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Master's Thesis Behavioural Economics

Sooner or Safer: How Do People Balance Risk Against Time?

Revisiting the probability-time constant trade-off hypothesis

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Abstract

Intertemporal and risky choices are two very important research streams in the behavioral literature. They both have received a lot of attention, especially since recurrent anomalies have been discovered with respect to Constant Discounting and Expected Utility models respectively. Prelec and Loewenstein (1991) have put forward that similar behavioral drivers may be operating behind risk and delays and have presented what they called a “common approach”. Keren and Roelofsma (1995) went beyond, showing that risk and delays, due to cross effects, must be considered jointly, in an integrated way.

In this context, to construct a performant model of decision making allowing for both risk and delay, it is highly relevant to determine how people balance risk against time. An important integrated model, the Probability-Time Trade-Off Model, assumes that this trade-off is constant (Baucells and Heukamp, 2012). This paper challenges the constant trade-off assumption out of consistency with previous findings and presents an alternative theoretical hypothesis. It also sketches an empirical pilot test of this alternative. Its results give ground for the alternative even if they are not statistically significant, reinforcing the critic of the probability-time constant trade-off hypothesis.

Keywords: Choice under risk, intertemporal choice, common approach, probability-time trade-off.

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"A bird in the hand is worth two in the bush"

Table of content

1. Introduction

Imagine you are the boss of a pharma company. With limited resources you must choose between two research programs. The first one offers the possibility to make EUR 1 billion within 5 years with a probability of 50%, the second one to make the same profit of EUR 1 billion with a higher probability, let's say 75%, but later, let's say within 10 years. Between "sooner but riskier" and "later but safer", what do you choose? In other words, "sooner or safer"?

If such a decision trade-off may appear as common for an investor, it is in fact a daily life consideration, as a tremendous number of decisions involve both risk and time. Let's take an example. Driverless cars will come on the market. We may assume that the first models will need to be further improved. Would you prefer to wait a little bit longer in order to buy a car with a lower probability of defaults or to enjoy more rapidly the car but with a higher risk of "infant" defaults?

To have a good descriptive or normative analysis of those phenomena, it is important to start with having a suited model of choice under risk and uncertainty. Choices under risk, on the one hand, and intertemporal choices, on the other hand, and their respective deviations from the Expected Utility and Constant Discounting models, have independently received a lot of attention, as behavioral economics textbooks might confirm (Camerer, 2004, Angner, 2016, Dhami, 2016).

There is a recent and scarce literature offering a framework combining risk and delays, a stream called, in this work, the **common approach literature**[1]. The interest for such an approach comes from two sources. First, both risk and time preference may derive from the same psychological drivers (Prelec and Loewenstein, 1991). Secondly, there is an interaction between them. Indeed, it has been established (Keren and Roelofsma, 1995) that the present bias is strongly affected by the probability level, with a sharp increase of the preference for today's pay-off in case of (almost) sure outcome. The surer the outcome, the stronger the preference for the present. On top of that, the sooner the pay-off, the stronger the certainty bias is. People are ready to give up much more in terms of pay-off in order to have an (almost) sure outcome when the outcome will come soon. When the outcome is long delayed people are less risk averse.[2]

Inside this new literature, Baucells and Heukamp (2012) have developed a model, the Probability and Time Trade-Off (PTT), that achieved to create a framework that responds to the aforementioned empirical findings. Yet, this model relies, among other assumptions, on a hypothesis of constant trade-off between probability and time.

This paper has three objectives. The first one is to highlight that that the constant trade-off assumption is not compatible with empirical findings of Ambrus et al. (2014) and of Green and Myerson (2004) presented hereunder. The second one is to offer alternative hypotheses about the risk/time trade-off. The third one is to offer a pilot survey with the aim of testing these hypotheses. The ambition is more generally to contribute to the modeling of choice under risk and delay. Such a contribution may have a strong impact on many economic and financial decisions. It has to be added that this is a largely uncharted territory. The interaction between risk and time has been surprisingly rarely studied. Gerber

[1] This name refers to the pioneer paper in that field of Loewenstein and Prelec (1991) titled: "Decision making over time and uncertainty: a common approach".
[2] This literature is close to the construal levels theory in social psychology (Trope and Liberman, 2010) presented in section 2.

and Rohde (2015, p. 57) states that "*There are only few studies … that analyze the tradeoff between risk and delay*".

The rest of the paper is organized as follows. Section 2 presents a selective and critical survey of the literature, in three parts: choices under risk, intertemporal choice, and the common approach literature. Section 3 puts forward alternative assumptions, challenging the Probability and Time Trade-Off model. Sections 4 and 5 respectively sketch the pilot test and present its results. Section 6 discusses the results and offers ways for further research. Finally, section 7 concludes.

2. Selective and critical review of the literature

The topic of this paper is intertemporal risky choices. In this section, we will review selectively the literature from behavioral economics in three steps. The first one will deal with risky choices. The second one will handle intertemporal choices. The third one will review the current state of the common approach literature, where those two dimensions, the risk one and the intertemporal one, are dealt with jointly. The first two steps will be covered without digging into the details, with the aim of offering a good start in the common approach literature. This is useful to better understand the importance of the PTT model and the interest of testing the probability-time constant trade-off hypothesis.

2.1. Choice under risk

It's nowadays a well-established fact that agent's real-life behavior may deviate from the standard prediction of the classical Expected Utility theory which does assume a certain rationality (procedure invariance, description invariance and, especially, independence axiom, also known as linear perception of probability[3]) from individuals (Abdellaoui, 2002; Dhami, 2016).

In this theory, the utility, written U, of an outcome x that can be received with a probability p is given as: $U(x;p) = p \times u(x)$ where the individual is supposed to choose the bundle offering the highest utility U(.) and where $u(x)$ is the classical, with certainty, utility function that subjectively evaluates the outcome independently from the probability. The classical utility function $u(x)$ will reflect through its shape the nature and the degree of risk aversion.

2.1.1. Anomalies of the Expected Utility Theory

There is evidence that the classical Expected Utility framework does not hold. In particular, the linearity of the utility in probabilities is challenged. Here is a common pedagogical **example**[4]:

Note first that we denote the bundle [outcome x with probability p] as (x;p).

If you ask to a group of people to choose firstly between (4000 ; 0.8) and (3000 ; 1) where "(3000 ; 1)" means "receiving 3000 for sure, and then between (4000 ; 0.2) and (3000 ; 0.25), a significant part of the group will, in general, choose the sure outcome of 3000 in the first case and the riskier of 4000 with a probability of 20% in the second one. To see intuitively that this violates the Expected Utility theory, using "$\prec$" as the symbol of preference for the right-hand side option, it can be expressed as follows:

[3] The interested reader will find the relationship between the indifference axiom and the linearity in the probability of the valuation function in Dhami (2016).

[4] Kahneman and Tversky (1979).

$$
\begin{aligned}
&(4000\ ;\ 0.8) \prec (3000\ ;\ 1) \\
&\leftrightarrow U(4000\ ;\ 0.8) < U(3000\ ;\ 1) \\
&\leftrightarrow 0.8 \times u(4000) < u(3000) \\
&\leftrightarrow 0.25 \times 0.8 \times u(4000) < 0.25 \times u(3000) \\
&\leftrightarrow 0.2 \times u(4000) < 0.25 \times u(3000) \\
&\leftrightarrow U(4000\ ;\ 0.2) < U(3000\ ;\ 0.25) \\
&\leftrightarrow (4000\ ;\ 0.2) \prec (3000\ ;\ 0.25)
\end{aligned}
$$

The Expected Utility (EU) theory predicts that people who prefer to get 3000 with a 100% probability against receiving 4000 with a probability of 80% will also prefer to get 3000 with a 25% probability against receiving 4000 with a probability of 20%. This prediction is largely invalidated by empirical findings.

This phenomenon is called the "***common ratio effect***". It means that multiplying the probability by a common factor may reverse the preferences, in contradiction with EU. A psychological explanation of this is proposed by Prelec and Loewenstein (1991) and presented hereunder in the common approach literature part of this section.

A particular case of the common ratio effect is when one of the outcomes is fully certain, as in the example hereabove. The additional value of certainty is called the "*certainty bias*".

Next to the aforementioned pedagogical example with a choice between two bundles with a single outcome, it is also possible to exhibit the limits of the EU theory with a choice between two bundles of multiple outcomes. A nice example of this is the so-called Allais paradox, also known as the "*common consequence effect*".

There exist also **other forms of violations** of the EU theory such as (Tversky and Thaler, 1990):

- The *violation of procedure invariance*, which means that the way the researcher will try to elicit the preferences of somebody may affect the result;
- The *violation of description invariance*, which means that the framing of the question may also affect the result. A well-known example is the Asian disease, which highlights that the choice may be altered by switching the framework from positive to negative (this is sometimes called the *reflection effect*). In this essay, we will deal only with positive outcomes.

2.1.2. The probability weighting function

Among multiple attempts to relax the linearity in probability[5] (such as those described in Dhami, 2016), one has received a particular attention, the probability weighting function (pwf). The idea is that probabilities are distorted in individuals' minds. This simple idea may have rather complex ramifications, as we will see hereunder. Kahneman and Tversky (1979) have implemented it in their **Prospect theory (PT)**.

This can be written: $PT(x;p) = w(p) \times u(x)$

[5] Also known as the independence axiom.

where w(.) denotes the probability-weighting function. The perception of probabilities is now subjective but is still separated from the evaluation of the outcome.

Importantly, the PT has also other features such as the reference point. This allows for mirroring behaviors when positive outcomes are turned negative, and therefore offers a response to the aforementioned reflection effect. Another feature is the diminishing sensitivity, which translates into a utility function being concave in gains and convex in losses. Another aspect is the loss aversion, with a utility function steeper in losses than in gains.

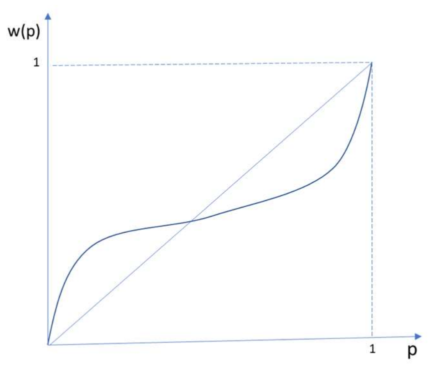
As far as the probability weighting function is concerned, Kahneman and Tversky (1979) have highlighted that agents overweight small probabilities and underweight large ones. The overweighting of small probabilities may be illustrated by the gambling behavior where the extremely low probability of gaining the super prize is perceived as being not that low.

In addition, agents seem to have also a tendency to overweight the **increases** (and decreases) of low and large probabilities compared to changes of medium ones. An intuition for this bias is provided by Gonzalez and Wu (1999, p. 129). As the authors say, "*If men have a 2% chance of contracting a disease and women have a chance of 1%, we perceive the risk for men as twice the risk for women. However, the same difference of 1% appears less dramatic when the chance of contracting the disease is near the middle of the probability scale, e.g., a 33% chance for men and a 32% chance for women may be perceived as a trivial sex difference.*" These authors show this tendency through an experiment. Individuals have to choose between two lotteries giving the same outcome x but with a probability of 5% for the first one and 30% for the other one. Then they get the right to increase one of the probabilities by 5 points of percentage, i.e. to get the first one from 5% to 10% or the second one from 30% to 35%. At this proposition, agents significantly (75% in the authors' study) prefer to increase the low probability lottery, while it should be around 50% under the EU theory. A next step of the study goes about a similar choice but with initial probabilities of 65% and 90%, respectively. In this situation, agents significantly (63% in the authors' study) prefer to increase the high probability lottery. This illustrates that increases in the subjective weighting function is steeper for low and high probabilities than for medium ones.

Together, these findings suggest that the probability weighting function draws a sort of inverted s-shape. This form is the most commonly used and will thus be the basis of this work even if, as we'll see further, some critics exist about this stylized curve.

For the sake of clarity, figure 1 illustrates such a curve relating perceived probabilities to effective probabilities.

Figure 1: *The inverted s-shaped curve - illustration*



It has the following key features:

- Low probabilities are overweighted;
- High probabilities are underweighted;
- The increase of probability starting from both a low and a high probability is overweighted compared to a medium starting probability.

A notable feature of such an inverted s-shape pwf is that it enhances risk aversion against bundles with high probabilities and lowers risk aversion for low probability ones, compared to the neutral, linear weighting.

Functional forms have already been designed for this curve like in Tversky and Kahneman (1992), Gonzalez and Wu (1999), and Prelec (1998). Some of them come along with a psychological narrative (Nardon and Pianca, 2015). This essay does not intend to parametrize the pwf and so we will not cover this literature. Yet, the Prelec function will be used as an example later, thus it can be interesting to see already its functional form:

$\omega(p) = \exp\left(-(-\ln(p))^{\alpha}\right)$ with: $\omega(1) = 1$; $\omega(0) = 0$ and $0 \leq a \leq 1$.

Researchers have tried to link probability distortions to either personal characteristics such as gender, mood state, etc. (see Duda, Schürer and Shubert, 2006) or the nature of the commodity at stake, noticing, in particular, that the fungibility of money could affect the pwf (Krawczyk, 2015). In this essay we will not cover this.

It can also be mentioned that an important body of the literature has already tried to extend the evaluation of risky outcomes to the evaluation of lotteries with multiple non-zero outcomes (Dhami, 2016). The most prominent model is the so-called *Cumulative Prospect theory* (Tversky and Kahneman, 1992) where the subjective evaluation of probabilities depends also on the rank of its paired outcome. Yet, this study will focus on the evaluation of single outcome prospects and then won't regard such aspects.

2.1.3. The criticism of the PT with inverted S-shape pwf

A first critic of the inverted s-shape function comes from al-Nowaihi and Dhami (2010). The authors claim that an additional pattern should be considered. They argue that there is evidence suggesting that extremely low probabilities are, in fact, underweighted and very large ones over-weighted. An illustration of this comes from the driving behavior. Jumping at the red light represents, for the authors, a small risk of accident but with a very large potential cost. If people were overestimating low probabilities, they would not run red lights (neither read their emails while driving). For the authors, there exists a boundary under which low probabilities are not over-weighted but underweighted, and high probabilities not underweighted but over-weighted. This last case is explained by the fact that very high probabilities are seen as a certainty. They have named this the *composite pwf*. There is only limited evidence about this, such that this paper won't deal with this effect, but keeping in mind that it may offer ground for further research.

A review of the different shapes of the pwf and some of their mathematical forms can be found in Nardon and Pianca (2015).

A second critic concerns the effect of the size of the bundle on the risk appetite/aversion. According to the Prospect Theory, agents should show more risk aversion for gains than for losses. Yet, agents may show risk appetite when the stake is positive but quite low and risk aversion when it is negative but low as well. This is called the "peanut effect" (Loewenstein and Prelec, 1991). The name gives well the intuition that it may arise from a lack of concern for low rewards as for low penalties. This has some support, a. o. from Peel, Cain and Law (2005) where authors analyzed the gambling behavior. However, this is contradicted by Etchart-Vincent (2004), who found that risk aversion for losses was in fact increasing with the value of the loss and not present mainly for small-stake loss as the PT with peanut effect would predict. The size effect deserves further research, but this essay won't regard it, and the empirical part will keep the size constant.

A third critic concern the s-shape pwf. In Krawczyk (2015) and in Wilcox (2015), the authors relate or run experiments designed to determine the shape of the pwf. Surprisingly, they reported results quite far from the inverted s-shape. This definitely deserves more scrutiny, but it goes beyond the scope of this paper, which will be based on the inverted s-shape pwf.

2.1.4. Conclusion regarding decision under risk

In a nutshell, the expected utility model fails to match various forms of evidence regarding the decision taking process under risk. The prospect theory with inverted s-shape pwf meets some of those anomalies, such as the common ratio effect (with its extreme case, the certainty bias) and the reflection effect. Yet, other anomalies remain unaddressed by this theory, such as the peanut effect and the procedure variance. Nevertheless, none of the identified but unaddressed by the PT anomalies deals with choices where outcomes are kept constant in terms of amount and nature. Given that this paper will handle choices between outcomes offering the same amount under the same nature (in case, monetary outcomes), these critics of the PT are not relevant, and the PT will be used here as framework for decision-making under risk.

2.2. Intertemporal choice

It's well-established that agent's real-life behavior may deviate from the standard prediction of the classical Constant Discounting theory (CD) which does assume that agents have a unique, constant discount factor. In this theory, the evaluation of an outcome to be received at time "t" is written:

$U\left(x;t\right)=\delta^{t}\times u(x)$ or $U\left(x;t\right)=e^{-rt}\times u(x)$ for the continuous framework.

Where (x;t) represents an outcome x to be received at the period t and where δ is a constant factor of patience and can be written $\frac{1}{(1+r)}$ where *r* is the discount rate.

The (more general) family of discounted utility models (DU after) can be written:

$$U\left(x;t\right)=f(t)\times u(x)$$

They, thus, still assume that the time provokes a change in the weighting (often a displeasure due to waiting, e.g. they assume impatience) but doesn't affect the instantaneous utility function u(.), reflecting the subjective valuation of the outcome.

In fact, when the utility function is constant over time, which is the case in the DU family, the instantaneous discount rate of any continuous (not necessarily constant) discount function can be defined as[6]:

$\rho = \frac{-f'(t)}{f(t)}$ where $f(t)$ is the discount function. This rate ρ is a measure of the impatience as the higher it is, the more future outcomes will be discounted. In the CD (for the continuous framework), ρ is equal to r, and therefore is a constant. To allow for a non-constant discount rate will better fit agents' behavior as described hereunder.

2.2.1. The common difference effect

The constant discounting model implies, among other assumptions**, stationarity**.

This assumption requires that preference between two bundles $(x;t_1)$ and $(y;t_2)$ where x and y are two potential outcomes and t_1, t_2 two different periods of consumption, is unaffected by a common delay:

$(x;t_1) \sim (y;t_2) \leftrightarrow (x;t_1+\Delta) \sim (y;t_2+\Delta)$ with $x \le y$ and $t_1 \le t_2$

where " $\sim$ " is the symbol of indifference between the two options.

A recurrent violation (Prelec and Loewenstein, 1997) of this hypothesis that received the name of **common difference effect** can be expressed as $(x;t_1) \sim (y;t_2) \rightarrow (x;t_1+\Delta) \prec (y;t_2+\Delta)$.

It can be interpreted as the fact that agents tend to accept more easily to wait longer when the initial period is further away in the future. So, the further the periods involved in the choice, the more agents are patient. When confronted with a choice between a gain today and a gain tomorrow, agents exhibit a strong preference for today while when confronted with a similar choice but between a gain next year or next year plus one day, they are less prone to go for the next year choice, i.e. less prone to impatience. This feature can be written:

$(x;t_1+\Delta) \sim (y;t_2+\Delta) \rightarrow (x;t_1) \succ (y;t_2)$

Where " $\succ$ " is the symbol of preference for the left-hand side option.

This allows to see that the common difference effect causes, by definition, that the closer involved periods are, the higher the degree of impatience is. When the current period is involved (at $t_1=0$), the impatience is then the strongest. This is known as the **present bias,** also called the ***immediacy effect***.

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[6] Laibson (1996)

2.2.2. Two main solutions for the common difference effect

The evidence against the CD theory has fueled research into **two main dimensions**.

The first and more prominent one is to consider that the discount function, rather than having an instantaneous discount rate (ρ) constant over time, has a discount rate declining over time (but still positive). It implies that outcomes very far into the future are less "discriminated" against the present than in the CD framework. In more general terms, we can talk about decreasing impatience discount functions.

Several forms of such functions exist. An example can be found in Loewenstein and Prelec (1992), with the form:

$f(t) = (1+at)^{-\frac{b}{a}}$ with a;b >0

Without going into details, we can compute its instantaneous discount rate:

$\rho = -\frac{f_t^{'}}{f} = \frac{b}{1+at}$ which does indeed decrease in t.

Another form of decreasing impatience weighting function has been proposed by Bleichrodt, Rohde and Wakker (2009):

$f(t) = ke^{-rt^{1-d}}$ with then $\rho = r(1-d)t^{-d}$ who satisfies decreasing impatience if $0 < d < 1$.

In this paper, the authors present some characteristics of decreasing impatience discounting functions. The one hereabove, for example, has a **constant relative level of decreasing impatience**. This refers to the following index that serve as a measure of the degree of relative decreasing impatience: $\gamma(t) = -t\frac{(\ln f(t))''}{(\ln f(t))'}$. Here, this index is equal to "d". This won't be developed into details in this work but will serve to construct the theoretical predictions in section 3.

Chabris, Laibson and Schuldt (2006) describe several empirical findings (of other studies) about the link between the degree of impatience and **some personal characteristics**. There is, for example, the fact that smokers exhibit a higher discount rate (more impatience). There is also that the young exhibit a higher discount rate than adults. This latter feature will be important for the research conducted here.

The **second** dimension of research**,** that has received less attention, concerns the utility function, and in particular that the utility function may not be stationary, what had been assumed so far. A notable example of this new venue is the CD but with baseline consumption that moves over time. An experiment of Ambrus et al. (2014) allows to disentangle two potential sources of preference for the present. The first one is impatience. The second one is the expected increase in income. Intuitively, if agents expect to have a higher income in the future (and then, as they face liquidity constraints, a higher baseline consumption), they will exhibit a stronger preference for present consumption. It implies that the instantaneous utility function is assumed to be *dependent* of the time.

One illustration of a DU model with instantaneous utility function dependent of the time through the baseline consumption is as follows:

$$U(x;t) = e^{-rt} \times u(x;t)$$

$$u(x;t) = \frac{(x+b_t)^{1-\gamma}}{1-\gamma} - \frac{b_t^{1-\gamma}}{1-\gamma}$$ where $u_t' = u_{b_t}' \times b_t'$; $u_{b_t}' < 0$ and $u_{b_t}'' > 0$

The impact of changes in expected baseline sheds an alternative light on the observation that young generations tend to be more impatient. As a matter of fact, the time preference is driven not only by the degree of impatience but also by expected changes in income and baseline consumption. It is rational that those who expect an increase in their baseline consumption appear to exhibit a higher discount rate.[7] If you are a low paid worker who just enjoyed a bequest and knowing that you will get a significant wage increase over 6 months, you wouldn't spread evenly the benefits of the bequest over your remaining lifetime without taking the forthcoming pay increase into consideration. The so called higher degree of impatience of the young as put forward by Chabris, Laibson and Schuldt (2006) is mislabeled. The young may not be more impatient but may simply anticipate on the expected increase in baseline consumption. Interestingly, the authors had also noticed that the elderly tended to be the least patient, opening the way to integrating additional features, including remaining life expectations, into the model.

The baseline consumption will be an important component of the model developed in section 3.

2.2.3. Other violations of (constant) discounted utility

With non-stationarity being the first one, **a second violation** of constant discounting is the **time inconsistency**. In a nutshell, this means that agents may take a given decision to be implemented at some point in the future and, as time has passed, change their mind.

The hypothesis of time-consistency could be written:

$$(x;t_1) \sim_t (y;t_2) \leftrightarrow (x;t_1) \sim_{t'} (y;t_2) \quad \text{with } t_1;t_2 > t;t'$$

where "$\sim_t$" is the symbol of preference at time "t".

This formulation allows for changing preferences over time. Agents may get back on their decision and turn to be time-inconsistent. It is frequently observed in real life. Let's think about all the occurrences of procrastination among students who, on Monday, are prone to commit themselves to do some homework over the next weekend and then on Friday decide to postpone the job by another couple of days.

The two proposed ways to address the non-stationarity issue, i.e. the decreasing impatience discount function and the baseline consumption, may be compatible with changes in preferences. The decreasing impatience discounting generates it automatically.[8] Unforeseen changes in baseline consumption may also provoke it.

[7] That agents appear to exhibit a higher discount rate does not mean per se that they have a higher discount rate as the time preference is influenced not only by the discount rate but also by the specification of the utility function.

[8] Dhami (2016).

This pattern of time-inconsistency and the discussion whether agents are aware or not of their own inconsistency (a discussion known as sophisticated versus naïve agents[9]) is, however, not relevant for this paper as it concentrates on static decision making.

A third violation of constant discounting will have an impact on the rest of the study, the so-called **magnitude effect**. As presented in Frederick, Loewenstein and O'Donoghue (2002), this refers to the agents' tendency to discount higher outcome at a lower rate. Intuitively, it means that agents are ready to wait longer for higher outcomes.

These are not the only anomalies. Here is a bunch of others coming from Frederick, Loewenstein and O'Donoghue (2002), Prelec and Loewenstein (1997), and Prelec and Loewenstein (1991). They are presented quickly as they are not relevant for this essay:

- Preference for improvement when sequences of outcomes are involved. Agents prefer a pattern of increasing outcomes than of decreasing outcomes. This violates even the strict impatience assumption but this paper deals with single outcomes, not with sequences;
- Interaction between cost and benefit. Under certain circumstances, such as holidays, most agents prefer to pay before enjoying the outcome. This also violates strict impatience, but this paper considers only gains, without payments;
- Preference for spreading outcomes when multiple similar outcomes are at play;
- Sign effect: gains are discounted at higher rate than losses, but this work consider only gains.
- The discount rate depends on the nature of the outcome, here we'll consider only money;
- Delay speed-up asymmetry: the amount needed to compensate somebody who will enjoy later a given outcome is higher than the amount the same person would be ready to give up to get back to the initial date for enjoying the outcome;
- Description variance, also called framing effect: in line with the aforementioned example of the Asian disease, preferences may be influenced by the description of the bundle.

Those anomalies have fueled **original propositions** of modeling such as the habit formation model, models with utility from anticipation and models with visceral influences. Those won't be regarded in this work as they are not suited for the model developed in section 3.

2.2.4. Conclusion of intertemporal choice

Among the anomalies with respect to constant discounting and dealing with single outcome bundles, which are the focus of this work, we can highlight the common difference effect (and its extreme case the present bias) and the magnitude effect. Those two effects are the most relevant, given the framework of the experiment developed in section 4 of this work. In order to address the common difference effect, the focus is on the decreasing impatience discounting function and on the baseline consumption models. Both elements will be of importance in section 3. A response to the magnitude effect is brought by the PTT model of the common approach literature presented hereunder.

[9] Chabris, Laibson and Schuldt (2006).

2.3. The common approach literature

A growing body of the literature intends to merge the two dimensions of risk and intertemporal preferences. It is motivated by theoretical and empirical findings that these considerations could interplay. These findings will be highlighted hereunder. This literature receives here the name of "**common approach literature**" in reference to the seminal work of Prelec and Loewenstein (1991) called: "*Decision making over time and uncertainty: a common approach*". It will be presented in first place

2.3.1. Multiple similarities and Common drivers

The paper of Loewenstein and Prelec (1991) brings (as mentioned in the title) a powerful tool to demonstrate that choices under risk and intertemporal choices might be strongly related. The main objective of the paper is (as the authors say) to "*draw attention to some remarkable parallels between EU and DU violations*"[10] (by DU the authors mean in fact what is called in this work constant discounting). By making those parallels, authors show that both choices may be driven by the same psychological factors.

The first step of the argumentation is to compare several anomalies and see that they seem to be "paired" between each others. These seemingly close anomalies are the change of perception between gains and losses, the effect of the amount at stake and the framing effect. More importantly, the authors state also a parallel between the immediacy and the certainty effect. This can give the intuition that both situations are driven by a common factor. Yet, it has to be underlined that the multiplication of probabilities as seen with the common ratio effect is different from the addition of delays as seen with the common difference effect.

This brings us to step two. The authors argue that choices under risk and intertemporal choices can be driven by two common drivers of preferences:

- **A decreasing absolute sensitivity (DAS)** that the authors define as: *"Increasing the absolute magnitude of all values of an attribute by a common <u>additive</u> constant decreases the weight of the attribute."*
- **An increasing proportional sensitivity (IPS)** that the authors define as: *"Increasing the absolute magnitude of all values of an attribute by a common <u>multiplicative</u> constant increases the weight of the attribute."*

To explain this, let's focus on the intertemporal choice.

Imagine that an agent is indifferent between an outcome x to be received in a period t_1 and a higher outcome y to be received in a further away period t_2. Note that as we assume strict impatience, y must be higher than x. Formally:

$$(x;t_1) \sim (y;t_2)$$

[10] Loewenstein and Prelec (1991, p. 770)

The decreasing absolute sensitivity (DAS), assumes that:

- Adding the same extra time to both bundles, the agent will focus less on the time and more on the outcome. As a result, the agent will be more patient, which is the definition of the common difference effect:
  → $(x;t_1) \sim (y;t_2) \leftrightarrow (x;t_1+\Delta) \prec (y;t_2+\Delta)$
- Adding the same extra amount to the outcome of both bundles, the agent will focus less on the outcome and more on the time. This assumption holds in fact already in any model where we assume a concave instantaneous utility function:
  → $(x;t_1) \sim (y;t_2)$

  $\leftrightarrow f(t_1) \times u(x) = f(t_2) \times u(y)$

  $\leftrightarrow f(t_1) \times u(x+z) > f(t_2) \times u(y+z)$ if z > 0 because of concavity of u(.)

  $\leftrightarrow (x+z;t_1) \succ (y+z;t_2)$

The increasing proportional sensitivity (IPS) assumes that:

- Multiplying the time by a factor greater than 1 in both bundles, the agent will focus more on the time and less on the outcome. The agent will then be less patient, which can be written:
  → $(x;t_1) \sim (y;t_2) \leftrightarrow (x;a \times t_1) \succ (y;a \times t_2)$
  This, according to the authors[11], has "*not been systematically tested, but we have little reason to doubt that it holds.*" The authors continue: "*For example, if one is indifferent between a large prize in two years and a smaller prize in one year, then one is likely to prefer the larger prize when months or days are substitutes for years in the comparison.*"
- Multiplying the amount of the outcome by a factor greater than 1 in both bundles, the agent will focus more on the outcome and less on the time. Formally:
  → $(x;t_1) \sim (y;t_2) \leftrightarrow (a \times x;t_1) \prec (a \times y;t_2)$
  This, in fact, correspond to the magnitude effect that is already documented in the literature as presented hereabove. As a reminder, this effect stipulates that higher outcomes are discounted with a smaller discount rate, i.e. that agents are more patient for bigger rewards.

The authors argue, in the paper, that the two same drivers DAS and IPS can explain the pattern of choice under risk (including the common ratio effect), but with a persistent shortcoming: the outcome doesn't seem to follow the IPS assumption. When the outcomes are multiplied, according to IPS, preferences should shift towards the bigger/riskier bundle, while the opposite, i.e. increasing risk aversion, is observed. They argue that agents have a sort of deception aversion. When facing relatively high stakes, the agent might go for the smaller but safer bundle in order to avoid the deception of having missed a high stake. This may be linked to the theory of disappointment aversion as presented in Dhami (2016, p.182). Whatever the explanation, the shortcoming of the classification of Prelec and Loewenstein (1991) remains. Called the increasing relative risk aversion, it will be addressed in the

[11] Loewenstein and Prelec (1991, p. 781)

model presented in section 3. The integration of risk and intertemporal approaches made by the authors is a significant and elegant step forward but there is room for an alternative framework.

2.3.2. The construal-level theory of psychological distance

Trope and Liberman (2010) offer also a framework joining risk and delay approaches through a psychological model, the construal-level theory. This theory of decision making stipulates that agents first "classify" outcomes with the scale reflecting a combination of various types of "distances". The sources of distance include the temporal distance and the so-called hypotheticality distance, the last one covering risk and uncertainty. On top, the approach allows for other dimensions, including the spatial distance, the social distance, and the distance coming from more abstract, schematic or decontextualized outcomes (Nan, 2007). An important feature of this model for the present essay is that distance is defined in a way that integrates (among other drivers) risk and time.

Once the classification is done, the theory stipulates that "distant" outcomes induce the agent to be driven by long term goals while "close" outcomes push him to be more short-term feasibility oriented.

From their confrontation with empirical research, the authors indicate that "*a stimulus in one dimension may affect its perceived distance on other dimensions*"[12]. This is what matters for our work as it shows that risk and time may not be handled independently. To handle different dimensions together does not mean that the treatment has to be equivalent. They give the intuition of this: "*Time is unidimensional and incontrollable (…) Spatial distance has three dimensions, is relatively stable, and can be controlled (…) Hypotheticality is also only partially controllable*"[13].

The idea that different sources of distance, while related, have to be handled with their respective specificities, is of importance. It will lead us to criticize the Probability and Time Trade-off model, presented hereunder, and to propose alternatives in section 3.

2.3.3. The comparison of weighting and discounting functions

Another important finding in the joint study of risk ad time is presented in Green and Myerson (2004). In this paper, the authors present the interesting findings that decreasing impatience discount function (used in reaction to the common difference effect, a violation of constant discounting) could be suited to generate an inverted s-shape function used to describe the behavior under risk.

Without going into the details, it's important to remark that the similitudes between the weighting functions should not hide the differences. In particular, Green and Myerson (2004, p. 16) state that "*despite the similarity in the mathematical form of the discounting functions, a number of variables have different effects on temporal and probability discounting*".

Out of the variables that may affect differently the weightings of time and of probability as listed by the authors, there are the amount and the cultural differences.

[12] Trope and Liberman (2010, p. 442).
[13] Trope and Liberman (2010, p. 444).

Amount

In intertemporal choice, high amounts are less discounted than small ones while it is the opposite in choice under risk. This finding is parallel to the decreasing proportional sensitivity highlighted in Prelec and Loewenstein (1991).

Cultural differences

The convergence between the two weighting functions does not imply that we could consider only one applicable separately to both the risk and the intertemporal dimensions. Cultural differences do confirm it. The authors relate an experiment conducted by Du et al. (2002) where it was observed that: "*the Chinese discounted delayed rewards more steeply than the Japanese but the Japanese discounted probabilistic rewards more steeply than the Chinese*"[14].

As with the need to handle differently different distances observed about the construal-level theory, we have here another criticism of a single approach. There is a need for an integrated approach of risk and time but not a single process. The Probability and Time Trade-off model, presented hereunder, is such an integrated approach but that fails to explain these cultural differences. It also fuels the case for the alternative framework to be presented in section 3.

2.3.4. Empirical evidence of cross effects

The superiority of the integrated approach is confirmed by empirical evidence. This subsection is devoted to two papers offering such evidence.

In **Keren and Roelofsma (1995)**, the authors tested if two well-known anomalies of CD and EU, respectively the immediacy effect and the certainty effect (that are in fact special cases of respectively the common difference and the common ratio effect) were sensitive to the initial level of the variable in the other dimension. It is: the immediacy effect is affected by the initial level of probability and the certainty effect by the initial level of delay.

Before Keren and Roelofsma, only unidimensional experiments had been performed: the immediacy effect had been analyzed only with sure outcome and the certainty effect had been tested only for no-delay reward. Among the experiments they ran, two offer great insight for this essay.

The first one is to study the immediacy effect with different probabilities. They started asking individuals to choose between $100 now (choice A) and $110 in 4 weeks (choice B). Then they asked to other individuals to choose between $100 in 26 weeks (choice C) and $110 in 30 weeks (choice D). The probability associated to the outcome was set initially at 1. As expected by the intertemporal choice and more particularly by the common difference effect, agents were relatively more patient when the initial delay was already large: 82% chose A and only 37% chose C, a difference of 45%. This 45% represent well the increased impatience when the present period is involved, i.e. the present bias or immediacy effect.

They re-ran the same experiments but with probabilistic outcomes such that agents could then get a lottery instead of a sure outcome. They did it with a probability of 0.9 and of 0.5. For example, choice A becomes: "$100 with 0.9 probability now" and B: "$110 with 0.9 probability in 4 weeks".

[14] Green and Myerson (2004, p. 18).

While there was a difference of 45% between the first (sure) A and C, what represent the immediacy effect, this difference drops dramatically to 29% when the probability is 0.9 and to 6% when the probability is 0.5. This means that the absolute value of the probability influences the time-outcome trade-off (here the immediacy effect). In particular, it shows that a lower probability decreases the impatience. It has an effect looking like a longer initial delay.

This feature means that preferences may not be represented through a disentangling model such as U(x;p;t)=w(p).h(t).u(x).

Note A' and B' the bundles A and B with a 0.9 probability.

$$\begin{aligned}
&\text{If } A' \succ B' \\
&\leftrightarrow U(A') > U(B') \\
&\leftrightarrow w(0.9)\times h(0)\times u(100) > w(0.9)\times h(4)\times u(110) \\
&\leftrightarrow w(1)\times h(0)\times u(100) > w(1)\times h(4)\times u(110) \\
&\text{Then } A \succ B
\end{aligned}$$

The model would predict that the proportion of preference for A should be independent from the probability, what is contradicted by the aforementioned evidence.

A second experiment tested the parallel effect of delays on the certainty effect. Here they ran a similar experiment by asking preference between, first, two bundles A and B, with B being certain, and then, two "re-scaled with lower probabilities" bundles, C and D respectively, to show the common ratio effect. It leads to a deviation from the Expected Utility model shown above, as agents who were preferring B to A shift towards preferring C to D. B was benefiting from the certainty bias. Then they submitted the same two choices but each of the four bundles were then delayed by one year. The preference for the sure outcome B in the first choice dropped then dramatically from 78% to 57%, what means that the added value of certainty is much enhanced when the bundle deals with present outcome. In other words, adding a delay has the same effect as re-scaling to the bottom the probabilities.

By the way, this experiment also invalidates a disentangling approach such as U(x;p;t)=w(p).h(t).u(x).

The two experiments show that there are cross effects between risk and delays. As a result, it is not satisfactory to only underline the similarities between risky and intertemporal choices. The common approach has to be restricted to an integrated approach.

Baucells and Heukamp (2012) have two major contributions for the common approach literature. They firstly present a synthesis of previous experiments that confirms the existence of the common difference and of the common ratio effect and that invalidates any disentangling approach. The second part of their paper presents a very interesting model, the probability and time trade-off model, that will be presented hereunder. It's this model that uses the probability-time fixed trade-off assumption that will be tested in this work.

Their synthesis table is provided hereunder, under heading Table 1.

Table 1: *Synthesis of Baucells & Heukamp (2012)*

Choices between Prospects A and B

	Prospect A	Vs.	Prospect B	Response	N
1.	**(€9, for sure, now)**	Vs.	(€12, with 80%, now)	**58%** vs. 42%	142
2.	(€9, with 10%, now)	Vs.	**(€12, with 8%, now)**	22% vs. **78%**	65
3.	(€9, for sure, 3 months)	Vs.	**(€12, with 80%, 3 months)**	43% vs. **57%**	221
4.	**(f100, for sure, now)**	Vs.	(f110, for sure, 4 weeks)	**82%** vs. 18%	60
5.	(f100, for sure, 26 weeks)	Vs.	**(f110, for sure, 30 weeks)**	37% vs. **63%**	60
6.	(f100, with 50%, now)	Vs.	**(f110, with 50%, 4 weeks)**	39% vs. **61%**	100
7.	**(€100, for sure, 1 month)**	Vs.	(€100, with 90%, now)	**81%** vs. 19%	79
8.	(€5, for sure, 1 month)	Vs.	**(€5, with 90%, now)**	43% vs. **57%**	79

Source. Baucells and Heukamp (2012). The authors mentioned:
"*Sources. Rows 1-3, Baucells and Heukamp (2010, Table 1); rows 4-6, Keren and Roelofsma (1995, Table 1) (f1 in 1995 equaled $0.6); rows 7 and 8, Baucells et al. (2009).*
Notes. Modal preferences are shown in bold. For rows 4-6, we do not know whether the authors employed real incentives. In all the rest, some subjects were selected at random and one of their choices was played out for real money. For rows 4 and 5, there is abundant evidence of the common difference effect using real incentives (Horowitz 1991, Loewenstein and Prelec 1992). Except for row 7, the modal preference was significantly higher than 50% using a binomial test. Hence, the five preference patterns are statistically significant. In all cases, the timing of resolution is set at t, except for row 6, in which it was not stipulated."

As the authors present:

- Comparison between choices 1 and 2 illustrates the (traditional or "at time 0") **common ratio effect**;
- Comparison between choices 4 and 5 illustrates the (traditional or "with certainty") **common difference effect**;
- Comparison between 1 and 3 illustrates that adding a common delay to a choice where delays were the same makes the choice to shift towards the riskier one. This suggests that adding a common delay has a similar effect as dividing both probabilities by a common factor or, as said by the authors, "time acts as probability". They call this effect the "**common ratio using delay"**;
- Comparison between questions 4 and 6 illustrates that dividing both probabilities of a choice by a common factor (starting from identical probabilities) makes the choice to shift towards the later and larger reward. This suggests that dividing both probabilities by a common factor has the same effect as adding a common delay to both choices or, as said by the authors, "probability acts as time". They call this effect the **"common difference using probabilities"**;
- A last comparison between 7 and 8 illustrates the magnitude effect or the fact that higher rewards are less time discounted than small ones. This effect is called in their paper "subendurance".

Two important concluding remarks can be made about the two papers presented in this subsection. The first one is that findings contradict a separate approach and appeal for a common, integrated approach. The second one is that the reason why probability and time are interplaying remains to be elucidated.

2.3.5. The Probability and Time Trade-Off Model

Baucells and Heukamp (2012) proposed a model of integrated approach of risk and time to overcome the aforementioned violations. The model is called "The Probability and Time Trade-Off model" (PTT after) and can be written as follows:

$$V(x;p;t) = \omega(pe^{-r_x t}) \times u(x) = e^{-d(-\ln p + r_x t)} \times u(x)$$

where:

$\omega(s) = e^{-d(-\ln s)}$	is a joint weighting function that looks like the pwf of Prelec (1998):
$d(.)$	is a psychological distance function assumed continuous and strictly increasing;
r_x	represents a discount parameter that can be made dependent on the amount x.

It may be noted that if the psychological distance function has the simple form: $d(s) = s^a$, at time "0" the weighting is exactly the Prelec pwf presented hereabove.

This model has several strengths[15]: on some conditions such as d(.) concave and r(.) decreasing in x, it may account for a bunch of observations, including:

- It can be consistent with the (classical) common ratio effect. An easy way to see it intuitively is to remark that, when using the simple form of the psychological distance presented hereabove, the model looks like the PT with a (Prelec 1998) inverted s-shape pwf, and is even perfectly identical with it at time 0;
- It can be consistent with the (classical) common difference effect which means that the instantaneous discount rate decreases in time (the initial argument for the decreasing impatience discounting);
- It can be consistent with common ratio using delay which means that adding delays increases the risk appetite;
- It can be consistent with common difference using probabilities which means that, once the outcome is no longer certain, the preference for an immediate reward is less acute;
- It can be consistent with the subendurance effect, which means that the instantaneous discount rate decreases with the amount of the outcome (the high amounts are less discounted than the small ones, i.e. the agent accepts to wait more for large outcomes).

Yet, this model faces a double shortcoming. Firstly, it imposes that the deviation from EU and from CD are driven by the same parameters, i.e. the same psychological function d(.). As a result, it fails to meet the evidence presented in Green and Myerson (2004) suggesting that agents could exhibit a degree of deviation from EU different from their degree of deviation from the CD. Their model is more easily parametrizable but, at the same time, is less accurate with respect to preferences under risk and uncertainty. It is an argument in favor of the PTT alternative put forward in section 3.

[15] All mathematical proofs can be found in Baucells and Heukamp (2012)

Secondly, the PTT imposes a constant trade-off between risk and time[16] given an outcome x. This can be written:

$$\text{For all } (x;p;t),\ (x;q;s) \text{ in the strictly positive domain, } \Delta \in (0;\infty), \text{ and } \theta \in (0;1),$$
$$\text{if } (x;p;t+\Delta) \sim (x;\theta p;t), \text{ then } (x;q;s+\Delta) \sim (x;\theta q;s)$$

Intuitively, for a given x, it exists a fixed trade-off between risk and delay, as shown here:

$$(x;p_1;t_1) \sim (x;p_2;t_2)$$
$$\leftrightarrow V\left(x;p_1;t_1\right) = V\left(x;p_2;t_2\right)$$
$$\leftrightarrow \omega(p_1 e^{-r_x t_1})u(x) = \omega(p_2 e^{-r_x t_2})u(x)$$
$$\leftrightarrow \frac{p_1}{p_2} = e^{r_x(t_1-t_2)}$$

So, the ratio between the two probabilities does not depend on the initial period of time but only on the time gap between the two outcomes and the time gap, in turn, depends only on the ratio of probabilities. Initial levels of probabilities and delays in the two outcomes are not relevant.

We highlight that the fixed trade-off has implications. A first one is as follows:

According to the PTT, if $(x;p_1;t_1) \sim (x;\overline{p};t_1+\Delta)$ and $(x;p_2;t_2) \sim (x;\overline{p};t_2+\Delta)$

Then, we should get $p_1 = p_2$, what means that the initial values of time ($t_1;t_2$) have no influence, only the additional delay, noted Δ, matters.

The core of our empirical work will be to test such a result, with the idea, if it is invalidated, to gain support for the alternative model we will present.

A second implication can be presented:

If $(x;p;t) \sim (x;\overline{p};t+\Delta)$ and $(x;q;t) \sim (x;\overline{q};t+\Delta)$, then we should get $\frac{p}{\overline{p}} = \frac{q}{\overline{q}}$ → ratios between probabilities should be equals.

Our empirical work will also test this, with the idea of differentiating among different alternatives to the PTT.

Baucells and Heukamp (2012) have tested the PTT and claim that the evidence does support it. Yet, their test may appear not entirely convincing as it goes about short additional delays, what may alter results. Our test will deal with extended additional delays.

To conclude on this, Baucells and Heukamp (2012), provide a very innovative and powerful model but that might be too simplistic to represent preferences under risk and time. Our section 3 will offer an alternative framework and section 4 will present a test related to both the PTT and our alternative.

[16] In Baucells and Heukamp (2012), the name of the hypothesis is “Probability-Time Trade-Off”. For the sake of clarity, we add to it the word “constant”.

A last point concerning the PTT is the existence of a literature that analyzed the neurological interactions between probability and time decision making (Arrondo et al., 2015). It's however beyond the scope of the present paper.

2.3.6. Another integrated approach: The Weighted Temporal Utility Model

Gerber and Rohde (2017) present another integrated approach model: the Weighted Temporal Utility (WTU after). This model represents the preferences by the following function:

$V(x;p;t) = w(p;t) \times u(x;t)$

Without going into the details, this model differs from the PTT on important aspects, presented hereunder:

- Their joint weighting function has a much more flexible form as there is no imposed proportionality between p and t. In particular, the probability-time fixed trade-off hypothesis is not required, even if we consider only the weighting function;
- Deviations from the EU and the CD are not necessary parallel;
- Their weighting function doesn't depend on the magnitude of the outcome x. The subendurance effect is allowed for by changes in the instantaneous utility function;
- The instantaneous utility function depends on the time. This is a major difference as it gives a lot of flexibility to the model. The proposition of the authors make sense in the light of the paper of Ambrus et al. (2015), presented hereabove. As a reminder, in this paper the authors put in evidence the importance of the baseline consumption in the deviation of CD. It's important to remark that thanks to this factor, even if the weighting function is similar to the one of the PTT model, as it would be the case with: $V(x;p;t) = \omega(pe^{-rt}) \times u(x;t)$, the changes in instantaneous utility function may invalidate the probability time fixed trade-off hypothesis, as it will be shown in the next section.

Taken together, because the weighting function of the WTU is more flexible and because the instantaneous utility function depends on the time, the WTU doesn't need to respect the probability-time fixed trade-off hypothesis.

2.4. Summary

If the probability weighting function and the time discounting function were known and independent from each other, the trade-off would be directly computable. Empirical research has shown that it is not the case, requiring a more sophisticated approach, called here an integrated approach.

The PTT offers an elegant framework for such an approach. It, however, faces shortcomings, in particular it requires a probability-time fixed trade-off. In case the trade-off would turn not to be fixed, the highly flexible approach of the WTU is appealing.

Two theoretical alternatives to the PTT, one fully in line with the WTU and the other one deviating slightly from the WTU, will be designed, and presented in section 3. They will then be tested along the lines presented in section 4.

3. The PTT and two alternative models

This section will present two alternatives to the PTT we have developed under the influence of the WTU. It will be structured as follows. We will start with the definition of the scope of our research in terms of which kind of decision types are considered, next offer a short overview of the PTT, and then present the two alternatives.

3.1. Which types of decision

The analysis is restrained to a sub-set of decision-making processes, the ones who meet the following rules:

- single decision period only, excluding therefore questions about dynamic consistency;
- there is definitely room for risk but probabilities are known in advance; there is therefore no ambiguity;
- outcomes are only positive; the gain/loss asymmetry is left aside;
- there are only monetary gains; there is no alteration due to the possibly different natures of the outcome;
- the payment is fully paid at the announced time; there is no interference due to multiple installments.

3.2. Overview of the PTT model

The Probability and Time Trade-Off model (PTT) of Baucells and Heukamp (2012) combines risk and time approaches, fueled by the empirical finding that they are interrelated. In particular, the initial level of time affects the probability-outcome trade-off and the initial level of probability affects the time-outcome trade-off. The model takes the following valuation form:

$$V(x;p;t)=\omega(pe^{-r_x t})\times u(x)=e^{-d(-\ln p+r_x t)}\times u(x)$$

This model, as seen in the previous section, imposes a fixed trade-off between probability and time:

$$(x;p_1;t)\sim(x;\overline{p};t+\Delta)$$

$$\leftrightarrow \frac{p_1}{\overline{p}}=e^{r_x\Delta}$$

It has several implications, that can be tested. We will focus on two of them. The first one is as follows.

If $(x;p_1;t_1)\sim(x;\overline{p};t_1+\Delta)$ and $(x;p_2;t_2)\sim(x;\overline{p};t_2+\Delta)$ with $t_1<t_2$.

Then: $p_1=p_2$ → the initial values of time ($t_1;t_2$) have no influence, only the additional time matters.

The second implication we will look at is:

If $(x;p;t)\sim(x;\overline{p};t+\Delta)$ and $(x;q;t)\sim(x;\overline{q};t+\Delta)$

Then: $\frac{p}{\bar{p}} = \frac{p'}{\bar{p}'}$ → the ratio between probabilities is fixed.

3.3. Alternative 1: Baseline consumption-enriched WTU

In this section, we will present a first alternative to the PPT. We will briefly introduce the model, and then look at the two implications along the ones of the PPT stressed hereabove, respectively when the initial delay is changed and when the initial probability is changed.

3.3.1. The model

As the PTT has a constant instantaneous utility function, it does not allow for the observation put forward by Ambrus et al. (2014) presented in the previous section, i.e. that the agent's intertemporal choice is influenced in a potentially non-negligible way by expected changes in future income, affecting the baseline consumption. The intuition comes from the observation that young people are more impatient. The idea would be that in fact they are not more impatient but they take into consideration their expected future income increase.

Gerber and Rohde (2017), next to the WTU model, have proposed a functional form with a simple weighting function similar to the PTT one and with an instantaneous utility function encompassing a baseline consumption component.

Their value function is $V(x;p;t) = \omega(pe^{-rt}) \times u(x;t)$

Instantaneous $u(.) = u(x;t) = u(x;b_t)$ where $u_{b_t}^{'} < 0$ and $u_{b_t}^{''} \geq 0$

A close functional form is: $u(x;t) = \frac{(x+b_t)^{1-\gamma}}{1-\gamma} - \frac{b_t^{1-\gamma}}{1-\gamma}$.

3.3.2. Identification of implication 1, with changing initial delay

Here, If $(x;p_1;t_1) \sim (x;\bar{p};t_1+\Delta)$ and $(x;p_2;t_2) \sim (x;\bar{p};t_2+\Delta)$ with $t_2 > t_1$

Then[17]:

$$p_1 = \frac{\omega^{-1}\left(\omega\left(\bar{p}e^{-r\Delta}e^{-rt_1}\right) \times A\right)}{e^{-rt_1}} \qquad \text{where } A = \frac{u(x;t_1+\Delta)}{u(x;t_1)}$$

[17] Proof in appendix A.1.

And similar for p_2:

$$p_2 = \frac{\omega^{-1}\left(\omega\left(\overline{p}e^{-r\Delta}e^{-rt_2}\right)\times B\right)}{e^{-rt_2}} \qquad \text{where } B = \frac{u(x;t_2+\Delta)}{u(x;t_2)}$$

From this, it appears that in the WTU framework the probability time trade-off has not to be constant. Being non-constant, the trade-off may take many forms. We will bring forward hereunder a hypothesis regarding the nature of this trade-off and offer, in section 4, a test of it.

Furthermore, let's remark that in case of constant instantaneous u(.) w.r.t. time, A and B would be equal to 1 and thus $p_1 = \overline{p}e^{-r\Delta}$ and $p_2 = \overline{p}e^{-r\Delta}$. Therefore, $p_1 = p_2$. This is no surprise because with u(.) constant, the model is then similar to the PTT (with the exception of the parameter r_x which depends on the outcome in the PTT but here the outcome is fixed such that it doesn't matter for the equality of the probability hereinabove).

In case of non-constant u(.), the probabilities p_1 and p_2 will be different from $\overline{p}$, the probability of the alternative bundle, for two reasons. Firstly, they have to be lower than $\overline{p}$ to compensate for the shortening of the delay within the weighting function (this is captured by $\overline{p}e^{-r\Delta}$). Secondly, they have to offset the potential change in instantaneous utility in relationship with the shape of u(.) with respect to time.

The direction of this second effect, a lowering or an increase of the two probabilities is, at this stage, undetermined. This indeterminacy will be lifted by the first of the two additional constraints here just under, coming on top of $u'_{b_t} < 0$ and of $u''_{b_t} \geq 0$

The first one does explicit the relationship between u(.) and time, and the second one goes further in defining the shape of this relationship.

The first additional restriction is that $b'_t > 0$ and $b''_t < 0$. It means that the baseline consumption increases concavely in time. This assumption will be discussed later. It allows to have the following features: $u'_t(x;t) < 0$ and $u''_t(x;t) > 0$[18]. As a result, the indeterminacy is lifted as we know for sure that p_1 and p_2 will be lower than $\overline{p}$. Yet, we still don't know if p_1 is smaller or not than p_2.

The second additional restriction comes from an intuition, to be seen as a parallel of the decreasing impatience, under the form of the following constraint: $\frac{\partial\left(-\frac{u'_t(t)}{u(t)}\right)}{\partial t} < 0$. This makes sense for agents expecting a sharp increase in baseline consumption over the coming period, and a slower increase afterwards. It is suited for young adults, which will be concerned by the pilot test.

With these two constraints, A<B[19] in the formula for p_1 and p_2.

[18] Proof in appendix A.2.
[19] Proof in appendix A.3.

If the weighting function had been linear, i.e., with w(s) = s, p_1 would for sure be lower than p_2 because then $p_1 = \bar{p}e^{-r\Delta} \times A$ and similarly for $p_2 = \bar{p}e^{-r\Delta} \times B$. Here again, the probabilities p_1 and p_2 must decrease compared to $\bar{p}$ to compensate for both the gain in time within the weighting function and the increase in instantaneous utility $u_t^{'}(x;t) < 0$. The compensation for the gain in the weighting function is the same for both probabilities ($\bar{p}e^{-r\Delta}$) but the gain in terms of instantaneous utility is higher for p_1 than for p_2 given that A < B, implying that p_1 must decrease more than p_2 compared to the alternative bundle's probability $\bar{p}$ to keep the indifference. That's why $p_1 < p_2$.

With a non-linear weighting function, p_1 is no longer for sure smaller than p_2. Here, we will not attempt to demonstrate that $p_1 < p_2$ but that it is likely to be the case. To get the intuition of it, first denote $c = \bar{p}e^{-r\Delta}$; $k_1 = e^{-rt_1}$ and $k_2 = e^{-rt_2}$.

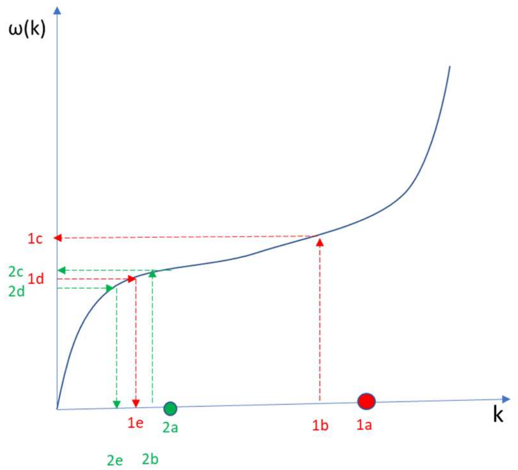
We can rewrite p1 as $p_1 = \dfrac{\omega^{-1}\left(\omega\left(ck_1\right) \times A\right)}{k_1}$ and p2 as $p_2 = \dfrac{\omega^{-1}\left(\omega\left(ck_2\right) \times B\right)}{k_2}$.

As t1 < t2, we know that $k_1 > k_2$. We can also see that: $c = \bar{p}e^{-r\Delta} < 1$.

We also know, as shown hereabove, that A < B.

From A<B, it comes that, ceteris paribus, p_1 would have to be smaller than p_2. However, we have to take into account the impact of k_1 on p_1 vs. the impact of k_2 on p_2. That $k_1 > k_2$ leads to a higher denominator for p_1 than for p_2, also inducing $p_1 < p_2$. Yet, the presence of k in the numerators goes into the opposite direction. This makes a unique prediction about p_1 being smaller or not than p_2 impossible. Yet, it appears that in most cases p_1 will indeed be smaller than p_2 with the exception of very high values of k. Figure 2 illustrates this feature.

Figure 2: *Relative size of probability changes in Alternative Model 1*



How to read figure 2?

- We make use of figure 1, the inverted s-shaped curve presented in section 2;
- The red color is related to p_1, the green one to p_2;
- To compute p_1, we follow the following process:

- Step 1 is to start with k_1 noted as 1a on the graph;
- Step 2 is to take the fraction c of k_1, noted 1b;
- Step 3 is to move to the perceived value, $\omega(c.k_1)$, noted 1c;
- Step 4 is to take the fraction A of the perceived value, A. $\omega(c.k_1)$, noted 1d;
- Step 5 is to get back to the real value ω^{-1} (A. $\omega(c.k_1)$), noted 1e;
- Step 6 is to take the ratio between this last value and the initial k_1, which is the formula of $p_1 = \frac{\omega^{-1}\left(\omega\left(ck_1\right)\times A\right)}{k_1}$, i.e. p_1= 1e/1a;

- We compute p_2 following the same process, with k_2 (2a on the graph) lower than k_1 (1a on the graph) and with B bigger than A, leading to p_2= 2e/2a
- Finally, we can compare the two ratios, p_1 and p_2.

On the figure, it is obvious that 1e/1a is lower than 2e/2a, i.e. that p_1 is lower than p_2. It can also be seen that it is when $c.k_1$ is very high, in the right-hand steep part of the $\omega(.)$ curve, that this relationship may not be observed, depending then on the relative magnitude of A and B. It may be remarked that if $c.k_1$ is small enough to be in the concave part of $\omega(.)$, we know for sure that p_1 is lower than p_2, without having to consider the relative magnitude of A and B.

As a result of this development, our first alternative model leads, in terms of its first implication, to the following hypothesis, that will be tested in section 5, when eliciting p_1 and p_2 in the following indifferences:

$(x;p_1;t_1)\sim(x;\overline{p};t_1+\Delta)$ and $(x;p_2;t_2)\sim(x;\overline{p};t_2+\Delta)$ with $t_2>t_1$

Hypothesis 1: *when eliciting the probabilities* p_1 *and* p_2 *such as presented above, young adults exhibit* $p_1<p_2$ *under the restriction of* $c.k_1$ *(=* $\overline{p}e^{-r\Delta-rt_1}$ *) being not too high.*

3.3.3. Identification of implication 2, with changing initial probabilities

After having looked at the implication of changing the initial delay in terms of preferences, we now turn to the implication of changing the initial probabilities. From the strict impatience assumption, we know that the agent is ready to accept a lower probability when the delay is shortened. The question here at stake is to know if the lower the initial probability, the more agents will be reluctant to go for a bundle with a lower probability. The PTT does assume that there is no such effect, and that the trade-off is independent from the initial probabilities.

In our model,

If $(x;p;t)\sim(x;\overline{p};t+\Delta)$ and $(x;q;t)\sim(x;\overline{q};t+\Delta)$ with $\overline{p}>\overline{q}$

Then[20]:

[20] The proof isn't presented in this work but is really close to the one of implication 1 of the first alternative, presented in appendix A.1.

$$\frac{p}{\overline{p}}=\frac{\omega^{-1}\left(\omega\left(\overline{p}e^{-r\Delta}e^{-rt}\right)\times A\right)}{\overline{p}e^{-rt}} \quad \text{and} \quad \frac{q}{\overline{q}}=\frac{\omega^{-1}\left(\omega\left(\overline{q}e^{-r\Delta}e^{-rt}\right)\times A\right)}{\overline{q}e^{-rt}} \qquad \text{where } A=\frac{u(x;t+\Delta)}{u(x;t)}$$

Our model does not preclude that the ratios $\frac{p}{\overline{p}}$ and $\frac{q}{\overline{q}}$ are equal.

As with the indetermination between p_1 and p_2 in implication 1, it is not possible to make a clear and unique prediction about $\frac{p}{\overline{p}}$ being lower than $\frac{q}{\overline{q}}$ or not. Nevertheless, we may venture that $\frac{p}{\overline{p}}$ is indeed smaller than $\frac{q}{\overline{q}}$. The rationale for it follows the same lines of reasoning as the one presented about implication 1.

Let's note: $l_1=\overline{p}e^{-rt}$, $l_2=\overline{q}e^{-rt}$ and $c=e^{-r\Delta}$.

We can see that $l_1>l_2$ because $\overline{p}>\overline{q}$.

We can rewrite: $\frac{p}{\overline{p}}=\frac{\omega^{-1}\left(\omega\left(cl_1\right)\times A\right)}{l_1}$ and $\frac{q}{\overline{q}}=\frac{\omega^{-1}\left(\omega\left(cl_2\right)\times A\right)}{l_2}$

We can see that both right-hand terms look similar to the ones presented w.r.t. implication 1. As there it led to putting forward that the first one (there, p_1) was smaller than the second one (there, p_2), here it leads to $\frac{p}{\overline{p}}<\frac{q}{\overline{q}}$.

There was a caveat with the statement $p_1 < p_2$ in implication 1, namely that $c.k_1$ was not too high. The same caveat does apply here, with $c.l_1$ being not too high , and is exacerbated. Indeed, in implication 1, the fact that A was smaller than B made the case for p_1 being smaller than p_2 stronger. In implication 2, we do not have this effect, and therefore miss this reinforcement.

As a result of this development, our first alternative model leads, in terms of its second implication, to the following hypothesis, that will be tested in section 5, when eliciting p and q in the following indifferences:

$(x;p;t)\sim(x;\overline{p};t+\Delta)$ and $(x;q;t)\sim(x;\overline{q};t+\Delta)$ with $\overline{p}>\overline{q}$

Hypothesis 2: *when eliciting the probabilities p and q such as presented above, young adults exhibit* $\frac{p}{\overline{p}}<\frac{q}{\overline{q}}$ *under the restriction of* $c.l_1$ *(*$=e^{-r\Delta}\overline{p}e^{-rt}$*) being not too high.*

3.4. Alternative 2: Flexible probability price of time

As mentioned above, the PTT model imposes deviations from EU and CD to be parallel. This might be too restrictive to represent preferences. Somebody may be fully rational in terms of probabilities (no deviation in terms of EU), i.e. exhibit a linear weighting of probabilities (instead of the inverted s-shaped curve) and at the same time exhibit a decreasing impatience, meaning that the outcome/time trade-off is decreasing instead of being constant (deviation from CD). The PTT model allows for deviations or "irrationality" but under the constraint that the degree of "irrationality" in terms of probability is strictly related to the degree of decreasing impatience.

We remind here the empirical evidence about the divergences between probabilistic distortions and time distortions put forward about the cultural difference effect by Green and Myerson (2004). As the lack of flexibility of the PTT model in terms of distortions is problematic, we offer as our second alternative a model with a flexible weighting function allowing for non-parallel deviations.

This section will present our second alternative model first, and then consider two implications, along the lines of the previous section, i.e. with changing initial delay and changing initial probability, respectively.

3.4.1. The model

The valuation function of this model is written:

$$V(x;p;t) = \omega(ph(t;x)) \times u(x)$$

$$\omega(s) = e^{-g(-\ln s)}$$

$h(t) = e^{-r_x t^{1-d}}$ is a constant relative decreasing impatience function (CRDI) presented in section 2.[21]

Then: $V(x;p;t) = e^{-g(-\ln p + r_x t^{1-d})} \times u(x)$

This joint weighting function can handle any cultural difference such as the one presented above because h(t) may generate more or less discount for intertemporal choice than for probabilistic choice.

Importantly, this weighting function encompasses the PTT model. Indeed, with d = 0, we are back to the original PTT. As here d may be different from 0, this model does extend the PTT. The parameter d allows for non-parallel deviations while, at the same time, it drives the intensity of the relationship between time and probability.

Let's consider the simple form of psychological distance $g(z) = z^a$; consider, then: $V(x;p;t) = e^{-(-\ln p + r_x t^{1-d})^a} \times u(x)$.

With d = 0, it is then clear that a is the only parameter that drives probabilistic distortions and the degree of decreasing impatience, forcing to have "parallel deviations". With d ≠ 0, this is not the case, and deviations do not have to be parallel.

Yet, boundaries for d do emerge. Let's consider, for simplicity, $r_x = 1$. On the one hand, d has to be smaller than 1 to accommodate for strict impatience. On the other hand, if d is larger than 1 – 1/a, we have decreasing impatience for every initial values of p and t.[22]

[21] At this stage, the degree of decreasing impatience is not yet specified and can be positive and negative.
[22] Proofs in appendix A.4.

It can be shown that the degree of outcome/time decreasing impatience is increasing in d.[23]

Starting with d=0 for the case of parallel deviations between probability distortions (from the EU) and time distortions (from CD), it can be seen from these findings that when the deviation in terms of time is more pronounced than the deviation in terms of probability, then d has to be higher than 0. In the opposite case, d will be lower than 0.

From this, we can move to the direct trade-off between probability and time. We will call this trade-off the **probability price of time**. In general terms, it reflects how much an agent is ready to give up in terms of probability in order to shorten the delay. The probability price of time has not to be a constant.

It has to be noticed that the form p. H(t) for the probability price of time looks like the outcome/time trade-off under certainty, and, more particularly, the constant relative decreasing impatience (CRDI) discounting proposed by Bleichrodt, Rohde and Wakker (2009) mentioned in section 2. There, d in excess of 0 means strictly decreasing impatience. Here, **d>0 does mean a decreasing probability price of time**, to be understood as the shorter the initial delay, the more the agent is ready to give up in terms of probability in absolute terms in order to shorten the delay by a fixed period of time. This feature will be made clear with implication one to be presented hereunder.

The beauty of the model resides in the following. We just stated that there is a direct, bi-directional relationship both between d and the degree of decreasing impatience relative to probabilistic distortion and between d and the intensity of the probability price of time. Therefore, there is a direct link between the degree of decreasing impatience relative to probabilistic distortions and the intensity of the probability price of time. In particular, a higher degree of time/outcome decreasing impatience will go hand in hand with a decreasing probability price of time.

3.4.2. Identification of implication 1, with changing initial delay

With this model, the probability-time fixed trade-off doesn't need to hold neither. Indeed[24]:

If $(x;p_1;t_1) \sim (x;\overline{p};t_1+\Delta)$ and $(x;p_2;t_2) \sim (x;\overline{p};t_2+\Delta)$ with $t_1 < t_2$

Then $p_1 = \overline{p}\dfrac{H(t_1+\Delta)}{H(t_1)}$ and similarly $p_2 = \overline{p}\dfrac{H(t_2+\Delta)}{H(t_2)}$

It appears therefore that p_1 may be different from p_2. More precisely, p_1 will be smaller than p_2 if H(t) follows a strictly decreasing impatience discounting.[25]

At this stage, we can venture an innovative hypothesis: young adults exhibit a relatively smaller probabilistic distortion compared to their time distortion. This hypothesis comes to fore without having had to rely on the baseline consumption theory. It emerges from the criticism of the PTT, with the advantage of being easier to parametrize than the baseline consumption model.

If young adults indeed exhibit such a higher time distortion than their probabilistic distortion, it means that d > 0. In that case, H(t) follows a strictly decreasing impatience discounting, and then $p_1 < p_2$.

[23] Proof in appendix A.4.
[24] Proof in appendix A.5.
[25] Proof in appendix A.5.

Incidentally, this hypothesis is in no way incompatible with the findings presented in Green and Myerson (2004) about cultural differences, what is not the case of the PTT. Noting the time distortion of group i TD_i and the probabilistic distortion of group i PD_i (by group, we mean agents sharing a same cultural feature, but not the age, such as being Japanese or Chinese), the PTT imposes that iff $TD_j > TD_k$, then $PD_j > PD_k$. This has been invalidated by Green and Myerson (2004), where $TD_{Chinese} > TD_{Japanese}$ while $PD_{Chinese} < PD_{Japanese}$. Our hypothesis about young adults is compatible with the kind of patterns exposed by Green and Myerson. For the sake of the illustration, we could have $TD_{Chinese} > TD_{Japanese} > PD_{Japanese} > PD_{Chinese}$.

As a result of this development, our second alternative model leads, in terms of its first implication, to the following hypothesis, that will be tested in section 5, when eliciting p_1 and p_2 in the following indifferences:

$(x; p_1; t_1) \sim (x; \bar{p}; t_1 + \Delta)$ and $(x; p_2; t_2) \sim (x; \bar{p}; t_2 + \Delta)$ with $t_2 > t_1$

Hypothesis 1: *when eliciting the probabilities* p_1 *and* p_2 *such as presented above, young adults exhibit* $p_1 < p_2$.

This hypothesis is almost the same as for implication 1 of our first alternative model, but here without the restriction of $c.k_1$ (= $\bar{p}e^{-r\Delta - rt_1}$) being not too high.

3.4.3. Identification of implication 2, with changing initial probability

Importantly, it may be seen that[26]:

If $(x; p; t) \sim (x; \bar{p}; t + \Delta)$ and $(x; p'; t) \sim (x; \bar{p}'; t + \Delta)$

Then: $\frac{p}{\bar{p}} = \frac{q}{\bar{q}}$.

→ the ratio between probabilities is fixed, as in the PTT; it does not depend on the initial absolute value of probabilities.

Hypothesis 2': *when eliciting the probabilities* p *and* p' *such as presented above, the agents, exhibit*

$\frac{p}{\bar{p}} = \frac{q}{\bar{q}}$.

Interestingly, this is not the same hypothesis as the one of our first alternative model (presented in subsection 3.3.3). It could therefore give a hint towards which of our two alternative models is the most relevant.

[26] The proof isn't presented in this work but is really close to the one of implication 1 of the second alternative, presented in appendix A.5.

3.5. Conclusion

In this section, three models involving the probability/time trade-off have been presented. We started with the PTT, that relies on a constant trade-off hypothesis. Then, we offered two alternatives: a version of the WTU with baseline consumption and a hybrid model combining features of the PTT and of the WTU, that we called the flexible probability price of time. The two alternative models are based on a mix of empirical findings and intuitions.

Both alternatives allow for the same unilateral deviation from the fixed trade-off as far as what we called the first implication is concerned, dealing with changing initial delay (under specific conditions concerning the WTU alternative). Regarding the second implication, with changing initial probability, on the contrary, the two alternatives lead to different predictions.

Section 4 will present the pilot test, and section 5 its results. Before moving to the test, an additional remark deserves to be made about our empirical sample. Before starting the study, we knew in advance that the sample will consist mostly of young adults close to completing university studies. The models and the hypotheses have been designed specifically for this population. This is a limit of this work that will be discussed in section 6.

4. The experiment

In this section, we present the empirical pilot test we performed to challenge the probability-time constant trade-off hypothesis and the two alternatives we proposed in section 3. We deliberately call it "pilot" as we already recognize that its design (in particular that the choices have no real consequences for the respondents) as well as the size and the composition of the sample do affect severely its statistical power.

4.1. The implications

In the previous sections, we highlighted that the fixed trade-off does impose constraints. A first one is as follows:

According to the PTT, if $(x;p_1;t_1) \sim (x;\overline{p};t_1+\Delta)$ and $(x;p_2;t_2) \sim (x;\overline{p};t_2+\Delta)$ Then, we should get $p_1 = p_2$, what means that the initial values of time ($t_1;t_2$) have no influence, only the additional delay, noted Δ, matters.

We saw in section 3 why and how this implication could be invalidated. We argued that for a public of young adults we should observe the following pattern: $p_1 < p_2$. This will be tested in the experiment presented hereunder.

Another constraint of the PTT model is that, if $(x;p;t) \sim (x;\overline{p};t+\Delta)$ and $(x;q;t) \sim (x;\overline{q};t+\Delta)$, then we should get $\frac{p}{\overline{p}} = \frac{q}{\overline{q}}$ → ratios between probabilities should be equal. We test also this equality in our test. This second implication is not central in the analysis and has received less weight in the empirical analysis. As a result, the statistical significance is affected and conclusions to be drawn do require extra caution.

4.2. Preliminary remarks about the elicitation

For the conduct of the experiment we have made two choices. Firstly, we have chosen to elicit probabilities (p_1 and p_2 in the first implication and p and q in the second one) and not delays. Secondly, we have chosen to elicit them in case of a shortening of the delay, and not of a lengthening of it. Why these two choices?

Elicit probability and not time

To elicit only probabilities is a deliberate choice made to simplify the survey for the respondents. We intuitively thought that it's easier for agents to think in terms of changing probabilities (*in casu*, to elicit the probability that compensates for a change in the delay) than in terms of changing delays (*in casu*, it would have been the extra delay that compensates for a given change in probability).

Elicit in case of a shortening, not a lengthening

The choice has been made to elicit probabilities that compensate for a shortening of the delay (called "to the bottom"), and not for a lengthening. As we deal only with positive gains, this is made intentionally to force a solution in terms of indifference between two bundles. In case of extending the delay, at some point, there will be no indifference possible any longer. If for a fixed outcome of EUR 100, the agent is indifferent between [50% probability; 1 year delay] and [90% probability; 5 year delay], it may be that no probability would made him indifferent, keeping the amount fixed, with a bundle to be received with a 10 year delay : the bundle [90% probability; 5 year delay] is likely to always dominate the bundle [100% probability; 10 year delay]. In case of shortening the delay, the agent will never go to the corner solution of a 0% probability. There will always be a strictly positive probability making the agent indifferent when offered a bundle with a shorter delay. A 0% probability option would be absurd, reminding that we deal only with gains.

It can be noted, however, that there might be an asymmetry between shortening the delay and lengthening it. Our paper deals only with shortenings of delays. Due to the shortening/ lengthening asymmetry of intertemporal choice, our results in absolute terms cannot be generalized to the lengthening cases, while the inequalities in our implications 1 and 2 would still be valid.

4.3. The questions

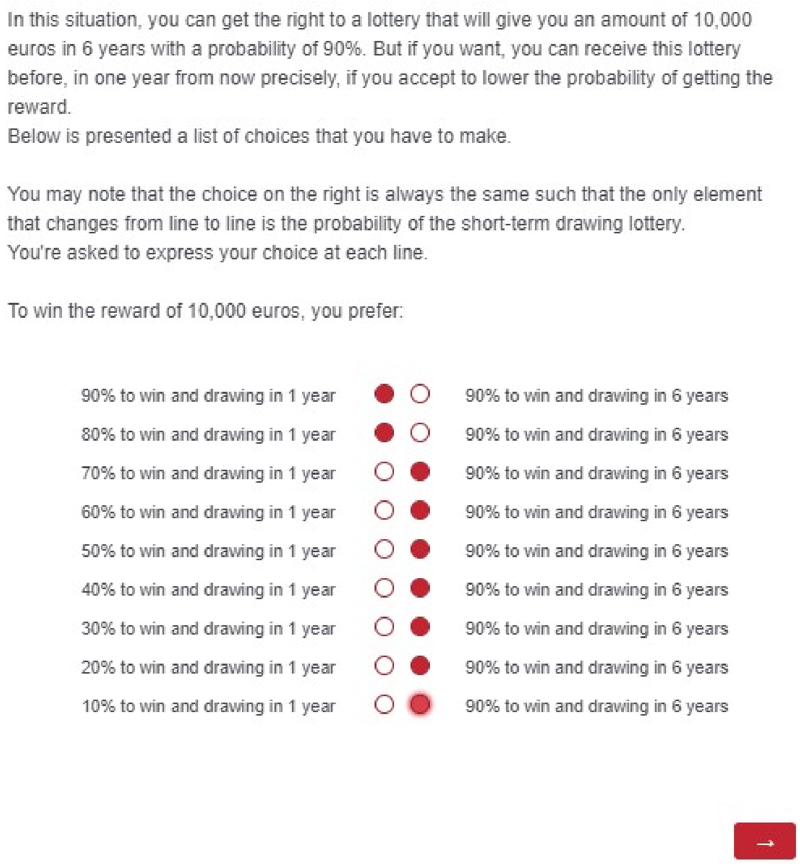
To collect the data, a questionnaire has been designed in Qualtrics[27] and sent to the entourage of the author fulfilling the criterion of being a young adult. Fifteen responses have been collected.

Four questions were presented to the participants. They were organized as a multiple-choice list, Figure 3 hereunder serves as an illustration. It must be noted that from a line to another the only changing parameter is the probability of winning the lottery. The outcome is fixed (at 10.000 euros) and equal for both sides. The delays for receiving the lottery are constant across lines but different from the left column to the right one.

The first question, called "short t large p" (STLP after), presented a list of choices between a lottery to be received in one year and the other one in 6 years. The probability of the second one was fixed at 90% chance to win, the probability of the first one was decreasing (from 90% to 10%, by range of 10 percentage points). Strict impatience assumption requires agents to choose the short-term lottery in the first case (when both lotteries offer a 90% chance to win). In the next choices of the list, agents could switch to the delayed but safer option. Monotonicity[28] of preferences does impose that agents switch maximum once[29]. This is this question that is presented in figure 3, here with a typical answer:

[27] The survey is in appendix B.
[28] Who can be defined as "the more the better".
[29] The proof isn't presented in this work.

Figure 3: *STLP question and a typical answer*



In this situation, you can get the right to a lottery that will give you an amount of 10,000 euros in 6 years with a probability of 90%. But if you want, you can receive this lottery before, in one year from now precisely, if you accept to lower the probability of getting the reward.
Below is presented a list of choices that you have to make.

You may note that the choice on the right is always the same such that the only element that changes from line to line is the probability of the short-term drawing lottery.
You're asked to express your choice at each line.

To win the reward of 10,000 euros, you prefer:

90% to win and drawing in 1 year	●	○	90% to win and drawing in 6 years
80% to win and drawing in 1 year	●	○	90% to win and drawing in 6 years
70% to win and drawing in 1 year	○	●	90% to win and drawing in 6 years
60% to win and drawing in 1 year	○	●	90% to win and drawing in 6 years
50% to win and drawing in 1 year	○	●	90% to win and drawing in 6 years
40% to win and drawing in 1 year	○	●	90% to win and drawing in 6 years
30% to win and drawing in 1 year	○	●	90% to win and drawing in 6 years
20% to win and drawing in 1 year	○	●	90% to win and drawing in 6 years
10% to win and drawing in 1 year	○	●	90% to win and drawing in 6 years

→

The probability that would make this agent indifferent between the two options is between 70 and 80%. We range him then into the category "75%". It can be already noticed that we should consider the numbers only as ordered categories. That's why this essay will rely on ordered probit models and on non-parametrical models.

Formally, this question elicits the borders of the probability p_{STLP} that makes agents indifferent between: $(10,000; p_{STLP}; 1 year)$ and $(10,000; 0.9; 6 years)$.

The second question, called "large t large p" (LTLP after), presented a similar list of choices but now the left-hand side lottery can be received in 10 years and the right-hand one in 15 such that the additional delay stays at 5 years, as in the previous question.

Formally, this question elicits the borders of the probability p_{LTLP} that makes agents indifferent between: $(10,000; p_{LTLP}; 10 years)$ and $(10,000; 0.9; 15 years)$.

The third question, called "t zero large p" (T0LP after), presented a similar list of choices but now the left-hand side lottery can be received now and the other one in 5 years such that the additional delay stays at 5 years, as in the two previous questions. The initial time is not that different from the STLP question large but it allows to capture any immediacy effect by comparing the two sets of results.

Formally, this question elicits the borders of the probability p_{TOLP} that makes agents indifferent between: $(10,000; p_{TOLP}; now)$ and $(10,000; 0.9; 5 years)$.

The fourth and final question, called "small t small p" (STSP after), presented a similar list of choices as the first question STLP, with the same periods of drawing, in one year time and in 6 years time, but now the fixed probability lottery offers a 70% chance to win instead of 90%. This question addresses what has been called implication 2 in section 3. It is not central to our analysis but allows to confront the two alternative models presented there.

Formally, this question elicits the borders of the probability p_{STSP} that makes agents indifferent between: $(10,000; p_{STSP}; 1year)$ and $(10,000; 0.7; 6years)$.

The flow of questions has been organized as follows. In order to control for a potential statistical bias coming from the ordering of the questions, when comparing STLP and LTLP, 50% of the respondents have had to start with STLP, and the other 50% with LTLP. In addition, between these two questions, we chose to ask the STSP question to separate the answers to STLP from the ones to LTLP in order to avoid a "too quick" answer at the second one. They are very similar such that agents could, by mistake, think that they face twice the same question. The change in initial probability of the STSP question would push agents to re-concentrate each time on the different options and not choose automatically. Then, the T0LP was asked in last position for everybody.

The respondents had then one of the following flows:

1) STLP – STSP – LTLP – T0LP
2) LTLP – STSP – STLP – T0LP

The reader may note that by design, the most robust comparison will be the one between STLP and LTLP. Indeed, it is the only bilateral comparison that has a perfectly opposite order. This is a deliberate choice as the test is focused on the so-called implication 1, about changing initial delay.

Furthermore, this flow has the additional advantage that half of the agents will have STLP as first question and the other half LTLP as first question. By nature, answers to the first question are not influenced by answers to other questions as they come in the first place. Thus, a study of the first answers of the agents may be interesting as it prevents the potential influence of other questions as we'll discuss in section 5.

4.4. Choice of values

The chosen values used in the survey are the result of a compromise.

The amount of 10.000 euros was chosen as it may be associated to the price of a new economy car and thus makes appeal for young adults, given their income expectations. In a nutshell, it is an amount that is big enough to be considered seriously without being too big, which could have run the risk of inflating risk aversion.

The fixed probabilities (0.7 for the STSP, 0.9 for the three other questions) had to be high enough to provide "room" for explicit elicitation below them. Otherwise, we would have run the risk of a compression of most answers in the lowest line (the one with 10% or less probability). On the other hand, as explained in section 3 regarding the first alternative model, $\overline{p}e^{-r\Delta-rt_1}$ cannot be too high, and therefore $\overline{p}$ neither.

The initial values of delay (now / 1 year / 10 years) had to be not too high in order to remain meaningful for respondents.

The difference in terms of initial delay between the T0LP, the STLP and the LTLP questions had to be large enough to allow for a significantly different behavior in terms of instantaneous utility function or in terms of probability price of time.

The shortening of the delay (by 5 years) had to be not too large in order to give enough "space" for the elicitation of probabilities making agents indifferent while being high enough to meet the restriction concerning the implications in our first alternative ($\overline{p}e^{-r\Delta - rt_1}$ cannot be too high, and therefore Δ has to be high enough).

For the 4 questions, the scale of the multiple-choice list (10 percentage points) had to be small enough to capture preferences in a not too rough way and, at the same time, not be too small to make the choice not too heavy for respondents. This is convenient for the test of the first implication but creates a problem for the test of the second one as we'll see hereunder.

5. Results

In this section we propose a treatment of the data that should go into the direction of either invalidating the probability-time constant trade-off assumption or reinforcing it. We first present some general remarks about the data set and the statistical models about to be used, then we present separately the tests performed about the two implications presented in the past sections.

5.1. General remarks

As far as the data set is concerned, we can directly highlight that all respondents are maximum 26 year old. None of them violates the monotonicity assumption which means that they all switch maximum once per question.

For the statistical test, because the data we have are ordinal, we chose to perform ordinal probit models. Furthermore, in this work we are interested in detecting an effect of the change in either the initial delay or the initial probability and not in computing the magnitude of such an effect. That's why we also run non-parametric tests. Such tests are more appropriate for small size samples like ours because they do require less assumptions over the underlying distributions of the data. Yet, those have often difficulties at dealing with ties which are frequent in our data set as it will be shown hereunder. For all statistical tests, it is important to note that the responses to the 4 questions are paired, allowing a comparison of the answers per individual (this is called a "within subject design" and is very powerful from a statistical point of view as it allows to perfectly account for individual heterogeneity). At the same time, such an approach faces the risks of respondents feeling obliged to show themselves as rational, therefore responding to the subsequent questions not on the basis of their effective preference but on the basis of what they answered to the first one. This can be linked to what is called the experimenter demand effect, presented in section 6. As mentioned in subsection 4.3 about the flow of the questions, an analysis of the first answers of the respondents (where 50% had the STLP question and the other 50% the LTLP one) will be performed without interference from this potential bias. It must be noted, however, that this analysis goes about unpaired answers, losing the advantage of paired answers in terms of heterogeneity.

Another remark is that we chose not to perform any power calculation as our pilot test has anyway quite a low statistical power due to the small size of the sample. Indeed, due to resources constraints, only 15 answers have been gathered.

The various tests and their results will be presented in the next two subsections, without going into much details. We'll just try to give the main hints. About the performed tests, the reader might find more information elsewhere.[30] Within each subsection, the intermediate conclusion should give enough information without having to go through the stats.

[30] See, for instance, UCLA: Institute for Digital Research and Education (https://stats.idre.ucla.edu/stata/whatstat/what-statistical-analysis-should-i-usestatistical-analyses-using-stata/).

5.2. Implication 1, with changing initial delay

As presented in the previous sections, we want to challenge the following PTT constraint: if $(x;p_1;t_1)\sim(x;\overline{p};t_1+\Delta)$ and $(x;p_2;t_2)\sim(x;\overline{p};t_2+\Delta)$ with $t_2>t_1$ then $p_1=p_2$, with the intuition that $p_1<p_2$. In practical term, this means that we expect as alternative $p_{TOLP}<p_{STLP}<p_{LTLP}$ instead of $p_{TOLP}=p_{STLP}=p_{LTLP}$.

As a first approach, we can consider some direct features of the data. Table 2 presents, for 3 of the 4 questions, the distribution of answers in terms of the central value of probability across the ranges submitted to respondents. For instance, in the column regarding the STLP question, the number "5" in front of probability 65 means that 5 of the 15 respondents give a probability leaving them indifferent between 60% and 70%. The forth question, the STSP one, is not included here because it goes only about implication 2.

Table 2: *Distribution of answers related to implication 1*

	Questions			Total
central probability	T0LP	STLP	LTLP	
35	1	0	0	1
45	0	1	1	2
55	2	2	2	6
65	5	5	3	13
75	4	3	2	9
85	3	4	6	13
95	0	0	1	1
Total	15	15	15	
median probability	65	65	75	

Firstly, we can look at **the medians** of the three questions. It appears that they fit the feature $p_{TOLP}<p_{STLP}<p_{LTLP}$ of the two alternative models better than the PTT. According to the later model, a respondent shouldn't change of probability from one question to another, such that the median should stay the same across the 3 questions. In our sample, the categories of the median probabilities are: 65% for T0LP and STLP and 75% for LTLP. The equality between STLP and T0LP means that medians are in fact in the same interval, not that they are exactly the same.

Secondly, we can look at individual paired answers in order to assess if more agents do give answers compatible with the PTT than with our alternatives. This is not represented in table 2 as this table aggregates individual answers. By comparing paired answers to STLP and LTLP questions, it appears that 7 agents are in an interval of lower probability for the STLP question than for the LTLP one, 4 are in the same interval, and 4 agents are in an interval of higher probability for the STLP question than for the LTLP one. It means that 4 agents contradict our two alternatives, while 11 agents contradict the PTT.

Turning now to the comparison of T0LP and STLP, it appears that 3 agents are in an interval of lower probability for the T0LP question than for the STLP one, 9 are in the same interval, and 3 agents are in an interval of higher probability for the T0LP question than for the STLP one. It means that 3 agents contradict our two alternatives, while 6 agents contradict the PTT.

Those first findings do, to some extent, back our alternatives, but we must discuss first their statistical significance. That's is the **second approach** of the analysis.

We first run an **ordinal probit** (oprobit later) model, with the pooled elicited probabilities as dependent variable and the questions as explanatory variables. Because we have only 15 respondents we can use dummies to control for individual fixed effects. Four different situations are overviewed, with each time the STLP question as baseline. Main results are presented in table 3.

Table 3: *The oprobit models related to implication 1*
(with STLP as baseline question)

	(1) oprobit with individuals dummies	(2) oprobit without individuals dummies	(3) oprobit without T0LP	(4) oprobit with the first answers only
T0LP	-0.233 (0.598)	-0.107 (0.777)		
LTLP	0.863 (0.061)	0.380 (0.325)	0.972 (0.052)	0.767 (0.184)
observations	45	45	30	15

p-values in parentheses

Lessons from table 3 are as follows:

- From column (1), it appears that, considering all the answers to the three questions and controlling for the individual heterogeneity:
    - The coefficient of the question T0LP is negative as forecasted but not statistically significant at a 10% significance level (p-value is 0.598 at a two-sided test);
    - The coefficient of the question LTLP is positive as forecasted and **statistically significant** at a 10% significance level (p-value is 0.061 for a two-sided test);
- From column (2), it appears that the significance of the parameters drops dramatically when the individual fixed effect is not accounted for; the rationale of this deserves probably more scrutiny.
- From column (3) it appears that, considering all the answers but only to the question STLP and LTLP, the coefficient of LTLP is positive as forecasted and **statistically significant** at a 10% significance level (p-value is 0.052 at a two-sided test).
- From column (4), it appears that, considering only the first answer of each respondent (7 for STLP and 8 for LTLP), the coefficient of LTLP is positive as forecasted but not significant at a 10% significance level (p-value is 0.184 at a two-sided test).

It may be noted that for columns 1 and 2, all the three answers of the 15 agents are used. That's why there are 45 observations. For column 3, answers to two questions are used, leading to 30 observations. For column 4, only the first answer of each agent is used, leading to 15 observations. The latest is performed to analyze data without the potential bias coming from the influence of one answer on the next ones.

We run also **three non-parametric tests for paired sample:**

- The Wilcoxon "signrank test" that can compare samples two by two by ranking the difference between paired values;
- The "signtest test" that considers the number of positive versus negative differences between two paired samples;
- The Friedman test that proposes a way to use the ranks of the differences as in the Wilcoxon test with more than two samples but without possibilities of one sided-tests.

At the Wilcoxon "signrank test":

- difference between STLP and LTLP isn't significant (p-values are 0.2625 at a two-sided test and 0.1312 at a one-sided test);
- difference between T0LP and STLP isn't significant (p-values are 0.9223 at a two-sided test and 0.4611 at a one-sided test).
- difference between T0LP and LTLP isn't significant (p-value are 0.2533 at a two-sided test and 0.1266 at a one-sided test).

At the "signtest test":

- difference between STLP and LTLP isn't significant (p-values are 0.5488 at a two-sided test and 0.2744 at a one-sided test);
- difference between STLP and T0LP isn't significant (p-values are 1 at a two-sided test and 0.6563 at a one-sided test);
- difference between T0LP and LTLP isn't significant (p-values are 0.3877 at a two-sided test and 0.1938 at a one-sided test).

At the "Friedman test", difference between the three questions isn't significant (p-value is 0.5397).

To complete this analysis, we perform **two other non-parametrical tests**:

- the "Mann-Whitney test" that compares the ranks between values of two sample as if they would be in a pooled one. Because it doesn't consider the pairing it cannot be used to test the whole dataset. Yet, this test is helpful to analyze the differences between the first answers to the STLP and LTLP questions;
- The "Jonckheere test" that proposes a rank-based test similar to the Mann-Whitney test but with multiple samples. It must be noted that this one, but not the Friedman test, offers the possibility to have a one-sided test but doesn't consider the samples as paired, what may affect results. This test is performed because we are not aware of any test of multiple paired samples that can offer a one-sided test.

At the "Mann-Whitney test", the difference between the first answers to the STLP and LTLP is not significant (p-value is 0.2051 at the two-sided test and 0.1026 at a one-sided test).

At the "Jonckheere test", the difference between groups is not significant (p-value is 0.2878 at a two-sided test and 0.1439 at a one-sided test).

Intermediary conclusion from Implication 1

Together, these results go into the direction of favoring our alternatives compared to the PTT. The medians are more favorable, and the number of agents complying with the expectations of the alternatives is larger. Yet there are statistical limitations. While only the ordered probit models give statistically significant results, it has to be noted that the small size of the sample may have had a dramatic influence on significance.

In a nutshell, results do invite to carry more research along the lines of the two proposed alternatives.

5.3. Implication 2, with changing initial probability

Another implication of the PTT model is: If $(x;p;t)\sim(x;\overline{p};t+\Delta)$ and $(x;q;t)\sim(x;\overline{q};t+\Delta)$ with $\overline{p}>\overline{q}$, then we should get $\frac{p}{\overline{p}}=\frac{q}{\overline{q}}$ → ratios between probabilities should be equal.

In section 3, we saw that our first alternative, the WTU with baseline consumption, was predicting $\frac{p}{\overline{p}}<\frac{q}{\overline{q}}$ while the other alternative, the flexible probability price of time was predicting a constant ratio: $\frac{p}{\overline{p}}=\frac{q}{\overline{q}}$.

We test this hypothesis by confronting data from the STLP and STSP questions. These questions elicited the probabilities that make agents indifferent between:

STLP: $(10000;p_{STSP};1)\sim(10000;0.9;6)$

And STSP: $(10000;p_{STSP};1)\sim(10000;0.7;6)$.

In practical term, we test as alternative: $\frac{p_{STLP}}{0.9}<\frac{p_{STSP}}{0.7}$.

First, we start at looking at the consistency of the answers. Results show that the median of the STSP question (in the 50-60% category) is lower than the one of the STLP question (in the 60-70% category) and that out of the 15 agents, 13 give a lower interval for the STSP question and 0 a higher one. These two features do assess that respondents have been consistent.

To have p_{STSP} lower than p_{STLP} is indeed quite rational as the elicited probability must be increasing in the fixed probability of the bundle it is compared to. Yet, according to the first alternative prediction, the p_{STSP} should decrease at a smaller rate such that the ratio does increase, while the ratio is constant both for the PTT and the second alternative.

Because we have intervals for the elicited probability we have also intervals for the ratios. Then, we could be sure that the ratios differ in the predicted way if the lower bound of $\frac{p_{STSP}}{0.7}$ is higher than the upper bound of $\frac{p_{STLP}}{0.9}$. This happens for only two individuals, those who gave the same intervals for both elicited probabilities. This gives the intuition that our experiment is not suited to test this hypothesis. With the method used to determine whether the ratio does increase, we will not be able to perceive such an increase as soon as p_{STSP} is in a lower interval than p_{STLP}.[31] We would have needed much smaller intervals than of 10 percentage points, but at the risk of being too cumbersome for respondents.

With only 2 non-constant ratios, to perform any statistical test would be meaningless.

Intermediary conclusion from Implication 2

Together, these results go into the direction of not providing much ground for our alternative 1, as only 2 respondents, out of 15, do exhibit the predicted pattern of that alternative. It could be seen as supporting the PTT and our second alternative, while rejecting alternative 1. Yet, caution is more than required before rejection alternative 1 on this basis. Indeed, implication 2 has been built with more restrictive assumptions, that alternative model 1 may not meet while still being relevant.

An additional argument for being especially cautious is that there are statistical limitations here as well. Next to the limited number of answers, the too large intervals may have had a dramatic influence on the results.

[31] Proof in Appendix C2.

6. Discussion and further developments

This work has ventured to study the agents' behavior under both risk and delays. Firstly, we confronted the PTT with contradictory empirical findings of previous studies. Secondly, we proposed, on the basis of those contradictions, two alternative models, the WTU with baseline consumption and the flexible probability price of time. Thirdly we studied the implications of the three models in two situations of probability/time trade-off. Finally, we offered a test of these implications with the objective of determining if our alternative models are more suited than the PTT to describe individuals' preferences. Those developments lead us to state that the PTT is dominated by our alternative models in terms of preference accuracy. Yet, all along our study, we made assumptions and imposed ourselves restrictions that may have an impact on our conclusions. Hereafter are offered elements to fuel additional discussions and developments.

Young adults as a starting point

As mentioned in the methodological part of section 3, we knew before developing the alternative models and the implications that the sample of our empirical pilot test would consist in its majority of young adults close to completing university studies. That influenced the construction of the alternative models and led us to design our hypothesis specifically for this group. An obvious refinement of this work is then to explore the implications of the models for another group of the population, defined on the basis of age, culture or another criterion. For instance, the remaining life expectation may be a relevant parameter for the probability/time trade-off. As indicated in subsection 2.2.2, Chabris, Laibson and Schuldt (2006) have put in light that the elderly (around 75 year old) shows the highest impatience pattern.

Limited scope of decision-making

As presented in section 3, we restricted our analysis to the evaluation of bundles with some characteristics: single period of decision only (agents cannot get back on their decision), no ambiguity, only positive outcomes, only monetary gains and no multiple installments of the payment. All of these features have been introduced purposely to keep the complexity of the theoretical development under control. Any attempt to relax one of those constraints and propose a "more global" framework would surely be of great interest for the behavior under risk and time literature.

Only two implications

The probability-time constant trade-off hypothesis of the PTT has many testable implications. In this work, we chose to focus on two of them. In both cases, we elicit only probabilities, we consider only a shortening of the delay, and the change in delay is constant. The difference between the two implications is that the first one deals with a change in initial delays, while the second one deals with a change in initial probabilities.

To investigate other implications would have made life more cumbersome for respondents, with a risk of no answer or of hasty answers.

Of course, to restrict our analysis and our tests to a sub-set of implications may prevent us from important findings. For instance, as mentioned in the methodological part of section 3, we can expect that the shortening/lengthening asymmetry of intertemporal choices has an effect on the elicitation of the probability price of time.

Alternative model 1 and its additional assumptions

In the alternative model 1, we presented a version of the WTU with a baseline consumption component. In addition to this, we argued firstly that young adults do anticipate a concavely increasing baseline consumption and secondly that this increase has a shape that generates a specific evolution of the utility function w.r.t. time. Formally it has been written:

$$b_t^{'} > 0 ; b_t^{''} < 0 \text{ and } \frac{\partial\left(-\frac{u_t^{'}(t)}{u(t)}\right)}{\partial t} < 0.$$

It can be noted that this assumption is not strictly necessary to make the prediction of hypothesis 1. Yet, it increases the likelihood of this hypothesis. As these assumptions are based on intuitions and not on empirical findings, one could question them.

Alternative model 2 and difference between probability and time weightings

When evaluating which of the probability and time weightings should deviate more from the "rational" behavior, we argued intuitively that young adults should exhibit more deviations at the time weighting. It would be of interest to test this hypothesis.

A new argument can be sketched here about the relative magnitude of the two distortions, with the idea that the bigger deviation in terms of time vs. probability is not a specificity of young adults but could be observed across the whole population. It is that delay per se creates risk. The simple fact that a reward will be received not instantaneously but, in the future, creates the risk of not receiving it. For example, the beneficiary may die before getting the reward or the person who promised the reward may go bankrupt. Therefore, adding a delay to an outcome generates not only a displeasure due to the impatience assumption but creates this additional risk. Thus, the discounting of a delay encompasses a component of risk aversion.

Experimenter demand effect

The survey is not a real-life choice. It had been asked to respondent to focus themselves on a choice. We cannot exclude that this could impact their answers. Here, some agents might think that their rationality is tested and opt for answers denoting a constant trade-off behavior even if this doesn't represent their preferences. Alternatively, some agents might have a clue about which results would "please" the researcher and answer accordingly. This effect has been partially controlled for by comparing the answers from respondents who had first the STLP question with those of respondents who had been assigned the LTLP question first. With this solution, however, the statistical power is diminished compared to the paired sample tests.

Interpretation fallacy

We have proposed two alternatives to the PTT. The fact that the results of the test goes rather against the PTT could be interpreted as supporting the alternatives. This, however, may be excessive, as the invalidation of the PTT may call for another alternative than one of the two presented here.

7. Conclusion

Economics goes by essence about trade-offs, but it is only recently that the one between probability and time came to the fore, while it is of utmost importance in a world of impatience and uncertainty. Will the consumer postpone the purchase of a driverless car in order to lower the probability of design default? Will the pharma company prefer to invest into a given program versus another one with a shorter time to market but with a lower probability of success? In other words, "sooner or safer" is a key social question.

There is a growing but still scarce literature attempting to develop a decision-taking framework dealing with risk and delay simultaneously. One model is the Probability-Time Trade off (PTT). While being an important and elegant step into the integration of risk and time, it relies on the strict hypothesis that the trade-off between risk and time follows a constant rule. This hypothesis stipulates that to leave the agent indifferent between any set of two bundles different by the delay and the probability (but not the outcome), a given shortening of the delay in absolute terms must come along with a constant divisor of the probability, and vice versa. The divisor being constant, we can talk about the probability/time constant trade-off hypothesis.

The PTT faces two drawbacks w.r.t. empirical findings. Firstly, the PTT gives no room for income expectations. Secondly, it supposes that an agent will exhibit "parallel" deviations with respect to the Expected Utility theory and to the Constant Discounting assumption. It means that agents differ from each other but that an agent who exhibits relatively linear preferences in terms of probabilities will exhibit a rather constant discounting pattern.

These two drawbacks call for improving the PTT model. In this paper, we present two such improvements, that we called alternatives. The first one integrates a baseline consumption evolving over time. We called it the baseline consumption-enriched WTU model, where WTU stands for weighted temporal utility. The second alternative is constructed on the idea of allowing for non-parallel deviations. We called it the flexible probability price of time. We studied then the predictions of the three models, the "basic" PTT and the two alternatives, in two situations of trade-off between risk and time (those situations have been called the implications).

The major theoretical finding of this work is that the two attempts ("alternatives") to improve the PTT in order to accommodate these drawbacks leads to a violation of the probability/time constant trade-off hypothesis.

We then designed a test regarding the nature, constant or not, of the probability/time trade-off. We performed a pilot of this test targeted to young adults. The first results, while being not all statistically significant, go rather against the constant trade-off and therefore do offer support for improving the PTT along the lines developed here.

The modeling of choice under risk and delay simultaneously, while being a major issue, had been described in the introduction as being largely an unchartered territory. The present conclusion does not alter the statement, but we ambition to have modestly contributed to clearing the field and fueled the interest for additional research.

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Appendix

Appendix A: Demonstrations of section 3

1. Trade-off under alternative 1: WTU with baseline consumption

$$(x;p_1;t_1) \sim (x;\overline{p};t_1+\Delta)$$
$$\leftrightarrow w(p_1;t_1)\times u(x;t_1) = w(\overline{p};t_1+\Delta)\times u(x;t_1+\Delta)$$
$$\leftrightarrow \omega(p_1\times e^{-rt_1})\times u(x;t_1) = \omega(\overline{p}\times e^{-rt_1})\times u(x;t_1+\Delta)$$

Then:

$$p_1 = \frac{\omega^{-1}\left(\omega\left(\overline{p}e^{-r\Delta}e^{-rt_1}\right)\times A\right)}{e^{-rt_1}} \qquad \text{where } A = \frac{u(x;t_1+\Delta)}{u(x;t_1)}$$

And similar for p_2:

$$p_2 = \frac{\omega^{-1}\left(\omega\left(\overline{p}e^{-r\Delta}e^{-rt_2}\right)\times B\right)}{e^{-rt_2}} \qquad \text{where } B = \frac{u(x;t_2+\Delta)}{u(x;t_2)}$$

2. The derivative of u(.) w.r.t. time under WTU with baseline consumption model

Because we assume $u'_{b_t} < 0$ and $u''_{b_t} \geq 0$.

If we also assume that $b'_t > 0$ and $b''_t < 0$, then $u'_t(x;t) < 0$ and $u''_t(x;t) > 0$:

$$u'_t(x;t) = u'_{b_t}\times b'_t < 0$$
$$u''_t(x;t) = u''_{b_t}\times b'_t + u'_{b_t}\times b''_t > 0$$

3. A<B in the alternative 1 model

$$A = \frac{u(x;t_1+\Delta)}{u(x;t_1)} < \frac{u(x;t_2+\Delta)}{u(x;t_2)} = B \qquad \text{iff } t_1 < t_2 \text{ and } \frac{\partial\left(-\frac{u'_t(t)}{u(t)}\right)}{\partial t} < 0$$

Indeed:

$$A < B \text{ iff}$$

$$\frac{\partial \frac{u(x;t+\Delta)}{u(x;t)}}{\partial t} > 0$$

$$\leftrightarrow \frac{u'(x;t+\Delta)\times u(x;t) - u(x;t+\Delta)\times u'(x;t)}{\left(u(x;t)\right)^2} > 0$$

$$\leftrightarrow \frac{u'(x;t+\Delta)}{u(x;t+\Delta)} > \frac{u'(x;t)}{u(x;t)}$$

$$\leftrightarrow -\frac{u'(x;t+\Delta)}{u(x;t+\Delta)} < -\frac{u'(x;t)}{u(x;t)} \text{ which holds by the above assumption}$$

4. Decreasing impatience in alternative 2: flexible probability price of time

1) Boundaries of d

Consider $f(t) = e^{-(-\ln p + t^{1-d})^a}$ for simplicity.

Then, $f_t^{'} = e^{-(-\ln p + t^{1-d})^a} \times (-1) \times a \times (-\ln p + t^{1-d})^{a-1} \times (1-d) \times t^{-d} \qquad < 0 \text{ iff } d < 1$

And thus: $\rho_t = \dfrac{-f'(.)}{f(.)} = a \times (1-d) \times (-\ln p + t^{1-d})^{a-1} \times t^{-d}$

Decreasing impatience happens when $\rho_t^{'} < 0$, while:

$$\rho_t^{'} = a(1-d) \times \left[(a-1)(-\ln p + t^{1-d})^{a-2}(1-d)t^{-d}t^{-d} - (-\ln p + t^{1-d})^{a-1}(-d)t^{-d-1} \right]$$

$$\rho_t^{'} = a(1-d) \times (-\ln p + t^{1-d})^{a-1} t^{-d-1} \left[(a-1)(-\ln p + t^{1-d})^{-1}(1-d)t^{1-d} - d \right]$$

Noting that $a(1-d)\times(-\ln p + t^{1-d})^{a-1} t^{-d-1} \geq 0$; $\qquad \rho_t^{'} < 0$ iff

$d > (a-1)(1-d)(-\ln p + t^{1-d})^{-1} t^{1-d}$;

Seeing that $0 \leq (-\ln p + t^{1-d})^{-1} t^{1-d} \leq 1 \qquad \rho_t^{'} < 0$ for sure if

$d > (a-1)(1-d) \leftrightarrow d > 1 - \dfrac{1}{a}$.

There is then decreasing but still strict impatience if $1 - \dfrac{1}{a} < d < 1$**.**

2) Influence of b on the degree of relative decreasing impatience

The degree of relative decreasing impatience is:

$$\gamma(t)=-t\frac{\left(\ln f(t)\right)''}{\left(\ln f(t)\right)'}$$

Where : $\ln f(t)=-(-\ln p+t^{1-d})^{a}$

$$\left(\ln f(t)\right)_t^{'}=-a(-\ln p+t^{1-d})^{a-1}(1-d)t^{-d}=-a(1-d)(-\ln p+t^{1-d})^{a-1}t^{-d}$$

$$\left(\ln f(t)\right)_t^{''}=-a(1-d)\left[(a-1)(-\ln p+t^{1-d})^{a-2}(1-d)t^{-d}t^{-d}+(-\ln p+t^{1-d})^{a-1}(-d)t^{-d-1}\right]$$
$$=-a(1-d)(-\ln p+t^{1-d})^{a-1}t^{-d}\left[(a-1)(1-d)(-\ln p+t^{1-d})^{-1}t^{-d}+(-d)t^{-1}\right]$$
$$=\left(\ln f(t)\right)_t^{'}\times\left[(a-1)(1-d)(-\ln p+t^{1-d})^{-1}t^{-d}+(-d)t^{-1}\right]$$

Thus : $\gamma(t)=-t\frac{\left(\ln f(t)\right)''}{\left(\ln f(t)\right)'}=-t\times\left[(a-1)(1-d)(-\ln p+t^{1-d})^{-1}t^{-d}+(-d)t^{-1}\right]$

$$\gamma(t)=d-(a-1)(1-d)\frac{t^{1-d}}{(-\ln p+t^{1-d})}$$

Note $\gamma(t)=d-(a-1)(1-d)\times K$ With $K=\frac{t^{1-d}}{\left(-\ln p+t^{1-d}\right)}$ → $0\le K\le 1$

Then $\gamma_d^{'}=1-(a-1)\left((1-d)K_d^{'}-K\right)$

Where : $K_d^{'}=\frac{t^{1-d}\times\ln(t)\times(-1)\times\left(-\ln p+t^{1-d}\right)-t^{1-d}\times t^{1-d}\times\ln(t)\times(-1)}{\left(-\ln p+t^{1-d}\right)^2}$

$$K_d^{'}=\frac{t^{1-d}}{\left(-\ln p+t^{1-d}\right)}\times\ln(t)\times(-1)\times\frac{\left(-\ln p+t^{1-d}\right)-t^{1-d}}{\left(-\ln p+t^{1-d}\right)}=K\frac{\ln(t+p)}{\left(-\ln p+t^{1-d}\right)}$$

Thus $\gamma_d^{'}=1-(a-1)\times K\times\left((1-d)A-1\right)$ where $A=\frac{\ln(t+p)}{\left(-\ln p+t^{1-d}\right)}$

With $A\ge 0$ iff $\ln(t+p)\ge 0\leftrightarrow t+p\ge 1$ because $\left(-\ln p+t^{1-d}\right)\ge 0$

Thus, if $t+p\ge 1$ → $A\ge 0$ and then:

$\to(1-d)A\ge 0$ because $(1-d)\ge 0$ by the strict impatience assumption.

$\to(1-d)A-1\ge -1$

$\to K\left[(1-d)A-1\right]\ge -1$ because $0\le K\le 1$

$\rightarrow (a-1)K\left[(1-d)A-1\right] \leq 1$ because $0 \leq a \leq 1$

And thus → $1-(a-1)K\left[(1-d)A-1\right] \geq 0$

→ $\gamma_d^{'} > 0$ If $t + p \geq 1$

This condition over t and p is quite surprising. Because we should (in this framework) be allowed to fix arbitrarily the scale of t the condition is always met. The analysis of this strange condition deserves probably more scrutiny.

For the moment we can conclude that the **degree of decreasing impatience grows with "d"!**

5. Trade-off under Flexible probability price of time

1) The value of p_1

$$
\begin{aligned}
&(x; p_1; t_1) \sim (x; \overline{p}; t_1 + \Delta) \\
&\leftrightarrow w(x; p_1; t_1) \times u(x) = w(x; \overline{p}; t_1 + \Delta) \times u(x) \\
&\leftrightarrow \omega(p_1 \times h(x; t_1)) = \omega(\overline{p} \times h(x; t_2)) \\
&\leftrightarrow p_1 \times h(x; t_1) = \overline{p} \times h(x; t_2) \quad \text{if monotonicity of } \omega(.) \text{ which is routine axiom.} \\
&\leftrightarrow p_1 = \overline{p} \frac{h(t_1 + \Delta)}{h(t_1)}
\end{aligned}
$$

2) Effect of initial period on p_1

The derivative of p_1 w.r.t. initial delay is:

$$
\begin{aligned}
&\frac{\partial p_1}{\partial t} = \overline{p} \frac{h'(t_1 + \Delta) \times h(t_1) - h(t_1 + \Delta) \times h'(t_1)}{\left(h(t_1)\right)^2} \\
&\geq 0 \text{ iff: } h'(t_1 + \Delta) \times h(t_1) - h(t_1 + \Delta) \times h'(t_1) \geq 0 \\
&\leftrightarrow \frac{h'(t_1 + \Delta)}{h(t_1 + \Delta)} > \frac{h'(t_1)}{h(t_1)} \\
&\leftrightarrow \frac{-h'(t_1 + \Delta)}{h(t_1 + \Delta)} < \frac{-h'(t_1)}{h(t_1)} \\
&\leftrightarrow \rho(t_1 + \Delta) < \rho(t_1)
\end{aligned}
$$

Where ρ is the instantaneous discount rate as presented in the overview of the intertemporal choice literature. The last inequality holds if H(.) is a decreasing impatient discount function. If d>0 with the functional form, $H(t) = ke^{-r_x t^{1-d}}$, we chose in this work.

Appendix B: Qualtrics survey

This is a paper version of the survey performed trough the Qualtrics website.

Start of Block: Block starting

Start Dear participant,   Thanks a lot for participating in my (short) experiment.   The survey will last for approximately 6 minutes. You will be asked to state your preferences between Lotteries that will differ in terms of probability of winning the lottery and in terms of year of getting the reward (if successful at the drawing).   The experiment is hypothetical but please take the time to think at the choices as if it would be for real.   Thank you again for participating!

End of Block: Block starting

Start of Block: Group 1, Block 1 - small t and large P)

Q1
In this situation, you can get the right to a lottery that will give you an amount of 10,000 euros in 6 years with a probability of 90%. But if you want, you can receive this lottery before, in one year from now precisely, if you accept to lower the probability of getting the reward.
Below is presented a list of choices that you have to make.

You may note that the choice on the right is always the same such that the only element that changes from line to line is the probability of the short-term drawing lottery.
You're asked to express your choice at each line.

To win the reward of 10,000 euros, you prefer:

	1 (1)	2 (2)	
90% to win and drawing in 1 year	○	○	90% to win and drawing in 6 years
80% to win and drawing in 1 year	○	○	90% to win and drawing in 6 years
70% to win and drawing in 1 year	○	○	90% to win and drawing in 6 years
60% to win and drawing in 1 year	○	○	90% to win and drawing in 6 years
50% to win and drawing in 1 year	○	○	90% to win and drawing in 6 years
40% to win and drawing in 1 year	○	○	90% to win and drawing in 6 years
30% to win and drawing in 1 year	○	○	90% to win and drawing in 6 years
20% to win and drawing in 1 year	○	○	90% to win and drawing in 6 years
10% to win and drawing in 1 year	○	○	90% to win and drawing in 6 years

End of Block: Group 1, Block 1 - small t and large P)

Start of Block: (Group 1, Block 3- small t and smaller P)

Q2
In this similar situation, you can get the right to a lottery that will give you the same amount of 10,000 euros in 6 years but with now a probability of 70%. Once again, if you want, you can receive this lottery before, in one year from now precisely, if you accept to lower the probability of getting the reward.
Below is presented a list of choices that you have to make.

You may note that, again, the choice on the right is always the same such that the only element that changes from line to line is the probability of the short-term drawing lottery.
You're asked to express your choice at each line.

To win the reward of 10,000 euros, you prefer:

	1 (1)	2 (2)	
70% to win and drawing in 1 year	○	○	70% to win and drawing in 6 years
60% to win and drawing in 1 year	○	○	70% to win and drawing in 6 years
50% to win and drawing in 1 year	○	○	70% to win and drawing in 6 years
40% to win and drawing in 1 year	○	○	70% to win and drawing in 6 years
30% to win and drawing in 1 year	○	○	70% to win and drawing in 6 years
20% to win and drawing in 1 year	○	○	70% to win and drawing in 6 years
10% to win and drawing in 1 year	○	○	70% to win and drawing in 6 years

End of Block: (Group 1, Block 3- small t and smaller P)

Start of Block: (group 1, block 2 - large t and large P)

Q3
In this third situation, you can, again, get the right to a lottery that will give you an amount of 10,000 euros but now in 15 years and with a probability of 90%. But if you want, you can receive this lottery before, in ten years from now precisely, if you accept to lower the probability of getting the reward. Below is presented a list of choices that you have to make.

You may note that, again, the choice on the right is always the same such that the only element that changes from line to line is the probability of the short-term drawing lottery.
You're asked to express your choice at each line.

To win the reward of 10,000 euros, you prefer:

	1 (1)	2 (2)	
90% to win and drawing in 10 years	○	○	90% to win and drawing in 15 years
80% to win and drawing in 10 years	○	○	90% to win and drawing in 15 years
70% to win and drawing in 10 years	○	○	90% to win and drawing in 15 years
60% to win and drawing in 10 years	○	○	90% to win and drawing in 15 years
50% to win and drawing in 10 years	○	○	90% to win and drawing in 15 years
40% to win and drawing in 10 years	○	○	90% to win and drawing in 15 years
30% to win and drawing in 10 years	○	○	90% to win and drawing in 15 years
20% to win and drawing in 10 years	○	○	90% to win and drawing in 15 years
10% to win and drawing in 10 years	○	○	90% to win and drawing in 15 years

End of Block: (group 1, block 2 - large t and large P)

Start of Block: Group 1, Block 4 - t=0 and large P

Q4
In this last situation, you can either get a lottery that will give you an amount of 10,000 euros in 5 years and with a probability of 90% or receive it today if you accept to lower the probability.
Below is presented a list of choices that you have to make.

You may note that the choice on the right is always the same such that the only element that changes from line to line is the probability of the short-term drawing lottery.
You're asked to express your choice at each line.

To win the reward of 10,000 euros, you prefer:

	1 (1)	2 (2)	
90% to win and drawing today	○	○	90% to win and drawing in 5 years
80% to win and drawing today	○	○	90% to win and drawing in 5 years
70% to win and drawing today	○	○	90% to win and drawing in 5 years
60% to win and drawing today	○	○	90% to win and drawing in 5 years
50% to win and drawing today	○	○	90% to win and drawing in 5 years
40% to win and drawing today	○	○	90% to win and drawing in 5 years
30% to win and drawing today	○	○	90% to win and drawing in 5 years
20% to win and drawing today	○	○	90% to win and drawing in 5 years
10% to win and drawing today	○	○	90% to win and drawing in 5 years

End of Block: Group 2, Block 4 - t=0 and large P

Start of Block: Block 5: demo

Q9 The questionnaire is almost finished.
Please fill in the following demographic questions and this will be the end.

Q10 Age

____

Q11 Gender

○ Male

○ Female

Q12 Nationality

____

Q13 Study or Occupation

____

End of Block: Block 5: demo

Appendix C: Support of section 5

Appendix C1: Statistical tables

Oprobit Models

```
Question_cat1 is TOLP
Question_cat2 is STLP
Question_cat3 is LTLP
```

```
And question 2 is the baseline group in all the models.
```

All three questions

```
. oprobit proba question_cat1 question_cat3 i.id

Iteration 0:   log likelihood = -72.699256
Iteration 1:   log likelihood = -44.395059
Iteration 2:   log likelihood = -41.181106
Iteration 3:   log likelihood = -40.681134
Iteration 4:   log likelihood = -40.583291
Iteration 5:   log likelihood = -40.565704
Iteration 6:   log likelihood = -40.563398
Iteration 7:   log likelihood = -40.563053
Iteration 8:   log likelihood = -40.562987
Iteration 9:   log likelihood = -40.562973
Iteration 10:  log likelihood =  -40.56297

Ordered probit regression                       Number of obs     =         45
                                                LR chi2(16)       =      64.27
                                                Prob > chi2       =     0.0000
Log likelihood =  -40.56297                     Pseudo R2         =     0.4420

------------------------------------------------------------------------------
       proba |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
question_cat1 |  -.2330552    .442485    -0.53   0.598     -1.10031    .6341995
question_cat3 |   .8632068   .4604933     1.87   0.061    -.0393435    1.765757
             |
          id |
          2  |  -9.076474   491.7599    -0.02   0.985    -972.9081    954.7552
          3  |  -7.637224   491.7597    -0.02   0.988    -971.4686    956.1942
          4  |  -8.717105   491.7599    -0.02   0.986    -972.5488    955.1146
          5  |  -6.27e-11    566.181    -0.00   1.000    -1109.694    1109.694
          6  |  -5.087986   491.7597    -0.01   0.992    -968.9192    958.7432
          7  |  -8.844057   491.7598    -0.02   0.986    -972.6756    954.9874
          8  |  -8.136383   491.7597    -0.02   0.987    -971.9676    955.6948
          9  |  -9.408681   491.7599    -0.02   0.985    -973.2404     954.423
         10  |  -8.717105   491.7599    -0.02   0.986    -972.5488    955.1146
         11  |   -7.63568   491.7598    -0.02   0.988    -971.4671    956.1958
         12  |  -5.856642   491.7596    -0.01   0.990    -969.6877    957.9744
         13  |   5.321775   488.9166     0.01   0.991    -952.9372    963.5807
         14  |  -5.856642   491.7596    -0.01   0.990    -969.6877    957.9744
         15  |   -7.63162   491.7598    -0.02   0.988    -971.4631    956.1998
-------------+----------------------------------------------------------------
       /cut1 |  -10.31053   491.7597                     -974.1419    953.5209
       /cut2 |  -9.747053   491.7597                     -973.5783    954.0842
       /cut3 |  -8.829358   491.7596                     -972.6604    955.0017
       /cut4 |  -7.146966   491.7594                     -970.9778    956.6838
       /cut5 |  -5.465008   491.7591                     -969.2952    958.3652
       /cut6 |   6.041251   488.9161                     -952.2166    964.2991
```

```
------------------------------------------------------------------------------
Note: 6 observations completely determined.  Standard errors questionable.
```

Questions 2 and 3 only

```
. oprobit proba question_cat3 i.id if(question !=1)

Iteration 0:   log likelihood = -47.395877
Iteration 1:   log likelihood = -25.330376
Iteration 2:   log likelihood = -22.328035
Iteration 3:   log likelihood =  -21.85205
Iteration 4:   log likelihood = -21.782152
Iteration 5:   log likelihood = -21.770023
Iteration 6:   log likelihood = -21.767558
Iteration 7:   log likelihood = -21.767205
Iteration 8:   log likelihood = -21.767152
Iteration 9:   log likelihood = -21.767141
Iteration 10:  log likelihood = -21.767139

Ordered probit regression                         Number of obs     =         30
                                                  LR chi2(15)       =      51.26
                                                  Prob > chi2       =     0.0000
Log likelihood = -21.767139                       Pseudo R2         =     0.5407

------------------------------------------------------------------------------
        proba |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
question_cat3 |   .9718192    .500546     1.94   0.052    -.0092329    1.952871
              |
           id |
           2  |  -9.312462   646.5737    -0.01   0.989    -1276.574    1257.949
           3  |  -7.559239   646.5733    -0.01   0.991     -1274.82    1259.701
           4  |  -9.392268   646.5738    -0.01   0.988    -1276.654    1257.869
           5  |   2.15e-11   786.7668     0.00   1.000    -1542.035    1542.035
           6  |   2.15e-11   786.7668     0.00   1.000    -1542.035    1542.035
           7  |   -9.11638   646.5735    -0.01   0.989    -1276.377    1258.144
           8  |  -6.626661   646.5732    -0.01   0.992    -1273.887    1260.633
           9  |   -10.7824   646.5739    -0.02   0.987    -1278.044    1256.479
          10  |  -9.392268   646.5738    -0.01   0.988    -1276.654    1257.869
          11  |  -7.533259   646.5734    -0.01   0.991    -1274.794    1259.727
          12  |  -5.622448   646.5731    -0.01   0.993    -1272.882    1261.638
          13  |   5.789442   638.3953     0.01   0.993    -1245.442    1257.021
          14  |  -5.622448   646.5731    -0.01   0.993    -1272.882    1261.638
          15  |  -8.490981   646.5736    -0.01   0.990    -1275.752     1258.77
-------------+----------------------------------------------------------------
        /cut1 |  -10.19687   646.5733                     -1277.457    1257.064
        /cut2 |  -8.978743   646.5732                     -1276.239    1258.281
        /cut3 |  -7.031511   646.5728                     -1274.291    1260.228
        /cut4 |  -5.313602   646.5724                     -1272.572    1261.945
        /cut5 |   6.275487   638.3947                     -1244.955    1257.506
------------------------------------------------------------------------------
Note: 6 observations completely determined.  Standard errors questionable.
```

First answer to questions 2 and 3 only

Group 1 has had STLP in first question.
Group 2 has had LTLP in first question.

```
. oprobit proba i.group
```

```
Iteration 0:   log likelihood = -19.844243
Iteration 1:   log likelihood = -18.952202
Iteration 2:   log likelihood = -18.951952
Iteration 3:   log likelihood = -18.951952

Ordered probit regression                         Number of obs     =         15
                                                  LR chi2(1)        =       1.78
                                                  Prob > chi2       =     0.1816
Log likelihood = -18.951952                       Pseudo R2         =     0.0450

------------------------------------------------------------------------------
       proba |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     2.group |   .7672298   .5770899     1.33   0.184    -.3638456    1.898305
-------------+----------------------------------------------------------------
       /cut1 |  -.4891801   .4591058                     -1.389011    .4106507
       /cut2 |  -.0617955     .43768                     -.9196325    .7960415
       /cut3 |   .8641822    .475291                     -.0673711    1.795735
------------------------------------------------------------------------------
```

Non-parametric models of paired samples

Signrank tests

```
   1)  signrank STLP = LTLP

Wilcoxon signed-rank test

        sign |      obs   sum ranks    expected
-------------+---------------------------------
    positive |        4          36          55
    negative |        7          74          55
        zero |        4          10          10
-------------+---------------------------------
         all |       15         120         120

unadjusted variance      310.00
adjustment for ties      -15.00
adjustment for zeros      -7.50
                     ----------
adjusted variance        287.50

Ho: STLP = LTLP
             z =  -1.121
    Prob > |z| =   0.2625

.
. di normal(r(z))
.13123784



   2)  signrank  T0LP = STLP

Wilcoxon signed-rank test

        sign |      obs   sum ranks    expected
-------------+---------------------------------
    positive |        3          36        37.5
    negative |        3          39        37.5
        zero |        9          45          45
-------------+---------------------------------
         all |       15         120         120
```

```
unadjusted variance      310.00
adjustment for ties       -2.50
adjustment for zeros     -71.25
                     ----------
adjusted variance        236.25

Ho: T0LP = STLP
             z =  -0.098
    Prob > |z| =   0.9223

.
. di normal(r(z))
.46112893

.
   3)  signrank T0LP = LTLP

Wilcoxon signed-rank test

        sign |      obs   sum ranks    expected
-------------+---------------------------------
    positive |        4        37.5          57
    negative |        8        76.5          57
        zero |        3           6           6
-------------+---------------------------------
         all |       15         120         120

unadjusted variance      310.00
adjustment for ties      -15.13
adjustment for zeros      -3.50
                     ----------
adjusted variance        291.38

Ho: T0LP = LTLP
             z =  -1.142
    Prob > |z| =   0.2533

.
. di normal(r(z))
.12664921
```

Signtest tests

```
   1)  signtest STLP = LTLP

Sign test

        sign |    observed    expected
-------------+------------------------
    positive |           4         5.5
    negative |           7         5.5
        zero |           4           4
-------------+------------------------
         all |          15          15

One-sided tests:
  Ho: median of STLP - LTLP = 0 vs.
  Ha: median of STLP - LTLP > 0
      Pr(#positive >= 4) =
         Binomial(n = 11, x >= 4, p = 0.5) =  0.8867

  Ho: median of STLP - LTLP = 0 vs.
  Ha: median of STLP - LTLP < 0
      Pr(#negative >= 7) =
```

```
         Binomial(n = 11, x >= 7, p = 0.5) =  0.2744

Two-sided test:
  Ho: median of STLP - LTLP = 0 vs.
  Ha: median of STLP - LTLP != 0
      Pr(#positive >= 7 or #negative >= 7) =
         min(1, 2*Binomial(n = 11, x >= 7, p = 0.5)) =  0.5488
```

2) signtest T0LP = STLP

```
Sign test

        sign |    observed    expected
-------------+------------------------
    positive |           3           3
    negative |           3           3
        zero |           9           9
-------------+------------------------
         all |          15          15

One-sided tests:
  Ho: median of T0LP - STLP = 0 vs.
  Ha: median of T0LP - STLP > 0
      Pr(#positive >= 3) =
         Binomial(n = 6, x >= 3, p = 0.5) =  0.6563

  Ho: median of T0LP - STLP = 0 vs.
  Ha: median of T0LP - STLP < 0
      Pr(#negative >= 3) =
         Binomial(n = 6, x >= 3, p = 0.5) =  0.6563

Two-sided test:
  Ho: median of T0LP - STLP = 0 vs.
  Ha: median of T0LP - STLP != 0
      Pr(#positive >= 3 or #negative >= 3) =
         min(1, 2*Binomial(n = 6, x >= 3, p = 0.5)) =  1.0000
```

3) signtest T0LP = LTLP

```
Sign test

        sign |    observed    expected
-------------+------------------------
    positive |           4           6
    negative |           8           6
        zero |           3           3
-------------+------------------------
         all |          15          15

One-sided tests:
  Ho: median of T0LP - LTLP = 0 vs.
  Ha: median of T0LP - LTLP > 0
      Pr(#positive >= 4) =
         Binomial(n = 12, x >= 4, p = 0.5) =  0.9270

  Ho: median of T0LP - LTLP = 0 vs.
  Ha: median of T0LP - LTLP < 0
      Pr(#negative >= 8) =
         Binomial(n = 12, x >= 8, p = 0.5) =  0.1938

Two-sided test:
  Ho: median of T0LP - LTLP = 0 vs.
  Ha: median of T0LP - LTLP != 0
      Pr(#positive >= 8 or #negative >= 8) =
         min(1, 2*Binomial(n = 12, x >= 8, p = 0.5)) =  0.3877
```

Friedman test

```
. friedman v1-v15

Friedman =   1.2333
Kendall =    0.0411
P-value =    0.5397
```

Non-parametrical test for unpaired samples

Ranksum test of the first answer to STLP and LTLP

```
. ranksum proba, by( group)

Two-sample Wilcoxon rank-sum (Mann-Whitney) test

       group |      obs    rank sum    expected
-------------+---------------------------------
           1 |        7        45.5          56
           2 |        8        74.5          64
-------------+---------------------------------
    combined |       15         120         120

unadjusted variance       74.67
adjustment for ties       -6.00
                     ----------
adjusted variance         68.67

Ho: proba(group==1) = proba(group==2)
             z =  -1.267
    Prob > |z| =   0.2051

. di normal(r(z))
.10255694
```

Jonckheere test of all pooled answers

```
. jonter proba , by ( question )

Jonckheere-Terpstra Test for Ordered Alternatives

       J  = 387
       J* = 1.063 (corrected for ties)


Pr(|Z| > |J*|) = 0.2878 (ordered alternative in either direction)
    Pr(Z > J*) = 0.8561 (descending ordered alternative)
    Pr(Z < J*) = 0.1439 (ascending ordered alternative)
```

Appendix C2: Demonstration linked to section 5

Consider that true probability at the STLP elicitation is x and the (10 percentage points length) interval is: $\left[x^{-};x^{+}\right]$. Similarly, for the STSP: $y \in \left[y^{-};y^{+}\right]$. Note that we should have $y \leq x$ by monotonicity but we wonder if $\frac{x}{0.9} < \frac{y}{0.7}$.

We know for sure that $\frac{x}{0.9} < \frac{y}{0.7}$ if $\frac{x^{+}}{0.9} < \frac{y^{-}}{0.7}$.

Let's suppose that the interval of y is just under the one of x. Then $y^{-} = x^{+} - 0.2$.

To fill the above condition, x^{+} must then satisfy: $\frac{x^{+}}{0.9} < \frac{y^{-}}{0.7} = \frac{x^{+} - 0.2}{0.7} \leftrightarrow x^{+} > 0.9$.

This means that as soon as y is in small enough to be in a lower interval than x, we'll never perceive the predicted increase in the ratio even if it exists.