome.

	All switch	None switch
Above $\underline{C}_{-i}$	Pure	Mixed
Above $\underline{C}^{\beta}$	Pure	Pure

Table 1: Equilibria and tie-sharing rules

When all borrowers switch, proposition four showed that a pure strategy equilibrium arises. When none switch, the equilibrium after a collateral floor is a mixed strategy equilibrium as long as $C_{floor} \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$, proven by proposition five.

Each combination of collateral floor and tie-sharing rule also gives rise to differences in profits and average collateral payable by the borrowers. Table two shows the effect of a collateral floor on the average collateral $\widetilde{C}_i$ provided by the borrowers in the portfolio i under both tie-sharing rules.

Tie sharing rule:	**All switch**		**None switch**	
C_{floor}	$\widetilde{C}_i$	$\widetilde{C}_{-i}$	$\widetilde{C}_i$	$\widetilde{C}_{-i}$
0.568	0	0.568	0.619	0.663
0.597	0	0.597	0.627	0.672
0.627	0	0.627	0.642	0.680
0.656	0	0.656	0.659	0.684
0.685	0	0.685	0.685	0.685
0.700	0	0.700	0.700	0.700

Table 2: Average collateral provided by the borrowers after installing a collateral floor[2].

When a mixed equilibrium arises after a collateral floor is imposed, borrowers will have to provide more collateral on average when $C_{floor} > \underline{C}_{-i}$. Contrary to this, a pure strategy equilibrium might make borrowers better off on average, given that the collateral floor is not too high. With the parameters set as in table two, the outside lender's break even collateral on borrowers in i's portfolio $\underline{C}_{-i} = 0.568$. The break even collateral on high risk borrowers $\underline{C}^{\beta} = 0.685$. Above the high risk break even collateral $\underline{C}^{\beta}$, a collateral floor results in a pure strategy equilibrium regardless of the borrowers' switching behaviour. Moreover, such a high collateral floor is always bad for borrowers. Lastly, under the all switch tie-sharing rule, a pure strategy equilibrium arises, so that the inside lender loses all his borrowers. Hence no collateral is provided to the inside lender and the column $\widetilde{C}_i$ under the all switch rule is empty.

Note also that, for the interval $C_{floor} \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$ the average collateral demanded is always smaller when all borrowers switch than when none switch.

[2] The colours indicate whether the amount is lower or higher than the average collateral provided on the free credit market. $\alpha_i = \beta_i = 1$, $r = 0.035$, $p = 0.95$, $q = 0.9$. Without a collateral floor, the average collaterals are $\widetilde{C}_i = 0.619$ and $\widetilde{C}_{-i} = 0.663$

When all borrowers switch, the outside lenders have the market power but not an informational advantage. They cannot have an informational rent but they do have a rent because of the collateral floor being higher than the break even collateral. Contrary to this, when non of the borrowers switch upon a tie, the inside lender has both market power and informational advantage. This makes that the inside lender can demand higher collaterals under the inside lender retains all tie-sharing rule. Consequently, the inside lender also makes a higher profit under this tie-sharing rule.

On a market free from collateral floors, lenders would not be able to make profits from competing on credit markets, except for the informational rent they obtain. This informational rent stems from the ability of an inside lender to distinguish between the quality of the borrowers in his portfolio. It allows an inside lender to demand from his low risk borrowers a collateral that is lower than what an outside lender could ask, but still high enough to make a profit on these lenders. When a sufficiently high collateral floor is introduced, the capacity of inside lenders to undercut their outside competitors diminishes.

The profits that a lender will expect to make are different for the different equilibria. Naturally, the case when all borrowers switch upon a tie puts a limit on a lender's market power. Whereas the second case, when no borrowers switch, enhances the market power of lenders. This is reflected by table three, which depicts the lenders' profits at varying levels of a collateral floor. The parameters of the model are the same as in table two.

Tie sharing rule:	**All switch**		**None switch**	
C_{floor}	i	$-i$	i	$-i$
0.568	0	0.0000	0.0117	0
0.597	0	0.0022	0.0131	0
0.627	0	0.0044	0.0146	0
0.656	0	0.0066	0.0160	0
0.685	0	0.0088	0.0175	0
0.700	0	0.0099	0.0198	0

Table 3: Expected profits for the individual lenders, after installing a collateral floor [3].

From table three, the following stands out. When all borrowers switch, the inside lender makes zero profit after installing a sufficiently high collateral floor. Under this switching assumption, the market power lies with the outside lenders, consequently they do make a positive profit. On the contrary, when no borrowers switch, the inside lender does not have to fear losing his clients when he ties with an outside lender. This increased market power is reflected by the fact that the inside lender makes positive profits under this tie-sharing rule.

[3] $\alpha_i = \beta_i = 1$, $r = 0.035$, $p = 0.95$, $q = 0.9$. Note that when the collateral floor is below $\underline{C}^{\beta}$ the inside lender always has zero profit on the high risk borrowers and outside lenders would make a loss on high risk borrowers. Without a collateral floor, the outside lenders have zero expected profit. The inside lender would have an expected profit of 0.0117.

Secondly, under both tie-sharing rules, the total level of profits increases with the level of the collateral floor. This is a logical consequence from the fact that the average collaterals increase with the level of the collateral floor. Since the amount of borrowers in the market is constant, when they have to provide more collateral on average, it must be that the expected profits of the lenders increase.

Thirdly, when all borrowers switch, the sum of the profits that the outside lenders make is smaller than the inside lender's profit under the alternative tie-sharing rule. This reflects the fact that a pure strategy equilibrium could lower the average collateral demanded, relative to what is demanded without a collateral floor.

Lastly, note that when $C_{floor} > \underline{C}_{-i}$ and the "all switch" tie-sharing rule applies the inside lender will make zero profits. Hence he is indifferent between making offers and not participating at all. If he does not participate, competition between the outside lenders will lead to the bertrand outcome and they both demand C_{floor}. In contrast, when the "None switch " tie-sharing rule applies, the outside lenders make zero profit and are indifferent to participating. However, when both outside lenders leave the market, the inside lender would be a monopolist and could obtain monopoly profits. It can be concluded that also in the adverse selection credit market, two lenders are needed to reach the competitive equilibrium, similar to a standard price competition model.

Altogether a collateral floor equal to the outside lender's break even collateral $\underline{C}_{-i}$ would be the best for borrowers. And it would be in the best interest of borrowers that they switch every time they receive an equally good offer from a competitor of their house bank.

When no indifferent borrowers switch, the informational rent of the inside lender is positively related to the level of the collateral floor. As explained in section five, the ECB has lowered the requirements for the collateral that commercial banks must provide, implicitely lowering the collateral floor. According to the results from this paper, this would mean that the informational rents of banks in the Eurozone have decreased. Making for example credit providing to SME's in the Eurozone less profitable.

Perhaps this can explain a recent trend in online banking. More and more banks are making loans to SME's via an online process, without ever consulting the entrepeneur in person. ABN AMRO bank offers loans up to one million euros in the Netherlands within two working days, via an online platform (BNR, 2017). ING, another Dutch bank, provides this service in France, Spain, and Italy, for amounts up to 100.000 euros within ten minutes (Economist, 2017). Usually, the process requires the entrepeneur to upload his firm 's income and expenses over the past months as well as his balance sheets. An algorithm then decides whether the borrower is a low risk or a high risk client. Obviously this is a cost-cutting technology, that could be a reaction to lower profitabilty of SME credit.

On the other hand, if the algorthims can perfectly distinghuish between low risk and high risk borrowers, there would no longer be an information asymmetry at all. The outside lenders would then be able to make competitive offers to both low risk and high risk borrowers just as the inside lender can.

8 Conclusion

Comparing sections five and six, as done in the results section above, conveys two important messages. Firstly, the analysis of the credit market including a collateral floor shows that a collateral floor can both increase as well as decrease the welfare of the borrowers. Secondly, whether the welfare effect is positive or negative depends on the switching behaviour of the borrowers in the market. There is not much evidence about borrower switching in case borrowers receive two similar offers. One reasonable assumption is that borrowers do not switch from their current credit provider to another lender if they receive an equally good contract offer. In that case, a collateral floor would always make borrowers worse off if it is higher than the break even collateral in a market without collateral floor.

Of course when the welfare of borrowers decreases, the other side of the coin is that the profits of the lenders increase. High collateral floors lead to high amounts of average collateral demanded, which raises lenders profits if the interest rate and default rates remain constant. For banks, a collateral floor might thus be a policy that they like, as it increases their informational rent as long as indifferent borrowers do not switch.

In a sense, a collateral floor also reduces competition among banks. If no borrowers switch, the uninformed banks will have a larger mass point at $\underline{C}^{\beta}$ when the collateral floor increases. Thus it is less likely that borrowers receive a better contract from the outside banks, softening competition. When all borrowers switch upon receiving equal offers, a pure strategy equilibrium arises. This implies that there is no competition on the amount of collateral demanded at all.

The model developed in section three relies on very few variables. This makes the model easily applicable to different credit market situations. Next to this, the same model could possibly also be applied analyse adverse selection in the insurance market. The model could be adapted to analyse the insurance industry instead of credit markets. The insurance markets also suffer from asymmetric information and very often adverse selection. What would a higher premium or a higher own risk do to the an adverse selection insurance market, and how does this differ with the characteristics of the clients in the insurance market? The health insurance market is a natural point of interest, however in the Netherlands it is forbidden by law to discriminate between clients when offering base health insurance contracts. Nonetheless, adverse selection is present in many other insurance markets and this model could improve the analysis.

Staying in the research area on credit markets, some additions could be interesting for further research. One of the adaptations would be to extend the model into more periods. Right now the credit market only exists of one period, in which all agents act simultaneously. Extending the periods to two, or possibly infintely many periods, could lead to different types of competition.

Lenders will then try to maximise the present value of their entire stream of profits, both in the current period and in future periods.

In this paper, no attention was given to the cost side of the banks. There could be a cost of funds for the bank, however this would mostly be a linear change to the break even collaterals. Most likely it would not affect the strategies of the lenders other than that they would all move to higher amounts of collateral. Also, one could consider fixed costs. Right now, free entry was assumed, implying perfect competition. But if there are sufficiently high fixed costs, it might be unprofitable for new lenders to enter the market. As they cannot obtain informational rents when they do not yet have a portfolio consisting of low and high risk borrowers. Moreover, a higher collateral floor can lead to higher profits for the lenders, thus triggering an inflow of new lenders despite fixed costs.

Another avenue for research would be to extend the analysis of the borrowers further. One could add more levels of riskiness, or perhaps a continuous stochastic distribution of riskiness. Next to this, the borrowers currently only differ in the dimension of riskiness. An interesting analysis might be when borrowers differ in more then one dimension, for example the amount of wealth they invest. Switching costs could also be added to the borrower side of the model.

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A Appendix 1

proof of lemma 1

This proof will show that a pure strategy equilibrium does not exist in the adverse selection loan market. The proof will use the following steps. Step one, the initial setting, in which the inside lender sets a pure strategy for both his low risk and high risk followers. Step two, the best response of the outside lenders to the inside lender's pure strategy. Showing that competition between the outside lenders will drive their demanded collateral down to the break even collateral $\underline{C}_{-i}$. Step three, shows the best response from the inside lender when both outside lenders demand collateral equal to the break even amount. His best response will be to marginally undercut the outside lenders and protect his low risk borrower pool. Step four, shows that the outside lenders make a loss in this case and will thus want to raise the amount of collateral demanded. Step five, to finalise, shows that this is not an equilibrium either. Hence the reasoning repeats itself from step two again in a loop.

Step one:

Assume that the lender i does not discriminate between his borrowers initially. Thus, he demands from all his borrowers, both low and high risk, a collateral $C_i > \underline{C}^{\beta}$ with certainty. More specifically he demands $C_i = \underline{C}^{\beta} + \varepsilon$, where ε can be any arbitrary positive number. In that case his expected payoff on both types of borrowers is positive. On his low risk borrowers he has an expected return of

$$v_i^{\alpha} = \left(p(1+r) + (1-p)\left(\underline{C}^{\beta} + \varepsilon\right) - 1\right)\alpha_i$$

Substitute $\underline{C}^{\beta} = 1 - \frac{qr}{1-q}$ to find

$$\begin{aligned} v_i^{\alpha} &= \left(p(1+r) + (1-p)\left(1 - \frac{qr}{1-q} + \varepsilon\right) - 1\right)\alpha_i \\ &= \left(\frac{(p-q)\,r}{(1-q)} + (1-p)\,\varepsilon\right)\alpha_i \\ &> 0\,\forall\varepsilon \end{aligned}$$

On his high risk borrowers he has an expected profit of

$$\begin{aligned} v_i^{\beta} &= \left(q(1+r) + (1-q)\left(\underline{C}^{\beta} + \varepsilon\right) - 1\right)\beta_i \\ &= \left(q(1+r) + (1-q)\left(1 - \frac{qr}{1-q} + \varepsilon\right) - 1\right)\beta_i \\ &= \beta_i(1-q)\varepsilon \\ &> 0\,\forall\varepsilon \end{aligned}$$

Moreover, his borrowers would switch to another lender if any of his competitors would demand an amount of collateral equal or less than C_i from his borrowers. This is due the tie-sharing rule that all borrowers switch upon a tie. The proof continues to step two.

Step two:
The two outside lenders knowing that the inside lender i uses the pure strategy from step one evaluate their best response to this strategy. The two outside lender cannot distinguish between low risk and high risk borrowers so they have to make a similar offer to both types of borrowers. First comes lender j who knows that if he demands an amount of collateral $C_j = \underline{C}^{\beta} + \varepsilon$ he will capture all borrowers at a profitable rate. He will effectively steal the borrowers and the expected profit of the inside lender as calculated in step one. Moreover if he demands a collateral above C_i he will not capture any borrowers and have zero profit.

The third outside lender, k, now observes that both his competitors demand a collateral equal to $\underline{C}^{\beta}+\varepsilon$. However, when two outside lenders tie, the borrowers flip a coin to choose which outside lender's offer they will take. Thus, the outside lender k has expected profits:

$$v_k = \begin{cases} \begin{bmatrix} (p(1+r)+(1-p)C_k-1)\,\alpha_i \\ +\,(q(1+r)+(1-q)C_k-1)\,\beta_i \end{bmatrix} & \text{if } C_k < \underline{C}^{\beta}+\varepsilon \\ \left(\frac{(p-q)r}{(1-q)}+(1-p)\,\varepsilon\right)\frac{\alpha_i}{2}+\frac{\beta_i}{2}(1-q)\varepsilon & \text{if } C_k = \underline{C}^{\beta}+\varepsilon \\ 0 & \text{if } C_k > \underline{C}^{\beta}+\varepsilon \end{cases}$$

In that case, lender k maximises his profit by demanding an amount of collateral marginally lower than $\underline{C}^{\beta}+\varepsilon$. He would just undercut his competitors and attract all borrowers while his profit is only slightly lower than what the inside lender made under step one. So $C_k < \underline{C}^{\beta} + \varepsilon$ but only by a minimal amount.

Of course lender j would realise this and he now faces the following expected profits:

$$v_j = \begin{cases} \begin{bmatrix} (p(1+r)+(1-p)C_j-1)\,\alpha_i \\ +\,(q(1+r)+(1-q)C_j-1)\,\beta_i \end{bmatrix} & \text{if } C_j < C_k \\ \begin{matrix} (p(1+r)+(1-p)C_k-1)\,\frac{\alpha_i}{2} \\ +\,(q(1+r)+(1-q)C_k-1)\,\frac{\beta_i}{2} \end{matrix} & \text{if } C_j = C_k \\ 0 & \text{if } C_j > C_k \end{cases}$$

Again, he maximises his profits by slightly undercutting his competitor. Then again he captures all the borrowers in the portfolio of lender i and his competitors make zero profit. This will trigger lender k to also undercut lender j by a very small amount. The best responses of both outside lenders to the pure strategy of the other outside lender are to undercut each other.

This undercutting will go on untill both outside lenders demand the portfolio break even rate $\underline{C}_{-i}$. When both outside lenders demand the portfolio break even rate their expected profit functions look as follows:

$$v_{-i} = \begin{cases} <0 & \text{if } C_{-i} < \underline{C}_{-i} \\ \begin{array}{c} \left(p(1+r)+(1-p)\underline{C}^{i}_{-i}-1\right)\frac{\alpha_i}{2} \\ +\left(q(1+r)+(1-q)\underline{C}^{i}_{-i}-1\right)\frac{\beta_i}{2} \end{array} = 0 & \text{if } C_{-i} = \underline{C}_{-i} \\ 0 & \text{if } C_{-i} > \underline{C}_{-i} \end{cases}$$

In this case, none of the outside lenders has an incentive to deviate as they can at most make zero profits. When the inside lender use a pure strategy and demands from his borrowers a collateral $C_i > \underline{C}^{\beta}$ competition between the two outside lenders will drive the amount of collateral they demand down to the break even rate $\underline{C}_{-i}$. This is similar to the bertrand outcome of price competition. However it ignores the response of the inside lender, step three continues here.

Step three:
The inside lender knows that both outside lenders demand a collateral $C_{-i} = \underline{C}_{-i}$. Inside lender i has an informational advantage as he can distinguish between the types of borrowers in his portfolio. Therefore he can demand from his low risk borrowers an amount of collateral $\underline{C}^{\alpha} < C_i^{\alpha} < \underline{C}_{-i}$ in that case he will retain all his low risk borrowers and make a profit on them.

Since C_i^{α} is below the high risk break even amount $\underline{C}^{\beta}$ the inside lender will not demand this amount of collateral from all his borrowers. The expected payoff on his high risk borrowers is:

$$v_i^{\beta} = \begin{cases} <0 & \text{if } C_i^{\beta} < \underline{C}_{-i} \\ 0 & \text{if } C_i^{\beta} \geq \underline{C}_{-i} \end{cases}$$

Thus the inside lender has no incentive to change the amount of collateral demanded from his high risk borrowers. Regarding his low risk borrowers, he will try to maximise his profit by demanding a collateral minimally smaller than $\underline{C}_{-i}$. Say that he demands from his low risk borrowers a collateral $C_i^{\alpha} = \underline{C}_{-i} - \varepsilon$ where ε is a very small but positive number. In this case the payoff on his low risk borrowers is:

$$\begin{aligned} v_i^{\alpha} &= \left(p(1+r)+(1-p)\left(\underline{C}_{-i}-\varepsilon\right)-1\right)\alpha_i \\ &= \left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)} - (1-p)\,\varepsilon\right)\alpha_i \\ &> 0 \;\forall\, \varepsilon < \frac{\beta_i(p-q)r}{(1-p)\left(\alpha_i(1-p)+\beta_i(1-q)\right)} \end{aligned}$$

When both outside lenders demand a collateral $C_{-i} = \underline{C}_{-i}$ the inside lender will use his informational advantage and discriminate between his borrowers.

This will give him zero profit on his high risk borrowers and positive profits on his low risk borrowers.

Step four:

Given that the inside lender simultaneously demands $C_i^\alpha < \underline{C}_{-i}$ from his low risk borrowers and $C_i^\beta > \underline{C}^\beta$ the outside lenders evaluate their expected profits. If they demand a collateral equal to $\underline{C}_{-i}$ they will each attract half ofthe high risk borrowers and since $\underline{C}_{-i} < \underline{C}^\beta$ this will give them a loss. If an outside lender demands a collateral equal or smaller than C_i^α he will attract all borrowers in the portfolio of lender i. However, since this is below the portfolio break even amount of collateral $\underline{C}_{-i}$ it will also give a loss.

Lender j's best response is to demand an amount of collateral at least larger than the other outside lender. In that case he will attract none of the borrowers in the portfolio of lender i and make zero profit. Lender k will realise that he only attracts high risk borrowers, thus by the same reasoning he wants to demand a collateral at least larger than what lender j demands. Now the best response of each outside lender is to try and overbid the other outside lender rather than to undercut as seen in step two. Thus when the inside lender demands a collateral $C_i^\alpha < \underline{C}_{-i}$ upward pressure will exist due to the competition between outside lenders.

Thus upward pressure will exist untill both outside lenders demand $C_{-i} = \underline{C}^\beta$. At this point, both outside lenders capture half of the β_i high risk borrowers and make zero profit on them. Furthermore, no outside lender has an incentive to demand more than $\underline{C}^\beta$ as then he captures no borrowers and makes zero profit as well:

$$v_{-i} = \begin{cases} <0 & \text{if } C_{-i} < \underline{C}^\beta \\ 0 & \text{if } C_{-i} \geq \underline{C}^\beta \end{cases}$$

Step five:

The last step is to see what the best response of the inside lender is when both outside lenders demand a collateral $\underline{C}^\beta$. With regards to his high risk borrowers, the payoff function of inside lender i becomes

$$v_i^\beta = \begin{cases} <0 & \text{if } C_i^\beta < \underline{C}^\beta \\ 0 & \text{if } C_i^\beta \geq \underline{C}^\beta \end{cases}$$

Since the inside lender retains no high risk borrowers when he demands from them a collateral equal or larger than the high risk break even collateral. making zero profit. On his low risk borrowers he can make a profit if he demands an amount slightly smaller than $\underline{C}^\beta$. If he demands from his low risk borrowers a collateral $C_i^\alpha = \underline{C}^\beta - \varepsilon$ where ε is a very small but positive number, his low risk expected payoff is:

$$
\begin{aligned}
v_i^{\alpha} &= \left(p(1+r) + (1-p)\left(\underline{C}^{\beta} - \varepsilon\right) - 1\right)\alpha_i \\
&= \left(\frac{(p-q)\,r}{(1-q)} - (1-p)\,\varepsilon\right)\alpha_i \\
&> 0 \;\; \forall\,\varepsilon < \frac{(p-q)r}{(1-p)\,(1-q)}
\end{aligned}
$$

Thus his best response is to slightly undercut the two outside lenders with the amount of collateral he demands from his low risk borrowers. Furthermore, the inside lender still has no incentive to change the amount of collateral he demands from his high risk borrowers because he will always lose them and make zero profit.

Step two has showed that the response of the outside lenders and competition between them will now make that they undercut the inside lender and demand a collateral $C_{-i} = \underline{C}_{-i}$. Thus this situation is not an equilibrium either, as the reasoning will start over at step two again only to continue in a loop each time. It is proven that no pure strategy equilibrium exists in an adverse selection credit market.

proof of proposition 2

The structure of this proof is as follows. Step one will derive the equilibrium strategy $L_i^*(C)$ for which the outside lenders always make zero profit. Step two will derive the equilibrium strategy $U_{-i}^*(C)$ of the outside lenders. Step three will verify that the expected profit of the outside lenders is indeed zero and that they have no incentive to change their strategies. Step four will show that the inside lender has no incentive to change his strategy and therefore the strategies are mutual best responses. Step five will show that the strategy of the inside lender with regard to his high risk borrowers $H_i^*(C)$, is unimportant as long as he demands a collateral $C_i^\beta \geq \underline{C}^\beta$.

Step one:

The inside lender i sets his strategy so that each individual outside lender makes zero expected profit for any amount of collateral he demands, as zero profits are assumed. Thus $v_{-i} = 0$:

$$0 = \left[\begin{array}{c} (1 - U(C))(1 - L(C))(p(1+r) + (1-p)C - 1)\alpha_i \\ + (1 - U(C))(1 - H(C))(q(1+r) + (1-q)C - 1)\beta_i \end{array} \right]$$

Lemma one saw that the inside lender will not demand from his high risk borrowers a collateral smaller than $\underline{C}^\beta$ since he can verify the risk type of his borrowers. When $C < \underline{C}^\beta$ it will be that $1 - H(C) = 0$. So solving for $L(C)$ gives:

$$\begin{aligned}
0 &= \left[\begin{array}{c} (1 - U(C))(1 - L(C))(p(1+r) + (1-p)C - 1)\alpha_i \\ + (1 - U(C))(1 - 0)(q(1+r) + (1-q)C - 1)\beta_i \end{array} \right] \\
&= \left[\begin{array}{c} (1 - U(C))(1 - L(C))(p(1+r) + (1-p)C - 1)\alpha_i \\ + (1 - U(C))(q(1+r) + (1-q)C - 1)\beta_i \end{array} \right] \\
1 - L(C) &= \frac{-(1 - U(C))(q(1+r) + (1-q)C - 1)\beta_i}{(1 - U(C))(p(1+r) + (1-p)C - 1)\alpha_i} \\
L_i^*(C) &= 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1 - q(1+r) - (1-q)C}{p(1+r) + (1-p)C - 1}\right)
\end{aligned}$$

At the bottom of the support $\underline{c}$, it must be that $L_i^*(\underline{c}) = 0$ and at $\overline{c}$ the top of the support $L_i^*(\overline{c}) = 1$:

$$\begin{aligned}
L_i^*(\underline{c}) &= 0 = 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1 - q(1+r) - (1-q)\underline{c}}{p(1+r) + (1-p)\underline{c} - 1}\right) \\
1 &= \frac{\beta_i}{\alpha_i}\left(\frac{1 - q(1+r) - (1-q)\underline{c}}{p(1+r) + (1-p)\underline{c} - 1}\right) \\
\underline{c} &= 1 - \frac{(\alpha_i p + \beta_i q)r}{\alpha_i(1-p) + \beta_i(1-q)} \\
\underline{c} &= \underline{C}_{-i}
\end{aligned}$$

and

$$
\begin{aligned}
L_i^*(\bar{c}) &= 1 = 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1-q(1+r)-(1-q)C}{p(1+r)+(1-p)C-1}\right) \\
0 &= \frac{\beta_i}{\alpha_i}\left(\frac{1-q(1+r)-(1-q)C}{p(1+r)+(1-p)C-1}\right) \\
\bar{c} &= 1 - \frac{qr}{1-q} \\
\bar{c} &= \underline{C}^{\beta}
\end{aligned}
$$

The derivation of $H_i^*(C)$ is postponed to step five, take from lemma one that the inside lender will not demand from his high risk borrowers a collateral smaller than $\underline{C}^{\beta}$. First, the proof continues with analysing the strategies of the outside lenders.

Step two:
Lemma one showed that the inside lender has an informational advantage because he can undercut the outside lenders to just below $\underline{C}_{-i}$. Since the outside lenders cannot go below $\underline{C}_{-i}$ without making a loss, the inside lender will at least make a profit on his low risk borrowers of size:

$$
\begin{aligned}
v_i^{\alpha} &= \left(p(1+r)+(1-p)\underline{C}_{-i}-1\right)\alpha_i \\
&= \left(p(1+r)+(1-p)\left(1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)-1\right)\alpha_i \\
&= \frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\alpha_i = \pi_i^{\alpha} \\
v_i^{\alpha} &= \pi_i^{\alpha}
\end{aligned}
$$

Where π_i^{α} is the informational rent equal to $\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\alpha_i$. The equilibrium strategy $U_{-i}^*(C)$ of the outside lenders ensures that lender i gets his informational rent π_i^{α} for any amount of collateral demanded from his low risk borrowers:

$$
\begin{aligned}
\pi_i^{\alpha} &= (1-U_j(C))(1-U_k(C))\left(p(1+r)+(1-p)C-1\right)\alpha_i \\
\pi_i^{\alpha} &= \left(1-U_{-i}^*(C)\right)^2\left(p(1+r)+(1-p)C-1\right)\alpha_i \\
\left(1-U_{-i}^*(C)\right) &= \left(\frac{\pi_i^{\alpha}}{\left(p(1+r)+(1-p)C-1\right)\alpha_i}\right)^{0.5} \\
U_{-i}^*(C) &= 1-\left(\frac{\pi_i^{\alpha}}{\left(p(1+r)+(1-p)C-1\right)\alpha_i}\right)^{0.5} \\
U_{-i}^*(C) &= 1-\left(\left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)\left(\frac{1}{\left(p(1+r)+(1-p)C-1\right)}\right)\right)^{0.5}
\end{aligned}
$$

with

$$
\begin{aligned}
U^*_{-i}\left(\underline{c}\right) &= 0 = 1 - \left(\left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)\left(\frac{1}{(p\left(1+r\right)+\left(1-p\right)\underline{c}-1)}\right)\right)^{0.5} \\
\underline{c} &= 1 - \frac{\left(\alpha_i p + \beta_i q\right) r}{\alpha_i(1-p)+\beta_i(1-q)} \\
\underline{c} &= \underline{C}_{-i}
\end{aligned}
$$

and

$$
U^*_{-i}\left(\underline{C}^{\beta}\right) = 1 - \sqrt{\frac{\beta_i(1-q)r}{\alpha_i(1-p)+\beta_i(1-q)}}
$$

When both outside lenders use strategy $U^*_{-i}(C)$ the inside lender's payoff on his high risk borrowers is:

$$
\begin{aligned}
v_i^{\beta} &= \left(1 - U^*_{-i}(C)\right)^2 \left(q\left(1+r\right) + \left(1-q\right)C_i^{\beta} - 1\right)\beta_i \\
&= \left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)\left(\frac{\left(q\left(1+r\right)+\left(1-q\right)C_i^{\beta}-1\right)}{\left(p\left(1+r\right)+\left(1-p\right)C_i^{\beta}-1\right)}\right)\beta_i
\end{aligned}
$$

Taking the derivative with respect to C_i^{β}:

$$
\begin{aligned}
\frac{\partial v_i^{\beta}}{\partial C_i^{\beta}} &= \left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)\beta_i\left(\frac{pr+qr}{\left(p\left(1+r\right)+\left(1-p\right)C_i^{\beta}-1\right)^2}\right) \\
&> 0\,\forall\, C_i^{\beta}
\end{aligned}
$$

So the best response of the inside lender is to demand from his high risk borrowers the highest collateral possible and maximise his profits.

Suppose lender j uses the strategy $U_j^*\left(C\right) = U^*_{-i}(C)$ as derived above. The inside lender uses $L_i^*(C)$ for his low risk borrowers while demanding the highest possible collateral from his high risk borrowers. It can be shown that the other outside lender k will want to use a different strategy than $U^*_{-i}(C)$. From step two it can be seen that when lender k demands $C_k \geq \underline{C}^{\beta}$, specifically $C_k =$

$\underline{C}^{\beta} + \varepsilon$ where $\varepsilon \geq 0$ the payoff of lender k is:

$$
\begin{aligned}
v_k &= \left(1 - U_j^*\left(C_k\right)\right)\left(1 - H_i(C_k)\right)\left(q\left(1+r\right) + \left(1-q\right)C_k - 1\right)\beta_i \\
&= \left(1 - \left(1 - \sqrt{\frac{\beta_i(1-q)r}{\alpha_i(1-p)+\beta_i(1-q)}}\right)\right)\left(1-0\right)\left(q\left(1+r\right) + \left(1-q\right)C_k - 1\right)\beta_i \\
&= \sqrt{\frac{\beta_i(1-q)r}{\alpha_i(1-p)+\beta_i(1-q)}}\left(q\left(1+r\right) + \left(1-q\right)\left(1 - \frac{qr}{1-q} + \varepsilon\right) - 1\right)\beta_i \\
&= \left(\sqrt{\frac{\beta_i(1-q)r}{\alpha_i(1-p)+\beta_i(1-q)}}\left(1-q\right)\varepsilon\right)\beta_i \\
v_k &> 0 \;\forall\, \varepsilon > 0
\end{aligned}
$$

So the lender k can make a profit by demanding a collateral higher than $\underline{C}^{\beta}$. Of course the same reasoning applies to lender j if he knows that lender k uses the strategy $U_k^*\left(C\right)$. He will want to demand a collateral higher than $\underline{C}^{\beta}$ and make a profit.

In fact $U_{-i}^*(C)$ can only be an equilibrium strategy if the outside lenders never demand a collateral higher than $\underline{C}^{\beta}$. Thus they have a mass point at $U_{-i}^*(\underline{C}^{\beta})$ and the mass point has size

$$
\begin{aligned}
1 - U_{-i}^*\left(\underline{C}^{\beta}\right) &= 1 - \left(1 - \sqrt{\frac{\beta_i(1-q)r}{\alpha_i(1-p)+\beta_i(1-q)}}\right) \\
&= \sqrt{\frac{\beta_i(1-q)r}{\alpha_i(1-p)+\beta_i(1-q)}}
\end{aligned}
$$

So that the payoff of each outside lender when demanding a collateral $C_{-i} \geq \underline{C}^{\beta}$ becomes:

$$
\begin{aligned}
v_{-i} &= \left(1 - U_{-i}^*\left(C_{-i}\right)\right)\left(1 - H(C_{-i})\right)\left(q\left(1+r\right) + \left(1-q\right)C_{-i} - 1\right)\beta_i \\
&= \left(1-1\right)\left(1 - H(C_{-i})\right)\left(q\left(1+r\right) + \left(1-q\right)C_{-i} - 1\right)\beta_i \\
&= 0\left(1 - H(C_{-i})\right)\left(q\left(1+r\right) + \left(1-q\right)C_{-i} - 1\right)\beta_i \\
&= 0
\end{aligned}
$$

Step three:

Suppose the outside lender demands a collateral $C_j < \underline{C}_{-i}$, more specifically he demands a collateral $C_j = 1 - \frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)} - \varepsilon$. Where ε is any arbitrary positive number. In that case the outside lender j knows his payoff function becomes

$$
\begin{aligned}
v_j &= \left[\begin{array}{c} \left(1-U_k^*\left(C_j\right)\right)\left(1-L_i^*\left(C_j\right)\right)\left(p\left(1+r\right)+\left(1-p\right)C_j-1\right)\alpha_i \\ +\left(1-U_k^*\left(C_j\right)\right)\left(1-H_i^*\left(C_j\right)\right)\left(q\left(1+r\right)+\left(1-q\right)C_j-1\right)\beta_i \end{array}\right] \\
&= \left[\begin{array}{c} \left(1-0\right)\left(1-0\right)\left(p\left(1+r\right)+\left(1-p\right)C_j-1\right)\alpha_i \\ +\left(1-0\right)\left(1-0\right)\left(q\left(1+r\right)+\left(1-q\right)C_j-1\right)\beta_i \end{array}\right] \\
&= \left[\begin{array}{c} \left(p\left(1+r\right)+\left(1-p\right)\left(1-\frac{\left(\alpha_i p+\beta_i q\right)r}{\alpha_i(1-p)+\beta_i(1-q)}-\varepsilon\right)-1\right)\alpha_i \\ +\left(q\left(1+r\right)+\left(1-q\right)\left(1-\frac{\left(\alpha_i p+\beta_i q\right)r}{\alpha_i(1-p)+\beta_i(1-q)}-\varepsilon\right)-1\right)\beta_i \end{array}\right] \\
&= \frac{0}{\alpha_i(1-p)+\beta_i(1-q)}-\varepsilon\left(\left(1-p\right)\alpha_i+\left(1-q\right)\beta_i\right) \\
&= -\varepsilon\left(\left(1-p\right)\alpha_i+\left(1-q\right)\beta_i\right)<0\ \forall\varepsilon>0
\end{aligned}
$$

As long as the other outside lender k does not demand a lower collateral than $C_j = 1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}-\varepsilon$ lender j will capture all borrowers from lender i and make a loss on them. Of course this simultaneously applies to lender k, if he were to demand less than $\underline{C}_{-i}$ he will attract all borrowers at a loss giving rate unless j undercuts him.

Demanding the break even amount of collateral $C_{-i}=\underline{C}_{-i}$ gives zero profit per definition. Next, lender j needs to evaluate what happens when he demands a collateral $C_j \in \left(\underline{C}_{-i},\underline{C}^{\beta}\right)$. Suppose that he demands a collateral $C_j = 1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}+\varepsilon$ with $\varepsilon \in \left(0,\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}-\frac{qr}{1-q}\right)$. In this case it is straightforward to find $H_i^*\left(C_j\right)=0$. Finding a value for $L_i^*(C_j)$:

$$
\begin{aligned}
L_i^*(C_j) &= 1-\frac{\beta_i}{\alpha_i}\left(\frac{1-q\left(1+r\right)-\left(1-q\right)C_j}{p\left(1+r\right)+\left(1-p\right)C_j-1}\right) \\
L_i^*(C_j) &= 1-\frac{\beta_i}{\alpha_i}\left(\frac{1-q\left(1+r\right)-\left(1-q\right)1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}+\varepsilon}{p\left(1+r\right)+\left(1-p\right)1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}+\varepsilon-1}\right) \\
L_i^*(C_j) &= 1-\frac{\beta_i}{\alpha_i}\frac{\alpha_i\left(p-q\right)r-\varepsilon\left(1-q\right)\left(\alpha_i(1-p)+\beta_i(1-q)\right)}{\beta_i\left(p-q\right)r+\varepsilon\left(1-p\right)\left(\alpha_i(1-p)+\beta_i(1-q)\right)}
\end{aligned}
$$

The outside lenders cannot verify the risk types of the borrowers in the portfolio of lender i. The expected payoff of outside lender j when demanding $C_j = 1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}+\varepsilon$ is:

$$
\begin{aligned}
v_j &= \left[\begin{array}{c} \left(1-U_k^*\left(C_j\right)\right)\left(1-L_i^*\left(C_j\right)\right)\left(p\left(1+r\right)+\left(1-p\right)C_j-1\right)\alpha_i \\ +\left(1-U_k^*\left(C_j\right)\right)\left(1-H_i^*\left(C_j\right)\right)\left(q\left(1+r\right)+\left(1-q\right)C_j-1\right)\beta_i \end{array}\right] \\
&= \left[\begin{array}{c} \left(1-U_k^*\left(C_j\right)\right)\left(1-L_i^*\left(C_j\right)\right)\left(p\left(1+r\right)+\left(1-p\right)1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}+\varepsilon-1\right)\alpha_i \\ +\left(1-U_k^*\left(C_j\right)\right)\left(1-0\right)\left(q\left(1+r\right)+\left(1-q\right)1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}+\varepsilon-1\right)\beta_i \end{array}\right]
\end{aligned}
$$

Substituting in $L_i^*(C_j) = 1-\frac{\beta_i}{\alpha_i}\frac{\alpha_i(p-q)r-\varepsilon(1-q)(\alpha_i(1-p)+\beta_i(1-q))}{\beta_i(p-q)r+\varepsilon(1-p)(\alpha_i(1-p)+\beta_i(1-q))}$:

$$\begin{aligned}
v_j &= \left[\begin{array}{c} (1-U_k^*(C_j))\left(\frac{\beta_i(q-p)\alpha_i r}{\alpha_i(1-p)+\beta_i(1-q)} - \varepsilon(1-q)\beta_i\right) \\ +(1-U_k^*(C_j))\left(\frac{\beta_i(q-p)\alpha_i r}{\alpha_i(1-p)+\beta_i(1-q)} + \varepsilon(1-q)\beta_i\right) \end{array} \right] \\
&= 0\,\forall\,\varepsilon \in \left(0, \frac{(\alpha_i p+\beta_i q)\,r}{\alpha_i(1-p)+\beta_i(1-q)} - \frac{qr}{1-q}\right)
\end{aligned}$$

The easy next step is to evaluate the outside lender j's payoff in case he demands a collateral $C_j \geq \underline{C}^\beta$ then $L_i^*(C_j) = 1$. The outside lender knows that

$$\begin{aligned}
v_j &= \left[\begin{array}{c} (1-U_k^*(C_j))(1-L_i^*(C_j))(p(1+r)+(1-p)C_j-1)\alpha_i \\ +(1-U_k^*(C_j))(1-H_i^*(C_j))(q(1+r)+(1-q)C_j-1)\beta_i \end{array} \right] \\
&= \left[\begin{array}{c} (1-1)(1-1)(p(1+r)+(1-p)C_j-1)\alpha_i \\ +(1-1)(1-H(C_j))(q(1+r)+(1-q)C_j-1)\beta_i \end{array} \right] \\
&= \left[\begin{array}{c} (0)(0)(p(1+r)+(1-p)C_j-1)\alpha_i + \\ (0)(1-H(C_j))(q(1+r)+(1-q)C_j-1)\beta_i \end{array} \right] \\
&= 0
\end{aligned}$$

The payoffs of the outside lenders are summarised by:

$$v_{-i} = \begin{cases} (-\varepsilon((1-p)\alpha_i+(1-q)\beta_i))(1-U(C)) \leq 0 & \text{if } C_{-i} \leq \underline{C}_{-i} \\ 0 & \text{if } C_{-i} \geq \underline{C}_{-i} \end{cases}$$

When the inside lender uses strategy $L_i^*(C)$ for his low risk borrowers and the outside lenders use $U_k^*(C)$ as their strategy, each individual outside lender is indifferent to demanding any amount of collateral $C_{-i} \geq \underline{C}_{-i}$. Since it will give zero expected profit. Therefore, none of the outside lenders has an incentive to change his strategy.

Step four:

Starting with the amount of collateral that the inside lender demands from his low risk borrowers. If he demands a collateral $C_i^\alpha \leq \underline{C}_{-i}$ he retains with certainty his low risk borrowers given $U_{-i}^*(C)$. So that:

$$\begin{aligned}
v_i^\alpha &= (1-U_{-i}^*(C_i^\alpha))^2(p(1+r)+(1-p)C_i^\alpha-1)\alpha_i \\
&= (1-0)^2(p(1+r)+(1-p)C_i^\alpha-1)\alpha_i \\
&= (p(1+r)+(1-p)C_i^\alpha-1)\alpha_i
\end{aligned}$$

which is linear in C_i^α so that lender i does best by demanding an amount of collateral as high as possible, $C_i^\alpha = \underline{C}_{-i}$. In this case his profit is

$$\begin{aligned}
v_i^\alpha &= \left(p(1+r)+(1-p)\left(1-\frac{(\alpha_i p+\beta_i q)\,r}{\alpha_i(1-p)+\beta_i(1-q)}\right)-1\right)\alpha_i \\
&= \frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\alpha_i \\
&= \pi_i^\alpha
\end{aligned}$$

Now suppose he demands a collateral $C_i^\alpha \in \left(\underline{C}_{-i}, \underline{C}^\beta\right)$, say $C_i^\alpha = 1 - \frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)} + \varepsilon$ with $\varepsilon \in \left(0, \frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)} - \frac{qr}{1-q}\right)$. In that case $U^*_{-i}\left(C_i^\alpha\right)$ becomes

$$
\begin{aligned}
U^*_{-i}\left(C_i^\alpha\right) &= 1 - \left(\left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)\left(\frac{1}{(p\,(1+r)+(1-p)\,C_i^\alpha - 1)}\right)\right)^{0.5} \\
&= 1 - \left(\left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)\left(\frac{1}{\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)} + (1-p)\,\varepsilon}\right)\right)^{0.5} \\
&= 1 - \left(\frac{\frac{\pi_i^\alpha}{\alpha_i}}{\frac{\pi_i^\alpha}{\alpha_i} + (1-p)\,\varepsilon}\right)^{0.5}
\end{aligned}
$$

So that the inside lender i has an expected profit on his low risk borrowers of

$$
\begin{aligned}
v_i^\alpha &= \left(1 - U^*_{-i}\left(C_i^\alpha\right)\right)^2 \left(p\,(1+r) + (1-p)\,C_i^\alpha - 1\right)\alpha_i \\
&= \frac{\frac{\pi_i^\alpha}{\alpha_i}}{\frac{\pi_i^\alpha}{\alpha_i} + (1-p)\,\varepsilon}\left(p\,(1+r) + (1-p)\,C_i^\alpha - 1\right)\alpha_i \\
&= \frac{\frac{\pi_i^\alpha}{\alpha_i}}{\frac{\pi_i^\alpha}{\alpha_i} + (1-p)\,\varepsilon}\left(\pi_i^\alpha + \alpha_i\,(1-p)\,\varepsilon\right) \\
&= \pi_i^\alpha\,\forall\,\varepsilon > 0
\end{aligned}
$$

If lender i demands from his low risk borrowers a collateral equal to $\underline{C}^\beta$ exactly he will have a positive probability of tying with the outside lenders since they both have a mass point and upon a tie he loses all his borrowers:

$$
\begin{aligned}
v_i^\alpha &= \left(1 - U^*_{-i}\left(\underline{C}^\beta\right)\right)^2 \left(p\,(1+r) + (1-p)\,\underline{C}^\beta - 1\right)\alpha_i \\
&= \left(\sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)}}\right)^2 \left(p\,(1+r) + (1-p)\left(1 - \frac{qr}{1-q}\right) - 1\right)0 \\
&= \frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)}0 \\
&= 0
\end{aligned}
$$

Now regarding the profit that the inside lender i makes on his low risk borrowers when he demands a collateral $C_i^\alpha > \underline{C}^\beta$, due to the masspoint at $U^{i*}_{-i}\left(\underline{C}^\beta\right)$ his payoff will be:

$$
\begin{aligned}
v_i^\alpha &= \left(1 - U^*_{-i}\left(C\right)\right)^2 \left(p\,(1+r) + (1-p)\,C_i^\alpha - 1\right)\alpha_i \\
&= (1-1)^2 \left(p\,(1+r) + (1-p)\,C_i^\alpha - 1\right)\alpha_i \\
&= 0\,\forall C_i^\alpha \geq \underline{C}^\beta
\end{aligned}
$$

So the inside lender's payoff on his low risk borrowers can be summarised as:

$$v_i^\alpha = \begin{cases} <\pi_i^\alpha & \text{if } C_i^\alpha < \underline{C}_{-i} \\ \pi_i^\alpha & \text{if } C_i^\alpha \in \left[\underline{C}_{-i}, \underline{C}^\beta\right) \\ 0 & \text{if } C_i^\alpha \geq \underline{C}^\beta \end{cases}$$

Also, now that the strategy of the outside lenders has been derived, their expected profits can be summarised by:

$$v_{-i}^i = \begin{cases} <0 & \text{if } C_{-i} < \underline{C}_{-i} \\ 0 & \text{if } C_{-i} \geq \underline{C}_{-i} \end{cases}$$

So the individual outside lenders can make at most zero profit on any point above C_{-i} and have no incentive to change their strategies. Moreover, the inside lender i has an expected profit equal to π_i^α for any amount of collateral $C_i^\alpha \in \left[\underline{C}_{-i}, \underline{C}^\beta\right)$ which corresponds with the support of $L_i^*(C)$. Thus he has no incentive to change his low risk strategy either.

Step five:

Lastly, the optimal strategy for the inside lender with respect to his high risk borrowers needs to be derived. The payoff of the inside lender on his high risk borrowers is:

$$v_i^\beta = \left(1 - U_{-i}^*(C_i^\beta)\right)^2 \left(q\,(1+r) + (1-q)\,C_i^\beta - 1\right)\beta_i$$

Due to the mass point at $U_{-i}^*(\underline{C}^\beta)$ the payoff function becomes:

$$\begin{aligned} v_i^\beta &= (1-1)^2 \left(q\,(1+r) + (1-q)\,C_i^\beta - 1\right)\beta_i \\ &= 0 \,\forall\, C_i^\beta \geq \underline{C}^\beta \end{aligned}$$

When the inside lender demands from his high risk borrowers a collateral smaller than $\underline{C}^\beta$ his expected profits will be negative. Assume that he demands a collateral $C_i^\beta = \underline{C}^\beta - \varepsilon$, with ε any positive number:

$$\begin{aligned} v_i^\beta &= \left(1 - U_{-i}^*(C_i^\beta)\right)^2 \left(q\,(1+r) + (1-q)\,C_i^\beta - 1\right)\beta_i \\ &= \left(1 - U_{-i}^*(C_i^\beta)\right)^2 \left(q\,(1+r) + (1-q)\left(1 - \frac{qr}{1-q} - \varepsilon\right) - 1\right)\beta_i \\ &= -\left(1 - U_{-i}^*(C_i^\beta)\right)^2 (1-q)\,\varepsilon \\ &< 0 \,\forall\, \varepsilon \end{aligned}$$

So the best that the inside lender can do is demand from his high risk borrowers a collateral equal or larger than $\underline{C}^\beta$ and have zero profit because all his high risk clients will switch to the outside lenders. Regardless of what the inside lender demands exactly from his high risk borrowers, the outside lenders will make zero profit. This was shown at the end of step three, no individual outside lender has an incentive to change his strategy. Due to the mass point at $U^*_{-i}(\underline{C}^\beta)$ no lender will obtain any borrowers if he demands a collateral higher than $\underline{C}^\beta$.

proof of proposition 3

Suppose that the collateral floor $C_{floor} \geq \underline{C}_{-i}$ is imposed on the three lender market. Step one will change the strategy $L_i(C)$ so that the shape is the same as under proposition two but that it has a mass point at the bottom of the strategy. Step two will show that the outside lenders have an incentive to change their strategies. Step three, will develop the best response of the inside lender.

Step one:

Suppose a collateral floor is introduced so that $C_{floor} = \underline{C}_{-i} + \varepsilon$, with ε being any positive number that determines the height of the collateral floor. The equilibrium strategy $L_i^*(C)$ that the inside lender sets for his high risk borrowers was derived under proposition two:

$$L_i^*(C) = 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1-q(1+r)-(1-q)C}{p(1+r)+(1-p)C-1}\right)$$
$$\text{on the support } C \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$$

with ε being greather than zero, the inside lender can no longer demand collateral on the part of the support $C \in \left[\underline{C}_{-i}, C_{floor}\right)$. Instead he will have a mass point at $L_i^*(C_{floor})$ and on the interval $C \in \left(C_{floor}, \underline{C}^{\beta}\right)$ the strategy after the collateral floor $L_i^{cf}(C)$ will be the same as $L_i^*(C)$:

$$L_i^{cf}(C) = \begin{cases} 1 - \frac{\beta_i}{\alpha_i}\frac{\alpha_i(p-q)r-\varepsilon(1-q)(\alpha_i(1-p)+\beta_i(1-q))}{\beta_i(p-q)r+\varepsilon(1-p)(\alpha_i(1-p)+\beta_i(1-q))} & \text{if } C = C_{floor} \\ 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1-q(1+r)-(1-q)C}{p(1+r)+(1-p)C-1}\right) & \text{if } C \in \left(C_{floor}, \underline{C}^{\beta}\right) \end{cases}$$

where the masspoint at $L_i^{cf}(C_{floor})$ is found by plugging $C_{floor} = \underline{C}_{-i} + \varepsilon$ into the equilibrium strategy $L_i^*(C)$ from proposition two:

$$\begin{aligned} L_i^*(C_{floor}) &= 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1-q(1+r)-(1-q)C_{floor}}{p(1+r)+(1-p)C_{floor}-1}\right) \\ &= 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1-q(1+r)-(1-q)\left(1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}+\varepsilon\right)}{p(1+r)+(1-p)\left(1-\frac{(\alpha_i p+\beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)}+\varepsilon\right)-1}\right) \\ &= 1 - \frac{\beta_i}{\alpha_i}\frac{\alpha_i(p-q)r-\varepsilon(1-q)(\alpha_i(1-p)+\beta_i(1-q))}{\beta_i(p-q)r+\varepsilon(1-p)(\alpha_i(1-p)+\beta_i(1-q))} \end{aligned}$$

Step two:

The outside lenders respond to this new strategy by demanding with certainty an amount of collateral $C_{-i} = C_{floor}$. To see this, evaluate the payoff function of the inside lenders with the collateral floor adjusted strategy $L_i^{cf}(C_{floor})$. It is prohibited to demand a collateral less than C_{floor}, firstly evaluate the payoff when the outside lender j demands a collateral $C_{-i} >$

C_{floor}. The strategy of the inside lender has not changed on the interval $C \in \left(C_{floor}, \underline{C}^{\beta}\right)$ hence the payoff is the same as under proposition three:

$$v_j = 0 \forall C_{-i} \in \left(C_{floor}, \underline{C}^{\beta}\right)$$

Now evaluate the payoff when the outside lender j demands $C_j = C_{floor}$. The tie-sharing rule is such that borrowers always switch upon a tie. So by demanding exactly the collateral floor he undercuts the inside lender in $1 - L_i^{cf}(C_{floor})$ percent of the cases. Moreover, he will tie with probability $L_i^{cf}(C_{floor})$ in which case the borrowers will also switch to the outside lender. Thus with certainty, all borrowers will leave the portfolio of the inside lender, the payoff of the outside lender is then only dependend on the strategy of the other outside lender k:

$$\begin{aligned}
v_j &= U_k(C_{floor}) \left[\begin{array}{c} (p(1+r)+(1-p)C_{floor}-1)\frac{\alpha_i}{2} \\ +(q(1+r)+(1-q)C_{floor}-1)\frac{\beta_i}{2} \end{array} \right] \\
&\quad + (1-U_k(C_{floor})) \left[\begin{array}{c} (p(1+r)+(1-p)C_{floor}-1)\alpha_i \\ +(q(1+r)+(1-q)C_{floor}-1)\beta_i \end{array} \right] \\
&= \left(1 - \frac{U_k(C_{floor})}{2}\right) \left[\begin{array}{c} \frac{\beta_i(p-q)\alpha_i r}{\alpha_i(1-p)+\beta_i(1-q)} + \varepsilon(1-p)\alpha_i \\ -\frac{\beta_i(p-q)\alpha_i r}{\alpha_i(1-p)+\beta_i(1-q)} + \varepsilon(1-q)\beta_i \end{array} \right] \\
&= \left(1 - \frac{U_k(C_{floor})}{2}\right)((1-p)\alpha_i + (1-q)\beta_i)\varepsilon \\
&> 0 \forall \varepsilon
\end{aligned}$$

Thus the outside lender does best by demanding an amount equal to C_{floor}, regardless of his competitor lender k. Naturally, this reasoning applies to lender k as well who does best to demand exactly the collateral floor, regardless of what lender j does. Thus they both demand with certainty an amound of collateral $C_{-i} = C_{floor}$ and have an expected payoff of:

$$v_{-i} = (p(1+r)+(1-p)C_{floor}-1)\frac{\alpha_i}{2} + (q(1+r)+(1-q)C_{floor}-1)\frac{\beta_i}{2}$$

No outside lender has a reason to deviate, as lemma one showed they want to try and undercut each other. However this is impossible when a collateral floor $C_{floor} \geq \underline{C}_{-i}$ is installed, so demanding a lower amount of collateral is impossible. Demanding a higher amount of collateral than C_{floor} means that the other outside lender will obtain all the borrowers and thus that profit is zero. The payoff function of the outside lenders is:

$$v_{-i} = \begin{cases} \left[\begin{array}{c} (p(1+r)+(1-p)C_{floor}-1)\frac{\alpha_i}{2} \\ +(q(1+r)+(1-q)C_{floor}-1)\frac{\beta_i}{2} \end{array} \right] & \text{if } C_{-i} = C_{floor} \\ 0 & \text{if } C_{-i} > C_{floor} \end{cases}$$

Step three:

The inside lender knows that the outside lenders demand a collateral of C_{floor} with certainty so that $U^*_{-i}(C) : C_{-i} = C_{floor}$. Then the inside lender evaluates his expected payoff function for his low risk borrowers

$$v_i^\alpha = (1 - U_j(C))(1 - U_k(C))(p(1+r) + (1-p)C - 1)\alpha_i$$

which can be reduced to

$$v_i^\alpha = \quad 0 \quad \text{if } C \geq C_{floor}$$

When the inside lender i demands a collateral equal to C_{floor} as well, he ties with the outside lenders and loses all his clients. When he demands a collateral higher than C_{floor} he will also lose all his clients. Since it is prohibited to demand less than C_{floor} the inside lender is indifferent between demanding any amount of collateral $C \geq C_{floor}$. Given that the outside lenders demand with certainty a collateral equal to the collateral floor.

Similarly, the payoff function on high risk borrowers can be reduced to

$$v_i^\beta = \quad 0 \quad \text{if } C \geq C_{floor}$$

And again the inside lender is indifferent between demanding any amount of collateral $C \geq C_{floor}$. Even if the collateral floor is below the break even collateral on high risk borrowers, as the inside lender always loses his borrowers and makes zero profit in any case.

Thus the equilibrium is

$$\begin{array}{l} L_i^*(C) : C_i^\alpha \geq C_{floor} \\ U_{-i}^*(C) : C_{-i} = C_{floor} \\ H_i^*(C) : C_i^\beta \geq C_{floor} \end{array}$$

with both outside lenders sharing the gained borrowers and the respective profits.

Proof of proposition 4

Changing the tie-sharing rule in favour of the inside lender gives him more market power. Under the tie-sharing rule that borrowers do not switch from the inside lender to an outside lender upon a tie, a mixed equilibrium arises. Step one will show that the pure strategy equilibrium from proposition three does not hold now. Step two derives the equilibrium strategy $L_i^*(C)$. Step three derives the equilibrium strategy of the outside lenders. Step four shows the payoff and best response of the outside lenders.

Step one:

It is straightforward to proof that no pure strategy equilibrium exists when the collateral floor is imposed on the interval $C_{floor} \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$. Consider the pure strategy as under proposition three, the outside lenders demand a collateral equal to the collateral floor. Where the collateral floor is $C_{floor} = 1 - \frac{(\alpha_i p + \beta_i q) r}{\alpha_i (1-p) + \beta_i (1-q)} + \varepsilon$ with $\varepsilon \in \left(0, \frac{(\alpha_i p + \beta_i q) r}{\alpha_i (1-p) + \beta_i (1-q)} - \frac{qr}{1-q}\right)$. If the inside lender i would also demand a collateral equal to C_{floor} from his low risk borrowers, he would retain all of them because of the new tie-sharing rule. This leaves the outside lenders with only the high risk borrowers and since $C_{floor} < \underline{C}^{\beta}$ they will make a loss on these borrowers, Their best response would thus be to demand a collateral equal to $\underline{C}^{\beta}$. Analogue to the proof of lemma two, there cannot be a pure strategy equilibrium for any amount of collateral $C \in \left[C_{floor}, \underline{C}^{\beta}\right)$. As it will result in endless undercutting by at least one lender.

Step two:

Strategy $L_i^*(C)$ must give the same expected payoff to the outside lenders on each point of its support. Assuming that profits are zero, the same function $L_i^*(C)$ can be conjectured as in a free market model. However, when the level of the collateral floor $C_{floor} > \underline{C}_{-i}$ it is forbidden to demand any amount of collateral smaller than C_{floor}.

Thus the mass that was previously dedicated to the part of $L_i^*(C)$ where $C \in \left[\underline{C}_{-i}, C_{floor}\right)$ needs to be shifted, as the total probability must amount to one. This makes that the equilibrium strategy $L_i^*(C)$ has a masspoint at the bottom of the support with size:

$$\begin{aligned}
L_i^*(C_{floor}) &= 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1 - q\,(1+r) - (1-q)\,C_{floor}}{p\,(1+r) + (1-p)\,C_{floor} - 1}\right) \\
&= 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1 - q\,(1+r) - (1-q)\left(1 - \frac{(\alpha_i p + \beta_i q) r}{\alpha_i (1-p) + \beta_i (1-q)} + \varepsilon\right)}{p\,(1+r) + (1-p)\left(1 - \frac{(\alpha_i p + \beta_i q) r}{\alpha_i (1-p) + \beta_i (1-q)} + \varepsilon\right) - 1}\right) \\
&= 1 - \frac{\beta_i}{\alpha_i}\frac{\alpha_i\,(p-q)\,r - \varepsilon\,(1-q)\,(\alpha_i(1-p) + \beta_i(1-q))}{\beta_i\,(p-q)\,r + \varepsilon\,(1-p)\,(\alpha_i(1-p) + \beta_i(1-q))}
\end{aligned}$$

this makes that the strategy $L_i^*(C)$ of the inside lender with regard to his low risk borrowers is :

$$L_i^*(C) = \begin{cases} 1 - \frac{\beta_i}{\alpha_i}\frac{\alpha_i(p-q)r - \varepsilon(1-q)(\alpha_i(1-p)+\beta_i(1-q))}{\beta_i(p-q)r+\varepsilon(1-p)(\alpha_i(1-p)+\beta_i(1-q))} & \text{if } C = C_{floor} \\ 1 - \frac{\beta_i}{\alpha_i}\left(\frac{1-q(1+r)-(1-q)C}{p(1+r)+(1-p)C-1}\right) & \text{if } C \in \left(C_{floor}, \underline{C}^{\beta}\right) \end{cases}$$

Step three:

Note that the inside lender i can always with certainty get a positive profit by demanding from his low risk borrowers a collateral $C_i^{\alpha} = C_{floor}$ with $C_{floor} = 1 - \frac{(\alpha_i p + \beta_i q)r}{\alpha_i(1-p)+\beta_i(1-q)} + \varepsilon$. Since he will always retain his borrowers upon a tie, the inside lender has enough market power to ensure his informational rent. The return of the inside lender when he strictly demands an amount of collateral equal to the collateral floor would be:

$$\pi_i^{\alpha} = \left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)} + (1-p)\,\varepsilon\right)\alpha_i$$

$U_{-i}^*(C)$ must ensure that the inside lender i receives at least π_i^{α}. Thus the equilibrium strategy of the outside lenders can be found by:

$$\begin{aligned} \pi_i^{\alpha} &= \left(1 - U_{-i}^*(C)\right)^2 \left(p\,(1+r) + (1-p)\,C - 1\right)\alpha_i \\ \left(1 - U_{-i}^*(C)\right)^2 &= \frac{\pi_i^{\alpha}}{\left(p\,(1+r) + (1-p)\,C - 1\right)\alpha_i} \\ U_{-i}^*(C) &= 1 - \left(\frac{\pi_i^{\alpha}}{\left(p\,(1+r) + (1-p)\,C - 1\right)\alpha_i}\right)^{0.5} \\ U_{-i}^*(C) &= 1 - \left(\left(\frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)} + (1-p)\,\varepsilon\right)\frac{1}{p\,(1+r)+(1-p)\,C-1}\right)^{0.5} \end{aligned}$$

analogue to step two of proposition two, there must be a mass point at $U_{-i}^*\left(\underline{C}^{\beta}\right)$, otherwise the outside lenders might have an incentive to change their strategy. The mass point has a mass:

$$1 - U_{-i}^*\left(\underline{C}^{\beta}\right) = \sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)} + \frac{(1-p)\,(1-q)\,\varepsilon}{(p-q)\,r}}$$

By construction the inside lender always has an expected payoff of π_i^{α} when he demands a collateral from his low risk borrowers $C_i^{\alpha} \in \left[C_{floor}, \underline{C}^{\beta}\right]$. When he demands $C_i^{\alpha} > \underline{C}^{\beta}$ he will lose all his low risk borrowers and make zero profit:

$$v_i^{\alpha} = \begin{cases} \pi_i^{\alpha} & \text{if } C_i^{\alpha} \in \left[C_{floor}, \underline{C}^{\beta}\right] \\ 0 & \text{if } C_i^{\alpha} > \underline{C}^{\beta} \end{cases}$$

So the inside lender i has no incentive to change his strategy $L_i^*(C)$. Also the outside lenders have no incentive to change their strategy $U_{-i}^*(C)$ as step four will show.

Step four:

From the viewpoint of the outside lender j nothing changes. When he demands a collateral $C = C_{floor}$ he still has the same probability of winning over the borrowers as before when $L_i^*(C)$ was an atomless function. With a collateral floor $C_{floor} > \underline{C}_{-i}$ imposed on the model, a masspoint at the bottom of the support exists. When the outside lender demands C_{floor} exactly, he has a positive chance of a tie with the inside lender, the probability being equal to the size of the masspoint $L_i^*(C_{floor})$. His payoff in case of a tie is zero as upon a tie all borrowers are retained by the inside lender. Still the outside lender j has a $1 - L_i^*(C_{floor})$ chance that he demands a lower collateral than the inside lender. His expected payoff when demanding $C_j = C_{floor}$ is:

$$\begin{aligned}
v_j &= \begin{bmatrix} L_i^*(C_j)0 + (1 - L_i^*(C_j))\,(1 - U_k^*(C_j))\,(p\,(1+r) + (1-p)\,C_j - 1)\,\alpha_i \\ +\,(1 - H_i^*(C_j))\,(1 - U_k^*(C_j))\,(q\,(1+r) + (1-q)\,C_j - 1)\,\beta_i \end{bmatrix} \\
&= \begin{bmatrix} (1 - L_i^*(C_{floor}))\,(1-0)\left(p\,(1+r) + (1-p)\left(1 - \frac{(\alpha_i p + \beta_i q) r}{\alpha_i (1-p) + \beta_i (1-q)} + \varepsilon\right) - 1\right)\alpha_i \\ +\,(1-0)\,(1-0)\left(q\,(1+r) + (1-q)\left(1 - \frac{(\alpha_i p + \beta_i q) r}{\alpha_i (1-p) + \beta_i (1-q)} + \varepsilon\right) - 1\right)\beta_i \end{bmatrix} \\
&= \begin{bmatrix} \beta_i \frac{\alpha_i (p-q) r - \varepsilon (1-q)(\alpha_i (1-p) + \beta_i (1-q))}{\alpha_i (1-p) + \beta_i (1-q)} \\ +\beta_i \frac{\alpha_i (q-p) r + \varepsilon (1-q)(\alpha_i (1-p) + \beta_i (1-q))}{\alpha_i (1-p) + \beta_i (1-q)} \end{bmatrix} \\
&= \begin{bmatrix} \beta_i \frac{\alpha_i (p-q) r - \varepsilon (1-q)(\alpha_i (1-p) + \beta_i (1-q))}{\alpha_i (1-p) + \beta_i (1-q)} \\ -\beta_i \frac{\alpha_i (p-q) r - \varepsilon (1-q)(\alpha_i (1-p) + \beta_i (1-q))}{\alpha_i (1-p) + \beta_i (1-q)} \end{bmatrix} \\
&= 0
\end{aligned}$$

Thus at the mass point, when $C_j = C_{floor}$ the outside lender has zero profit. On the part where $C_j > C_{floor}$ by construction lender j has zero profit as well. That strategy $L_i^*(C)$ ensures zero expeceted profit to the outside lenders was proven by proposition two. This gives that the outside lenders payoff function is described by:

$$v_j = \left\{ \begin{array}{ll} 0 & \text{if } C \in \left[C_{floor}, \underline{C}^{\beta}\right) \end{array} \right.$$

Making the outside lender j indifferent to demanding any amount of collateral on the support of $L_i^*(C)$.

Proof of collorary 5

Step one will show that the average collateral demanded by the inside lender is higher when a collateral floor is installed. Step two will show the same for the average collateral demanded by the outside lender. Remember that the tie-sharing rule has been changed to 'the inside lender retains all' tie-sharing rule.

Step one:

As proven by proposition four, if no indifferent borrowers switch, a mixed strategy equilibrium arises for $C_{floor} \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$. In case $\underline{C}_{-i} < C_{floor} < \underline{C}^{\beta}$ the strategy $L_i^*(C)$ has a mass point at the bottom of the support which size is increasing with C_{floor}. Also the masspoint at the top of $U_{-i}^*(C)$ is increasing with C_{floor}. This makes that the average collaterals are larger than without a collateral floor. The average collateral that the inside lenders demand is

$$\begin{aligned}\widetilde{C}_i &= L_i^*(C_{floor})C_{floor} + \int_{C_{floor}}^{\underline{C}^{\beta}} l_i^*(C)CdC \\ &= L_i^*(C_{floor})C_{floor} + \int_{C_{floor}}^{\underline{C}^{\beta}} l_i^*(C)CdC\end{aligned}$$

$L_i^*(C_{floor})$ is a masspoint and was derived by pluggin C_{floor} into the equilibrium strategy $L_i^*(C)$ of proposition two. Therefore it can be written as

$$\int_{\underline{C}_{-i}}^{C_{floor}} l_i^*(C)dC$$

so that

$$\widetilde{C}_i = C_{floor} \int_{\underline{C}_{-i}}^{C_{floor}} l_i^*(C)dC + \int_{C_{floor}}^{\underline{C}^{\beta}} l_i^*(C)CdC$$

Note that C_{floor} is a constant so that it can be placed outside the integration.

Comparing the average collateral that the inside lender demands with a collateral floor and without a collateral floor, it is easy to see that average collateral demanded is higher after a floor is installed. That is under the assumption that all borrowers are retained upon a tie. The left hand side below gives the average collateral demanded with a collateral floor imposed, and the right hand side is without a collateral floor.

$$\begin{aligned}
\text{With collateral floor} \quad &> \quad \text{Without collateral floor} \\
C_{floor}\int_{\underline{C}_{-i}}^{C_{floor}} l_i^*(C)dC + \int_{C_{floor}}^{\underline{C}^{\beta}} l_i^*(C)CdC \quad &> \quad \int_{\underline{C}_{-i}}^{\underline{C}^{\beta}} l_i^*(C)CdC \\
C_{floor}\int_{\underline{C}_{-i}}^{C_{floor}} l_i^*(C)dC + \int_{C_{floor}}^{\underline{C}^{\beta}} l_i^*(C)CdC \quad &> \quad \int_{\underline{C}_{-i}}^{C_{floor}} l_i^*(C)CdC \\
&\qquad + \int_{C_{floor}}^{\underline{C}^{\beta}} l_i^*(C)CdC \\
C_{floor}\int_{\underline{C}_{-i}}^{C_{floor}} l_i^*(C)dC \quad &> \quad \int_{\underline{C}_{-i}}^{C_{floor}} l_i^*(C)CdC
\end{aligned}$$

Step two:

Now for the average lowest collateral demanded by the outside lenders.The masspoint at $U_{-i}^*\left(\underline{C}^{\beta}\right)$ is higher with a collateral floor than without as can be seen from comparing proposition four with proposition two:

$$\begin{aligned}
\text{With collateral floor} \quad &> \quad \text{Without collateral floor} \\
\sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)} + \frac{(1-p)(1-q)\varepsilon}{(p-q)\,r}} \quad &> \quad \sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)}}
\end{aligned}$$

This means that after a collateral floor is installed $\underline{C}^{\beta}$ is demand more often, increasing the average collateral demanded. Secondly, without a collateral floor, the outside lenders demanded with positive probability collateral on the interval $C \in \left[\underline{C}_{-i}, C_{floor}\right]$. This is now impossible because of the collateral floor, and since the function $U_{-i}^*(C)$ shifts to the right after a collateral floor is installed the average collateral will be higher.

B Appendix 2

Consider now the following thie sharing rule: upon an inside-outside lender tie, all borrowers stay with the inside lender. This is the exact opposite of the tie-sharing rule that was used in section three to five. This appendix serves to show that the equilibrium as stated under proposition three holds, regardless of the tie-sharing rule. Thus the tie-sharing rule is unimportant on a credit market without a collateral floor. Section six in the main text will show that the tie-sharing rule does change the equilibrium when a collateral floor is imposed.

The structure of this proof is as follows. Step one will show that again no pure strategy equilibrium is possible. Step two will show why the tie-sharing rule is unimportant for the mixed strategy equilibrium

Step one:

As the proof of lemma one showed, the outside lenders try to undercut each other untill the portfolio break even collateral $\underline{C}_{-i}$. The inside lender, due to his informational advantage could undercut the outside lenders slightly below $\underline{C}_{-i}$ and still make a profit on his low risk borrowers. However, when the tie-sharing rule changes, lender i does not even have to undercut his competitors. He can match them at $\underline{C}_{-i}$ and as the tie-sharing rule says, he retains his low risk borrowers.

In that case, the outside lenders again only obtain high risk borrowers at a loss giving rate since $\underline{C}_{-i} < \underline{C}^{\beta}$. Thus they want to demand a collateral equal to the high risk break even collateral $\underline{C}^{\beta}$. Similar to lemma one, this cannot be an equilibrium either because the inside lender will now also want to demand a collateral equal to $\underline{C}^{\beta}$. In that case he retains all his low risk borrowers making a large profit on them, and he loses his high risk borrowers. As under lemma one, the reasoning starts over from this point and the outside lenders would want to undercut each other down to $\underline{C}_{-i}$. Hence a pure strategy equilibrium does not exist either under this tie-sharing rule.

Step two:

The equilibrium as under proposition three is:

$$L_i^*(C) = 1 - \tfrac{\beta_i}{\alpha_i}\left(\tfrac{1-q(1+r)-(1-q)C}{p(1+r)+(1-p)C-1}\right)$$
$$\text{on the support } C \in \left[\underline{C}^{i}_{-i}, \underline{C}^{\beta}\right]$$

$$U^*_{-i}(C) = 1 - \left(\left(\tfrac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\right)\left(\tfrac{1}{p(1+r)+(1-p)C-1}\right)\right)^{0.5}$$
$$\text{on the support } C \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$$

with mass point

$$1 - U^*_{-i}(\underline{C}^{\beta}) = \sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)}}$$

and

$$H_i^*(C) : C \geq \underline{C}^\beta$$

Consider that the inside lender demands C_i^α from his low risk borrowers. The inside lender i's expected payoff is:

$$v_i^\alpha = \begin{cases} (p\,(1+r) + (1-p)\,C - 1)\,\alpha_i < \pi_i^\alpha & \text{if } C_i^\alpha < \underline{C}_{-i} \\ \pi_i^\alpha & \text{if } C_i^\alpha \in \left[\underline{C}_{-i}, \underline{C}^\beta\right] \\ 0 & \text{if } C_i^\alpha \geq \underline{C}^\beta \end{cases}$$

Where $\pi_i^\alpha = \frac{\beta_i(p-q)r}{\alpha_i(1-p)+\beta_i(1-q)}\alpha_i$ is the informational rent that the inside lender obtains. From his high risk borrowers he demands a collateral C_i^β so that his expected payoff is:

$$v_i^\beta = \begin{cases} (q\,(1+r) + (1-q)\,C - 1)\,\beta_i < 0 & \text{if } C_i^\beta < \underline{C}^\beta \\ 0 & \text{if } C_i^\beta \geq \underline{C}^\beta \end{cases}$$

For the outside lenders the expected payoff is

$$v_{-i} = \begin{cases} <0 & \text{if } C_{-i} < \underline{C}_{-i} \\ 0 & \text{if } C_{-i} \geq \underline{C}_{-i} \end{cases}$$

The only point where two lenders have a positive probability of a tie is at $\underline{C}^\beta$ where both outside lenders have a mass point. Knowing that $L_i^*(\underline{C}^\beta) = 1$ and $H_i^*(\underline{C}^\beta) = 0$ the outside lenders both have an expected payoff at $\underline{C}^\beta$ of zero, see for example the payoff of lender j:

$$\begin{aligned}
v_j &= \left[\begin{array}{c} \left(1 - L_i^*(\underline{C}^\beta)\right)\left(1 - U_k^*(\underline{C}^\beta)\right)\left(p\,(1+r) + (1-p)\,\underline{C}^\beta - 1\right)\alpha_i \\ + \left(1 - H_i^*(\underline{C}^\beta)\right)\left(1 - U_k^*(\underline{C}^\beta)\right)\left(q\,(1+r) + (1-q)\,\underline{C}^\beta - 1\right)\beta_i \end{array}\right] \\
&= \left[\begin{array}{c} (1-1)\sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)}}\left(p\,(1+r) + (1-p)\,\underline{C}^\beta - 1\right)\alpha_i \\ + (1-0))\sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)}}\left(q\,(1+r) + (1-q)\,\underline{C}^\beta - 1\right)\frac{\beta_i}{2} \end{array}\right] \\
&= \sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)}}\left(q\,(1+r) + (1-q)\left(1 - \frac{qr}{1-q}\right) - 1\right)\frac{\beta_i}{2} \\
&= \sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)}}\,(0)\,\frac{\beta_i}{2} \\
&= 0
\end{aligned}$$

And the same applies for lender k were he to demand a collateral equal to the high risk break even collateral. Of course the tie-sharing rule did not change for a tie between two outside lender. Consider next what would happen if the inside lender would demand $\underline{C}^{\beta}$. He would have a positive probability of tying with at least one of the outside lenders:

$$\begin{aligned}
v_i^{\alpha} &= \left(1 - U_{-i}^*(\underline{C}^{\beta})\right)^2 \left(p\,(1+r) + (1-p)\,\underline{C}^{\beta} - 1\right)\alpha_i \\
&= \left(\sqrt{\frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)}}\right)^2 \left(p\,(1+r) + (1-p)\left(1 - \frac{qr}{1-q}\right) - 1\right)\alpha_i \\
&= \frac{\beta_i(1-q)}{\alpha_i(1-p)+\beta_i(1-q)} \frac{(p-q)\,r}{(1-q)} \\
&= \frac{\beta_i\,(p-q)\,r}{\alpha_i(1-p)+\beta_i(1-q)} = \pi_i^{\alpha}
\end{aligned}$$

So the only difference that the alternative tie-sharing rule has made is that the expected payoff for the inside lender now equals π_i^{α} at $\underline{C}^{\beta}$ as well. Whereas under the inside lender loses all tie-sharing rule, he had an expected profit of zero when demanding $\underline{C}^{\beta}$. The difference is also visible in the support of $L_i^*(C)$, under the inside lender loses all tie-sharing rule, the support is $C \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right)$. With the inside lender retains all tie-sharing, the support of $L_i^*(C)$ is $C \in \left[\underline{C}_{-i}, \underline{C}^{\beta}\right]$.

Without a collateral floor the tie-sharing rule does not change the equilibrium strategies nor does it change the equilibrium payoffs to all lenders. The mixed strategy nature of the equilibrium arises because the inside lender still has an informational advantage and could undercut the outside lenders further than they can retaliate, that is below $\underline{C}_{-i}$.

As proposition three in section five showed, a collateral floor prevents the inside lender from undercutting the outside lenders below C_{floor}. Thus his informational advantage vanishes when $C_{floor} > \underline{C}_{-i}$. Section six in the main text shows that the tie-sharing rule is crucial for the type of equilibrium in this case.