Should Investors Reassess the Traditional View of the Risk-Return Trade-off?

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Abstract

In this thesis I investigate whether the volatility managed portfolio strategy implemented on German and European single factors and multifactor portfolios leads to attractive risk-adjusted returns. Empirical evidence suggests that certain German and European managed factors present large and significant alphas and high Sharpe ratios, whereas all German and European multifactor portfolios outperform their peers. Moreover, when using the GARCH (1,1) approach instead of the realized variance approach, the volatility managed portfolio strategy offers superior and higher risk-adjusted returns only for the German and European momentum factor.

Key words: mean-variance trade-off, volatility persistence, realized variance, market, size, value and momentum risk factors, GARCH.

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1 Introduction

The ability to manage risk and make sound forward-looking financial decisions has always been an essential component for boosting financial performance in an economy. As Bernstein suggests (1998, p. 4) “today, we rely less on superstition and tradition than people did in the past, not because we are more rational, but because our understanding of risk enables us to make decisions in a rational way”. In the same vein, since Markowitz (1952) mathematically demonstrated a direct link between mean and variance of stock returns, much of the financial literature has extensively investigated the relationship between risk and expected return.

Models such the Capital Asset Pricing Model (CAPM), developed by Sharpe (1964), assume a positive linear relationship between systematic risk and expected return. A traditional investment is often considered as one, where the expected return is either an exact compensation for the risk an investor is bearing or where expected return exceeds what the markets would consider to be a fair compensation. Various asset pricing models have been used to approximate the trade-off between risk and return. Nevertheless, empirical findings do not seem to be unanimous of whether the relation of risk and return is both either positive or negative, and linear or nonlinear. This ambiguity may exist because volatility fluctuates over time.

Data from several studies suggest that these contradictions might be the result of traditional assumptions such as (i) variations in expected returns can only be explained by variations in volatility and (ii) expected returns are high even when business conditions are persistently risky. In fact, recent time-series studies on the mean-variance trade-off show that the relation between risk and return tends to be uncertain especially during extreme periods of volatility. For example, in response to an unexpected increase in stock market volatility, Muir (2016) suggests that “in volatile markets, there is a lot of additional risk that investors are exposed to, and if they are not being adequately compensated for that risk, then the right thing to do is to exit the market” (see, e.g., Rosenberg, 2016). This argument is largely explained by Moreira and Muir (2016a). The authors show that the mean-variance trade-off is weak, especially in volatile times.

To demonstrate this, they find that past volatility does poorly predict current expected returns, whereas past volatility predicts current volatility in turbulent times. Furthermore, the authors find that movements of volatility are persistent, but to a less extent than movements in expected returns. By combining their findings, the authors suggest that after a rise in volatility on stock markets the better alternative is to time volatility. In other words, they recommend to reduce risk market exposure when volatility is high and to take position again when markets start to recover. To do so the authors time risk i.e. manage volatility by using their volatility managing portfolio strategy (VMP hereafter).

The VMP strategy is computed by scaling monthly single factors and multifactor portfolios by the inverse of its respective realized variance of the previous month. Once the factors and multifactor portfolios are scaled they are called managed factors and managed multifactor portfolios. The realized variance is the key variable of the strategy and is calculated with daily data. If the realized variance in the previous month i.e. the one-month-lagged realized variance is higher than expected, then the risk exposure to the market in the following month should be reduced.

The study by Moreira and Muir (2016a) shows that managing the volatility of a variety of U.S risk factors and U.S. multifactor portfolios produce positive and significant alphas, large Sharpe ratios and utility gains from a mean-variance investor perspective. In other words, this simple strategy offers to the average investor an attractive opportunity to earn superior risk-adjusted returns in real time. This pattern is robust when controlling for other risk factors, after transaction costs and across different country indices.

The main contribution of my research to the existing literature is the investigation whether the VMP strategy outperforms when using international data. More specifically, I manage the German version of the three risk factors of Fama and French (1992) namely market, size and value and the momentum risk factor of Carhart (1997). The sample period investigated is from January 1990 to June 2016. Moreover, I use different combinations of risk factors to create multifactor portfolios. I manage four monthly German multifactor portfolios during the same sample period used in the factor analysis. Considering specific particularities of the German stock market the empirical results are interpreted in single and multifactor settings.

The evidence presented in this thesis supports the hypothesis that managing German risk factors, the managed momentum factor is the only one that presents optimal results. In other words, managed momentum shows a large and statistically significant alpha as well as a higher Sharpe ratio compared to the German non-managed momentum factor. This is also true when controlling for momentum using other German factors and two different subsample periods. Applying the VMP strategy on multifactor portfolios, I find that the alphas for all German managed multifactor portfolios are positive and significant. The managed Sharpe ratios are moderately higher than those for the non-managed Sharpe ratios.

I also evaluate the benefits of the VMP strategy at a regional level. Previous studies which explored both country- and regional-specific risk factors show that country-specific factors outperform regional-specific factors. The reason is that data used to construct such factor returns should have a high degree of capital market integration, which is usually the case only in domestic capital markets. In my research, however, I analyze a highly integrated regional capital market; the Euro area. I manage the European Fama-French three risk factors; market, size and value, as well as the European Carhart's momentum factor.

The analysis on both European single factors and European multifactor portfolios covers the period from July 1990 to December 2016. I provide evidence that all factors except the European managed market factor present positive and significant alphas and higher managed Sharpe ratios than the European non-managed factors. For the case of European multifactor portfolios, all managed multifactor portfolios outperform the non-managed ones.

The German and the European analysis suggest that the market factor does not present statistical significance when applying the VMP strategy. However, the German and European managed momentum factor generally presents superior risk-adjusted returns. The overall findings are somewhat at odds with the existing literature, which suggests that country-specific factors perform better than regional-specific ones. Furthermore, I find that the European time-series analysis on single factors generally presents higher and statistically significant alphas than the German single factor analysis.

To my best knowledge, this is the first master thesis that replicates the main analysis of Moreira and Muir (2016a) by using all U.S. risk factors as in the original paper. One factor, however is excluded; the carry factor. The carry factor is not used in the analysis because its returns belong to other different asset class than equities. The objective is to replicate the VMP strategy on factors and multifactor portfolios and compare my findings to those of the authors. Moreover, the study of Moreira and Muir (2016a) is a country-specific analysis; therefore, it will serve as benchmark to make respective comparisons with the German analysis of this thesis.

For the U.S study, I consider a similar sample period as in Moreira and Muir's (2016a) research. It starts on January 1927 and ends in December 2016. The U.S. non-managed risk factors are: market, size, value, momentum, investment[1], profitability, return on equity and investment[2]. For the U.S. multifactor portfolios, I manage the Fama and French (1993) three- and five- (2015) factor models, the Carhart (1997) four-factor model, and the four-factor model of Hou, Xue and Zhang (2014).

I find that the alphas earned by the single and multifactor analysis are positive except for the alpha of the managed size factor. Moreover, the largest alpha value belongs again to the managed momentum factor in the single regression analysis and to the Fama and French three-factor model for the multifactor analysis. In most cases, the alphas are also statistically significant and managed Sharpe ratios are substantial.

In addition, I examine whether another measure for the conditional variance could be a better proxy than the one-month-lagged realized variance. I forecaste variance by modelling the simplest form of the Generalized Auto Regressive Conditional Heteroscedasticity model: the GARCH (1,1). I find that managing German and European single factors with the GARCH (1,1) approach leads to larger and statistically significant alphas and higher Sharpe ratios than managing factors compared to the realized variance approach, though only for the German and European managed momentum factor. Consequently, implementing a forecasted variance, such as one-month-lagged GARCH (1,1), enhances the performance of the VMP strategy when managing momentum.

[1] This factor is the difference between the returns of conservative and aggressive investment stocks (CMA).
[2] This factor is the difference between returns of low and high investment stocks (IA).

The remainder of this thesis is structured as follows: The following section 2 shows the literature review related to the mean-variance criterium suggesting the existence of a non-dynamic and intertemporal relationship between risk and return. Next, I explain some volatility timing strategies and the key intuition behind the volatility managed portfolio strategy. Then, I present the underlying literature which serves to develop my three hypotheses. Section 3 describes the data collected and the empirical description of the variables used in the regressions. Section 4 shows the methodology, strategy construction and performance measures. Section 5 presents the regression results based on the realized variance approach of single factors and multifactor portfolios. Moreover, section 5 shows the output of single factor regressions when implementing the GARCH (1,1) approach. Finally, section 6 summarizes, draws conclusions and shows possible limitations of my research.

2 Literature review

The Modern Portfolio Theory of Markowitz (1952) states that there exists a direct relationship between mean and variance of stock returns. In the Sharpe (1967) and Lintner´s (1965) static approach, the Capital Asset Pricing Model (CAPM) goes beyond by assuming that the mean-variance criterion is characterized by a linear relation between systematic risk and expected returns. Since then, asset pricing literature, such as Frama and French (1989) and Harvey (1989) investigate the intertemporal interaction between mean and variance. While Campbell (1987), Nelson (1991) and Glosten, Jagannathan and Runkle (1993) find evidence of volatility being negatively related to expected returns, French, Schwert and Stambaugh (1987) and Campbell and Hentschel (1992) find a positive relationship. These mixed empirical results suggest a great ambiguity to justify a traditional mean-variance approach across time.

2.1 Volatility timing

Fleming, Kirby and Ostdiek (2001) demonstrate that implementing volatility timing strategies across asset allocations can generate substantial economic benefits for mean-variance investors.

As Busse (1998) shows, mutual fund managers can efficiently time market volatility. He also finds a significant increase in risk-adjusted returns when investors decrease their market risk exposure during periods of high expected volatility.

In the context of risk factor investing, Barroso and Santa-Clara (2015) suggest that momentum´s volatility is highly variable but predictable over time. Hence, the dynamic of momentum´s volatility can be managed, eliminating potential exposure to crashes. Moreira and Muir (2016a) use a similar methodology as Barroso and Santa-Clara (2015) but they address the asset allocation problem evaluating several systematic factors and multifactor models, which gives a better information about the evolution of the aggregate mean-variance trade-off in the economy.

2.1.1 Volatility managed portfolio strategy

Moreira and Muir (2016a) base their study on the intertemporal dynamic of the trade-off between risk and return. The authors show that while past volatility captures enough information to predict future volatility, it cannot explain expected returns, especially during periods of high market volatility. Hence the traditional mean-variance trade-off might be a weak reference for portfolio allocation. Furthermore, the authors suggest that fluctuations in volatility and expected returns are counter-cyclical. They observe that in response to a volatility shock, volatility initially spikes but expected returns do not follow the same increasing pattern. Subsequently, while volatility decreases returning to its mean[3], expected return stay elevated for a prolonged period. This means that volatility is indeed persistent[4] though to a lower extent than expected returns.

By combining these findings, that (i) volatility poorly explains expected returns, (ii) past volatility predicts future volatility and (iii) volatility is less persistent[5] than expected returns, the authors suggest that when stock markets are volatile the better alternative for an average investor is to time volatility. By doing so they apply their VMP strategy. For example, they recommend that suggest that when stock markets are volatile the better alternative for an average investor is to time

[3] Engle and Patton (2001) and Hillebrand (2003) describe in detail the mean reversion in volatility.
[4] Volatility persistence is a well-established phenomenon of volatility, see, e.g., Fama (1965), Chou (1988), Schwert (1989) and Nelson (1991).
[5] For a more detailed study on the different persistence of German and European factor returns in this thesis refers to Appendix A 7.3.

volatility. By doing so they apply their VMP strategy. For example, they recommend that investors should reduce their risk exposure by around 50% on a volatility shock on the market. The reason is that investors might not be adequately compensated for the risk that are bearing.

More specifically, the strategy consists of reducing risk exposure during periods when market conditions are volatile, because the mean-variance trade-off becomes initially more uncertain deteriorating the optimal portfolio allocation. The next strategical move is that investors should re-take positions as volatility starts diminishing. The more the stock market starts to recover i.e. the more volatility decreases, the more the mean-variance trade-off gradually ameliorates.

Moreira and Muir (2016a) suggest that average investors can obtain attractive risk-adjusted returns in real time by implementing the VMP strategy. To illustrate the benefits of the strategy the authors manage a broader set of U.S. systematic risk factors and multifactor portfolios. The strategy is computed by scaling monthly factors and multifactor portfolios by the inverse of the respective realized variance of the previous month (one-month-lagged RV_t^2 or RV_{t-1}^2, hereafter). If the RV_t^2 in the preceding month is higher (lower) than expected, the risk exposure to the market in the following month should be reduced (increased). The authors measure the superior strategy´s performance through positive and significant alphas and positive Sharpe ratios.

2.2 Hypothesis development

By studying the business conditions and behavior between stock volatility and expected stock return, Fama and French, (1989) support the traditional view that expected returns are higher when economic conditions are risky. Moreira and Muir (2016a), however, contradict this financial wisdom; by suggesting that during turbulent times, the mean-variance tradeoff is weak because expected returns adjust more slowly than volatility. Therefore, investors are not adequately compensated for the risk they are taking. In fact, the authors' suggested strategy differs from strategies; such as the buy-and-hold strategies or even strategies that maintain constant risky holdings, especially in recessions.

Concerning market risk premiums, French, Schwert and Stambaugh (1987) suggest the existence of a direct and positive relation between risk premium and volatility. In the work of Backus and

Gregory (1993), the risk premium on the market portfolio is shown to be lower during volatile times. This latter evidence is consistent with the authors of the VMP strategy.

Muir's (2016) assumes that mean-variance investors might not be efficiently rewarded for the extra risk that investors are exposed to during risky periods. The reason is that during volatile times, volatility in capital markets is affected by much more additional risk than only systematic risk[6]. Therefore, a smart step would be exit the market. Thereby, the intuition of the VMP strategy has led to a growing debate among academics and in financial media. Motivated by this debate, I investigate empirically the feasibility and performance of the VMP strategy using German and European factor returns.

2.2.1 German managed factors

Moreira and Muir (2016a) find significant and positive alphas across U.S. single factors and divers U.S. multifactor portfolios by implementing their VMP strategy. While well-diversified U.S. risk factors, such as market, size and value from Fama and French (1992), are recognized worldwide in various empirical studies, there are empirical evidences that these U.S. factors are country-specific. For example, Griffin (2002) advocates that other domestic versions of the Fama-French three-factor model fail to explain stock returns. Hence, my motivation is to examine the performance of the VMP strategy´s validity by applying other country-specific factors.

In this thesis, I attempt to answer the question whether the VMP strategy is also suitable for factors consisting in German stock returns. The German economy and its capital market are of special interest for several reasons. First, the German stock market is one of the most important international capital markets and the second largest European market by market capitalization[7]. Second, in contrast with its European neighbors, Germany´s economy maintained its strong economic power during the last 2009 great recession. Despite a severe contraction of the GDP

[6] To see Muir, T., (2016) interview: https://www.cnbc.com/2016/03/23/when-markets-get-scary-panicking-is-smart-yale-profs.html.
[7] Statistics of the German market capitalization can be obtained in the World Bank data set from 1975 to 2017: https://data.worldbank.org/indicator/CM.MKT.LCAP.CD?locations=FR-DE-GB&name_desc=false.

between 2008 and 2009, Germany showed a low increase in unemployment and a stable export-import balance (see, e.g., Dustmann, Fitzenberger, Schönberg, and Spitz-Oener, 2014). Finally, Schildbach, (2017) demonstrates that the German stock market has largely outperformed its European peers despite the 2011 European sovereign debt crisis.

One of the major inconveniences of conducting international factor analysis is the lack of reliable data. In my thesis, I use a reliable and publicly available dataset for German risk factors. Therefore, to test the VMP strategy´s performance, I construct a country-specific model managing four German risk factors: market, size, value and momentum. Considering specific characteristics of the German stock market explained by Ziegler, Eberts, Schröder, Schulz and Stehle (2007) and Stehle, Brückner, Lehmann, Schmidt (2015a), I expect that:

- Hypothesis 1: The managed German factors and multifactor portfolios present positive and significant alphas and substantial Sharpe ratios in comparison with the non-managed German factors and multifactor portfolios.

2.2.2 European managed factors

To construct an adequate risk factor model, Fama and French (1998) show that such a model should be based on a high degree of capital market integration. This assumption is consistent with the time-series study of Griffin (2002), which shows that domestic factor models outperforms global ones. Fama and French (2012) complement this research by suggesting that regional risk factors generate better results than global risk factors.

Since countries have more integrated capital markets than financial or economic regions, it is not surprising that very few studies provide empirical comparisons between country-specific and region-specific data. However, the regional case study of the European capital market, represents an exception to the rule. According to Hardouvelis, Malliaropulos and Priestley (1999), European countries have gradually become a highly integrated stock market region, due to the progressive elimination of economic, financial and trade barriers. Therefore, I expect that managing European factors would provide acceptable results when implementing the VMP strategy, though to a lesser magnitude than the German country-specific model. Following some assessments criteria of Moerman (2005), I suggest that:

- Hypothesis 2: Managed European factors and multifactor portfolios outperform the non-managed European factors and multifactor portfolios with positive and significant alphas and substantial Sharpe ratios though to a lower extent than in the German case.

2.2.3 Volatility managed portfolio strategy: GARCH (1,1) approach

Moreira and Muir (2016a) suggest that when managing factors by other variance forecasting measures, such as those proposed by Andersen and Bollerslev (1998), the VMP strategy offers similar outputs. Nevertheless, the results cannot be compared because they have not been published. Andersen and Bollerslev (1998) examine the implications of the Generalized Auto Regressive Conditional Heteroscedasticity (GARCH) using different data frequencies. They conclude that GARCH (1,1) does provide a good volatility forecast. Therefore, my objective in this thesis is to examine whether implementing GARCH (1,1) model provides a similar or superior performance than that of the realized variance approach, when constructing the VMP strategy.

Previous research has concluded that GARCH is a parsimonious forecast measure. It also shows better forecasting power to model time-series data when it exhibits heteroskedasticity and volatility clustering[8] (see, e.g., Akgiray, 1989). Moreover, Hansen and Lunde (2005) conclude that none relative competing GARCH-based model outperforms the accuracy forecast of the GARCH (1,1). Therefore, I assume that:

- Hypothesis 3: Managing German and European factors with the one-month-lagged GARCH (1,1) provides a better performance in terms of alphas and Sharpe ratios than managing German and European factors with one-month-lagged RV_t^2.

[8] Volatility clustering captures the volatility shock magnitudes i.e. big (small) volatility shocks are followed by big (small) volatility shocks.

3 Data

3.1 Data description

To analyze the VMP strategy's implementation, I use a data set which contains four German risk factors. I use daily and monthly German factor returns. The factors are the German version of Fama-French three risk factors and the German Carhart's momentum factor. The factors are defined as GMKTRF, GSMB, GHML, and GMOM for market, size, value and momentum, respectively. I gather the dataset from Professor Stehle´s website at Humboldt University Berlin[9]. These risk factors capture specific peculiarities of the German stock market and cover the period from January 1990 to June 2016[10].

The risk factors include data from the largest German security exchange; the Frankfurt Stock Exchange (FSE). The German MVE multifactor portfolios are GMKMO, GHMO, GFF3 and GC. GMKMO and GHMO are the portfolios that combine the non-managed market and momentum factors for the former and non-managed value and momentum factors for the latter portfolio. GFF3 is the Fama and French three-factor portfolio, and the GC is the Carhart four-factor portfolio.

The European data consists of daily and monthly factor returns. The factors are European market (EMKTRF), size (ESMB), value (EHML) and momentum (EMOM). I obtained this data from the website of Ken French and the sample covers the period from July 1990 to December 2016. Although two out of sixteen countries considered in the data set do not belong to the Euro Area, their financial markets are highly integrated to the Euro Area members[11]. The European MVE multifactor portfolios are the combination of European market and momentum factor (EMKMO), the combination of European value and momentum factor (EHMO), the European Fama and French three-factor model (EFF3) and the European four-factor model of Carhart (EC).

[9] Brückner, Lehmann, Schmidt and Stehle (2015b) provide a detailed description and construction of the German data.
[10] The German risk factors can be downloaded from the University Humboldt website: http://www.wiwi.hu-berlin.de/professuren/bwl/bb/data/fama-french-factors-germany. The dataset belongs to the ALL (BP: TOP) group with tax credit.
[11] Non-EU countries: Norway and Switzerland. Norway belongs to the European Economic Area (EEA). Although, Switzerland is neither an EU nor EEA member, it is part of the European single market.

I downloaded U.S. daily and monthly factors from Ken French's website namely market (MKTRF), size (SMB), value (HML), momentum (MOM), profitability (RMW), and investment (CMA). MKTRF, SMB and HML form the original Fama and French three-factor model (FF3). With the addition of RMW and CMA factors, I form the Fama and French (2015) five-factor model (FF5). I also combine factors to create the U.S. Carhart four-factor model (C). The sample periods for MKTRF, SMB, HML and MOM start on January 1927, and for RMW and CMA starts on July 1963. These factor returns are calculated from data from listed firms in the NYSE, AMEX, and NASDAQ exchanges.

Additionally, I collect U.S. daily and monthly factors on investment (IA), and on returns on equity (ROE) described in Hou, Xue and Zhang, (2014)[12]. IA and ROE are two of the four-factors model of Hou, Xue and Zhang (2014). This U.S. four-factor model is noted as HXZ. The HXZ is combined with the momentum factor to create the HXZMO portfolio. The sample period of the factors ROE and IA and the multifactor portfolios HXZ and HXMO start on January 1967. The ending dates for all the U.S. factors and multifactor portfolios is December 2016[13].

3.2 Summary statistics

Table I shows the summary statistics for all non-managed risk factors and non-managed multifactor portfolios. All factor returns are in percentages. Panel A reports the statistics of the four non-managed German factors, $GMKTRF^{nm}$, $GSMB^{nm}$, $GHML^{nm}$ and $GMOM^{nm}$, each of them accounts for 318 monthly observations. Panel B shows the four European non-managed risk factors, $EMKTRF^{nm}$, $ESMB^{nm}$, $EHML^{nm}$ and $EMOM^{nm}$. Each European factor has also 318 monthly observations. In panel C, each of the U.S. factors: $MKTRF^{nm}$, SMB^{nm}, HML^{nm} and MOM^{nm} consists of 1,080 monthly observations, except of RMW^{nm} and CMA^{nm} (642 monthly observations) as well as ROE^{nm} and IA^{nm} (600 monthly observations). Panel D, E and F report the mean-variance efficient (MVE) multifactor portfolios for Germany, Europe, and the United States, respectively.

[12] I am grateful to Professor Lu Zhang for providing me the data described in Hou, Xue and Zhang, (2014).
[13] I am thankful to Professor Alan Moreira to give me some insights how to collect the U.S. data.

The non-managed MVE portfolios for Germany are $GMKMO^{nm}$, $GHMO^{nm}$ GFF3 and GC^{nm} and those for Europe are $EMKMO^{nm}$, $EHMO^{nm}$, $EFF3^{nm}$ and EC^{nm}. Each of these MVE portfolios has 318 monthly observations. The U.S. multifactor portfolios $FF3^{nm}$ and C^{nm} have 1,080 monthly observations. The data sample of the $FF5^{nm}$ and the portfolio conformed by $FF5^{nm}$ plus momentum ($FF5MO^{nm}$) account for 641 monthly observations. The last two MVE multifactor portfolios in Panel F are: HXZ^{nm} and $HXZMO^{nm}$. Both portfolios consist on 600 monthly observations.

At first glance, all average factor returns (mean) are positive, with exception for $GSMB^{nm}$ (-0.56) and $ESMB^{nm}$ (-0.007). The values are lower than the one, with exception for $GMOM^{nm}$ (1.22). The mean of the monthly market factor differs considerably among the German (0.39), European (0.45) and U.S. (0.65) data. The means of the momentum factor are the largest factor means among the three types of data: $GMOM^{nm}$ (1.22), $EMOM^{nm}$ (0.89) and U.S. MOM^{nm} (0.66). The standard deviation sheds light on the historical average volatility. Among the three different data sets, market and momentum factors are on average the riskier factors. For example, the standard deviations of U.S., German and European market factor are 5.38, 5.37, 4.96, respectively. Momentum volatility ranges between 4.03 for $EMOM^{nm}$ and 5.41 for $GMOM^{nm}$.

One way to model the mean-variance framework is using normal distributed returns. The normal distribution on returns has a relevant implication because it shapes the statistical estimation results considerably. In theory, it suffices to observe the balance between the mean and variance of normal distributed returns to explain the portfolio problem for a mean-variance investor. If returns obey a normal distribution approximately, they should have a skewness qual to zero and a kurtosis equal to three. The summary statistics show that none of the factor and multifactor portfolio distributions present symmetry around their means (skewness $\neq 0$). Moreover, all return distributions present leptokurtic, i.e. positive excess kurtosis. The presence of non-normal distribution in my data is a typical phenomenon of financial data.

While most of the U.S. factors present positive skewness, i.e. the probability of large gains is higher than the probability of large losses, most of the German and European factors have negative skewness, i.e. the probability of large losses is higher than the probability of large gains. In the

Table I

Summary statistic of monthly non-managed factors and MVE multifactor portfolios

Table I shows the summary statistics of return distributions for the non-managed factors and MVE multifactor portfolios. The first moment, i.e. the mean denotes the average value of the factor and multifactor return. The standard deviation (Std. Dev.) is the square root of the second moment. The third and fourth moments are skewness and kurtosis, respectively. Skewness captures the asymmetry of the distribution and kurtosis depicts the tail thickness of the distribution. None of the factor and multifactor portfolio distributions are characterized by a normal distribution. * denotes that the sample period of RMW^{nm} and CMA^{nm} factors starts on July 1963 (642 observations). ** denotes that the sample period of ROE and IA factors starts on January 1967 (600 observations). *** and **** denotes that data sample of $FF5^{nm}$ and $FF5MO^{nm}$ start on July 1963 and that of HXZ^{nm} and $HXZMO^{nm}$ start on January 1967.

Non-managed single factors

	Mean	Std. Dev.	Skewness	Kurtosis
Panel A: German factors, Jan. 1990-Jun. 2016 (318 observations)				
$GMKTRF^{nm}$	0.39	5.37	-0.54	4.58
$GSMB^{nm}$	-0.56	3.55	-0.04	3.78
$GHML^{nm}$	0.50	3.11	0.45	6.00
$GMOM^{nm}$	1.22	5.41	-1.28	11.7
Panel B: European factors, Jul. 1990-Dec. 2016 (318 observations)				
$EMKTRF^{nm}$	0.45	4.96	-0.58	4.52
$ESMB^{nm}$	-0.00	2.24	-0.08	3.94
$EHML^{nm}$	0.34	2.40	0.36	5.87
$EMOM^{nm}$	0.89	4.02	-1.25	10.30
Panel C: U.S. factors, Jan. 1927-Dec. 2016 (1,080 observations)				
$MKTRF^{nm}$	0.64	5.38	0.19	10.74
SMB^{nm}	0.21	3.22	1.93	22.32
HML^{nm}	0.40	3.51	2.16	22.03
MOM^{nm}	0.66	4.73	-3.05	30.58
RMW^{nm}*	0.24	2.23	-0.35	16.21
CMA^{nm}*	0.31	2.00	0.28	4.64
ROE^{nm}**	0.54	2.54	-0.69	7.65
IA^{nm}**	0.41	1.87	0.11	4.44

Non-managed MVE multifactor portfolios

	Mean	Std. Dev.	Skewness	Kurtosis
Panel D: German MVE portfolios, Jan.1990-Jun. 2016 (318 observations)				
$GMKMO^{nm}$	0.70	3.61	0.28	3.99
$GHMO^{nm}$	0.96	3.63	-1.03	8.21
$GFF3^{nm}$	0.92	3.53	-1.19	11.8
GC^{nm}	0.57	2.02	-1.25	8.58
Panel E: European MVE portfolios, Jul.1990-Dec. 2016 (318 observations)				
$EMKMO^{nm}$	0.75	2.73	-0.76	5.54
$EHMO^{nm}$	0.60	1.95	-0.10	6.94
$EFF3^{nm}$	0.33	2.08	-0.09	4.02
EC^{nm}	0.60	1.77	-0.23	4.40
Panel F: U.S. MVE portfolios, Jan.1927-Dec. 2016 (1,080 observations)				
$FF3^{nm}$	0.48	3.23	2.14	26.24
C^{nm}	0.55	1.94	-0.50	7.08
$FF5^{nm}$***	0.30	1.01	0.10	5.43
$FF5MO^{nm}$***	0.36	1.02	-0.36	7.74
HXZ^{nm}****	0.45	1.04	-0.07	6.10
$HXZMO^{nm}$****	0.45	1.05	-0.18	6.52

case of MVE multifactor portfolios or well-diversified systematic portfolios, investors seek to diversify their portfolios to avoid a high probability of large losses. Paradoxically, the statistics for most of the MVE multifactor portfolios exhibit negative skewness. Levy and Post (2005a) propose a possible explanation for this paradox. They assume that the correlation between returns tends to rise during high volatility periods. This in turn diminishes the benefits of diversification.

All factor and multifactor portfolio present high kurtosis, meaning that all return distributions have fat tails, or rather that there is a high probability of extreme returns. For example, the German, European and U.S. MOM^{nm} factor distributions have their left tails extremely fat ranging from 11.7 for $GMOM^{nm}$ to 30.58 for MOM^{nm}. The fattest tail among portfolios has the value of 26.24 for $FF3^{nm}$ portfolio. The high values of kurtosis reflect that extreme negative returns can occur with a much more frequency than expected. This might be explained by the fact that variance is not constant over time (see, e.g., Levy and Post, 2005b).

Overall, the descriptive statistics reveals that my data set is not normally distributed, confirmed by high levels of significance of the Jarque-Bera test statistics as shown in Appendix A, section 7.1. A common problem in time-series studies is that even if a normal distribution were used to model the mean-variance investors' problem, investors might underestimate the level and magnitude of recessions and booms. The problem is that the normal distribution quantifies isolated outlier events as extremely rare. However, these extreme outlier events are more frequently observable in capital markets than the normal distribution predicts (see, e.g., Reinhart and Rogoff, 2009).

3.3 Correlation matrix

Table II reports Pearson's correlation matrix for the non-managed factors at monthly frequency. Pearson's correlation coefficient (ρ) helps to identify linear relationships among variables. Panel A and B show the correlation coefficients for the German and European risk factors, respectively. Panel C reports the correlations for U.S risk factors. The criteria for correlation degree are: high correlation degree $\pm 0.50 < \rho < \pm 1$, moderate correlation degree $\pm 0.30 < \rho < \pm 0.49$ and low correlation degree $\pm 0.01 < \rho < \pm 0.29$. Most of the German coefficient correlations are low and negative, except of the correlation between $GMKTRF^{nm}$ and $GSMB^{nm}$ (-0.50).

Table II

Pearson`s correlation coefficients of non-managed factors

Table II reports the correlation matrix for German, European and U.S. risk factors at monthly frequency. The criteria for the correlation degree are: high correlation ± 0.50 < ρ < ± 1, moderate correlation ± 0.30 < ρ < ± 0.49 and low correlation ± 0.01 < ρ < ± 0.29. For most of the factors, the correlation coefficients are low and negative. * specifies the significance level of correlation coefficients at 5%.

Panel A: German factor correlation coefficients				
	$GMKTRF^{nm}$	$GSMB^{nm}$	$GHML^{nm}$	$GMOM^{nm}$
$GMKTRF^{nm}$	1.00			
$GSMB^{nm}$	-0.50*	1.00		
$GHML^{nm}$	-0.02	-0.17*	1.00	
$GMOM^{nm}$	-0.27*	-0.14*	0.09	1.00

Panel B: European factor correlation coefficients				
	$EMKTRF^{nm}$	$ESMB^{nm}$	$EHML^{nm}$	$EMOM^{nm}$
$EMKTRF^{nm}$	1.00			
$ESMB^{nm}$	-0.17*	1.00		
$EHML^{nm}$	0.18*	-0.06	1.00	
$EMOM^{nm}$	-0.28*	0.09	-0.28*	1.00

Panel C: U.S factor correlation coefficients								
	$MKTRF^{nm}$	SMB^{nm}	HML^{nm}	MOM^{nm}	RMW^{nm}	CMA^{nm}	ROE^{nm}	IA^{nm}
$MKTRF^{nm}$	1.00							
SMB^{nm}	0.32*	1.00						
HML^{nm}	0.24*	0.12*	1.00					
MOM^{nm}	-0.33*	-0.14*	-0.41*	1.00				
RMW^{nm}	-0.23*	-0.40*	0.07	0.10*	1.00			
CMA^{nm}	-0.38*	-0.16*	0.69*	-0.01	-0.03	1.00		
ROE^{nm}	-0.20*	-0.37*	-0.13*	0.50*	0.66*	-0.08*	1.00	
IA^{nm}	-0.38*	-0.25*	0.67*	0.02	0.09*	0.91*	0.03	1.00

The output of Panel B shows that all correlation coefficients of the European factors are low. While most of the European factor correlations are negative, the coefficient correlations between $EMKTRF^{nm}$ and $EHML^{nm}$, and between $ESMB^{nm}$ and $EMOM^{nm}$ are positive. The coefficient results are mixed when observing U.S. correlations. Panel C shows that while just over half of the correlation outcomes are low and negative, there are also very high positive correlations. The two highest coefficient values belong to the correlation between CMA^{nm} and IA (ρ= 0.91) and HML^{nm} and CMA^{nm} (ρ= 0.69).

4 Methodology

4.1 Mean-variance framework

The starting point to understand the VMP strategy is to study such strategy through the perspective of a mean-variance investor. This investor evaluates her portfolio allocation according to the intertemporal attractiveness of the risk-return trade-off. According to Moreira and Muir (2016a), the mean-variance trade-off is given by:

$$\frac{\mu_t}{\sigma_t^2} \qquad (1)$$

where μ_t is the expected excess return and σ_t^2 is the conditional variance. Since variance is not directly observable, it should be estimated as $\widehat{\sigma_t^2}$.

4.2 Portfolio construction

Moreira and Muir (2016a) use the following methodology to construct the realized variance ($\mathrm{RV_t^2}$):

$$RV_t^2 = \widehat{\sigma_t^2} = \sum_{d=1/22}^{1} (F_{t+d}^{nm} - (\frac{\sum_{d=1/22}^{1} F_{t+d}^{nm}}{22}))^2 \qquad (2)$$

where $\mathrm{F_{t+d}^{nm}}$ is the return of the non-managed factor. $\mathrm{RV_t^2}$ is the sum of the squared deviation of each daily factor return on day d and the daily average return. The value 22 is the average number of daily observations per month. From this equation, one can obtain the one-month-lagged $\mathrm{RV_t^2}$ (or $\mathrm{RV_{t-1}^2}$) i.e. the proxy of the conditional variance of the non-managed factor. The square root of $\mathrm{RV_t^2}$ is its realized volatility ($\mathrm{RV_t}$). Once $\mathrm{RV_{t-1}^2}$ is obtained, the authors construct the managed factor ($\mathrm{F_t^m}$) with the following model:

$$F_t^m = \frac{c}{RV_{t-1}^2} F_t^{nm}. \qquad (3)$$

F_t^m is obtained by scaling monthly non-managed factor by the inverse of its respective RV_{t-1}^2. The constant c serves to control the average risk exposure of the strategy. In other words, c is set in such a way that the unconditional standard deviation of the managed factor is the same as the unconditional standard deviation of the non-managed factor.

4.3 Mean-variance weakness

To validate the weakness of the mean-variance framework, Moreira and Muir (2016a) assume that while there is a strong relation between past volatility and current volatility, there is a weak relation between past volatility and average returns during periods of high volatility. To verify these predictions, I estimate both, the average impact of past volatility on current returns and the average impact of past volatility on current volatility.

4.3.1 Volatility and return predictions

By estimating the relation between volatility and return I run a time-series regression of monthly log factor return on its previous monthly log realized volatility. The following equation approximates this relation:

$$log(F_t^{nm}) = a + blog(RV_{t-1}) + \epsilon_t \qquad (4)$$

where one-moth-lagged RV_t is the factor past realized volatility. Further, I run a time-series regression of monthly realized volatility on the one-month-lagged RV_t . For this purpose, I take logs of both past and current volatility of factor returns. The regression equation is:

$$log(RV_t) = a + blog(RV_{t-1}) + \epsilon_t \qquad (5)$$

In the original paper of Moreira and Muir (2016a), the authors sort the factor returns according to the level of factor realized volatility. More specifically, they form low to high volatility groups to calculate the risk-return predictions. However, I do not sort the data into low to high factor volatility. Therefore, both regressions are not intended to accurately capture the intertemporal prediction during economic contraction and expansion periods. Nonetheless, I empirically approach the average relation between one-month-lagged volatility and returns and the relation between one-month-lagged volatility and current volatility. The coefficient of the one-month-lagged realized volatility (b) will be reported for both equations 4 and 5. The larger and the more statistically significant the past realized volatility coefficient, the stronger is the power of prediction.

4.4 Managed single factor and multifactor portfolio

4.4.1 Single managed factor

The VMP's methodology consists of running a time-series regression of each managed factor on its respective non-managed factor:

$$F_t^m = \alpha + \beta F_t^{nm} + \epsilon_t. \qquad (6)$$

A positive alpha means that the Sharpe ratio of the managed factor is relatively higher than the Sharpe ratio of the non-managed factor. The factors used in this thesis are highly diversified so that total risk of each factor coincides with its respective systematic risk. Moreover, every non-managed factor captures a different dimension of systematic risk. For instance, GMKTRF explains German market risk and GSMB captures German firm size risk.

4.4.2 Managed multifactor portfolio

I create different combinations of monthly non-managed factors to create monthly non-managed multifactor portfolios. The multifactor portfolio is created to be a mean-variance efficient (MVE) portfolio. The constant weights of the monthly non-managed factors are calculated in such a way that they produce the maximal Sharpe ratio of the MVE portfolio. This means that the MVE portfolio is an optimal portfolio in the mean-variance space when a risk-free rate is available. The non-managed MVE portfolio is:

$$MVE\_p_t^{nm} = \sum_{t=1}^{i} W_i F_{t,i}^{nm} \qquad (7)$$

where W_i is the monthly weight of the non-managed factor i ($F_{t.i}^{nm}$). To construct the managed MVE portfolio, the non-managed MVE portfolio is scaled by the inverse of its one-month-lagged realized variance ($RV_{MVE,t-1}^2$). $RV_{MVE,t-1,}^2$ uses the same methodology as RV_{t-1}^2. The only difference is that the daily MVE portfolio returns are constructed using daily weights at a given risk-free rate. The managed MVE portfolio is:

$$MVE\_p_t^{m} = \frac{c}{RV_{MVE,t-1,}^2} MVE\_p_t^{nm}, \qquad (8)$$

where c is a constant that normalizes the variance of the managed MVE portfolio to be equal as the non-managed MVE portfolio. The time-series equation of the managed MVE portfolio is:

$$MVE\_p_t^{m} = \alpha + \beta\, MVE\_p_t^{nm} + \epsilon_t. \quad (9)$$

By running a time-series regression of the managed MVE portfolio on the non-managed MVE portfolio, a positive alpha implies that the VMP strategy increases the Sharpe ratio of the MVE portfolio compared to the best Sharpe ratio of any MVE non-managed multifactor portfolio. This means that a positive alpha expands the mean-variance frontier.

4.5 Performance measurement of the VMP strategy

To evaluate the managed portfolio performance Moreira and Muir (2016a) construct the managed Sharpe ratio (SR^m). The SR^m is the square root of the sum of the squared non-managed Sharpe ratio ($SR^{nm\,2}$) and the squared appraisal ratio ($(\alpha/\sigma_\varepsilon)^2$). The SR^{nm} is defined as the expected excess factor return divided by the realized standard deviation of the factor. The managed Sharpe ratio is calculated as following:

$$SR^m = \sqrt{(SR^{nm})^2 + \left(\frac{\alpha}{\sigma_\varepsilon}\right)^2} \qquad (10)$$

where σ_ε is the root mean squared error (RMSE) and measures the accuracy of (multi)factor return estimations relative to the historical (multi)factor average returns (see, e.g., Verbeek, 2012). The appraisal ratio (AR) will be reported in annualized terms (√12 * AR). When using different MVE portfolios, this term indicates how much the dynamic of VMP strategy expands the mean-variance efficient frontier compared to the non-managed multifactor strategy[14] i.e. passive strategies.

4.6 Forecasting variance

Most of the performance of the VMP strategy comes from the technique to proxy conditional variance by using RV_{t-1}^2. Although RV_{t-1}^2´s construction seems to involve a simple methodology, I investigate, whether a forecasted variance measure can be a more accurate proxy for the conditional variance than the RV_{t-1}^2. I approximate conditional variance by using the one-month-lagged Generalized Auto Regressive Conditional Heteroscedasticity process (one-month-lagged GARCH (1,1) or σ_{t-1}^2 hereafter).

4.6.1 The GARCH (1,1) forecasting model

The traditional GARCH (p,q) which allows for different lags can be written as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i}^2 + \sum_{i=1}^{q} \beta_i \dot{\sigma}_{t-\iota}^2 \qquad (11)$$

where σ_t^2 is the forecasted variance which depends on the weighted average of past squared returns (R_{t-i}^2), on its own previous lags ($\dot{\sigma}_{t-\iota}^2$) and on the parameters ω, α and β. If the aim is to forecast variance on a short-term, the simplest version of GARCH(p,q), the GARCH (1,1), suffices. This means only one lag of squared returns and one lag of the variance itself are looked at. I use the following standard GARCH (1,1) process:

$$\sigma_t^2 = \omega + \alpha R_{t-1}^2 + \beta\, \dot{\sigma}_{t-1}^2, \qquad (12)$$

[14] In the paper of Moreira and Muir (2016a), "Volatility Managed Portfolios", the portfolio formation and empirical methodology are explained in the section 2. My thesis uses the same methodology, strategy construction and criterium of performance for the analysis of the single factors and multifactor portfolios. However, most of the equation terms are changed to avoid repetitive notations.

where $\omega = \theta\sigma_L^2$. The parameter θ is the weight assigned to the long-run average variance, σ_L^2. To ensure non-negativity of GARCH (1,1), the equation (12) requires that $\theta + \alpha + \beta = 1$. The GARCH (1,1) is constructed using daily factor returns and the parameters are calculated using the method of Maximum Likelihood Estimation (see, e.g., Hull, 2012)

4.6.2 Managed GARCH (1,1) factor construction

I scale the monthly non-managed factors[15] (F_t^{ng}) by the inverse of its respective one-month-lagged GARCH (1,1) or σ_{t-1}^2 , to construct the managed GARCH (1,1) factor (F_t^g). F_t^g can be obtained by the following model:

$$F_t^g = \frac{c}{\sigma_{t-1}^2} F_t^{ng}. \tag{13}$$

The constant c serves to control the average risk exposure of the strategy. Once F_t^g is obtained, I run a time-series regression of the managed GARCH (1,1) factor on the non-managed factor (F_t^{ng})[16]. Similarly, as in the realized variance approach, a positive and significant alpha and a large Sharpe ratio imply the superior performance of the strategy.

5 Empirical results

5.1 German risk factors

5.1.1 Predictions of volatility and returns for Germany factors

In the study of Moreira and Muir (2016a), the trade-off between mean and variance is explained by the assumption that past volatility does not explain average returns, emphasizing that this

[15] F_t^{ng} is the same as the F_t^{nm} i.e monthly non-managed factors. The "g" denotes only that the GARCH (1,1) approach is applied in computation.
[16] To model monthly GARCH (1,1) from daily GARCH (1,1) I use an estimation window to aggregate daily GARCH (1,1) observations (in average 22 observations per month) for each month. (see, e.g., Liu and Tse, 2013, p 5). Moreover, it is important to reiterate that the monthly one-month-lagged GARCH (1,1) is noted as σ_{t-1}^2 whereas one lag of the variance using daily data is $\sigma_{t-1}^{\dot{2}}$.

relation is especially weak during periods of high volatility. The authors illustrate two scenarios: One where the risk-return trade-off is strong and another where it is weak. In both cases, the volatility is high. If the relation between risk and return is strong, volatility should predict average returns. This intuitively leads to obtain potential high expected returns if investors increase their risk exposure on the market. However, if the risk-return relation is weak, volatility cannot predict average returns, because stock volatility varies even more in volatile periods. Therefore, an adequate compensation for risk-taking is not achievable.

Table III

Average returns and current volatility predicted by past volatility

Panel A reports the coefficients for time-series regressions of monthly non-managed factor returns on its previous one-month-lagged realized volatility ($\log(F_t^{nm}) = a + b\log(RV_{t-1}) + \epsilon_t$). Panel C shows the coefficients for time-series regressions of monthly factor volatility on its one-month-lagged realized volatility ($\log(RV_t) = a + b\log(RV_{t-1}) + \epsilon_t$). Panel B presents Person´s correlation coefficients between non-managed factor return and its respective one-month-lagged realized volatility. Panel D shows Person´s correlation coefficients between factor volatility and its respective one-month-lagged realized volatility. The sample period for all German factor returns starts in January 1990 and ends in June 2016. I perform the two-sided test and t-values are in parentheses, t-values are based in standard errors which are adjusted for heteroscedasticity. *, **, and *** denote the 10%, 5% and 1% significance level, respectively.

Panel A: Regression of factor returns on lagged realized volatility				
	$GMKTRF^{nm}$	$GSMB^{nm}$	$GHML^{nm}$	$GMOM^{nm}$
RV_{t-1}	-0.02	-0.04	0.01	-0.26
(t)	(-0.64)	(-1.12)	(0.27)	(-1.16)
Observations	317	317	317	317
R^2	0.00	0.00	0.00	0.01
Panel B: Correlation between factor returns and lagged realized volatility				
RV_{t-1}	-0.04	-0.06	0.01	-0.13**
Panel C: Regression of factor realized volatility on lagged realized volatility				
RV_{t-1}	0.63***	0.68***	0.58***	0.72***
(t)	(14.18)	(15.52)	(11.65)	(17.67)
Observations	317	317	317	317
R^2	0.40	0.47	0.33	0.52
Panel D: Correlation between factor realized volatility and lagged realized volatility				
RV_{t-1}	0.63**	0.68**	0.58**	0.72**

To study the power of past volatility to predict average returns, I run a time-series regression of the log average return on the log one-month-lagged realized volatility for each German risk factor. The regressions are at monthly frequency. In Table 3, Panel A reports the coefficients of the one-month-lagged realized volatility. I find that all coefficients except that of the value factor volatility are negative. None of the past realized volatilities have a statistically significant impact on factor returns. Therefore, the one-month-lagged volatility of market, size, value, and momentum appear not to predict factor average returns.

Panel B complements the previous regressions by showing the Pearson's correlation coefficients between the one-month-lagged realized volatility and average return. While the correlation coefficient for market and size factor volatility is negative and insignificant, this is positive though also insignificant for value's volatility. The lagged realized volatility of momentum is negatively correlated with the momentum average returns at the 5% level.

Moreira and Muir (2016a) also argue that the relation between past realized volatility and current realized volatility is strong. To investigate this finding, I run a time series regression of the log realized volatility on the log one-month-lagged realized volatility. Regarding Panel C, I find that all coefficients for the lagged realized volatility are positive and highly significant at the 1% level. Therefore, past volatility has a high power of prediction for current volatility at the short term. This output is highly relevant because the power of explanation of past volatility on current volatility is a key piece when implementing the VMP strategy. Panel D reports the correlation coefficients between lagged realized volatility and current realized volatility. All correlation coefficients are highly positive and significant at the 5% level, especially for the momentum factor.

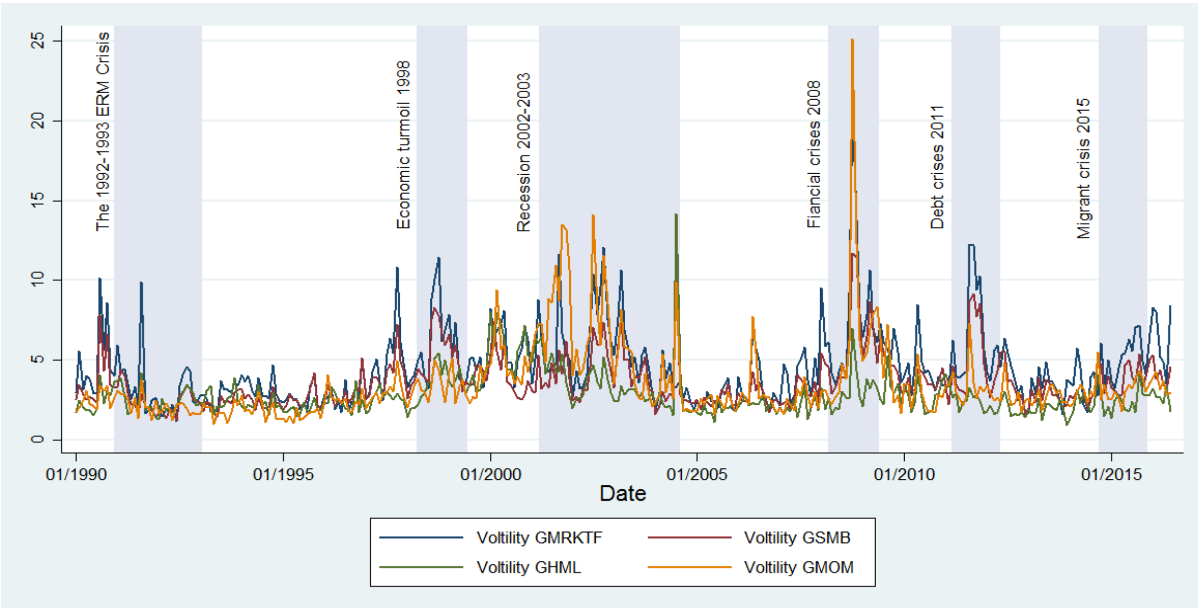
5.1.2 Time-series German factor volatility

Figure I depicts the monthly time-series realized volatility of German factors from January 1990 to June 2016. Bars in shaded grey present the OECD based recession and crisis periods for the German economy during the last 26 years. At first glance, the increase in volatility for all German factors coincide with the turbulent periods, showing that factors' volatility is strongly associated with the business cycle pattern. During the last 26 years, the German stock market was hit by

Figure I

German factor volatility and business cycle, 1990-2016

The y-axis shows the German monthly time-series realized volatility of the non-managed factors: $GMKTRF^{nm}$, $GSMB^{nm}$, $GHML^{nm}$ and $GMOM^{nm}$ from January 1990 to June 2016. Bars in shaded grey present OECD based recession and crisis periods for the Germany economy, namely the ERM crisis in 1992-1993, deterioration of business sentiment in 1998, recession in 2002-2003, financial crisis in 2008-2009, 2011 debt crisis, and 2015 migrant crisis. All German factors present a similar business cycle pattern, i.e. the movements across factors follow a similar volatility pattern over time.



Source: OECD economic indicators.

several economic contractions, such as the Exchange-rate mechanism (ERM) crisis in 1992-1993, the deterioration of business sentiment in 1998, the recession of 2002-2003, the financial crisis in 2008-2009, the debt crisis 2011 and the migrant crisis 2015.

One can observe that German factors, especially market and momentum, have high volatility in the middle of the analyzed period, which includes the recession from the start of mid-2002 until mid-2003, after the introduction of the Euro. Thereafter, volatility dropped significantly until the global financial crisis 2008 reached Germany. Throughout 2008 until the second quarter of 2009, factors' volatility increased dramatically, especially for market and momentum factors. From then onwards volatility followed a downward trend with some fluctuations until the second quarter of 2011. During the debt crisis period of 2011-2013, volatility of market, size and momentum raised steeply and declined again after the crisis.

A striking feature of this analysis is that momentum volatility peaked during the 2008 financial crisis. The profitability of the momentum factor had large drawdowns, causing the crash of the German momentum strategy in the third quarter of 2009. There is some evidence that market participants such as mutual funds and foreign investors traded aggressively on this strategy during that time due to the high profitability of the German momentum factor (see, e.g., Baltzer, Jank and Smajlbegovic, 2015). Nevertheless, when German momentum reversed, less sophisticated investors suffered great losses. Therefore, the VMP strategy on momentum and other momentum trading strategies proposed by Barroso and Santa-Clara (2014) and Daniel and Moskowitz (2016) propose that during this volatile episode the best alternative is to reduce exposure to momentum, especially for non-sophisticated investors.

5.1.3 German managed single factors

To investigate, whether the VMP strategy works by using German factors, I run a time-series regression of the managed factor on its respective non-managed factor. Panel A in Table IV illustrates the regression outputs of the market, size, value, and momentum factors. The alphas are positive and statistically insignificant for the managed market and value factors. The annualized alpha of the size factor is statistically insignificant as well though has a negative sign. The German momentum factor is the only factor that reports a highly positive and statistically significant annualized alpha of 8.14%. The latter finding is consistent with the work of Moreira and Muir (2016) who also find that managing momentum produces high and significant risk-adjusted returns.

Panel A also reports root mean squared errors (RMSE) and annualized appraisal ratios ($\sqrt{12}$ * AR= $\sqrt{12}$ * $\alpha/\sigma_\varepsilon$). AR allows to measure the expansion of the slope of the mean-variance efficient frontier by managing factors in comparison with the non-managed factors. The managed momentum factor has the highest annualized AR of 0.73, which produces an increase in its Sharpe ratio (Sh^m = 0.27) compared with its non-managed Sharpe ratio (Sh^{nm} = 0.17). Controlling for the Fama and French three-factors, the output is relatively unchanged. Panel B shows that while sign of the annualized alpha of the market factor turns from positive to negative, those of size, value and momentum stay

Table IV

Single managed factors for Germany

Panel A shows the monthly time-series regressions for the German managed factors on their respective non-managed factors. The regression equation is: $F_t^m = \alpha + \beta F_t^{nm} + \epsilon_{t+1}$, where F_t^m is the managed factor which is calculated by scaling its respective monthly non-managed factor (F_t^{nm}) by the inverse of its one-month-lagged realized variance (c/RV_{t-1}^2). C is a scalar so that the managed factor has the same unconditional standard deviation as the non-managed factor. Each factor is annualized in percentage per year by multiplying monthly factors by 12. Managed and non-managed Sharpe ratios, RMSE, and annualized AR are reported in the table. Panel B reports the three factors of Fama and French as control variables for each regression. Panel C shows the annualized alphas for two different subperiods as robustness check. The whole sample period extends from January 1990 to June 2016. I perform the two-sided test and t-values are in parentheses and t-values are based in standard errors which are adjusted for heteroscedasticity. *, **, and *** denote the 10%, 5% and 1% significance level, respectively.

Panel A: Individual regressions				
	$GMKTRF^m$	$GSMB^m$	$GHML^m$	$GMOM^m$
Alpha	0.46	-0.36	1.15	8.14***
(t)	(0.27)	(-0.59)	(1.25)	(3.48)
$GMKTRF^{nm}$	0.35***			
(t)	(8.50)			
$GSMB^{nm}$		0.31***		
(t)		(15.47)		
$GHML^{nm}$			0.47***	
(t)			(9.36)	
$GMOM^{nm}$				0.36***
(t)				(6.92)
Sh^{nm}	0.07	-0.23	0.07	0.17
Sh^m	0.07	0.23	0.09	0.27
RMSE	28.76	10.61	17.41	38.21
Annualized. AR	0.05	-0.11	0.22	0.73
Observations	317	317	317	317
R^2	0.38	0.61	0.50	0.28
Panel B: Alphas controlling for the Fama and French three factors				
Alpha	-0.06	-0.26	1.05	7.35***
(t)	(-0.04)	(-0.44)	(1.12)	(3.07)
Panel C: Alphas for subsample periods				
1990 – 2000	-1.26	-1.70*	0.40	0.68
(t)	(-0.36)	(-1.86)	(0.30)	(0.17)
2001 – 2016	1.52	0.55	1.57	8.68***
(t)	(1.19)	(0.72)	(1.32)	(4.83)

the same. The alphas of market, size and value stay statistically insignificant and the momentum's alpha is again large and statistically significant.

Moreira and Muir (2016s) state that during volatile periods alphas should be positive and significant, because the performance of the VMP strategy is based on a high degree of variation in volatility. If movements on volatility were minimal or constant alphas would be close to zero. Therefore, I perform a robustness check across two subsample periods: a less risky period from 1990 to 2000 and a risky period from 2001 to 2016. Panel C shows that the annualized alphas for the period of 1990-2000 are either low or negative. The only statistically significant alpha belongs to the managed size factor (alpha = -1.70). The negative or low alphas might be explained by the fact that in Germany the period from 1990 to 2000 was less volatile than the period from 2001 to 2016. This is consistent with Figure I, which shows that during the 90s, two out six OECD based economy contractions were present in this decade. By regressing managed factors across the 2001-2016 subperiod, all alphas increase and are positive. However, only the annualized alpha for managed momentum factor is large (alpha=8.68%) and statistically significant (t= 4.83). Overall, it seems that implementing the VMP strategy to German factors, the German momentum factor is the only managed factor leading to high risk-adjusted expected returns.

5.1.4 German mean-variance efficient multifactor portfolios

By adding more uncorrelated systematic factors to a single factor portfolio, the mean-variance investor can obtain the benefits of diversification with any combination of the systematic or mean-variance efficient (MVE) portfolios. In other words, the lower the correlation between the systematic portfolios the higher the gain of diversification and the mean-variance frontier shift outward. This is the case for the factors I use in this thesis. As mentioned in Table II, the correlation among non-managed German factors are very low. The MVE multifactor portfolios are: $GMKMO^{nm}$, $GHMO^{nm}$, $GFF3^{nm}$ and $GC4^{nm}$.

Portfolio theory says that a linear relation between expected average return and risk is obtained when a risk-free rate is available in the mean-variance space. This linear relation is called Capital Market Line (CML). To obtain an efficient portfolio, investors can allocate their wealth in different proportions by mixing risky portfolios and the risk-free rate. In equilibrium, the tangency point between the MVE frontier and the CML represents the optimal efficient portfolio. Similarly, as Moreira and Muir (2016a), I use constant weights of the monthly non-managed factors. The weights are calculated in such a way that they maximize the slope of the CML. This slope is also known as the maximum Sharpe ratio of the MVE portfolio. This means, the MVE portfolio is the optimal efficient portfolio. A particularity of the MVE multifactor portfolio is that it is the only portfolio which invests 100% of the wealth in risky portfolios and 0% in the risk-free asset. For instance, Figure II illustrates, how I construct the non-managed mean-variance efficient GFF3 portfolio. The $GFF3^{nm}$ invests 100% in German risky factors and 0% in the German risk-free rate. Allowing for short-selling, the monthly $GFF3_t^{nm}$ is constructed using the following equation:

$$GFF3_t^{nm} = W_m GMKTRF_t^{nm} + W_s GSMB_t^{nm} + W_H GHML_t^{nm} , \qquad (14)$$

where W_m, W_s and W_H are the constant weights of the monthly market, size and value factors for the period January 1990 to June 2016. The average German risk-free rate for the whole period is 3.20%[17]. This rate is, however, too high considering that since the financial crisis of 2008, the German risk-free rate merely has arrived at 100bps. In fact, throughout the last year of the analyzed period, the German risk-free rate has raged between 40 bps to -37 bps. Therefore, I consider rather a more realistic scenario setting the German risk-free rate at 0%[18].

Moreira and Muir (2016a) highlight that the non-managed Sharpe ratios might be overstated compare to the real ones since the non-managed MVE portfolios are constructed in-sample. This in turn might underestimate the calculations for managed Sharpe ratios. Therefore, I deduce that the German non-managed Sharpe ratios might also be slightly inflated as well. Furthermore, since the German non-managed MVE portfolios are constructed using constant-weight returns for a historical 26-year period, the VMP strategy ought to be taken as a theoretical reference only.

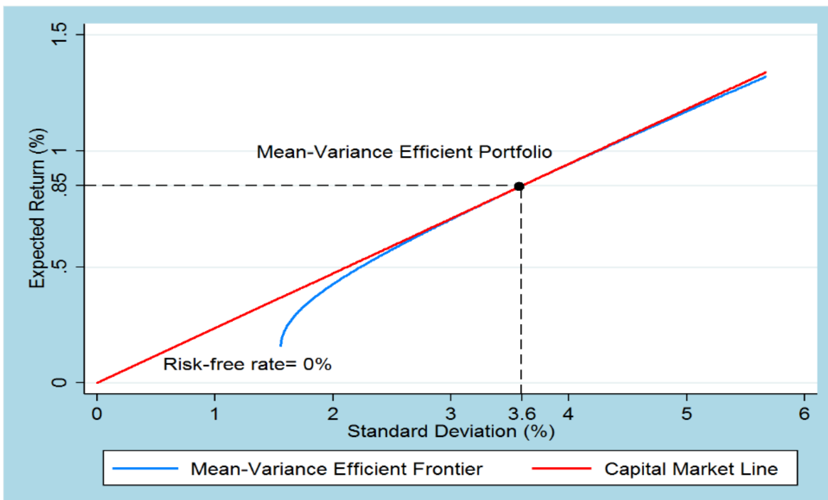
[17] To obtain the proxy of the time-series risk-free rate for Germany, I follow the same procedure of Stehle and Schmidt (2013, p. 24-25). I use the monthly one-month money market rates (Monatsgeld) from January 1990 to May 2012. Then, I use the one-month EURIBOR (Einmonatsgeld) from June 2012 to June 2016. The German data is publicly available in the Deutsche Bundesbank databank.

[18] Daily German risk-free rate is also set up at 0% to construct daily MVE portfolios.

Figure II

Mean-variance efficient frontier and Capital market line

Figure II shows the mean-variance efficient frontier along with the Capital Market Line (CML). Following the portfolio theory, incorporating a risk-free rate to the mean-variance framework, a linear relation between expected average return and risk is obtained (CML). By identifying the tangency portfolio between the mean-variance efficient frontier and the CML, it is possible to obtain the optimal mean-variance efficient (MVE) portfolio. In this figure, the non-managed MVE portfolio ($GFF3_t^{nm}$) is constructed by using the monthly $GMKTRF^{nm}$, $GSMB^{nm}$ and $GHML^{nm}$ factor as follows: $GFF3_t^{nm} = W_m GMKTRF_t^{nm} + W_s GSMB_t^{nm} + W_H GHML_t^{nm}$. The monthly factor returns cover the period from January 1990 to June 2016**.** Allowing for short-selling, the weights are $W_m = 0.20$, $W_s =$ -0.20 and W_H =1.0 for of $GMKTRF^{nm}$, $GSMB^{nm}$ and $GHML^{nm}$ respectively. The monthly German risk-free rate is set at 0%.



5.1.5 German managed MVE multifactor portfolio regressions

Table V contains the annualized alphas of the time-series regressions of the managed MVE portfolios on the non-managed MVE portfolios. I restate the managed market factor's results in the first column for a simple comparison. Panel A shows that all German managed MVE portfolios have positive and statistically significant alphas. While the highest alpha belongs to the $GFF3^m$ portfolio (alpha = 4.44%), the lowest belongs to the $GC4^m$ portfolio (alpha = 0.84%). Moreira and Muir (2016a) also find that all managed MVE portfolios outperform all non-managed MVE portfolios, where the annualized alpha of the Fama and French three-factor model outperforms the most.

Table V

Managed MVE multifactor portfolios for Germany

Panel A shows the monthly output of the time-series regressions of the managed MVE portfolios on the non-managed MVE portfolios from January 1990 to June 2016. The managed MVE portfolio is the non-managed MVE portfolio scaled by the inverse of its respective MVE-one-month-lagged realized variance ($c/RV^2_{MVE,t-1}$). C is a scalar so that the managed MVE portfolio has the same unconditional variance as the non-managed MVE portfolio. Managed and non-managed Sharpe ratios, RMSE and annualized AR are reported in the table. The annualized AR is calculated by $\sqrt{12} * (alpha/RMSE)$. Panel B reports the alpha results when regressing across two subsample periods. I perform the two-sided test and t-values are in parenthesis, t-values (t) are based on standard errors that are adjusted for heteroscedasticity. Each non-managed MVE portfolio is annualized in percentage per year by multiplying monthly non-managed MVE portfolio by 12. *, **, and *** denotes the 10%, 5% and 1% significance level, respectively.

Panel A: Multifactor regressions					
	$GMKTRF^m$	$GMKMO^m$	$GHMO^m$	$GFF3^m$	$GC4^m$
Alpha	0.46	0.86*	1.72***	4.44***	0.84***
(t)	(0.27)	(1.65)	(3.50)	(2.62)	(2.51)
$GMKTRF^{nm}$	0.35***				
(t)	(8.50)				
$GMKMO^{nm}$		0.15***			
(t)		(1.65)			
GHMO			0.12***		
(t)			(6.54)		
$GFF3^{nm}$				1.03***	
(t)				(13.92)	
GC^{nm}					0.21***
(t)					(10.13)
SR^{nm}	0.07	0.49	0.47	0.23	0.53
SR^m	0.07	0.50	0.51	0.26	0.55
RMSE	28.76	7.72	7.81	35.24	5.14
Annualized AR	0.05	0.38	0.76	0.43	0.56
Observations	317	317	317	317	317
R^2	0.38	0.41	0.31	0.62	0.50
Panel B: Alphas for subsample periods					
1990-2000	-1.26	-0.44	-0.73	3.37	-0.13
(t)	(-0.36)	(-0.46)	(-1.02)	(1.30)	(-0.28)
2001-2016	1.52	1.53***	2.19***	5.13***	1.36***
(t)	(1.19)	(4.13)	(5.21)	(2.31)	(3.84)

The annualized AR of multifactor portfolio regressions are considerable higher in comparison with most of the annualized AR of the single factor regressions. The MVE annualized AR range between 0.38 to 0.76. Panel B shows the regressions across two subsamples, 1990-2000 and 2001-2016. As expected, none of the MVE alphas is statistically significant and most of them are negative across the low-volatility 1990-2000 subperiod. Across the risky 2001-2016 subperiod, I obtain positive and statistically significant alphas for all MVE multifactor portfolios. By interpreting the reward to the total volatility trade-off, I expect to obtain larger managed Sharpe ratios. Indeed, all managed Sharpe ratios are higher than the non-managed ones. However, the managed Sharpe ratios show a modest increase, which might be an indicator that they are understated.

5.2 European risk factors

5.2.1 Predictions of volatility and returns for European factors

To assess whether the power of past volatility predicts average return for European factors, I a run time-series regression of the log average return on the log one-month-lagged realized volatility. As seen in Table VI, Panel A reports low, negative, and statistically insignificant volatility coefficients for the market, value, and momentum factors. The size factor is also negative, though statistically significant at the 5% level (t = -2.29). Hence, while the one-month-lagged volatility of market, value and momentum seems not to predict average returns, the one-month-lagged volatility of the size factor has a low and negative relation its average returns.

Panel B shows the results of the Pearson's correlation coefficients between the one-month-lagged realized volatility and the average return. Past realized volatility has a low and negative coefficient correlation with average return for all European factors. When regressing current volatility on one-month-lagged realized volatility, Panel C shows positive and statistically significant lagged volatility coefficients. Analyzing Panel D, the correlation coefficients between past and current realized volatility are large and positive. Thus, I conclude that the power of prediction of past realized volatility on current volatility is strongly positive for European risk factors.

Table VI

Predictions of European factor returns and volatility

Panel A reports the coefficients for the time-series regression of monthly factor return on its one-month-lagged realized volatility ($\log(F_t^{nm}) = a + b\log(RV_{t-1}) + \epsilon_t$). Panel C shows the coefficients for time-series regression of monthly factor volatility on its previous one-month-lagged realized volatility. All the independent and dependent variables are in logarithms ($\log(RV_t) = a + b\log(RV_{t-1}) + \epsilon_t$). The sample period for all factors starts in July 1990 and end in December 2016. Panels B and D present the Person´s correlation coefficients between factor return and its one-month-lagged realized volatility and the factor volatility and its one-month-lagged realized volatility, respectively. The two-sided test is performed, and t-values are in parentheses, t-values are based on standard errors that are adjusted for heteroscedasticity. Each non-managed European factor is annualized in percentage per year by multiplying monthly non-managed European factors by 12. *, **, and *** denotes the 10%, 5% and 1% significance level, respectively.

Panel A: Regression of factor returns on lagged volatility				
	$EMKTRF^{nm}$	$ESMB^{nm}$	$EHML^{nm}$	$EMOM^{nm}$
RV_{t-1}	-0.01	-0.12**	-0.13	-0.24
(t)	(-0.64)	(-2.29)	(-1.30)	(-1.20)
Observations	316	316	316	316
R^2	0.002	0.01	0.004	0.020
Panel B: Correlation between factor returns and lagged volatility				
RV_{t-1}	-0.04	-0.10	-0.06	-0.15**
Panel C: Regression of factor volatility on lagged volatility				
RV_{t-1}	0.31***	0.36***	0.80***	0.82***
(t)	(13.58)	(19.54)	(25.09)	(28.49)
Observations	316	316	316	316
R^2	0.41	0.54	0.65	0.67
Panel D: Correlation between factor volatility and lagged volatility				
RV_{t-1}	0.64**	0.73**	0.80**	0.82**

5.2.2 European single managed factors

Table VII provides the results obtained from the time-series regressions of the European managed factors on the non-managed factors. Panel A shows that all annualized alphas of managed factors are positive and statistically significant, except for the annualized alpha of market factor. The largest alpha belongs to managed momentum (alpha =19.72% and t=5.17). The annualized alphas for the managed size and value factor are 1.22% (t=2.03) and 4.08% (t=2.14), respectively.

The values of RMSE are large and range from 10.71 for the size factor to 67.91 for the momentum factor. Regarding the annualized AR, I obtain that the slope of the mean-variance efficient frontier is expanded by 0.39, 0.40 and 0.99 for the managed size, value, and momentum factor, respectively. In terms of Sharpe ratios, all managed Sharpe ratios are twice as large in contrast to the non-managed Sharpe ratios.

Panel B shows the regression results after adding the European version of the Fama-French three factors as control variables. The findings are relatively unchanged. The managed size, value and momentum factors continue to be positive and statistically significant whereas the managed market factor is still positive though statistically insignificant. For the robustness check, I split out the whole sample in two subsamples: a less risky one from 1990 to 2000 and a risky subsample from 2001 to 2016. As in the case of Germany in Table IV, the OECD based recession and crisis periods are used to categorize a volatile- and less-volatile period in the European analysis. The explanation for this is that most of the OECD based economic contractions which affected the German economy affected first the Euro Area as well.

Panel C reports the negative and statistically insignificant alphas of market, size, and value across the less volatile 1990-2000 subperiod. It is not surprising that the alpha of managed momentum factor is positive and significant across this period. However, the striking result is that it is even higher (alpha=21.93%) when comparing with the annualized alpha for the momentum factor of the whole period. It could be that even during the less volatile period, the movements of momentum's volatility were large and fluctuating. Panel C also reports the robustness check for the risky 2001-2016 subperiod. As expected, all managed factors have annualized alphas which are positive and statistically significant, except for the managed market factor. Overall, it seems that implementing the VMP strategy on the European factors leads to high risk-adjusted expected returns for size, value and momentum factors but not for the market factor.

Table VII

Single managed factors for Europe

From Table VII, one can observe the results of the monthly time-series regressions for the European managed factor (F_t^m) on their respective non-managed factor (F_t^{nm}). F_t^m is calculated by scaling its respective non-managed factor by the inverse of its one-month-lagged realized variance (c/RV_{t-1}^2). C is a scalar so that the managed factor has got the same unconditional standard deviation as the non-managed factor. R^2, root mean squared errors (RMSE) and annualized appraisal ratios (AR) are reported in the table. Annualized AR is calculated by $\sqrt{12} * (\text{alpha}/\text{RMSE})$. Panel B reports the regression results when using the three factors of Fama and French as control variables. Panel C shows the annualized alphas for two different subperiods. Moreover, t-values are in parenthesis and based on standard errors which are adjusted for heteroscedasticity. The sample period extends from July 1990 to December 2016. Each factor is annualized in percentage per year by multiplying monthly factors by 12. *, **, and *** denote significance at the 10%, 5% and 1% significance level, respectively.

Panel A: Individual regressions				
	$EMKTRF^m$	$ESMB^m$	$EHML^m$	$EMOM^m$
Alpha	1.21	1.22**	4.08**	19.72***
(t)	(0.92)	(2.03)	(2.14)	(5.17)
$EMKTRF^{nm}$	0.29***			
(t)	(7.30)			
$ESMB^{nm}$		0.45***		
(t)		(12.82)		
$EHML^{nm}$			0.87***	
(t)			(8.47)	
$EMOM^{nm}$				0.63***
(t)				(5.96)
Sh^{nm}	0.01	-0.10	0.05	0.17
Sh^m	0.05	0.15	0.12	0.33
RMSE	21.40	10.71	34.70	67.91
Annualized. AR	0.19	0.39	0.40	0.99
Observations	317	317	317	317
R^2	0.40	0.56	0.35	0.17
Panel B: Alphas controlling for Fama and French three factors				
Alpha	1.22	1.06*	3.72**	17.27***
(t)	(0.94)	(1.75)	(1.98)	(4.65)
Panel C: Alphas for subsample periods				
1990 – 2000	-0.20	-0.10	-3.90	21.93***
(t)	(-0.07)	(-0.11)	(-1.14)	(2.89)
2001 – 2016	1.64	2.00***	9.36***	17.18***
(t)	(1.46)	(2.58)	(4.48)	(5.43)

5.2.3 European managed MVE multifactor portfolio regressions

The European non-managed MVE portfolios are constructed using the same technique as in the case of the German analysis. Panel A in Table VIII presents the evidence that all annualized alphas of the European managed MVE multifactor portfolios are large and statistically significant. The largest annualized alpha belongs to the $EHMO^m$ (alpha=3.01%, t=6.85), whereas the lowest annualized alpha belongs to the $EFF3^m$ (alpha=0.95%, t=2.07). The annualized AR are also high and range from 1.19 for $EHMO^m$ to 0.39 for $EFF3^m$. All managed Sharpe ratios present a slightly increase than the non-managed Sharpe ratios.

Panel B reports the annualized alphas when regressing across subsample periods. While I expect across the less volatile subperiod either negative or close-to-zero alphas, most of the MVE portfolios' results present positive and statistically significant annualized alphas, especially for $EMKMO^m$ (alpha=2.73, t=3.84) and $EHMO^m$ portfolio (alpha=2.77, t= 4.44). Only the $EFF3^m$ portfolio turns out negative and statistically insignificant (alpha = - 0.89). Managed EC^m portfolio generates the lowest positive annualized alpha of 0.71% (t=1.68).

These results might indicate that the less risky subperiod presented a high variation in the European stock volatility. Analyzing with more detail, however, all combinations of portfolios that present positive and significant alphas are those including the momentum factor. As shown in Table VII, the annualized alpha of managed momentum factor shows a very large percentage, suggesting that the 90s were also characterized by high variations in momentum's volatility in the European markets. As a result, the contribution of momentum volatility in the MVE portfolio construction is significant. As expected, all alphas are positive and significant across the subsample 2001-2016.

5.3 Replication of the VMP strategy: U.S. factors

I replicate the main results of Moreira and Muir (2016a) by using the same U.S. factors. The same methodology to construct managed and non-managed factors and MVE portfolios is implemented. The objective is to compare my results of the single factor and multifactor portfolio regressions to the findings of the original paper. Furthermore, since the work of Moreira and Muir (2016a) is a

Table VIII

Managed MVE multifactor portfolios for Europe

Panel A contains the monthly output of the time-series regressions of the managed MVE portfolios on the non-managed MVE portfolios from July 1990 until December 2016. The managed MVE portfolio is the non-managed MVE portfolio scaled by the inverse of its respective MVE-one-month-lagged realized variance ($c/RV^2_{MVE,t-1}$). C is a scalar so that the managed MVE portfolio has the same unconditional variance as the non-managed MVE portfolio. Managed and non-managed Sharpe ratios, RMSE and annualized AR are reported in the table. The annualized AR is calculated by $\sqrt{12} * (\text{alpha}/\text{RMSE})$. Panel B reports the alphas when running regression across two subsample periods. I perform the two-sided test and t-values are in parentheses, t-values (t) are based on standard errors that are adjusted for heteroscedasticity. Each non-managed MVE portfolio is annualized in percentage per year by multiplying monthly non-managed MVE portfolio by 12. *, **, and *** denotes the 10%, 5% and 1% significance level, respectively.

Panel A: Multifactor regressions					
	$EMKTRF^m$	$EMKMO^m$	$EHMO^m$	$EFF3^m$	EC^m
Alpha	1.21	2.06***	3.01***	0.95***	1.36***
(t)	(0.92)	(5.27)	(6.85)	(2.07)	(4.34)
$EMKTRF^{nm}$	0.29***				
(t)	(7.30)				
EMKMO		0.17***			
(t)		(9.45)			
EHMO			0.20***		
(t)			(7.15)		
$EFF3^{nm}$				0.26***	
(t)				(9.91)	
EC^{nm}					0.25***
(t)					(10.17)
SR^{nm}	0.01	0.42	0.46	0.19	0.50
SR^m	0.05	0.51	0.57	0.22	0.54
RMSE	21.40	6.97	8.68	8.35	6.32
Annualized AR	0.19	1.02	1.19	0.39	0.74
Observations	317	317	317	317	317
R^2	0.40	0.40	0.23	0.38	0.43
Panel B: Alphas for subsample periods					
1990-2000	-0.20	2.73***	2.77***	-0.89	0.71*
(t)	(-0.07)	(3.84)	(4.44)	(-1.29)	(1.68)
2001-2016	1.64	1.65***	2.58***	2.16***	1.74***
(t)	(1.46)	(3.74)	(5.18)	(3.74)	(4.28)

country-specific study, it will serve as basis for the comparisons with the German analysis of this thesis.

5.3.1 Time-series volatility by factor using U.S. data

Figure III plots the U.S. monthly time-series realized volatility of the non-managed factors $MKTRF^{nm}$, SMB^{nm}, HML^{nm}, MOM^{nm}, RMW^{nm} and CMA^{nm}. Bars in shaded grey present some NBER based recession and great depression periods. All factors, especially market and momentum presented exceptionally high realized volatility at the beginning of the analysis, which includes the Great Depression. Thereafter volatility dropped significantly but increased at the end of the 1930's.

Throughout almost the next 40 years, the volatility of the factors varied to a much lesser extent, which coincided with a relatively less volatile period of the capital market in the U.S. Thereafter, several economy contractions appear to have affected the American economy, which in turn caused variations of the volatility of the risk factors in large magnitude. As in the case of Germany, the movements on U.S. factor volatility follows a similar business cycle pattern over time.

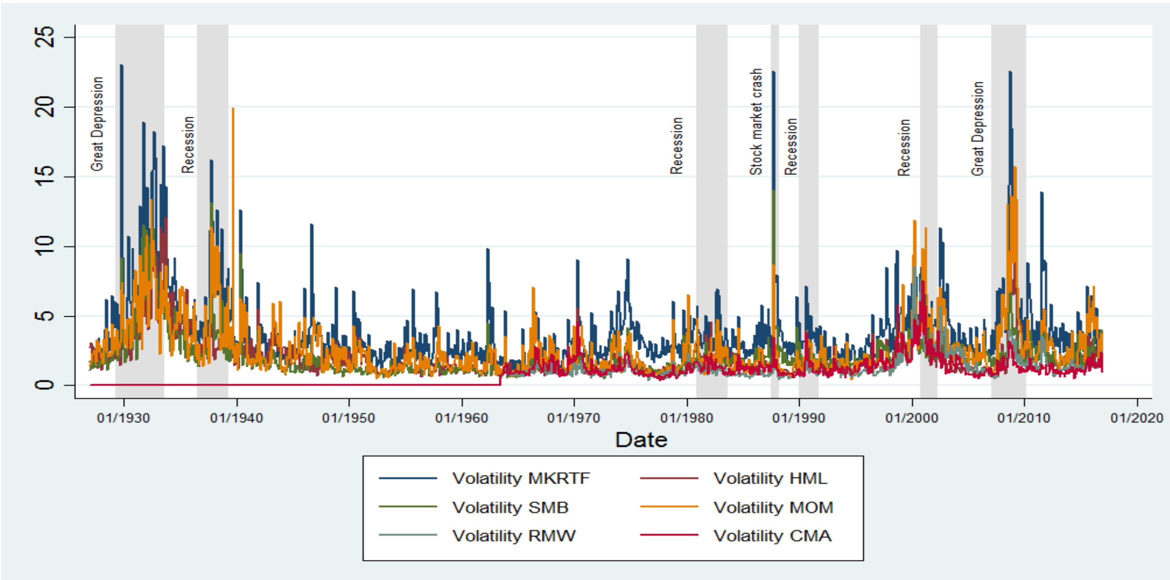
In general, U.S. market and momentum factors are the riskiest ones. This is consistent with Panel C in Table 1, where the standard deviations of these U.S factors are the largest for the entire period. Similarly, (see, Figure 1) the German market and momentum factors are also on average the riskier factors during contractions. Since the German managed momentum factor is the only one presenting statistically significant alphas, I find interesting to compare the momentum volatility movements in both countries.

Despite the dates of the U.S. NBER business cycle differ strongly from the German OECD expansion and contraction dates, the financial crisis 2008-2009 affected both countries almost at the same time. While German and U.S momentum strategy performed remarkably well in the past, it performed extremely poor during the last financial crisis. In 2009, the U.S. momentum factor experienced one of the most meaningful crashes. Likewise, the German momentum factor saw large drawdowns, which caused the crash of the German momentum in the same year. This is consistent with the work of Baltzer, Jank, and Smajlbegovic, (2015). They find that the course and

Figure III

U.S. factor volatility and business cycle

Figure III shows the U.S. monthly time-series realized volatility of the factors $MKTRF^{nm}$, SMB^{nm}, HML^{nm} and MOM^{nm} from January 1927 to December 2016. The time-series realized volatility of the factors RMW^{nm} and CMA^{nm} starts in July 1963 and finishes on December 2016. Bars in shaded grey present some NBER based recession and great depression periods. All U.S. factors present a similar business cycle pattern, i.e. the volatility movements across factors follow a similar pattern. Note that the U.S. NBER business cycle dates differ strongly from the German OECD expansions and contraction dates.



Source: NBER based recession indicators.

magnitude of U.S. and German momentum crashes coincided significantly during the last financial crisis.

5.3.2 U.S. single managed factors

In Panel A of Table IX, I replicate the U.S. factor regressions of Moreira and Muir (2016a) from January 1927 until December 2016. Moreira and Muir (2016a) find that all annualized alphas are positive except the one for the managed size factor. In my regressions, I obtain the same expected signs for the annualized alphas for all managed factors. The authors also find that most of the alphas are significant. In my regressions, five out of eight alphas are statistically significant namely the alpha for the market, momentum, profitability, return of equity and investment factor.

With respect to the alpha magnitudes, most of my annualized alphas are higher than those reported by Moreira and Muir (2016). The authors highlight the regression results of the market factor because managing this factor would be easily available for the average investor. They obtain a managed market's alpha of 4.86%, whereas I find a statistically significant alpha equal to 2.89% (t = 3.34).

Furthermore, the authors point out that the managing momentum factor would be difficult strategy for the average investor. This is consistent with an extensive body of literature which assumes that investing in momentum needs the professional experience of investment managers (see, e.g., Sias, 2005). Whereas I find a statistically significant alpha for the momentum factor equal to 15.24%, Moreira and Muir (2016a) report an annualized alpha of 12.51%. In my results, most of the RMSE's values are higher than those of the original regressions. Panel B shows the annualized alphas when controlling for Fama and French three factors. My results change slightly compared to the alphas of Panel A. Again, most of the annualized alphas are positive and statistically significant.

To facilitate comparison between the U.S. and the German single factor regressions, in Panel C, I use the same sample period as in the German analysis in Table IV. More specifically, I use the period from January 1990 to June 2016 to regress the U.S. managed market, size, value, and momentum factor on their respective non-managed factor[19]. The expected signs are obtained. The U.S alpha of managed size factor is negative and those of the managed market, value and momentum factor are positive.

The main differences lie in the level of significance and magnitude. For example, while the German alpha for the managed market factor is very low (alpha = 0.46%) and insignificant (t = 0.27), the U.S. alpha for market is higher (alpha = 2.39%) and statistically significant (t = 2.30). The alphas of the U.S and German momentum are statistically significant, although both presents a very different magnitude (U.S. alpha= 13.28% and German alpha= 8.14%).

[19] To make comparison feasible, I run a time-series regression of U.S. managed factors on U.S. non-managed factors by using the same German sample period and compering the results only for market, value, size and momentum.

Table IX

U.S. single risk factor regressions

Panel A shows the monthly time-series regressions of the U.S. managed on their respective non-managed factors. Each factor is annualized in percentage per year by multiplying monthly factors by 12. RMSE, and annualized AR are reported in the table. Panel B reports the alpha results when the three factors of Fama and French are used as controlling variables. Panel C shows the annualized alphas from the period January 1990 to June 2016 to make the respective comparisons with the German single factor analysis. I perform the two-sided test and t-values are in parentheses, t-values are based in standard errors which are adjusted for heteroscedasticity. *, **, and *** denote the 10%, 5% and 1% significance level, respectively.

Panel A: Individual regressions								
	$MKTRF^m$	SMB^m	HML^m	MOM^m	RMW^m	CMA^m	ROE^m	IA^m
alpha	2.89***	-0.85	2.18	15.24***	4.41***	0.09	7.85***	1.60*
(t)	(3.34)	(-0.68)	(1.57)	(7.41)	(2.93)	(0.09)	(5.21)	(1.95)
$MKTRF^{nm}$	0.35***							
(t)	(12.67)							
SMB^{nm}		0.87***						
(t)		(7.63)						
HML^{nm}			0.76***					
(t)			(7.58)					
MOM^{nm}				0.57***				
(t)				(7.05)				
RWM^{nm}					1.02***			
(t)					(7.03)			
CMA^{nm}						1.02***		
(t)						(14.04)		
ROE^{nm}							1.01***	
(t)							(10.08)	
IA^{nm}								0.92***
(t)								(13.53)
RMSE	28.71	42.85	47.74	61.15	37.68	25.89	36.65	20.75
Annualized AR	0.59	-0.06	0.15	0.86	0.40	0.01	0.74	0.26
N	1079	1079	1079	1079	641	641	599	599
R^2	0.39	0.38	0.31	0.22	0.35	0.48	0.42	0.50
Panel B: Alphas controlling for the Fama and French three factors								
Alpha	3.26***	-0.46	3.15**	12.84***	5.88***	-0.47	8.43***	1.00
(t)	(3.77)	(-0.37)	(2.26)	(6.83)	(3.86)	(-0.47)	(5.53)	(1.19)
Panel C: Individual regressions, January 1990-June 2016								
Alpha	2.39**	-0.61	0.18	13.28***	2.77*	-0.33	8.95***	1.08
(t)	(2.30)	(-0.60)	(0.08)	(4.69)	(1.77)	(-0.27)	(5.02)	(1.01)

5.3.3 U.S. MVE managed multifactor portfolios

Moreira and Muir (2016a) construct six different combinations of factors to form the non-managed MVE multifactor portfolios. The authors show that managing these portfolios, all regressions produce positive and significant alphas and substantial Sharpe. I run time-series regressions using almost the same sample period and I construct the same six U.S. MVE portfolios as in the original paper.

Panel A, in Table X shows that all alphas of the U.S. managed MVE portfolios are positive and statistically significant, except the one for the Fama and French five-factor portfolio (alpha= 0.13, t=1.43). Moreover, the magnitudes of all alphas are much lower compared to the original results[20]. While the original annualized AR range from 0.33 to 0.91, my regressions present annualized AR which range from 0.35 to 0.82. As mentioned in section 5.1.4. the non-managed Sharpe ratios might be overstated relative to the real ones, therefore managed Sharpe ratios could have very low values[21]. My results show that the managed Sharpe ratios for all MVE portfolios increase though at a modest level.

To compare the German and U.S. MVE portfolio regressions, I use the same sample period from January 1990 to June 2016 as in the German equivalent analysis. Looking at Panel B, it is only possible to compare the alphas for the German and U.S. Fama and French three-factor portfolio ($GFF3^m$ vs $FF3^m$) and the German and U.S. Carhart four-factor portfolio (GC^m vs C^m).

While the alpha of German managed Fama and French three-factor portfolio is large and statistically significant (alpha = 4.44), the U.S equivalent alpha is 1.33% and statistically insignificant (t=0.70). In the case of the Carhart four-factor portfolio, the magnitude of German alpha is much lower (alpha = 0.84%) than the U.S alpha (alpha = 1.84%). Both annualized alphas' results are statistically significant.

[20] See Table 2 in the original paper of Moreira and Muir (2016): "Volatility Managed Portfolios".

[21] This point is neither theoretically nor mathematically explained by the authors. Therefore, my assumption of understated managed Sharpe ratios just follows the referent insights of Moreira and Muir (2016a).

Table X

U.S. MVE multifactor regressions

Panel A reports the monthly output for the time-series regressions of the managed MVE portfolios on the non-managed MVE portfolios using U.S data. Managed and non-managed Sharpe ratios, RMSE and annualized AR are reported in the table. The annualized AR is calculated by $\sqrt{12} * (\text{alpha}/\text{RMSE})$. Panel B reports the annualized alpha results from the period January 1990 to June 2016 to compare them with the German alphas of Table VIII. I perform the two-sided test and t-values are in parentheses and based on standard errors that are adjusted for heteroscedasticity. Each non-managed MVE portfolio is annualized in percentage per year by multiplying monthly non-managed MVE portfolio by 12. *, **, and *** denotes the 10%, 5% and 1% significance level, respectively.

Panel A: Multifactor regressions							
	$MKTRF^m$	$FF3^m$	C^m	$FF5^m$	$FF5MO^m$	HXZ^m	$HXZMO^m$
alpha	2.89***	4.11***	2.04***	0.13	0.51***	0.39***	0.42***
(t)	(3.34)	(3.39)	(7.13)	(1.43)	(4.17)	(5.44)	(5.31)
$MKTRF^{nm}$	0.35***						
(t)	(12.67)						
$FF3^{nm}$		0.49***					
(t)		(6.71)					
C^{nm}			0.25***				
(t)			(13.13)				
$FF5^{nm}$				0.21***			
(t)				(12.11)			
$FF5MO^{nm}$					0.20***		
(t)					(10.31)		
HXZ^{nm}						0.13***	
(t)						(11.71)	
$HXZMO^{nm}$							0.13***
(t)							(11.36)
SR^m	0.20	0.21	0.42	0.40	0.50	0.58	0.50
SR^{nm}	0.11	0.19	0.38	0.40	0.48	0.54	0.45
RMSE	28.71	41.05	10.27	2.66	2.98	1.69	1.76
Annualized AR	0.59	0.35	0.69	0.17	0.60	0.79	0.82
N	1079	1079	1079	641	641	599	599
R^2	0.39	0.17	0.25	0.48	0.40	0.48	0.47
Panel B: Multifactor regressions January 1990-June 2016							
Alpha	2.39**	1.33	1.84***	0.21*	0.53***	0.45***	0.48***
(t)	(2.30)	(0.70)	(4.15)	(1.90)	(3.70)	(5.04)	(4.89)

5.4 Managed GARCH (1,1) regressions

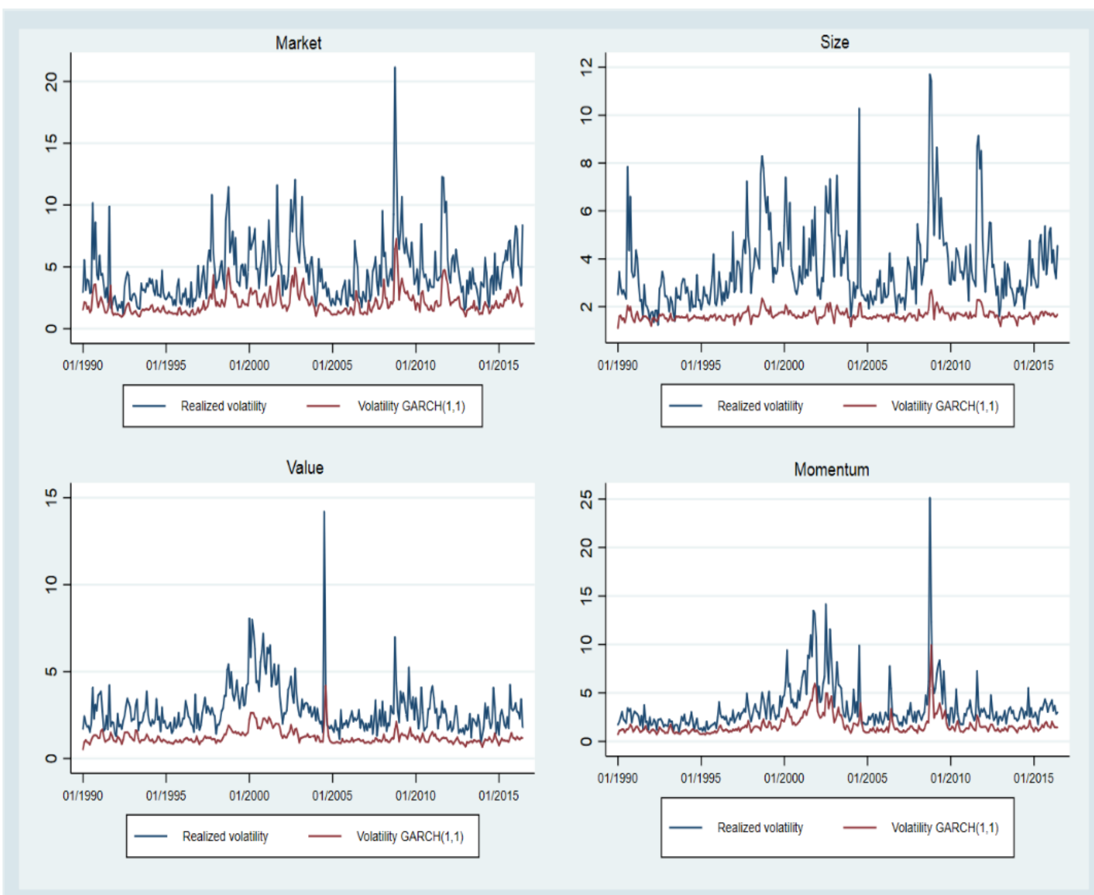
5.4.1 Forecasted volatility vs. realized volatility

The forecasted volatility GARCH (1,1) takes on significantly lower values than the realized volatility does during the 318-month period. This means that even if the GARCH (1,1) volatility follows a similar pattern as the realized volatility, the movements of the GARCH (1,1) volatility

Figure IV

GARCH (1,1) volatility vs. realized volatility

Figure IV illustrates the time-series of the monthly GARCH (1,1) volatility vs. the realized volatility of the German market, size, value and momentum factors. The volatility analyzed covers the 26-year period. It can be clearly seen that the GARCH (1,1) volatility and the realized volatility follow a similar pattern, but the GARCH (1,1) volatility tends to have lower spikes.



present less dramatic fluctuations, especially during recessions. However, the movements of the GARCH (1,1) volatility and the realized volatility of the German size factor are different. While the realized volatility of size shows more spikes, the GARCH (1,1) volatility is smoother. This is clearly depicted in Figure IV, where the red line represents the GARCH (1,1) volatility and the blue line depicts the realized volatility of the German market, size, value and momentum factor returns.

5.4.2 Single managed GARCH (1,1) factors

I assess the performance of the VMP strategy by scaling the non-managed factors by the inverse of the one-month-lagged GARCH (1,1) rather than the inverse of the one-month-lagged realized variance. Expecting better results than those when using the realized variance approach (see Table IV and VII), I manage the German and European market, size, value and momentum factors.

The German (Table XI) and the European (Table XII) regression results show that compared with market, value and size, the managed momentum factor presents the highest and the only significant alpha. The German and European managed GARCH (1,1) alphas of momentum reach values of 32 % and 61%, respectively. These alphas' values triple the alphas of the German and European managed momentum when using the realized variance approach.

Under the GARCH (1,1) analysis, all root mean squared errors are large, and the German and European annualized appraisal ratios for momentum are 0.91 and 1.32, respectively. The managed Sharpe ratios present, however, a slight increase. The managed Sharpe ratios of momentum are 0.31 and 0.41 for Germany and Europe, respectively. These results suggest that managing the volatility of momentum implementing the GARCH (1,1) approach offers higher risk-adjusted returns compared to managing momentum using the realized variance procedure in the German and European stock markets.

Table XI

German single managed factors, GARCH (1,1) approach

Table XI shows the monthly time-series regressions for the German managed factors on their respective non-managed factors. In the GARCH (1,1) approach, the GARCH (11) managed factor (F_t^g) is calculated by scaling the monthly non-managed factor (F_t^{ng}) by the inverse of its one-month-lagged GARCH (1,1) or σ_{t-1}^2, such as c/σ_{t-1}^2. C is a scalar so that the managed factor has got the same unconditional standard deviation as the non-managed factor. R^2, root mean squared errors (RMSE) and annualized appraisal ratios (AR) are reported in the table. Annualized AR is calculated by $\sqrt{12} * (\text{alpha}/\text{RMSE})$. The respective t-values are in parenthesis and based on standard errors which are adjusted for heteroscedasticity. The sample period extends from January 1990 to June 2016. Each factor is annualized in percentage per year by multiplying monthly factors by 12. *, **, and *** denote 10%, 5% and 1% significance level, respectively.

	Single regressions			
	$GMKTRF^g$	$GSMB^g$	$GHML^g$	$GMOM^g$
Alpha	3.16	0.63	2.47	32.40***
(t)	(0.83)	(0.89)	(1.06)	(4.67)
$GMKTRF^{ng}$	1.37***			
(t)	(13.51)			
$GSMB^{ng}$		1.28***		
(t)		(47.37)		
$GHML^{ng}$			1.98***	
(t)			(13.12)	
$GMOM^{ng}$				1.44***
(t)				(7.63)
Sh^{ng}	0.07	-0.23	0.07	0.17
Sh^g	0.08	0.23	0.09	0.31
RMSE	66.26	12.83	42.46	122.48
Annualized AR	0.16	0.16	0.20	0.91
Observations	317	317	317	317
R^2	0.64	0.94	0.75	0.41

Table XII

European single managed factors, GARCH (1,1) approach

From Table XII can be observed monthly time-series regressions for the European managed factors on their respective non-managed factors. R^2, root mean squared errors (RMSE) and annualized appraisal ratios (AR) are reported in the table. Annualized AR is calculated by $\sqrt{12} * (\text{alpha}/\text{RMSE})$. The sample period extends from July 1990 to December 2016. Each factor is annualized in percentage per year by multiplying monthly factors by 12. *, **, and *** denote 10%, 5% and 1% significance level, respectively.

Single regressions				
	$EMKTRF^g$	$ESMB^g$	$EHML^g$	$EMOM^g$
Alpha	4.03	-0.13	2.34	61.22***
(t)	(1.11)	(-0.05)	(0.65)	(6.37)
$EMKTRF^{ng}$	1.17***			
(t)	(10.60)			
$ESMB^{ng}$		2.22***		
(t)		(14.52)		
$EHML^{ng}$			5.15***	
(t)			(16.57)	
$EMOM^{ng}$				2.45***
(t)				(7.09)
Sh^{ng}	0.01	-0.10	0.05	0.17
Sh^g	0.06	0.01	0.06	0.42
RMSE	60	45.70	68.44	159.06
Annualized AR	0.23	-0.01	0.12	1.32
Observations	317	317	317	317
R^2	0.57	0.63	0.82	0.35

5.5 Discussion

The VMP strategy can be easily implemented, by scaling risk factors and MVE multifactor portfolios by the inverse of the one-month-lagged realized variance. Since risk exposure is not constant over time (see, e.g., Schwert, 1989) and volatility is persistent, though less persistent than expected returns (see, e.g., Moreira and Muir, 2016a), the VMP strategy seems to offer attractive risk-adjust returns to investors. In fact, investors should reassess the non-dynamic view of the traditional risk-return trade-off despite the conventional financial wisdom. Selling when equity stocks are expected to be volatile and buying when the market starts to recover seems to be a smart volatility timing strategy.

The VMP strategy depends on the value of the prior month realized variance of each factor to increase or decrease exposure in the next month on the stock market. Hence, the realized variance is the key variable in my research. The evidence in this thesis suggests that the German managed momentum factor is by far the only strategy which provides attractive risk-adjusted returns. Institutional investors who hold the momentum factor should apply the VMP strategy when the German capital market experiences large movements in volatility. For example, exiting the German market during the second and third quarter in 2009, the VMP strategy would have prevented a loss of 42.01% in terms of cumulative returns (see, e.g., Baltzer, Jank, Smajlbegovic, 2015).

While the analysis on German single factors is a country-specific study, the case of European factors is a regional-specific one. Therefore, a direct comparison might not be a rigorous analysis. However, drawing a theoretical comparison between country and regional risk factors, might help investors to obtain gains from international diversification. I find that most managed European factors outperform non-managed European factors. Therefore, my results entail the fact that timing risk factors from a highly integrated region, such as the Euro Area, benefits investors as well.

As in the German. case, investing in the European managed momentum factor provides the best performance among factors. This is true not only for the entire sample analyzed but also for the controlling regressions and robustness checks. My results corroborate the findings of Moreira and

Muir (2016a) as well as results of similar volatility timing strategies on momentum such as those of Barroso and Santa-Clara (2015).

Despite of the pervasive performance when managing momentum, timing momentum deserves a special attention. Since the VMP strategy requires monthly rebalancing, replicating and timing momentum would require large amount of capital and professional investment management. Hence, the suggestion that managing factors is a practicable strategy for average investors should be interpreted with caution. Curiously, one of my findings is somewhat at odds with the results of Moreira and Muir (2016a). While the U.S. market factor shows superior performance across different regressions, my data provide no evidence that managing market factor offers significant risk-adjusted returns. Even when using control variables or regressing across different subsample periods, the alphas of managed market factor continues to be either close-to-zero or negative and statistically insignificant.

Moreira and Muir (2016a) show mathematically that the time-series of the alpha is a measure of opposite movements between risk-return trade-off and variance. This suggests that when the relation between returns and variance is strong alphas should be close-to-zero in volatile periods. However, when the risk-return relation is weak, alphas should be positive even in risky periods. Since the alphas are increasing in the volatility of volatility, the alphas should present higher values during risky times than those alphas evaluated in less turbulent periods.

Finally, the alphas of the German and European managed momentum factor are higher when scaling the non-managed momentum factor with the one-month-lagged GARCH (1,1) rather than using the realized variance approach. This means that the performance of momentum when using the VMP strategy is pervasive across German and European data and when implementing different measures of variance.

6 Conclusion

The time-series analysis in this thesis demonstrates only the German managed momentum factor and most of the European managed factors present large and statistically significant alphas as well as higher managed Sharpe ratios than their non-managed peers. Moreover, all managed German and European MVE multifactor portfolios outperform passive strategies, expanding the mean-variance efficient frontier.

Two findings are notable: First, the managed market factor leads to inconsistent results and shows only close-to-zero and insignificant alphas. Second, the managed momentum factor presents pervasively large and statistically significant alphas and substantial Sharpe ratios. In addition, my study provides some suggestions: The VMP strategy should be applied only by institutional investors when considering to time momentum's volatility in the German and European markets. Furthermore, when using the GARH (1,1) approach, only the German and European momentum managed factor provides better performance in terms of annualized alphas and Sharpe ratios than managing German and European factors with the realized variance approach.

Although this thesis research has been carefully conducted, I am still aware of some limitations and potential sources of bias. First, the lack of prior research studies on the VMP strategy impedes comparison and economic interpretation of the positive alphas. Second, the small size of the monthly German and European factor data might affect the significance of the regression results. This issue can be addressed by a careful constructing of risk factors for longer periods. Third, since the VMP strategy might be improved using more realistic volatility measures, other sophisticated GARCH models than the GARCH (1,1) could be implemented.

7 Appendix A

7.1 Test for normality

To test whether the German, European, and U.S factor and MVE portfolio returns follow a normal distribution, I use the Jarque-Bera (JB) test (see, e.g., Jarque and Bera, 1980). Referring to the results in Table XIII, all the P-values for the German, European and U.S factors and MVE portfolios are below 0.05, except for the German size factor. Thus, I can reject the null hypothesis of normality for all risk factors and MVE portfolios, with the exception of GSMB. The χ^2_2 statistic has been adjusted because of the small sample size of German and European factors. Critical values that are extremely large are not reported and are signalled by “-“.

7.2 Test for heteroscedasticity

To test the existence of heteroscedasticity on the distribution of factor returns, I use the test signaled by Breusch and Pagan (1980). The null hypothesis that the authors propose implies that in a regression model, the variance of error term or residual is constant, i.e. the variance presents homoscedasticity. I find a very clear rejection of the null hypothesis for all German and European factors, except for the German size factor ($GSMB^{nm}$). The results of the Breusch-Pagan test are not reported. It should be noted that the heteroscedasticity of all regressions using the realized variance and GARCH approach, has been corrected by adjusting the standard errors.

7.3 Risk and return dynamics of the German and European market factors

I investigate the dynamics of risk and return of the non-managed market factor. To do so, I use the impulse-response analysis by using a vector autoregression with one lag or VAR (1), which allows analyzing whether the monthly one-month-lagged realized variance predicts factor returns. Figure V shows the impulse-response effect after one-volatility shock i.e. one-standard deviation increase of the log of the one-month-lagged realized variance. Panel A and Panel B report the impulse-response analysis of the log of the one-month-lagged RV^2 of the market factor on the log of the market returns for Germany and Europe, respectively.

Table XIII

Normal distribution test

Table XIII presents the Jarque-Bera test for normality. This a goodness-of-fit test which jointly combines the skewness and kurtosis summary's outputs in one test. If the P-value is larger or equal to 0.05, the null hypothesis is not rejected. χ^2_2 values have been adjusted because the German and European datasets are based on a small data sample (318 observations). Panel A shows the statistical results for the German, European and U.S factor returns. Panel B reports the test outputs for the MVE portfolios. "-" denotes that the values are extremely large and they are not reported.

	Observations	Prob. Skewness	Prob. Kurtosis	Adjusted χ^2_2	Prob. > χ^2_2
Panel A: Individual portfolios					
$GMKTRF^{nm}$	318	0.00	0.00	23.39	0.00
$GSMB^{nm}$	318	0.72	0.01	5.79	0.05
$GHML^{nm}$	318	0.00	0.00	30.23	0.00
$GMOM^{nm}$	318	0.00	0.00	-	0.00
$EMKTRF^{nm}$	318	0.00	0.00	24.14	0.00
$ESMB^{nm}$	318	0.53	0.00	7.26	0.02
$EHML^{nm}$	318	0.00	0.00	26.77	0.00
$EMOM^{nm}$	318	0.00	0.00	-	0.00
$MKTRF^{nm}$	1,080	0.00	0.00	-	0.00
SMB^{nm}	1,080	0.00	0.00	-	0.00
HML^{nm}	1,080	0.00	0.00	-	0.00
MOM^{nm}	1,080	0.00	0.00	-	0.00
RMW^{nm}	642	0.00	0.00	-	0.00
CMA^{nm}	642	0.00	0.00	28.95	0.00
ROE^{nm}	600	0.00	0.00	-	0.00
IA^{nm}	600	0.24	0.00	19.46	0.00
Panel B Multifactor portfolios					
$GMKMO^{nm}$	318	0.00	0.00	65.35	0.00
$GHMO^{nm}$	318	0.00	0.00	-	0.00
$GFF3^{nm}$	318	0.03	0.00	-	0.00
GC^{nm}	318	0.00	0.00	10.74	0.00
$EMKMO^{nm}$	318	0.00	0.00	38.90	0.00
$EHMO^{nm}$	318	0.43	0.00	28.71	0.00
$EFF3^{nm}$	318	0.48	0.00	8.05	0.01
EC^{nm}	318	0.07	0.00	13.19	0.00
$FF3^{nm}$	1,080	0.00	0.00	-	0.00
C^{nm}	1,080	0.00	0.00	-	0.00
$FF5^{nm}$	642	0.25	0.00	34.54	0.00
$FF5MO^{nm}$	642	0.00	0.00	69.58	0.00
HXZ^{nm}	600	0.47	0.00	39.97	0.00
$HXZMO^{nm}$	600	0.06	0.00	46.82	0.00

The top graphs of Panels A and B show that realized variance of the German and European factors spikes fist and then decreases in a consecutive series after one-volatility shock. This effect diminishes after seven or eight months approximately. This is consistent with the theory that variance is mean reverting. The impact on realized returns differs substantially from the impact onto variance. The bottom graph of Panel A shows that after one-volatility shock, the increase of German returns is by far weaker than that of its respective realized variance. The returns rise marginally and then decrease constantly. This effect dies out after approximately four or five months. In the bottom graph of Panel B, one can see that returns of the European market factor decrease marginally after an impact of one-volatility shock. After that, however, returns increase modestly and stay elevated for a longer period.

Overall, I present evidence that (i) the realized variance and realized returns of the German and European market factors are counter-cyclical. Furthermore, (ii) the realized variance is mean reverting. I show as well that after one-volatility shock (iii) German returns rise in a much lower magnitude than variance (iv) though this effect dies out after some months. The effect on European returns is quite different namely (v) the European returns decrease marginally, (vi) but then recover staying elevated for several months.

All my results concerning the dynamics of the realized variance are consistent with Moreira and Muir's (2016) findings, except for the outputs (iv) and (v). The authors show that U.S market returns increase much less on volatility impact but stay elevated for a longer period, whereas I find that the German and European market returns behave quite differently. Hence a possible reason why the alpha of German and European managed factors underperforms when comparing it with the alpha U.S managed market factor.

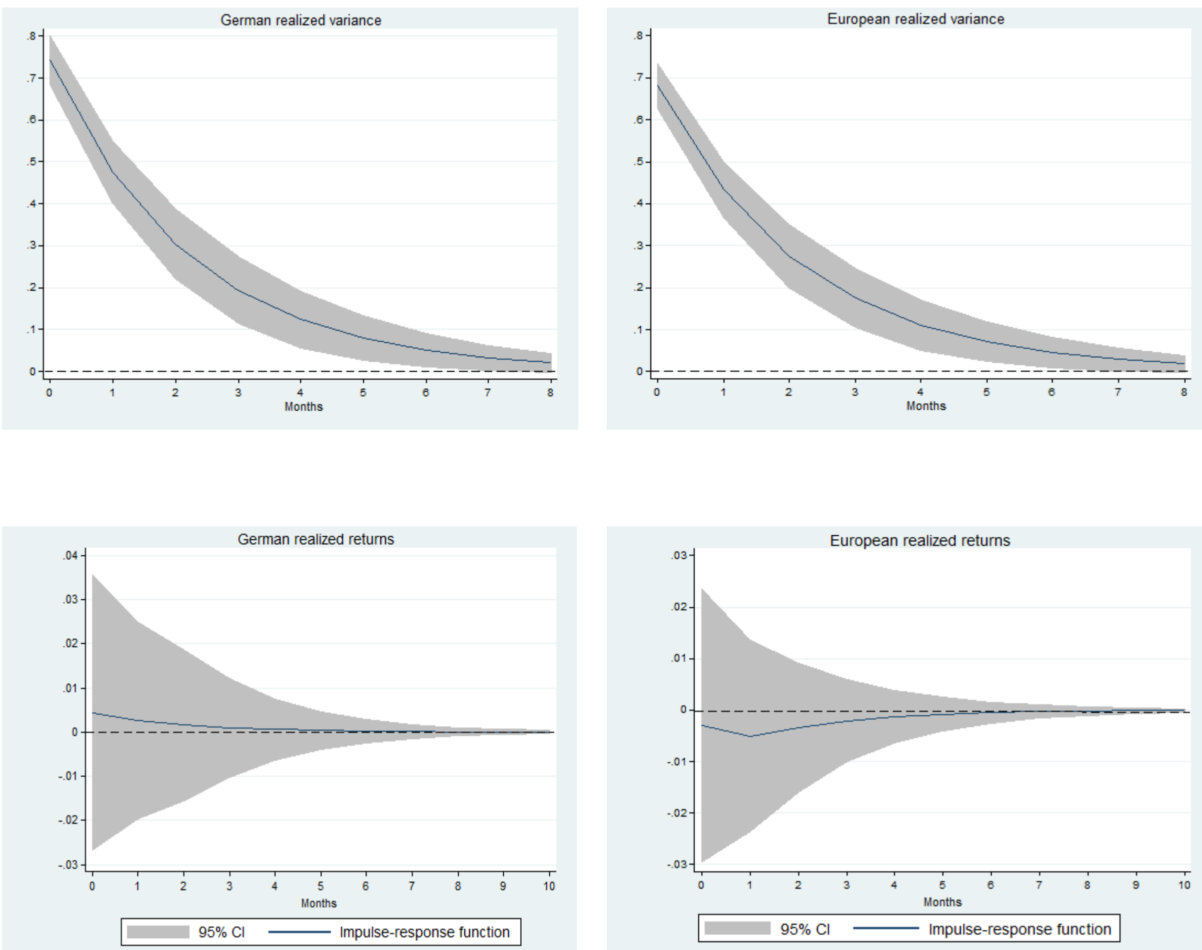
Figure V

Risk and return dynamics after one-volatility shock

Figure V depicts the impulse-response of the realized variance and realized returns of monthly German and European market factors after one- volatility shock to the realized variance. The impulse-response analysis uses a VAR (1) of realized variance and realized returns. The x-axis is in months. The shady bands represent the lower and upper bounds at 95% of confidence interval (CI). In both cases, the movements of German and European realized variance and realized returns are counter-cyclical. Moreover, the German and European market factor variances are mean reverting. The European market returns are more persistent than its variance.

Panel A: Variance and returns of the German market factor

Panel B: Variance and returns of the European market factor



7.4 Average realized volatility by subsample period

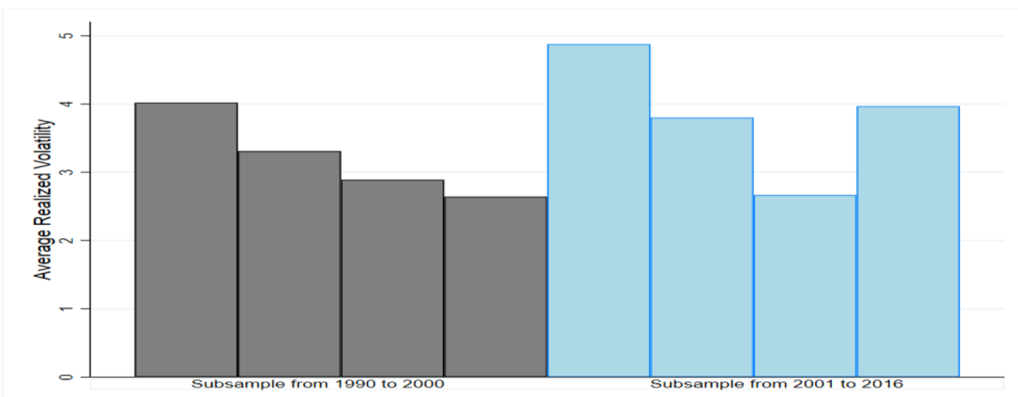
From left gray (blue) to right gray (blue) bar, Figure VI shows the average realized volatility of the market, size, value and momentum factors for each subperiod. The gray bars depict the less risky subperiod, whereas the blue bars represent the risky one. Figure VI reveals that the period from 1990 to 2000 is less volatile than the period from 2001 to 2016 on average.

Figure VI

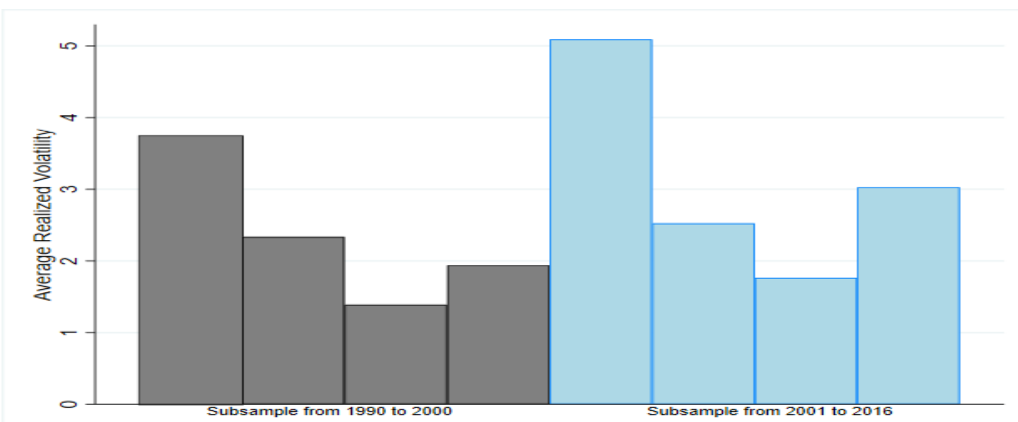
Average realized volatility per factor

Figure VI illustrates the average realized volatility of each risk factor for each subsample period. In Panels A and B, the less volatile (1990-2000) subperiod is differentiated by gray bars and the risky period (2001-2016) by blue bars. The first, second, third and fourth bar of each subperiod represents the average realized volatility of market, size, value and momentum, respectively.

Panel A: Average realized volatility of German risk factors



Panel B: Average realized volatility of European risk factors



7.5 Test for volatility clustering in the residuals

Since the seminal studies of Mandelbrot (1963) und Fama (1965), volatility clustering is a well-known phenomenon in the finance literature. This phenomenon describes that periods of high (low) volatility are followed by periods of high (low) volatility. I compute the residuals for each German (Table IV), European (Table VII) and U.S (Table IX) managed single factor regression, to determine whether the residuals are characterized by volatility clustering. I find that residuals for every regression model displays volatility clustering. For instance, Figure VII plots the monthly residual estimations obtained from the monthly time-series regression of managed momentum factor on the non-managed momentum factor for the German, European and U.S. data set.

Figure VII

Volatility clustering in residuals

Figure VII illustrates the residual estimations obtained from the time-series regression of the monthly managed momentum factor on the non-managed momentum factor. All models are estimated from 1990 to 2016. The upper, middle, and lower graphs show the residuals of the German, European and U.S regression models, respectively. The three graphs clearly depict the existence of periods with high volatility followed by periods of high volatility as well as periods of low volatility followed by periods of low volatility. The average monthly change is sometimes higher and sometimes close to zero, especially from 2000 onwards. The distribution of non-managed momentum factor is characterized by fat tails, which is reflected in the high significance of the JB test.

8 Appendix B

8.1 Acronyms

AR	Appraisal ratio
AMEX	American stock exchange
C	U.S. Carhart four factor portfolio
CI	Confidence interval
CAPM	Capital asset pricing model
CMA	U.S. Fama and French investment factor
CML	Capital market line
EEA	European Economic Area
EFF3	European Fama and French three factor portfolio
EHML	European value factor
EHMO	European value factor + European momentum factor
EMKMO	European market factor + European momentum factor
EMKTRF	European market factor
EMOM	European momentum factor
ESMB	European size factor
EU	European Union
FF3	U.S. Fama and French three factor portfolio
FF5	U.S. Fama and French five factor portfolio
FSE	Frankfurt Stock Exchange
GC	German Carhart four factor portfolio
GDP	Gross Domestic Product
GFF3	German Fama and French three factor portfolio
GHML	German value factor
GHMO	German value factor + German momentum factor
GMKMO	German market factor + German momentum factor
GMKTRF	German market factor
GMOM	German momentum factor
(G)ARCH	(Generalized) autoregressive conditional heteroscedasticity
GSMB	German size factor
HML	U.S. value factor
HXZ	Hou-Xue-Zhang four factor portfolio
HXZMO	Hou-Xue-Zhang four factor portfolio + U.S. momentum factor
IA	U.S. investment factor
JB	Jarque-Bera test
MKTRF	U.S. market factor
MOM	U.S. momentum factor
MVE	Mean-variance efficient
NASDAQ	National association of securities dealers automated quotations
NYSE	New York stock exchange
LM	Lagrange multiplier test
OECD	Organization for economic co-operation and development
RMSE	Root mean squared error
ROE	U.S. return on equity factor
RMW	U.S profitability factor
SMB	U.S. size factor
U.S.	Unites States
VAR	Vector autoregressive
VIX	U.S. volatility index
VMP	Volatility managed portfolios

8.2 List of tables and figures

8.2.1 Tables

8.2.2 Figures

9 References

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