Erasmus University Rotterdam
Erasmus School of Economics
22 August 2018

Outperforming the Global Market Portfolio with GDP-weighting?
A Matter of Factor Exposures

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A thesis submitted for the degree of
Master of Science in Financial Economics

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Abstract

This paper examines the performance of GDP-weighted indexes relative to their cap-weighted equivalents after adjusting for Size, Value, Momentum, Betting-Against-Beta, Quality-Minus-Junk, Profitability and Investment factor exposures. In addition, this paper includes a detailed overview of the annual factor exposures per unit of risk for these indexes. I find that the higher cumulative excess total returns and Sharpe ratios of GDP- relative to cap-weighted indexes are just a reflection of factor exposures. GDP-weighted indexes are significantly more positively exposed to Size, Value, Betting-Against-Beta and Profitability factors, while significantly more negatively exposed to the Quality-Minus-Junk and Investment factor. When adjusting the returns of a GDP-weighted index for these risk factor exposures, the GDP-weighted index does not generate positive alpha and therefore loses its superiority over cap-weighted indexes.

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Part I

Introduction

1 Motivation

Since the introduction of the CAPM model by Sharpe et al. in 1964, the implication that a market-capitalization-weighted index is the most mean-variance efficient[1] proxy of the market portfolio (Siegel, 2003; Arnott, Hsu, and Moore, 2005) formed the traditional investment approach supported by the majority of the financial industry (Hamza, Kortas, L'Her, and Roberge, 2007; Arnott, Kalesnik, Moghtader & Scholl, 2010).

However, investor interest increased in alternatively-weighted indexes based on economic size - measured by Gross Domestic Product (GDP), since these schemes historically outperformed the traditional capitalization-weighted index (Jun and Malkiel, 2007; MSCI Barra, 2010). As GDP-weighted indexes assign more weight to emerging markets (MSCI Barra, 2010; Deutsche Asset & Wealth Management, 2014; FTSE Russell, 2014), they have different risk factor exposures than cap-weighted indexes and potentially contribute to the mean-variance superiority of the GDP-weighted index.

This study provides an answer to the following main question: 'Do GDP-weighted indexes still significantly outperform their cap-weighted equivalents after adjusting returns for risk-factor exposures?'

The remainder of this paper is organized as follows. Section 2 provides the theoretical background for my research and is divided in five sub-sections: Section 2.1 describes Modern Portfolio Theory, Section 2.2 describes the Capital Asset Pricing Model, Section 2.3 describes the Efficient Market Hypothesis, Section 2.4 describes important market anomalies, and Section 2.5 introduces alternatively-weighted indexes. Next, Section 3 summarizes the benefits of GDP-weighted indexes compared to cap-weighted indexes. Section 4 summarizes the relevant research papers that I used to derive my hypotheses. In Section 5, I present my research questions and hypotheses. Section 6 and 7 contain the description of the data and methodology I used for conducting my research. In section 8-12, I test my hypotheses and discuss the results. Finally, Section 13 concludes and in Section 14 I discuss the limitations of my research.

[1] Mean-variance efficiency is defined as the optimal trade-off between risk and return (Markowitz, 1952)

2 Background

2.1 Modern Portfolio Theory

The popularity of cap-weighted indexes and the rise in alternatively weighted investment portfolios dates back to the year 1952 when Harry Markowitz published his article on Modern Portfolio Theory (MPT). MPT formed one of the first and oldest frameworks within the empirical field of investments used for portfolio construction and builds on the observations that investors are willing to invest their money in assets, of which there are several options they can choose from, and they do so according to their preferences (Versijp, 2018).

Modern Portfolio Theory assumes the typical investor is non-satiated, risk-averse and rational. Moreover, the investor's investment choice should be influenced solely by the asset's expected return, the changes in the price of the asset, and the relation between the assets. In other words, the investor should care about mean, variance and covariance only. In addition, the model assumes markets are not distorted by trading costs, inflation or taxes, all information is available at no cost, and investments are infinitely divisible (Markowitz, 1952; Versijp, 2018).

The investment decision as a trade-off between return and risk or mean and variance, forms the central tenet of Modern Portfolio Theory. It implies investors maximize their utility by minimizing risk at a given level of return or maximize returns at a given level of risk according to their level of risk aversion (Markowitz, 1952; Sharpe, 1964; Haugen and Baker, 1991).

For a single investor and an investment environment conforming with the MPT assumptions, MPT prescribes the efficient set of weight combinations of risky assets within an investment portfolio that are optimal for different levels of risk aversion, as graphically represented by the Efficient Frontier. The Efficient Frontier shows that diversification[2] increases investor utility.

2.2 The Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) - introduced by Sharpe (1964) - draws further upon the diversification and utility maximization concepts of Modern Portfolio Theory by introducing a risk-free asset to the model. As investors dislike risk, they can obtain an even higher utility by proportionally combining the portfolio of risky assets with a risk-free asset such that this combination matches their level of risk tolerance, as long as the MPT plus the following additional CAPM assumptions hold: All investors have the same expectations regarding the

[2] Diversification is defined as obtaining a greater return per unit of risk, or less risk per unit of return by adding more assets to the portfolio.

mean (returns), variances (risk) and covariances (diversification), they can lend and borrow at the risk-free rate, and capital markets are characterized by perfect competition (Versijp, 2018).

If these assumptions hold in reality, this implies all investors hold the same portfolio of risky assets. As utility is maximized when diversification is optimal, the CAPM claims all investors end up allocating their assets in proportion to the investable universe, which is proxied best by the capitalization-weighted market portfolio.

The capitalization-weighted market portfolio represents optimal diversification and therefore contains only non-diversifiable, systematic risk. For this reason, the CAPM claims it to be mean-variance efficient, which implies it should have the highest Sharpe ratio an investor could ever obtain (Hsu, 2006). Therefore, the market portfolio is also the tangency portfolio on the Efficient Frontier.

Risk-averse investors will allocate their capital amongst the risk-free asset and this tangency portfolio in such a way that it matches their level of risk tolerance. The line drawn between the investor's capital allocation to the risk-free asset and the tangency portfolio on the Efficient Frontier is called the Capital Market Line (CML).

An investor that chooses to allocate part of his capital towards risky assets other than the market portfolio should therefore compare the riskiness of his investments with a mean-variance efficient benchmark portfolio containing only systemic risk when assessing the value of the risky assets.

In the first place, taking on systemic risk requires compensation. When investing in the market portfolio, investors therefore require a market risk premium over the risk-free rate (Sharpe, 1964). Taking on more or less risk than that of the market portfolio by investing in other risky assets, should yield a return that is proportional to the riskiness of that asset relative to market risk. In the CAPM model, "beta" represents the measurement for the asset's relative riskiness with that of the market.

When the risk-free rate, the market risk premium and the asset's beta are known, one can calculate the asset's risk-adjusted required return on an asset by using the CAPM formula:

$$r_i = r_f + \beta_i (r_m - r_f) \tag{2.1}$$

where r_i is the investment's required return, r_f is the risk-free rate of return, β_i is the asset's beta, and r_m is the rate of return of the market and r_m - r_f is the market risk premium.

The relationship between the required return and the asset's beta is represented by the Security Market Line (SML), which plots the asset's corresponding required return for given levels of systemic risk. If the actual return on an asset is different from the required return for that asset as calculated by the CAPM formula, the investment strategy entails positive or negative “alpha”. Alpha reflects the investor’s ability to outperform the market portfolio in terms of a higher return per unit of risk, so a higher Sharpe ratio, than that of the market portfolio. When alpha is different from zero, the CAPM formula is as follows:

$$r_i = r_f + \beta_i\,(r_m - r_f) + \alpha_i \tag{2.2}$$

where r_i is the investment’s required return, r_f is the risk-free rate of return, β_i is the asset’s beta, r_m is the rate of return of the market, r_m - r_f is the market risk premium, and α_i represents the investment’s positive or negative alpha.

2.3 The Efficient Market Hypothesis

Thanks to the Capital Asset Pricing Model, investors are now able to determine the fair value of their investments. By assuming investors are rational and all information is fully available to investors at no cost, all investors should therefore value their assets correctly by using the CAPM. The Efficient Market Hypothesis (EMH) - introduced by Eugene Fama in 1970 - incorporates these views and claims markets are efficient, reflect all available information and fundamental asset values, and follow a random walk.

The main idea of the EMH is that no investment strategy can earn excess risk-adjusted returns[3] as captured by alpha, since deviations from fair asset values will be arbitraged away by rational investors in the direction of their fundamental, fair value. However, “In the real world of investments, there are obvious arguments against the EMH. There are investors who have beaten the market – Warren Buffett”.

2.4 Market Anomalies & Multi-factor models

As the true mechanism of capital markets was different from the way described by the EMH, academics performed more research on market anomalies[4]. In 1977 the value factor was discovered by Basu. The value anomaly challenged the EMH, since it revealed that stocks trading at a relatively high earnings yield systematically – therefore not randomly - outperform stocks that trade at a low earnings yield. After the value anomaly, Banz (1981) discovered the size factor, which implies stocks with smaller market capitalizations systematically outperform

[3] Excess risk-adjusted returns are defined as the return on an asset that is higher than the required return the CAPM formula prescribes for a given level of that asset’s riskiness.

[4] Market anomalies are defined as the occurrence when an asset or combination of assets perform different from the way they are prescribed to behave by the Efficient Market Hypothesis.

stocks with larger market caps. Several well-documented research papers on value and size factors showed these market inefficiencies were not traded away by rational investors, and therefore caused 'noise' in capital markets that could neither be explained by the single-factor[5] Capital Asset Pricing Model, nor the Efficient Market Hypothesis.

In 1993, these size and value factors where accounted for in the new Fama and French three-factor model, which basically is a multi-factor CAPM:

$$r_i = r_f + \beta_i (r_m - r_f) + \beta_{Size,i} Size + \beta_{Value,i} Value + \alpha_i \quad (2.4)$$

where r_i is the investment's required return, r_f is the risk-free rate of return, β_i is the asset's beta, r_m is the rate of return of the market, r_m - r_f is the market risk premium, $Size$ represents the size factor, $Value$ represents the value factor, and α_i represents the investment's positive or negative alpha.

The inclusion of the size and value factors in the model increased its explanatory power significantly.

During that same year, Jegadeesh and Titman (1993) published their research paper on the momentum factor, better known as the finding that stocks which have done well in the past outperform stocks that have done poorly over that same time frame. The momentum factor was later added to the three-factor model by Carhart (1997), adapting it into the four-factor model:

$$r_i = r_f + \beta_i (r_m - r_f) + \beta_{Size,i} Size + \beta_{Value,i} Value + \beta_{UMD,i} Momentum + \alpha_i \quad (2.5)$$

where r_i is the investment's required return, r_f is the risk-free rate of return, β_i is the asset's beta, r_m is the rate of return of the market, r_m - r_f is the market risk premium, $Size$ represents the size factor, $Value$ represents the value factor, *Momentum* represents the momentum factor, and α_i represents the investment's positive or negative alpha.

In 2013, Asness, Frazzini, and Pedersen (2013) and Frazzini and Pedersen (2014) expanded Carhart's four-factor model into a six-factor model by adding two new factors to it called "Betting-Against-Beta" (BAB) and "Quality Minus Junk"- (QMJ). The authors showed that these factors historically and empirically generated significant positive excess risk-adjusted returns. The BAB-factor reflects the fact that low-beta assets outperform high-beta assets and the QMJ-

[5] Single-factor refers to systematic risk (captured the market risk premium) as the only factor that influences an asset's required return.

factor shows that high-quality[6] stocks outperform low-quality ("Junk") stocks. Frazzini, Kabiller and Pedersen (2013) wrote a working paper called Buffett's Alpha, in which they introduced their six-factor model as follows:

$$r_i = r_f + \beta_i (r_m - r_f) + \beta_{Size,i}Size + \beta_{Value,i}Value + \beta_{UMD,i}Momentum + \beta_{QMJ,i}QMJ + \beta_{BAB,i}BAB + \alpha_i \quad (2.6)$$

where r_i is the investment's required return, r_f is the risk-free rate of return, β_i is the asset's beta, r_m is the rate of return of the market, r_m - r_f is the market risk premium, $Size$ represents the size factor, $Value$ represents the value factor, *Momentum* represents the momentum factor, *QMJ* represents the Quality-Minus-Junk factor, *BAB* represents the Betting-Against-Beta factor, and α_i represents the investment's positive or negative alpha.

Creating an asset pricing model that explains asset returns as consistent as possible, Fama and French (2015) expanded their model once again into their five-factor model by adding their own quality factors like "Robust-Minus-Weak" (RMW) and "Conservative-Minus-Aggressive" (CMA). The RMW or profitability factor reflects the anomaly that companies with a high operating profitability (referred to as "robust") perform better than stocks with low operating profitability (referred to as "weak"). The CMA factor reflects the anomaly that stocks of companies that invest conservatively (referred to as "conservative") outperform stocks of companies that invest aggressively (referred to as "aggressive")[7] (Fama & French, 2014; Robeco, 2016). These two new Fama and French (2014) factors are similar to the QMJ and BAB-factors (Asness, Frazzini, Israel, Moskowitz & Pedersen, 2018) and adding them to the Fama & French (1993) three-factor model increased its explanatory power even more. The Fama & French five-factor model is expressed as follows:

$$r_i = r_f + \beta_{MKT,i} (r_m - r_f) + \beta_{Size,i}Size + \beta_{Value,i}Value + \beta_{RMW,i}RMW + \beta_{CMA,i}CMA + \alpha_i \quad (2.7)$$

where r_i is the investment's required return, r_f is the risk-free rate of return, β_i is the asset's beta, r_m is the rate of return of the market, r_m - r_f is the market risk premium, $Size$ represents the size factor, $Value$ represents the value factor, *RMW* represents the profitability factor, *CMA* represents the investment factor, and α_i represents the investment's positive or negative alpha.

[6] High-quality stocks are defined as stocks that are safe, profitable, growing, and well managed (Asness, Frazzini & Pedersen, 2013).
[7] Also defined as 'stocks of companies that invest with high total asset growth have below average returns' (Robeco, December 2016)

2.5 Alternatively-weighted equity indexes

As the multi-factor models produced a higher explanatory power relative to the traditional CAPM, the market-capitalization-weighted equity index as a mean-variance efficient proxy of the market came under fire. Throughout time, the financial industry developed interest in alternatively weighted indexes based on the abovementioned risk factors. It was the beginning of a new investing trend known as "Smart-Beta-indexing", or "rules-based-indexing" (Haugen and Baker, 1991; Hsu, 2004; Arnott et al., 2005).

Smart Beta indexes are constructed from the same investable universe as a market-cap-weighted index, but the rules used to select the stocks included in the index and the way the weights of the underlying assets are assigned to them differs from the traditional market-cap-weighting methods. Smart Beta strategies can be classified as risk-focused, return-focused or simple reweighting methods (Podkaminer, 2015).

Risk-focused Smart Beta indexes are constructed by selecting the underlying assets out of the investable universe according to their contribution to portfolio risk or diversification-augmenting characteristics. Subsequently, each stock is then weighted in such a way that the portfolio reflects minimum volatility or, for example, a maximum Sharpe ratio. This index weighting method fully deviates from market-capitalization weighting and can be applied for both national as global equity portfolios.

Return-aware Smart Beta indexes are typically constructed by selecting the underlying assets out of the investable universe according to fundamental-, quality-, value-, momentum-, size-, dividend yield and other stock specific factors, and weigh each stock according to their specific factor exposure. This index weighting method fully deviates from market-capitalization weighting and can be applied for both national as global equity portfolios.

For simple-reweighting, however, according to MSCI's global indexing methodology, one first constructs national equity indexes by selecting the underlying assets from the global investable universe based on their national market capitalization, and then reweights each country index based on a specific factor, like economic size, as measured by Gross Domestic Product (GDP) (Deutsche Asset & Wealth Management, 2014). For obvious reasons, this weighting method can only be applied to global equity indexes. In section 3, I will elaborate on the advantages an investor can benefit from by investing in GDP-weighted indexes over cap-weighted ones.

3 Benefits of GDP- versus cap-weighting

3.1 Cap-weighted indexing

The traditional market-cap-weighted approach to indexing is based on share prices and the corresponding amount outstanding, a characteristic that, as stated by Vanguard (2015, p.2), "captures all potential factors[8] that all investors collectively use to determine a stock's price".

These indexes offer investors a low-cost, passive investment strategy that automatically rebalances as security prices fluctuate (Hamza et al., 2007; Hsu, 2006) and requires trading only in case of a constituent security replacement in the portfolio (Arnott et al., 2005).

In addition, as market capitalization is highly correlated with trading liquidity, these indexes tend to emphasize the more heavily traded stocks (Arnott et al., 2005).

Moreover, through its broad exposure to the investable universe of stocks (Hamza et al., 2007; Podkaminer, 2015), the cap-weighted index has the lowest tracking error relative to the equity market (Arnott et al., 2010).

Despite the many appealing features of the market-cap-weighted index (Siegel, 2003; Arnott et al., 2005), its mean-variance efficiency is based on strong CAPM assumptions[9], some of which have been shown not to hold in real market conditions by academics like Mayers (1976) and Markowitz (2005) (Haugen and Baker, 1991; Deutsche Asset & Wealth Management, 2014).

Moreover, Treynor (2005), Hsu (2006) as well as Siegel (2006) argued that the Efficient Market Hypothesis (Fama, 1970) does not hold in reality and that cap-weighted indexes are prone to overvaluation bias. When prices do not reflect fundamentals due to market inefficiencies, market-cap-weighted indices tend to overweight overvalued stocks and underweight undervalued stocks, influencing performance negatively (Vanguard, 2015).

During asset price bubbles, strong momentum effects arise as market-cap weightings rise strongly with ballooning stock prices, followed by drastically lower country weights when the bubble bursts (Deutsche Asset & Wealth Management, 2014). Real examples are the 1991-1992 Japanese housing bubble, the 2000-2002 tech bubble and the 2007-2008 financial crisis that left investors who linked their portfolio returns to the cap-weighted index with large losses and made them curious about better ways to capture the market's return without experiencing the asset price turbulence that characterized the last bubble (Siegel, 2006).

[8] See Figure 5 in the appendix for an overview of these factors (Vanguard, 2015: Figure 2, p.3)
[9] The CAPM assumes that all investors have the same expectations about the risk and return for all securities and can short-sell these without restriction, markets are not distorted by trading costs, inflation or taxes, and the investable universe is restricted to securities in the cap-weighted index.

3.2 GDP-weighted indexing

The strong CAPM assumptions, sensitivity to overvaluation bias, momentum and asset price bubbles inherent to cap-weighted indexes led Haugen and Baker (1991), Hsu (2006) and Arnott et al. (2005) to reject the idea that these indices are a good market proxy and that they are mean variance-efficient.

These academics argue that investors could obtain a more efficient alternative within the global efficient set of stocks through breaking the link between asset allocation and a country's market capitalization. More specifically, as inspired by numerous academic studies (Fama, 1990; Cheung and Ng, 1998; Flannery and Protopapadakis, 2002) that have shown an existing positive link between GDP and stock returns (Hamza et al., 2007; Singh, Mehta and Varsha, 2011), by weighting the index according to a country's Gross Domestic Product (GDP), its asset allocation reflects more the country's economic size than its market size. According to Bilson, Brailsford & Hooper (2001) the positive relation between economic size and stock returns is also widely accepted intuition in the financial industry.

With the country factor as an important driver of equity market returns (MSCI Barra, 2010), the matter is whether the countries within a global equity index are best represented by their market capitalization or economic size. As a cap-weighted indexes do not consider the economic value added by unlisted companies within the economy, GDP is a more encompassing measure, as it captures the aggregated value added by all companies within a country. In the end, 'all companies within a country contribute to the state of the economy, whether or not they are listed, available to foreign investors, private or public' (MSCI Barra, 2010).

Another important difference between these measures is that a cap-weighted index overweights countries that have better developed equity markets or rely more on equity-financing relative to countries that have less developed equity markets, more privately or government owned companies or rely more on bank-financing (FTSE Russell, 2014; MSCI Barra, 2010). On the other hand, a GDP-weighted index gives more weight to emerging markets when compared to the cap-weighted index, as their economic weight is generally greater than their market capitalization weight (MSCI Barra, 2010).

Since emerging market returns are generally more volatile and therefore have a higher equity risk premium relative to developed markets, investing in an index that is more exposed to this, like the GDP-weighted index, potentially generates higher returns (Salomons and Grootveld, 2003). Research performed by Hamza et al., (2007), MSCI Barra (2010), Deutsche Asset & Wealth Management (2014) and FTSE Russell (2014) shows this is indeed the case. In addition, the higher returns for GDP-weighted indexes are partly due to size and value premia, as overweighting emerging markets automatically increases stocks with smaller market capitalization (Deutsche Asset & Wealth Management, 2014; Vanguard, 2015)

Interestingly, by overweighting emerging markets stocks, the GDP-weighted index volatility does not increase proportionally with the increase in returns (Hamza et al., 2007; MSCI Barra, 2010; Deutsche Asset & Wealth Management, 2014).

An explanation for this is provided by Errunza (1983) and Harvey (1995) who state that emerging markets have low correlations with developed markets. Hence, increasing exposure to emerging markets leads to better diversification of country risks compared to the more concentrated cap-weighted indexes (FTSE Russell, 2014). This is supported by Hamza et al. (2007) and Deutsche Asset & Wealth Management (2014), who examined the country concentration within both indexes by measuring their relative Herfindahl-Hirschman Index (HHI)[10](Rhoades, 1993; Hamza et al, 2007) and found the GDP-weighted index is generally less concentrated[11] than the cap-weighted index. This suggests that country diversification or higher HHI levels have a "moderating effect"[12] on the GDP-weighted index's riskiness, which is and should be higher than that of the cap-weighted index through its higher exposure to emerging markets (Salomons & Grootveld, 2003).

Moreover, as GDP-weighted indexes need to be rebalanced on an annual basis, momentum and overvaluation bias are tempered, which makes these indexes more stable (Hamza et al., 2007).

In addition to the stabilizing merits of annual rebalancing, it potentially improves returns as well. For instance, Vanguard (2015, p.8) states that "this rebalancing enhances portfolio returns by systematically selling overvalued and buying undervalued securities in a process that exploits perceived mean reversion in pricing errors across different market segments". Vanguard's statement is supported by Deutsche Asset & Wealth Management (2014), Hsu (2014) and Perold and Sharpe (1995), who examined the benefits of rebalancing and conclude that rebalanced portfolios outperform unrebalanced portfolios when market patterns are characterized more by mean-reversion than by trends.

However, from their regressions Vanguard (2015) finds evidence against these arguments and conclude that simple-reweighting does not add to the portfolio returns over time. Although this is an interesting discussion, I will not go into more details on the effects of

[10] The Herfindahl-Hirschman Index (HHI) is a commonly accepted measure of market concentration (Hamza, Kortas, L'Her & Roberge, 2007).
[11] A less concentrated index is characterized by weights more equally spread amongst countries included in the index, or simply through increasing the variety of countries included in the index.
[12] The" moderating effect" is defined as lowering risk through increased diversification. By taking into account the correlations between developed and emerging markets risk levels are lower than what they would have been when only taking into account the higher volatility of emerging markets.

rebalancing, as the focus of my research is on differences in risk factor exposures between GDP- and cap-weighted indexes.

All in all, the improved country diversification and stabilization combined with higher returns for the GDP-weighted index results in a higher Sharpe ratio[13] than is the case for cap-weighted indexes (Hsu and Campollo, 2006; Hamza et al., 2007; MSCI Barra, 2010; Deutsche Asset & Wealth Management, 2014). Hence, in terms of return per unit of risk, one could claim a GDP-weighted equity index outperforms the cap-weighted market portfolio and is therefore superior in terms of mean-variance efficiency.

[13] The Sharpe ratio (Sharpe, 1966; Morningstar, 1993) is a measure of reward to variability or the excess return of an asset or portfolio over the risk-free rate, divided by the standard deviation of the asset or portfolio.

4 Relevant Research Papers

4.1 Hamza, Kortas, L'Her & Roberge (2007)

The least recent but elaborate article with a similar focus as my research is the one written by Hamza, Kortas, L'Her & Roberge from 2007. Within the 1970-2004 period, they compare the performance of the MSCI EAFE cap-weighted index with a hypothetical GDP-weighted and an Equally-weighted (EW) index, which they constructed themselves from the countries included in the MSCI EAFE index. They report summary statistics for the MSCI EAFE index consisting of its annualized return, standard deviation, return-to-volatility ratio, skewness and excess kurtosis. Next, they performed analysis on the country weight differences for the three indexes and report their annual Herfindahl-Hirschman index values for the 1970-2004 horizon. Not surprisingly, the GDP-weighted index has a much lower country concentration relative to the market-cap index. Whereas for the cap-weighted index, Japan and the UK get the highest weights, the spread is more equal within the GDP-weighted index (France, Germany, Italy, Japan and the UK as the five biggest shares). They emphasize the intuition that by investing more in smaller and less liquid markets through EW and GDP-weighting relative to cap-weighting, these indexes might reflect a size or 'lack of liquidity' premium. They provide the reader with another overview of the returns, volatilities, Sharpe ratios, and alpha values of the three indexes from regressions they ran before adjusting and after adjusting for these factors and considering the effect of Japan's housing bubble. For all three regressions, the Sharpe ratios for the 10-, 20-, 30- and 35-year horizons of the GDP-weighted index are higher than those for the cap-weighted index. When testing for alpha, they find significant[14] values for the 10- and 20 years in history when not correcting for size and liquidity premia. However, when excluding Japan from the index or correcting for size and liquidity premia, the alpha values become insignificant.

Moreover, making use of randomized portfolios, they test two hypotheses[15] on potential rebalancing and low country concentration benefits. They find that rebalanced portfolios, whatever the frequency is, always outperform unrebalanced portfolios and that indices with lower concentration perform better than their more concentrated counterparts, as the return per unit of risk increases with lower concentration levels. From this they conclude that 'weighting a portfolio according to market-cap might actually be the worst one could possibly do'.

[14] Significant the 5% level.

[15] H1: The outperformance of the EW index is attributable in part to the rebalancing bonus. H2: The outperformance of the EW index is attributable in part to its low concentration.

4.2 MSCI Barra (2010)

Three years later, MSCI (2010) published their article about the relative performance of the MSCI ACWI, MSCI World index, MSCI Emerging markets index and their GDP-weighted equivalents over the 1969-2009[16] and 1988-2009[17] periods. They show that for the GDP-weighted MSCI ACWI the top overweights are represented by China, Germany, Italy, Russia and Mexico, and the five underweights include the USA, UK, Switzerland, Canada and Australia. Moreover, they show the historical evolution of the weight differences between the GDP-weighted ACWI index and the cap-weighted one and find that the Emerging markets GDP weight has been growing significantly faster than the market capitalization weight, and the GDP-weight for the US has been decreasing strongly over the past. They also emphasize the effect of Japan's asset price bubble that results in a change from a negative difference between GDP and market cap, into a positive difference after the bubble bursted. In addition, they report the returns, volatilities, return/risk ratios and annualized relative performance for the six indexes and find the GDP-weighted equivalents outperform the cap-weighted indexes based on their risk-return ratios and annualized relative performance. Moreover, MSCI reports their market-cap to GDP ratios and find the lowest ratios for emerging markets countries like Turkey, Poland, Indonesia, Russia, Mexico, and China, and the highest ratios for the more developed countries like Switzerland, Taiwan, Australia, the UK, USA, and Canada. Finally, MSCI states the following: 'Our analysis shows no significant value or other fundamental factor bias in the GDP-weighted indices and very little industry bias'. However, they do not provide the reader with any details on their exact analysis. In their conclusion, they relax this statement by writing that 'it is not clear that there are fundamental reasons for the outperformance of this strategy or if it will continue to outperform in the future'.

4.3 Deutsche Asset & Wealth Management (2014)

Interestingly, in their paper from august 2014, Deutsche Asset & Wealth Management mentions that the exposure to size and value factors can be potential benefits of GDP-weighting, but they do not run any tests on these. In this paper, they construct their own GDP-weighted portfolio of country ETFs, which they compare with the MSCI ACWI and GDP-weighted MSCI ACWI based on their returns, volatility, and Sharpe ratios within the 2000-2013 horizon. Similar to MSCI Barra's 2010 article, they find the GDP-weighted ETF portfolio has a better return and Sharpe ratio than the MSCI ACWI. More specifically, they also take a closer look at the differences between the country weights of their GDP-weighted ETF portfolio and those of the MSCI ACWI and again find

[16] For the MSCI ACWI and GDP ACWI.
[17] For the MSCI World, EM, GDP World and GDP EM indexes.

similar results as reported by MSCI Barra (2010). Deutsche Asset & Wealth Management also shows the HHI values and find their GDP-weighted portfolio to have a decreasing and much lower HHI. Just like in the paper of Hamza et al. from 2007, they mention that GDP-weighted indices may bring potential additional relative returns from its annual rebalancing. All in all, they find their GDP-weighted ETF portfolio to consistently outperform the MSCI ACWI even after adjusting for costs.

4.4 FTSE Russell (2014)

Shortly after Deutsche Asset & Wealth Management's august 2014 publication, FTSE Russell followed with their article from 2014 on their own FTSE All-World, Developed markets, Emerging markets indexes, and the GDP equivalents of these. Over the period of 2001-2014, they find that, for the FTSE Developed markets and Emerging markets indexes, the GDP-version outperforms the cap-weighted indexes. From their attribution analysis, they conclude that most of the outperformance of the FTSE Emerging GDP weighted index was due to its overweight position in China. Moreover, they give an overview of the largest country weightings and weighting differences between the cap-weighted and GDP-weighted indexes and report individual countries equity market capitalizations to GDP ratios and find the top five highest values for Switzerland, South Africa, the UK, US, Korea. However, they do not give an overview of the indexes' Sharpe ratios or ran tests on factor exposures.

4.5 Vanguard (2015)

A year later, Vanguard (2015) published their research paper that most resembles the focus of my research. Although they compare performances across many smart beta indices in general, the MSCI World GDP-weighted index is included as well. Vanguard runs regressions with Fama & French's (1993) size and value factors and find no significant alphas for the GDP-weighted index after adjusting for these risk premia. From these regressions they also express doubts about the existence of any rebalancing premium, which contradicts the statements from Hamza et al. (2007) and Deutsche Asset & Wealth Management (2014) about an existing rebalancing bonus. More specifically, Vanguard states that 'the simple act of rebalancing at the security level does not systematically add to the returns of the portfolio over time'. Their strongest argument for this comes from Blitz, Van der Grient, and Van Vliet (2010), who showed the rebalancing effect is time-period dependent and influenced by the month chosen to rebalance (Vanguard, 2015).

4.6 Podkaminer (2015)

Finally, the most recent related paper that comes closest to my research, is the one by Podkaminer (2015). Similar to Vanguard's 2015 paper, Podkaminer analyzes the performance of a broad set of smart beta indices like the MSCI USA minimum volatility index, MSCI USA index, Russell Fundamental US index, Russell 3000 index, S&P500 EW index and the S&P500, but most importantly, also the MSCI EAFE GDP-weighted index. Podkaminer runs regressions to test the MSCI USA minimum volatility index, S&P500 EW index, Russell Fundamental US index, FTSE RAFI US 1000 index etc. on their exposure to the Fama and French (1993) size and value factors plus Carhart's (1997) momentum factor. However, they did not include the GDP-weighted index in this analysis. Podkaminer (2015) does run another regression on the MSCI EAFE GDP-weighted index with the Book-to-Market, Earnings-to-Price, Cash Earnings-to-Price and Dividend-to-Price ratio and finds significant exposure to the Book-to-Market ratio, which implies a significant value premium for the GDP-weighted index. Moreover, the Performance and Risk statistics overview shows contradicting findings in that the MSCI EAFE GDP-weighted index has a lower return, higher volatility, lower Sharpe ratio and negative alpha value over the 2003-2013 period relative to the MSCI EAFE index, which is cap-weighted! This suggests the GDP-weighted index is not as superior as suggested by less recent research.

5 My Research & Hypotheses

5.1 Research Questions

The relevant literature I summarized in Section 4 neither agrees about the significance of a GDP-weighted index's alpha, nor does it show a detailed overview of the risk factor exposures that potentially affect the performance of the GDP-weighted index.

To the best of my knowledge, my research paper is the first to provide the financial industry with a detailed overview of these risk factor exposures by testing the excess returns from MSCI and FTSE's GDP- and cap-weighted indexes on their potential different exposures to the most recent and well-documented risk factors. The factors included as independent variables in the linear regressions I run are the Fama and French (2015) size, value, profitability and investment factors[18], Jigadeesh & Titman's (1993) Momentum factor, the Quality Minus Junk and Betting Against Beta factors by Asness, Frazzini & Pedersen (2013, 2014).

Moreover, as MSCI Barra (2010) shows its GDP-weighted All-Countries World index has systematically outperformed its cap-weighted equivalent on a risk-return basis, I test whether these returns, standard deviations, and Sharpe ratios are significantly different from each other. By means of the results from the abovementioned tests, I aim to answer my main research question:

'Do GDP-weighted indexes still significantly outperform their cap-weighted equivalents after adjusting returns for risk-factor exposures?'

In addition, the following sub-questions guide the reader of my paper towards the answer on my main questions:

(1) Are the mean excess total returns, variances and Sharpe ratios between GDP-and cap-weighted indexes really different from each other?

(2) Do GDP-weighted indexes generate significant alpha?

(3) Do GDP- weighted indexes have different factor exposures than their cap-weighted equivalents, and if so, is the superior performance[19] of the GDP-weighted index attributable to these?

To answer each of these sub-questions, I compare the hypotheses presented in Section 5.2 with the results I find from the tests and regressions in Part 3.

[18] Market, Size and Value, Investment and Profitability factors

[19] Superior performance" is equivalent to: More mean-variance efficient, generating a higher Sharpe ratio.

5.2 Hypotheses

I derive my hypothesis from the literature I discussed in Section 3 and 4 of this paper.

5.2.1 Descriptive Statistics

Sub-question 1: *Are the mean excess total returns, variances and Sharpe ratios between GDP-and cap-weighted indexes really different from each other?*

MSCI Barra (2010) and Deutsche Asset & Wealth Management (2014) report higher mean returns for the GDP-weighted index than its cap-weighted equivalent, however, they do not test whether these returns are significantly different from each other. Given this information, I test the following hypothesis about the difference in mean returns of the MSCI and FTSE GDP-weighted indexes relative to their cap-weighted equivalents:

Hypothesis 1: The mean returns of the MSCI and FTSE GDP-weighted indexes are different from the mean returns of their cap-weighted equivalents, with:

$$H_0: \bar{r}_{CAP} = \bar{r}_{GDP} \text{ versus } H_1: \bar{r}_{CAP} \neq \bar{r}_{GDP}$$

where $\bar{r}_{CAP}$ represents the mean return of the MSCI and FTSE cap-weighted indexes and $\bar{r}_{GDP}$ represents the mean return of the MSCI and FTSE GDP-weighted indexes.

As mentioned earlier, the higher returns reported for the GDP-weighted indexes come at a higher volatility, as measured by their standard deviation (MSCI Barra, 2010; Deutsche Asset & Wealth Management, 2014). MSCI Barra and DeAWM do not test these standard deviations for being significantly different from each other. Therefore, I test the following hypothesis about the standard deviations of the MSCI and FTSE GDP-weighted indexes relative to their cap-weighted equivalents:

Hypothesis 2: The standard deviations of the MSCI and FTSE GDP-weighted indexes are different from the standard deviations of their cap-weighted equivalents, with:

$$H_0: \sigma_{CAP} = \sigma_{GDP} \text{ versus } H_1: \sigma_{CAP} \neq \sigma_{GDP}$$

where σ_{CAP} represents the standard deviation of the MSCI and FTSE cap-weighted indexes and σ_{GDP} represents the standard deviation of the MSCI and FTSE GDP-weighted indexes.

The third important metric reported by MSCI Barra (2010) and Deutsche Asset & Wealth Management (2014) is the higher Sharpe ratio for the GDP-weighted index relative to that of the cap-weighted equivalent. The Sharpe ratios are not tested for being significantly different from each other. Therefore, I test the following hypothesis about the Sharpe ratios of the MSCI and FTSE GDP-weighted indexes relative to their cap-weighted equivalents:

Hypothesis 3: The Sharpe ratios of the MSCI and FTSE GDP-weighted indexes are different from the Sharpe ratios of their cap-weighted equivalents, with:

$$H_0: S_{CAP} = S_{GDP} \quad \text{versus } H_1: S_{CAP} \neq S_{GDP}$$

where S_{CAP} represents the Sharpe ratio of the MSCI and FTSE cap-weighted indexes and S_{GDP} represents the Sharpe ratio of the MSCI and FTSE GDP-weighted indexes.

5.2.2. Alpha

Sub-question 2: *Do GDP-weighted indexes generate significant alpha?*

Hamza, Kortas, L'Her & Roberge (2007) find a positive and significant alpha for the MSCI EAFE index over the 1994-2004 and 1984-2004 time-frame when not adjusting for the size factor. Based on this finding, I will test the following hypothesis about the MSCI and FTSE GDP-weighted indexes:

Hypothesis 4: The alphas of the MSCI and FTSE GDP-weighted indexes are different from zero when not adjusting for other factors than the market risk premium, with:

$$H_0: \alpha_{GDP1} = 0 \text{ versus } H_1: \alpha_{GDP1} \neq 0$$

where α_{GDP1} represents the value of alpha for the MSCI and FTSE GDP-weighted indexes when adjusting for one risk factor only, which is the CAPM market risk premium.

However, when adjusting for size and value factors as in Fama & French's (1993) three-factor model, Vanguard (2015) finds that the alpha values for the MSCI World GDP-weighted index become insignificant. Based on this finding I will test the following hypothesis about the MSCI and FTSE GDP-weighted indexes:

Hypothesis 5: When adjusting the alphas of the MSCI and FTSE GDP-weighted indexes for size and value premia, the Fama & French (1993) three-factor alphas are zero, with:

$$H_0: \alpha_{GDP3} = 0 \text{ versus } H_1: \alpha_{GDP3} \neq 0$$

where α_{GDP3} represents the value of alpha for the MSCI and FTSE GDP-weighted indexes when adjusting for the Fama and French (1993) three risk factors: Market, Size and Value.

Adding more risk factors to the model increases its explanatory power. Therefore, I expect that the value of alpha for the MSCI and FTSE GDP- and cap-weighted indexes, as denoted by the intercept of the regression, decreases or even gets negative the more factors are added. However, it is more straightforward to find out which specific factors are drivers of the alphas of the GDP- and cap-weighted indexes.

5.2.3 Factor Exposures

Sub-question 3: *Do GDP-weighted indexes have different factor exposures than their cap-weighted equivalents, and if so, is the superior performance[20] of the GDP-weighted index attributable to these?*

Based on the findings from Vanguard (2015) on the greater exposure of the GDP-weighted to size and value factors than the cap-weighted equivalent, I derive my sixth hypothesis.

Hypothesis 6:
(a) GDP-weighted indexes are significantly *positively* exposed to size and value factors.
(b) GDP-weighted indexes are significantly *more* exposed to size and value factors than their cap-weighted equivalents.

As the cap-weighted index is not rebalanced every year, stock prices that keep following a rising trend will gain a greater weight in this index, whereas in the GDP-weighted index, these stocks will be sold due to rebalancing. Therefore, the GDP-weighted index is more stable, and less prone to momentum (Deutsche Asset & Wealth Management, 2014). I expect the cap-weighted index to have a stronger exposure towards this factor than the GDP-weighted index. This leads to the following hypothesis:

[20] Superior performance" is equivalent to: More mean-variance efficient, generating a higher Sharpe ratio.

Hypothesis 7: Cap-weighted indexes are significantly *more* exposed to the momentum factor than their GDP-weighted equivalents.

Moreover, as the GDP-index overweights emerging markets countries, one could argue it assigns more weights to the smaller, lower quality, more volatile, "junky" firms and the cap-weighted index assigns more weights to the larger, higher quality stocks that are typically more stable. I therefore expect a cap-weighted index to be positively and significantly more exposed to Quality Minus Junk (QMJ) and Betting Against Beta (BAB) factors from Asness, Frazzini & Pedersen (2014) than a GDP-weighted index. This leads to the following hypotheses:

Hypothesis 8:
(a) Cap-weighted indexes are significantly *positively* exposed to the QMJ factor.
(b) Cap-weighted indexes are significantly *more* exposed to the QMJ factor than GDP-weighted indexes.

Hypothesis 9:
(a) Cap-weighted indexes are significantly *positively* exposed to the BAB factor.
(b) Cap-weighted indexes are significantly *more* exposed to the BAB factor than GDP-weighted indexes.

In their five-factor model, Fama & French (2015) added their own quality factors like the profitability (RMW) and investment (CMA) factor. As Asness, Frazzini, Israel, Moskowitz & Pedersen (2015) argue that the QMJ and BAB factors pick up the information on the Fama and French (2014) quality factors, I expect the cap-weighted index again to be the positively and significantly more exposed to the RMW and CMA factors. This leads to the following hypotheses:

Hypothesis 10:
(a) Cap-weighted indexes are significantly *positively* exposed to the RMW factor.
(b) Cap-weighted indexes are significantly *more* exposed to the RMW factor than GDP-weighted indexes.

Hypothesis 11:
(a) Cap-weighted indexes are significantly *positively* exposed to the CMA factor.
(b) Cap-weighted indexes are significantly *more* exposed to the CMA factor than GDP-weighted indexes.

The next section describes how I will test these hypotheses empirically.

Part II

Data and Methodology

6 Data

I took daily total return data for the MSCI ACWI, MSCI ACWI GDP-weighted, FTSE All-World, FTSE All-World GDP-weighted indices from Datastream going back in time as long as there was data available. For the MSCI this means a January 2nd, 2001-June 29th, 2018 time frame, and for the FTSE a January 1st, 2002-June 29th, 2018 time frame. In addition, I downloaded daily time series for the Fama & French market, size, value, profitability, and investment premia from 2001-2018 on Kenneth French's website. Moreover, I downloaded daily time series data on the momentum, Quality Minus Junk and Betting Against Beta factors for the 2001-2018 horizon from AQR's database. Finally, I contacted MSCI and FTSE to request annual data on the country weights they used for their indexes.

While most investors rather have a monthly instead of a daily investment horizon, I initially used monthly total return time series, as MSCI Barra (2010) and Hamza et. al (2007) did as well in their research. However, as for the MSCI ACWI GDP-weighted index total return data was available only from 2001 on and for the FTSE All-World GDP-weighted index only from 2002 on in Datastream, the amount of my observations, or excess total returns, was rather moderate. My preference for daily data stems from empirical over practical reasons, as it increases the amount of excess total return observations, and therefore the degrees of freedom for my regressions. This might be useful, as my regressions include more factors than the ones performed by previous research.

7 Methodology

7.1 Relative Performance

First of all, I give a graphical representation of the cumulative excess total returns of the MSCI GDP- and cap-weighted indexes over the 2001-2018 time frame and for the FTSE GDP- and cap-weighted indexes over the 2002-2018 time frame, as presented in Figure 1 and 2 in Section 8.

7.2 Country Weights

Next, I present an overview of the country weights for the MSCI and FTSE GDP- and cap-weighted indexes over the 2016-2018 period, as represented by Table 1 and 2 in Section 9. Moreover, I calculate the country weight differences between the MSCI and FTSE GDP- and cap-weighted indexes in percentage points by subtracting the percentage country weights of the cap-weighted index from the percentage country weights of the GDP-weighted index, as presented in Table 3 and 4 in Section 9. A positive percentage point difference in country weights means the GDP-weighted index overweights that country relative to its cap-weighted equivalent. A negative percentage point difference in country weights means the GDP-weighted index underweights that country relative to the cap-weighted index. Lastly, I rank the percentage point country weight differences from largest overweight to largest underweight in the GDP-weighted index relative to its cap-weighted equivalent, as presented in Table 5 and 6 in Section 9. Since FTSE Russell and MSCI can provide students with only a limited amount of data for research, I obtained the MSCI and FTSE country weights over 2016-2018 period instead of the complete 2001-2018 time-frame.

Next, by using the country weights as inputs, I calculate the Herfindahl-Hirschman Index (HHI) values for the MSCI and FTSE cap- and GDP-weighted indexes over the 2016-2018 period. The HHI is a measure of the country concentration of an index and I calculate this according to the formula used in the paper of Hamza et al. (2007):

$$HHI = w_1^2 + w_2^2 + w_3^2 + \ldots + w_N^2 = \Sigma w_i^2 \qquad (7.1)$$

where w_i is the country's weight in the index multiplied by 100.

7.3 Descriptive Statistics

The descriptive statistics include the mean, standard deviation, minimum, maximum, Sharpe ratio and Information ratio calculated from the excess total returns of the MSCI and FTSE indexes in Stata. The Sharpe ratio is the measure of mean-variance efficiency (excess return per unit of risk) and is expressed as follows:

$$S = \frac{\bar{r}_i - \bar{r}_f}{\sigma_i} \tag{7.2}$$

where S represents the Sharpe ratio, $\bar{r}_i$ is the average total return of the index, $\bar{r}_f$ is the mean risk-free rate, $\bar{r}_i - \bar{r}_f$ denotes the mean excess total return of the index, and σ_i is the standard deviation of the excess total return of the index.

The Information ratio is a measure of the risk-adjusted returns of active investment strategies, where the GDP-weighted index represents the active investment strategy relative to a passive benchmark, which is represented by the cap-weighted equivalent. The information ratio is expressed by the following formula:

$$I = \frac{\bar{r}_{GDP} - \bar{r}_{CAP}}{\sigma_{G-C}} \tag{7.3}$$

where I represents the Information ratio, $\bar{r}_{GDP}$ is the mean total return of the GDP-weighted index, $\bar{r}_{CAP}$ is the mean total return of the cap-weighted index, $\bar{r}_{GDP} - \bar{r}_{CAP}$ denotes the mean "active return" or the mean excess total return difference between the GDP-weighted index minus that of the cap-weighted index and σ_{G-C} is the standard deviation of the active return or "tracking error".

7.4 Tests for Differences in Descriptive Statistics

7.4.1 Variance Ratio Test

To test my first three hypotheses, I first need to find out whether the excess total return variances of the MSCI and FTSE Cap-weighted indexes are significantly different from their GDP-weighted equivalents. Therefore, I perform a variance ratio-test in Stata. The variance ratio in the case of the MSCI indexes is:

$$\frac{(MSCI\ ACWI\ variance)}{(MSCI\ GDP{-}weighted\ ACWI\ variance)} = \frac{s_{CAP}^2}{s_{GDP}^2} = F \quad (7.4)$$

And for the FTSE indexes:

$$\frac{(FTSE\ All{-}World\ index\ variance)}{(FTSE\ GDP{-}weighted\ All{-}World\ index\ variance)} = \frac{s_{CAP}^2}{s_{GDP}^2} = F \quad (7.5)$$

The variance ratio test in Stata is a function that tests the equality of the variances of two random samples assuming the data is normally distributed. For example, when testing the equality of the variances for the MSCI indexes, sample one is the Cap-weighted index time series, and sample two is the GDP-weighted index time series data. The ratio of the variances of the two samples, denoted by $s_{CAP}^2$and s_{GDP}^2, is expressed by the F-statistic:

$$F = \frac{s_{CAP}^2}{s_{GDP}^2} \quad (7.6)$$

The degrees of freedom of this test are the number of observations (4564 for the MSCI and 4303 for the FTSE indexes) of each sample minus 1, so 4563 and 4302 degrees of freedom, respectively. The variance ratio test in Stata tests these three hypotheses:

1. H_0: $F = s_{CAP}^2 / s_{GDP}^2 \geq 1$ versus H_1: $F = s_{CAP}^2 / s_{GDP}^2 < 1$
2. H_0: $F = s_{CAP}^2 / s_{GDP}^2 = 1$ versus H_1: $F = s_{CAP}^2 / s_{GDP}^2 \neq 1$
3. H_0: $F = s_{CAP}^2 / s_{GDP}^2 \leq 1$ versus H_1: $F = s_{CAP}^2 / s_{GDP}^2 > 1$

7.4.2 Two-sample T-test on means

Based on the outcome of the variance ratio test, I perform a two sample T-test assuming equal or unequal variances in Stata to test for the equality in mean excess total returns over the 2001-2018 horizon for the MSCI Cap and GDP-weighted indices and over the 2002-2018 time-frame for the FTSE Cap- and GDP-weighted indices. If the variances of the two samples are equal, I perform a Student T-test that calculates the T-statistic as follows:

$$T = (\bar{r}_{CAP} - \bar{r}_{GDP}) / \sqrt{(s_P^2/n_{CAP} + s_P^2/n_{GDP})} \quad (7.7)$$

where $\bar{r}_{CAP}$ is the mean excess total return of the cap-weighted index and $\bar{r}_{GDP}$ is the mean total excess return of the GDP-weighted index. s_P^2 is a pooled estimate of the similar variance of the cap- and GDP-weighted index excess total returns, n_{CAP} is the number of observations of the cap-weighted index time series, and n_{GDP} is the number of observations of the GDP-weighted index. s_P^2 is calculated as a weighted average of the sample variances of the cap- and GDP-weighted index excess total returns:

$$s_P^2 = [(n_{CAP}-1)\, s_{CAP}^2 + (n_{GDP} -1)\, s_{GDP}^2\,] \,/\, (n_{CAP} + n_{GDP} - 2) \tag{7.8}$$

s_{CAP}^2 and s_{GDP}^2 are calculated as follows:

$$s_{index} = 1/(n_{index}-1)\, \Sigma (r_i - \bar{r}) \tag{7.9}$$

where n_{index} is the number of the cap-or GDP-weighted index excess total returns as denoted by n_{CAP} or n_{GDP}, r_i is the i-th excess total return, and $\bar{r}$ is the mean excess total return of the index.

The degrees of freedom used for the Student T-test are calculated by:

$$d_f = n_{CAP} + n_{GDP} -2 \tag{7.10}$$

In the case of unequal variances, I perform the Welch T-test, where the T-statistic is calculated as follows:

$$T = (\bar{r}_{CAP} - \bar{r}_{GDP}) / \sqrt{(s_{CAP}^2/n_{CAP} + s_{GDP}^2/n_{GDP})} \tag{7.11}$$

Welch's T-test uses the following degrees of freedom calculation:

$$d_f = \frac{\frac{s_{CAP}^2}{n_{CAP}} + \frac{s_{GDP}^2}{n_{GDP}}}{\frac{\left[\frac{s_{CAP}^2}{n_{CAP}}\right]^2}{(n_{CAP}-1)} + \frac{\left[\frac{s_{GDP}^2}{n_{GDP}}\right]^2}{(n_{GDP}-1)}} \tag{7.12}$$

The T-test in Stata tests three hypotheses:

1. H_0: $m_{CAP} - m_{GDP} \geq 0$ versus H_1: $m_{CAP} - m_{GDP} < 0$
2. H_0: $m_{CAP} - m_{GDP} = 0$ versus H_1: $m_{CAP} - m_{GDP} \neq 0$
3. H_0: $m_{CAP} - m_{GDP} \leq 0$ versus H_1: $m_{CAP} - m_{GDP} > 0$

7.4.3 Jobson-Korkie test on Sharpe ratios

Finally, I calculated the Sharpe ratios in Excel according to the formula below:

$$S = (\bar{r}_i - \bar{r}_f)/\sigma_i \tag{7.13}$$

where S stands for Sharpe ratio, $\bar{r}_i$ denotes the mean excess total return of the MSCI ACWI, MSCI ACWI GDP-weighted, FTSE All-World or FTSE All-World GDP-weighted index, $\bar{r}_f$ denotes the mean one-month Treasury bill rate from Ibbotson Associates, and σ_i denotes the standard deviation of the MSCI ACWI, MSCI ACWI GDP-weighted, FTSE All-World or FTSE All-World GDP-weighted index excess total returns.

In addition, I tested the Sharpe ratios of the MSCI and FTSE GDP-weighted and cap-weighted indexes for equality. To do so, I performed the Jobson-Korkie (1981), (JKM) test in Excel, adjusted by Memmel (2003), as explained by Lankinen & Ahmed (2018).

The JKM test is performed with a Z-test that assumes the Z-statistic is normally distributed with a mean of zero and standard deviation of 1. The Z-statistic is defined as follows:

$$Z = s(\Delta)/\Delta \tag{7.14}$$

where $s(\Delta)$ is the standard error of the differences between the Sharpe ratios, as denoted by Δ, of the cap- and GDP-weighted indexes.

$s(\Delta)$ is expressed as follows:

$$s(\Delta) = [T^{-1}\{2\text{-}2\,\rho_{CG} + \tfrac{1}{2}(S_{CAP}^2 + S_{GDP}^2 - 2 \times S_{CAP} \times S_{GDP} \times \rho_{CG}^2)\}]^{1/2} \tag{7.15}$$

where T is the number of excess total returns for each index, ρ_{CG} is the correlation between the excess total returns of the cap- and GDP-weighted indexes and S_{CAP} is the Sharpe ratio of the cap-weighted index, S_{GDP} is the Sharpe ratio of the GDP-weighted index.

The Z-statistic is tested against the following hypotheses:

1. H_0: $S_{GDP} \geq S_{CAP}$ *versus* H_1: $S_{GDP} < S_{CAP}$
2. H_0: $S_{GDP} = S_{CAP}$ *versus* H_1: $S_{GDP} \neq S_{CAP}$
3. H_0: $S_{GDP} \leq S_{CAP}$ *versus* H_1: $S_{GDP} > S_{CAP}$

Finally, I construct a relative performance time series by subtracting the daily cap-weighted excess total returns from the daily GDP-weighted excess total returns. Next, I calculate the summary statistics for the relative performance series just as I did with the MSCI and FTSE cap- and GDP-weighted index time series. However, the ratio of the relative performance mean divided by the relative performance standard deviation, I now define as the Information ratio (Treynor and Black in 1973). This ratio reflects the GDP-weighted index's 'active' return per unit of active return volatility to the cap-weighted index.

7.5 Regressions on Alpha and Factor Exposures

To test whether the MSCI and FTSE GDP-weighted indexes generate significant alphas, I run several regressions.

7.5.1 Capital Asset Pricing Model

First of all, I examine whether the alphas of the MSCI and FTSE GDP-weighted indexes are different from zero when not adjusting for other factors than the market risk premium (hypothesis 4) by running four CAPM regressions with the indexes as the dependent variables and the market risk premium as the independent variable:

$$r_i - r_f = \alpha_i + \beta_{MKT,i}(r_m - r_f) + \varepsilon_i \qquad (7.16)$$

where r_i is the total excess return of the index, r_f is the risk-free rate of return, α_i is the model's intercept that represents the positive or negative value for alpha of the index , $\beta_{MKT,i}$ is the index's market beta, r_m is the market rate of return, r_m - r_f is the market risk premium, and ε_i is the error term.

7.5.2 Fama & French three-factor model

To test whether alpha disappears after correcting for market, size and value factors and examine whether the GDP-weighted index has a greater exposure to size and value factors (hypothesis 5), I run four three-factor regressions with the MSCI and FTSE GDP-weighted and cap-weighted indexes as the independent variables and the market, size and value factors as the independent variables:

$$r_i - r_f = \alpha_i + \beta_{MKT,i}(r_m - r_f) + \beta_{SMB,i}(SMB) + \beta_{HML,i}(HML) + \varepsilon_i \qquad (7.17)$$

where r_i is the total excess return of the index, r_f is the risk-free rate of return, α_i is the model's intercept that represents the positive or negative value for alpha of the index , $\beta_{MKT,i}$ is the index's market beta, r_m is the market rate of return, r_m - r_f is the market risk premium, $\beta_{SMB,i}$ is the size coefficient, *SMB* is the size premium, $\beta_{HML,i}$is the value coefficient, *HML* is the value premium, and ε_i is the error term.

7.5.3 Carhart four-factor model

To test whether alpha disappears after correcting for market, size value and momentum factors and whether the cap-weighted index has a greater exposure to the momentum factor (hypothesis 6), I run four four-factor regressions with the MSCI and FTSE GDP-weighted and cap-weighted indexes as the independent variables and the market, size, value and momentum factors as the independent variables:

$$r_i - r_f = \alpha_i + \beta_{MKT,i}(r_m - r_f) + \beta_{SMB,i}(SMB) + \beta_{HML,i}(HML) + \beta_{UMD,i}(UMD) + \varepsilon_i \quad (7.18)$$

where r_i is the total excess return of the index, r_f is the risk-free rate of return, α_i is the model's intercept that represents the positive or negative value for alpha of the index , $\beta_{MKT,i}$ is the index's market beta, r_m is the market rate of return, r_m - r_f is the market risk premium, $\beta_{SMB,i}$ is the size coefficient, *SMB* is the size premium, $\beta_{HML,i}$ is the value coefficient, *HML* is the value premium, $\beta_{UMD,i}$ is the momentum coefficient, *UMD* is the momentum premium, and ε_i is the error term.

7.5.4 Buffett six-factor model

To test whether alpha disappears after correcting for market, size, value, momentum, QMJ and BAB factors and examine whether the cap-weighted index has a greater exposure to the QMJ and BAB factors (hypothesis 7), I run four six-factor regressions with the MSCI and FTSE GDP-weighted and cap-weighted indexes as the independent variables and the market risk premium, size, value, momentum, QMJ and BAB factors as the independent variables.

$$r_i - r_f = \alpha_i + \beta_{MKT,i}(r_m - r_f) + \beta_{SMB,i}(SMB) + \beta_{HML,i}(HML) + \beta_{UMD,i}(UMD) + \beta_{QMJ,i}(QMJ) + \beta_{BAB,i}(BAB) + \varepsilon_i \quad (7.19)$$

where r_i is the total excess return of the index, r_f is the risk-free rate of return, α_i is the model's intercept that represents the positive or negative value for alpha of the index , β_{MKT} is the index's market beta, r_m is the market rate of return, r_m - r_f is the market risk premium, $\beta_{SMB,i}$ is the size coefficient, SMB is the size premium, $\beta_{HML,i}$ is the value coefficient, HML is the value premium, $\beta_{UMD,i}$ is the momentum coefficient, UMD is the momentum premium, $\beta_{QMJ,i}$ is the Quality-Minus-Junk coefficient, QMJ is the Quality-Minus-Junk premium, $\beta_{BAB,i}$ is the Betting-Against-Beta coefficient, BAB is the Betting-Against-Beta premium, and ε_i is the error term.

7.5.5 Fama & French five-factor model

To test whether alpha disappears after correcting for market, size, value, RMW and CMA factors and examine whether the cap-weighted index has a greater exposure to the RMW and CMA factors (hypothesis 8), I run four five-factor regressions with the MSCI and FTSE GDP-weighted and cap-weighted indexes as the independent variables and the market risk premium, size, value, momentum, RMW and CMA factors as the independent variables.

$$r_i - r_f = \alpha_i + \beta_{MKT,i}(r_m - r_f) + \beta_{SMB,i}(SMB) + \beta_{HML,i}(HML) + \beta_{RMW,i}(RMW) + \beta_{CMA,i}(CMA) + \varepsilon_i \qquad (7.20)$$

where r_i is the total excess return of the index, r_f is the risk-free rate of return, α_i is the model's intercept that represents the positive or negative value for alpha of the index , $\beta_{MKT,i}$ is the index's market beta, r_m is the market rate of return, r_m - r_f is the market risk premium, $\beta_{SMB,i}$ is the size coefficient, SMB is the size premium, $\beta_{HML,i}$ is the value coefficient, HML is the value premium, $\beta_{RMW,i}$ is the profitability coefficient, RMW is the profitability premium, $\beta_{CMA,i}$ is the investment coefficient, CMA is the investment premium, and ε_i is the error term.

7.5.6 Annual Factor Exposures per Unit of Risk

To measure which portion of the index returns can be attributed to size, value, momentum, Betting-Against-Beta, Quality-Minus-Junk, profitability and investment factor exposures, for each regression model[21], I calculate the annual factor exposures per unit of risk, as explained by Ronen Israel and Adrienne Voss from AQR in the March 2016 Institutional Investor Sponsored report.

Firstly, I multiply each factor coefficient (denoted by β_i) - as given by the regression output - by its respective average risk premium. The average risk premium is the mean of the factor time series data, calculated over the 2001-2018 time frame for the MSCI and over the 2002-2018 time frame for the FTSE. I will refer to this multiplication as the "notional factor exposure". For example, if the MSCI GDP-weighted index 5-factor regression gives a coefficient

[21] I run five regressions per index tested (so 4 x 5 regressions) based on the CAPM model, the Fama & French 3-factor model, Carhart's 4-factor model, the 6-factor model used in the article titled "Buffett's Alpha" by Frazzini, Kabiller & Pedersen (2013).

of 0.09449 for the size factor, and the average daily size premium is 0.00010, the daily notional size factor exposure is 0.09449 * 0.00010 = 0.00099%. The annual notional size factor exposure is therefore $[(0.0000099+1)^{252} - 1]$ = 0.24927%.

To account for differences in the volatility of the indexes, I express each annual notional factor exposure in terms of exposure per unit of risk, where risk is measured by the index's standard deviation of the excess total returns. Suppose the annual standard deviation of the MSCI GDP-weighted excess total returns is 15.9%. The annual size factor exposure per unit of risk for the MSCI GDP-weighted index is then: $\frac{0.24927\%}{15.9\%}$ = 1.57%.

If the MSCI cap-weighted index's annual size factor exposure per unit of risk based on the corresponding 5-factor regression is -2.57%, the Relative Performance annual factor exposure per unit if risk is 1.57% - (-2.57%) = 4.14%. This indicates that the MSCI GDP-weighted index is 4.14% points per unit of risk more exposed to the size factor than the MSCI cap-weighted index.

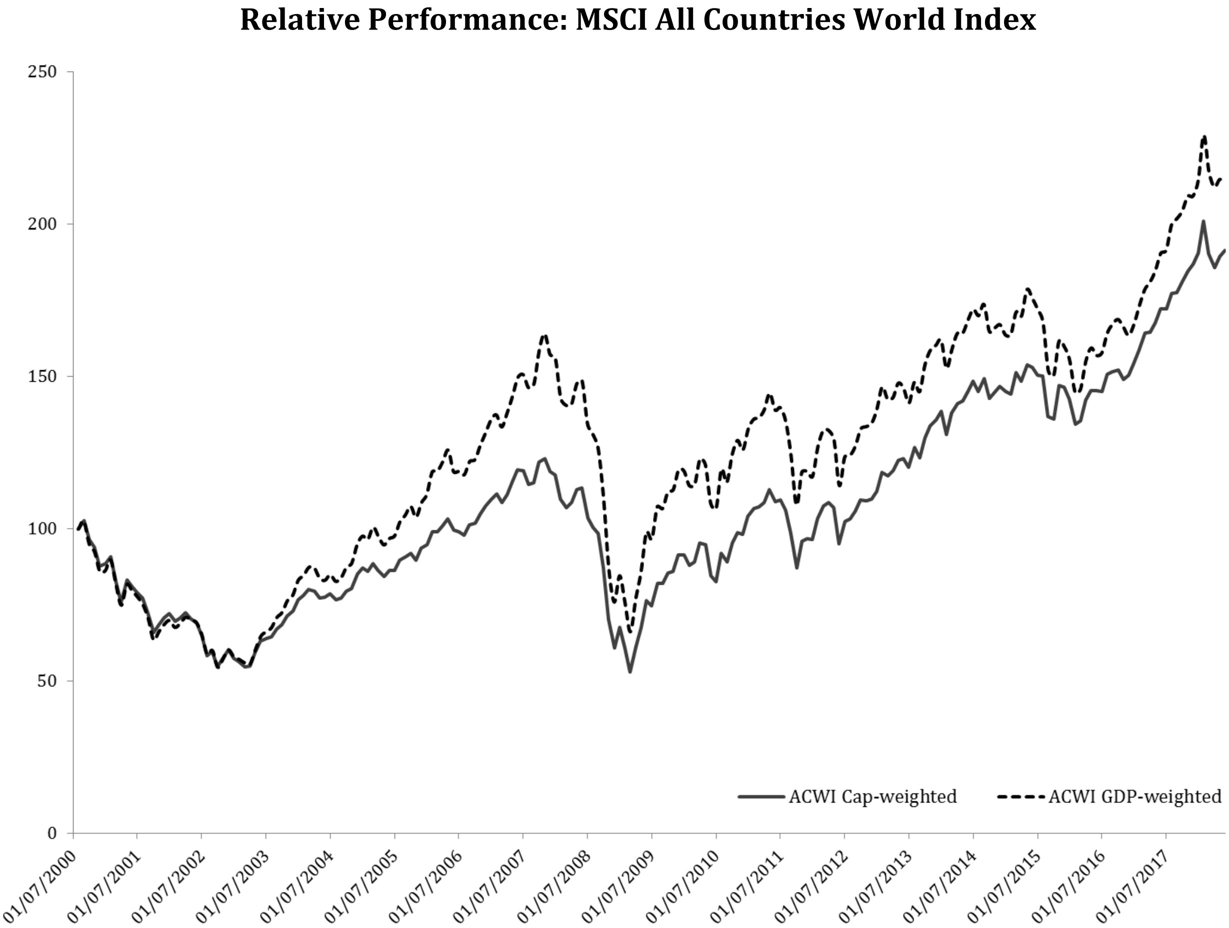
Part III

Results

8 Relative Performance

Figure 1 and 2 show the relative performance of the MSCI and FTSE cap- versus GDP-weighted indexes, measured by their monthly cumulative excess total returns[22]. For a graphical representation of the daily cumulative excess total returns of the MSCI and FTSE cap- and GDP-weighted indexes, please refer to Figure 3 and 4 in the Appendix. Clearly, the monthly and daily cumulative excess total returns for the GDP-weighted indexes in Figure 1, 2, 3 and 4 are higher than the cumulative excess total returns for the cap-weighted indexes.

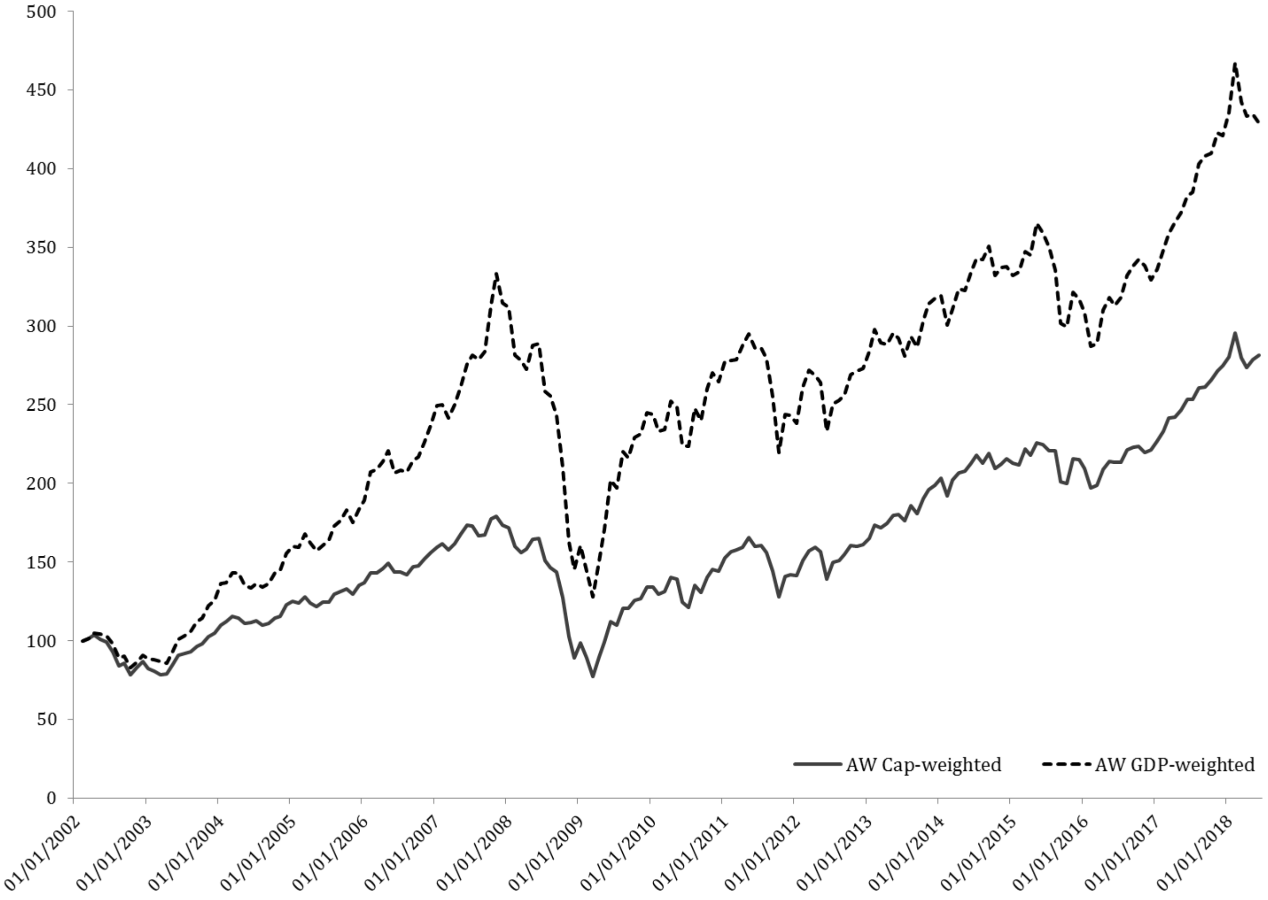
Figure 1



[22] For aesthetic purposes I chose to graphically represent the monthly cumulative excess total returns over the daily ones in this section. This does not cause any differences to my conclusions.

Figure 2

Relative Performance: FTSE All-World Index



9 Country Weights

Table 1 and 2 show the country weights in percentages for the MSCI and FTSE cap- and GDP weighted indexes over the 2016-2018 period in alphabetical order. Table 3 and 4 show the percentage point differences between the MSCI and FTSE GDP- versus cap-weighted indexes in alphabetical order, as measured by the percentage weight of the GDP-weighted index minus the percentage weight of the cap-weighted index. This means that countries with a positive percentage point weight difference are overweighted in the GDP-weighted index relative to the cap-weighted equivalent. Countries with a negative percentage point difference, however, are underweighted in the GDP-weighted index relative to the cap-weighted index. Table 5 and 6 show the percentage point differences between the MSCI and FTSE GDP- and cap-weighted indexes ranked from the largest overweights to the largest underweights. Finally, table 7 shows the Herfindahl-Hirschman Index values for the MSCI and FTSE cap-and GDP-weighted indexes over the 2016-2018 period.

Table 1

Country weights in %: MSCI All Countries World Index

	2016		2017		2018	
Country	Cap	GDP	Cap	GDP	Cap	GDP
Australia	2.37	2.09	2.26	1.75	2.17	1.76
Austria	0.06	0.62	0.08	0.69	0.08	0.55
Belgium	0.49	0.89	0.39	0.59	0.33	0.57
Brazil	0.64	2.85	0.76	3.06	0.72	2.49
Canada	3.14	2.55	3.03	2.11	3.05	2.20
Chile	0.12	0.35	0.13	0.37	0.13	0.37
China	2.38	12.87	3.08	18.35	3.68	19.10
Colombia	0.05	0.48	0.05	0.43	0.05	0.42
Czech Republic	0.02	0.27	0.02	0.24	0.02	0.29
Denmark	0.68	0.57	0.59	0.37	0.55	0.42
Egypt	0.02	0.35	0.01	0.37	0.01	0.36
Finland	0.33	0.41	0.34	0.36	0.33	0.34
France	3.32	4.21	3.52	3.69	3.52	3.48
Germany	2.99	5.47	3.18	5.16	3.06	4.68
Greece	0.05	0.22	0.04	0.26	0.03	0.23
Hong Kong	1.09	0.39	1.17	0.48	1.20	0.48
Hungary	0.03	0.27	0.04	0.20	0.03	0.16
India	0.84	3.19	0.98	3.23	0.98	3.11
Indonesia	0.26	1.28	0.28	1.35	0.23	1.17
Ireland	0.17	0.41	0.15	0.31	0.16	0.41
Israel	0.23	0.44	0.21	0.34	0.16	0.41
Italy	0.70	2.73	0.73	2.61	0.75	2.58
Japan	7.74	6.79	7.67	6.06	7.95	7.40
Korea	1.50	1.97	1.74	2.41	1.78	2.04
Malaysia	0.32	0.43	0.27	0.39	0.28	0.41
Mexico	0.42	1.77	0.39	1.52	0.31	1.16
Netherlands	1.01	1.33	1.18	1.15	1.14	1.07
New Zealand	0.06	0.36	0.05	0.23	0.06	0.25
Norway	0.20	0.63	0.20	0.54	0.22	0.57
Pakistan	0.00	0.00	0.00	0.00	0.01	0.23
Peru	0.04	0.30	0.04	0.29	0.05	0.34
Philippines	0.15	0.43	0.13	0.36	0.11	0.36
Poland	0.12	0.62	0.14	0.79	0.13	0.58
Portugal	0.05	0.33	0.05	0.30	0.05	0.29
Qatar	0.09	0.25	0.08	0.22	0.07	0.18
Russia	0.39	2.23	0.38	1.89	0.41	1.89
Singapore	0.43	0.39	0.43	0.43	0.43	0.44
South Africa	0.70	0.43	0.76	0.49	0.75	0.39
Spain	1.07	1.87	1.16	1.87	0.94	1.47
Sweden	0.95	0.79	0.97	0.73	0.82	0.64
Switzerland	3.06	0.99	2.98	0.96	2.40	0.82
Taiwan	1.21	0.72	1.36	0.88	1.35	0.74
Thailand	0.23	0.54	0.24	0.59	0.28	0.66
Turkey	0.14	1.08	0.12	0.93	0.09	0.88
United Arab Emirates	0.09	0.56	0.08	0.40	0.07	0.41
United Kingdom	6.54	4.01	5.95	3.83	5.77	3.62
United States	53.55	28.24	52.56	26.41	53.27	27.58

Notes: This table shows the percentage country weights for the MSCI Cap- and GDP-weighted All Countries World Indexes over the 2016-2018 period. The MSCI data contained herein is the property of MSCI Inc. (MSCI). MSCI, its affiliates and its information providers make no warranties with respect to any such data. The MSCI data contained herein is used under license and may not be further used, distributed or disseminated without the express written consent of MSCI. *Source: MSCI.*

Table 2

Country weights in %: FTSE All-World Index

Country	2016 Cap	2016 GDP	2017 Cap	2017 GDP	2018 Cap	2018 GDP
Australia	2.56	1.14	2.58	1.14	2.25	1.06
Austria	0.05	0.38	0.06	0.37	0.09	0.37
Belgium	0.51	0.47	0.43	0.46	0.39	0.45
Brazil	0.65	3.46	0.82	2.62	0.96	2.76
Canada	2.74	1.60	2.96	1.50	2.69	1.53
Chile	0.13	0.42	0.13	0.43	0.15	0.40
China	2.00	22.02	2.13	22.78	3.44	23.22
Colombia	0.04	0.71	0.04	0.70	0.05	0.67
Czech Republic	0.02	0.36	0.02	0.32	0.02	0.34
Denmark	0.68	0.24	0.55	0.24	0.58	0.24
Egypt	0.01	1.08	0.01	1.08	0.02	1.37
Finland	0.35	0.21	0.35	0.21	0.35	0.21
France	3.26	2.47	3.26	2.56	3.38	2.43
Germany	3.07	3.65	3.18	3.61	3.17	3.52
Greece	0.02	0.30	0.02	0.27	0.04	0.24
Hong Kong	1.19	0.41	1.24	0.40	1.30	0.40
Hungary	0.03	0.27	0.03	0.23	0.04	0.26
India	0.96	9.62	1.05	10.35	1.12	9.98
Indonesia	0.24	3.00	0.25	3.30	0.23	3.02
Ireland	0.11	0.25	0.08	0.31	0.08	0.31
Israel	0.22	0.26	0.16	0.27	0.16	0.27
Italy	0.77	2.00	0.78	2.07	0.91	1.95
Japan	8.06	4.21	8.08	4.16	8.67	4.38
Korea	1.48	1.97	1.58	1.91	1.80	1.89
Malaysia	0.38	0.90	0.28	0.90	0.35	0.92
Mexico	0.51	2.25	0.39	2.31	0.37	2.17
Netherlands	1.09	0.80	1.10	0.80	1.18	0.80
New Zealand	0.08	0.17	0.07	0.16	0.07	0.16
Norway	0.22	0.33	0.21	0.32	0.22	0.32
Pakistan	0.00	0.95	0.00	0.97	0.01	1.08
Peru	0.02	0.42	0.04	0.38	0.04	0.41
Philippines	0.14	0.87	0.10	0.89	0.14	0.87
Poland	0.12	1.13	0.11	0.99	0.15	0.95
Portugal	0.05	0.28	0.05	0.28	0.05	0.28
Qatar	0.00	0.00	0.06	0.30	0.07	0.31
Russia	0.36	3.39	0.41	3.31	0.46	3.34
Singapore	0.47	0.48	0.46	0.46	0.45	0.46
South Africa	0.77	0.74	0.75	0.64	0.88	0.62
Spain	1.05	1.55	1.11	1.67	1.07	1.52
Sweden	0.99	0.46	0.96	0.46	0.89	0.44
Switzerland	3.08	0.44	3.00	0.44	2.60	0.43
Taiwan	1.22	1.11	1.29	1.05	1.38	1.05
Thailand	0.23	1.09	0.30	1.13	0.39	1.11
Turkey	0.13	1.70	0.09	1.59	0.12	1.82
United Arab Emirates	0.08	0.63	0.07	0.61	0.09	0.62
United Kingdom	6.827	2.493	6.192	2.499	5.87	2.48
United States	53	17.31	53.16	16.54	51.27	16.54

Notes: This table shows the percentage country weights for the FTSE Cap- and GDP-weighted All Countries World Indexes over the 2016-2018 period. *Source: FTSE*

Table 3

Country weight differences in % points: MSCI All Countries World Index

Country	*2016*	*2017*	*2018*
Australia	-0.28	-0.51	-0.41
Austria	0.56	0.61	0.47
Belgium	0.40	0.20	0.24
Brazil	2.21	2.29	1.77
Canada	-0.59	-0.92	-0.85
Chile	0.23	0.24	0.24
China	10.49	15.27	15.41
Colombia	0.43	0.38	0.37
Czech Republic	0.26	0.22	0.27
Denmark	-0.11	-0.22	-0.13
Egypt	0.33	0.36	0.35
Finland	0.08	0.02	0.00
France	0.89	0.18	-0.04
Germany	2.48	1.98	1.62
Greece	0.17	0.21	0.20
Hong Kong	-0.69	-0.68	-0.71
Hungary	0.24	0.16	0.13
India	2.35	2.26	2.12
Indonesia	1.02	1.07	0.94
Ireland	0.24	0.16	0.25
Israel	0.21	0.13	0.25
Italy	2.04	1.89	1.83
Japan	-0.94	-1.61	-0.55
Korea	0.48	0.66	0.26
Malaysia	0.12	0.12	0.13
Mexico	1.35	1.12	0.85
Netherlands	0.32	-0.03	-0.07
New Zealand	0.30	0.18	0.19
Norway	0.44	0.34	0.35
Pakistan	0.00	0.00	0.22
Peru	0.26	0.25	0.29
Philippines	0.28	0.23	0.25
Poland	0.50	0.65	0.45
Portugal	0.27	0.24	0.24
Qatar	0.16	0.14	0.11
Russia	1.84	1.51	1.48
Singapore	-0.04	0.00	0.01
South Africa	-0.27	-0.27	-0.36
Spain	0.80	0.71	0.53
Sweden	-0.16	-0.24	-0.18
Switzerland	-2.07	-2.02	-1.58
Taiwan	-0.49	-0.48	-0.61
Thailand	0.31	0.35	0.38
Turkey	0.94	0.81	0.79
United Arab Emirates	0.47	0.32	0.33
United Kingdom	-2.53	-2.13	-2.15
United States	-25.31	-26.15	-25.69

Notes: This table shows the country weight differences in percentage points between the MSCI GDP- and Cap-weighted All Countries World Index over the 2016-2018 period. The difference is measured as follows: GDP-weighted index percentage country weight minus Cap-weighted index percentage country weight. The MSCI data contained herein is the property of MSCI Inc. (MSCI). MSCI, its affiliates and its information providers make no warranties with respect to any such data. The MSCI data contained herein is used under license and may not be further used, distributed or disseminated without the express written consent of MSCI. *Source: MSCI.*

Table 4

Country weight differences in % points: FTSE All-World Index

Country	*2016*	*2017*	*2018*
Australia	-1.42	-1.44	-1.19
Austria	0.33	0.31	0.28
Belgium	-0.04	0.03	0.06
Brazil	2.82	1.80	1.80
Canada	-1.15	-1.46	-1.16
Chile	0.29	0.30	0.25
China	20.02	20.65	19.78
Colombia	0.67	0.66	0.62
Czech Republic	0.34	0.30	0.32
Denmark	-0.44	-0.31	-0.34
Egypt	1.07	1.07	1.35
Finland	-0.15	-0.14	-0.14
France	-0.78	-0.70	-0.95
Germany	0.58	0.43	0.35
Greece	0.28	0.25	0.20
Hong Kong	-0.79	-0.83	-0.90
Hungary	0.24	0.20	0.22
India	8.66	9.30	8.86
Indonesia	2.76	3.05	2.79
Ireland	0.14	0.23	0.23
Israel	0.04	0.11	0.11
Italy	1.23	1.29	1.04
Japan	-3.85	-3.92	-4.29
Korea	0.49	0.32	0.09
Malaysia	0.51	0.61	0.57
Mexico	1.74	1.92	1.80
Netherlands	-0.29	-0.29	-0.38
New Zealand	0.09	0.08	0.09
Norway	0.11	0.10	0.10
Pakistan	0.95	0.97	1.07
Peru	0.40	0.34	0.37
Philippines	0.72	0.79	0.73
Poland	1.01	0.87	0.80
Portugal	0.23	0.23	0.23
Qatar	0.00	0.24	0.24
Russia	3.03	2.91	2.88
Singapore	0.02	0.00	0.01
South Africa	-0.03	-0.11	-0.26
Spain	0.50	0.56	0.45
Sweden	-0.53	-0.50	-0.45
Switzerland	-2.65	-2.56	-2.17
Taiwan	-0.11	-0.24	-0.33
Thailand	0.86	0.82	0.72
Turkey	1.56	1.49	1.70
United Arab Emirates	0.55	0.54	0.53
United Kingdom	-4.33	-3.69	-3.39
United States	-35.69	-36.62	-34.73

Notes: This table shows the country weight differences in percentage points between the FTSE GDP- and Cap-weighted All-World Index over the 2016-2018 period. The difference is measured as follows: GDP-weighted index percentage country weight minus Cap-weighted index percentage country weight.
Source: FTSE.

Table 5

Ranked country weight differences in % points: MSCI All Countries World Index

Country	*2016*	*Country*	*2017*	*Country*	*2018*
China	10.49	China	15.27	China	15.41
Germany	2.48	Brazil	2.29	India	2.12
India	2.35	India	2.26	Italy	1.83
Brazil	2.21	Germany	1.98	Brazil	1.77
Italy	2.04	Italy	1.89	Germany	1.62
Russia	1.84	Russia	1.51	Russia	1.48
Mexico	1.35	Mexico	1.12	Indonesia	0.94
Indonesia	1.02	Indonesia	1.07	Mexico	0.85
Turkey	0.94	Turkey	0.81	Turkey	0.79
France	0.89	Spain	0.71	Spain	0.53
Spain	0.80	Korea	0.66	Austria	0.47
Austria	0.56	Poland	0.65	Poland	0.45
Poland	0.50	Austria	0.61	Thailand	0.38
Korea	0.48	Colombia	0.38	Colombia	0.37
United Arab Emirates	0.47	Egypt	0.36	Norway	0.35
Norway	0.44	Thailand	0.35	Egypt	0.35
Colombia	0.43	Norway	0.34	United Arab Emirates	0.33
Belgium	0.40	United Arab Emirates	0.32	Peru	0.29
Egypt	0.33	Peru	0.25	Czech Republic	0.27
Netherlands	0.32	Portugal	0.24	Korea	0.26
Thailand	0.31	Chile	0.24	Ireland	0.25
New Zealand	0.30	Philippines	0.23	Israel	0.25
Philippines	0.28	Czech Republic	0.22	Philippines	0.25
Portugal	0.27	Greece	0.21	Belgium	0.24
Peru	0.26	Belgium	0.20	Chile	0.24
Czech Republic	0.26	New Zealand	0.18	Portugal	0.24
Ireland	0.24	France	0.18	Pakistan	0.22
Hungary	0.24	Hungary	0.16	Greece	0.20
Chile	0.23	Ireland	0.16	New Zealand	0.19
Israel	0.21	Qatar	0.14	Malaysia	0.13
Greece	0.17	Israel	0.13	Hungary	0.13
Qatar	0.16	Malaysia	0.12	Qatar	0.11
Malaysia	0.12	Finland	0.02	Singapore	0.01
Finland	0.08	Singapore	0.00	Finland	0.00
Pakistan	0.00	Pakistan	0.00	France	-0.04
Singapore	-0.04	Netherlands	-0.03	Netherlands	-0.07
Denmark	-0.11	Denmark	-0.22	Denmark	-0.13
Sweden	-0.16	Sweden	-0.24	Sweden	-0.18
South Africa	-0.27	South Africa	-0.27	South Africa	-0.36
Australia	-0.28	Taiwan	-0.48	Australia	-0.41
Taiwan	-0.49	Australia	-0.51	Japan	-0.55
Canada	-0.59	Hong Kong	-0.68	Taiwan	-0.61
Hong Kong	-0.69	Canada	-0.92	Hong Kong	-0.71
Japan	-0.94	Japan	-1.61	Canada	-0.85
Switzerland	-2.07	Switzerland	-2.02	Switzerland	-1.58
United Kingdom	-2.53	United Kingdom	-2.13	United Kingdom	-2.15
United States	-25.31	United States	-26.15	United States	-25.69

Notes: This table shows the country weight differences in percentage points between the MSCI GDP- and Cap-weighted All Countries World Indexes over the 2016-2018 period ranked from largest overweight to largest underweight in the GDP-index relative to the cap-weighted index. The difference is measured as follows: GDP-weighted index percentage country weight minus Cap-weighted index percentage country weight. A positive difference represents an overweight and a negative difference represents an underweight in the GDP-weighted index relative to its cap-weighted equivalent. The MSCI data contained herein is the property of MSCI Inc. (MSCI). MSCI, its affiliates and its information providers make no warranties with respect to any such data. The MSCI data contained herein is used under license and may not be further used, distributed or disseminated without the express written consent of MSCI. *Source: MSCI.*

Table 6

Ranked country weight differences in % points: FTSE All-World Index

Country	*2016*	*Country*	*2017*	*Country*	*2018*
China	20.02	China	20.65	China	19.78
India	8.66	India	9.30	India	8.86
Russia	3.03	Indonesia	3.05	Russia	2.88
Brazil	2.82	Russia	2.91	Indonesia	2.79
Indonesia	2.76	Mexico	1.92	Brazil	1.80
Mexico	1.74	Brazil	1.80	Mexico	1.80
Turkey	1.56	Turkey	1.49	Turkey	1.70
Italy	1.23	Italy	1.29	Egypt	1.35
Egypt	1.07	Egypt	1.07	Pakistan	1.07
Poland	1.01	Pakistan	0.97	Italy	1.04
Pakistan	0.95	Poland	0.87	Poland	0.80
Thailand	0.86	Thailand	0.82	Philippines	0.73
Philippines	0.72	Philippines	0.79	Thailand	0.72
Colombia	0.67	Colombia	0.66	Colombia	0.62
Germany	0.58	Malaysia	0.61	Malaysia	0.57
United Arab Emirates	0.55	Spain	0.56	United Arab Emirates	0.53
Malaysia	0.51	United Arab Emirates	0.54	Spain	0.45
Spain	0.50	Germany	0.43	Peru	0.37
Korea	0.49	Peru	0.34	Germany	0.35
Peru	0.40	Korea	0.32	Czech Republic	0.32
Czech Republic	0.34	Austria	0.31	Austria	0.28
Austria	0.33	Czech Republic	0.30	Chile	0.25
Chile	0.29	Chile	0.30	Qatar	0.24
Greece	0.28	Greece	0.25	Portugal	0.23
Hungary	0.24	Qatar	0.24	Ireland	0.23
Portugal	0.23	Portugal	0.23	Hungary	0.22
Ireland	0.14	Ireland	0.23	Greece	0.20
Norway	0.11	Hungary	0.20	Israel	0.11
New Zealand	0.09	Israel	0.11	Norway	0.10
Israel	0.04	Norway	0.10	New Zealand	0.09
Singapore	0.02	New Zealand	0.08	Korea	0.09
Qatar	0.00	Belgium	0.03	Belgium	0.06
South Africa	-0.03	Singapore	0.00	Singapore	0.01
Belgium	-0.04	South Africa	-0.11	Finland	-0.14
Taiwan	-0.11	Finland	-0.14	South Africa	-0.26
Finland	-0.15	Taiwan	-0.24	Taiwan	-0.33
Netherlands	-0.29	Netherlands	-0.29	Denmark	-0.34
Denmark	-0.44	Denmark	-0.31	Netherlands	-0.38
Sweden	-0.53	Sweden	-0.50	Sweden	-0.45
France	-0.78	France	-0.70	Hong Kong	-0.90
Hong Kong	-0.79	Hong Kong	-0.83	France	-0.95
Canada	-1.15	Australia	-1.44	Canada	-1.16
Australia	-1.42	Canada	-1.46	Australia	-1.19
Switzerland	-2.65	Switzerland	-2.56	Switzerland	-2.17
Japan	-3.85	United Kingdom	-3.69	United Kingdom	-3.39
United Kingdom	-4.33	Japan	-3.92	Japan	-4.29
United States	-35.69	United States	-36.62	United States	-34.73

Notes: This table shows the country weight differences in percentage points between the FTSE GDP- and Cap-weighted All-World Indexes over the 2016-2018 period ranked from largest overweight to largest underweight in the GDP-index relative to the cap-weighted index. The difference is measured as follows: GDP-weighted index percentage country weight minus Cap-weighted index percentage country weight. A positive difference represents an overweight and a negative difference represents an underweight in the GDP-weighted index relative to its cap-weighted equivalent. *Source: FTSE*

According to Table 5, the top six overweights for the MSCI GDP-weighted index over the 2016-2018 period are: China, Germany, India, Brazil, Italy and Russia. The top six underweights are: The United States, United Kingdom, Japan, Switzerland, Canada and/or Hong Kong and/or Taiwan. According to Table 6, the top six overweights for the FTSE GDP-weighted index over the 2016-2018 period are: China, India, Russia, Brazil and/or Indonesia and/or Mexico. The top six underweights are: The United States, United Kingdom, Japan, Switzerland, Australia, and Canada. From year to year the rankings between these countries change, as their country weights are dependent on that year's GDP. However, this is not the case for China and the Unites States. Over the 2016-2018 period and within both the MSCI and FTSE GDP-weighted indexes, China is consistently ranked as the top overweight, while the United States remains the largest underweight. Strikingly, the top over- and underweights are more extreme for the FTSE GDP-weighted index than for the MSCI GDP-weighted index: +20-21% points for China and -35-37% points for the U.S. in the FTSE GDP-weighted index versus +10-15% points for China and -25-26% points for the U.S. in the MSCI GDP-weighted index.

Table 7

Herfindahl-Hirschman Index

	MSCI			FTSE		
	2016	*2017*	*2018*	*2016*	*2017*	*2018*
Cap	3032.45	2925.91	3002.28	2980.94	2992.48	2806.23
GDP	1137.09	1185.96	1278.22	986.70	1004.70	1017.01

Notes: This table shows the Herfindahl-Hirschman Index (HHI) values for the FTSE and MSCI Cap- and GDP-weighted indexes over the 2016-2018 period. The HHI values are calculated from the % country weights presented in table 1 and 2, according to the HHI formula in section 7.2 of this paper. *Source: MSCI and FTSE*

The results in table 7 show that the MSCI and FTSE GDP-weighted indexes have a much lower HHI value relative their cap-weighted equivalents. This means that the GDP has a much lower country concentration, and hence a much higher country diversification.

10 Descriptive Statistics

Table 8 and 9 show the descriptive statistics for the excess total return time series data of the MSCI ACWI and FTSE All-World cap- and GDP-weighted indexes and their relative performance. The values for the mean excess total returns, standard deviations and Sharpe ratios of the indexes are in line with MSCI Barra's 2010 paper and the theory I summarized: Higher mean excess returns accompanied by higher volatility, and a higher Sharpe ratio for the GDP-weighted index relative to those of the cap-weighted index. Table 10 in Section 11 provides more insight about the significance of these differences between the cap- and GDP-weighted indexes, as it presents the p-values of the statistical tests on the equality of variances, means and Sharpe ratios.

Table 8

Descriptive statistics: MSCI All Countries World Index

	Daily			Annual		
	Cap-weighted	*GDP-weighted*	*Relative Performance*	*Cap-weighted*	*GDP-weighted*	*Relative Performance*
Mean	0.021%	0.024%	0.003%	5.555%	6.342%	0.745%
Standard Deviation	0.985%	1.003%	0.245%	15.630%	15.927%	3.889%
Minimum	-7.099%	-7.008%	-1.579%			
Maximum	9.313%	9.032%	1.333%			
Sharpe Ratio	0.022	0.024		0.355	0.398	
Information Ratio			0.012			0.192

Notes: This table shows descriptive statistics of the excess returns (index return - risk free rate) of the MSCI Cap-weighted All Countries World Index, the MSCI GDP-weighted All Countries World Index, and the Relative Performance of the two indexes from January 2001 - June 2018. Relative performance is measured by: (MSCI Cap-weighted All Countries World Index excess returns) - (MSCI GDP-weighted All Countries World Index excess returns). The Sharpe ratio is the ratio of excess return of the cap- or GDP-weighted index per unit of risk (standard deviation). The Information ratio is the Relative Performance mean return per unit of risk (tracking error).

Table 8 shows that the annual mean total excess return of the MSCI ACWI GDP-weighted index of 6.342% is less than one percentage point higher than the one of the cap-weighted index (5.555%). Its standard deviation is also slightly higher: 15.927% versus 15.630. Dividing the means by the standard deviations, this results in an annual Sharpe ratio for the MSCI GDP-weighted index of 0.398 versus 0.355 for the cap-weighted one. The higher Sharpe ratio for the

MSCI GDP-weighted index suggests this index is more mean-variance efficient than its cap-weighted equivalent. In addition, the relative performance results show the annual GDP-weighted index excess total returns are on average 0.745% higher. Moreover, dividing the mean of 0.745% by the standard deviation of the relative performance time series (3.889%)[23], I find an information ratio of 0.192.

Table 9

Descriptive statistics: FTSE All-World Index

	Daily			Annual		
	Cap-weighted	*GDP-weighted*	*Relative Performance*	*Cap-weighted*	*GDP-weighted*	*Relative Performance*
Mean	0.028%	0.038%	0.010%	7.248%	10.035%	2.599%
Standard Deviation	0.978%	1.009%	0.489%	15.530%	16.018%	7.762%
Minimum	-7.099%	-7.008%	-1.579%			
Maximum	9.313%	9.032%	1.333%			
Sharpe Ratio	0.028	0.038		0.467	0.626	
Information Ratio			0.021			0.335

Notes: This table shows descriptive statistics of the excess returns (index return - risk free rate) of the FTSE Cap-weighted All-World Index, the FTSE GDP-weighted All-World Index, and the Relative Performance of the two indexes from January 2002 - June 2018. Relative performance is measured by: (FTSE Cap-weighted All-World Index excess returns) - (FTSE GDP-weighted All-World Index excess returns). The Sharpe ratio is the ratio of excess return of the cap- or GDP-weighted index per unit of risk (standard deviation). The Information ratio is the Relative Performance mean return per unit of risk (tracking error).

Table 9 shows that the percentage point difference of about 3% for the annual mean total excess return of the FTSE All-World GDP- (10.035%) versus its cap-weighted equivalent (7.248%) is greater than the difference between the MSCI cap- and GDP-weighted indexes (6.342% versus 5.555%). Like for the MSCI indexes, the standard deviation of the FTSE GDP-weighted index (16.018%) is also higher than the standard deviation of the FTSE cap-weighted index (15.530%). Dividing the FTSE means by the standard deviations, this results in an annual Sharpe ratio for the FTSE GDP-weighted index of 0.626 versus 0.467 for the cap-weighted one. Just as for the MSCI GDP-weighted index, the higher Sharpe ratio for the FTSE GDP-weighted index suggests it is more mean-variance efficient than its cap-weighted equivalent. The relative performance results show the annual FTSE GDP-weighted index excess returns are on average 2.599% higher than those of the cap-weighted index. Moreover, dividing this mean by the

[23] The standard deviation of the difference in excess returns of the indexes is also called the 'tracking error'.

standard deviation of the relative performance time series (7.762%), I find an information ratio of 0.335.

The descriptive statistics of both the MSCI and FTSE indexes suggest the GDP-weighted index is superior to the cap-weighted index in terms of mean variance efficiency.

When comparing the MSCI and FTSE GDP-weighted indexes with each other, the much higher mean and slightly higher standard deviation of the FTSE GDP-weighted index relative to the MSCI GDP-weighted index[24] result in a much better Sharpe ratio of 0.626 of the FTSE GDP-weighted index relative to 0.398 for the MSCI GDP-weighted index. These results suggest the FTSE is the most mean-variance efficient index out of the four indexes tested. However, these differences could be due to the difference in time frames of the tests, with a 2001-2018 horizons for the MSCI indexes and 2002-2018 horizon for the FTSE indexes.

Part of the FTSE GDP-weighted index's higher Sharpe ratio could be explained by its higher information ratio of 0.335 compared to the 0.192 information ratio for the MSCI GDP-weighted index. As the information ratio represents the risk-adjusted active return for the FTSE GDP-weighted index, part of the differences in Sharpe ratios between the MSCI and FTSE GDP-weighted indexes might be due to differences in the way the MSCI and FTSE GDP-weighted indexes are "actively" reweighted every year, but it could also be due to the difference in time frames of the time series data.

[24] FTSE mean of 10.035% versus MSCI mean of 6.342% and FTSE standard deviation of 16.018% versus MSCI standard deviation of 15.927%

11 Tests for Differences in Descriptive Statistics

Table 10 presents the results from statistical tests for the equality of the excess total return means, variances, and Sharpe ratios of the MSCI and FTSE GDP- versus cap-weighted indexes. These results shine a different light on the suggested superiority of GDP-weighted indexes over their cap-weighted equivalents.

Table 10

Statistical tests for equality: Variances, Means, Sharpe Ratios

Variances			Means			Sharpe ratios		
H1:	*MSCI*	*FTSE*	*H1:*	*MSCI*	*FTSE*	*H1:*	*MSCI*	*FTSE*
Cap/GDP < 1	0.102	0.021**	*Cap-GDP < 0*	0.444	0.317	*Cap < GDP*	0.536	0.517
Cap/GDP ≠ 1	0.204	0.042**	*Cap-GDP ≠ 0*	0.887	0.635	*Cap ≠ GDP*	0.928	0.966
Cap/GDP > 1	0.898	0.979	*Cap-GDP > 0*	0.556	0.683	*Cap > GDP*	0.464	0.483

Notes: This table shows the p-values from tests on the equality of excess return variances, means and Sharpe ratios between the Cap- and GDP-weighted indexes from MSCI and FTSE. The variances are tested with an F-test. The means are tested with a T-test. The Sharpe ratios are tested with the Jobson-Korkie test. The majority of the results do not show significant differences between the variables. However, the FTSE p-values show the Cap- and GDP-weighted variances are significantly different from each other at the 5% level (denoted by **). Therefore, FTSE excess return means are tested with a two-sample T-test assuming unequal variances, and the MSCI excess returns are tested with a two-sample T-test assuming equal variances.

11.1 Hypotheses 1-3

From the variance ratio test results on the left of table 10, I conclude the differences in the variances of the MSCI GDP- versus cap-weighted indexes are not significant. However, for the FTSE indexes, at the 5% level, I find the GDP-weighted index variance to be significantly higher than that of the cap-weighted equivalent. Hypothesis 2 - which predicts different variances for cap- and GDP-weighted indexes - is therefore correct only for the FTSE indexes.

For my first and third hypotheses that predict different mean excess returns and Sharpe ratios for the GDP- versus cap-weighted indexes, I do neither find significant evidence for the difference in means and Sharpe ratios for the MSCI GDP-and cap-weighted indexes, nor for those of the FTSE, as the p-values of the T-test for equality of means and the Jobson-Korkie test for equality of Sharpe ratios are all above the 10% level. I therefore reject hypothesis 1 and 3.

12 Regressions

12.1 Hypotheses 4-5

This section presents CAPM and Fama & French three-factor regression results for the significance of alpha and factor exposures of the MSCI and FTSE cap- and GDP-weighted indexes (Table 11 and 12). Moreover, I show how much each factor contributes to the excess total annual return of the MSCI and FTSE indexes per unit of risk (Table 13 and 14). Table 11 shows the MSCI indexes' daily and annual alpha values and factor exposures from a CAPM regression that includes only the market risk premium as the independent variable, and a three-factor regression with additional Size and Value factors from Fama & French (1993). Table 12 shows results of similar regressions for the FTSE indexes. Table 13 shows how much each factor premium's contribution to the annual excess return of the MSCI indexes is based on the CAPM and three-factor regression results. Table 14 shows similar results for the FTSE indexes like for those of the MSCI in Table 13.

Table 11

Regressions I-III: MSCI All Countries World Index

	Cap-weighted		GDP-weighted		Relative Performance	
	CAPM	*3-factor*	*CAPM*	*3-factor*	*CAPM*	*3-factor*
β_{MKT}	1.01206***	0.97602***	0.99131***	1.01038***	-0.02076***	0.03437***
	(0.003)	(0.003)	(0.005)	(0.006)	(0.004)	(0.004)
β_{SMB}		-0.15295***		0.09449***		0.24744***
		(0.007)		(0.014)		(0.009)
β_{HML}		-0.00110		0.13321***		0.13431***
		(0.008)		(0.014)		(0.010)
$Alpha_{daily}$	-0.00002	0.00001	0.00002	-0.00002	0.00003	-0.00002
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$Alpha_{annual}$	-0.00443	0.00171	0.00419	-0.00388	0.00866	-0.00558
Observations	4,564	4,564	4,564	4,564	4,564	4,564
R-squared	0.966	0.969	0.892	0.896	0.007	0.186
Adj. R-squared	0.966	0.969	0.892	0.896	0.006	0.185

Notes: This table presents the CAPM and Fama & French 3-factor regressions results of the excess returns (index return - risk free rate) of the MSCI Cap-weighted All Countries World Index, the MSCI GDP-weighted All Countries World Index, and the Relative Performance of the two indexes from January 2001 - June 2018. Relative performance is measured by: (MSCI Cap-weighted All Countries World Index excess returns) - (MSCI GDP-weighted All Countries World Index excess returns). β_{MKT} represents the market risk premium coefficient, β_{SMB} represents the size factor coefficient and β_{HML} represents the value factor coefficient. $Alpha_{daily}$ represents the regression intercept. $Alpha_{annual}$ is the annualized regression intercept. Standard errors in parentheses; *, **, *** denotes significance at the 10, 5 and 1% level, respectively.

I regress the MSCI ACWI cap- and GDP-weighted index returns solely on the market risk premium, as reflected by the CAPM regression results in the first column on the left of Table 11 and find that the MSCI GDP-weighted index does not generate significant alpha.

The three-factor regressions in Table 12 show that the alpha for the MSCI GDP-weighted index remains insignificant when size and value factors are included in the model. However, the value of alpha changed from positive to negative, as part of the excess return is now explained by significant size and value factor exposures.

Table 12

Regressions I-III: FTSE All-World Index

	Cap-weighted		GDP-weighted		Relative Performance	
	CAPM	*3-factor*	*CAPM*	*3-factor*	*CAPM*	*3-factor*
β_{MKT}	1.00856***	0.97723***	0.93056***	0.98468***	-0.07800***	0.00745
	(0.003)	(0.003)	(0.008)	(0.009)	(0.006)	(0.007)
β_{SMB}		-0.13845***		0.27231***		0.41075***
		(0.008)		(0.020)		(0.016)
β_{HML}		-0.00824		0.14595***		0.15419***
		(0.009)		(0.024)		(0.019)
$Alpha_{daily}$	-0.00001	0.00001	0.00011	0.00006	0.00012**	0.00005
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$Alpha_{annual}$	-0.00352	0.00212	0.02821	0.01449	0.03184	0.01235
Observations	4,303	4,303	4,303	4,303	4,303	4,303
R-squared	0.964	0.967	0.772	0.783	0.033	0.179
Adj. R-squared	0.964	0.967	0.771	0.783	0.033	0.178

Notes: This table presents CAPM and Fama & French 3-factor regression results of the excess returns (index return - risk free rate) of the FTSE Cap-weighted All-World Index, the FTSE GDP-weighted All-World Index, and the Relative Performance of the two indexes from January 2002 - June 2018. Relative performance is measured by: (FTSE Cap-weighted All-World Index excess returns) - (FTSE GDP-weighted All-World Index excess returns). β_{MKT} represents the market risk premium coefficient, β_{SMB} represents the size factor coefficient and β_{HML} represents the value factor coefficient. $Alpha_{daily}$ represents the regression intercept. $Alpha_{annual}$ is the annualized regression intercept. Standard errors in parentheses; *, **, *** denotes significance at the 10, 5 and 1% level, respectively.

From the CAPM regressions in Table 12 I do neither find a significant alpha for the FTSE GDP-weighted index. Based on the results in Table 11 and 12, I therefore reject hypothesis 4, which predicts that a CAPM regression shows a positive and significant alpha for GDP-weighted indexes. The results from the CAPM regressions in Table 11 and 12 suggest that the cap-weighted index cannot be outperformed by simply reweighting it based on GDP.

The 3-factor regressions in Table 12 show that the alpha for the FTSE GDP-weighted index remains insignificant when size and value factors are included in the model. While insignificant, I do find that alpha decreased in value, but does not turn negative. Therefore, hypothesis 5 about an insignificant alpha after adjusting for size and value factors is therefore true. Nonetheless, alpha was already insignificant in the first place.

Other interesting results are the values of R^2 for the MSCI and FTSE regressions in Table 11 and 12. R^2 is a goodness-of-fit measure[25] that represents what percentage of the dependent variable's variation is explained by the linear model (in the above case the CAPM or Fama & French three-factor model). Both the MSCI and FTSE cap- and GDP-weighted regressions show very high values for R^2 (77-97%), which implies almost all of the variability of the cap- and GDP-weighted index's excess total returns are explained by the factor exposures. The R^2 of the relative performance CAPM and three-factor regressions is much lower: The three-factor regressions in Table 11 and 12 show that only 18-19% of the relative performance variability of the GDP-weighted index versus that of the cap-weighted index is explained by the Fama & French factors. For the CAPM regressions in Table 11 and 12, R^2 is only 1-3%! Although the R^2 for the relative performance regressions is much lower than for the cap- and GDP-weighted index regressions, this does not matter for the significance of the factor coefficients. It only shows how good the model fits the data.

12.2 Hypothesis 6

To find out whether the GDP-weighted index is positively and more exposed to size and value factors relative to its cap-weighted equivalent, I analyze the results from the MSCI and FTSE relative performance 3-factor regressions in table 13 and 14. I find that the MSCI and FTSE GDP-weighted indexes are indeed significantly positively and more exposed to size and value factors relative to their cap-weighted equivalents.

Table 13 and 14 show the contribution of each factor premium to the excess total returns of the MSCI and FTSE cap- and GDP-weighted indexes measured by the annual factor exposure per unit of risk.

[25] R^2 is also referred to as the coefficient of (multiple) determination.

Table 13

Annual Factor Exposure per Unit of Risk I-III: MSCI All Countries World Index

	Cap-weighted		GDP-weighted		Relative Performance	
	CAPM	*3-factor*	*CAPM*	*3-factor*	*CAPM*	*3-factor*
β_{MKT}	**38.55%**	**37.14%**	**37.03%**	**37.77%**	**-1.52%**	**0.63%**
β_{SMB}		**-2.57%**		**1.57%**		**4.14%**
β_{HML}		-0.02%		**2.81%**		**2.83%**
Alpha	-2.84%	1.09%	2.68%	-2.48%	5.52%	-3.57%
Σ	35.71%	35.64%	39.71%	39.67%	4.00%	4.03%

Notes: This table presents the annual factor exposure per unit of risk for the excess returns (index return - risk free rate) of the MSCI Cap-weighted All Countries World Index, the MSCI GDP-weighted All Countries World Index, and the Relative Performance of the two indexes from January 2001 - June 2018. Relative performance is measured by: (Cap-weighted annual factor exposure per unit of risk) - (GDP-weighted annual factor exposure per unit of risk). β_{MKT}, β_{SMB}, β_{HML} represent the annualized factor exposure per unit of risk, calculated as follows: market, size and value coefficients from table 11 multiplied by the mean market risk, size and value premia, which are then converted to annualized exposures and divided by the standard deviation of the cap- or GDP-weighted index. Alpha represents the annualized regression intercept from table 11. Σ represents the total annual expected excess return, which is the sum of Alpha and the factor exposures per unit of risk. The bold numbers are significant factor exposures.

From the results of the MSCI relative performance three-factor regression (right-most column) in Table 13, I find that on an annual-per-unit-of-risk-basis, relative to the MSCI cap-weighted index, the MSCI GDP-weighted index is significantly more exposed to size and value factors with 4.14% points and 2.83% points, respectively.

Table 14

Annual Factor Exposure Per Unit of Risk I-III: FTSE All-World Index

	Cap-weighted		GDP-weighted		Relative Performance	
	CAPM	*3-factor*	*CAPM*	*3-factor*	*CAPM*	*3-factor*
β_{MKT}	**49.11%**	**47.53%**	**43.80%**	**46.44%**	**-5.30%**	-1.08%
β_{SMB}		**-2.04%**		**3.91%**		**5.96%**
β_{HML}		-0.12%		**2.02%**		**2.15%**
Alpha	-2.27%	1.36%	18.17%	9.33%	**20.43%**	7.97%
Σ	46.84%	46.73%	61.97%	61.72%	15.13%	14.99%

Notes: This table presents the annual factor exposure per unit of risk for the excess returns (index return - risk free rate) of the FTSE Cap-weighted All-World Index, the FTSE GDP-weighted All-World Index, and the Relative Performance of the two indexes from January 2001 - June 2018. Relative performance is measured by: (Cap-weighted annual factor exposure per unit of risk) - (GDP-weighted annual factor exposure per unit of risk). β_{MKT}, β_{SMB}, β_{HML} represent the annualized factor exposure per unit of risk, calculated as follows: market risk, size and value coefficients from table 12 multiplied by the mean market, size and value premia, which are then converted to annualized exposures and divided by the standard deviation of the cap- or GDP-weighted index. Alpha represents the annualized regression intercept from table 12. Σ represents the total annual expected excess return, which is the sum of Alpha and the factor exposures per unit of risk. The bold numbers are significant factor exposures.

From the results of the FTSE relative performance three-factor regression (right-most column) in table 14, I find that on an annual-per-unit-of-risk-base, relative to the FTSE cap-weighted index, the FTSE GDP-weighted index is significantly more exposed to size and value factors with 5.96% points and 2.15% points, respectively.

I therefore conclude that hypothesis 6 - which predicts that the GDP-weighted index is (a) positively and (b) more exposed to size and value factors - is correct. This means that the GDP-weighted index thanks part of its higher returns to a higher exposure to size and value factors. The reason for this is the fact that the GDP-weighted index assigns more weight to emerging markets relative to a cap-weighted index, and therefore automatically overweights stocks with a smaller market capitalization (as represented by the size factor) and stocks that are undervalued (as represented by the value factor) relative to a cap-weighted index.

12.3 Hypotheses 7-11

This section provides results of 4-, 5-, and 6-factor regressions for the MSCI FTSE cap- and GDP-weighted indexes significance of factor exposures. In addition to the market, size and value factors, momentum (UMD), Quality minus Junk (QMJ, Betting Against Beta (BAB), Profitability (RMW) and Investment (CMA) factors are included in the regressions presented below. Moreover, I show how much each factor contributes to the excess total annual return of the index per unit of risk. Table 15 shows the MSCI indexes' alpha values and factor exposures from regressions based on Carhart's four-factor model, the six-factor model from the 2013 paper on Buffett's Alpha by Frazzini, Kabiller and Pedersen, and the Fama and French five-factor model. Table 16 shows results of similar regressions for the FTSE indexes. Table 17 shows how much each factor contributes to the annual excess return of the MSCI based on the four-, five- and six-factor regression results. Table 18 shows similar results for the FTSE indexes like for the MSCI indexes in Table 17.

Table 15

Regressions IV-VI: MSCI All Countries World Index

	Cap-weighted			GDP-weighted			Relative Performance		
	4-factor	*6-factor*	*5-factor*	*4-factor*	*6-factor*	*5-factor*	*4-factor*	*6-factor*	*5-factor*
β_{MKT}	0.97327***	0.96022***	0.96807***	1.00658***	0.97903***	0.99123***	0.03331***	0.01880***	0.02315***
	(0.003)	(0.005)	(0.004)	(0.006)	(0.008)	(0.007)	(0.004)	(0.006)	(0.005)
β_{SMB}	-0.14923***	-0.16604***	-0.15616***	0.09964***	0.06214***	0.08384***	0.24887***	0.22818***	0.24000***
	(0.007)	(0.008)	(0.007)	(0.014)	(0.014)	(0.013)	(0.009)	(0.010)	(0.009)
β_{HML}	-0.00276	-0.01999**	0.02929***	0.13091***	0.09246***	0.25722***	0.13367***	0.11245***	0.22793***
	(0.008)	(0.008)	(0.009)	(0.014)	(0.015)	(0.016)	(0.010)	(0.010)	(0.011)
β_{UMD}	-0.01436***	-0.00986**		-0.01989**	-0.01205		-0.00553	-0.00219	
	(0.004)	(0.005)		(0.008)	(0.009)		(0.005)	(0.006)	
β_{QMJ}		-0.06986***			-0.15436***			-0.08450***	
		(0.012)			(0.021)			(0.015)	
β_{BAB}		0.02776***			0.06721***			0.03945***	
		(0.006)			(0.010)			(0.007)	
β_{RMW}			0.01110			0.13231***			0.12121***
			(0.011)			(0.019)			(0.013)
β_{CMA}			-0.07443***			-0.27166***			-0.19723***
			(0.011)			(0.020)			(0.014)
$Alpha_{daily}$	0.00001	0.00002	0.00	-0.00001	0.00000	-0.00001	-0.00002	-0.00001	-0.00006
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$Alpha_{annual}$	0.00276	0.00454	0.00306	-0.00243	0.00087	-0.00343	-0.00518	-0.00366	-0.00647
Observations	4,564	4,564	4,564	4,564	4,564	4,564	4,564	4,564	4,564
R-squared	0.969	0.969	0.969	0.896	0.898	0.901	0.186	0.195	0.236
Adj. R-squared	0.969	0.969	0.969	0.896	0.897	0.901	0.185	0.194	0.235

Notes: This table shows 4-factor, 6-factor and 3-factor regression results of the excess returns (index return - risk free rate) of the MSCI Cap-weighted All Countries World Index, the MSCI GDP-weighted All Countries World Index, and the Relative Performance of the two indexes from January 2001 - June 2018. Relative performance is measured by: (MSCI Cap-weighted All Countries World Index excess returns) - (MSCI GDP-weighted All Countries World Index excess returns). The 4-factor model is based on Carhart's paper (1997), the 6-factor model on Frazzini, Kabiller and Pedersen's paper (2013), and the 5-factor model on the paper by Fama & French (2015). $\beta_{MKT,}$ β_{SMB}, β_{HML}, β_{UMD}, β_{QMJ}, β_{BAB}, β_{RMW}, β_{CMA} represent the Market, Size, Value, Momentum, Quality Minus Junk, Betting Against Beta, Profitability and Investment factor regression coefficients. $Alpha_{daily}$ represents the regression intercept. $Alpha_{annual}$ is the annualized regression intercept. Standard errors in parentheses; *, **, *** denotes significance at the 10, 5 and 1% level, respectively.

Table 16

Regressions IV-VI: FTSE All-World Index

	Cap-weighted			GDP-weighted			Relative Performance		
	4-factor	*6-factor*	*5-factor*	*4-factor*	*6-factor*	*5-factor*	*4-factor*	*6-factor*	*5-factor*
β_{MKT}	0.97298***	0.95731***	0.96166***	0.98120***	0.93974***	0.94720***	0.00822	-0.01756*	-0.01446*
	(0.003)	(0.005)	(0.004)	(0.009)	(0.013)	(0.010)	(0.007)	(0.010)	(0.008)
β_{SMB}	-0.12991***	-0.14912***	-0.14548***	0.27929***	0.23029***	0.24547***	0.40920***	0.37941***	0.39094***
	(0.008)	(0.008)	(0.008)	(0.021)	(0.022)	(0.020)	(0.016)	(0.017)	(0.016)
β_{HML}	-0.02355**	-0.04442***	0.01865*	0.13342***	0.07969***	0.32091***	0.15698***	0.12411***	0.30227***
	(0.009)	(0.010)	(0.010)	(0.024)	(0.026)	(0.025)	(0.019)	(0.020)	(0.020)
β_{UMD}	-0.03206***	-0.02639***		-0.026231**	-0.00992		0.00583	0.01647	
	(0.005)	(0.005)		(0.013)	(0.014)		(0.010)	(0.011)	
β_{QMJ}		-0.07880***			-0.20280***			-0.12399***	
		(0.012)			(0.033)			(0.026)	
β_{BAB}		0.02767***			0.06500***			0.03823***	
		(0.006)			(0.016)			(0.013)	
β_{RMW}			-0.03045**			0.15675***			0.18719***
			(0.012)			(0.031)			(0.024)
β_{CMA}			-0.11977***			-0.48432***			-0.36455***
			(0.013)			(0.033)			(0.026)
$Alpha_{daily}$	0.00002	0.00003	0.00003	0.00007	0.00009	0.00008	0.00005	0.00006	0.00005
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$Alpha_{annual}$	0.00463	0.00674	0.00637	0.01657	0.02268	0.01919	0.01189	0.01584	0.01273
Observations	4,303	4,303	4,303	4,303	4,303	4,303	4,303	4,303	4,303
R-squared	0.967	0.967	0.967	0.783	0.786	0.797	0.179	0.184	0.234
Adj. R-squared	0.967	0.967	0.967	0.783	0.786	0.796	0.178	0.183	0.233

Notes: This table shows 4-factor, 6-factor and 3-factor regression results of the excess returns (index return - risk free rate) of the FTSE Cap-weighted All-World Index, the FTSE GDP-weighted All-World Index, and the Relative Performance of the two indexes from January 2002 - June 2018. Relative performance is measured by: (FTSE Cap-weighted All-World Index excess returns) - (FTSE GDP-weighted All-World Index excess returns). The 4-factor model is based on Carhart's paper (1997), the 6-factor model on Frazzini, Kabiller and Pedersen's paper (2013), and the 5-factor model on the paper by Fama & French (2015). β_{MKT}, β_{SMB}, β_{HML}, β_{UMD}, β_{QMJ}, β_{BAB}, β_{RMW}, β_{CMA} represent the Market, Size, Value, Momentum, Quality Minus Junk, Betting Against Beta, Profitability and Investment factor regression coefficients. $Alpha_{daily}$ represents the regression intercept. $Alpha_{annual}$ is the annualized regression intercept. Standard errors in parentheses; *, **, *** denotes significance at the 10, 5 and 1% level, respectively.

First of all, both the MSCI and FTSE relative performance four-factor regressions in Table 15 and 16 show that the significantly higher size and value factor exposures of the GDP- relative to the cap-weighted index found in the MSCI and FTSE relative performance three-factor regressions are robust when a momentum (UMD) factor is added.

Secondly, the relative performance four-factor regressions in Table 15 and 16 show the GDP-weighted index is not significantly more exposed to the momentum factor than the cap-weighted index. This means that the cap-weighted index does not have greater momentum exposure and therefore is not more prone to momentum or bubbles than a GDP-weighted index. Therefore, as I do not find significant supportive evidence, I reject hypothesis 7, which predicts that the cap-weighted index has greater exposure towards the momentum factor than a GDP-weighted index. Moreover, rejecting hypothesis 7 implies that a GDP-weighted index is not significantly more stable than a cap-weighted index. Hence, a suggestion for further research could be to find out whether annual rebalancing and country diversification have a stabilizing effect on the performance of GDP-weighted indexes relative to cap-weighted indexes.

Looking at the six-factor MSCI and FTSE Cap- and GDP-weighted regressions in table 15 and 16, I find a significant negative Quality-Minus-Junk (QMJ) coefficient. Moreover, the relative performance regressions in Table 15 and 16 show that the GDP-weighted index is significantly negatively and more exposed to the QMJ factor relative to a cap-weighted index. Based on these results, I therefore reject hypothesis 8, which predicts that the cap-weighted index is significantly (a) positively and (b)more exposed to the QMJ factor. It is difficult to explain why there is a significant negative correlation with the QMJ factor for the cap-weighted index, as one would expect a cap-weighted index to have a large exposure to highly priced, quality stocks and therefore move in the same - not the opposite - direction as the QMJ factor. The negative correlation of the cap-weighted index to the QMJ factor, could be due to the fact that the MSCI and FTSE cap-weighted indexes include emerging markets stocks, whereas the QMJ factor is built solely from developed markets stocks. The inclusion of emerging markets stocks in the MSCI and FTSE indexes could cause these indexes to move in opposite directions from the QMJ factor, as emerging markets stocks can generally be classified as the smaller, lower quality, more volatile, “junky” stocks. Moreover, a potential explanation for the fact that the GDP-weighted index is relatively more negatively correlated to the QMJ factor than the cap-weighted index, could be that a GDP-weighted index overweights emerging markets stocks relative to a cap-weighted index, and therefore automatically assigns a higher weight to these “junky” stocks within the GDP-weighted index relative to its cap-weighted equivalent.

Contrary to the negative QMJ exposure, I find from the six-factor regression results in Table 15 and 16 that the MSCI and FTSE cap- and GDP-weighted indexes are significantly positively correlated to the Betting-Against-Beta (BAB) factor. When looking at the relative performance, I find that the GDP-weighted index is significantly more exposed to the BAB factor than its cap-weighted equivalent. Based on these results, I therefore conclude hypothesis 9(a) - which predicts that cap-weighted indices are significantly positively exposed to the BAB factor – is correct. However, I do reject hypothesis 9(b), which predicts that cap-weighted indices are more exposed to the BAB factor. The annual rebalancing and higher country diversification of the GDP-weighted index could potentially have a strong stabilizing effect, making the GDP-weighted index less sensitive to market risk and is therefore positively and more correlated with the BAB factor than a cap-weighted index.

For the MSCI cap-weighted index five-factor regression in table 15, I find it is not significantly exposed to the profitability (RMW) factor. This suggests there is no clear correlation between the MSCI cap-weighted index and high or low operating profitability stocks. For the FTSE cap-weighted index five-factor regression in Table 16, however, I find a significant negative RMW factor exposure. This implies the FTSE cap-weighted index includes more low operating profitability stocks in the index than high operating profitability stocks. I therefore reject hypothesis 10(a), which predicts that cap-weighted indexes are significantly positively exposed to the profitability factor.

Looking at the GDP-weighted index and relative performance five-factor regression in Table 15 and 16, I find that the GDP-weighted index, surprisingly, is significantly positively and more exposed to the RMW factor than the cap-weighted index. While theory predicts the cap-weighted index would include the more mature, quality stocks with higher operating profitability, it is the GDP-weighted index that shows to have a greater exposure to this kind of stocks. I therefore reject hypothesis 10(b), which predicts that cap-weighted indexes are significantly more exposed to the profitability factor.

Moreover, from the MSCI and FTSE five-factor regressions in Table 15 and 16, I find that the MSCI and FTSE cap-weighted indexes have a significant negative exposure towards the investment factor, which implies it includes more stocks of firms that invest aggressively relative to stocks of companies that invest conservatively. This could be due to the fact that the MSCI and FTSE cap-weighted indexes include emerging markets stocks, which are usually smaller, growing stocks which invest more aggressively than the stocks of more mature companies in developed markets that invest more conservatively. The CMA factor, however, is constructed from developed markets stocks only. I therefore reject hypothesis 11(a), which predicts that cap-weighted indexes are significantly positively exposed to the investment factor.

Considering the relative performance five-factor regression in Table 15 and 16, I find that the MSCI and FTSE GDP-weighted indexes are even more negatively exposed to the CMA factor. This sounds intuitively logical, as the GDP-weighted index has a greater exposure towards emerging markets stocks, and therefore the more growing and heavily investing stocks. However, this implies that the cap-weighted index includes less aggressively investing companies than the GDP-weighted index. Nevertheless, I reject hypothesis 11(b), which predicts that, in absolute terms, cap-weighted indexes are more exposed towards the investment factor.

Other interesting results are the values of R^2 for the MSCI and FTSE regressions in Table 15 and 16. Both the MSCI and FTSE cap- and GDP-weighted regressions show very high values for R^2 (78-97%), which implies almost all of the variability of the cap- and GDP-weighted index's excess total returns are explained by the factor exposures. The R^2 of the relative performance regressions is much lower: The five-factor regressions show that only 24% of the relative performance variability of the GDP-weighted index versus that of the cap-weighted index is explained by the Fama & French five-factor exposures. For the four- and six-factor regressions in table 15 and 16, R^2 is only 18-20%! However, compared to the CAPM and Fama & French three-factor relative performance regressions in Table 11 and 12, this means the four-, five- and six-factor linear models fit the data slightly better. In other words, up to 24% of the variability in the relative performance of the GDP-weighted index versus the cap-weighted index is now explained by the factor exposures in the four-, five- and six-factor regressions, while the single- and three-factor exposures only explained up to 19%.

Table 17

Annual Factor Exposure per unit of risk IV-VI: MSCI All Countries World Index

	Cap-weighted			GDP-weighted			Relative Performance		
	4-factor	*6-factor*	*5-factor*	*4-factor*	*6-factor*	*5-factor*	*4-factor*	*6-factor*	*5-factor*
β_{MKT}	**37.03%**	**36.52%**	**36.83%**	**37.62%**	**36.56%**	**37.03%**	**0.59%**	**0.04%**	**0.20%**
β_{SMB}	**-2.51%**	**-2.79%**	**-2.63%**	**1.65%**	**1.03%**	**1.39%**	**4.16%**	**3.82%**	**4.02%**
β_{HML}	-0.05%	**-0.43%**	**0.63%**	**2.77%**	**1.95%**	**5.45%**	**2.81%**	**2.38%**	**4.82%**
β_{UMD}	**-0.59%**	**-0.41%**		**-0.81%**	-0.25%		-0.21%	0.15%	
β_{QMJ}		**-2.32%**			**-5.01%**			**-2.70%**	
β_{BAB}		**2.14%**			**5.09%**			**2.96%**	
β_{RMW}			0.24%			**3.10%**			**2.86%**
β_{CMA}			**-1.43%**			**-5.10%**			**-3.67%**
Alpha	1.76%	2.90%	1.96%	-1.56%	0.55%	-2.19%	-3.32%	-2.35%	-4.15%
Σ	35.64%	35.61%	35.59%	39.67%	39.92%	39.67%	4.03%	4.31%	4.08%

Notes: This table presents the annual factor exposure per unit of risk for the excess returns (index return - risk free rate) of the MSCI Cap-weighted All Countries World Index, the MSCI GDP-weighted All Countries World Index, and the Relative Performance of the two indexes from January 2001 - June 2018. Relative performance is measured by: (Cap-weighted annual factor exposure per unit of risk) - (GDP-weighted annual factor exposure per unit of risk). β_{MKT}, β_{SMB}, β_{HML}, β_{UMD}, β_{QMJ}, β_{BAB}, β_{RMW}, β_{CMA} represent the annualized factor exposure per unit of risk, calculated as follows: Market, Size, Value, Momentum, Quality Minus Junk, Betting Against Beta, Profitability and Investment factor coefficients from table 15 multiplied by the mean corresponding factor premia, which are then converted to annualized exposures and divided by the standard deviation of the cap- or GDP-weighted index. Alpha represents the annualized regression intercept from table 15. Σ represents the total annual expected excess return, which is the sum of Alpha and the factor exposures per unit of risk. The bold numbers are significant factor exposures.

Table 18

Annual Factor Exposure per unit of risk IV-VI: FTSE All-World Index

	Cap-weighted			GDP-weighted			Relative Performance		
	4-factor	*6-factor*	*5-factor*	*4-factor*	*6-factor*	*5-factor*	*4-factor*	*6-factor*	*5-factor*
β_{MKT}	**47.31%**	**46.53%**	**46.74%**	**46.27%**	**44.25%**	**44.62%**	-1.04%	**-2.27%**	**-2.13%**
β_{SMB}	**-1.92%**	**-2.20%**	**-2.15%**	**4.01%**	**3.31%**	**3.53%**	**5.93%**	**5.51%**	**5.67%**
β_{HML}	**-0.34%**	**-0.63%**	**0.27%**	**1.85%**	**1.10%**	**4.46%**	**2.19%**	**1.74%**	**4.19%**
β_{UMD}	**-1.32%**	**-1.08%**		**-1.04%**	-0.14%		0.27%	0.95%	
β_{QMJ}		**-2.38%**			**-5.93%**			**-3.55%**	
β_{BAB}		**2.13%**			**4.92%**			**2.79%**	
β_{RMW}			**-0.64%**			**3.21%**			**3.85%**
β_{CMA}			**-1.63%**			**-6.38%**			**-4.75%**
Alpha	2.98%	4.34%	4.10%	10.67%	14.60%	12.36%	7.69%	10.27%	8.25%
Σ	46.72%	46.69%	46.69%	61.76%	62.12%	61.78%	15.04%	15.43%	15.09%

Notes: This table presents the annual factor exposure per unit of risk for the excess returns (index return - risk free rate) of the FTSE Cap-weighted All-World Index, the FTSE GDP-weighted All-World Index, and the Relative Performance of the two indexes from January 2002 - June 2018. Relative performance is measured by: (Cap-weighted annual factor exposure per unit of risk) - (GDP-weighted annual factor exposure per unit of risk). β_{MKT}, β_{SMB}, β_{HML}, β_{UMD}, β_{QMJ}, β_{BAB}, β_{RMW}, β_{CMA} represent the annualized factor exposure per unit of risk, calculated as follows: Market, Size, Value, Momentum, Quality Minus Junk, Betting Against Beta, Profitability and Investment factor coefficients from table 16 multiplied by the mean corresponding factor premia, which are then converted to annualized exposures and divided by the standard deviation of the cap- or GDP-weighted index. Alpha represents the annualized regression intercept from table 16. Σ represents the total annual expected excess return, which is the sum of Alpha and the factor exposures per unit of risk. The bold numbers are significant factor exposures.

Table 17 and 18 show the factor exposures per unit of risk for the FTSE and MSCI indexes per multi-factor regression. The relative performance columns give a good impression of the magnitude of the significant differences in factor exposures between cap- and GDP-weighted indexes. Summarizing the differences between the MSCI and FTSE cap- and GDP-weighted indices as presented by Table 17 and 18, the size factor positively contributes for about 4-6% points and the value factor for 2-4% points more to the excess returns of the GDP-weighted index relative to the cap-weighted index. The momentum factor does not play a significant role in the higher returns for the GDP-weighted index. On the other hand, the GDP-weighted index does have a greater exposure to junk stocks of about 2.7-3.5% points relative to the cap-weighted index, which worsens the GDP-weighted index returns. However, this is compensated by the fact that the GDP-weighted index is more stable, as represented by its positive and higher exposure to the BAB factor of about 3% points more than the cap-weighted index. In addition, the GDP-weighted indexes' higher excess returns are partly due to their 3-4% points higher exposure to high operating profitability stocks. However, this is canceled out completely by the 4-5% greater exposure to companies that invest aggressively, which worsens the excess returns of GDP-weighted indexes versus cap-weighted ones.

12.4 Summary of Findings

The results of the tests for differences based on hypotheses 1-3 provide no significant evidence to conclude that the mean excess total returns, variances (except for the FTSE GDP indexes) and Sharpe ratios differ between cap- and GDP-weighted indexes. Moreover, the results of the CAPM and Fama & French three-factor regressions based on hypotheses 4-5 indicate that GDP-weighting does not generate significant positive alpha. Finally, I find that GDP-weighted indexes differ from their cap-weighted weighted equivalents in terms of factor exposure. The MSCI and FTSE relative performance columns show that next to the size and value factors, the BAB and profitability factors have a strong positive effect on the annual total expected excess returns for the GDP-weighted index, whereas the greater negative exposures to QMJ and CMA factors worsen its performance relative to the cap-weighted index. Based on these annual factor exposures per unit of risk, I therefore conclude that a big part of the GDP-weighted index superior performance[26] is due to its higher exposure towards smaller, undervalued, low beta and high operating profitability companies. However, this comes at the cost of its exposure to "junky" stocks and companies that invest aggressively.

[26] "Superior performance" is equivalent to: More mean-variance efficient, generating a higher Sharpe ratio.

Part IV

Conclusion & Limitations

13 Conclusions

This study is the first paper within the related literature on GDP-weighted indexing that examines the performance of GDP-weighted indexes relative to cap-weighted equivalents after adjusting for size, value, momentum, Betting-Against-Beta, Quality-Minus-Junk, Profitability and Investment factor exposures. Moreover, I provide the financial industry with a detailed overview of the annual factor exposures per unit of risk for the MSCI and FTSE cap- and GDP-weighted indexes.

Whereas well-known index providers like MSCI Barra (2010) and FTSE Russell (2015) claim GDP-weighted indexes outperformed their cap-weighted equivalents based on cumulative returns and Sharpe ratios, I find these descriptive statistics not to be significantly different from each other [27].

Moreover, in contrast to the findings of Hamza et al. (2007), my regression results show that GDP-weighted indexes do not generate significant positive alpha. GDP-weighted indexes' higher Sharpe ratios can be deceiving, as my regressions and annual factor exposures per unit of risk indicate this is simply because of the GDP-weighted index's higher positive exposure to size and value factors.

Furthermore, I find that GDP-weighted indexes are not less prone to momentum, but do have a significant higher exposure to low beta stocks than their cap-weighted equivalents. This evidence partly supports the claim made by Hamza et al. (2007) that GDP-weighted indexes are more stable relative to cap-weighted indexes. However, my suggestion for further research is to find out whether the higher exposure to the low-beta factor can be explained by the lower country concentration and annual rebalancing of the GDP-weighted index.

However, I also find GDP-weighted indexes to be significantly more exposed to "junky" stocks and companies that invest aggressively, which has a declining effect on the expected return of GDP-weighted indexes, but it does not completely cancel out the positive contributions of the size, value, Betting-Against-Beta and Profitability exposures.

When adjusting GDP-weighted returns for these risk factors, I conclude that GDP-weighted indexes do not outperform the traditional cap-weighted index. After all, the at first sight superior performance of GDP-weighted indexes is just a reflection of risk factor exposures.

[27] Except for the FTSE GDP-weighted index variance, which I find to be significantly different from its cap-weighted equivalent at the 5% level.

14 Limitations

Every study has its limitations. For my research, it is important to consider the fact that I analyzed daily instead of monthly excess total returns for the cap- and GDP-weighted indexes, while in practice, most investors have a longer investment horizon than daily.

Another important fact is that I have not considered the costs involved with following (or investing in ETFs[28] that track) cap- or GDP-weighted indexes. However, as the GDP-weighted index needs to be rebalanced annually, while a cap-weighted index does not, including the effect of costs in the analysis would - in addition to the adjustment for factor exposures - deteriorate the GDP-weighted index's performance even more.

Moreover, in my research and regressions I have not accounted for the potential effects of annual rebalancing on the GDP-weighted index's returns. Hence, adding a rebalancing factor to the models I tested would be a suggestion for further research.

In addition, it would also be interesting to add a liquidity factor to the regressions, as emerging markets stocks tend to be less liquid compared to developed markets stocks. This implies that the overweighting of emerging markets stocks by the GDP-weighted index relative to a cap-weighted one, could result in a higher exposure to less liquid stocks relative to a cap-weighted index. One would expect this liquidity factor to be subsumed by the RMW and QMJ factors, so I suggest adding the liquidity factor as an interaction term to see whether the RMW and QMJ coefficients change.

All in all, I expect these limitations not to have a crucial effect on my conclusions, but I kindly dare others to extend my study.

[28] ETF stands for Exchange-Traded Fund

Part V

Appendices

Figure 3

Daily relative performance: MSCI All Countries World Index

Figure 4

Daily relative performance: FTSE All-World Index

Figure 5

Factors Captured by Market Capitalization

The all-factor strategy

Because current price reflects every factor used by any investor to estimate a company's value, a market-cap-weighted index also represents a true multifactor approach—that is, an all-factor approach (see **Figure 2**)—to investing and an ex ante (forward-looking), theoretically mean-variance-efficient portfolio. One could argue that the only "smart-beta" index available to investors is one that tracks the aggregate capital in the market, since it reflects the aggregate wisdom embedded in prices.

Figure 2. A cap-weighted index is *the* 'smart-beta' strategy

Source: Vanguard.

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