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Master Thesis

A Joint Optimisation of Demand Responsive Transport and Fixed Line and Schedule Bus Transport

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Abstract

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Master of Science

A Joint Optimisation of
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by Jessica van der Zee

Demand responsive transport (DRT) systems were originally used as a complement to traditional fixed line and schedule (FLS) transport. However, in some cases the cost of using DRT can be competitive, or even lower than the cost of FLS transport. This thesis researches the optimal design of public transport systems in areas that are currently served by FLS transport. The proposed method considers replacing and complementing (parts of) FLS lines with DRT. The problem is formulated as a mixed integer linear problem with the objective to maximise profit for the operator. The method does not assume any area characteristics such as street or transit patterns, as most other research in DRT design does, and is thus applicable to all sorts of areas. Customer choice modelling is used to evaluate customer behaviour when different travel options are offered. The method is tested on a case study in the Netherlands which proves not to be interesting for DRT in the current circumstances. However, aspects like outsourcing or subsidising DRT could change that outcome. Two of the main advantages of the model are that it is very easy to personalise to the characteristics of an area and that the model works for all kinds of areas. A disadvantage of our implementation of the model is that several assumptions are required. We show how this disadvantage can be removed or alleviated.

Keywords: demand responsive transport; modeling public transport; network design; flexible transport services.

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Contents

List of Abbreviations

DARP	**D**ial-**A**-**R**ide **P**roblem
DRT	**D**emand **R**esponsive **T**ransport
FLS	**F**ixed **L**ine and **S**chedule
IVTT	**I**n-**V**ehicle **T**ravel **T**ime
MILP	**M**ixed **I**nteger **L**inear **P**roblem
OVT	**O**ut-of-**V**ehicle **T**ime
VOT	**V**alue **O**f **T**ime

Chapter 1

Introduction

Demand Responsive Transport (DRT) is a form of public transport in which customers can order small/medium sized vehicles which are shared between flexible pick-up and drop-off locations. There is no timetable or bus line, instead transport is arranged according to customers' needs. Historically, this service is better known as dial-a-ride, which usually focused on transport for disabled and elderly people. Dial-a-ride services, or paratransit, grew mainly due to government regulations that required transport for mobility impaired and disabled persons (Nelson et al., 2010). Since conventional Fixed Line and Schedule (FLS) transport services could not supply this, DRT systems were offered. These traditional DRT systems often ran against high cost per passenger compared to FLS transport. When only a few customer trips are shared, cost per customer can raise up to the level of taxi services.

Originally, DRT systems were mainly used as a complement to FLS transport. When FLS transport fails to supply transport for disabled and elderly people, DRT systems offer these services. However, DRT systems can in some cases also be considered to partly replace FLS transport instead of just complement it. In some areas, the cost of using DRT systems to provide public transport services can be competitive, or even lower than the cost of FLS transport. This will most likely occur in areas with lower demand, such as rural or even suburban areas. These areas could be considered for introducing DRT with unrestricted usage. However, often these areas might already be served by FLS transport. Introducing DRT as substitute for FLS is difficult, since it has unclear implications for the operator, as it is hard to predict the difference in costs. Furthermore, due to the difference between DRT and FLS services, customer behaviour can change.

In this thesis, we investigate the optimal design of bus transport in areas that are currently served by FLS transport. There are no assumptions about area characteristics and thus this method works for all sorts of areas. We consider DRT systems to partly replace or complement FLS transport. Instead of offering DRT services from address to address, we consider fixed DRT stops such that customers can travel from any DRT stop to any other DRT stop. Enlarging the service area generally incurs additional cost for the operator and therefore DRT from address to address is not considered. The optimisation objective is from a transport operator's perspective and therefore to maximise profit. Not only could DRT save costs, but other advantages could be a lower environmental impact and higher customer satisfaction.

The proposed method consists of a Mixed Integer Linear Problem (MILP) that decides per stop whether it should stay an FLS stop, become a DRT stop or should be a stop to transit between FLS and DRT. Area characteristics are given as input and estimated customer behaviour towards different travel options is used to make a prediction on customer flows. Our methods are tested on a case study in the Netherlands.

The main purpose of this thesis is for transport operators to get an indication of which areas to consider for introducing DRT. So far, no general approaches to designing DRT areas with heterogeneous characteristics have been developed. The method discussed in this thesis can be applied to any area.

The remainder of this thesis is organised as follows. In the next chapter, relevant literature is discussed. Chapter 3 contains a problem description. In Chapter 4, a MILP is formulated and more information about additional simulations and the dial-a-ride problem is given. The MILP is tested on a case study in Chapter 5. Finally, the thesis is concluded in Chapter 6.

Chapter 2

Literature Review

When operating a DRT area, there are several aspects that should be considered. First of all, the design of the DRT area has to be determined. This consists of the location of the DRT stops, which is the main focus of this thesis. To operate the DRT area, an algorithm is needed for dispatching the vehicles, also known as the Dial-A-Ride Problem (DARP).

In this thesis, we optimise the design of bus systems by maximising profit from the operator's perspective. This is a relatively new topic and there is limited research about this topic. Besides literature about the design of DRT systems, other topics are also relevant for this thesis. Even though we focus on the design of the DRT area, the expected fleet size is needed to determine the costs and this depends on the technique used for the DARP. The expected cost are also dependant on the number of customers travelling and to determine the expected behaviour of customers, choice modelling theory is used. Therefore we also need information about the DARP and customer choice modelling. For an overview of the research about the DARP, we refer to Cordeau and Laporte (2007) for all progress up to 2007 and Ho et al. (2018) for the progress from 2007 up to 2018. Previous research about the design of DRT systems and customer choice modelling will be discussed in the remainder of this chapter.

2.1 DRT systems

The design of DRT systems can have drastic impacts on the performance of the system as shown by Li and Quadrifoglio (2009). In general, the larger the DRT area, the longer vehicles have to travel and the more trips are eligible for DRT. Smaller areas have to deal with shorter distances but also less trips, in general.

There have been some studies about the design of DRT areas with homogeneous characteristics such as grid street patterns, even distribution of stops and specific patterns of transit stations. Aldaihani et al. (2004) study a method to design DRT systems in areas with gridded street patterns. They assume the service area to be rectangular and optimally divide the area into $n \times n$ subareas. Within each subarea, DRT serves as a feeder system towards the transit station of that subarea. All transit stations of the subareas should be connected through FLS transport. Li and Quadrifoglio (2009) developed a method to determine the optimal DRT design for feeder systems that are rectangular and connected on one end to a transit network. The optimal number of DRT areas is determined as well as the optimal length and width. Both of these papers' results are only applicable to areas with very specific characteristics. In 2014, Edwards presented a master thesis in which he describes a method to optimally design DRT feeder systems. By using isochrones instead of geometric shapes centred around transit stations, performance is improved. Even though this method works for all sorts of areas, it does not consider substituting part of FLS bus transport by DRT and only considers DRT as feeder systems.

So far, research in zone optimisation often simplifies circumstances by making assumptions about street patterns, demand distribution and transit patterns or using DRT purely as feeder systems. Therefore the techniques discussed in this research cannot be applied to areas with heterogeneous characteristics where DRT is used as a substitute for regular bus transport. To the best of the authors knowledge, there has not been any research that focuses on designing DRT areas with heterogeneous circumstances.

2.2 Customer choice modelling

When customers are offered various options to travel, choice modelling can be used to determine the expected customer behaviour. We use discrete choice modelling to determine the expected number of customers that travel with DRT and FLS. For a detailed overview of the methods of discrete choice analysis and their applications in the modelling of transportation systems we refer to Ben-Akiva and Lerman (1985). More recently, Train (2009) describes the new generation of discrete choice methods while focusing on the many advances that are made possible by simulation.

In this thesis, we use similar methods for customer choice modelling as introduced by Atasoy et al. (2015). They introduce an innovative transport concept called Flexible Mobility on Demand. Passengers are provided with a choice between different forms of transportation: taxi, shared taxi (DRT equivalent) or mini-bus. The assortment is optimised per customer. They assume customers choose among the products in the assortment based on a logit model. Robenek et al. (2016) focus on improving the planning process of passenger railway services. When solving the Passenger Centric Train Timetabling Problem, they take into account customer satisfaction. Where Atasoy et al. (2015) determine the customer utility of different travel options by taking into account the travel time, price, type of service and schedule passenger delay, Robenek et al. (2016) also use the number of transfers and waiting time.

Chapter 3

Problem Description

This thesis considers the cost-benefit of substituting (parts of) Fixed Line and Schedule (FLS) lines with Demand Responsive Transport (DRT). In this chapter, the approach to designing the bus system is discussed, as well as the objective of this problem and choice modelling theory to determine expected customer behaviour.

3.1 Design

The FLS transport we consider in this thesis consists of vehicles that drive fixed routes on fixed times, and stop at fixed bus stops. DRT, on the other hand, consists of DRT vehicles that bring customers to their desired destination on demand, without the need for transfers. Customers are required to order a vehicle and depending on the other customers at that moment, customers can share the same vehicle, which can result in a detour for some customers. DRT trips can either be allowed to start and end at any address in a specific area or at a set of stops such that customers can travel between any two DRT stops. The first option results in more flexibility towards the customer, the second option results in customer service comparable to FLS transport. In this thesis, we optimise bus systems that can consist of DRT and FLS transport. Since we consider replacing FLS stops by DRT stops, the second option is used in which DRT can be ordered from and to designated DRT stops only. In general, operating public transport becomes profitable due to the subsidy that transport operators receive from the government. The government assures that public transport is affordable for the public and subsidises transport operators that serve areas according to specific demands. Therefore, servicing larger areas than is strictly required by the government will most likely result in less profit for the operator. This is the reason that offering DRT from address to address is not considered. Furthermore, a disadvantage of offering

DRT from address to address is that combining trips becomes harder and less efficient. For the same reason, it is also not considered to add stops to the existing FLS network.

For every stop that is part of the FLS network, it is considered whether or not it should be part of the DRT network and whether or not FLS lines should still visit it. Every stop has to be served by at least one service (DRT or FLS). When some stops are removed from FLS lines, these FLS lines can be rerouted such that the total duration and length of that bus line decreases. This way, changes to FLS lines are limited to removing stops from existing bus lines and rerouting bus lines by adding new stops is not an option. Line planning is a different problem and although it is ideally solved simultaneously with the design of the DRT area, it is not considered here due to its complexity. Ceder and Wilson (1986) describe the bus network design problem and different approaches to solving it. More recently, Ibarra-Rojas et al. (2015) review the literature on planning and network design of urban transport focusing on bus service. If some specific changes to FLS lines are considered, their impact can be evaluated by comparing the output of the model with and without the changes.

Area characteristics are given as input to the model. Area characteristics consist of the route and frequencies of the FLS lines in the system. Furthermore, the origin-destination matrix of all stops is taken as input. The average distance and duration of every trip is also needed. Besides these area characteristics, other important information that is input to the MILP is the costs of personnel and vehicles as well as the price of bus tickets. The output on the other hand consists of which FLS lines stop at which stops and which stops are part of the DRT network. Also, the expected fleet size and the expected number of drivers is determined.

3.2 Objective

The objective is from the transport operator's perspective and is therefore to maximise profit. Note that the profit can also be negative, since often transport operators are subsidised by the government and without subsidy the service would not be profitable. Customer satisfaction is not taken into account in the objective but can be maintained by implying constraints.

The profit directly related to bus transport consists of revenue from bus tickets minus the costs directly related to bus transport: vehicle cost and personnel (driver) cost. The vehicle and personnel cost are variable and depend on the bus system and customers. Two types of vehicles are considered: large vehicles used for FLS transport and small vehicles used for DRT. In the case study, standard 12m buses with 40 seats are considered for FLS transport and 8-person vans for DRT. In reality, multiple types of vehicles can be used for FLS transport (and DRT) and fleet dispatching is a relevant problem for transport operators. However, fleet dispatching is not the focus of this thesis. As write-off costs are generally smaller than personnel costs, assuming only two types of vehicles is a reasonable assumption. Revenue, on the other hand, comes from bus tickets. In general, FLS ticket prices consist of a fixed price plus a fee per kilometre. DRT tickets often have a fixed price, independent of the origin and destination.

For FLS transport, it is quite straightforward to determine personnel costs and bus costs. Fuel and maintenance costs can be calculated per kilometre and since it is a fixed schedule and fixed line system, the number of kilometres driven can be calculated. For DRT, on the other hand, it is not as straightforward. We can analyse the number of trips that are expected to be executed with DRT but do not know which trips will be combined or where the vehicles are when they will receive the request for a trip. An estimation of the DRT fleet size and the total distance that the vehicles cover is needed. From DRT systems already in operation, estimations can be made on the number of vehicles needed and the expected distance covered. These simulations will be described in Section 4.3.

3.3 Choice modelling

To model the behaviour of customers when choosing if and how to travel, we use choice modelling theory. The utility of a customer is determined for each of the transport options the customer can choose from:

- DRT: Flexible transport from origin to destination without transfers or timetables. However, a detour can be made to pick-up or drop-off other customers sharing the vehicle.
- FLS: Transport via fixed routes with or without transfers and according to a fixed timetable. The route is generally not direct but follows the FLS network.
- Reject: Reject DRT and FLS transport.

To determine the utility function we use the same approach as Atasoy et al. (2015). The utility of a customer is influenced by the in-vehicle travel time, the price of the ticket, the type of service, the number of transfers and the waiting time outside the vehicle (Atasoy et al., 2015, Robenek et al., 2016). Note that since we are looking at aggregated customer flows, schedule delay is not taken into account.

The utility of a DRT trip from stop o to stop d depends on the price of the trip (pr_{od}^{DRT}) and the In-Vehicle Travel Time (IVTT), which consists of the direct driving time (ti_{od}) plus the added duration of a possible detour that can be made when the ride is shared (Δti_{od}). The utility of an FLS trip from stop o to stop d depends on the price of the trip (pr_{od}^{FLS}), the in-vehicle travel time (ti_{od}^{IVTT}) which can consist of multiple rides with transfers between them, the Out-of-Vehicle Time (OVT) in between these rides (ti_{od}^{OVT}) and the number of transfers (T_{od}). The utility of the reject option is influenced by the distance of the trip (di_{od}).

The deterministic utilities U_{od}^{DRT}, U_{od}^{FLS} and U_{od}^{reject} of a trip from stop o to stop d are calculated as

$$
\begin{aligned}
U_{od}^{\text{DRT}} &= \beta_{\text{DRT}} - pr_{od}^{\text{DRT}} - \beta_{\text{VOT}}^{\text{IVTT}} \times (ti_{od} + \Delta ti_{od}),\\
U_{od}^{\text{FLS}} &= \beta_{\text{FLS}} - pr_{od}^{\text{FLS}} - \beta_{\text{VOT}}^{\text{IVTT}} \times ti_{od}^{\text{IVTT}} - \beta_{\text{VOT}}^{\text{OVT}} \times ti_{od}^{\text{OVT}} - \beta_{\text{transfer}} \times T_{od},\\
U_{od}^{\text{reject}} &= -\beta_{\text{dist}} \times di_{od}.
\end{aligned}
$$

The utility functions are normalised in monetary units which can be considered as dividing by the price parameter $\beta_{\text{price}} > 0$. Constants β_{DRT} and β_{FLS} project the differences between DRT and FLS in terms of vehicle comfort and service. The option to reject is considered as a reference and therefore its constant is fixed to zero. Other coefficients are $\beta_{\text{VOT}}^{\text{IVTT}} > 0$ and $\beta_{\text{VOT}}^{\text{OVT}} > 0$, which display the monetary Value Of Time (VOT) of the in-vehicle travel time and out-of-vehicle time respectively. Coefficient $\beta_{\text{transfer}} > 0$ is the monetary value of a transfer. The reject option displays the travel alternatives other than FLS or DRT. When the travel distance decreases, customers will be less likely to use bus transport (Atasoy et al., 2015). For example, when a customers needs to travel 10 km or 2 km, he will be less likely to use bus transport for the trip of 2 km. When the travel distance increases to very large distances, eventually the reject option will become more appealing again. Therefore this reject utility function is a valid assumption for relatively small areas. The coefficient $\beta_{\text{dist}} > 0$ displays the monetary value of distance. These utilities are used in a deterministic setting and not in real time, therefore taking stochastic variables like delay into account is not possible.

Based on the transport modes stated above, there are three types of choice options that customers can experience:

- $\mathcal{A}^1 = \{DRT\}$, the only option to travel is by using DRT.
- $\mathcal{A}^2 = \{FLS\}$, the only option to travel is by using FLS.
- $\mathcal{A}^3 = \{DRT, FLS\}$, the customer can choose to travel with DRT or with FLS. This happens when both origin and destination are DRT stops as well as connected FLS stops.

The probability that a customer chooses a transport option depends on the choices that are available. These probabilities are calculated by using a logit choice model (Atasoy et al., 2015). The probability pi_{od}^m that a customer chooses transport mode $m \in \mathcal{A}^i$ for the trip from stop o to stop d when choice option $i \in \{1, 2, 3\}$ is offered, can now be calculated for every feasible option as

$$pi_{od}^m = \frac{\exp(\beta_{\text{price}} U_{od}^m)}{\exp(\beta_{\text{price}} U_{od}^{\text{reject}}) + \sum_{x \in \mathcal{A}^i} \exp(\beta_{\text{price}} U_{od}^x)}. \tag{3.1}$$

Since the utility functions are normalised in monetary units, they are adjusted by the scale parameter β_{price}. These choice modelling techniques allow us to predict customer behaviour when considering which transport modes to offer.

The choice of parameters is backed up by various literature:

- Atasoy et al. (2015) consider taxi, shared-taxi (DRT equivalent) and mini-bus (FLS equivalent) transport and for all services the same vehicle is used. They consider equal alternative specific constants (similar to our β_{DRT} and β_{FLS}) for shared-taxi and minibus and the constant for taxi is \$2 higher. In our case, DRT and FLS transport use different vehicles, specifically DRT uses small vans with a capacity of eight customers and there will never be more than eight customers in the van so a seat is guaranteed. FLS uses large 12m buses with about forty seats where seating is not guaranteed. Therefore DRT is considered to be little more convenient and customers are assumed to be willing to pay €1 more for DRT than FLS when all other factors are kept equal. Similar values of β_{DRT} and β_{FLS} as in Atasoy et al. (2015) are chosen: $\beta_{\text{DRT}} =$ €8, $\beta_{\text{FLS}} =$ €7.
- The scale parameter β_{price} is assumed to be 0.5/\$ = 0.44/€, just as the price parameters estimated by Koppelman and Bhat (2006).

- For the value of time for IVTT, we use similar estimations as Atasoy et al. (2015): $\beta_{\text{VOT}}^{\text{IVTT}} = €0.18/\text{minute}$. Note that since aggregated customer flows are considered, the average value is used instead of a probability distribution.
- The value of time for OVT ($\beta_{\text{VOT}}^{\text{OVT}}$) is considered to be 1.7 times $\beta_{\text{VOT}}^{\text{IVTT}}$ (Atasoy et al., 2015).
- A transfer is considered to have the same value as 10 minutes IVTT (de Keizer, Geurs, & Haarsman, 2012), so $\beta_{\text{transfer}} = 10 \times \beta_{\text{VOT}}^{\text{IVTT}}$.
- The monetary value of distance (β_{dist}) for the reject option is estimated to be €0.002/metre (Atasoy et al., 2015).
- To determine the expected detour time (Δti_{od}), we perform simulations derived from real-life DRT systems. Our simulation shows that on average the detour time is $0.2 \times ti_{od}$. Our approach to these simulations will be thoroughly discussed in Section 4.3 and the results in Section 5.1.2.

The reason customer choice modelling is introduced, is to make sure that when different travel options are offered, customer behaviour changes according to the travel options. For example, when the ticket price is set very high, it would be unrealistic for customer behaviour not to change accordingly. Different parameters would not directly cause a very different solution.

Now that we have set out the problem conditions together with the choice modelling theory and objective, in the next chapter the problem will be formulated as a mathematical model.

Chapter 4

Problem Formulation

In this chapter, the problem as described in Chapter 3 is formulated as a mathematical problem. First we will discuss the mathematical model, afterwards the approach that is used to solve the dial-a-ride problem is explained. Simulations with DRT data, that are used to estimate certain aspects in the mathematical model, are thoroughly discussed in the final section of this chapter.

4.1 Mathematical formulation

To determine the design of the DRT system, linear programming is used. First, some background information about linear programming is discussed.

4.1.1 Background in linear programming

When translating the problem into a mathematical formulation, we come across a lot of non-linear constraints that have to be linearised to be used in the linear problem, many of which can be formulated with a minimum or maximum operator. Some linearisation techniques are discussed that specifically apply to the model that is discussed later.

Consider the maximum $m_1 \in \mathbb{B}$ of n binary variables $x_1, ..., x_n$:

$$m_1 = \max\{x_1, ..., x_n\}. \tag{4.1}$$

This maximum operator can be linearised by

$$m_1 \geq x_i \qquad \forall i = 1, ..., n, \tag{4.2}$$

$$m_1 \leq \sum_{i=1}^{n} x_i, \tag{4.3}$$

where (4.2) ensures that m_1 will be at least as big as the highest value of $x_1, \ldots, x_n$ and (4.3) ensures m_1 to be zero when $x_1 = \ldots = x_n = 0$ and finally m_1 is bounded by 1 since it is a binary variable. Together these constraints form a linear way to formulate a maximum of binary variables. This method can be easily transformed to linearise a minimum $m_2 \in \mathbb{B}$ of binary variables $x_1, \ldots, x_n$. Consider

$$m_2 = \min\{x_1, \ldots, x_n\}, \tag{4.4}$$

m_2 can be transformed to a maximum operator of binary variables by

$$\begin{aligned} m_2 &= 1 - \max\{1 - x_1, \ldots, 1 - x_n\}, \\ 1 - m_2 &= \max\{1 - x_1, \ldots, 1 - x_n\}. \end{aligned}$$

Next we can use the same technique as shown before, to obtain the constraints

$$\begin{cases} 1 - m_2 & \geq 1 - x_i \qquad \forall i = 1, \ldots, n, \\ 1 - m_2 & \leq \sum_{i=1}^{n} 1 - x_i, \end{cases} \iff \begin{cases} m_2 & \leq x_i \qquad \forall i = 1, \ldots, n, \\ m_2 & \geq 1 - n + \sum_{i=1}^{n} x_i. \end{cases}$$

These techniques only apply to binary variables $x_1, \ldots, x_n$, integer variables require different techniques. Consider the case of a binary variable m_3 that indicates when at least one of the variables $y_1, \ldots, y_n$ is larger than zero with $y_i \in \{0, \ldots, U_i\}$ where U_i, $i = 1, \ldots, n$, is a positive integer. This can be formulated as

$$m_3 = \min\Big\{1,\ \max\{y_1, \ldots, y_n\}\Big\}. \tag{4.5}$$

This non-linear constraint can be linearised by

$$m_3 \geq \frac{1}{U_i}\, y_i \qquad \forall i = 1 \ldots, n, \tag{4.6}$$

$$m_3 \leq \sum_{i=1}^{n} y_i, \tag{4.7}$$

where (4.6) ensures m_3 to be one when at least one of $y_1, \ldots, y_n$ is larger than zero and (4.7) ensures m_3 to be zero when $y_1 = \ldots = y_n = 0$. A similar case is a binary variable m_4 that is zero when at least one of the variables $y_1, \ldots, y_n$ is larger than zero.

$$m_4 = \max\Big\{0,\ \min\{1 - y_1, \ldots, 1 - y_n\}\Big\}. \tag{4.8}$$

This is closely related to m_3:

$$m_4 = \max\Big\{0,\ \min\{1-y_1,\dots,1-y_n\}\Big\},$$
$$m_4 = \max\Big\{0,\ 1-\max\{y_1,\dots,y_n\}\Big\},$$
$$m_4 = 1-\min\Big\{1,\ \max\{y_1,\dots,y_n\}\Big\} = 1-m_3,$$

and can be linearised as

$$\begin{cases} 1-m_4 & \geq \frac{1}{U_i}\, y_i \qquad \forall i = 1\dots,n, \\ 1-m_4 & \leq \sum_{i=1}^{n} y_i, \end{cases} \iff \begin{cases} m_4 & \leq 1-\frac{1}{U_i}\, y_i \qquad \forall i = 1\dots,n, \\ m_4 & \geq 1-n+\sum_{i=1}^{n} 1-y_i. \end{cases}$$

Note that these are just a few of the possibilities for linearisation and only the techniques relevant to our model are shown. One more case will be considered where the binary variable m_5 should be zero when at least one of the variables $y_1,\dots,y_n$ equals zero and 1 otherwise. This can be formulated as

$$m_5 = \min\{1, y_1, \dots, y_n\}. \tag{4.9}$$

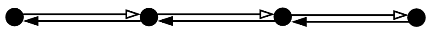
It is not possible to linearise this specific constraint without adding additional variables, specifically $n-1$ indicator variables $\mathcal{I}_1,\dots,\mathcal{I}_{n-1}$ are needed to linearise this, with $\mathcal{I}_i \in \mathbb{B}$, for $i = 1,\dots,n-1$. This can be formulated as

$$m_5 \leq y_i \qquad \forall i = 1,\dots,n,$$
$$m_5 \geq \frac{1}{U_i} y_i - (1-\mathcal{I}_i)M \qquad \forall i = 1,\dots,n-1,$$
$$m_5 \geq \frac{1}{U_n} y_n - (\sum_{i=1}^{n-1} \mathcal{I}_i)M,$$

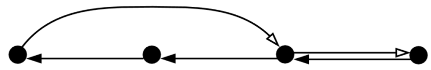
where M is a sufficiently large number. This way m_5 will always be smaller than all y_i, $i = 1,\dots,n$, and it will be bigger than $\frac{1}{U_n} y^{\min}$, where $y^{\min}$ is the minimum of $y_1,\dots,y_n$.

4.1.2 Mixed integer linear problem

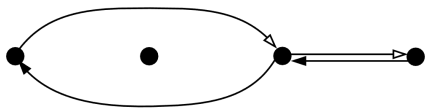
Using these linearisation techniques, the problem is formulated as a Mixed Integer Linear Problem (MILP) in this section. This model is designed to be used for a time frame with constant origin-destination flows of passengers and constant FLS frequencies. Therefore time should be discretised into such time frames and the model should be run for each of these time frames. The model is also designed to consider one DRT



(a) The original situation before introducing DRT.



(b) When circuits are not used, it can happen that the bus line visits a stop only in one direction which is undesirable.



(c) When circuits are used, it can happen that the FLS line skips a stop in both directions, this is acceptable since this stop can then be served by DRT.

FIGURE 4.1: Example FLS line that goes back and forth over the line that is used to explain the use of circuits.

area, so customers can travel from any DRT stop to any other DRT stop regardless of how far apart these stops are. This is done to restrict the complexity of the model and keep it manageable. If it is considered to use separate DRT systems in the region, the model can be run multiple times for different subregions of the total region. More information about this approach can be found in Section 4.1.3.

Consider the set $\mathcal{S}$ that contains all bus stops and the set $\mathcal{B}$ that contains all FLS lines. All bus lines that go back and forth between two stations are considered as circuits instead of two one-way routes. Otherwise, it could happen that a stop is only visited by the bus in one direction. This is visualised in Figure 4.1. Presenting bus lines as circuits ensures the undesired situation in Figure 4.1b never happens. Instead, the situation in Figure 4.1c could happen, this is acceptable since the stop that is not visited by the FLS line can be visited by DRT.

To visualise the use of variables, the example in Figure 4.2 is used. The figure visualises FLS line b that starts and ends at stop A and visits the stops in the order of the arrows. Consider a potential bus design as in Figure 4.3. Decision variable $y_{bs} = 1$ if stop $s \in \mathcal{S}$ is a stop on bus line $b \in \mathcal{B}$ and 0 otherwise. In Figure 4.3, $y_{bs} = 1$ for $s \in${A,B,E,F,G,H} and $y_{bs} = 0$ for $s \in${C,D}. Decision variable x_s indicates if stop $s \in \mathcal{S}$ is a DRT-stop. In Figure 4.3, $x_s = 1$ for $s \in${A,C,D,E,F} and $x_s = 0$ for $s \in${B,G,H}. A DRT stop can also be a stop of an FLS line. Besides deciding on which stops should be FLS or DRT, the MILP also determines the expected number of vehicles and drivers needed. Furthermore, in order to calculate fuel cost, the expected distance that the vehicles drive is determined.

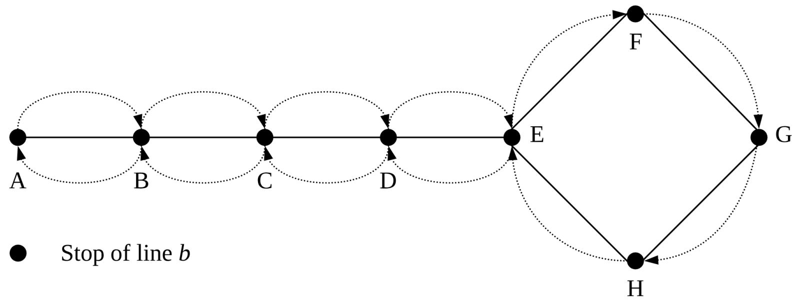


FIGURE 4.2: Example FLS line b that consecutively visits stations A, B, C, D, E, F, G, H, E, D, C, B and back to A.

In the next sections, the objective function and constraints of the MILP are discussed. Instead of displaying the large MILP all at once, the constraints are divided in sections to improve readability. Relevant decision variables and parameters are mentioned before introducing the constraints. To obtain a clear and structured model, all variables have names of one character and all parameters have names consisting of two characters. There are also non-linear constraints in the MILP. The linearisation techniques that are needed to linearise these constraints have been discussed in Section 4.1.1 and the exact details of linearisation are not shown. In the appendix, the complete MILP can be found in its entirety.

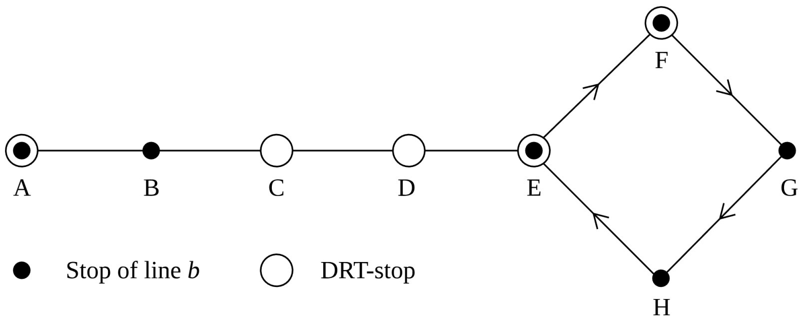


FIGURE 4.3: Example outcome of the MILP on original bus line b where a combination of DRT and FLS transport is used.

Objective

The objective of the MILP is from the transport operators perspective and is therefore to maximise profit. Customer choice is taken into account when estimating customer behaviour. However, customer satisfaction is not taken into account in the objective function. Customer utility could be incorporated in the objective, but it is more common to use constraints to bound customer satisfaction. For example, Atasoy et al. (2015) use constraints to bound the probability that customers reject all available travel options.

The costs and revenue are both calculated per time unit for ease of interpretation. Additionally, variables and parameters that consider time or distance are respectively in time and distance units. This way, these variables and parameters can be displayed as integer variables. Logical units to choose for time and distance for this specific problem, are seconds and metres respectively.

Consider the revenue r_{od} which represents the revenue from all tickets for trips from stop o to stop d, for $o, d \in \mathcal{S}$. The costs consist of salary costs that are dependent on the number of employees needed for both DRT and FLS vehicles together (w). Another component is the travel costs and we let d^{DRT} and d^{FLS} represent the expected total distance to be driven with DRT and FLS vehicles respectively. Finally, b^{DRT} and b^{FLS} represent the expected number of DRT and FLS vehicles that are needed to run the DRT and FLS system, respectively. The costs for these variables, are given by cs, cd^{DRT}, cd^{FLS}, cw^{DRT} and cw^{FLS} which represent the salary cost per driver per time unit, the fuel and maintenance cost per distance unit for DRT and FLS and the write-off cost per time unit for DRT and FLS, respectively. We assume DRT and FLS drivers receive the same salary.

Variable	Explanation
r_{od}	Expected revenue from all trips from origin o to destination d
w	Expected number of drivers needed to run this system
d^{DRT}	Expected total distance driven with DRT vehicles in distance units
d^{FLS}	Expected total distance driven with FLS vehicles in distance units
b^{DRT}	Expected number of vehicles that is used for the DRT system
b^{FLS}	Expected number of vehicles that is used for the FLS system
Parameter	
cs	Salary per time unit for drivers
cd^{DRT}	Fuel and maintenance price per distance unit per DRT vehicle
cd^{FLS}	Fuel and maintenance price per distance unit per FLS vehicle
cw^{DRT}	Write-off cost per time unit per DRT vehicle
cw^{FLS}	Write-off cost per time unit per FLS vehicle

$$\max \quad \sum_{o\in\mathcal{S}}\sum_{d\in\mathcal{S}} r_{od} - \Big(cs \times w + cd^{\text{DRT}} \times d^{\text{DRT}} + cd^{\text{FLS}} \times d^{\text{FLS}} + cw^{\text{DRT}} \times b^{\text{DRT}} + cw^{\text{FLS}} \times b^{\text{FLS}}\Big) \tag{4.10}$$

The objective of the model is to maximise profit, which results in the maximisation of the revenue minus the personnel costs, distance costs and write-off costs for both DRT and FLS vehicles as in (4.10). Transport operators often receive subsidy for providing public transport. Therefore, the objective can be negative since subsidy is not part of the objective.

Demands

As explained before, each stop should be served by either DRT and/or FLS transport. Furthermore, rerouting FLS transport is not considered. However, removing stops from FLS lines is an option. Parameter yo_{bs} indicates if stop s used to be part of the original bus line b, for $s \in \mathcal{S}$ and $b \in \mathcal{B}$.

Variable	Explanation
y_{bs}	1 if bus line b stops at stop s, 0 otherwise
x_s	1 if stop s is part of the DRT network, 0 otherwise
Parameter	
yo_{bs}	1 if bus line b stops at stop s in the original situation, 0 otherwise

$$x_s + \sum_{b \in \mathcal{B}} y_{bs} \geq 1 \qquad \forall s \in \mathcal{S} \quad (4.11)$$

$$y_{bs} \leq yo_{bs} \qquad \forall s \in \mathcal{S}, b \in \mathcal{B} \quad (4.12)$$

To make sure that each stop is served by at least one service, either DRT and/or FLS, (4.11) is introduced. Constraint (4.12) makes sure that FLS lines cannot be rerouted, the only option is to remove stops from the bus line.

Travel options

The constraints in this section determine the expected number of customers that travel with DRT and FLS for each trip (o, d), $\vartheta_{od}^{\text{DRT}}$ and $\vartheta_{od}^{\text{FLS}}$ respectively. To determine these variables, four indicator variables are needed: $\alpha_{od}, \beta_{od}, \gamma_{od}$ and δ_{od}. In some cases DRT is the only option ($\beta_{od} = 1$), in some cases a transfer between FLS and DRT is necessary ($\alpha_{od} - \beta_{od} = 1$), there is the possibility that a customer can choose between DRT and FLS ($\gamma_{od} = 1$) and finally there are cases when FLS is the only option ($\delta_{od} = 1$). The choices that customers experience depend on the decision variables x_s and y_{bs}. In this section, the outcome of the customer choice modelling analysis is also used, remember that pi^m_{od} is the probability that a customer chooses the transport mode $m \in \mathcal{A}^i$ for trip (o, d) with $o, d \in \mathcal{S}$ when choice option $i \in \{1, 2, 3\}$ is offered (see Section 3.3). Parameter qt_{od} represents the average number of customers that want to travel from stop o to stop d for $o, d \in \mathcal{S}$.

Variable	Explanation
y_{bs}	1 if bus line b stops at stop s, 0 otherwise
x_s	1 if stop s is part of the DRT network, 0 otherwise
α_{od}	1 if DRT is required for trip (o, d), that is, when at least one of o and d is not part of the FLS network, 0 otherwise
β_{od}	1 if choice option 1: {DRT} is offered, that is, when stops o and d are both part of the DRT network and at least one of o and d is not part of the FLS network, 0 otherwise
γ_{od}	1 if choice option 3: {DRT,FLS} is offered, that is, when both stop o and d are part of the DRT and FLS network, 0 otherwise
δ_{od}	1 if choice option 2: {FLS} is offered, that is, when both stop o and d are part of the FLS network but at least one of o and d is not part of the DRT network, 0 otherwise
$\vartheta^{\text{DRT}}_{od}$	Expected number of trips from stop o to stop d in the DRT network
$\vartheta^{\text{FLS}}_{od}$	Expected number of trips from stop o to stop d in the FLS network
Parameter	
qt_{od}	Number of customers that want to travel from origin o to destination d (average) per time unit
$p1^{\text{DRT}}_{od}$	Probability that a customer will choose DRT to travel from stop o to stop d when choice option 1 is offered: {DRT}
$p2^{\text{FLS}}_{od}$	Probability that a customer will choose FLS to travel from stop o to stop d when choice option 2 is offered: {FLS}
$p3^{\text{DRT}}_{od}$	Probability that a customer will choose DRT to travel from stop o to stop d when choice option 3 is offered: {DRT, FLS}
$p3^{\text{FLS}}_{od}$	Probability that a customer will choose FLS to travel from stop o to stop d when choice option 3 is offered: {DRT, FLS}

$$\alpha_{od} = 1 - \min\Big\{1, \sum_{b \in \mathcal{B}} y_{bo}, \sum_{b \in \mathcal{B}} y_{bd}\Big\} \qquad \forall o, d \in \mathcal{S} \quad (4.13)$$

$$\beta_{od} = \min\{x_o, x_d, \alpha_{od}\} \qquad \forall o, d \in \mathcal{S} \quad (4.14)$$

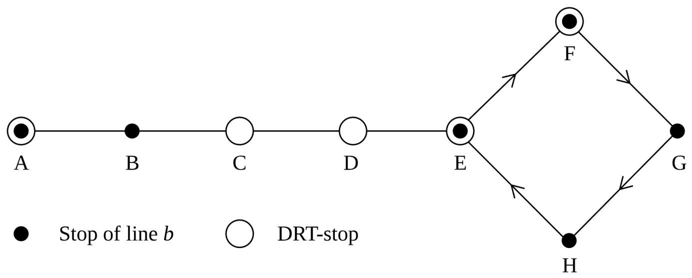
$$\gamma_{od} = \min\{x_o, x_d, 1 - \alpha_{od}\} \qquad \forall o, d \in \mathcal{S} \quad (4.15)$$

$$\delta_{od} = 1 - \max\{\alpha_{od}, \gamma_{od}\} \qquad \forall o, d \in \mathcal{S} \quad (4.16)$$

$$\vartheta^{\text{DRT}}_{od} = qt_{od}\big(p1^{\text{DRT}}_{od} \times \beta_{od} + p2^{\text{FLS}}_{od}(\alpha_{od} - \beta_{od}) + p3^{\text{DRT}}_{od} \times \gamma_{od}\big) \qquad \forall o, d \in \mathcal{S} \quad (4.17)$$

$$\vartheta^{\text{FLS}}_{od} = qt_{od}\big(p2^{\text{FLS}} \times (\delta_{od} + \alpha_{od} - \beta_{od}) + p3^{\text{FLS}}_{od} \times \gamma_{od}\big) \qquad \forall o, d \in \mathcal{S} \quad (4.18)$$

Variables α_{od} and β_{od} are determined in non-linear constraints (4.13) and (4.14). Variable α_{od} indicates if DRT needs to be used on trip (o, d), this can be in combination with FLS transport. α_{od} is 1 if and only if at least one out of the origin and destination is not a FLS stop. Figure 4.4 displays an example bus system together with corresponding values of α_{od}. We refer to this example to clarify the use of α_{od}. Non-linear constraint (4.13) contains a minimum operator similar to m_5 in (4.9) in the previous section, therefore we can use the same technique for linearisation. For this linearisation $|S|^2$ binary indicator variables are needed.



o	d	α_{od}	β_{od}	γ_{od}	δ_{od}
B	G	0	0	0	1
A	B	0	0	0	1
A	E	0	0	1	0
B	C	1	0	0	0
A	C	1	1	0	0
C	D	1	1	0	0

FIGURE 4.4: Example outcome of the MILP on original bus line b with corresponding values for α_{od}, β_{od}, γ_{od} and δ_{od} to explain the use of these variables.

Variable β_{od} indicates if DRT is the only option for trip (o, d) and thus FLS can not be used, it is 1 if and only if both origin and destination are DRT stops and at least one of them is not part of the FLS network. For β_{od}, some values are displayed in Figure 4.4 as well. In the non-linear constraint (4.14), β_{od} is of the same form as m_2 in (4.4). The same technique can be used for linearisation.

When $\gamma_{od} = 1$, customers can choose between both travel options DRT and FLS when travelling from stop o to stop d. This is ensured in the non-linear constraint (4.15). For linearisation a similar technique is used as for m_2 in (4.4). The final indicator variable $\delta_{od} = 1$, when the only option for travelling is FLS and 0 otherwise. Non-linear constraint (4.16) ensures this and can be linearised with the same technique as m_1 in (4.1). Some values of γ_{od} and δ_{od} are displayed in Figure 4.4.

Finally, in (4.17) and (4.18) the expected number of customers travelling from stop o to stop d is determined for DRT and FLS transport, respectively.

Characteristics of DRT system

In this section, the duration of the longest allowed DRT trip (s) and the expected total distance driven by DRT vehicles (d^{DRT}) is determined. The longest allowed DRT trip is used to estimate the DRT fleet size in the next section. The total distance driven by DRT vehicles is used to determine the expected fuel and maintenance cost. The determination of the total distance driven by DRT vehicles is difficult, since this depends on the location of the (idle) vehicles when they are assigned trips and the combination of trips. Therefore it is stochastic. An estimation of the total distance can be made by using experience from real-life DRT systems. To estimate this, we use the ratio between the total distance driven by DRT vehicles and the sum of the direct distance of all served trips. By multiplying the direct length of a trip (di_{od}) by this factor (fc), an estimate is obtained of the average distance needed to cover this trip. More information about this estimate and details about the relocation policy can be found in Section 4.3.

Variable	Explanation
s	Duration of the longest trip in the DRT network, this is an indication for the size of the DRT network
β_{od}	1 if choice option 1: {DRT} is offered, that is, when stops o and d are both part of the DRT network and at least one of o and d is not part of the FLS network, 0 otherwise
d^{DRT}	Expected total distance driven with DRT vehicles in distance units
$\vartheta_{od}^{\text{DRT}}$	Expected number of trips from stop o to stop d in the DRT network
Parameter	
ti_{od}	Duration of the trip from stop o to stop d when driving directly
fc	Factor by which the distance of a DRT-trip is multiplied to estimate the distance needed to cover that trip
di_{od}	Distance of trip (o, d) when driving directly

$$s = \max_{o\in\mathcal{S},d\in\mathcal{S}} \{ti_{od} \times \beta_{od}\} \tag{4.19}$$

$$d^{\text{DRT}} = fc \sum_{o\in\mathcal{S}} \sum_{d\in\mathcal{S}} di_{od} \times \vartheta_{od}^{\text{DRT}} \tag{4.20}$$

In (4.19) the longest trip in the DRT system is determined. The longest trip is an indication to the size of the DRT system and is used to determine the amount of DRT vehicles needed. This non-linear constraint can be easily linearised by assuring that s

is larger than all DRT trips in the system. Introducing an upperbound is not necessary since the larger s is, the more vehicles are needed and the higher the cost. This will never happen since the model is designed to minimise costs.

Constraint (4.20) estimates the distance driven by DRT vehicles for all trips in the DRT system. As explained before, the distance DRT vehicles drive can be estimated by multiplying the sum of the distance of every trip by a factor fc. This factor is determined through simulation with data from existing DRT systems and is explained in Section 4.3.

Number of DRT vehicles

It is hard to determine the DRT fleet size for a DRT system. This is highly stochastic and depends on many variables. However, it is possible to make an estimation by using experience from other DRT systems. By using this experience, we try to explain the fleet size using two variables: the longest (allowed) trip in the DRT-network and the average number of trips per time unit. Simulation was used to determine the required fleet size to reach a 90% acceptance rate for different values of the two explanatory variables: the longest (allowed) trip in the DRT-network and the average number of trips per time unit. The details of this simulation are presented in Section 4.3. The constraints in this section select the right expected number of vehicles by looking up in which category the longest trip and trips per hour belong. Parameters bo_i^{length} and bo_j^{trips} display the boundaries of the categories for the longest trip and the number of trips, respectively. Variables ϕ_i^{length} and ϕ_j^{trips} then indicate in which category, respectively, the longest DRT trip s and the number of trips per time unit $\sum_{o\in\mathcal{S}}\sum_{d\in\mathcal{S}}\vartheta_{od}^{\text{DRT}}$ belong. Finally, parameter bv_{ij} is the number of DRT vehicles for category (i, j) and variable Φ_{ij} indicates if (i, j) is the corresponding category.

Variable	Explanation
s	Duration of the longest trip in the DRT network, this is an indication for the size of the DRT network
$\vartheta_{od}^{\text{DRT}}$	Expected number of trips from stop o to stop d in the DRT network
Φ_{ij}	Binary variable that indicates which type of DRT-area we consider
ϕ_i^{length}	Binary variable that indicates in which category the longest trip belongs
ϕ_j^{trips}	Binary variable that indicates in which category the number of trips belongs
b^{DRT}	Expected number of vehicles that is used for the DRT system

Parameter	Explanation
bo_i^{length}	Upper bound for category i of the longest trip
bo_j^{trips}	Upper bound for category j of the trips
bv_{ij}	Number of DRT vehicles needed to achieve a 90% acceptance rate in category (i, j)
M	Sufficiently large number

$$s \leq bo_i^{\text{length}} \times \phi_i^{\text{length}} + M(1 - \phi_i^{\text{length}}) \qquad \forall i \in \{1, ..., n\} \tag{4.21}$$

$$s > bo_i^{\text{length}} \times \phi_{i+1}^{\text{length}} \qquad \forall i \in \{1, ..., n-1\} \tag{4.22}$$

$$\sum_{o \in \mathcal{S}} \sum_{d \in \mathcal{S}} \vartheta_{od}^{\text{DRT}} \leq bo_j^{\text{trips}} \times \phi_j^{\text{trips}} + M(1 - \phi_j^{\text{trips}}) \qquad \forall j \in \{1, ..., m\} \tag{4.23}$$

$$\sum_{o \in \mathcal{S}} \sum_{d \in \mathcal{S}} \vartheta_{od}^{\text{DRT}} > bo_j^{\text{trips}} \times \phi_{j+1}^{\text{trips}} \qquad \forall j \in \{1, ..., m-1\} \tag{4.24}$$

$$\Phi_{ij} \leq \frac{1}{2} \times (\phi_i^{\text{length}} + \phi_j^{\text{trips}}) \qquad \forall i \in \{1, ..., n\}, j \in \{1, ..., m\} \tag{4.25}$$

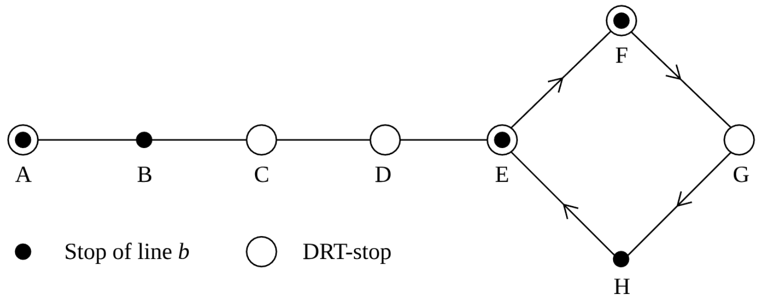
$$\sum_{i=1}^{n} \sum_{j=1}^{m} \Phi_{ij} = 1 \tag{4.26}$$

$$b^{\text{DRT}} = \sum_{i=1}^{n} \sum_{j=1}^{m} \Phi_{ij} bv_{ij} \tag{4.27}$$

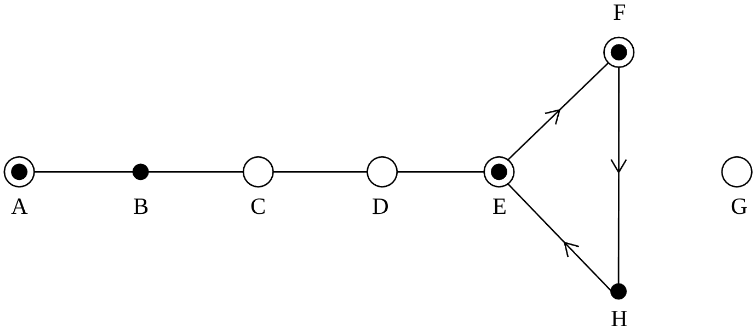
The binary variables ϕ_i^{length}, ϕ_j^{trips} and Φ_{ij} are specified in (4.21) to (4.26). The expected fleet size for DRT is determined in (4.27).

Characteristics of FLS bus system

To determine the costs associated with the FLS system, the constraints in this section determine the duration and length of each bus line (t_b and ℓ_b, respectively). The number of FLS vehicles needed (b^{FLS}) is determined as well as the total distance driven by FLS vehicles (d^{FLS}) in order to calculate personnel, fuel and maintenance cost. To determine the duration and distance of each bus line, the stops that are part of the bus line and the route are critical. This is not as straightforward as it might seem. Consider the example in Figure 4.5. Since stop G is not a part of bus line b anymore, the bus line should be rerouted as in Figure 4.5b. The length of the bus line is the sum of the length between any two consecutive stops and similarly for the duration. Parameter ol_i^b displays the original order of bus line $b \in \mathcal{B}$, $ol_i^b = s$ if stop $s \in \mathcal{S}$ was the i^{th} stop of bus line b. The variable n_{ij}^b is then introduced that indicates if bus line b stops at the original j^{th} stop (ol_j^b), directly after stopping at the original i^{th} stop (ol_i^b).



(a) Outcome of the MILP with the original route of the bus line.

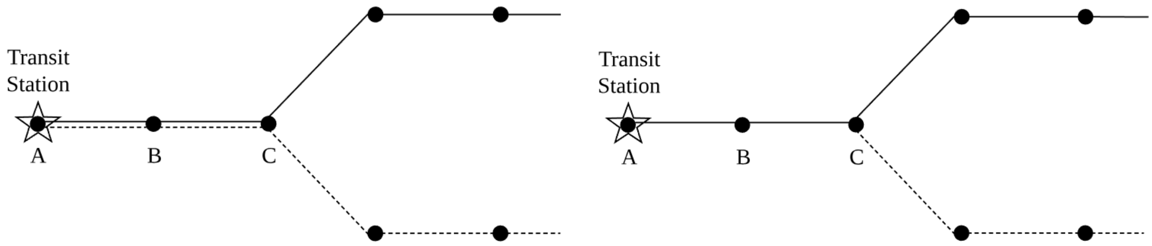


(b) Outcome of the MILP with the rerouted route to decrease the duration and distance of bus line b.

FIGURE 4.5: Example outcome of the MILP on original bus line b with rerouting of FLS line b.

There are also some constraints in this section that bound the flexibility of the FLS transport. A consequence of this formulation is that, to minimise costs, FLS lines will have no overlap. In other words, the result will be an unconnected network of bus lines which is undesired. To fix this problem, three methods are used. First of all, it is ensured that each FLS line has at least one transfer possibility to another FLS line. The variable $z_{bs} = 1$ if stop $s \in \mathcal{S}$ is a transfer stop of bus line $b \in \mathcal{B}$, 0 otherwise. This helps to prevent cases with no connection between FLS lines, however it is still possible to obtain multiple unconnected FLS networks.

Another consequence is visualised by an example in Figure 4.6a where two bus lines (dotted and filled line) come together two stops before reaching a big transit station. A possible outcome of the formulation would be as in Figure 4.6b where the dotted bus line would go no further than stop C (since every stop is served and adding stops increases cost). This can be undesired since it is common that the endpoint of a bus line is at the bigger transit station A. Therefore a parameter sc^b_{od} is introduced, with $sc^b_{od} = 1$ if stop d should be visited if stop o is visited by bus line b and 0 otherwise. This allows the user to specify certain if-then statements concerning stops. In this specific example, $sc^b_{CA} = sc^b_{BA} = 1$ would make sure that stop A will always be visited when either stop B or C is visited by bus line b.



(a) The original situation in which both bus lines visit the transit station.

(b) An undesired outcome of the MILP where the dotted bus line ends at C, this can be prevented with proposed constraints.

FIGURE 4.6: Example bus system with two FLS lines that originally come together at stop C before reaching transit station A.

The final method is the most drastic. We ensure that for every FLS stop, either all original bus lines continue stopping there or none at all. This can be applied to all stops (in that case the previous method is redundant) or for example only to the stops of two bus lines that continuously overlap. The set of stops for which this constraint applies is displayed as $\mathcal{S}^* \subset \mathcal{S}$. These three methods allow the user to customise the MILP to their demands.

Variable	Explanation
y_{bs}	1 if bus line b stops at stop s, 0 otherwise
n_{ij}^b	1 if bus line b stops at stop $o\ell_j^b$ directly after stop $o\ell_i^b$, 0 otherwise
t_b	Duration of bus line b in time units
ℓ_b	Length of bus line b in distance units
z_{bs}	1 if stop s is a transfer station of bus line b, 0 otherwise
b^{FLS}	Expected number of vehicles that is used for the FLS system
d^{FLS}	Expected total distance driven with FLS vehicles in distance units
Parameter	
$o\ell_i^b$	Original order of bus line b, $o\ell_i^b = s$ if stop s is the i^{th} stop of bus line b
ti_{od}	Duration of the trip from stop o to stop d when driving directly
ee_s^b	Expected duration of customers entering and exiting the FLS vehicle at stop s on bus line b
di_{od}	Distance of trip (o, d) when driving directly
sc_{od}^b	1 if stop d needs to be part of bus line b when stop o is part of bus line b, 0 otherwise
yo_{bs}	1 if bus line b stops at stop s in the original situation, 0 otherwise
fo_b	Original frequency of bus line b (trips per time unit)

$$n_{ij}^b = \max\left\{0, \min\left\{y_{b,o\ell_i^b},\ y_{b,o\ell_j^b},\ 1 - \sum_{s=i+1}^{j-1} y_{b,o\ell_s^b}\right\}\right\} \quad \forall i,j \in \{0,\ldots,|o\ell^b|\}, b \in \mathcal{B} \quad (4.28)$$

$$t_b = \sum_{i=0}^{|ol^b|}\sum_{j=0}^{|ol^b|} \left(ti_{o\ell_i^b,o\ell_j^b} + ee_i^b\right) \times n_{ij}^b \quad \forall b \in \mathcal{B} \quad (4.29)$$

$$\ell_b = \sum_{i=0}^{|ol^b|}\sum_{j=0}^{|ol^b|} di_{o\ell_i^b,o\ell_j^b} \times n_{ij}^b \quad \forall b \in \mathcal{B} \quad (4.30)$$

$$y_{bd} \geq sc_{od}^b \times y_{bo} \quad \forall o,d \in \mathcal{S}, b \in \mathcal{B} \quad (4.31)$$

$$\sum_{s\in\mathcal{S}} z_{bs} \geq 1 \quad \forall b \in \mathcal{B} \quad (4.32)$$

$$y_{bs} \geq z_{bs} \quad \forall s \in \mathcal{S}, b \in \mathcal{B} \quad (4.33)$$

$$\sum_{x\in\mathcal{B}\setminus\{b\}} y_{xs} \geq z_{bs} \quad \forall s \in \mathcal{S}, b \in \mathcal{B} \quad (4.34)$$

$$yo_{b_1s} \times yo_{b_2s}\left(y_{b_1s} - y_{b_2s}\right) = 0 \quad \forall s \in \mathcal{S}^*,\ b_1,b_2 \in \mathcal{B} \quad (4.35)$$

$$b^{\text{FLS}} \geq \sum_{b\in\mathcal{B}} fo_b \times t_b \quad (4.36)$$

$$d^{\text{FLS}} = \sum_{b\in\mathcal{B}} fo_b \times \ell_b \quad (4.37)$$

To determine the total duration and distance of FLS lines, the order of stops is needed. Using the original order of bus stops $o\ell^b$ and the variable y_{bs} that indicates if stop b is still part of bus line b. Variable n_{ij}^b indicates if the j^{th} stop of bus line b ($o\ell_j^b$) comes directly after the i^{th} stop of bus line b ($o\ell_i^b$). Hence $n_{ij}^b = 1$, if $y_{b,o\ell_i^b} = y_{b,o\ell_j^b} = 1$ and all stops that used to be in between $o\ell_i^b$ and $o\ell_j^b$ are not part of the bus line anymore ($\sum_{s=i+1}^{j-1} y_{b,o\ell_s^b} = 0$). This is expressed in (4.28). To linearise this constraint, a similar technique is used as for m_4 in (4.8).

Constraints (4.29) and (4.30) determine the duration and distance respectively of the FLS lines, using the variable n_{ij}^b that was previously determined. The duration of bus line b is determined by summing the duration of the trips between any two consecutive stops and the expected time for customers entering or exiting the vehicle at every stop. The distance is determined similarly.

As explained before, some constraints are introduced to ensure a connected network. Constraint (4.31) makes sure that beforehand determined stops are either served together or not at all. By constraints (4.32), (4.33) and (4.34), every bus line has at least one transfer stop (or more if desired). For the stops in set $\mathcal{S}^*$, it is assured that either all the original bus lines stop at this stop or none at all by (4.35). Together these constraints result in a connected FLS network.

An approximation on the fleet size to run the FLS system is made in (4.36) by multiplying per bus line the frequency by the duration and summing this for all bus lines. An example of this calculation is given in Table 4.1. In this case $\sum_{b\in\mathcal{B}} fo_b \times t_b = 1\frac{1}{2}h$ and therefore $b^{\text{FLS}} \geq 1\frac{1}{2}$. So at least two vehicles are needed to run the system in this example. The total distance that is driven for running the system is determined similarly in (4.37).

TABLE 4.1: Some example frequencies and durations of two imaginary bus lines that are used to illustrate the calculation in (4.36).

b	1	2
fo_b	$2/h$	$4/h$
t_b	$25\ min = 5/12\ h$	$10\ min = 1/6\ h$

Revenue and costs

In this section, the revenue and costs of the system are determined. The constraints below are adapted to the current pricing policy for FLS and DRT services of the case study. However, this can easily be adapted to different pricing strategies. The price for FLS trips consists of a fixed price plus a price per distance unit. DRT trips have a fixed price, independent of the distance. It is assumed that when a passenger needs to transfer between DRT and FLS, they only pay the fixed price for DRT.

Variable	Explanation
r_{od}	Expected revenue from all trips from origin o to destination d
$\vartheta_{od}^{\text{DRT}}$	Expected number of trips from stop o to stop d in the DRT network
$\vartheta_{od}^{\text{FLS}}$	Expected number of trips from stop o to stop d in the FLS network
w	Expected number of drivers needed to run this system
t_b	Duration of bus line b in time units
b^{DRT}	Expected number of vehicles that is used for the DRT system
b^{FLS}	Expected number of vehicles that is used for the FLS system
Parameter	
pr^{DRT}	Set price for bus ticket in DRT system
pr^{dist}	Price per distance unit for FLS bus tickets
pr^{fixed}	Fixed price for FLS bus tickets
di_{od}	Distance of trip (o, d) when driving directly

$$r_{od} = \left(pr^{\text{DRT}} \times \vartheta_{od}^{\text{DRT}} + (pr^{\text{dist}} \times di_{od} + pr^{\text{fixed}})\vartheta_{od}^{\text{FLS}}\right) \quad \forall o, d \in \mathcal{S} \quad (4.38)$$

$$w = b^{\text{DRT}} + b^{\text{FLS}} \quad (4.39)$$

In (4.38), the expected revenue of all trips is calculated. When DRT is used somewhere on the trip, the customer pays a fixed price pr^{DRT} for the whole trip. When DRT is not used, the customer pays a fixed price pr^{fixed}, plus a factor pr^{dist} dependant on the distance of the bus trip. The expected number of customers that use DRT or FLS is given by $\vartheta_{od}^{\text{DRT}}$ and $\vartheta_{od}^{\text{DRT}}$, respectively. In (4.39) the expected number of drivers needed to run this system, is calculated by adding the number of DRT vehicles (and thus DRT drivers) by the number of FLS vehicles (and thus FLS drivers). Note that this is just an approximation, crew rostering will most likely result in slightly higher personnel cost.

Range of variables

Finally, the variables are declared in (4.40) to (4.48). All variables are either binary or natural numbers.

$$x_s \in \mathbb{B} \qquad \forall s \in \mathcal{S} \quad (4.40)$$

$$y_{bs}, z_{bs} \in \mathbb{B} \qquad \forall b \in \mathcal{B}, s \in \mathcal{S} \quad (4.41)$$

$$n_{ij}^b \in \mathbb{B} \qquad \forall i, j \in \{0, \ldots, |o\ell^b|\}, b \in \mathcal{B} \quad (4.42)$$

$$t_b, \ell_b \in \mathbb{N} \qquad \forall b \in \mathcal{B} \quad (4.43)$$

$$r_{od}, \alpha_{od}, \beta_{od}, \mathcal{I}_{od}, \in \mathbb{B} \qquad \forall o, d \in \mathcal{S} \quad (4.44)$$

$$\Phi_{ij} \in \mathbb{B} \qquad \forall i \in \{1, \ldots, |\phi^{\text{length}}|\}, j \in \{1, \ldots, |\phi^{\text{trips}}|\} \quad (4.45)$$

$$\phi_i^{\text{length}} \in \mathbb{B} \qquad \forall i \in \{1, \ldots, |\phi^{\text{length}}|\} \quad (4.46)$$

$$\phi_j^{\text{trips}} \in \mathbb{B} \qquad \forall j \in \{1, \ldots, |\phi^{\text{trips}}|\} \quad (4.47)$$

$$w, s, b^{\text{DRT}}, b^{\text{FLS}}, f^{\text{DRT}}, f^{\text{FLS}}, d^{\text{DRT}}, d^{\text{FLS}} \in \mathbb{N} \quad (4.48)$$

4.1.3 Subregions

As discussed briefly in the beginning of the previous section, the MILP considers only one DRT area. Customers can travel from any DRT stop to any other DRT stop. However, when operators consider introducing multiple separate DRT areas, the model should be run multiple times.

When there is already a clear separation in the service area, such as multiple villages, the approach is straightforward: run the model for every subregion. This way every subregion is evaluated independently and there is the possibility for multiple DRT regions. When there is, however, no clear separation in the service area, for example when the service area consists of one city, the approach gets a bit more challenging. As a first step, the operator should run the MILP for the entire region. If there is a profitable DRT area, the operator can run the MILP again for the entire region with the optimal DRT area cut off. When the outcome of the second MILP results in another DRT area, the same procedure can be repeated until no more DRT areas are found. This is a heuristic procedure and optimality can not be guaranteed. However, in most scenarios, operators already have some assumptions about the locations of the DRT areas or have a maximum size or number of vehicles and the heuristic procedure does not have to be used.

4.2 Dial-a-ride problem

The optimisation of the dial-a-ride problem (DARP) is not the focus of this thesis. However, in order to be able to run the simulations with data from other DRT areas, there is still a method needed to simulate the dispatching of vehicles. The method that is used for that in this thesis is discussed in this section.

4.2.1 Formulation

The DARP is formulated similarly as in Cordeau (2006). The formulation is presented in this section.

The DARP is defined on a complete directed graph $G = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ is the set of vertices that represent the pick-up locations, drop-off locations and the depot and $\mathcal{A}$ is the set of arcs that represent the routes between these vertices. Let n be the number of trip requests. Then subsets $\mathcal{P} = \{1, ..., n\}$ and $\mathcal{D} = \{n+1, ..., 2n\}$ represent the set of pick-up locations and drop-off locations respectively, while nodes 0 and $2n+1$ are the origin and destination depot, such that $\mathcal{V} = \mathcal{P} \cup \mathcal{D} \cup \{0, 2n+1\}$. Let $\mathcal{K}$ be the set of vehicles. Every vehicle $k \in \mathcal{K}$ has a capacity Q_k and a maximum duration of being on the road of T_k (this can be due to driver contracts, fuel range or battery range). With every trip request, and thus every node $i \in \mathcal{V}$, is associated a load q_i (number of passengers), such that $q_0 = q_{2n+1} = 0$ and $q_i = -q_{n+i}$ for $i = 1, ..., n$. The service duration d_i for every node $i \in \mathcal{V}$ represents the non-negative service duration at that node which can consists of for example entry and exit time of the customers, such that $d_0 = d_{2n+1} = 0$. A time window $[e_i, \ell_i]$ is associated with each node $i \in \mathcal{V}$ such that e_i is the earliest time at which service may begin at node i and ℓ_i the latest. For every arc $(i, j) \in \mathcal{A}$, there is a cost c_{ij} and a travel time t_{ij}. The maximum ride time of a customer $i \in \mathcal{P}$ is denoted by L_i.

Now we define the binary decision variable x^k_{ij} for $(i, j) \in \mathcal{A}$ and $k \in \mathcal{K}$, $x^k_{ij} = 1$ if and only if vehicle k travels over arc (i, j). Next, for every $k \in \mathcal{K}$ and $i \in \mathcal{V}$, let variable $B^k_i = 1$ be the time at which vehicle k starts service at node i and let Q^k_i be the load of vehicle k after visiting node i. Finally, let L^k_i be the ride time of user i on vehicle k.

Now we can formulate the model for the DARP as follows:

$$\min \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} c_{ij} x_{ij}^k, \tag{4.49}$$

subject to:

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{V}} x_{ij}^k = 1 \qquad \forall i \in \mathcal{P}, \tag{4.50}$$

$$\sum_{j \in \mathcal{V}} x_{ij}^k - \sum_{j \in \mathcal{V}} x_{n+i,j}^k = 0 \qquad \forall i \in \mathcal{P}, k \in \mathcal{K}, \tag{4.51}$$

$$\sum_{j \in \mathcal{P} \cup \{2n+1\}} x_{0j}^k = 1 \qquad \forall k \in \mathcal{K}, \tag{4.52}$$

$$\sum_{i \in \mathcal{D} \cup \{0\}} x_{i,2n+1}^k = 1 \qquad \forall k \in \mathcal{K}, \tag{4.53}$$

$$\sum_{j \in \mathcal{V}} x_{ji}^k - \sum_{j \in \mathcal{V}} x_{ij}^k = 0 \qquad \forall i \in \mathcal{P} \cup \mathcal{D}, k \in \mathcal{K}, \tag{4.54}$$

$$B_j^k \geq (B_i^k + d_i + t_{ij}) x_{ij}^k \qquad \forall i, j \in \mathcal{V}, k \in \mathcal{K}, \tag{4.55}$$

$$Q_j^k \geq (Q_i^k + q_j) x_{ij}^k \qquad \forall i, j \in \mathcal{V}, k \in \mathcal{K}, \tag{4.56}$$

$$L_i^k = B_{n+i}^k - (B_i^k + d_i) \qquad \forall i \in \mathcal{P}, k \in \mathcal{K}, \tag{4.57}$$

$$t_{i,n+i} \leq L_i^k \leq L_i \qquad \forall i \in \mathcal{P}, k \in \mathcal{K}, \tag{4.58}$$

$$B_{2n+1}^k - B_0^k \leq T_k \qquad \forall k \in \mathcal{K}, \tag{4.59}$$

$$e_i \leq B_i^k \leq \ell_i \qquad \forall i \in \mathcal{V}, k \in \mathcal{K}, \tag{4.60}$$

$$\max(0, q_i) \leq Q_i^k \leq \min(Q^k, Q^k + q_i) \qquad \forall i \in \mathcal{V}, k \in \mathcal{K}, \tag{4.61}$$

$$x_{ij}^k \in \{0, 1\} \qquad \forall i, j \in \mathcal{V}, k \in \mathcal{K}. \tag{4.62}$$

The objective in (4.49) minimises costs. Then (4.50) makes sure that all customers are served. The origin as well as the destination of every trip has to be visited by the same vehicle, this is ensured in (4.51). Constraints (4.52), (4.53) and (4.54) ensure that all vehicles start and end at the depot. The consistency of arrival and departure time and load of all vehicles is ensured in (4.55) and (4.56). For each trip, the duration is determined in (4.57) and it is bounded in (4.58). The duration that vehicles are on the road is bounded in (4.59). Constraint (4.60) ensures that the service begins in the specified time windows. Finally, (4.61) imposes the required capacity constraints.

The model is currently non-linear because of constraints (4.55) and (4.56). By introducing constants M_{ij}^k and W_i^k, the model can be linearised as follows:

$$M_{ij}^k = \max(0, \ell_i + d_i + t_{ij} - e_j) \qquad \forall i, j \in \mathcal{V}, k \in \mathcal{K},$$
$$W_i^k = \min(Q^k, Q^k + q_i) \qquad \forall i, j \in \mathcal{V}, k \in \mathcal{K},$$
$$B_j^k \geq B_i^k + s_i + t_{ij} - M_{ij}(1 - x_{ij}^k) \qquad \forall i, j \in \mathcal{V}, k \in \mathcal{K},$$
$$Q_j^k \geq Q_i^k + q_j - W_i^k(1 - x_{ij}^k) \qquad \forall i, j \in \mathcal{V}, k \in \mathcal{K}.$$

4.2.2 Parameters

The DARP as explained in the above section, is used under conditions specified by the operator of the case study. The maximum duration of the detour is set at 0.5 times the duration of the direct trip from origin to destination, such that $L_i = 1\frac{1}{2} \times t_{i,n+i}$ for $i \in \mathcal{N}$. For the entrance and exit time 30 seconds are expected, so $d_i = 30s$ for $i \in \{1, ..., 2n\}$. The maximum capacity of the vehicles $Q^k = 8$ passengers for all vehicles $k \in \mathcal{K}$. There is no maximum duration T_k taken into account. Furthermore, there is a maximum on the duration of the waiting time of the customer. Customers are only allowed to specify a desired pick-up time *or* drop-off time. For every customer $i \in \mathcal{P}$ with desired pick-up time e_i^*, the earliest pick-up time $e_i = e_i^*$, the latest pick-up time $\ell_i = e_i + 22$ minutes. This results in an earliest drop-off time $e_{n+i} = e_i + t_{i,n+i}$ and a latest drop-off time $\ell_{n+i} = \ell_i + 1\frac{1}{2} \times t_{i,n+i}$. A similar technique is used for customers that specify a desired drop-off time instead of pick-up time. The costs c_{ij} in the objective, consist not only of financial costs but also incorporates customer satisfaction. The DARP is solved in real time whenever a trip request comes in.

4.3 Experience from other DRT systems

Here we will present all the details of the simulations from historic data from other DRT areas. The reason these simulations are used, is to get an impression of the costs of introducing DRT. It is unfortunately not so straightforward to determine the expected DRT fleet size, the expected distance they need to cover or the expected time the customer will be in the vehicle. To estimate these numbers, experience from other DRT areas is used. The specific methods are discussed in this section. The DRT area that is used to calculate these estimations will be referred to as the *simulation area*.

For all these estimations, it is assumed that DRT vehicles start and end the day at the depot. When the vehicles are idle, they wait at the location of their last drop-off for a new trip.

4.3.1 Number of DRT vehicles

The DRT fleet size is dependant on many (stochastic) variables, like the size of the DRT area, the spread of stops, the customer flows and most of all on the real time situation at the time of the request (such as location of vehicles, other trips in the system). To determine the expected fleet size for a specific DRT area, this specific DRT area can be simulated. However, since we want to be able to have an estimate inside the MILP, this needs to be done without using simulation in the MILP. Therefore, a different approach is needed.

After analysing some results with real data from other DRT areas, we found that the fleet size can be determined from the maximum trip distance (indicator for the size of the DRT area) and the average number of trips per hour. The maximum trip distance does not necessarily correlate directly with average trip distance or trip combination factor, but is also influenced by other area specific characteristics. However, it is a reasonable assumption and the methods works for any area. Furthermore, the method is consistent. Since we will use these estimates while solving the MILP, we calculated the DRT fleet size for some combinations of maximum length and trips per hour and use these as estimations in the model. So, when the model considers a specific DRT area, it can "look up" what number of DRT vehicles is expected to be needed for it.

A different approach to estimating the fleet size for a DRT area that does not rely on simulation is given by Diana et al. (2006). They determine the number of vehicles needed for a set of trips. The fleet size also depends on service constraints like the maximum waiting and detour time. However, we need to be able to determine the fleet

Table 4.2: Example outcome of simulation results for the expected DRT fleet size for different characteristics of the DRT area.

Average #trips/h	**Max. length between two DRT-stops (km)**						
	0	5	6	7	8	9	10
0	0	0	0	0	0	0	0
2	0						
4	0				⋮		
6	0	⋯	⋯	⋯	$bv_{8,6}$		
8	0						
10	0						

size in the MILP without having a specific set of trips, just expected customer flows and therefore this technique is not applicable in our situation. Both techniques are suitable for different situations.

From the simulation area, trips are filtered to obtain an area with the required number of trips and the required size. Then by simulation, it is tested how many vehicles are needed to be able to accept 90% of the trip requests. This is done multiple times to obtain consistent results. The threshold of 90% is chosen, because that way there is a small margin for extremely busy periods, while at the same time customer satisfaction is kept at a reasonable level. One should also note that this percentage is about first time requests. So when trip requests get denied, the customer can simply try again a few minutes later and usually gets accepted eventually. This way, the actual percentage of customers that get accepted is even higher.

To illustrate the method, see an example of simulation results in Table 4.2. When a DRT area with 6 trips per hour and a maximum length of DRT-trips of 8 km is considered, it is expected that $bv_{8,6}$ vehicles are needed to be able to accept 90% of the trip requests.

4.3.2 Expected travel distance of DRT vehicles

When a trip is assigned to a DRT vehicle, the vehicle drives (possibly empty) to the origin of the trip, then takes the customer towards their destination (possibly with a detour) and then may or may not return to the depot afterwards. Especially when trips are combined, it is quite hard to determine what distance was covered for a specific trip. However, to make an estimate, we can use the ratio between the total distance driven by the DRT vehicles and the sum of the direct travel distances of all trips that were executed in the simulation area. Next, when needing an estimate of the required distance for an incoming trip, the direct distance between origin and destination of a trip can be multiplied by this ratio to obtain an estimate of the required distance to cover that trip. When many trips are combined, this ratio can also be smaller than one.

4.3.3 In-vehicle travel time of DRT customers

To determine the expected In-Vehicle Travel Time (IVTT), we use an estimation based on the average time customers spent in vehicles in the simulation area. For every trip from the simulation area, the ratio is calculated between direct travel time and IVTT and averaged over all trips. The result is a ratio that we can use to multiply the direct travel time with.

Chapter 5

Results

In this chapter, the input and results of the model described in the previous chapters are discussed. First of all, information about the cost and pricing strategy and the simulation area that is used for the estimations of DRT characteristics is given. Then, the correctness of the model is validated by using a test case which shows very clearly what the impact of the price of DRT tickets is on the outcome of the MILP. The case study is then introduced, accompanied by its results. Finally, an analysis of customer behaviour is given. All data that was used to run the simulations as well as the case study is provided by *Transdev*, which is the operator of the case study.

5.1 Input

This section shows the pricing and costs together with the simulation results from other DRT areas. These values are used for the case study that is located in the Netherlands and are also used for the test case.

5.1.1 Pricing and costs

The price for FLS bus transport in the Netherlands consists of a fixed price per trip of €0.90. Additionally, there is a price per kilometre which varies per region. In the case study a price of €0.15 per kilometre is used. DRT trips in the Netherlands often have a fixed price, independent of the distance of the trip. Depending on the location of the DRT area, the tickets are all priced between €3 and €4.

The salary of drivers can be considered €25 per active hour (this price includes breaks outside of active hours and aspects like holidays). For DRT vehicles, standard 8-person vans are considered. The write-off costs for these vehicles come down to €2,400 per

TABLE 5.1: Pricing strategy for the case study which is also used for the test case.

Parameter	Value
Price per DRT ticket	€ 3 – € 4
Fixed price for FLS tickets	€ 0.90
Price per km for FLS tickets	€ 0.15/km
Salary driver	€ 25/h
Write-off cost for DRT vehicle	€ 2,400/year
Write-off cost for FLS vehicle	€ 22,700/year
Maintenance and fuel cost per km for DRT vehicle	€ 0.21/km
Maintenance and fuel cost per km for FLS vehicle	€ 0.53/km

year. Standard 12m buses with 40 seats and 44 standing spots are considered for all FLS transport, which cost €22,700 in write-off yearly. Finally, the costs for maintenance and fuel per kilometre are €0.21 and €0.53 for the DRT vehicles and FLS vehicles, respectively. An overview of these parameters is given in Table 5.1. All parameters are provided by *Transdev*.

In order to obtain the time and distance matrices for all stops part of the model, a shortest path algorithm is used. The travel distances are obtained from *TomTom*, while *Speedprofiles* are used to get the corresponding travel times. The matrices are customised to the time windows that they are used for.

5.1.2 Simulation area

The DRT area *Breng Flex* is used as a simulation area for the case study as well as the test case. The set of trips consists of 140,000 trips from December 2016 until September 2018. For all simulations, the dial-a-ride problem as described in Section 4.2 is used for dispatching. First, the ratio between the average time customers spend in the vehicle to the direct travel time is determined. This ratio equals 1.2. This means that on average, customers have detours of 20% of the direct travel time. The ratio between the sum of all direct travel distances and the distance that the vehicles have covered is 1.3. So for example, when the direct travel distance of a trip is 5 km, on average the DRT vehicle covers 6.5 km to bring the customer to their desired destination. These extra 1.5 km can either be driven empty or with the customer(s). These numbers relate as expected. The difference of 0.1 is influenced by vehicles driving around empty, by combining trips or from converting between time and distance.

TABLE 5.2: The expected DRT fleet size for different combinations of trips per hour and maximum length between two DRT stops with the lowest and highest ratio of vehicles:trips circled.

Average	**Max. length between two DRT-stops (km)**											
#trips/h	0	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	2	2	2	2	2	2	(3)	3	3	3	3
4	0	3	3	4	4	4	4	4	4	4	4	4
6	0	4	4	4	4	4	4	4	5	5	5	5
8	0	5	5	5	5	6	6	6	6	6	6	6
10	0	5	5	6	6	6	6	6	6	7	7	7
12	0	6	6	7	7	7	7	7	7	7	7	8
14	0	7	7	7	7	8	8	8	8	8	8	8
16	0	7	7	8	9	9	9	9	9	9	9	9
18	0	8	8	8	9	9	9	9	10	10	10	10
20	0	8	8	9	9	10	10	10	10	10	11	11
22	0	9	9	9	10	10	11	11	12	12	12	12
24	0	(9)	9	10	11	11	12	12	12	12	12	12

Finally, the estimations of the number of DRT vehicles for different area characteristics is given in Table 5.2. The ratio between number of vehicles and trips per hour is 9:24 at best (see circled value). This is partly due to the fact that we consider the *average* number of trips per hour. So during a weekday in the simulation, it could be that there are more trips in rush hours than during the rest of the day. This also explains the quite extraordinary ratio of 3:2 where you need three vehicles to service an average of two trips per hour (see circled value). Overall, the required number of vehicles is quite high, resulting in high cost for the operator. The total running time of all the simulations used to obtain the values in Table 5.2 is thirty minutes.

5.2 Test case

The test case consists of three bus lines as displayed in Figure 5.1. Every stop is located on the crossing of two grid lines and the distance and duration between two consecutive lines of the grid is assumed to be 1 km and 4 minutes, respectively. FLS transport follows the marked bus lines, however, it is assumed that DRT does not need to follow the bus lines but can drive to the destination directly in a straight line. Most trips take place to or from the transit station. All stops on bus line 1 have 5 customers travelling to and from the transit station per hour and bus line 1 is thereby the busiest bus line. The stops on bus line 3 have one customer travelling to and from the transit station every hour, hence bus line 3 is the least crowded bus line. Finally, the customer flows on bus line 2 to and from the transit station are according to a discrete uniform distribution between 1 and 5. See also Table 5.3 for all the details about customer flows. Besides the trips to and from the transit station, a randomly selected 5% of all possible trips gets an additional passenger per hour. The FLS lines 1, 2 and 3 have frequencies corresponding to the customer flows of 3, 2 and 1 bus(es) per hour, respectively. For the test case, utility theory is not taken into account and instead it is assumed that for all trips $p1_{od}^{\text{DRT}} = p2_{od}^{\text{FLS}} = 0.7$ and $p3^{\text{DRT}} = p3_{od}^{\text{FLS}} = 0.4$ with $o, d \in \mathcal{S}$. This is done to focus on the effect of DRT price on the solution when customer behaviour does not change. All parameters such as personnel costs, vehicle costs and DRT fleet size are the same as the values for the case study as discussed in the previous section.

5.2.1 Results

Since the customer utility is fixed for the test case and is thus not dependent on price, the expected behaviour of the MILP is that when the price rises, the size of the DRT area will too. Eventually, DRT will be so attractive that the MILP wants to make all stops part of the DRT network. However, when the total number of trips per hour in the area exceeds the number of trips per hour in Table 5.2, this is not possible. So,

Table 5.3: Characteristics of the bus lines of the test case.

Bus line	Customers to/from the transit station from every station on the bus line	Frequency
1	5/h	3/h
2	Discrete uniform distribution (1,5)	2/h
3	1/h	1/h

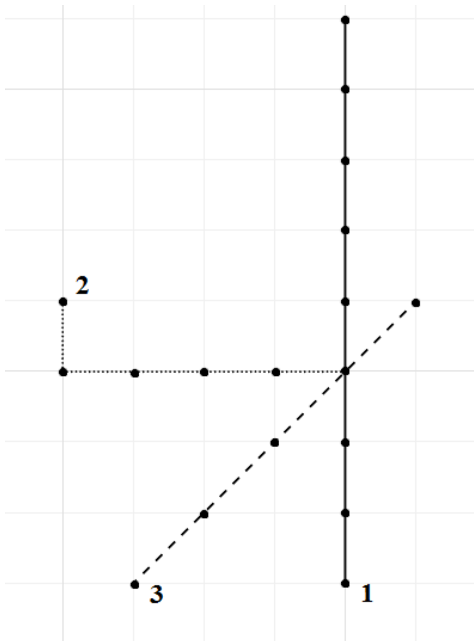
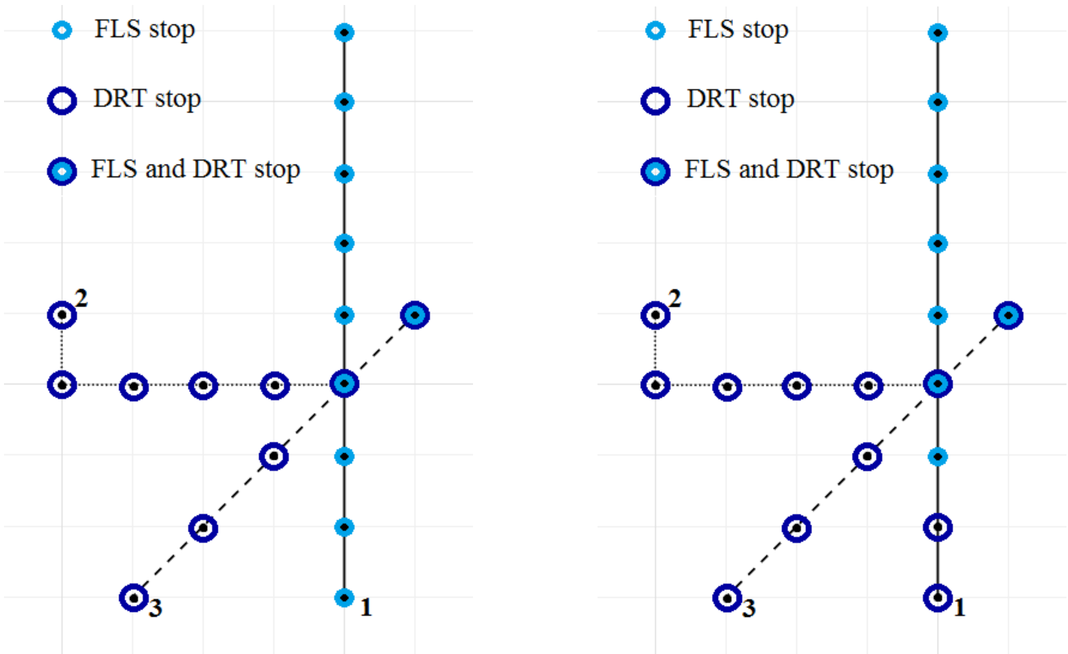


FIGURE 5.1: The lay-out of the test case that consists of three bus lines (1, 2 and 3), that intersect at a transit station.

there will be a DRT price, where the DRT area will have the maximum number of trips possible and from there the DRT area can not grow anymore. If this happens for realistic DRT prices in realistic settings, it is an indication that the simulation values in Table 5.2 are not sufficient and should be expanded.

When varying the price of DRT from €1 to €10, with steps of €1, at first DRT is not profitable and is never chosen. At a price of €4, DRT is introduced as in Figure 5.2a. In this scenario there are five DRT and four FLS vehicles needed. The DRT vehicles serve a total of 10 trips per hour. This remains the optimal system, until the price reaches a level of €8 and the optimal system changes as in Figure 5.2b. In this case nine DRT vehicles are needed that serve 24 trips per hour. Additionally, three FLS buses are needed in this scenario.

In the scenario in Figure 5.2a, bus line 1 is the only bus line that is not part of the DRT network. This is as expected, since bus line 1 is the busiest bus line of the three and therefore the least likely to become DRT. When the DRT price is €8, the DRT system runs at its maximum capacity. In this scenario there are already 24 trips per hour, so for meaningful results at a price of €8 or higher, the simulation results should be expanded. However, what can be concluded from this final scenario, is that DRT price is now so high that even introducing DRT on the busiest bus line becomes profitable.



(a) Optimal bus system for DRT prices €4, €5, €6 and €7.

(b) Optimal bus system for DRT prices €8, €9 and €10.

FIGURE 5.2: Test case with optimal bus system for different prices of DRT tickets.

Note that since this test area was made up purely to test the correctness of the MILP, the numerical values of the DRT price thresholds at which the optimal solution changes are meant for illustrational purposes only and have no further meaning.

5.3 Case study

In this section, the case study is discussed. First some area characteristics are given and the data set is discussed. Then, the results are given and explained.

5.3.1 Characteristics

The case study that is used for this thesis is the area of *Gooi en Vechtstreek*. The area is quite diverse and consists of rural areas as well as a medium-sized city, Hilversum (90,000 inhabitants), and a few villages. The area is located in the Netherlands, east of the capital Amsterdam. The population density ranges from less than 500 inhabitants per square kilometre to over 2,500 inhabitants per square kilometre as can be seen in Figure 5.3. The current FLS bus network consists of nineteen bus lines and three-hundred bus stations, which interconnect the cities and villages with one another and Amsterdam. A map of the bus network can be seen in Figure 5.4. The operator of *Gooi en Vechtstreek* is *Connexxion* which is a subsidiary of *Transdev*. On average 10,000 bus trips are made daily using the FLS network from *Connexxion* in *Gooi en Vechtstreek*. This region has been chosen to be the case study of this research, since the government of *Gooi en Vechtstreek* has shown interest in introducing DRT there.

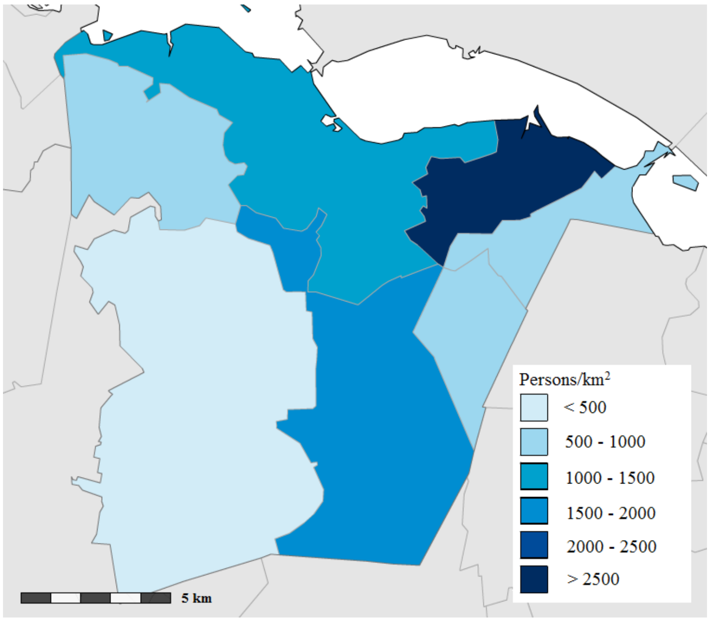


FIGURE 5.3: Population density of *Gooi en Vechtstreek* (Centraal Bureau voor de Statistiek, 2017).

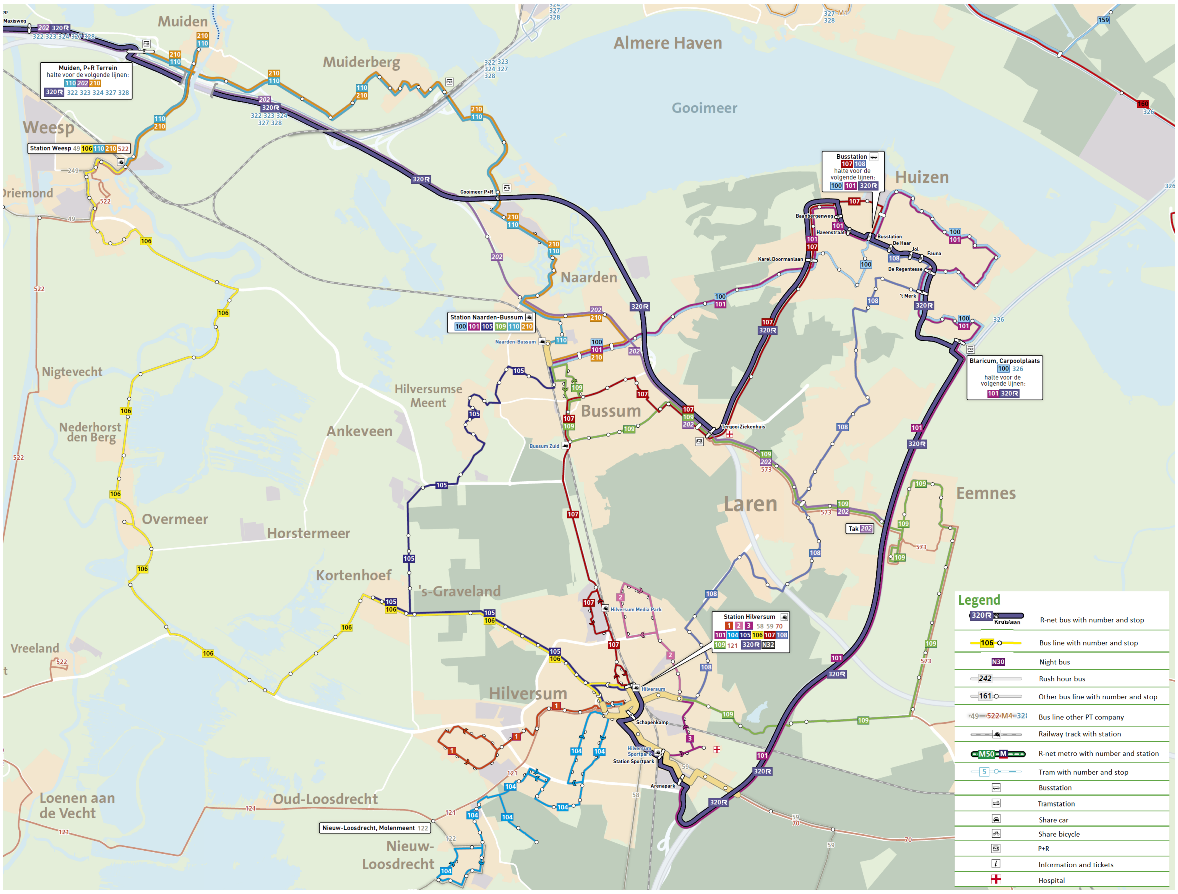


FIGURE 5.4: Bus network of *Gooi en Vechtstreek* (Carto Studio, 2017).

5.3.2 Data

Most of the input values for the MILP of the case study are given in Section 5.1. The expected price for DRT tickets in *Gooi en Vechtstreek* is €3.50. The origin-destination matrix is constructed by using a data set that contains all trips that were made in the FLS network in *Gooi en Vechtstreek* in 2017. For every trip, the origin and destination are registered by a card (*OV chipkaart*), that is used for checking in at the origin and checking out at the destination. When customers transfer between two buses, these trips are linked and considered as a single trip. In some rare cases no data of the check out of a trip is available, which happens when customers forget to check out. These trips are not considered here. In total there were 3,959,258 trips in 2017. Converting the raw data from *Connexxion* to timetables and an origin-destination matrix was a very lengthy process.

We distinguish three major time windows in the timetable for the region *Gooi en Vechtstreek*. These time windows are chosen such that in each period FLS bus lines have more or less consistent timetables. The time windows are as follows:

1. Monday to Friday from 07:00 to 17:00 and Saturday from 10:00 to 16:00
2. Monday to Sunday from 19:00 to 22:00
3. Sunday and public holidays from 11:00 to 17:00

The periods in between these time windows are usually inconsistent regarding timetables. Note that for this case study, the analysis is meant to give an indication on what would be an optimal DRT area. However, our model does not prevent using additional time windows. Since for this case study it is not realistic to change the DRT area every hour in terms of customer service, the week is divided in time windows during which the timetables are currently consistent. The DRT design in the time windows that are not analysed can be inferred from the DRT designs of the analysed time windows.

As explained in Section 4.1.2, there are three methods to ensure a connected FLS network. In this particular case study, we ensure that every bus line has at least one transfer possibility. Additionally, for every stop that is part of a transit station with train transport, we assure that when bus lines come as close as three stops from the station (or closer), they have to visit the station as well (see Figure 4.6). Finally, for all stops that are visited by exactly two FLS lines, we add the constraint that they are either visited by both of the bus lines or by neither of the bus lines.

When analysing the data of *Gooi en Vechtstreek*, it unfortunately seems like DRT will very likely be more expensive than FLS. Even in the best case scenario, there is still a need of 3 DRT vehicles for every 8 trips per hour (see Table 5.2). While in comparison, an FLS vehicle can seat up to 40 people plus an additional 44 that are standing. Even though DRT vehicle costs are much lower and they consume less fuel, when a lot more vehicles are needed to supply the same number of customers compared to FLS transport, DRT will not be profitable.

5.3.3 Results

To analyse the profitability of DRT in *Gooi en Vechtstreek*, a series of models have been run. The MILP was run for prices from €3 to €10 for all three of the time windows and in all scenarios the results are homogeneous: *DRT is never chosen.* As explained before, these results are predictable considering the input of the model.

A phenomenon that did occur was that certain *ghost* stops were made DRT stops. There are a few stops in the area that have never been visited in the data set. The MILP decides that these stops should be DRT stops, such that the FLS lines can possibly get a shorter route while the DRT service is never used and thus no vehicles are needed. This obviously does not make sense, although it might be interesting for the operator to analyse ghost stops, since visiting these ghost stops seems like a waste of time and money as they have never been used in an entire year. As expected, in time window 3 there are more ghost stops than in time window 1 and 2.

Another phenomenon that occurs is that when the price of DRT gets higher, the chance of customers choosing DRT gets lower. Eventually, for every trip there is a threshold for which DRT is (almost) always rejected by the customer. In that case, the MILP does choose some stations to be DRT stops. But there are no trips at all at these stops, since DRT is always rejected by the customer and thus there are also no DRT vehicles needed. This is not a realistic outcome, since increasing the price until there are no more customers is not realistic in this scenario. This can easily be prevented by introducing constraints on the reject probability. Note that this only happens with high prices (above €7) and these prices are currently not realistic for public transport in the Netherlands.

Given the current input and characteristics of the model, the region *Gooi en Vechtstreek* proves not to be interesting for introducing DRT. However, there are some factors that could improve the profitability of DRT. First of all, combining more trips results in higher vehicle-trip ratios. Right now there are constraints on customer detour time and waiting time and when these constraints are loosened up, more trips will be combined which will reduce cost and lower customer satisfaction.

Another possibility that can influence the profitability of DRT is outsourcing the DRT. Currently there are quite a few areas in the Netherlands where the DRT customers are transported by the local taxi company. The taxi company is paid per active driver hour. This can be profitable for both parties since for large periods of time the DRT vehicles are idle and waiting for trips. The taxi company can however execute other trips in these periods. The price that is paid to the taxi companies is around €25 per active driver hour. An advantage of this method for the transport operator is that there is the possibility to hire more drivers in rush hours and less during the rest of the day. This is not as easy when you employ the drivers, since you need to comply with labour rules. However, paying per active driver hour is only slightly cheaper in costs per hour.

In The Netherlands, public transport is highly subsidised and is made affordable for the customers. It is not uncommon that the government supports new services, like DRT, and subsidises companies that will execute DRT. This is the case in the region *Breng Flex* for example, where DRT is offered besides the regular FLS transport. When the operator receives (enough) subsidy for every DRT trip, DRT can be profitable. This is very easily incorporated and is a powerful tool for transport operators. In some cases operators might want to introduce DRT even though it is not profitable. This might give them an advantage above other operators that are competing for the same contract. Note that this can also be very easily incorporated in the MILP.

To conclude, DRT is not profitable in the current circumstances at *Gooi en Vechtstreek*. This could be changed by loosening the constraints with regards to detour and waiting time or by outsourcing DRT. When governments subsidise DRT, the MILP can be used to analyse the impact on costs.

Computational complexity

The number of variables of the MILP is in the order of magnitude of $2|\mathcal{S}|^2$ and the number of constraints is in the order of magnitude of $(|\mathcal{B}| + 18)|\mathcal{S}|^2 + |\mathcal{B}|^2|\mathcal{S}|$, where $|\mathcal{S}|$ is the number of stops and $|\mathcal{B}|$ is the number of FLS lines. The size of the service area has a crucial impact on the running time. The test case takes less than a minute to solve while the running time of the case study with 19 bus lines and 300 stops varies between one and three hours. All the results were conducted on a Dell laptop with an Intel i5-5300U processor and 8GB RAM. Areas that are interesting for DRT are usually not that big. Therefore, the scalability of the model should not be an issue for areas for which DRT is considered, especially with a more advanced machine.

5.4 Customer behaviour analysis

Currently, the objective of the MILP has been to maximise profit for the transport operator, given the service constraints. Expected customer behaviour is determined by using the utilities for different travel options. However, the expected customer behaviour itself is not analysed, only the expected costs. In this section, an analysis of customer behaviour is given. We are interested in the effect of different travel options on customer satisfaction. An analysis of the shift in choice between FLS and DRT compared to price is discussed. Note that this analysis is conducted on an imaginary situation in which at all stops DRT and FLS is offered, such that for every trip the

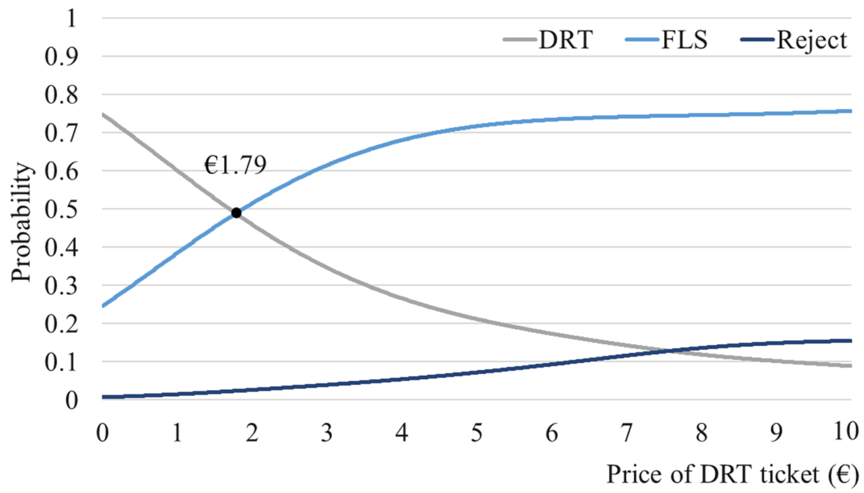


FIGURE 5.5: Probabilities of choosing DRT or FLS or rejecting, displayed against the price of DRT tickets. These probabilities are averaged over all customers in the test region.

customer has a choice between DRT and FLS. This is not realistic since as discussed in the results in the previous section, it is never optimal to have DRT stops in our case study. However, since we are interested in customer behaviour, the comparison between DRT and FLS is relevant.

5.4.1 Effect of DRT price

To determine the shift in choice between DRT and FLS for different prices, we look at the rural region in the west of *Gooi en Vechtstreek*. This region seems most suitable for DRT since it has the lowest demand while covering quite a big area. For all demand in this rural region, we look at the average chance of choosing DRT or FLS or rejecting both travel options. This trade-off is made while varying the price of DRT and the price of FLS transport is kept constant. Interesting is the switch point in customer choice between FLS and DRT.

The region we consider consists of bus lines 105, 106, 110 and 210 in the west of *Gooi en Vechtstreek*, see Figure 5.4. Within this region, we evaluate the probabilities of choosing DRT, FLS or reject both travel options while varying the price for a DRT ticket. The average probability is calculated for all customers that travel in this area. The results can be found in Figure 5.5. This figure is made by increasing the price of DRT by one euro cent every time and evaluating the probabilities for that price. As expected, the probability that customers choose DRT goes down when the price for DRT tickets goes up. The chance of choosing FLS goes up consequentially. The

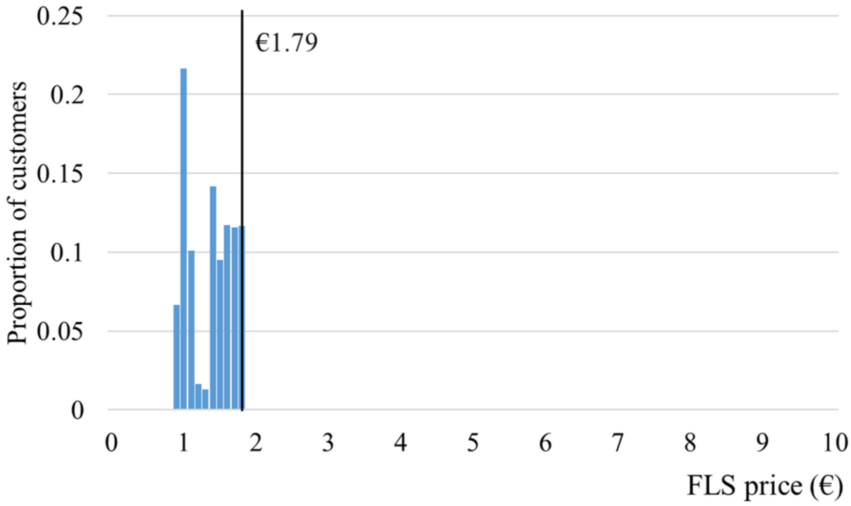


Figure 5.6: The FLS prices customers experience in the test region compared to the switch point between FLS and DRT.

interception between both lines happens at a price of €1.79. In Figure 5.6, the prices of the FLS tickets are displayed against the proportion of customers that experienced that price. The threshold of €1.79 is on the right hand side of the prices customers experience in the area. Recall that the price sensitivity is 0.5 (see Section 3.3), so an increase in price of €1 will result in a decrease of 0.5 in the utility function.

It is interesting that even when DRT is free, there are still over 20% of customers that are expected to travel with FLS. This can be explained as there are few roads in the area, so the expected travel time by FLS transport will most likely be quite similar to the direct travel time. DRT however, is still assumed to have a detour of 20% of the direct travel time, so FLS might actually be shorter. Since this analysis does not use simulation or linear programming, the results are obtained very quickly.

Chapter 6

Conclusion and Discussion

In this chapter, we will discuss the key findings of our research and the significance to transport operators. Furthermore, an elaborate discussion is presented.

6.1 Key findings

In this thesis we have proposed a method to optimise the public transport system in areas with heterogeneous characteristics by considering substituting FLS transport by DRT. Currently, there has not been much research in the design of DRT systems and especially little for heterogeneous areas. To the best of the author's knowledge, the joint optimisation of FLS and DRT has not been researched. However, many transport operators who currently operate FLS transport are interested in the relatively new DRT service. We have developed a MILP that, given the characteristics of the area, determines which stops should stay FLS stops, which should become DRT stops or which should be a combination of both services in order to minimise operator costs. The MILP uses characteristics of the area as input, such as travel times and most importantly historic trip data from the FLS transport in the area to create an origin-destination matrix. Customer choice modelling is used to predict customer behaviour when different services are offered. The MILP is designed such that it works for a time frame in which passenger flows are (more or less) constant and FLS frequencies are similar throughout the time frame.

One of the advantages of this model is that it is very easy to personalise to the characteristics of the area. Different pricing strategies or different characteristics of the DRT and FLS system leave quite some room for personalisation. When the transport operator has some requirements on the DRT area, like specific stops that should be part of the DRT network, this can easily be incorporated. The main disadvantage of our implementation of the model is that several assumptions are required. Even though the model is designed to analyse all costs as well as possible, it is still a simplification of reality. This effect can be minimised by determining the optimal DRT area according to the MILP and then running an additional simulation in which the costs of the final DRT area are assessed before applying. The MILP was tested on a case study *Gooi en Vechtstreek*. This areas proves to be unsuitable for DRT in the current circumstances. This is discussed more thoroughly in the discussion.

This research contributes to the literature by providing a method to optimise the design of DRT areas in combination with FLS which has not been done before. So far, most research about DRT was focused on the dispatching of vehicles when the design of the area is already given. One of the biggest challenges faced in this research was to design a method that is suitable for all types of areas. Another challenge was to be able to estimate the DRT fleet size that would be needed without using simulation. Finally, it was a challenge to put all this in a linear model.

6.2 Significance to transport operators

The main goal of the model is to give transport operators a calculated indication of which areas are suitable for DRT. Currently, the design of DRT areas is usually done without using an optimisation model. Our model can be used for comparing the costs of FLS with the costs of DRT, but also to analyse the impact of subsidy on the profitability of DRT. So far, research into DRT mostly focuses on the optimisation of the dial-a-ride problem and handles the design of the DRT area as given. When the design of DRT areas is researched, it is either for homogeneous circumstances or for DRT feeder areas around transit stations. This research is a first step into the optimisation of the application of DRT in combination with FLS and is meant as an indicator to the operator. After transport operators have designed a bus system, additional simulation can be used to evaluate the costs in more detail.

6.3 Discussion

Given the current circumstances, the case study is not at all suitable for DRT. However, when the operator receives subsidy from the government specifically for DRT, this could change. The impact of the subsidy can easily be analysed using the MILP. Another reason for the implementation of DRT, even though it is not profitable, is that it has a competitive value compared to other transport operators competing for an operator contract. If, by offering DRT the operator can secure the contract, offering DRT might be profitable after all.

The outcome that DRT is not profitable in the case study, is highly dependent on the customer flows and the expected number of DRT vehicles that are needed. Currently, in the best case scenario there is still a need for 9 DRT vehicles for 24 trips per hour on average and the ratio gets as high as 3 DRT vehicles for 2 trips per hour. If this ratio could be improved, this would mean a direct difference in expected costs of DRT systems and DRT will more likely be chosen.

As a final comment on the profitability of DRT in *Gooi en Vechtstreek*, we would like to mention the possibility of outsourcing DRT transport. This offers both the opportunity to save personnel costs as well as to vary the number of DRT drivers throughout the day.

Currently, the MILP only considers a fixed set of stops that all have to be served. This is justified, since from a transport operators point of view, adding more stops to the network that is currently served is in general not profitable as they will not receive subsidy for servicing more than the required area. The MILP does offer the possibility to add new stops, but this is difficult as customer flows are currently deterministic. When not every stop *has* to be served, customer flows should depend on the decision variables. This would require a method to estimate customer flows and it has to be incorporated in the model. This is a possibility for future research. Another limitation of the model is that it only considers to offer DRT transport from and to designated DRT stops instead of from and to any address in the DRT area. For the same reasons as for adding stops, this is generally not profitable. Also, combining customer trips will become harder en less efficient when customers can travel from and to any address in the DRT area. When this approach is desired by the operator or government, the MILP should be extended.

Even though we currently assume that customer flows do not shift from one stop to another when the bus service changes partly from FLS to DRT, it is not unthinkable that customers choose to change stops. When customers live in between two stops and one of the two changes from FLS to DRT, customers will make a trade-off between the two stops. Maybe they used to catch the bus at stop A because that was slightly closer to their origin (house, work, etc.), but now take DRT transport from stop B since they appreciate the DRT service or their destination is a DRT stop, even though B is further away. This also offers interesting future research. In order to execute this change, more data is needed. Especially data about the origin and destination of all trips is crucial.

When determining customer utility functions for different travel options, one thing that is not taken into account is the frequency of the trip. When customers want to travel with DRT, they can order the trip any time they want, while FLS transport is limited to the timetable. There are some studies about the impact of service frequency to customer utility, see for example Eboli and Mazzulla (2008). They create a multinomial logit model to identify the importance of service quality attributes. One of these attributes was the dichotomous variable that represents service frequency, which they varied from once every hour to once every fifteen minutes. Frequency had a very significant impact, however when using the utility value from their research in our utility function, frequency would have by far the most impact out of all factors. Since this did not seem appropriate, it was not incorporated. However, when research would be conducted in which they take into account all relevant factors to this research like service frequency, IVTT, OVT, transfers and price, frequency could be incorporated.

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Appendix A

Complete MILP

Variable	Explanation
b^{DRT}	Expected number of vehicles that is used for the DRT system
b^{FLS}	Expected number of vehicles that is used for the FLS system
d^{DRT}	Expected total distance driven with DRT vehicles in distance units
d^{FLS}	Expected total distance driven with FLS vehicles in distance units
ℓ_b	Length of bus line b in distance units
n_{ij}^b	1 if bus line b stops at stop $o\ell_j^b$ directly after stop $o\ell_i^b$, 0 otherwise
r_{od}	Expected revenue from all trips from origin o to destination d
s	Duration of the longest trip in the DRT network, this is an indication for the size of the DRT network
t_b	Duration of bus line b in time units
w	Expected number of drivers needed to run this system
x_s	1 if stop s is part of the DRT network, 0 otherwise
y_{bs}	1 if bus line b stops at stop s, 0 otherwise
z_{bs}	1 if stop s is a transfer station of bus line b, 0 otherwise
α_{od}	1 if DRT is required for trip (o, d), that is, when at least one of o and d is not part of the FLS network, 0 otherwise
β_{od}	1 if choice option 1: {DRT} is offered, that is, when stops o and d are both part of the DRT network and at least one of o and d is not part of the FLS network, 0 otherwise
γ_{od}	1 if choice option 3: {DRT,FLS} is offered, that is, when both stop o and d are part of the DRT and FLS network, 0 otherwise
δ_{od}	1 if choice option 2: {FLS} is offered, that is, when both stop o and d are part of the FLS network but at least one of o and d is not part of the DRT network, 0 otherwise
$\vartheta_{od}^{\text{DRT}}$	Expected number of trips from stop o to stop d in the DRT network
$\vartheta_{od}^{\text{FLS}}$	Expected number of trips from stop o to stop d in the FLS network

Φ_{ij}	Binary variable that indicates which type of DRT-area we consider
$\phi_{ij}^{\text{length}}$	Binary variable that indicates in which category the longest trip belongs
ϕ_{ij}^{trips}	Binary variable that indicates in which category the number of trips belongs

Parameter	Explanation
bo_i^{length}	Upper bound for category i of the longest trip
bo_j^{trips}	Upper bound for category j of the trips
bv_{ij}	Number of DRT vehicles needed to achieve a 90% acceptance rate in category (i, j)
cs	Salary per time unit for drivers
cd^{DRT}	Fuel and maintenance price per distance unit per DRT vehicle
cd^{FLS}	Fuel and maintenance price per distance unit per FLS vehicle
cw^{DRT}	Write-off cost per time unit per DRT vehicle
cw^{FLS}	Write-off cost per time unit per FLS vehicle
di_{od}	Distance of trip (o, d) when driving directly
et_{bs}	Expected duration of customers entering and exiting the FLS vehicle at stop s on bus line b
fc	Factor by which the distance of a DRT-trip is multiplied to estimate the distance needed to cover that trip
fo_b	Original frequency of bus line b (trips per time unit)
M	Sufficiently large number
$o\ell_{ij}^b$	Original order of bus line b, $o\ell_i^b = s$ if stop s is the i^{th} stop of bus line b
$p1_{od}^{\text{DRT}}$	Probability that a customer will choose DRT to travel from stop o to stop d when choice option 1 is offered: {DRT}
$p2_{od}^{\text{FLS}}$	Probability that a customer will choose FLS to travel from stop o to stop d when choice option 2 is offered: {FLS}
$p3_{od}^{\text{DRT}}$	Probability that a customer will choose DRT to travel from stop o to stop d when choice option 3 is offered: {DRT, FLS}
$p3_{od}^{\text{FLS}}$	Probability that a customer will choose FLS to travel from stop o to stop d when choice option 3 is offered: {DRT, FLS}
pr^{DRT}	Set price for bus ticket in DRT system
pr^{dist}	Price per distance unit for FLS bus tickets
pr^{fixed}	Fixed price for FLS bus tickets
qt_{od}	Number of customers that want to travel from origin o to destination d (average) per time unit
sc_{od}^b	1 if stop d needs to be part of bus line b when stop o is part of bus line b, 0 otherwise
ti_{od}	Duration of the trip from stop o to stop d when driving directly
yo_{bs}	1 if bus line b stops at stop s in the original situation, 0 otherwise

Objective

$$\max \quad \sum_{o\in\mathcal{S}}\sum_{d\in\mathcal{S}} r_{od} - \Big(cs \times w \quad + cd^{\text{DRT}} \times d^{\text{DRT}} + cd^{\text{FLS}} \times d^{\text{FLS}} + cw^{\text{DRT}} \times b^{\text{DRT}} + cw^{\text{FLS}} \times b^{\text{FLS}}\Big)$$

Demands

$$x_s + \sum_{b\in\mathcal{B}} y_{bs} \geq 1 \qquad \forall s \in \mathcal{S}$$

$$y_{bs} \leq yo_{bs} \qquad \forall s \in \mathcal{S}, b \in \mathcal{B}$$

Travel options

$$\alpha_{od} = 1 - \min\Big\{1, \sum_{b\in\mathcal{B}} y_{bo}, \sum_{b\in\mathcal{B}} y_{bd}\Big\} \qquad \forall o, d \in \mathcal{S}$$

$$\beta_{od} = \min\{x_o, x_d, \alpha_{od}\} \qquad \forall o, d \in \mathcal{S}$$

$$\gamma_{od} = \min\{x_o, x_d, 1 - \alpha_{od}\} \qquad \forall o, d \in \mathcal{S}$$

$$\delta_{od} = 1 - \max\{\alpha_{od}, \gamma_{od}\} \qquad \forall o, d \in \mathcal{S}$$

$$\vartheta_{od}^{\text{DRT}} = qt_{od}\big(p1_{od}^{\text{DRT}} \times \beta_{od} + p2_{od}^{\text{FLS}}(\alpha_{od} - \beta_{od}) + p3_{od}^{\text{DRT}} \times \gamma_{od}\big) \qquad \forall o, d \in \mathcal{S}$$

$$\vartheta_{od}^{\text{FLS}} = qt_{od}\big(p2^{\text{FLS}} \times (\delta_{od} + \alpha_{od} - \beta_{od}) + p3_{od}^{\text{FLS}} \times \gamma_{od}\big) \qquad \forall o, d \in \mathcal{S}$$

Characteristics of DRT system

$$s = \max_{o\in\mathcal{S}, d\in\mathcal{S}} \{ti_{od} \times \beta_{od}\}$$

$$d^{\text{DRT}} = fc \sum_{o\in\mathcal{S}}\sum_{d\in\mathcal{S}} di_{od} \times \vartheta_{od}^{\text{DRT}}$$

Number of DRT vehicles

$$s \leq bo_i^{\text{length}} \times \phi_i^{\text{length}} + M(1 - \phi_i^{\text{length}}) \qquad \forall i \in \{1, \ldots, n\}$$

$$s > bo_i^{\text{length}} \times \phi_{i+1}^{\text{length}} \qquad \forall i \in \{1, \ldots, n-1\}$$

$$\sum_{o\in\mathcal{S}}\sum_{d\in\mathcal{S}} \vartheta_{od}^{\text{DRT}} \leq bo_j^{\text{trips}} \times \phi_j^{\text{trips}} + M(1 - \phi_j^{\text{trips}}) \qquad \forall j \in \{1, \ldots, m\}$$

$$\sum_{o\in\mathcal{S}}\sum_{d\in\mathcal{S}} \vartheta_{od}^{\text{DRT}} > bo_j^{\text{trips}} \times \phi_{j+1}^{\text{trips}} \qquad \forall j \in \{1, \ldots, m-1\}$$

$$\Phi_{ij} \leq \frac{1}{2} \times (\phi_i^{\text{length}} + \phi_j^{\text{trips}}) \qquad \forall i \in \{1, \ldots, n\}, j \in \{1, \ldots, m\}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\Phi_{ij}=1$$

$$b^{\text{DRT}}=\sum_{i=1}^{n}\sum_{j=1}^{m}\Phi_{ij}bv_{ij}$$

Characteristics of FLS bus system

$$n_{ij}^{b}=\max\left\{0,\min\left\{y_{b,o\ell_i^b},\ y_{b,o\ell_j^b},\ 1-\sum_{s=i+1}^{j-1}y_{b,o\ell_s^b}\right\}\right\} \qquad \forall i,j\in\{0,\ldots,|o\ell^b|\},b\in\mathcal{B}$$

$$t_b=\sum_{i=0}^{|ol^b|}\sum_{j=0}^{|ol^b|}\left(ti_{o\ell_i^b,o\ell_j^b}+ee_i^b\right)\times n_{ij}^b \qquad \forall b\in\mathcal{B}$$

$$\ell_b=\sum_{i=0}^{|ol^b|}\sum_{j=0}^{|ol^b|}di_{o\ell_i^b,o\ell_j^b}\times n_{ij}^b \qquad \forall b\in\mathcal{B}$$

$$y_{bd}\geq sc_{od}^b\times y_{bo} \qquad \forall o,d\in\mathcal{S},b\in\mathcal{B}$$

$$\sum_{s\in\mathcal{S}}z_{bs}\geq 1 \qquad \forall b\in\mathcal{B}$$

$$y_{bs}\geq z_{bs} \qquad \forall s\in\mathcal{S},b\in\mathcal{B}$$

$$\sum_{x\in\mathcal{B}\setminus\{b\}}y_{xs}\geq z_{bs} \qquad \forall s\in\mathcal{S},b\in\mathcal{B}$$

$$yo_{b_1s}\times yo_{b_2s}\big(y_{b_1s}-y_{b_2s}\big)=0 \qquad \forall s\in\mathcal{S}^*,\ b_1,b_2\in\mathcal{B}$$

$$b^{\text{FLS}}\geq\sum_{b\in\mathcal{B}}fo_b\times t_b$$

$$d^{\text{FLS}}=\sum_{b\in\mathcal{B}}fo_b\times\ell_b$$

Revenue and costs

$$r_{od}=\left(pr^{\text{DRT}}\times\vartheta_{od}^{\text{DRT}}+(pr^{\text{dist}}\times di_{od}+pr^{\text{fixed}})\vartheta_{od}^{\text{FLS}}\right) \qquad \forall o,d\in\mathcal{S}$$

$$w=b^{\text{DRT}}+b^{\text{FLS}}$$

Range of variables

$$x_s\in\mathbb{B} \qquad \forall s\in\mathcal{S}$$

$$y_{bs},z_{bs}\in\mathbb{B} \qquad \forall b\in\mathcal{B},s\in\mathcal{S}$$

$$n_{ij}^b\in\mathbb{B} \qquad \forall i,j\in\{0,\ldots,|o\ell^b|\},b\in\mathcal{B}$$

$$t_b,\ell_b\in\mathbb{N}^+ \qquad \forall b\in\mathcal{B}$$

$$r_{od}, \alpha_{od}, \beta_{od}, \mathcal{I}_{od}, \in \mathbb{B} \qquad \forall o, d \in \mathcal{S}$$

$$\Phi_{ij} \in \mathbb{B} \qquad \forall i \in \{1, ..., |\phi^{\text{length}}|\}, j \in \{1, ..., |\phi^{\text{trips}}|\}$$

$$\phi_i^{\text{length}} \in \mathbb{B} \qquad \forall i \in \{1, ..., |\phi^{\text{length}}|\}$$

$$\phi_j^{\text{trips}} \in \mathbb{B} \qquad \forall j \in \{1, ..., |\phi^{\text{trips}}|\}$$

$$w, s, b^{\text{DRT}}, b^{\text{FLS}}, f^{\text{DRT}}, f^{\text{FLS}}, d^{\text{DRT}}, d^{\text{FLS}} \in \mathbb{N}^{+}$$