Master Thesis in Economics and Business

Financial Economics

Equity Volatility Targeting Strategy

Smoothing the Volatility of Volatility

P.W. van der Velde

373186

Supervisor

dr. R. Quaedvlieg - Erasmus University Rotterdam

Second Reader

dr. E. Smajlbegovic - Erasmus University Rotterdam

Monday 13th August, 2018

Abstract

This paper builds on the work of Moreira and Muir (2016). Their investment strategy takes less risk when volatility is relatively high and more risk when volatility is relatively low. The contribution to the literature of this paper is that this investment strategy is tested for individual stocks as well as jointly for the constituents of the Dow Jones Industrial Average index. Several different volatility and multivariate volatility models will be used to forecast the (co)variance (matrix). This strategy has an enormous theoretical potential for outperformance. However, this paper documents that in practice scaling the market exposure based on the level of risk generally does not outperform the buy-and-hold strategy in terms of alphas and Sharpe ratios.

Keywords: Volatility Timing, Volatility Targeting, Volatility Scaling, Market Timing, Volatility Forecasting, Covariance Forecasting

Contents

1 Introduction

Ever since the seminal paper of Engle (1982) a lot of research has been dedicated to volatility. There is so much interest in volatility because of the importance of volatility in many financial decision problems (e.g. option pricing and portfolio construction). Volatility is roughly defined as the dispersion of returns for a given security. One of the problems associated with volatility is that volatility is latent. That is, volatility cannot be observed directly. Therefore, a model is needed to estimate volatility. Over the years many different types of volatility models have been introduced, such as ARCH-type (Engle, 1982; Bollerslev, 1986), stochastic (Heston, 1993) and realised volatility models (Barndorff-Nielsen and Shephard, 2004). All of these models are extended to be able to handle multiple assets. These extensions made them interesting for practice as well as academia.

Investors are mainly concerned about two concepts, namely return and risk (usually measured as volatility). The old adage in finance is that 'there is no free lunch'. This roughly means that the only way to get more return is to accept more risk. However, in practice outperformance is possible. There are two possible ways to achieve outperformance, either by security selection (cross-sectional strategy: Fama and French (1993); Jegadeesh and Titman (1993)) or by market timing (time-series strategy: Pesaran and Timmermann (1995); Moskowitz, Ooi and Pedersen (2012)). This paper focuses on the latter. That is, the implemented strategy invests more in the market when the risk-reward trade-off is favorable than when the risk-reward trade-off is unfavorable. Andrew Lo calls this strategy the 'cruise control' strategy (Lo, 2017). The cruise control is set at a specific speed (target volatility). When the car goes down hill the brakes are automatically applied (reduce exposure: put some money in the bank) and when the car goes up hill the car automatically applies some extra throttle (increase exposure: use borrowed funds to invest extra). It should be highlighted that this strategy does not alter the cross-sectional weights of the constituents in the market. The weights could be seen as 'given'. For example, the constituents could be weighted by their market capitalization or in the case of the Dow Jones Industrial Average (DJIA) index the constituents are price-weighted. Lo (2015) explains that this cruise control strategy separates active risk management from active investment management. Another important aspect of this strategy concerns the 'relative' risk compared to a buy-and-hold portfolio. The unconditional risk of this strategy is (by design) very similar to the buy-and-hold portfolio. However, the conditional 'relative' risk in every period can be quite large (i.e. high tracking error).

An empirical observation of daily asset returns is that volatility clusters, meaning that periods of high and low volatility alternate. It is likely to be beneficial for an investor to forecast volatility if it is predictable to some extent. Moreira and Muir (2016) show that for factors an increase in volatility is not offset by a proportional change in expected returns.[1] Therefore, the Sharpe ratio of their strategy is higher than the benchmark. They do this for a wide range of factors.[2] This paper uses the constituents of the DJIA index. Hence, it extends the paper of Moreira and Muir (2016) by looking at individuals stocks. Furthermore, the constituents are jointly modelled to test whether multivariate variance forecasts help to improve the performance. The univariate models used in this research are the AutoRegressive Conditional Heteroskedasticity (ARCH) model (Engle, 1982), the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model (Bollerslev, 1986), the Heterogeneous AutoRegressive (HAR) model (Corsi, 2009) and the High frEquency bAsed VolatilitY (HEAVY) model (Shephard and Sheppard, 2010). The multivariate models used are the Constant Conditional Correlation (CCC) GARCH model (Bollerslev, 1990), the Dynamic Conditional Correlation (DCC) GARCH model (Engle and Sheppard, 2001), the vech-HAR model (Chiriac and Voev, 2011) and the multivariate HEAVY model (Noureldin, Shephard and Sheppard, 2012). A horse race will be held between all of these uni- and multivariate models. The first step is to make the forecasts of the (co)variance (matrix) according the model. Then the stock or portfolio will be scaled according to the strategy. The performances of the different implementations will be compared for some economic measures. The hypothesis is that the risk-adjusted returns for this strategy are higher than for the buy-and-hold strategy. Because, also for equities an increase in volatility will not be offset by a proportional change in expected returns. A backtest is a reasonably safe way to test the performance of this strategy (Cooper, 2010).

This research cannot be compared to factor investing, and more specifically to the low volatility and betting-against-beta factors (Blitz and Van Vliet, 2007; Frazzini and Pedersen, 2014). The low volatility factor states that on average low volatility assets earn higher risk-adjusted returns than high volatility assets. The rationale behind the betting-against-beta factor is that the securities market line (SML) is too flat relative to the capital asset pricing model (CAPM). Both factors are cross-sectional strategies. As mentioned before, the research ob-

[1]The Appendix contains the replicating results of Moreira and Muir (2016). The Financial Times wrote an article about this paper, available at: https://www.ft.com/content/397937d6-e491-11e5-bc31-138df2ae9ee6.

[2]Ken French library, available at: mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

jective of this paper is to create and maintain a trading strategy where the volatility of the portfolio will be 'timed' in the time-series dimension. This means that the volatility of volatility will be smoothed, in contrast to return smoothing where the volatility of returns is smoothed (Hallerbach, 2012).[3] This strategy is also different from the delta-hedging strategy of Black and Perold (1992). Their strategy reduces the (equity) exposure when the market falls. The strategy considered in this paper reduces the equity exposure because of an increase of the volatility.

This strategy is going against current economic theory on portfolio allocation. The strategy takes relatively less risk in periods of high volatility and relatively more risk in period of low volatility. A complete explanation why this strategy should be able to outperform the benchmark is beyond the scope of this research and left for future research. However, several different possible explanations are briefly mentioned. The first explanation assumes that some investors are slow to trade. So, an increase (or decrease) in volatility is not immediately met with a proportional change in the expected return. The second possible explanation is that investors have leverage constraints. Therefore, they are not able to use this strategy. The third possible explanation is that portfolio managers are compared to a benchmark and that they are not allowed to deviate too much from the benchmark holdings (i.e. low tracking error).

The motivation for this research is twofold. First, this topic is relevant for academia, because theory states that it is only possible to obtain a higher return if the associated risk is higher. Moreira and Muir (2016) show that volatility timing yields outperformance at an asset class level. If there is also outperformance within an asset class some theory might possibly have to be revisited. Secondly, it is also a relevant topic for asset managers. Because, if outperformance is possible they probably want to implement this strategy into their portfolio. As mentioned before, the paper of Moreira and Muir (2016) is used as a starting point. The contribution of this paper will be to elaborate Moreira and Muir (2016) on the following aspects:

[3] A basic introduction to volatility scaling is provided by Man AHL. Available at: https://www.man.com/ahl-explains-volatility-scaling.

- Does this trading strategy also work for individual stocks?
- Do different univariate volatility models yield (very) different results?
- Does this trading strategy also work for the index?
- Do different multivariate volatility models yield (very) different results?

These four questions focus on performance. The last research questions check the robustness of the results:

- Is this strategy still feasible if transaction costs are taken into account?
- Is this strategy still feasible if constraints on the amount of leverage are considered?
- Does the performance of the strategy change (substantially) if volatility is used instead of variance?

This paper finds that volatility targeting is possible in theory. However, in practice this strategy is generally not profitable for individual stocks and the DJIA index. Also, multivariate models do not perform better than the index. Imposing leverage constraints and using volatility instead of variance do not alter the conclusions. Transaction costs decrease the performance substantially.

The rest of this paper is organized as follows: section 2 provides an overview of the existing literature regarding this topic. Section 3 explains the data used in this research, followed by methodology in section 4. Section 5 presents the results of the research and section 6 concludes this paper.

2 Literature Review

Volatility timing, sometimes in combination with market timing, is and has been a topic of great interest by many academic and corporate researchers. This section will provide an overview of the literature on volatility timing.

2.1 Theory

Markowitz (1952) started modern finance with his paper on portfolio selection. He demonstrates how an investor should allocate his wealth based on the first and second (co)moment of the assets under consideration. Given the assumptions of the model all investors should scale the tangency (market) portfolio based on their risk aversion. The investors' optimal portfolio allocation changes over time. For example, in periods of high returns investors should allocate more to stocks compared to the risk-free rate. Hallerbach (2012) provides a theoretical derivation for the optimality of volatility weighting over time. He concludes that the better the conditional volatility forecasts, the more constant the volatility of the (normalized) market will be and the higher the timing information ratio.

2.2 Volatility Managed Portfolios

The paper of Moreira and Muir (2016) forms the basis of this thesis. Therefore, it will be explained in some depth.[4] They document for several well-known factors (market, value, momentum, profitability, return on equity, investment and betting-against-beta factors, and the currency carry trade) that volatility timing increases the Sharpe ratios and produces large alphas. A (short-term) mean-variance investor will find an utility gain of around 65%. The reason that this outperformance can be achieved, is that changes in volatility are not offset by proportional changes in expected returns.

Moreira and Muir (2017) conduct research to volatility timing for long term investors. They assume an investor who allocates between a risky and a riskless asset. Also, they assume that volatility and expected returns are time-varying. The conclusion is that volatility timing is also beneficial for long term investors.

[4]Their methods will be explained in detail in Section 4.1.

Lo (2015) implements a similar strategy for the daily CRSP value-weighted index. He documents that the Sharpe ratio for the volatility managed portfolio increased by 33% compared to the buy-and-hold portfolio. Also, the cumulative return is almost 300% higher. Harvey et al. (2018) confirm these results for the US equities market based on Ken French data and based on S&P500 futures.

2.3 Volatility Trading Rules

One strand of literature backtests trading strategies based on the level or change in volatility. Copeland and Copeland (1999) decide whether to invest in value and large cap firms or growth and small cap firms based on the one day percentage change in the VIX. They conclude that value and large cap portfolios outperform growth and small cap portfolios when volatility increases, and vice versa.

The idea behind the paper of Cooper (2010) and Moreira and Muir (2016) is similar. The weights (leverage) in the portfolio are determined by the level of risk. Cooper (2010) has two strategies that are relevant for this paper, the 'constant volatility strategy' (CVS) and the 'optimal volatility strategy' (OVS). The CVS uses volatility as measure of risk and the OVS uses the variance as measures of risk. Depending on the level of leverage they choose the corresponding leveraged exchange traded funds (ETF's).

2.4 Mean-Variance Framework with Constant Returns

Another strand of literature on volatility timing assumes a mean-variance investor. Furthermore, it is assumed that expected returns are constant. Merton (1971) showed that forecasting (daily) returns is (far) more difficult than forecasting (co)variances. Hence, the weights of the assets in the portfolio are only determined by the covariance matrix. Fleming et al. (2001) manage a portfolio across different asset classes (stocks, bonds, gold and cash) at a daily frequency. They assume a short horizon mean variance investor. They find that volatility timing strategies outperform static portfolios that have the same target expected return and volatility.

2.5 Incorporating High-Frequency Data

Recently, models that incorporate high-frequency data to forecast the (co)variance have been developed (Chiriac and Voev, 2011; Noureldin, Shephard and Sheppard, 2012). The idea behind Fleming et al. (2003) is the same as Fleming et al. (2001). But, here they incorporate high-frequency data instead of daily data. They document that using intraday returns to estimate the conditional covariance instead of daily data is substantial. Hautsch et al. (2013) also conclude that: 'high-frequency based covariance forecasts outperform low-frequency approaches over investment horizons of up to a month'.

De Pooter, Martens and Van Dijk (2005) investigate the optimal sampling frequency for the constituents of the S&P100. They conclude that the optimal frequency ranges from 30 to 65 minutes. Bandi et al. (2008) also investigate the optimal sampling frequency. However, their objective is to select a time-varying optimal sampling frequency for each entry in the covariance matrix based on the mean squared error (MSE). Liu (2009) tries to minimize the tracking error of following the S&P500 index with the 30 DJIA index stocks. He concludes that an investor will only use high-frequency data if the investor rebalances his portfolio daily or if the investor has less than six months of historical data.

3 Data

The decision which benchmark and corresponding investment universe are used to backtest a strategy has a large impact on the results of the research (Goltz and Campani, 2011; Kidd, 2012). This research uses the constituents of the DJIA index and the DJIA index itself as the investment universe and the corresponding buy-and-hold portfolio is used as the benchmark.

The DJIA index was founded in 1885. However, this research needs high-frequency (intraday) data for some methods. The possible investment universe is therefore limited to more recent years. Moreover, Wurgler (2010) concludes that the correlation between two assets increases when both are included in an index. Therefore, to create an unbiased analysis this research uses a period where the constituents of DJIA index do not change. The longest and fairly recent period where the constituents of the DJIA index do not change is from June 8, 2009 until September 24, 2012. There are 833 daily observations in total. The data is explained in more detail in the next section.

3.1 Wharton Research Data Services

The Wharton Research Data Services (WRDS) has two relevant databases for this research.[5] The Center for Research in Security Prices (CRSP) database provides the daily returns of the constituents of the DJIA index. The Trade and Quote (TAQ) database provides the intraday data.

CRSP

CRSP provides the daily data for the thirty constituents of the DJIA index. Table 3 provides some descriptive statistics for the constituents and the index between June 8, 2009 and September 24, 2012.[6] The returns are adjusted for stock splits and dividends. The weights

[5] Available at: wrds-web.wharton.upenn.edu/

[6] The constituents of the DJIA index between June 8, 2009 and September 24, 2012 are: 3M Company (MMM), Alcoa Inc. (ARNC (AA)), American Express Company (AXP), AT&T Inc. (T), Bank of America Corporation (BAC), The Boeing Company (BA), Caterpillar Inc. (CAT), Chevron Corporation (CVX), Cisco Systems (CSCO), The Coca-Cola Company (KO), E.I. du Pont de Nemours & Company (DD), Exxon Mobil Corporation (XOM), General Electric Company (GE), Hewlett-Packard Company (HPQ), The Home Depot, Inc. (HD), Intel Corporation (INTC), International Business Machines Corporation (IBM), Johnson & Johnson (JNJ), JPMorgan Chase & Co. (JPM), Kraft Foods Inc. (MDLZ (KFT)), McDonald's Corporation (MCD), Merck & Co., Inc. (MRK), Microsoft Corporation (MSFT), Pfizer Inc. (PFE), The Procter & Gamble Company (PG), The Travelers Companies, Inc. (TRV), United Technologies Corporation (UTX), Verizon Communications Inc. (VZ), Wal-Mart Stores, Inc. (WMT) and The Walt Disney Company (DIS).

are determined by their respective daily close prices.[7] The volatility index for the DJIA index measures the implied volatility of the index (ticker: VXD). The methodology for the VXD is the same as for the VIX (volatility index of the S&P500).

Trade and Quote

The TAQ database contains all tick data for the thirty stocks. The following steps are used to clean the trade data:

- only select trades between exchange hours
- only select trades from NYSE and NASDAQ
- delete trades with price or volume equal to zero
- delete trades with abnormal sales condition
- delete trades with abnormal correction indicator
- merge trades with same time-stamp
- extract five minutes prices by means of last trade in the corresponding interval

The DJIA index is not traded. The high-frequency data of the constituents are used to obtain the high-frequency data for the DJIA index. The five minute returns are multiplied with their respective weight (determined at the end of the previous day) in the index.

3.2 Ken French Library and Global Financial Data

Global Financial Data (GFD) provides the daily DJIA index level and volatility index level data.[8] To replicate the results of Moreira and Muir (2016) factor premiums are needed. These are obtained from the Ken French library.[9] The daily factors used are the market factor, value (HML) factor, size (SMB) factor and momentum (MOM) factor. The risk-free rate is also obtained from the Ken French library.

[7]The weights of the stocks are determined by their price, and not by their market capitalization. Hence, a higher price means a larger weight in the index.

[8]Available at: https://www.globalfinancialdata.com/

[9]Available at: mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

4 Methods

This section explains the methods used in this research. The methods are based on the ideas of Moreira and Muir (2016). Therefore, the first subsection explains their research methodology. The second and third subsections explain the portfolio scaling procedure and the general set-up of all the models. Then the conditional (co)variance models are explained. The last two subsections explain the robustness checks and the performance measures, respectively.

4.1 Methodology of Moreira and Muir (2016)

Moreira and Muir (2016) construct a volatility managed portfolio by scaling an excess return by the inverse of its conditional variance:

$$f_{t+1}^{\sigma} = \frac{c}{\widehat{\sigma_t^2}(f)} f_{t+1}, \tag{1}$$

where f_{t+1}^{σ} is the return of the volatility managed portfolio, f_{t+1} is the buy-and-hold portfolio excess return, $\widehat{\sigma_t^2}(f)$ is a proxy for the portfolios conditional variance and c controls the average exposure of the strategy. The portfolio f is already determined in this set-up (Ken French factor). They use the variance of last month as a proxy for the conditional variance:

$$\widehat{\sigma_t^2}(f) = \sum_{d=1/22}^{1} \left(f_{t+d} - \frac{\sum_{d=1/22}^{1} f_{t+d}}{22} \right)^2 \tag{2}$$

Multi-factor portfolios are constructed such that the multi-factor portfolio is mean-variance efficient for the set of factors used. If F_{t+1} denotes a vector of factor returns, b denotes the static weights that produce the maximum in-sample Sharpe ratio, then the mean-variance efficient portfolio is calculated as $f_{t+1}^{MVE} = b' F_{t+1}$. Again, it is stressed that this trading strategy does not alter the cross-sectional weights of the portfolio.

4.2 Equity Volatility Targeting Strategy

The first two research questions of this paper are directly related to equation 1. The first question can be answered by using the constituents of the DJIA index (and the DJIA index itself) for f_{t+1} instead of factors. The second question can be answered by considering different volatility models for $\widehat{\sigma_t^2}(f)$. Moreira and Muir (2016) use the variance of last month for $\widehat{\sigma_t^2}(f)$, this paper intends to use forecasts for $\widehat{\sigma_t^2}(f)$. To make this explicit in the notation this paper will use $\widehat{h_{t+1|t}}$. Similar as to Moreira and Muir (2016) the constant c is chosen such that the unconditional variances of the managed portfolio and the buy-and-hold portfolio are ex-ante equal. The formula for this paper is:

$$r_{t+1}^{\sigma} = \frac{c}{\widehat{h_{t+1|t}}} r_{t+1}, \tag{3}$$

where r_{t+1}^{σ} is the return of the volatility scaled stock, c controls the average exposure, r_{t+1} is the return of the buy-and-hold strategy and $\widehat{h_{t+1|t}}$ is the forecasted (conditional) variance. The multivariate equivalent of equation 3 is:

$$r_{t+1}^{\sigma,P} = w_t \boldsymbol{r}_{t+1}^{\sigma} = w_t \frac{c}{\widehat{h_{t+1|t}^{P}}} \boldsymbol{r}_{t+1}, \tag{4}$$

where $r_{t+1}^{\sigma,P}$ is the return of the volatility managed portfolio, w_t denotes the weights of the constituents in the DJIA index, $\boldsymbol{r}_{t+1}^{\sigma}$ is the n times 1 vector of volatility managed returns, c is a scaler controlling the average exposure of the strategy, $\widehat{h_{t+1|t}^{P}}$ denotes the variance of the entire portfolio (of stocks in the DJIA index) and $\boldsymbol{r}_{t+1}$ is the vector of returns of the buy-and-hold portfolio. The conditional variance of the portfolio is calculated as follows:

$$\widehat{h_{t+1|t}^{P}} = w_t' \widehat{H_{t+1|t}} w_t, \tag{5}$$

where $\widehat{H_{t+1|t}}$ denotes the forecast of the covariance matrix.

The different univariate models make forecasts for $\widehat{h_{t+1|t}}$ and the multivariate models for $\widehat{H_{t+1|t}}$. The first forecast for the variance is made for January 3, 2012. So, the estimation period is from June 8, 2009 to December 30, 2011. All models make use of an expanding window. Some models use maximum likelihood estimation (MLE) to estimate the parameters. These models need starting values h_0 or H_0 for the (co)variance. Often, the unconditional (co)variance of the entire sample is used. However, to eliminate the look-ahead bias, the unconditional (co)variance is estimated with observations up to and including $t-1$. In total 184 forecasts of the daily variance are made from January 3, 2012 to September 24, 2012.

4.3 General Set-Up

Univariate Volatility Models

The general set-up for the univariate volatility models is as follows:

$$\begin{aligned} r_t &= \mu + \epsilon_t, \\ \epsilon_t &= z_t\sqrt{h_t}, \\ h_t &= f(I_{t-1}), \end{aligned} \tag{6}$$

where μ is the mean daily (simple) return (assumed to be constant), z_t is assumed to have a standard normal distribution, I denotes the information set, h_t is some function depending of the specific model on the information set at time $t-1$ and ϵ_t is the error term with the following properties:

$$\begin{aligned} E[\epsilon_t|I_{t-1}] &= 0, \\ E[\epsilon_t^2|I_{t-1}] &= h_t, \\ E[\epsilon_t^2] &= \sigma^2, \end{aligned} \tag{7}$$

where E denotes the expectation and σ^2 denotes the unconditional variance. The first equation states that the conditional mean is zero and the second equation states that there is a time-varying conditional variance. A direct consequence is that:

$$\begin{aligned} E[r_t|I_{t-1}] &= \mu, \\ V[r_t^2|I_{t-1}] &= h_t, \end{aligned} \tag{8}$$

where V denotes the variance.

All models are estimated using variance targeting. This reduces the number of parameters to be estimated with one. Also, the mean return is set to the unconditional mean (up until $t-1$). The parameter(s) θ of the MLE models can be estimated by optimizing the quasi log likelihood function. This is the sum of all quasi log likelihoods:

$$l_t(\theta) \propto -ln(h_t) - \frac{\epsilon_t^2}{h_t}, \tag{9}$$

where l_t denotes the quasi log likelihood. The one-step ahead forecast can directly be obtained by iterating one step forward.

Multivariate Volatility Models

The multivariate set-up is similar to the univariate one:

$$\begin{aligned} \boldsymbol{r_t} &= \boldsymbol{\mu} + \boldsymbol{\epsilon_t}, \\ \boldsymbol{\epsilon_t} &= H_t^{1/2}\boldsymbol{z_t}, \\ H_t &= f(I_{t-1}), \end{aligned} \tag{10}$$

where $\boldsymbol{\mu}$ is the n times 1 vector of mean returns, $\boldsymbol{z_t}$ is assumed to have a multivariate standard normal distribution, H_t is a function depending of the specific model on the information set at time $t-1$ and $\boldsymbol{\epsilon_t}$ has the following properties:

$$\begin{aligned} E[\boldsymbol{\epsilon_t}|I_{t-1}] &= \mathbf{0}, \\ E[\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t'}|I_{t-1}] &= H_t, \\ E[\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t'}] &= \Sigma, \end{aligned} \tag{11}$$

where Σ is the unconditional covariance matrix. The DCC GARCH and multivariate HEAVY models will be represented in their BEKK parametrization (with $K=1$, $q=1$ and $p=1$):

$$H_t = C'C + A\boldsymbol{\epsilon_{t-1}}\boldsymbol{\epsilon_{t-1}'}A + BH_{t-1}B \tag{12}$$

All models will be estimated with covariance targeting, which decreases the number of parameters to be estimated substantially. Also, the mean vector will be estimated by the unconditional mean vector. In this paper A and B are assumed to be scalers.

The sample covariance (or correlation) matrix cannot be estimated properly when the concentration ratio (N/T) is larger than 1/100. In the case of this paper this ratio is 30/833 (at best). Therefore, a linear shrinkage estimator will be applied to the unconditional covariance (or correlation) matrix (Engle at el., 2017). The linear shrinkage estimator is expressed as:

$$\tilde{\Sigma} = \sum_{i=1}^{n}[\rho\bar{\lambda} + (1-\rho)\lambda_i]u_i u_i', \tag{13}$$

where $\tilde{\Sigma}$ is the shrunken sample covariance matrix, ρ is the shrinkage intensity (a number between zero and one), λ is the vector of eigenvalues of the sample covariance matrix $\hat{\Sigma}$, u_i's are the corresponding eigenvectors and $\bar{\lambda}$ is the cross-sectional average of the eigenvalues.

The shrinkage intensity ρ will be optimised within the maximum likelihood procedure.

The quasi log likelihood for the multivariate models at time t is:

$$l_t(\theta) \propto -ln(det(H_t)) - (\epsilon_t' H_t^{-1} \epsilon_t), \tag{14}$$

where det denotes the determinant. Again, forecasts for the covariance matrix can be directly obtained through iteration.

4.4 Univariate Variance Models

ARCH(1) Model

The ARCH model was introduced by Engle (1982). The conditional variance is modelled as a linear function of the squared past shocks. This paper uses the following ARCH(1) representation:

$$h_t = (1 - \alpha)\widehat{\sigma^2} + \alpha\epsilon_{t-1}^2, \tag{15}$$

Because h_t is a (conditional) variance it should be non-negative. Therefore, α should be between zero and one. This parameter can be estimated by maximizing the log likelihood function.

GARCH(1,1) Model

The GARCH(1,1) specification of Bollerslev (1986) for the conditional variance is:

$$h_t = (1 - \alpha - \beta)\widehat{\sigma^2} + \alpha\epsilon_{t-1}^2 + \beta h_{t-1} \tag{16}$$

It extends the ARCH specification by adding the lagged conditional variance. Again, to ensure that h_t is non-negative all parameters should be non-negative and their sum should be smaller than one. The parameters are estimated by means of maximum likelihood estimation.

HAR Model

The HAR model of Corsi (2009) depends on high-frequency data. The first step is to calculate the realised variance:

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2, \tag{17}$$

where RV_t is the realised variance on day t, $r_{t,j}^2$ denotes the squared return of the j^{th} period of length $1/M$ during day t. The trading hours for the DJIA index are between 09.30 and 16.00. The length of the period is equal to five minutes. Hence, M is equal to 78. The realised variance is an unbiased and consistent estimator of the true variance. So, volatility becomes directly observable, i.e. it is not latent anymore. It is therefore possible to model volatility directly. The model proposed by Corsi (2009) with variance targeting is:

$$h_t = \overline{RV}_{t-1} + \beta_1(RV_{t-1} - \overline{RV}_{t-1}) + \beta_2(RV_{t-1,5} - \overline{RV}_{t-1,5}) + \beta_3(RV_{t-1,22} - \overline{RV}_{t-1,22}) + \epsilon_t, \tag{18}$$

where $RV_{t-1,L} = \frac{1}{L}\sum_{j=1}^{L} RV_{t-j}$ and the bar denotes the time-series average up until time t. The lagged realised variances RV_{t-1}, $RV_{t-1,5}$ and $RV_{t-1,22}$ represent the daily, weekly and monthly realised variances, respectively. The parameters can be estimated with OLS.

HEAVY Model

The HEAVY model of Shephard and Sheppard (2010) 'applies' the realised variance to the GARCH(1,1) model. Their idea was to replace ϵ_{t-1}^2 with the more accurate measure RV_{t-1}. This leads to the following model:

$$h_t = (1 - \alpha\kappa - \beta)\widehat{\sigma^2} + \alpha RV_{t-1} + \beta h_{t-1}, \tag{19}$$

where κ is a factor to adjust α. Because, due to overnight effects the RV_t is likely to be a biased downward measure of r_t^2. Therefore, it is estimated as:

$$\kappa = \frac{\frac{1}{T}\sum_{t=1}^{T} RV_t}{\frac{1}{T}\sum_{t=1}^{T} r_t^2} \tag{20}$$

To ensure that the conditional variance is non-negative the parameters have the same restrictions as for the GARCH(1,1) model. The parameters are estimated with MLE. A note should be made about the two models that incorporate high-frequency data. The HAR model models the open-to-close variance, while the HEAVY model models the close-to-close variance.

4.5 Multivariate Variance Models

There are many multivariate models for modeling and forecasting the (conditional) covariance matrix. However, most models suffer from the curse of dimensionality. Hence, most models restrict the number of parameters to make them feasible in higher dimensions. The multivariate models considered here are the multivariate extensions of the models in the previous section. First, two multivariate extensions of the GARCH model are explained. Then the multivariate HAR and HEAVY models are explained.

CCC GARCH Model

The CCC GARCH model assumes that the covariance matrix H_t can be modelled as:

$$H_t = D_t R D_t, \tag{21}$$

where D_t is a matrix with conditional standard deviations $\sqrt{h_{ii,t}}$ on the diagonal and R is the correlation matrix. The individual elements of H_t can be modelled as:

$$h_{ii,t} = (1 - \alpha_{ii} - \beta_{ii})\widehat{\sigma^2_{ii}} + \alpha_{ii}\epsilon^2_{i,t-1} + \beta_{ii}h_{ii,t-1}, \qquad \text{for } i = 1, ..., N \tag{22}$$

$$h_{ij,t} = \rho_{ij}\sqrt{h_{ii,t}}\sqrt{h_{jj,t}} \qquad \text{for all } i \neq j \tag{23}$$

Equation 22 models the individual conditional variances as GARCH(1,1) processes. Equation 23 is the formula for the covariance between i and j. The model is estimated as follows:

- Estimate univariate GARCH(1,1) models for the individual variances
- Calculate the standardized residuals
- The correlation matrix is estimated by means of the unconditional correlation matrix of the standardized residuals

DCC GARCH Model

The DCC GARCH model is similar to the CCC GARCH model, except that the correlation matrix is now time-varying. So, the covariance matrix can be formulated as:

$$H_t = D_t R_t D_t \tag{24}$$

The conditional variances are modelled in the same way as equation 22. The conditional correlations are modelled as follows:

$$Q_t = (1 - \alpha - \beta)\tilde{Q} + \alpha \widehat{z_{t-1}} \widehat{z_{t-1}}' + \beta Q_{t-1}, \tag{25}$$

where Q_t is the conditional pseudo-correlation matrix, $\tilde{Q}$ is the unconditional correlation matrix with the shrinkage estimator applied and z_{t-1} are the standardized residuals. The correlation matrix is obtained by:

$$R_t = Diag(Q_t)^{-1/2} Q_t Diag(Q_t)^{-1/2} \tag{26}$$

Multivariate HAR Model

The realised covariance matrix is multivariate equivalent of the realised variance. It is calculated as follows:

$$RC_t = \sum_{j=1}^{M} r_{t,j} r'_{t,j}, \tag{27}$$

where $r_{t,j}$ is a vector of returns of the j^{th} period of length $1/M$ during day t. The next step is to take the Cholesky decomposition of RC_t, which is denoted by P_t:

$$P'_t P_t = RC_t \tag{28}$$

Then let X_t denote half-vectorization of P_t, such that:

$$X_t = vech(P_t) \tag{29}$$

The daily volatility X_t is modelled as:

$$X_{t,d} = \overline{X}_{t-1,d} + \beta_d (X_{t-1,d} - \overline{X}_{t-1,d}) + \beta_w (X_{t-1,w} - \overline{X}_{t-1,w}) + \beta_m (X_{t-1,m} - \overline{X}_{t-1,m}) + \omega_{t,d}, \tag{30}$$

where d denotes the daily volatility, w denotes the weekly (5 days) volatility and m denotes the monthly (22 days) volatility. Again, the bar denotes the time-series average. Chiriac and Voev (2011) use 20 days for the monthly specification. However, to maintain internal consistency with the univariate HAR model this paper uses 22 days for the monthly volatility. The parameters β_d, β_w and β_m are scalers. The independent variables $X_{t,(.)}$ are averages of past values of X_t.

The covariance matrix can be obtained as follows:

$$H_t = invvech(X_t)' \, invvech(X_t), \tag{31}$$

where *invvech* denotes the inverse of the half-vectorization operation.

Multivariate HEAVY Model

The multivariate HEAVY model uses the following specification for the covariance matrix:

$$H_t = (I - A\boldsymbol{\kappa} - B)\tilde{\Sigma} + \alpha RC_{t-1} + \beta H_{t-1}, \tag{32}$$

where $A = \alpha I$, $B = \beta I$, I is the identity matrix and $\boldsymbol{\kappa}$ is the multivariate equivalent of κ:

$$\boldsymbol{\kappa} = E[RC_t]^{1/2} E[H_t]^{-1/2} \tag{33}$$

4.6 Robustness

Borrowing and Transaction Costs

To trade in financial markets some costs have to be incurred. This robustness test checks whether this strategy is viable if transactions and borrowing costs are incurred. The risk-free rate is used for the borrowing costs or yield if the leverage is smaller than one. It is assumed that the transactions costs are 5 basis points. The management fee for the DJIA index is assumed to be 10 basis points per year.

Leverage Constraint and Volatility Scaling

This paper considers two strategies to reduce the turnover of this strategy. First, the portfolio leverage is limited between 0.8 and 1.2:

$$0.8 \leq \frac{c}{\widehat{h_{t+1|t}}} \leq 1.2 \tag{34}$$

Secondly, volatility scaling is used instead of the variance scaling. The time-series weight is calculated as: $\frac{c}{\sqrt{\widehat{h_{t+1|t}}}}$. Hence, the changes in time-series weights are lower.

Random Walk Forecasts

The random walk model is a 'naive' model that uses the current level as the forecast for the next period. However, in practice it is a difficult model to beat. The first random walk model uses the realised variance of day $t-1$ as a forecast for day t. The second model uses the level of the volatility index in stead of the realised variance.

Ex-Post Optimal Time-Series Weighting

The theoretical optimal strategy is to use the realised variance of day t to as a forecast for day t. This strategy is not feasible in practice. However, it gives the theoretical 'maximum' performance of the strategy. Also, comparing the different forecasting models with the ex-post optimal scaling gives an indication how the different models perform.

Ex-Post Optimal Gamma

This strategy scales the stock or portfolio based on the variance or volatility. These two are relatively arbitrarily chosen. It might be optimal to scale by a different power. First, equation 3 is rewritten to:

$$r_{t+1}^{\sigma} = c\,\widehat{h_{t+1|t}}^{-\gamma}\, r_{t+1},$$

where γ is a scaling parameter. So, γ is assumed to be one for the variance and one half for the volatility. However, ex-post can be determined whether this value for γ is optimal in terms of the Sharpe ratio or alpha. An optimal γ larger than one indicates that the strategy should adapt much more strongly to changes in the level of the forecast than it currently does. The optimization of γ for the multivariate models is done in the same way.

4.7 Performance Measures

Sharpe Ratio

The first performance measure is the Sharpe ratio (Sharpe, 1994). It measures the excess return per unit of risk:

$$S = \frac{E(r - r_f)}{\sqrt{Var(r - r_f)}} \tag{35}$$

Where r the return of the (managed) asset or portfolio and r_f denotes the risk-free rate. The expected return is calculated as the geometric average.

Time-Series Regression

The second performance measure uses a time-series regression to test the (out)performance:

$$r_t^{\sigma} - r_{f,t} = \alpha + \beta(r_{m,t} - r_{f,t}) + \epsilon_t, \tag{36}$$

where r_t^{σ} is the return of the managed portfolio, $r_{m,t}$ is the market (DJIA index) return and $r_{f,t}$ is the risk-free rate. A significantly positive α indicates outperformance.

Timing Ability

This strategy is performing well when the portfolio exposure is smaller than 100% when there is a negative return and when the portfolio exposure is larger than 100% if there is a positive return. The hit ratio is an intuitive way to measure this.

Kurtosis

The distribution of (equity) returns is not normal. The distribution has fat tails (extreme returns happen more often) and is negatively skewed (large negative returns happen more often large positive returns). Volatility targeting tries to 'normalize' the returns. So, another intuitive measure is to compare the kurtosis of the unscaled and the scaled returns.

Rolling Volatility

This strategy with volatility scaling targets a specific level of volatility. Therefore, the volatility of volatility should be small. The level of volatility should be stable for the scaled portfolio when the volatility is calculated based on a rolling window of 20 days. The volatility of the unscaled portfolio 'should' change more over time.

5 Results

The results are presented in the following sections. The first subsection will contain the main results, then the results for the univariate and multivariate models are presented. More detailed results can be found in the Appendix.

5.1 Main Results

Volatility targeting with the ex-post realised variance works exceptionally well. This shows that this strategy has a large potential for outperformance. However, all univariate and multivariate models perform poorly. None of them do better than the buy-and-hold strategy in terms of Sharpe ratio or alpha. The random walk forecast with the realised variance also performs poorly. The HAR model is the only model that is able to outperform the buy-and-hold portfolio for the DJIA index based on alpha and Sharpe ratio. However, the outperformance is not significant.

The forecasted variances of the index based on the univariate models are most of the time a bit too high compared to the realised variances. The time-series weights for the DJIA index are much 'spikier' for the realised variance than all univariate models. The parameter estimates for the ARCH, GARCH and HAR model are quite stable over time. Only the HEAVY parameters vary slightly near the end of the forecasting period.

Modelling the constituents of the DJIA index jointly does not perform better than the buy-and-hold strategy. All models have lower Sharpe ratios, lower returns and negative alphas. The betas and time-series weights are both close to one. The MHEAVY model performs better than the other multivariate models. The CCC GARCH and DCC GARCH models perform better than the univariate GARCH model for the DJIA index. This is also true for MHEAVY model, but not for the MHAR model.

The parameters of all multivariate models are stable over time. Again, the forecasted variances are too high compared to the realised variance. The time-series weights change much more gradually over time compared to the time-series weights for the univariate models. The rolling volatility is not more stable for the managed portfolios than for the buy-and-hold portfolio.

5.2 Univariate Results

Table 1 presents the performance measures for the univariate models.

Table 1: Performance Univariate Models

Ticker	Sharpe B&H	ARCH S.	ARCH A.	GARCH S.	GARCH A.	HAR S.	HAR A.	HEAVY S.	HEAVY A.
DJIA	1.45	1.37	-0.93	0.70	-8.22	1.65	3.44	1.14	-1.91
MSFT	1.24	1.31	1.83	1.28	1.46	1.17	-0.81	1.23	0.77
KO	1.14	1.06	-1.00	1.39	3.77	1.27	3.01	1.39	4.63
DD	1.03	0.91*	-2.03*	0.25*	-13.36*	1.01	0.06	0.46	-10.22
XOM	0.91	0.88	-0.36	0.33	-8.34	0.80	-1.28	0.97	8.68
GE	1.84	1.82	-0.37	1.38	-7.55	1.61	-3.69	1.47	-5.36
IBM	1.06	1.13	1.32	0.98	-0.54	0.87	-2.14	0.30	-11.24
CVX	1.00	1.02	0.36	0.82	-2.05	1.23	5.26	0.74	-3.32
UTX	0.74	0.69	-1.04	0.58	-2.83	0.48	-4.74	0.70	0.95
PG	0.72	0.68	-0.36	0.51	-2.69	0.84	1.75	0.97	4.23
CAT	0.09	0.07	-0.42	-0.44	-13.96	0.04	-0.80	0.06	-0.15
BA	-0.20	-0.24	-0.80	-0.77	-11.46	-0.35	-2.96	-0.70	-10.09
PFE	1.76	1.75	-0.01	1.82	1.09	1.70	-0.46	1.82	2.93
JNJ	1.15	0.85	-2.61	0.69	-3.67	1.12	0.17	0.39*	-6.94
MMM	1.55	1.42	-1.77	0.69*	-11.59*	1.27	-3.65	0.51*	-14.30
MRK	1.92	2.03	1.80	2.02	2.60	2.00	1.63	1.88	-0.06
AA	0.26	0.15	-3.10	-0.08	-9.52	0.18	-1.75	0.33	2.97
DIS	2.72	2.75	1.03	2.64	-0.04	2.51	-2.71	2.64	0.95
HPQ	-1.70	-1.41	10.33	-1.15	21.54	-1.80	-4.46	-1.79	-5.51
MCD	-0.45	-0.40	0.71	-0.50	-0.76	-0.25	2.80	0.00	7.06
JPM	1.05	1.03	0.32	0.98	2.08	1.35	14.02	1.42	17.08
WMT	2.05	2.68	13.73*	2.34	9.47	1.83	-2.26	1.39	-9.81
AXP	1.40	1.26	-2.66	0.92*	-9.3*	1.69	8.18	1.55	6.94
INTC	-0.25	-0.31	-1.13	-0.34	-1.93	-0.56	-6.30	0.18	11.08
BAC	1.65	1.57	-1.27	1.54	3.89	1.30	-9.88	1.41	-3.54
TRV	1.42	1.51	2.18	1.03	-5.80	1.72	6.26	1.36	1.68
VZ	1.73	1.84	1.45	1.42	-3.89	1.76	0.63	1.44	-3.62
T	2.88	2.77	-1.08	2.25	-6.68	2.89	1.11	2.40	-4.72
HD	2.72	2.35	-6.12	2.55	0.19	3.25	12.57*	3.10	13.07
CSCO	0.25	0.20	-1.37	0.20	-1.37	0.20	-1.29	0.03	-5.74
KFT	1.25	1.32	1.30	0.66*	-8.32*	1.29	0.97	1.22	0.14
#Outperformance		0	1	0	0	0	1	0	0

Note: this table contains the performance measures for the individual variance models. The Sharpe ratio (S.) and alpha (A.) are annualized. 'B&H' denotes buy-and-hold. The last row counts the number of times the model performs significantly better than the index. An asterisk denotes a significant difference from the buy-and-hold performance. The significance of alpha is determined by the t-statistic of the constant of the time-series regression. The significance of the difference for the Sharpe ratios is determined with methodology of Ardia and Boudt (2015).

None of the univariate models is able to significantly outperform the buy-and-hold strategy. Incorporating high-frequency data does not yield any benefit compared to the models that use daily data in terms of the number of times of significant outperformance. These results do match the results of the always long time-series momentum strategy of Moskowitz, Ooi and Pedersen (2012). They document that their strategy has a positive alpha for ninety percent of the assets and for 26% of these assets this alpha is significant.

The ARCH model has a higher Sharpe ratio than the buy-and-hold portfolio and a positive alpha for about one third of the assets. None of the assets have a significantly higher Sharpe ratio than buy-and-hold portfolio and one asset a significantly lower Sharpe ratio. There is one asset with a significantly better Sharpe ratio and one asset with a significantly worse Sharpe ratio. There are a couple of 'extreme' results in terms of alpha. There are five constituents for which alpha is lower than minus two, the lowest being -6.12%. Three constituents have an alpha larger than two, two of them have alphas larger than ten percent. The GARCH model generally performs worse than the ARCH model. Only for six out of the 31 assets it performs better than the buy-and-hold portfolio in terms of Sharpe ratio and only four stocks have a higher Sharpe ratio than the corresponding ARCH model. It also has many large negative alphas (16) and large positive alphas (6). The largest being -13.36% and 21.54%.

The HAR model performs roughly similar as the ARCH model. The HEAVY model performs slightly worse. The HAR and HEAVY models perform worse than buy-and-hold strategy based on Sharpe ratios (18 and 21, respectively) and alphas (16 and 16, respectively). The HEAVY model performs better than the HAR model for fourteen constituents based on alphas. The HEAVY model has more extreme negative alphas (13 vs 10) and more extreme positive alphas (10 vs 8).

Figures 2, 3, 6 and 8 show the parameter estimates, forecasted variances and time-series weights for the different univariate models. The ARCH parameter is roughly 0.10 for the DJIA index. This implies that the forecasts are dominated by the unconditional variances. Hence, the time-series weights do not deviate much from one. The estimation period was a period with higher volatility than the backtesting period. The forecasted variances are generally too high, because the unconditional variance changes slowly over time (especially for an expanding window). Although, it should be noted that all models (except the HAR and MHAR models) model the close-to-close variance while the realised variance measures the open-to-close variance. The beta of the GARCH model is well above 0.8, indicating the persistence of the volatility over time. The forecasted variances change much more over time than for the ARCH model. This can also be seen in the dispersion of the time-series weights.

The parameters of the HAR model are all stable over time. The negative value for the weekly variance is difficult to interpret. The variance forecasts made by the HAR model follow the pattern of the realised variances quite well. Only the level is generally a bit too high. This leads to lower spikes in the amount of leverage. In the beginning of the period both HEAVY parameters hover around 0.6. Near the end of the period alpha becomes a bit higher while beta decreases a bit. The sum of the parameters is larger than one, which is usually problematic for volatility models. But, the κ for the DJIA index is about 0.60 to compensate for this.

Table 6 presents the results for the robustness strategies. The random walk forecast with the volatility index does not perform well. This strategy increases the Sharpe ratio for only one stock and only for four stocks is the alpha positive. Moreover, the DJIA index itself does not benefit by scaling with the volatility index level. The random walk forecast with the realised variance also performs poorly. It does increase the Sharpe ratio for thirteen stocks. However, none of them are significant. There is only one significant improvement in terms of alpha. Scaling by the ex-post realised variance performs extremely well. For 26 stocks is the alpha positive and nine of them are significant. This strategy significantly increases the Sharpe ratio for six stocks.

Tables 7, 9 and 11 contain the results of the various robustness tests. Putting constraints on the amount of leverage does not change the overall performance much compared to the unconstrained strategy. As expected, the alphas are becoming less extreme (both positive and negative). The same holds for scaling with the volatility instead of the variance of the portfolio. When transaction costs are incorporated the performance deteriorates even further. The ARCH model is able to outperform the buy-and-hold portfolio for six assets, the GARCH model for five, the HAR model for seven and the HEAVY model for eight in terms of Sharpe ratios. However, none of them are significant.

Tables 13, 14, 15 and 16 show the optimal ex-post gamma and the corresponding value. A notable result are the extreme optimal gamma's, both in terms of the dispersion and in terms of the absolute level. Even more surprising, for multiple assets is the optimal gamma negative. Hence, implying that when volatility is high the time-series weight should be relatively high. For the DJIA index is the optimal gamma (very) negative for the ARCH, GARCH and HEAVY model. Only the HAR model has a positive optimal gamma.

Figures 11 and 12 show the average return and realised volatility sorted on the realised volatility of the previous period. Similar as to Moreira and Muir (2016) the realised volatility is slightly increasing over the buckets. However, in contrast to Moreira and Muir (2016) the returns are not flat over the buckets. Even more extreme, the return after a period of high volatility is very high.

5.3 Multivariate Results

The results for the multivariate models can be found in table 2.

Table 2: Performance Multivariate Models

	B&H DJIA Index	CCC GARCH	DCC GARCH	MHAR	MHEAVY
Annual Return	18.54	14.50	12.85	16.83	17.66
Unc. Variance	0.55	0.58	0.63	0.53	0.54
Annual Sharpe	1.45	1.12	0.96	1.34	1.40
Kurtosis	0.88	1.25	1.85	0.84	0.79
Annual Alpha		-3.65	-5.55	-0.83	-0.57
Alpha T-Stat		-1.55	-1.85	-0.27	-0.77
Beta		1.02	1.05	0.96	0.99
Timing Ability		0.49	0.49	0.51	0.49
Mean TS		1.03	1.05	1.04	1.00

Note: this table contains the performance measures for the multivariate variance models. The Sharpe ratios and alphas are annualized. 'B&H' denotes buy-and-hold. An asterisk denotes a significant difference from the buy-and-hold performance. The significance of alpha is determined by the t-statistic of the constant of the time-series regression. The significance of the difference for the Sharpe ratios is determined with methodology of Ardia and Boudt (2015).

None of the multivariate models is able to outperform the DJIA index in terms of Sharpe ratio, return or alpha. The backtesting period had a very high return of 18.5% annualized. Resulting in a Sharpe ratio of almost 1.5. All models have similar unconditional variances, betas and mean time-series weights. The CCC GARCH performs better than the DCC GARCH model. But, both perform worse than the univariate ARCH model. The MHAR model performs worse than the HAR model, while the MHEAVY performs better than the HEAVY model. A possible explanation for the 'disappointing' performance of the strategy is the low kurtosis in combination with high returns of the index. During the backtest period there were not enough extreme returns. Hence, the strategy had no change to scale back when needed and scale up when the volatility is low.

Figures 1, 4, 5, 7, 9 and 10 show the cumulative return, parameters estimates, forecasted variances, time-series weights and rolling volatility for the different multivariate models. The parameter estimates are all stable over time. The alpha of the DCC GARCH model is roughly 0.40 and the beta is about 0.05. Hence, the conditional correlation matrix is mostly a combination of the unconditional correlation matrix and the standardized shock. The linear shrinkage intensity is always zero for the DCC GARCH and MHEAVY models. A possible reason why the shrinkage intensity is 'optimised' at zero is the large dispersion in eigenvalues. Tables 4 and 5 show the summary statistics for the eigenvalues. Ledoit and Wolf (2012) state that the linear shrinkage estimator is not optimal in this case. Although, linear shrinkage should still be better than not applying shrinkage at all.

Again, all forecasts are in general too high compared to the realised variance. The forecasts made with the CCC GARCH and DCC GARCH models are similar, except that the DCC GARCH model experiences a couple of large spikes. The MHAR variance forecasts are generally lower than the MHEAVY forecasts. It holds for all models that their time-series weights are only gradually changing over time. Furthermore, they do not spike like the weights for the realised variance. All models have lower cumulative returns than the index from May till September. Furthermore, there is no clear pattern in the rolling volatility. None of the models show a constant level of volatility.

Tables 8, 10 and 12 show the robustness results for the multivariate models. Imposing a leverage constraint or scaling with volatility improves the performance for all models. Incorporating transaction costs makes the performance obviously worse.

Table 17 shows the ex-post optimal gamma. All multivariate models have large negative optimal gammas. Again, this could be due to low volatility in the market during the backtest period. The only positive optimal gamma is the one for the ex-post optimal strategy.

6 Conclusion

This paper backtests the investment strategy of Moreira and Muir (2016) to a new investment universe. The investment universe considered are the constituents of the DJIA index and the DJIA index itself. Moreira and Muir (2016) propose an investment strategy that intends to time the volatility of the portfolio. This paper uses volatility models to forecast the volatility.

The first obtained result of this paper is that the trading strategy does not generally work for individual stocks. Most Sharpe ratios are lower than a 'simple' buy-and-hold strategy. Different models do yield some different results, but there are no extreme differences. Setting constraints on the amount of leverage does not change the performance. The second obtained result is that volatility timing of the DJIA index does not generally work. Again, different models do yield some different results, but the differences are small. The third obtained result is that modelling the covariance of constituents of the DJIA index jointly does not yield any benefit. None of the multivariate models provide evidence that they are able to outperform the buy-and-hold strategy.

To conclude, volatility timing for individual stocks and the DJIA index does seem to be possible in theory. However, in practice this strategy does not work well. This research implies that investors should not try to time volatility for equities.

6.1 Discussion of Research

This report shows that the trading strategy of Moreira and Muir (2016) does not work for individual stocks and for the DJIA index. The obvious question is why this trading strategy does not work? A possible explanation is that this strategy is only beneficial if volatility itself is volatile and that during the period considered in this research the volatility is not so volatile. Hence, other time periods might show more promising results for this strategy.

There are many other possible extensions for further research. First of all, the question raised above: why does this strategy work for factors but not for individual stocks? Also, the methodology can be adjusted in several ways. This research predicts one day ahead, it might be beneficial to make multi-period forecasts (either direct or by iteration). This strategy could also be tested with a monthly horizon instead of a daily one. The high-frequency data could be subsampled instead of using intervals. And, would it then be more suitable to use a rolling or an expanding window? Also, quote data next to trade data can be used. This research could be extended to other indices, other asset classes or by implementing trough derivatives. The current strategy could be enhanced to lower the transaction costs, e.g. do not trade when the signal changes slightly. It could be beneficial to winsorize large absolute returns for better parameter estimation. It could be advantageous to use log returns and hence model the log volatility. Log returns are somewhat better behaved than normal returns. Another possible extension is to create a bottom-up portfolio of individually volatility scaled stocks. Then the overall portfolio could also be scaled. The current strategy forces the market exposure to be 100% on average. If this strategy is combined with a cross-sectional strategy if might be beneficial to loosen this restriction. It could also be beneficial to only have positions when markets are open, so close out at the end of the day. Yet another possible extension is to model returns as well. This could improve the (co)variance forecasts. An example would be the Factor-GARCH model.

References

Ardia, D. and Boudt K. (2015). *Testing Equality of Modified Sharpe Ratios*, Finance Research Letters 13, pp. 97–104.

Bandi, F.M., Russell, J.R., and Zhu, Y. (2008). *Using high-frequency data in dynamic portfolio choice*, Econometric Reviews 27 (1-3), pp. 163–198.

Barndorff-Nielsen O.E. and Shephard, N. (2004). *Econometric Analysis of realised Covariation: High Frequency Based Covariance, Regression, and Correlation in Financial Economics*, Econometrica 72 (3), pp. 885–925.

Black, F. and Perold, A.F. (1992). *Theory of constant proportion portfolio insurance*, Journal of Economic Dynamics and Control 16 (3), pp. 403–426.

Bollerslev, T. (1986). *Generalized autoregressive conditional heteroskedasticity*, Journal of Econometrics 31, pp. 307–327.

Bollerslev, T. (1990). *Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model*, Review of Economics and Statistics 72, pp. 498–505.

Blitz, D. and van Vliet, P. (2007). *The Volatility Effect: Lower Risk Without Lower Return*, Journal of Portfolio Management Fall 2007, pp. 102–113

Chiriac, R. and Voev, V. (2011). *Modelling and forecasting multivariate realised volatility*, Journal of Applied Econometrics 26 (6), pp. 922–947.

Clements, A. and Silvennoinen, A. (2012). *Volatility Timing and Portfolio Selection: How best to forecast volatility*, NCER Working Paper.

Cooper, T. (2010). *Alpha Generation and Risk Smoothing Using Managed Volatility*, Available at SSRN: https://ssrn.com/abstract=1664823

Copeland, M.M. and Copeland, T. (1999). *Market Timing: Style and Size Rotation Using the VIX*, Financial Analysts Journal 55 (2), pp. 73–81.

Corsi, F. (2009). *A Simple Approximate Long-Memory Model of realised Volatility*, Journal of Financial Econometrics 7 (2), pp. 174–196.

De Pooter, M., Martens, M., and Van Dijk, D.J.C. *Predicting the Daily Covariance Matrix for S&P 100 Stocks Using Intraday Data - But Which Frequency to Use?*, Available at SSRN: https://ssrn.com/abstract=829545 or http://dx.doi.org/10.2139/ssrn.829545

Engle, R.F. (1982). *Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation*, Econometrica 50, pp. 987–1008.

Engle, R.F. and Kroner, K.F. (1995). *Multivariate simultaneous generalized ARCH*, Econometric theory 11 (1), pp. 122–150.

Engle, R.F., Ng, V.K. and Rothschild, M. (1990). *Asset pricing with a factor-ARCH covariance structure: empirical estimates for treasury bills*, Journal of Econometrics 45, pp. 213-238.

Engle, R.F. and Sheppard, K. (2001). *Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH*, National Bureau of Economic Research w8554

Engle, R.F., Ledoit, O. and Wolf, M. (2017). *Large Dynamic Covariance Matrices*, Journal of Business and Economic Statistics, DOI: 10.1080/07350015.2017.1345683

Fama, E.F. and French, K.R. (1993). *Common Risk Factors in the Returns of Stocks and Bonds*, Journal of Financial Economics 33, pp. 3–56.

Fleming, J., Kirby, C., and Ostdiek, B. (2001). *The economic value of volatility timing*, The Journal of Finance 56(1), pp. 329-352.

Fleming, J., Kirby, C., and Ostdiek, B. (2003). *The economic value of volatility timing using realised volatility*, Journal of Financial Economics 67, pp. 473-509.

Frazzini, A. and Pedersen L.H. (2014). *Betting against beta*, Journal of Financial Economics 111, pp. 1–25.

Goltz, F., Campani, C.H. (2011). *A Review of Corporate Bond Indices: Construction Principles, Return Heterogeneity, and Fluctuations in Risk Exposures*, Available at EDHEC Risk: $http://docs.edhecrisk.com/mrk/000000/Press/EDHEC_Publication_Corporate_Bond_Indices.pdf$

Hallerbach, W.G. (2012). *A proof of the optimality of volatility weighting over time*, Journal of Investment Strategies, Vol. 1, No. 4, pp. 87-99.

Harvey, C.R., Hoyle, E., Korgaonkar, R., Rattray, S., Sargaison, M. and Van Hemert, O. (2018). *The Impact of Volatility Targeting*, Available at SSRN: https://ssrn.com/abstract=3175538 or http://dx.doi.org/10.2139/ssrn.3175538

Hautsch, N., Kyj, L.M. M and Malec, P. (2013). *Do high-frequency data improve high-dimensional portfolio allocations?*, SFB 649 Discussion Paper No. 2013-014.

Heston, S.L. (1993). *A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options*, The Review of Financial Studies 6 (2), pp. 327-343.

Israel, R., Palhares, D. and Richardson, S.A. (2015). *Common Factors in Corporate Bond and Bond Fund Returns*, Available at SSRN: http://ssrn.com/abstract=2576784

Jegadeesh, N., and Titman, S. (1993). *Returns to buying winners and selling losers: Implications for stock market efficiency*, The Journal of finance 48 (1), pp. 65–91.]

Kidd, D. (2012). *Understanding Fixed-Income Benchmark Risk*, Available at: https://www.cfainstitute.org/learning/products/publications/irpn/Pages/irpn.v2012.n1.13.aspx

Ledoit, W. and Wolf M. (2012). *NONLINEAR SHRINKAGE ESTIMATION OF LARGE-DIMENSIONAL MATRICES*, The Annals of Statistics 40 (2), pp. 1024–1060.

Liu, Q. (2009). *On portfolio optimization: How and when do we benefit from highfrequency data?*, Journal of Applied Econometrics 24 (4), pp. 560–582.

Lo, A.W. (2015). *What is an Index?*, Available at SSRN: https://ssrn.com/abstract=2672755.

Lo, A.W. (2017). *Adaptive Markets: Financial Evolution at the Speed of Thought*, Princeton University Press, 2017.

Markowitz, H. (1952). *Portfolio selection*, The journal of finance 7 (1), pp. 77–91.

Merton, R. (1971). *Optimum Consumption and Portfolio Rule in Continuous Time*, Journal of Economic Theory 3, pp. 373–413.

Merton, R. (1980). *On estimating the expected return on the market: An exploratory investigation*, Journal of Financial Economics 8, pp. 323–361.

Moreira, A. and Muir, T. (2016). *Volatility Managed Portfolios*, National Bureau of Economic Research No. w22208.

Moreira, A. and Muir, T. (2017). *Should Long-Term Investors Time Volatility?*, Available at SSRN: https://ssrn.com/abstract=2879234

Moskowitz, T.J., Ooi, Y.H. and Pedersen L.H. (2012). *Time series momentum*, Journal of Financial Economics 104, pp. 228–250.

Noureldin D., Shephard, N. and Sheppard, K. (2012). *Multivariate highfrequencybased volatility (HEAVY) models*, Journal of Applied Econometrics 27 (6), pp. 907–933.

Pesaran M.H. and Timmermann, A. (1995). *Predictability of stock returns: robustness and economic Significance*, Journal of Finance 50, pp. 1201-1228.

Sharpe, W.F. (1994). *The Sharpe Ratio*, The Journal of Portfolio Management, Vol. 21, No. 1, pp. 49-58.

Shephard N. and Sheppard, K. (2010). *Realising the future: forecasting with highfrequency-based volatility (HEAVY) models*, Journal of Applied Econometrics 25 (2), pp. 197–231.

Welch, I. (2017). *Stock Return Outliers*, Available at SSRN: https://ssrn.com/abstract=3068347

Wurgler, J. (2010). *On the Economic Consequences of Index-Linked Investing*, Available at: http://www.nber.org/papers/w16376

7 Appendix

7.1 Descriptive Statistics of DJIA

Table 3: Descriptive Statistics of DJIA

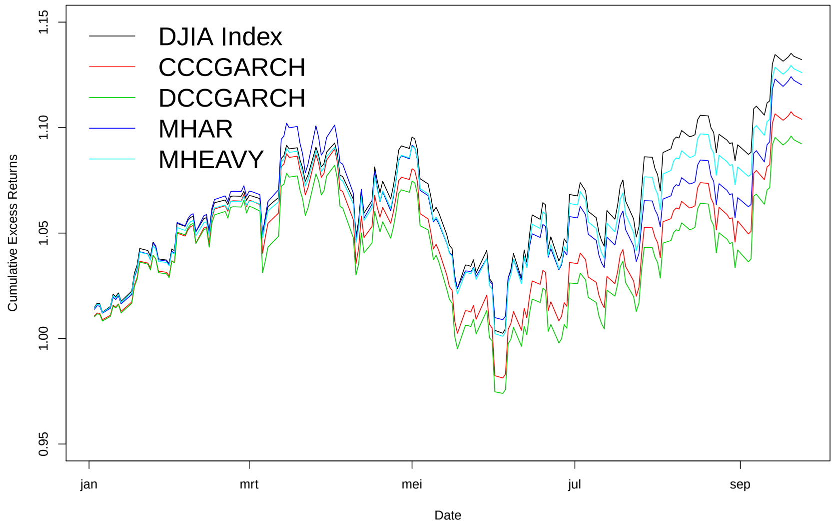
Ticker	Min.	Mean	Max.	Variance	Mean Weight
DJIA	-5.50	0.07	4.24	1.13	
MSFT	-8.26	0.06	5.65	2.05	1.83
KO	-4.08	0.07	3.92	0.96	4.10
DD	-7.15	0.11	6.01	2.88	2.90
XOM	-6.19	0.05	5.22	1.61	5.01
GE	-6.54	0.09	7.11	3.19	1.16
IBM	-4.95	0.09	5.67	1.43	10.31
CVX	-7.54	0.09	5.58	1.97	5.99
UTX	-8.76	0.06	5.15	2.14	4.92
PG	-4.54	0.05	4.24	0.88	4.18
CAT	-9.22	0.14	8.11	4.59	5.25
BA	-7.23	0.06	8.36	3.20	4.38
PFE	-4.75	0.09	5.64	1.78	1.26
JNJ	-3.22	0.04	5.38	0.74	4.24
MMM	-6.25	0.07	7.36	1.94	5.59
MRK	-6.62	0.09	6.42	1.82	2.38
AA	-11.42	0.02	9.46	6.40	0.83
DIS	-9.11	0.11	5.95	2.59	2.42
HPQ	-20.03	-0.07	7.22	3.86	2.57
MCD	-4.64	0.07	4.69	0.97	5.16
JPM	-9.41	0.05	8.44	4.78	2.68
WMT	-4.66	0.06	4.21	0.92	3.73
AXP	-8.83	0.12	11.28	3.92	2.98
INTC	-4.46	0.07	7.80	2.47	1.47
BAC	-20.32	0.01	16.74	8.64	0.82
TRV	-7.59	0.07	7.66	1.96	3.65
VZ	-5.51	0.09	3.78	1.12	2.31
T	-4.25	0.08	4.15	1.04	1.93
HD	-5.89	0.13	5.56	2.13	2.38
CSCO	-16.21	0.01	15.95	3.67	1.38
KFT	-5.87	0.07	5.01	1.11	2.17

Note: this table shows the descriptive statistics of the constituents of the DJIA index and the DJIA index itself between June 8, 2009 and September 24, 2012. The returns and variance are in percentages. The mean return and the mean weight are the time-series averages.

7.2 Additional Results

Cumulative Returns

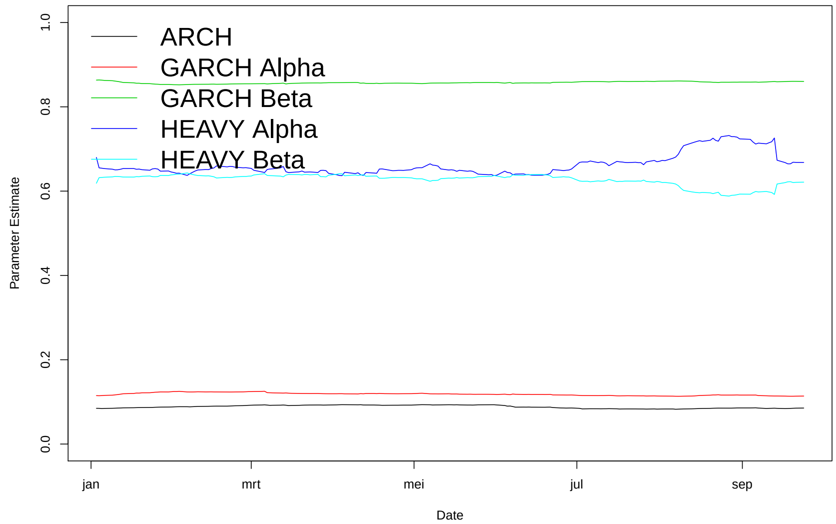
Figure 1: Cumulative Returns for the Multivariate Models



Note: this figure shows the cumulative returns for the different multivariate models.

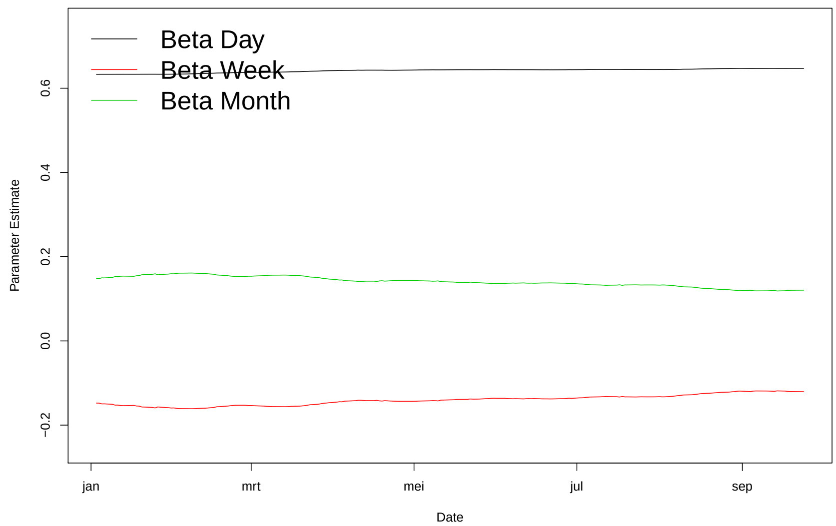
Parameter Estimates

Figure 2: Univariate Maximum Likelihood Parameter Estimates



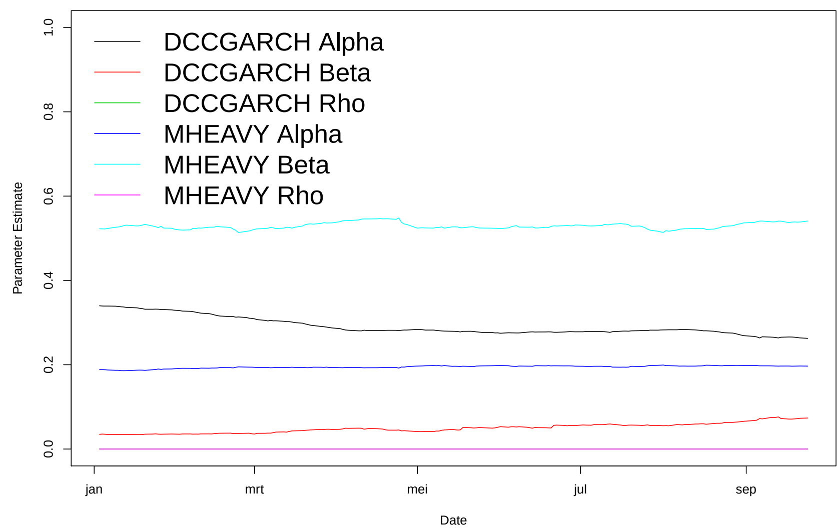
Note: this figure shows the MLE parameters for the different univariate model specifications over time.

Figure 3: Univariate OLS Parameter Estimates



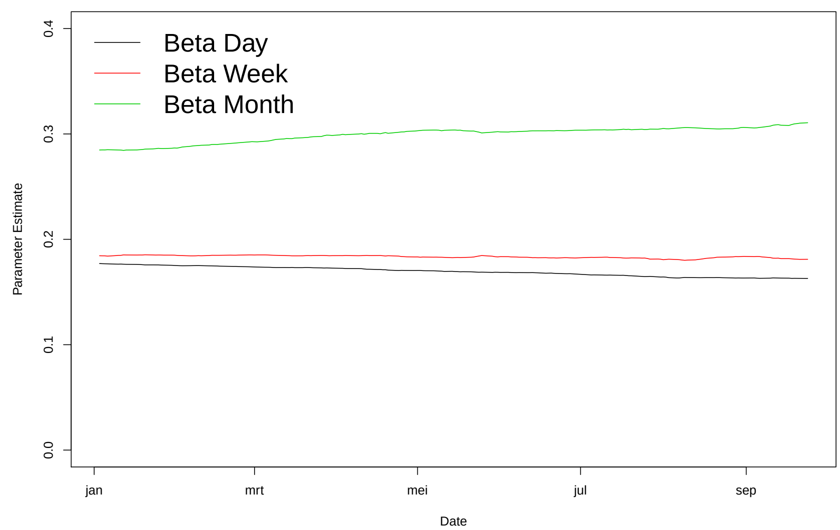
Note: this figure shows the OLS parameters for the HAR model over time.

Figure 4: Multivariate Maximum Likelihood Parameter Estimates



Note: this figure shows the MLE parameters for the different multivariate model specifications over time.

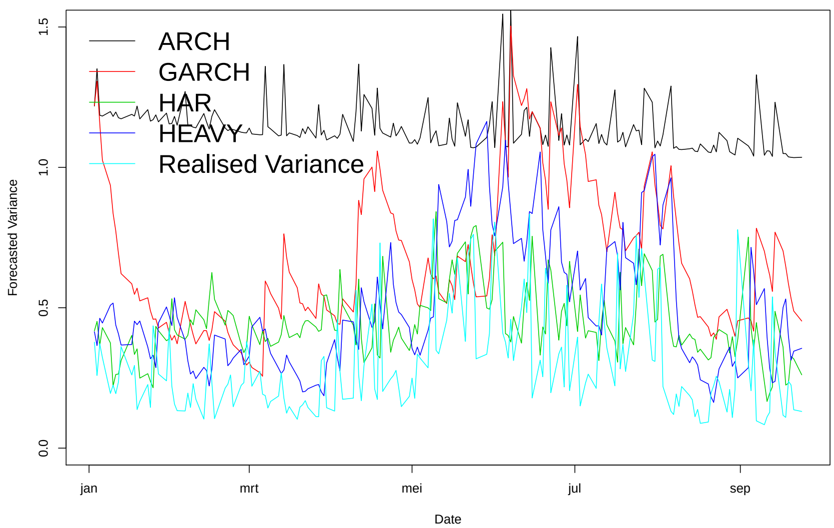
Figure 5: Multivariate OLS Parameter Estimates



Note: this figure shows the OLS parameters for the MHAR model over time.

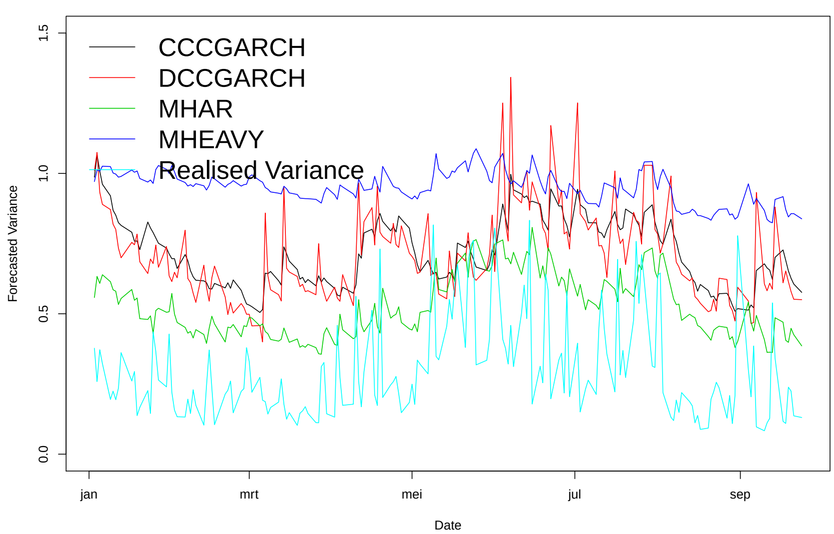
Forecasted Variance

Figure 6: Forecasted Variance of the DJIA for the Univariate Models



Note: this figure shows forecasted variances of the DJIA for the different univariate models.

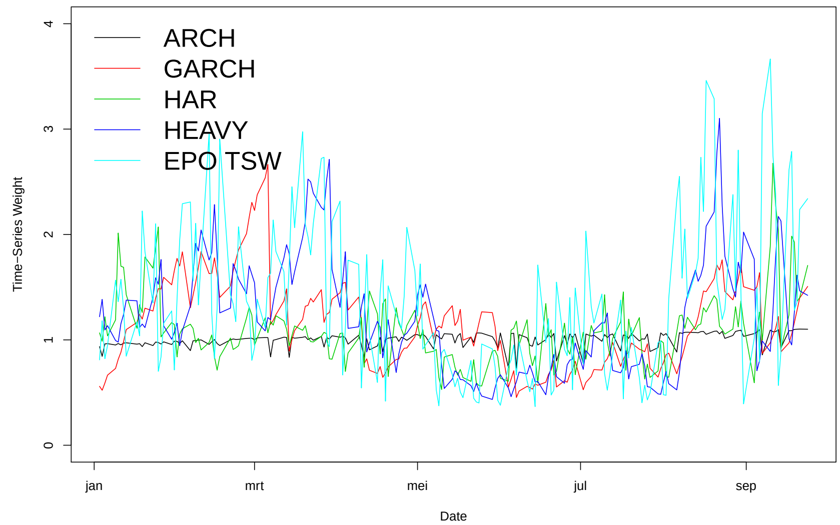
Figure 7: Forecasted Variance of the DJIA for the Multivariate Models



Note: this figure shows forecasted variances of the DJIA for the different multivariate models.

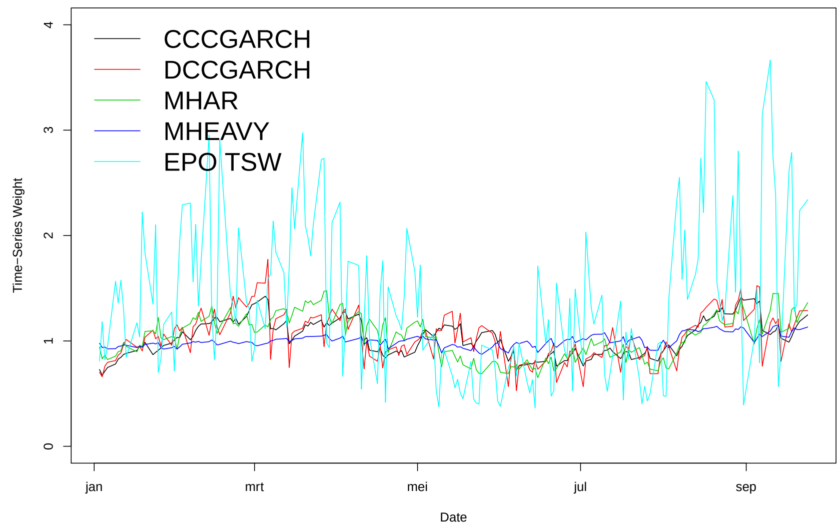
Time-Series Weights

Figure 8: DJIA Index Time-Series Weights for the Univariate Models



Note: this figure shows time-series weights for the DJIA for the different univariate models.

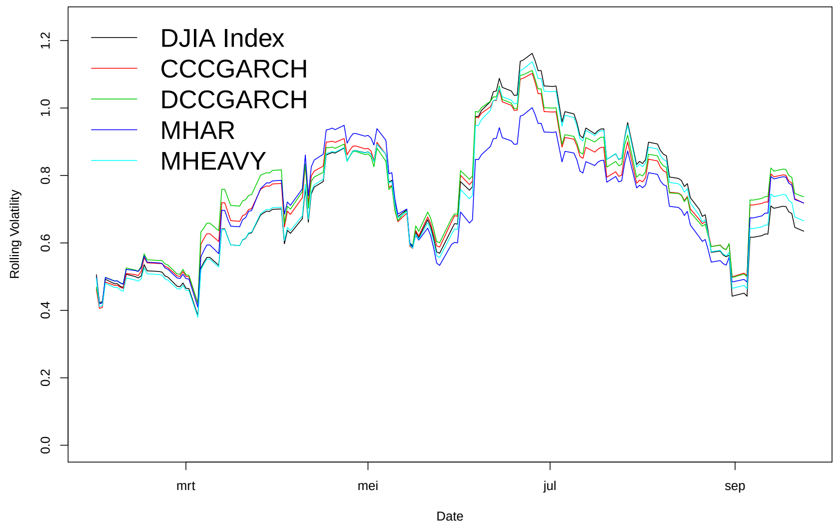
Figure 9: DJIA Index Time-Series Weights for the Multivariate Models



Note: this figure shows time-series weights for the DJIA for the different multivariate models.

Rolling Volatility

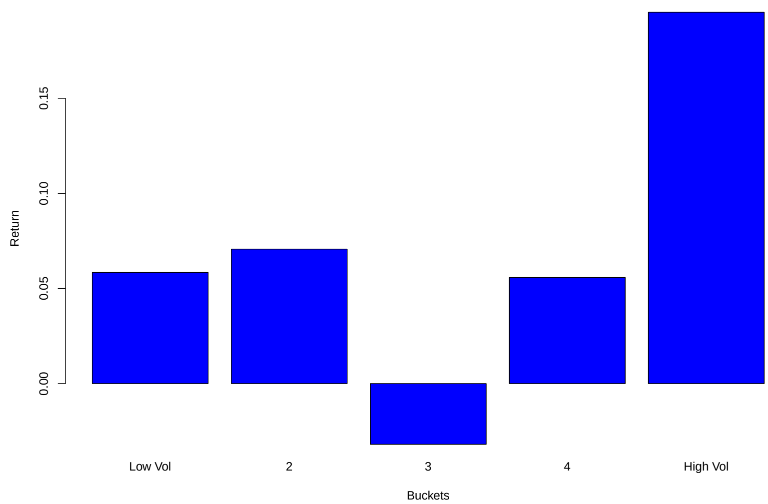
Figure 10: Rolling Volatility for the Multivariate Models



Note: this figure shows the rolling volatility for the DJIA for the different multivariate models with volatility scaling.

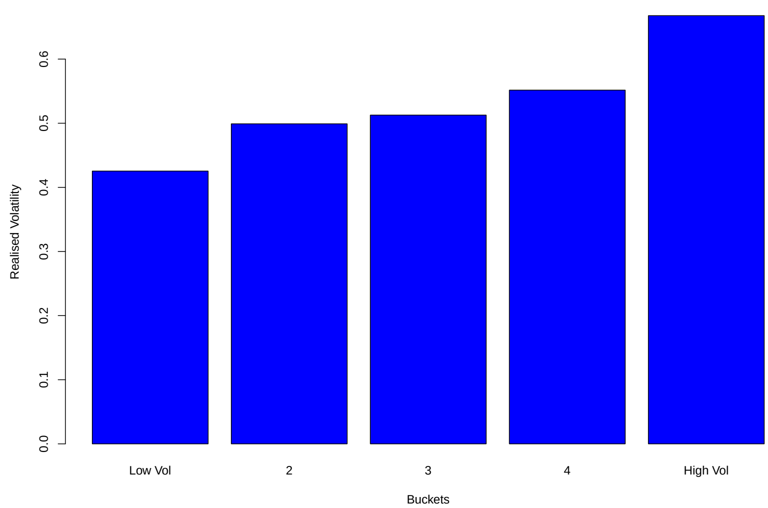
Time-Series Sorts on Realised Volatility

Figure 11: Average Return after Sorting on Previous Day Realised Vol.



Note: this figure shows the returns of the DJIA index after sorting the realised volatility on the previous day into five buckets. Hence, 'Low Vol' indicates that the realised volatility on the previous belongs to lowest 20% of the distribution.

Figure 12: Average Realised Vol. after Sorting on Previous Day Realised Vol.



Note: this figure shows the returns of the DJIA index after sorting the realised volatility on the previous day into five buckets. Hence, 'Low Vol' indicates that the realised volatility on the previous belongs to lowest 20% of the distribution.

Summary Eigen Values for Linear Shrinkage

Table 4: DCC GARCH Model Correlation Matrix Eigen Values Summary

Nr.	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1	13.70	13.81	13.86	13.89	13.94	14.26
2	1.51	1.55	1.56	1.56	1.57	1.60
3	1.03	1.04	1.04	1.04	1.05	1.05
4	0.94	0.95	0.96	0.96	0.97	0.98
5	0.86	0.87	0.88	0.88	0.88	0.89
6	0.84	0.85	0.85	0.85	0.85	0.86
7	0.76	0.77	0.78	0.78	0.78	0.79
8	0.71	0.72	0.73	0.73	0.75	0.76
9	0.63	0.64	0.64	0.64	0.65	0.66
10	0.61	0.62	0.62	0.62	0.63	0.64
11	0.58	0.58	0.59	0.59	0.60	0.61
12	0.55	0.56	0.56	0.56	0.57	0.57
13	0.53	0.55	0.55	0.55	0.55	0.56
14	0.51	0.53	0.53	0.53	0.54	0.55
15	0.50	0.52	0.52	0.52	0.52	0.53
16	0.48	0.49	0.50	0.50	0.50	0.50
17	0.47	0.48	0.48	0.48	0.49	0.49
18	0.45	0.45	0.46	0.46	0.47	0.47
19	0.44	0.44	0.44	0.44	0.44	0.45
20	0.41	0.43	0.43	0.43	0.43	0.43
21	0.40	0.40	0.41	0.41	0.41	0.41
22	0.38	0.39	0.39	0.39	0.40	0.40
23	0.34	0.37	0.37	0.37	0.38	0.38
24	0.34	0.34	0.35	0.35	0.35	0.36
25	0.30	0.31	0.31	0.31	0.32	0.32
26	0.29	0.29	0.30	0.30	0.30	0.30
27	0.25	0.26	0.26	0.26	0.26	0.27
28	0.22	0.23	0.23	0.23	0.23	0.24
29	0.17	0.19	0.20	0.19	0.20	0.21
30	0.16	0.17	0.18	0.17	0.18	0.19

Note: this table contains summary results for the eigen values of the unconditional correlation matrix in the DCC GARCH model.

Table 5: MHEAVY Model Covariance Matrix Eigen Values Summary

Nr.	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1	44.53	46.04	46.68	47.16	48.27	50.69
2	4.78	4.89	4.96	4.94	5.00	5.09
3	2.74	2.81	2.86	2.87	2.93	3.01
4	2.40	2.44	2.45	2.46	2.47	2.54
5	1.99	2.02	2.05	2.05	2.08	2.14
6	1.88	1.90	1.92	1.92	1.94	2.00
7	1.51	1.54	1.57	1.58	1.60	1.68
8	1.42	1.47	1.48	1.49	1.50	1.57
9	1.31	1.36	1.38	1.38	1.41	1.46
10	1.15	1.17	1.18	1.18	1.19	1.24
11	1.11	1.12	1.13	1.15	1.17	1.21
12	1.08	1.10	1.11	1.12	1.13	1.16
13	0.99	1.01	1.01	1.02	1.04	1.06
14	0.94	0.97	0.99	0.99	1.00	1.03
15	0.87	0.88	0.90	0.90	0.91	0.92
16	0.81	0.83	0.85	0.85	0.86	0.88
17	0.75	0.76	0.77	0.77	0.78	0.80
18	0.72	0.73	0.74	0.75	0.76	0.77
19	0.69	0.70	0.71	0.71	0.73	0.75
20	0.60	0.61	0.62	0.62	0.64	0.65
21	0.59	0.60	0.61	0.61	0.62	0.64
22	0.59	0.59	0.60	0.61	0.62	0.63
23	0.54	0.55	0.56	0.56	0.57	0.58
24	0.47	0.48	0.49	0.49	0.50	0.51
25	0.40	0.41	0.42	0.42	0.43	0.43
26	0.40	0.40	0.41	0.41	0.42	0.42
27	0.36	0.37	0.37	0.37	0.37	0.38
28	0.29	0.30	0.30	0.30	0.31	0.32
29	0.26	0.26	0.26	0.26	0.27	0.27
30	0.24	0.24	0.24	0.25	0.25	0.25

Note: this table contains summary results for the eigen values of the unconditional covariance matrix in the MHEAVY model.

7.3 Robustness Results

Performance Robustness Strategies

Table 6: Performance Robustness Strategies

Ticker	Sharpe B&H	RWF VXD S.	RWF VXD A.	EPO S.	EPO A.	RWF RV S.	RWF RV A.
DJIA	1.45	1.26	-1.96	2.72*	22.41*	1.11	-1.78
MSFT	1.24	0.97	-5.22	1.17	1.24	1.33	7.64
KO	1.14	1.06	-0.86	2.23	21.59*	1.43	9.68
DD	1.03	0.87	-2.65	1.04	2.85	0.53	-8.00
XOM	0.91	0.74	-2.34	1.34	10.35	1.10	8.12
GE	1.84	1.64	-3.31	2.45	21.82	0.87	-16.58
IBM	1.06	1.03	-0.26	1.14	4.59	0.73	-3.07
CVX	1.00	0.84	-2.43	2.01*	24.05*	1.50	15.83
UTX	0.74	0.59	-2.79	0.61	-0.46	0.22	-10.59
PG	0.72	0.71	-0.08	0.96	5.38	1.09	8.62
CAT	0.09	-0.12	-5.49	0.87	29.32	0.33	13.79
BA	-0.20	-0.49	-5.45	-0.14	1.20	-0.47	-5.92
PFE	1.76	1.69	-0.67	1.62	1.34	1.46	-1.77
JNJ	1.15	0.97	-1.52	0.89	-1.02	0.62	-4.05
MMM	1.55	1.32	-2.86	1.93	11.13	0.49	-14.56
MRK	1.92	1.72	-2.72	1.56	-0.92	1.86	4.01
AA	0.26	-0.06	-8.86	-0.01	-7.11	0.41	13.40
DIS	2.72	2.45	-4.13	2.55	4.65	1.45*	-20.8
HPQ	-1.70	-1.98	-8.94	-1.60	-3.55	-1.60	-0.41
MCD	-0.45	-0.36	1.15	2.08*	45.09*	0.27	13.37
JPM	1.05	1.04	0.17	1.06	8.53	1.69	43.99
WMT	2.05	1.63*	-6.33*	2.78	18.23	1.81	0.96
AXP	1.40	1.31	-1.40	2.23	30.83	1.81	19.05
INTC	-0.25	-0.67*	-8.49*	1.48*	49.65*	-0.65	-9.57
BAC	1.65	1.44	-7.52	1.42	1.28	0.89	-20.82
TRV	1.42	1.36	-0.77	1.95	14.00	1.97	16.91
VZ	1.73	1.37	-4.59*	2.75	19.4*	1.68	2.37
T	2.88	2.54*	-4.13*	4.17*	32.63*	2.87	6.95
HD	2.72	2.72	0.74	4.36*	56.02*	3.49	30.74*
CSCO	0.25	0.24	-0.20	1.41	39.11*	0.07	-3.37
KFT	1.25	1.25	0.32	1.21	3.67	1.10	0.89
#Outperformance		0	0	6	9	0	1

Note: this table contains the performance measures for the robustness models. The Sharpe ratio (S.) and alpha (A.) are annualized. 'B&H' denotes buy-and-hold, 'RWF' denotes Random Walk Forecast, 'EPO' denotes Ex-Post Optimal and 'RV' denotes realised variance. The last row counts the number of times the model performs significantly better than the index. An asterisk denotes a significant difference from the buy-and-hold performance. The significance of alpha is determined by the t-statistic of the constant of the time-series regression. The significance of the difference for the Sharpe ratios is determined with methodology of Ardia and Boudt (2015).

Performance with Transaction Costs

Table 7: Performance Univariate Models with Transaction Costs

Ticker	Sharpe B&H	ARCH S.	ARCH A.	GARCH S.	GARCH A.	HAR S.	HAR A.	HEAVY S.	HEAVY A.
DJIA	1.44	1.31*	-1.54*	0.59	-9.50	1.44	0.90	0.97	-4.11
MSFT	1.24	1.24	0.46	1.25	0.87	1.06	-2.93	1.12	-1.53
KO	1.13	0.96	-2.12	1.34	3.21	1.01	-0.71	1.24	2.52
DD	1.02	0.89*	-2.41*	0.20*	-14.18*	0.92	-1.55	0.32	-12.97
XOM	0.90	0.85	-0.83	0.27	-9.06	0.71	-2.59	0.79	4.25
GE	1.84	1.79	-0.77	1.34*	-8.1	1.52	-5.24	1.38	-6.92
IBM	1.05	1.07	0.42	0.91	-1.61	0.66	-5.23	0.10	-14.54
CVX	0.99	1.00	0.25	0.76	-2.94	1.08	2.59	0.68	-4.32
UTX	0.73	0.67	-1.37	0.53	-3.76	0.37	-7.05	0.61	-1.30
PG	0.71	0.57	-1.75	0.41	-4.12	0.73	0.40	0.80	1.88
CAT	0.09	0.05	-1.02	-0.48	-14.73*	-0.06	-3.66	0.01	-1.58
BA	-0.21	-0.29	-1.58	-0.82	-12.25*	-0.44	-4.38	-0.85	-12.87
PFE	1.75	1.72	-0.40	1.76	0.45	1.61	-1.57	1.71	1.45
JNJ	1.14	0.73*	-3.51*	0.58	-4.67	0.96	-1.21	0.12*	-9.41*
MMM	1.55	1.38*	-2.29*	0.65*	-12.05*	1.20*	-4.54*	0.36*	-16.38*
MRK	1.91	1.97	1.01	1.96	1.72	1.93	0.60	1.81	-1.20
AA	0.26	0.13	-3.72	-0.10	-10.01	0.11	-3.90	0.29	1.82
DIS	2.71	2.67	-0.37	2.59	-0.90	2.43	-4.17	2.55	-0.85
HPQ	-1.70	-1.52	6.20	-1.19	19.77	-1.85	-5.70	-1.88	-8.26
MCD	-0.46	-0.48	-0.39	-0.53	-1.11	-0.42	0.35	-0.20	3.84
JPM	1.04	0.97	-1.48	0.95	1.02	1.25	10.27	1.38	15.83
WMT	2.04	2.54	11.15	2.23	7.20	1.67	-5.00	1.25*	-11.94*
AXP	1.39	1.22*	-3.38	0.88*	-10.08*	1.56	5.10	1.42	3.33
INTC	-0.26	-0.32	-1.19	-0.37	-2.43	-0.64	-7.71	0.03	7.32
BAC	1.64	1.52	-3.44	1.51	2.58	1.23	-12.55	1.36	-5.64
TRV	1.41	1.39	0.35	0.98	-6.48	1.55	3.46	1.23	-0.79
VZ	1.73	1.79	0.85	1.33	-4.88	1.66	-0.64	1.34	-4.89
T	2.87	2.71	-1.79	2.19*	-7.44	2.75	-0.57	2.32	-5.77
HD	2.72	2.24*	-7.96*	2.48	-1.20	3.13	10.02	2.92	9.21
CSCO	0.25	0.20	-1.44	0.20	-1.43	0.14	-2.80	-0.05	-7.74
KFT	1.24	1.23	0.07	0.60*	-9.04*	1.17	-0.58	1.12	-1.14
#Outperformance		0	0	0	0	0	0	0	0

Note: this table contains the performance measures for the individual variance models with transaction costs. The Sharpe ratio (S.) and alpha (A.) are annualized. 'B&H' denotes buy-and-hold. The last row counts the number of times the model performs significantly better than the index. An asterisk denotes a significant difference from the buy-and-hold performance. The significance of alpha is determined by the t-statistic of the constant of the time-series regression. The significance of the difference for the Sharpe ratios is determined with methodology of Ardia and Boudt (2015).

Table 8: Performance Multivariate Models with Transaction Costs

	B&H DJIA Index	CCC GARCH	DCC GARCH	MHAR	MHEAVY
Annual Return	18.42	13.82	11.03	15.67	17.30
Unc. Variance	0.55	0.58	0.63	0.53	0.54
Annual Sharpe	1.44	1.07	0.83*	1.25	1.37
Kurtosis	0.88	1.25	1.87	0.84	0.79
Annual Alpha		-4.13	-6.97	-1.73	-0.78
Alpha T-Stat		-1.75	-2.32	-0.57	-1.05
Beta		1.02	1.05	0.96	0.99
Timing Ability		0.49	0.49	0.51	0.49
Mean TS		1.03	1.05	1.04	1.00

Note: this table contains the performance measures for the multivariate variance models with transaction costs. The Sharpe ratios and alphas are annualized. 'B&H' denotes buy-and-hold. An asterisk denotes a significant difference from the buy-and-hold performance. The significance of alpha is determined by the t-statistic of the constant of the time-series regression. The significance of the difference for the Sharpe ratios is determined with methodology of Ardia and Boudt (2015).

Performance with Leverage Constraint

Table 9: Performance Univariate Models with Leverage Constraint

Ticker	Sharpe B&H	ARCH S.	ARCH A.	GARCH S.	GARCH A.	HAR S.	HAR A.	HEAVY S.	HEAVY A.
DJIA	1.45	1.37	-0.91	1.17	-3.01	1.45	0.17	1.45	0.23
MSFT	1.24	1.31	1.68	1.28	1.20	1.24	0.34	1.19	-0.74
KO	1.14	1.09	-0.59	1.22	1.18	1.09	-0.40	1.09	-0.47
DD	1.03	0.91*	-2.03*	0.69	-5.83	1.00	-0.25	0.90	-2.12
XOM	0.91	0.89	-0.29	0.56	-4.99	0.80	-1.40	0.49*	-5.86*
GE	1.84	1.82	-0.37	1.54	-5.06	1.60	-4.04	1.74	-1.35
IBM	1.06	1.12	1.04	1.03	-0.27	0.95	-1.50	0.79	-3.91
CVX	1.00	1.02	0.36	0.99	0.10	1.04	1.05	0.70	-4.71
UTX	0.74	0.69	-1.04	0.65	-1.48	0.64	-1.87	0.73	0.21
PG	0.72	0.68	-0.45	0.59	-1.70	0.76	0.52	0.74	0.40
CAT	0.09	0.07	-0.37	-0.10	-5.01	-0.06	-4.02	0.26	5.26
BA	-0.20	-0.26	-1.18	-0.42	-4.19	-0.34	-2.71	-0.28	-1.47
PFE	1.76	1.75	-0.01	1.80	0.67	1.72	-0.34	1.99	3.47
JNJ	1.15	0.96	-1.68	0.76	-3.37	1.17	0.37	0.77*	-3.37
MMM	1.55	1.42	-1.75	1.03*	-7.08*	1.28	-3.53	1.29	-3.30
MRK	1.92	2.02	1.59	1.85	-0.65	1.93	0.38	1.90	0.02
AA	0.26	0.19	-2.11	0.02	-6.84	0.07	-5.37	0.26	0.33
DIS	2.72	2.73	0.49	2.68	-0.12	2.53	-2.79	2.72	0.74
HPQ	-1.70	-1.63	2.52	-1.67	0.69	-1.68	-0.04	-1.77	-2.76
MCD	-0.45	-0.42	0.44	-0.43	0.23	-0.32	1.84	-0.27	2.51
JPM	1.05	0.94	-2.82	1.02	-0.25	1.22	6.06	1.07	1.12
WMT	2.05	2.32	5.42*	2.16	2.45	1.87	-2.49	1.71	-5.16
AXP	1.40	1.26	-2.62	1.04*	-7.07*	1.63	5.62	1.34	-0.74
INTC	-0.25	-0.31	-1.13	-0.40	-3.09	-0.49	-4.94	-0.19	1.35
BAC	1.65	1.54	-3.31	1.48	-5.55	1.60	-0.82	1.53	-3.56
TRV	1.42	1.57	2.94	1.13	-4.42	1.61	3.38	1.43	0.64
VZ	1.73	1.84	1.43	1.54	-2.38	1.71	-0.07	1.52	-2.58
T	2.88	2.82	-0.62	2.64	-2.74	2.94	1.23	2.57	-3.48
HD	2.72	2.56	-2.54	2.65	-0.51	3.11	8.12*	2.77	1.66
CSCO	0.25	0.20	-1.44	0.20	-1.44	0.09	-4.19	0.15	-2.68
KFT	1.25	1.36	1.84	0.93*	-4.49*	1.35	1.70	1.28	0.82
#Outperformance		0	1	0	0	0	1	0	0

Note: this table contains the performance measures for the individual variance models with leverage constraint. The Sharpe ratio (S.) and alpha (A.) are annualized. 'B&H' denotes buy-and-hold. The last row counts the number of times the model performs significantly better than the index. An asterisk denotes a significant difference from the buy-and-hold performance. The significance of alpha is determined by the t-statistic of the constant of the time-series regression. The significance of the difference for the Sharpe ratios is determined with methodology of Ardia and Boudt (2015).

Table 10: Performance Multivariate Models with Leverage Constraint

	B&H DJIA Index	CCC GARCH	DCC GARCH	MHAR	MHEAVY
Annual Return	18.54	15.36	15.59	16.89	17.66
Unc. Variance	0.55	0.56	0.58	0.52	0.54
Annual Sharpe	1.45	1.21	1.20	1.36	1.40
Kurtosis	0.88	1.03	1.13	0.69	0.79
Annual Alpha		-2.69	-2.76	-0.77	-0.57
Alpha T-Stat		-1.42	-1.48	-0.34	-0.77
Beta		1.00	1.02	0.96	0.99
Timing Ability		0.49	0.49	0.51	0.49
Mean TS		1.02	1.03	1.02	1.00

Note: this table contains the performance measures for the multivariate variance models with leverage constraint. The Sharpe ratios and alphas are annualized. 'B&H' denotes buy-and-hold. An asterisk denotes a significant difference from the buy-and-hold performance. The significance of alpha is determined by the t-statistic of the constant of the time-series regression. The significance of the difference for the Sharpe ratios is determined with methodology of Ardia and Boudt (2015).

Performance with Volatility

Table 11: Performance Univariate Models with Volatility Scaling

Ticker	Sharpe B&H	ARCH S.	ARCH A.	GARCH S.	GARCH A.	HAR S.	HAR A.	HEAVY S.	HEAVY A.
DJIA	1.45	1.41	-0.48	1.10	-3.74	1.55	1.35	1.35	-0.69
MSFT	1.24	1.28	0.88	1.27	0.77	1.20	-0.68	1.26	0.59
KO	1.14	1.10	-0.54	1.26	1.63	1.20	1.07	1.25	1.67
DD	1.03	0.97*	-1.02*	0.67*	-6.30*	1.06	0.69	0.72	-5.20
XOM	0.91	0.90	-0.19	0.62	-4.07	0.86	-0.71	0.84	-0.38
GE	1.84	1.83	-0.19	1.62	-3.79	1.73	-1.94	1.68	-2.57
IBM	1.06	1.10	0.71	1.03	-0.29	0.98	-1.08	0.77	-4.14
CVX	1.00	1.01	0.18	0.92	-0.98	1.13	2.61	0.86	-2.01
UTX	0.74	0.71	-0.52	0.67	-1.29	0.60	-2.72	0.69	-0.53
PG	0.72	0.70	-0.17	0.61	-1.44	0.80	1.04	0.83	1.59
CAT	0.09	0.08	-0.23	-0.15	-6.37	0.05	-1.04	0.09	0.16
BA	-0.20	-0.22	-0.36	-0.47	-5.17	-0.27	-1.43	-0.44	-4.52
PFE	1.76	1.76	-0.01	1.79	0.49	1.72	-0.39	1.88	2.09
JNJ	1.15	0.98	-1.44	0.87	-2.34	1.12	-0.17	0.74*	-3.64
MMM	1.55	1.49	-0.91	1.12*	-5.74*	1.41	-1.83	1.08	-6.05
MRK	1.92	1.98	0.90	1.99	1.25	1.97	0.86	1.92	0.12
AA	0.26	0.20	-1.68	0.08	-5.15	0.22	-1.03	0.28	0.98
DIS	2.72	2.75	0.58	2.69	-0.14	2.63	-1.39	2.72	0.67
HPQ	-1.70	-1.53	5.16	-1.44	8.35	-1.77	-2.61	-1.79	-3.58
MCD	-0.45	-0.42	0.38	-0.47	-0.34	-0.36	1.21	-0.22	3.16
JPM	1.05	1.07	0.99	1.05	1.10	1.19	5.24	1.24	6.95
WMT	2.05	2.48	8.00*	2.29	5.20	1.98	-0.75	1.73	-4.85
AXP	1.40	1.33	-1.34	1.16*	-4.63	1.57	4.00	1.49	2.73
INTC	-0.25	-0.28	-0.57	-0.30	-1.08	-0.41	-3.18	-0.02	5.10
BAC	1.65	1.63	-0.29	1.61	-0.04	1.53	-3.70	1.52	-3.39
TRV	1.42	1.46	0.81	1.22	-3.05	1.55	2.45	1.39	0.23
VZ	1.73	1.79	0.74	1.59	-1.79	1.77	0.53	1.60	-1.61
T	2.88	2.83	-0.62	2.56	-3.59	2.90	0.55	2.65	-2.53
HD	2.72	2.52	-3.48	2.66	-0.32	3.02	5.99*	2.97	5.89
CSCO	0.25	0.23	-0.67	0.23	-0.67	0.23	-0.70	0.14	-2.95
KFT	1.25	1.28	0.51	0.95*	-4.14*	1.28	0.50	1.26	0.28
#Outperformance		0	1	0	0	0	1	0	0

Note: this table contains the performance measures for the individual variance models with volatility scaling. The Sharpe ratio (S.) and alpha (A.) are annualized. 'B&H' denotes buy-and-hold. The last row counts the number of times the model performs significantly better than the index. An asterisk denotes a significant difference from the buy-and-hold performance. The significance of alpha is determined by the t-statistic of the constant of the time-series regression. The significance of the difference for the Sharpe ratios is determined with methodology of Ardia and Boudt (2015).

Table 12: Performance Multivariate Models with Volatility Scaling

	B&H DJIA Index	CCC GARCH	DCC GARCH	MHAR	MHEAVY
Annual Return	18.54	16.43	15.59	17.45	18.08
Unc. Variance	0.55	0.55	0.57	0.52	0.54
Annual Sharpe	1.45	1.29	1.21	1.40	1.42
Kurtosis	0.88	0.96	1.14	0.61	0.82
Annual Alpha		-1.79	-2.69	-0.43	-0.29
Alpha T-Stat		-1.55	-1.87	-0.28	-0.79
Beta		1.00	1.01	0.97	0.99
Timing Ability		0.50	0.48	0.50	0.48
Mean TS		1.01	1.01	1.01	1.00

Note: this table contains the performance measures for the multivariate variance models with volatility scaling. The Sharpe ratios and alphas are annualized. 'B&H' denotes buy-and-hold. An asterisk denotes a significant difference from the buy-and-hold performance. The significance of alpha is determined by the t-statistic of the constant of the time-series regression. The significance of the difference for the Sharpe ratios is determined with methodology of Ardia and Boudt (2015).

Performance with Optimal Gamma

Table 13: ARCH Model Optimal Gamma

Ticker	Opt. Gamma for Alpha	Opt. Alpha	Opt. Gamma for Sharpe	Opt. Sharpe
DJIA	-2.50	1.69	-2.50	3.00
MSFT	2.50	1.40	2.50	5.79
KO	-2.50	1.39	-2.50	5.98
DD	-2.50	1.32	-2.50	5.63
XOM	-2.50	0.97	-2.50	1.13
GE	-2.50	1.87	-2.50	1.20
IBM	2.50	1.18	2.50	2.78
CVX	2.50	1.05	2.50	0.96
UTX	-2.50	0.87	-2.50	2.79
PG	-0.27	0.73	-2.50	3.84
CAT	-2.50	0.16	-2.50	2.16
BA	-1.27	-0.18	-1.25	0.44
PFE	-0.41	1.76	-2.50	0.11
JNJ	-2.50	1.94	-2.50	16.95
MMM	-2.50	1.91	-2.50	5.59
MRK	2.50	2.15	2.50	4.78
AA	-2.50	0.69	-2.50	16.28
DIS	1.01	2.75	2.50	2.20
HPQ	1.92	-1.31	2.50	72.86
MCD	2.50	-0.35	2.50	1.58
JPM	0.37	1.07	0.50	0.99
WMT	1.95	2.79	2.50	63.88
AXP	-2.50	1.71	-2.50	7.66
INTC	-2.50	-0.10	-2.50	3.06
BAC	0.06	1.65	0.09	0.02
TRV	2.50	1.66	2.50	9.17
VZ	2.50	1.96	2.50	3.48
T	-1.53	2.98	-2.50	7.60
HD	-2.29	3.60	-2.50	80.34
CSCO	-2.50	0.35	-2.50	2.68
KFT	2.50	1.42	2.50	5.88

Note: this table contains the optimal ex-post gamma for the ARCH model and the corresponding optimal value.

Table 14: GARCH Model Optimal Gamma

Ticker	Opt. Gamma for Alpha	Opt. Alpha	Opt. Gamma for Sharpe	Opt. Sharpe
DJIA	-2.20	2.09	-2.50	54.71
MSFT	1.04	1.28	2.50	3.32
KO	2.50	1.76	2.50	16.33
DD	-2.50	1.85	-2.50	44.09
XOM	-2.07	1.50	-2.50	33.03
GE	-2.50	2.49	-2.50	33.42
IBM	-0.22	1.06	-0.72	0.26
CVX	-0.69	1.03	-2.50	15.63
UTX	-1.30	0.82	-2.50	7.20
PG	-0.99	0.90	-2.50	25.72
CAT	-2.50	0.59	-2.50	33.14
BA	-2.33	0.35	-2.50	24.59
PFE	2.50	1.87	2.50	3.85
JNJ	-2.34	2.31	-2.50	115.85
MMM	-1.93	2.59	-2.50	53.39
MRK	2.40	2.05	2.50	10.54
AA	-2.50	1.13	-2.50	71.70
DIS	-0.33	2.72	2.50	1.82
HPQ	2.50	-0.35	2.50	243.62
MCD	-2.50	-0.34	-2.50	1.63
JPM	0.25	1.06	2.50	20.59
WMT	1.00	2.34	2.50	65.23
AXP	-2.50	2.17	-2.50	32.65
INTC	-2.50	0.05	-2.50	12.99
BAC	-0.54	1.66	2.50	156.53
TRV	-2.25	2.02	-2.50	35.74
VZ	-1.46	1.92	-2.50	6.12
T	-1.16	3.30	-2.50	106.88
HD	-0.30	2.73	2.50	9.20
CSCO	-2.50	0.35	-2.50	2.68
KFT	-1.31	1.78	-2.50	42.46

Note: this table contains the optimal ex-post gamma for the GARCH model and the corresponding optimal value.

Table 15: HAR Model Optimal Gamma

Ticker	Opt. Gamma for Alpha	Opt. Alpha	Opt. Gamma for Sharpe	Opt. Sharpe
DJIA	2.50	1.90	2.50	24.64
MSFT	-2.49	1.45	2.50	1.31
KO	2.15	1.37	2.50	38.05
DD	0.40	1.06	0.47	0.69
XOM	-1.80	1.02	-2.50	7.76
GE	-2.33	2.17	-2.50	19.02
IBM	-0.42	1.09	-0.95	1.40
CVX	2.05	1.30	2.50	20.73
UTX	-1.29	1.27	-2.50	3442.94
PG	2.01	0.87	2.50	3.49
CAT	2.50	0.12	2.50	13.33
BA	-2.50	0.01	-2.50	9.15
PFE	-2.38	1.99	2.50	1.04
JNJ	2.50	1.22	2.50	5.11
MMM	-2.06	1.88	-2.50	9.55
MRK	2.43	2.04	2.50	3.98
AA	-1.37	0.33	-2.50	5.40
DIS	-1.32	2.84	-2.50	12.61
HPQ	-2.50	-0.43	-2.50	109.40
MCD	2.50	0.12	2.50	12.92
JPM	2.47	1.64	2.50	114.85
WMT	-0.07	2.05	-0.17	0.06
AXP	1.77	1.78	2.50	71.98
INTC	-2.50	0.32	-2.50	22.52
BAC	-0.22	1.66	-0.81	2.13
TRV	2.50	2.14	2.50	35.99
VZ	0.63	1.77	0.89	0.64
T	0.52	2.90	2.50	2.92
HD	2.25	3.48	2.50	48.09
CSCO	-0.56	0.26	-2.50	9.53
KFT	1.12	1.29	2.50	1.88

Note: this table contains the optimal ex-post gamma for the HAR model and the corresponding optimal value.

Table 16: HEAVY Model Optimal Gamma

Ticker	Opt. Gamma for Alpha	Opt. Alpha	Opt. Gamma for Sharpe	Opt. Sharpe
DJIA	-0.26	1.46	-2.50	18.68
MSFT	0.42	1.26	1.00	0.77
KO	2.50	1.75	2.50	39.34
DD	-1.42	2.01	-2.50	18809.83
XOM	1.32	1.01	2.50	19525.91
GE	-1.25	2.02	-2.50	31.96
IBM	-0.48	1.15	-0.73	2.52
CVX	-2.33	1.41	-2.50	40.19
UTX	1.53	0.71	2.50	47.99
PG	2.50	1.44	2.50	39.21
CAT	0.26	0.09	0.50	0.16
BA	-2.13	0.35	-2.50	81.12
PFE	0.59	1.89	1.01	2.93
JNJ	-1.65	2.35	-2.50	210.13
MMM	-1.20	2.07	-2.50	210.67
MRK	0.22	1.92	0.44	0.12
AA	2.50	0.46	2.50	18.49
DIS	0.27	2.73	1.07	0.95
HPQ	2.27	-1.67	-2.50	2120.31
MCD	2.50	0.40	2.50	43.29
JPM	2.41	1.68	2.50	152.39
WMT	-1.55	2.58	-2.50	43.95
AXP	1.35	1.57	2.50	191.56
INTC	2.50	0.52	2.50	69.46
BAC	-1.07	1.81	2.50	45.02
TRV	-0.37	1.42	2.50	22.47
VZ	-0.86	1.83	-1.38	2.20
T	-1.28	3.17	-2.50	38.56
HD	1.48	3.13	2.50	105.28
CSCO	-2.50	0.64	-2.50	47.52
KFT	0.34	1.26	0.57	0.28

Note: this table contains the optimal ex-post gamma for the HEAVY model and the corresponding optimal value.

Table 17: Optimal Gamma for Multivariate Models

	Opt. Gamma for Alpha	Opt. Alpha	Opt. Gamma for Sharpe	Opt. Sharpe
Random Walk	-0.34	1.48	-2.50	47.08
VXD	-2.50	1.75	-2.50	7.19
Ex-Post Optimal	1.76	3.02	2.50	307.87
CCC GARCH	-2.50	1.98	-2.50	10.61
DCC GARCH	-2.50	2.29	-2.50	19.63
MHAR	-0.95	1.48	-2.50	3.56
MHEAVY	-2.50	1.57	-2.50	1.66

Note: this table contains the optimal ex-post gamma for the multivariate models and the corresponding optimal value.

7.4 Replicating Results of Moreira and Muir Study

Table 18: Results of Replicating Moreira and Muir Individual Factors

	Alpha	Alpha SE	Beta	Beta SE
MKT	0.20	0.12	0.60	0.03
MKT Annualized	2.47			
MKT Results Moreira and Muir	4.86	1.56	0.61	0.05
SMB	0.01	0.09	0.76	0.03
SMB Annualized	0.13			
SMB Results Moreira and Muir	-0.58	0.91	0.62	0.08
HML	0.09	0.07	0.59	0.03
HML Annualized	1.10			
HML Results Moreira and Muir	1.97	1.02	0.57	0.07
RMW	0.16	0.05	0.44	0.02
RMW Annualized	2.00			
RMW Results Moreira and Muir	2.44	0.83	0.62	0.08
CMA	0.02	0.05	0.57	0.02
CMA Annualized	0.29			
CMA Results Moreira and Muir	0.38	0.67	0.68	0.05
MOM	0.65	0.10	0.37	0.02
MOM Annualized	8.08			
MOM Results Moreira and Muir	12.51	1.71	0.47	0.07

Note: this table contains the performances measures for the replicating study of the results of Moreira and Muir (2016). 'SE' denotes standard error. 'MKT' denotes the market return. 'SMB' the size factor. 'HML' the value factor. 'RMW' the profitability factor. 'CMA' the investment factor. 'MOM' denotes the momentum factor. 'Annualized' denotes the annualized alpha. 'Results Moreira and Muir' are the results presented in Moreira and Muir (2016).

Table 19: Results of Replicating Moreira and Muir Multiple Factors

	Sharpe	Sharpe Annualized	Sharpe M&M	Alpha	Alpha Annualized	Alpha M&M
FF3 Managed	0.15	0.53	0.69	0.10	1.21	4.99
FF3 MVE	0.17	0.61	0.52			
FF3M Managed	0.34	1.17	1.09	0.30	3.60	4.04
FF3M MVE	0.27	0.93	0.98			
FF5 Managed	0.32	1.10	1.20	0.16	1.90	1.34
FF5 MVE	0.28	0.96	1.19			
FF5M Managed	0.39	1.36	1.42	0.22	2.69	2.01
FF5M MVE	0.31	1.08	1.34			

Note: this table contains the performance measures for the replicating study of the results of Moreira and Muir (2016). 'M&M' denotes Moreira and Muir. 'Managed' denotes volatility timed. 'MVE' denotes mean-variance efficient. 'FF3' denotes the Fama-French three factor model. 'FF3M' denotes the Fama-French three factor model including momentum. 'FF5' denotes the Fama-Fench five factor model. 'FF5M' denotes the Fama-French five factor model including momentum.