How to prevent pension funds from becoming a sinking giant?

The impact of a dynamic investment strategy on the position of pension funds with a Defined Benefit plan.

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Abstract

A sinking giant is a pension fund which has a low coverage ratio and is not able to recover anymore without applying curtailments. This research uses simulated scenarios to analyse the impact of fund maturation and investment strategies on the probability of becoming a sinking giant. Fund maturation and a high equity exposure both increase the probability of becoming a sinking giant. The initial coverage ratio is of more importance for funds with relatively more pension payments on the short term; the initial premium coverage ratio is of more importance for the probability of becoming a sinking giant for funds with relatively more contribution inflow. Applying curtailments keeps the coverage ratio on a decent level and thus decrease the probability of becoming a sinking giant. The coverage ratio does not indicate the level of pension payments when curtailments are applied. The pension result is introduced as a measure which indicates if profits and losses are well shared among all generations. A dynamic investment strategy is applied to optimise the average pension result. It shows that the asset allocation of funds should be more defensive when the initial financial position is better; more risk taking is not rewarded.

Key words: pension funds, coverage ratio, sinking giant, curtailments, investment strategy, pension result, optimisation

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Contents

1 Introduction

In the second pillar of the Dutch pension system, occupational pension entitlements are being built up and managed. A pension fund manages the pension capital of the plan members. There are several pension schemes, but in this thesis the Defined Benefit pension scheme is used as a base. That is, the pension fund promises a specific amount of pension benefits and charges the member (or its employer) a contribution which can vary over time. The entitlements are promised to the member, provided that the fund has enough funds to cover the pension benefits. It is therefore essential to monitor the coverage ratio; the ratio to which the pension fund's assets cover the present value of the liabilities. If the fund has ample assets to cover all liabilities, the fund may index the pension entitlements to compensate for e.g. price inflation. If the fund has insufficient assets to cover all liabilities, the fund is in coverage deficit. A fund has several measures to take to recover from coverage deficit. As a measure of last resort, the fund may curtail the pension benefits. When a fund applies a curtailment, it decreases the expected pension payments to scale it to the current value of the assets. The decrease in liabilities will increase the coverage ratio.

A decrease in coverage ratio can be the result of several causes. Because the coverage ratio depends on the assets, liabilities and the interest rate term structure, changes in one or more of these aspects affect the coverage ratio. First, pension payments and newly accrued pension entitlements impact the coverage ratio because they both affect the assets and the liabilities. Pension payments have a negative effect on the coverage ratio when the fund is in coverage deficit. A fund has less assets than the present value of the liabilities, so the coverage ratio will decrease when the pension benefits are still paid at 100%. This effect is stronger for mature funds than for younger funds, as their short-term pension payments are a relatively big part of the total pension entitlements. Contribution inflow has a negative effect on the coverage ratio when the ratio to which the contribution covers the present value of the newly accrued entitlements, called the premium coverage ratio, is lower than the coverage ratio. Besides, a decrease in the coverage ratio can also be the result of causes independent of the initial coverage ratio. Examples are an increase in life expectancy, disappointing investment results and a lower interest rate term structure. An increase in life expectancy leads to higher liabilities for the pension funds; disappointing investment results decrease the value of the assets and a lower interest rate term structure raises the present value of the liabilities. These three aspects came together in the two financial crises of this century. The crises together with an aging society, which is the result of the increasing life expectancy and the baby-boom after World War II, had a big impact on the value of the coverage ratios of pension funds and the aftermath is still visible, Bovenberg (2014); Pino and Yermo (2010).

A fund has several measures to take to prevent or recover from a coverage deficit. Increasing the amount of contribution while keeping the newly accrued entitlements unchanged (or keeping the amount of contribution unchanged while decreasing the newly accrued entitlements) has a positive effect on the coverage ratio. This paper shows that this can be an effective tool for young funds, which still receive a lot of contribution in the coming years and most of the pension payments are on the long term. For mature funds this measure is less effective as they have relatively less contribution inflow, while they have large pension payment obligations in the short term. A fund which already used the measures to raise the contributions and stopped applying indexation can only recover on their own strength by earning a high risk premium. A high risk premium has a positive effect on the coverage ratio. This positive effect should offset the constantly growing shortage as a result of pension payments. The risk premium to be earned for several consecutive years therefore depends among others on the initial coverage ratio. When the initial coverage ratio is very low and curtailments are not applied, hence pensions are still paid at 100%, the risk premium to be earned could be unrealistically high to cover the constantly growing shortage, Kocken and Potters (2010).

When the outflow has a bigger negative effect on the coverage ratio than the positive effect of the investment return and possibly the inflow, the coverage ratio continues to sink. It is therefore possible that a pension fund does not recover anymore without applying benefit curtailments. We call such funds 'sinking giants'. Rauh (2009, 2010) revealed the worrying situation of pension funds in America and stated that several funds run out of money in the coming decade. The tilting point depends on the recovery strength of a fund and is dependent on the age structure of its member population, rates of return on investments and expected in- and outflow of members and corresponding pension contribution and entitlements. Kocken and Potters (2010) showed that mature funds are more vulnerable to shocks in the coverage ratio than young funds. If a fund continues with paying pension entitlements in full until no assets are left, there is a generation who paid pension contribution but will not receive their pension entitlements. The oldest cohorts then receive pension at the expense of the youngst cohorts. Hence, when the coverage ratio is low, at some point the fund needs to curtail the pension entitlements to prevent it from becoming a sinking giant. This illustrates the unequal treatment between young and old members of a sinking giant.

To prevent funds from becoming a sinking giant, there are legal limits to the coverage ratio. The Dutch Pension law[1] states that the fund has to curtail the pension entitlements unconditionally when the coverage ratio is too low.

This analysis shows the differences in probabilities of becoming a sinking giant between a mature and a young fund, which are called the 'Grey' and 'Green' fund, respectively. For this analysis the KNW-model of Koijen et al. (2009) is used to simulate scenarios of the stock and bond market, the interest rate term structure and the price inflation. Each scenario consists of 80 values of stock returns, bond returns and price inflation, and 80 interest rate term structures, corresponding with 80 years. These scenarios are used to calculate the probability that the fund is a sinking giant, for a given investment strategy, initial coverage ratio and premium coverage ratio, which is the ratio of the pension contribution to the present value of the newly accrued entitlements. We distinguish cases based on whether new pension entitlements are accrued and on whether curtailments are applied. The analysis shows that the Grey fund has a higher probability of becoming a sinking giant when the initial coverage ratio is below 100%, because the relatively big pension payments have a negative effect on the coverage ratio. When the initial coverage ratio is above 100%, the pension payments have a positive effect on the coverage ratio, which works to the advantage of the Grey fund. Besides, we see that a defensive investment strategy decreases the probability of sinking. Applying curtailments will obviously decrease the probability of sinking.

When curtailments are applied, the coverage ratio no longer indicates whether a good pension for the member is provided. Hence, a fund with a decent coverage ratio which incurred several curtailments may be worse for members than a fund with a relatively low coverage ratio which never incurred a curtailment. Indexations and curtailments influence the level of pension benefits and therefore the purchasing power conservation of the members. To determine the purchasing power conservation, we introduce the pension result. The legal definition of the pension result is a per scenario expressed percentage with the sum of the expected payments of pension entitlements in the numerator and in the denominator the sum of the expected payments of pension entitlements without the application of curtailments and with the application of annual indexation at the level of the scenario price inflation. It indicates if profits and losses

[1]Source: http://wetten.overheid.nl/BWBR0020809/2018-07-28

are well shared among all members. Applying indexation and curtailments and the investment strategy influence the pension result. When the oldest cohorts receive a pension at the expense of the youngest cohorts, the pension result will be low because the nominator will include the low expected pension of the youngest cohorts. When the fund always applies indexation with the amount of the scenario price inflation, the pension result will be 1.

The pension fund's goal is to find the optimal balance between the amount of pension payments and recovery, such that every member receives a fair pension compared to the contribution paid and the pension capital is well-balanced distributed. In this analysis we assume the pension fund aims to optimise the average pension result. That is, the average of pension results obtained from the simulations. The average pension result is optimised by applying a dynamic investment strategy according to a linear glide path. Three parameters indicate the glide path; the initial weight in equity, after how many years the fund starts to decrease its weight in equity and the slope with which the fund decreases its weight in equity. The optimisation shows that funds in a good financial position should have a lower equity exposure. Additional risk-taking is not rewarded because indexation is maximised at the price inflation, but when big losses are incurred the curtailments can be substantial. It is therefore optimal to take less risk and maintain the current good position. For funds in a bad financial position, the Grey fund should initially take more risk than Green funds, but also decrease its weight in equity faster than Green funds. A higher initial weight in equity for Grey funds may sound counter-intuitive, but for Grey funds the need for investment results is higher.

This research is relevant because it is a very topical issue and it is designed with assumptions close to reality. In contrast to Kocken and Potters (2010), who made their calculations with several strong assumptions and pointed at the worrying situation of some funds in America, this research is based on real data of Dutch funds and makes use of scenarios constructed in the same way as the scenarios which are used by pension funds in the feasibility test. Also, the existing pension rules are included to see what the long-term effect of the current policy is. The results can contribute to the evaluation of the FTK (Financial Assessment Framework[2]). Dutch politicians tend to adjust the law when curtailments come close. An example for this is the stretching of the period in which the fund should recover from 3 to 5 years, Donner (2009). A system could not be evaluated when its rules are not first complied. Constantly postponing curtailments lead to an unrestricted system. This paper shows the long-term forecast of pension

[2]Source: http://wetten.overheid.nl/BWBR0020871/2018-07-01

funds in different scenarios, with the distinction in applying the existing law as a restriction or not.

The remainder of this paper is structured as follows. Section 2 describes the relevant literature. Section 3 gives an overview of the current Dutch pension system. The data we use is described in Section 4. Section 5 elaborates on the used methods to conduct the research. The results from this research are described in 6. Section 7 and 8 conclude the paper and provide suggestions for further research, respectively.

2 Literature review

The pension system of the Netherlands receives a lot of praise worldwide. Nevertheless, the domestic debate is in full swing. The concerns about the sustainability of the Dutch pension system and the corresponding legislation are an ongoing and current political topic, Goudswaard et al. (2010); Rutte et al. (2017).

The sustainability of the pension system is at risk, among others due to the aging society. Due to the baby boom after World War II, the average age in society increased over the last decades. According to Statistics Netherlands (CBS[3]), the average age in society increased from 30.8 to 41.6 in the period from 1950 to 1970. Aging affects society in several areas, such as the costs of labour, net wages, pension contributions and employment, Bovenberg and Knaap (2005). Logically, the average age of pension fund members increases, which has impact on the risks and decisions of pension funds. For example, the amount of accrued pension capital increased considerably over the last decades. Due to the rising amount of pension capital, pensions become relatively more dependent on the financial market. The contribution of newly accrued pension entitlements becomes a relatively smaller part of the total pension capital. For funds in coverage deficit, increasing the pension contribution may not be enough to absorb shocks in the financial market, Van Riel (2016). When changing the price of pension is not effective anymore, changing the asset allocation is the last possible measure before a curtailment is inevitable.

How the management of a pension fund changes the asset allocation is of great importance for the sustainability of the fund. The members' interest is on the long-term, hence the members long for a well pension. However, management may have short-term goals, for example when they are judged on and implicitly rewarded for high short-term investment results. It could

[3]Source: http://statline.cbs.nl/StatWeb/publication/?VW=TDM=SLNLPA=37296nedD1=aD2=0,10,20,30,40,50,60,(l -1),lHD=130605-0924HDR=G1STB=T

happen that the management's decision making for asset allocation is not in line with the members' interest, Sappington (1991). Because employees are required to accrue pension at a specific pension fund, and thus not have a free choice of fund, the mismatch between members' and management's interest is not easily solved, Veen (2013). This undermines the trust in the pension funds, Boot et al. (2014). The possibility of intergenerational risk sharing even increases the risk tolerance of fund managers, Bovenberg and Knaap (2005) and Cui et al. (2011). Blake et al. (1999) states that the asset allocation may not be efficient due to management incentives; revising the asset allocation could improve the investment results substantially.

How to determine the optimal asset allocation is a popular research field. A pension fund needs to generate excess returns, i.e. earn risk premium over the risk-free rate, to be able to index the pensions, but is not willing to take too much risk. The balance between risk and return may be described by a utility function. For an individual, traditional literature agree that the closer you get to retirement age, the more defensive your asset allocation should be, Jagannathan and Kocherlakota (1996). Heaton and Lucas (1997) even recommend young people to invest leveraged in equity because the time to retirement is long. These findings are based on a Constant Relative Risk Aversion (CRRA). Gomes et al. (2008) find the same conclusion with an Epstein-Zin utility function. In these analyses human capital is assumed to be risk-free. Cocco et al. (2005) supports this assumption, as they showed that labor income risk is not highly correlated with stock market risk and that labor income acts as a substitute for risk-free asset holdings. Assuming human capital as risk-free is a strong assumption. Benzoni et al. (2007) relax this assumption and studied portfolio choice when labor income and dividends are cointegrated. They find human capital is more stock-like for young participants, which implies that young participants should have low weight in equity. Human capital of old participants is more bond-like because old participants are closer to retirement. As a consequence, the cointegration does not have enough time to act. These effects create a hump-shaped life-cycle investment strategy. In this research the assumption of risk-free human capital is maintained.

For collective pension funds, a dynamic investment strategy with decreasing weight in equity as liability duration decreases should also be optimal. In fact, the investment strategy of a collective should be the sum of the investment preferences of the individuals. Bikker et al. (2017) showed a correlation between the average age of fund members and the equity exposure in the asset allocation; a higher average age of the fund members correlates with a less risky asset allocation.

Many research papers wrote about optimising the asset allocation. The objective can be the utility, described by a utility function. The utility function of individuals is often described by a CRRA utility function, but for firms or collectives utility functions are defined differently among research papers. For example, Chen (2016) optimizes the social welfare function, defined as the sum of the expected discounted utilities of future generations; Wang et al. (2018) minimizes the combination of benefit risk and intergenerational transfers. In this analysis a new objective is introduced; the average pension result. The average pension result is chosen as objective because the coverage ratio does not indicate whether a good pension for the member is provided when curtailments are applied. The coverage ratio is more informative over time when curtailments are not applied. Based on the coverage ratio, Kocken and Potters (2010) and Kocken (2012) showed that Grey funds are more vulnerable to shocks in the financial markets than Green funds. Pension payments have a negative effect on the coverage ratio when the fund is in coverage deficit. For Grey funds, pension payments are a relatively bigger part of the total liabilities and thus a Grey fund in coverage deficit is more vulnerable than a Green fund. Grey funds face a bigger risk to become a sinking giant, because the risk premium to recover, that should be earned for several consecutive years to recover, is much higher for Grey funds than for Green funds. In this research we analyse the effect of the asset allocation on the probability of becoming a sinking giant.

For this analysis we use simulated scenarios. There are several possible ways to construct simulated scenarios. In papers about pensions in which simulated scenarios are used, Monte-Carlo simulations are commonly used, Wernekinck (2013); Cong and Oosterlee (2016); Goossens et al. (2016). However, Dutch funds have to work with a uniform scenario set to calculate their financial position. To stay close to reality, we decided to simulate the scenarios in the same way as the scenarios which are used in the feasibility test of pension funds, according to the model of Koijen et al. (2009) with the estimated parameter values of Draper (2014). It is possible to extend this model to create a scenario set which is even more accurate, for example by adding jumps in the interest rate process and the inflation rate process, Schutte (2018). Extensions and possible improvements to the scenario set depend on many, possibly biased, assumptions and is beyond the scope of this research.

We show the Dutch situation for Green and Grey funds and calculate the the probability of becoming a sinking giant. The curtailments as prescribed by Dutch law are applied when we determine the asset allocation with the average pension result as objective function.

3 The Dutch pension system

The Dutch pension system consists of three pillars. The first pillar of the Dutch pension system considers state pension, which is called AOW in the Netherlands. The third pillar can be considered as any form of private savings. This paper is about the second pillar, which considers occupational pension entitlements. Employees build up pension entitlements while being employed. About 80% of the working population build up pension[4]. Usually, the employee and employer both pay a part of the pension contribution. The pension contribution will be invested by either the pension fund or an insurer. Of the total pension, a relatively small part consists of the paid contribution; the remaining part is obtained by investment results. The part of the total pension obtained by investment results is approximated by several pension funds at two-thirds[5], Hannema (2014). From retirement age, the member receives a constant monthly payment until death. This is called an annuity.

The underlying pension contract can be different among employers and industries. We can distinguish many pension contracts, but the main difference is whether the contribution component or the benefit component is fixed. The former variant is the Defined Contribution (DC) contract. In this variant, the contribution is fixed and there is no guaranteed annuity the member receives at retirement. The accumulated wealth at retirement is used to purchase an annuity at the then applicable interest rates. The height of the annuity is therefore dependent on the investment results during pension accrual and the interest rates at retirement. The risks of not meeting the desired annuity are at the member. The latter variant is a Defined Benefit (DB) contract, which promises a certain annuity after retirement. Approximately 95% of the pensions in the Netherlands are built up in a DB contract, Bruil et al. (2015). The paid contributions are invested and the returns should be sufficient to pay the promised pension entitlements. The accrual of pension entitlements can be based on average pay or final pay. Most funds with a DB contract use pension accrual based on average pay. Newly accrued pension entitlements based on average pay are calculated as the accrual percentage (maximised at 1.875% in 2018) times the pensionable salary (maximised at 105.075 in 2018), which is the salary minus the franchise (13.344 in 2018).

If the investment returns are well, the pensions can be indexed to inflation. If the investment returns are bad, the pension fund has several measures to prevent curtailments, such as adapting

[4]Source: https://www.dnb.nl/nieuws/nieuwsoverzicht-en-archief/statistisch-nieuws-2017/dnb369221.jsp
[5]Source: http://www.spov.nl/Over-SPOV/Helder-over-het-pensioenfonds/Zo-wordt-uw-pensioen-betaald

the contribution amount or the accrual percentage. However, in some cases it can turn out that the promises made were too generous and the pensions cannot be fully paid. In that case, the pension fund is allowed (or sometimes forced) to curtail the pension entitlements. This paper is about the risks involved in a DB pension contract.

For the remainder of this paper, the investor of pension contributions and the pension provider will be referred to as the pension fund. A person for whom pension is built up will be referred to as a member. Members can be distinguished in workers, deferred members and pensioners. Deferred members are not yet retired, but also do not build up pension anymore in a specific fund, e.g. due to taking a different job.

To track the health of the pension fund which offers a DB pension, the fund needs to calculate and report the coverage ratio (CR, in Dutch: dekkingsgraad) to the Dutch Central Bank (DNB).

$$CR_t = \frac{A_t}{PV_t(L_t)} \tag{1}$$

where A_t means Assets, L_t means Liabilities, which are the pension entitlements corrected for mortality rates, and PV_t means the Present Value at time t.

As shown in Equation 1, the CR is calculated by dividing the total assets of the fund by the present value of the liabilities. A change in CR over time is therefore the consequence of changes in either assets, liabilities or the interest rate term structure ($IRTS$), which is published monthly by DNB and used to calculate the present value of the liabilities. Pension funds face interest rate risk, as a decreasing $IRTS$ leads to a decreasing CR. Besides interest rate risk, pension funds face longevity risk. As the life expectancy increases, the pension funds has to pay pensions for a longer period. This increases the liabilities and thus lowers the CR. Disappointing investment results lower the assets and thus the CR. Pension payments and pension accrual affect both the asset side as the liability side. Whether this has a positive or negative effect on the CR depends on the CR and the premium coverage ratio (PCR).

When pension payments are done, both the asset side and the liability side of the balance sheet decrease with the amount paid. When the CR is above 100%, pension payments have a positive effect on the CR, and vice versa.

Example 1: *When a pension benefit payment is made, both the Assets-side and the Liabilities-side decrease. When the current CR is $\frac{95}{100}$, and a benefit payment of 5 is made, the new CR will become $\frac{90}{95}$, which is lower than 95%. When the current CR is $\frac{105}{100}$, and the same benefit* *payment is made, the new CR will become $\frac{100}{95}$, which is higher than 105%.*

When current or new members build up entitlements, the liabilities increase with the amount of the new expected payments. The expected payments are the pension entitlements corrected for the mortality rates. The contribution paid for the new entitlements are added to the assets. A PCR above the CR has a positive effect on the CR, as the assets relatively increase more than the present value of the liabilities. A PCR below the CR has a negative effect on the CR. A PCR of 100% indicates the actuarially required contribution.

On top of the nominal entitlements, a fund wants to apply conditional indexation. The indexation is funded from excess returns. Investing in equity on average leads to higher returns, but also involves downward risk. To be able to promise the nominal entitlements and strive for indexing the pensions with inflation, the fund should hold a buffer and the PCR should be higher than 100%. The cost-effective PCR is therefore higher than the actuarially required contribution and may differ among funds. When a fund always charges the cost-effective contribution, the contributions can differ every year. To smooth the contribution amount, funds are allowed to use their expected return in discounting the value of the new entitlements. The contribution amount is therefore less sensitive to the interest rates. As a consequence, the PCR can deviate from 100%. Note, a PCR lower than 100% indicates that excess returns are needed for meeting the nominal pension obligations. The excess return needed for meeting the nominal pension obligation can then not be used for applying conditional indexation, or where relevant, recovery.

A fund is in coverage deficit when its CR is too low. Applying indexation is not possible for a fund in coverage deficit and may even risk benefit curtailments. A fund has several possibilities to take recovery measures. First, the fund can make the purchase of a pension more expensive, by increasing the contribution and/or reducing the accrual percentage. The price of pension increases, hence the PCR increases. The assets proportionally increase more than the liabilities, hence the CR increases. This measure is more effective for Green funds than for Grey funds, as Grey funds already have a lot of pension capital and the expected contribution inflow is relatively low. Green funds have relatively little pension capital and expect a lot of contribution inflow in the coming decades.

Example 2: *Assume the initial CR is 100% and the PCR is 120%. The present value of the liabilities of the new pension accrual is 10, and thus the contribution is 12. The Green fund's assets and liabilities are worth 100. The newly accrued pension entitlements increases* *the liabilities with 10 and the assets with 12. The coverage ratio becomes* $\frac{112}{110} = 101,8\%$*. The Grey fund's assets and liabilities are worth 1000. The newly accrued pension entitlements lead to a* CR *of* $\frac{1012}{1010} = 100,2\%$*.*

Second, the fund can adapt its investment mix. An aggressive investment mix may lead to high expected returns. But, this also leads to a high expected shortfall. The Dutch law states that a fund in coverage deficit is only allowed to change the investment mix when it does not increase its overall risk profile; gambling on recovery is not allowed.

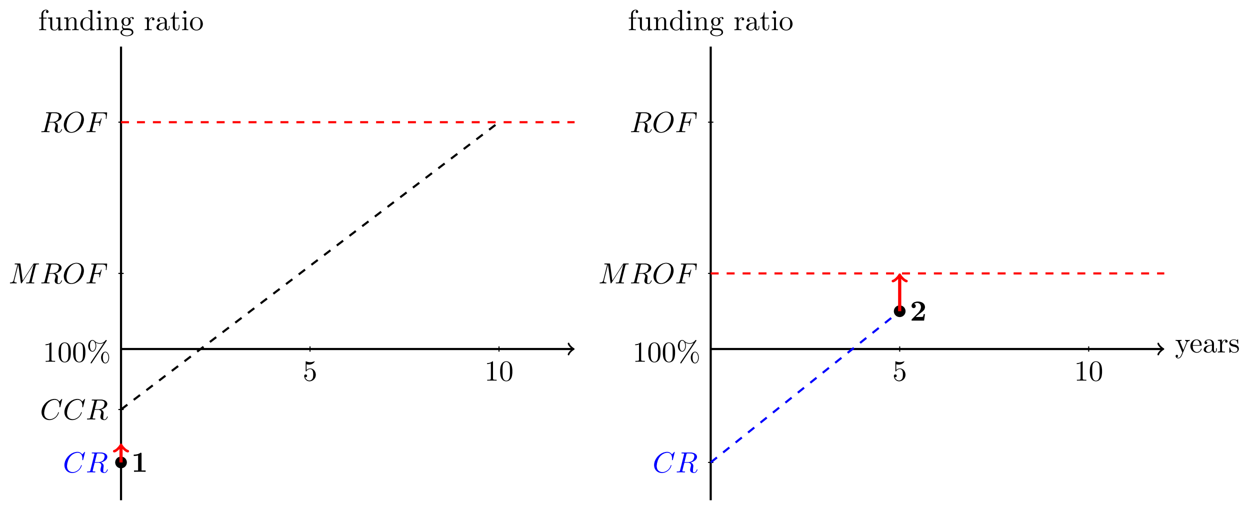
A third and last option for the pension fund to increase the CR is the least popular option; curtail the pension entitlements. The pension entitlements decrease for all members.

3.1 Curtailments

The required level a fund's CR should have is called the Required Own Funds (ROF, in Dutch: Vereist Eigen Vermogen). When a fund's CR is below the ROF, the fund is in reserve deficit and it needs to submit a recovery plan to DNB. In this plan, the fund shows its ability to increase the CR to at least the ROF, within ten years. If the fund is not expected to be able to let the CR meet the ROF in ten years, the fund needs to curtail the benefits immediately and unconditionally. The lowest level from which the fund could recover, calculated based on the expected returns, is called the Critical Coverage Ratio (CCR). If the CR is below the CCR, the fund needs to curtail the pension entitlements immediately. The fund is allowed to spread the curtailment over ten years; the CR needs to increase directly with at least a tenth of the difference between the CR and the CCR. The first curtailment is unconditional. The nine conditional curtailments of the following years expire when the fund submits a new recovery plan next year. If the fund's CR is still under the CCR next year, it unconditionally needs to curtail the pension entitlements again with at least a tenth of the new difference between the CR and the CCR. This is shown by number **1** in Figure 1a.

The third boundary ratio is the Minimum Required Own Funds ($MROF$). The $MROF$ is the ratio under which the fund's CR may not be for five consecutive years. A fund with its CR under $MROF$ is in coverage deficit and if this is the case for five consecutive years, the fund needs to curtail the pension liabilities unconditionally, which means a pension curtailment for all members. The curtailment may be spread over at most ten years, but the curtailment has to be applied immediately, such that the CR meets the $MROF$. This is shown by number **2** in Figure 1b.

The boundary ratios are fund specific and depend on the fund's risk profile. The size of the *ROF* is determined such that, when the fund's *CR* starts at the *ROF*, with a probability of 97.5% the fund's *CR* is above 100% on a one-year horizon[6]. The *CCR* is calculated with the fund's expected returns. The expected return is maximised at a certain percentage which is determined by the Parameter Committee[7]. The *MROF* is calculated based on the fund's investment risk and whether the fund has fixed its administrative expenses[8]. The exact calculation will not be elaborated in this research, but it is about 105%.



(a) When the fund's *CR* is below the *CCR*, which means the fund cannot recover to *ROF* within ten years, the fund needs to curtail unconditionally at least a tenth of the difference between the *CR* and the *CCR*.

(b) When the fund's *CR* is under the *MROF* for five consecutive years, the fund needs to curtail unconditionally. The *CR* after applying the curtailment needs to be at least the *MROF*.

Figure 1: Visualization of applying unconditional curtailments

3.2 Indexation

When the financial position of the pension fund is good, the board may decide to apply indexation. This means that, ideally, pension entitlements and rights are increased annually by a percentage corresponding to the development of negotiated wages (prosperity index-linked) or consumer prices (price index-linked)[9]. A fund must have a policy in place with respect to applying future-proof indexation. That means that the expectation should be that the now applied indexation can also be applied in the future. Dependent on the financial position of the fund, the indexation can or cannot be applied. When the fund's *CR* is below 110%, the fund

[6]Source: http://www.toezicht.dnb.nl/2/50-202134.jsp
[7]Source: https://zoek.officielebekendmakingen.nl/blg-308035.pdf
[8]Source: http://www.toezicht.dnb.nl/2/50-202132.jsp
[9]Source: http://www.toezicht.dnb.nl/en/2/51-201852.jsp

is not allowed to apply indexation. When the fund's CR is between 110% and the ROF, the fund may apply partial indexation. When the fund's CR is above ROF, the fund may apply full indexation, according to their own policy.

When the fund's CR is above the ROF, the fund may decide to apply an incidental benefit adjustment. That is an increase in entitlements to compensate for curtailments and for not or not fully applied indexation in previous years. A fund is only allowed to apply an incidental benefit adjustment when its CR is above the ROF. One-fifth of the difference between the CR and the ROF may be used for incidental benefit adjustment. A fund cannot apply more indexation when it fulfilled its own policy and compensated for the not applied indexation in the previous years.

4 Data description

In this research two fictional funds are used. These fictional funds are constructed using real data from Dutch pension funds. 20 funds with a high liability duration aggregated form the "Green fund". 20 funds with a low liability duration aggregated form the "Grey fund". Data of real Dutch funds, available at DNB, is used. The assets and liabilities, liability duration, coverage ratio and the asset allocation mix are characteristics which are used in aggregated form, which means weighted to the fund's technical reserves.

Fund	CR	PCR	$\omega_{E,0}$	Duration
Green	98.9%	94.2%	43%	27.1
Grey	115.7%	108.9%	43%	15.3

Table 1: Characteristics of aggregated funds "Green fund" and "Grey fund". The characteristics are weighted to the technical reserves of funds. $\omega_{E,0}$ is the last measured weight in equity.

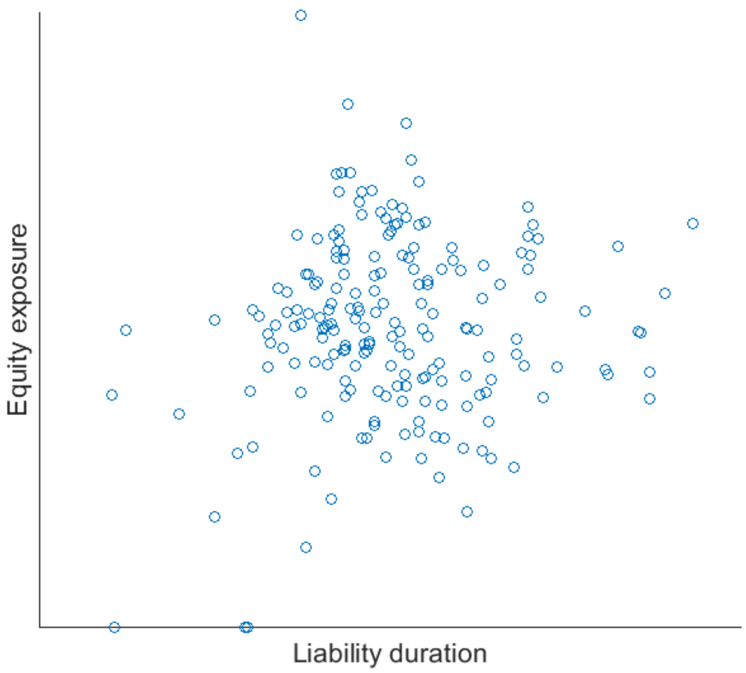
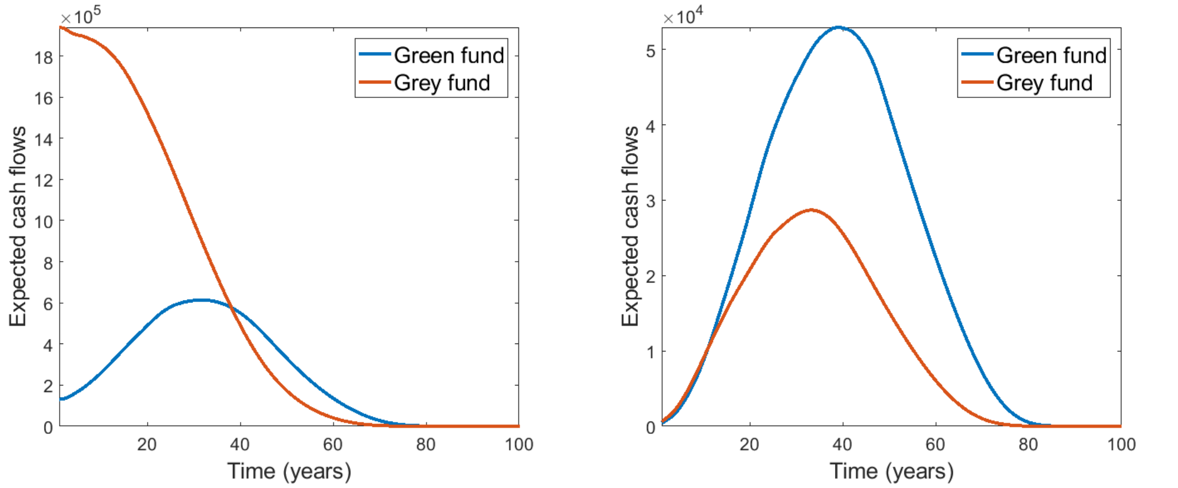


Figure 2: Duration versus equity exposure
The values at the axes have been removed such that the dots cannot be linked to specific pension funds, for the sake of confidentiality.

Figure 2 shows the Dutch pension funds' liability duration against their equity exposure. There is no clear pattern between duration and the exposure to equity. This is striking because a positive correlation would be expected. The lower the duration becomes, the more vulnerable a fund is to shocks in the financial market, hence the lower the exposure to equity should be. This correlation is not visible between Dutch pension funds. From this cross-sectional figure we cannot conclude the funds do not follow a glide path. If a fund does follow a glide path linked to the duration, a time-series analysis would show a clear pattern. Because we do not see a pattern in the cross-sectional figure, we can only conclude that funds do not follow a similar investment strategy depending on the duration.



(a) Already accrued pension entitlements

(b) Accrual of new pension entitlements for next year

Figure 3: Expected cash flows for the Green fund and the Grey fund.

Figure 3a shows the expected future cash flows of the fictional funds. These expected cash flows are the accrued entitlements corrected for the mortality rates. Mortality rates are fund specific. The data of cash flows of funds, which are used in this research, are available at DNB.

The Green fund has liabilities for a longer period than the Grey fund, with its peak after 32 years. The peak of the Grey fund's pension benefits has already passed. Note, these are the already accrued entitlements. Every year pension payments are done, which shifts the entire line to the left. When entitlements are indexed or curtailed, the line shifts up or down, respectively. Every year new entitlements are accrued by existing or new members. These will be added to the liabilities. The newly accrued entitlements for the coming year, also corrected for mortality rates, are shown in Figure 3b.

5 Methodology

This research is divided into five steps. In every step, the probability of becoming a sinking giant is assessed.

Definition Sinking Giant: A sinking giant is a pension fund which has a low CR and is unable to recover without benefit curtailments.

Continuing with paying entitlements at 100% when the CR is lower has a negative effect on the CR. When the fund's liability duration is low (hence, a Grey fund), the fund's investment returns may not undo the negative effect of the high pension payments. Without applying curtailments, the fund's CR continues to sink. We call the fund a sinking giant.

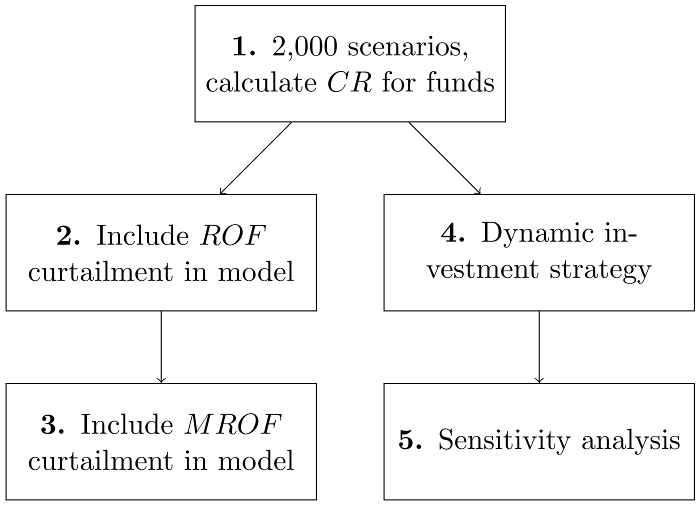


Figure 4: Research structure

Figure 4 shows the five steps of this research. The first step is to simulate 2,000 scenarios of stock returns, bond returns, interest rates and price inflation for 80 years. This simulation will be done with the model created by Koijen, Nijman and Werker (KNW-model), Koijen et al. (2009). For every scenario, the CR will be calculated at every point in time. This shows the development of the CR. The percentage of scenarios in which the CR is below 100% after 80 years will be assumed to be the probability of becoming a sinking giant. The ROF and $MROF$ rules are meant to prevent the CR from sinking. When these measures are included (Step 2 and 3), the CR will increase. Curtailments may prevent the fund's CR from sinking but are not desirable. To prevent curtailments, the fund needs to invest carefully. The investment strategy which fairly distributes the pension capital among generations is calculated by optimising the average pension result (Step 4). Finally, a sensitivity analysis is conducted on the effect of the PCR and on the results of the optimal investment mix (Step 5).

Due to curtailments the CR may be on a reasonable level, but when the fund had incurred a lot of curtailments, the members who receive pension after the curtailments are applied, suffer a loss in pension entitlements. Therefore, the CR no longer indicates whether a good pension for the member is provided. For example, members in a fund with a CR of 90% which never had to incur a curtailment before may be better off than members in a fund with a CR of 110% which incurred one or more curtailments. Therefore, the pension result is introduced, as defined in Dutch law[10].

[10] Source: http://wetten.overheid.nl/BWBR0020917/2018-04-11, Article 30, 1e

Definition Pension result: a per scenario expressed percentage with the sum of the expected payments of pension entitlements in the numerator and in the denominator the sum of the expected payments of pension entitlements, without the application of curtailments and with the application of annual indexation at the level of the scenario price inflation. These entitlements are expressed in real terms, which means discounted for the scenario price inflation.

The pension result gives an indication of the extent to which level the pension wealth is spread over several generations of members. A good pension for the current pensioners at the expense of the current and new workers, or the other way around, will not result in the best possible pension result. Funds have the responsibility to provide a reasonable pension for members now and in the future. The average pension result could therefore be a policy guideline. Given the initial CR and PCR, the average pension result can be optimized by changing the investment mix. This optimisation will be executed for cases with and without accruing new entitlements.

5.0.1 Open and closed funds

An open fund means that the workers still pay pension contribution and accrue pension entitlements in return. The fund is open for new inflow of members. The expected entitlements of funds for next year are known by DNB. These are entitlements of current members. In this research, the new entitlements are supposed to be the same for all coming years, which indirectly simulates new member inflow. This may be an underestimation for a fund which expects a lot of new inflow or an overestimation for a fund which expects less inflow, but to avoid making other strong assumptions on the contribution inflow this assumption is made. The ratio to which the contribution covers the present value of the newly accrued entitlements is called the PCR. The initial PCR indicates the contribution paid for the accrual of pension entitlements. The amount of contribution is assumed constant over time as funds use smoothing techniques with the aim of keeping paid-in contributions more or less constant. Because the $IRTS$, which is used to discount the newly accrued pension entitlements, fluctuates over time, the PCR is also fluctuating over time.

A closed fund is a fund in which no new pension entitlements are accrued. The already accrued entitlements will be paid out in the coming years, but there will be no inflow of new members and current workers do not accrue new entitlements (at this fund) anymore.

In this research, both open and closed funds are considered and the differences in results are addressed.

5.1 Simulation

The simulation will be done with the KNW-model. In this section the model specifications are shown, as stated by Draper (2014). The model provides simulations of the stock and bond market, the interest rate term structure and the price inflation. The uncertainty of the financial market is simulated by Z_t. Z is a four-dimensional Brownian motion which captures four sources of uncertainty: uncertainty about the real interest rate, uncertainty about the instantaneous expected inflation, uncertainty about unexpected inflation and uncertainty about the stock return.

The underlying state of the economy is defined by two state variables, collected in vector X and defined as

$$dX_t = -KX_t + \Sigma'_X dZ_t \tag{2}$$

where K is a 2x2 matrix and Σ'_X represents the correlation matrix of X and is defined as $[I_{2x2} 0_{2x2}]$. Note that the states of the economy only depend on the first two elements of Z; uncertainty about the real interest rate and uncertainty about the expected inflation.

The expected inflation π is affected by the states X. The expected inflation is defined as

$$\pi_t = \delta_{0\pi} + \delta'_{1\pi} X_t \tag{3}$$

The price inflation Π, which is determined by the expected inflation π and uncertainty in the economy, is defined as

$$\frac{d\Pi_t}{\Pi_t} = \pi_t dt + \sigma'_\Pi dZ_t \tag{4}$$

The stock return is a function of the instantaneous interest rate R_t, a risk premium η_S and uncertainty. The instantaneous interest rate R_t is affected by state variables X. R_t is defined as

$$R_t = R_0 + R'_1 X_t \tag{5}$$

The stock returns are defined as

$$\frac{dS_t}{S_t} = (R_t + \eta_S)dt + \sigma'_S dZ_t \tag{6}$$

The bond prices are simulated for several maturities τ. $P^{F\tau}$ determines the price for a bond with maturity τ. P^{F0} is the short rate and consists therefore only of a drift term, which is dependent on R_0 and R_1. $P^{F\tau}$ consists of a Brownian motion term. The bond returns are defined as

$$\frac{dP^{F\tau}}{P^{F\tau}} = (R_t + B^N(\tau)'\Sigma_X'\Lambda_t)dt + B^N(\tau)'\Sigma_X' dZ_t \tag{7}$$

where Λ_t is the price of risk, which is defined as

$$\Lambda_t = \Lambda_0 + \Lambda_1 X_t \tag{8}$$

$B^N(\tau)$ can be obtained by solving the partial differential equation

$$P^N(X_t, t, t+\tau) = exp(A^N(\tau) + B^N(\tau)'X_t) \tag{9}$$

The solutions for $A^N(\tau)$ and $B^N(\tau)$ according to Draper (2014) are

$$B^N(\tau) = (K' + \Lambda_1'\Sigma_X)^{-1}[exp(-(K' + \Lambda_1'\Sigma X)\tau) - I_{2x2}]R_1 \tag{10}$$

$$A^N(\tau) = \int_0^\tau \dot{A}^N(s)ds \tag{11}$$

Under the assumption the markets are complete, the stochastic discount factor determines the value of all assets. The parameter restrictions that are imposed to make sure that the expected value of the discounted stock price does not change over time are defined as

$$\sigma_S'\Lambda_0 = \eta_S \tag{12}$$

$$\sigma_S'\Lambda_1 = 0 \tag{13}$$

The model can be written as a multivariate Ornstein-Uhlenbeck process with the log equations of the price index, stock index and bond wealth index.

$$
d\begin{bmatrix} X \\ ln\Pi \\ lnS \\ lnP^{F0} \\ lnP^{F\tau} \end{bmatrix} = \left(\begin{bmatrix} 0 \\ \delta_{0\pi} - \frac{1}{2}\sigma'_{\Pi}\sigma_{\Pi} \\ R_0 + \eta_S - \frac{1}{2}\sigma'_S\sigma_S \\ R_0 \\ R_0 + B^N(\tau)'\Sigma'_X\Lambda_0 - \frac{1}{2}B^{N\prime}\Sigma'_X\Sigma_X B^N \end{bmatrix} + \begin{bmatrix} -K & 0 \\ \delta'_{1\pi} & 0 \\ R'_1 & 0 \\ R'_1 & 0 \\ R'_1 + B^N(\tau)'\Sigma'_X\Lambda_1 & 0 \end{bmatrix} \begin{bmatrix} X \\ ln\Pi \\ lnS \\ lnP^{F0} \\ lnP^{F\tau} \end{bmatrix} \right) dt + \begin{bmatrix} \Sigma'_X \\ \sigma'_{\Pi} \\ \sigma'_S \\ 0 \\ B^N(\tau)'\Sigma'_X \end{bmatrix} dZ_t \tag{14}
$$

The parameter values used for simulation are the estimation results of the Netherlands from Draper (2014) and are displayed in the Appendix, Table 12.

5.2 Indexation and curtailments

The CR is the guideline for applying indexation or curtailments and is calculated in every scenario i for every point in time t. The CR is calculated as shown in Equation 1. In more detail, $CR_{i,t}$ is the ratio of the assets and the present value of the liabilities.

$$
CR_{i,t} = \frac{A_{i,t}}{\sum_{j=1}^{80} \frac{L_{i,t,j}}{(1+IRTS_{i,t,j})^j}} \tag{15}
$$

where $L_{i,t,j}$ is the pension payment that is expected in scenario i at time t, to be made j years after t, $A_{i,t}$ is the value of the assets in scenario i at time t and $IRTS_{i,t,j}$ is the value on the $IRTS$ in scenario i at time t for yield j.

As explained in Section 3, a fund applies indexation in scenario i at time t when $CR_{i,t}$ is above 110%. The fund has to curtail when $CR_{i,t}$ is below the CCR and when $CR_{i,t}$ is below $MROF$ for five consecutive years. In this research, indexation and curtailments are applied uniformly on the liabilities present at that time, i.e. we abstract from the possibility to smooth benefit curtailments over time for the sake of computational simplicity. Every year will be evaluated whether the fund can apply indexation or has to apply curtailments. The liabilities will be updated by multiplying with $index_{i,t}$.

$$L^{updated}_{i,t,j} = L_{i,t,j} * index_{i,t} \qquad \forall j \tag{16}$$

The value for $index_{i,t}$ differs at different levels of the CR.

$$index_{i,t} = \begin{cases} 1 + \frac{d\Pi_{i,t}}{\Pi_{i,t}} & \text{if } CR_{i,t} \geq ROF \\ 1 + \frac{CR_{i,t}-110}{ROF-110} * \frac{d\Pi_{i,t}}{\Pi_{i,t}} & \text{if } 110 < CR_{i,t} < ROF \\ 1 & \text{if } MROF < CR_{i,t} \leq 110\% \\ 1 & \text{if } CCR < CR_{i,t} < MROF \wedge CR_{i,t-1} \vee ... \vee CR_{i,t-4} \geq MROF \\ \frac{CR_{i,t}}{MROF} & \text{if } CR_{i,t} \wedge CR_{i,t-1} \wedge ... \wedge CR_{i,t-4} < MROF \\ \frac{CR_{i,t}}{CR_{i,t}+\frac{1}{10}*(CCR-CR_{i,t})} & \text{if } CR_{i,t} < CCR \end{cases} \tag{17}$$

The first two lines of Equation 17 show the *index* when the fund applies indexation. When the CR is above ROF, the fund applies full indexation based on the price inflation. Incidental benefit adjustments, which is a compensation for curtailments and for not or not fully applied indexation in previous years is not taken into account in this research. When the CR is between 110% and ROF, the fund applies partial indexation. All liabilities are increased with the same percentage. As a result, the CR will decrease when the price inflation is positive and increase when the price inflation is negative.

The two bottom lines of Equation 17 show the *index* when the fund has to curtail the pension entitlements. All liabilities are curtailed with the same percentage to increase the CR. The value in the denominator is the level to which the CR has to increase. In case of the $MROF$ curtailment, the level to which the CR has to increase is the $MROF$. In case of the ROF curtailment, the fund needs to curtail a tenth of the difference between the current CR and the CCR.

In every scenario, on every point in time, the liabilities will be updated with $index_{i,t}$, as stated in Equation 16, before the liabilities on $t + 1$ are calculated. In this research, the ROF is assumed on 125%, the $MROF$ on 105% and the CCR on 90%.

Besides the evaluation whether indexation or curtailments are applied, pension payments of this year will be done and newly accrued entitlements will be added to the liabilities. When a pension payment is done, both the asset side and the liability side decrease with the amount of the pension payment. When new pension entitlements are accrued, the liability side increases with the new entitlements, corrected for mortality rates; the asset side increases with the premium paid. At $t+1$, the assets and liabilities will be

$$A_{i,t+1} = A_{i,t} * R_{i,t} - L_{i,t,0} + PCR * \sum_{j=1}^{80} \frac{L_j^a}{(1 + IRTS_{i,0,j})^j} \tag{18}$$

$$L_{i,t+1,j} = L_{i,t,j+1}^{updated} + L_j^a \qquad \forall j \tag{19}$$

where $R_{i,t}$ is the return on the investments, depending on the weight in equity and the returns on bonds and equity, $L_{i,t,0}$ is the amount of pension payments done at t and the last term of Equation 18 is the amount of premium, which depends on the PCR and the present value of the new accrued pension entitlements at $t = 0$. Note that the contribution amount is independent of t. $L_{i,t,j}$ is the expected pension payment in scenario i at time t, to be made j years after t and L_j^a is the amount of newly accrued pension entitlements j years after evaluation moment, corrected for mortality rates.

5.3 Dynamic investment strategy

A pension fund invests its assets according to an investment strategy. There are several asset classes a pension fund can invest in. In this research, the choice of a pension fund is restricted to two asset classes; bonds and equity. The in- and outflow of pension entitlements, developments in the financial market and the investment strategy over time lead to a fluctuating CR. The literature showed that the utility of an individual (pension) investor is optimized for an investment mix that becomes more defensive as the investor got older. To ensure a proper pension without taking too much risk, the risk exposure should decline as the retirement age comes closer. The same idea applies to pension funds. A fund with a high duration of pension liabilities should have a different equity exposure as a fund with a low duration of pension liabilities. When no new pension entitlements are accrued, the duration of the liabilities decreases over time until all obligations have been met. Hence, the weight in equity should decrease over time and follow a so-called 'glide path'. Although we expect the glide path to have a negative slope, we do not impose a restriction to it. A positively sloped glide path would also be a possible result.

The static investment strategy will be replaced by a dynamic investment strategy, where the weight in equity follows a glide path. The goal is to find the glide path that maximises the average pension result.

The glide path to follow has to be determined in advance and will be followed for the entire lifetime of the fund. The position in the asset classes will be rebalanced yearly such that the weight in the asset classes corresponds with the desired percentage. There is no reconsidering possibility. This analysis can be done for different starting values of the CR and PCR, with and without accrual of new pension entitlements. The ROF and $MROF$ rule are taken into account.

The glide path is constructed on the basis of three parameters, as shown in Equation 20.

$$\omega_{E,t} = max(0, \omega_{E,0} - max(0, t - \alpha - 1) * \beta) \tag{20}$$

where t is the time in years, $\omega_{E,t}$ is the weight in equity at time t, α is the number of years after which the weight in equity will decrease linearly with β percentage point per year. Note, a negative value of β, which leads to a rising glide path, would be a possible result. Although a linear glide may not be globally optimal, we chose for this functional form because it allows positively and negatively shaped glide paths and for the sake of computational feasibility.

If the accrual of new pension entitlements is included, the duration of the liabilities is higher. The new inflow, as displayed in Figure 3b, is assumed to be the same every year. Note, this analysis does not take discontinuity risk into account.

Given the 2,000 scenarios, the start CR and the PCR, the pension result is calculated for every scenario. The average pension result will be optimised over the three variables of Equation 20, using the Nelder-Mead simplex method.

5.3.1 Sensitivity analysis

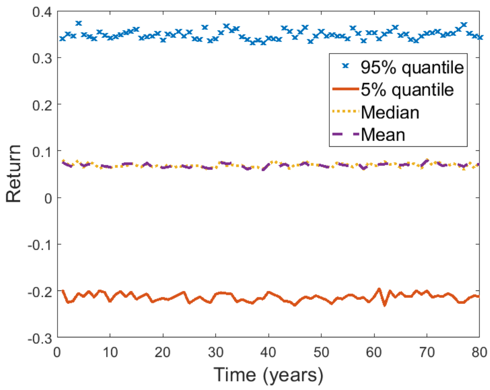
Finally, a sensitivity analysis is included. In steps 1 and 2 of Figure 4 the PCR is set at 100%. In the sensitivity analysis results are shown when the PCR deviates from 100%. Besides, histograms of the pension results are shown when the optimal investment strategy is applied.

6 Results

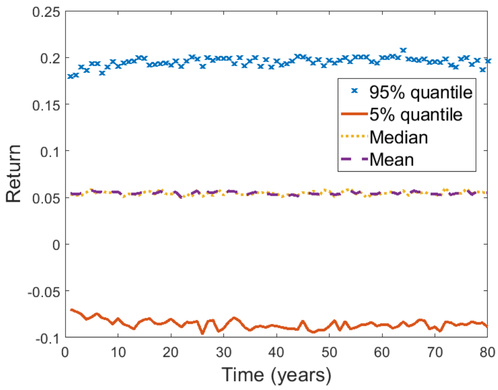
In this section, the obtained results are displayed. First, the simulations are shown and the course of the CR for the funds when the initial values are as displayed in Table 1 (Step 1 of Figure 4). Second, the financial position of the Green fund and the Grey fund is shown for every simulated scenario, in cases where curtailments are and are not applied (Step 2 and 3 of Figure 4). The percentage of scenarios in which the coverage ratio is below 100% after 80 years is shown for different start coverage ratios and weights in equity. Third, the investment strategy that maximises the average pension result is calculated (Step 4 of Figure 4).

6.1 Scenarios

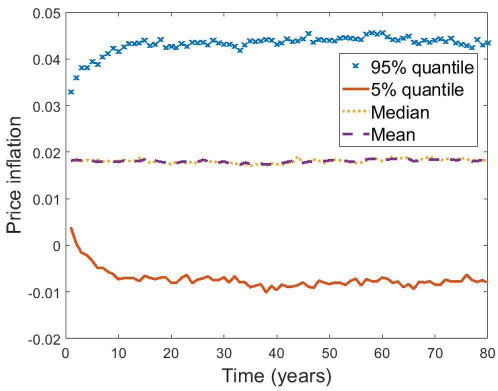
The 2,000 scenarios are simulated with the KNW-model. This resulted in 2,000 simulations of the stock returns, bond returns, interest rates and price inflation. Every simulation contains 80 years. For the returns and inflation rate, there is one simulation point for every simulated year. The interest rate term structure is a curve of 120 yields, which is constructed in every scenario and at every point in time. Figure 5 shows the 5% and 95% quantiles of an execution of the 2,000 scenarios for the stock returns, bond returns and the price inflation. With these



(a) Stock returns



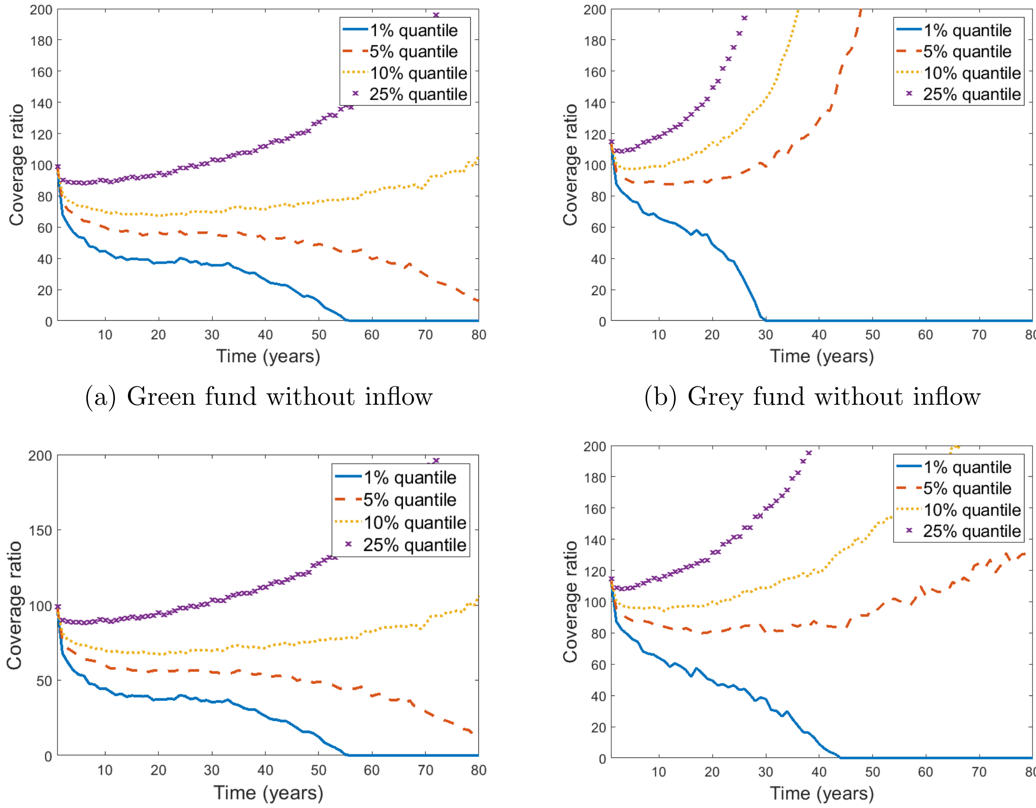
(b) Bond returns



(c) Price inflation

Figure 5: 5% and 95% quantiles, mean and median of stock returns, bond returns and price inflation

simulations the current financial position of the pension funds is calculated, in case there are no curtailments. A scenario with high stock returns and high interest rate term structures will lead



(a) Green fund without inflow

(b) Grey fund without inflow

(c) Green fund with inflow

(d) Grey fund with inflow

Figure 6: 1%, 5%, 10% and 25% quantiles of the CR paths in 2,000 scenarios for the Green and Grey fund, when no curtailments are applied. The funds do apply conditional indexation.

to high coverage ratios, and vice versa. For a situation with indexation but without curtailments, the Green fund's CR development is shown in Figure 6a and Figure 6c. The development of the Grey fund's CR is shown in Figure 6b and Figure 6d. The initial CR and PCR are as shown in Table 1. Without inflow all pension payments are done within 80 years. The fund does not apply curtailments, so it will pay out until its out of assets. It therefore makes sense the CR is very high or zero after 80 years.

Example: *Example: If you have 99 euros to divide to 100 people but continue to pay out one euro per person, the last person will receive nothing. If you would have 101 euros, the last person would receive double.*

The Grey fund initially has a higher CR than the Green fund. However, in the worst scenarios, the Grey fund's CR sinks to 0 even faster than the Green fund's. When new inflow is included, the liability duration is higher. Every year new entitlements are added, so after 80 years there are still future entitlements. In the worst case scenarios, the CR still sinks to 0, but it will take longer. New inflow can keep the CR above 0 for a while and delay the sinking. When the pension payment is higher than the available assets and new premium inflow together, the CR still becomes 0. Note that the fund's initial values are different (according to Table 1). To address the effect of fund maturity on the probability of becoming a sinking giant, the initial values should be identical.

Section 6.2 will evaluate what happens with the funds when, e.g. due to a financial crisis, the CR drops to a very low level. The sinking giant problem will be evaluated by comparing the funds when their initial values are the same. The financial situation and recovery strength of the funds are addressed for several starting values.

6.2 Sinking Giants

First, the sinking giant problem will be illustrated in a situation without accrual of new pension entitlements. The probability that the fund's CR is under 100% after 80 years is shown in Table 2 and 3, for several starting values of the CR and weights in equity. The percentages of scenarios in which the fund's CR is under 100% during the 80 years are shown in the Appendix, Table 15 and 16. Note that when there is no new accrual of pension entitlements, after 80 years all pension entitlements are paid out. The equity weight is assumed to be constant over time.

	No curtailments							Including ROF curtailments						
	Initial CR							Initial CR						
W_e	70	80	90	100	110	120	130	70	80	90	100	110	120	130
0%	37.9	13.3	4.5	1.4	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25%	29.9	15.4	7.1	3.1	1.7	0.9	0.5	0.2	0.1	0.1	0.0	0.0	0.0	0.0
50%	35.5	25.9	18.7	13.8	10.0	8.0	6.2	3.9	3.4	2.9	2.2	1.7	1.4	1.1
75%	43.8	37.3	30.7	25.4	21.3	18.9	16.7	9.7	8.9	7.9	7.1	6.3	5.6	5.3
100%	51.2	46.0	41.6	37.4	33.7	30.2	27.9	17.2	16.3	15.4	14.2	13.1	11.9	11.1

Table 2: Green fund; percentage of scenarios with a CR below 100% after 80 years. There is no inflow.

	No curtailments							Including ROF curtailments						
	Initial CR							Initial CR						
W_e	70	80	90	100	110	120	130	70	80	90	100	110	120	130
0%	90.6	47.8	10.3	1.1	0.1	0	0	11.5	2.8	0.6	0.1	0.0	0.0	0.0
25%	69.9	34.6	11.3	2.7	0.7	0.1	0	6.8	2.8	0.7	0.2	0.0	0.0	0.0
50%	59.7	38.5	23.6	14.4	8.3	5.4	3.4	13.6	9.1	6.7	4.6	2.8	2.0	1.4
75%	59.5	45.8	34.8	26.5	20.1	15.6	12.7	22.4	17.7	13.9	10.6	8.8	7.2	6.0
100%	62.0	52.2	44.2	36.6	31.1	27.5	23.2	31.7	27.1	23.2	20.0	17.0	15.0	13.2

Table 3: Grey fund; percentage of scenarios with a CR below 100% after 80 years. There is no inflow.

For both the Green and Grey fund, the probability of ending up with a low CR will decline as the risk exposure decreases. Only when the initial CR is very low, hence 70%, there is an opportunity of 'gambling for resurrection'. Taking no risk is then worse than taking risk, even when risk-taking increases the probability of sinking further away. In all other cases, taking less risk leads to lower probabilities of a low CR.

For the Grey fund in coverage deficit, the probability of sinking is higher than for Green funds in coverage deficit. The Green fund still receives a relatively big amount of contribution in contrast to the Grey fund, for which the pension contribution is relatively small compared to the already accrued pension capital. The Grey fund experiences the negative effects of pension payments on the CR. Grey funds with a high equity exposure faces a big risk of sinking.

In contrast, the Grey fund experiences the positive effects of pension payments on the CR when its CR is above 100%. When curtailments are not applied and the CR is high, the Grey fund's probability of sinking is less than the Green fund's probability. Because the Grey fund has relatively more pension payments on the short-term, this positive effect is stronger for the Grey fund than for the Green fund.

When ROF curtailments are applied, the fund's CR can still sink to 0. The ROF curtailment may delay the moment that the CR drops to 0, but still the pension payment can become bigger than the remaining assets; pensions cannot be paid out anymore. Applying the ROF curtailment is a weak instrument for Grey funds, but a surprisingly strong instrument for Green funds. The probabilities of sinking for a Green fund with an initial CR of 70 roughly correspond with the probabilities of sinking for a Grey fund with initial CR between 100 and 110. This indicates that the ROF curtailment may be a sufficient measure to prevent the Green fund from sinking, but is insufficient for the Grey fund.

The CR increases as a consequence of curtailments. In Table 4 and 5 we show the impact of the curtailments. These tables show the number of scenarios in which at least one curtailment is applied, the average number of times a curtailment is applied per scenario and the average impact of the curtailment, expressed in percentage points, e.g. a curtailment which increases the CR from 90% to 105% is a curtailment of 15 percentage point.

		Including ROF curtailments						
		Initial CR						
	W_e	70	80	90	100	110	120	130
Number of scenarios in which at least one curtailment is applied.	0%	2000	2000	1938	1176	757	463	263
	25%	2000	2000	1945	1219	855	597	379
	50%	2000	2000	1947	1331	1063	854	691
	75%	2000	2000	1953	1455	1233	1074	933
	100%	2000	2000	1961	1550	1405	1283	1170
Average number of curtailments in scenarios in which at least one curtailment is applied.	0%	9.79	7.10	4.90	3.63	2.85	2.45	2.27
	25%	10.08	7.68	5.67	4.75	4.06	3.59	3.41
	50%	13.22	10.96	8.98	8.86	8.37	8.14	7.75
	75%	17.20	15.09	13.25	13.93	13.91	13.87	13.90
	100%	21.51	19.66	18.11	19.39	19.08	18.83	18.75
Average impact of the applied curtailments in percentage points.	0%	1.36	1.08	0.92	0.83	0.74	0.71	0.65
	25%	1.47	1.20	1.06	0.96	0.88	0.86	0.84
	50%	2.17	1.98	1.99	1.98	1.98	1.99	1.95
	75%	3.11	3.00	3.11	3.21	3.26	3.31	3.37
	100%	4.08	4.01	4.16	4.22	4.26	4.31	4.35

Table 4: Green fund; the frequency and intensity of the applied curtailments. There is no inflow.

		Including ROF curtailments						
		Initial CR						
	W_e	70	80	90	100	110	120	130
Number of scenarios in which at least one curtailment is applied.	0%	2000	2000	1915	741	236	76	13
	25%	2000	2000	1933	775	295	109	36
	50%	2000	2000	1951	1010	641	416	266
	75%	2000	2000	1965	1217	932	750	591
	100%	2000	2000	1971	1364	1147	995	866
Average number of curtailments in scenarios in which at least one curtailment is applied.	0%	22.66	10.93	4.85	2.65	1.68	1.26	1.15
	25%	17.43	10.21	5.44	3.88	2.75	2.43	2.17
	50%	20.88	14.99	10.80	11.95	10.98	11.04	10.96
	75%	25.36	20.44	16.45	18.76	19.00	18.78	18.89
	100%	30.31	25.91	22.37	25.98	25.96	26.00	25.52
Average impact of the applied curtailments in percentage points.	0%	2.72	1.73	1.12	0.74	0.45	0.34	0.25
	25%	2.41	1.79	1.26	0.86	0.58	0.54	0.52
	50%	3.66	3.29	3.40	3.40	3.42	3.54	3.65
	75%	4.87	4.62	4.70	4.74	4.86	4.92	5.00
	100%	5.83	5.71	5.83	5.88	5.89	5.87	5.90

Table 5: Grey fund; the frequency and intensity of the applied curtailments. There is no inflow.

We see that the number of scenarios in which curtailments apply, the average number of curtailments per scenario and the average impact of the curtailments all increase with the weight in equity, for every initial CR. The number of scenarios in which curtailments apply decrease with the initial CR. It makes sense that the better the financial position, the less often the fund has to curtail. The impact of the applied curtailments is also decreasing in the initial CR, except for cases where the weight in equity is high. We then see that the impact of the curtailments can be higher when the initial CR is high. This can be explained by the fact that the fund with a low initial CR has to apply curtailments in all of the scenarios. With a high equity exposure, the impact of the curtailments will differ a lot. There may be many curtailments with a low impact, which lowers the average impact. A fund with a high initial CR will only have to curtail in the worst scenarios. The combination of a worse scenario and a high equity exposure will make the curtailment extensive. The average impact may therefore be higher for funds with a high initial CR than for funds with a low initial CR when the equity exposure is high.

When also the $MROF$ curtailment would be included, the fund's CR cannot sink anymore. If the CR is below the $MROF$ for five consecutive years, the $MROF$ curtailment brings the CR back at the $MROF$. The scenarios in which the CR is below 100% after 80 years would therefore only be the consequence of the last four years. According to the definition of a sinking giant, there are no sinking giants when $MROF$ curtailments are applied.

6.2.1 Inflow included

The probabilities that the fund's CR is below 100%, when inflow of newly accrued pension entitlements is included with an initial PCR of 100, are shown in Table 6 and 7.

	No curtailments						
	Initial CR						
W_e	70	80	90	100	110	120	130
0%	10.5	7.7	5.9	4.7	3.2	2.7	2.2
25%	9.9	7.8	6.7	5.5	4.8	3.7	3.3
50%	17.3	15.1	12.9	11.4	10.6	9.8	9.0
75%	27.8	26.2	24.0	22.0	20.2	18.8	17.7
100%	38.1	36.4	35.0	33.5	32.1	30.8	30.2

Table 6: Green fund; percentage of scenarios with a CR below 100% after 80 years, in the case that the initial PCR is 100% and no curtailments are applied.

	No curtailments						
	Initial CR						
W_e	70	80	90	100	110	120	130
0%	60.6	24.7	5.9	1.3	0.4	0.1	0.0
25%	43.8	20.0	7.3	2.6	1.4	0.7	0.3
50%	43.1	28.9	18.3	12.0	7.8	5.8	3.7
75%	50.3	39.2	30.1	23.8	20.1	17.0	13.7
100%	56.6	48.6	42.3	37.3	32.2	28.1	25.3

Table 7: Grey fund; percentage of scenarios with a CR below 100% after 80 years, in the case that the initial PCR is 100% and no curtailments are applied.

We see here as well that lowering the weight in equity is an efficient tool to prevent the CR from sinking. For the Grey fund it may be optimal to take risk when the initial CR is low. A low weight in equity has less downward risk but also less upward potential. In a bad financial situation, taking risk for upward potential offsets the corresponding downward risk.

For both Green and Grey funds the probabilities of sinking are lower when inflow is included compared to the situation without inflow in Section 6.2. It depends on the CR whether new inflow has a positive or negative effect on the CR. Example 1 in Section 3 shows the effect of pension payments, but also holds for contribution inflow. Hence, contribution inflow with a PCR of 100% positively affects the CR when the CR is below 100% and negatively affects the CR when the CR is above 100%. When the CR is below 100% at time t, the contribution inflow positively effects the CR on $t + 1$. This indicates that it takes longer for the CR to sink to 0. The probability of sinking is lower for both Green and Grey funds with inflow than the probability of sinking without inflow.

6.3 Dynamic investment strategy

When curtailments are applied, the CR no longer indicates whether a good pension for the member is provided. Applying curtailments raises the CR but decreases the pensions. Therefore we take the pension result as decision variable. The goal is to optimise the average pension result by changing the investment strategy. To consider many possible investment strategies, a dynamic investment strategy is introduced. If a static investment strategy turns out to be optimal, the parameter β will return a 0. To stay close to reality, this analysis is done in case of both the ROF and the $MROF$ curtailments are applied. The pension result is calculated for every scenario. For the Green and Grey fund the analysis is done in both the closed and open situation, for several starting values for the CR and PCR.

	Initial CR		
	80	100	120
$\omega_{E,0}$	0.2006	0.1297	0.1270
α	1.2563	1.3834	1.3995
β	0.0023	0.0024	0.0023
Average pension result	0.6591	0.8082	0.9217

Table 8: Green fund; weights for which the pension result is optimal, in the case that indexation and curtailments are applied. There is no inflow.

	Initial CR		
	80	100	120
$\omega_{E,0}$	0.2521	0.1812	0.1721
α	3.7758	1.4446	1.3466
β	0.0025	0.0018	0.0021
Average pension result	0.7033	0.8695	0.9647

Table 9: Grey fund; weights for which the pension result is optimal, in the case that indexation and curtailments are applied. There is no inflow.

Table 8 and 9 show the optimal glide paths for the Green and Grey fund for different initial values of the CR, when there is no inflow. For both the Green and Grey funds the weight in equity decreases over time. When no inflow is included, the duration of the liabilities declines over time. After about 80 years all entitlements have been paid and the fund is to liquidate. It therefore makes sense to decrease the weight in equity over time. The slope with which the weight of equity decreases is denoted by β. β seems small but certainly decreases the weight in equity over time. For the Green fund with an initial CR of 80, the weight in equity after 80 years is 0.0218. When the initial CR is 100 or 120, the weight in equity is decreased to 0 after 60 and 61 years, respectively. For the Grey fund with an initial CR of 80, 100 and 120 the weight in equity after 80 years are 0.0640, 0.0416 and 0.0090, respectively.

It turns out to be optimal to invest more defensive when the financial position is good. When the fund's financial position is good, the fund should take less risk to maintain its position and do not risk curtailments. Assumed in this analysis is that the maximum indexation a fund can give is the price inflation. When the fund's CR is above the ROF, the fund fully applies indexation. There is no incentive for additional risk-taking, because a higher CR will not lead to more indexation. Therefore, when the financial situation is good, risk-taking is not worthwhile. Investing more wealth in equity is therefore not rewarded with indexation, but the risk of curtailments does increase. We can therefore conclude that the fund should have a lower risk exposure when its financial position is better, to maintain its good position and not to take unnecessary risks.

A Grey fund in coverage deficit has the challenge to recover. The fund needs upward potential, but investing more in equity also increases the downside risk. The fund's challenge is to prevent becoming a sinking giant. A counter-intuitive result is the higher equity exposure for Grey funds compared to Green funds. The explanation for this result consists of two parts. First, the Grey fund has big pension payments on the short-term, which means that the assets will decrease fast in the coming years. The investment returns are the most effective when the asset mass is greatest. For Grey funds this is in the immediate years; for Green funds the peak has yet to come. Second, if the additional risk-taking leads to a loss, the $MROF$ curtailment provides the solution. When we assume there are no political and management risks and the legal curtailments are applied correctly when necessary, the Grey fund may use the $MROF$ curtailment as motivation for risk-taking. The upward potential pays off in indexation; the downward risk is limited by applying the curtailment. When the curtailment is applied, the CR is above 100%. From that moment the pension payments positively affect the CR. Because the positive effect of pension payments is higher for Grey funds than for Green funds, the Grey fund may allow themselves to take more risk.

6.3.1 Inflow included

When the accrual of pension entitlements and corresponding inflow of contribution is included, the PCR is of influence on the results. The optimal glide paths for different initial values of the CR and PCR are shown in Table 10 and Table 11. A visualisation of the glide paths is shown in the Appendix, Figure 9 and 10.

		Initial CR		
Initial PCR		80	100	120
80	$\omega_{E,0}$	0.4751	0.2180	0.1798
	α	0.0000	1.0979	1.4149
	β	0.0136	0.0024	0.0019
	Average pension result	0.7632	0.8165	0.8686
100	$\omega_{E,0}$	0.4787	0.1684	0.1589
	α	0.0745	1.3501	1.3940
	β	0.0205	0.0021	0.0021
	Average pension result	0.8119	0.8618	0.9073
120	$\omega_{E,0}$	0.4576	0.1335	0.1288
	α	1.8575	1.4145	1.4295
	β	0.0355	0.0023	0.0022
	Average pension result	0.8498	0.9075	0.9735

Table 10: Green fund; weights for which the pension result is optimal, in the case that indexation and curtailments are applied.

		Initial CR		
Initial PCR		80	100	120
80	$\omega_{E,0}$	0.6351	0.1703	0.1507
	α	0.1910	1.4642	1.3986
	β	0.0677	0.0020	0.0023
	Average pension result	0.7776	0.8923	0.9680
100	$\omega_{E,0}$	0.6235	0.1623	0.1554
	α	0.0000	1.3754	1.3320
	β	0.0584	0.0021	0.0025
	Average pension result	0.7913	0.8999	0.9709
120	$\omega_{E,0}$	0.5416	0.1460	0.1372
	α	0.0000	1.2820	1.3427
	β	0.0530	0.0027	0.0023
	Average pension result	0.8026	0.9075	0.9735

Table 11: Grey fund; weights for which the pension result is optimal, in the case that indexation and curtailments are applied.

For the Green fund we see a clear pattern in the results. The better the financial position of the fund, the less risk exposure the funds should have. This is in line with the explanation in Section 6.3. An additional result is the effect of the PCR. When the PCR is higher than the CR, the contribution inflow has a positive effect on the CR, and vice versa. It is therefore obvious that the higher the PCR, the better the financial position, hence the lower the risk exposure. As explained earlier, a bad financial position indicates more risk-taking as optimal glide path. When the initial CR and PCR are both 80%, the initial weight in equity should be 47,51%. Note, the slope with which this weight in equity decreases is much steeper than the slope of the glide path of funds in a better financial position. This indicates that initially risk-taking is rewarded, but the risk-taking should fast be decreased. The Green fund should decrease its equity weight in 36 years to zero. Risk-taking when a fund is in a bad financial position pays off because the implemented $MROF$ curtailment absorbs the downward risk by applying a curtailment. After the curtailment, the CR is on a decent level and the pension payments affect the CR positively. The fund decreases its risk exposure fast to maintain the obtained financial position. This strategy leads to the highest average pension result.

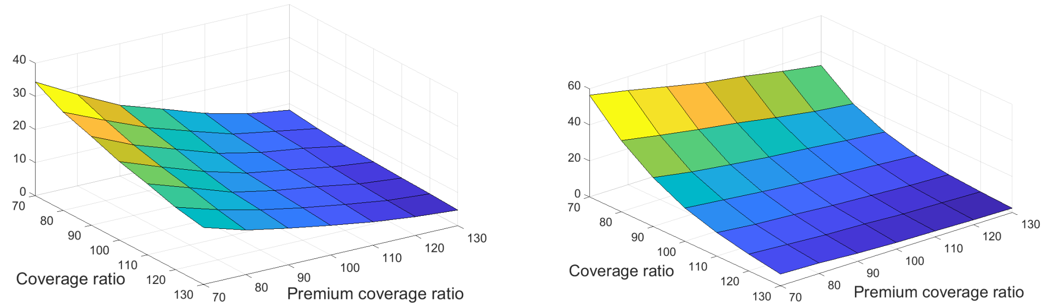
For the Grey fund we see similar results. The better the financial position of the fund, the more defensive the fund should invest. We see that the Grey fund with an initial CR of 80 has a high initial weight in equity. The weight in equity also decreases fast. When the initial PCR is 80, the weight in equity decreases to 0 in 11 years. When the initial PCR is 100 or 120, the weight in equity decreases in 12 years to 0. For higher initial CR, the slope is less steep but still clearly visible in the glide paths.

When we compare the results of the Grey fund with the results of the Green fund, both in a bad financial situation, we see that the initial weights in equity for the Grey fund are higher than for the Green fund. This is striking because Section 6.2 showed that the probability of sinking is higher for the Grey fund. Hence, in this optimisation curtailments are applied, contrary to the results of Section 6.2. The intuition of this result can be that the Grey fund's assets will decrease fast in the coming decade. The effect of investment results is the greatest when the fund's assets are at its peak. For the Green fund, the peak has yet to come; for the Grey fund the peak has already been. The investment results are therefore more important for Grey funds than for Green funds. In case of a bad financial position, the Grey fund may invest more in equity than the Green fund. However, the Grey fund decreases its weight in equity faster. The $MROF$ curtailment prevents the fund's CR from sinking further. Therefore, the fund needs the upward potential most when its financial position is bad. For the Grey fund the $MROF$ curtailment is necessary to prevent the fund from becoming a sinking giant. Besides, the $MROF$ curtailment may be a motivation for additional risk-taking. If investment returns are bad, the fund needs to incur one (considerable) curtailment, but from that moment the CR is on a decent level and the pension payments have a positive effect on the CR. It could therefore be optimal for Grey funds to apply the $MROF$ earlier than after five years. The quantification of such a measure is an interesting topic for further research.

The question may arise why investing risky is preferred over investing less risky or risk-free when the fund's financial situation is bad. A fund could also invest solely in bonds, apply the $MROF$ curtailment if necessary and maintain the CR above 100%. The explanation for this is that the returns on equity are positive on average. On average, the risk incurred pays off in higher returns. From an optimisation perspective, it is therefore optimal to invest risky.

6.4 Sensitivity analysis

In Section 6.2 the assumption is made that the initial PCR is 100%. In Figure 7 the sensitivity of this assumption is shown. For the Green fund and the Grey fund is shown what the impact of the initial PCR is on the probability of having a CR below 100% after 80 years. When we

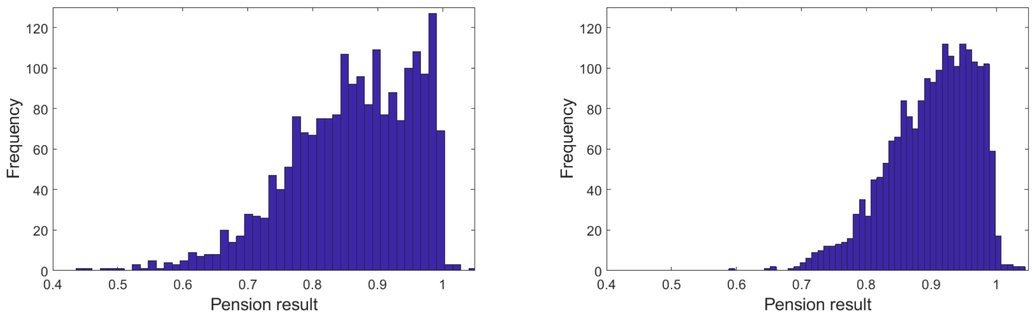


(a) Green fund's sensitivity (b) Grey fund's sensitivity

Figure 7: Sensitivity of the initial PCR and the initial CR on the probability of having a CR below 100% after 80 years when the weight in equity is 50%.

compare Figure 7a with Figure 7b, we first see that for the Grey fund, the probability of sinking is higher than for the Green fund. This which was already displayed for a PCR of 100% in Table 6 and 7. Besides, we see that a higher PCR leads to a lower probability of sinking, because the higher the PCR the more often the contribution inflow works in favour of the CR. We can conclude that the impact of the initial PCR is higher for the Green fund than for the Grey fund. The effect of the PCR on the probability of sinking is indicated by the slope, which is steeper for the Green fund. The Green fund benefits more from a high initial PCR, as the contribution inflow is a relatively bigger part of the total pension capital. For the Grey fund the initial CR is of bigger importance.

Figure 8 shows the histograms of the pension results for the Green and Grey fund when both the initial CR and the initial PCR are 100% and the optimal glide path is applied. When the Grey fund's CR is above 100%, it profits more from pension payments than the Green fund. Note that the CR of the funds are above 100% at least once in five years, due to a possible



(a) Green fund's histogram of pension results (b) Grey fund's histogram of pension results

Figure 8: Histogram of the pension results for the glide path which produces the optimal average pension result.

curtailment. The median of the pension results of the Green and Grey fund are 0.8725 and 0.9099, respectively. These values are higher than the average pension result due to the longer left tail. Some scenarios return a pension result higher than 1. That is possible when the fund does not apply indexation in years where the price inflation is negative and does apply in years where the price inflation is positive. When the fund always applies indexation according to the price inflation, whether the price inflation is positive or negative, the pension result is 1.

7 Conclusion

From this research, several conclusions can be drawn. The main topic of this research is the sinking giant problem. To prevent a fund from a sinking CR, curtailments can be applied. However, applying curtailments is undesirable for current members. In reality, it turns out that funds and politicians try to avoid or postpone curtailments by changing the law. Especially pensioners oppose abrupt and big curtailments because they immediately see the effect in their lower pension entitlements. If curtailments are continued to be postponed, this leads to a system where no curtailments are applied at all. An unrestricted system contains big risks for pension funds. Especially Grey funds in coverage deficit are vulnerable to shocks in the CR. When, e.g. due to a financial crisis, the CR drops to a level of 70% and keeps its weight in equity at 25%, in more than two third of the scenarios the fund does not recover. The current and future working members of the fund pay pension contribution for their entire career, but face a big risk that they will not receive any pension. The current pensioners may not like curtailments, but maybe curtailments are necessary to provide a fair pension for every worker who paid contribution. When no curtailments are applied and the fund's objective is to prevent the CR from sinking, it is optimal to invest only in bonds.

There is an exception in the cases with and without inflow and with or without applying the ROF curtailment. When the fund's CR is very low, hence 70% or 80%, and the fund wants to minimize the probability of sinking, it is optimal to invest some weight in equity instead of investing solely in bonds, which is optimal in all other cases. Taking risk as the last attempt to survive can be called 'gambling for resurrection'. Still, the fund's CR can sink to 0, but taking risk on average pays off. This holds for Grey funds with a low CR and for Green funds with a low CR where no curtailments are applied and no inflow is taken into account. The side effect of additional risk taking is that the probability of sinking increases.

The sensitivity analysis showed that the initial CR has more impact on the probability of sinking for the Grey fund than for the Green fund. The Grey fund's CR fluctuates more as a consequence of the relatively large pension payments. A pension payment has a positive effect on the CR when the CR is above 100%, and a negative effect when the CR is below 100%. These effects are bigger for Grey funds than for Green funds. The PCR will have more impact on the probability of sinking for the Green fund than for the Grey fund as the Green fund has relatively more contribution inflow.

If the ROF curtailment is applied, the probability of sinking decreases. However, it is still possible to sink to 0. This happens when the pension payment in a specific year is bigger than the remaining assets and corresponding investment returns. The ROF curtailment can keep the CR above 0 for a while but in some bad case scenarios, this measure does not hold. A Grey fund in coverage deficit is more likely to sink than a Green fund. The higher the weight in equity, the bigger the fraction of scenarios in which the CR is under 100% after 80 years. Applying only the ROF curtailments is an insufficient measure for Grey funds. Applying the $MROF$ curtailments is necessary to prevent from becoming a sinking giant. For Green funds, applying the ROF curtailments is an efficient tool to prevent from becoming a sinking giant.

Besides the probabilities that the CR is below a certain level, this research shows what the optimal investment mix is by optimising the average pension result. We can conclude that funds in a good financial position should have a low equity exposure. Additional risk-taking is not rewarded in the pension result because indexation is maximised at the price inflation, but punished with curtailments when big losses are incurred. It is therefore optimal to take less risk and maintain a good position once the fund has reached it.

For funds in a bad financial position, Grey funds should take more risk than Green funds. This may sound counter-intuitive, but for Grey funds the need for investment results is higher. The effect of investment results is the greatest when the fund's assets are at its peak. The Grey fund's peak has already been. To achieve the upward potential to recover from coverage deficit, the fund should take some risk. The downside risk is captured by the curtailments. When the Grey fund applies an $MROF$ curtailment such that the CR is 105%, the pension payments are in favour of the CR. Because the Grey fund has relatively more pension payments in the short term, it profits more from a positive CR than the Green fund. Because the average return of equity is positive, risk-taking is preferred over taking no risk.

The weighted average of the Dutch funds resulted in a weight in equity of 43% for both the Green and Grey fund. It turns out it is optimal to invest less weight in equity and decrease the weight over time. Besides the mathematical optimisation, investing carefully is well explainable to members and should be a good standard when other people's money is invested.

8 Discussion

Every model is by definition a simplified version of reality. A research is based on assumptions and therefore knows its limitations. In this research, the scenarios are simulated with the KNW-model according to the parameters of Draper (2014). These scenarios are quite optimistic; the risk premium is supposed to be 4.5% and the unconditional expected inflation is supposed to be 1.8%. These percentages are based on historical data but have not been met in recent years. It is not a given fact that the risk premium and inflation return to these levels. The parameters and corresponding scenarios are used in the feasibility test of pension funds. If the parameters are overestimated, the funds prospect their return too favourable. A suggestion for further research would be to change the parameters and execute the research with different scenarios. This could show what the fund's recovery strength and weaknesses are in case the average inflation and risk premium are not as favourable.

For computational purposes, the ROF, $MROF$ and CCR are fixed in this research. In reality, these boundary ratios are fund specific and depend on the risk exposure of the fund. When the boundary ratios are calculated for every fund specific, it may be that the CCR for the Grey fund should be higher than for the Green fund, because with the same investment mix their risk of sinking is higher. If that is the case, it could be a strong recommendation for Grey funds to apply curtailments earlier, to prevent members from large and unnecessary curtailments. As shown in this research, a Grey fund profits more from a positive CR than a Green fund because of the relatively big pension payments. Because the funds are different and has different recovery strength, the measures to recover should also be different. Further research could show if it is optimal for Grey funds to apply curtailments earlier than five years.

The glide path which is used in the optimisation of the average pension result, is assumed to be linear. This may not be the optimal shape of the glide path. Other monotonic functions (e.g. a logistic function) could be considered. Further research could take different shapes of the glide path into account, for example a hump-shaped glide-path, as proposed by Benzoni et al. (2007). Besides, this research suggests that the fund determines its investment strategy in advance and will execute it entirely according to the glide path. The glide path is constructed based on an optimisation of the average pension result. If the fund finds itself in an extremely good or extremely bad situation, it may want to deviate from the previously determined glide path. This research does not take the possibility to reconsider into account. In reality, a fund always has the opportunity to reconsider. When the fund finds itself in an unexpectedly good or bad scenario, the board probably will not stick to the previously optimal glide path, Yao et al. (2016).

The glide path is negative sloping with the lifetime of the fund. Also when inflow is included, the glide path's slope is negative. It would be interesting to not link the weight in equity to the fund's lifetime but to the state of the economy. Bikker et al. (2010) showed that the investment policies of pension funds are partially driven by the cyclical performance of the stock market. A fund may want to invest less in equity in a bear market and more in equity in a bull market. In the used scenarios the state variable is a mean-reverting process. A Markov switching model could be used to predict future states and whether the fund finds itself in a bull or a bear market at that time, Kole and Van Dijk (2017). Then the investment strategy can be linked to the state of the economy. The link between the state of the economy and the investment strategy is not taken into account in this research but is a suggestion for further research.

References

Benzoni, L., Collin-Dufresne, P., and Goldstein, R. S. (2007). Portfolio choice over the life-cycle when the stock and labor markets are cointegrated. *The Journal of Finance*, 62(5):2123–2167.

Bikker, J. A., Broeders, D. W., and De Dreu, J. (2010). Stock market performance and pension fund investment policy: Rebalancing, free float, or market timing? *International Journal of Central Banking*, 6(2).

Bikker, J. A., Broeders, D. W., and Hollanders, D. A. (2017). Pension fund investment policy, risk-taking, ageing and the life-cycle hypothesis. In *Pension Fund Economics and Finance*, pages 157–178. Routledge.

Blake, D., Lehmann, B. N., and Timmermann, A. (1999). Asset allocation dynamics and pension fund performance. *The Journal of Business*, 72(4):429–461.

Boot, A., Cools, K., Bovenberg, L., van Ewijk, C., and Nijman, T. (2014). Governance en pensioenstelsel. *Preadviezen van de Koninklijke Vereniging voor de Staathuishoudkunde*, (2014).

Bovenberg, A. L. and Knaap, T. (2005). Ageing, funded pensions and the Dutch economy.

Bovenberg, L. (2014). Pensioeninnovatie in nederland en de wereld: Nederland kampioen in pensioen. *TPE-Digitaal*, 8(4):163–185.

Bruil, A., Schmitz, C., Gebraad, J., and Bhageloe-Datadin, R. (2015). Totale pensioenaanspraken van Nederland in beeld. *Centraal Bureau voor de Statistiek, De Nederlandse Economie*, 2.

Chen, D. H. J. (2016). Essays on collective funded pension schemes.

Cocco, J. F., Gomes, F. J., and Maenhout, P. J. (2005). Consumption and portfolio choice over the life cycle. *The Review of Financial Studies*, 18(2):491–533.

Cong, F. and Oosterlee, C. W. (2016). Multi-period mean–variance portfolio optimization based on monte-carlo simulation. *Journal of Economic Dynamics and Control*, 64:23–38.

Cui, J., De Jong, F., and Ponds, E. (2011). Intergenerational risk sharing within funded pension schemes. *Journal of Pension Economics & Finance*, 10(1):1–29.

Donner, J. P. H. (2009). Brief regering d.d. 20 februari 2009 - J.P.H. Donner, minister van Sociale Zaken en Werkgelegenheid - Korte termijn herstelplannen pensioenfondsen.

Draper, N. (2014). A financial market model for the Netherlands.

Gomes, F. J., Kotlikoff, L. J., and Viceira, L. M. (2008). Optimal life-cycle investing with flexible labor supply: A welfare analysis of life-cycle funds. *American Economic Review*, 98(2):297–303.

Goossens, J., de Haan, J., van Leuvensteijn, M., and Sender, S. (2016). *Quantification of the Discontinuity Risk of Pension Funds*. PhD thesis, Tilburg University.

Goudswaard, C., Beetsma, R., Nijman, T. E., and Schnabel, P. (2010). Een sterke tweede pijler. *Commissie Toekomstbestendigheid Aanvullende Pensioenregelingen.*

Hannema, E. (2014). Een goed pensioen is niet gratis: Kosten in context. *Financial Investigator*, (2).

Heaton, J. and Lucas, D. (1997). Market frictions, savings behavior, and portfolio choice. *Macroeconomic Dynamics*, 1(1):76–101.

Jagannathan, R. and Kocherlakota, N. R. (1996). Why should older people invest less in stocks than younger people? *Federal Reserve Bank of Minneapolis. Quarterly Review-Federal Reserve Bank of Minneapolis*, 20(3):11.

Kocken, T. P. (2012). Pension liability measurement and intergenerational fairness: two case studies.

Kocken, T. P. and Potters, J. J. (2010). Sinking giants.

Koijen, R. S. J., Nijman, T., and Werker, B. J. M. (2009). When can life-cycle investors benefit from time-varying bond risk premia?

Kole, E. and Van Dijk, D. (2017). How to identify and forecast bull and bear markets? *Journal of Applied Econometrics*, 32(1):120–139.

Pino, A. and Yermo, J. (2010). The impact of the 2007-2009 crisis on social security and private pension funds: A threat to their financial soundness? *International Social Security Review*, 63(2):5–30.

Rauh, J. D. (2009). Are state public pensions sustainable? In *Train Wreck: A Conference on America's Looming Fiscal Crisis," sponsored by the Urban-Brookings Tax Policy Center and the USC-Caltech Center for the Study of Law and Politics.(January 15, 2010)*. Citeseer.

Rauh, J. D. (2010). The day of reckoning for state pension plans. https://kelloggfinance.wordpress.com/2010/03/22/the-day-of-reckoning-for-state-pension-plans/.

Rutte, M., Van Haersma Buma, S., Pechtold, A., and Segers, G. (2017). Vertrouwen in de toekomst: Regeerakkoord voor de periode 2017-2021. https://www.rijksoverheid.nl/documenten/publicaties/2017/10/10/regeerakkoord-2017-vertrouwen-in-de-toekomst.

Sappington, D. E. (1991). Incentives in principal-agent relationships. *Journal of economic Perspectives*, 5(2):45–66.

Schutte, H. (2018). Feasibility test for Defined Contribution pension schemes.

Van Riel, B. (2016). SER-verkenning persoonlijk pensioenvermogen en collectieve risicodeling. *Geron*, 18(4):57–60.

Veen, M. (2013). Het echte governance probleem in de pensioensector wordt niet benoemd. *Financial Investigator*, (2).

Wang, S., Lu, Y., and Sanders, B. (2018). Optimal investment strategies and intergenerational risk sharing for target benefit pension plans. *Insurance: Mathematics and Economics*, 80:1–14.

Wernekinck, F. (2013). The holistic balance sheet.

Yao, H., Chen, P., and Li, X. (2016). Multi-period defined contribution pension funds investment management with regime-switching and mortality risk. *Insurance: Mathematics and Economics*, 71:103–113.

Appendix

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
$\delta_{0\pi}$	1.81%	$\sigma_{\Pi(1)}$	0.02	$\Lambda_{0(1)}$	0.403
$\delta_{1\pi(1)}$	-0.63%	$\sigma_{\Pi(2)}$	-0.01%	$\Lambda_{0(2)}$	0.039
$\delta_{1\pi(2)}$	0.14%	$\sigma_{\Pi(3)}$	0.61%	$\Lambda_{1(1,1)}$	0.149
R_0	2.40%	$\sigma_{\Pi(4)}$	0.00%	$\Lambda_{1(1,2)}$	-0.381
$R_{1(1)}$	-1.48%	$\sigma_{S(1)}$	-0.53%	$\Lambda_{1(2,1)}$	0.089
$R_{1(2)}$	0.53%	$\sigma_{S(2)}$	-0.76%	$\Lambda_{1(2,2)}$	-0.083
κ_{11}	0.08	$\sigma_{S(3)}$	-2.11%	$\Lambda_{1(3,1)}$	0.00
κ_{12}	0.00	$\sigma_{S(4)}$	16.59%	$\Lambda_{1(3,1)}$	0.00
κ_{21}	-0.19	η_S	4.52%	$\Lambda_{1(4,1)}$	0.009*
κ_{22}	0.35			$\Lambda_{1(4,2)}$	-0.16*

Table 12: Estimation results for the Netherlands, Draper (2014). These parameter values are used in the simulation with the KNW-model.

* These values follow from the restrictions in Equation 12 and 13

Tables 13 till 16 show the percentage of scenarios in which the CR is below 100% after X years, for several values of X.

	No curtailments							Including ROF curtailments						
	Initial CR							Initial CR						
X	70	80	90	100	110	120	130	70	80	90	100	110	120	130
20	67.0	45.6	30.7	19.1	11.7	8.4	5.6	29.1	25.3	22.2	17.7	14.4	12.1	10.1
30	61.5	41.3	25.9	15.2	9.60	6.5	4.3	17.0	14.3	12.0	10.3	8.8	7.2	6.2
40	60.5	39.3	24.3	14.5	8.6	5.6	3.4	9.4	8.1	7.0	6.1	5.1	3.9	3.3
50	59.8	38.7	23.6	14.4	8.3	5.4	3.4	6.0	5.1	4.4	3.4	2.7	2.2	2.0
60	59.7	38.5	23.6	14.4	8.3	5.4	3.4	4.4	3.9	3.3	2.3	1.9	1.6	1.2
70	59.7	38.5	23.6	14.4	8.3	5.4	3.4	3.9	3.5	2.9	2.2	1.8	1.4	1.1
80	59.7	38.5	23.6	14.4	8.3	5.4	3.4	3.9	3.4	2.9	2.2	1.7	1.4	1.1

Table 13: Green fund; percentage of scenarios with a CR below 100% after X years. There is no inflow. The weight in equity is 50%.

	No curtailments							Including ROF curtailments						
	Initial CR							Initial CR						
X	70	80	90	100	110	120	130	70	80	90	100	110	120	130
20	67.0	45.6	30.7	19.1	11.7	8.4	5.6	37.8	29.1	21.1	13.9	9.3	6.6	4.3
30	61.5	41.3	25.9	15.2	9.6	6.5	4.3	24.8	18.3	13.1	8.8	5.8	3.9	2.4
40	60.5	39.3	24.3	14.5	8.6	5.6	3.4	18.0	12.4	8.4	5.7	3.5	2.3	1.6
50	59.8	38.7	23.6	14.4	8.3	5.4	3.4	14.3	9.8	7.3	4.9	3.1	2.2	1.4
60	59.7	38.5	23.6	14.4	8.3	5.4	3.4	13.8	9.3	6.7	4.8	2.8	2.1	1.4
70	59.7	38.5	23.6	14.4	8.3	5.4	3.4	13.6	9.1	6.7	4.7	2.8	2.0	1.4
80	59.7	38.5	23.6	14.4	8.3	5.4	3.4	13.6	9.1	6.7	4.6	2.8	2.0	1.4

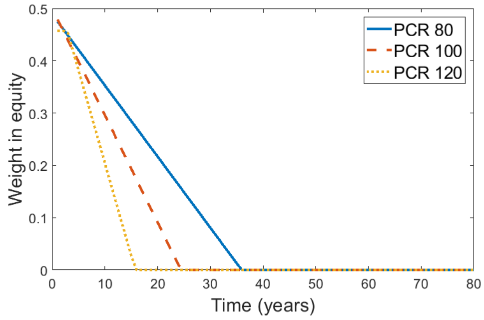
Table 14: Grey fund; percentage of scenarios with a CR below 100% after X years. There is no inflow. The weight in equity is 50%.

	No curtailments						
	Initial CR						
X	70	80	90	100	110	120	130
20	29.1	25.3	22.2	17.7	14.4	12.1	10.1
30	17.0	14.3	12.0	10.3	8.8	7.2	6.2
40	9.4	8.1	7.0	6.1	5.1	3.9	3.3
50	6.0	5.1	4.4	3.4	2.7	2.2	2.0
60	4.4	3.9	3.3	2.3	1.9	1.6	1.2
70	3.9	3.5	2.9	2.2	1.8	1.4	1.1
80	3.9	3.4	2.9	2.2	1.7	1.4	1.1

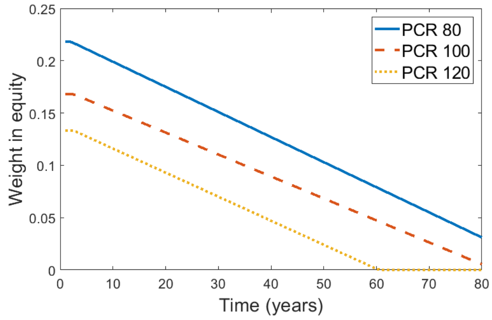
Table 15: Green fund; percentage of scenarios with a CR below 100% after X years, in the case that the initial PCR is 100% and no curtailments are applied. The weight in equity is 50%.

	No curtailments						
	Initial CR						
X	70	80	90	100	110	120	130
20	37.8	29.1	21.1	13.9	9.4	6.6	4.3
30	24.8	18.3	13.1	8.8	5.8	3.9	2.4
40	18.0	12.4	8.4	5.7	3.5	2.3	1.6
50	14.3	9.8	7.3	4.9	3.1	2.2	1.4
60	13.8	9.3	6.7	4.8	2.8	2.1	1.4
70	13.6	9.1	6.7	4.7	2.8	2.0	1.4
80	13.6	9.1	6.7	4.6	2.8	2.0	1.4

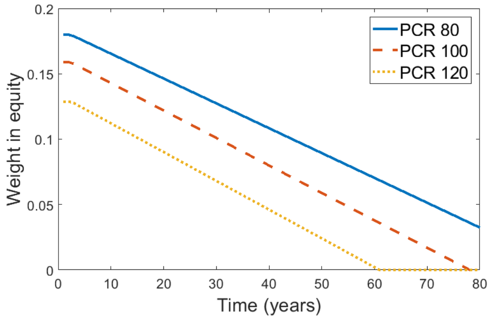
Table 16: Grey fund; percentage of scenarios with a CR below 100% after X years, in the case that the initial PCR is 100% and no curtailments are applied. The weight in equity is 50%.



(a) CR 80

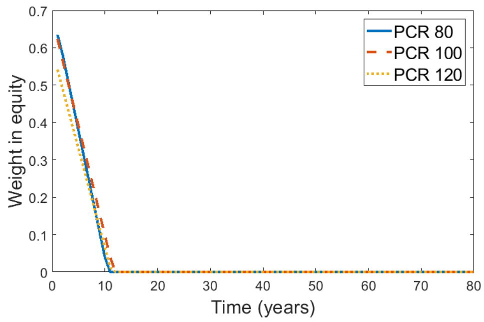


(b) CR 100

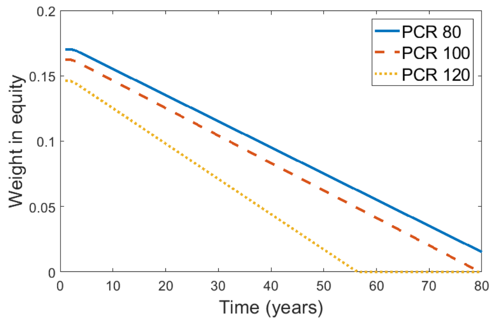


(c) CR 120

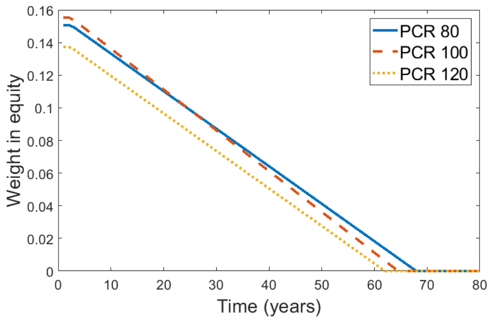
Figure 9: Optimal glide paths for the Green fund



(a) CR 80



(b) CR 100



(c) CR 120

Figure 10: Optimal glide paths for the Grey fund