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Master's Thesis

Secular Stagnation and Inequality

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Abstract

In line with the growing strand of literature on the *secular stagnation hypothesis*, this paper expands the seminal work of Eggertsson, Mehrotra, and Robbins (2017) to allow for income inequality in a two-household type setting and argue that the aforementioned is among the *secular forces* that have been steadily driving real interest rates down for the past three decades. Additionally, by exploring the AD/AS environment, this paper finds that the previous result has a real impact on the aggregate economy, where for a high-enough level of income inequality, a permanent, uniquely determined *secular stagnation* equilibrium arises. Lastly, this paper looks both at Monetary and Fiscal Policies to understand how conventional stabilization measures could aid in solving *secular stagnation*. On the one hand, Monetary Policy is deemed to be fairly ineffective in the presence of rising inequality, due to the aggravation of the so-called *timidity-trap*. On the other hand, Fiscal Policies are revealed to have a very relevant role in stimulating the economy. Particularly, this paper finds that there is considerable scope for redistributive policies to pull the economy out of a *secular stagnation equilibrium*.

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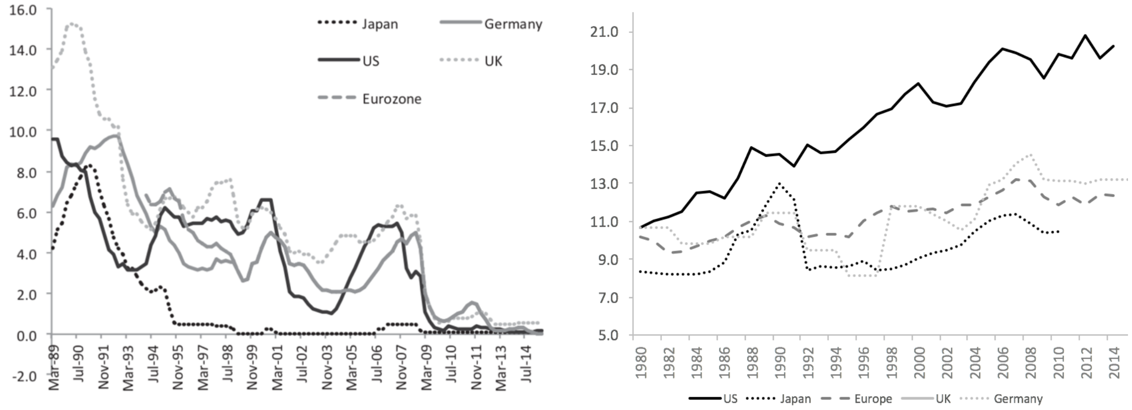
1 Introduction

Figure 1: Key economic indicators

(a) Short-term Nominal Interest Rates (b) Top 1% National Income Share



Source: (a) - Eggertsson, Mehrotra, and Robbins (2017); (b) - World Inequality Database (2018)

When Summers (2013) resurrected the concept of *secular stagnation*[1] at the annual IMF economic forum, his comments struck a chord amongst the economic community and fueled a heated debate on the topic[2]. At the time, both the US and Europe faced unprecedented sluggish recoveries, nominal interest rates were at the zero lower bound (ZLB) for the fifth consecutive year and potential GDP estimates were showing a permanently depressed output-slump very similar to that of Japan's lost decade in the 1990's (Baldwin and Teulings, 2014; Summers, 2014b). However, five years have passed, and although Europe's recovery continues to be shy (ECB, 2018a)[3], across the Atlantic, the Federal Reserve (Fed) is already doubling down on its efforts to regain fire-power over the US economy[4], while economic performance seems to remain unhindered: inflation is hovering its 2% target, unemployment is reported below 4% in 2018 (U.S. Bureau of Labor Statistics, 2018) and GDP growth is projected at 2.5% annually for the next three

[1]The term *secular stagnation* dates back to the aftermath of the Great Depression, when the then President of the American Economic Associations, Alvin Hansen (1939), introduced the concept as a possible explanation for the sluggish growth of the 1930's. However, the concept was abandoned swiftly in the following years, as World War II and the *"baby boom"* that followed would rapidly expand demand and give rise to unprecedented economic growth over the upcoming decades.

[2]See Krugman (2014), Summers (2014a; 2014b; 2016) for some examples or Baldwin and Teulings (2014) for a review of the issues.

[3]The ECB has stated that nominal interest rates on the main refinancing operations are expected to remain negative *"well-past the horizon of the net asset purchases"* (ECB, 2018a), which will still be running at €30bn per month until September 2018 and at €15bn monthly until December 2018, when it is scheduled to end (ECB, 2018b).

[4]The Fed has started early in 2018 the 2-years, \$1.5tr, balance-sheet normalization procedure (FED, 2014), while also increasing the federal funds rate to a healthy 2.25% (FED, 2018).

years (U.S. Bureau of Economic Analysis, 2018). These figures, together with a booming stock market, show an encouraging economic performance that might be interpreted as the return to the pre-crisis prosperity and stability. For that reason, the question arises - *"Is the secular stagnation hypothesis still something to be concerned about?"*

First of all, and specially since *secular stagnation* has proved to be *"an economist's Rorschach test"* (Eichengreen, 2015), it is important to start by clearly defining what this concept stands for. Indeed, as Summers (2018) recently wrote, many economists have come to believe that the *secular stagnation hypothesis* is the apocalyptic perception that developed economies are doomed to remain stagnant at high levels of unemployment. However, instead, this hypothesis claims that due to these societies' structural characteristics (e.g. demographics, financial frictions or income inequality), the "normal times" full-employment real interest rate - or, hereafter, the natural rate of interest (NRI) - that provides the necessary stimulus to attain potential output, might have fallen below (or close to) what conventional monetary policy regimes can withstand. As a consequence, a permanently crippled private aggregate demand arises and periods of a binding ZLB become exponentially more likely in the absence of exogenous forces, such as government intervention or assets/credit bubbles[5] (Summers, 2013; Krugman, 2014; Baldwin and Teulings, 2014).

Thus, secondly, one can now look both at the evolution of the NRI, as well as at the overall economic performance of developed countries, in order to understand whether the empirical observations corroborate or refute the previous hypothesis and provide an answer to the first paragraph's question. Accordingly, looking at the NRI of the mentioned economies, Krugman (2014), IMF (2014), Pichelmann (2015), Laubach and Williams (2016) and Kiley and Roberts (2017), identify that it is no coincidence that the ZLB bound for so long and with such severity during the last recession. In fact, as Panel (a) of Figure 1 illustrates[6], these authors highlight that the NRI of developed economies has consistently decreased over the last 30 years. Indeed, as Summers (2014a) first noted, a negative interest rate most likely arose somewhere during the last decade but remained hidden behind the historical pilling of unsustainable consumer (in the US) and state (in Europe) debt that supported aggregate demand throughout the post-dot-com bubble years (Illing,

[5] As Illing, Ono, and Schlegl (2018), as well as Summers (2014a) explain, a credit-boom artificially sustains consumption, thus temporarily sustaining growth and interest rates. This issue is again touched upon later in this section. As Baldwin and Teulings (2014) and Koo (2014) explain, (rational) asset-bubbles arise as an alternative form of holding savings in the event of a very low real rate of return. Therefore, as long as there is some scarcity-related pressure that maintains future expectations above the current rate, these bubbly assets produce windfalls and/or rents which boost consumption and alleviate the interest rate market.

[6] Note that it is not the NRI that is depicted in this graph, but rather the nominal interest rates. Given that the inflation targets during the last decades have had little or no change, the nominal interest rate is, thus, an imperfect reflection of the NRI.

Ono, and Schlegl, 2018). The same happened in Japan, but during the 1990's, when after the burst of the Japanese asset-bubble that artificially inflated aggregate demand in the pre-crisis period, the country's extremely fast aging-population and tighter borrowing constraints revealed a very low (privately-sustained) NRI that plunged the country into a decades-long recession (Illing, Ono, and Schlegl, 2018; Romei, 2018; Okazaki and Sudo, 2018; Sudo and Takizuka, 2018). Hence, both observations explored above are highly consistent with the *secular stagnation* hypothesis, which claims that slow-moving forces linked to structural characteristics have been endogenously driving these societies' NRI downwards for years. Finally, this paper highlights the recent empirical work of Meyer (forthcoming), who found that at least 60% of the interest rate decline in the US over the last decade can be directly attributed to *secular* changes in demographics and inequality alone[7], which provides further support to the referred theory.

One the other hand, as Summers (2018) notes, although the economic performance of the past couple of years should be seen as encouraging, it still corresponds to *"(...) fairly ordinary growth with extraordinary policy and financial conditions"*. Indeed, if one looks at the massive institutional responses that followed the 2008 financial crisis - a \$4.5tr Quantitative Easing (QE) program, 8 years at the ZLB and \$789bn in fiscal stimulus in the US, and a €2.4tr QE program, a still binding ZLB, as well as a €500bn investment program in the EU (Le Moigne, Saraceno, and Villemot, 2016; ECB, 2018a) - an analogous rapid and strong recovery of the economy would be expected. Instead, developed economies have only experienced either mild (US) or very shy (EU) recoveries over the past couple of years, which is again consistent with the idea that a crippled private aggregate demand might be hampering what would otherwise be an exceptional recovery. Ultimately, as this paper's title hints, and as Summers (2018) recently wrote, the answer to the question made in the first paragraph ought to be a convincing *"yes"*. For that reason, it is due to the economic community to deepen the understanding of this issue, in order to face the challenges of managing a perhaps prolonged *"new normal"*, where ZLB periods will occur much more often and private sector-sustained growth will most likely become the exception rather than the norm.

In this spirit, this paper is going to look at income inequality, which is often referred to as one of the key contributers to *secular stagnation*, together with demographic changes and financial frictions (Eggertsson and Mehrotra, 2014; Gordon, 2014; Summers, 2014b; Storm, 2017). Moreover, due to its wide propagation during the great moderation and emphasized increase during the Great Recession (Forster, Chen, and Llenanozal, 2011; Pichelmann, 2015) - Panel (b) of Figure 1 -, income inequality has become one of the

[7] Other explanatory factors are shifts related to factors outside of the US, as well as supply-side arguments such as decrease in the long-run labor productivity growth and rate of technological advancements.

hottest topics in the economic debate, with many different theories explaining the causes and the impacts of such malaise (Kuznets, 1955; Atkinson, Piketty, and Saez, 2011; Cynamon and Fazzari, 2015; Pichelmann, 2015; Piketty (2015); Piketty, Saez, and Zucman, 2017). More specifically, in the context of this paper, this issue becomes of ever-growing importance to policy-makers and economists around the world, who face the challenge of effectively stimulating the economy out of potential *secular stagnation equilibrium*, without truly understanding how underlying forces such as income inequality might be hampering conventional stabilizing measures.

For that reason, this paper adds to the existing discussion by providing an in-depth expansion of Eggertsson, Mehrotra, and Robbins (2017) (hereafter EMR) 3-period OLG model to allow for income inequality in a two-household type setting. By doing so, this paper formally defines, for the first time, an income inequality induced ZLB period, while exploring in detail the mechanics behind such phenomenon. Additionally, the aggregate demand-supply framework is explored, in order to show how rising income inequality promotes the creation of an endogenous *secular stagnation equilibrium*, where involuntary unemployment, a persistent output-gap and deflation arise. This paper hopes to shed some light into what is still a relatively unexplored topic by showing both how rising inequality can hinder important economic dynamics - particularly the fragile consumption-savings equilibrium -, but also by exploring how specific Monetary and Fiscal policies might provide a way out of the previous equilibrium.

The rest of this paper is organized as follows. In section 2, a thorough review of the *secular stagnation/liquidity trap*, as well as income inequality and growth literature is provided. Subsequently, EMR's simple endowment model is reproduced fully and analyzed in section 3.1, while endowment inequality is introduced - still in the spirit of EMR - in section 3.2. From then onwards, in section 4, this paper proceeds to define a more comprehensive framework where a nominal price level (4.1), a supply side (4.2), nominal rigidities (4.3), and a monetary rule (4.4) are considered and, in section 5, a steady-state analysis of the aggregate supply (5.1) and demand (5.2) is derived, in order to be able to assess and analyze the making of a *"normal"* full-employment equilibrium and of a *secular stagnation* inequality-induced one (5.3). Finally, this paper dives into the realm of stabilization policies with the intent of understanding how rising inequality might affect both Monetary (section 6) and Fiscal (section 7) policies' effectiveness. Lastly, section 8 wraps-up the paper, highlighting the main insights and conclusions.

2 Literature Review

At the heart of the *secular stagnation hypothesis* lies the possibility that the natural rate of interest (NRI) of an economy - i.e. the real interest rate (RIR) that stimulates a consumption-savings equilibrium to meet full-employment at the targeted level of inflation - is structurally determined by key slow-moving factors, such as population growth, income inequality or tighter borrowing constraints (Krugman, 2014; Summers, 2014b; Eggertsson, Mehrotra, and Robbins, 2017). Although this idea is neither new, nor revolutionary (Samuelson, 1958), the plethora of new-Keynesian (NK) models that have been extensively used during the past decades remain completely oblivious to such mechanism. Instead, these models assume that the RIR is simply given by the inverse of the household's discount rate. Among others, this assumption underpins the contemporary macroeconomic view that *booms and busts* are temporary deviations from a predictable path of growth and voids the contemplation of an indefinitely binding ZLB[8]. Furthermore, as Cochrane (2016) discusses, NK models have been shown to produce contradicting results compared to the recent economic performance, as well as to structurally suffer from the so-called *"forward guidance puzzle"*, as Del Negro, Giannoni, and Patterson (2012) analyze[9]. However, as Galí (2018)[10] reviews, despite a considerable amount of criticism, NK models remain today the powerhouse of modern macroeconomics. In fact, the scope of these models has been substantially widened, as new specifications and different assumptions are incorporated in order to cope with the shortcomings of earlier versions.

In this context, Eggertsson, Mehrotra, and Robbins (2017) formally modeled the *secular stagnation hypothesis* for the first time[11], by re-framing the consumer maximization problem, in the spirit of Samuelson (1958), and developing a series of OLG models that incorporated many of the key financial and nominal frictions that have become the center of Great-Recession-focused analysis (Eggertsson and Krugman, 2012; Schmitt-Grohé and Uribe, 2016). By doing so EMR explored the impacts of a long ZLB in an array of dif-

[8]ZLB episodes in a new-Keynesian framework occur due to a shock to the household's discount rate, which implies that the NRI temporarily falls enough so that the necessary nominal interest rates that would attain the inflation target would be negative. However, as Eggertsson, Mehrotra, and Robbins (2017) stress, the household's problem becomes ill-defined if the shock is other than temporary, which limits the scope, duration and impact of a ZLB episode under NK economics.

[9]The forward guidance puzzle is a mathematically derived property of infinitely lived agents, where small changes to the interest rate, in a very far future, produce unrealistically large results in the present, due to the the compounding factor in the model (Del Negro, Giannoni, and Patterson, 2012).

[10]Galí (2018) provides a very interesting review on how NK models have evolved since the crisis, to incorporate "hybrid" specifications - such as the present one - as a response to NK models' inability to deal with the ZLB. The author also reviews the pre and post-crisis NK literature, as well as the many arguments against and for this body of work.

[11]Although in a different context, the first microfounded *secular stagnation* model appeared in Ono (1994) and later in Ono (2001), where by exploring insatiable liquidity preferences a permanent *liquidity-trap* arises.

ferent degrees of complexity, while inaugurating a new growing strand of macroeconomic models (where this paper is included) on the *secular stagnation hypothesis*. Furthermore, these authors perfectly captured both the empirical observations[12] and the theoretical predictions of this *hypothesis*, where if a cocktail of structural characteristics come together to consistently deliver an incongruent bundle of an NRI and an inflation target, a permanently depressed output level and involuntary employment arise. Although the authors acknowledge other possible explanations for an indefinitely long ZLB period, such as self-fulfilling expectations (Benigno and Fornaro, 2017), falling land prices (Kocherlakota, 2013) or a scarcity of safe-assets (Caballero and Farhi, 2017), they also highlight that their model provides a new broad conceptual environment, where the previous arguments and other structural characteristics might be encompassed.

Nevertheless, as it was previously mentioned, it is once again important to note that changing the mechanisms governing an economy's natural rate of interest is not the same as repelling earlier models' insights altogether. In fact, as it will become evident throughout the paper, this model owes most of its mechanics to the vast *liquidity-trap* literature inaugurated by Hicks (1937) and continuously expanded until today (see Krugman, Dominquez, and Rogoff (1998), Gauti and Woodford (2003) or Eggertsson and Krugman (2012) for some recent examples). As mentioned above, both EMR and this paper alike closely follow Eggertsson and Krugman (2012) in introducing financial frictions to the households' problem, as well as on choosing a particular inflation-targeting monetary regime. Moreover, on the supply-side of the economy, a normal accelerationist (vertical) Friedman-Phelps Phillips curve (Phelps, 1967; Friedman, 1968) was derived for periods of positive inflation (Taylor, 1979; Forder, 2010), while a nominal wage downward rigidity as in Schmitt-Grohé and Uribe (2016) was used to give rise to the upward slopping Phillips curve (commonly used in the *liquidity-trap* literature - see for instance Akerlof et al., 1996, Benigno and Ricci, 2011 or Auclert and Rognlie, 2018). For that reason, many of the takeaways from modern NK models that touch upon low-interest rates economics can be (and will be) identified throughout this paper.

Lastly, ever since Summers (2013) first revived the issue and Eggertsson and Mehrotra (2014) published EMR's first draft, several papers have looked into the *secular stagnation hypothesis*. Amongst them, this paper highlights the work of Carvalho, Ferrero, and Nechio (2016), Gagnon, Johannsen, and Lopez-Salido (2016), as well as that of Ikeda and Saito (2014) in exploring the role of demographics on the decrease of real interest rates in the OECD, the US and Japan, respectively. These authors unanimously found a significant relationship between the aging population trends of developed societies and

[12] EMR go on to produce a quantitative model of the US economy, where the model's mechanics are shown to be highly consistent with the economic behavior of the past decades.

the respective three-decades-long decrease in RIRs. Moreover, the work of Eggertsson, Mehrotra, and Summers (2016) is also highlighted, as the OLG setting herein was expanded to allow for open-economy considerations. These authors found that (i) *secular stagnation* enhances the benefits from policy coordination, as the interdependence between economies is emphasized, and (ii) in the presence of external players, only Fiscal polices are effective in stimulating the economy. Finally, this paper highlights the work of Pichelmann (2015), Lancastre (2016), Auclert and Rognlie (2018) as the three leading references on the very scarce body of literature relating *secular stagnation* and inequality. In the first paper, Pichelmann (2015) starts by throughly reviewing the relevant literature on both topics, to then perform a qualitative analysis on the inter-linkage between the empirical trends of a declining NRI and an increasing income inequality in Europe. In the second paper, Lancastre (2016) used a very similar framework to that of Eggertsson, Mehrotra, and Robbins (2017) to find a negative relationship between rising inequality and the natural rate of interest of an economy. Finally, Auclert and Rognlie (2018), used a life-cycle model to analyze the transmission between income inequality and output. In line with this paper's and Lancastre (2016) findings, Auclert and Rognlie (2018) argued that rising income inequality translated into a lower output, due to richer individual's lower marginal propensity to consume, when compared to their poorer counterparts.

When it comes to income inequality *per se*, there are two main discussions that are relevant for this paper. First of all, this paper will approach the decades-long debate on how to measure and quantify income inequality. Secondly, given that this paper focuses on the impacts of income inequality on aggregate output, unemployment and inflation, surveying the literature on these topics is central to the discussion.

Therefore, since Lorenz (1905) and Gini (1912)[13], up until Atkinson (1970), and more recently Cowell (2011), economists have spent several decades debating on the intricate process of measuring, discussing and analyzing inequality in a consistent and unbiased way. As Cowell (2011) writes *"Inequality is in itself an awkward word [...] that [...] can trigger quite a number of different ideas [...] depending on [a reader's] training and prejudices, obviously suggest[ing] a departure from some idea of equality"*. Indeed, Dalton (1920), Atkinson (1970), as well as Rothschild and Stiglitz (1973), all noted that producing a statement on inequality is always a subjective exercise that builds upon a normative judgment on the preferred social welfare function (SWF), which - ultimately - determines whether a particular distribution is *"more equal"* than another. Moreover, even if the previous issue is disregarded for a moment, a brief analysis of the most popular inequal-

[13] Although the original work of Gini (1912) is herein referenced, the reader would be better off reading Ceriani and Verme (2012), which is written in English and provides an extensive review of the first publication.

ity measurements[14] quickly reveals that these are often impractical and inadequate. On the one hand, the majority of these dimensions are exclusively tailored for continuous income distributions. On the other hand, whenever discrete distribution measures are indeed fit to be applied, these are hard to explicitly introduce in the model, given that the underlying mechanism do not necessarily follow the indicators' structure.

Hence, given that building a comprehensive and intuitive framework is the corner stone of a sound theoretical analysis, it becomes extremely unappealing to choose a particular inequality parameter at the expense of mathematical simplicity and generalization power. For this reason, this paper looked at the work of Dalton (1920) and Allison (1978), and particularly at the *principle of transfers*, to argue that a relative *"flow"* analysis of income inequality might produce a more consistent, unbiased and engaging approach than its absolute parametric counterpart. Indeed, this concept - stating that a wealth or income transfer from the poorest to the richest ought to increase any measurement of inequality - allows for a perspective shift, where the impacts of inequality can be studied by looking at a relative increase/decrease in such characteristic, instead of focusing on the parameter in itself. In other words, by drawing from the fact that a redistribution shock from the bottom to the top of the income distribution constitutes an increase in inequality, regardless of the underlying measure, this paper is able to assess the impacts of income inequality without neither loosing consistency with the relevant literature, nor having to succumb to the costly trade-offs previously mentioned.

Secondly, in order to navigate the seemingly confusing plethora of ideas surrounding the impacts of income inequality on growth, this paper will borrow some of the main insights from the comprehensive reviews provided by Furman and Stiglitz (1998), Aghion, Caroli, and Garcia-Penalosa (1999) and Dynan, Skinner, and Zeldes (2004). In fact, as these authors do, this paper highlights the first and foremost observation that has become this subject's powerhouse during the past decades, which is the idea that an individual's marginal propensity to consume (MPC) decreases with income. Although this stylized-fact dates back to Brady and Friedman (1947) and Kuznets (1955), this concept withstood the test of time (see Huggett and Ventura (2000), Alvarez-Cuadrado and El-Attar Vilalta (2012) or Carroll et al. (2017) for some empirical analysis on the matter) and is now the basis for several of today's inequality frameworks (e.g. Mankiw (2000)). Moreover, despite the many empirical and theoretical studies that argue in either direction regarding the hindering (Persson and Tabellini, 1994; Sabot, Ross, and Birdsall, 2016) or the stimulating (Li and Zou, 1998) effects of inequality on growth, this paper emphasizes the importance of the underlying mechanisms governing each income's group MPCs, which underpins this model's key negative relationship between inequality and ouput.

[14] Gini Coefficient, Atkison coefficient, standard deviation/variance, among others (Cowell, 2011).

3 Endowment economy - modeling secular stagnation

3.1 3-period OLG model with a binding borrowing constraint for the young

In order to understand the key determinants of the real interest rate, Eggertsson, Mehrotra, and Robbins (2017) start by considering a simple 3-period overlapping generations endowment economy with a binding borrowing constraint in the first period. In this model, a representative utility-maximizing household lives for 3 periods $j = \{0, 1, 2\}$ - for young (y) in $j = 0$, middle-aged (m) in $j = 1$, and old (o) in $j = 2$ - and receives an endowment w^i_{t+j} in periods 1 and 2, with $i = \{y, m, o\}$. Moreover, the authors assume that households have no initial wealth, that no outstanding savings can remain in period $j = 2$ and that borrowing and lending takes place through a risk-free 1-period-long bond with a return of $(1 + r_t)$[15]. Thus, for a representative household born at time t, the following budget constraints arise:

$$c^y_t = -s^y_t \tag{1}$$

$$c^m_{t+1} + s^m_{t+1} = w^m_{t+1} - (1 + r_t)s^y_t \tag{2}$$

$$c^o_{t+2} = w^o_{t+2} + (1 + r_{t+1})s^m_{t+1}, \tag{3}$$

where c^i_{t+j} and s^i_{t+j}[16] are, respectively, the household's consumption and savings at life-phase i and at time $t+j$. Interestingly, looking at the household's budget constraints (1), (2) and (3), two particularities of this model can be identified right away: (i) due to no income in the first period, young households finance consumption solely through savings, and (ii) old households are *"hand-to-mouth"* consumers, meaning that they consume all of the remaining income in the last period and will not participate in the borrowing market ($s^o_t = 0$). This last property also means that consumption in the last-period can simply be given as a function of previous consumption decisions, life-time income and current and previous interest rates, which will be particularly useful to solve the household's maximization problem ahead.

Additionally, consider that the same representative household aims to maximize a

[15]Note that although individual households hold savings, the model does not allow for aggregate wealth, given that all savings should be held as loans and consequentially absorbed by a borrower. For that reason, any instances of physical capital are structurally excluded by the model.

[16]In the original paper (Eggertsson, Mehrotra, and Robbins, 2017), B^i_{t+j} denotes *bonds held by the individual* and is used to represent the symmetric of the household's saving positions. However, in this paper, in line with most literature, the more common designation for savings s^i_{t+j} was used to facilitate the reader's adaptation to the nomenclature.

general time-separable utility function of the form $U = \sum_{j=0}^{2} \beta^j u(c_{t+j})$, where $\beta = \frac{1}{(1+\delta)}$ and $0 < \delta < 1$ is the household's discount rate, and $u(\cdot)$ is a linear instantaneous utility function given by

$$u(c_t) = \ln(c_t). \tag{4}$$

Lastly, in the spirit of Eggertsson and Krugman (2012), the authors imposed a time-varying exogenous debt constraint D_t so that:

$$(1 + r_t)s_t^i \geq -D_t \tag{5}$$

$$s_t^i \geq -\frac{D_t}{(1 + r_t)}. \tag{6}$$

Intuitively, this borrowing constraint states that at life-phase i, in period t, households can never borrow more than $\frac{D_t}{(1+r_t)}$. However, note that, for the borrowing constraint to be binding in period $j = i$, $s_{i,t}^* \leq -\frac{D_t}{(1+r_t)}$, where $s_{i,t}^*$ is the optimum amount of savings in a standard, unrestricted, no-borrowing-constraint model[17]. In this case, since the authors assume that the borrowing constraint binds for the young ($j = 0$), this corresponds to $D_t \leq \frac{1}{1+\beta+\beta^2}\left[w_{t+1}^m + w_{t+2}^o/(1 + r_{t+1})\right]$, which the authors find to be the case in their numerical experiments[18].

With the borrowing constraint in place, the new budget constraints for the representative household are:

$$c_t^y = \frac{D_t}{(1 + r_t)} \tag{7}$$

$$c_{t+1}^m + s_{t+1}^m = w_{t+1} - D_t \tag{8}$$

$$c_{t+2}^o = w_{t+2} + (1 + r_{t+1})s_{t+1}^m. \tag{9}$$

Finally, the household's maximization problem can be formalized as:

$$\begin{aligned} &\max_{(c_{t+1}^m, c_{t+2}^o)} \ln\left[\frac{D_t}{(1 + r_t)}\right] + \beta \ln(c_{t+1}^m) + \beta^2 \ln(c_{t+2}^o) \\ &\text{s.to} \\ &c_{t+1}^m + s_{t+1}^m = w_{t+1} - D_t \\ &c_{t+2}^o = w_{t+2} + (1 + r_{t+1})s_{t+1}^m. \end{aligned} \tag{10}$$

Given that the utility function is time and state separable and exhibits well-behaved properties, households are at an interior solution of the maximization problem and the

[17]Refer to appendix A for the model's full derivation. For reference, this is a standard 3-period OLG model, where households spread the available income equally amongst periods in concordance with the Permanent Income Hypothesis (Friedman, 1957).

[18]EMR build a quantitative analysis upon the endowment model herein defined. In this section, the authors found that the borrowing constraint introduced is in line with the what is empirically observed.

famous Euler equation (or tangency condition) - in its general form $u'(c_t) = (1+r_t)u'(c_{t+1})$ - can be applied to find the individual's optimum consumption responses between periods 1 and 2:

$$c^o_{t+2} = (1 + r_{t+1})\beta c^m_{t+1}. \tag{11}$$

Consequently, the problem can be solved to find[19]

$$c^m_{t+1} = \frac{1}{1+\beta}\left[w_{t+1} - D_t + \frac{w_{t+2}}{(1 + r_{t+1})}\right] \tag{12}$$

$$s^m_{t+1} = \frac{1}{1+\beta}\left[\beta(w_{t+1} - D_t) - \frac{w_{t+2}}{(1 + r_{t+1})}\right] \tag{13}$$

$$c^o_{t+2} = \frac{\beta}{1+\beta}\left[(1 + r_{t+1})(w_{t+1} - D_t) + w_{t+2}\right]. \tag{14}$$

Looking at the household's savings patterns in particular, one can observe that a capital market can now be established, as the demand for borrowings (L^d_t) from young households,

$$L^d_t = -s^y_t N_t = \frac{D_t}{(1 + r_t)}N_t, \tag{15}$$

interacts with the middle-aged household's savings supply (L^s_t),

$$L^s_t = s^m_t N_{t-1} = \frac{1}{1+\beta}\left[\beta(w^m_t - D_{t-1}) - \frac{w^o_{t+1}}{(1 + r_t)}\right]N_{t-1}, \tag{16}$$

where N^i_t is total size of the generation born at time t. Assuming no population growth ($N_{t-1} = N_t$)[20], the equilibrium clearing ($L^d_t = L^s_t$) real interest rate is then defined as

$$(1 + r_t) = \frac{(1 + \beta)D_t + w^o_{t+1}}{\beta w^{m,d}_t}, \tag{17}$$

where $w^{m,d}_t = w^m_t - D_{t-1}$ is the disposable income at time t for middle-aged individuals. Thus, under EMR's setting, the real interest rate now depends (i) positively on the current debt limit D_t, once relaxing it ($\uparrow D_t$) allows for further borrowing and hence demand expansion, (ii) negatively on middle-age available income $w^{m,d}_t$, where optimizing individuals seek to smooth consumption by saving current available income for the next period, (iii) positively with old-age endowment for the opposite reason (i.e. higher-income old households reduce savings to meet their consumption smoothing path) and, lastly, (iv) positively on the household's discount rate (negatively on the discount factor β), where a

[19] Refer to appendix B for the full derivation.

[20] In the original paper, population growth is highlighted as one of the most important *secular forces* driving the equilibrium real interest rate. However, given that (1) this variable it is not key to this paper's analysis, (2) Eggertsson, Mehrotra, and Robbins (2017) already provide an extensive analysis on the former and (3) it considerably simplifies the mathematical equations at hand by avoiding the proliferation of growth terms, population growth was thus assumed to be inexistent.

higher degree of utility impatience ($\downarrow \beta$) expands the demand for loans.

Accordingly, by moving away from the representative agent's framework, EMR were able to determine the real interest rate as a function of slow-moving secular forces. Indeed, the mechanism herein can be seen as the central piece of this model, given that it completely changes the nature, scope and impact of a potential fall in the real interest rate. At this point, as portrayed in the *secular stagnation hypothesis*, it is possible to account for the possibility that a cocktail of structural characteristics might sufficiently depress $(1 + r_t)$, so that an arbitrary long ZLB period arises, without any endogenous built-in forces to restore a normal equilibrium. In other words, equation (17) captures the underpinnings of *secular stagnation* by allowing the real interest rate to reflect the economy's key determinants, rather than a specific - temporary - shock to a parameter (as in NK economics).

To further explore the mechanisms above, consider, for instance, a temporary negative shock to the borrowing constraint D_t[21] and denote the shocked variable by D_t'. First of all, due to its immediate effects on the demand for savings (15), one can start by observing that if the shocked borrowing constraint becomes small enough so that

$$D_t' \leq \frac{\beta \left[w_t^m - D_{t-1} \right] - w_{t+1}^o}{(1 + \beta)}, \tag{18}$$

the economy immediately falls into a ZLB state. This result is in line with what is thought to have happened in the months following the financial crisis, where due to a significant shock to the collateral constraint, the demand for loans (15) immediately collapses and the excess savings pressure real interest rates down.

Secondly, due to its lagged effects on the supply of savings (16), an otherwise one period shock becomes propagated to the future, with potentially important implications. Intuitively, this can be understood as the consequence of middle-aged individuals' disposable income ($w_t^{m,d}$) dependence on the debt contracted in the first period (D_{t-1}). Thus, if due to an external shock young individuals borrow less in $t - 1$, when these migrate to t, the now middle-age households will have a lower debt burden - i.e. excess savings. For that reason, two main results arise. On the one hand, if one considers an economy's recovery after a full deleveraging cycle - i.e. D_t' is restored to D_t -, it might still be the case that the currently expanded supply for loans, due to $(t - 1)$'s still heavily shocked borrowing constraint (16), dominates over the demand expansion (15), corresponding to D_t's recovery. On the other hand, if one considers a shock that is not large enough to throw the economy off its balance, at first, it might be the case that, due to its persistence, the former might indeed occur in the following period. Equation (19), illustrates

[21] Following a previous work of Eggertsson and Krugman (2012), the authors argue that this shock can model the 2008-crisis deleveraging process and offer a particularly interesting insight on how a temporary shock might throw an otherwise healthy economy into *secular stagnation*

how much smaller a persistent shock has to be in order to induce a ZLB period, compared to (18):

$$D'_t \leq \frac{\beta w^m_{t+1} - w^o_{t+2}}{(1+2\beta)}, \tag{19}$$

3.2 Introducing endowment inequality

In order to understand the impact of inequality on the equilibrium real interest rate previously defined, Eggertsson, Mehrotra, and Robbins (2017) expanded their 3-period OLG model with a borrowing constraint to allow for a two-household type setting, where high *(h)* and low *(l)* income individuals are now faced with different middle-age endowments[22].

In this setting, all households receive income in the second and third periods of life, but now high-income households are awarded with $w^{h,m}_{t+1}$ and behave as in (7), (12) and (14), whereas poorer households receive a small enough middle-age endowment ($w^{m,l}_{t+1}$) so that the borrowing constraint (5) binds in the second period as well:

$$w^{m,l}_{t+1} \leq D_t + \frac{w^{o,l}_{t+2} - (1+\beta)D_{t+1}}{\beta(1+r_{t+1})}. \tag{20}$$

In other words, low-income households borrow throughout middle-age in order to finance consumption and roll over their first period's debt, while repaying any outstanding debts, as well as consuming any remaining income, in the last period. For that reason, budget constraints (21), (22) and (23) already provide all the information about the household's optimal behaviors as they cannot do otherwise

$$c^{y,l}_t = \frac{D_t}{(1+r_t)} \tag{21}$$

$$c^{m,l}_{t+1} = \frac{D_{t+1}}{(1+r_{t+1})} + w_{t+1} - D_t \tag{22}$$

$$c^{o,l}_{t+2} = w^{o,l}_{t+2} - D_{t+1}. \tag{23}$$

With respect to the household's savings decisions, the previous equations can be translated in:

$$\begin{aligned} s^{y,l}_t &= -\frac{D_t}{(1+r_t)} \\ s^{m,l}_{t+1} &= -\frac{D_{t+1}}{(1+r_{t+1})} \\ s^{o,l}_{t+2} &= 0. \end{aligned} \tag{24}$$

Once again, looking at the household's aggregate savings, a well-defined capital

[22] For consistency, this paper will follow EMR in assuming a uniform old-age endowment (w^o_{t+2}) across income groups. However, it is noted that this simplification has no impact whatsoever over the equilibrium real interest, given that only $w^{o,h}_{t+2}$, regardless of its value, plays a role in (27).

market can be established as a result of the interaction between the demand for loans (L_t^d) from low-income households and youngsters,

$$L_t^d = -N_t s_t^y - \eta N_{t-1} s_t^{m,l} = \frac{D_t}{(1+r_t)} N_t + \eta \frac{D_t}{(1+r_t)} N_{t-1}, \tag{25}$$

and the supply of savings (L_t^s), which is now solely derived from high-income individuals,

$$L_t^s = (1-\eta) N_{t-1} s_t^{m,h} = \frac{(1-\eta)}{1+\beta} \left[\beta (w_t^{m,h} - D_{t-1}) - \frac{w_{t+1}^{o,h}}{(1+r_t)} \right] N_{t-1}, \tag{26}$$

where $0 \leq \eta \leq 1$ denotes the fraction of the population that is borrowing-constrained during $j = 1$. Finally, equations (25) and (26) can be combined together with $N_{t-1} = N_t$ to define the new equilibrium real interest rate as

$$(1+r_t) = \frac{(1+\eta)}{(1-\eta)} \frac{(1+\beta) D_t}{\beta (w_t^{m,h} - D_{t-1})} + \frac{w_{t+1}^{o,h}}{\beta (w_t^{m,h} - D_{t-1})}. \tag{27}$$

Looking at equation (27), and comparing it with its representative agent counterpart (17), there is one implicit and three explicit main differences that ought to be analyzed in order to have a fuller understanding of this new real interest rate equilibrium.

First of all, explicitly, one can start by noticing that the real interest rate is now solely dependent on the richer households' income profile and particular on the amount of disposable income in $j = 1$. This result arises from the fact that, in this setting, only high-income earners participate in the supply of loans (26) and poorer households, as well as younger ones, are bounded to borrow $\frac{D_t}{(1+r_t)}$ (25)(exogenously defined). For that reason, the market becomes solely dependent on how much high-income earners spare in excess savings in their middle-period of life ($j = 1$), without any direct connection to the low-income earners' endowment. More importantly, this result is at the core of *Proposition 1*, regarding the role of increasing income inequality, defined as an endowment redistribution from lower to higher income households[23] ($w_{t+1}^{m,h} \uparrow, w_{t+1}^{m,l} \downarrow$). By closely analyzing (27), one can clearly identify that an increase in inequality translates into a surplus of savings from richer high-income households, without any compensating demand side force, which pushes the real interest rate downwards. In other words, the equilibrium equation (27) shows that, apart from the secular forces already approached in section 3.1[24], increasing inequality might also be one of the slowly evolving forces that have been steadily driving the real interest rate down - as in line with the empirical evidences[25]. *Proposition 1* summarizes this insight explicitly:

[23] The discrete equivalent of the commonly used mean-preserving spread of the income distribution.

[24] As well as population growth and aging dynamics (Eggertsson, Mehrotra, and Robbins, 2017).

[25] Empirically, the 30 years-long trend of rising inequality (Forster, Chen, and Llenanozal, 2011; Piketty, 2015) can be broadly paired with the also 30 years-long trend in decreasing real interest rates (Krugman, 2014; Pichelmann, 2015; Laubach and Williams, 2016). Although further empirical research should be performed in order to assess the a clear relationship between the variables, this paper highlights the consistency of this result with the observable data.

Proposition 1: *Rising income inequality is among the secular forces consistent with the empirically observable, 30 years-long decrease in the real interest rates.*

Secondly and thirdly, also explicitly, one can look at the newly introduced η, and more particularly, at the quotient $\frac{(1+\eta)}{(1-\eta)}$ in (27), to draw two main insights. On the one hand, quite straightforwardly, it is possible to observe that the equilibrium real interest rate now depends positively on the former, since for $\eta < 1$, $\frac{(1+\eta)}{(1-\eta)} > 1$. On the other hand, an increase in the same parameter η seems to bring a higher level of volatility to the economy, as (28) shows. These results arise from the simultaneous expansion of (25) and contraction of (26) - the demand and supply for loans, respectively -, occurring when middle-age individuals shift from net savers to net borrowers, due to an increase in η. The former, not only pressures the real interest rate upwards, but also means that more people are exposed to a shock to the collateral constraint - i.e. higher volatility - as shown below.

$$\frac{\delta(1+r_t)^{ineq}}{\delta D_t} = \frac{(1+\eta)}{(1-\eta)}\frac{\delta(1+r_t)^{noineq}}{\delta D_t} \Rightarrow \frac{\delta(1+r_t)^{ineq}}{\delta D_t} \geq \frac{\delta(1+r_t)^{noineq}}{\delta D_t}, \forall \eta \in [0,1], \tag{28}$$

Although, at first sight, this result could imply that an increase in inequality would stimulate the real interest rate, section's 2 discussion on the measurements of inequality should once again be summoned up to argue that it is not the amount of borrowing-constrained people *per se* that define inequality, but rather the relationship between the relative wealth and size of both groups. Furthermore, an important insight to highlight for future sections is that, compared with the representative agent scenario (3.1), societies where a larger share of the population face a binding borrowing constrained for two consecutive periods react more strongly to an exogenous shock and thus are more vulnerable to persistent episodes of ZLB and *secular stagnation*.

Lastly, implicitly, and in line with section's 3.1 remarks, assuming that D_t binds simultaneously for the young and for low-income middle-aged households, establishes an upper-bound that can be given by $D_t \leq \frac{(1+\beta)^2}{(1+(1+\beta)\beta)}\frac{(w^{o,l}_{t+2}-D_{t+1})}{(1+r_{t+1})}$, which compared to its analogous representative agent embodies two main differences: (i) the upper-bound is much smaller than before, and (ii) it now depends on the endowment of the poorest individuals. Thus, although this restriction has no actual effects in the model, given that D_t is exogenous and assumed to bind regardless, an endogenous analysis to D_t would deem that a drop in the poorer household's endowment (increasing inequality) would produce a decrease in $(1+r_t)$. This is a reinforcing mechanism to the hindering effects of a redistribution shock, as already approached ($w^{m,h}_{t+1} \uparrow, w^{m,l}_{t+1} \downarrow$), but only by endogenizing this parameter the fuller dynamics of this upper-bound would become clearer. However, this paper will leave this discussion as a suggestion for future research.

4 Moving away from an endowment economy

Moving away from an endowment economy and towards a more comprehensive framework, this paper will now consider a plethora of extensions to 3, with the intent of capturing the main economic implications of the defined *secular stagnation* model.

4.1 Price Level

First of all, the introduction of a perfectly flexible nominal price level P_t will be considered. To do so, assume that instead of holding real bonds, household's now hold a one-period nominal risk-free bond denominated in currency that yields a nominal return controlled by the central bank of $(1 + i_t)$[26]. Furthermore, consider, as usual, that the nominal interest rate cannot take negative values (the ZLB binds),

$$i_t \geq 0, \tag{29}$$

and that the Fisher relation (assuming perfect foresight) holds at all times,

$$(1 + r_t) = \frac{(1 + i_t)}{\Pi_t}, \tag{30}$$

where $\Pi_t = \frac{P_{t+1}}{P_t}$ and denotes the inflation rate.

At this point, and in line with the literature on monetary neutrality[27], introducing a perfectly flexible nominal price level, while assuming perfect foresight, does not change the household's behaviors whatsoever. Indeed, and considering that, instead of exogenous endowments (w^i_{t+j}), households now earn a nominal labor income $(W^i_{t+j} l^i_{t+j})$ - where W^i_{t+j} is the nominal income and l^i_{t+j} is the amount of labor hours worked by an individual i at time $t + j$ -, indeed, the nominal version of the agent's maximization problem (10) can be simply written by substituting every variable for its nominal counterpart (denoted in

[26]The authors explicitly extend their model in Appendix F of EMR to allow for a micro-founded demand for money. As the results are not considerably different from the framework herein explored, this paper will not pursue such analysis.

[27]See for example Lucas Jr (1972) and Lucas Jr (1996) for an interesting review of the issue.

capital letters for differentiation[28]):

$$\begin{aligned}
&\max_{(C^m_{t+1},C^o_{t+2})} \ln\left[\frac{\Pi_t}{(1+i_t)}\frac{D_t}{P_t}\right] + \beta\ln\left(\frac{C^m_{t+1}}{P_{t+1}}\right) + \beta^2\ln\left(\frac{C^o_{t+2}}{P_{t+2}}\right)\\
&\text{s.to}\\
&C^m_{t+1} + S^m_{t+1} = W^m_{t+1}l^m_{t+1} - D_t\\
&C^o_{t+2} = W^o_{t+2}l^o_{t+2} + \frac{\Pi_t^{\phi_\pi - 1}}{\Gamma^*}S^m_{t+1}.
\end{aligned} \tag{31}$$

From (31), the nominal equivalent of Euler equation (11) can also be derived to confirm the previous proposition:

$$\frac{1}{C^m_{t+1}} = \beta\frac{1}{C^o_{t+2}}\frac{(1+i_t)}{\Pi_t}. \tag{32}$$

Hence, in the same spirit, the household's optimal responses, as well as the equilibrium real interest rate under fully flexible prices are already defined by equations (7), (12) and (14) - for the representative agent and for the high-income households -, by (21), (22) and (23) - for low-income households -, and by (17) and (27) for, respectively, the capital market clearing conditions for the no-inequality (section 3.1) and inequality settings (section 3.2).

On the other hand, taking a closer look at the newly introduced nominal interest and inflation rates, and particularly at the Fisher equation (30), one can note that a lower-bound for inflation is established by simply substituting $(1 + i_t)$ for the ZLB condition (29) in (30):

$$\underline{\Pi} \geq \frac{1}{(1+r_t)}, \tag{33}$$

where $\underline{\Pi}$ denotes the *natural lower bound on inflation*. As the authors discuss, if for a positive real interest rate the inflation's natural lower bound is of little relevance[29], when $(1 + r_t)$ becomes negative, inflation needs to raise above $\underline{\Pi}$ to sustain an equilibrium. Although the question of interest in this analysis will become "*what would happen if the central bank refused to raise the inflation target above* $\underline{\Pi}$?", as of now, it can not yet be answered, given that under fully flexible prices, relationship (33) *must hold*[30]. However, as nominal rigidities are introduced later in this paper, central banks will be able to determine a specific inflation target and the impacts of these decisions will be assessed later on.

[28]Note that the connotation equivalence holds for Latin letters only and rate variables are excluded (i.e. i_t, r_t).

[29]For a positive real interest rate, (33) shows that inflation's natural lower bound is negative and thus much smaller than any inflation targets upheld by central banks and authorities.

[30]The model does not admit any other equilibrium.

4.2 Firms

In order to define the supply side of this model, consider an economy where firms are perfectly competitive, take prices as given and aim to maximize period-by-period profits (Z_t) by producing a generic good Y and by using labor according to a standard diminishing marginal returns to scale production function

$$Y_t = L_t^{\alpha}. \tag{34}$$

Hence, the firms' profit maximization problem can be formalized as

$$\begin{aligned} &\max_{L_t} \quad Z_t = P_t Y_t - W_t L_t \\ &\text{s.to} \\ &Y_t = L_t^{\alpha}, \end{aligned} \tag{35}$$

from where the firm's inverse labor demand[31] immediately follows

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1}. \tag{36}$$

Naturally, equation (36) arises from the firm's optimization problem together with the no-profit condition yielded by a perfect competition setting, which forces firms to equate the marginal productivity of labor to the real wage rate at all times.

4.3 Downward Nominal Wage Rigidity

In line with the new-Keynesian literature, nominal rigidities to this model will be adopted in order to depart from the previously frictionless economy. In particular, this paper will follow Schmitt-Grohé and Uribe (2016) in adopting a downward nominal wage rigidity (DNWR), which has been the focus of intense empirical and theoretical research since at least Tobin (1972). Indeed, this author, and more recently Akerlof et al. (1996), Babecky et al. (2010), Daly and Hobijn (2015) and Fallick, Lettau, and Wascher (2016), for instance, argue that households exhibit high-levels of resistance to downward nominal adjustments both in normal times and in periods of high-unemployment and major labor market distress[32]. Moreover, given that no consensual conclusion is found in the literature regarding whether different income groups exhibit DNWR disparities[33],

[31] With respect to the real wage $\frac{W_t}{P_t}$ rather than L_t^d directly, which yields the actual labor demand.

[32] In fact, the former is thought to be especially prevalent during high-unemployment periods. Tobin (1972) first highlighted this phenomena by looking at the 1970's oil crisis and arguing that not only the DNWR existed, but also that it was accentuated during the crisis. More recently, Great-Recession-focused empirical studies, such as Fallick, Lettau, and Wascher (2016) or Daly and Hobijn (2015) unequivocally confirmed this hypothesis, also registering a more sever level of DNWR during the crisis.

[33] Babecky et al. (2010) and Fallick, Lettau, and Wascher (2016) observed in their studies on, respectively, the EU and the US labor markets, that: (1) (white-color) high-income individuals usually exhibit a higher downward nominal wage rigidity than their (blue-collar) low-income counterparts; (2) labor

this paper will follow a single wage norm that closely tracks EMR's representative agent version as displayed below:

$$\begin{aligned} &W_t = max\left\{\tilde{W}_t, W_t^{flex}\right\} \\ &\qquad\quad for \\ &\tilde{W}_t = \psi W_{t-1} + (1-\psi)W_t^{flex} \\ &W_t^{flex} = P_t \alpha \bar{L}^{\alpha-1}. \end{aligned} \tag{37}$$

In a more intuitive manner, the previous equations can be rewritten to obtain:

$$W_t = \begin{cases} W_t^{flex} = P_t \alpha \bar{L}^{\alpha-1} \quad if \quad \Pi \geq 1 \\ \tilde{W}_t = \psi W_{t-1} + (1-\psi)W_t^{flex} \quad if \quad \Pi < 1, \end{cases} \tag{38}$$

where $\tilde{W}_t$ is a *"wage norm"* stating the minimum wage accepted by households under deflation. As the former is a weighted average of the previous period's wage (W_{t-1}) and today's fully-flexible (W^{flex}) counterpart, where the relative weights are given by $0 < \psi < 1$, this parameter can then be interpreted as the wage-rigidity degree of the economy[34].

The wage regime herein outlined, as EMR highlight, is a comprehensive theoretical approach to model the empirical particularities of the modern new-Keynesian supply schedule and, particularly, the low-inflation dynamics that this paper is interested in. As it will become clearer in section 5.1, when the aggregate supply schedule is derived explicitly, the chosen functional form is able to simultaneously capture both the accelerationist Friedman-Phelps Phillips curve for periods of positive inflation (Phelps, 1967, Friedman, 1968), widely consensual among economists (Taylor, 1979, Forder, 2010), as well as the upward slopping Phillips curve commonly used for low inflation periods (see for instance Akerlof et al., 1996, Benigno and Ricci, 2011). Indeed, looking at the upper branch of system (38), the perfectly flexible adjustment of wages under inflation means that there are no long-run trade-offs between higher inflation and unemployment: workers get their salaries adjusted in order to maintain real wages constant. On the other hand, the bottom branch of (38), shows that households exhibit nominal resistance in having their wages

intensive industries (usually associated with lower-income individuals) are identified as more rigid than others; and (3) labor protection laws and minimum wage requirements would create almost perfect wage rigidity for the bottom of the income distribution. Hence, although not consensual, the most convincing conclusion would be that low-income workers would have a higher degree of wage rigidity than richer individuals. However, given that Schmitt-Grohé and Uribe (2016) already estimated a wage rigidity close to unity and that the extent to which income group plays a relevant role in the DNWR level remains unclear, this paper argues that the impacts of inequality on such rigidity wouldn't be appreciable.

[34]Note that this specification is able to encompass a wide range of wage rigidity levels - from when $\psi = 0$ and wages are fully flexible to when $\psi = 1$ and wages are fully downwardly rigid -, which will be of interest later on.

adjusted downwards, which not only creates a wedge between how much companies are prepared to offer in real terms (36) and the wage norm previously defined (38), but also deems nominal changes highly consequential and voids the monetary neutrality[35] (Akerlof et al., 1996).

4.4 Monetary Policy Rule

Lastly, to close this system of equations, consider that the Central Bank follows a standard Taylor rule given by

$$(1+i_t) = max\left(1, (1+i^*)\left[\frac{\Pi_t}{\Pi^*}\right]^{\phi_\Pi}\right), \tag{39}$$

where Π^* and i^* are policy rule parameters set by the Central Bank and held constant, denoting, respectively, the inflation rate and nominal interest rate targets. $\phi > 1$ can be interpreted as the responsiveness of the Central Bank to deviations from the targeted inflation, where a higher ϕ will mean stronger nominal interest rate adjustments (bigger overshoot) to correct inflation towards its target.

As EMR interestingly note, the particulars of the Taylor rule specified do not substantially affect the results at hand. Instead, this rule aims to mimic an inflation targeting regime - common to most central banks - in order to study the effects of setting an unreachably low inflation target under a ZLB episode. Note that the this also means that the nominal interest rate target will be set either according to the Fisher interaction between the NRI of the economy and the inflation rate target under a non-binding ZLB or, if $i^* < 0$, the ZLB binds and the interest rate takes the value of 1:

$$(1+i^*) = max\left(1, (1+r^f)\Pi^*\right) \tag{40}$$

Hence, after setting the inflation rate target (Π^*), the central bank looks at the determinants of the economy that define the NRI, to then decide on the nominal interest rate target $(1+i^*)$ - and, in turn, on the actual nominal interest rate $(1+i)$ - that yield an implicit real rate of return in line with the NRI so that full employment can be attained.

5 Steady-State

Finally, as the economy is now fully defined, an equilibrium for the inequality model derived in 3.2 and 4 can then be written as follows:

[35] It is highlighted that as monetary neutrality disappears, so does the model's inability to admit a different inflation target smaller than $\underline{\Pi}$, as discussed in 4.1. Accordingly, the issue raised in 4.1 becomes addressable as central authorities can set an inflation target at their discretion, as defined in section 4.4.

Definition 1: *A competitive equilibrium is a sequence of quantities* $\{C_t^{y,l}, C_t^{m,l}, C_t^{o,l}, S_t^{y,l}, S_t^{m,l}, C_t^{y,h}, C_t^{m,h}, C_t^{o,h}, S_t^{y,h}, S_t^{m,h} L_t, Y_t, Z_t\}$ *and prices* $\{P_t, W_t, W_t^{flex}, r_t, i_t\}$ *that satisfy (7), (3), (11), (21), (22), (23), (27), (29), (35), (36), (38), (39) and (40) given an exogenous process for* $\{D_t, l^l, l^h\}$ *and initial values for* W_{-1} *and* $S_{-1}^{m,h}$.

Accordingly, this paper will start by fully analyzing the model's steady-state by deriving, plotting and discussing the relationships between output and inflation - i.e. aggregate supply (AS) and aggregate demand (AD) - to then proceed to analyze the economy's actual equilibrium and draw comparative statics to fully understand the impacts of income inequality on the *secular stagnation model*.

5.1 Aggregate Supply

In order to derive the AS curve in the steady-state, one has to look at the firm's production function (34) under the conditions that equilibrate both the labor and capital markets simultaneously[36]. Thus, and given that the wage rigidities introduced in (37) define two distinct *"wage regimes"* for $\Pi > 1$ *and* $\Pi < 1$, the AS curve will also feature these particularities, having two distinct segments with a kink at $\Pi = 1$. Accordingly, for the case of inflation ($\Pi > 1$):

$$W = W^{flex} = P\alpha\bar{L}^{\alpha-1}$$

Naturally, as households accept exactly the same as firms are willing to offer, the labor market clears at any level $\bar{L}$. Hence, the economy's output can be derived by substituting $L^* = \bar{L}$ in the firm's production function (34) to yield

$$Y^s = \bar{L}^\alpha = Y^f,$$

where Y^s denotes the steady-state aggregate output supplied by firms and Y^f designates the full-employment output, which, in this case, match. Note that, as discussed in 4.3, under rising prices, output production is completely independent of the inflation rate and it is only determined by the amount of labor supplied (Friedman-Phelps Phillips curve). This phenomenon can be identified in Figure 2 as the vertical section of the AS curve[37].

Contrarily, in the event of a deflationary episode ($\Pi < 1$), the wage norm $\tilde{W}_t$ binds, meaning that households no longer accept nominal wages to be downwardly adjusted to meet the constant real wage rate offered by firms (36). This means that the lowest real wage accepted by households rises above the marginal productivity of labor, which,

[36] Note that *equilibrate the market* should not be read as *"clearing the market"*, given that the labor market will not be able to clear under deflation as explained below.

[37] Note that the normalization of $\bar{L} = 1$ in Figure 2 was adopted for simplicity and clarity purposes. In turn, this assumption yields $Y^f = 1$, which is very practical for interpretation purposes. This assumption has no relevant impacts other than providing a stylized view of the economy.

thus, contracts the labor demanded below the $L^s = \bar{L}$ and pushes the market towards a persistent disequilibrium with involuntary unemployment $(\bar{L} - L^*)$. Mathematically, one can use the wage norm (37) to derive that under a constant deflation rate Π, the minimum steady-state real wage rate accepted by households is

$$\frac{W_t}{P_t} = \frac{(1-\psi)\Pi\alpha}{\Pi - \psi}\bar{L}^{\alpha-1}, \tag{41}$$

which can be substituted in the demand for labor to derive $L^* < \bar{L}$:

$$L^* = \left[\frac{(1-\psi)\Pi}{\Pi - \psi}\right]^{\frac{1}{\alpha-1}} \bar{L}. \tag{42}$$

Given that the equilibrium amount of labor is now defined, the production function (34) can once again be used to derive the AS curve under $\Pi < 1$:

$$\begin{gathered} wrt \quad to \quad \Pi \\ \frac{\psi}{\Pi} = 1 - (1-\psi)\left(\frac{Y^s}{Y^f}\right)^{\frac{1-\alpha}{\alpha}} \\ or \quad wrt \quad to \quad Y \\ Y^s = \left(\frac{1-\frac{\psi}{\Pi}}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} Y^f, \end{gathered} \tag{43}$$

which is, as EMR note, a non-linear Phillips curve modeling the long-run relationship between low-inflation and output as previously discussed in 4.3 (bottom section of the AS curve in Figure 2[38]).

The intuition behind such relationship is intimately linked to the nominal wage rigidity introduced. Given that prices are sticky in nominal terms only, a rising inflation will effectively depreciate real wages, allowing companies to hire more labor and thus produce more. Hence, naturally, in this lower part of the AS curve (43), aggregate production will increase with inflation. Moreover, one can also observe that the slope of the supply schedule also depends positively on the degree of wage flexibility, where full wage rigidity $(\psi \to 1)$ would push the AS towards an horizontal line at $\Pi = 1$ and perfectly flexible wages $(\psi \to 0)$ would create a vertical line at Y^f .

5.2 Aggregate Demand

On the Aggregate Demand side of this economy, one can look at the sum of each living household's i consumption to derive the steady-state output demand as

$$Y^d = \sum_i^N C^i. \tag{44}$$

[38]In order to plot the AS curve $\Pi = \frac{\psi}{1-(1-\psi)Y^{\frac{1-\alpha}{\alpha}}}$ was used, where $\alpha = 0.5$ and $\psi = 0.7$.

Assuming that low-income households receive labor income both trough middle and old-age, while rich individuals only work in period $j = 1$[39], (44) can be expanded to show

$$Y^d = \eta N \left[W^{m,l} l^{m,l} + W^{o,l} l^{o,l} \right] + (1-\eta) N W^{m,h} l^{m,h}, \tag{45}$$

which simply states that Y^d is given by the sum of all labor income earned in this economy at a given time. Further considering that all households receive the same wage[40], that each household is exogenously awarded a limited life-time labor endowment - l^l and l^h - according to its income group and that poorer households receive theirs evenly across periods ($l^{m,l} = l^{o,l}$), the aggregate labor supply (L^s) in this economy can then be defined as

$$L^s = \underbrace{\eta N l^l}_{L^l} + \underbrace{(1-\eta) N l^h}_{L^h}, \tag{46}$$

where L^l and L^h denote the aggregate labor supplied by each income group. Accordingly, (46) can be used in (45) to simplify aggregate demand as the marginal wage multiplied by the total labor supplied in the economy[41]:

$$Y^d = W L^s. \tag{47}$$

Finally, denoting by ξ and $(1-\xi)$, respectively, the fraction of the total supply of labor L^s awarded to the rich and poor households on aggregate such that

$$\begin{gathered} \underbrace{L^h/L^s}_{\xi} + \underbrace{L^l/L^s}_{(1-\xi)} = 1 \\ \xi > \eta, \end{gathered} \tag{48}$$

where the restriction on ξ simply states that rich(er) households *(h)* should have a higher life-time income than poor(er) individuals *(l)*[42]. Additionally, note that this parameter can be interpreted as an inequality proxy for this economy, since an increase in the former corresponds to a transfer of relative wealth - via the labor endowments - from the bottom

[39] In EMR's representative agent model, no old-age income ($W^o L^o = 0$) is assumed and thus output demand is solely dependent on middle-age aggregate nominal income ($W^m L^m$), which considerably simplifies the derivation. However, by analyzing the low-income households budget constraints (21), (22) and (23), one can observe that it is structurally impossible to follow suit for the poorer households: as these individuals are borrowing-constrained for two consecutive periods, last period's outstanding debts will need to be covered by some sort of old-age endowment. Hence, a compromise between EMR's approach and the one admitted by the model was chosen with $W^{o,h} L^{o,h} = 0$.

[40] All households receive the same wages given that labor is homogeneously productive regardless of income group, as equation (34) shows.

[41] Note that (47) is a standard AD definition that would arise regardless of the wage/labor supply setting chosen. The former derivation is in place to allow for an intuitive step-by-step introduction to the particulars of this model.

[42] Note that this restriction is structurally, as well as intuitively, important for this the model, ensuring that the model derived above can be consistently applied. In fact, without this restriction, condition (20) would apply to rich households as it does for the poor, making the former equal in every aspect to the latter and thus voiding the application of our model.

of to the top of the distribution (as discussed in section 2)[43]. The former is in all aspects equivalent to the previously explored endowment shock ($w^{m,h}_{t+1} \uparrow, w^{m,l}_{t+1} \downarrow$) in section 3.2.

Nevertheless, finally, by combining the capital market's clearing condition (17) with the ZLB condition (29), the Fisher equation (30), the monetary rule (39), and taking advantage of (46), (47) and (48), two-regimes for the AD schedule can be identified: one where $(1+i_t) > 1$ and the ZLB condition does not bind and another for $(1+i_t) = 1$ where the ZLB does bind. Therefore, for the first situation, the following AD schedule can be derived:

$$Y^d = \frac{N}{\xi}\left[(1-\eta)D + (1+\eta)\frac{(1+\beta)D}{\beta}\frac{\Gamma^*}{\Pi^{\phi_{pi}-1}}\right], \tag{49}$$

where $\Gamma^* = \frac{(\Pi^*)^{\phi_\pi}}{(1+i^*)}$ is a composite parameter of the monetary policy reaction function set by the central bank. Taking a closer look at (49), one can observe that this is a rather standard negatively sloped AD curve[44], where an inflation targeting central authority overshoots nominal interest rates according to the Taylor rule (39)(since $\phi > 1$) as a response to increasing inflation. Thus, since nominal interest rates increase faster than inflation, real interest rates raise accordingly, pushing individuals to shift consumption towards the future - i.e. higher savings - (intertemporal substitution effect)[45]. For that reason, as inflation increases, AD follows with an output reduction as the top part of Figure 2[46] shows.

On the other hand, when the ZLB does bind, the real interest rate is given by $(1+r_t) = \frac{1}{\Pi_t}$, which in combination with (17), (30), (39), (46), (47) and (48), yields

$$Y^d = \frac{N}{\xi}\left[(1-\eta)D + (1+\eta)\frac{(1+\beta)D}{\beta}\Pi\right]. \tag{50}$$

Interestingly, contrarily to (49), (50) is, in fact, a positively sloped AD curve, where

[43]Note that the underlying redistribution dynamics of the actual labor endowments attributed to each group (l^l and l^h) do not maintain a linear relationship with ξ. In fact, a 1 percentage point (pp) increase in ξ means that $\Delta l^h = \frac{1}{(1-\eta)}$ and $\Delta l^l = -\frac{1}{\eta}$.

[44]It is noted that for $\xi = 1$ and $\eta = 0$ the original equation as derived in EMR is retrieved. This observation also occurs for the upward slopping part of the AD. This highlights the present model's consistency with regards to its original counterpart.

[45]Note that at an individual level, a positive or negative wealth effect from an increase in the real interest rate could also occur to, respectively, net savers/net borrowers due to higher interests proceeds received/payed. However, on aggregate, these forces cancel each other out, given that every borrower has a lender and hence any positive wealth effects on one side are offset by the other.

[46]In line with EMR, the following parametric values where chosen for the graphs (unless explicitly mentioned otherwise): $D = 0.265, \beta = 0.985, N = 1, \phi_\pi = 2$; according to the ECB's inflation target (ECB, 2018a), $\Pi^* = 2\%$ was preferred rather than the 1% Π^* chosen by EMR; and lastly looking at the work of Grant (2007) $\eta = 0.3$ was set, given that this author estimated that around 26% to 31% of American households should be credit constrained. For this particular example, $\xi = 0.85$. The actual equations rearranged to show Π as the dependent variable follow: $\Pi = \frac{1}{\xi}D\left[(1-\eta) + (1+\eta)\frac{(1+\beta)}{\beta}\frac{(\Pi^*)^2}{(1+i^*)}\right]\frac{1}{Y}$ for (49) and $\Pi = \beta\frac{\xi Y - (1-\eta)D}{(1+\eta)(1+\beta)D}$ for (50).

a raising inflation increases output demanded. The former relationship is very common in the liquidity trap literature (Eggertsson and Krugman, 2012) and arises due to the central bank's inability to accompany the low inflation rate with low(er) nominal interest rates (due to the binding ZLB - $(1+i_t) = 1$), which then means that inflation is simply given by the inverse of the real interest rate as shown in (33). Hence, rising inflation drives the real interest rate down and, through the same mechanism as in (49), this pushes individuals to shift towards higher present consumption, thus increasing AD. This phenomenon is illustrated by the bottom, positively slopping part of the AD curve in Figure 2.

Lastly, to fully define the AD curve as presented in Figure 2, it is important to understand that the kink separating the upper (49) from the bottom part (50) of the curve is a direct consequence of the central bank's monetary policies (39) (Π^* and ϕ_π), together with the real interest rate that is required to attain full employment output (r^f)(52). Indeed, by computing the inflation rate under which (39) would prescribe a nominal interest rate $(1+i_t) = 1$, and by substituting the nominal interest rate target as in (40), the kink of the AD can then be derived as

$$\Pi_{kink} = \begin{cases} \left(\frac{(\Pi^*)^{\phi_\pi - 1}}{1+r^f}\right)^{\frac{1}{\phi_\pi}} & if \quad i^* > 0 \\ \Pi^* \quad if \quad i^* = 0, \end{cases} \tag{51}$$

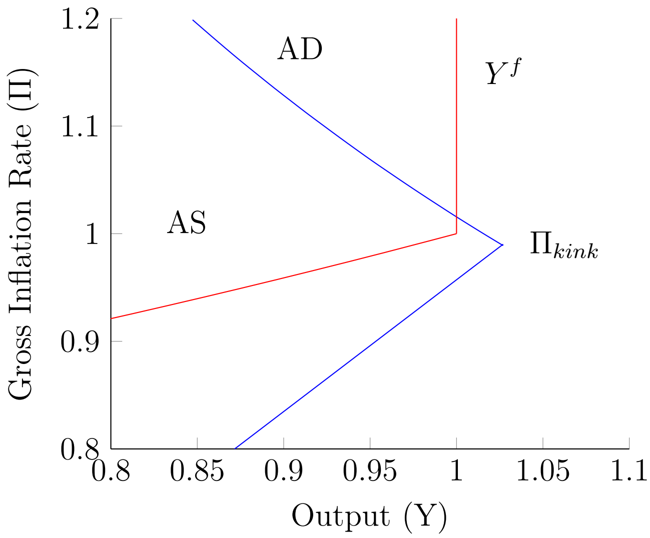
illustrating how, for a given particular combination of (r^f, Π^*), Π_{kink} is the lowest inflation rate from when, to recover to Π^*, the nominal interest rate prescribed by the Taylor rule would fall to a non-positive value. Accordingly, given that this prescription is not possible due to the ZLB, the bottom - positively sloped ($\Pi = \frac{1}{1+r}$) - section of the AD kicks in precisely at the kink. Furthermore, if to attain the desired combination (r^f, Π^*), $i^* = 0$, then the kink assumes the value of the inflation target, given that once $\Pi = \Pi^*$ nominal interest rates cannot become lower. In other words, under this situation the economy will not be able to reach full employment as there is no nominal rate that achieves r_f together with the inflation target. In Figure 2, the kink can be easily identified by the intersection of the AD with the dashed line, while in Figure 3 the shift from the upper to the bottom equation of (51) is observable. Furthermore, the natural rate of interest previously mentioned can be obtained by analyzing (27) at full employment as follows:

$$(1+r^f) = \frac{(1+\beta)}{\beta}\frac{(1+\eta)D}{\xi Y^f - (1-\eta)D} \tag{52}$$

5.3 Equilibrium, Inequality and Secular Stagnation

Finally, the equilibrium output and inflation rate can be given by equalizing the AD and AS schedules, which is illustrated in Figure 2 as the intersection between the two curves. In this figure, the upper part of the AD crosses the vertical section of the

Figure 2: Steady-state aggregate demand (AD) and supply (AS) schedules



AS curve, creating a unique full-employment equilibrium at the targeted level of inflation Π^*. This is possible due to a positive natural rate of interest rate (NRI), which in turn allows for the inflation target to be accommodated and for labor markets to clear (no rigidities in play) so that full employment output is attained (Y^f). As it naturally arises from the the AD/AS derivation performed above, as long as the NRI (52) is large enough so that $\Pi_{kink} < \Pi^*$[47], the underlying dynamics at play in this economy are no different from any other standard new-Keynesian macroeconomic model: in order to respond to an exogenous shock, nominal interest rates adjust accordingly so that consumption is either incentivized/disincentivize and full output is met trough adjustments along the vertical segment of the supply curve. However, under a biding ZLB - i.e. $(1 + r^f) < (\Pi^*)^{-1}$ - the same dynamics do not apply.

Indeed, consider that, as formulated in *Proposition 1*, a sufficiently strong increase in the wealth distribution factor ξ pushes the NRI below the inverse of the inflation target, so that the ZLB becomes binding. More intuitively, consider that due to the central bank's refusal to accommodate a higher inflation target[48], there will be no nominal interest rate that can be set so that to generate r^f and, consequently, nominal interest rates will be set to 0. Additionally, take into account that, as equation (50) shows, an increase in parameter ξ also translates into a depressed output demand curve, given that high-income individuals, who choose to allocate excess income between consumption and savings (32), become richer, while low-income households, who use their available income fully to fuel

[47]Note that in the event of $\Pi_{kink} = \Pi^*$ a secular stagnation equilibrium is not necessarily in place. However, the mechanisms at play are no longer governed by the rules of standard macroeconomics. Section 6 sheds some further light on this issues.

[48]Which translates to $\Pi^* < \underline{\Pi}$ as defined in (33).

current consumption (22)(23), become poorer[49]. Note that the mechanisms herein can be compared with the wide-spread Keynesian idea of an inversely proportional relationship between income and propensity to consume. In fact, these dynamics can be interpreted as a theoretical microfounded equivalent to the referred concept, which aligns the working of this model with the relevant outstanding literature[50].

Accordingly, in the face of the above-mentioned negative demand shock, given that nominal interest rates can no longer be used to thrust consumption back to its pre-shocked level (Y^f), the economy will then shift towards a uniquely defined and determined *secular stagnation equilibrium*[51] (from AD_1 to AD_2, in Figure 3)[52], where, absent of other forces, it will remain without being able to attain full employment or output. In other words, as monetary policy becomes ineffective in reestablishing Y^f, an increase in income inequality will produce an inadequately low aggregate demand that fails to provide enough pressure on the goods market so that a positive inflation can occur. Thus, due to the nominal wage rigidities in place, the market will not be able to frictionlessly adjust nominal wages to reflect the deflationary pressures and, as a result, real wages will rise above the marginal productivity of labor. In turn, firms' labor demand will be reduced as a consequence of higher real wages, hence creating involuntary unemployment and finally culminating in a contracted output production that defines the determinate *secular stagnation equilibrium* (intersection at Figure 3).

Lastly, although upon the removal of the previous shock the economy would recover back to its full-employment output - i.e. AD_1 is recovered in Figure 3 -, it can be noted that forces like income inequality are fairly slow-moving phenomena, which means that in the absence of any government or other exogenous intervention, the economy does not possess any endogenous stabilizing mechanism to automatically restore full employment. Instead, the former equilibrium would persist indefinitely, perfectly embodying the spirit of the *secular stagnation hypothesis*, where involuntary unemployment, the ZLB and an inadequate AD arise naturally from the economy's structure. Ultimately, the mechanisms

[49] As Figure 3 illustrates, graphically, this also means that the kink of the AD schedule will be pushed inwards to meet Π^* below Y^f.

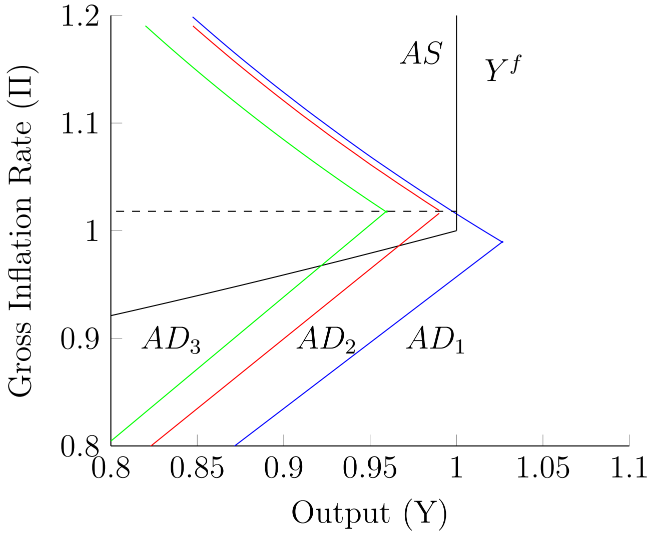
[50] Mathematically, this mechanism can be identified by looking at the steady-state aggregate output demand's first derivative - (49) and (50) - with respect to ξ and noticing the negative sign in both equations:

$$\frac{\delta Y^d}{\delta \xi} = \begin{cases} -\frac{N(1-\eta)D}{\xi^2}\left[1 - \frac{(\Pi^*)^{\phi_\pi}}{\Pi^{\phi_\pi}}\right] & for \quad i^* > 0 \\ -\frac{N}{\xi^2}\left[(1-\eta)D + (1+\eta)\frac{(1+\beta)D}{\beta}\Pi\right] & for \quad i^* = 0. \end{cases} \tag{53}$$

[51] As this model closely follows EMR, and given that the inequality mechanisms introduced are theoretically and mathematically consistent with this model, the discussion of the determinacy of the secular stagnation equilibrium is referred to the original paper.

[52] From AD_1 to AD_2, the inequality parameter was shocked from $\xi = 0.85$ to $\xi = 0.93$, respectively. From AD_2 to AD_3 the parameter is further shocked to $\xi = 0.95$

Figure 3: Increasing inequality ($\xi \uparrow$) - the formation of a secular stagnation equilibrium



herein and the derived *secular stagnation equilibrium* allows this paper to unequivocally formulate *Proposition 2*, on the role of income inequality as a secular force in the economy.

Proposition 2: *Rising income inequality can be responsible for the creation of a secular stagnation equilibrium.*

6 Monetary Policy

In order to escape a *secular stagnation* equilibrium in the event of a ZLB episode, the only expansionary monetary policy tool available to central banks would be to credibly commit[53] to a higher inflation target. By doing so, the real interest rate effectively depreciates and households are instigated to consume more in the current period, which, as it will be shown below, can help to overcome an AD slump. This technique is illustrated in Figure 4, where an outward and upward expansion of the kink is visible as the inflation's target rises by 1pp[54] and 6pp from AD_1 to AD_2 and AD_3, respectively. Indeed, this result arises from the fact that, for the same NRI, at any given inflation rate, $\Pi^* \uparrow$ allows for a higher corresponding nominal interest rate target (51), which shifts the upper section of the AD curve along its bottom part.

Nevertheless, as EMR readily note, the success of such a policy is in large contingent on the size of the inflation target increase, given that for a small increment only, the ex-

[53]The issue of credibility is key for monetary policy as Krugman, Dominquez, and Rogoff (1998) or Eggertsson (2006) explore. However, this issue is linked with individuals' inflation expectations, which are absent from our model. Thus, the omission of this dimension to the model can be read as if the central bank's target is automatically taken as credible and attainable.

[54]Percentage points.

pansionary pull in the economy might not suffice to drag the economy out of the *secular stagnation equilibrium*. Indeed, this phenomenon, first discussed by Krugman, Dominquez, and Rogoff (1998) and again by Krugman (2014), is a comparable mechanism to the so-called *"timidity trap"* or *"law of the excluded middle"*, where an increase in the inflation target might be disregard altogether if it is not seen as a credible push towards a different equilibrium. This situation is perfectly illustrated within the intricacies of this model, where in Figure 3 a 1pp increase in the inflation target moves AD_1 to AD_2, without being able to restore full employment.

On the other hand, if the central bank commits to a large enough increase in the inflation rate target, then the expansionary policy in mind might produce the desired effects. However, as the shift from AD_1 to AD_3 in Figure 3 illustrates, the 6pp increase in Π^* not only allows for a full-employment equilibrium at the target, but also forms two other interception points along the AS schedule. Although, one of the intersections (where $i^* = 0$ and $\Pi < \Pi^*$) is deemed as locally indeterminate and the other one ($Y^* = Y^f$ but $\Pi < \Pi^*$) can, arguably, be avoided by setting a high enough nominal interest rates (which the central authority would), EMR provide a fuller explanation on the implications of the (apparent) multiplicity of equilibriums. Nonetheless, they conclude, this is a valuable insight to understand that under *secular stagnation* the full capacity of Monetary Policy might not be readily available to policy-makers as other models would suggest - e.g. Gauti and Woodford (2003).

When it comes to the impact of income inequality on the effectiveness of Monetary Policy, the previous characteristic clearly relates to the first issue herein approached. Indeed, by analyzing the impacts of the an increase in ξ in the NRI, it is observable that the problem of the *timidity trap* is emphasized under growing inequality. Mathematically speaking, the increase in the inflation target is considered just high enough if the $Y^{AD}(\Pi_{kink}) = 1$ (if the kink sits at the vertical segment of the AS curve) or, with regards to interest rates, if the ZLB becomes binding exactly at the kink. Hence, by evaluating (40) to show $(1 + i^*) = 1$, the now famous ZLB condition arises once again:

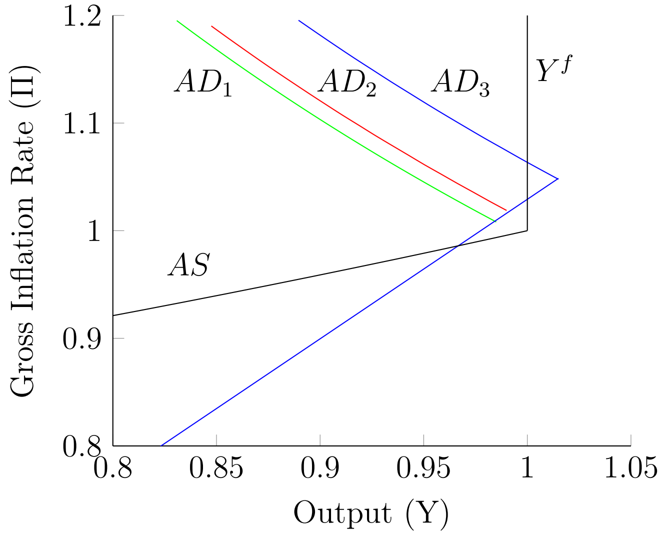
$$\Pi^* = \frac{1}{(1 + r^f)_{(\xi)}}, \tag{54}$$

where the subscript (ξ) denotes the dependence of $(1 + r^f)$ on ξ. Accordingly, by taking the derivative of the inflation target rate with respect to ξ,

$$\frac{\delta \Pi^*}{\delta \xi} = \frac{\beta}{(1 + \beta)(1 + \eta)D}, \tag{55}$$

it is possible to see that the inflation target needed to accommodate the same NRI is higher. In other words, an increase in inequality shifts the AD kink inwards, meaning

Figure 4: Monetary Policy - increasing the inflation target (Π^*)



that a lager increase in the inflation target is deemed when compared to a more equal economy. Although this result follows naturally from the 5.3, explicitly in the context of monetary policy, (55) highlights that under a more unequal society, central banks face increasing difficulties to surpass the aforementioned *timidity trap*.

Proposition 3: *Rising income inequality emphasizes the so-called timidity-trap (or low of the excluded middle), hindering the effectiveness of monetary policy.*

7 Fiscal Policy

In order to study how Fiscal Policy could provide a viable solution to exit *secular stagnation*, this paper explores the role of lump-sum taxes charged to working individuals in the nominal version of the model above. In line with the previously chosen setting, this corresponds to adding a nominal lump-sum tax $T_{t+j}^{i,\{h,l\}}$ that is paid to the government on the left-hand side of the middle-aged and old-age low-income individuals', as well as middle-age high-income households' budget constraints. Accordingly, three new budget constraints will arise to substitute (8), (22) and (23), while the remaining - (7), (9), (21) - are kept unchanged

$$C_{t+1}^{m,l} + T_{t+1}^{m,l} = \frac{D_{t+1}}{(1+r_{t+1})} + W_{t+1}\frac{l_{t-1}^{l}}{2} - D_t \tag{56}$$

$$C_{t+2}^{o,l} + T_{t+2}^{o,l} = W_{t+2}\frac{l_{t-2}^{l}}{2} - D_{t+1} \tag{57}$$

$$C_{t+1}^{m,h} + S_{t+1}^{m,h} + T_{t+1}^{m,l} = W_{t+1}^{m} l_{t+1}^{m} - D_t \tag{58}$$

In its most general case, the government is now allowed to borrow from private individuals by participating in the exchange of nominal bonds with return of $(1 + i_t)$ as any other player. Accordingly, a new loans demand (L_t^d) can be derived as follows[55]:

$$L_t^d = N_t S_t^y - \eta N_{t-1} S_t^{m,l} + N_{t-1} S_t^g = N_t \frac{D_t}{(1+r_t)} + N_{t-1}\eta \frac{D_t}{(1+r_t)} - N_{t-1} S_t^g, \tag{59}$$

where S_t^g denotes nominal government's savings normalized to the middle-aged generation. Additionally, the supply of savings (L_t^s) can once again be obtained by looking at the excess savings from high-income individuals, which considering tax payments to the government, reads

$$L_t^s = N_{t-1}(1-\eta) S_t^{m,h} = N_{t-1} \frac{(1-\eta)}{1+\beta} \Big[\beta (W_t l_t^{m,h} - D_{t-1} - T_t^{m,h}) \Big]. \tag{60}$$

Finally, the government budget constraint closes the systems of equations, and is assumed to hold at all times[56]

$$(1-\eta) T_t^{m,h} + \eta (T_t^{m,l} + T_t^{o,l}) + S_t^g = G_t + (1+r_t) S_{t-1}^g, \tag{61}$$

where G_t denotes government spending normalized to the middle-age generation.

Looking at (60) and (59), it is possible to highlight that the government can now influence the real interest rate trough any of the newly introduced variables. Although, none of the mechanisms explored in section 3.2 suffer from any significant changes, it is worthwhile noting that by either changing the amount of government debt, public spending or by shifting the relative composition of the tax schedule, the government is able to bend the economy's NRI with direct implications for the AD schedule. In EMR, the authors dive into these mechanisms to provide an extensive overview on the steady-state government spending multiplier (GSM) and explain that an increase in public spending is only beneficial up to the extent that it tackles the 'savings-glut' behind the formation of the *secular stagnation*. Indeed, the authors show that introducing a debt-financed public spending shock produces the highest multiplier of all, given that the government directly absorbs some of the excess savings in the economy, channeling it fully towards aggregate consumption. Contrarily, if a shock is tax-financed, the increase in public spending can have all sorts of impacts in the economy: if middle-aged individuals are taxed, the GSM is positive but smaller than before, given that the state will fully consume income that would otherwise be split between savings and consumption; if young-individuals are taxed, the GSM sits at 0, given that the private reduction in consumption exactly matches

[55]Note that for the capital market equilibrium, the Fisher equation (30) holds at all times and thus the $(1+r_t)$ can interchangeably be used as $(1+i_t)/\Pi$. In order to to avoid the proliferation of $(1+i_t)/\Pi$ terms and to follow the same structure as in the sections above, the latter is replaced by the former.

[56]Population growth was once again set to zero.

the increase in public spending; lastly, if old-age individuals are taxed, the GSM will become negative, given that old-age individuals will forego exactly the same amount of consumption that the state will increase and middle-age individuals will increase their savings, aggravating the initial problem.

Ultimately, EMR show that by using adequate fiscal policies it is possible to tackle and solve the 'savings-glut' that is behind the *secular stagnation equilibrium*. In Appendix D, this paper explicitly derives and analyzes both EMR's and the current setting's GSMs for the financing schemes approached above, in order to show that both frameworks yield similar results and that the mechanisms herein explained remain unchanged. Additionally, it is also shown that, as inequality increases, both the debt-financed and the middle-aged tax-financed GSMs increase, in line with the idea that as the 'savings-glut' in the economy is emphasized, the benefits of a state intervention considerably increase. Lastly, in Panel (b) of Figure 5, a stylized increase of the government expenditure as a percentage of the GDP[57] financed trough a proportional tax increase on high-income and low-income individuals (hybrid situation)[58] is portrayed, in order to illustrate one of the possible policies that sufficiently stimulates the economy towards full-employment.

For that reason, the central question in this section is no longer whether there are policies that can be put into place in order to propel the economy towards full-employment, but rather weather the previous equilibrium can be attained without increasing public spending altogether. Arguably, in today's highly indebted developed societies, deploying adequate fiscal stimulus during recessions can be an arduous process[59], which means that carefully studying how a government can shuffle the tax system in order to stimulate the economy might be of invaluable importance to policy-makers and economists alike.

Accordingly, this paper will now explore the role of redistributive taxes by starting to define a debt-preserving fiscal regime. To do so, it is assumed that the government runs a balanced budget where all public expenditures are financed trough the lump-sum taxes previously introduced:

$$S^g = 0. \tag{62}$$

Moreover, consider that the government sets two types of taxes contingent on the individual's income group and that, for poorer households, the referred tax is charged in two installments, so as to follow the spread of the labor endowment defined in 5.2. In

[57] Θ is soon to be introduced below.

[58] For a fuller understanding of this shock's tax-financing refer to the next paragraphs, where these relationships are introduced.

[59] It is noted that apart from actually fiscally constrained countries such as Portugal, Greece, Spain or Italy, who are currently in the post-crisis deleveraging progress (Bacchiocchi, Borghi, and Missale, 2011), many other countries struggle to adopt adequate fiscal stimulus due to political unwillingness to adopt these programs during crisis periods (Odendahl, 2014).

other words, assume that $T_t^{m,l} + T_t^{o,l} = T_t^l$[60] and $T_t^{m,h} = T_t^h$ so that the new government budget constraint is given by

$$(1-\eta)T_t^h + \eta T_t^l = \bar{G}_t, \tag{63}$$

where the total amount of public expenditure is exogenous and denoted by $\bar{G}_t$. Furthermore, consider that τ and $(1-\tau)$ denote the share of the total government expenditure financed trough taxes collected, respectively, from the richest and poorest households so that:

$$\underbrace{(1-\eta)T_t^h/\bar{G}_t}_{\tau} + \underbrace{\eta T_t^l/\bar{G}_t}_{(1-\tau)} = 1. \tag{64}$$

At this point it is important to note that if $(1-\tau) < 0$[61], the state is effectively redistributing wealth from the richest to the poorest, by making the latter bear all the costs of government spending, as well as the negative transfer (subsidy) received by poorer households. Moreover, note that similarly to ξ, a 1% increase in τ translates to: $\Delta\tau = 1\% \rightarrow \left(\Delta T^h = \frac{1}{1-\eta} \wedge \Delta T^l = -\frac{1}{\eta}\right)$, meaning that this parameter does not maintain a linear relationship with the actual lump-sum taxes charged to individuals.

Lastly, to complete this specification, consider that G is endogenized by defining $0 \leq \Theta < 1$ as the share of public expenditure to total output set by the government as shown below:

$$\Theta = \frac{G}{Y}. \tag{65}$$

Note that with the redistributive taxes in place, the aggregate and individual wealth of rich households can now be given by the terms $(\xi - \tau\Theta)Y$ and $\frac{(\xi-\tau\Theta)Y}{(1-\eta)}$, respectively. This means that, while looking at ξ still provides an insight on how the wealth is allocated to the different income groups, only by looking at term $(\xi - \tau\Theta)$ actual post-tax inequality in this economy is assessed.

Accordingly, the new tax-rich model is now fully defined and, in line with previous sections, the capital markets clearing condition can be computed by equating the new supply (60) and demand (59) for loans, together with the government budget constraint (63) and the equations defining τ (64) and Θ (65) to arrive at

$$(1+r^f) = \frac{(1+\beta)}{\beta} \frac{(1+\eta)D}{(\xi - \tau\Theta)Y^f - (1-\eta)D}. \tag{66}$$

[60] Note that in order to maintain proportionality with regards to the labor endowment, which is constant throughout periods, $T_t^{m,l} = T_t^{o,l} = \frac{T_t^l}{2}$. However, on the aggregate level, given that these individuals are *hand-to-mouth* consumers, it is irrelevant how taxes are charged throughout the life-time profile of these individuals; only the total amount T^l has an impact. For that reason, the previous relationship is relaxed in order to encompass a more general setting, where no specific taxing pattern is chosen.

[61] The only restriction on this parameter is $\tau < (1-\eta) - \frac{L^s}{G}\left[(1-\eta)(1-\xi) - \eta\xi\right]$, which ensures, as before, that the rich are wealthier than the poor (note that $\xi > \eta$ is still in place).

where the previous equation is evaluated at full employment output in order to directly show the economy's NRI. Note that by comparing (66) with its predecessor (52), it is possible to observe that the redistributive taxes introduced have a positive effect on the natural rate of interest of the economy. Indeed, by reducing rich people's available lifetime income, the government is able to interfere on the supply side of the economy, contracting it. As a consequence, loans become scarcer and their price - $(1+r)$ - increases correspondingly.

Furthermore, in line with the process in 5.2, the AD for this new setting can be derived by combining (30), (39), (46), (47), (48) and the newly derived (66) for both the upper and the bottom part of the curve. Respectively,

$$Y^d = \underbrace{\frac{N}{\xi - \tau\Theta}}_{A} \underbrace{\left[(1-\eta)D + (1+\eta)\frac{(1+\beta)D}{\beta}\frac{\Gamma^*}{\Pi^{\phi_{pi}-1}}\right]}_{B} \tag{67}$$

$$Y^d = \underbrace{\frac{N}{\xi - \tau\Theta}}_{A} \underbrace{\left[(1-\eta)D + (1+\eta)\frac{(1+\beta)D}{\beta}\Pi\right]}_{C}. \tag{68}$$

Likewise, comparing (49) and (50) to (67) and (68), respectively, it is possible to identify that not only the terms B and C remain completely unchanged, but also that the term A, present in both equations, is merely the new after-tax counterpart to the inverse of the wealth accumulation measurement previously given by ξ. Accordingly, the underpinning relationships between all variables in the model remain substantially the same and the focus can now be turned on exploring the redistributive fiscal stimulus proposed above.

In order to understand the impacts of a shift on the relative tax burden between income groups, one can start by looking at the first derivative of both (67) and (68) with respect to τ:

$$\frac{\delta Y^d}{\delta \tau} = \begin{cases} \frac{\Theta}{(\xi-\tau\Theta)^2} NB & for \quad i^* > 0 \\ \frac{\Theta}{(\xi-\tau\Theta)^2} NC & for \quad i^* = 0. \end{cases} \tag{69}$$

Accordingly, it can be identified that a positive effect on the aggregate demand arises from an increase in τ - $(\xi > 0 \wedge \tau > 0) \Rightarrow \frac{\delta Y^d}{\delta \tau} > 0, \forall \tau \neq \frac{\xi}{\tau}$. In line with the previous section's insights (5.3), this result is strongly consistent with the idea that increasing inequality reduces consumption by redistributing wealth from *hand-to-mouth* consumers - i.e. lower-income households - to individuals with low marginal propensity to consume - i.e. richer households. In fact, in this situation, the same mechanisms are applied in reverse, with the particularity that, instead of reaching low-income households directly, the taxed wealth first goes to the state, which in turn first covers public expenditures[62] and then channels the rest of the available income to the poor. Accordingly, from an aggregate point of view, the government behaves exactly as a *hand-to-mouth* agent, which means that increasing τ produces a similar result to that of reducing ξ. Panel (a) of figure 5 graphically represents the mechanisms previously described, where a strong increase in the referred parameter expands the AD sufficiently so that the economy escapes *secular stagnation*. *Proposition 4* sums up the previous insight on the role of redistributive taxes.

Proposition 4: *Redistributive taxes can provide a sufficiently large stimulus to pull the economy out of a secular stagnation equilibrium.*

Lastly, by taking a closer look at (67) and at (68), it is possible to notice that not all combinations of (ξ, τ, Θ) are feasible, given that the restriction $Y^d > 0$ structurally binds and that it is not possible to divide by zero, $(\xi - \tau\Theta) \neq 1$. However, since (69) is an increasing function, then it can be said that the optimal tax regime in this economy is given by $(\xi - \tau\Theta) \to 0$, which translates to

$$\tau\Theta \to \xi. \tag{70}$$

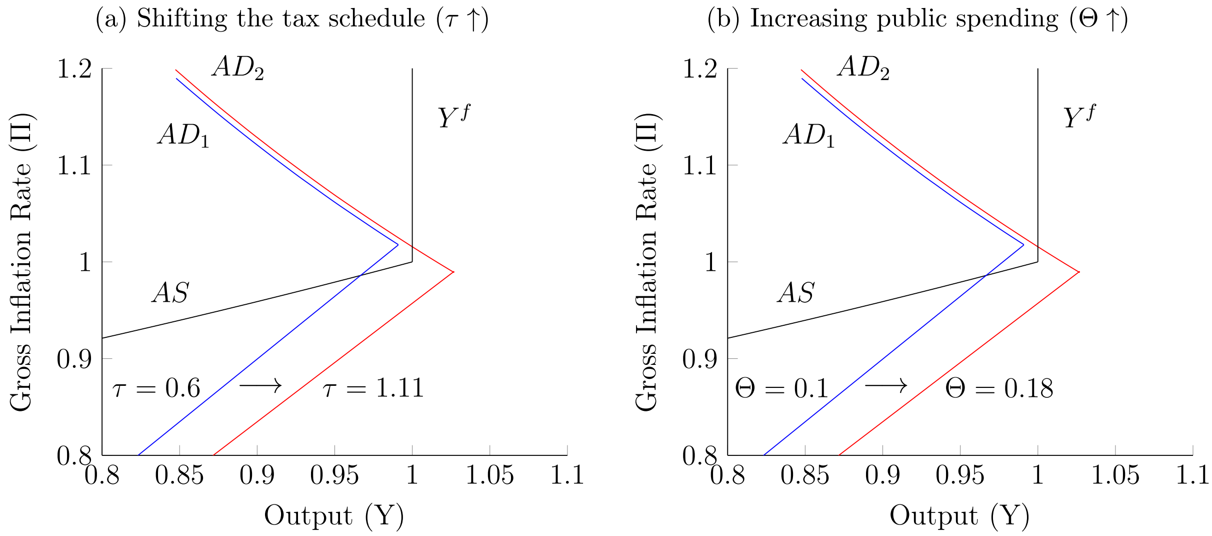
Interestingly, note that the previous result together with restriction (48) $(\xi > \eta)$, provide a valid guideline for both the debt-to-GDP (Θ) and the tax burden attributed to the rich (τ) parameters to be taken endogenously. Intuitively, it is also a fairly appealing result as the result (70) defines (τ, Θ) - given $(\xi > \eta)$ - that restores full equality.

8 Conclusion

Ever since Summers (2013) first resurrected the *secular stagnation hypothesis*, this theory has gained a fair amount of well-deserved attention. The simple idea that a very low natural rate of interest, caused by structural characteristics of developed societies, could be the missing piece in explaining the sluggish recoveries of the early 2010's, reasoned well with the economic community and led to the creation of a body of literature in which this paper is included. Five years have passed, and although the economic performance of the biggest economic blocs (US and EU) has slightly improved, it is still at best anemic, when compared to the massive stimulus that has been in place for years. Furthermore, as economists look at the years leading up to the crisis, as well as to the Japanese decades-

[62] Hitherto, government spending and private consumption are in every sense equivalent, given that in the closed system described above, a lump-sum tax transfer from the private to the public sector is completely frictionless. For that reason, the total output demanded in the economy can still be given by the aggregate labor income - $Y^d = WL^s$ -, which is consistent with the notion that lump-sum taxes do not produce any distortions to the household's maximization problem. For a comprehensive analysis on the topic, read Jacobs (forthcoming).

Figure 5: Fiscal Policy- debt-preserving redistribution stimulus



long *liquidity-trap*, this hypothesis seems to offer a new perspective that fits well both with the relevant literature and the empirical observations.

In line with the previous remarks, this paper extends the *secular stagnation* model of Eggertsson, Mehrotra, and Robbins (2017) to allow for a two-households type setting, where the role of income inequality can be analyzed. By deriving the equilibrium real interest rate as the result of structural characteristics, this paper identifies that an increase in income inequality produces an expansionary effect on the economy's supply of savings. This result arises from high-income individual's lower propensity to consume, when compared to their poorer counterparts. As a consequence, in the presence of a sufficient increase in income inequality, a significant contraction of the equilibrium interest rate is shown to occur, leading the economy towards an indefinitely long ZLB period. Furthermore, this paper explores how the previous mechanisms also underpins the creation of a uniquely defined *secular stagnation equilibrium*, where a persistent output-gap, involuntary unemployment and deflation arise endogenously.

Lastly, contrarily to Eggertsson, Mehrotra, and Robbins (2017), who found *"policy recommendations (...) that were considered vices rather than virtues in macroeconomic theory"*, such as permanently increasing the inflation target or the debt-to-GDP ratio, this paper is able to offer an alternative body of solutions related to the mitigation of income inequality, while still being able to contemplate the policy 'vices' aforementioned. Indeed, by exploring the underlying mechanisms of a redistibutive tax system, this paper shows that a sufficiently progressive shift towards offsetting inequality can produce a sufficient expansion of the aggregate demand so that the economy permanently exits *secular stagnation* and restores full-employment.

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Appendix

A 3-period OLG without financial constraints

In this setting, household's follow a standard 3-period OLG model without any financial frictions. Accordingly, for individuals that receive an endowment both in middle-age (w^m_{t+1}) and old-age (w^o_{t+1}), the following budget constraints arise:

$$c^y_t + s^y_t = 0 \tag{A.1}$$

$$c^m_{t+1} + s^m_{t+1} = w^m_{t+1} + (1 + r_t)s^y_t \tag{A.2}$$

$$c^o_{t+2} = w^o_{t+2} + (1 + r_{t+1})s^m_{t+1}. \tag{A.3}$$

Furthermore, considering that each individual maximizes the following utility function,

$$U(c_{t+j}) = \sum_{j=0}^{2} \beta^j \ln(c_{t+j}), \tag{A.4}$$

the household's optimization problem can be given by:

$$\begin{aligned}
&\max_{(c^y_t, c^m_{t+1}, c^o_{t+2}, s^y_t, s^m_{t+1})} \ln(c^y_t) + \beta \ln(c^m_{t+1}) + \beta^2 \ln(c^o_{t+2}) \\
&\text{s.to} \\
&c^y_t = -s^y_t \\
&c^m_{t+1} + s^m_{t+1} = w_{t+1} + (1 + r_t)s^y_t \\
&c^o_{t+2} = w_{t+2} + (1 + r_{t+1})s^m_{t+1}.
\end{aligned} \tag{A.5}$$

In order to solve the former the following Lagrangian arises:

$$\begin{aligned}
\mathcal{L} &= \ln(c^y_t) + \beta \ln(c^m_{t+1}) + \beta^2 \ln(c^o_{t+2}) \\
&- \lambda^y_t(c^y_t + s^y_t) \\
&- \lambda^m_{t+1}(c^m_{t+1} + s^m_{t+1} - w_{t+1} - (1 + r_t)s^y_t) \\
&- \lambda^o_{t+2}(c^o_{t+2} - w_{t+2} - (1 + r_{t+1})s^m_{t+1}),
\end{aligned} \tag{A.6}$$

From the the first order conditions can be derived:

$$\begin{aligned}
c_t^y &: \frac{1}{c_t^y} = \lambda_t^y \\
c_{t+1}^m &: \beta \frac{1}{c_{t+1}^m} = \lambda_{t+1}^m \\
c_{t+2}^o &: \beta^2 \frac{1}{c_{t+2}^o} = \lambda_{t+2}^o \\
s_{t+1}^m &: \lambda_{t+1}^m = (1+r_{t+1})\lambda_{t+2}^o \\
s_t^y &: \lambda_t^y = (1+r_t)\lambda_{t+1}^m.
\end{aligned} \tag{A.7}$$

Simplifying, the Euler equation - $u'(c_t) = (1+r_t)u'(c_{t+1})$ in its general form - arises for all periods which can then be simplified to get:

$$\begin{aligned}
c_{t+1}^m &= (1+r_t)\beta c_t^y \\
c_{t+2}^o &= (1+r_{t+1})\beta c_{t+1}^m.
\end{aligned} \tag{A.8}$$

By looking at the household's budget constraints for each period - (A.1), (A.2) and (A.3) - the Intertemporal Budget Constraint (IBC) can be constructed as follows:

$$c_t^y + \frac{c_{t+1}^m}{(1+r_t)} + \frac{c_{t+2}^o}{(1+r_t)(1+r_{t+1})} = \frac{w_{t+1}}{(1+r_t)} + \frac{w_{t+2}}{(1+r_t)(1+r_{t+1})}. \tag{A.9}$$

Substituting and rearranging,

$$c_t^y = \frac{1}{(1+\beta+\beta^2)(1+r_t)}\left[w_{t+1} + \frac{w_{t+2}}{(1+r_{t+1})}\right], \tag{A.10}$$

which, considering the Euler equations, in turn means

$$c_{t+1}^m = \frac{\beta}{(1+\beta+\beta^2)}\left[w_{t+1} + \frac{w_{t+2}}{(1+r_{t+1})}\right] \tag{A.11}$$

$$c_{t+2}^o = \frac{\beta^2}{(1+\beta+\beta^2)}\left[(1+r_{t+1})w_{t+1} + w_{t+2}\right]. \tag{A.12}$$

Naturally, these results are consistent with the Permanent Income Hypothesis (Friedman, 1957). Lastly, looking once more at (A.1), (A.2) and (A.3), the optimum savings behaviors can be derived:

$$s_t^y = -\frac{1}{(1+\beta+\beta^2)(1+r_t)}\left[w_{t+1} + \frac{w_{t+2}}{(1+r_{t+1})}\right] \tag{A.13}$$

$$s_{t+1}^m = \frac{1}{(1+\beta+\beta^2)}\left[\beta^2 w_{t+1} - (1+\beta)\frac{w_{t+2}}{(1+r_{t+1})}\right]. \tag{A.14}$$

B 3-period OLG with financial constraints for the young

Apart from the model derived in A, now consider that the young face a borrowing constraint, such that:

$$\begin{aligned}(1+r_t)s_t^i &\geq -D_t \\ s_t^i &\geq -\frac{D_t}{(1+r_t)}\end{aligned} \tag{B.1}$$

Accordingly, a new set of budget constraints can be derived:

$$c_t^y + s_t^y = 0 \tag{B.2}$$

$$c_{t+1}^m + s_{t+1}^m = w_{t+1}^m - (1+r_t)s_t^y \tag{B.3}$$

$$c_{t+2}^o = w_{t+2}^o + (1+r_{t+1})s_{t+1}^m \tag{B.4}$$

.

Considering the same utility function as before,

$$U(c_{t+j}) = \sum_{j=0}^{2} \beta^j \ln(c_{t+j}), \tag{B.5}$$

again the maximization problem can be given by

$$\begin{aligned}&\max_{(c_{t+1}^m, c_{t+2}^o, s_{t+1}^m)} \ln\left[\frac{D_t}{(1+r_t)}\right] + \beta\ln(c_{t+1}^m) + \beta^2\ln(c_{t+2}^o) \\ &\text{s.to} \\ &c_{t+1}^m + s_{t+1}^m = w_{t+1} - D_t \\ &c_{t+2}^o = w_{t+2} + (1+r_{t+1})s_{t+1}^m.\end{aligned} \tag{B.6}$$

Similarly, the following Lagrangian arises,

$$\begin{aligned}\mathcal{L} = &\ln\left(\frac{D_t}{(1+r_t)}\right) + \beta\ln(c_{t+1}^m) + \beta^2\ln(c_{t+2}^o) \\ &- \lambda_{t+1}^m(c_{t+1}^m + s_{t+1}^m - w_{t+1} - D_t) \\ &- \lambda_{t+2}^o(c_{t+2}^o - w_{t+2} - (1+r_{t+1})s_{t+1}^m),\end{aligned} \tag{B.7}$$

from the first order conditions yield:

$$\begin{aligned}c_{t+1}^m &: \beta\frac{1}{c_{t+1}^m} = \lambda_{t+1}^m \\ c_{t+2}^o &: \beta^2\frac{1}{c_{t+2}^o} = \lambda_{t+2}^o \\ s_{t+1}^m &: \lambda_{t+1}^m = (1+r_{t+1})\lambda_{t+2}^o.\end{aligned} \tag{B.8}$$

Simplifying, the Euler equation - $u'(c_t) = (1+r_t)u'(c_{t+1})$ in its general form - arise, which can then be simplified to get:

$$c^o_{t+2} = (1 + r_{t+1})\beta c^m_{t+1}. \tag{B.9}$$

By looking at the household's budget constraints, the Intertemporal Budget Constraint can once again be constructed as:

$$D_t + c^m_{t+1} + \frac{c^o_{t+2}}{(1 + r_{t+1})} = w_{t+1} + \frac{w_{t+2}}{(1 + r_{t+1})}. \tag{B.10}$$

Substituting and rearranging,

$$c^m_{t+1} = \frac{1}{1+\beta}\left[w_{t+1} - D_t + \frac{w_{t+2}}{(1 + r_{t+1})}\right], \tag{B.11}$$

which means that:

$$s^m_{t+1} = \frac{1}{1+\beta}\left[\beta(w_{t+1} - D_t) - \frac{w_{t+2}}{(1 + r_{t+1})}\right]. \tag{B.12}$$

From the maximization we can finally get:

$$c^o_{t+2} = \frac{\beta}{1+\beta}\left[(1 + r_{t+1})(w_{t+1} - D_t) + w_{t+2}\right]. \tag{B.13}$$

Looking at the bonds market, the borrowing needs from the young have to be balanced by the supply of savings by the mid generations. All the equations from above are applied with time indexes adjustments. The market clearing condition can thus be translated into:

$$s^m_t = -s^y_t. \tag{B.14}$$

Substituting,

$$\frac{1}{1+\beta}\left[\beta(w^m_t - D_{t-1}) - \frac{w^o_{t+1}}{(1 + r_t)}\right] = \frac{D_t}{(1 + r_t)}. \tag{B.15}$$

Rearranging,

$$(1 + r_t) = \frac{D_t(1 + \beta) + w^o_{t+1}}{\beta(w^m_t - D_{t-1})}, \tag{B.16}$$

which finally defines the equilibrium real interest rate.

C 3-period OLG with financial constraints for the young and endowment inequality

Given that in this setting the high-income households behave as previously derived in (7), (12) and (14) and given that the low-income households face two consecutive borrowing constraints, these households will borrow - up until the borrowing constraint - in the first and second period, repaying its debt in the last period. For that reason, the budget constraints already provide all the information about the household's behavior as the restrictions structurally imposed don't allow him/her to do otherwise. Thus, the new budget constraints for the low income individuals can be derived as follows:

$$c_t^{y,l} = -s_t^{y,l} = \frac{D_t}{(1+r_t)} \tag{C.1}$$

$$c_{t+1}^{m,l} = -s_{t+1}^{m,l} + w_{t+1}^{m,l} - D_t = \frac{D_{t+1}}{(1+r_{t+1})} + w_{t+1} - D_t \tag{C.2}$$

$$c_{t+2}^{o,l} = w_{t+2}^{o,l} - D_{t+1}. \tag{C.3}$$

In turn the previous equations can be written with respect to each individual's savings behaviors to yield:

$$\begin{aligned} s_t^{y,l} &= -\frac{D_t}{(1+r_t)} \\ s_{t+1}^{m,l} &- \frac{D_{t+1}}{(1+r_{t+1})} \\ s_{t+2}^{o,l} &= 0. \end{aligned} \tag{C.4}$$

Looking at the bond's market, the new equilibrium is then given by the balance between supply for savings from high-income households and youngsters' and low-income households' demand for loans. Again all the time subscripts were adjusted and the equilibrium reads:

$$s_t^{m,h} = -s_t^{y} - s_t^{m,l}. \tag{C.5}$$

Denoting by η the fraction of the population that is borrowing constrained during the middle-period, the real interest rate is then given by:

$$(1+r_t) = \frac{1}{\beta}\left[\frac{(1+\eta)(1+\beta)D_t}{(1-\eta)(w_t - D_t)} + \frac{w_{t+1}}{(w_t - D_t)}\right]. \tag{C.6}$$

D Government spending multipliers

This appendix picks-up on equation (61) of section 7, in order to compute the GSM for the inequality setting. Accordingly, looking at the new demand for loans (59) and supply of savings (60) in the steady-state, the equilibrium real interest rate can be once again computed:

$$(1+r) = \frac{(1+\eta)}{(1-\eta)} \frac{(1+\beta)D}{\beta(Wl^{m,h} - D - T^{m,h}) + (1+\beta)S^g}, \tag{D.1}$$

where both the lump-sum taxes on the rich middle-aged individuals, as well as the government's savings parameter can be identified. Moreover, similarly to section 5.2, a new

AD curve can be calculated to incorporate (D.1):

$$Y^d = \frac{1}{\xi}\left[(1-\eta)(D_{t-1}+T_t^{m,h}) - \frac{(1-\eta)(1+\beta)}{\beta}S^g + \frac{(1+\eta)(1+\beta)D}{\beta}\Pi\right]. \tag{D.2}$$

Given that the all else remains constant, Table 1 looks at the GSM computed by EMR (MEMR) and compares it with this specifications' multipliers (MPR) (for simplicity denoted by the initials of my name - PR). This paper follows EMR's small increase approach, as well as the linearization of the model that is performed in Appendix C of EMR, in order to be able to compute MPR. Similarly, κ denotes the slope of the linearized AS curve, which is the same for both papers[63] and, thus, is given by:

$$\kappa = \frac{1-\alpha}{\alpha}\frac{1-\psi}{\psi}. \tag{D.3}$$

However, contrarily to EMR, the slope of the AD is directly substituted to show each model's differences. Therefore:

Table 1: Government spending multipliers - MEMR v MPR

Financing	MEMR	MPR	MEMR<MPR
Public debt	$\frac{1+\beta}{\beta-(1+\beta)D\kappa}$	$\frac{(1-\eta)(1+\beta)}{\xi\beta-(1+\eta)(1+\beta)D\kappa}$	for $\frac{1-\xi}{\eta} > \beta - 2(1+\beta)D\kappa$
Tax on the young	0	0	0
Tax on the poor middle-aged	-	0	-
Tax on the poor old-aged	-	0	-
Tax on the rich middle-aged	$\frac{\beta}{\beta-(1+\beta)D\kappa}$	$\frac{(1-\eta)\beta}{\xi\beta-(1+\eta)(1+\beta)D\kappa}$	for $\frac{1-\xi}{\eta} > 1 - \frac{2(1+\beta)D\kappa}{\beta}$

where $(MEMR < MPR)$ indicates the relationship between the *per capita* wealth of poor-income households and the model's other determinants that yields $MEMR < MPR$. Given that the relationship between both GSMs is neither intuitive nor clear, and that the many parameters involved in the computation of these multipliers significantly blurs their analysis, this paper went on to assign to each parameter its value on the *secular stagnation equilibrium* derived in Figure 3 in order to portray a particular example as in Table 2.

Thus, taking a closer look at the results obtained, it is possible to start by observing that, as mentioned in the main text, the broad properties of these multipliers remain the same throughout settings, given that the key mechanisms at play do not change. Accordingly, the debt-financed GSM for the public spending shock remains the highest of all, while the GSM for taxing rich middle-aged individuals is positive but smaller than the previous one. Moreover, it can be identified that increasing government spending by taxing either young individuals or poor households at any life stage yields no benefits at all. This result is an expansion of the already neutral MEMR for the young, given

[63] In EMR ψ is denoted by γ.

Table 2: Government spending multipliers - MEMR v MPR - particular example

Financing	MEMR	MPR	MEMR v MPR
Public debt	2.6	2.35	larger
Tax on the young	0	0	0
Tax on the poor middle-aged	-	0	-
Tax on the poor old-aged	-	0	-
Tax on the rich middle-aged	1.3	1.16	larger

Parameters: $\beta = 0.985$, $\eta = 0.3$, $D = 0.265$, $\xi = 0.9$, $\alpha = 0.5$, $\psi = 0.7$ $\kappa = 0.43$

that now, similarly to youngsters, both middle-aged and old-age low-income households are *hand-to-mouth* consumers, meaning that the increase in public spending will exactly match the decrease in private consumption.

Secondly, and given that these multipliers are computed solely for the ZLB - where $\xi \geq 0.88$ -, it is possible to note that the comparison relationship in Table 1 ($MEMR < MPR$) deems that: $MEMR > MPR, \forall\,(\eta \in [0,1], \xi \in [0.88,1])$. In fact, the previous relationship can be extended to say that EMR's GSM provides the upper-bound for this model's multiplier as for $(\xi \to 1 \wedge \eta \to 0) \Rightarrow MPR \to MEMR$, which intuitively states that as the current model approaches EMR's representative agent framework, MPR tends to MEMR. However, although the present comparison is undoubtedly useful to be to bound MPRs to the representative agent model and understand the parallel analysis that can be preformed, an inequality-equality comparison cannot be attained by comparing MEMR and PR (the conceptual discussed of section 2 and the specific example in 3.2 can be referred to further information).

Thus, finally looking at the role of inequality by evaluating an increase in ξ, it possible to identify that both MPRs maintain a positive relationship with this parameter, which is strongly in line with the idea that as the 'savings glut' in the economy is emphasized, the benefits of a state intervention increase considerably.