

# **Momentum and low volatility factor returns over the period 1800 – 2017**

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## **Abstract:**

In this study, I explore factor investing strategies based on momentum and low volatility in returns of publicly traded common stocks in three samples: the U.S. and the U.K. over the period 1800 – 1926 and the U.S. over the period 1926 – 2017. I find strong evidence for the time-invariant performance of momentum. In the historical samples, I find no evidence for a low volatility effect based on idiosyncratic volatility. Portfolios consisting of stocks with low systematic volatility generate higher risk-adjusted returns than their counterparts in all samples, although the factor yields low risk-adjusted returns. Starting from 1960 onwards does a low idiosyncratic volatility factor strategy start to generate positive returns, but in earlier periods there is no evidence for the existence of this phenomenon. Furthermore, I find that momentum and low-volatility have low correlation. Overall, my findings suggest that low-volatility is a relatively new factor that requires further empirical research on its emergence and relation to behavioral explanations.

# Table of Contents

<b>1</b>	<b>Introduction</b> .....	<b>5</b>
<b>2</b>	<b>Literature</b> .....	<b>7</b>
2.1	Momentum .....	7
2.2	Low volatility anomaly .....	8
<b>3</b>	<b>Data</b> .....	<b>10</b>
3.1	Filtering the datasets .....	10
3.2	Calculation of returns .....	11
3.3	Stock market returns .....	16
<b>4</b>	<b>Methodology</b> .....	<b>16</b>
4.1	Factor construction .....	17
4.1.1	Momentum .....	17
4.1.2	Low volatility .....	18
4.1.3	Double-sorted factors .....	19
4.1.4	Combo factor .....	19
4.1.5	Residual factors .....	19
4.2	Performance measurement .....	20
4.3	Factor loadings .....	21
4.3.1	Mutual factor loadings .....	21
4.3.2	Market states .....	21
4.3.3	Liquidity and economic effects .....	22
<b>5</b>	<b>Results</b> .....	<b>23</b>
5.1	Factor performance .....	23
5.1.1	Momentum .....	23
5.1.2	Low volatility .....	26
5.1.3	Double-sorted factors .....	30
5.1.4	Combo factor .....	32
5.1.5	Residual factors .....	33
5.2	Factor loadings .....	40
5.2.1	Mutual factor loadings .....	40
5.2.2	Market states .....	42
5.2.3	Liquidity and economic effects .....	45
<b>6</b>	<b>Conclusion</b> .....	<b>46</b>
<b>7</b>	<b>Bibliography</b> .....	<b>49</b>
<b>8</b>	<b>Appendix</b> .....	<b>52</b>

## Index of Tables

Table 1 Number of stocks in original and cleaned historical datasets.....	11
Table 2 Minimum number of stocks per month in historical datasets.....	13
Table 3 Minimum number of stocks per month in cleaned datasets .....	14
Table 4 Equity premia in the original historical datasets .....	15
Table 5 Equity premia in the cleaned and CRSP datasets.....	15
Table 6 Momentum summary statistics .....	23
Table 7 Momentum performance statistics .....	25
Table 8 Low volatility summary statistics .....	26
Table 9 Low volatility performance statistics .....	28
Table 10 Betting against Beta summary statistics.....	29
Table 11 Betting against Beta performance statistics.....	30
Table 12 Double-sorted factor summary statistics .....	31
Table 13 Double-sorted factor performance .....	32
Table 14 Combo factor summary statistics .....	33
Table 15 Combo factor performance statistics.....	33
Table 16 Residual momentum summary statistics .....	33
Table 17 Residual momentum performance statistics.....	35
Table 18 Residual low volatility summary statistics .....	36
Table 19 Residual low volatility performance statistics.....	38
Table 20 Residual double-sorted factor returns .....	39
Table 21 Residual double-sorted factor performance statistics.....	40
Table 22 Factor returns regression .....	41
Table 23 Factor correlations and market states .....	42
Table 24 Market states and factor performance .....	43
Table 25 Residual factor correlations and market states .....	44
Table 26 Residual factor performance and market states.....	44
Table 27 Regression of economic variables on factors .....	45
Table 28 Momentum portfolios summary statistics (Robust) .....	52
Table 29 Low volatility summary statistics (Robust).....	53
Table 30 Double-sort summary statistics (Robust) .....	54
Table 31 Market returns on all data samples.....	55
Table 32 U.S. Momentum 50-year period yearly returns and alphas .....	55
Table 33 U.K. Momentum 50-year period yearly returns and alphas .....	56
Table 34 CRSP Momentum 50-year period yearly returns and alphas .....	56
Table 35 U.S. Low volatility portfolios 50-year period yearly returns and alphas .....	57
Table 36 U.K. Low volatility portfolios 50-year period yearly returns and alphas.....	58
Table 37 CRSP Low volatility portfolios 50-year period yearly returns and alphas.....	59

## Index of Figures

Figure 1 Log-transformed cumulative market returns U.S.....	60
Figure 2 Log-transformed cumulative market returns U.K. ....	60
Figure 3 Log-transformed cumulative market returns CRSP .....	61
Figure 4 Log-transformed cumulative momentum portfolio returns U.S.....	61
Figure 5 Log-transformed cumulative momentum portfolio returns U.K.....	62
Figure 6 Log-transformed cumulative momentum portfolio returns CRSP .....	62
Figure 7 Log-transformed cumulative low volatility portfolio returns U.S. ....	63
Figure 8 Log-transformed cumulative low volatility portfolio returns U.K.....	63
Figure 9 Log-transformed cumulative low volatility portfolio returns CRSP.....	64

Figure 10 Log-transformed cumulative Betting against Beta returns U.S. .... 64  
Figure 11 Log-transformed cumulative Betting against Beta returns U.K. .... 65  
Figure 12 Log-transformed cumulative Betting against Beta returns CRSP ..... 65  
Figure 13 Log-transformed cumulative double-sorted portfolio returns U.S. .... 66  
Figure 14 Log-transformed cumulative double-sort double-sorted portfolio returns U.K. .... 66  
Figure 15 Log-transformed cumulative double-sorted portfolio returns CRSP ..... 67  
Figure 16 Mean-variance plot of factor returns U.S. .... 67  
Figure 17 Mean-variance plot of factor returns U.K. .... 68  
Figure 18 Mean-variance plot of factor returns CRSP ..... 68  
Figure 19 Cumulative inflation rates of the pound and dollar ..... 69

# 1 Introduction

Factor investors aim to reap returns from exposure to a systematic risk-related factor that has earned a long-term premium, which has been empirically demonstrated in academic literature. The first factor to be defined as such is the market factor in the Capital Asset Pricing Model (CAPM), which demonstrates that market exposure is a significant determinant of stock returns (Bender, Briand, Melas, & Subramanian, 2015). Other factors have been discovered since, most notably by Fama and French (1993), who documented factors such as value and size. Regarding the nature of the factor premium, Fama and French (2015) argue that factors are able to explain returns because they capture components that are directly related to firm value. These components include profitability, investments and horizon effects in the term investment or expected returns structure. However, momentum and low volatility are factors based on returns and are at most indirectly related to any of these components and therefore stand out from other factors.

Jegadeesh and Titman (1993) first demonstrated momentum. Jegadeesh and Titman (2001) replicated their original study to refute criticism on data mining by proving its out-of-sample existence. Since then, momentum has been demonstrated in different markets and time samples going back to the 19<sup>th</sup> century (Asness, Moskowitz, & Pedersen, 2013; Chabot, Ghysels, & Jagannathan, 2009; Geczy & Samonov, 2016). Low volatility strategies, on the other hand, are broadly defined in literature and range from strategies based on systematic risk exposure measured by beta to strategies on idiosyncratic volatility in stock returns. Frazzini and Pedersen (2014) define the Betting against Beta (BAB) factor, which shows that high beta stocks underperform low beta stocks, whereas Ang et al. (2006) and Blitz and Van Vliet (2007) show profitable strategies for stocks with low idiosyncratic volatility. Explanations for both factors often originate from the behavioral field. Momentum returns have been related to investor reactions to news events and investor overconfidence. Blitz and Van Vliet (2013) argue that high volatility stocks are overpriced due to investors over-attention and by leverage-constrained fund managers seeking to beat a benchmark, which induces them to buy risky stocks. However, for low volatility research on its performance over long-run samples is sparse, as is literature on its relation to momentum. These points are relevant, because time-invariant performance is relevant to investors who want to invest in it for future periods. Furthermore, insights on the relation between the factor strategies offer potential diversification benefits for investors.

In this study, I test the time-invariant performance of the forenamed factor strategies in long-run samples in U.S. and U.K. markets for publicly traded common stock. This study contributes to existing literature by analyzing factor returns in historical stock markets in the U.S. and the U.K. over the period 1800 – 1926 in comparison to U.S. stock returns from CRSP from 1926 to 2017. Previous studies on momentum focused on either historical performance in the U.S. market or in the U.K. market, while in this study results from both markets are compared. This yields insights on the long-run factor performance of momentum and low volatility and allows for comparison over different markets. Two different low volatility strategies are tested, namely the idiosyncratic volatility factor portfolio and the BAB factor. The former is used for further comparison with momentum. Factor performance is measured on CAPM alpha and Sharpe ratio relative to the markets. To explore the diversification benefits from momentum and low volatility, a combo factor portfolio holding 50/50 in both factors and double-sorted factor portfolios on momentum and low volatility are tested on performance. Momentum and low volatility strategies are also formed on residual returns obtained from 36-month rolling CAPM-regressions to determine whether factor performance can be improved by decreased market exposure. Furthermore, I examine factor performance, measured in returns and alphas, and correlations during different market states. Next to this, factor returns are regressed on the other factor plus the market factor to determine factor loadings. The sensitivity of factor performance to economic variables including liquidity shocks, GDP-growth, and market states is measured in an OLS-regression. Finally, an overview of the cumulative returns of momentum and low volatility is shown in figures 4 to 15 in the appendix and the mean-variance performance across all samples in figures 16 to 18.

This study shows that momentum performance yields positive risk-adjusted returns in all samples. Momentum performance decreases during market down states and displays low correlation with low volatility returns, although in a regression both factors load positively on each other. In the historical samples, there is no evidence for a low volatility phenomenon based on overall volatility. It is from 1960 onwards that the low idiosyncratic volatility factor becomes more profitable, but in earlier periods there is no evidence for the existence of this phenomenon. Portfolios consisting of stocks with low systematic volatility generate higher risk-adjusted returns than their counterparts in all samples, although the factor yields low risk-adjusted returns. Double-sorted and combo factor portfolios profit from increased returns relative to low volatility and lower

drawdown than momentum. No significant relation between either factor and liquidity and GDP-growth is found across all samples, although this could be caused by the limitations on liquidity measurement in the historical samples.

The outline of this paper is as follows: in section 2, I examine the literature on the factors and their mutual relation, section 3 and 4 discuss respectively the data and methodology. Section 5 presents the results and section 6 concludes and discusses recommendations for future research.

## 2 Literature

This section presents the literature on the momentum and low volatility factors regarding their backgrounds, formation, and explanations for their existence.

### 2.1 Momentum

Jegadeesh and Titman (1993) first demonstrated the momentum factor. Their momentum strategy is defined as a zero-investment portfolio, which shorts portfolios of losing stocks and buys winning stocks. Stocks are sorted on cumulative returns during formation periods, which vary from three to twelve months. They found that this portfolio type yields positive risk-adjusted returns, with short-term reversals over the period of one week to one month, and long-term reversals after three to five years. Since their publication, the momentum factor sparked a debate regarding the origins of momentum and its behavioral explanations, some of which already predates their publications.

Conrad and Kaul (1998) put forward the idea that cross-sectional variation in expected stock returns causes the profitability of momentum strategies since some stocks consistently attract higher returns than others. Jegadeesh and Titman (2001) reject this hypothesis, based on their observation of the reversal phenomenon in stock returns. However, evidence has been put forward that momentum portfolios composition can be related to industries (Moskowitz & Grinblatt, 1999) and that small firms are overrepresented in the extreme portfolios of momentum strategies (D. Blitz, Huij, & Martens, 2011; Fama & French, 2012). An important contribution to this debate is by Blitz et al. (2011), who demonstrated the dependence of momentum portfolios on factor loadings. They increased risk-adjusted returns for momentum strategies by selecting and shorting stocks based on their residual returns after controlling for the effects of beta, size and value. This effect also helped to explain the time-variation in the profitability of momentum strategies.

Several explanations have also been given for the reversal phenomenon, since the duration and persistence of momentum returns can be linked to capital constraints and liquidity, which prevents traders from using arbitrage (Asness, Moskowitz, & Pedersen, 2013; Chabot et al., 2009; Shleifer & Vishny, 1997; Korajczyk & Sadka, 2017). Momentum returns seem to be most strongly affected by shifting market states because momentum tends to overload on high-beta stocks during market up states and low-beta stocks during market down states (Chabot et al., 2009). This effect is strongest when markets start to rise after down states (Asness et al., 2013; Barroso & Santa-Clara, 2015; Cooper, Gutierrez, & Hameed, 2004). The duration of market states exacerbates this risk, because of increased beta exposure (Geczy & Samonov, 2016).

Explanations for the existence of the momentum anomaly often originate from the behavioral perspective. Before Jegadeesh and Titman (1993), Debondt and Thaler (1985) observed the persistence and reversal in the performance of winning and losing stocks, and attributed its cause to market overreaction to news events. The evolution of momentum returns is also attributed to investor conservatism about news, leading investors to update their beliefs too slowly and thereby creating the momentum and reversal pattern (Barberis, Shleifer, & Vishny, 1997). Hong & Stein (1999) on the other hand attribute this effect to momentum style traders extrapolating price changes by news watchers, thereby shifting prices to strongly. Stocks that have performed well generally attract attention and therefore are an attractive target for investors experiencing search costs in selecting stocks, which can further boost returns (Sirri & Tufano, 1998). Another explanation is related to overconfidence caused by the self-attribution bias. This bias leads investors to overinvest in stocks with positive past performance, which they attribute to their stock-picking skills. Prices are thereby pushed above their fundamental values, up to the point whereby they finally revert (Daniel, Hirshleifer, & Subrahmanyam, 1998). Cooper, Gutierrez and Hameed (2004) provide evidence for this hypothesis by demonstrating that up markets positively affect momentum profitability and cycles since this market state decreases risk aversion and increase overconfidence.

## 2.2 Low volatility anomaly

Classic portfolio theory states that investors should receive compensation only for systematic risk, because it is undiversifiable. Idiosyncratic risk, however, is diversifiable and should therefore not be priced. Yet in practice, the relation between risk and return is less straightforward than the CAPM predicts (D. C. Blitz & Van Vliet, 2007). Already in the 1970s, researchers found that



stocks' expected returns are not proportional to the beta of stocks, and this effect is aggravated when investors face borrowing restrictions (Black, 1972; Black, Jensen, & Scholes, 1972). Later research found that beta and stock returns were not positively related and that beta had little explanatory power for past returns (Fama & French, 1992). High beta stocks were also consistently underweighted in minimum-variance portfolio despite potential diversification benefits (Clarke, de Silva, & Thorley, 2011; Clarke, Silva, & Thorley, 2007). Based on these findings on the lack of compensation for systematic risk, Frazzini and Pedersen (2014) identified the Betting against Beta (BAB) factor. They proxied factor returns by forming a beta-neutral portfolio shorting high-beta stocks and buying low-beta stocks. They demonstrated the risk-adjusted profitability of the BAB factor in over four asset classes and eighteen international equity markets.

Traditional conceptions on portfolio theory are put further under pressure by research from Blitz and Van Vliet (2007). They demonstrate that after sorting stocks in deciles on idiosyncratic volatility, high volatility portfolios tend to have significantly lower risk-adjusted returns than low volatility portfolios. Blitz and Van Vliet (2013) documented a similar effect in emerging markets. Furthermore, Ang et al. (2006) found that stocks with high idiosyncratic volatility significantly underperformed their counterparts after controlling for factor loadings from the Fama and French three-factor model. These findings held over different markets, economic circumstances and holding periods (Ang, Hodrick, Xing, & Zhang, 2009; Cao & Han, 2016). However, some argue that stocks with high idiosyncratic volatility are correlated to cross-sectional skewness (Malagon, Moreno, & Rodríguez, 2015) or exposed to a latent risk factor unaccounted for in the three-factor model (Malagon et al., 2015). Malagon et al. (2015) found that the idiosyncratic volatility effect disappears after controlling for the factors profitability and investment from the Fama and French five-factor model.

As with momentum, explanations for the existence of a low volatility anomaly are often related to the behavioral field and in particular agency issues. For example, volatile stocks tend to receive more news attention, which can inflate stock prices and decrease future returns (Barber & Odean, 2008). As noted before, idiosyncratic volatility tends to be related to skewness (Malagon et al., 2015). Asset managers that face mandates to outperform their benchmark and are subject to option-like incentive structures under which they receive bonuses for outperformance might be incentivized to overweight high-risk stocks, thereby creating agency issues (Blitz et al., 2014). The

same goes for investment funds since exceptionally positive performing funds receive disproportionately more cash inflows than badly performing funds tend to lose (Sirri & Tufano, 1998), which also creates option-like incentives for the fund-owners (Blitz et al., 2014). Performance measures compared to a benchmark may also favor high beta stocks with low alpha compared to stocks with low beta and high alpha (Baker, Bradley, & Wurgler, 2011). Even when managers perceive that low-risk stocks yield superior risk-adjusted returns, they are often prevented from exploiting this opportunity because of leverage constraints (Baker & Haugen, 2012; Frazzini & Pedersen, 2014).

### 3 Data

Three separate datasets are tested in this study. The first two are datasets on historical stock returns for the U.K. and the U.S. markets over the period of 1800 – 1926. Data on these historical markets is collected from two sources: the first is the International Centre for Finance (ICF) from the Yale School of Management, which includes data for the London Stock Exchange based on the Investor Monthly Manual (IMM) for the years 1869-1929, and data for the New York Stock Exchange (NYSE) 1815-1925. The second source is Global Financial Data (GFD), which contains information on stock prices, splits, and dividends for common stock over the period 1800 – 1926 for the U.S. and the U.K. The third dataset is retrieved from CRSP and covers the period 1926 – 2017. All stocks returns are monthly. In this section, I discuss the filtering of the datasets, the construction of stock returns and show the summary statistics on market returns and equity premia for each dataset.

#### 3.1 Filtering the datasets

The final samples use only publicly traded common stocks. However, as Chabot et al. (2009) point out, the word ‘stock’ has an ambiguous meaning in the 19<sup>th</sup> century, since it denominates bonds, while the term common stock refers to equity. Security descriptions were manually checked in all historical datasets. Securities are excluded if they include bond or preferred equity characteristics such as ‘scripture’, ‘debenture’, ‘preferred’, ‘par’, ‘preference’, ‘debenture’, ‘deferred’, ‘guaranteed’ and ‘convertible’ or ‘stock’, if it is not accompanied by the adjective ‘ordinary’, ‘common’ or ‘limited’. Over-the-counter stocks are excluded as well. Next to this, all types of specifically specified share classes ranging from ‘A’ to ‘D’ are excluded. The reason ‘B’ shares are also excluded, is that there are occurrences where these shares pay guaranteed dividends

and therefore it cannot be guaranteed that this type of stock refers to common equity. The result of the data filtering on stock characteristics can be seen in table 1:

**Table 1 Number of stocks in original and cleaned historical datasets**

This table lists the number of stocks in the original datasets from ICF and GFD for the U.S. and the U.K. and the number of stocks left in the cleaned files after dropping duplicates, non-common stocks, and OTC stocks. The number of dropped stocks are shown in the column on the right.

<b>Dataset</b>	<b>Original file</b>	<b>Cleaned file</b>	<b>Dropped</b>
ICF U.S.	671	549	122
ICF U.K.	11983	3709	8274
GFD U.S.	10103	6274	3829
GFD U.K.	4893	3960	933

The cleaned datasets from GFD and ICF for the U.S. and the U.K. are merged using automatic and manual checks. First names are matched based on the likelihood of a potential match. These potential matches were then manually verified. The remaining non-matched stocks from both datasets are then compared manually. In some cases, matches with nearly identical names are rejected, because these securities can be paired with multiple securities in the other dataset. An example of this is the stock ‘Merchant’s Bank’ in the U.S. ICF dataset and variations of ‘Merchant’s Bank’ stock related to specific branches in different cities in the GFD dataset. Matches are finally verified by preliminary merging stocks after which equity returns are recalculated and checked for sudden spikes or crashing at the date of matching. This is done to double check examples where security descriptions are very similar such as ‘Coca Cola’ and ‘Coca Cola Ltd.’, but still differ in price and price movements and likely are different securities. In total, 126 stocks are matched in the historical U.S. samples and 1,951 stocks in the historical U.K. samples. After merging the total historical U.S. sample contained a total number of 6,709 stocks and the total U.K. sample 5,277 different stocks. Stocks are included over their full lifespan and are still used if they were delisted during the timespan of the sample in order to prevent a survivorship bias.

### 3.2 Calculation of returns

In the CRSP dataset, monthly returns are downloaded directly. GFD also provides monthly returns, but these often include errors with returns exceeding 10,000% or returns fluctuating strongly, because of shifting commas in price notation. Returns are therefore recalculated in all historical datasets based on monthly capital gains from the difference in closing prices, stock splits and dividends. If closing prices are missing they were replaced by next month’s opening price if possible. To counter the effect of fluctuating commas, observations are dropped if a stock has

returns exceeding 500% in the current period and returns lower than 80% in the previous period or the other way around. In the ICF datasets, however, dividends paid occasionally exceed last month's closing price without the stock price falling in the next month, which casts suspicion on the validity of the data. Returns higher than 850% percent are therefore excluded as well. Periods in which a rights distribution occurred for a stock are excluded because in several cases the investor received bonds for his investment, which makes the calculation of returns infeasible. Furthermore, there are many observations in the historical samples where returns equal zero percent, either in one month or several periods in a row. Returns of zero are an indication that the stocks were not traded, and therefore these observations are excluded from the analysis. In the historical U.S. dataset, this led to 117,691 observations being dropped out of 526,642 observations after filtering on common stock and in the U.K. 340,550 out of 924,220 observations are dropped. The impact of this filter shows that a large number of stocks were probably not traded in multiple periods, and this also indicates that liquidity was low in the historical periods.

Following Chabot et al. (2009) capital calls are treated as negative dividends. Information on capital calls is only indirectly available in the ICF dataset for U.K. stocks. Back then, firms could issue shares without requiring shareholders to pay the full or 'nominal' amount of the share, and would instead call upon shareholders later to make up for the difference between the 'paid' or 'par' value and the nominal value of a share. In this study, capital calls are calculated as a decrease in the difference between the paid and nominal value of stock. Chabot et al. (2009) mention the occurrence of negative share prices due to expected capital calls, but this does not occur in the ICF data. Capital calls also affected the value of dividends paid per share, because dividends were calculated as a percentage of paid/par share value instead of nominal value, so this method is applied to the estimation of dividends paid as well. However, suspicious movements occur in the nominal and paid value per share, such as simultaneous movements in the par and paid value, or decreases in the paid value per share without prior share price movements. In these cases, capital calls could not be calculated.

In order to address the problems regarding the validity of the historical data, limitations were imposed on the observations. Stocks with returns exceeding 850% are excluded from the analysis because it is likely this is due to some error in price notation or dividend calculations and returns below -100% in cases where capital calls occurred were also excluded from the analysis.

The reason for this is that Chabot et al. (2009) report negative prices originating from capital calls, but the returns in some cases are too extreme, even below -200%, to be assumed valid. Next to this, in order to be included in factor portfolios, stocks had to have full observations over the past 36 months. If these stocks prices increased more than 100% in one month or lost more than -70% over two consecutive months in this period they are excluded as well. These three constraints are relaxed as a robustness check for the momentum and volatility sorted factors, and the results are reported in tables 28 and 29 in the appendix. To avoid microstructure problems in factor portfolios, stocks in all datasets minimally have to be worth an inflation-adjusted dollar or pound from the year 1800. Data for pound inflation for the years 1949-2017 onwards is available on the Office for National Statistics and from 1800 to 1948 from the Bureau of National Statistics<sup>1</sup>. Data for dollar inflation is obtained from Alioth Finance (2019)<sup>2</sup>. Compounded inflation can be seen for both currencies in graph 16 in the appendix. Inflation was stable over the 19<sup>th</sup> century for both currencies, but increased in the 20<sup>th</sup> century with one dollar in 1800 being worth about twenty dollars in the 2000s. Inflation for the pound was even more severe, but only starting in the middle of the 20<sup>th</sup> century, so this did not affect the historical U.K. dataset. However, the minimum value of a stock ranging from five to twenty dollars over the course of the total CRSP is sufficiently reasonable to use as a price requirement. The effect of the stock characteristics and return filters on the minimum number of stocks available for factor portfolios over ten-year periods in the historical datasets is shown in tables 2 and 3, where the former describes the minimum number of stocks before filtering and the latter after filtering.

**Table 2 Minimum number of stocks per month in historical datasets**

This table displays the monthly minimum amount of common stocks in the cross-section per ten year period in the GFD and ICF datasets for the U.S. and the U.K. before these datasets are filtered on stock characteristics, returns, and rights distributions.

Period	GFD U.S.	GFD U.K.	ICF U.S.	ICF U.K.
1800	6	3		
1810	31	27	18	
1820	60	36	30	
1830	137	21	38	
1840	159	38	32	
1850	105	149	35	
1860	48	206	47	689
1870	99	875	5	705
1880	290	990	3	935

<sup>1</sup> <https://www.ons.gov.uk/ons/rel/elmr/economic-trends--discontinued-/no--604--march-2004/economic-trends.pdf>

<sup>2</sup> <https://www.officialdata.org/>

1890	700	738	3	838
1900	2233	1254	2	2437
1910	195	22	1	1613
1920	2286	1229	1	2734

**Table 3 Minimum number of stocks per month in cleaned datasets**

This table displays the monthly minimum amount of stocks in the cross-section per ten year period in the GFD and ICF datasets for the U.S. and the U.K. and the total datasets per country. Stocks are filtered on stock characteristics and the requirements on previous stock returns and observations for a stock to be selected for a factor strategy in order to determine the minimum number of stocks available for factor strategies.

Period	Total U.S.	Total U.K.	GFD U.S.	GFD U.K.	ICF U.S.	ICF U.K.
1800	1	1	1	1		
1810	3	3	3	3	2	
1820	14	6	11	6	14	
1830	36	4	24	4	30	
1840	41	9	27	9	11	
1850	61	36	26	36	12	
1860	24	56	20	55	19	
1870	30	219	30	221	1	1
1880	79	366	72	320	3	98
1890	201	329	188	309	3	62
1900	263	445	263	344	2	232
1910	74	10	61	11	1	198
1920	600	239	599	206	1	99

What stands out in table 3 is the sharp drop in shares in the period of 1910 – 1920 in the minimum number of shares in both total historical datasets for the U.S. and the U.K. This drop coincides with the occurrence of the First World War, which not only reduced the number of stocks, but also led to observations being missing more frequently, and because the factor strategies require full 36-month observations, this significantly reduces the number of stocks available for factor portfolios.

Returns for all samples are compared using equity premia. These were calculated by taking the difference between a stock's return and monthly yield from a bond series that substitutes the risk-free rate in the historical sample. In the U.S. sample, the Long-Term Bond Yield is used as a proxy for the risk-free rate, which is an index composed of government bonds obtained from GFD. In the U.K. sample, the British Consol is used, which was a perpetuity issued by the Bank of England, and is also obtained from GFD. The maturity of these bonds is longer than desirable because it exceeds ten years. However, there were no short-term government-related bond indices available for the 19<sup>th</sup> century in both countries. For the CRSP data sample, the market risk premium

was directly obtained from CRSP. Equity premia in the original historical datasets and the final historical samples and CRSP are shown in respectively tables 4 and 5:

**Table 4 Equity premia in the original historical datasets**

Displayed are the average annual equity premia calculated as the annualized monthly stock returns minus monthly long-term government bonds yields for the original datasets before filtering on stock and return characteristics. In the case of the U.S. Long-Term Bond Yield was used as a proxy for the risk-free rate and for the U.K. the British Consol, which were perpetuities issued by the Bank of England.

Period	GFD U.S.	GFD U.K.	ICF U.S.	ICF U.K.
1800	4.7	12.2		
1810	-1.6	6.8	-7.7	
1820	1	14.6	13.1	
1830	2.1	24.1	1.3	
1840	10.1	8.1	9.2	
1850	11.6	20.2	-3.2	
1860	12.1	18.6	11.1	15.3
1870	16	25.2	-6.4	11
1880	15.6	18.3	3	5.2
1890	10	19.2	6.1	5.1
1900	8.4	15.9	-1.2	0.7
1910	8.5	17.8	22.3	1.6
1920	12.7	10.9	3.8	-0.6

**Table 5 Equity premia in the cleaned and CRSP datasets**

Displayed are the average annualized equity premia calculated as annualized monthly stock return minus monthly long-term government bonds yields per ten year period for the historical datasets filtered on stock characteristics and returns. Equity premia for CRSP are directly obtained from CRSP by using the market risk premium. In the case of the U.S. Long-Term Bond Yield was used as a proxy for the risk-free rate and for the U.K. the British Consol, which were perpetuities issued by the Bank of England.

Period	Total US	Total UK	GFD US	GFD UK	ICF US	ICF UK	Period	CRSP
1800	6.34	9.45	6.34	9.45			1926	14.73
1810	8.26	8.78	2.06	8.78	27.19		1930	4.87
1820	9.3	17.93	8.34	17.93	10.61		1940	10.18
1830	3.61	18.69	5.51	18.67	0.49		1950	17.58
1840	14.19	11.75	16.37	11.75	10.79		1960	5.25
1850	15.7	18.27	23.31	18.36	-2.38		1970	1.67
1860	15.87	18.75	18.09	18.83	11.94		1980	7.98
1870	26.02	18.78	27.34	26.71	-11.44	8.27	1990	13.29
1880	17	13.55	17.32	18.97	9.06	4.43	2000	-0.81
1890	17.65	11.6	17.98	18.01	4.27	7.47	2010	14.45
1900	20.08	8.86	20.86	15.26	-3.07	6.2		
1910	16.14	8.88	15.8	15.27	23.97	10.25		
1920	15.12	6.2	15.55	11.43	2.3	0.29		

There are significant swings in the equity premia in the ICF U.S. dataset while the equity premia in the GFD sample for the U.S. remain stable, but this is most likely due to the low number

of stocks in the cleaned ICF sample. Equity premia for the merged datasets are similar for the U.S. and the U.K., with slightly larger returns in the 19<sup>th</sup> century for the U.K., but this effect reverses in the 20<sup>th</sup> century. However, compared to the CRSP dataset equity premia are much higher during the historical periods. This could be due to a number of factors such as lower liquidity in the historical samples, fewer stocks per period or differences in risk aversion.

### 3.3 Stock market returns

In the case of the CRSP sample, value-weighted market returns are obtained from CRSP. GFD also offers historical stock market returns, but these are not used because not all stocks in this dataset match the criteria for common stock and it does not include all stocks from ICF. It was not possible in the historical samples to use value-weighted stock returns, because data on shares outstanding is mostly missing. Historical stock market returns are therefore calculated using a price-weighted index (Goetzmann, Ibbotson, & Peng, 2001):

$$r_t^m = \frac{\sum_{i=1}^N P_t^i}{\sum_{j=1}^N P_{t-1}^j} \quad (1)$$

Average stock market returns can be seen in table 31 in the appendix and log-transformed cumulative market returns for the historical and CRSP samples in graphs 1 to 3 in the appendix. Cumulative returns in graph 2 for the historical U.K. market display little volatility, but this effect has been documented by others as well (Acheson, Hickson, Turner, & Ye, 2009). Price-weighted market returns are less volatile and higher than those reported by Goetzmann et al. (2001), especially for the early 19<sup>th</sup> century, but they only use 600 stocks over the period 1815 to 1925, while this study includes 6,709 stocks over the full sample.

## 4 Methodology

This section discusses the construction of the different variations on the momentum and low volatility factors. Furthermore, I elaborate on performance measures for factor portfolios together with the corresponding hypotheses. Lastly, the measurements of factor loadings, sensitivities to market states and economic variables are discussed together with their respective hypotheses.



## 4.1 Factor construction

### 4.1.1 Momentum

Jegadeesh and Titman (1993) use several criteria to construct momentum strategies. Momentum strategies are built by ranking stocks on past returns over a certain formation period ranging from ‘J’ months ago to either the most recent month or skipping ‘S’ of the last months. The desirability of skipping previous months is outlined by Asness et al. (2013), who argue that it avoids illiquidity and/or microstructure effects, and it also mitigates the effect of short-term reversals. Therefore all momentum strategies exclude last month’s return from the formation period. Stocks are sorted into equally-weighted tercile portfolios on cumulative return during the formation period. The usage of terciles is preferable since the minimum amount of monthly observations in the historical samples is sparse throughout the 19<sup>th</sup> century. The momentum portfolio is generated using a zero-cost investment portfolio that buys the winning portfolio and shorts the losing. Various momentum portfolios are generated based on formation periods ‘K’. In this study, four variations of the momentum strategy are tested, with formation periods ‘J’ being either six or twelve and holding periods ‘K’ one or three. These momentum variations are compared on annualized returns.

The fifty-year period average annualized returns and alphas are shown for momentum in the appendix in tables 32 to 34. Cumulative momentum returns are shown in graphs 4 to 6 in the appendix. The robust summary returns for momentum are shown in table 30 in the appendix, and since this version of momentum does not require full 36-month observations, the following formula is used to calculate cumulative returns during the formation period if observations are missing:

$$r_{form. period} = \frac{(\prod_{T-J}^{T-S} split_t) * close_{T-S} + \sum_{T-J}^{T-S} div_t * \prod_{T-J}^t split_{T-J} - close_{T-J}}{close_{T-J}} \quad (2)$$

Here the formation period starts ‘J’ months before the current period ‘T’, and ends ‘S’ months before the current period. Stocks are bought at closing prices in the previous month. Capital gains over the formation period consist of the closing price of the stock at time ‘T’ – ‘S’, multiplied by the sum product of all stock splits during the formation period plus the sum of all dividends received during the formation period, which is calculated as the nominal amount of dividend received multiplied by the sum product of all stock splits from the start of the formation period up to time ‘t’ at dividend payout. This method expands the number of stocks available for selection, especially in the first half of the 19<sup>th</sup> century when observations are frequently missing.

## 4.1.2 Low volatility

### 4.1.2.1 Volatility sorted portfolios

The standard measure for low volatility in this study is by Blitz and Van Vliet (2007). They rank stocks on total volatility in weekly returns over the past three years and sort these in equally weighted decile portfolios. In this study, monthly volatility is used due to the limitations of the historical dataset, and stocks are sorted in tercile portfolios instead of deciles. Three formation periods are tested: 12, 24 and 36 months. Portfolios are rebalanced every month as in Blitz and Van Vliet (2007). The low volatility effect is measured by first leveraging the volatility of the portfolios to that of the market. The difference in return between the levered portfolios is used as the factor return. Portfolios are not allowed to be shorted or bought more than one time the value of the portfolio, so if the ratio for the volatility of the portfolio and volatility of the market was larger than two or smaller than half, the leverage is capped. The following equation, similar to equation 4 by Frazzini and Pedersen (2014), is used taking into account the previous constraints:

$$r_{t+1}^{factor} = \frac{Vol_t^{low}}{Vol_t^{market}} (r_{t+1}^l - r_t^f) - \frac{Vol_t^{high}}{Vol_t^{market}} (r_{t+1}^h - r_t^f) \quad (3)$$

Stocks are required to have full observations during the full formation period in order to be included in a portfolio. A robust version of the low volatility effect is presented in table 30 of the appendix, where stocks are still required to have had full returns over the formation period, but not over the past 36 months, and return requirements are relaxed as well. Annualized average fifty-year period returns for the low volatility factor are shown in tables 35 to 37 in the appendix and cumulative returns over the total samples in graphs 7 to 9.

### 4.1.2.2 Betting against Beta

The systematic variation of the low volatility effect is based on the BAB factor by Frazzini and Pedersen (2014). They calculate stock betas using five-year market correlations as well as rolling one-year volatilities because the former is less stable. Stocks are sorted on weighted betas in two portfolios. This study makes three alterations on their methodology. I use monthly instead of daily returns, three-year market correlations instead of five, consistent with the method that will be applied for the residual strategies, and stocks in portfolios are equal-weighted instead of beta-weighted. Frazzini and Pedersen (2014) calculate the BAB return as the difference in return between each portfolio with their beta levered to one, which yields the following formula:

$$r_{t+1}^{BaB} = \frac{1}{B_t^l} (r_{t+1}^l - r_t^f) - \frac{1}{B_t^h} (r_{t+1}^h - r_t^f) \quad (4)$$

Here too leverage for shorting or buying is subject to the same restrictions as in equation 3. In the case of the BAB factor, there are no robustness results in the appendix because the calculation of the beta already requires full 36-month observations over the formation period.

#### 4.1.3 Double-sorted factors

Double-sort portfolios are constructed based on the momentum  $J=12$  and  $K=1$  variation and the volatility sorted factor with a formation period of 36 months. Stocks are first sorted into terciles on cumulative return after which these terciles are sorted in quantiles on volatility. This process is repeated by sorting stocks on low volatility and then on cumulative returns. In total six portfolios are created, but only the outer portfolios, those with the highest cumulative returns and lowest stock volatility and those with the lowest cumulative returns and highest stock volatility, and factor portfolios are presented. The double-sort factor return is calculated using the methodology from equation 3, using the same constraints on leverage.

#### 4.1.4 Combo factor

In addition to the double-sorted portfolios, a combo factor is created following Asness et al. (2013). They argue that this measure yields valuable information on the correlation structure between two factors, which should be displayed in the combo factor's Sharpe ratio. The combo factor is constructed as an equal-weighted portfolio of the volatility sorted factor with a 36-month formation period and the momentum portfolio with the 12-month formation period and one month holding period.

#### 4.1.5 Residual factors

In order to control for market factor loadings, momentum and low volatility portfolios are also formed based on residual returns. This method has been applied by Blitz et al. (2011) for momentum and similarly by Ang et al. (2007) for low volatility. Residual returns are constructed following Blitz et al. (2011). They regress stock returns on the market, value and size factor to obtain the residual returns of a given stock  $i$  at each time  $t$  in rolling regressions over 36-month periods, based on weekly observations. The authors use these residuals in a momentum strategy by sorting stocks in deciles on residual returns from the past twelve to last months. However, since data on market capitalization and book value of public companies is lacking for the historical periods, only the market factor is included. It is therefore possible that the resulting factors have latent exposure to these omitted factors. Stock betas are calculated per period as follows, where  $s$  denotes the stock and  $m$  denotes the market:

$$\widehat{B}_t = \frac{\rho_{s,m(t-36 \text{ to } t-1)} * \sigma_{s(t-36 \text{ to } t-1)}}{\sigma_{m(t-36 \text{ to } t-1)}} \quad (5)$$

Alphas are calculated following Chabot et al (2009). RMRF stands for excess market returns and  $r^f$  for the bond yield proxying the risk free rate:

$$\alpha_t = \left[ \sum_{k=t-36}^{t-1} (r_k - r^f - \widehat{B}_t (RMRF_k)) \right] / 36 \quad (6)$$

The residuals  $\varepsilon_{i,t}$  for each stock  $i$  at time  $t$  are then obtained using the results from above and the stock's individual return  $r_{i,t}$ :

$$r_{i,t} = \alpha_{t,i} + r^f + \widehat{B}_{t,i} RMRF_t + \varepsilon_{i,t} \quad (7)$$

Variations on the momentum factor are constructed based on the cumulative residuals over the past formation periods and the low volatility portfolios on the volatility in these residuals. Traditional momentum is sensitive to crash risk arising from market downturns or shifts in market states, and because the residual approach filters the effect of the stock's beta it should make momentum less sensitive to systematic risk and make it less volatile. This effect should be represented in a higher Sharpe ratio for the residual strategy.

#### 4.2 Performance measurement

Performance is measured for all sorted and factor portfolios using several metrics. First, summary statistics display information on the annualized mean return, standard deviation, minimum and maximum return and the number of observations in years. Furthermore, annualized performance statistics are presented, which include the portfolio's alpha, the t-statistic of the alpha, beta, Sharpe ratio and the z-statistic applied by Blitz and Van Vliet (2007) to measure the difference with the Sharpe ratio of the market portfolio:

$$Z = \frac{SR_1 - SR_2}{\sqrt{\frac{1}{T} [2(1 - \rho_{1,2}) + \frac{1}{2}(SR_1^2 + SR_2^2 - SR_1 SR_2 (1 - \rho_{1,2}^2))]} \quad (8)$$

The following hypotheses on factor performance are tested:

H1: Factor performance is constant over markets.

H2: Factors do not outperform the market based on Sharpe ratios.

H3: The construction of a combo factor increases the Sharpe ratio relative to the original factors.

H4: Residual variations increase a factor's Sharpe ratio.

### 4.3 Factor loadings

This study compares momentum and low volatility factor returns using the momentum  $J=12$  and  $K=1$  variation and the volatility sorted factor portfolio with a 36-month formation as benchmarks. The volatility factor variation is chosen because it is most in line with the method applied by Blitz and Van Vliet (2007) and Blitz et al. (2011) point out that it  $J=12$  and  $K=1$  is the most common approach in momentum studies.

#### 4.3.1 Mutual factor loadings

Factor loadings are first analyzed by regressing momentum and low volatility factor returns on the other factor, excess market returns and a constant. This regression is performed across all three samples. This regression provides information on the relation between factor returns and market risk premia, while it also indicates whether both factors are positively related. Because the low volatility factor capitalizes on less volatile stocks, which are likely to have low betas, and because momentum stocks have been found to have low betas (Chabot et al., 2009), I expect no significant load on excess market returns. Further on, momentum stocks buy winning stocks, which are likely to attract attention, while low volatility strategies capitalize on stocks with less attention and volatility. For this reason, I also expect no significant relation between both factors.

H6: Momentum and low volatility have no significant load on each other.

H7: Excess market returns do not significantly impact momentum and low volatility.

#### 4.3.2 Market states

As literature shows, momentum returns are influenced by market states, with returns increasing in up markets and decreasing during bear markets or after market swings. Chabot et al. (2009) define market states based on their trailing 36-month cumulative returns. However, they find that trailing 36-month market returns are seldom negative and therefore define a market Down state as a period with trailing returns in the bottom 15% of the sample and market up states as periods in the top 85%. Because trailing market returns are negative in fewer than 5% of the cases in the historical samples in this study, the method by Chabot et al. (2009) is used instead of looking at the sign of trailing market returns. Based on the findings on momentum and market states, and the possibility of down market states increasing risk aversion and thereby potentially making less volatile stocks more attractive, my hypothesis is that momentum and low volatility are negatively

correlated during market states. The results on factor correlations are complemented by statistics on factor performance during different market states. Annualized alphas and factor returns are calculated over different market states and are used to complement the results on correlations.

Correlations are also compared between the residual momentum and low volatility strategies. The residual approach in this study should filter out the effect of a stock's market beta and therefore decrease the sensitivity of factor returns to market circumstances. The following hypotheses are tested:

H8: Correlation between low volatility and momentum decreases during market down states.

H9: Momentum alpha decreases relative to low volatility during market down states.

H10: Correlations between residual momentum and the low volatility effect is not affected by market states.

#### 4.3.3 Liquidity and economic effects

Asness et al. (2013) find that liquidity loadings of value and momentum explain their low correlation because momentum loads positively on liquidity shocks while value stocks do not. This finding is supportive of the point that liquidity increases capital available for arbitrage and can decrease factor returns (Korajczyk & Sadka, 2017; Shleifer & Vishny, 1997). Low volatility can be especially vulnerable to capital abundance because lower risk allows for more use of leverage, which makes these stocks a suitable target for arbitrage. Capital availability and liquidity can be related to the state of the market, and therefore the latter is used as a control variable. GDP-growth is added as well since it is likely to influence financial markets and because low volatility performance is linked to economic cycles (Kochard & Sullivan, 2014).

Shocks to funding liquidity are measured based on Asness et al. (2013), who use market and funding liquidity. In this thesis market liquidity could not be measured, because no data on and off the run government bonds is available for the historical periods. Fortunately, Asness et al. (2013) do not find significant effects for market liquidity on momentum. Funding liquidity is measured as the spread between the corporate bond yield and the risk-free rate. In the case of the U.S. (the historical and CRSP sample) the Moody's Corporate AAA yield is used as the corporate yield and for the U.K. the private discount rate is used. Both yields are obtained from GFD, and in the former is based on an index. For the U.S. yield observations start in 1815 and go to 2017,

whereas the private discount rate covers the full U.K. sample. Shocks to liquidity are measured as the residuals from an AR(2)-model on the spread between the corporate and government bond yield. I then regress the factors on the other factor's return, liquidity shocks, a dummy variable indicating the market state, GDP-growth, which is obtained from the Maddison Project (Bolt, Inklaar, Jong, & Zanden, 2018), and a constant. The following hypotheses are tested:

H11: Momentum returns increase with shocks to liquidity.

H12: Low volatility returns increase with liquidity shocks.

## 5 Results

### 5.1 Factor performance

#### 5.1.1 Momentum

Table 6 shows the summary statistics on the momentum portfolios for all samples. The results on the returns of the momentum variations are consistent over all samples. Increases in the formation period increase the mean return of the momentum portfolio. Holding periods decrease the standard deviation of the momentum portfolios, and in the CRSP and U.S. samples, it also increases the mean return. Maximum drawdowns displayed in the minimum return are also lower for all momentum strategies using an increased formation period compared to its respective holding period.

**Table 6 Momentum summary statistics**

Annualized summary statistics are reported for returns of momentum portfolios and portfolios sorted in terciles on cumulative returns over formation periods 'J' of 6 and 12 months with varying holding periods 'J' of 1 to 3 months. For all strategies, last month's return is skipped in the formation period. The momentum factor is constructed as a zero-cost investments portfolio, which buys and shorts respectively the portfolios containing stocks with the highest and lowest cumulative returns over the formation period. All stocks had to have full observations over the past 36 months and were not allowed to have returns exceeding 100% in one month or cumulative returns lower than -70% over two consecutive months during this period. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017. Returns are reported in percentages.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	Win J6 K1	24.71	32.81	-22.73	153.88	99
	Win J6 K3	14.55	21.81	-26.53	82.11	98
	Win J12 K1	24.02	31.38	-16.38	158.33	92
	Win J12 K3	15.59	20.65	-19.82	101.7	91
	Middle J6 K1	6.95	12.15	-26.47	50.52	95
	Middle J6 K3	4.71	12.16	-24.3	41.88	93
	Middle J12 K1	7.48	14.89	-30.99	47.72	76
	Middle J12 K3	3.58	12.23	-21.95	34.26	75
	Lose J6 K1	6.36	25.52	-56.55	111.89	98
	Lose J6 K3	-2.82	21.57	-60.11	54.83	97

	Lose J12 K1	3.72	25.75	-55.49	71.97	92
	Lose J12 K3	-4.02	22.88	-69.44	58.16	88
	Momentum J6 K1	16.33	30.78	-55.27	148.17	98
	Momentum J6 K3	19.44	23.22	-24.18	129.71	97
	Momentum J12 K1	18.83	32.63	-34.06	191.55	92
	Momentum J12 K3	23.53	28.47	-23.84	204.14	88
U.K.	Win J6 K1	19.98	24.72	-25.69	181.27	85
	Win J6 K3	11.27	19.24	-30.25	106.51	84
	Win J12 K1	21.45	21.98	-32.92	150.88	81
	Win J12 K3	15.82	18.59	-33.65	112.29	80
	Middle J6 K1	3.44	13.94	-42.12	55.49	52
	Middle J6 K3	0.95	10.99	-40.41	27.44	52
	Middle J12 K1	1.26	9.8	-32.24	22.29	71
	Middle J12 K3	-0.49	8.95	-28.05	22.32	71
	Lose J6 K1	0	19.85	-44.05	107.78	85
	Lose J6 K3	-6.18	13.42	-50.57	30.25	84
	Lose J12 K1	-2.63	18.98	-34.54	127.27	81
	Lose J12 K3	-8.18	14.87	-53.12	29.55	80
	Momentum J6 K1	19.77	19.95	-50.4	121.54	85
	Momentum J6 K3	18.54	17.19	-44.63	91.08	84
	Momentum J12 K1	24.67	19.14	-57.18	117.22	81
	Momentum J12 K3	26.56	16.95	-19.54	86.31	80
CRSP	Win J6 K1	15.47	25.85	-42.07	73.67	90
	Win J6 K3	10	25.37	-48.47	83.49	89
	Win J12 K1	18.14	26.42	-45.89	107.96	89
	Win J12 K3	12.29	25.56	-48.8	112.27	88
	Middle J6 K1	13.27	23.72	-46.76	91.62	90
	Middle J6 K3	9.16	22.96	-56.69	98.45	89
	Middle J12 K1	12.87	23.68	-47.28	99.26	89
	Middle J12 K3	9.31	23.42	-56.48	111.65	88
	Lose J6 K1	10.5	32.22	-51.64	148.93	90
	Lose J6 K3	2.02	30.87	-61.46	149.05	89
	Lose J12 K1	8.27	30.08	-55.35	86.63	89
	Lose J12 K3	0.51	29.23	-65.35	103.35	88
	Momentum J6 K1	4.48	16.22	-53.35	51.75	90
	Momentum J6 K3	9.4	13.88	-35.7	66.38	89
	Momentum J12 K1	8.69	16.93	-59.18	46.49	89
	Momentum J12 K3	13.52	16.17	-37.34	65.93	88

The observations from table 6 are in accordance with the performance statistics in table 7. In all cases, both the alpha and Sharpe ratio of the momentum portfolio increase with holding period and formation period. Momentum portfolios outperform their counterpart winner portfolios with respect to formation and holding period on alpha in all samples. Momentum betas are negative as in Chabot et al. (2009), who also find higher betas for losers than winners. The consistency in



results across markets and time support the hypothesis on invariant performance. Of all momentum portfolios, only the J=12 and K=3 variation in the CRSP sample reports a significant positive z-statistic, which is the only observation rejecting the market performance hypothesis.

**Table 7 Momentum performance statistics**

Shown are the annualized performance statistics for all momentum portfolios and tercile portfolios sorted on cumulative returns over formation periods ‘J’ of 6 and 12 months with holding periods ‘K’ of 1 and 3 months. For all strategies last month’s return is skipped in the formation period. Alphas and Betas are obtained from a CAPM-regression. The t-statistic stands for the significance of the portfolio’s alpha, whereas the z-statistic measures the significance of the difference between the Sharpe ratio of the portfolio and the Sharpe ratio of the market. Sharpe ratios are calculated using the respective proxy for the risk-free rate in the different samples tested. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

Portfolio	Market	Alpha	(T-stat)	Beta	Sharpe	(Z-stat)
U.S.	Win J6 K1	7.78	3.6	1.6	0.9	-2,69
	Win J6 K3	5.99	5.29	0.63	0.93	-1,97
	Win J12 K1	7.09	3.5	1.46	0.91	-2,51
	Win J12 K3	6.36	6.41	0.59	1.11	-0,55
	Middle J6 K1	-3.33	-3.3	1.02	0.31	-7,85
	Middle J6 K3	-2.18	-3.16	0.52	0.16	-8,42
	Middle J12 K1	-3.65	-3.48	1.07	0.3	-7,77
	Middle J12 K3	-3.36	-4.93	0.55	0.02	-9,59
	Lose J6 K1	-9.21	-4.75	1.78	0.08	-9,62
	Lose J6 K3	-11.67	-10.13	0.77	-0.6	-14,66
	Lose J12 K1	-11.44	-5.4	1.81	-0.01	-10,02
	Lose J12 K3	-13.11	-10.05	0.78	-0.64	-14,47
	Momentum J6 K1	13.32	4.72	-0.17	0.48	-4,68
	Momentum J6 K3	14.83	11.21	-0.13	1.18	-0,16
	Momentum J12 K1	15.77	5.43	-0.35	0.52	-4,26
	Momentum J12 K3	17.16	12.4	-0.19	1.32	0,66
U.K.	Win J6 K1	7.23	3.96	1.56	0.83	-0,29
	Win J6 K3	5.22	5.36	0.57	0.84	0,13
	Win J12 K1	9.59	5.58	1.48	1	1,26
	Win J12 K3	8.72	10.13	0.48	1.33	4,1
	Middle J6 K1	-1.8	-1.02	0.8	0.14	-5,42
	Middle J6 K3	-2.54	-3.3	0.35	-0.1	-7,36
	Middle J12 K1	-3.63	-3.51	0.71	0	-7,3
	Middle J12 K3	-2.9	-4.3	0.33	-0.18	-8,19
	Lose J6 K1	-8.49	-4.86	1.33	-0.06	-8,32
	Lose J6 K3	-10.76	-10.85	0.57	-0.71	-13,61
	Lose J12 K1	-11.06	-6.87	1.24	-0.25	-9,99
	Lose J12 K3	-12.48	-13.6	0.55	-0.95	-15,93
	Momentum J6 K1	13.18	5.25	0.23	0.61	-1,66
	Momentum J6 K3	13.89	10.27	0.01	1.09	1,64
	Momentum J12 K1	18.85	7.65	0.25	0.91	0,27
	Momentum J12 K3	19.95	16.69	-0.07	1.8	5,88
CRSP	Win J6 K1	2.67	3.06	1.05	0.57	2,43

Win J6 K3	1.35	1.13	0.42	0.36	-0,8
Win J12 K1	5.33	6.24	1.05	0.71	5,57
Win J12 K3	3.87	3.23	0.42	0.56	1,09
Middle J6 K1	0.6	0.81	1.09	0.47	0,36
Middle J6 K3	0.92	0.79	0.42	0.33	-1,08
Middle J12 K1	0.58	0.79	1.08	0.46	0,3
Middle J12 K3	1.29	1.1	0.42	0.36	-0,8
Lose J6 K1	-4.3	-2.99	1.36	0.24	-3,8
Lose J6 K3	-8.01	-5.35	0.55	-0.23	-6,58
Lose J12 K1	-5.55	-3.69	1.35	0.18	-4,54
Lose J12 K3	-8.86	-5.9	0.54	-0.28	-7,08
Momentum J6 K1	3.76	2.39	-0.31	0.08	-1,98
Momentum J6 K3	6.52	7.3	-0.12	0.66	1,2
Momentum J12 K1	7.83	4.47	-0.29	0.33	-0,65
Momentum J12 K3	10.15	10.64	-0.12	1.05	3,36

## 5.1.2 Low volatility

### 5.1.2.1 Volatility sorted portfolios

Table 8 displays the summary statistics of the returns on the portfolios sorted on past volatility with formation periods of 12, 24 and 36 months. In contrast to momentum an increased formation period does not increase factor returns and performance over formation periods is inconsistent. The low volatility factor portfolios have maximum drawdowns similar to momentum, with maximum losses varying between minus thirty and minus forty percent. Annualized mean returns are negative for all low volatility factors, except for the formation period of 36 months in the CRSP sample, despite using leverage to increase the returns of low volatility stocks relative to high volatility stocks.

**Table 8 Low volatility summary statistics**

Annualized summary statistics are reported for returns of the low volatility factor portfolios and tercile portfolios sorted on idiosyncratic volatility in stock returns over varying formation periods of 12, 24 and 36 months. The returns of the low volatility factor are calculated by levering the returns of the low- and high-volatility portfolios using the ratio of the portfolio's volatility to that of the market, with maximum and minimum leverage restricted to 2 and 0.5. The difference between the levered low and high volatility portfolios their return minus the risk-free rate multiplied by the leverage is used as the factor return. All stocks had to have full observations over the past 36 months and were not allowed to have returns exceeding 100% in one month or cumulative returns lower than -70% over two consecutive months during this period. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017. Returns are reported in percentages.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	High Volatility 12 months	21.92	35.13	-53.32	114.01	104
	High Volatility 24 months	21.06	35	-55.34	119.61	104
	High Volatility 36 months	20.96	34.93	-54.4	116.67	104
	Low Volatility 12 months	9.37	12.9	-13.17	79.72	86
	Low Volatility 24 months	10.13	14.6	-7.81	97.02	85
	Low Volatility 36 months	8.33	10.15	-11.32	47.22	85

	Mean Volatility 12 months	7.64	13.64	-25.86	42.14	87
	Mean Volatility 24 months	8.32	13.94	-24.53	38.83	87
	Mean Volatility 36 months	9.77	15.76	-30.35	65.71	88
	Volatility Factor 12 months	-2.26	13.3	-43.15	26.46	71
	Volatility Factor 24 months	-1.03	14.04	-32.09	41.13	71
	Volatility Factor 36 months	-1.81	12.09	-29.47	28.49	71
U.K.	High Volatility 12 months	15.93	38.2	-51.5	278.67	114
	High Volatility 24 months	14.4	30.91	-46.79	169.55	114
	High Volatility 36 months	14.7	32.86	-49.5	220.62	114
	Low Volatility 12 months	5.71	15.44	-20.14	137.62	101
	Low Volatility 24 months	5.93	10.86	-24.66	66.82	100
	Low Volatility 36 months	5.51	10.44	-22.37	72.36	100
	Mean Volatility 12 months	5.7	15.46	-29.43	103.16	99
	Mean Volatility 24 months	7.09	17.26	-25.18	90.58	104
	Mean Volatility 36 months	7.23	16.57	-23.08	91.47	106
	Volatility Factor 12 months	-2.29	12.32	-38.76	42.97	83
	Volatility Factor 24 months	-1.69	11.86	-38.87	42.29	82
	Volatility Factor 36 months	-1.62	11.71	-38.94	43.74	82
CRSP	High Volatility 12 months	12.41	29.76	-51.91	98.38	90
	High Volatility 24 months	12.29	30.4	-51.61	97.93	90
	High Volatility 36 months	14.34	28.51	-53.72	90	87
	Low Volatility 12 months	7.2	17.47	-37.94	56.58	90
	Low Volatility 24 months	7.36	16.97	-41.89	55.17	90
	Low Volatility 36 months	13.16	17.48	-45.09	52.82	87
	Mean Volatility 12 months	14.39	23.48	-49.68	103.27	88
	Mean Volatility 24 months	14.25	23.26	-46.6	104.85	88
	Mean Volatility 36 months	15.39	23.97	-46.73	113.28	87
	Volatility Factor 12 months	-3.12	11.05	-28.43	17.94	88
	Volatility Factor 24 months	-2.55	10.94	-29.52	16.65	88
	Volatility Factor 36 months	4.86	10.43	-31.78	30.5	84

Table 9 shows the performance statistics for the volatility sorted portfolios. In the historical samples, the portfolios with the lowest volatility are all outperformed on alpha and Sharpe ratio by the respective high volatility portfolio. Alphas for the low volatility factor are negative in the historical samples and their Sharpe performance is lower than that of the market. However, in the CRSP sample, low volatility portfolios have higher alphas and Sharpe ratios than high volatility portfolios. However, it is still outperformed by the mean volatility portfolio, except for the 36-month formation period. These findings contradict the low volatility effect. The only factor portfolio with a positive alpha is the CRSP 36-month formation period variation, but this portfolio still has a lower Sharpe ratio than the market. Low volatility performance in the CRSP and historical samples is sufficiently different to reject the hypothesis on time-invariant performance,

and based on z-statistics the hypothesis that the low volatility factor outperforms the market cannot be rejected.

**Table 9 Low volatility performance statistics**

Shown are the annualized performance statistics for all low volatility factor portfolios and tercile portfolios sorted on idiosyncratic volatility in stock returns over varying formation periods of 12, 24 and 36 months. Alphas and Betas are obtained from a CAPM-regression. The t-statistic stands for the significance of the portfolio's alpha, whereas the z-statistic measures the significance of the difference between the Sharpe ratio of the portfolio and the Sharpe ratio of the market. Sharpe ratios are calculated using the respective proxy for the risk-free rate in the different samples tested. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

Portfolio	Market	Alpha	(T-stat)	Beta	Sharpe	(Z-stat)
U.S.	High Volatility 12 months	1.12	0.52	2.06	0.63	-5.14
	High Volatility 24 months	0.31	0.14	2.06	0.6	-5.47
	High Volatility 36 months	0.33	0.16	2.05	0.6	-5.44
	Low Volatility 12 months	2.4	1.66	0.63	0.45	-5.55
	Low Volatility 24 months	2.38	1.54	0.65	0.44	-5.64
	Low Volatility 36 months	0.56	0.43	0.66	0.37	-6.31
	Mean Volatility 12 months	-3.75	-4.11	1.19	0.35	-8.24
	Mean Volatility 24 months	-3.08	-3.43	1.2	0.44	-7.43
	Mean Volatility 36 months	-2.53	-2.68	1.21	0.48	-6.93
	Volatility Factor 12 months	-1.91	-1.17	-0.7	-0.54	-8.99
	Volatility Factor 24 months	-0.63	-0.36	-0.69	-0.42	-8.35
	Volatility Factor 36 months	-2.02	-1.43	-0.69	-0.63	-9.36
U.K.	High Volatility 12 months	2.02	0.95	1.68	0.5	-3.3
	High Volatility 24 months	1.7	0.84	1.62	0.48	-3.42
	High Volatility 36 months	1.87	0.89	1.62	0.48	-3.44
	Low Volatility 12 months	1.36	0.81	0.49	0.23	-4.67
	Low Volatility 24 months	2.05	1.59	0.41	0.32	-3.98
	Low Volatility 36 months	1.44	1.29	0.42	0.32	-4.09
	Mean Volatility 12 months	-2.03	-1.95	1.11	0.29	-5.72
	Mean Volatility 24 months	-0.97	-0.71	1.16	0.34	-4.75
	Mean Volatility 36 months	-0.9	-0.64	1.16	0.34	-4.71
	Volatility Factor 12 months	-3.08	-2.19	-0.41	-0.38	-7.16
	Volatility Factor 24 months	-3.31	-2.78	-0.46	-0.49	-7.63
	Volatility Factor 36 months	-3.09	-2.63	-0.47	-0.48	-7.54
CRSP	High Volatility 12 months	-2.44	-2.23	1.37	0.33	-2.71
	High Volatility 24 months	-2.65	-2.36	1.38	0.32	-2.86
	High Volatility 36 months	-1.27	-1.19	1.43	0.35	-2.46
	Low Volatility 12 months	-1.42	-1.87	0.63	0.28	-2.69
	Low Volatility 24 months	-1.26	-1.73	0.63	0.3	-2.51
	Low Volatility 36 months	2.44	4.36	0.79	0.57	3.04
	Mean Volatility 12 months	0.92	1.18	1.06	0.49	0.86
	Mean Volatility 24 months	0.9	1.13	1.05	0.49	0.79
	Mean Volatility 36 months	1.71	2.28	1.15	0.49	1
	Volatility Factor 12 months	-4.73	-5.23	-0.17	-0.68	-6.37
	Volatility Factor 24 months	-4.37	-4.88	-0.15	-0.63	-6.19

Volatility Factor 36 months | 1.2      1.31      -0.02      0.13      -1.87

### 5.1.2.2 Betting against Beta

Table 10 shows that the BAB portfolios generate positive returns in all samples. Still, compared to the low and high beta portfolios the BAB factor does not offer higher returns, and the benefits from lower volatility are limited in the CRSP and U.K. sample.

**Table 10 Betting against Beta summary statistics**

Annualized summary statistics are reported for returns of the BAB factor portfolio and the two portfolios composed of equally-weighted stocks with high and low betas. Betas are obtained in rolling 36-month regressions. The factor return is calculated by levering the beta of each portfolio to one, with maximum and minimum leverage restricted to 2 and 0.5. The difference between the levered low and high beta portfolios their return minus the risk free rate multiplied by the leverage is used as the factor return. All stocks had to have full observations over the past 36 months and were not allowed to have returns exceeding 100% in one month or cumulative returns lower than -70% over two consecutive months during this period. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017. Returns are reported in percentages.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	Low Beta	11.09	13.66	-15.07	75.03	113
	High Beta	16.31	30.66	-39.21	139.37	113
	BAB Factor	0.61	17.1	-47.17	67.88	107
U.K.	Low Beta	9.35	9.66	-10.77	42.25	90
	High Beta	11.94	13.67	-28.83	62.45	90
	BAB Factor	2.75	9.62	-21.23	25.86	79
CRSP	Low Beta	12.89	19.34	-45.32	55.07	87
	High Beta	13.47	25.58	-52.76	75.71	87
	BAB Factor	3.55	8.61	-22.72	33.49	84

Table 11 shows that the BAB factor does not generate a significant positive alpha or outperforms the market on Sharpe ratio in any of the samples. However, the low beta portfolios generate a higher alpha than the high beta portfolios in all samples, which does indicate that a BAB effect is present in the samples. The hypothesis that results are similar across all samples is not rejected. However, the BAB factor in this study does not manage to outperform the market on either alpha or Sharpe ratio.

**Table 11 Betting against Beta performance statistics**

Shown are the annualized performance statistics for all beta sorted portfolios and the BAB factor. Alphas and Betas are obtained from a CAPM-regression. The t-statistic stands for the significance of the portfolio's alpha, whereas the z-statistic measures the significance of the difference between the Sharpe ratio of the portfolio and the Sharpe ratio of the market. Sharpe ratios are calculated using the respective proxy for the risk free rate in the different samples tested. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

<b>Portfolio</b>	<b>Market</b>	<b>Alpha</b>	<b>(T-stat)</b>	<b>Beta</b>	<b>Sharpe</b>	<b>(Z-stat)</b>
US	Low Beta	1.9	2.22	0.72	0.62	-5.65
	High Beta	1.33	0.8	1.61	0.55	-6.76
	BAB Factor	-2.73	-1.72	-0.21	-0.25	-9.78
UK	Low Beta	1.41	2.45	0.58	0.77	-1.62
	High Beta	1.15	1.2	0.9	0.67	-2.68
	BAB Factor	-1.1	-1.09	0.05	-0.11	-6.55
CRSP	Low Beta	1.28	2.17	0.92	0.48	0.86
	High Beta	-1.29	-1.52	1.39	0.36	-2.83
	BAB Factor	0.39	0.5	-0.06	-0.02	-2.68

### 5.1.3 Double-sorted factors

Summary returns on the double-sort factors are reported in table 12. In the historical U.K. and CRSP samples, the portfolio that double-sorts first on momentum and then volatility generates higher mean returns, while in the U.S. sample the difference is negligible. Interesting is that the Mom Vol variation has the strongest load on momentum. Yet the double-sort factor returns are still not as high as momentum but do constitute a significant improvement towards the low volatility factor. Except for the U.S. sample, the double-sort factors have much smaller drawdown than momentum, with maximally -10.25% in the U.K. sample and -23.12% in CRSP, which is an improvement relative to momentum.

**Table 12 Double-sorted factor summary statistics**

Annualized summary statistics are reported for returns of the double-sorted factor portfolios and outer portfolios sorted in terciles, which are then double-sorted in quantiles. Stocks are either sorted on cumulative returns over the past 12 months skipping last month and then on volatility over the past 36 months. This process is repeated in reverse order. The returns of the double-sort factors is calculated by leveraging the returns of the outer portfolios consisting of the least volatile winning stocks and the most volatile losing stocks, or the least volatile stocks with the highest returns and the most volatile stocks with the lowest returns, based on the ratio of the portfolio's volatility to that of the market. Maximum and minimum leverage is restricted to 2 and 0.5. The difference between the levered outer portfolios their return minus the risk-free rate multiplied by the leverage is used as the factor return. All stocks had to have full observations over the past 36 months and were not allowed to have returns exceeding 100% in one month or cumulative returns lower than -70% over two consecutive months during this period. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017. Returns are reported in percentages.

<b>Market</b>	<b>Strategy</b>	<b>Mean</b>	<b>Std.Dev</b>	<b>Min</b>	<b>Max</b>	<b>N</b>
U.S.	Win Mom Vol	15.56	15.81	-9.51	76.08	92
	Lose Mom Vol	6.52	37.3	-68.03	128.11	87
	Factor Mom Vol	4.28	16.66	-59.51	47.03	72
	Win Vol Mom	7.48	7.19	-5.78	25.04	28
	Lose Vol Mom	6.7	36.02	-72.65	134.73	90
	Factor Vol Mom	5.08	18.99	-37.9	44.48	16
U.K.	Win Mom Vol	13.62	13.53	-29.99	45.99	81
	Lose Mom Vol	-2.6	26.09	-58.38	157.62	81
	Factor Mom Vol	12.21	13.24	-16.22	55.21	71
	Win Vol Mom	6.94	6.18	-8.29	27.97	42
	Lose Vol Mom	-2.16	24.39	-51.3	131.67	81
	Factor Vol Mom	6.45	8.18	-10.25	23.86	37
CRSP	Win Mom Vol	18.03	20.85	-41.23	85.27	87
	Lose Mom Vol	9.06	27.65	-53.93	73.37	87
	Factor Mom Vol	10.94	11.26	-23.12	45.47	84
	Win Vol Mom	15.41	17.73	-33.14	56.1	87
	Lose Vol Mom	10.3	28.68	-54.28	78	87
	Factor Vol Mom	7.13	10.76	-15.36	47.79	87

Table 13 exhibits double-sorted factor performance. All double-sorted factors have negative betas, similar to momentum. The double-sort factor portfolios deliver significant positive alpha in all samples, with strongest risk-adjusted performance measured in Sharpe ratio and alpha for the momentum volatility sorted factor portfolio. However, only in the CRSP sample does this variation have a positive z-statistic for its Sharpe ratio performance compared to the market. All double-sorted factor portfolios have positive alphas, which supports the hypothesis on performance consistency. Yet given the z-statistic, the hypothesis on market outperformance cannot be rejected.

**Table 13 Double-sorted factor performance**

Reported are the annualized performance statistics for all the outer double-sorted portfolios and the factor portfolios. Alphas and Betas are obtained from a CAPM-regression. The t-statistic stands for the significance of the portfolio's alpha, whereas the z-statistic measures the significance of the difference between the Sharpe ratio of the portfolio and the Sharpe ratio of the market. Sharpe ratios are calculated using the respective proxy for the risk-free rate in the different samples tested. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

<b>Portfolio</b>	<b>Market</b>	<b>Alpha</b>	<b>(T-stat)</b>	<b>Beta</b>	<b>Sharpe</b>	<b>(Z-stat)</b>
U.S.	Win Mom Vol	2.5	2.34	1.08	0.9	-2.59
	Lose Mom Vol	-11.35	-3.13	2.26	0.08	-8.6
	Factor Mom Vol	6.62	2.68	-0.76	0.03	-6.09
	Win Vol Mom	-0.46	-0.61	0.41	0.31	-5.83
	Lose Vol Mom	-11.92	-4.06	2.39	0.1	-8.95
	Factor Vol Mom	5.79	2.92	-1.04	-0.1	-6.18
U.K.	Win Mom Vol	5.99	4.84	0.74	0.81	-0.19
	Lose Mom Vol	-12.19	-3.83	1.93	-0.06	-7.62
	Factor Mom Vol	10.22	7.09	-0.32	0.69	-0.69
	Win Vol Mom	2.73	3.01	0.23	0.44	-2.43
	Lose Vol Mom	-11.1	-4.45	1.77	-0.06	-7.95
	Factor Vol Mom	6.82	3.95	-0.71	0.22	-2.85
CRSP	Win Mom Vol	6.3	8.75	0.85	0.79	7.25
	Lose Mom Vol	-5.75	-4.22	1.42	0.16	-5.45
	Factor Mom Vol	7.73	5.99	-0.01	0.69	1.48
	Win Vol Mom	4.88	7.94	0.75	0.74	6.46
	Lose Vol Mom	-4.93	-3.73	1.46	0.2	-4.97
	Factor Vol Mom	6.38	6.72	-0.3	0.41	-0.19

#### 5.1.4 Combo factor

Tables 14 and 15 report the summary and performance statistics for the combo factor. The combo factor shows positive mean returns and very low maximum drawdown in the U.K. sample of only -2%. Alphas for the combo factor are positive and significant, which improves upon the low volatility portfolio, with the highest alphas in the historical samples. Performance measured in alphas and Sharpe ratios is similar to the double-sorted factor. The results show consistency in performance in all samples, and confirm the hypothesis on performance consistency across markets. However, the z-statistics show that the factors Sharpe ratios are not higher than that of the market, which does not reject the hypothesis that the factor does not outperform the market. Based on this results, and that Sharpe ratios are similar to that of momentum, the hypothesis that the combo factor increases the Sharpe ratio is rejected. In conclusion, the combo factor profits from the low correlation between both factor strategies, although momentum has higher, though equally significant, alphas and similar Sharpe ratios.



**Table 14 Combo factor summary statistics**

Summary statistics are presented for annualized combo factor returns. The combo factor is created as a 50/50 holding portfolio of the momentum J=12, K=1 variation and the low volatility factor portfolio with the 36-month formation period. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017. Returns are reported in percentages.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	Combo Factor	8.02	15.85	-30.08	72.18	68
U.K.	Combo Factor	12.24	10.51	-2.09	72.86	75
CRSP	Combo Factor	6.83	9.49	-26.83	34.58	84

**Table 15 Combo factor performance statistics**

Reported are the annualized performance statistics for the combo factors consisting of a 50/50 holding portfolio in the momentum J=12 and K=1 variation and the low volatility factor with the 36-month formation period. Alphas and Betas are obtained from a CAPM-regression. The t-statistic stands for the significance of the portfolio's alpha, whereas the z-statistic measures the significance of the difference between the Sharpe ratio of the portfolio and the Sharpe ratio of the market. Sharpe ratios are calculated using the respective proxy for the risk-free rate in the different samples tested. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

Market	Portfolio	Alpha	(T-stat)	Beta	Sharpe	(Z-stat)
U.S.	Combo Factor	8.39	4.47	-0.69	0.2	-4,99
U.K.	Combo Factor	7.4	5.64	-0.04	0.61	-1,3
CRSP	Combo Factor	4.35	4.08	-0.15	0.31	-0,71

### 5.1.5 Residual factors

#### 5.1.5.1 Momentum

Summary returns and performance statistics on residual momentum are presented in tables 16 and 17. Formation and holding periods have a positive effect on the mean return, except for the U.K. sample where the effect of holding periods is negative. Holding periods still negatively affect volatility. Maximum drawdowns have not decreased in the historical U.S. sample, but have in the other two samples.

**Table 16 Residual momentum summary statistics**

Annualized summary statistics are reported for returns of residual momentum portfolios and portfolios sorted in terciles on cumulative residual returns over formation periods 'J' of 6 and 12 months with varying holding periods 'J' of 1 to 3 months. For all strategies last month's return is skipped in the formation period. The momentum factor is constructed as a zero-cost investments portfolio, which buys and shorts respectively the portfolios containing stocks with the highest and lowest residual cumulative returns over the formation period. Residual returns are obtained in 36-month rolling CAPM-regressions. All stocks had to have full observations over the past 36 months and were not allowed to have returns exceeding 100% in one month or cumulative returns lower than -70% over two consecutive months during this period. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017. Returns are reported in percentages.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	Win J6 K1	14.32	28.75	-36.27	132.68	53
	Win J6 K3	9.67	25.72	-36.06	116.19	52

	Win J12 K1	13.54	27.29	-40.28	123.85	53
	Win J12 K3	9.96	23.39	-42.67	90.93	52
	Middle J6 K1	9.8	17.82	-28.36	48.58	53
	Middle J6 K3	7.82	15.91	-26.56	44.36	52
	Middle J12 K1	12.33	19.99	-26.34	55.86	53
	Middle J12 K3	7.55	17.34	-24.74	48.02	52
	Lose J6 K1	13.09	29.74	-37.25	114.51	53
	Lose J6 K3	6.04	25.5	-33.8	80.96	52
	Lose J12 K1	11.76	29.56	-39.3	118.3	53
	Lose J12 K3	5.75	25.33	-35.71	73.57	52
	Momentum J6 K1	0.21	19	-38.96	51.57	53
	Momentum J6 K3	3.79	14.6	-29.16	43.74	52
	Momentum J12 K1	0.52	20.39	-61.77	50.51	53
	Momentum J12 K3	4.72	16.4	-40.24	45.55	52
U.K.	Win J6 K1	25.87	21.22	-10.27	92.49	43
	Win J6 K3	17.56	17.74	-27.29	66.47	42
	Win J12 K1	26.45	21.45	-22.82	94.32	43
	Win J12 K3	18.64	17.06	-28.97	65.19	42
	Middle J6 K1	14.16	15.1	-28.06	51.95	43
	Middle J6 K3	12.42	13.49	-26.85	41.52	42
	Middle J12 K1	14.5	15.33	-14.15	51.38	43
	Middle J12 K3	12.6	14.59	-24.81	46.68	42
	Lose J6 K1	12.01	21.08	-31.17	69.23	43
	Lose J6 K3	11.27	18.03	-21.88	49.33	42
	Lose J12 K1	10.99	19.57	-25.69	69.39	43
	Lose J12 K3	10.07	18.61	-21.67	61.68	42
	Momentum J6 K1	12.92	19.44	-23.69	44.21	43
	Momentum J6 K3	6.22	13.27	-24.57	34.91	42
	Momentum J12 K1	14.02	18.3	-17.3	51.3	43
	Momentum J12 K3	8.5	13.27	-21.84	38.23	42
CRSP	Win J6 K1	13.84	21.64	-48.29	63.61	87
	Win J6 K3	11.32	21.61	-51.74	76.44	86
	Win J12 K1	16.75	23.85	-50.02	100.47	87
	Win J12 K3	13.35	23.47	-47.76	114.77	86
	Middle J6 K1	12.96	20.24	-43.93	59.74	87
	Middle J6 K3	10.81	19.75	-49.41	69.9	86
	Middle J12 K1	12.48	20.62	-46	65.37	87
	Middle J12 K3	10.47	19.94	-51.59	77.42	86
	Lose J6 K1	12.32	24.02	-47.99	75.62	87
	Lose J6 K3	8.55	23.15	-54.3	88.95	86
	Lose J12 K1	9.73	22.16	-52.93	53.98	87
	Lose J12 K3	6.78	22.08	-57.39	57.47	86
	Momentum J6 K1	1.19	10.56	-31.17	25.95	87
	Momentum J6 K3	2.81	7.94	-12.7	27.05	86
	Momentum J12 K1	6.11	12.46	-26.33	55.24	87
	Momentum J12 K3	6.49	10.99	-21.06	40.72	86

Table 17 shows that residual momentum strategies do not outperform standard momentum based on either alpha or Sharpe ratio. Alphas are insignificant and even negative in the historical U.S. sample and in CRSP for the six months formation period. Alphas are lower than standard momentum and none of the residual momentum portfolios manages to significantly outperform the market measured in z-statistics. Based on the mixed effects of holding periods and the varying performance of the residual momentum factor across markets, the hypothesis that the performance in all samples is constant is rejected, while the hypothesis on market performance is not rejected. The hypothesis that it increases the Sharpe ratio is also rejected.

**Table 17 Residual momentum performance statistics**

Reported are the annualized performance statistics for residual momentum and the portfolios sorted on cumulative returns over formation periods J=6 or J=12 skipping last month's residual return with monthly holding periods K=1 or K=3. Alphas and Betas are obtained from a CAPM-regression. The t-statistic stands for the significance of the portfolio's alpha, whereas the z-statistic measures the significance of the difference between the Sharpe ratio of the portfolio and the Sharpe ratio of the market. Sharpe ratios are calculated using the respective proxy for the risk-free rate in the different samples tested. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

Portfolio	Market	Alpha	(T-stat)	Beta	Sharpe	(Z-stat)
U.S.	Win J6 K1	-7.85	-3.26	1.97	0.54	0.54
	Win J6 K3	-1.11	-0.67	0.65	0.41	-0.33
	Win J12 K1	-8.44	-3.76	1.82	0.46	-0.03
	Win J12 K3	-0.61	-0.38	0.59	0.41	-0.28
	Middle J6 K1	-6.96	-2.84	1.46	0.34	-0.8
	Middle J6 K3	-1.06	-0.82	0.52	0.38	-0.48
	Middle J12 K1	-7.89	-4.27	1.64	0.47	0.03
	Middle J12 K3	-2.46	-1.97	0.58	0.29	-1.05
	Lose J6 K1	-13.29	-4.41	2.23	0.28	-1.32
	Lose J6 K3	-7.14	-3.56	0.76	-0.04	-3.08
	Lose J12 K1	-12.33	-3.44	2.29	0.31	-1.09
	Lose J12 K3	-6.62	-3.25	0.77	0	-2.82
	Momentum J6 K1	2.18	0.67	-0.25	-0.03	-2.52
	Momentum J6 K3	2.43	1.36	-0.1	0.08	-1.94
	Momentum J12 K1	0.43	0.12	-0.47	-0.17	-3.22
	Momentum J12 K3	2.37	1.33	-0.18	0.02	-2.24
U.K.	Win J6 K1	12.88	5.14	0.89	1.19	3.52
	Win J6 K3	10.06	7.18	0.33	1.37	4.39
	Win J12 K1	13.36	5.45	0.86	1.25	3.8
	Win J12 K3	11.06	8.46	0.29	1.58	5.31
	Middle J6 K1	5.89	3.73	0.65	0.93	2.44
	Middle J6 K3	6.38	6.14	0.27	1.18	3.59
	Middle J12 K1	5.66	3.7	0.72	0.98	2.77
	Middle J12 K3	6.45	6.95	0.3	1.35	4.56
	Lose J6 K1	0.32	0.14	1	0.43	-0.33
	Lose J6 K3	3.92	2.89	0.37	0.7	1.12
	Lose J12 K1	0	0	0.96	0.39	-0.59

	Lose J12 K3	2.84	1.9	0.38	0.51	0.14
	Momentum J6 K1	9.08	3.21	-0.11	0.45	-0.18
	Momentum J6 K3	2.43	1.67	-0.03	0.19	-1.38
	Momentum J12 K1	9.87	3.59	-0.1	0.52	0.14
	Momentum J12 K3	4.67	3.17	-0.09	0.41	-0.34
CRSP	Win J6 K1	1.05	1.35	1.11	0.45	4.77
	Win J6 K3	2.62	2.2	0.41	0.44	1.7
	Win J12 K1	3.31	4.28	1.13	0.57	7.75
	Win J12 K3	4.44	3.66	0.42	0.58	3
	Middle J6 K1	0.53	0.83	1.06	0.43	4.89
	Middle J6 K3	2.63	2.4	0.39	0.46	1.92
	Middle J12 K1	0.13	0.21	1.05	0.41	4.1
	Middle J12 K3	2.29	2.11	0.39	0.44	1.66
	Lose J6 K1	-1.45	-1.54	1.27	0.33	1.58
	Lose J6 K3	-0.39	-0.29	0.47	0.21	-0.52
	Lose J12 K1	-3.52	-3.79	1.25	0.24	-0.7
	Lose J12 K3	-1.97	-1.5	0.47	0.1	-1.59
	Momentum J6 K1	-0.81	-0.75	-0.16	-0.19	-2.58
	Momentum J6 K3	-0.34	-0.54	-0.05	-0.13	-2.3
	Momentum J12 K1	3.57	3.2	-0.11	0.27	0.02
	Momentum J12 K3	3.05	4.65	-0.04	0.46	1.15

#### 5.1.5.2 Volatility sorted portfolios

Table 18 shows the summary statistics for residual volatility sorted portfolios. Mean returns are negative for all factor portfolios in the historical samples, but positive in the CRSP sample. The effect of increased formation periods on mean returns is mixed, and only increases returns in the historical U.K. sample.

**Table 18 Residual low volatility summary statistics**

Annualized summary statistics are reported for returns of the residual low volatility factor portfolios and tercile portfolios sorted on idiosyncratic volatility in stock returns over varying formation periods of 12, 24 and 36 months. Residual returns are obtained as the error term in rolling CAPM-regression over 36-month periods. The return of the residual low volatility factor is calculated by levering the returns of the low- and high- residual volatility portfolios using the ratio of the portfolio's volatility to that of the market, with maximum and minimum leverage restricted to 2 and 0.5. The difference between the levered low and high residual volatility portfolios their return minus the risk-free rate, multiplied by the leverage, is used as the factor return. All stocks had to have full observations over the past 36 months and were not allowed to have returns exceeding 100% in one month or cumulative returns lower than -70% over two consecutive months during this period. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017. Returns are reported in percentages.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	Low Volatility 12 months	9.69	14.62	-24.18	58.79	53
	Mean Volatility 12 months	11.7	21.8	-28.28	57.74	53
	High Volatility 12 months	15.49	42.12	-43.38	159.46	53
	Low Volatility 24 months	9.18	12.35	-22.74	30.87	53
	Mean Volatility 24 months	12.01	22.39	-25.56	76.75	53
	High Volatility 24 months	15.66	42.18	-43.17	160.76	53

	Low Volatility 36 months	8.44	12.1	-25.34	33.53	53
	Mean Volatility 36 months	11.92	25.49	-28.74	99.79	53
	High Volatility 36 months	15.17	40.23	-43.1	162.25	53
	Volatility Factor 12 months	-3.47	11.16	-32.57	22.67	49
	Volatility Factor 24 months	-3.41	11.88	-32.55	22.44	49
	Volatility Factor 36 months	-3.46	11.83	-31.8	22.59	49
U.K.	Low Volatility 12 months	8.99	10.7	-21.29	34.62	43
	Mean Volatility 12 months	16.57	16.96	-17.2	60.04	43
	High Volatility 12 months	26.68	29.56	-18.08	119.68	43
	Low Volatility 24 months	9.78	11.4	-21.15	49.58	43
	Mean Volatility 24 months	17.18	15.59	-16.06	68.12	43
	High Volatility 24 months	25.27	30.38	-22.07	109.07	43
	Low Volatility 36 months	10.24	11.37	-22.72	47.97	43
	Mean Volatility 36 months	17.2	16.37	-12.98	68.39	43
	High Volatility 36 months	24.99	30.61	-23.75	118.61	43
	Volatility Factor 12 months	-5.54	9.63	-33.79	8.05	37
	Volatility Factor 24 months	-3.73	12.13	-28.07	33.11	37
	Volatility Factor 36 months	-2.81	12.75	-29.45	38.37	37
CRSP	Low Volatility 12 months	13.07	18.29	-42.03	55.98	87
	Mean Volatility 12 months	13.85	21.8	-46.62	74.62	87
	High Volatility 12 months	12.72	26.94	-53.96	80.61	87
	Low Volatility 24 months	12.99	18.57	-43.3	53.57	87
	Mean Volatility 24 months	13.92	21.82	-47.96	67.17	87
	High Volatility 24 months	12.69	26.83	-52.03	75.15	87
	Low Volatility 36 months	12.9	18.37	-43.22	51.33	87
	Mean Volatility 36 months	13.45	21.49	-47.97	67.94	87
	High Volatility 36 months	13.25	27.54	-52.06	74.96	87
	Volatility Factor 12 months	4.04	8.83	-28.95	24.75	84
	Volatility Factor 24 months	4.08	8.9	-31.43	24.86	84
	Volatility Factor 36 months	3.73	9.21	-31.21	24.36	84

Table 19 shows that for all low volatility factors in the historical samples the alphas are negative, and positive in the CRSP sample but insignificant based on the t-statistic. None of the residual low volatility factors manages to outperform the market on Sharpe ratio. The residual approach did not increase the performance of volatility sorted portfolios, and its performance is inconsistent across markets. However, Sharpe ratios are higher than standard low-volatility, which confirms the hypothesis that a residual strategy increases low volatility performance.

**Table 19 Residual low volatility performance statistics**

Shown are the annualized performance statistics for all residual low volatility factor portfolios and tercile portfolios sorted on volatility in residual stock returns over varying formation periods of 12, 24 and 36 months. Alphas and Betas are obtained from a CAPM-regression. The t-statistic stands for the significance of the portfolio's alpha, whereas the z-statistic measures the significance of the difference between the Sharpe ratio of the portfolio and the Sharpe ratio of the market. Sharpe ratios are calculated using the respective proxy for the risk free rate in the different samples tested. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

<b>Portfolio</b>	<b>Market</b>	<b>Alpha</b>	<b>(T-stat)</b>	<b>Beta</b>	<b>Sharpe</b>	<b>(Z-stat)</b>
U.S.	Low Volatility 12 months	-5.19	-3.29	1.17	0.42	-0.28
	Mean Volatility 12 months	-10.98	-5.31	1.91	0.34	-0.97
	High Volatility 12 months	-11.4	-2.59	2.62	0.37	-0.7
	Low Volatility 24 months	-5.79	-4.07	1.13	0.34	-0.89
	Mean Volatility 24 months	-9.48	-4.41	1.85	0.37	-0.75
	High Volatility 24 months	-15.46	-3.61	2.84	0.26	-1.47
	Low Volatility 36 months	-6.25	-4.59	1.1	0.26	-1.51
	Mean Volatility 36 months	-10.82	-5.08	2.05	0.35	-0.93
	High Volatility 36 months	-12.52	-2.85	2.5	0.25	-1.48
	Volatility Factor 12 months	3.62	1.93	-1	-0.6	-4.41
	Volatility Factor 24 months	3.56	1.94	-1.01	-0.62	-4.48
	Volatility Factor 36 months	2.78	1.61	-0.93	-0.65	-4.65
U.K.	Low Volatility 12 months	2.87	2.57	0.45	0.72	1.31
	Mean Volatility 12 months	5.01	2.85	0.88	0.87	2.16
	High Volatility 12 months	10.19	3.52	1.22	0.96	2.45
	Low Volatility 24 months	2.12	1.61	0.52	0.58	0.52
	Mean Volatility 24 months	6.7	3.8	0.73	0.94	2.48
	High Volatility 24 months	8.65	3.12	1.28	0.93	2.35
	Low Volatility 36 months	2.25	1.71	0.55	0.61	0.69
	Mean Volatility 36 months	5.76	3.2	0.81	0.89	2.3
	High Volatility 36 months	10.11	3.58	1.08	0.94	2.33
	Volatility Factor 12 months	-6.47	-4.17	-0.37	-0.96	-5.96
	Volatility Factor 24 months	-5.42	-3.23	-0.32	-0.77	-5.21
	Volatility Factor 36 months	-4.5	-2.52	-0.35	-0.66	-4.71
CRSP	Low Volatility 12 months	1.51	2.65	0.93	0.49	6.56
	Mean Volatility 12 months	0.7	1.02	1.15	0.44	5.11
	High Volatility 12 months	-1.97	-1.93	1.38	0.32	1.21
	Low Volatility 24 months	1.5	2.68	0.91	0.49	6.57
	Mean Volatility 24 months	0.7	0.98	1.16	0.44	4.93
	High Volatility 24 months	-1.97	-1.92	1.39	0.32	1.22
	Low Volatility 36 months	1.48	2.68	0.9	0.49	6.58
	Mean Volatility 36 months	0.37	0.49	1.16	0.42	4.25
	High Volatility 36 months	-1.62	-1.55	1.4	0.33	1.56
	Volatility Factor 12 months	0.26	0.31	0.02	0.07	-1.25
	Volatility Factor 24 months	0.35	0.41	0.03	0.09	-1.13
	Volatility Factor 36 months	0.17	0.19	0.02	0.04	-1.38

### 5.1.5.3 Double-sorted

Tables 20 and 21 describe the results on the double-sorted residual factor portfolios show a discrepancy between results in the historical and CRSP samples. Both double-sort factors generate low returns in the historical samples but achieve significant alphas and market outperformance in the CRSP sample. Performance is clearly not stable across all samples. Furthermore, in none of the markets does the double-sorted residual portfolio manage to outperform the market on Sharpe ratio, and therefore does not constitute an improvement relative to standard double-sorted factor portfolios.

**Table 20 Residual double-sorted factor returns**

Annualized summary statistics are reported for returns of the residual double-sorted factor portfolios and outer portfolios sorted in terciles, which are then double-sorted in quantiles. Residuals are obtained in 36-month rolling regression on the CAPM factor. Stocks are either sorted on cumulative residual returns over the past 12 months skipping last month and then on residual volatility over the past 36 months. This process is repeated in reverse order. The returns of the double-sort factors are calculated by levering the returns of the outer portfolios consisting of the least volatile winning stocks and the most volatile losing stocks, or the least volatile stocks with the highest returns and the most volatile stocks with the lowest returns, all based on residuals. The ratio of the portfolio's volatility to that of the market determines leverage, which is capped at maximally 2 and minimally 0.5. The difference between the levered outer portfolios their return minus the risk-free rate multiplied by the leverage is used as the factor return. All stocks had to have full observations over the past 36 months and were not allowed to have returns exceeding 100% in one month or cumulative returns lower than -70% over two consecutive months during this period. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017. Returns are reported in percentages.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	Win Mom Vol	11.39	16.5	-22.56	55.61	53
	Lose Mom Vol	13.57	40.93	-62.93	186.08	52
	Factor Mom Vol	-2.29	11.64	-29.89	20.78	49
	Win Vol Mom	10.99	12.86	-23.36	50.44	50
	Lose Vol Mom	14.1	41.07	-63.87	163.57	53
	Factor Vol Mom	0.19	16.62	-37.75	59.7	50
U.K.	Win Mom Vol	17.44	15.6	-22.3	73.67	43
	Lose Mom Vol	13.13	34.06	-42.24	145.93	43
	Factor Mom Vol	3.44	12.39	-29.04	25.9	37
	Win Vol Mom	11.36	11.85	-26.29	38.72	43
	Lose Vol Mom	15.37	30.51	-38.58	96.14	43
	Factor Vol Mom	-0.8	12.07	-26.45	17.37	43
CRSP	Win Mom Vol	15.31	21.3	-51	78.67	87
	Lose Mom Vol	8.65	25.47	-55.34	69.56	87
	Factor Mom Vol	7.16	10.38	-24.58	31.76	84
	Win Vol Mom	14.38	19.27	-38.54	65.08	87
	Lose Vol Mom	9.08	27.45	-57.24	67.78	87
	Factor Vol Mom	6.33	11.55	-37.45	40.41	87

**Table 21 Residual double-sorted factor performance statistics**

Reported are the annualized performance statistics for all the outer residual double-sorted portfolios and the factor portfolios. Alphas and Betas are obtained from a CAPM-regression. The t-statistic stands for the significance of the portfolio's alpha, whereas the z-statistic measures the significance of the difference between the Sharpe ratio of the portfolio and the Sharpe ratio of the market. Sharpe ratios are calculated using the respective proxy for the risk-free rate in the different samples tested. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

<b>Portfolio</b>	<b>Market</b>	<b>Alpha</b>	<b>(T-stat)</b>	<b>Beta</b>	<b>Sharpe</b>	<b>(Z-stat)</b>
U.S.	Win Mom Vol	-2.81	-1.5	1.23	0.61	1
	Lose Mom Vol	-8.27	-1.16	2.46	0.3	-1.05
	Factor Mom Vol	3.42	1.43	-0.92	-0.45	-3.88
	Win Vol Mom	-2.61	-1.91	1.04	0.65	1.39
	Lose Vol Mom	-14.14	-2.27	3	0.29	-1.16
	Factor Vol Mom	3.44	1.02	-0.96	-0.27	-3.53
U.K.	Win Mom Vol	8.91	4.93	0.56	1.08	3.03
	Lose Mom Vol	0.72	0.2	0.99	0.3	-1.01
	Factor Mom Vol	2.73	1.49	-0.24	0.07	-1.69
	Win Vol Mom	5.31	3.76	0.5	0.91	2.27
	Lose Vol Mom	2.42	0.69	1.15	0.43	-0.37
	Factor Vol Mom	-1.3	-0.75	-0.33	-0.32	-3.65
CRSP	Win Mom Vol	2.91	3.7	0.99	0.55	6.52
	Lose Mom Vol	-5.39	-4.35	1.41	0.18	-1.88
	Factor Mom Vol	3.95	3.49	0.02	0.43	1.03
	Win Vol Mom	2.89	4.27	0.88	0.57	7.15
	Lose Vol Mom	-5.56	-4.07	1.46	0.17	-1.81
	Factor Vol Mom	4.68	4.69	-0.24	0.29	0.14

## 5.2 Factor loadings

### 5.2.1 Mutual factor loadings

Mutual factor loadings of the momentum J=12 and K=1 variation and low volatility factors with a 36-month formation period are measured in an OLS-regression, which regresses factor returns on the other factor, excess market returns and a constant. The regressions are reported in table 22. In all samples, both factors positively impact the other factor's return implying a positive relation between both factors. This rejects the hypothesis that both factors are not related. This positive relation might be caused by an external factor that simultaneously influences momentum and low volatility returns. Excess market returns have a significant negative impact on momentum returns in the historical U.S. and CRSP sample. Low volatility returns are also significantly negatively impacted by excess market returns in both historical samples. The negative effect is consistent with the low betas of both factors but is stronger than expected. The hypothesis that



excess market returns do not affect factor returns is rejected. However, the regressions have little overall explanatory power given the low adjusted R-squared levels ranging from 0.06 to 0.14.

**Table 22 Factor returns regression**

This table displays the results of a regression of the annualized momentum J=12, K=1 variation and the low volatility factor with the 36-month formation period on the other respective factor's return, annualized market returns and a constant. P-values are in parentheses. \* stands for significance at the 10% level, \*\* at the 5% level, and \*\*\* at the 1% significance level. Furthermore, the amount of observation in the regression and adjusted R-squared are also reported. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

U.S.		
	Momentum	Low Volatility
Momentum		0.099*** (0.001)
Low Volatility	0.122*** (0.001)	
Excess Market Returns	-0.278*** (0.003)	-0.660*** (0.000)
Constant	0.018*** (0.000)	0.000 (0.828)
Observations	911	911
Adj. R-Squared	0.028	0.085
U.K.		
	Momentum	Low Volatility
Momentum		0.092*** (0.003)
Low Volatility	0.091*** (0.003)	
Excess Market Returns	-0.053 (0.303)	-0.437*** (0.000)
Constant	0.019*** (0.000)	-0.001 (0.323)
Observations	1080	1080
Adj. R-Squared	0.010	0.075
CRSP		
	Momentum	Low Volatility
Momentum		0.114*** (0.000)
Low Volatility	0.122*** (0.000)	
Excess Market Returns	-0.114*** (0.000)	-0.007 (0.630)
Constant	0.010*** (0.000)	0.003*** (0.001)
Observations	1029	1029
Adj. R-Squared	0.066	0.014

### 5.2.2 Market states

This section investigates factor correlation and performance during different market states. Table 23 reports correlation during market states and overall factor correlation together with p-values to measure equality between overall correlation and during a market state. Market Down states display stronger shifts in correlation than Up states, because it constitutes 15% of overall observations, while market Up states contain 85% of overall observations. Tables 23 shows that correlation is only significantly impacted by market states in the historical samples. Yet in the historical U.K. sample correlation increases while in the historical U.S. sample it decreases. In the CRSP sample correlations. In all samples overall correlations are low, which underlines the diversification possibilities of combined strategies. Given the positive impact of a market Down state in the historical U.K. sample and the insignificant impact in the CRSP sample, the hypothesis that correlations decrease during down states is rejected.

**Table 23 Factor correlations and market states**

Displayed are the correlations between the momentum  $J=12$ ,  $K=1$  and low volatility 36-month formation period factors during market Up and Dow states and overall correlations for all three datasets. Next to the overall correlations, the p-values for equality between the overall factor correlation and correlation during the respective market states are shown. In the latter column, the number of observations is shown. A market down state is defined as the bottom 15% of trailing market returns and market up state as the top 85% of market states. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

Market	State	Correlation	Overall Correlation	P-value	N
U.S.	Down	-0.210	0.141	0.000	226
	Up	0.175	0.141	0.367	1287
U.K.	Down	0.386	0.103	0.000	226
	Up	0.055	0.103	0.198	1287
CRSP	Down	0.135	0.125	0.897	165
	Up	0.125	0.125	0.999	940

Factor performance measured in annualized returns and alphas during market Up and Down states is shown in table 24. Momentum returns and alphas remain constant across market states in the historical U.S. sample, while alphas increases during the Down state in the U.K. sample. However, as can be seen in table 31, the maximum market loss in the historical samples is below 20%, which might explain the limited impact of market Down state on momentum. Surprisingly, in CRSP momentum performance is highest during down states, although it should be noted that shifting market states are not taken into account, and therefore momentum return and alphas might

have been affected these shifts. Low volatility returns and alphas increase with market Down states in all samples, but performance is still meager. The hypothesis that momentum alpha decreases relative to low volatility during market down states is rejected based on the results in the historical U.K. and CRSP sample.

**Table 24 Market states and factor performance**

This table reports annualized mean returns and alphas for the momentum J=12 and K=1 variation and low volatility factor with a 36 formation period in all three samples during market up and down states, based on 36-month trailing cumulative market returns. The bottom 15% is defined as market down states and the top 85% as up-states. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

<b>Panel A: Monthly returns during Up-markets (85% of observations)</b>				
		U.S.	U.K.	CRSP
Momentum	Mean Return	22.09	29.66	5.1
	Alpha	16.48	29.74	10.85
Low Volatility	Mean Return	-6.39	-2.35	3.4
	Alpha	-11.57	-10.7	0.04
<b>Panel B: Monthly returns during Down-markets (15% of observations)</b>				
		U.S.	U.K.	CRSP
Momentum	Mean Return	20.52	22.55	14.24
	Alpha	18.66	22.44	8.88
Low Volatility	Mean Return	-2.77	-1.96	4.65
	Alpha	2.48	0.29	1.27

Table 25 reports residual factor correlations, which should have reduced market exposure and therefore less sensitive to market states. However, in both historical samples correlations decrease significantly during Down states, which does not this expectation. In the CRSP sample correlations do not differ significantly during both states. The hypothesis that the residual strategy decreases the sensitivity of factor correlations to the state of the market is rejected.

**Table 25 Residual factor correlations and market states**

Displayed are the correlations between the residual variations of momentum J=12, K=1 and low volatility 36-month formation period factors during market Up and Down states and overall correlations for all three datasets. Next to the overall correlations, the p-value for equality between the overall factor correlation and correlation during the respective market states are shown together with the number of observations. A market Down state is defined as the bottom 15% of trailing market returns and a market Up state as the top 85% of market states. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

Market	State	Correlation	Overall Correlation	P-value	N
U.S.	Down	-0.641	0.226	0.000	226
	Up	0.240	0.226	0.702	1287
U.K.	Down	-0.252	0.022	0.000	226
	Up	0.040	0.022	0.641	1287
CRSP	Down	0.019	-0.035	0.523	165
	Up	-0.050	-0.035	0.734	939

The performance of residual factors during market states are reported in table 26. In the historical U.S. sample, residual momentum performance measured in alpha strongly increases and slightly increases for low-volatility, although both factor returns decrease. However in the U.K. both factor returns and alpha decrease with market down states. In the CRSP sample, factor performance proves to be most stable. Yet due to the conflicting nature of the results, no consistent increase in performance relative to the original factors can be attributed to the residual approach.

**Table 26 Residual factor performance and market states**

This table reports annualized mean returns and alphas for the residual momentum J=12 and K=1 variation and residual low volatility factor with a 36 formation period during market Up and Down states. Per month 36-month trailing cumulative market returns are calculated and the month in the bottom 15% is defined as market down states and the top 85% as up-states. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

<b>Panel A: Monthly returns following Up-markets (85% of observations)</b>					
			U.S.	U.K.	CRSP
Momentum	Mean Return		10.15	12.5	5.94
	Alpha		-20.9	7.12	5.64
Low Volatility	Mean Return		7.34	2.3	5.06
	Alpha		2.96	-4.06	-1.45
<b>Panel B: Monthly returns following Down-markets (15% of observations)</b>					
			U.S.	U.K.	CRSP
Momentum	Mean Return		4.48	7.12	6.19
	Alpha		2.95	4.59	2.29
Low Volatility	Mean Return		-2.99	-4.24	3.49
	Alpha		3.9	-1.2	-0.11

### 5.2.3 Liquidity and economic effects

Table 27 shows the results of the regression of momentum and low volatility returns on the other factor, liquidity shocks, excess market returns and a constant. No significant relation is found between liquidity shocks and factor returns in any of the samples. The lack of a significant relation might be due to the measurement of the liquidity spread in this study. In the U.S. historical sample, the risk-free nature of the long term government bond yield is debatable, while the long term maturity of the corporate index could make it sensitive to yield shocks. Market Down states negatively affect momentum returns in the U.S. and SCRSP historical samples and are significant at the 1% level, which confirms earlier findings on momentum returns and market states. In the historical U.S. sample market Down states and GDP growth also negatively impact low volatility returns at the 5% and 1% significance levels, but this effect is not observed in the other samples and therefore might be spurious. Based on these findings, the hypotheses that liquidity shocks improve momentum or low volatility performance are rejected.

**Table 27 Regression of economic variables on factors**

The table shows OLS-regressions for all separate markets of the momentum and low volatility factor returns on economic variables such as liquidity shocks and GDP-growth and market down states. Liquidity shocks are measured as the residuals in an AR(2)-regression of the spread of corporate over government bond-yields. P-values are in parentheses. \* stands for significance at the 10% level, \*\* at the 5% level, and \*\*\* at the 1% significance level. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

U.S.		
	Momentum	Low Volatility
Liquidity Shock	-0.103 0.83	0.163 0.596
GDP Growth	2.327 0.858	-35.575** 0.027
Market Down State	-0.152** 0.04	-0.090** 0.049
Constant	0.307** 0.029	0.389** 0.014
Observations	88	71
Adj. R-squared	0.017	0.086
U.K.		
	Momentum	Low Volatility
Liquidity Shock	0.587 0.214	0.542 0.188
GDP Growth	-21.565 0.125	-16.217 0.207
Market Down State	-0.029 0.545	-0.026 0.507

Constant	0.357***	0.076
	0	0.336
Observations	80	82
Adj. R-squared	0.018	0.011
<hr/>		
CRSP		
	Momentum	Low Volatility
Liquidity Shock	0.329	-0.431
	0.323	0.149
GDP Growth	0.957	3.598
	0.751	0.113
Market Down State	-0.148***	-0.024
	0.001	0.493
Constant	0.218***	0.008
	0.001	0.872
Observations	88	84
Adj. R-squared	0.103	0.030

## 6 Conclusion

In this study, momentum and low volatility strategies are analyzed over the years 1800 – 2017 in the historical U.S. and U.K. stock markets and U.S. stocks from CRSP, based on the consistency of factor returns, factor correlations, sensitivity towards market circumstances and exposure to liquidity and economic variables. I find that momentum is a consistent and proven factor in all samples. Increased momentum formation and holdings period improve performance measured in returns and volatility. Alphas for momentum are significant and positive, but momentum does not consistently outperform the market on Sharpe ratio in the samples. A residual strategy forming momentum portfolios on residual returns from a 36-month rolling CAPM-regression does not increase momentum performance nor does it significantly increase minimum returns in all samples. Momentum has negative exposure to the market factor measured in beta, and returns are negatively affected by market down states in the U.S. samples, although alpha performance does not decrease during these states.

Results on low volatility strategies on beta and idiosyncratic volatility are mixed. The portfolio consisting of low volatility stocks is consistently outperformed in both historical samples by the high volatility portfolio, but not in the CRSP dataset. The low volatility factor portfolio delivers negative returns for all formation periods in the historical samples and in the CRSP sample, only the 36-month formation period low volatility reports positive mean returns. The BAB factor generates no significant outperformance based on alpha or Sharpe ratio in any of the samples. This

can be due to the construction and measurement of the factor, which differs from the original paper by Frazzini and Pedersen (2014). However, low-risk portfolios sorted on beta do outperform high beta portfolios in all samples based on alpha. The residual strategy improves factor performance of low volatility, although the original factor already has poor performance. The low volatility factor portfolio has low to negative correlation with momentum. Combo and double-sorted factor strategies generate significantly positive risk-adjusted returns, with higher minimum returns than momentum, although it does not increase the Sharpe ratio relative to momentum. Market down states have no consistent effect on factor correlations in the samples. The regressions show no significant relation between factor returns and liquidity shocks and GDP-growth, except for the U.S. before 1926 in the latter case.

Based on these results, there is strong evidence supporting the consistency and time-invariance of momentum. However, low volatility requires further research on why idiosyncratic volatility is not a factor in the historical samples, while stocks with low exposure to systematic risk have a tendency to outperform their counterparts over the long run on risk-adjusted returns. An indication of a turning point for idiosyncratic low volatility performance is found in table 37, which presents performance over 50-year periods for low volatility in the CRSP sample. The 36-month low-volatility factor's mean return increases per 50-year period.

There are several limitations regarding factor analysis in the historical samples. The number of stocks per month in the historical samples is sometimes sparse and future research could be limited to periods with a minimum number of stocks in the cross-section to avoid extreme results in the factor strategies. Furthermore, transaction costs are not taken into account in the historical samples. Information was probably traveling at different speeds in the 19<sup>th</sup> century, making arbitrage potentially costly and infeasible, even if capital may have been abundant at times. In addition, liquidity is significantly lower in the historical samples, which is exemplified by the large number of stocks not being traded at multiple periods. Furthermore, historical stock markets had few different industries in the index. Goetzman et al. (2001) point out that in the early 19<sup>th</sup> century financial companies almost composed the entire index, while at the beginning of the 20<sup>th</sup> century they had been nearly entirely replaced by transportation companies and industrials. Further research could explore momentum and low volatility effects within industries to control for this effect. Another problem in the historical samples is the absence of a short-term government bond

that can proxy the risk-free rate. This absence hampers measurements of equity premia and estimation of excess market returns and liquidity measures.

Future research on the low volatility effect could use weekly observations on U.S. stocks from the CRSP sample to see how the profitability of low volatility develops during the 20<sup>th</sup> century. Given the ambiguity on the performance of low volatility strategies and because explanations for the low volatility effect mostly originate in the behavioral field, it is worthwhile to look at the history of stock markets and analyze whether investors backgrounds have changed over time shifting from proprietary investors to a reliance on fund managers, which can introduce agency issues on the stock markets. Empirical evidence for the behavioral explanations could provide insights on the causes of the low volatility phenomenon as well on future factor performance.



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## 8 Appendix

**Table 28 Momentum portfolios summary statistics (Robust)**

This table shows the monthly summary statistics on robust momentum portfolios that did not require full 36-month stock observations and allowed returns of more than 100% or two periods cumulative returns lower than -70% during the formation period. Included are mean returns, standard deviations, minimum and maximum returns and the number of observations. The momentum factor is constructed by taking the difference in returns between the tercile portfolios containing stocks with respectively the highest and lowest performance during the formation period. 'J' stands for formation period of the respective momentum strategy, and 'K' for holding period. For all strategies last month's return is skipped in the formation period. In order to be included in the portfolio, stocks had to have full observations over the past 36 months. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	Win J6 K1	23.45	21.4	-7.78	113.58	95
	Win J6 K3	16.18	18.35	-14.54	100.21	95
	Win J12 K1	28.47	24.81	-2	126.02	94
	Win J12 K3	19.71	19.37	-9.05	104.33	94
	Middle J6 K1	6.73	9.91	-17.97	41.06	95
	Middle J6 K3	4.84	8.99	-12.19	43.14	95
	Middle J12 K1	6.23	9.93	-10.53	47.71	94
	Middle J12 K3	4.34	8.18	-12.79	37.3	94
	Lose J6 K1	9.17	19.95	-27.92	79.53	95
	Lose J6 K3	0.64	16.65	-33.12	41.82	95
	Lose J12 K1	7.38	20.14	-34.17	91.62	94
	Lose J12 K3	-0.59	16.46	-35.74	55.82	94
	Momentum J6 K1	13.28	21.27	-35.12	71.42	95
	Momentum J6 K3	16.48	17.85	-26.65	61.3	95
	Momentum J12 K1	20.18	26.14	-31.46	90.13	94
	Momentum J12 K3	21.67	20.87	-19.4	95.28	94
U.K.	Win J6 K1	17.25	25.01	-30.89	116.58	88
	Win J6 K3	8.79	15.99	-37.4	61.74	88
	Win J12 K1	16.43	20.13	-36.22	97.96	87
	Win J12 K3	11.07	15.93	-37.58	70.73	87
	Middle J6 K1	12.14	13.9	-26.92	51.66	81
	Middle J6 K3	7.8	10.58	-23.84	46.29	81
	Middle J12 K1	11.59	11.16	-24.53	42.16	85
	Middle J12 K3	8.36	9.71	-24.29	37.83	85
	Lose J6 K1	10.57	20.75	-40.43	116.14	88
	Lose J6 K3	2.76	15.83	-49.46	43.81	88
	Lose J12 K1	10.07	18.33	-26.04	83.94	87
	Lose J12 K3	2.71	14.72	-42.48	44.69	87
	Momentum J6 K1	4.62	15.76	-41.62	87.52	88
	Momentum J6 K3	6.24	13.31	-16.79	92.07	88
	Momentum J12 K1	5.57	18.77	-86.77	47.6	87
	Momentum J12 K3	8.14	11.99	-38.09	38.87	87
CRSP	Win J6 K1	17.03	26.41	-41.1	88.35	90
	Win J6 K3	12.01	25.48	-46.48	72.96	89
	Win J12 K1	19.6	27.3	-47.53	104.12	89

Win J12 K3	13.54	26.13	-51.53	105	88
Middle J6 K1	14.54	24.76	-45.08	116.49	90
Middle J6 K3	11.12	24.45	-56.25	124.12	89
Middle J12 K1	14.54	24.67	-44.99	115.12	89
Middle J12 K3	11.38	25.21	-55.71	134.45	88
Lose J6 K1	12.73	32.94	-52.3	203.33	90
Lose J6 K3	7.66	31.56	-61.51	194.22	89
Lose J12 K1	9.49	28.44	-55.77	121.61	89
Lose J12 K3	5.69	28.56	-65.07	138.37	88
Momentum J6 K1	4.16	15.6	-69.1	39.6	90
Momentum J6 K3	5.17	11.98	-52.59	37.53	89
Momentum J12 K1	8.63	14.88	-54.93	43.11	89
Momentum J12 K3	8.43	13.08	-35.47	56.32	88

**Table 29 Low volatility summary statistics (Robust)**

This table describes the performance of variations on the robust low volatility factor, which is constructed by sorting stocks into tercile portfolios over varying formation periods, namely 12, 24 and 36 months, and taking the difference in returns between the tercile portfolio with respectively the highest and lowest volatility after leveraging the portfolio's volatility to that of the market, with leverage not exceeding the size of the portfolio. The robust version allowed stock returns higher than 100% and lower than cumulative -70% during two consecutive periods. All statistics displayed are monthly, included are mean returns, standard deviations, minimum and maximum returns and the number of observations, denoted by N. The markets analyzed are the cleaned historical datasets for the U.S. and the U.K. over the period 1800 to 1926, which are both composed over the filtered and merged datasets taken from ICF and GFD, and CRSP returns over the period 1926 until 1927. Formation periods are stated in the left column. All stocks had to have full 36-month observations to be included into a portfolio. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	High Volatility 12 months	32.49	39.41	-48.98	200.62	90
	High Volatility 24 months	34.53	44.93	-50.79	268.7	85
	High Volatility 36 months	34.76	47.69	-42.11	250.44	82
	Low Volatility 12 months	6.8	9.56	-22.6	48.75	90
	Low Volatility 24 months	7.17	8.72	-14.68	47.78	85
	Low Volatility 36 months	7.17	9.05	-14.9	45.66	82
	Mean Volatility 12 months	8.2	14.04	-24.77	53.76	90
	Mean Volatility 24 months	8.85	16.09	-29.84	72.85	85
	Mean Volatility 36 months	10.01	17.4	-31.03	69.71	82
	Volatility Factor 12 months	-8.92	13.39	-40.54	34.84	83
	Volatility Factor 24 months	-9.68	12.84	-48.6	20.45	79
	Volatility Factor 36 months	-9.65	13.45	-47.58	16.8	78
U.K.	High Volatility 12 months	24.62	24.65	-53.45	100.41	82
	High Volatility 24 months	27.69	21.22	-7.56	101.29	74
	High Volatility 36 months	28.82	23.23	-20.56	116.16	67
	Low Volatility 12 months	6	8.33	-16.09	41.79	82
	Low Volatility 24 months	5.67	8.82	-21.92	41.53	74
	Low Volatility 36 months	4.94	7.74	-18.57	27.01	67
	Mean Volatility 12 months	9.17	14.37	-25.96	67.33	82
	Mean Volatility 24 months	8.19	13.36	-25.97	65.04	74

	Mean Volatility 36 months	8.9	13.49	-25.77	68.99	67
	Volatility Factor 12 months	-5.48	10.47	-63.24	33.01	75
	Volatility Factor 24 months	-7.14	12.17	-33.88	67.65	65
	Volatility Factor 36 months	-8.14	7.4	-29.69	4.57	59
CRSP	High Volatility 12 months	15.59	36.07	-53.91	144.47	89
	High Volatility 24 months	16.37	35.62	-54.57	140.93	88
	High Volatility 36 months	17.82	34.98	-55.51	136.88	87
	Low Volatility 12 months	13.54	19.98	-38.97	81.71	89
	Low Volatility 24 months	13.72	19.6	-41.53	83.33	88
	Low Volatility 36 months	14.23	18.78	-42.77	79.57	87
	Mean Volatility 12 months	15.32	26.89	-50.31	119.8	89
	Mean Volatility 24 months	16.33	26.8	-49.25	125.62	88
	Mean Volatility 36 months	16.94	26.05	-48.38	120.91	87
	Volatility Factor 12 months	8.89	18.61	-38.43	102.06	88
	Volatility Factor 24 months	9.53	20.39	-40.46	107.5	87
	Volatility Factor 36 months	10.02	21.45	-37.83	118.24	86

**Table 30 Double-sort summary statistics (Robust)**

This table reports summary returns for the robust outer double-sort portfolios and factors. Stocks did not require full 36-month observations and returns of more than 100% and lower than cumulative -70% during two consecutive periods. Stocks are sorted first in terciles based on 36-month volatility and then further sorted into quantiles based on cumulative returns over the past twelve months. This process is repeated the other way around by sorting on cumulative returns and then on volatility. The factor is created by leveraging the volatility of the outer portfolios to that of the market, with leverage not exceeding the size of the portfolio, and taking the difference in returns. The three markets include the total U.K. and U.S. historical markets over the period 1800 – 1926 as well as CRSP returns for U.S. stocks from 1926 to 2017.

Market	Strategy	Mean	Std.Dev	Min	Max	N
US.	Win Mom Vol	14.32	14.48	-7.2	89.14	79
	Lose Mom Vol	9.83	45.64	-68	164.82	59
	Factor Mom Vol	0.32	15.4	-37.04	37.01	37
	Win Vol Mom	9.93	9.36	-7.07	44.55	83
	Lose Vol Mom	14.06	45.72	-64.49	218.51	86
	Factor Vol Mom	-0.22	19.12	-45.78	51.55	66
U.K.	Win Mom Vol	11.1	9.96	-12.32	43.43	61
	Lose Mom Vol	17	22.25	-21.93	100.28	62
	Factor Mom Vol	1.59	8.01	-17.64	17.73	49
	Win Vol Mom	7.5	7.93	-12.67	40.47	72
	Lose Vol Mom	24.02	30.69	-50.19	139.31	77
	Factor Vol Mom	-6.25	9.01	-29.69	11.37	52
CRSP	Win Mom Vol	19.43	21.89	-42.17	95.54	87
	Lose Mom Vol	11.87	32.78	-52.46	141.18	87
	Factor Mom Vol	13.43	16.22	-34.48	75.32	86
	Win Vol Mom	16.48	18.57	-32.75	83.52	87
	Lose Vol Mom	13.84	35.67	-53.74	143.44	87
	Factor Vol Mom	17.42	30.97	-32.84	177.35	86

**Table 31 Market returns on all data samples**

This table describes summary statistics on the annualized market returns for the historical U.S. and U.K. datasets and CRSP. In the case of CRSP market returns are value-weighted, whereas market returns for the historical datasets are price-weighted.

Market	Strategy	Mean	Std.Dev	Min	Max	N
U.S.	Market Return	10.76	8.88	-7.86	40.19	125
U.K.	Market Return	9.63	10.37	-16.42	62.55	125
CRSP	Market Return	11.77	20.07	-44.39	57.41	91

**Table 32 U.S. Momentum 50-year period yearly returns and alphas**

This table displays average annualized and alphas for momentum portfolios over 50 year periods in the historical U.S. sample over the years 1810 - 1926. The momentum variations shown include formation periods 'J' of six and twelve months and holding periods 'K' of one and three months.

Portfolio	Performance	Period	Portfolio variations			
			J = 6		J = 12	
			K = 1	K = 3	K = 1	K = 3
Momentum	Return	1810	20.68	20.76	21.55	25.11
			Alpha	17.46	16.39	13.51
	Alpha	1860	11.34	14.44	12.37	14.59
			12.7	11.54	17.52	13.65
		1910	25.62	26.48	28.3	29.6
			23.77	24.1	31.21	26.93
Win	Return	1810	22.6	14.91	20.69	14.63
			Alpha	10.6	7.01	6.33
	Alpha	1860	25.04	13.68	22.5	13.66
			0.36	-0.18	0.76	1.21
		1910	28.19	19.32	31.67	22.72
			-1.99	4.3	2.33	6.93
Middle	Return	1810	5.33	3.84	6.65	2.85
			Alpha	-0.66	-1.8	-1.75
	Alpha	1860	8.57	6.05	8.6	5.4
			-6.75	-4.34	-9.2	-5.99
		1910	12.44	8.24	9.06	6.13
			-7.33	-3.19	-10.58	-5.42
Lose	Return	1810	1.62	-4.92	-0.71	-8.46
			Alpha	-10.23	-12.38	-10.87
	Alpha	1860	11.79	-0.67	9.1	-0.84
			-14.57	-14.31	-17.74	-14.37
		1910	2.09	-5.77	2.68	-5.42
			-24.24	-19.5	-25.51	-19.33

**Table 33 U.K. Momentum 50-year period yearly returns and alphas**

This table displays average annualized and alphas for momentum portfolios over 50 year periods in the historical U.K. sample over the years 1810 - 1926. The momentum variations shown include formation periods 'J' of six and twelve months and holding periods 'K' of one and three months.

Portfolio	Performance	Period	Portfolio variations			
			J = 6		J = 12	
			K = 1	K = 3	K = 1	K = 3
Momentum	Return	1810	16.94	17.64	22.19	20.56
			10.73	18.72	12.7	26.13
	Alpha	1860	18.89	18.6	26.57	27.1
			15.52	13.93	24.38	24.81
	1910	18.59	14.85	22.63	20.08	
		9.01	8.79	11.55	13.02	
Win	Return	1810	24.45	12.63	26.11	15.14
			3.88	2.56	6.63	10.48
	Alpha	1860	17.58	10.54	20.35	14.36
			6.43	3.58	10.29	8.66
	1910	18.94	13.82	20.2	16.49	
		5.37	6.59	5.69	9.12	
Middle	Return	1810	6.82	2.83	4.17	4.49
			-5.8	-11.37	-4.8	-6.84
	Alpha	1860	4.02	2.08	2.84	0.57
			-3.8	-3.03	-4.72	-4.04
	1910	10.2	2.31	1.84	-0.62	
		-3.27	-0.21	-7.11	-4.11	
Lose	Return	1810	6.43	-4.41	2.93	-4.53
			-9.34	-16.67	-8.48	-15.46
	Alpha	1860	-1.12	-6.89	-5.01	-10.21
			-10.67	-11.85	-14.14	-15.74
	1910	0.3	-0.65	-1.99	-3.2	
		-7.2	-5.85	-9.05	-7.3	

**Table 34 CRSP Momentum 50-year period yearly returns and alphas**

This table displays average annualized and alphas for momentum portfolios over 50 year periods in the CRSP sample over the years 1926 - 2017. The momentum variations shown include formation periods 'J' of six and twelve months and holding periods 'K' of one and three months.

Portfolio	Performance	Period	Portfolio variations			
			J = 6		J = 12	
			K = 1	K = 3	K = 1	K = 3
Momentum	Return	1926	1.84	5.28	6.26	8.43
			-0.23	3.73	3.11	6.71
	Alpha	1960	6.77	12.21	10.73	16.07
			1.15	7.06	5.29	11.14
	2010	3.42	5.22	8.47	10.99	
		2.96	3.07	3.44	6.43	



Win	Return	1926	16.87	11.32	19.87	13.14
			Alpha	1.93	4.27	4.12
		1960	14.8	6.55	17.83	9.63
			Alpha	3.13	-0.49	5.99
		2010	11.24	6.72	13.98	10.32
			Alpha	-1.79	-0.59	-0.87
Middle	Return	1926	15.46	9.21	15.03	9.16
			Alpha	2.42	2.98	1.94
		1960	12.21	6.63	12.15	7.2
			Alpha	1.63	-0.38	1.83
		2010	10.83	9.71	12.24	10.62
			Alpha	-0.4	4.47	-0.2
Lose	Return	1926	14.78	5.76	12.88	4.37
			Alpha	1.08	-0.55	-0.06
		1960	7.56	-5.09	6.48	-5.61
			Alpha	-3.28	-11.9	-4.51
		2010	7.58	1.43	5.12	-0.6
			Alpha	-4.87	-3.8	-4.42

**Table 35 U.S. Low volatility portfolios 50-year period yearly returns and alphas**

This table displays average annualized and alphas for idiosyncratic volatility factor portfolios over 50 year periods in the historical U.S. sample over the years 1810 - 1926. The variations shown include formation periods of 12, 24 and 36 months.

Portfolio	Period	Period	Portfolio variations		
			J = 12	J = 24	J = 36
Factor	Return	1810	-3.51	-0.86	-3.16
			Alpha	-6.82	-5.66
		1860	-2.75	-1.72	-3.46
			Alpha	6.16	7.59
		1910	-0.05	-0.27	0.41
			Alpha	7.15	7.14
Low Volatility	Return	1810	6.66	6.9	4.81
			Alpha	0.12	0.93
		1860	14.79	15.69	13.2
			Alpha	1.13	2.01
		1910	12.44	10.64	10.89
			Alpha	3.24	1.76
Middle Volatility	Return	1810	6.49	7.42	8.38
			Alpha	-2.14	-1.98
		1860	8.79	9.39	9.69
			Alpha	-11.22	-10.17
		1910	12.91	14.88	15.56
			Alpha	-7.9	-6.31

High Volatility	Return	1810	19.16	17.17	17.34
	Alpha	1860	7.86	7.06	6.68
			21.23	21.22	21.1
	1910	-7.14	-8.22	-8.19	
		16.28	14.89	14.38	
	-17.1	-18.07	-18		

**Table 36 U.K. Low volatility portfolios 50-year period yearly returns and alphas**

This table displays average annualized and alphas for idiosyncratic volatility factor portfolios over 50 year periods in the historical U.K. sample over the years 1810 - 1926. The variations shown include formation periods of 12, 24 and 36 months.

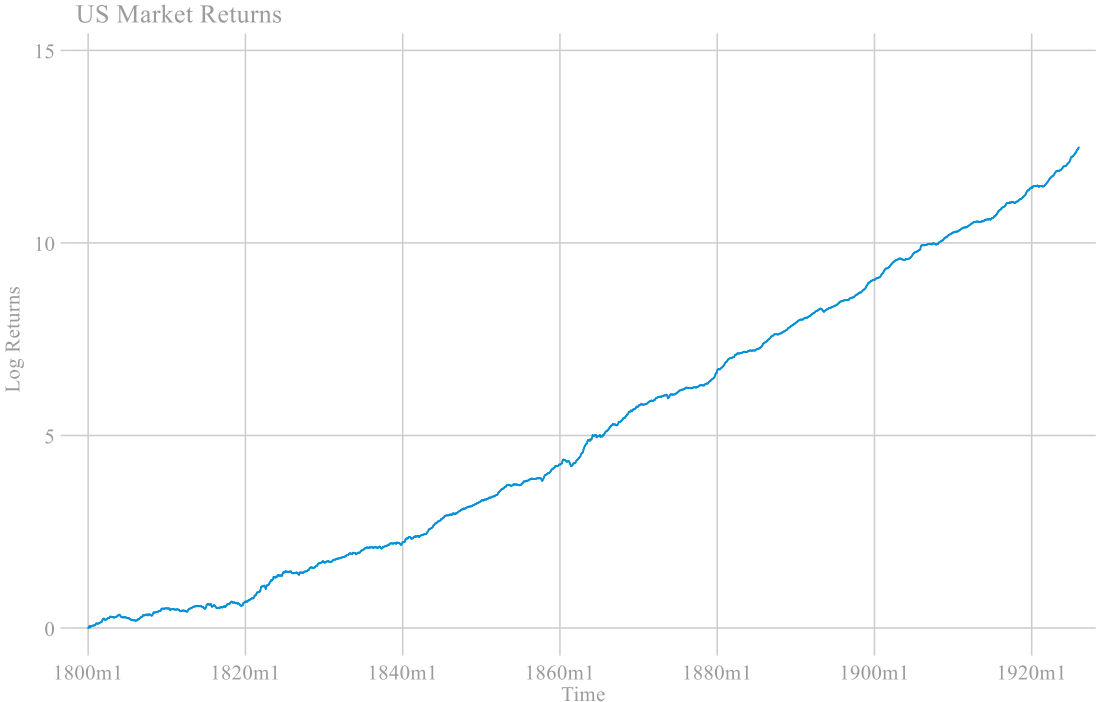
Portfolio	Period	Period	Portfolio variations		
			J = 12	J = 24	J = 36
Factor	Return	1810	1.14	-1.89	-1.29
	Alpha	1860	-1.27	-2.73	-2.32
			-1.47	-0.6	-0.68
	1910	-0.17	0.53	0.33	
		-9.45	-8.76	-8.63	
	-6.65	-6.27	-6.01		
Low Volatility	Return	1810	12.35	11.81	10.48
	Alpha	1860	2.35	3.34	3
			5.47	6.08	5.88
	1910	0.62	1.05	1	
		0.73	0.86	0.86	
	-3.74	-3.72	-3.75		
Middle Volatility	Return	1810	8.59	12.58	12.1
	Alpha	1860	-1.56	-1.06	-1.15
			5.53	5.66	5.97
	1910	-3.13	-3.01	-3.13	
		8.39	9.1	10.5	
	-1.65	-1.33	-0.65		
High Volatility	Return	1810	22.01	21.1	22.11
	Alpha	1860	7.21	6.3	7.01
			9.05	8.46	8.17
	1910	-5.27	-5.72	-5.58	
		13.36	11.68	11.27	
	-3.67	-4.14	-4.78		

**Table 37 CRSP Low volatility portfolios 50-year period yearly returns and alphas**

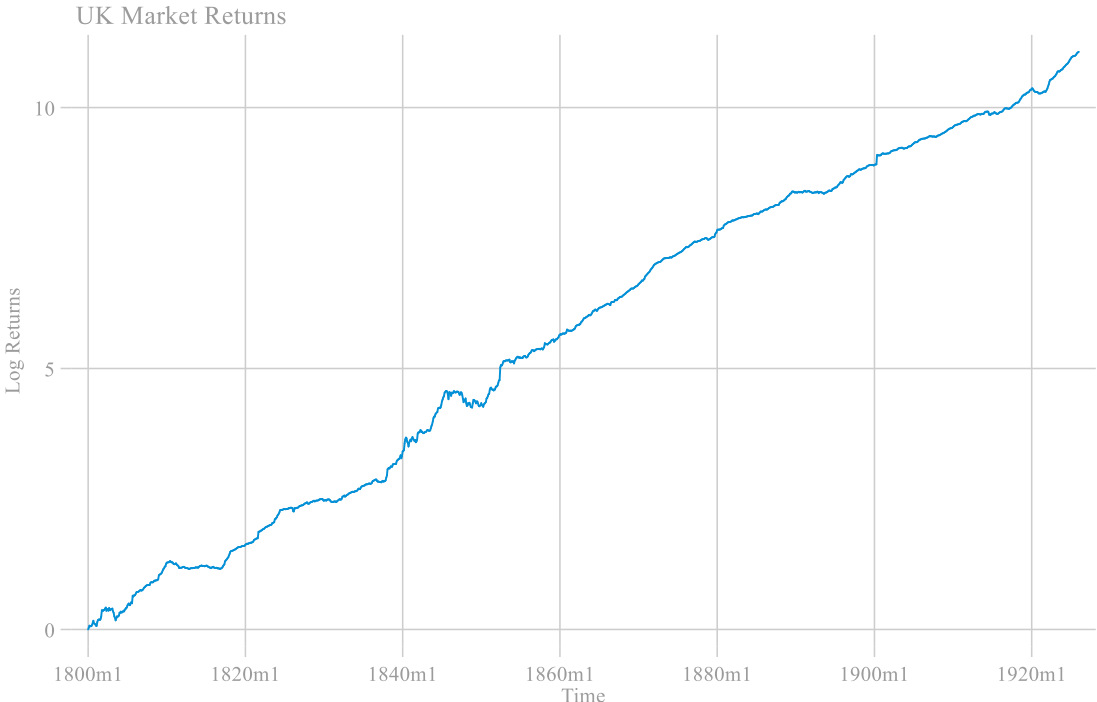
This table displays average annualized and alphas for idiosyncratic volatility factor portfolios over 50 year periods in the CRSP sample over the years 1926 - 2017. The variations shown include formation periods of 12, 24 and 36 months.

Portfolio	Period	Period	Portfolio variations		
			J = 12	J = 24	J = 36
Factor	Return	1926	2.36	2.3	3.12
			Alpha	1960	0.57
	1926	-6.33			-5.39
		Alpha	1960	-8.33	-7.4
	1926			2010	-2.01
		Alpha	2010		1.26
Low Volatility	Return			1926	12.23
		Alpha	1960		1.2
	1926			4.33	4.78
		Alpha	1960	-2.13	-1.74
	1926			2010	3.93
		Alpha	2010		-0.77
Middle Volatility	Return			1926	15.36
		Alpha	1960		1.56
	1926			12.45	12.08
		Alpha	1960	2.85	2.76
	1926			2010	12.44
		Alpha	2010		0.01
High Volatility	Return			1926	17.32
		Alpha	1960		0.29
	1926			9.64	9.36
		Alpha	1960	-1.88	-2.22
	1926			2010	10.75
		Alpha	2010		-3.62

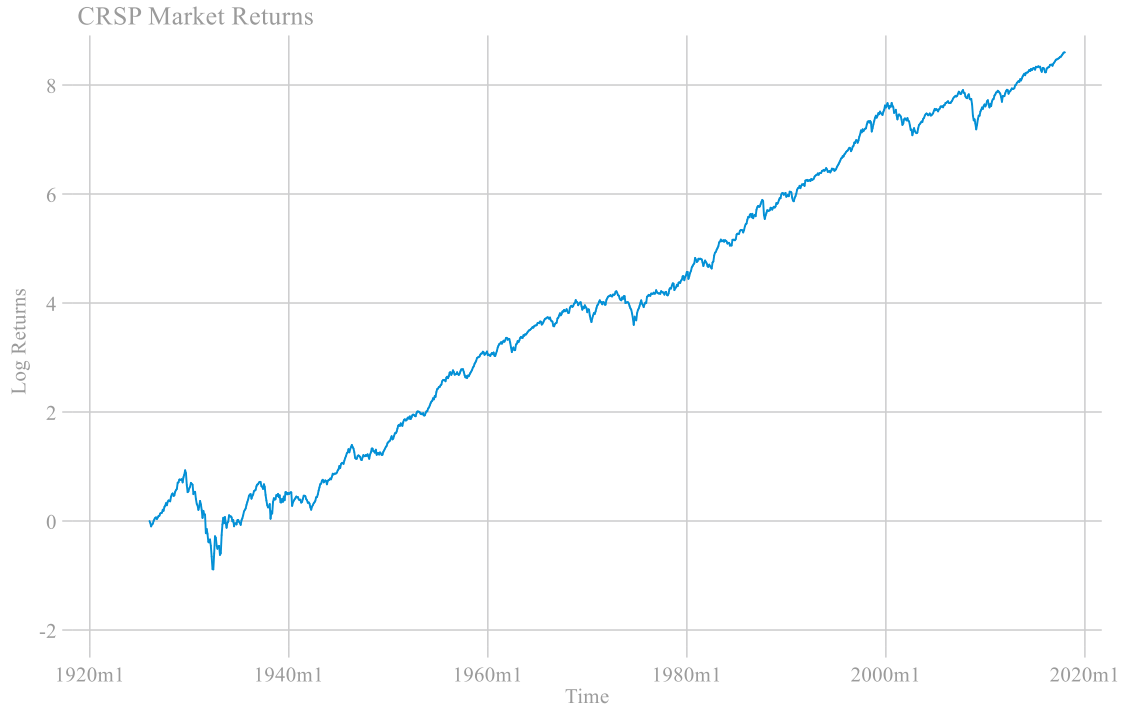
**Figure 1 Log-transformed cumulative market returns U.S.**



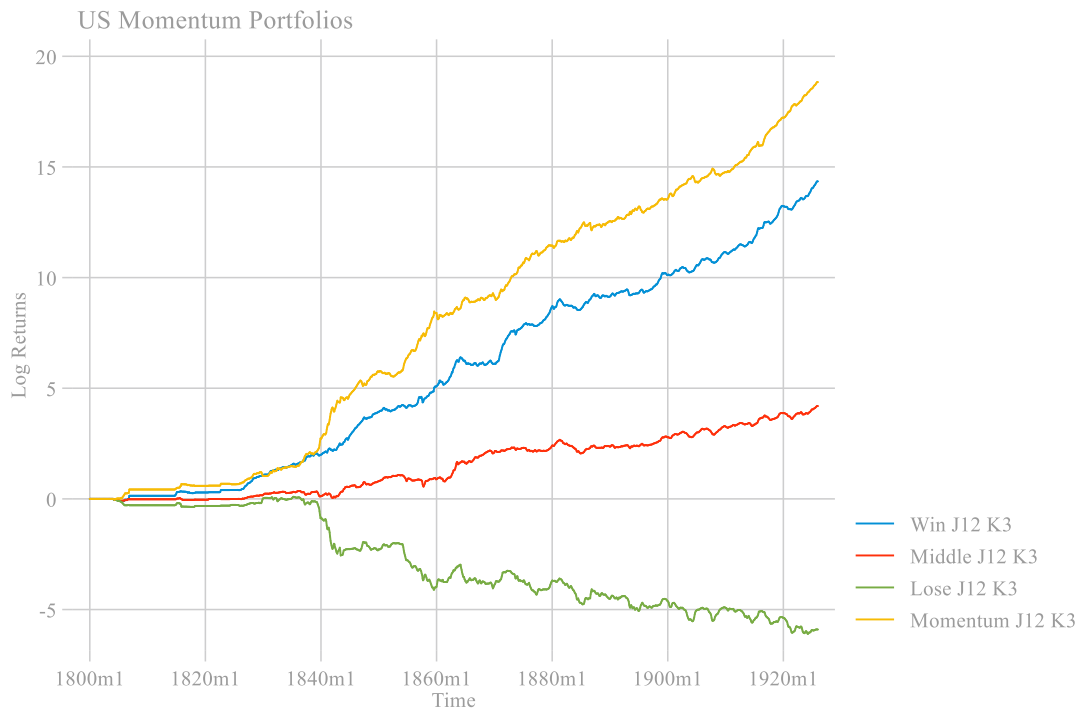
**Figure 2 Log-transformed cumulative market returns U.K.**



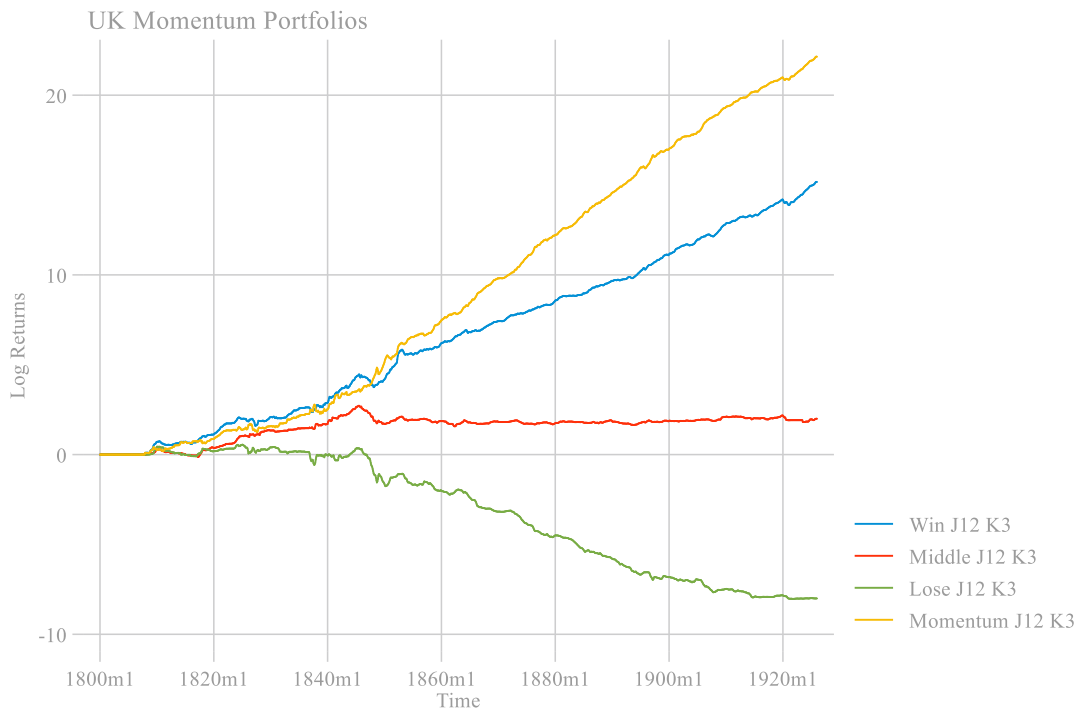
**Figure 3 Log-transformed cumulative market returns CRSP**



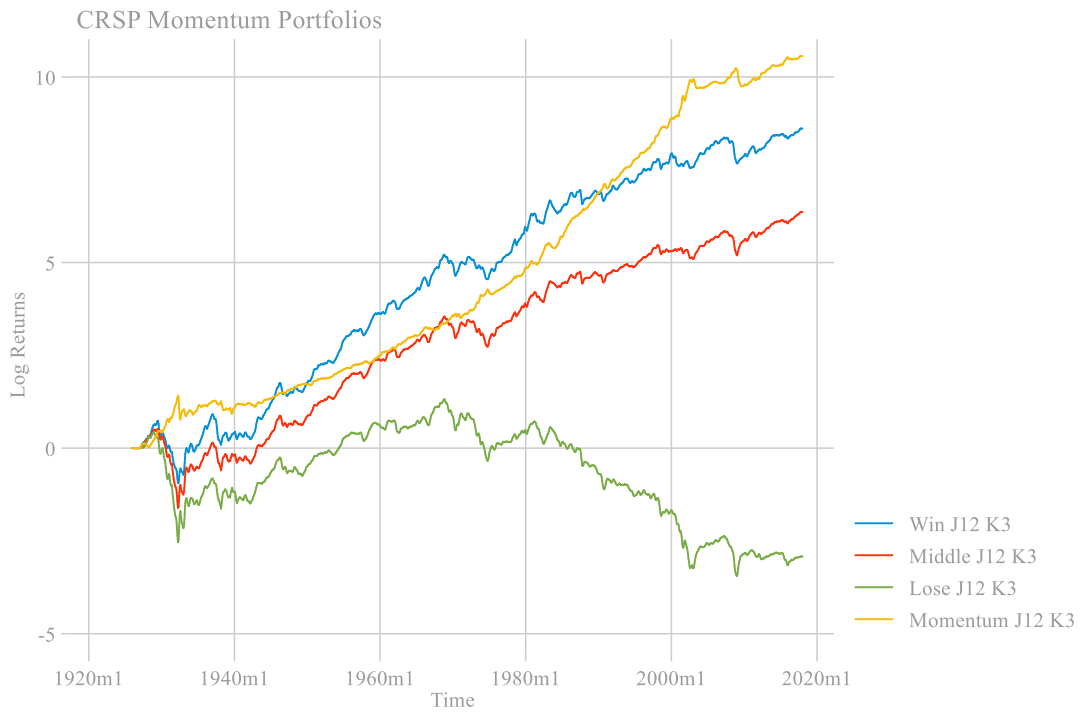
**Figure 4 Log-transformed cumulative momentum portfolio returns U.S.**



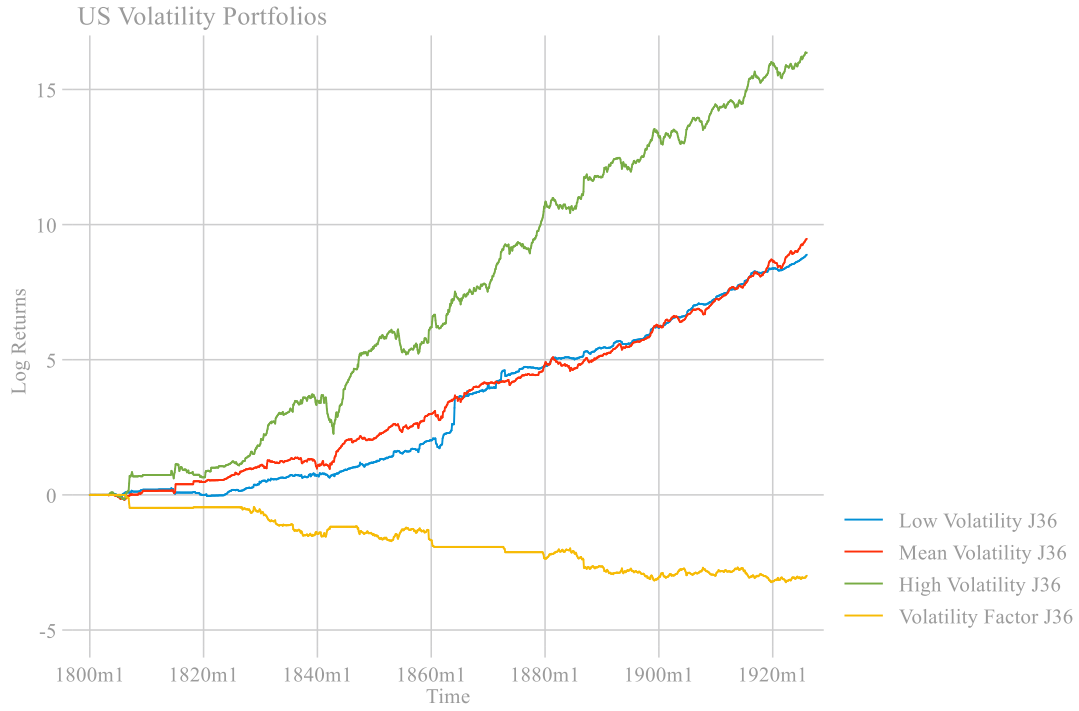
**Figure 5 Log-transformed cumulative momentum portfolio returns U.K.**



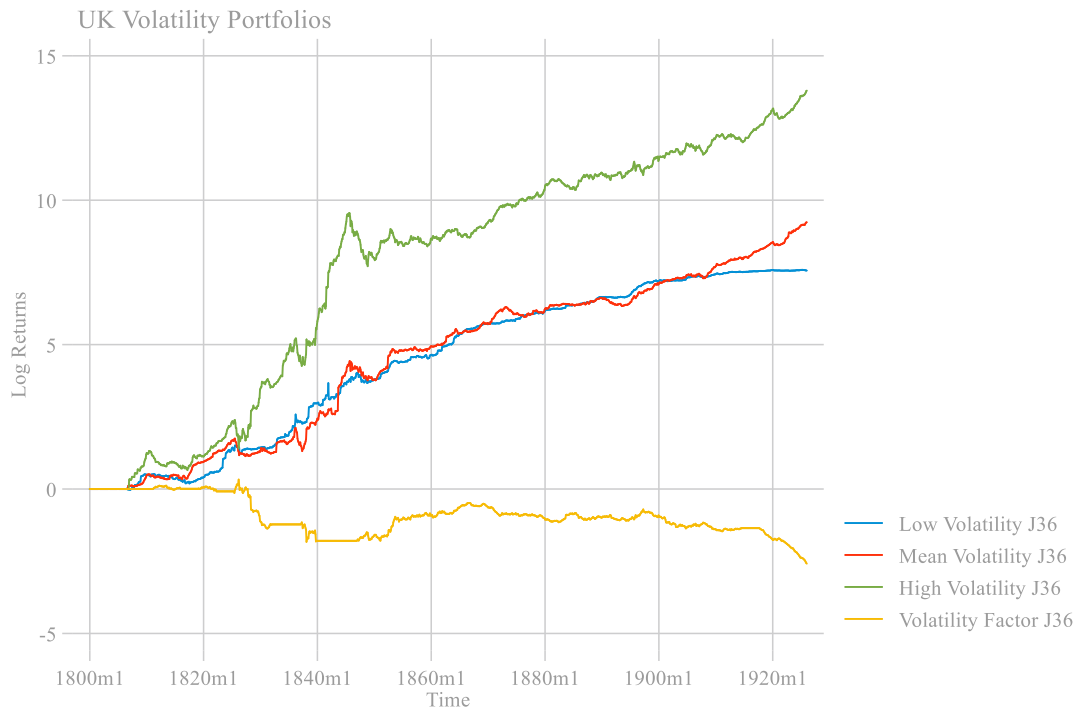
**Figure 6 Log-transformed cumulative momentum portfolio returns CRSP**



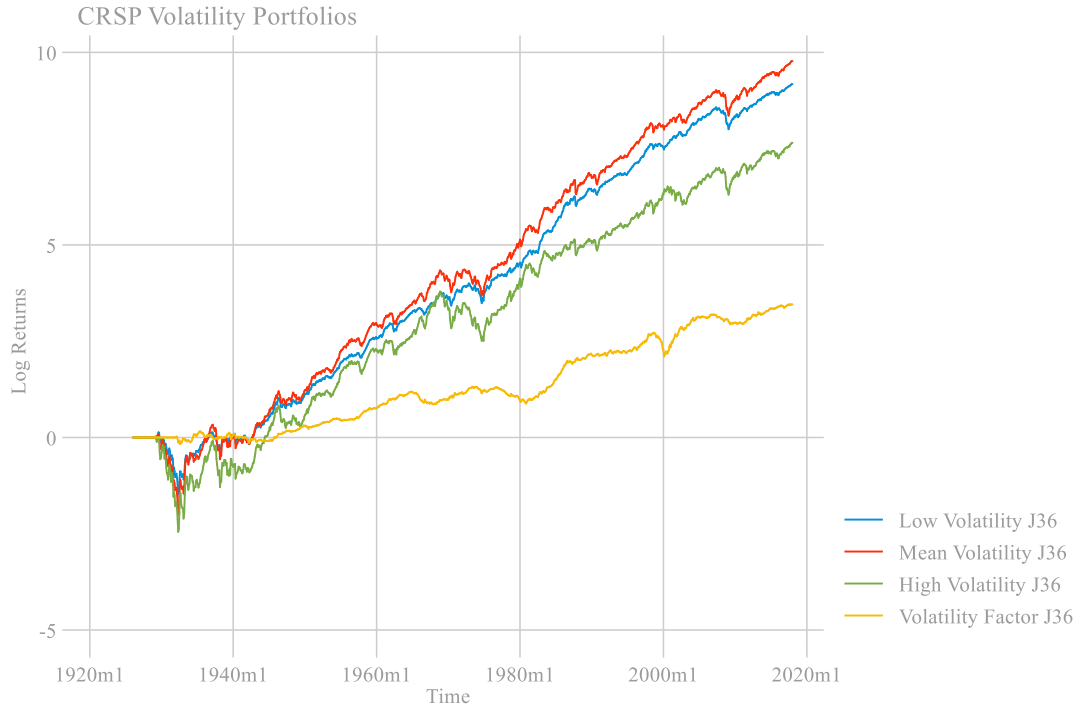
**Figure 7 Log-transformed cumulative low volatility portfolio returns U.S.**



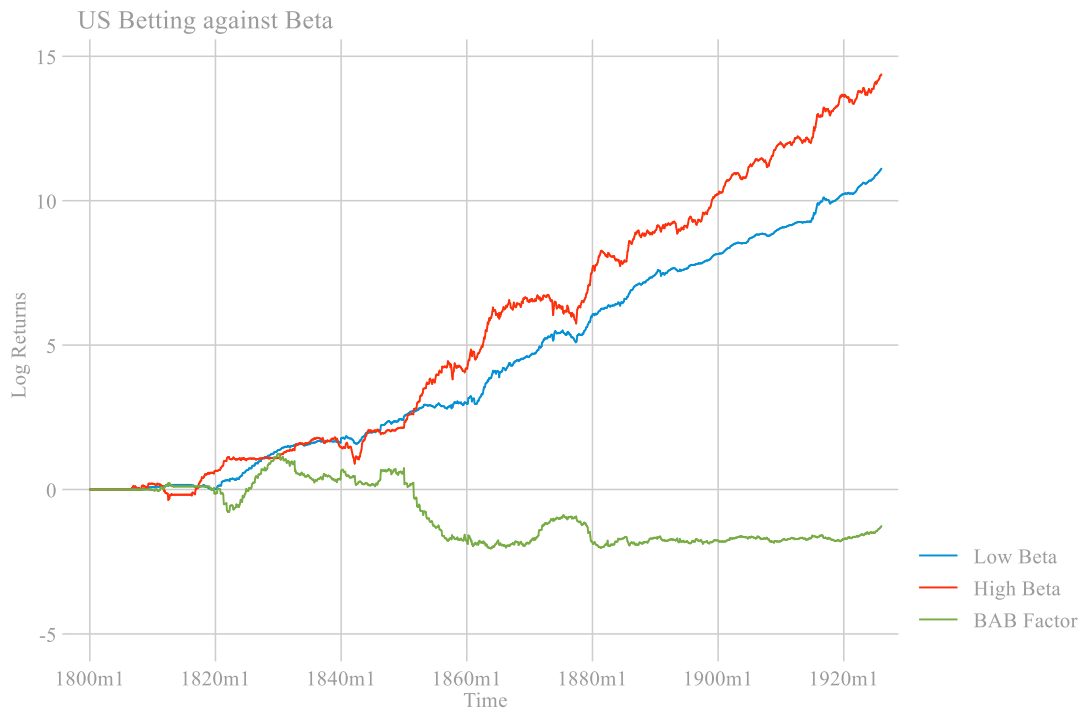
**Figure 8 Log-transformed cumulative low volatility portfolio returns U.K.**



**Figure 9 Log-transformed cumulative low volatility portfolio returns CRSP**

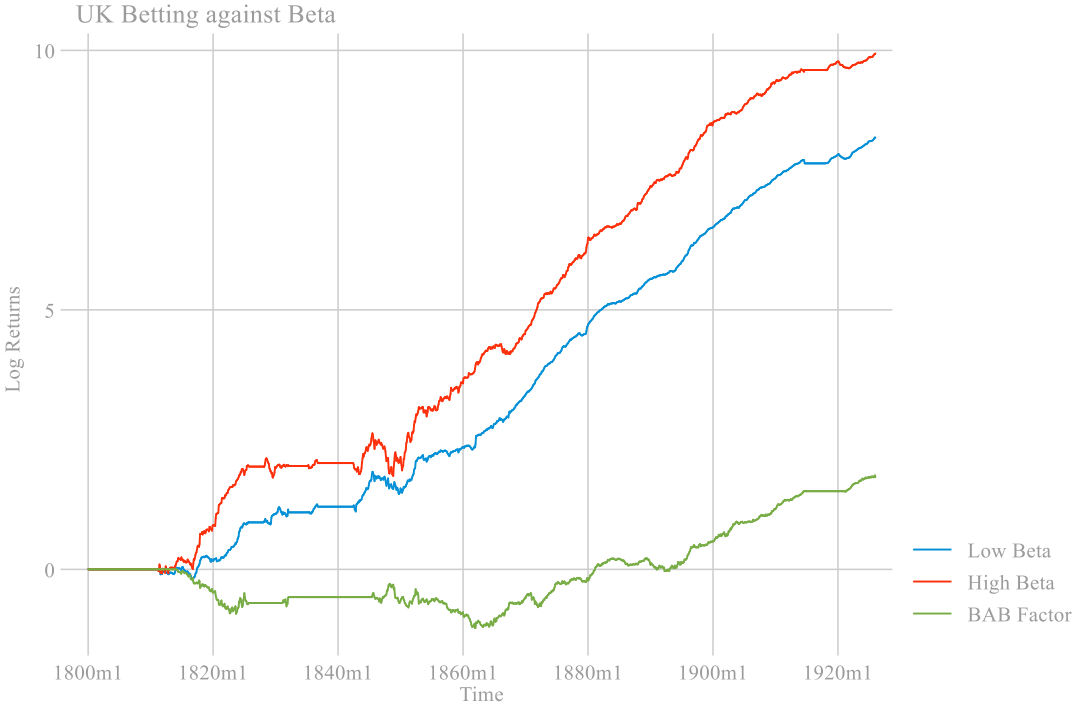


**Figure 10 Log-transformed cumulative Betting against Beta returns U.S.**

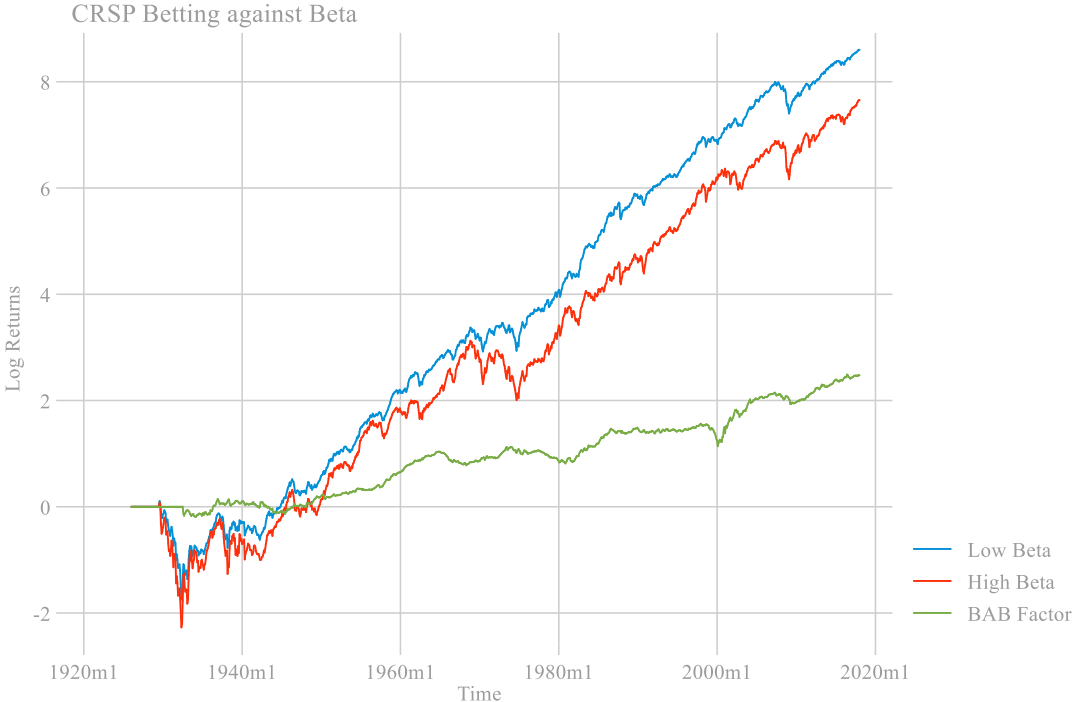




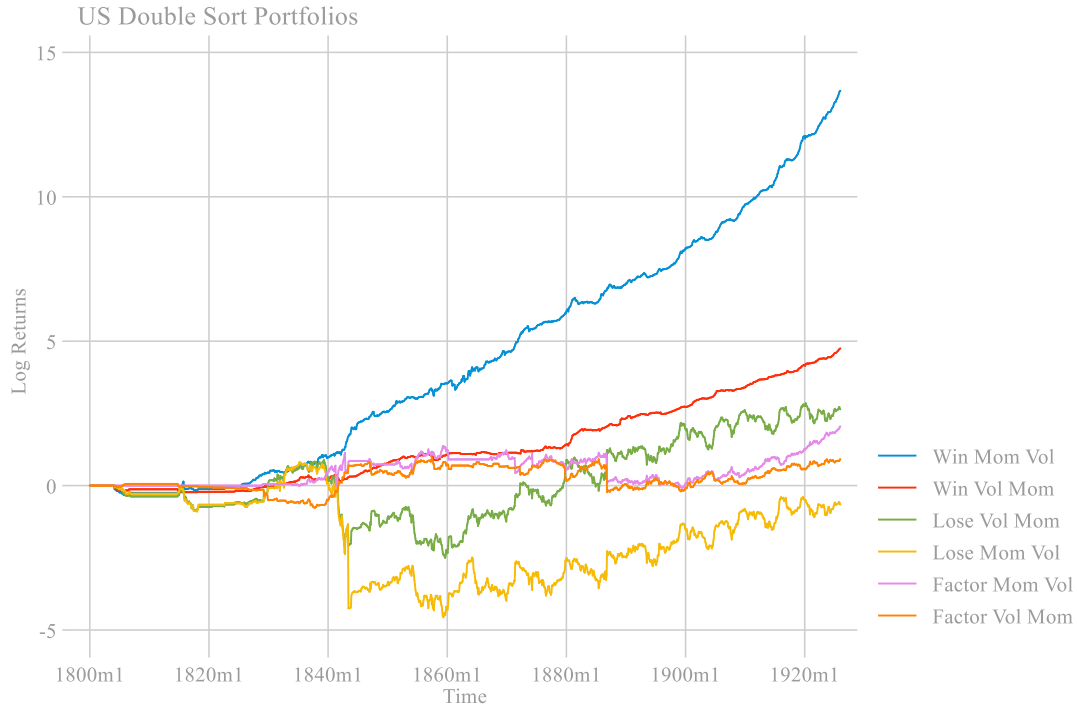
**Figure 11 Log-transformed cumulative Betting against Beta returns U.K.**



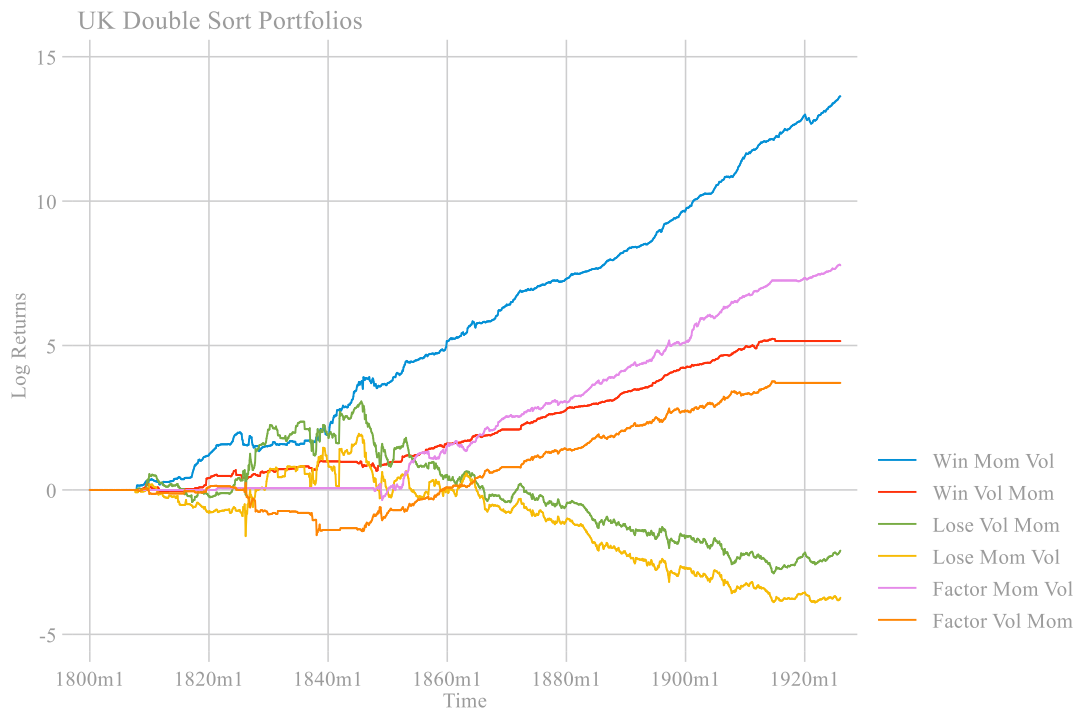
**Figure 12 Log-transformed cumulative Betting against Beta returns CRSP**



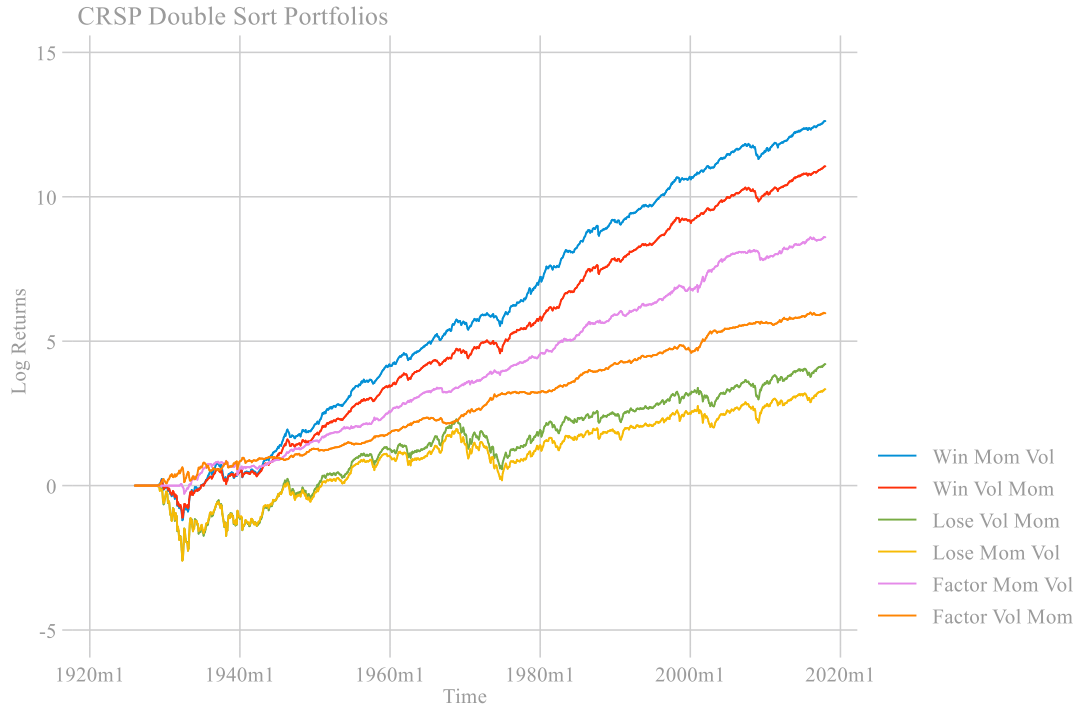
**Figure 13 Log-transformed cumulative double-sorted portfolio returns U.S.**



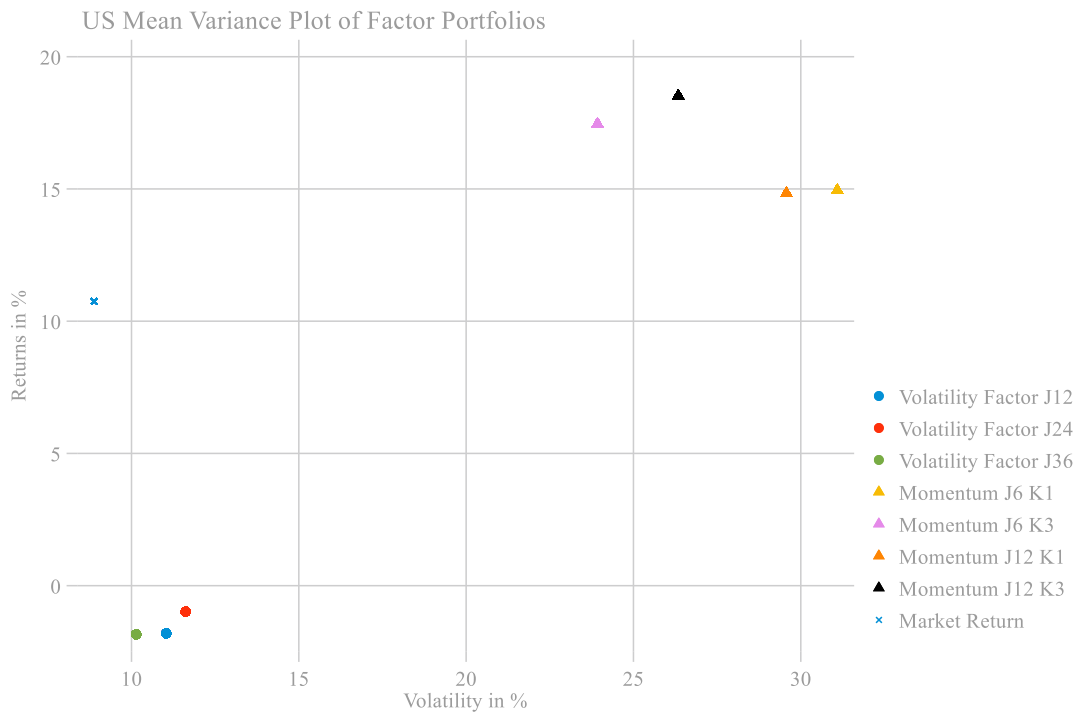
**Figure 14 Log-transformed cumulative double-sort double-sorted portfolio returns U.K.**



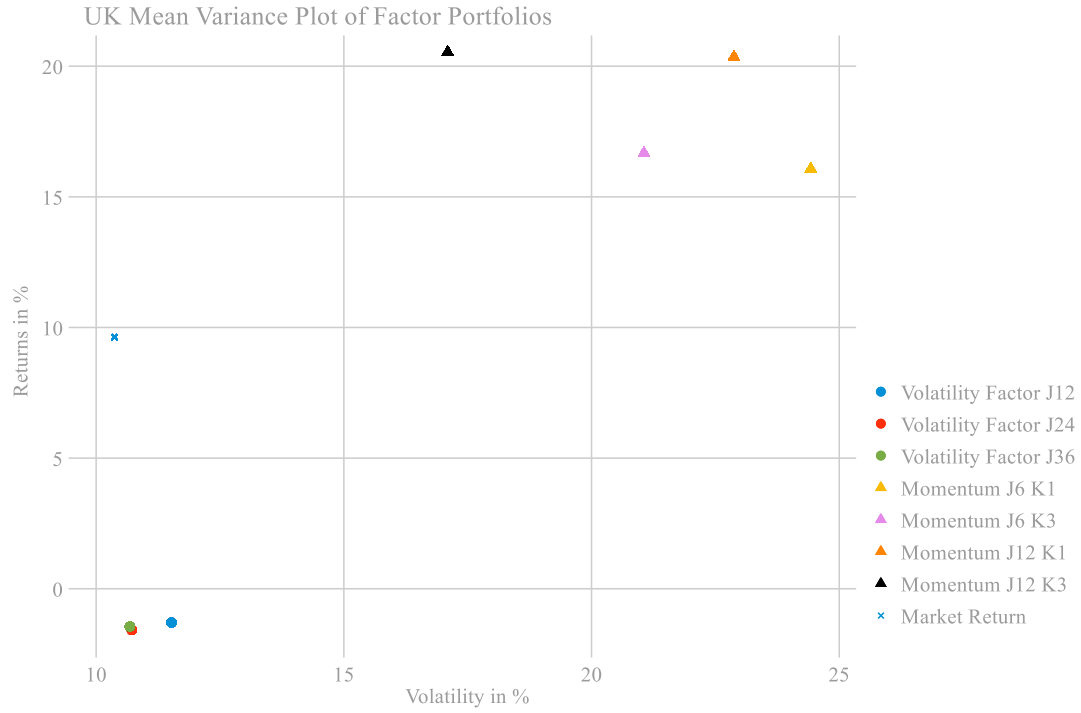
**Figure 15 Log-transformed cumulative double-sorted portfolio returns CRSP**



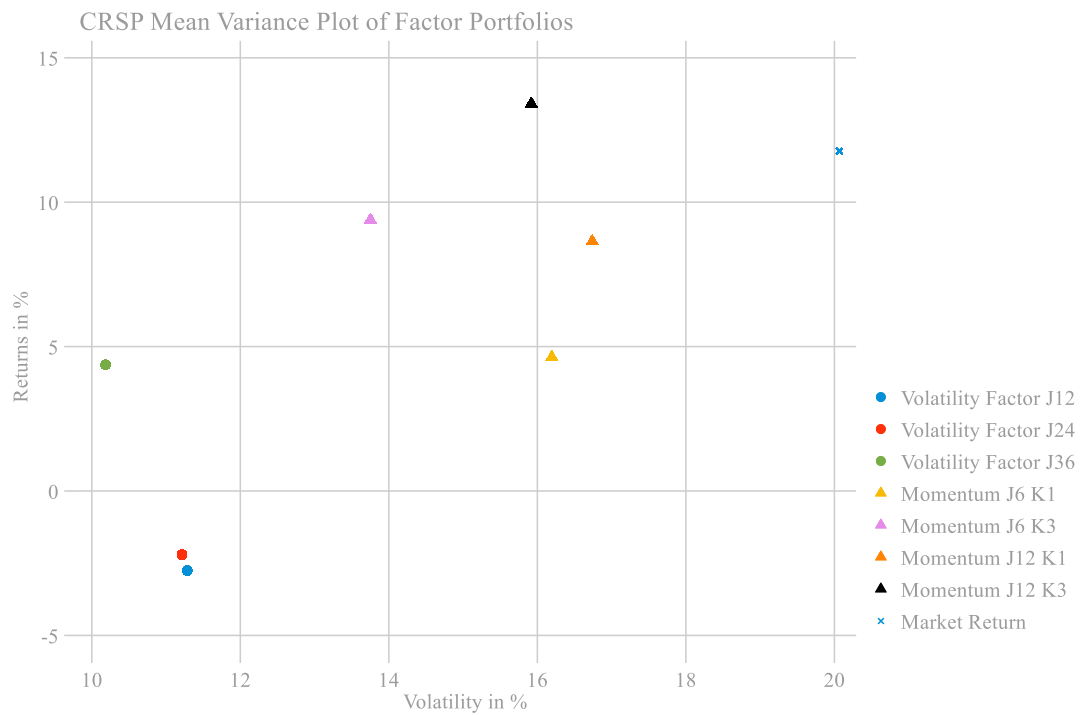
**Figure 16 Mean-variance plot of factor returns U.S.**



**Figure 17 Mean-variance plot of factor returns U.K.**



**Figure 18 Mean-variance plot of factor returns CRSP**



**Figure 19 Cumulative inflation rates of the pound and dollar**

