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To Switch or Not to Switch: Return Prediction and Financial Cycles

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Abstract

This paper emphasizes the importance of identifying changes in financial cycles when predicting monthly US excess stock returns for the period 1977 - 2017. Incorporating regime switching into the predictive models improves the quality of the excess return forecasts in terms of market timing ability, economic value and stability. The Markov Switching models consisting of predictor variables selected based on their performance during bull and bear markets performs especially well. A mean-variance investor would be willing to pay several hundreds basis points to switch from the static benchmark portfolios to one of these portfolio strategies.

Keywords: Return predictability, Markov Switching models, factor models, variable selection, Lagrange Multiplier test, market timing, economic value

Contents

1 Introduction

Forecasting stock returns has been a fascinating endeavor with a long history and has been an important field of financial research for many decades. Numerous studies have attempted to predict excess stock returns using a variety of different predictor variables and forecasting techniques. Most of the literature has focused on the predictability of stock returns using traditional valuation ratios such as the dividend-price, earnings-price and book-to-market ratios among others within a predictive regression framework. Some of these studies have shown positive forecasting results such as in Fama and French (1988) and Campbell and Shiller (1988a,1988b) among others. However, several authors have also expressed their concerns regarding the predictability of stock returns, arguing that most forecasting models have both poor in-sample and out-of-sample predictive power compared to a simple metric such as the historical average. Goyal and Welch (2008) therefore concluded that “the profession has yet to find some variable that has a meaningful and robust empirical equity premium forecasting power”.

Additionally, several studies such as Devpura et al. (2018), have also documented the instability in the relation between stock returns and the predictor variables and find substantial variation across subsamples for their forecasting accuracy. The appearance of structural breaks to the model parameters substantially affects the forecasting performance of the predictive model as the parameter estimates obtained from the historical sample do not stay constant over time. In fact, studies such as Paye and Timmermann (2006), Rapach and Wohar (2006) and Ravazazzolo, van Dijk, Paap, and Franses (2008) among others have confirmed this phenomenon of varying relationship between predictor variables and excess stock returns.

This paper aims to tackle both problems by analyzing the predictability of excess stock returns using a variety of predictor variables and by accounting for changes in the financial cycle. Predictive regression models consisting of three types of explanatory variables, namely: (1) traditional financial variables and valuation ratios, (2) macroeconomic variables and (3) technical indicators are used to forecast excess stock returns. The state of the equity market i.e. bullish and bearish states is incorporated to improve the forecasting accuracy of the predictive regression model. The forecasting power of these variables are assessed during both market states. Ultimately, the aim of this paper is to link the explanatory performance of certain predictor variables to bull and bear markets and to use them in regime switching models, accordingly.

Considering macroeconomic information to predict the equity premium seems profound as the stock market is assumed to be linked to business conditions. However, while many macroeconomic variables are available, only a select number of them seem to have positive explanatory power in predicting stock returns. Few examples are the consumption-wealth ratio and survey-based measures of expected business conditions as proposed by Lettau and Ludvigson (2001) and Campbell and Diebold (2009), respectively. Cakmakli and van Dijk (2016) show that combining the information of a large set of macroeconomic predictor variables improves the forecasting power for stock returns. They propose a factor-augmented predictive regression model where they extract factors from a large set of macroeconomic variables in order to forecast US excess stock returns. They find that this macroeconomic factor based model improve upon benchmark models that include valuation ratios and interest rate related variables and naive predictors such as the historical average.

From a practitioners point of view, trading strategies based on technical indicators have shown profitable results. Therefore, it seems interesting to consider technical indicators as possible predictor variables for excess stock returns. Although technical trading rules are widespread used among practitioners, literature regarding its explanatory power of the equity risk premium is rather scarce. Neely et al (2013) show that technical indicators have significant forecasting power in predicting excess stock returns and that they capture a different type of relevant information than macroeconomic variables. Therefore, it seems advantageous to build a forecasting model where different types of predictor variables, whether it be valuation ratios, macroeconomic variables or technical indicators, are combined in order to adequately describe and predict excess stock returns.

This paper will focus on factor-augmented predictive models as in Cakmakli and van Dijk (2016) to predict excess stock returns. Given the large set of macroeconomic variables, a dynamic factor approach is considered in order to account for model uncertainty, parameter estimation uncertainty, and structural instability jointly. To be more specific, a principal component analysis is used to extract a small amount of factors from the large set of macroeconomic variables, which are then used in the factor-augmented regression model. The same approach is used to construct predictive models based on technical indicators, using factors obtained from 14 commonly used technical indicators based on moving averages, momentum, and volume as discussed in Neely et al, (2013). Predictive regression models based on traditional financial variables are constructed using valuation ratios and interest-rate variables as in Goyal and Welch (2008).

Changes in the financial cycle are captured using regime switching models consisting of several explanatory variables. This type of model enables to specify the relation between the explanatory variables and stock returns during different market conditions. By doing so, implementing regime switching to the predictive models could improve the stability of the return forecasts. Following Kole and van Dijk (2017), parametric Markov Switching models are used to identify and predict future market states as this method works best out-of-sample. Markov Switching models consisting of predictor variables are considered, which is supported by their finding that including macro-financial predictor variables improves the forecasting performance of the Markov Switching models. The variables included in the Markov Switching models are selected based on their forecasting performance during periods of bull and bear markets. A Lagrange Multiplier test for omitted variables as discussed in Hamilton (1996) is used to obtain this set of best performing variables for both market states. This method is used to evaluate the marginal increase in the likelihood function when a set of explanatory variables is included to a Markov Switching model consisting of only a level parameter. By doing so, the regressor variables are linked to a specific state based on their explanatory power in describing excess stock returns. These variables are then used in Markov Switching models to obtain excess return forecasts during the corresponding market state.

An empirical analysis of the predictive power of the Markov Switching models and factor-augmented predictive regression models for monthly US excess stock returns over the period January 1977 until December 2017 is conducted. The forecasting accuracy of these models are assessed both in statistical and economic terms and are tested against several static benchmark portfolio strategies. The directional accuracy of the excess return forecasts is used to evaluate the market timing ability of the predictive regression models. The economic value of the predicted excess returns are assessed by including them in active mean-variance investment strategies. Next to Sharpe ratios, a utility-based metric is considered to evaluate the amount a mean-variance investor is willing to pay to switch from the static benchmark portfolios to active investment strategies based on excess return forecasts obtained from the predictive regression models.

The results show that incorporating regime switching into the models improves the forecasting accuracy in several ways. First, the Markov Switching models have superior market timing ability over the factor-augmented predictive regression models. Second, the economic value of active investment strategies based on the excess return forecasts obtained from the Markov Switching models are considerably higher compared to models without regime switching. A mean-variance investor would be willing to pay an annual performance fee of up to c. 200 basis points to switch from the static benchmark portfolio strategies to the predictions obtained from the Markov Switching models. In addition, implementing regime switching slightly improves the stability of the return forecasts over the sub periods.

The results of the Lagrange Multiplier test demonstrates that certain variables are preferred in predicting excess stock returns during periods of bull and bear markets. In general, macroeconomic variables and technical indicators appear to be good performers during both bull and bear markets, in particular the Federal Funds rate and the trading rule based on the on-balance volume, respectively. This paper demonstrates that the performance of Markov Switching models improve even further when predictor variables are included based on their performance during the market states. A mean-variance investor would now be willing to pay an annual performance fee of up to 370 basis points to switch from the same static benchmark portfolios to the return forecasts obtained from the Markov Switching model with selected predictor variables.

The results of this research is both relevant as an addition to the financial literature regarding equity premium forecasting, as well as for practitioners who are seeking to enhance investment performances. From the standpoint of practitioners in finance, the ability to improve stock return forecasts becomes even more relevant. Especially now, with the continued pressure on the asset management industry due to the increasing popularity of passive investing strategies, being able to produce significant positive alpha is of crucial importance.

Several studies have analyzed the effect of regime switching on stock return predictability and found positive results. For example, Jacobsen et al. (2012) demonstrates that economically important industrial metals have positive state-switching return predictability for stock returns and Hammerschmid and Lohre (2018) show that macroeconomic and technical information used to predict equity risk premia demonstrate profitable predictive power along different market states and periods. To the best of my knowledge, this is the first paper to comprehensively evaluate and compare the forecasting performance of traditional financial variables, macroeconomic variables and technical indicators during periods of bull and bear markets and link variables to market states based on their explanatory power.

2 Literature review

The first academic study to asset return forecasting started almost a century ago with a paper written by Cowles (1933). In this paper, the aggregated US stock market return is forecasted using technical analysis. In the 1960's, the empirical research further developed into examining whether individual stocks could be predicted using filter rules such as the moving average. However, none of these paper showed any ability to accurately forecast excess returns (Elliot and Timmermann, 2013).

The first real evidence on predictability of aggregated stock returns came in the late 1970's and 1980's when numerous economic predictor variables, such as the dividend-price ratio and earnings-price ratio, were used within a predictive regression framework to show that they where able to capture return predictability as in Fama and French (1988) and Campbell and Shiller (1988a,1988b) among others. Chen et al. (1986) explored the effect of several economic state variables on stock market returns and found that several of them, such as industrial production, changes in risk premium and twists in the yield curve, among others were significant in explaining expected stock returns. Furthermore, Campbell and Thompson (2008) among others, pointed out that yields on short and long term treasury and corporate bonds are correlated with stock returns.

In the early 2000's, a number of authors expressed their concerns over the predictability of stock returns. Goyal and Welch (2008) reexamined the performance of variables that have been suggested by the academic literature to be good predictors of the equity premium. They argue that by large, these models have predicted poorly both in-sample as well as out-of-sample compared to the historical average, are unstable, and would not have helped investors with access only to available information to profitably time the market. The authors therefore concluded that "the profession has yet to find some variable that has meaningful and robust empirical equity premium forecasting power".

In the following years, a collection of studies actually showed that certain forecasting approaches could improve the forecasting performance in such a manner that the historical average was beaten. Examples of these approaches are using a factor approach (Cakmakli and van Dijk, (2016)), incorporating technical indicators (Neely et al, (2013)) and creating forecast combinations (Rapach et al., (2010)). Cakmakli and van Dijk (2016) show that factors extracted from a large set of macroeconomic variables contain information that can be useful for predicting excess stock returns. Their results show that factor-augmented predictive regression models improve upon benchmark models that include only valuation ratios and interest rate related variables, and possibly individual macro variables, as well as the historical average excess return. These improvements are both statistically and economically significant due to the stability of their forecasting accuracy.

Neely et al. (2013) utilizes technical indicators to directly predict excess stock returns and find statistically and economically significant in-sample and out-of-sample forecasting power. Also, they show that technical indicators and macroeconomic variables capture different types of information which are relevant for predicting stock returns. Technical indicators (macroeconomic variables) better detect decline (rise) in the equity risk premium near business-cycle peaks (troughs) and therefore combining this information produces superior stock return forecasts. Rapach et al. (2010) argues that combining forecasts delivers statistically and economically significant out-of-sample gains relative to the historical average consistently over time. They provide two empirical explanations for the benefits of forecast combinations: (i) the incorporation of information from numerous economic variables while substantially reducing forecast volatility, (ii) combination forecasts of the equity premium are linked to the real economy.

Several studies such as Pesaran and Timmermann (1995), Ang and Bekaert (2007) and Goyal and Welch (2008) have documented the instability in the relation between stock returns and several predictor variables and find substantial variation across subsamples in the coefficients of return prediction models and in the degree of return predictability. Building on this evidence, I want to analyse whether the forecasting accuracy of the proposed methods in this research improve when the state of the equity market is incorporated. To identify current and predict future market states, I will use the methodology proposed by Kole and van Dijk (2017). They find that using Markov switching models are preferred to forecast future states of the market out-of-sample compared to semi-parametric rule-based methods. Arguing that, as Markov switching models use both the mean and variance to infer the states, they produce superior forecasts and lead to significantly better out-of-sample performance than rule-based methods.

3 Data

The analysis throughout this paper is based on monthly excess returns on the S&P 500 index for the sample period from January 1967 until December 2017. The risk-free rate will be proxied using the one-month T-bill rate obtained from the updated data set of Goyal and Welch (2008)

, which will be gathered from the Federal Reserve Bank of St. Louis.

Table 1: Summary statistics of the monthly excess return on the S&P 500 index.

Mean	Variance	Minimum	Maximum	Kurtosis	Skewness
0.26%	0.19%	-22.27%	15.68%	4.79	-0.45

Table 1 shows the summary statistics of the monthly excess returns for the complete sample period, with mean 0.26% and variance 0.19%. The minimum and maximum monthly excess return are equal to -22.27% and 15.68%. The data exhibits a slightly negative skewness and a somewhat higher kurtosis than under a normal distribution, with skewness and kurtosis equal to -0.45 and 4.79.

In this research, three types of explanatory variables will be considered to describe and forecast excess stock returns, namely financial variables, macroeconomic variables and technical indicators. The dataset of Goyal and Welch (2008) will be used as it is widely applied in research for predicting stock returns. This set of financial predictor variables consist of the dividend yield, price-earnings ratio, risk-free rate and its first lag, and the default spread defined as the difference between Moody's Baa and Aaa corporate bond yields. The FRED monthly dataset will be used to obtain a wide variety of macroeconomic variables consisting of several categories such as: Output and Income, Employment and Hours, Sales, Consumption, Housing starts and Sales, Inventories, Orders, Exchange rates, Money and credit quantity aggregates, Interest Rates and Spreads, Price indexes and Average hourly earnings, consisting of 128 macroeconomic variables in total. See appendix **A** for more information regarding the individual macroeconomic variables, data transformations and outlier treatment. To avoid look-ahead biases, the macroeconomic variables will be lagged by one month such that it does not contain any information that was not available at the time the forecast is made.

The technical indicators used in this research consist of the 14 variables as described in Neely et al. (2013). These technical indicators are based on three popular strategies i.e. moving-average (MA), momentum and trading volume. The MA rule compares two different moving averages to generate trading signals at the end of time t, with $TS_{MA,t} = 1$ and $TS_{MA,t} = 0$ representing a buy and sell signal, respectively. The moving averages are computed as follows:

$$MA_{j,t} = \frac{1}{j} \sum_{i=0}^{j-1} SP_{t-i} \text{ for } j = d, e \tag{1}$$

with SP_t representing the level of the S&P 500 index at time t and d (e) the length of the short (long) moving average ($d < e$). The moving-average trading rule which is used to identify buy and sell signals is formulated as follows:

$$TS_{MA,t} = \begin{cases} 1, & \text{if } MA_{d,t} \geq MA_{e,t} \\ 0, & \text{if } MA_{d,t} < MA_{e,t} \end{cases} \tag{2}$$

Logically, the short MA is more sensitive to movements in the S&P 500 index as the inclusion of new information has a stronger impact on a smaller sample. As an example, if the S&P 500 index begins an upward trend, then the SMA will increase more than the LMA, generating a buy signal as it indicates a bullish breakout and that the trend is shifting up. Monthly MA rules with $d = 1, 2, 3$ and $e = 9, 12$ will be considered in this research.

The Momentum-based trading rule is based on the empirical findings that rising asset prices tend to rise further. A buy or sell signal is generated using the following formula:

$$TS_{M,t} = \begin{cases} 1, & \text{if } SP_t \geq SP_{t-h} \\ 0, & \text{if } SP_t < SP_{t-h} \end{cases} \tag{3}$$

The intuition behind this expression is that if the current stock price is higher than its level h periods ago, a positive momentum is indicated and therefore higher expected excess returns. The momentum indicator which is denoted by $MOM(q)$ compares SP_t to SP_{t-h} with monthly signals for $h = 9, 12$.

The volume-based trading rule is based on the empirical finding that assets with rising prices and high trading volumes tend to rise further. Combining trading volume information with past prices can therefore be used to identify market trends. The strategy used in this research is based on the "on-balance" volume as discussed in Granville (1963) and is defined as follows:

$$OBV_t = \sum_{k=1}^{t} VOL_k D_k, \tag{4}$$

with VOL_k and D_k denoting a measure of the trading volume during period k and a binary variable which equals one if $SP_k - SP_{k-1} \geq 0$ and -1 otherwise, respectively. A buy or sell signal is generated using the following expression:

$$TS_{VOL,t} = \begin{cases} 1, & \text{if } MA_{s,t}^{OBV} \geq MA_{l,t}^{OBV} \\ 0, & \text{if } MA_{s,t}^{OBV} < MA_{l,t}^{OBV} \end{cases}, \quad \text{with } MA_{j,t}^{OBV} = \frac{1}{j} \sum_{i=0}^{j-1} OBV_{t-i} \quad \text{for j=d,e.} \tag{5}$$

The trading indicator is denoted by $VOL(d, e)$ where monthly trading signals are computed for $d = 1, 2, 3$ and $e = 9, 12$.

All the aforementioned trading indicators have a binary output and therefore their first differences are also considered as these might contain more information. The first differences are expressed as follow:

$$\begin{aligned} D_{MA,t} &= MA_{d,t} - MA_{e,t} \\ D_{SP,t} &= \log SP_t - \log SP_{t-h} \\ D_{VOL,t} &= MA_{d,t}^{OBV} - MA_{e,t}^{OBV} \end{aligned} \tag{6}$$

with $D_{MA,t}$, $D_{SP,t}$ and $D_{VOL,t}$ representing the difference in the strategies based on the moving average, momentum and on-balance volume.

Due to the large size of the datasets, a principal component analysis will be used to extract a small amount of factors from the macroeconomic variables and technical indicator/difference variables. The scree plot in figure **1** shows the eigenvalues for the first six principal components. For all three datasets, the scree plot illustrates that the first principal components is good for explaining the bulk of the variance in the datasets.

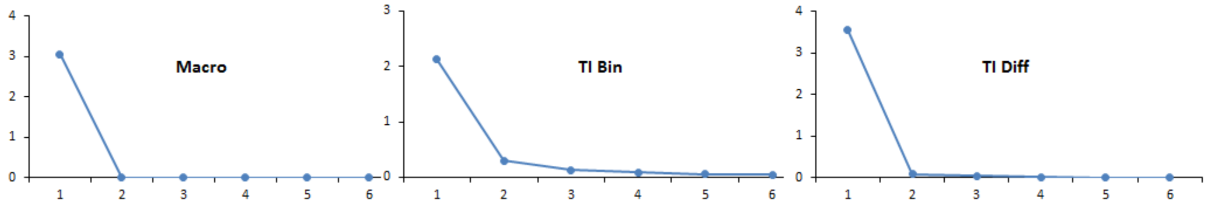

Figure 1: **Scree plot for principal components.** Notes: The graphs illustrate the eigenvalues for the first 6 principal components. Abbreviations are as follows: Macro stands for macroeconomic variables. TI Bin stands for technical indicator variables and TI Diff stands for the difference of the technical indicator variables as in equation **6**.

4 Methodology

This section describes the methodology used to forecast excess stock returns using predictive regression models. The forecasts are obtained using a recursive approach, meaning that all models are specified using only historical information up to and including period t to forecasts excess returns in period $t + 1$. Also, a moving window approach with a window length of 10 years will be used to obtain the excess return forecasts. First, individual predictive regressions will be conducted in order to evaluate the performance of the variables separately. Thereafter, the forecasting accuracy when combining the information across the variables will be analyzed. Next, the inclusion of the state of the equity market using Markov switching models will be discussed. This section will also describe the procedure to link certain predictor variables to bull and bear market based on their explanatory power during the market states. Finally, the last section will discuss the forecasting evaluation methods to compare the forecasting accuracy of the models proposed above to the benchmark model.

4.1 Equity premium forecasting using different types of explanatory variables

The first model to be evaluated consists of traditional financial predictor variables as in Goyal and Welch (2008) and is denoted as:

$$r_{t+1} = \beta_0 + \beta_v' v_t + \epsilon_{t+1} \tag{7}$$

with v_t a $(l \times 1)$ vector of financial predictor variables as described in the data section

A different approach is considered for the models consisting of macroeconomic variables and technical indicators. As the data consist of a large set of variables, including all of them in the predictive model could result in a significant higher parameter estimation uncertainty and therefore deteriorate the forecasting accuracy. Several approaches exist to avoid this issue i.e. parameter selection procedures, model averaging and forecast combinations among others. In order to take all the information embedded in the set of variables into account, a statistical factor model as in Cakmali and van Dijk (2016) is considered. In this approach, it is assumed that the variables obey a factor structure of the form:

$$z_t = \Lambda f_t + e_t \tag{8}$$

With z_t a $(N \times 1)$ vector of variables, f_t a $(c \times 1)$ vector of common factors, given that the number of elements c is significantly smaller than the number of variables N. The factors f_t are assumed to be mutually orthogonal and are ordered such that the first factors captures the bulk of the variation in the predictor variables z_t. The factors considered in this approach are latent, but can be consistently estimated using a principal component analysis, as was discussed by Stock and Watson (2002a,b), among others.

After obtaining factor estimates using principal component analysis on the macroeconomic variables, a factor-augmented predictive regression to forecast excess returns can be constructed as follows:

$$r_{t+1} = \beta_0 + \beta_v' v_t + \beta_{f,m}' f_{m,t} + \epsilon_{t+1} \tag{9}$$

with $f_{m,t}$ representing the factors obtained from the set of macroeconomic variables and v_t the financial predictor variables. A factor-augmented predictive model consisting of only macroeconomic factors will also be considered, which is obtained by using equation **9** with $\beta_v = 0$.

Using this factor-based approach to forecast excess stock returns has the benefit of exploiting the available information in all the predictive variables. However, several studies such as those by Boivin and Ng (2006) and Bai and Ng (2008) have shown that pre-selecting the variables used in the construction of the factors based on their individual predictive ability can improve the forecasting performance of the factor-augmented model. This, as the bulk of the variation in the variables captured by the first principal components need not be the most relevant information for predicting excess returns. This procedure will only be considered for the macroeconomic case because the set of macroeconomic predictor variables is large enough for a pre-selection method to make sense. By using a pre-selection method, the set of macroeconomic predictor variables could be reduced to a smaller set consisting of variables that exhibit individual predictive power in explaining excess stock returns. The research of Cakmakli and van Dijk (2016) confirms the finding of Bai and Ng (2008) by showing that implying pre-selection methods to the set of macroeconomic variables indeed adds more economic value than simply using all available macroeconomic variables. Therefore, it seems useful to implement pre-selection procedures when constructing factors in order to enhance the predictive power of the factor model.

The **hard** thresholding method will be considered to pre-select the macroeconomic variables, which is based on the individual predictive power of the macroeconomic variable m_{it} for excess return r_{t+1}. To determine this, r_{t+1} is regressed onto each macroeconomic variable m_{it}, $i = 1, 2, ..., n$ separately. The initial set of macroeconomic variables will reduced to a new set consisting of b predictor variables with $b < n$ for which the estimated coefficient $\beta_{m,i}$ is significantly different from zero, given a significance level α. Now, the factors will be constructed by using principal components as before, but on a smaller set of predictors obtained using the hard thresholding rule. The new factor-augmented predictive regression model can be constructed by replacing $f_{m,t}$ from equation **9** with the factors obtained from the set of macroeconomic variables after applying the hard thresholding method.

Factor estimates for the set of technical indicator variables and their first difference are obtained using principal components as mentioned before. The factor-augmented predictive models are then constructed similar to the case including macroeconomic variables, using equation **9** with and without v_t. Additionally, predictive regression models are constructed using financial variables v_t together with 3 technical indicator variables. These variables are selected based on their predictive power for excess returns. The procedure to obtain these variables is similar to the hard thresholding method, with the 3 variables selected based on the magnitude of their t-statistics.

As a significant amount of literature questions the predictive power of the aforementioned variables, the combined information of financial, macroeconomic and technical indicator variables are considered to predict excess stock returns. Factor estimates are obtained using principal components on this complete variable set. The factor augmented predictive regression model is represented as follow:

$$r_{t+1} = \beta_0 + \beta'_{ALL} f_{ALL,t} + \epsilon_{t+1} \tag{10}$$

With $f_{ALL,t}$ denoting the factors obtained using a principal components analysis on the entire data set.

Additionally, predictive regression models are constructed based on the individual predictive power of all the considered variables with respect to excess returns. The process is similar to the **hard** thresholding method in the sense that the variables with the highest t-statistics for their estimated coefficients are selected.

4.2 Analysing the explanatory power of predictor variables during financial cycles

As several studies have documented the instability in the relation between stock returns and the predictor variables, the state of the equity market will be incorporated to improve the forecasting accuracy of the predictive regression models. In order to do so, a Markov Switching model in the style of Hamilton (1989, 1990) and as in Kole and van Dijk (2017) is used in order to both identify and predict financial cycles. By doing so, it is assumed that the state of the economy is proxied by the state of the equity market, which follows a first-order Markov chain. A Markov Switching model with two regimes is considered i.e. bullish state ($S_t = 1$) and bearish state ($S_t = 2$).

I build a model that switches from the set of best performing explanatory variables during specific market states. First, the performance of different regressor variables are evaluated during bull and bear markets. Thereafter, these results will be incorporated in the Markov Switching regression model to describe the return process in each state. For example, if variable set 1 and 2 are best in describing excess stock returns during bull and bear markets respectively, then the return process in the Markov Switching regression model will be described using the model containing variable set 1 for bull markets and variable set 2 for bear markets.

To select the variables that are used to predict excess returns during bull and bear markets, a Lagrange Multiplier test for omitted variables is applied as in Hamilton (1996). First, the score of the simple Markov Switching model with respect to the mean parameters will be computed. By doing so, a state-specific score is obtained as the mean parameters are linked to a specific market state. The score for the more general model can then be obtained by multiplying the score of the Simple Markov Switching model with the value of the corresponding explanatory variables. This score is then applied in the Lagrange Multiplier test in order to verify the marginal increase in the likelihood function if the constraints were relaxed. The restriction is then rejected if the marginal effects are too large, indicating that the additional variable have significant explanatory power in describing excess stock returns. The magnitude of the statistic is used to select which set of explanatory variables perform better during periods of bull and bear markets. For example, suppose one want to compare the performance of all the proposed variables during bull markets. Then, these variables will separately be used in the Lagrange Multiplier test as described above. The one with the highest test statistic is the variable that performs best compared to the others during bull markets and will therefore be used in the Markov Switching regression model to describe the dynamics of stock returns during bull markets.

Let S_t denotes an unobserved random variable that reflects the state of the market during time t, with $S_t = 1$ and $S_t = 2$ indicating that the process is in a bull and bear market, respectively. The state of the equity market follows a first-order Markov chain with transition probabilities calculated as:

$$p_{ij} \equiv P[S_t = j | S_{t-1} = i], \quad i, j \in \{0, 1\}, \tag{11}$$

The Markov Switching models consisting of explanatory variables are used to characterize the time-series behaviors in both regimes as follows:

$$r_t = \begin{cases} \mu_1 + \beta_1' x_{1,t-1} + \sigma_1 \epsilon_t, & \text{if } S_t = 1 \\ \mu_2 + \beta_2' x_{2,t-1} + \sigma_2 \epsilon_t, & \text{if } S_t = 2 \end{cases} \tag{12}$$

with $\beta_j = (\beta_{j,1}, .., \beta_{j,k})'$ a $(k \times 1)$ vector representing the coefficients of the variables that describe the return process during bull and bear markets. The excess stock returns have state-specific coefficients (μ_j, β_j) and variances σ_j^2 for both states. As the volatility is higher during bear markets, the states are ordered based on the estimated volatility with restriction $\sigma_1 < \sigma_2$ to ensure that the first state $(S_t = 1)$ is labeled as a bull market. The function $f(r_t|x_t, S_t; \theta)$ is used to denote the normal pdf with state-dependent parameters:

$$f(r_t|x_{t-1}, S_t; \theta) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left(-\frac{(r_t - \mu_j - \beta_j' x_{j,t-1})^2}{2\sigma_j^2}\right) \tag{13}$$

with θ a $((2k+4) \times 1)$ vector consisting of the regression coefficients and variances

$$\theta = (\mu_1, \beta_1, \mu_2, \beta_2, \sigma_1^2, \sigma_2^2)' \tag{14}$$

In order to analyze the performance of the explanatory variables during bull and bear markets, a Lagrange Multiplier test for omitted variables will be used as in Hamilton (1996). The simple Markov Switching model will be used as the base model for all the Lagrange Multiplier tests, which is denoted as the model in equation **12** consisting of only the level parameters μ_j. The score of the Markov Switching model is defined as the derivative of the **observed** conditional log-likelihood of the tth observation with respect to the parameter vector θ. To obtain the **observed** likelihood from the specification containing unobservables (13), the transition probabilities that governs changes in the states (11) are needed. The **observed** (log)likelihood for the Markov Switching model in equation **12** is parameterized by λ, which consists of both the parameters θ and the transition probabilities, $\lambda' = (\theta', p')$ and can be written as follows:

$$\begin{aligned}
\mathcal{L}(\mathbf{r}_T|\mathbf{x}_{T-1};\lambda) &= f(r_1, r_2, ..., r_T|x_1, x_2, ..., x_{T-1};\lambda) = \prod_{t=1}^{T} f(r_t|\mathbf{r}_{t-1}, \mathbf{x}_{t-1};\lambda) \\
&= \prod_{t=1}^{T} [f(r_t|\mathbf{r}_{t-1}, \mathbf{x}_{t-1}, S_t = 1;\theta)P(S_t = 1|\mathbf{r}_{t-1}) + f(r_t|\mathbf{r}_{t-1}, \mathbf{x}_{t-1}, S_t = 2;\theta)P(S_t = 2|\mathbf{r}_{t-1})] \\
\ell(\mathbf{r}_T|x_{t-1};\lambda) &= \sum_{t=1}^{T} \log[f(r_t|\mathbf{r}_{t-1}, \mathbf{x}_{t-1}, S_t = 1;\theta)P(S_t = 1|\mathbf{r}_{t-1}) + f(r_t|\mathbf{r}_{t-1}, \mathbf{x}_{t-1}, S_t = 2;\theta)P(S_t = 2|\mathbf{r}_{t-1}
\end{aligned} \tag{15}$$

At first, it seems hard to obtain the derivative of the **observed** log likelihood function for each time t as the states S_t in equation **13** are unobserved. However, this problem can be solved by considering the unobserved states S_t as being part of the dataset. Then, using the Expectation Maximization (EM) algorithm as proposed by Hamilton (1989), the EM estimator $\tilde{\lambda}$ of the parameter vector λ can be obtained, see appendix **B** for a detailed overview of the procedure.

The score of the tth observation is denoted by the vector-valued function $h_t(\tilde{\lambda})$ and is obtained by taking the derivative of the log of the **observed** likelihood (15) with respect to the parameter vector θ. Appendix **C** shows that the score with respect to θ for the **observed** likelihood specification as in 15 is given by:

$$h_t(\tilde{\lambda}) = \frac{\partial \log f(r_t|\mathbf{r}_{t-1}, \mathbf{x}_{t-1};\lambda)}{\partial \theta} = \sum_{j=1}^{2} \psi_{t,j} P[S_t = j|\Omega_t] + \sum_{\tau=1}^{t-1} \sum_{j=1}^{2} \psi_{\tau,j} \left(P[S_\tau = j|\Omega_t] - P[S_\tau = j|\Omega_{t-1}]\right) \tag{16}$$

where $t = 1, 2, ..., T$ and

$$\psi_{t,j} = \frac{\partial \log f(r_t|x_{t-1}, S_t = j;\theta)}{\partial \theta} \tag{17}$$

To evaluate **16**, the inferred and forecasted state probabilities are needed which can be obtained by estimating the Markov Switching model using the EM algorithm. Note that in order to compute the score of the models, these state probabilities are calculated for the **simple** Markov Switching model only.

The scores with respect to the variance and the transition probabilities are not required for evaluating the performance of the explanatory variables during different market states. The focus will be on the score with respect to the mean parameters during bull and bear markets. The main idea behind this procedure is to address the size of the LM test statistic when relaxing the restriction on the regressor parameters in the more general models containing explanatory variables as in equation **12**. The resulting return process is then equivalent to that of the **simple** Markov Switching model, containing only a level parameter which corresponds to the mean of the excess returns. The estimated value of the mean for both market states is obtained using the EM algorithm. First, the scores of the **simple** Markov Switching model with respect to the mean parameters are computed using equation 16 with

$$\psi_{t,j} = \frac{(r_t - \mu_j)}{\sigma_j^2}, \quad \text{for } j = 1, 2 \tag{18}$$

Thereafter, restrictions are imposed on the more general model containing explanatory variables as in equation **12**. Under the null hypothesis, the additional variables $x_{j,t}$ do not add significant explanatory value in describing excess stock returns. Therefore, the restriction $\beta_j = 0$, with β_j a $(\kappa \times 1)$ vector, is imposed to test the marginal effects on the likelihood function when relaxing this constraint. Note that under this restriction, the regression form as in the **simple** Markov Switching case is obtained. Calculations similar to those above reveal that the score of the more general Markov Switching model with respect to the restricted parameter β_1 is obtained using:

$$\begin{aligned} \frac{\partial \log f(r_t|\mathbf{r}_{t-1}, \mathbf{x}_{t-1}; \lambda, \beta_j)}{\partial \beta_j}\Big|_{\beta_j=0} &= \sum_{j=1}^{2} \psi_{t,j} P(S_t = j|\Omega_t; \lambda) \\ &+ \sum_{\tau=1}^{t-1} \sum_{j=1}^{2} \psi_{\tau,j} [P(S_\tau = j|\Omega_t; \lambda) - P(S_\tau = j|\Omega_{t-1}; \lambda)] \end{aligned} \tag{19}$$

where

$$\psi_{t,j} = \frac{\partial \log f(r_t|x_{t-1}, S_t; \theta, \beta}{\partial \beta_1}\Big|_{\beta_1=0} \propto \frac{(r_t - \mu_j - \beta_1 x_{t-1})x_{t-1}}{\sigma_j^2}\Big|_{\beta_1=0} = \frac{(r_t - \mu_j)x_{t-1}}{\sigma_j^2} \tag{20}$$

Let $\tilde{\lambda}$ denote the EM estimator of $\lambda' = (\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, p_{11})$ as described before in the simple Markov Switching case and thus ignoring the effects of the explanatory variables. Let λ^* denote the $((5+\kappa) \times 1)$ parameter vector of the more general model including explanatory variables $\lambda^* = (\lambda', \beta)'$ with associated constrained EM estimator $\tilde{\lambda}^* = (\tilde{\lambda}, 0)'$. Under the null hypothesis, the score for the more general model $h_t(\tilde{\lambda}^*)$ is obtained by multiplying the score for the basic model $h_t(\tilde{\lambda})$ with the value of the explanatory variables at time

t (x_t). $h_t(\tilde{\lambda}^*)$ is then used in the Lagrange Multiplier test to evaluate the effect of the explanatory variables as follow:

$$\left[T^{-\frac{1}{2}}\sum_{t=1}^{T}h_t(\tilde{\lambda}^*)\right]'\left[\frac{1}{T}\sum_{t=1}^{T}\sum_{t=1}^{T}[h_t(\tilde{\lambda}^*)][h_t(\tilde{\lambda}^*)]'\right]^{-1}\left[T^{-\frac{1}{2}}\sum_{t=1}^{T}h_t(\tilde{\lambda}^*)\right]\sim\chi^2(l) \quad (21)$$

By doing so, a χ^2 statistic is obtained with k (number of parameter restrictions) degrees of freedom under the null hypothesis that x_t does not add significant explanatory power in describing excess stock returns.

This test is easy to implement. One needs to estimate the model under the null hypothesis first, in this case the **simple** Markov Switching model. The inferred and predicted state probabilities are obtained when estimating this model with the EM algorithm using the restricted parameters $\lambda' = (\mu_1, \mu_2, \sigma_1^2, \sigma_2^2)$ as described in appendix **B** . After obtaining this, the score $h_t(\tilde{\lambda})$ of the simple Markov Switching model is computed. Then, the score of the general model containing explanatory variables with respect to the restricted parameters $\beta_1 = 0$ is computed using the smoothed probabilities obtained from the restricted model (simple Markov Switching model). Finally, the score of the model under the null hypothesis $h_t(\tilde{\lambda})$ is obtained by multiplying the score of the restricted model with the explanatory variables x_t and plugged into 21.

4.3 Equity premium forecasting by implementing a latent state variable

The variable sets obtained using the Lagrange Multiplier test are used in Markov Switching models to predict the returns during the corresponding market states. As mentioned before, these variables are selected based on their explanatory power during periods of bull and bear markets. The best performing variable set during bull and bear markets are denoted by x_1 and x_2, respectively and implemented in the Markov Switching model of equation **12**.

Using this approach, the state of the equity market is incorporated to improve the forecasting accuracy. More specifically, the model predicts upcoming bull and bear markets and uses the predicted state probabilities to weight the excess return forecasts obtained from the best performing explanatory variables during this market states as follows:

$$\hat{r}_{t+1} = [P(S_{t+1} = 1|\Omega_t), P(S_{t+1} = 2|\Omega_t)] \begin{bmatrix} \hat{\mu}_1 + \hat{\beta}_1' x_{1,t} \\ \hat{\mu}_2 + \hat{\beta}_2' x_{2,t} \end{bmatrix} \tag{22}$$

with $P(S_{t+1} = j|\Omega_t)$ for $j = 1, 2$, the predicted state probability obtained using the Hamilton filter as shown in appendix **D**. By doing so, the problem of structural breaks in the relation between stock returns and explanatory variables is avoided as the model switches dynamically from best performing regressors based on the predicted state probabilities. Next to this, this model can be interpreted as a forecast combination as it uses the predictive state probabilities to weight the obtained return forecasts from both specifications.

Additionally, Markov Switching models are constructed using the same set of variables as in the factor-augmented predictive regression models (section **4.2**) to predict excess returns in both bull and bear markets. By comparing the return predictions from this Markov Switching model with the corresponding factor-augmented predictive regression models, the stand-alone effect of regime switching on the return predictions will be assessed.

4.4 Forecast evaluation

Two different evaluation methods are used to assess the performances of the predictive regressions models. First, the forecasting power will be evaluated in statistical terms by considering the market timing ability of the predicted returns. Traditional statistics such as the mean squared prediction error (MSPE) are not used throughout this research as Pesaran and Timmerman (1995), among others, argued that the outcome of such measures are not necessarily linked to profitable investment decisions. Therefore, the market timing ability is used to evaluate the return forecasts in a statistical manner as this is generally better in capturing their economic value in terms of the performance of the corresponding investment strategies. Second, the return forecasts are evaluated based on their economic value explicitly when used in active mean-variance investment strategies. Therefore, a mean-variance investor with a monthly horizon is considered who allocates a proportion of her wealth to stocks and a riskless asset based on its excess return predictions. Following Cakmakli and van Dijk (2016), a utility-based metric is used to evaluate the amount that an investor would be willing to pay in order to switch from the static benchmark strategies to the strategies using the return predictions obtained from the predictive regression models as proposed in this paper.

4.4.1 Market timing ability

The Market timing ability is assessed by means of the hit ratio, which is defined as the proportion of correctly predicted signs of the monthly excess return forecasts and represented as:

$$\hat{P} = \frac{1}{N}\sum_{t=1}^{N} I[r_{t+1} \times \hat{r}_{t+1} > 0], \tag{23}$$

with N being the number of predictions obtained in the forecasting period and $I[x]$ an indicator function which equals one when the corresponding argument is positive and zero otherwise. In order to assess the market timing ability of the forecasts, the null hypothesis of no market timing ability will be tested by evaluating whether the empirical hit ratio $\hat{P}$ is significantly higher than the expected hit ratio when assumed that the signs of the actual and forecasted value are independent. This directional accuracy (DA) test statistic was proposed by Pesaran and Timmermann (1992) and is defined as:

$$DA = \frac{\hat{P} - \hat{P}^{*}}{\sqrt{\hat{V}(\hat{P}) - \hat{V}(\hat{P}^{*})}} \sim N(0,1), \tag{24}$$

With $\hat{P}^{*}$ being the expected hit ratio under the independence assumption and computed as $\hat{P}^{*} = \hat{P}_r\hat{P}_{\hat{r}} + (1-\hat{P}_r)(1-\hat{P}_{\hat{r}})$. $\hat{P}_r$ and $\hat{P}_{\hat{r}}$ are efficient estimators of the probability that the actual (r) and forecasted ($\hat{r}$) returns are positive and is represented as the proportion of months for which r and $\hat{r}$ are positive, respectively. Hence, $\hat{P}_r = \frac{1}{N}\sum_{t-1}^{N} I[r_{t+1}]$ and $\hat{P}_{\hat{r}} = \frac{1}{N}\sum_{t-1}^{N} I[\hat{r}_{t+1}]$. The variance estimates of $\hat{P}$ and $\hat{P}^{*}$ are computed as $\hat{V}(\hat{P}) = \frac{1}{N}\hat{P}^{*}(1-\hat{P}^{*})$ and $\hat{V}(\hat{P}^{*}) = \frac{1}{N}(2\hat{P}_r - 1)^2\hat{P}_{\hat{r}}(1-\hat{P}_{\hat{r}}) + \frac{1}{N}(2\hat{P}_{\hat{r}} - 1)^2\hat{P}_r(1-\hat{P}_r) + \frac{4}{N^2}\hat{P}_r\hat{P}_{\hat{r}}(1-\hat{P}_r)(1-\hat{P}_{\hat{r}})$. See Pesaran and Timmermann (1992) for more detail regarding the derivations of these expressions.

As mentioned before, the directional accuracy (DA) test is a statistical method for analyzing the directional predictive power of the forecasting models. However, a positive outcome of this test does not necessarily mean that the models also provide positive economic value. For example, the DA statistic can imply that a certain model is good in predicting the direction of the returns correctly. However, it could still be the case that the losses due to incorrect direction forecasts can exceed the gains obtained when the directions are predicted correctly. Therefore, it is important to consider this statistic in combination with the economic value of the excess return forecasts.

4.4.2 Economic value

In this section, the performances of the trading strategies constructed using the return forecasts will be evaluated based on its economic value explicitly. As in Cakmakli and van Dijk (2016), a utility-based metric will be used in order to assess how much an investor would pay in order to switch from using the predictions obtained from the benchmark models to the predictions obtained from the predictive regression models as proposed in this research. A mean-variance investor with a monthly horizon is considered with her position in stocks and the risk-free rate (T-bills) determined by using the return forecasts in the following objective function:

$$\max_{w_{t+1}} E_t[r_{p,t+1}] - \frac{1}{2}\gamma \mathrm{Var}_t[r_{p,t+1}], \tag{25}$$

with γ representing the relative risk aversion (RRA) and $E_t[r_{p,t+1}]$ and $Var_t[r_{p,t+1}]$ the expected value and variance of the portfolio return $r_{p,t+1}$ conditional on the information available in period t. Following Cakmakli and van Dijk (2016), the value for the relative risk aversion is set equal to 6 throughout this research. The portfolio return is obtained by:

$$r_{p,t+1} = r_{f,t+1} + w_{t+1} r_{t+1}, \tag{26}$$

where the risk-free rate and portfolio weight in period $t+1$ is denoted by $r_{f,t+1}$ and w_{t+1}. The mean-variance investor determines the portfolio weights by solving the objective function in equation 25, such that the optimal weight to invest in stocks is given by:

$$w^*_{t+1} = \frac{E_t[r_{t+1}]}{\gamma Var_t[r_{t+1}]} \tag{27}$$

As mentioned before, the mean-variance investor determines the fraction invested in stocks based on the return predictions. Therefore, in order to compute the portfolio weights, the investor uses its return forecasts as estimates for the conditional expectation given in 27. The conditional variance is estimated using the realized variance over month t, which is computed using daily return as recent literature such as Andersen, Bollerslev, Christoffersen & Diebold (2006), among others, has indicated the gain in accuracy of the volatility estimates due to higher frequencies. Therefore, the conditional variance used to compute the portfolio weights is obtained using the following formula for the realized variance:

$$\sigma_t^2 = \sum_{t=1}^{n_t} (r_{i,t} - \bar{r}_t)^2 [1 + \frac{2}{n_t} \sum_{j=1}^{n_t} (n_t - j) \hat{\phi}_t^j] \tag{28}$$

with $r_{i,t}$ the daily return over month t, n_t the number of trading days during the corresponding month, $\bar{r}$ the monthly mean over the daily returns and ϕ_t the first-order autocorrelation of the daily returns. Considering a case where short selling and leveraging are prohibited, the portfolio weights are restricted to be between 0 and 1, thus $w_{t+1}^* \in [0, 1]$. Also, new return and volatility predictions become available each month and the investor rebalances its portfolio accordingly. Therefore, the realized portfolio returns are assessed net of transaction costs, where transaction costs are defined as a fixed proportion of the investment when changing the allocation to stocks from w_t to w_{t+1}. As the investment space is defined such that it consist of stocks and risk-free assets only, the investor pays transaction costs twice when rebalancing as its position in stocks and the risk-free asset is adjusted simultaneously. Hence, the loss in gain in the form of transaction costs is defined as:

$$c_{t+1} = 2c|w_{t+1} - w_t|, \tag{29}$$

where c is the fixed proportion of wealth invested and set equal to 0.1%. The gross portfolio return net of transaction costs is then defined as $R_{p,t+1} = 1 + r_{p,t+1} - c_{t+1}$. Assuming that the investor obeys a quadratic utility function, the expected utility obtained by the investor can consistently be estimated by using the average realized utility as follows:

$$\bar{U} = \frac{W}{n} \sum_{t=0}^{n-1} \left(R_{p,t+1} - \frac{\gamma}{2(1+\gamma)} R_{p,t+1}^2 \right), \tag{30}$$

where W is the given level of initial wealth and set equal to 1 as it is not relevant for computing the relative performance fees. The expected utility will be used to assess the economic value of the predictive regression models by considering the amount that an investor would be willing to pay in order to switch from using the benchmark strategy to the trading strategy based on the return forecasts. This performance fee can be computed by setting the expected utility obtained from using the benchmark models equal to the expected utility obtained when using the trading strategy based on the return forecasts which are subject to an annual expense of Δ. As both strategies would yield the same utilities, the performance fee Δ could be interpreted as the maximum amount an investor would be willing to pay in order to switch from the benchmark strategy to the trading strategy constructed using the predictive regression models. The performance fee Δ can be computed by solving the following equation:

$$\sum_{t=0}^{n-1}\left((R^a_{p,t+1}-\Delta)-\frac{\gamma}{2(1+\gamma)}(R^a_{p,t+1}-\Delta)^2\right)=\sum_{t=0}^{n-1}\left(R^b_{p,t+1}-\frac{\gamma}{2(1+\gamma)}(R^b_{p,t+1})^2\right), \quad (31)$$

with $R^a_{p,t+1}$ and $R^b_{p,t+1}$ denoting the portfolio returns obtained from the trading and benchmark strategies, respectively. The economic value from the predictive regression models are compared with the benchmark buy-and-hold strategy. For completion, the average returns, standard deviations and Sharpe ratios are computed for each portfolio. The standard errors of the performance fees are obtained using the delta method as proposed in Ledoit and Wold (2018). The trick is to express Δ as a function of its first and second moments:

$$\Delta=\frac{(\eta\mu^a-1)\pm\sqrt{(\eta\mu^a-1)^2-2\eta(\mu^b-\mu^a-\frac{\eta}{2}(S^b-S^a))}}{\eta}, \text{ with } \eta=\frac{\gamma}{(1+\gamma)} \quad (32)$$

Let $z=[\mu^a,\mu^b,S^a,S^b]$ be a (1×4) vector consisting of the first and second moments for both return series $R^a_{p,t+1}$ and $R^b_{p,t+1}$. Then, Δ can be expressed as $\Delta=f(z)$ and the standard error can be computed using the delta method as follows:

$$SE(\hat{\Delta})=\sqrt{\frac{\nabla' f(z)V(z)\nabla f(z)}{T}} \quad (33)$$

with $\nabla' f(z)$ the gradient of $f(z)$ and $V(z)$ the 4×4 covariance matrix of $z=[\mu^a,\mu^b,S^a,S^b]$. See appendix **E** for more detail.

5 Results

5.1 Model construction and variable selection

In order to construct the factor-augmented predictive regressions as mentioned in section 4.1, a factor approach is used to adequately describe the large number of macroeconomic variables and technical indicators. Consequently, the obtained principal components are then used in factor-augmented predictive regressions to forecasts excess stock returns. Figure 2 summarizes the included number of factors for each predictive regression model which are selected using the Bayesian Information Criteria (BIC). For most windows, the BIC only includes the first principal component. Multiple factors are more often selected when hard thresholding is applied to the macro economic variables, which is in line with the findings of Cakmakli and van Dijk (2016). More information regarding the number and most frequent selected variables after applying the hard thresholding method can be found in appendix **E**.

In order to analyze the stand-alone effect of regime switching on the return forecasts, Markov Switching models are constructed using the same set of variables to predict the returns during both bull and bear markets. The only difference compared to the factor-augmented predictive models is the ability to detect regimes and to use the corresponding parameter estimates during that regime to generate return forecasts. Table 2 summarizes the parameter estimates for each Markov Switching model during bull and bear markets for the complete sample period from Jan 1967 to Dec 2017. In most cases, the parameter estimates differ substantially during different market states, indicating that their relation to excess returns are regime dependent. For example, the magnitudes of the parameter estimates for the risk free rate and its first lag, $\hat{\beta}_3$ and $\hat{\beta}_4$, in the model consisting of only financial variables ("Fin") are roughly 7 and 3 times higher during bull markets. In some cases, even the sign of the parameter estimates changes during the regimes, indicating that the relation between certain variables and excess returns can be positive or negative depending on the state of the market. For example, the parameter estimate for the default spread ($\hat{\beta}_5$) in the model consisting of both financial variables and macroeconomic factors ("FM"), is equal to -0.039 and 2.423 during bear and bull markets.

Figure 3 illustrates the ability of these models to identify bull and bear markets for

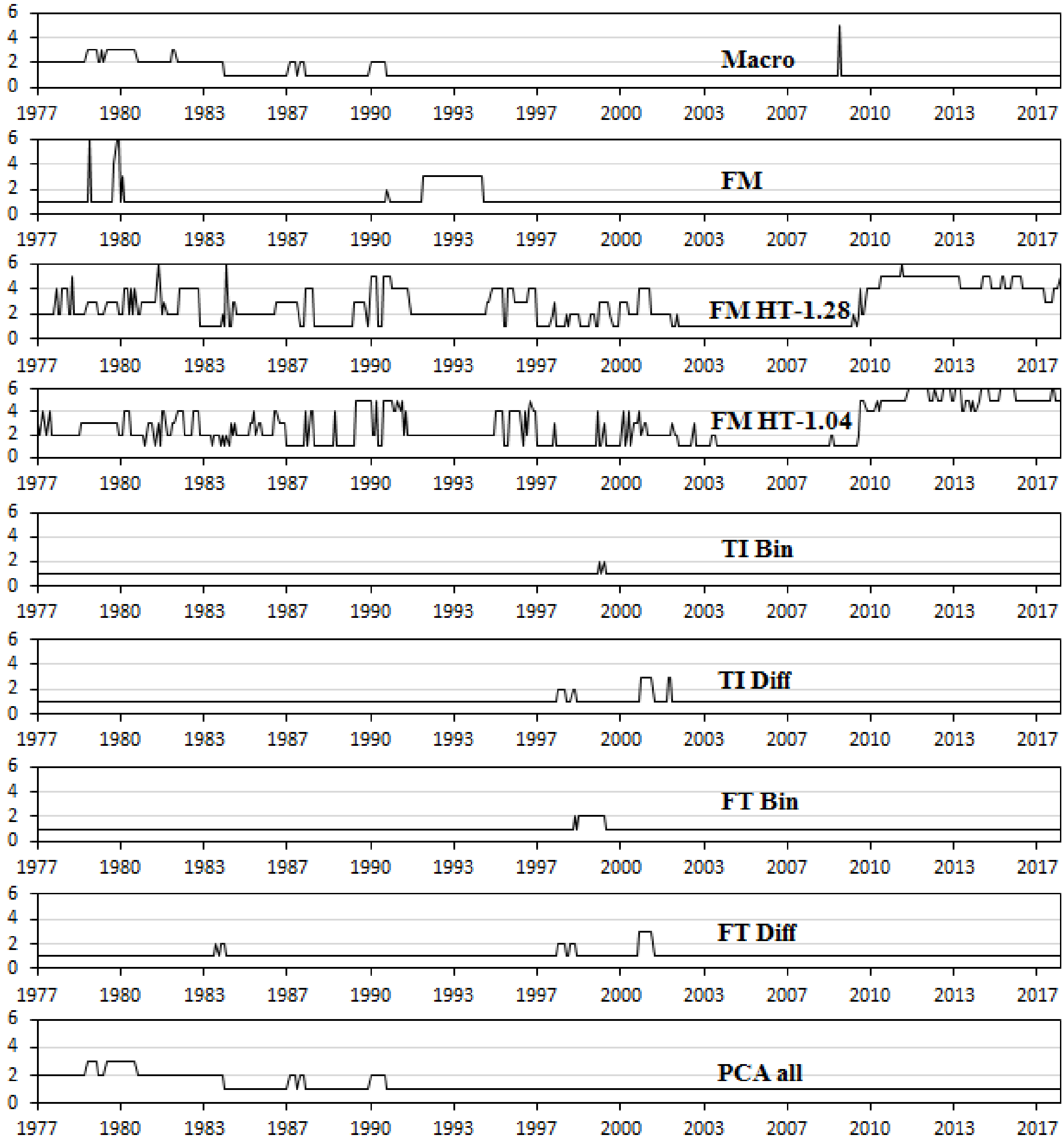

Figure 2: **Numbers of factors used in each period.** Notes: The graphs represent the numbers of factors included in the predictive regression models for excess returns consisting of financial variables and one to six PCA factors. The factors are selected using the BIC and are computed for a rolling window of 120 months. The model abbreviations are described in table 5

the complete sample period. The graphs show that, in most cases, the proposed Markov Switching models are relatively good in identifying bull and bear markets. Almost all of them correctly identified the stock market crashes during 2000-2002 and 2008-2009 as bear markets. Also, the models seems to quite accurately capture the relative smaller crashes in the beginning of the sample, such as in 1969-1970 and 1972-1973.

Table 2: Parameter estimates of Markov Switching models during bull and bear markets

	Fin		**Macro**		**FM**		**FM HT-1.28**		**FM HT-1.04**		**TI Bin**	
	Bear	Bull	Bear	Bull	Bear	Bull	Bear	Bull	Bear	Bull	Bear	Bull
$\hat{\beta}_0$	-0.001	0.009	-0.017	0.009	-0.001	0.008	-0.002	0.020	-0.002	0.021	-0.007	0.008
	(0.012)	(0.006)	(0.005)	(0.002)	(0.018)	(0.006)	(0.015)	(0.008)	(0.015)	(0.008)	(0.006)	(0.002)
$\hat{\beta}_1$	-1.948	-0.738	0.059	-0.001	-1.783	-0.746	-1.912	-0.764	-1.912	-0.782	0.969	0.291
	(0.941)	(0.340)	(0.002)	(0.001)	(0.925)	(0.341)	(0.951)	(0.391)	(0.951)	(0.392)	(0.288)	(0.151)
$\hat{\beta}_2$	0.287	-0.094	0.071	0.161	0.222	-0.094	0.298	-0.101	0.301	-0.098	-	-
	(0.385)	(0.122)	(0.089)	(0.051)	(0.379)	(0.122)	(0.040)	(0.134)	(0.397)	(0.134)	-	-
$\hat{\beta}_3$	-2.776	-18.788	-	-	-4.569	-19.186	-2.578	-13.064	-2.871	-12.668	-	-
	(8.935)	(5.118)	-	-	(8.795)	(5.152)	(11.987)	(6.134)	(12.106)	(6.160)	-	-
$\hat{\beta}_4$	6.803	19.828	-	-	8.841	20.274	7.313	13.514	7.567	13.180	-	-
	(9.036)	(5.082)	-	-	(8.902)	(5.124)	(11.442)	(6.053)	(11.506)	(6.063)	-	-
$\hat{\beta}_5$	-0.098	2.395	-	-	-0.039	2.423	-0.293	17.246	-0.258	1.686	-	-
	(1.015)	(0.508)	-	-	(0.996)	(0.510)	(1.489)	(0.655)	(1.501)	(0.656)	-	-
$\hat{\beta}_6$	-	-	-	-	0.055	0.007	0.070	-0.084	0.068	-0.082	-	-
	-	-	-	-	(0.002)	(0.001)	(0.160)	(0.083)	(0.160)	(0.083)	-	-
$\hat{\beta}_7$	-	-	-	-	-	-	0.033	-0.520	0.059	-0.547	-	-
	-	-	-	-	-	-	(0.722)	(0.288)	(0.729)	(0.288)	-	-

	TI Diff		**FT Bin**		**FT Diff**		**FT 3 Bin**		**FT 3 Diff**		**PCA all**	
	Bear	Bull	Bear	Bull	Bear	Bull	Bear	Bull	Bear	Bull	Bear	Bull
$\hat{\beta}_0$	-0.011	0.009	0.006	0.004	-0.001	0.008	-0.014	0.012	-0.004	0.006	-0.017	0.008
	(0.005)	(0.002)	(0.012)	(0.006)	(0.012)	(0.006)	(0.013)	(0.008)	(0.012)	(0.008)	(0.005)	(0.002)
$\hat{\beta}_1$	0.594	0.144	-1.564	-0.594	-0.768	-0.330	-1.685	-0.784	-0.926	-0.492	0.006	0.000
	(0.003)	(0.001)	(0.930)	(0.348)	(0.922)	(0.343)	(0.957)	(0.350)	(1.007)	(0.402)	(0.002)	(0.001)
$\hat{\beta}_2$	-	-	0.309	-0.086	-0.293	-0.199	0.324	-0.083	-0.212	-0.139	0.078	0.165
	-	-	(0.376)	(0.122)	(0.375)	(0.123)	(0.383)	(0.123)	(0.440)	(0.135)	(0.089)	(0.051)
$\hat{\beta}_3$	-	-	-8.356	-19.387	-8.440	-17.654	-7.591	-18.693	-6.619	-18.879	-	-
	-	-	(8.967)	(5.113)	(8.695)	(5.129)	(9.078)	(5.114)	(8.732)	(4.328)	-	-
$\hat{\beta}_4$	-	-	10.733	20.002	9.641	18.756	10.103	19.869	9.264	19.849	-	-
	-	-	(8.946)	(5.068)	(8.801)	(5.086)	(9.083)	(5.070)	(8.794)	(4.289)	-	-
$\hat{\beta}_5$	-	-	0.008	2.445	1.949	2.628	0.081	2.308	1.312	2.098	-	-
	-	-	(0.992)	(0.507)	(1.175)	(0.508)	(1.013)	(0.511)	(1.211)	(0.512)	-	-
$\hat{\beta}_6$	-	-	0.843	0.300	0.952	0.268	0.007	-0.021	0.044	0.020	-	-
	-	-	(0.310)	(0.157)	(0.000)	(0.000)	(0.204)	(0.085)	(0.001)	(0.001)	-	-
$\hat{\beta}_7$	-	-	-	-	-	-	-0.015	0.202	-1.788	-1.064	-	-
	-	-	-	-	-	-	(0.194)	(0.086)	(0.023)	(0.010)	-	-
$\hat{\beta}_8$	-	-	-	-	-	-	0.192	-0.020	1.405	0.877	-	-
	-	-	-	-	-	-	(0.148)	(0.054)	(0.001)	(0.002)	-	-

Notes: This table shows the parameter estimates of the Markov Switching models consisting of the same set of variables as used in the predictive regression models for the complete sample period. The same variables are used to predict returns during both bull and bear markets. The numbers in parentheses under the parameter estimates represents the corresponding standard errors. The model abbreviations are as follows: Fin denotes the 5 financial variables. Macro denotes the set of PCA factors obtained using all macroeconomic variables. FM consists of financial variables and the same factors as in Macro. FM HT-1.28 and FM HT-1.04 represents macroeconomic factors obtained after the hard thresholding rule of $|t|$=1.28 and 1.04. TI-Bin and TI-Diff consists of technical indicator and difference variables. FT 3 Bin and FT 3 Diff consists of financial variables and 3 best performing technical binary and difference variables. FT Bin and FT Diff consists of financial variables and the same factors as in TI Bin and TI Diff. PCA-all stands for the factors obtained from all the variables.

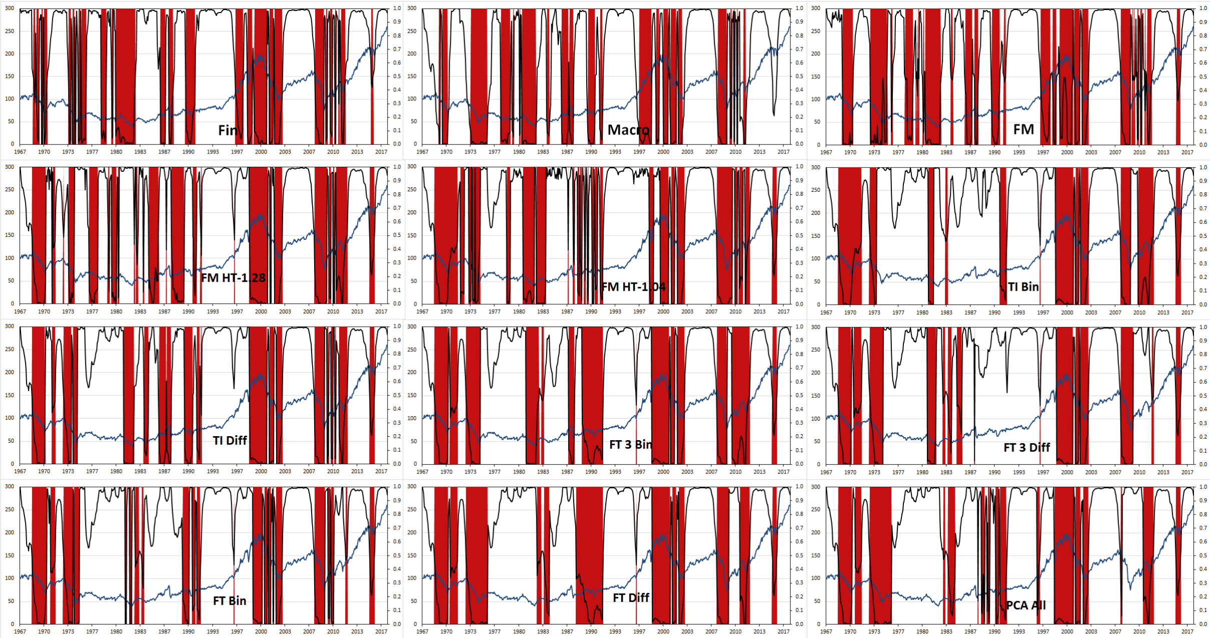

Figure 3: **Identification of bull and bear markets using Markov Switching models** Notes: The graphs represent the identification of bull and bear markets using Markov Switching model with the regressor variables as described in table 2 for the complete sample period. The blue line denotes the indexed excess stock returns on the S&P 500. The red area indicates bear markets based on the smoothed probability of a bear market, with a bear market prevailing when its corresponding smoothed probability exceeds 0.5. The black line indicates the smoothed probability for a bull market.

In addition to the proposed factor-augmented predictive models, a variable selection method is considered to construct predictive regression models consisting of variables that are best in describing the excess stock returns. Table 3 summarizes the variables with the strongest predictive power based on the size of their t-statistics for each variable set during the complete sample period. Macroeconomic variables appear to exhibit the strongest explanatory power in describing future excess stock returns, with t-statistics ranging from 3.26 to 3.50 for the 6 and 3-Month Treasury Bill (“TB6MS” and “TB3MS”). From these results, five predictive regression models are constructed consisting of 1 to 5 best performing explanatory variables. The models are labeled accordingly with “M1” denoting the model consisting of only the variable with the highest t-statistic (“TB3MS”), “M2” denoting the model consisting of both “TB3MS” and the variable with the second highest t-statistic (“GS5”), and so on until “M5”.

To analyze the explanatory power of the variables during bull and bear markets, a Lagrange Multiplier test for omitted variables is used for each variable separately.

Table 3: Variable selection based on t-statistics

Fin		**Macro**		**PC Macro**		**TI Bin**	
Riskfree	2.040	TB3MS	3.349	PC6	3.100	OBV3-12	2.652
Riskfree-1	1.653	GS5	3.331	PC2	2.565	OBV2-12	2.612
Default spread	0.808	GS1	3.323	PC3	1.762	OBV1-12	2.381
Div Yield	0.208	GS10	3.286	PC1	1.067	MA2-12	2.371
PE ratio	0.139	TB6MS	3.260	PC5	0.540	MA1-12	2.345
TI Diff		**PC TI Bin**		**PC TI Diff**		**PCA all**	
OBV3-12	1.812	PC1	2.265	PC1	1.748	PC2	2.614
OBV3-9	1.794	PC5	0.667	PC6	1.372	PC4	1.796
OBV1-12	1.724	PC6	0.622	PC4	0.918	PC3	1.751
OBV2-12	1.709	PC2	0.550	PC2	0.309	PC5	1.355
OBV1-9	1.643	PC3	0.159	PC3	0.071	PC6	1.160

Notes: This table summarizes the five variables with the highest t-statistic for each variable set for the complete sample period. All the variables within a certain sets are individually used in a regression and sorted based on the size of their t-statistics. The model abbreviations are as in table 2.

Table 4 summarizes the results for the Lagrange Multiplier test for the complete sample period where the five variables with the highest individual $\chi^2(1)$ statistic are displayed together with the $\chi^2(l)$ statistic when these variables are combined in a descending order. In general, macroeconomic variables appear to better explain excess stock returns during both bull and bear markets compared to other variables. This finding is in line with the case without regime switching (table 3) in the sense that they both select macroeconomic variables as best describers of excess returns. However, table 4 shows that when accounting for changes in financial cycles, the choice of the macroeconomic variables differ. For example, the variable "FEDFUNDS" and "PC4 HT-1.28" appear to have the strongest explanatory power during bull and bear markets ($\chi^2(1)$ of 10.37 and 7.29), while table 3 showed that without regime switching, "TB3MS" was best in describing excess stock returns. The column "ALL" denotes the best performing explanatory variables across all the variable sets. The second till the fifth variable are selected based on the highest possible $\chi^2(l)$ statistic when combining the first variable with any other variable, the first two variables with any other, and so on. To be more specific, no other combination of two variables than "FEDFUNDS" and "NDMANEMP" could yield a higher $\chi^2(2)$ statistic than 16.91 during periods of bull markets. The values 28.11 and 24.57 represent the $\chi^2(5)$ statistic for bull and bear markets when all five variables are included in the Markov

Switching model. Based on these findings, five Markov Switching models are constructed in a similar way as in the case without regime switching. The models are labeled accordingly with “MS1” indicating the Markov Switching model where “FEDFUNDS” and “PC4 HT-1.28” describes the return process during bull and bear markets. Similarly, “MS5” will represents the Markov Switching model consisting of the five best variables as presented in the column “ALL”.

Table 4: Lagrange Multiplier test for variables omitted from the mean in bull and bear markets.

μ_1	**Fin**			**Macro**			**PC Macro**			**HT-1.28**			**HT-1.04**		
	Riskfree	0.47		FEDFUNDS	10.37***		PC4	5.50**		PC3	6.84***		PC3	6.82***	
	Riskfree-1	0.29	6.17*	TB6MS	7.35***	10.55***	PC1	4.49**	10.82***	PC2	1.58	8.45**	PC2	1.70	8.53**
	Div Yield	0.04	7.35*	NDMANEMP	7.21***	17.74***	PC6	2.99*	13.10**	PC6	0.12	8.52*	PC6	0.10	8.58*
	Default spread	0.04	8.71*	GS1	6.04**	18.47***	PC5	1.27	13.26**	PC5	0.05	8.52*	PC4	0.04	9.61*
	PE ratio	0.01	8.76	CP3Mx	5.64**	19.87***	PC3	0.82	14.05**	PC4	0.04	9.56*	PC3	0.03	9.61*
μ_2															
	Riskfree	0.29		GS5	6.83***		PC6	2.95*		PC4	7.29***		PC4	7.28***	
	Default spread	0.25	1.96	GS10	6.26***	6.83*	PC2	2.87*	6.12**	PC1	4.02**	8.91**	PC1	4.02**	8.90**
	Riskfree-1	0.13	5.37	GS1	6.05**	6.97*	PC3	1.88	7.29*	PC5	0.65	10.34**	PC3	0.59	9.24**
	Div Yield	0.12	7.22	CES1021000001	6.01**	10.41*	PC5	0.29	7.55	PC3	0.59	10.60**	PC5	0.55	10.36**
	PE ratio	0.06	7.34	AAA	5.58**	10.50*	PC1	0.13	7.81	PC6	0.17	11.74*	PC6	0.21	10.69*
μ_1	**TI Bin**			**TI Diff**			**TI PCA**			**TI Diff PCA**			**ALL**		
	Momentum12	0.77		MA1-9	0.41		PC6	1.32		PC5	2.48		FEDFUNDS	10.37***	
	OBV1-12	0.51	0.82	MA1-12	0.26	0.97	PC1	1.11	2.22	PC4	0.58	3.01	NDMANEMP	7.21***	16.91***
	MA2-12	0.44	1.32	OBV2-12	0.21	3.19	PC3	0.79	2.83	PC6	0.44	3.53	PC3 HT1.04	6.82***	21.18***
	OBV3-9	0.44	1.34	OBV3-12	0.19	3.40	PC4	0.56	3.22	PC3	0.34	3.54	CES2000000008	1.92	24.47***
	MA3-9	0.42	1.35	MM12	0.18	4.20	PC5	0.10	3.34	PC2	0.25	4.09	CLAIMSx	1.79	28.11***
μ_2															
	OBV3-12	3.06*		OBV3-9	0.78		PC1	1.54		PC6	3.18*		PC4 HT1.28	7.29***	
	OBV2-12	2.41	3.14	OBV3-12	0.66	0.85	PC6	1.23	2.70	PC4	0.90	3.91	CUSR0000SAD	4.25**	12.36***
	OBV1-12	2.01	3.14	OBV1-9	0.55	0.85	PC2	0.75	3.40	PC1	0.49	4.80	OBV3-12	3.06*	16.84***
	MA2-12	1.90	3.17	OBV1-12	0.53	1.81	PC3	0.31	3.67	PC3	0.12	4.91	M2SL	2.53*	21.08***
	MA1-12	1.65	3.29	OBV2-9	0.52	2.67	PC5	0.25	4.39	PC2	0.03	4.93	OBV3-9	0.68	24.57***

Note: Lagrange Multiplier (LM) test statistics for variables omitted from the mean with respect to bull and bear states for the complete sample period. The model under the null hypothesis is the Simple Markov Switching model consisting of only a level parameter as described in section **4.2**. The LM test statistic is χ^2 distributed with l degrees of freedom and l denoting the number of parameter restrictions in the alternative model specification. See table 2 for model abbreviations. For each variable set, the 5 variables with the highest individual $\chi(1)^2$ statistic are represented in a descending order together with its corresponding $\chi(1)^2$ next to it. The third column of each variable set represent the $\chi^2(l)$ statistic when 2,3,4 or 5 variables are included in the Markov Switching model. The variables under “ALL” are the ones with the highest LM test statistic across all variable sets. The first variable is simply the variable with the highest $\chi(1)^2$ statistic. The second till the fifth variable are selected based on the highest possible $\chi(l)^2$ statistic when combining the first variable with any other variable, the first two variables with any other, and so on. *, ** and *** denote the 10%, 5% and 1% significance level, respectively.

5.2 Market timing ability

Table 5 shows the market timing ability for the excess return forecasts obtained from the factor-augmented predictive regression models and the corresponding Markov Switching models using the same variables. The hit ratios, together with their statistical significance obtained using the DA test, are computed for the complete out-of-sample period from January 1977 to December 2017 and for the first and second halves. PCS^{MS} and DA^{MS} represents the statistics for the Markov Switching models. Panel A demonstrates that all models exhibit positive market timing abilities during the complete out-of-sample period except for the factor-augmented predictive model consisting of factors based on all variables "PCA all". Also, almost all models consisting of both financial variables and additional factors demonstrate higher market timing ability than the model consisting of only financial variables "Fin". This suggests that macroeconomic variables and certain technical indicators contain additional information which seems to be relevant for market timing.

The hit ratios for models with and without regime switching in panel A range between 42% to 58%. In general, the hit ratios improve when regime switching is incorporated with the exception of "FT 3 Diff" and "FT Diff". For example, the hit ratio for "PCA all" increased from 42% to 54% during the complete out-of-sample period. Also, incorporating different financial cycles seems to improve the hit ratios in statistical terms as well, with hit ratios obtained from 5 Markov Switching models being significant instead of 2 from factor-augmented predictive models, given a 10% and 5% significance level. For example, the DA statistics for "Macro" and "FT 3 Bin" reported in panel A increased from 1.24 to 1.53 and 1.21 to 1.56.

Comparing the hit ratios in the sub samples, it seems that the marking timing ability of the factor-augmented predictive models are fairly stable over time. Incorporating regime switching seems to slightly improve the stability of the hit ratios, with the average absolute difference of the hit ratios between the two sub samples decreasing from 4.7% to 3.0%. The models consisting of both financial variables and technical trading rules seems to result in the most instable hit ratios. For example, the hit ratio obtained from "FT 3 Bin" in the case without and with regime switching increased from 50% and 51% in Panel B to 59% and 58% in panel C.

Table 5: Market timing for factor-augmented predictive regression models and corresponding Markov Switching models

	Macro	FM	FM HT-1.28	FM HT-1.04	TI Bin	TI Diff	FT 3 Bin	FT 3 Diff	FT Bin	FT Diff	PCA all	Fin
Panel A: Jan 1977 - Dec 2017												
PCS	0.554	0.554*	0.532	0.538	0.580**	0.548	0.542	0.538	0.523	0.509	0.420	0.530
DA	1.24	1.28	0.79	1.06	1.85	0.90	1.21	1.14	0.33	-0.31	0.85	0.35
PCS^{MS}	0.562*	0.566*	0.538	0.541	0.581**	0.554	0.547*	0.503*	0.533	0.506	0.542	0.538
DA^{MS}	1.53	1.59	0.86	1.05	1.88	1.24	1.56	1.44	0.33	-0.31	1.06	0.74
Panel B: Jan 1977 - Dec 1997												
PCS	0.576**	0.551*	0.531	0.527	0.588**	0.531	0.498	0.486	0.482	0.469	0.433	0.494
DA	2.15	1.57	0.67	0.63	1.89	0.74	0.33	-0.11	-0.30	.71	na	0.01
PCS^{MS}	0.581**	0.557*	0.535	0.543	0.584**	0.547	0.514	0.502	0.498	0.482	0.551	0.510
DA^{MS}	2.21	1.63	0.75	1.08	1.78	1.13	0.60	0.30	0.05	0.50	1.24	0.21
Panel C: Jan 1997 - Dec 2017												
PCS	0.533	0.557	0.533	0.549	0.573	0.565	0.585*	0.589*	0.565	0.549	0.407	0.565
DA	-0.86	-0.24	0.42	0.80	0.77	0.28	1.29	1.56	0.51	-0.04	0.82	0.16
PCS^{MS}	0.543	0.575	0.542	0.539	0.579	0.561	0.579	0.504**	0.569	0.530	0.533	0.567
DA^{MS}	-0.79	0.01	0.24	0.92	0.79	0.44	1.07	1.68	0.18	-0.06	0.79	0.67

Notes: This table evaluates the marking timing abilities of the monthly excess stock return forecasts. PCS and PCS^{MS} stands for proportion of signs predicted correctly for the factor-augmented predictive regression models and the corresponding Markov Switching models. Directional Accuracy (DA/DA^{MS}) represents the test statistic for market timing ability as described in equation **24**. Both models consist of the same variables as stated in the first row with abbreviations as in table 2. * 10% Significance level, ** 5% Significance level, *** 1% Significance level.

Figure 4 gives a more detailed overview of the (in)stability in the market timing ability of the corresponding factor-augmented predictive regression models. The graph displays the hit ratios for a rolling window of 5 years together with the corresponding expected hit ratio under independence. Several models such as “TI Bin”, “FM” and “FM HT-1.04”, among others, have hit ratios above the expected hit ratio under independence for the majority of time, indicating positive market timing ability. However, as opposed to table 5, the graphs show that the performance of most models are actually not very stable over time. For example, the hit ratios obtained from “FM” and “Fin” ranges between c.40% - 75% and c.35% - 75%, Also most models demonstrate a decline in performance around similar time periods, such as between 1984 - 1989, 2000 - 2003, and during the recent financial crisis. These results may indicate that the performance of certain predictor variables are subject to changes in regimes and give more support to introducing models that account for different financial cycles.

Table 6 summarizes the market timing results for the five predictive regression models and Markov Switching models during the full out-of-sample period and sub samples. In addition to the 5 MS models, the results for the Simple Markov Switching model are also demonstrated to analyze whether including predictor variables to the Markov Switching model affects the market timing ability. Also, the performances of the Markov Switching models during bull and bear markets are shown for the complete out-of-sample period. Interestingly, table 6 shows that all Markov Switching models achieved higher hit ratios than the predictive regression models during the complete out-of-sample period. This indicates that incorporating the state of the market and selecting the variables based on their performances during the corresponding states, positively affects the market timing ability of the models. For example, the hit ratio increased from 56% for "M2" to 60% for "MS2". The Markov Switching model consisting of four variables ("MS4") in panel A achieved the highest hit ratio of 60% and is significant on a 1% level. Next to improving the hit ratios, allowing for regime switching also leads to more stable results across the two sub periods, with "MS2" showing the highest deviation which is only an increase of c.1.4% from panel B to C. Table 6 also shows that in general, MS models with predictor variables have higher hit ratios than the Simple Markov Switching model, indicating that these variables exhibit information that are relevant for market timing. Surprisingly, all the MS models with predictor variables achieved higher hit ratios during bear markets compared to bull markets, which may indicate that the information embedded in the predictor variables are especially useful for timing the market during bear states. For example, "MS1" achieved hit ratios of 57% and 59% during periods of bull and bear markets.

Figure 5 gives a more detailed overview of the (in)stability in the market timing ability of the Markov Switching models. The Simple Markov Switching model appeared to be quite unstable over time with strong dips during 2000 - 2003 and the recent financial crisis. These declines are also visible in the other MS models but with substantial smaller magnitudes, especially during the latter period. This observation confirms that adding predictor variables to the Markov Switching models improves the ability to time the market during bear markets. Compared to figure 4, all the Markov Switching models expect for "MS3" obtained hit ratios well above the expected hit ratio under independence, indicating positive market timing ability. Therefore, it seems that in general, accounting for regime switching and selecting the variables based on their performance during these regimes, significantly improved the ability to time the market and their stability over time. To assess whether this also translates into more profitable investment strategies, their relative performance fees with respect to the three static benchmark portfolios are evaluated in the Economic Value section.

Table 6: Market timing for predictive regression models based on hard thresholding and Markov Switching models based on LM results

Panel A: Jan 1977 - Dec 2017						
Without regime switching		**M1**	**M2**	**M3**	**M4**	**M5**
PCS		0.574**	0.556*	0.550*	0.558*	0.552*
DA		1.94	1.44	1.10	1.57	1.38
With regime switching	**MS Simple**	**MS1**	**MS2**	**MS3**	**MS4**	**MS5**
PCS	0.572**	0.580*	0.602***	0.567***	0.604***	0.587**
DA	2.00	1.41	2.23	2.68	2.78	1.86
Bull regime						
PCS	0.587**	0.565	0.599	0.556	0.599**	0.577
DA	2.29	0.370	0.950	1.14	1.97	0.63
Bear regime						
PCS	0.556	0.593	0.604	0.579**	0.609**	0.596
DA	0.21	1.17	0.53	2.28	1.80	0.78
Panel B: Jan 1977 - Dec 1997						
Without regime switching		**M1**	**M2**	**M3**	**M4**	**M5**
PCS		0.588***	0.559**	0.563**	0.571***	0.551**
DA		2.50	1.87	1.91	2.17	1.47
With regime switching	**MS Simple**	**MS1**	**MS2**	**MS3**	**MS4**	**MS5**
PCS	0.588***	0.575**	0.595	0.565*	0.610*	0.591**
DA	2.47	1.80	0.94	1.42	1.56	1.64
Panel C: Jan 1997 - Dec 2017						
Without regime switching		**M1**	**M2**	**M3**	**M4**	**M5**
PCS		0.561	0.553	0.537	0.545	0.553
DA		0.41	0.30	0.89	0.34	0.25
With regime switching	**MS Simple**	**MS1**	**MS2**	**MS3**	**MS4**	**MS5**
PCS	0.557	0.585	0.609**	0.569***	0.598***	0.583*
DA	0.04	1.01	1.89	2.37	2.46	1.46

Notes: This table evaluates the market timing abilities of the monthly excess stock return forecasts using predictive regression models and Markov Switching models. The predictive regression models consist of 1 to 5 variables which are selected based on the magnitude of their t-statistics as displayed in table 3. The same holds for the Markov Switching models which consists of variables with the highest LM statistic as shown in table 4. MS Simple represents the Simple Markov Switching model as in equation 12.
* 10% Significance level, ** 5% Significance level, *** 1% Significance level.

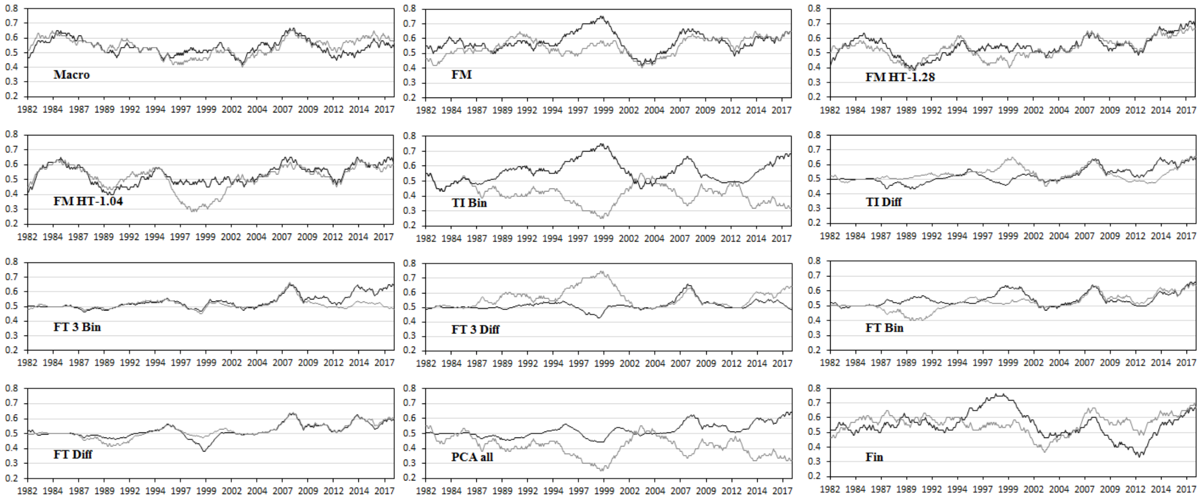

Figure 4: **Hit ratios for excess returns over five-year moving windows.** Notes: The black line represents the hit ratio for the sign of the monthly excess stock returns obtained from the factor-augmented predictive regression models over the five-year moving window ending at the date as shown in the horizontal axis. The gray line indicates the expected hit ratio under the assumption of independence between the actual and predicted signs. The model abbreviations are as in table 5

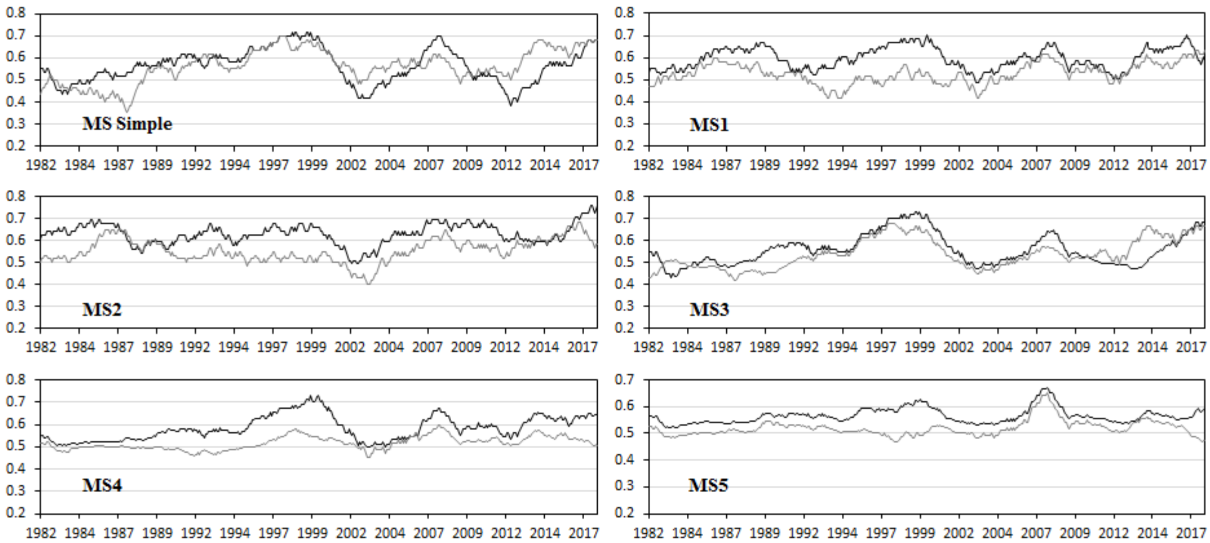

Figure 5: **Hit ratios for excess returns over five-year moving windows.** Notes: The black line represents the hit ratio for the sign of the monthly excess stock returns obtained from the Markov Switching models consisting of 1 to 5 variables based on the LM test as described in table **4** over the five-year moving window ending at the date as shown in the horizontal axis. The gray line indicates the expected hit ratio under the assumption of independence between the actual and predicted signs.

5.3 Economic value

Table 7 summarizes the economic value of the factor-augmented predictive regression models and the corresponding Markov Switching models relative to three buy-and-hold strategies for the complete out-of-sample period from January 1977 to December 2017. The results in panel A show that almost all investment strategies based on the predictive regression models outperform the buy-and-hold benchmark strategies, with the exception of "TI Diff", "FT 3 Diff" and "FT Diff". For example, an investor would be willing to pay an annual performance fee of 138 basis points to switch from the portfolio consisting of 100% stocks (P_{100}) to the portfolio strategy based on "FT 3 Bin". The mixed portfolio with a constant weight of 50% in stocks (P_{50}) appeared to be somewhat harder to outperform, with relative performance fees for the factor-augmented models ranging from -95 to 116 basis points instead of -74 to 138 basis points relative to P_{100}.

Panel B illustrates that the performance fees with respect to the benchmark models increased substantially for all models when regime switching was introduced.[1] For example, the relative performance fee for "FM" and "PCA all" with respect to the P_{100} benchmark portfolio increased from 116 to 203 and 37 to 202. Not only did introducing regime switching affected the magnitudes of the relative performance fees, it also affected the model choice that resulted in the highest relative performance fee. For example, "FT 3 Bin" achieved the highest performance fee for the factor-augmented predictive models, which changed to "FM" when regime switching was introduced, with annual performance fees of 138 and 203 basis points relative to the P_{100} portfolio. Next to this, the achieved performance fees improved in statistical terms as well. For example, Δ_{100} achieved by 8 Markov Switching models appear to be significant on a 1% level, instead of only 4 in the case without regime switching. Incorporating changes in the financial cycle also improve the excess return forecasts in terms of the achieved Sharpe ratios, which ranged between 0.20-0.45 for the factor-augmented predictive models and 0.32-0.52 for the Markov Switching models.

[1]The performance fees for the active trading strategies are not computed with respect to each other as they have a linear relationship with the presented performance fees

Table 7: Performance of active trading strategies for the period Jan 1977 - Dec 2017. RRA = 6, transaction costs = 0.1%, $w_{t+1} \in[0,1]$

	μ	σ	SR	Δ_{100}		Δ_{50}		Δ_{0}	
Passive portfolio strategies									
100% market	9.49%	14.71%	0.33						
50% market	7.06%	7.37%	0.32						
0% market	4.68%	1.05%	-						
Active portfolio strategies									
Without regime switching									
Fin	8.68%	8.85%	0.45	137***	(47)	115**	(56)	263***	(70)
Macro	7.69%	9.23%	0.33	33	(59)	12	(34)	159**	(76)
FM	8.75%	9.82%	0.42	116***	(45)	94***	(33)	242***	(79)
FM HT-1.28	8.58%	9.39%	0.42	112***	(43)	91***	(34)	239***	(73)
FM HT-1.04	8.26%	9.84%	0.36	69	(59)	47	(45)	195***	(78)
TI Bin	7.94%	9.23%	0.35	57	(70)	36	(53)	183***	(72)
TI Diff	6.58%	9.33%	0.20	-74	(69)	-95**	(57)	50	(48)
FT 3 Bin	8.70%	8.87%	0.45	138***	(57)	116***	(46)	264***	(71)
FT 3 Diff	7.17%	9.54%	0.26	-25	(30)	-46	(58)	100	(79)
FT Bin	7.73%	9.06%	0.34	42	(38)	20	(28)	167**	(73)
FT Diff	7.40%	8.70%	0.31	20	(32)	-1	(9)	145**	(66)
PCA all	7.72%	9.20%	0.33	37	(49)	15	(24)	162**	(76)
With regime switching									
Fin	9.16%	8.61%	0.52	187***	(49)	172***	(56)	320***	(79)
Macro	8.32%	9.02%	0.40	147***	(46)	134***	(44)	280***	(79)
FM	8.82%	9.36%	0.44	203***	(56)	186***	(55)	336***	(82)
FM HT-1.28	8.70%	9.35%	0.43	182***	(56)	167***	(52)	315***	(77)
FM HT-1.04	8.57%	9.64%	0.40	148***	(40)	134***	(56)	282***	(76)
TI Bin	8.03%	9.25%	0.36	113**	(66)	98**	(56)	247***	(81)
TI Diff	7.86%	9.52%	0.33	70	(64)	55	(59)	202***	(78)
FT 3 Bin	8.82%	8.76%	0.47	173***	(61)	158***	(46)	306***	(76)
FT 3 Diff	7.72%	9.53%	0.32	102	(93)	87*	(57)	236***	(77)
FT Bin	7.50%	8.62%	0.33	71*	(54)	57	(60)	205***	(64)
FT Diff	7.79%	9.08%	0.34	70*	(49)	55	(56)	202***	(79)
PCA all	8.78%	8.90%	0.46	202***	(60)	187***	(58)	336***	(71)

Note: Performance fees for active mean-variance portfolios based on the return predictions from the factor-augmented predictive regression models and the corresponding Markov Switching models during Jan 1979 - Dec 2017, obtained using equation 31. The columns μ and σ denote the percentage annualized mean and standard deviation of the portfolio returns. SR denotes the Sharpe ratio and Δ the annualized performance fees (in basis points) for switching from the strategy indicated by the subscript in the column to the strategy indicated by the corresponding row. The standard errors in parentheses are stated next to the corresponding performance fees and are computed using the delta method as explained in appendix **E**. The model abbreviations are as in table 2.

* 10% Significance level, ** 5% Significance level, *** 1% Significance level.

As demonstrated before in figure 4, the market timing ability of the factor-augmented predictive regression models were quite unstable over time. Figure 6 provides a detailed illustration of their instability in economic value compared to the static benchmark portfolios for a moving window of 5 years. The shaded areas correspond to US recession dates as reported by the NBER business cycle dating committee, which is included in the graphs to analyze whether the performances are dependent to certain states of the market.[2] The performances of all the portfolio strategies based on the predictive regression models vary substantially over time and even ranges between -800 and 1300 basis points in the case of “FM HT-1.04” with respect to the P_0 benchmark portfolio. Interestingly, all the portfolio strategies based on the predictive regression models seem to substantially outperform the benchmark portfolio consisting of only stocks (P_{100}) during the recent financial crisis. For example, the “FM HT-1.28” and “TI Diff” predictive regression model realized annualized performance fees over the P_{100} benchmark model of around 1500 basis points. These result might indicate that the predictive power of certain regressor variables are subject to changes in the state of the equity market.

To quantify this, table 8 summarizes the economic value for the factor-augmented predictive models and Markov Switching models during the two sub samples. For both cases, the relative performance fees with respect to the static portfolios tend to differ quite substantially during the two sub samples. For example, an investor would be willing to pay a performance fee of 79 and 274 basis points to switch from P_{100} to the predictive model consisting of financial variables (“Fin”) during the first sub period and second sub period. This difference is slightly lower in the case with regime switching, where an investor would be willing to pay 96 and 277 basis points to switch in the same scenario. In general, including regime switching improves the stability across the sub samples, with the average of the absolute deviations in performance fees for the predictive models decreasing from 144 to 107 for the regime switching models. Incorporating the effect of the state of the equity market do, in most cases, also result in higher performance fees

[2] Although this paper focuses on the behavior of certain variables during bull and bear markets, US recession dates are added to the graphs for two reasons. First, recessions exhibit strong correlations with prolonged periods of bear markets and therefore, the graph may provide an indicative illustration of the behavior during bearish states. Second, the dates are fixed by the NBER business cycle dating committee.

compared to the general predictive regression models in both sub samples. The biggest difference can be seen in the first sub sample where the performance fee for “TI Diff” with respect to the P_{100} portfolio increased from -129 to 84 basis points when regime switching was introduced. Based on tables 7 and 8, it can be concluded that by accounting for changes in the financial cycle, the relative performance fees tend to increase in almost all cases and (sub)periods. However, while it does improve upon the predictive models, implementing regime switching does not necessarily lead to stable results as the relative performance fees continue to differ quite substantially during both sub periods for some model specifications.

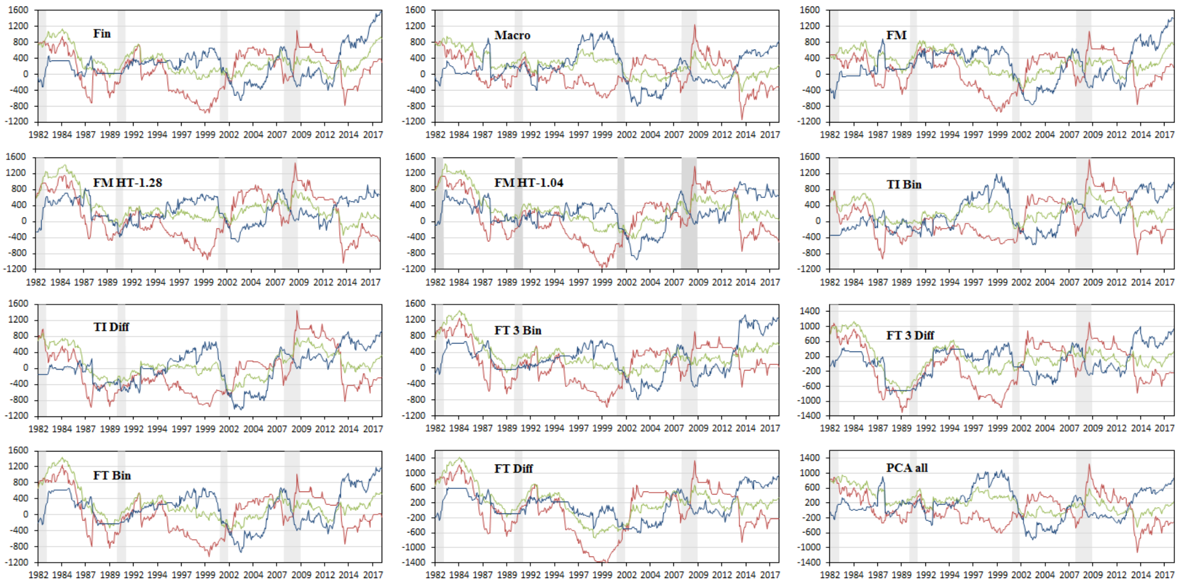

Figure 6: **Performance fees for excess returns forecasts over five-year moving window.** Notes: The figures represent the annualized performance fees in bps (Δ) computed using equation 31 for a rolling window of 5 years. The model abbreviations are as in table 2. Each figure represents the performance fees of the corresponding predictive model relative to the equity only portfolio (red line), 50-50 portfolio (green line) and the portfolio that only invest in a riskless asset (blue line). The shaded area corresponds to US recession dates as reported by the NBER business cycle dating committee

Table 9 summarizes the economic value for the five predictive regression models and Markov Switching models during the full out-of-sample period. This table provides a clear representation of the added economic value of regime switching as all the Markov Switching models achieved substantial higher performance fees than the predictive regression models, relative to the static benchmark portfolios. For example, the performance fee relative to P_{100} for the model consisting of 5 predictor variables increased from -9 to 288 basis points when regime switching was applied. In particular, the Markov Switching model consisting of 4 explanatory variables ("MS4") show favorable results, where an investor would be willing to pay an annual fee of 370 and 469 basis points to switch from the all stock portfolio "P_{100}" and T-bill only portfolio "P_0" to the "MS4" portfolio, respectively. Next to this, almost all the performance fees obtained from the Markov Switching models with predictor variables are significant on a 1% level. Also, all the Markov Switching models consisting of one or more explanatory variables outperformed the Simple Markov

Table 8: Performance of active trading strategies during sub samples, with and without regime switching. RRA = 6, transaction costs = 0.1%, $w_{t+1} \in [0,1]$

Panel A: Jan 1977 - Dec 1997												
	Δ_{100}		Δ_{50}		Δ_0		Δ_{100}		Δ_{50}		Δ_0	
	Without regime switching						With regime switching					
Fin	79	(48)	63	(30)	212***	(81)	96***	(34)	80***	(30)	229***	(99)
Macro	240**	(118)	224***	(81)	375***	(115)	233***	(92)	217***	(72)	366***	(113)
FM	182*	(56)	165***	(40)	315***	(107)	194***	(46)	178***	(71)	326***	(118)
FM HT-1.28	178*	(50)	162***	(69)	312***	(112)	171***	(48)	155***	(53)	304***	(118)
FM HT-1.04	169*	(52)	153**	(59)	303***	(110)	164***	(47)	150**	(66)	297***	(107)
TI Bin	-30	(20)	-46	(52)	101	(99)	-19**	(9)	-35	(32)	113	(112)
TI Diff	-129	(76)	-145**	(73)	1	(7)	84***	(33)	68*	(42)	217***	(75)
FT 3 Bin	149	(54)	133**	(70)	282***	(71)	152***	(42)	136***	(57)	285***	(107)
FT 3 Diff	-133	(35)	-149**	(78)	-3	(4)	20***	(7)	4	(8)	152**	(82)
FT Bin	58	(50)	43	(41)	191***	(79)	39***	(15)	23	(25)	172**	(79)
FT Diff	-17	(15)	-33	(31)	114*	(78)	17	(14)	1	(10)	150	(118)
PCA all	249**	(119)	232***	(80)	383***	(115)	296***	(98)	281***	(82)	429***	(85)
Panel B: Jan 1998 - Dec 2017												
	Without regime switching						With regime switching					
Fin	274**	(129)	247***	(79)	396***	(113)	277***	(91)	264***	(82)	411***	(105)
Macro	26	(37)	-1	(7)	145*	(101)	61**	(31)	47	(80)	194**	(113)
FM	163***	(57)	136**	(60)	283***	(115)	212***	(53)	198***	(65)	346***	(119)
FM HT-1.28	187***	(51)	160***	(55)	307***	(94)	192***	(52)	178***	(74)	326***	(105)
FM HT-1.04	111***	(41)	84**	(47)	230**	(112)	133***	(47)	120**	(59)	267***	(108)
TI Bin	224*	(144)	197**	(86)	345***	(108)	245***	(99)	231***	(88)	379***	(117)
TI Diff	62*	(41)	36	(48)	182*	(115)	55*	(42)	41	(46)	189*	(139)
FT 3 Bin	208***	(87)	181**	(79)	329***	(124)	193***	(68)	179***	(63)	326***	(110)
FT 3 Diff	165**	(82)	138*	(85)	285***	(120)	185***	(67)	171***	(64)	318***	(130)
FT Bin	102	(126)	75	(84)	221**	(123)	104	(94)	90	(92)	238**	(103)
FT Diff	115	(94)	88	(84)	235**	(109)	123*	(81)	111*	(72)	257***	(105)
PCA all	26	(37)	-1	(2)	145*	(101)	107**	(48)	94***	(35)	241**	(113)

Note: Performance fees for active mean-variance portfolios based on the return predictions from the factor-augmented predictive regression models and the corresponding Markov Switching models during the sub samples, obtained using equation 31. The columns μ and σ denote the percentage annualized mean and standard deviation of the portfolio returns, respectively. SR denotes the Sharpe ratio and Δ the annualized performance fees (in basis points) for switching from the strategy indicated by the subscript in the column to the strategy indicated by the corresponding row. The standard errors in parentheses are stated next to the corresponding performance fees and are computed using the delta method as explained in appendix **E**. The model abbreviations are as in table 2.

* 10% Significance level, ** 5% Significance level, *** 1% Significance level.

Switching model in terms of Sharpe ratios and economic value during the complete out-of-sample period. This is in line with the results based on the market timing ability in table 6, indicating that including certain predictor variables in the Markov Switching models improves the quality of the forecasts, both in terms of market timing ability and economic value. This table shows that an investor could outperform all the considered static benchmark portfolios and predictive regression models by using Markov Switching models consisting of variables selected based on their performances during specific market conditions.

Table 9: Performance of active trading strategies for Jan 1977 - Dec 2017. RRA = 6, Transaction costs = 0.1%, $w_{t+1} \in$[0,1]

	μ	σ	SR	Δ_{100}		Δ_{50}		Δ_0	
Passive portfolio strategies									
100% market	9.49%	14.71%	0.33						
50% market	7.06%	7.37%	0.32						
0% market	4.68%	1.05%	-						
Active portfolio strategies									
Without Regime switching									
M1	8.48%	8.82%	0.43	119***	(37)	97**	(45)	245***	(69)
M2	7.84%	8.83%	0.36	58*	(36)	37	(35)	184***	(70)
M3	7.62%	9.22%	0.32	27	(34)	6	(33)	152**	(73)
M4	6.62%	8.72%	0.22	-54*	(35)	-75**	(35)	71	(70)
M5	7.13%	8.87%	0.28	-9	(25)	-30	(35)	116*	(71)
With regime switching									
MS Simple	8.02%	8.74%	0.38	77	(96)	56	(53)	177***	(66)
MS1	8.46%	8.91%	0.42	126**	(60)	103***	(61)	226**	(107)
MS2	8.79%	9.03%	0.46	211***	(67)	187***	(55)	312***	(69)
MS3	8.69%	9.33%	0.43	187***	(65)	165***	(54)	289***	(94)
MS4	11.52%	9.56%	0.72	370***	(92)	346***	(59)	469***	(80)
MS5	10.30%	9.77%	0.57	288***	(75)	263***	(59)	388***	(94)

Note: Performance fees for active mean-variance portfolios based on the return predictions from the predictive regression models and the Markov Switching models during Jan 1979 - Dec 2017, obtained using equation 31. The model abbreviations are as in table 6. The columns μ and σ denote the percentage annualized mean and standard deviation of the portfolio returns, respectively. SR denotes the Sharpe ratio and Δ the annualized performance fees (in basis points) for switching from the strategy indicated by the subscript in the column to the strategy indicated by the corresponding row. The standard errors in parentheses are stated next to the corresponding performance fees and are computed using the delta method as explained in appendix **E**.

6 Conclusion

This paper analyses the predictability of excess stock returns and evaluates whether incorporating changes in the financial cycle improves the quality of the forecasts in terms of market timing ability and economic value. Based upon an empirical analyses, this paper finds evidence that introducing regime switching to the factor-augmented predictive regression models significantly improves the predictive power for monthly S&P 500 excess returns between January 1977 and December 2017. The Markov Switching models have superior market timing ability and economic value over the factor-augmented predictive models, such that a mean-variance investor would be willing to pay an annual performance fee of up to c. 200 basis point to switch from the buy-and-hold strategies to the predictions obtained from the Markov Switching models.

Next to this, the explanatory power of all variables are evaluated during periods of bull and bear markets using a Lagrange Multiplier test in order to create a regime switching model consisting of variables exhibiting the strongest predictive power during the corresponding states. In general, certain macroeconomic variables and technical indicators appears to perform particularly well during bull and bear markets, such as the federal funds rate and the composite price index for durables, respectively. Using specific explanatory variables in Markov Switching models lead to remarkable results, where the market timing ability not only improved but also seemed to be more stable over time. As for the achieved economic value when using the excess stock return forecasts in a dynamic portfolio strategy, an investor would now be willing to pay an annual performance fee of up to 370 basis points in order to switch to this strategy from an buy-and-hold portfolio consisting of stocks only. Next to that, this paper also demonstrates that including certain predictor variables improves the forecasts obtained from the Markov Switching models compared to the Simple Markov Switching model. The information embedded in these predictor variables seem to be especially useful in timing the market during periods of bear regimes

In short, this paper demonstrates that by using regime switching models in combination with a careful selection of certain predictor variables, an investor would have been able to obtain significant positive alpha over the market portfolio during the assessed period.

The focus in this research is put on monthly excess return forecasts, which is common in the financial literature on return prediction. However, using higher frequency could result into more compelling results as Kole and van Dijk (2017) argue that using weekly observations lead to more precise estimates of the switches between the regimes in the Markov Switching models. Next to this, different methods could have been used to deal with model uncertainty and parameter instability such as variable selection based on individual predictive power and forecast combinations. The drawback of these methods is the consideration of only a relative small amount of variables compared to factor-augmented predictive regression model, which uses information from a large set of predictor variables. The hard thresholding method as used in this paper could have a possible drawback as Campbell and Yogo (2006) showed that the distribution of the t-test can be non-standard when the predictor variables are persistent and therefore, could lead to possible over-rejection of the null hypothesis. This procedure could be improved by considering their pre-test to determine whether the conventional t-test lead to misleading inferences. Additionally, Markov Switching models consisting of more than two regimes could have been considered to predict excess stock returns. However, limiting the states to either a bull or bear market keeps the model relatively simple to estimate, while maintaining strong forecasting power for the equity premium.

A Appendix: Macroeconomic variables

This section provides a detailed description of the transformations applied to the macroeconomic variables used in the predictive regressions. First, the raw data is transformed to ensure stationary by using first and second level differences, log-levels, first and second log-level differences and first difference of percentage changes in accordance with the research paper of the FRED-MD: A Monthly Database for Macroeconomic Research. Then, outliers are detected and replaced using an EM algorithm as described in Stock and Watson (2002): First, outliers are defined as deviations of more than 10 interquartile ranges from the median value of the sample. Then, the algorithm is initialized by filling in missing the unconditional mean of the series, demeaning and standardizing the updated dataset, estimating factors from this demeaned and standardized dataset, and then using these factors to predict the dataset. The algorithm then proceeds as follows: update missing values using values predicted by the latest set of factors, demean and standardize the updated dataset, estimate a new set of factors using the demeaned and standardized updated dataset, and repeat the process until the factor estimates do not change.

Table 10: Set of macroeconomic variables used for extracting factors

Short name	Transf.	Description
Output and Income		
RPI	5	Real Personal Income
W875RX1	5	Real Personal Income ex transfer receipts
INDPRO	5	IP: Index
IPFPNSS	5	IP: Final Products and Nomindustrial Supplies
IPFINAL	5	IP: Final Products (Market Group)
IPCONGD	5	IP: Consumer Goods
IPDCONGD	5	IP: Durable Consumer Goods
IPNCONGD	5	IP: Nondurable Consumer Goods
IPBUSEQ	5	IP: Business Equipment
IPMAT	5	IP: Materials
IPDMAT	5	IP: Durable Materials
IPNMAT	5	IP: Nondurable Materials
IPMANSICS	5	IP: Manufacturing (SIC)
IPB51222S	5	IP: Residential Utilities
IPFUELS	5	IP: Fuels
CUMFNS	2	Capacity Utiliziation: Manufacturing
Labor Market		
HWI	2	Help-Wanted Index for United States
HWIURATIO	2	Ratio of Help Wanted / No. Unemployed

Table 10: Set of macroeconomic variables used for extracting factors

Short name	Transf.	Description
CLF16OV	5	Civilian Labor Force
CE16OV	5	Civilian Employment
UNRATE	2	Civilian Unemployment Rate
UEMPMEAN	2	Average Duration of Unemployment (Weeks)
UEMPLT5	5	Civilians Unemployed - Less Than 5 Weeks
UEMP5TO14	5	Civilians Unemployed for 5-14 Weeks
UEMP15OV	5	Civilians Unemployed - 15 Weeks and Over
UEMP15T26	5	Civilians Unemployed for 15-26 Weeks
UEMP27OV	5	Civilians Unemployed for 27 Weeks and Over
CLAIMSx	5	Initial Claims
PAYEMS	5	All Employees: Total nonfarm
USGOOD	5	All Employees: Goods-Producing Industries
CES1021000001	5	All Employees: Mining and Logging: Mining
USCONS	5	All Employees: Construction
MANEMP	5	All Employees: Manufacturing
DMANEMP	5	All Employees: Durable Goods
NDMANEMP	5	All Employees: Non durable Goods
SRVPRD	5	All Employees: Service-Providing Industries
USTPU	5	All Employees: Trade, Transportation & Utilities
USWTRADE	5	All Employees: Wholesale Trade
USTRADE	5	All Employees: Retail Trade
USFIRE	5	All Employees: Financial Activities
USGOVT	5	All Employees: Government
CES0600000007	1	Avg Weekly Hours: Goods-Producing
AWOTMAN	2	Avg Weekly Overtime Hours: Manufacturing
AWHMAN	1	Avg Weekly Hours: Manufacturing
CES0600000008	6	Avg Hourly Earnings: Goods-Producing
CES2000000008	6	Avg Hourly Earnings: Construction
CES3000000008	6	Avg Hourly Earnings: Manufacturing
Housing		
HOUST	4	Housing Starts: Total New Privately Owned
HOUSTNE	4	Housing Starts, Northeast
HOUSTMW	4	Housting Starts, Midwest
HOUSTS	4	Housing Starts, South
HOUSTW	4	Housing Starts, West
PERMIT	4	New Private Housing Permits (SAAR)
PERMITNE	4	New Private Housing Permits, Northeast (SAAR)
PERMITMW	4	New Private Housing Permits, Midwest (SAAR)
PERMITS	4	New Private Housing Permits, South (SAAR)
PERMITW	4	New Private Housing Permits, West (SAAR)
Consumption, orders and inventories		
DPCERA3M086SBEA	5	Real Personal Consumption Expenditures
CMRMTSPLx	5	Real Manu. And Trade Industries Sales
RETAILx	5	Retail and Food Services Sales

Table 10: Set of macroeconomic variables used for extracting factors

Short name	Transf.	Description
ACOGNO	5	New Orders for Consumer Goods
AMDMNOx	5	New Orders for Durable Goods
ANDENOx	5	New Orders for Nondefence Capital Goods
AMDMUOx	5	Unfilled Orders for Durable Goods
BUSINVx	5	Total Business Inventories
ISRATIOx	2	Total Business: Inventories to Sale Ratio
UMCSENTx	2	Consumer Sentiment Index
Money and credit		
M1SL	6	M1 Money Stock
M2SL	6	M2 Money Stock
M2REAL	5	Real M2 Money Stock
AMBSL	6	St. Louis Adjusted Monetary Base
TOTRESNS	6	Total Reserves of Depository Institutions
NONBORRES	7	Reserves of Depository Institutions
BUSLOANS	6	Commercial and Industrial Loans
REALLN	6	Real Estate Loans at All Commercial Banks
NONREVSL	6	Total Nonrevolving Credit
CONSPI	2	Nonrevolving Consumer Credit to Personal Income
MZMSL	6	MZM Money Stock
DTCOLNVHFNM	6	Consumer Motor Vehicle Loans Outstanding
DTCTHFNM	6	Total Consumer Loans and Leases Outstading
INVEST	6	Securities in Bank Credit at All Commercial Banks
Interest and exchange rates		
FEDFUNDS	2	Effective Federal Funds Rate
CP3Mx	2	3-Month AA Financial Commercial Paper Rate
TB3MS	2	3-Month Treasury Bill
TB6MS	2	6-Month Treasury Bill
GS1	2	1-Year Treasury Rate
GS5	2	5-Year Treasury Rate
GS10	2	10-Year Treasury Rate
AAA	2	Moody's Seasoned Aaa Corporate Bond Yield
BAA	2	Moody's Seasoned Baa Corporate Bond Yield
COMPAPFFx	1	3-Month Commercial Paper Minus FEDFUNDS
TB3SMFFM	1	3-Month Treasury Minus FEDFUNDS
TB6SMFFM	1	6-Month Treasury C Minus FEDFUNDS
T1YFFM	1	1-Year Treasury C Minus FEDFUNDS
T5YFFM	1	5-Year Treasury C Minus FEDFUNDS
T10YFFM	1	10-year Treasury C Minus FEDFUNDS
AAAFFM	1	Moody's Aaa Corporate Bond Minus FEDFUNDS
BAAFFM	1	Moody's Baa Corporate Bond Minus FEDFUNDS
TWEXMMTH	5	Trade Weighted U.S. Dollar Index: Major Currencies
EXSZUSx	5	Switzerland / U.S. Foreign Exchange Rate
EXJPUSx	5	Japan / U.S. Foreign Exchange Rate
EXUSUKx	5	U.S. / U.K. Foreign Exchange Rate

Table 10: Set of macroeconomic variables used for extracting factors

Short name	Transf.	Description
EXCAUSx	5	Canada / U.S. Foreign Exchange Rate
Prices		
WPSFD49207	6	PPI: Finished Goods
WPSFD49502	6	PPI: Finished Consumer Goods
WPSID61	6	PPI: Intermediate Materials
WPSID62	6	PPI: Crude Materials
OILPRICEx	6	Crude Oil, spliced WTI and Cushing
PPICMM	6	PPI: Metals and Metal Products
CPIAUCSL	6	CPI: All Items
CPIAPPSL	6	CPI: Apparel
CPITRNSL	6	CPI: Transportation
CPIMEDSL	6	CPI: Medical Care
CUSR0000SAC	6	CPI: Commodities
CUSR0000SAD	6	CPI: Durables
CUSR0000SAS	6	CPI: Services
CPIULFSL	6	CPI: All Items Less Food
CUSR0000SA0L2	6	CPI: All Items Less Shelter
CUSR0000SA0L5	6	CPI: All Items Less Medical Care
PCEPI	6	Personal Cons. Expend: Chain Index
DDURRG3M086SBEA	6	Personal Cons. Exp: Durable Goods
DNDGRG3M086SBEA	6	Personal Cons. Exp: Nondurable Goods
DSERRG3M086SBEA	6	Personal Cons. Exp: Services

B Appendix: Parameter estimation of the Markov Switching model

In order to apply the Expectation Maximization (EM) algorithm to the basic Markov Switching model as proposed in equation **12**, the joint likelihood of the states and data is needed:

$$\begin{aligned} f(r_{1:T}, s_{1,T}; \theta) &= \prod_{t=1}^{T} f(r_t, s_t | r_{1:t-1}, s_{1:t-1}; \theta) \\ &= \prod_{t=1}^{T} f(r_t, s_t | s_{t-1}; \theta) \qquad (34) \\ &= \left[\prod_{t=2}^{T} f(r_t | s_t; \theta) P(S_t = s_t | s_{t-1}; \theta) \right] f(r_1, s_1 | \theta) \end{aligned}$$

(35)

For the two-state Markov chain with normally distributed observations, four possibilities are available for $f(r_t, s_t|s_{t-1};\theta)$:

$$f(r_t, s_t|s_{t-1};\theta) = \begin{cases} \phi(r_t;\mu_1,\sigma_1^2)p_{11}, & \text{if } s_t = 1 \text{ and } s_{t-1} = 1 \\ \phi(r_t;\mu_1,\sigma_1^2)(1-p_{22}), & \text{if } s_t = 1 \text{ and } s_{t-1} = 2 \\ \phi(r_t;\mu_2,\sigma_2^2)(1-p_{11}), & \text{if } s_t = 2 \text{ and } s_{t-1} = 1 \\ \phi(r_t;\mu_2,\sigma_2^2)p_{22}, & \text{if } s_t = 2 \text{ and } s_{t-1} = 2 \end{cases} \tag{36}$$

Therefore, the joint log likelihood is represented as:

$$\begin{aligned} \log f(r_t, s_t|s_{t-1};\theta) &= \mathrm{I}[S_t = 1]\mathrm{I}[S_{t-1} = 1]\log[p_{11}\phi(r_t;\mu_1,\sigma_1^2)] \\ &= +\mathrm{I}[S_t = 1]\mathrm{I}[S_{t-1} = 2]\log[(1-p_{22})\phi(r_t;\mu_1,\sigma_1^2)] \\ &= +\mathrm{I}[S_t = 2]\mathrm{I}[S_{t-1} = 1]\log[(1-p_{11})\phi(r_t;\mu_2,\sigma_2^2)] \\ &= +\mathrm{I}[S_t = 2]\mathrm{I}[S_{t-1} = 2]\log[p_{22}\phi(r_t;\mu_2,\sigma_2^2)] \end{aligned} \tag{37}$$

The E-step can be computed by applying the expectation operator $\tilde{E}$ to the joint log likelihood of the data as follows:

$$\begin{aligned} \tilde{\mathrm{E}}[\log f(r_{1:T}, s_{1:T}|\theta)] &= \tilde{\mathrm{E}}[\log f(r_1, s_1|\theta)] + \sum_{t=2}^{T} \tilde{\mathrm{E}}[\log f(r_t, s_t|s_{t-1};\theta)] \\ &= \tilde{\mathrm{E}}[\log f(r_1, s_1|\theta)] + \sum_{t=2}^{T} \Big[P[S_t = 1, S_{t-1} = 1|r_{1:T})\log[p_{11}\phi(r_t;\mu_1,\sigma_1^2)] \\ &\quad + P(S_t = 1, S_{t-1} = 2|r_{1:T})\log[(1-p_{22})\phi(r_t;\mu_1,\sigma_1^2)] \\ &\quad + P(S_t = 2, S_{t-1} = 1|r_{1:T})\log[(1-p_{11})\phi(r_t;\mu_2,\sigma_2^2)] \\ &\quad + P[S_t = 2, S_{t-1} = 2|r_{1:T})\log[p_{22}\phi(r_t;\mu_2,\sigma_2^2)]\Big] \end{aligned} \tag{38}$$

In order to evaluate the joint smoothed probabilities $P(S_t = i, S_{t-1} = j|r_{1:T})$, the original two-state system must be augmented to a four-state representation as the output of the Kim Smoother based on a two-state system gives a single probability i.e. $P(s_t = i|r_{1:T})$. Therefore, the solution is to set up the Hamilton filter and smoother as a four state problem which works as follows:

$$\begin{aligned}
&\text{State 1 } (\tilde{S}_t = 1): \quad && S_t = 1 \text{ and } S_{t-1} = 1 \\
&\text{State 2 } (\tilde{S}_t = 2): \quad && S_t = 1 \text{ and } S_{t-1} = 2 \\
&\text{State 3 } (\tilde{S}_t = 3): \quad && S_t = 2 \text{ and } S_{t-1} = 1 \\
&\text{State 4 } (\tilde{S}_t = 4): \quad && S_t = 2 \text{ and } S_{t-1} = 2
\end{aligned} \tag{39}$$

With the transition probabilities for the four-state problem defined as follows:

$$P = \begin{bmatrix} p_{11} & p_{11} & 0 & 0 \\ 0 & 0 & 1-p_{22} & 1-p_{22} \\ 1-p_{11} & 1-p_{11} & 0 & 0 \\ 0 & 0 & p_{22} & p_{22} \end{bmatrix} \tag{40}$$

Excess stock returns are now described as follows:

$$r_t = \begin{cases} \mu_1 + \sigma_1 \epsilon_t, & \text{if } \tilde{S}_t = 1 \\ \mu_1 + \sigma_1 \epsilon_t, & \text{if } \tilde{S}_t = 2 \\ \mu_2 + \sigma_2 \epsilon_t, & \text{if } \tilde{S}_t = 3 \\ \mu_2 + \sigma_2 \epsilon_t, & \text{if } \tilde{S}_t = 4 \end{cases} \tag{41}$$

Now, after running the Hamilton filter and Kim Smoother, the joint conditional probability $p^*_{ij}(t) = P(S_t = i, S_{t-1} = j|\Omega_T)$ is obtained. The expectation step as in equation 38 can be computed by replacing $P(S_t = i, S_{t-1} = j|r_{1:T})$ with p^*_{ij}. The parameter estimated are obtained by optimizing analytically over all the parameters.

The EM-algorithm is as follows: First, initialize by drawing $p^*_{i,j}(t)$ randomly while ensuring that $p^*_{11}(t) + p^*_{12}(t) + p^*_{21}(t) + p^*_{22}(t) = 1$ for each t. Apply the maximization step by treating the $p_{ij} * (t)$ as given and fixed and optimise over the parameters. Finally, apply the estimation step by treating the parameter estimates as given and fixed and run the four-state Hamilton filter and smoother to compute the four probabilities $p^*_{ij}(t)$ for each time t. Repeat the M-step and E-step until convergence.

C Appendix: Score of the Markov Switching model

This appendix derives equation **16**, which is the score of the **basic** Markov Switching model with respect to θ for the **observed** likelihood specification.

Let $R_t \equiv (r_t, r_{t-1}, ..., r_1)$, $X_t \equiv (x_t, x_{t-1}, ..., x_1)$ and $S_t \equiv (s_t, s_{t-1}, ..., s1)$ denote the complete history of the excess stock returns, explanatory variables and the state of the market, respectively. Further, the summation of the two possible values of S_t is denoted by $\int dS_t$ and is expressed as:

$$\int g(S_t)dS_t \equiv \sum_{s_1=1}^{2}\sum_{s_2=1}^{2}...\sum_{s_t=1}^{2} g(s_t, s_{t-1}, ..., s_1) \tag{42}$$

Under the assumption that s and r are uncorrelated, the **observed** likelihood of observation 1 through t is written as:

$$f(R_t|X_t;\lambda) = \int f(R_t|X_t, S_t;\theta)f(S_t;p)dS_t \tag{43}$$

with

$$f(R_t|X_t, S_t;\theta) = \prod_{\tau=1}^{t} f(r_\tau|c_\tau, s_\tau;\theta) \tag{44}$$

$$f(S_t;p) = f(s_1;p)\prod_{\tau=2}^{t} f(s_\tau|s_{\tau-1};p) \tag{45}$$

Using these expressions, the derivative of the **observed** log likelihood of the first t observations is given by:

$$\frac{\partial \text{log} f(R_t|X_t;\lambda)}{\partial\theta} = \frac{1}{f(R_t|X_t;\lambda)}\int \frac{\partial f(R_t|X_t, S_t;\theta)}{\partial\theta} f(S_t|p)dS_t \tag{46}$$

$$= \int \frac{\partial \text{log} f(R_t|X_t, S_t;\theta)}{\partial\theta}\frac{f(R_t|X_t, S_t;\theta)f(S_t;p)}{f(R_t|X_t;\lambda)} \tag{47}$$

$$= \int \frac{\partial \text{log} f(R_t|X_t, S_t)}{\partial\theta} f(S_t|R_t, X_t;\lambda)dS_t \tag{48}$$

Using 44 and 46, the expression is written as:

$$\frac{\partial \text{log} f(R_t|X_t, S_t;\lambda)}{\partial\theta} = \sum_{\tau=1}^{t}\sum_{s_t=1}^{2}\frac{\partial \text{log} f(r_\tau|x_\tau, s_\tau;\theta)}{\partial\theta} f(s_\tau|\Omega_t) \tag{49}$$

For $t = 1$, it can be seen that $Y_1 = y_1$ and $X_1 = x_1$ and therefore, 49 is written as:

$$\frac{\partial \text{log} f(r_1|X_1;\lambda)}{\partial\theta} = \sum_{s_1=1}^{2}\frac{\partial \text{log} f(r_1|x_1, s_1;\theta)}{\partial\theta} f(s_1|\Omega_1) \tag{50}$$

with

$$\frac{\partial \log f(R_t|X_t;\lambda)}{\partial\theta} = \sum_{\tau=1}^{t} \frac{\partial \log f(r_\tau|x_\tau;\lambda)}{\partial\theta} \tag{51}$$

From, 49 and 51, the score of observation t is deduced and given by:

$$h_t(\tilde{\lambda}) = \frac{\partial \log f(r_t|x_t;\lambda)}{\partial\theta} = \sum_{j=1}^{2} \psi_{t,j} P[S_t = j|\Omega_t] + \sum_{\tau=1}^{t-1}\sum_{j=1}^{2} \psi_{\tau,j} \left(P[S_\tau = j|\Omega_t] - P[S_\tau = j|\Omega_{t-1}]\right) \tag{52}$$

for $t = 2, ..., T$

D Appendix: Hamilton filter and Kim smoother

The state probability forecasts are computed recursively using the Hamilton filter as proposed in Hamilton (1989). By the law of conditional probability the following holds:

$$\begin{aligned} P[S_t = s_t|\Omega_{t-1}] &= \sum_{s_{t-1}} P[S_t = s_t, S_{t-1} = s_{t-1}|\Omega_{t-1}] \\ &= \sum_{s_{t-1}} P[S_t = s_t|S_{t-1} = s_{t-1}, \Omega_{t-1}] P[S_{t-1} = s_{t-1}|\Omega_{t-1}] \\ &= \sum_{s_{t-1}} P[S_t = s_t|S_{t-1} = s_{t-1}] P[S_{t-1} = s_{t-1}|\Omega_{t-1}] \end{aligned} \tag{53}$$

It can easily be seen that the first term in expression 53 is the transition probability of the Markov process. The second term is the inference probability and can be written as follows:

$$P[S_{t-1} = s_{t-1}|\Omega_{t-1}] = \frac{f(r_{t-1}, S_{t-1} = s_{t-1}|\Omega_{t-2}}{f(r_{t-1}|\Omega_{t-2})} \tag{54}$$

Let $\hat{\xi}_{t+1|t} \equiv [P(S_{t+1} = 1|\Omega_t), P(S_{t+1} = 2|\Omega_t)]'$ denote the predicted states obtained using the Hamilton filter with $\hat{\xi}_{t+1|t} = P\hat{\xi}_{t|t}$. The vectors $\hat{\xi}_{t+1|t}$ and $\hat{\xi}_{t|t}$ represent the one-step ahead forecasted state probabilities and estimated state probabilities of the current state, respectively and P the transition probability matrix. In order to update the beliefs regarding the current state, only the rules of the conditional probabilities are needed. The

Hamilton updating step is therefore derived as follow:

$$\begin{aligned}
\hat{\xi}_{t|t} &\equiv \begin{bmatrix} P(S_t = 1|\Omega_{t-1}) \\ P(S_t = 2|\Omega_{t-1}) \end{bmatrix} = \begin{bmatrix} P(S_t = 1|\Omega_{t-1}, r_t) \\ P(S_t = 2|\Omega_{t-1}, r_t) \end{bmatrix} \\
&= \frac{1}{P(r_t|\Omega_{t-1})} \begin{bmatrix} P(S_t = 1, r_t|\Omega_{t-1}) \\ P(S_t = 2, r_t|\Omega_{t-1}) \end{bmatrix} \\
&= \frac{1}{P(r_t|\Omega_{t-1})} \begin{bmatrix} f(r_t|S_t = 1, \Omega_{t-1})P(S_t = 1|\Omega_{t-1}) \\ f(r_t|S_t = 2, \Omega_{t-1})P(S_t = 2|\Omega_{t-1}) \end{bmatrix} \\
&= \frac{1}{P(r_t|\Omega_{t-1})} \begin{bmatrix} f(r_t|S_t = 1) \\ f(r_t|S_t = 2) \end{bmatrix} \odot \hat{\xi}_{t|t-1} \\
&= \frac{\begin{pmatrix} f(r_t|S_t = 1) \\ f(r_t|S_t = 2) \end{pmatrix} \odot \hat{\xi}_{t|t-1}}{\begin{pmatrix} 1 & 1 \end{pmatrix} \left[\begin{pmatrix} f(r_t|S_t = 1) \\ f(r_t|S_t = 2) \end{pmatrix} \odot \hat{\xi}_{t|t-1} \right]}
\end{aligned} \tag{55}$$

To initialise the Hamilton filter, the unconditional probability of being in each state is used. This can be found by solving for the eigenvector problem of P to obtain the following:

$$\begin{pmatrix} p11 & 1-p22 \\ 1-p11 & p22 \end{pmatrix} \begin{pmatrix} \frac{1-p22}{2-p11-p22} \\ \frac{1-p11}{2-p11-p22} \end{pmatrix} = \begin{pmatrix} \frac{1-p22}{2-p11-p22} \\ \frac{1-p11}{2-p11-p22} \end{pmatrix} \tag{56}$$

The elements in the eigenvector can be interpreted as a long-term percentage of time spent in states 1 and 2, respectively and therefore the unconditional probability of being in state 1 is given by $\frac{1-p22}{1-p11-p22}$.

To estimate the parameters of the Markov Switching model, the Expectation Maximization (EM) algorithm is used as the Maximum Likelihood Estimator (ML) may get stuck in a local maximum. Therefore, the smoothed probabilities $\xi_{t|t+1}$ are needed which are obtained using the Kim smoothing equation denoted as:

$$\begin{aligned}
\hat{\xi}_{t|t+1} &= \begin{bmatrix} P(S_t = 1|\Omega_{t+1}) \\ P(S_t = 2|\Omega_{t+1}) \end{bmatrix} = \begin{bmatrix} P(S_t = 1|\Omega_t, r_{t+1} \\ P(S_t = 2|\Omega_t, r_{t+1}) \end{bmatrix} \\
&= \frac{1}{P(r_{t+1}|\Omega_t)} \begin{bmatrix} P(S_t = 1, r_{t+1}|\Omega_t) \\ P(S_t = 2, r_{t+1}|\Omega_t) \end{bmatrix} \\
&= \frac{1}{P(r_{t+1}|\Omega_t)} \begin{bmatrix} f(r_{t+1}|S_t = 1, \Omega_t)P(S_t = 1|\Omega_t) \\ f(r_{t+1}|S_t = 2, \Omega_t)P(S_t = 2|\Omega_t) \end{bmatrix} \\
&= \frac{1}{P(r_{t+1}|\Omega_t)} \begin{bmatrix} f(r_{t+1}|S_t = 1) \\ f(r_{t+1}|S_t = 2) \end{bmatrix} \odot \begin{bmatrix} P(S_t = 1|\Omega_t \\ P(S_t = 2|\Omega_t) \end{bmatrix} \\
&= \frac{1}{P(r_{t+1}|\Omega_t)} P' \begin{bmatrix} f(r_{t+1}|S_t = 1) \\ f(r_{t+1}|S_t = 2) \end{bmatrix} \odot \hat{\xi}_{t|t} \\
&= \hat{\xi}_{t|t} \odot P' \left(\begin{bmatrix} f(r_{t+1}|S_t = 1) \\ f(r_{t+1}|S_t = 2) \end{bmatrix} / P(r_{t+1}|\Omega_t) \right) \\
&= \hat{\xi}_{t|t} \odot P'(\hat{\xi}_{t+1|T} \oslash \hat{\xi}_{t+1|t})
\end{aligned} \tag{57}$$

Where $\odot$ and $\oslash$ representing element-wise multiplication and division, respectively. The elements of the smoothed probability vector $\hat{\xi}_{t+t+1}$ is then used to construct the score of the basic Markov Switching model in 18.

E Appendix: Delta method for standard errors of the performance fees

The standard errors for the performance fees are computes using the delta method as in Ledoit and Wolf (2018). The performance fee Δ can be computed by solving the following equation:

$$\sum_{t=0}^{n-1} \left((R^a_{p,t+1} - \Delta) - \frac{\gamma}{2(1+\gamma)} (R^a_{p,t+1} - \Delta)^2 \right) = \sum_{t=0}^{n-1} \left(R^b_{p,t+1} - \frac{\gamma}{2(1+\gamma)} (R^b_{p,t+1})^2 \right), \tag{58}$$

with $R^a_{p,t+1}$ and $R^b_{p,t+1}$ denoting the portfolio returns obtained from the trading and benchmark strategies, respectively. The trick is to derive Δ and write it as a smooth function of its population moments. See below a full derivation of Δ:

$$\sum_{t=0}^{n-1} R^a_{p,t+1} - n\Delta - \frac{\gamma}{2(1+\gamma)}\sum_{t=0}^{n-1} R^{a,2}_{p,t+1} + 2\frac{\gamma}{2(1+\gamma)}\Delta\sum_{t=0}^{n-1} R^a_{p,t+1} - \frac{\gamma}{2(1+\gamma)}n\Delta^2 = \sum_{t=0}^{n-1} R^b_{p,t+1} - \frac{\gamma}{2(1+\gamma)}\sum_{t=0}^{n-1} R^{b,2}_{p,t+1} \tag{59}$$

$$n\mu^a - n\Delta - \frac{\gamma}{2(1+\gamma)}nS^a + 2\frac{\gamma}{2(1+\gamma)}\Delta n\mu^a - \frac{\gamma}{2(1+\gamma)}n\Delta^2 = n\mu^b - \frac{\gamma}{2(1+\gamma)}nS^b \tag{60}$$

$$\mu^a + (2\frac{\gamma}{2(1+\gamma)}\mu^a - 1)\Delta - \frac{\gamma}{2(1+\gamma)}S^a + -\frac{\gamma}{2(1+\gamma)}\Delta^2 = \mu^b - \frac{\gamma}{2(1+\gamma)}S^b \tag{61}$$

$$\frac{\gamma}{2(1+\gamma)}\Delta^2 - (\frac{\gamma}{(1+\gamma)}\mu^a - 1)\Delta + (\mu^b - \mu^a) - \frac{\gamma}{2(1+\gamma)}(S^b - S^a) = 0 \tag{62}$$

$$\Delta = \frac{(\frac{\gamma}{(1+\gamma)}\mu^a - 1) \pm \sqrt{(\frac{\gamma}{(1+\gamma)}\mu^a - 1)^2 - \frac{2\gamma}{(1+\gamma)}(\mu^b - \mu^a - \frac{\gamma}{2(1+\gamma)}(S^b - S^a))}}{\frac{\gamma}{(1+\gamma)}} \tag{63}$$

Δ is a function of the first and second moments of return series R^a and R^b. Let $z = [\mu^a, \mu^b, S^a, S^b]$, then Δ can be expressed as $\Delta = f(z)$. Assuming that $\sqrt{T}(\hat{z} - z) \Rightarrow N(0, V(z))$, with $\Rightarrow$ denoting convergence in distribution, $\hat{z}$ the estimator of z and $V(z)$ the 4×4 variance matrix of $z = [\mu^a, \mu^b, S^a, S^b]$, the delta method implies that:

$$\begin{aligned} \sqrt{T}(\hat{\Delta} - \Delta) &\Rightarrow N(0, \nabla' f(z)V(z)\nabla f(z)) \\ SE(\hat{\Delta}) &= \sqrt{\frac{\nabla' f(z)V(z)\nabla f(z)}{T}} \end{aligned} \tag{64}$$

With $\nabla' f(z)$ the gradient of $f(z)$.

The heteroskedasticity and autocorrelation robust (HAC) kernel estimation is used to obtain a consistent estimate of $V(z)$. The HAC kernel estimate for $V(z)$ is given by:

$$\begin{aligned} \hat{V}(z) &= \frac{T}{T-2M}\sum_{j=-T+1}^{T-1} k\left(\frac{j}{S_T}\right)\hat{\Gamma}_T(j), \quad \text{where} \\ \hat{\Gamma}_T(j) &= \begin{cases} \frac{1}{T}\sum_{t=j+1}^{T} \hat{y}_t\hat{y}'_{t-j} & \text{for j} \geq 0 \\ \frac{1}{T}\sum_{t=-j+1}^{T} \hat{y}_{t+j}\hat{y}'_t & \text{for j} < 0 \end{cases}, \quad \text{where} \\ \hat{y}'_t &= [r_{t,a} - \mu^a, r^2_{t,a} - S^a, r_{t,b} - \mu^b, r^2_{t,b} - S^b] \end{aligned} \tag{65}$$

$k(\cdot)$ represents the Parzen kernel function and S_T the selected bandwidth. The standard error as in equation 64 can be computed by plugging in the variance estimate $\hat{V}(z)$ for Vz.

F Appendix: Hard thresholding results

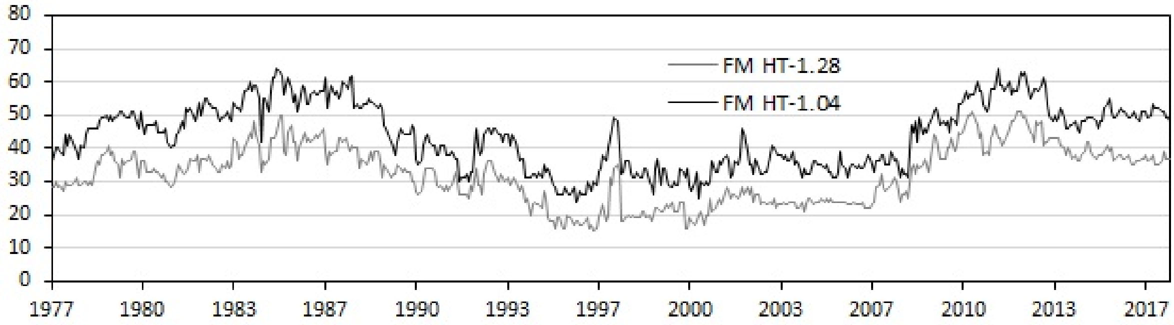

Figure 7: **Numbers of macroeconomic variables selected using the hard thresholding rule.** Notes: The graphs represent the numbers of macroeconomic variables that are selected using the hard thresholding approach, with significance levels of 0.10 and 0.15 for FM HT-1.28 and FM HT-1.04, respectively.

Table 11: Variables selected most frequently using variable selection methods

FM HT-1.28		**FM HT-1.04**		**TI Bin**		**TI Diff**	
CES0600000007	0.76	CES0600000007	0.84	MA1,9	0.46	MA1,9	0.41
AWHMAN	0.74	AWHMAN	0.81	MA2,12	0.34	MAOBV3,12	0.37
TB3MS	0.73	TB3MS	0.78	MA3,9	0.32	MA1,12	0.31
TB3SMFFM	0.62	IPBUSEQ	0.71	MAOBV3,12	0.27	MM12	0.27
PERMITMW	0.58	TB3SMFFM	0.68	MA2,9	0.24	MA3,12	0.26
IPDMAT	0.55	PERMITMW	0.67	MM12	0.22	MA2,9	0.26
IPBUSEQ	0.52	IPDMAT	0.65	MM9	0.20	MM9	0.22
PERMITW	0.51	PERMITW	0.62	MAOBV3,9	0.20	MAOBV2,12	0.21
DSERRG3M086SBEA	0.51	OILPRICEx	0.61	MAOBV2,12	0.20	MAOBV3,9	0.17
COMPAPFFx	0.50	USTRADE	0.60	MA1,12	0.16	MAOBV1,9	0.16
HWIURATIO	0.49	DSERRG3M086SBEA	0.60	MAOBV1,9	0.15	MA3,9	0.11
USTRADE	0.49	PERMITNE	0.59	MAOBV1,12	0.11	MAOBV1,12	0.11
OILPRICEx	0.48	HWIURATIO	0.58	MA3,12	0.07	MAOBV2,9	0.07
PERMITNE	0.48	RETAILx	0.56	MAOBV2,9	0.06	MA2,12	0.05
WPSID61	0.47	TB6SMFFM	0.55				

Notes: The variable names displayed in this table correspond to the short names provided in appendix **A**. FM HT-1.28 and FM HT-1.04 stand for the factor-augmented predictive regressions where the factors are constructed after employing hard thresholding rules with t-statistics of 1.24 and 1.04, respectively. TI Bin and TI Diff stand for the predictive regressions where technical variable are used with Bin and Diff indicating binary and differences, respectively. The numbers next to the variables indicate the proportion of times the variables are selected throughout the complete out-of-sample period.

G Appendix: Parameter estimates for the 5 Markov Switching models

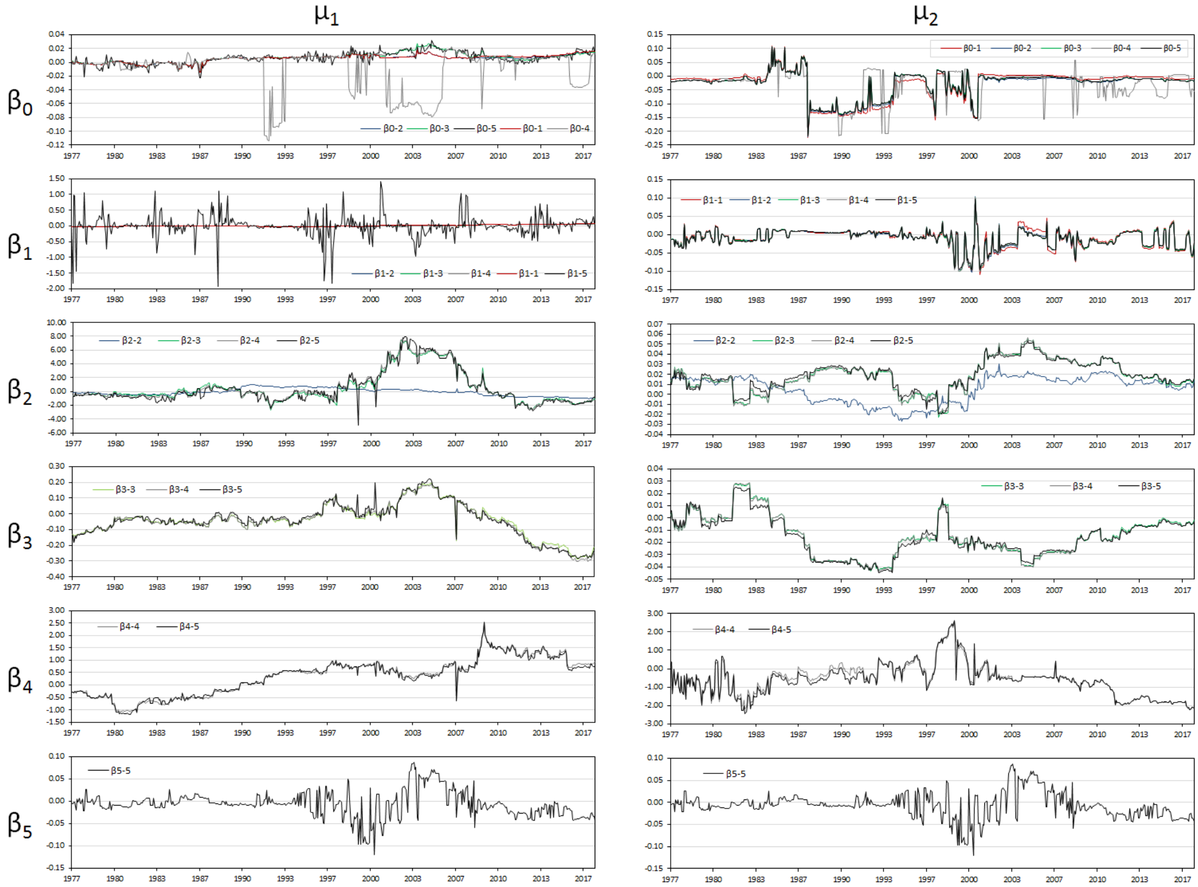

Figure 8: **Parameter estimates for the Markov Switching model consisting of selected predictor variables** Notes: The graphs represent the evolution of the 6 parameters over time. The first graphs represent the level parameter estimates for each model over time. The second graph denotes the estimate of the first predictor variable for all models, and so on.

H Appendix: Performance of Markov Switching models during sub samples

Table 12: Performance of active trading strategies based on Markov Switching models: Transaction costs = 0.1%, RRA=6, w_{t+1} ∈[0,1]

	μ	σ	SR	Δ_{100}	Δ_{50}	Δ_0
Panel A: Jan 1977 - Dec 2017						
MS Simple	8.02%	8.74%	0.38	77	54	177
MS1	8.46%	8.91%	0.42	126	103	226
MS2	8.79%	9.03%	0.46	211	187	311
MS3	8.69%	9.33%	0.43	188	165	288
MS4	11.52%	9.56%	0.72	370	347	469
MS5	10.30%	9.77%	0.57	288	263	387
Panel B: Jan 1977 - Dec 1997						
MS Simple	9.21%	9.18%	0.19	2	-11	75
MS1	10.02%	9.41%	0.27	98	85	171
MS2	10.42%	9.52%	0.31	176	163	249
MS3	10.30%	9.97%	0.29	143	130	216
MS4	13.27%	10.11%	0.58	296	283	369
MS5	12.67%	10.36%	0.50	238	225	311
Panel C: Jan 1998 - Dec 2017						
MS Simple	6.82%	8.29%	0.58	152	118	279
MS1	6.91%	8.41%	0.58	154	120	281
MS2	7.16%	8.54%	0.60	245	211	372
MS3	7.08%	8.70%	0.58	233	199	360
MS4	9.77%	9.02%	0.86	442	408	569
MS5	7.92%	9.15%	0.65	336	302	463

Note: Performance fees for active mean-variance portfolios during complete sample period and sub periods, obtained using equation 58. The columns μ and σ denote the percentage annualized mean and standard deviation of the portfolio returns, respectively. SR denotes the Sharpe ratio and Δ the annualized performance fees (in basis points) for switching from the strategy indicated by the subscript to the strategy indicated by the corresponding row. The model abbreviations are as in Table **??**

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