# Dependence between the stock market and foreign exchange markets in the Middle East

A GARCH-EVT-Copula approach

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## Abstract

This thesis investigates the dependence structure between a stock market and the foreign exchange rate for four Middle East countries: Egypt, Iran, Israel and Turkey. Exchange rates are examined with respect to both the US Dollar and the Euro. We model dependence using Gaussian, Student's t-, Clayton, Gumbel and Frank copulas, with AR-GJR-GARCH marginal models. We estimate the copulas with daily data spanning between 13 and 18 years up to 2017:9, depending on the series. Our major findings are (i) unanimous absence of lower tail dependence between every economy's stock market and the home currency. This asymmetry provides attractive features for international investors. (ii) Small but significant upper tail dependence for the frontier market, Egypt, and the emerging market of Iran. The findings have useful implications for diversification and risk management purposes.

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# 1 Introduction

The dependence between financial asset classes is more relevant than ever. With globally integrated financial markets and masses of available information, simple correlation, comovement and volatility statistics are widely and instantly obtainable. However, amongst this group, a fundamental measure of the entire dependence between variables is amiss. The aforementioned measures do not include the tail comovements between variables. For stock index returns and exchange rate returns, the tail comovements are crucial for risk measurement, diversification opportunities and prediction purposes. This paper specifically investigates the dependence structure, as a whole, between pairs of stock index returns and foreign exchange rate returns for four selected economies in the Middle East.

In practice, it is necessary to study the tail dependence between stock index returns and foreign exchange rate returns. From a macroeconomic and policy-making perspective, changes in the exchange rate directly affect aggregate demand via their impact on net exports. Meanwhile, changes in a country's stock index price affect aggregate demand indirectly via the wealth channel. Therefore, accurate measurement of aggregate demand and equilibrium in extreme periods requires knowledge of tail dependences between stock index returns and foreign exchange rate returns. Furthermore, knowledge of the comovement of pairs of stock index returns and exchange rate returns in different economic periods are beneficial for risk management and portfolio diversification. A portfolio of assets exhibiting negative dependence in the tails would be well-diversified and would not require hedging in extreme periods.

The four economies analysed in this paper are Egypt, Israel, Iran and Turkey. The selection is attributable to regional significance, interesting market features and gaps in research. Firstly, Egypt is classified as a frontier market by Russell Investments, characteristically offering low-risk investment opportunites. Specifically, Blanco (2013) recommends frontier markets to investors seeking long-run returns and Berger, Pukthuanthong, et al. (2011) proposes investing in frontier markets as a means of diversification due to the low correlations that the markets exhibit with other markets. Speidell and Krohne (2007) examines this low correlation and explains that frontier market volatility is not driven by the same factors that typically drive volatility in developed markets. This paper will add insight by analysing a frontier market by means of the Egypt stock market and the Egyptian exchange rate.

Secondly, Israel is classified as a developed economy by MSCi, indicating that the markets are highly accessible and liquid. With a market capitalisation of $400bn, the Tel Aviv Stock Exchange includes many strong-performing and low-volatility indices and is attractive to global investors. Hence, the presence of significant dependence between the Israeli stock market and Israel's foreign exchange rate may prove insightful for a wide scope of investors.

Thirdly, the dependence structure for both Iran and Turkey will provide insight into emerging markets. Previous research by Lin (2011) finds that for two emerging markets (South Korea and Indonesia), the pairs of stock and currency markets exhibit asymmetric tail dependences. Specifically, investors are more likely to exhibit large losses together in both markets than large gains. This paper will add to the research on the diversification benefits/risks of investing in emerging markets by investigating both Iran and Turkey.

Motivated by the empirical findings above, the primary goal of this paper is to investigate whether investing in the selected Middle Eastern stock markets can provide any diversification benefits for investors from the US and the Eurozone. Hence, the research questions are:

- *How does the dependence structure between each country's stock market and foreign exchange market offer diversification benefits for investors?*

- *Are there significant tail dependences (both with respect to the US Dollar and the Euro) for the selected Middle East countries?*

- *Furthermore, if the tail dependences exist, are they symmetric or assymetric?*

- *Which copula model captures the dependence with the best fit? How do the different models compare in effectiveness?*

In order to estimate the copula parameters and gain insight into the research questions, the two-stage inference function for margins approach of Joe and Xu (1996) is used. First, the return series are fitted with $AR(1) - GJR - GARCH(1,1)$ models to capture three relevant stylised facts of financial returns. In practice, the GARCH model is widely used to capture the volatility clustering of financial time series, which Mandelbrot (1963) explains as the observation that large (small) changes tend to be followed by similarly large (small) changes. Meanwhile, the AR term captures the serial dependence of the error terms, which evolve in a non-random manner. The GJR adaptation of the classic GARCH models, from Glosten, Jagannathan, et al. (1993), allows for innovations to have asymmetric effects on the returns in terms of both sign and magnitude. As such, GJR-GARCH is ideal for capturing the commonly observed leverage effect. There are many choices for modelling the standardised residuals from the AR-GJR-GARCH models, such that they are identically and independently distrivuted on $[0, 1]$. This paper uses a semi-parametric approach, combining the classic Gaussian kernel density estimator with extreme value theory for the tails.

Second, the dependence structure is modelled. To estimate the copula parameters, correctly specified marginal distributions are required. Hence, statistical tests are implemented on the probability transforms to check for mis-specification and the presence of heavy-tails. With correct marginal inputs, the parameters for the Gaussian, Student's t-, Clayton, Gumbel and Frank copulas are estimated. The Gaussian copula acts as benchmark, while the Student's t-copula captures heavy-tailedness. The Gumbel and Clayton copulas exhibit greater dependence in the positive and negative tails respectively and the Frank copula permits both forms of tail dependence. For the class of Archimedean copulas, the parameter estimates are directly linked to the coefficients of tail dependence, which we derive and interpret.

Third, we compare the different copula estimates using several goodness-of-fit measures. The log-likelihood and Akaike information criterion are used first. We extend this by applying the Cramér-von Moses test with parametric bootstrapping. This method is shown to be most powerful in Genest, Rémillard, et al. (2009) when compared to Kolmogorov-Smirnov, its variations, and all tests relying on Rosenblatt's transform[*]. By using a parametric bootstrap, $p$-values for the test statistic are approximated, offering a formal insight into whether the selected model is appropriate. Given the extensive data period and the naturally different dependent structures that each country's stock market exhibits with its foreign exchange market, we expect some copulas to perform better for specific pairs of returns.

Fourth, we review the insights that the copula estimates provide in confluence with the results of the goodness-of-fit tests. We hope that the results prove insightful for international investors and portfolio managers alike, who can both reap benefits from diversification opportunities.

The remainder of the study is organised as follows: the literature review provides a summary of both prominent and recent works relating to dependence structures (between stock and forex markets), copula theory, goodness-of-fit tests and other key elements to our methodology. Following this review, the core theories and mathematical definitions are outlined. We then explain how and where the data was gathered from, and provide a step-by-step explanation of the methodology used to obtain empirical results. Lastly, we gather insights from these results and offer suggestions for future studies.

[*]From equation (5) of Genest, Rémillard, et al. (2009).

## 1.1 Literature review

Extensive research has been conducted on the relationship between a country's equity and currency markets. An important model in explaining the relationship is the Dornbusch and Fischer (1980) flow-oriented model of exchange rates, which states that a country's current account is the important determinant of the exchange rate. The theory stems from a macroeconomic perspective and relies on the efficient market hypothesis. Dornbusch and Fischer (1980) postulates that any event that affects a firm's cash flow will be reflected in the firm's respective stock price, given that the market is efficient. Fluctuations in the exchange rate are an example of such an event, and should theoretically cause changes in stock prices. On the other hand, the stock-oriented model of exchange rates, introduced in Branson (1983), suggests that exchange rates are determined by the demand and supply of capital assets. Therefore, stock-oriented models emphasise the capital account as the main determinant of exchange rates. Although they stem from different theoretical linkages, both models support that there is indeed a relationship between equity and currency markets. The portfolio balance models, also used in Branson (1983), are a sub-set of stock-oriented models, stating that an increase in stock prices leads to an increase in the domestic interest rate, which drives the exchange rate down. From a microeconomic perspective, contrasting results have been found; the appreciation of local currency leads to a competitive disadvantage for exporting firms and a drop in their stock prices, suggesting a negative relationship between the markets. However, by the same line of logic, importing firms can benefit and the value of their stocks will rise, suggesting a positive relationship between the markets.

These theoretical models of the causal relationship have thus paved the way for research into the entire dependence structure between stock returns and exchange rate returns. Another aspect worth investigating is the presence of dependence within the respective tails of their distributions. Tail dependence is indicative of how the returns behave together when they both exhibit values far away from their means. When one chooses to investigate this relationship, linear measurements do not provide the greatest insights, and are deficient in many ways. Due to the non-stationarity of many financial time series, linear trends are few and far between. Eubank and LaRiccia (1992) explains the problem of the correlation breakdown pattern, whereby linear correlation measures fail to capture extreme dependence. With fat tails and excess kurtosis, financial asset returns are prone to exhibiting extreme fluctuations. This should be captured by a sound dependence measure, especially when investigated from a risk management and empirical research perspective. Embrechts, McNeil, et al. (2002) highlights that linear correlation ignores key elements of dependence, namely co-monotonicity and rank correlation, which should be understood and considered when measuring risk.

Much of the recent and prominent studies of dependence within finance, and particularly of tail dependence, are based on copula functions. A $d$-dimensional copula is a joint distribution function that lies in $[0, 1]^d$ and contains standard uniform marginal distributions. In line with the theorem of Sklar (1973), every multivariate distribution can be described using its marginal distributions and a selected copula. Many copulas exist that allow for modeling of the entire dependence structure of multiple variables, in particular allowing for jointly extreme negative and/or positive shocks. Studies using copulas to examine the dependence structure between pairs of equities include Hu (2006), Rodriguez (2007) and Chollete, Heinen, et al. (2009), while Michelis and Ning (2010) similarly examine dependence between the stock market and currency markets.

Aside from obtaining empirical results regarding the structural dependence between a country's stock market and its foreign exchange market, copulas are important for prediction purposes as well. Many portfolios are constructed with a variety of assets and/or foreign exchange rates/futures. Copulas provide a $d$-dimensional distribution function, from which sets of $d$ returns can be simulated. In the bivariate case, one can model the dependence structure between two assets from a combined portfolio. Simulations from their associated copula provides pairs of returns that share the dependence from the interiors of the respective distributions through to the extreme values in the tails. Hence, copulas have been widely used to predict the value at risk of portfolios. Examples include Hsu, Huang, et al. (2012), Nurrahmat, Noviyanti, et al. (2017) and Bob (2013). This paper focuses on the empirical dependence structure of the markets within the selected countries. The results aim to provide insight for future practitioners who may choose to use the associated copulas to form hypothetical portfolios and predict value at risk.

This paper is similar to the aforementioned research in that it also aims to model the dependence between financial market returns. However, this research is different and contributes to existing literature in many ways. Firstly, the countries and the period that the data spans through are different. Secondly, the opportunities for portfolio diversification are examined from the perspective of two types of investors: US and Eurozone. Most research on the dependence between equity and currency markets is from the perspective of the US investor, analysing the foreign exchange rate returns of a market with respect to the US Dollar only. This paper delves deeper and looks at the exchange rates with respect to the US Dollar and the Euro. As such, the findings aim to be more useful to a broader pool of readers and investment specialists.

As stated previously, Michelis and Ning (2010) examines the co-movements of real TSX returns (as a proxy for the Canada stock market) and the USD/CAD returns in the tails of their distributions. Their data is monthly and their methodology is derived from Sklar's theorem and the Joe and Xu (1996) two-step method to estimate the parameters of the copulas. This study will use daily data over a longer time period. In their first step, Michelis and Ning (2010) create marginal models for the rates of return, allowing for AR and GARCH terms to capture serial dependence and volatility clustering. Their research emphasises the importance of correct marginal results prior to fitting the copula. Michelis and Ning (2010) utilise the Lagrange Multiplier test as a goodness-of-fit test for serial independence. This study includes visual tests by inspecting quantile-quantile plots, ensuring that the transformed marginals have fat tails.

The research in Michelis and Ning (2010) is unique in that it only uses one copula function: the Symmetrized Joe-Clayton copula. However, it expands upon the single copula by including time-varying dynamics. The results point to statistically significant asymmetric tail dependence, both statically and dynamically. The asymmetry is such that there is larger dependence in extreme downside regimes and less dependence in the extreme upside. As an MSCi developed economy, Canada exhibits some similar market features to Israel, and we hypothesise that the tail dependence between the TA125 returns and the returns of the USD/ILS will be also be similarly asymmetric.

Our paper also uses a similar approach to Lin (2011), by analysing the tail dependences in a geographical region of the world. Lin (2011) estimates the tail dependence between stock index returns and foreign exchange rate returns for five East Asian economies (Hong Kong, Indonesia, Singapore, South Korea and Taiwan). We extend upon the methodology of Lin (2011) by incorporating a semi-parametric approach after extracting and transforming iid residuals from the GARCH models, using extreme value theory. The heavy-tailed distribution used in this paper is the Generalised Pareto Distribution. We hope that by modelling the tails of the residuals with EVT, this study will be more robust to mis-specification prior to the copula estimation. In similar fashion to most work revolving around copula functions, Lin (2011) also uses the two-step estimation approach of Joe and Xu (1996) to estimate the parameters of the copulas. In terms of a goodness-of-fit test, the Ljung-Box Q-statistic is used.

Lin (2011) uses three copula functions in her analysis: Student's t-copula, Clayton copula and the Symmetrised Joe-Clayton copula. A key conclusion of Lin (2011) is that the emerging markets (Indonesia and South Korea) exhibit asymmetric tail dependence, with higher dependence in the lower tail. If similar results are found in this study, with respect to Iran and Turkey, it can be recommended to those invested in the Iranian and Turkish stock markets to hedge their investments with currency derivatives in order to avoid periods in which both stocks exhibit unusually large and simultaneous losses.

Both Michelis and Ning (2010) and Lin (2011) use log-likelihood and information criterion variations to compare their copula estimates. Genest, Rémillard, et al. (2009) conduct a large Monte Carlo experiment on goodness-of-fit tests that they call "blanket" tests. This generally involves bootstrapping and extracting a deterministic $p$-value. These tests can be easily compared to each other and can be used on datasets regardless of sample size, smoothing parameters, windows, etc. A general observation is that for a bootstrap to be efficient, the number of repetitions $m$ must be larger than the sample size $n$. Furthermore, when compared to Kolmogorov-Smirnov and it's variations, the Cramér-von Mosis proves almost invariably more powerful. Genest, Rémillard, et al. (2009) specifically recommends the Cramér-von Mosis single bootstrap method, especially when the null hypothesis is the fit of a copula from the Archimedean class.

# 2 Theoretical Framework

Stochastic models in finance must aim to reproduce as many as necessary and feasible of the stylized facts commonly found in financial time series. Mandelbrot (1963) formally introduced many of these regular patterns in the case of log daily stock returns. We begin by defining the stylized facts that are principal in our theoretical framework:

- *Volatility clustering* is explained in Mandelbrot (1963) as the observation that large (small) changes tend to be followed by similarly large (small) changes. It is prominent in financial returns as we observe that volatility is found largely in bunches.

- *Leptokurtotic* describes a distribution with fat tails and excess peakedness around the mean, such as the Student's t- and logistic distributions. Financial returns often have fat tails due to extreme movements.

- The *leverage effect* is explained by Cont (2001) as the inverse relationship between an asset's volatility and its returns.

## 2.1 GARCH models

The Autoregressive Conditional Heteroscedasticity (ARCH) model was introduced by Engle (1982) to better forecast the variance of future financial returns. The model uses a fixed window of historical data to estimate a weighted average of past squared residuals. The weights are parameters to be estimated and then used in forecasting. The general $ARCH(q)$ model has the form:

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2, \tag{1}$$

$$\epsilon_t | I_{t-1} \sim N(0, \sigma_t^2), \tag{2}$$

where $\sigma_t^2$ is the variance of a stochastic process, say financial returns, at time $t$. $\omega > 0$ and $\alpha_j \geq 0$ for $j = 1, ..., q$, keeping the conditional variance positive. However, $q$ often needs to be very large to best fit the data.

The Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model is the parsimonious generalisation of ARCH, with continuously declining weights that asymptote to zero. GARCH models have been immensely popular in financial econometrics, and the associated literature is extensive. Defining similarly to Bollerslev (1986), the $GARCH(p, q)$ process is given by:

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2, \tag{3}$$

where $\omega > 0$, $p \geq 0$, $q > 0$, $\alpha_i \geq 0$ and $\beta_i \geq 0$ for $i = 1, ..., p$ and $j = 1, ..., q$. If $p = 0$ there is reduction to an ARCH process and if $p = q = 0$ the process is defined as white noise. Volatility clustering is captured by the $GARCH(p, q)$ model, which clearly imposes that the squared conditional volatility, $\sigma^2$, is a function of both past squared residuals and squared conditional volatilities. However, the model is symmetric in that positive and negative past innovations and volatilities have the same effect on time $t$ variance. This is inconsistent with the leverage effect.

An extension of the GARCH model, which takes asymmetries into account, is the GJR-GARCH model of Glosten, Jagannathan, et al. (1993). Formally, the process is:

$$\sigma_t^2 = \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 + \sum_{k=1}^{r} \gamma_k I_{t-k} \epsilon_{t-k}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2, \tag{4}$$

with $\alpha_j > 0$, $\beta_i > 0$, $\beta_i + \gamma_k \geq 0$ and $\sum_{j=1}^{q} \alpha_k + \sum_{k=1}^{r} \gamma_k + \sum_{i=1}^{p} \beta_i < 1$. The indicator variable, $I_{t-k}$, is defined as:

$$I_{t-k} = \begin{cases} 1, & \epsilon_{t-k} < 0, \\ 0, & \epsilon_{t-k} \geq 0. \end{cases} \tag{5}$$

We observe that the $GJR-GARCH(p,q)$ model captures the leverage effect by allowing for the true sign of the lagged residuals to affect the process differently.

## 2.2 Extreme Value Theory (EVT)

Extreme value theory is the field that addresses extreme data points that deviate largely from the mean and median. Hence, EVT is naturally applied to the tails of leptokurtotic distributions in financial returns. A distribution, $F$, is defined as heavy-tailed in the right tail if it satisfies the following:

$$\lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}, \tag{6}$$

where $\alpha$ is called the tail index. Many studies (Gençay, Selçuk, et al. (2003), Embrechts, Kluppelberg, et al. (1997) and McNeil and Frey (2000), to name but a few) look to EVT to model heavy-tailed distributions in finance.

### 2.2.1 Semi-parametric modeling

A common approach in conjunction with EVT is semi-parametric modeling. Many random variables can be found to reflect eminent distributions in their interiors whilst their tails exhibit irregular patterns. In this scenario, a kernel density estimator may be applicable to the interior of the distribution. The right and left tails can then be modeled separately. Kernel density estimators are non-parametric; they have no fixed structure and rely on the data points alone to determine estimates. The estimator smooths locally around each data point, originally to eliminate the common problem of bin widths in histograms, as explained in Hwang, Lay, et al. (1994). In a kernel estimate of a point $x^*$, the contribution of any data point $x^{(i)}$ is determined by the distance between them. The weight of this contribution relative to all other data points is dependent on the chosen kernel function and its associated "bandwidth". As in Hwang, Lay, et al. (1994), assuming a given kernel function $K$ and its associated bandwidth $h$, the estimated density at any point $x^*$ is given by:

$$\hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} K\left(\frac{x^* - x^{(i)}}{h}\right), \tag{7}$$

where $\int K(t)dt = 1$, imposing that the estimates $\hat{f}(x)$ integrate to 1.

A common next step is to parametrically model the tails. Embrechts, Kluppelberg, et al. (1997), amongst many more, use the Generalised Pareto Distribution (GPD henceforth). The tail estimator is:

$$F(\hat{z}) = \begin{cases} \frac{k^l}{n}\xi^l \left(\frac{v^l - z}{\beta^l}\right)^{-\frac{1}{\xi^l}}, & \text{for } z < v^l, \\ 1 - \frac{k^r}{n}\xi^r \left(\frac{v^r - z}{\beta^r}\right)^{-\frac{1}{\xi^r}}, & \text{for } z > v^r, \end{cases} \tag{8}$$

where $l$ and $r$ denote the left and right tails respectively. $n$ is the total number of observations and $k^r$ ($k^l$) is the number of observations beyond a threshold $v^r$ ($v^l$). $\beta$ and $\xi$ are scale and shape parameters for each tail of the distribution.

## 2.3 Copulas

Copulas are multivariate distribution functions that capture the dependence structure of a joint distribution, independent of the marginal distributions. In the case of finance, copulas are useful to join univariate returns together and better capture their co-movements. Copulas have been widely used recently in academia, with Patton (2006), Michelis and Ning (2010) and Lin (2011) as some notable works.

Copula theory was introduced in Sklar (1959), which defines a $d$-dimensional copula as a distribution function, $C$, mapping $[0,1]^d$ onto $[0,1]$, such that:

$$F(y_1, ..., y_d) = C(F_1(y_1), ..., F_d(y_d)), \tag{9}$$

where $F(y_1, ..., y_d)$ is a continuous and $d$-variate cumulative distribution function with univariate margins $F_i(y_i)$ for $i = 1, ..., d$. Differentiating the expression with respect to $y_1, y_2, ..., y_d$ sequentially leads to the canonical representation:

$$f(y_1, ..., y_d) = c(F_1(y_1), ..., F_d(y_d)) \prod_{i=1}^{d} f_i(y_i), \tag{10}$$

where $c$ is the copula density. Sklar (1959) explains that the joint density function is obtained by:

$$\frac{d^d F(y_1, ..., y_d)}{dy_1, ..., y_d} = \prod_{i=1}^{d} F_i(y_i) \frac{d^d C(F_1(y_1), ..., F_d(y_d))}{dF_1(y_1), ..., F_d(y_d))}. \tag{11}$$

Using the joint density function, we obtain the definition of a copula as a multivariate distribution function with standard uniform $[0, 1]$ margins, such that:

$$C(u_1, ..., u_d) = F(F_1^{-1}(u_1), ..., F_d^{-1}(u_d)), \tag{12}$$

where $u_i = F_i(y_i) \sim U[0, 1]$ for $i = 1, ..., d$.

### 2.3.1 Elliptial copulas

Frahm, Junker, et al. (2003) explains that elliptical copulas are simply those copulas that are derived from multivariate and elliptically contoured distributions. The most common of these distributions are the Gaussian (or normal) and the Student's t-distribution, which lend themselves to the namely Gaussian copula and Student's t-copula.

#### Gaussian copula

Nurrahmat, Noviyanti, et al. (2017) defines the Gaussian copula of a $d$-dimensional standard normal distribution with linear correlation matrix, $\rho$, as the distribution of the random vector $(\psi(X_1), ..., \psi(X_d))$, such that:

$$C_{\rho}^{Ga} = P(\psi(X_1) \leq u_1, ..., \psi(X_d) \leq u_d) = \psi_{\rho}^{d}(\psi^{-1}(u_1), ..., \psi^{-1}(u_d)), \tag{13}$$

where $\psi$ is the univariate standard normal distribution function and $X \sim N_d(0, \rho)$. Hence, $\psi_{\rho}^{d}$ is the distribution function of $X$. The Gaussian copula has found a place as a benchmark in academic work involving copulas (see Huang (2014) and Michelis and Ning (2010)).

#### Student's t-copula

Frahm, Junker, et al. (2003) defines the Student's t copula of a $d$-dimensional standard student's t distribution with $v \geq 0$ degrees of freedom and linear correlation matrix, $\rho$, as the distribution of the random vector $(t_v(X_1), ..., t_v(X_d))$, such that:

$$C_{v,\rho}^{t} = P(t_v(X_1) \leq u_1, ..., t_v(X_d) \leq u_d) = t_{v,\rho}^{d}(t_v^{-1}(u_1), ..., t_v^{-1}(u_d)). \tag{14}$$

where $t_v$ is the univariate standard student's t distribution function and $X \sim t^d(0, \rho, v)$. Hence, $t^d_{v,\rho}$ is the distribution of $X$. For $v = 1$ we have the Cauchy copula. For $v > 1$ but limited (to around 30), the copula has stronger peaks at the tails and more of a star shape than the Gaussian copula.

### 2.3.2 Archimedean copulas

The Archimedean class consists of many copulas that have favourable analytical properties and explicit formulas. The copulas are generated by a decreasing, continuous and convex function, $\varphi$, called the generator. Formally, the copula is defined as:

$$C(u_1, ..., u_d) = \varphi(\varphi^{-1}(u_1) + \cdots + \varphi^{-1}(u_d)), \tag{15}$$

where the generator is such that $\varphi(u) \in C^2$ with $\varphi(1) = 0$, $\varphi'(u) < 0$ and $\varphi''(u) > 0$ for $0 \leq u \leq 1$. The inverse of the generator, $\varphi^{-1}(u)$, must be monotonic on $[0, \infty)$. One attractive feature is the single parameter $\theta$ that allows for the modelling of dependence in high dimensions without an increasing number of parameters.

#### Clayton copula

The Clayton copula is asymmetric, exhibiting more dependence in the left tail than the right. It can be used in finance to model extreme downside risk in portfolio optimisation. The generator is given by $\varphi(u) = u^{-\theta} - 1$, hence $\varphi^{-1}(t) = (t+1)^{-1/\theta}$, and is monotonic for $\theta > 0$. The Clayton $d$-copula is:

$$C^{Cl}_\theta(u_1, ..., u_d) = \sum_{i=1}^{d} (u_i^{-\theta} - d + 1)^{-1/\theta}, \tag{16}$$

with $\theta > 0$. If $\theta = 0$, we have the independence copula and for $\theta \to \infty$, the co-monotonicity copula. The parameter $\theta$ can be used to obtain the coefficient of lower tail dependence, such that:

$$\tau_L = 2^{-\frac{1}{\theta}}. \tag{17}$$

#### Gumbel copula

The generator is $\varphi(u) = (-\ln(u)^\theta)^\theta$, hence $\varphi^{-1}(t) = \exp(-t^{1/\theta})$, and is monotonic for $\theta > 1$. The Gumbel $d$-copula is:

$$C_\theta^{Gu}(u_1, ..., u_d) = \exp\left\{- \left[\sum_{i=1}^{d} - \ln(u_i)^\theta\right]^{1/\theta}\right\}, \tag{18}$$

with $\theta > 1$. If $\theta = 1$, we have the independence copula and again, for $\theta \to \infty$, the comonotonicity copula. The Gumbel copula imposes greater dependence in the right tail, and the coefficient of upper tail dependence is defined as:

$$\tau_U = 2 - 2^{\frac{1}{\theta}}. \tag{19}$$

**Frank copula**

The generator is $\varphi(u) = \ln\left(\frac{\exp(-\theta u)-1}{\exp(-\theta)-1}\right)$, hence $\varphi^{-1}(t) = -\frac{1}{\theta}\ln(1 + \exp(t)(\exp(-\theta) - 1)$, and is monotonic for $\theta > 0$. The Frank $d$-copula is:

$$C_\theta^{F}(u_1, ..., u_d) = -\frac{1}{\theta}\ln\left\{1 + \frac{\prod_{i=1}^{d}(\exp(-\theta u_i) - 1}{(\exp(-\theta) - 1)^{d-1}}\right\}, \tag{20}$$

with $\theta > 0$ when $n \geq 3$.

### 2.3.3 Dependence measures

The dependence structure between random variables is described by their joint distribution function. For the purpose of this paper, we take the definition of Müller and Scarsini (2005):

*"Dependence is a matter of association between X and Y along any measurable function, i.e. the more X and Y tend to cluster around the graph of a function, either y = f(x) or x = g(y), the more they are dependent."*

This section summarises linear correlation as well as other measures of association that are directly linked to copula theory.

**Linear correlation**

For non-degenerate and square-integrable random variables $X$ and $Y$, Pearson's linear correlation coefficient $\rho$ is defined as:

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}. \tag{21}$$

$\rho$ is the extent to which a linear relation can describe the dependency between $X$ and $Y$. However, the linear correlation coefficient is not invariant under increasing and non-linear transformations.

**Concordance**

More commonly used ways to investigate dependence are measures that fulfill the scale invariance property, such that they remain unchanged given strictly increasing transformations of the random variables. Two widely known measures fulfilling this pre-requisite are Kendall's Tau and Spearman's Rho, which both measure concordance. Assuming that $(x_i, y_i)$ and $(x_j, y_j)$ are two observations from a random vector of continuous random variables $(X, Y)$. Then, $(x_i, y_i)$ and $(x_j, y_j)$ are concordant if $x_i < x_j$ and $y_i < y_j$, or if $x_i > x_j$ and $y_i > y_j$. Alternatively, this can be formulated as $(x_i - x_j)(y_i - y_j) > 0$. $(x_i, y_i)$ and $(x_j, y_j)$ are discordant if $(x_i - x_j)(y_i - y_j) < 0$.

**Kendall's tau**

For an i.i.d. pair of random vectors, $(X_1, Y_1)$, $(X_2, Y_2)$, with joint distributions $H$, Kendall's Tau is defined as:

$$\tau_{X,Y} = P((X_1 - X_2)(Y_1 - Y_2) > 0) - P((X_1 - X_2)(Y_1 - Y_2) < 0). \tag{22}$$

From the earlier definitions of concordance and non-concordance, we observe that Kendall's Tau is formally defined as the difference between the probability of concordance and the probability of non-concordance. To link Kendall's Tau to the theory of copulas, we define a concordance function, $Q$, as this difference in probabilities for two vectors $(X_1, Y_1)$ and $(X_2, Y_2)$ of continuous random variables with joint distributions $H_1$ and $H_2$, which may be different, and common margins $F$ and $G$. Then,

$$Q(C_1, C_2) = 4 \int\int_{I^2} C_2(u, v) dC_1(u, v) - 1, \tag{23}$$

where $C_1$ and $C_2$ are the copulas of $(X_1, Y_1)$ and $(X_2, Y_2)$ respectively, such that $H_1(x, y) = C_1(F(x), G(y))$ and $H_2(x, y) = C_2(F(x), G(y))$. This formulation shows that $Q$ depends on the the two vectors only through their respective copulas.

### Spearman's rho

Assuming that $(X_1, Y_1), (X_2, Y_2), (X_3, Y_3)$ are three independent random vectors with joint distribution functions $H$, common margins $F$ and $G$ and a copula function $C$. Then, Spearman's rho is proportional to the probability of disconcordance subtracted from the probability of concordance for the pairs $(X_1, Y_1)$ and $(X_2, Y_3)$, with the former's components being jointly distributed via $H$ and the latter's being independent. Formally:

$$\rho_{X,Y} = 3(P((X_1 - X_2)(Y_1 - Y_3) > 0) - P((X_1 - X_2)(Y_1 - Y_3) < 0)). \tag{24}$$

Spearman's Rho for the continuous variables $X$ and $Y$ is linked to their copula $C$ via the relation

$$\rho_{X,Y} = \rho_C = 12 \int\int_{I_2} C(u, v) du dv - 3. \tag{25}$$

### Tail dependence

Archimedean copulas particularly provide insight into the *tail dependence* between a pair of random variables. This is a specific measure of the comovements within the tails of their distributions, a characteristic not captured by simple correlations (Hartmann, Straetmans, et al. (2004)). The measure is used in conjunction with extreme value theory, and is prevalent in some recent financial studies. Pairs of financial returns have often been shown to exhibit tail dependence, as in Hu (2006) and Lin (2011).

$\tau_U$ and $\tau_L$ are the coefficients of upper and lower tail dependence. The upper tail dependence coefficient between a pair of random variables, $X$ and $Y$, is defined as:

$$\tau_U = \lim_{q \to 1^-} Pr(Y > F_Y(q) | X > F_X(q)). \tag{26}$$

Similarly, the lower tail dependence coefficient is:

$$\tau_L = \lim_{q \to 0^+} Pr(Y \leq F_Y^{-1}(q) | X \leq F_X^{-1}(q)). \tag{27}$$

### 2.3.4 Estimation

There are numerous methods to estimate a copula and its parameters, ranging from maximum likelihood and its varieties to a number of method-of-moment approaches.

### Maximum Likelihood (ML)

Looking back to equation (8), maximum likelihood estimation consists of choosing $C$ and $F_1, ..., F_n$, such that the probability of observing the given data is maximised. Assuming a data set consisting of realisations $(x_{1t}, ..., x_{nt})$, the ML estimator finds $\theta$, which maximises the following likelihood function:

$$l(\theta) = \prod_{t=1}^{T} \left( c(F_1(x_{1t}), ..., F_n(x_{nt}); \theta) \prod_{i=1}^{n} f_i(x_{it}); \theta) \right). \tag{28}$$

$\theta$ denotes the vector of parameters, depending on the chosen class of copula, and also maximises the log-likelihood function:

$$\log l(\theta) = \sum_{t=1}^{T} \log c(F_1(x_{1t}), ..., F_n(x_{nt}); \theta) + \sum_{t=1}^{T} \sum_{i=1}^{n} \log f_i(x_{it}); \theta). \tag{29}$$

Cherubini, Luciano, et al. (2004) explains that this function is computationally more convenient. The functions $l(\theta)$ and $\log l(\theta)$ are maximised by the vector $\theta$, such that the ML estimator is given by

$$\hat{\theta}_{ML} := \arg\max_{\theta} l(\theta). \tag{30}$$

The primary drawback of estimating using an exact MLE method is that given high dimensions, the process is computationally intensive, since both the margins and copula parameters are jointly estimated.

### Inference for Margins (IFM)

The IFM method overcomes the issue above. Joe and Xu (1996) propose that the vector of parameters, $\theta$, be estimated in two steps. First, the parameters of the margins are estimated, and second, the copula parameters. Reverting back to the log-likelihood function of equation (28), the two parts to be estimated are:

$$\hat{\theta}_1 := \arg\max_{\theta_2} \sum_{t=1}^{T} \sum_{i=1}^{n} \log f_i(x_{it}); \theta_1) \tag{31}$$

$$\hat{\theta}_2 := \arg\max_{\theta_1} \sum_{t=1}^{T} \log c(F_1(x_{1t}), ..., F_n(x_{nt}); \theta_1, \theta_2) \tag{32}$$

A common application in finance using the inference function for margins is to first fit a pre-determined marginal model and obtain the marginal parameters. Next, the parameters of the marginals are fixed and the copula parameters of equation (29) are estimated using ML. Joe and Xu (1996) show that the IFM provides a consistent estimator.

**Canonical Maximum Likelihood (CML)**

The CML approach skips the need to specify the marginal distributions of data. The original sample data is transformed from $\{x_{1t}, ..., x_{nt}\}_{t=1}^{T}$ to uniform variates $\{u_{1t}, ..., unt\}_{t=1}^{T}$ using the empirical distribution, $\hat{F}_1, ..., \hat{F}_n$. The final step consists of ML estimation, such that

$$\hat{\theta}_{CML} := \arg\max_{\theta} \sum_{t-1}^{T} \log c(\hat{F}_1(x_{1t}), ..., \hat{F}_n(x_{nt}); \theta) \tag{33}$$

is the CML estimate of the vector of copula parameters.

### 2.3.5 Goodness-of-fit

When it comes to comparing the fits of different copulas with the same bivariate inputs, a common approach is to compare the log-likelihood and/or an information criterion value, which are arbitrary statistics affected by sample size and number of estimated parameters. Beyond this, there are certain "blanket" tests as in Genest, Rémillard, et al. (2009), which are goodness-of-fit tests that are applicable to all copula structures. One can formally assess whether a specific copula is the appropriate model by inspection of a $p$-value, see Ane and Kharoubi (2003).

The premise behind the goodness-of-fit tests is to test the null hypothesis that $H_0 : C \in C_0$ for some class $C_0$ of copulas. By definition, the underlying copula $C$ of a random vector is invariant to continuous and strictly increasing transformations of its components. Hence, the testing of $H_0$ can be viewed as a function of some collection of pseudo-observations, $\boldsymbol{U_1} = (U_{11}, ..., U_{1d}), ..., \boldsymbol{U_n} = (U_{n1}, ..., U_{nd})$. These observations can be interpreted as random samples from the underlying $C$. Genest, Rémillard, et al. (2009) formally defines the

$\boldsymbol{U_1}, ..., \boldsymbol{U_n}$ as the maximally invariant statistics (i.e. the ranks). The study compares five blanket tests in terms of power, finding that the Cramér-von Mises test based on a specific bootstrap method performs best for their sample. $H_0$ can be tested using the empirical transformation. The information contained in the pseudo-observations is summarised by the associated empirical distribution, such that for a $d$-variate copula:

$$C_n(\boldsymbol{u}) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(U_{i1} \leq u_1, ..., U_{id} \leq u_d), \tag{34}$$

where $\boldsymbol{u} = u_1, ..., u_d \in [0, 1]^d$. Hence, a natural goodness-of-fit test compares a "distance" between $C_n$ and an estimation $C_{\theta_n}$ of $C$ under $H_0$. Genest, Rémillard, et al. (2009) propose to measure the distance using a rank-based version of the popular Cramér-von Mises test:

$$S_n = \int_{[0,1]^d} \left(\sqrt{n}(C_n(\boldsymbol{u}) - C_{\theta_n}(\boldsymbol{u}))\right)^2 \mathrm{d}C_n(\boldsymbol{u}). \tag{35}$$

This has generally led to more powerful tests than the Kolmogorov-Smirnov test. Large values of this test statistic lead to the rejection of $H_0$. $S_n$ is a consistent statistic; if $C \notin C_0$, $H_0$ is rejected with probability 1 as $n \to \infty$. Approximate $p$-values depend on the asymptotic behaviour of the process $\sqrt{n}(C_n(\boldsymbol{u}) - C_{\theta_n}(\boldsymbol{u})$, and are estimated using Monte Carlo methods. Genest, Rémillard, et al. (2009) suggests a tailored procedure for a (semi)-parametric bootstrap method leading to a $p$-value for $S_n$. This procedure is different based on the number of dimensions and whether an analytical expression for $C_\theta$.

# 3 Methodology and Results

## 3.1 Data

The data contains daily closing prices of stock index returns and foreign exchange rate returns with respect to both the US Dollar and the Euro, for each of the four Middle East countries. The data were gathered from Yahoo! Finance[†]. Proxies for the stock index of each country are the EGX30 for Egypt, TEPIX for Iran, TA125 for Israel and BIST100 for Turkey. The exchange rates are USD/EGP and EUR/EGP for Egypt, USD/IRR and EUR/IRR for Iran, USD/ILS and EUR/ILS for Israel, and USD/TRY and EUR/TRY for Turkey. Daily log returns, $r_t$, were calculated using the relation $r_t = 100 \ln(s_t/s_{t-1})$. Hence there are three time series per country, resulting in twelve vectors of returns that are presented in Figure 1.

[†]Apart from the three series for Iran, which were gathered from the Tehran Stock Exchange website and investing.com.

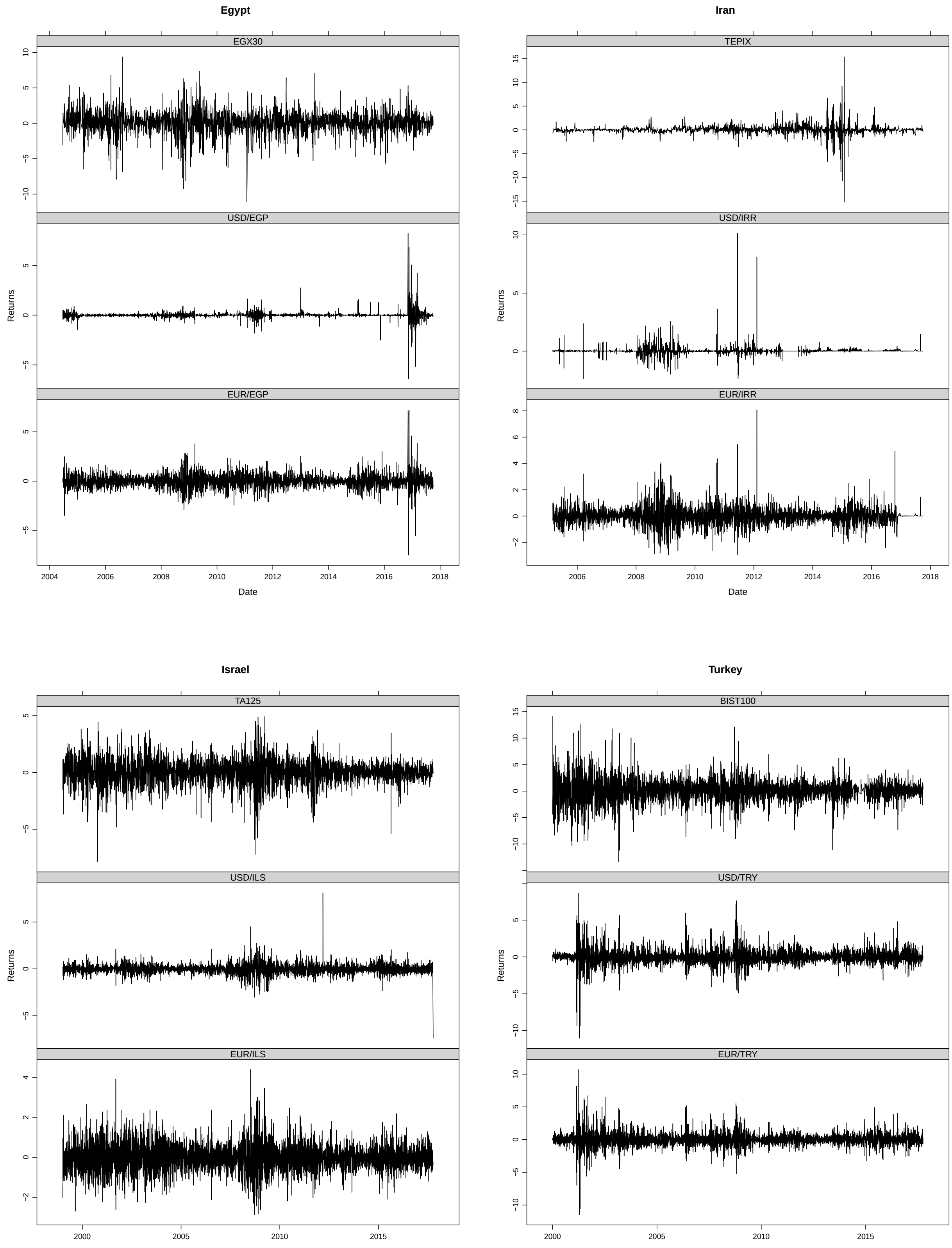


FIGURE 1 – Daily log returns of each return series.

### 3.1.1 Preliminaries

The summary statistics in Table 1 show signs of non-normality in the returns. All of the stock index returns are negatively skewed and the exchange rate returns are positively skewed. The skewness shows asymmetries around the mean and the excess kurtosis in all series, apart from EUR/ILS, is a sign of fat tails. We implement the Jarque-Bera test as a check and goodness-of-fit test on the data being normally distributed. The estimates are from the approximation of Lilien, Kotler, et al. (1995) and are also presented in Table 1. The large values verify that the returns are not normal, since we reject the null hypothesis that skewness and excess kurtosis are both equal to zero.

TABLE 1 – Summary statistics of daily log returns.

| Country | Asset | Start Date | N | Mean | Std. Dev | Skewness | Kurtosis | Jarque-Bera | Ljung-Box | Ljung-Box-S |
|---|---|---|---|---|---|---|---|---|---|---|
| Egypt | EGX30 | 22/06/2004 | 3465 | 0.062 | 1.553 | -0.631 | 4.715 | 3442.600 | 0.000 | 0.000 |
| | USD/EGP | 22/06/2004 | 3465 | 0.010 | 0.407 | 3.319 | 147.105 | 2922800.000 | 0.000 | 0.000 |
| | EUR/EGP | 22/06/2004 | 3465 | 0.009 | 0.733 | 0.254 | 14.871 | 31983.000 | 0.000 | 0.000 |
| Iran | TEPIX | 02/03/2005 | 3542 | 0.055 | 0.894 | -0.527 | 72.386 | 770230.000 | 0.236 | 0.000 |
| | USD/IRR | 02/03/2005 | 3542 | 0.017 | 0.327 | 13.326 | 380.950 | 21432000.000 | 0.000 | 0.000 |
| | EUR/IRR | 02/03/2005 | 3542 | 0.010 | 0.669 | 1.191 | 12.268 | 22959.000 | 0.000 | 1.000 |
| Israel | TA125 | 05/01/1999 | 4887 | 0.029 | 1.043 | -0.524 | 3.990 | 3470.000 | 0.000 | 0.000 |
| | USD/ILS | 05/01/1999 | 4887 | -0.003 | 0.492 | 0.353 | 29.240 | 174350.000 | 0.190 | 0.002 |
| | EUR/ILS | 05/01/1999 | 4887 | -0.003 | 0.646 | 0.283 | 2.246 | 1096.200 | 0.063 | 0.000 |
| Turkey | BIST100 | 04/01/2000 | 4630 | 0.040 | 2.030 | -0.016 | 4.877 | 4589.900 | 0.000 | 0.000 |
| | USD/TRY | 04/01/2000 | 4630 | 0.032 | 0.998 | 0.006 | 14.355 | 39756.000 | 0.000 | 0.000 |
| | EUR/TRY | 04/01/2000 | 4630 | 0.036 | 0.998 | 0.322 | 15.489 | 46360.000 | 0.000 | 0.000 |

*All series end on 01/10/2017. We present the first four moments, the Jarque-Bera test statistic, Ljung-Box p-values, and the p-values of the Ljung-Box test on squared log returns, denoted by Ljung-Box-S in the last column.*

For a visual check, we look at QQ plots, which display the the empirical quantiles of the returns versus the quantiles of the normal distribution as the reference distribution. For normally distributed returns, we would expect to observe linearity throughout the plots. Figure 2 presents the QQ plots for the four stock index returns[‡]. The returns show similarities to the linearity of the normal quantiles in the centre of the distributions. However, there is clear non-linearity in the quantiles of the tails. This observation paves the way to our choice of semi-parametric fit in the following subsections.

From Figure 1, we observe that the returns do not appear to be identically and independently distributed since the volatility clustering is noticeable. To check for serial correlation, we use the $p$-values belonging to the test of Ljung and Box (1978). The null hypothesis is that

[‡]See Appendix for the QQ plots belonging to the eight foreign exchange return series.

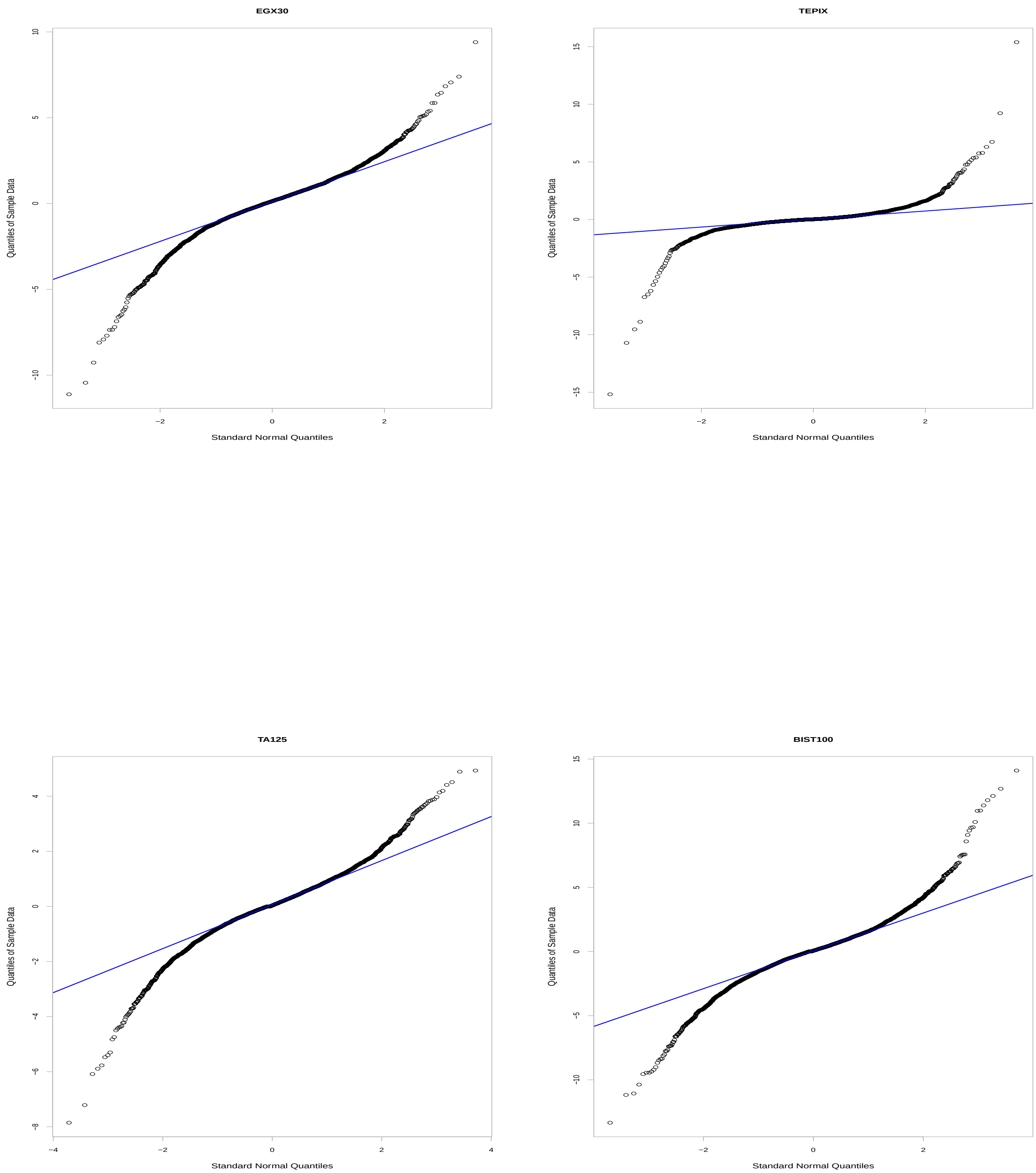


FIGURE 2 – QQ plots of the daily log returns of the four stock market returns.

of no autocorrelation. Looking back at Table 1, the null hypothesis is rejected and the autocorrelation is present, for nine of the twelve return series at a 99% confidence level. We also perform Ljung-Box tests on the squared returns, which rejects the null for eleven of the series at a 98% confidence level[§]. This implies that autocorrelation exists in most of the squared returns and autoregressive conditional heteroscedasticity is present.

### 3.1.2 Correlation coefficients

Before estimating copula parameters, we look for some insight by calculating three correlation coefficients. We calculate Pearson's linear correlation coefficient, and Kendall's Tau and Spearman's Rho, which both fall under the category of dependence measures based on concordance.

TABLE 2 – Correlation coefficients

| Country | Asset Pair | Pearson's Rho | Kendall's Tau | Spearman's Rho |
|---|---|---|---|---|
| Egypt | EGX30 USD/EGP | -0.0532 | -0.0330 | -0.0499 |
| | EGX30 EUR/EGP | 0.0241 | 0.0156 | 0.0235 |
| Iran | TEPIX USD/IRR | **0.0033** | **-0.0033** | **-0.0050** |
| | TEPIX EUR/IRR | 0.0286 | 0.0232 | 0.0348 |
| Israel | TA125 USD/ILS | -0.2171 | -0.1463 | -0.2161 |
| | TA125 EUR/ILS | -0.1221 | -0.0821 | -0.1217 |
| Turkey | BIST100 USD/TRY | **-0.3609** | **-0.2471** | **-0.3598** |
| | BIST100 EUR/TRY | -0.3073 | -0.2097 | -0.3063 |

*Smallest and largest correlations are in bold.*

The values of the linear Pearson's Rho measure of correlation, and the two rank correlations, are consistent across the board, other than for the Iranian stock market and Iranian Rial (US Dollar) pair. For the developed economy of Israel and the emerging market of Turkey, we see large and negative dependence and correlation from all measures. This indicates that the increase (decrease) of the national stock market is associated with the decrease (increase) of the local currency. In the case of Turkey, this is contradictory to emerging markets finance. In both cases, this pattern is appealing for international investors, as these stock markets offer some investment opportunities that do not require hedging. We find that the local stock

[§]The $USD/IRR$ series is void of any serial correlation.

market with the lowest dependence with its currency is that of Iran. This may be partially explained by anomalous external factors such as sanctions by the US, and further explained by odd exchange rate policies that have included pegging and attempts of redenomination and market manipulation.

## 3.2 Marginal modelling

The marginal distribution for each return series is modelled separately. The results are then used to obtain the probability integral transforms, which will be inputs to the copula estimation. For any pair, these will be $u$ and $v$, and used to estimate the copula parameters as in equation (12). With correlations in both the means and variances of the returns, the $AR(1) - GJR - GARCH(1,1)$ is an appropriate choice of model. We estimate using both innovations from the normal and the Student's t-distribution to begin with. The parameters are estimated by maximum likelihood and are reported in Table 3.

### 3.2.1 AR-GJR-GARCH fitting

We see that the $AR(1) - GJR - GARCH(1,1)$ models with Student's t-distributed innovations are a superior fit in all the return series. The log-likelihood is larger and the Akaike Information Criterion is unanimously smaller in every series. This result was expected after the observed fat tails in the returns, which the Student's t-distribution is better capable of capturing. We proceed with the models with Student's t innovations from here onwards.

Looking at the parameter estimates, we first observe that the $AR(1)$ term, $\varphi$, is significant for all of the stock index series, and not for the following currencies: EUR/EGP, EUR/IRR, USD/ILS and USD/TRY. This implies that the stock returns have larger tendencies of spillover effects from their returns in the previous period than the currency returns do. Second, we observe that all ARCH parameters $\alpha$ and GARCH parameters $\beta$ are statistically significant at the highest level implying that all returns experience ARCH and GARCH effects. Third, we notice that the GJR term $\gamma$ to capture the leverage effect is positive for all four stock returns. This means that there is indeed a leverage effect present in the stock returns, whereby negative shocks in the previous period have a larger effect than positive shocks to the current period. Conversely, the series USD/EGP, USD/IRR, EUR/ILS, USD/TRY and EUR/TRY all have negative values for $\gamma$. For these currency returns, the effect of negative shocks (bad news) is still larger than the effect of positive shocks (good news) but not to the degree of the stock returns. It must be noted that not all leverage terms are significant.

Next, we extract the standardised residuals from the fitted models. We denote these as $z_t = \varepsilon_t/\sigma_t$, where $\varepsilon_t$ is the residual at time $t$ and $\sigma_t$ is its associated standard error. We perform the Ljung-Box test (unreported) on the $z_t$ and $z_t^2$ and all series pass the test at a 99% confidence level. This indicates that the $AR(1) - GJR - GARCH(1,1)$ models with Student's t innovations sufficiently capture the autocorrelation and heteroscedasticity effects in the return series. The $z_t$ are now i.i.d. and we proceed to modelling the marginal distributions of these residuals.

### 3.2.2 EVT modelling

We apply EVT to the residuals, as described in section 2.2. Following observations from the QQ plots of Figure 2, we choose a semi-parametric distribution for the $z_t$. This consists of modelling the CDF of $z$ by amalgamating three components: the left tail, the interior and the right tail. For the interior of the distribution, we model non-parametrically using the Gaussian kernel density estimator:

$$K(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right). \tag{36}$$

$$\tag{37}$$

$K(z)$ is combined with the density estimator of equation (6). The tails are modelled parametrically using the Generalised Pareto Distribution of equation (7). We follow Neftci (2000) and Huang (2014), that both suggest the upper and lower 5% of the data as the respective thresholds between the interior and tails. As such, the first equation models the bottom 5% and the second equation models the top 5% of the data. The combined semi-parametric model for the CDF of $Z$ can therefore be written as:

$$\hat{F}(z) = \begin{cases} \frac{k^l}{n}\xi^l \left(\frac{v^l - z}{\beta^l}\right)^{-\frac{1}{\xi^l}}, & \text{for } z < v^l, \\ \frac{1}{n}\sum_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z_i^2\right)\left(\frac{z - z_i}{h}\right), & \text{for } v^l < z < v^r, \\ 1 - \frac{k^r}{n}\xi^r \left(\frac{v^r - z}{\beta^r}\right)^{-\frac{1}{\xi^r}}, & \text{for } z > v^r. \end{cases} \tag{38}$$

The parameter estimates of each series' residuals is presented in Table 4. For visual purposes, the semi-parametric CDF for two of the returns for Turkey are presented in Figure 3.

Table 3 – Estimates of $AR(1) - GJR - GARCH(1,1)$ parameters

| Country | Asset | Distribution | Mean | | Volatility | | | | Log-Likelihood | AIC |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\mu$ | $\varphi$ | $\omega$ | $\alpha$ | $\beta$ | $\gamma$ | | |
| Egypt | EGX30 | Normal | 0.073 (0.032) | 0.405 (0.017) | 0.190 (0.026) | 0.140 (0.022) | 0.649 (0.028) | 0.269 (0.038) | -5658.164 | 3.272 |
| | | Student's t | 0.085 (0.030) | 0.400 (0.017) | 0.209 (0.032) | 0.147 (0.028) | 0.617 (0.034) | 0.314 (0.049) | **-5582.045** | **3.229** |
| | USD/EGP | Normal | 0.000 (0.002) | -0.256 (0.023) | 0.001 (0.000) | 0.286 (0.021) | 0.828 (0.007) | -0.229 (0.021) | 1355.945 | -0.780 |
| | | Student's t | 0.000 (0.001) | -0.197 (0.018) | 0.001 (0.000) | 0.417 (0.042) | 0.616 (0.027) | -0.068 (0.050) | **2809.152** | **-0.619** |
| | EUR/EGP | Normal | 0.010 (0.010) | -0.020 (0.018) | 0.007 (0.002) | 0.062 (0.012) | 0.927 (0.011) | -0.003 (0.011) | -3342.771 | 1.935 |
| | | Student's t | 0.007 (0.009) | -0.028 (0.017) | 0.002 (0.001) | 0.040 (0.008) | 0.948 (0.006) | 0.018 (0.010) | **-3190.274** | **1.847** |
| Iran | TEPIX | Normal | 0.015 (0.015) | 0.707 (0.014) | 0.015 (0.002) | 0.531 (0.031) | 0.491 (0.020) | -0.046 (0.041) | -1856.985 | 1.058 |
| | | Student's t | 0.015 (0.010) | 0.665 (0.014) | 0.010 (0.002) | 0.566 (0.041) | 0.419 (0.030) | 0.033 (0.054) | **-1383.980** | **0.790** |
| | USD/IRR | Normal | 0.041 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.050 (0.000) | 0.900 (0.001) | 0.050 (0.000) | 1032.167 | -0.583 |
| | | Student's t | 0.000 (0.0000 | -0.037 (0.010) | 0.000 (0.000) | 0.440 (0.029) | 0.656 (0.009) | -0.204 (0.042) | **6381.830** | **-3.619** |
| | EUR/IRR | Normal | 0.0169 (0.007) | 0.030 (0.019) | 0.031 (0.000) | 0.057 (0.011) | 0.921 (0.007) | 0.042 (0.012) | -2981.775 | 1.696 |
| | | Student's t | 0.001 (0.001) | 0.025 (0.017) | 0.000 (0.000) | 0.098 (0.012) | 0.864 (0.008) | 0.073 (0.019) | **-2505.692** | **1.427** |
| Israel | TA125 | Normal | 0.017 (0.015) | 0.274 (0.015) | 0.023 (0.004) | 0.066 (0.011) | 0.855 (0.014) | 0.120 (0.019) | -6236.245 | 2.555 |
| | | Student's t | 0.035 (0.014) | 0.276 (0.014) | 0.014 (0.003) | 0.058 (0.012) | 0.874 (0.015) | 0.118 (0.020) | **-6145.544** | **2.518** |
| | USD/ILS | Normal | -0.009 (0.006) | 0.037 (0.017) | 0.004 (0.001) | 0.110 (0.012) | 0.883 (0.009) | 0.011 (0.015) | -2949.434 | 1.210 |
| | | Student's t | -0.010 (0.005) | 0.024 (0.014) | 0.003 (0.001) | 0.080 (0.014) | 0.909 (0.012) | 0.009 (0.014) | **-2528.137** | **1.038** |
| | EUR/ILS | Normal | -0.007 (0.008) | -0.030 (0.015) | 0.001 (0.000) | 0.034 (0.003) | 0.962 (0.012) | 0.004 (0.006) | -4416.354 | 1.810 |
| | | Student's t | -0.014 (0.007) | -0.038 (0.014) | 0.002 (0.000) | 0.041 (0.004) | 0.957 (0.002) | -0.001 (0.008) | **-4362.706** | **1.788** |
| Turkey | BIST100 | Normal | 0.081 (0.023) | 0.068 (0.016) | 0.034 (0.007) | 0.061 (0.009) | 0.902 (0.009) | 0.065 (0.012) | -9085.875 | 3.932 |
| | | Student's t | 0.093 (0.022) | 0.059 (0.015) | 0.036 (0.010) | 0.064 (0.011) | 0.896 (0.012) | 0.069 (0.016) | **-8946.821** | **3.872** |
| | USD/TRY | Normal | 0.018 (0.010) | -0.006 (0.017) | 0.018 (0.003) | 0.172 (0.016) | 0.858 (0.011) | -0.090 (0.016) | -5476.931 | 2.371 |
| | | Student's t | 0.003 (0.009) | -0.026 (0.015) | 0.010 (0.002) | 0.142 (0.018) | 0.882 (0.013) | -0.061 (0.018) | **-5215.490** | **2.258** |
| | EUR/TRY | Normal | 0.031 (0.010) | -0.030 (0.017) | 0.022 (0.004) | 0.150 (0.018) | 0.869 (0.015) | -0.084 (0.016) | -5633.314 | 2.439 |
| | | Student's t | 0.010 (0.009) | -0.046 (0.015) | 0.019 (0.004) | 0.145 (0.020) | 0.875 (0.017) | -0.087 (0.018) | **-5401.714** | **2.339** |

*The parameters correspond to the model in equation 4 with the inclusion of $\mu$ and $\varphi$, the $AR(1)$ parameters. Estimates are obtained using ML.*

TABLE 4 – Estimates of the semi-parametric distribution of the residuals.

| Country | Asset | Left Tail | | | Right Tail | | |
|---|---|---|---|---|---|---|---|
| | | $\nu^l$ | $\xi^l$ | $\beta^l$ | $\nu^r$ | $\xi^r$ | $\beta^r$ |
| Egypt | EGX30 | -1.227 | -0.106 | 0.669 | 1.201 | 0.137 | 0.471 |
| | USD/EGP | -1.062 | 0.435 | 0.574 | 1.082 | 0.444 | 0.647 |
| | EUR/EGP | -1.237 | 0.028 | 0.503 | 1.19 | 0.126 | 0.541 |
| Iran | TEPIX | -1.179 | 0.302 | 0.596 | 1.307 | 0.215 | 0.686 |
| | USD/IRR | -0.709 | 0.694 | 0.790 | 1.589 | 1.108 | 0.887 |
| | EUR/IRR | -1.247 | 0.034 | 0.678 | 1.323 | 0.368 | 0.590 |
| Israel | TA125 | -1.232 | 0.048 | 0.626 | 1.186 | 0.006 | 0.480 |
| | USD/ILS | -1.136 | 0.118 | 0.539 | 1.176 | 0.127 | 0.575 |
| | EUR/ILS | -1.204 | -0.061 | 0.536 | 1.251 | -0.017 | 0.625 |
| Turkey | BIST100 | -1.223 | 0.083 | 0.572 | 1.141 | 0.073 | 0.537 |
| | USD/ILS | -1.119 | 0.143 | 0.416 | 1.211 | 0.110 | 0.651 |
| | EUR/ILS | -1.112 | 0.016 | 0.496 | 1.193 | 0.184 | 0.603 |

*Parameters are estimated using ML estimation.*

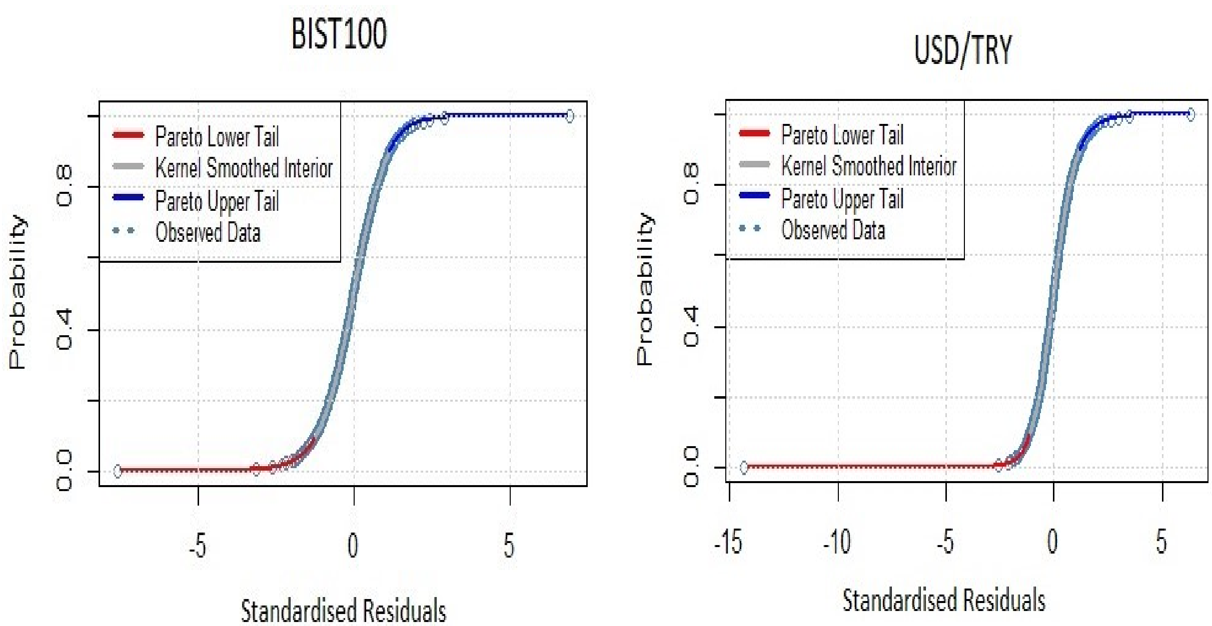


FIGURE 3 – Estimated semi-parametric CDFs for Turkey's BIST100 and USD/TRY.

## 3.3 Dependence modelling

### 3.3.1 Copula estimation

Parameter estimates of the Gaussian, Student's t-, Clayton, Gumbel and Frank copulas are reported in Table 5.

TABLE 5 – Copula parameter estimates

| **Country** | **Asset Pair** | **Gaussian** | | | **Student's t** | | | **Clayton** | | | **Gumbel** | | | **Frank** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\rho$ | Log-lik | AIC | $\rho$ | Log-lik | AIC | $\rho$ | Log-lik | AIC | $\rho$ | Log-lik | AIC | $\rho$ | Log-lik | AIC |
| Egypt | EGX30 * USD/EGP | -0.066 (0.021) | 4.769 | **-7.537** | -0.048 (0.026) | -37.940 | 77.874 | **-0.055** (0.019) | 3.734 | -5.468 | **1.004** (0.015) | 2.177 | -2.354 | -0.364 (0.126) | 4.148 | -6.297 |
| | EGX30 * EUR/EGP | 0.043 (0.018) | 3.001 | -4.003 | 0.019 (0.049) | -34.430 | 70.868 | **0.043** (0.019) | 2.875 | -3.751 | **1.029** (0.026) | 9.104 | **-16.207** | 0.145 (0.106) | 0.936 | 0.127 |
| Iran | TEPIX * USD/IRR | -0.007 (0.014) | 0.119 | **-0.270** | -0.005 (0.018) | -63.560 | 129.118 | 0.000 (0.007) | 0.000 | 1.439 | 1.000 (0.010) | 0.000 | 2.000 | -0.030 (0.077) | 0.076 | 1.849 |
| | TEPIX * EUR/IRR | 0.036 (0.021) | 1.415 | -0.830 | 0.030 (0.030) | -71.620 | 145.237 | 0.000 (0.010) | 0.083 | 2.000 | **1.006** (0.016) | 0.132 | 1.735 | 0.223 (0.122) | 1.666 | **-1.332** |
| Israel | TA125 * USD/ILS | -0.231 (0.014) | 124.400 | **-246.714** | -0.214 (0.024) | 98.760 | -195.520 | 0.000 (0.004) | 0.000 | 2.000 | 1.000 (0.004) | 0.000 | 2.000 | -1.402 (0.090) | 119.800 | -237.582 |
| | TA125 * EUR/ILS | -0.127 (0.014) | 37.040 | **-72.086** | -0.115 (0.018) | -6.293 | 14.586 | 0.000 (0.005) | 0.000 | 2.000 | 1.000 (0.005) | 0.000 | 2.000 | -0.757 (0.089) | 35.810 | -65.629 |
| Turkey | BIST100 * USD/TRY | -0.354 (0.012) | 367.000 | **-732.06** | -0.360 (0.016) | 343.70 | -685.404 | 0.000 (0.011) | 0.000 | 2.000 | 1.000 (0.008) | 0.000 | 2.000 | -2.439 (0.095) | 327.700 | -653.333 |
| | BIST100 * EUR/TRY | -0.338 (0.013) | 260.700 | **-519.462** | -0.311 (0.017) | 252.000 | -502.042 | 0.000 (0.006) | 0.000 | 2.000 | 1.000 (0.008) | 0.000 | 2.000 | -2.049 (0.094) | 234.40 | -466.749 |

*This table presents the results for the five copulas. $\rho$ is the dependence measure and $\theta$ is the parameter. The numbers in parenthesis are the asymptotic standard errors. Log-lik and AIC are the log-likelihood value and Akaike's information criterion respectively. In bold we highlight the best fitting copula under AIC for each pair. We also bold the Clayton and Gumbel parameter estimates for 3 pairs that exhibit tail dependence. The remaining 5 series reduce to the independence copula for their associated tail dependences.*

The dependence measure $\rho$ in the Gaussian copula and Student's t-copula are similar to the linear Spearman's Rho estimates in Table 2. This is unsurprising as these are the two elliptical copulas and the estimation of the parameters is the same as the linear correlation, just given the elliptical margins. The Gaussian copula has the best fit by measures of log-likelihood and AIC for six of the eight asset pairs. We observe that the copula estimates for Israel and Turkey pairs are generally more significant with smaller standard errors than those of Egypt and Iran. Tail dependence is present for the Egypt and Iran pairs, which we examine in section 3.4. For the more developed economy of Israel, there is no evidence of tail dependence in either tail. From the AIC values, the Clayton and Student's t-copula are never the best fitting between the selection. The lack of fit of the Clayton copula along with its parameter estimates indicate no real lower tail dependence between any of the pairs. The Student's t-copula's poor performance indicates an absence of significant dependence in the highly extreme values and in both tails for our data.

## 3.4 Tail dependence

The only sign of any lower tail dependence is for both Egypt pairs, where the tail dependences are significant at 90%, but very small ($\lambda_l = 0.001$). In conjunction with the poor performance of the Clayton copula across the board, our results point towards no lower tail dependence between the stock and foreign exchange markets for any of the Middle East countries investigated.

For Egypt's stock and currency (Euro) markets, we observe the largest upper tail dependence ($\lambda_u = 0.039$). For this pair the Gumbel copula was also the best fit by AIC. The other Egypt pair ($\lambda_u = 0.006$) and the Iran stock and currency pair ($\lambda_u = 0.008$) exhibit smaller but similarly asymmetric tail dependence. The upper tail dependence coefficients are all relatively small but highly significant at a 99% confidence level. Hence, between these markets, there is a significant and positive probability that the stock index exceeds a specific and extreme $q$-quantile, given that the exchange rate exceeds the same $q$-quantile. This is indicative of potentially double gains, since both markets have a positive likelihood of booming together.

Blanco (2013) and Berger, Pukthuanthong, et al. (2011) advocate investing in frontier economies due to their low-risk characteristics and low correlations with other markets. Even though the upper tail dependence coefficients are small, the absence of lower dependence may be appealing to investors to investors looking at the Egypt stock market. There is a small potential for high-reward to accompany the typically low-risk market features. Additionally, the linear Pearson's Rho is negative for the relationship between the stock market and currency in Egypt. As a naturally hedged opportunity, the presence of upper tail dependence and absence of any in the lower tails makes Egypt an appealing investment.

The same evaluation applies to the pair of Iran (stock and currency relative to the Euro). TEPIX, and stocks that fall it in, has the same potential for double gains due to the positive upper tail dependence it exhibits with the Iranian Rial. With no evidence pointing to lower tail dependence, this could be appealing to investors, though the dependence is very small.

## 3.5 Goodness-of-fit

We compare the goodness-of-fit between the different estimated copulas for the pairs of returns using the Cramér-von Mises test. The test statistic, $S_n$ of equation (35), and associated $p$-values are estimated using simulation. Following observations from Genest, Rémillard, et al. (2009), we simulate using the parametric bootstrap methods. When $p$-values are close to 0, the null hypothesis $H_0 : C \in C_0$ for some class $C_0$ of copulas is rejected.

By combining Akaike's information criterion with the "blanket" Cramér-von Mises test, we can compare results to formally assess how appropriate each copula model is to the data. Tables 5 and 6 show that the results of fit are mostly in line. Hence, the estimates are robust to differences in sample size and the number of parameters to be estimated.

We observe low $p$-values across the board for the Student's t- and Clayton copulas, adding further verification to the high AIC values that indicated the poor fit of these copulas to our data. Specifically, the other Cramér-von Mises $p$-values suggest that the Gumbel copula is most appropriate for any Egypt and Iran pair, as shown in bold in Table 6. This adds robustness and strength to any conclusions of upper tail dependence that we draw from the results in Table 5.

# 4 Observations and Recommendations

Based on the methodology in section 3, and the results that followed, both general and specific observations were made. These pave the way for recommendations that we can make to interested readers of this thesis, potential investors, international portfolio holders and risk practitioners.

From a general perspective, this paper analyses tail dependence between national stock and currency markets in one geographical region of the world, the Middle East. For all four stock markets, we observe an absence of lower tail dependence with the national currency from both a Euro and US Dollar perspective. Hence we can conclude that international investors with a home currency of either the Euro or US Dollar, seeking diversification into Egypt, Iran, Israel and Turkey stock markets, are unlikely to experience double losses (one in the stock market itself, and another in the currency market when converting into home currency returns). Thus, these investments would not require hedging. Future studies could investigate whether this trend lies specifically in these four economies, or whether it can be extended to other Middle East economies also.

From a more specific perspective, we make several observations. First, Egypt, which is defined as a frontier market, has a stock market that offers an attractive dependence structure with the national currency that is the Egyptian Pound. The presence of upper tail dependence means that investors from the Eurozone and the US could potentially exhibit double gains (in the stock market itself, and then in translating returns into home currency), without any risk of double losses. This is due to the asymmetry in tail dependence. Egypt is a particularly appealing market for US investors. The correlation coefficients are negative between the EGX30 and the USD/EGP (Pearson's Rho = -0.0532), meaning that investments in Egyptian stocks are naturally hedged against currency movements.

Second, we observe that the stock market of Iran has a very small dependence with the Iranian Rial. All correlation coefficients are close to zero, and there is an absence of lower tail dependence. There is a small chance of double gains for Eurozone investors looking into Iran for diversification ($\lambda_u = 0.008$), and overall, currency hedging is not required for this market.

Third, we look at the results for Israel for insight into a developed economy. Michelis and Ning (2010) finds lower tail dependence between the Canadian stock market and the Canadian Dollar, while Lin (2011) finds no tail dependence between these markets for the developed economies of Hong Kong and Singapore in East Asia. Similarly to Lin (2011), this study finds no evidence of any tail dependence between the return series for Israel. Hence, for investments made in this market, currency hedging is again not required. The same conclusion can be drawn for the emerging market of Turkey.

For risk practitioners and those interested in calculating the value at risk of hypothetical national stock-currency portfolios, we refer to Tables 5 and 6. For Israel and Turkey, where no tail dependence is present, the Gaussian copula is best fitting in terms of log-likelihood, AIC and the bootstrap Cramér-von Mises $p$-values. For the Iran stock market's dependence with the Iranian Rial, from a US investor perspective, the Cramér-von Mises test contradicts the AIC and shows the Gumbel copula to be the best fitting. Genest, Rémillard, et al. (2009) specifically recommends the Cramér-von Mises test over information criterion in the case of Archimedean copulas with a large sample size. The Gumbel copula is also the best fit for the EGX30 dependence with EUR/EGP.

# 5 Summary and Conclusions

In this thesis, the dependence structure between the stock market and different foreign exchange markets is investigate using copula theory. Five dependence structures are examined, namely the Gaussian and Student's t-copula from the elliptical class, and the Clayton, Gumbel and Frank copulas from the Archimedean family. For each country, two pairs are considered. Specifically, each country's stock index is paired with its foreign exchange market to the Dollar and its foreign exchange market to the Euro. The major findings are: (i) for the developed economy of Israel and the emerging market of Turkey, there is no evidence of tail dependence between stock and currency returns. (ii) For the frontier market of Egypt, we observe the most asymmetric tail dependence, with no dependence in the lower tail and some small but significant upper tail dependence. (iii) No lower tail dependence between national stock and currency markets for any of the four Middle East economies.

These results can provide useful and important direction for investors who consider diversification into economies in the Middle East region. Investment in any of the four stock markets does not require currency hedging against extreme downside events, since there is no evidence of double losses. For international investors seeking diversification into Egypt and Iran markets, there is a positive probability of experiencing double gain. Coupled with the absence of lower tail dependence, these markets may be attractive to some Eurozone and US investors.

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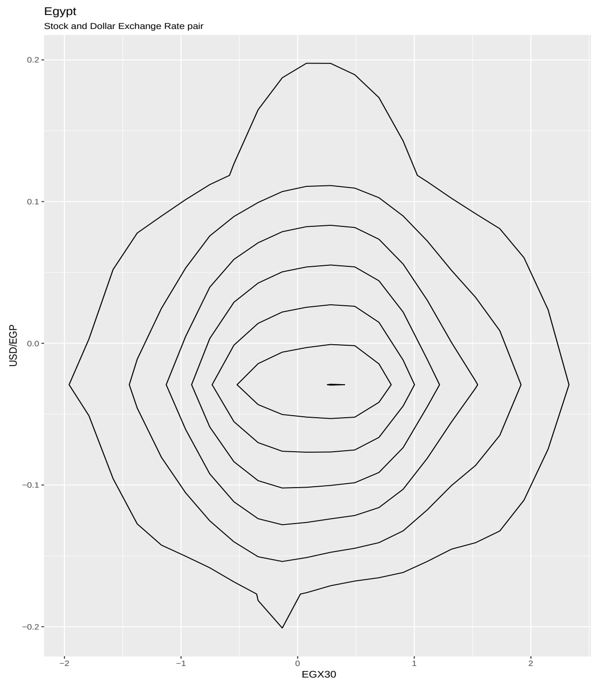
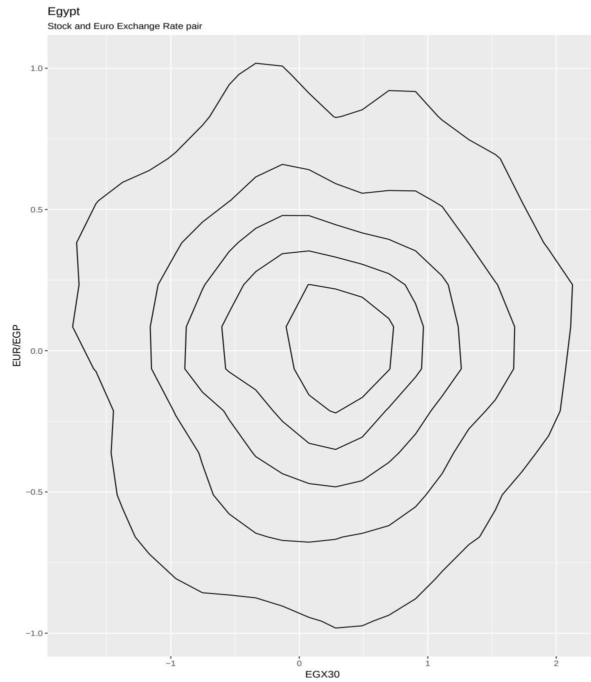
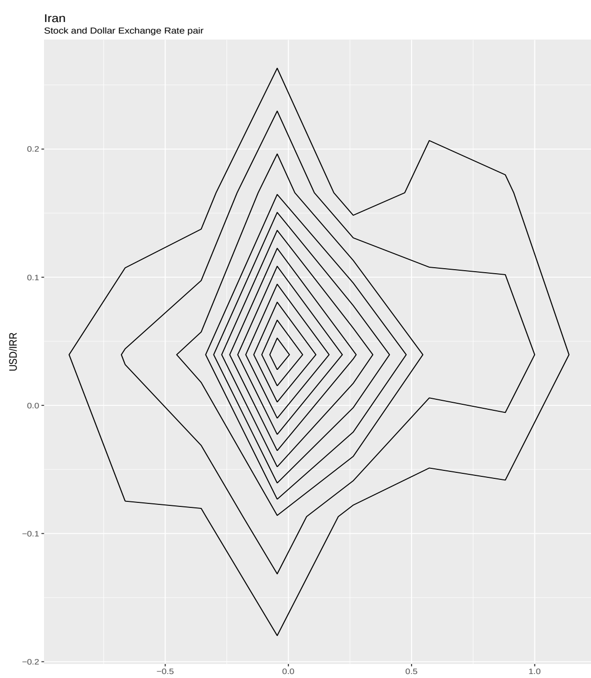
# 6 Appendix



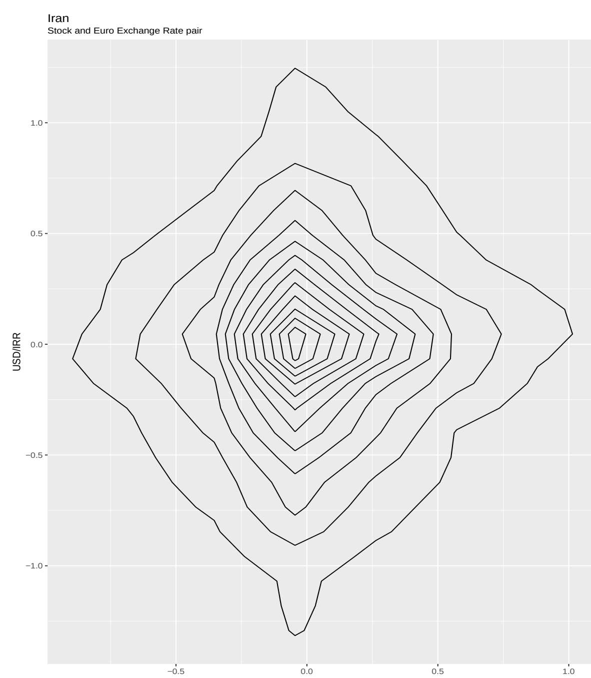
Egypt
Stock and Dollar Exchange Rate pair
USD/EGP
EGX30

Egypt
Stock and Euro Exchange Rate pair
EUR/EGP
EGX30

Iran
Stock and Dollar Exchange Rate pair
USD/IRR
TEPIX

Iran
Stock and Euro Exchange Rate pair
USD/IRR
TEPIX

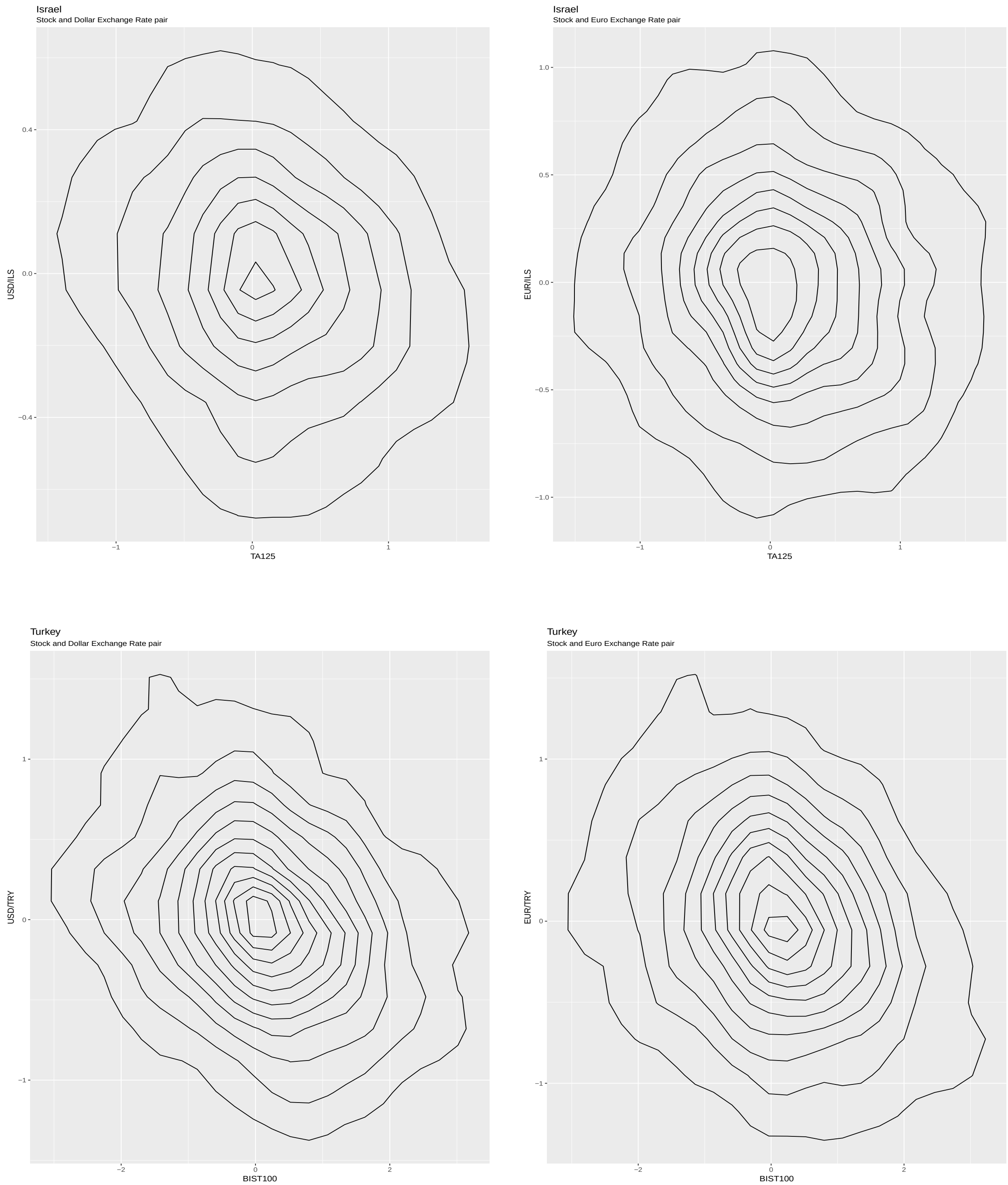



FIGURE 4 – Bivariate contour plots

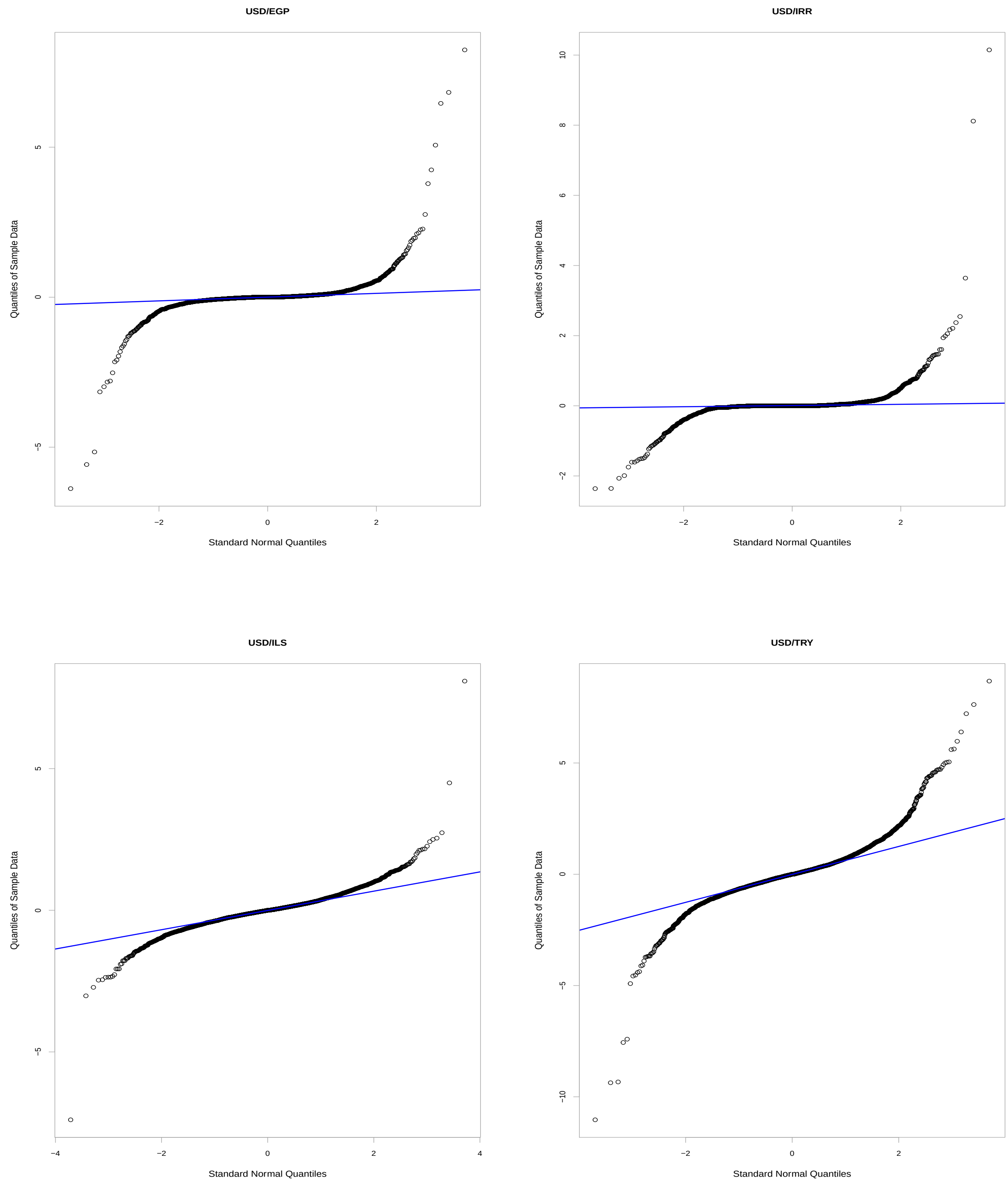



FIGURE 5 – QQ plots of the USD/EGP, USD/IRR, USD/ILS and USD/TRY daily log returns.

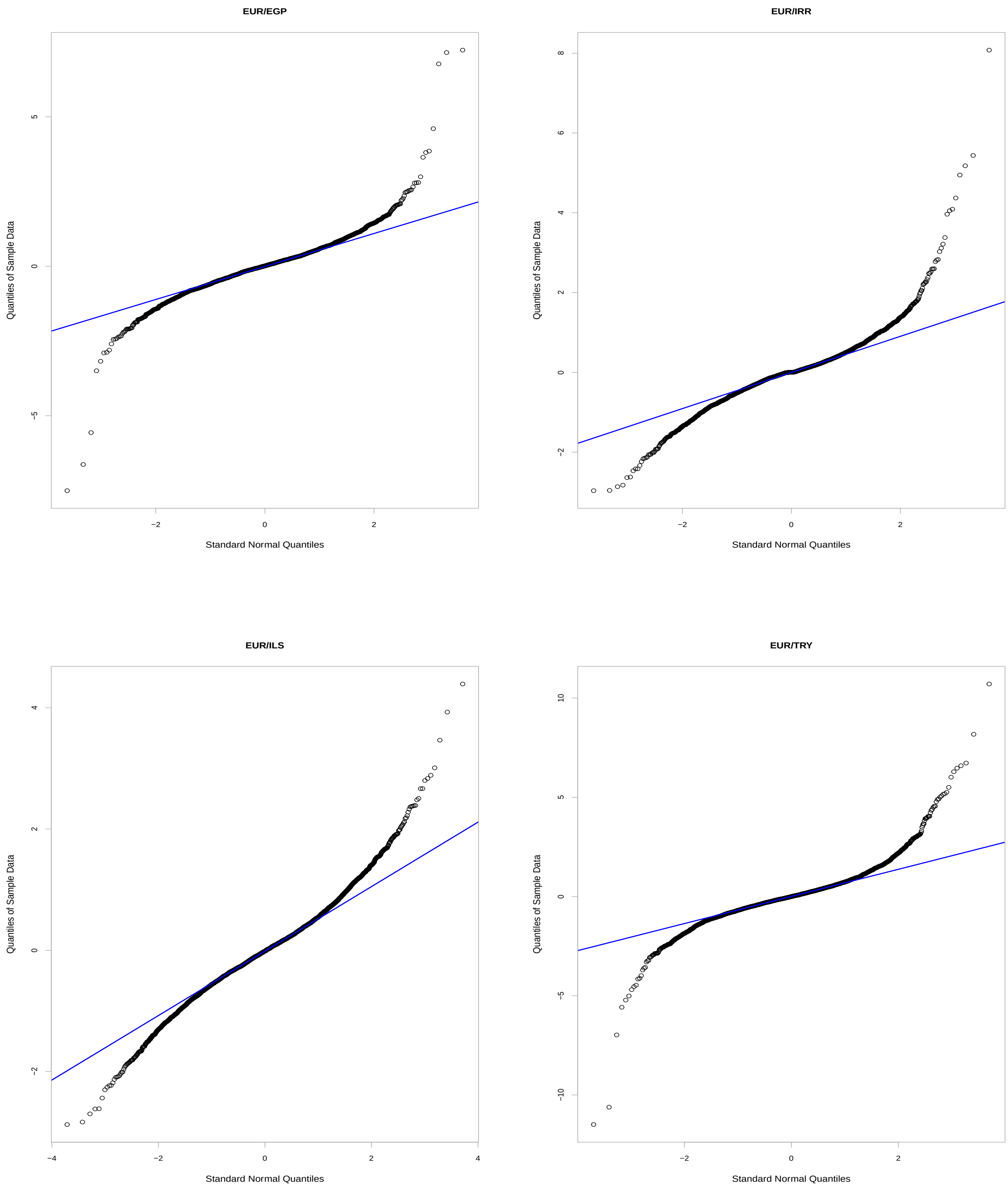



FIGURE 6 – QQ plots of the EUR/EGP, EUR/IRR, EUR/ILS and EUR/TRY daily log returns.

Table 6 – Cramér-von Mises test statistic and $p$-values

| **Country** | **Asset Pair** | **Gaussian** | | **Student's t** | | **Clayton** | | **Gumbel** | | **Frank** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $S_n$ | $p$-value | $S_n$ | $p$-value | $S_n$ | $p$-value | $S_n$ | $p$-value | $S_n$ | $p$-value |
| Egypt | EGX30 * USD/EGP | 0.044 | **0.090** | 0.072 | 0.000 | 0.043 | 0.071 | 0.101 | 0.005 | 0.043 | 0.005 |
| | EGX30 * EUR/EGP | 0.049 | 0.003 | 0.060 | 0.000 | 0.039 | 0.054 | 0.005 | **0.066** | 0.039 | 0.012 |
| Iran | TEPIX * USD/IRR | 0.016 | 0.678 | 0.046 | 0.003 | 0.019 | 0.000 | 0.016 | **0.712** | 0.016 | 0.584 |
| | TEPIX * EUR/IRR | 0.018 | 0.555 | 0.086 | 0.000 | 0.008 | 0.000 | 0.044 | 0.025 | 0.016 | **0.564** |
| Israel | TA125 * USD/ILS | 0.035 | **0.050** | 0.070 | 0.000 | 0.012 | 0.000 | 2.216 | 0.000 | 0.061 | 0.000 |
| | TA125 * EUR/ILS | 0.038 | **0.021** | 0.074 | 0.000 | 0.004 | 0.000 | 0.717 | 0.000 | 0.038 | 0.012 |
| Turkey | BIST100 * USD/TRY | 0.028 | **0.124** | 0.062 | 0.001 | 0.021 | 0.000 | 6.005 | 0.000 | 0.061 | 0.000 |
| | BIST100 * EUR/TRY | 0.035 | **0.050** | 0.044 | 0.003 | 0.026 | 0.000 | 4.290 | 0.000 | 0.038 | 0.015 |

*We run $m = 5000$ repetitions of the parametric bootstrap. Estimates are obtained using maximum pseudo-likelihood. $S_n$ is the test statistic. The p-values range from 0 to 0.712. Those p-values higher than 0.05 pass the goodness-of-fit test at 5% significance level, indicating that they are appropriate models. The best fits are in bold.*