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Master Thesis Report

Operations Research and Quantitative Logistics

Optimal maintenance policies for wind turbines under time-varying costs

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Abstract

In this thesis we introduce a new modelling approach for maintenance optimisation under time-varying costs. We extend the standard age replacement policy (ARP), block replacement policy (BRP) and modified block replacement policy (MBRP) to be able to include time-varying costs. We find that, via a discretisation of time, the optimal ARP can be found using a linear programming (LP) formulation. The optimal BRP and MBRP can be found using a mixed integer programming (MIP) formulation. For the BRP we introduce a sequential optimisation algorithm, a genetic algorithm and a memetic algorithm to decrease computation time for multi-component problems. We compare the performance of the heuristics to the performance of the MIP formulation. We apply the heuristics to a wind farm with 10 wind turbines with a turbine capacity of 9.5 MW and show that we can make significant cost savings using our extended policies.

Keywords: Offshore wind turbine maintenance, age replacement policy, block replacement policy, genetic and memetic algorithm

Preface

This thesis has been submitted as the final requirement for the Master of Science degree in Operations Research and Quantitative Logistics at the Erasmus University Rotterdam. First of all, I would like to thank Rommert Dekker for his ideas and guidance throughout the project. We have had interesting discussions on topics in operations research, renewable energy and other matters. I would like to thank my friends and Pitt for their support and inspiration. Special thanks to my family for their support, love and wise lessons. At last, thanks to Sena Eruguz for her time to read through and value this report.

Contents

1 Introduction

The renewable energy sector is a fast growing sector due to the energy crisis. As a result of this crisis there is a global tendency to move towards more sustainable ways to generate electricity. One of the most efficient ways to generate sustainable energy is by the use of wind turbines. By 2020 the Netherlands wants to be able to generate 6 GW of power in offshore wind farms, which is approximately 6% of the demand of electricity. As a result large part of the energy sector will be governed by offshore wind farms. Nowadays the operation and maintenance costs make up around 25-30% of the total life cycle costs for an offshore wind farm Röckmann et al. (2017). Since larger wind turbines are developed every year to be able to generate more electricity, the operations and maintenance costs will also increase. It is therefore important that the maintenance operations are optimised to keep the costs of wind energy at a low level.

The challenge in general maintenance optimisation problems is to find a cost-efficient balance between corrective and preventive maintenance. When components break down due to deterioration, corrective maintenance is needed, which is typically more expensive. We want to prevent that this maintenance is needed too often and we thus schedule enough moments of preventive maintenance. Scheduling preventive maintenance too frequently can also increase the costs. Finding this optimal policy is a common problem in maintenance optimisation, where block-based and age-based maintenance are examples of maintenance policies that are considered. We assume that deterioration is not visible, but that we do know how the failures (or break-down) are distributed via the lifetime distribution. In the literature it is typically assumed that expected maintenance costs are constant in time. We abandon this assumption in this thesis and generalise block-based and age-based maintenance policies for time-varying maintenance costs. The cost of shutting down wind turbines for maintenance namely depends on the strength of the wind, which itself depends on the time of the year. When there is a lot of wind during a maintenance action, the wind turbine misses a lot of potential income and this causes the maintenance to be more expensive. In summer in the Netherlands, maintenance costs are lower due to lower wind yields, while in winter there is more wind and maintenance is relatively expensive.

1.1 Goals of the project

The problem that we focus on is finding a long-run cost optimal age-based and block-based maintenance policy. We wish to generalise these policies for problems where maintenance costs follow a seasonal pattern, such as with wind turbines. This leaves us with the following main research questions:

- RQ-1 How can we generalise age replacement, block replacement and modified block replacement policies for time-varying costs in a single component and multi-component setting?
- RQ-2 How can we find optimal policies using these generalisations and what is the computation time?
- RQ-3 What heuristic approaches can be used to save computation time and find close to optimal policies for settings with many components?
- RQ-4 What are the cost savings by considering different cost fluctuations compared to assuming constant costs for different failure distributions and cost parameters?
- RQ-5 What are the cost saving if we apply these heuristics to an offshore wind farm setting?

1.2 Thesis outline

We give an overview of research that has been done on this topic and related topics in Section 2. In Section 3.1 we describe our modelling choices and assumptions that we make. In Section 3.2 we asses the maintenance costs in our model and the lifetime distribution. We use a Weibull distribution for the lifetime and use different cost functions for the costs of maintenance over the year. These are related to the wind speed distribution, which is assumed to be season dependent and historic wind speed data is used to predict this distribution. In Section 3.3 we elaborate on the generalisations of maintenance policies for single components and answer RQ-1 and RQ-2 for single components. Via a discretisation of time, we show how a Markov decision process can be used to describe the statistical behaviour of the components. Given this model we can optimise the costs using a linear programming (LP) formulation. For the block-based and modified-block based maintenance extra constraints and variables can be added to the LP formulation to obtain a mixed integer programming (MIP) formulation. In Section 3.4 we use a similar approach for multi-component systems for which we can also obtain an LP for the age-based maintenance and MIPs for the (modified) block-based maintenance and answer RQ-1 and RQ-2 for the multi-component setting. Solving the MIP is very time consuming for three or more components. To accommodate this problem we introduce (meta)heuristics that can find low cost solutions faster in Section 3.5 and answer RQ-3. We consider sequential optimisation, a genetic and memetic algorithm. In Sections 4.1 and 4.2 we generate results for different sets of parameters for the single component and multi-component setting and answer RQ-4. In Section 4.3 we show the results of the heuristics in an offshore wind turbine park example and answer RQ-5.

2 Literature review

Maintenance models are widely discussed in the literature and applications are found in many industries. A new advancing industry in which maintenance is very expensive is the wind energy industry. The operations and maintenance costs make up 30% percent of the total life-cycle costs for the newest offshore wind parks Blanco (2009). An equal percentage therefore contributes to the price of electricity and it is thus important to keep these costs at a low level. Finding a cost effective balance between preventive maintenance and corrective maintenance for wind turbines is challenging. In the literature small onshore turbines are discussed as well as the relatively new offshore turbines. The tallest offshore wind turbines are nowadays well over 200 m high and almost reach 10 MW at full power. At this power output they can deliver for over 30000 households in the Netherlands at average demand in 2017, see Centraal Bureau voor de Statistiek (2019). In the literature that deals with this problem many different assumptions are made, that lead to different optimisation approaches.

2.1 Challenges in offshore wind park maintenance

It is a big challenge to optimise all operations that are needed to build and maintain a wind park to be able to keep the electricity price low. Shafiee (2015) give an overview of the logistic challenges that are encountered in optimising all aspects of a wind park and present research that has been performed on many different topics. Design, infrastructure, transportation, operations and maintenance all need to be optimised to be able to compete with alternative ways to generate electricity.

Lumbreras and Ramos (2013) give an overview on the research that has been performed on the optimisation of the design of a wind park. They discuss which optimisation models can be used to decide at which sites it is economical to build a wind park and how large the wind park should be. Besnard et al. (2013) show how we can decide how many maintenance teams are available at all times and the number of vessels that should be available for transportation of the maintenance team. Besnard et al. (2013) and Seyr and Muskulus (2019) give an overview of the different maintenance strategies that are used for the maintenance of offshore wind park. In this thesis we extend some of these maintenance strategies to be able to include time-varying costs.

Many research that is performed on maintenance optimisation of wind turbine component is based on the assumption that sensor data of components is available. In these applications the deterioration of components can be measured. Wind turbines typically have sensors that measure the deterioration, see Ciang et al. (2008). They discuss the different types of measurement data. If this data of the deterioration of the components is available, we are able to predict more accurately when components will break down. Basing the maintenance decision on deterioration data is called condition-based maintenance (CBM). Maillart (2006), for example, show how a Markov process with multiple deterioration states can be used to describe the deterioration of a single component system. A method to find an optimal solution and a heuristic are presented. An application in the wind energy industry is given by Besnard and Bertling (2010). They derive different condition-based maintenance policies for the blades of wind turbines. Again a Markov process is used to describe the deterioration of the system.

For our problem we wish to use a model that can include multiple components, since we focus on the maintenance of the wind turbine or a wind park as a whole. CBM can also be applied in a multi-component setting. Byon and Ding (2010) show how we can account for failures of multiple components by adding states to the state space of the Markov process. They show that this Markov process with more states can be used to describe the deterioration of all parts in the system. Dynamic programming is used to find the minimum cost maintenance policy. Tian et al. (2011) use an artificial neural network to determine the deterioration of each component and find the optimal deterioration level at which maintenance should be performed. They apply their model in a wind park setting of 5 turbines, where four components per turbine are considered; gearbox, generator, rotor and main bearing. The costs in these maintenance models are typically computed using the renewal reward theorem, see Tijms (2003).

These methods can be applied to our problem, but all require that we have measurements on the deterioration of the system. These measurements are however not always available and if these measurements are, there is still a lot of uncertainty in the deterioration level of the system. We therefore focus on models that use a deterioration process of which we know the statistical behaviour, but where failures occur suddenly and we can thus not measure the deterioration using sensors. Not being able to measure the deterioration of the system significantly changes the problem. Well-known maintenance strategies associated with this problem are the (modified) block-based maintenance policies (or (modified) block replacement policies) and the age-based maintenance policies (or age replacement policies). Both will be explained in the next section.

2.2 Block-based and age-based maintenance

In the literature, block replacement policies (BRP), modified block replacement policies (MBRP) and age replacement policies (ARP) are commonly used under the assumption that we only know the lifetime distribution of components and the costs of maintenance/repair/replacement. In these models the probability of failure only depends on the age of the components, since we do not have any information on the deterioration. The costs of preventive and corrective maintenance are known (in expectation) and independent of time. We generalise this assumption to time-varying costs in this thesis, which is done in Section 3.

Block-based maintenance is a maintenance strategy in which preventive maintenance is planned beforehand and we know in advance when it is scheduled. The maintenance operations can therefore be planned in advance and are scheduled periodically, which is convenient for the maintenance crew. Under the block-based model preventive maintenance is done after every fixed time-intervals of length T, so at kT for $k \in \mathbb{N}$. Barlow and Proschan (1996) show how this T can be optimised, using the renewal reward theorem. Block maintenance is convenient because the maintenance team knows when it needs to perform preventive maintenance beforehand. The downside is that the maintenance is also performed when the component has just been maintained correctively. In this scenario it could have been better not to maintain and wait for the next opportunity. Berg and Epstein (1976) come up with a modified block-based maintenance policy to account for this problem. Maintenance can still be done at intervals of time T, but if the components are still younger than t for some $0 < t \leq T$ they will not be maintained and costs will be saved. They also show the improvement of the modified block-based maintenance in a numerical comparison.

The probability of failure only depends on the age, so it makes sense to let our decision of doing maintenance depend on the age as well. In an age-based maintenance policy this is done and units are maintained every time they have reached a certain age t. If we know the failure rate and costs are constant, we can use the renewal-reward theorem to come up with the optimal maintenance age t, see Barlow and Proschan (1996). Under this policy, every time a failure occurs we thus have to change the preventive maintenance schedule, which might be inconvenient for the maintenance crew. The optimal age-based maintenance policy is the global optimal policy. It is therefore at least as good as the block-based maintenance policy, but we ignore costs associated to rescheduling the maintenance schedule for the age-based maintenance.

In the literature a lot of research is performed on single-component systems. The aforementioned strategies are applied in many fields where maintenance is required one of them being offshore wind turbines. The analysis for single component systems is straightforward, under constant costs, but difficulties arise when we move towards multi-component systems. We can then do the maintenance individually, on the whole system, or something in between. Hameed and Vatn (2012) discuss how certain maintenance activities can be grouped in the offshore maintenance optimisation problem for wind turbines. Similar activities are grouped together and performed at the same time, such that an economic advantage is obtained via a reduction of set-up costs. Shafiee and Finkelstein (2015) investigate age-based group maintenance policies for a system which consists of multiple components and apply this to wind turbine maintenance. All these models however assume that costs of maintenance are constant over the year, which is a simplification of the problem. Seasonality in wind patterns has a significant influence on the corrective and preventive maintenance costs, which are thus time-varying. In the next section we describe the model that we use to solve the problem for time-varying costs first for single component systems and then for multiple component systems.

3 Methodology and analysis

In this section we describe our modelling approach and introduce methods that are used to find policies. In Section 3.1 we give a description of the important aspects of our maintenance model. In Section 3.2 we show how we can determine the lifetime distribution parameters of the components and the costs of the maintenance actions. We introduce a linear programming formulation in Section 3.3.1 for the optimal age-based maintenance strategy for a single component setting. This is extended to a mixed integer programming formulation for block-based and modified block-based maintenance strategies in Sections 3.3.2 and 3.3.3. These three formulations are new and can be used to find the optimal age-based, block-based and modified block-based maintenance policies. In section 3.4 we extend the methods for a single component system to a multi-component system. Methods to solve these problems are known, but can be computationally very intensive, especially when there are integer variables that need to be optimised. We therefore introduce heuristic approaches for the block-based maintenance policies to decrease computation time in Section 3.5. In this section we show how we can apply sequential optimisation, a genetic and a mememtic algorithm.

3.1 Maintenance model description

In this section we wish to come up with a policy that depends on the failures in the system. For a single component system with constant preventive and corrective maintenance costs it is proven that the age-based maintenance policy is cost-optimal in the long-run, see Özekici (1985). This single component system can be used if there is one component that dominates the maintenance costs. For the wind turbine this is not the case, but the single component model is a useful building step towards multi-component models.

We will use the four features of Dekker (1996) to describe the model that we will use. In short this gives the following.

1. The technical system that we will consider is a wind turbine park, which can be seen as a multi-component system. The turbines operates continuously and components need to be repaired immediately after a failure, since the turbine stops operating when one component is broken down and income is missed.

2. The deterioration of each component is assumed to be invisible. The lifetimes of these components are distributed according to a Weibull distribution with shape parameter $\alpha > 0$, and scale parameter $\beta > 1$. When components fail the wind turbine stops operating, inducing a downtime of the wind turbine. The failed components must be maintained immediately.

3. The costs for corrective and preventive maintenance are time-varying and stochastic, but the lifetime distribution is known. Corrective maintenance is performed whenever a component fails. We use an age-based, block-based and modified block-based maintenance policy with an extension. The age of maintenance can vary over the year, as a result of the costs varying over the year.

4. We wish to minimise the long-run average costs. The maintenance policies that we consider can be solved using a linear and mixed integer programming formulation. This is described in detail in Section 3.3.

Note that the model that we will introduce later is able to include other distributions as long as failure rate increases over time. The models that we use in Sections 3.3 and 3.4 use Markov decision processes to describe the state of the components. Puterman (1990) show that for Markov decision processes we only need to use the expected costs in each state if we wish to minimise the long-run expected costs. We therefore only use the expected preventive and corrective maintenance costs in our model.

3.2 Model parameters

Important parameters that influence the policy are the distribution parameters that describe the lifetimes of the components. When components fail more often on average, preventive maintenance should be done more frequently. Section 3.2.1 describes the lifetime distribution and the corresponding parameters. Other important parameters are the costs of preventive and corrective maintenance over the year. When costs of corrective maintenance is relatively large, preventive maintenance should be done more frequently. Section 3.2.2 describes how we can determine the functions that describe the time-varying expected costs.

3.2.1 Lifetime distribution

The lifetime of components are often described by a continuous Weibull distribution. This distribution is able to capture an increasing failure rate and the random variables only take positive values. Suppose that wind turbine components fail according to a continuous Weibull distribution with parameters α in years and β. Then the failure time T^c is distributed according to the following distribution function.

$$F^c(T^c) = \mathbb{P}(T^c \leq x) = 1 - \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}. \tag{1}$$

Since we discretise time in our model, we need to use a discrete version of the Weibull distribution, which was first described by Nakagawa and Osaki (1975). This discretised version takes values on $\{n \cdot \Delta T : \cdot n \in \mathbb{N}\}$ and the probability mass of the distribution is shifted to the right. Let T be distributed according to a discrete Weibull distribution with parameters $\alpha > 0$ and $\beta > 0$. Then we write $T \sim \text{Weibull}(\alpha, \beta)$ and equations (2a) and (2b) give the probability mass and distribution function for $x \in \mathbb{N}_{>0}$.

$$f(x) = \mathbb{P}(T = x) = \exp\left\{-\left(\frac{x-1}{\alpha}\right)^{\beta}\right\} - \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \tag{2a}$$

$$F(x) = \mathbb{P}(T \leq x) = 1 - \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \tag{2b}$$

where x and α have the same units of time. Using the probability mass function we can compute the mean failure time $\mu = \mathbb{E}(T)$ as follows.

$$\mathbb{E}(T) = \sum_{x=1}^{\infty} x\mathbb{P}(T = x), \tag{3a}$$

$$= \sum_{x=1}^{\infty} x \cdot \left[\exp\left\{-\left(\frac{x-1}{\alpha}\right)^{\beta}\right\} - \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}\right], \tag{3b}$$

$$= \sum_{x=0}^{\infty} \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \tag{3c}$$

$$\approx \sum_{x=0}^{n} \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\} \quad \text{for } n \text{ large enough.} \tag{3d}$$

Similarly we can compute the second moment.

$$\mathbb{E}\left(T^2\right) = \sum_{x=0}^{\infty}(2x+1)\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \tag{4a}$$

$$\approx \sum_{x=0}^{n}(2x+1)\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \quad \text{for } n \text{ large enough.} \tag{4b}$$

The probability of a failure given that the component has age $x-1$ can also be computed from equation (2) and is given as follows:

$$p_x = \frac{\mathbb{P}(X = x)}{\mathbb{P}(X \geq x)}, \tag{5a}$$

$$= \frac{\mathbb{P}(X = x)}{1 - \mathbb{P}(X \leq x-1)}, \tag{5b}$$

$$= \frac{\exp\left\{-\left(\frac{x-1}{\alpha}\right)^{\beta}\right\} - \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}}{\exp\left\{-\left(\frac{x-1}{\alpha}\right)^{\beta}\right\}}, \tag{5c}$$

$$= 1 - \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta} + \left(\frac{x-1}{\alpha}\right)^{\beta}\right\}. \tag{5d}$$

This p_x gives the probability of a failure before the component reaches the age of x, which is a strictly increasing function for $\beta > 1$, constant for $\beta = 1$ and strictly decreasing for $\beta < 1$. The discrete Weibull distribution with $\alpha > 0$, $\beta > 1$ will be used as the lifetime distribution throughout the rest of this thesis. For these distribution we have that

$$p_{\infty} = \lim_{x \to \infty} p_x = 1, \tag{6}$$

which is an important result that we will use later.

The theorems and modelling approach can be done for other distribution types as well as long as the failure rate increases and the distribution is 1-arithmetic, which is defined below in Definition 1. The proofs that we give will be done for arbitrary 1-arithmetic distributions with increasing failure rate, but in the results we consider the Weibull distribution only.

Definition 1 (Arithmetic)**.** *A non-negative random variable X (and its distribution) are called d-arithmetic for $d > 0$, if d is the maximum number for which $P(X \in d \cdot \mathbb{Z}) = 1$. If X is not d-arithmetic for any $d > 0$, it is called non-arithmetic.*

Note that discrete distributions are always d-arithmetic for some d.

3.2.2 Time-varying expected maintenance costs

The costs of the maintenance policies depend on the preventive and corrective maintenance costs that we allow to be stochastic and time-varying.

For both preventive and corrective maintenance we can split the costs in three parts.

1. Set-up costs, consisting of transportation costs, scaffolding costs, and personnel costs.
2. Costs of the maintenance action itself, consisting of building material costs, costs of components and personnel costs.
3. Missed income due to the downtime of the wind turbine.

The set-up costs and costs of the maintenance action itself are assumed to be constant over time. The expected missed income due to maintenance depends on the time of the year. It can be computed using the expected downtime of the components, the power output curve and historical wind speed data. For each maintenance action we assume that we know the downtime of the component, which is assumed to be constant in expectation. The expected missed income equals the expected power output integrated over the downtime. The expected power output can be computed using historical data. For each time of the year we can compute the average power output, using the power output function and the density function of the wind speed. The wind speed at the rotor is not equal to the wind speed at sea level for which we typically have data available. We thus also need to upscale the measured wind speeds.

Due to the seasonality in the wind patterns, we abandon the constant cost assumption. We assume that there is a seasonality in the costs due to the seasonality in the wind. We will come up with policies in which the maintenance decision depends on the time of the year. Intuitively one might expect that we wish to maintain more when there is not a lot of wind. The preventive maintenance is namely cheaper when there is less wind, due to the lower lost revenue, making preventive maintenance more attractive. On the other hand, the corrective maintenance will also be cheaper in this part of the year. As a result, a failure is less expensive, which makes preventive maintenance less attractive.

Wind speeds around the coast of the Netherlands can vary a lot on a daily basis, but also over the year. The monthly mean wind speeds can differ more than 20% from the yearly mean, see Koninklijk Nederlands Meteorologisch Instituut (2018). We can find historic daily wind speeds for different places in the Netherlands at heights of 10 m. In for example IJmuiden, a coastal town in the Netherlands, in 2017 the average wind speed was 7.4 m/s. The May average was lowest with 5.9 m/s (20.3% lower) and in December the average wind was highest with 9.2 m/s (24.3% higher). The wind speed at the height of the rotor, 138 m, is larger than the wind speed at the measured height of 10 m. Tennekes (1973) describes the logarithmic wind profile as follows.

$$u(h) = \frac{u_*}{\kappa}(\log h - \log h_0), \tag{7}$$

where the wind speed u is a function of the height h. κ is the Karman constant and u_* is the so-called friction velocity, which is also constant. h_0 is the surface roughness length, which is 0.0001 for sea water, see Kubik et al. (2011). If we know the wind velocity at some height h_1 we can compute it at h_2 as follows.

$$\frac{u(h_2)}{u(h_1)} = \frac{\log h_2 - \log h_0}{\log h_1 - \log h_0}. \tag{8}$$

Using $h_0 = 0.0001$ m, $h_1 = 10$ m and $h_2 = 138$ m we obtain $u(138) = 1.181u(10)$. This means that the average wind speed in IJmuiden at the hub of the wind turbine is 8.7 m/s instead of 7.4 m/s, the percentage fluctuations are still the same.

The power output and thus the income is non-linear in the wind speed, see Figure 1. This figure gives the approximated power output for a V164 9.5 MW turbine, the newest and most powerful Vestas wind turbine. The data is obtained from Commissie voor de milieueffectrapportage (2016), which contains the power output data for a V164 8.0 MW turbine. The cut-in wind speed for the 9.5 MW turbine is at 3.5 m/s instead of at 4 m/s for the 8.0 MW turbine and the maximum power output is obtained for 14m/s instead of 13 m/s for the 8.0 MW turbine. The power output for the 9.5 MW turbine is approximated by linearly scaling the given data for the 8.0 MW turbine. That is, we scale the x-values from $[4, 13]$ to $[3.5, 14]$ and the y-values from $[0, 8]$ to $[0, 9.5]$.

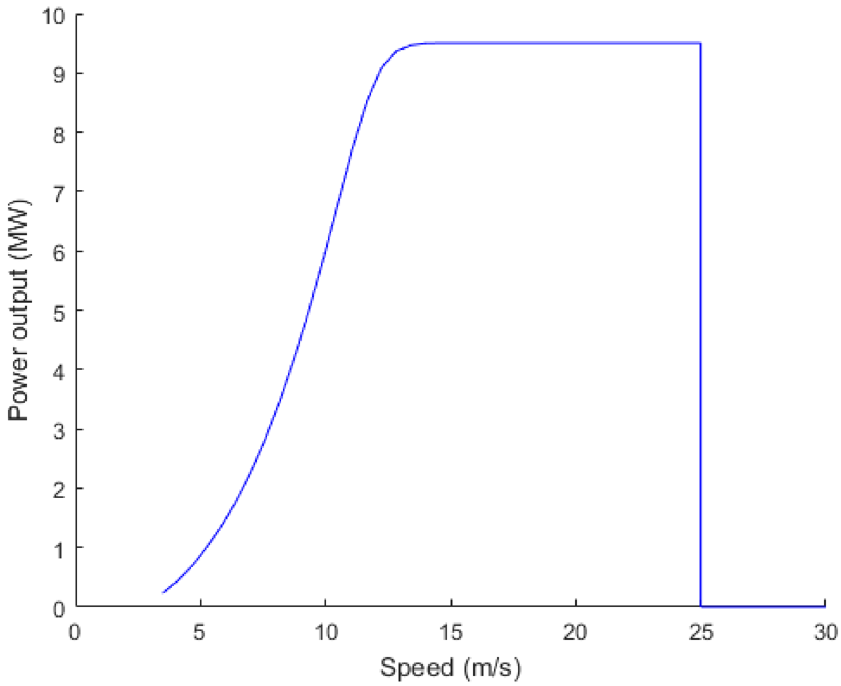


Figure 1: This figure shows the approximated power output of a Vestas V164-9.5 MW wind turbine as a function of the wind speed.

This turbine will in the future be able to reach a power output of 10 MW, which will be available by 2021, see Wind Business Intelligence (2018).

For relatively low wind speeds the power output increases faster and at larger wind speeds (>14 m/s) this power output stagnates. When the wind is larger than 25 m/s, which is Beaufort 10, the wind turbine is shut down for safety purposes.

For 5.9 m/s at 10 m (i.e. 7.0 m/s at 138 m), the average wind speed over the worst month May, the power output is 2.23 MW. For 9.2 m/s at 10 m (i.e. 10.9 m/s at 138 m), the average wind speed over December, the power output is 7.37 MW. The expected missed income at these power outputs is €3205.- and €10614.- per day, under an electricity price of €0.06/kWh. This is the guaranteed price in a 2018 German project, see Ten Brinck (2018). We thus see that the constant cost assumption is not valid, especially if these opportunity costs are a large proportion of the total maintenance costs.

Note that the average power output cannot be computed from the mean wind speed over the month, since the relation between power output and wind speed is non-linear. We use the density function of the wind as follows.

$$\mathbb{E}(P) = \int_{3.5}^{25} P(v) \mathrm{d}\mathbb{P}(V = v), \tag{9a}$$

$$= \int_{3.5}^{25} P(v) f_V(v) \mathrm{d}v, \tag{9b}$$

where V is the stochastic wind speed, f_V is the density function of the wind speed and $P(v)$ is the power output at wind speed v. The data from Koninklijk Nederlands Meteorologisch Instituut (2018) will be used to estimate the power output on. This contains the daily average wind speeds in IJmuiden from 1971-2018. For every day we approximate the power output by

$P(\overline{v})$ in which $\overline{v}$ is the mean wind speed over the day. Taking the average over all years for each day of the year approximates the average historic power output for each day of the year. This historic power output will be used as an estimate for the expected future power output.

3.2.3 A note on the cost parameters

In our planning problem we assume that we know the distribution of the wind for each day far ahead. We can simply use historical wind speeds to estimate the parameters of the wind distribution on. Subsequently, we can compute the expected downtime costs for maintenance in each time period of the year. Suppose we do this on a weekly basis. We do then, however, not take short-term weather forecasts into account and cannot choose the day in the week at which we maintain. In practise, we could possibly plan maintenance during a certain week in our long-term planning and specify the exact day later. For the decision of the exact day we can take short-term weather forecasts into account and choose a day at which low wind is expected. By choosing the day of the week smartly we can save expected downtime costs. We do however demand flexibility from the maintenance team, which possibly leads to extra costs.

Suppose it is Sunday evening, just before a week during which maintenance is planned and suppose the maintenance team is ready to perform the maintenance any day of the week. We could decide to maintain immediately on Monday, but if there is a lot of wind on Monday it might be smarter to postpone the maintenance to Tuesday. On Tuesday the same decision can be made. It could be that we keep postponing the decision until we have arrived at Sunday and we have to maintain. Given that we have some weather forecasts for each day of the week this short-term optimisation can be solved by for example dynamic programming.

There are a few underlying assumptions that are tricky. First in this approach it is assumed that the maintenance team is ready for the whole week and that there are no costs associated to letting them wait for a few days. We can simply decide to maintain the evening before maintenance. Obviously letting a maintenance crew wait normally costs money. We also assume that building materials and components can be stored and that scaffolding and transportation can be arranged the day before, but this might all lead to extra costs.

Now suppose that a delay of one day cost c_d extra. Then we can first plan the whole maintenance action on Monday. If the downtime costs for Tuesday are more than c_d lower than for Monday we should postpone the maintenance. If the difference is less than c_d, we might want to do the maintenance, but this depends on the forecasts of Wednesday as well. To be able to model this, we should make assumptions on how weather forecast evolve in time. We should also take into account that transportation to the offshore turbine is only allowed if the wind speed is not above a certain value. This problem becomes even more complex if we take into account that there is correlation between the missed income over the week. If the wind is larger than expected on Tuesday, it might be larger than expected on Wednesday as well.

Solving this problem gives an optimal strategy for each week and correspondingly an updated expected cost for each week of the year. These updated expected costs will be lower than the expected downtime costs for this week. These updated expected costs can be plugged in our long-term planning model to find the optimal maintenance schedule.

All in all, this is a rather complex problem itself that might save costs with respect to simply maintaining the components at Monday. If c_d is large with respect to the downtime costs, we will never postpone the maintenance and this method gives no advantage, but if c_d is small we might have large cost savings. Also if the maintenance takes a couple of days to complete the advantage of this method is smaller, since it is more difficult to predict the weather a couple of days ahead. Using this method will give costs of maintenance for each week of the year, that might be lower than the original costs, but there will still be seasonality in the costs. Therefore the modelling approach that we will introduce in the next section for the long-term maintenance planning stays the same. Throughout the rest of the paper we use the original expected costs as discussed in Section 3.2.2. The policies that we compute give a baseline of costs on which we might be able to improve if we do take short-term maintenance planning into account. The optimisation of this short-term planning is an interesting topic for future research.

3.3 Maintenance policies for a single component systems

Wind turbines consist of many components. Still, single component models can be applicable if there is only one component that fails occasionally or if there is one component that dominates the maintenance costs. This is generally not the case, but these models can be used as a building block towards multi-component models. In Section 3.3.1 we introduce an approach by which we can solve these single component maintenance problems for time-varying costs. We find the theoretical optimal policies which are age-based maintenance policies. In Section 3.3.2 we make a slight modification to the approach, such that we can find block-based maintenance policies. At last in Section 3.3.3 we discuss modified block-based maintenance policies.

3.3.1 Age-based maintenance

In this section we extend the theory for age-based maintenance for time-varying costs. We propose a maintenance policy in which the maintenance decision is based on the age of the component and the time of the year. To simplify the problem slightly, we will discretise time to contain N decision epochs during a year. If for example $N = 52$ we make the decisions on a weekly basis and for $N = 12$ we make the decision on a monthly basis.

Under constant costs it is optimal to maintain for a constant critical maintenance age (i.e. the age from which we will perform maintenance). Always maintaining from a certain critical age gives a constant age-based maintenance policy or age repair/replacement policy (ARP). An example is given in Example 1.

Example 1 (Standard ARP)**.** *Suppose we have a component with a discrete Weibull lifetime distribution, with $\alpha = 12$ months and $\beta = 2$ (i.e. $T \sim$ Weibull$(12, 2)$ with T the lifetime). Let the costs of preventive and corrective maintenance be given by $c_p = 10$ and $c_f = 50$. Nakagawa and Osaki (1977) use the renewal reward theorem to compute the long-run costs as a function of the critical age t. The yearly costs are:*

$$C(t) = \frac{c_p F(t) + c_f(1 - F(t))}{\sum_{s=0}^{t-1}(1 - F(s))}, \\ = \frac{10 + 40 \exp\left(-\left(\frac{t}{12}\right)^2\right)}{\sum_{s=0}^{t-1} \exp\left(-\left(\frac{s}{12}\right)^2\right)},$$

where $F(t) = \mathbb{P}(T \leq t)$ is the distribution function of T. An alternative way to compute the costs is given in Appendix D.1.

In Figure 2 we plot the yearly costs as a function of the critical maintenance age using the above formula.

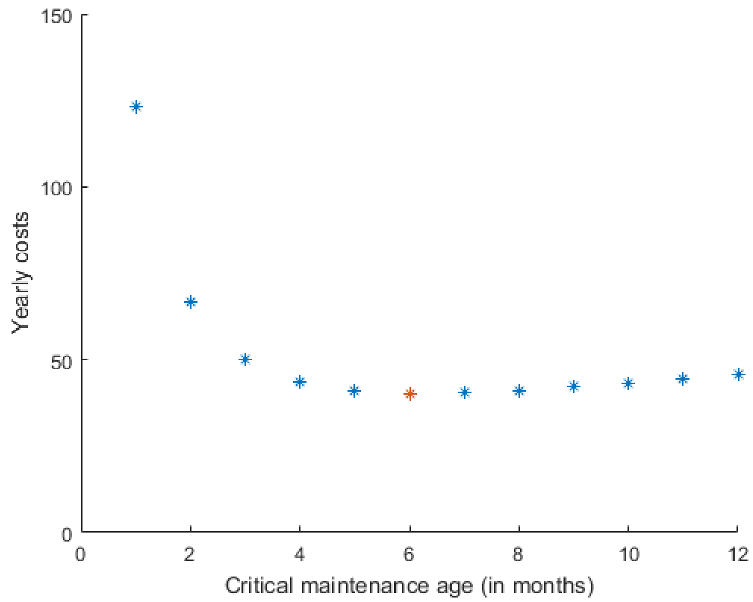


Figure 2: This figure shows the yearly maintenance costs as function of critical maintenance age where $c_f = 5c_p$ constant and $T \sim \text{Weibull}(12, 2)$.

We see that the minimum of 40.098 is obtained for $t^* = 6$ months and so preventive maintenance is performed if the component reaches this age in an optimal policy. For maintenance age $t = 5$ and $t = 7$ we obtain costs 40.938 and 40.260 and these are 2.09% and 0.40% more expensive.

Note that we can also compute the costs from the renewal reward theorem if no maintenance would be performed. This gives:

$$C(\infty) = \frac{c_f}{\mathbb{E}(T)} = 53.885.$$

When the costs are time-varying, the optimal policy is not necessarily a policy with a constant critical maintenance age over the year. Intuitively, it might make sense to have a lower critical maintenance age in periods where preventive maintenance is relatively cheap. As a result, the optimal critical maintenance age is then a function of the time in the year. Since we discretise time, we have the following for the optimal critical maintenance age $t^* : \{1, 2, \ldots, N\} \mapsto \mathbb{N}$. In Definition 2 we give the formal definition of this new ARP.

Definition 2 (ARP)**.** *An ARP is a policy in which the preventive maintenance decision depends on the time of the year and the age of the component only. When preventive maintenance is performed for age t during a period, so it is for ages $s \geq t$.*

Throughout the rest of the thesis we use ARP to denote age replacement/repair/maintenance policies in which the critical age can vary over the year. The ARP in which the critical age is constant is referred to as the standard ARP, which is common in the literature and used for problems for which maintenance costs are constant.

The expected costs of preventive and corrective maintenance are a function of the time of the year only and given by $c_p : \{1, 2, \ldots, N\} \mapsto \mathbb{R}$ and $c_f : \{1, 2, \ldots, N\} \mapsto \mathbb{R}$. We are interested in minimising the long-run maintenance costs and over the long-run the costs converge to the expectation. Throughout the rest of this report we denote the means, minimum and maximum over the year of these costs by

$$\bar{c}_p := \sum_{i_0=1}^{N} \frac{c_p(i_0)}{N}, \tag{10a}$$

$$\bar{c}_f := \sum_{i_0=1}^{N} \frac{c_f(i_0)}{N}, \tag{10b}$$

$$\min\{c_p\} := \min_{i_0 \in \mathcal{I}_0} \{c_p(i_0)\}, \tag{10c}$$

$$\min\{c_f\} := \min_{i_0 \in \mathcal{I}_0} \{c_f(i_0)\}, \tag{10d}$$

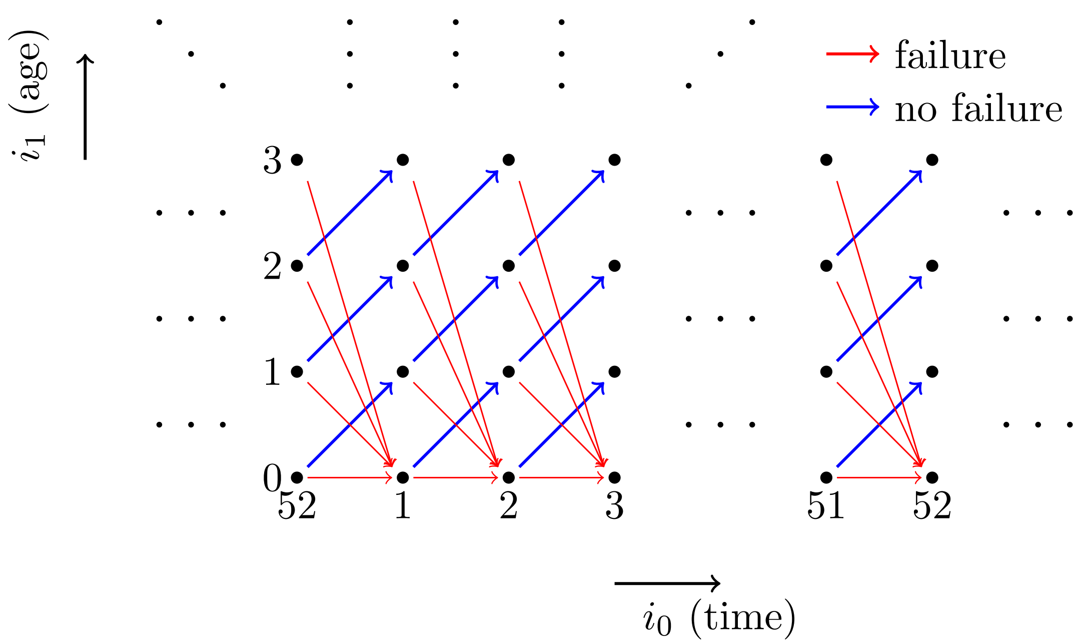
$$\max\{c_p\} := \max_{i_0 \in \mathcal{I}_0} \{c_p(i_0)\}, \tag{10e}$$

$$\max\{c_f\} := \max_{i_0 \in \mathcal{I}_0} \{c_f(i_0)\}. \tag{10f}$$

We describe the state, an ordered pair of the component age and time of the year, by a Markov decision process. We can decide to maintain or not when the component is not broken, which influences the state of the wind turbine. We can only make this decision at the beginning of a time interval for example the beginning of the month or week. The set of decision epochs is then given by $\mathbb{N}$, every decision epoch we can decide to maintain or not. The state space is given by $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1$. $\mathcal{I}_0 = \{1, 2, \ldots, N\}$ represents the set of periods in a year (this can for example represent months if $N = 12$ or weeks if $N = 52$). $\mathcal{I}_1 = \mathbb{N}$ represents the set of possible ages of the component, where we represent being broken by 0. The actions that we can take is to either not do maintenance $a = 0$ or to do maintenance $a = 1$. We use this $a = 1$ for both preventive and corrective maintenance, thus $\mathcal{A} = \{0, 1\}$ is the set of possible actions. When the component is broken (i.e. $i_1 = 0$) at the end of a period we have to maintain correctively (i.e. take action $a = 1$) at the beginning of the next period. If the component is not broken the choice is free and will depend on the state (i.e. time of the year and age of the component). In Figure 3 we show how each state can be reached from other states, when no preventive maintenance is done.

Figure 3: State Space

This figure shows part of the state space of the Markov process, where no preventive maintenance is done. $N = 52$ meaning that time is discretised to weeks.



This figure illustrates the two possible directions in which the Markov process evolves at every week and age. Either the component fails and the process moves to the state with $i_1 = 0$ weeks or there is no failure and the component age will increase by one week. The week of the year always simply increases with one, except after week 52 when we move back to week 1.

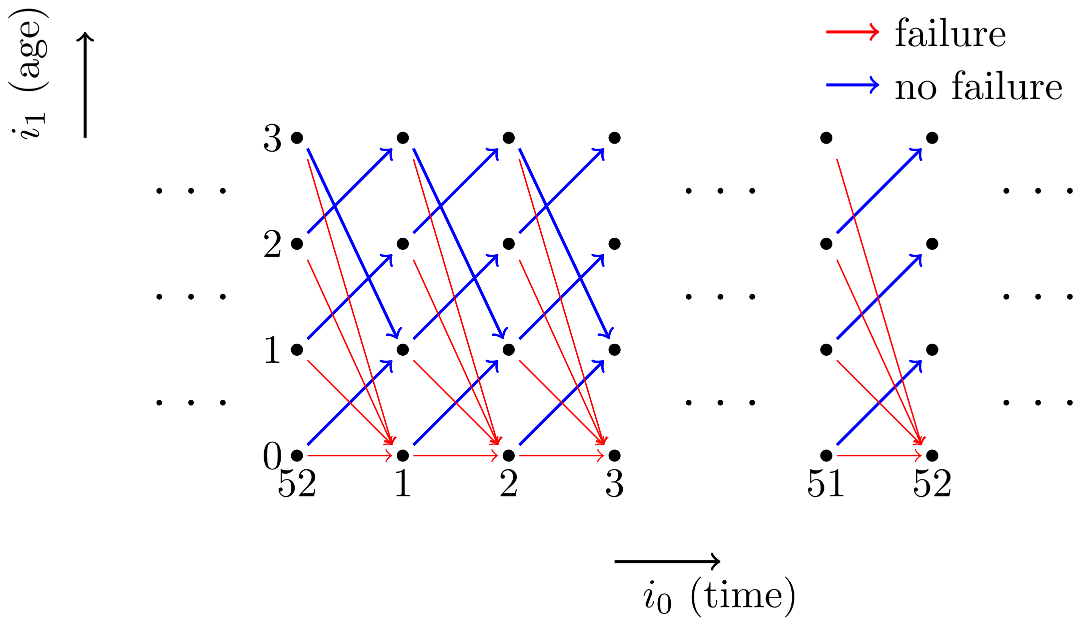
When we decide to do maintenance for certain nodes, the Markov process can still evolve in two directions for these nodes, independent of whether this was preventive or corrective maintenance. After maintenance, which is assumed to take negligible time, the component can namely fail before the end of the week. In case of a failure after the maintenance action the process moves to the state with $i_1 = 0$ weeks and has to be maintained the next week again. Otherwise, which is more likely to happen, the component moves to the state $i_1 = 1$ week.

Note that in a real wind turbine setting maintenance can take several days. If maintenance for example takes a full week we can set the failure probability to be zero for the first week. In this way the component always survives the first week after maintenance, since it was maintained the full week.

Suppose that we do maintenance every week when the component reaches the age of $i_1 = 3$ weeks. Figure 4 shows what the state space would look like in this scenario.

Figure 4: State Space

This figure shows part of the state space of the Markov process, where no preventive maintenance is done.



We see that we can never reach ages above $i_1 = 3$ weeks as a result of the maintenance policy. The resulting state space is thus finite.

Let us denote the set of possible policies by $\mathcal{R}$, then for $R \in \mathcal{R}$ we have $R : \mathcal{I} \mapsto \mathcal{A}$. For each $R \in \mathcal{R}$ we can compute the equilibrium probabilities and associated costs. Derman (1970) show that an optimal stationary policy exists when the state space and the number of decisions are both finite. Stationary policies are policies that for each $i \in \mathcal{I}$ take the same decision every time i is reached, this decision is independent of the past and deterministic. Let us denote the set of stationary policies be denoted by $\mathcal{R}_s \subset \mathcal{R}$. Finding this optimal stationary policy can be done in several ways. Hordijk and Kallenberg (1979) show that we can rewrite this problem to a linear programming formulation if the state space is finite. We could then also use policy iteration, see Howard (1960), or value iteration, see Bertsekas (1976), to come up with the optimal or ϵ-optimal policy, respectively. The linear programming approach is fast, easy to implement and guarantees optimality. If we want to find specific types of policies, such as a BRP or MBRP we can simply add constraints and variables to the linear programming formulation and find these policies. For convenience we therefore proceed with the LP.

In the general case this state space needs not be finite. If the lifetime distribution has infinite support the age of the component can, namely, be arbitrary large. Given a policy, this can however only happen if we do not maintain at all. We therefore wish to come up with a sufficient condition for the maintenance time to be finite. In the continuous time problem with constant costs a sufficient condition for the optimal maintenance time to be finite is that the failure rate is increasing and $c_f > c_p$, see Özekici (1985).

For discrete lifetime distributions Nakagawa and Osaki (1977) show that the optimal maintenance age is finite if and only if $c_f > c_p$ and

$$p_\infty = \lim_{x\to\infty} p_x > \frac{\Delta t}{\mathbb{E}(T)} \frac{c_f}{c_f - c_p}, \tag{11}$$

where p_x is the probability of failure between the ages $x-1$ and x.

Applying this condition for a Weibull distribution and using equation (6) gives us the following. For discrete Weibull lifetime distribution the optimal maintenance age is finite if and only if $\beta > 1$, $c_f > c_p$ and

$$\frac{c_p}{c_f} < 1 - \frac{\Delta t}{\mathbb{E}(T)}. \tag{12}$$

Suppose we have that components fail once a year on average (i.e. $\mathbb{E}(T) = 1$ year) and that the costs are given by $c_f = 10$, $c_p = 9$. As long as $\Delta t < 0.1$ year we have a finite optimal maintenance age. When we discretise to months we have a finite optimal maintenance age, since $\frac{1}{12} < 0.1$. If we, however, discretise to periods of 2 months this assumption is not valid, since $\frac{2}{12} > 0.1$. We must thus choose a fine enough discretisation. If we make our discretisation finer $\Delta t \to 0$, we have that $\frac{\Delta t}{\mathbb{E}(T)} \to 0$. As a result, condition (12) converges to the condition for continuous lifetime distributions (i.e. $c_f > c_p$ and increasing failure rate) for $\Delta t \to \infty$.

Condition (12) shows us that under constant costs $c_f > c_p$ we can bound the dimension of the state space corresponding to the age of the component if we choose our discretisation fine enough. We can then set $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1$, where $\mathcal{I}_1 := \{0, 1, \ldots, M\}$ for some large M. This M is the maximum age that we allow in our model. If the age of M is reached, we must maintain preventively. If we choose M larger than the optimal maintenance age, this does not give rise to any problem. We can simply check if the optimal age is below M after solving. If this is not the case, we simply increase M until this is the case.

We wish to extend this result to the case where the maintenance costs are time-varying. To be able to do this we first present Lemma 1, which is used as a building block towards the result for time-varying maintenance costs.

Lemma 1. *Let $C(T)$ denote the long-run maintenance costs for a policy, where we maintain at age $T \in \mathbb{N}_{>0}$ for all $i_0 \in \mathcal{I}_0$. Let $c_p(i_0)$ and $c_f(i_0)$ for $i_0 \in \mathcal{I}_0$ denote the preventive and corrective maintenance costs. Then we have that $C(T) = \overline{C}(T)$, where $\overline{C}(T)$ is the long-run average costs* *for the problem where the maintenance costs are replaced by the means over the year (i.e. $\overline{c}_p$ and $\overline{c}_f$).*

Proof. The proof can be found in Appendix B. $\square$

Using Lemma 1 we can extend the result of Nakagawa and Osaki (1977) for time-varying costs and obtain Theorem 1.

Theorem 1. *Let p_x denote the failure probabilities of a component between the ages $x-1$ and x and let $p_\infty = \lim_{x\to\infty} p_x > \frac{\Delta t}{\mathbb{E}(T)} \frac{\overline{c}_f}{\overline{c}_f - \overline{c}_p}$. Then the optimal ARP has a finite maintenance age for at least one period.*

Proof. The proof can be found in Appendix B. $\square$

We can thus simply obtain the condition for time-varying maintenance costs by replacing the costs in the condition for constant maintenance costs by the means over the year $\overline{c}_f$ and $\overline{c}_p$.

For discrete Weibull distributions the condition can be written as follows

$$\frac{\overline{c}_p}{\overline{c}_f} < 1 - \frac{\Delta t}{\mathbb{E}(T)}. \tag{13}$$

If this equation holds we can limit one dimension of the state space, namely to $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1$, where $\mathcal{I}_1 := \{0, 1, \ldots, M\}$ as long as we choose M large enough and Δt small enough. When the age reaches M we will always maintain preventively, but this age is typically not reached if we choose M large enough. We use the notation $\mathcal{I}^b = \mathcal{I}_0 \times \{0\}$ for the states representing a broken component at the start of the time period.

If the condition in Theorem 1 is not met, it might be that the zero maintenance policy is optimal. In this policy no preventive maintenance is performed (i.e. the maintenance age $t^* = \infty$ over the whole year). Combining the renewal reward theorem and Lemma 1 gives us that:

$$C(\infty) = \frac{\overline{c}_f}{\mathbb{E}(T)}. \tag{14}$$

We can use the corrective maintenance moments as regeneration epoch. The cycle costs are then $\overline{c}_f$ and the expected cycle length is $\mathbb{E}(T)$.

Given a certain stationary policy $R \in \mathcal{R}_s$ we can define $x_{i,a}$ the long-run fraction of time that the system is in state $i \in \mathcal{I}$ at the beginning of the period and decision $a \in \mathcal{A}$ is taken under policy R. The $x_{i,a}$'s are the decision variables in the linear programming formulation. The long-run average costs are given below.

$$\sum_{i\in\mathcal{I}\setminus\mathcal{I}^b} c_p(i_0)x_{i,1} + \sum_{i\in\mathcal{I}^b} c_f(i_0)x_{i,1}.$$

This equation will be the objective function that we wish to minimise in the linear programming formulation. To make sure that the inflow equals the outflow we introduce the following constraint.

$$\sum_{a\in\mathcal{A}(j)} x_{ja} = \sum_{i\in\mathcal{I}} \sum_{a\in\mathcal{A}(i)} \pi_{ij}(a)x_{ia} \quad \forall j \in \mathcal{I},$$

where $\pi_{ij}(a)$ is the transition probability of moving from state $i = (i_0, i_1)$ to state $j = (j_0, j_1)$ under action $a \in \mathcal{A}$. These are given below.

$$\pi_{ij}(0) = \begin{cases} 1 - p_{j_1} & \text{for } j_0 = i_0 + 1,\ j_1 = i_1 + 1,\ j_1 \neq 1, \\ p_{j_1} & \text{for } j_0 = i_0 + 1,\ j_1 = 0, \\ 0 & \text{else.} \end{cases} \tag{15a}$$

$$\pi_{ij}(1) = \begin{cases} 1 - p_1 & \text{for } j_0 = i_0 + 1,\ j_1 = 1, \\ p_1 & \text{for } j_0 = i_0 + 1,\ j_1 = 0, \\ 0 & \text{else.} \end{cases} \tag{15b}$$

p_{j_1} is the probability of failure between $j_1 - 1 \in \mathcal{I}_1$ and $j_1 \in \mathcal{I}_1$. We assume that we know this probability distribution, which can take any form as long as the condition in Theorem 1 is met (e.g. Weibull distribution with scale parameter $\beta > 1$ and fine enough discretisation).

The long-run probability of being in one of the states corresponding to a certain period $i_0 \in \mathcal{I}_0$ is $\frac{1}{N}$, since there are N periods in a year. The corresponding constraints are given below.

$$\sum_{i_1 \in \mathcal{I}_1} \sum_{a \in \mathcal{A}(i)} x_{ia} = \frac{1}{N} \quad \forall i_0 \in \mathcal{I}_0.$$

We must always perform maintenance when the wind turbine fails. Also when the maximum age M is reached we must perform maintenance. The following constraints make sure that we do not wait in these scenarios, but immediately maintain.

$$x_{i0} = 0 \quad \forall i = (i_0, i_1) \in \mathcal{I} : i_1 \in \{0, M\}.$$

Finally, the decision variables must be non-negative, since they represent a long-run fraction time. This is given in the following constraint.

$$x_{ia} \geq 0 \quad \forall i \in \mathcal{I},\ a \in \mathcal{A}.$$

The linear programming problem that we obtain by combining the objective function that we wish to minimise with the constraints is the following.

$$\text{(ARP1)} \qquad \text{minimise} \quad \sum_{i \in \mathcal{I} \setminus \mathcal{I}^b} c_p(i_0) x_{i,1} + \sum_{i \in \mathcal{I}^b} c_f(i_0) x_{i,1} \tag{16a}$$

such that:

$$\sum_{a \in \mathcal{A}(i)} x_{ia} - \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}(i)} \pi_{ij}(a) x_{ia} = 0 \qquad \forall j \in \mathcal{I} \tag{16b}$$

$$x_{i0} = 0 \qquad \forall i = (i_0, i_1) \in \mathcal{I} : i_1 \in \{0, M\} \tag{16c}$$

$$\sum_{i_1 \in \mathcal{I}_1} \sum_{a \in \mathcal{A}(i)} x_{ia} = \frac{1}{N} \qquad \forall i_0 \in \mathcal{I}_0 \tag{16d}$$

$$x_{ia} \geq 0 \qquad \forall i \in \mathcal{I},\ a \in \mathcal{A} \tag{16e}$$

We see that the number of decision variables is linear with M and we thus wish to choose M as small as possible. We will solve this linear program for different cost functions and failure distributions using CPLEX 12.8.0 in Java and the results will be given in Section 4.

Note that we do need to be careful in using the LP formulation. A problem arises if the states that can be reached according to the linear programming formulations form multiple recurrent classes, see Tijms (2003). In this scenario the initial state of our Markov chain can have an influence on the long-run costs. Dependent on the initial state we end up in one of the recurrent classes with some probability and the long-run costs converge to the long-run costs of this recurrent class. A sufficient condition for this not to happen is the weak unichain assumption, given in Definition 3.

Definition 3 (Weak unichain assumption)**.** *For each cost optimal stationary policy the Markov chain contains one recurrent class (and possibly multiple transient states).*

Lemma 2 states that this assumption is satisfied for ARP.

Lemma 2. *For 1-arithmetic lifetime distributions any ARP induces a Markov chain that contains one recurrent class and is therefore unichain.*

Proof. The proof can be found in Appendix B. □

If LP formulation (16) finds an ARP, the ARP is optimal and we find that the weak unichain assumption is satisfied for this ARP. The LP approach then finds the cost optimal ARP.

For general distributions, it could be the case that the LP finds a policy for which we maintain for age t during a period, but there exists an $s \geq t$ for which we do not maintain. Then it does not find an ARP. For example if components can fail on even ages only, we never maintain for uneven ages, because we would rather wait one period and this does not give an ARP. In the results in Section 4 for Weibull distributions we only find ARPs.

3.3.2 Block-based maintenance

In the model that we discuss in this section we make the same model assumptions as in the previous section. The only difference is the maintenance policies that we consider. We now consider an extension of the standard block-based maintenance policy instead of the age-based maintenance policy of previous section. In a standard block-based maintenance policy components are maintained preventively every multiple of time T (i.e. maintenance at $T, 2T, 3T, \ldots$). Under constant costs we can compute the optimal block time T^* as illustrated in Example 2.

Example 2 (Standard block-based maintenance)**.** *Suppose we have a component with a discrete Weibull failure distribution, with $\alpha = 12$ months and $\beta = 2$. Let the costs of preventive and corrective maintenance be given by $c_p = 10$ and $c_f = 50$. Note that these parameters are the same as in Example 1. To be able to compute the costs we need $M(t)$, which denotes the renewal function, or $m(t)$, the renewal density.*

We can use the renewal reward theorem to compute the long-run costs as a function of the block t. Nakagawa (1984) give an expression for the costs that we will use.

$$
\begin{aligned}
C(t) &= \frac{c_p + c_f M(t)}{t}, \\
&= \frac{c_p + c_f \sum_{s=1}^{t} m(s)}{t}, \\
&= \frac{c_p + c_f \sum_{s=1}^{t} \sum_{j=1}^{s} f^{(j)(s)}}{t}.
\end{aligned}
$$

where $f^{(j)}$ is the density function of the sum of j independent variables with density f. We can use the recursive formula $f^{(j)}(s) = \sum_{i=1}^{s-1} f(i) f^{(j-1)}(s-i)$ to compute this value. An alternative *way to compute the costs is given in Appendix D.1.*

In Figure 5 we plot the yearly costs as a function of the block time where we use the above formula.

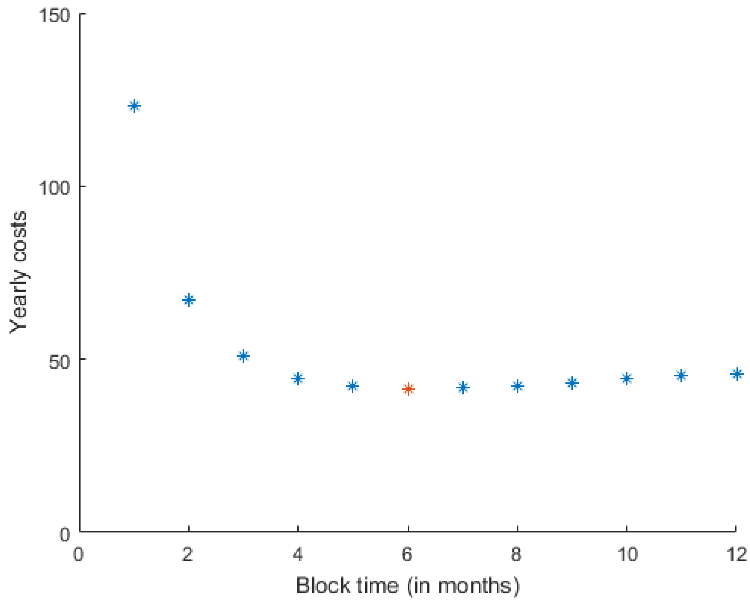


Figure 5: This figure shows the yearly maintenance costs as function of the block size, where $c_f = 5c_p$ constant and $T \sim \text{Weibull}(12, 2)$.

We see that the minimum of 41.501 is obtained for $t = 6$ months and so in the optimal BRP we maintain preventively every 6 months. In this example the optimal maintenance block is the same as the optimal maintenance age. The costs are 3.50% larger for the BRP than for the ARP.

Since the costs for wind turbine maintenance vary in time we allow the block size to vary in time as well. Using the varying block sizes we can thus maintain at $T_1, T_2, T_3, \ldots$, where $T_i \in \mathbb{N}\ \forall i$. To limit the scope of possible policies we restrict ourselves to policies that repeat themselves every m years for some $m \in \mathbb{N}$. In this way we generalise block maintenance policies.

Definition 4 (BRP). *A BRP is a policy in which the preventive maintenance decision depends on the time of the year and year number only (i.e. we maintain preventively independent of age at $T_1, T_2, T_3, \cdots$, where $T_i \in \mathbb{N}\ \forall i$).*

Throughout the rest of this thesis we use BRP to denote block replacement/repair/maintenance policies in which the block sizes vary over the years. The BRP in which the block size is constant is referred to as the standard BRP, which is common in the literature and used for problems for which maintenance costs are constant.

The Markov decision process that we discussed in Section 3.3.1 can still be used for these block-based maintenance policies. We use the same formulation of the Markov decision process and we will come up with a mixed integer programming (MIP) formulation, which is an LP formulation (16) with extra binary decision variables and constraints. Before we can proceed, we need to realise that it might be that no preventive maintenance can be optimal under certain conditions. When preventive maintenance is for example equally or more expensive than corrective maintenance we will never perform preventive maintenance. For the continuous constant cost maintenance problem the condition for the optimal maintenance block time T to be finite is given by Nakagawa (1984).

$$\frac{c_p}{c_f} < \frac{1}{2}(1 - c_T^2), \tag{17}$$

where c_T is the coefficient of variation of T, the lifetime random variable. We wish to come up with a condition for the discrete problem with varying maintenance costs. First, in Lemma 3 we give a sufficient condition for the discrete problem with constant maintenance costs. We can simply replace the coefficient of variation of the continuous lifetime distribution by the coefficient of variation of the discrete lifetime distribution and use the same condition.

Lemma 3. *Let the lifetime distribution of components T be distributed such that $\frac{c_p}{c_f} < \frac{1}{2}(1 - c_T^2)$, where c_f and c_p are the constant corrective and preventive maintenance costs. Then the optimal block-based maintenance time is finite.*

Proof. The proof can be found in Appendix B. □

Lemma 3 shows that a same sufficient condition can be formulated for the discrete time case as for the continuous time case, under constant costs. In Theorem 2 we generalise this result for time-varying costs and show that a similar condition is sufficient.

Theorem 2. *Let the lifetime distribution of components T be distributed such that $\frac{\bar{c}_p}{\bar{c}_f} < \frac{1}{2}(1 - c_T^2)$, where c_f and c_p are the time-varying corrective and preventive maintenance costs with yearly means $\bar{c}_p$, $\bar{c}_f$. Then the optimal block-based maintenance time is finite.*

Proof. The proof can be found in Appendix B. □

Note that the coefficient of variation is 0.523 and 0.363 for Weibull distributions with $\beta = 2$ and $\beta = 3$. This means that the condition becomes $\frac{\bar{c}_p}{\bar{c}_f} < x$ with $x \approx 0.239$ for $\beta = 2$ and $x \approx 0.319$ for $\beta = 3$. If for example the corrective maintenance costs are 4 times the preventive maintenance costs, this condition is not satisfied for $\beta = 2$, but are satisfied for distributions with $\beta = 3$. This makes sense, since the failures are more predictable for larger β.

The condition in Theorem 2 is independent of the cost fluctuations, since it depends only on the mean of the costs. However, the stronger the variation in the costs the more we can pick cheaper moments to do maintenance. One might expect that we maintain preventively when the preventive maintenance costs are cheap and that we can formulate a condition that depends on $\min\{c_p\} := \min_{i_0 \in \mathcal{I}_0}\{c_p(i_0)\}$, the minimal cost moment for preventive maintenance, instead of $\bar{c}_p$. We can do this, but the condition does not simply change to

$$\frac{\min\{c_p\}}{\bar{c}_f} < \frac{1}{2}(1 - c_T^2).$$

Maintaining at cheap moments, might induce that the failures occur at more expensive moments on average. If we maintain preventively at the cheapest moment once every k years, we indeed only pay $\min\{c_p\}$ as preventive maintenance costs and $\bar{c}_p$ becomes irrelevant. The average corrective maintenance costs are however not $\bar{c}_f$, since $m(t)$ (i.e. the renewal density) is not constant if the failure rate increases for increasing ages. We therefore need an adjustment and we define a new parameter below.

$$b = \limsup_{n \to \infty} \left\{ \sum_{t=1}^{nN} m(t) \left(\frac{c_f(t_{min} + t)}{\bar{c}_f} - 1 \right) \right\}, \tag{18}$$

where the lim sup is taken over numbers $n \in \mathbb{N}$ only. The parameter b depends on the distribution but is finite for Weibull distributions. As long as the renewal density converges to $\frac{1}{\mu}$, with μ

the mean of the interarrival times, fast enough, b is finite. The convergence needs to be of the order $O(t^{-a})$ for some $a > 1$. Using b we give the adjusted condition for which the block-based maintenance time is finite in Theorem 3.

Theorem 3. *Let the lifetime distribution of components T be distributed such that $\frac{\min\{c_p\}}{\bar{c}_f} + b < \frac{1}{2}(1 - c_T^2)$, where b is given by equation (18). Then the optimal block-based maintenance time is finite.*

Proof. The proof can be found in Appendix B. □

Note that the parameter b can be either negative or positive. If maintenance costs are constant over the year, we have $b = 0$, $\min\{c_p\} = \bar{c}_p$ and the condition is the same as the condition in Lemma 3.

Generally, we have no explicit formula for the renewal density $m(t)$ and therefore no explicit expression for b. When the interarrival are distributed according to an Erlang distribution we do know the renewal density explicitly and we can determine b. This is illustrated in Example 3.

Example 3. *Suppose that the time-varying costs are given as follows*

$$c_p(t) = \bar{c}_p + \Delta \cdot \bar{c}_p \cos(2\pi t),$$
$$c_f(t) = \bar{c}_f + \Delta \cdot \bar{c}_f \cos(2\pi t),$$

where t is the time in years and $\Delta \in [0, 1]$. Suppose the interarrival times are distributed according to an Erlang-2 distribution.

Note that the density of an Erlang-2 distribution is given by

$$f(t) = \lambda^2 t e^{-2t}, \tag{19}$$

and has mean $\mu = \frac{2}{\lambda}$.

The renewal density can be computed analytically and Varsei and Samimi (2009) show that

$$m(t) = \frac{\lambda}{2} - \frac{\lambda}{2} e^{-2\lambda t} = \frac{1}{\mu} - \frac{1}{\mu} e^{-\frac{4t}{\mu}}. \tag{20}$$

Theorem 3 is given for discrete distributions. Letting $\Delta t \to 0$ in our discretisation gives that

$$
\begin{aligned}
b &= \limsup_{n\to\infty} \int_0^n \left(\frac{\lambda}{2} - \frac{\lambda}{2}e^{-2\lambda t}\right) \cdot -\Delta\cos(2\pi t)dt,\\
&= \limsup_{n\to\infty} \left(-\int_0^n \frac{\lambda}{2}\Delta\cos(-2\pi t)dt + \int_0^n \frac{\lambda}{2}e^{-2\lambda t}\Delta\cos(-2\pi t)dt\right),\\
&= \limsup_{n\to\infty} \int_0^n \frac{\lambda\Delta}{2}e^{-2\lambda t}\cos(-2\pi t)dt,\\
&= \limsup_{n\to\infty} \frac{\lambda\Delta}{4}\int_0^n e^{-2\lambda t}\left(e^{2\pi t i} + e^{-2\pi t i}\right)dt,\\
&= \limsup_{n\to\infty} \frac{\lambda\Delta}{4}\int_0^n \left(e^{2\pi t i - 2\lambda t} + e^{-2\pi t i - 2\lambda t}\right)dt,\\
&= \limsup_{n\to\infty} \frac{\lambda\Delta}{4}\left(\frac{1}{2\pi i - 2\lambda}\left[e^{2\pi t i - 2\lambda t}\right]_0^n + \frac{1}{-2\pi i - 2\lambda}\left[e^{-2\pi t i - 2\lambda t}\right]_0^n\right),\\
&= \frac{\lambda\Delta}{4}\left(\frac{1}{2\pi i - 2\lambda}\left[e^{2\pi t i - 2\lambda t}\right]_0^\infty + \frac{1}{-2\pi i - 2\lambda}\left[e^{-2\pi t i - 2\lambda t}\right]_0^\infty\right),\\
&= -\frac{\lambda\Delta}{8}\left(\frac{1}{\pi i - \lambda} + \frac{1}{-\pi i - \lambda}\right),\\
&= -\frac{\lambda\Delta}{8}\frac{-2\lambda}{\lambda^2 + \pi^2},\\
&= \frac{\lambda^2\Delta}{4(\lambda^2 + \pi^2)},\\
&= \frac{\Delta}{(2 + \pi^2\mu^2)},
\end{aligned}
$$

which is always non-negative. If $\frac{\min\{c_p\}}{\bar{c}_f} + \frac{\Delta}{(2+\pi^2\mu^2)} < \frac{1}{2}(1 - c_T^2)$ is satisfied we are sure there exists a finite optimal T^* for the BRP.

Assuming the conditions of Theorem 2 or Theorem 3 are met, we can come up with our MIP formulation. We can use the same formulation as previously, but have to introduce some extra constraints to make sure that we obtain a block-based maintenance policy. First we introduce the following variables that help decide in which period we do the preventive maintenance.

$$
y_{i_0} = \begin{cases} 1, & \text{if we maintain preventively in period } i_0 \in \mathcal{I}_0, \\ 0, & \text{else.} \end{cases} \tag{21}
$$

For block maintenance we have that the maintenance decision depends on the time of the year and is independent of the age of the component. If we maintain preventively in period $j_0 \in \mathcal{I}_0$ (or do not maintain preventively), we are not allowed to take action $a = 0$ (or $a = 1$) for any age $i_1 > 0$.

Remark: we can force this by the following inequalities.

$$
\begin{aligned}
x_{ia} &\leq 1 - y_{i_0} && \forall i = (i_0, i_1) \in \mathcal{I} : i_1 > 0, a = 0,\\
x_{ia} &\leq y_{i_0} && \forall i = (i_0, i_1) \in \mathcal{I} : i_1 > 0, a = 1.
\end{aligned}
$$

Proof.
Let $i_0 \in \mathcal{I}_0$ be an arbitrary age. We distinguish two cases.

(i) There exists an $i_1 > 0$, such that for $i = (i_0, i_1)$ we have $x_{i,1} > 0$.

(ii) There exists no $i_1 > 0$, such that for $i = (i_0, i_1)$ we have $x_{i,1} > 0$.

Assume that (i) holds. Then $y_{i_0} \geq x_{i1} > 0$ and thus $y_{i_0} = 1$. As a result for all $i' \in \{(i_0, j_1) : 0 < j_1 \in \mathcal{I}_1\}$ we have that $x_{i'0} \leq 1 - y_{i_0} \leq 0$ and thus $x_{i'0} = 0$. We thus maintain preventively at i_0 for all ages (ignoring $i_1 = 0$ for which we always maintain correctively).

Assume that (ii) holds. Then we never maintain preventively at i_0. In conclusion, we either maintain preventively for all ages or we do not maintain preventively for any age and the decision thus only depends on i_0. □

We obtain the following MIP by combining these extra constraints with the linear programming formulation (16).

(BRP1) minimise
$$\sum_{i \in \mathcal{I} \setminus \mathcal{I}^b} c_p(i_0) x_{i,1} + \sum_{i \in \mathcal{I}^b} c_f(i_0) x_{i,1} \tag{22a}$$

such that:

$$\sum_{a \in \mathcal{A}(i)} x_{ia} - \sum_{j \in \mathcal{I}} \sum_{a \in \mathcal{A}(j)} \pi_{ji}(a) x_{ja} = 0 \qquad \forall i \in \mathcal{I} \tag{22b}$$
$$x_{i0} = 0 \qquad \forall i = (i_0, i_1) \in \mathcal{I} : i_1 \in \{0, M\} \tag{22c}$$
$$x_{ia} + y_{i_0} \leq 1 \qquad \forall i = (i_0, i_1) \in \mathcal{I} : i_1 > 0, a = 0 \tag{22d}$$
$$x_{ia} - y_{i_0} \leq 0 \qquad \forall i = (i_0, i_1) \in \mathcal{I} : i_1 > 0, a = 1 \tag{22e}$$
$$\sum_{i_1 \in \mathcal{I}_1} \sum_{a \in \mathcal{A}(i)} x_{ia} = \frac{1}{N} \qquad \forall i_0 \in \mathcal{I}_0 \tag{22f}$$
$$x_{ia} \geq 0 \qquad \forall i \in \mathcal{I},\ a \in \mathcal{A} \tag{22g}$$
$$y_{i_0} \in \{0, 1\} \qquad \forall i_0 \in \mathcal{I}_0 \tag{22h}$$

In this model we restrict ourselves to policies in which we perform maintenance during the same time periods every year. These policies thus have a periodicity of one year and we need to perform maintenance at least once a year. Constraint (22c) makes sure that we cannot let the component be arbitrarily old and thus makes sure that maintenance is done at least once a year. Depending on the parameters in the model it might however be optimal to maintain only once in several years, in which case it would be better to consider policies with periodicity of multiple years. This can be done by increasing the state space of our Markov decision process. That is, we use the following $\mathcal{I}_0$ in the new state space.

$$\mathcal{I}_0 = \{1, 2, \cdots, Nm\}, \tag{23}$$

where $m \in \mathbb{N}$ is the number of years considered. It can be that it is optimal to plan preventive maintenance on a yearly basis, then it suffices to consider $m = 1$ only. This depends on the cost and distribution parameters of the model. Considering larger m can only improve our solution, but does increase computation time, since then the amount of decision variables increases. We therefore wish to choose m large to be close to optimal, but not to large to keep the computation time short.

Note that these block maintenance policies are a special case of age-based maintenance, where the critical maintenance age is 1 for the moments that we maintain and infinity for other moments. The costs induced by any block maintenance policy are thus at least the costs induced by the optimal age-based maintenance policy.

We will solve this mixed integer program for different parameters and different m using CPLEX 12.8.0 in Java and the results will be given in Section 4.

3.3.3 Modified block-based maintenance

Modified block-based maintenance policies are policies in which we still plan maintenance in blocks, but when components have just been maintained correctively they will not be maintained again. In this way we keep the plannability of the block-based maintenance, but we are closer to the optimal age-based maintenance in costs. We namely save extra maintenance costs for relatively new components. In this section we show that we can use a similar approach to previous sections to come up with the optimal modified block-based maintenance policy.

In the model that we discuss in this section we make the same model assumptions as in previous sections. The only difference is the maintenance policies that we consider. We now consider an extension of the modified block-based maintenance policy instead of the age-based or block-based maintenance policy of previous sections. In a standard modified block-based maintenance policy components are maintained preventively every multiple of time T (i.e. maintenance at $T, 2T, 3T, \ldots$). We skip the maintenace if the age of the component is below the minimal age t for $0 \leq t \leq T$.

Under constant costs we can compute the optimal block time T^* and minimal age t^* as illustrated in Example 4.

Example 4 (Standard MBRP)**.** *Suppose we have a component with a discrete Weibull failure distribution, with $\alpha = 12$ months and $\beta = 2$. Let the costs of preventive and corrective maintenance be given by $c_p = 10$ and $c_f = 50$. Note that these parameters are the same as in Example 1.*

We can use the renewal reward theorem to compute the long-run costs as a function of the block T and minimal age $t \leq T$, see Appendix D. In Figure 6 we plot the yearly costs as a function of the block time T where t is optimised given T.

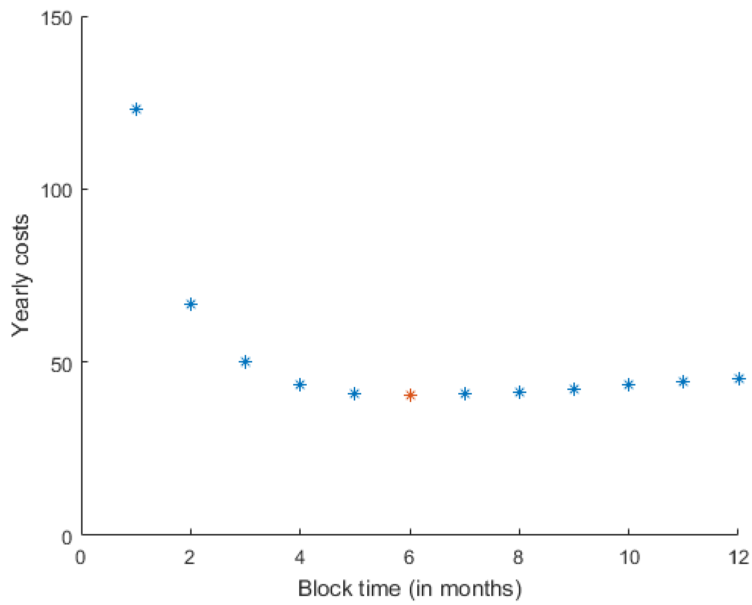


Figure 6: This figure shows the yearly maintenance cost as function of the block size, where $c_f = 5c_p$ constant and $T \sim \text{Weibull}(12, 2)$.

We see that the minimum of 40.310 is obtained for $T^* = 6$ months with $t^* = 4$ months and so in the optimal MBRP we maintain preventively every 6 months if the components are at least 4 months old. For $T = 5$ we have $t = 5$ and costs 40.880, which is 1.41% more expensive than the *optimal MBRP. For $T = 7$ we have $t = 4$ and costs 40.675, which is 0.91% more expensive than the optimal MBRP.*

In this example the optimal maintenance block is the same as the optimal maintenance block for the BRP. The costs for the optimal ARP are 0.53% lower than for the MBRP and the BRP costs are 2.95% larger than for the MBRP. The MBRP bridges approximately 85% of the gap between ARP and BRP for these parameters.

Since the costs for wind turbine maintenance vary in time we allow the block size and this age to vary in time as well. Using the varying block sizes we can thus maintain at $T_1, T_2, T_3, \ldots$, where $T_i \in \mathbb{N}\ \forall i$. We only maintain if the component reaches ages $t_1, t_2, t_3 \ldots$, where $t_i \leq T_i - T_{i-1}$ (i.e. the minimal age at an opportunity cannot be larger than the time between this opportunity and the previous opportunity). The MBRP is formally defined in Definition 5.

Definition 5 (MBRP)**.** *An MBRP is a policy in which we have preventive maintenance opportunities at $T_1, T_2, T_3, \cdots$ where $T_i \in \mathbb{N}\ \forall i$. We skip the maintenance at opportunity i if a minimal age t_i is not reached. Furthermore $t_i \leq T_i - T_{i-1}\ \forall i$.*

Throughout the rest of this thesis we use MBRP to denote modified block replacement/repair/maintenance policies in which the block time and minimal age can vary over the years. The MBRP in which the block size is constant is referred to as the standard MBRP, which is common in the literature and used for problems for which maintenance costs are constant. To limit the scope of possible policies we restrict ourselves to policies that repeat themselves every m years for some $m \in \mathbb{N}$ and we consider different m.

Similar to the ARP and BRP we wish to come up with sufficient conditions for the MBRP to have a finite optimal t and T. For constant costs, this condition is given in Theorem 4.

Theorem 4. *Let p_x denote the failure probabilities of a component between the ages $x-1$ and x and let $p_\infty = \lim_{x\to\infty} p_x > \frac{\Delta t}{\mathbb{E}(T)} \frac{c_f}{c_f - c_p}$. Then the optimal MBRP under has a finite optimal maintenance age t and block T.*

Proof. The proof can be found in Appendix B. □

The condition for the optimal t and T to be finite is the same as with the ARP. If the optimal age T^* in the ARP is finite we know that maintenance for age T^* is cheaper than no maintenance. As a result, maintenance from this $t = T^*$ onward every $T = T^*$ time periods is also cheaper than no maintenance at all. A similar condition holds for time-varying maintenance costs.

Theorem 5. *Let p_x denote the failure probabilities of a component between the ages $x-1$ and x and let $p_\infty = \lim_{x\to\infty} p_x > \frac{\Delta t}{\mathbb{E}(T)} \frac{\bar{c}_f}{\bar{c}_f - \bar{c}_p}$. Then the optimal MBRP has finite block times and minimal ages.*

Proof. The proof can be found in Appendix B. □

Again the condition is the same as for the ARP. Assuming the optimal maintenance ages and blocks are finite we can proceed and find the MIP formulation.

Below we introduce extra variables that decide for which period and which age we maintain.

$$z_i = z_{i_0 i_1} = \begin{cases} 1, & \text{if we maintain for age } i_1 \in \mathcal{I}_1 \text{ in period } i_0 \in \mathcal{I}_0, \\ 0, & \text{else.} \end{cases} \tag{24}$$

First of all, if we maintain for age i_1 in a certain period i_0 then we must maintain for age $j_1 > i_1$ as well in this period. We thus have that:

$$z_{i_0 i_1} \leq z_{i_0 j_1} \quad \forall i_0 \in \mathcal{I}_0 \text{ and } \forall i_1, j_1 \in \mathcal{I}_1 : i_1 < j_1. \tag{25}$$

Clearly, we must also have that:

$$z_{i_0 i_1} \leq y_{i_0} \quad \forall i_0 \in \mathcal{I}_0, i_1 \in \mathcal{I}_1. \tag{26}$$

where y_{i_0} is defined in (21).

Since there is no constraint that makes sure that $t_i \leq T_i - T_{i-1}$, any age-based maintenance policy is now feasible. Now let t_{i_0} be the minimum age at which we maintain for period i_0. Then the age should satisfy the following constraints for all $i_0, j_0 \in \mathcal{I}_0$ with $i_0 \neq j_0$.

$$t_{i_0} \leq i_0 - j_0 y_{j_0} + N(1 - y_{j_0}), \qquad \text{if } j_0 < i_0, \tag{27}$$

$$t_{i_0} \leq N + i_0 - j_0 y_{j_0}, \qquad \text{if } j_0 > i_0. \tag{28}$$

when $y_{i_0} = 0$, t_{i_0} can take any value, since maintenance is never performed in period $i_0 \in \mathcal{I}_0$.

The following constraints should be added to the problem to make sure that indeed $t_i \leq T_i - T_{i-1}$.

$$z_i = z_{i_0 i_1} = \begin{cases} y_{i_0}, & \text{if } i_1 \geq t_{i_0}, \\ 0, & \text{elsewhere.} \end{cases} \tag{29}$$

Note that this constraint is non-linear, since it contains an if statement. Using the relatively large number M (the maximum age) we can solve this and add the following linear constraints for all $i_0 \in \mathcal{I}_0$ and $i_1 \in \mathcal{I}_1$.

$$M(y_{i_0} - z_{i_0 i_1}) + (1 + i_1 - t_{i_0}) \leq M, \tag{30}$$

$$M z_{i_0 i_1} + (t_{i_0} - i_1) \leq M. \tag{31}$$

We add the above introduced constraints to linear programming formulation (16) and obtain the following MIP.

$$\text{(MPRP1)} \qquad \text{minimise} \quad \sum_{i \in \mathcal{I} \setminus \mathcal{I}^b} c_p(i_0) x_{i,1} + \sum_{i \in \mathcal{I}^b} c_f(i_0) x_{i,1} \tag{32a}$$

such that:

$$\sum_{a \in \mathcal{A}(i)} x_{ia} - \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}(i)} \pi_{ij}(a) x_{ia} = 0 \qquad \forall j \in \mathcal{I} \tag{32b}$$

$$x_{i0} = 0 \qquad \forall i = (i_0, i_1) \in \mathcal{I} : i_1 \in \{0, M\} \tag{32c}$$

$$\sum_{i_1 \in \mathcal{I}_1} \sum_{a \in \mathcal{A}(i)} x_{ia} = \frac{1}{N} \qquad \forall i_0 \in \mathcal{I}_0 \tag{32d}$$

$$z_{i_0 i_1} - y_{i_0} \leq 0 \qquad \forall i_0 \in \mathcal{I}_0,\ i_1 \in \mathcal{I}_1 \tag{32e}$$

$$t_{i_0} + j_0 y_{j_0} + N y_{j_0} \leq N + i_0 \qquad \forall i_0, j_0 \in \mathcal{I}_0 : j_0 < i_0 \tag{32f}$$

$$t_{i_0} + j_0 y_{j_0} \leq N + i_0 \qquad \forall i_0, j_0 \in \mathcal{I}_0 : j_0 > i_0 \tag{32g}$$

$$M y_{i_0} - M z_{i_0 i_1} - t_{i_0} \leq M - 1 - i_1 \qquad \forall i_0 \in \mathcal{I}_0,\ i_1 \in \mathcal{I}_1 \tag{32h}$$

$$M z_{i_0 i_1} + t_{i_0} \leq M + i_1 \qquad \forall i_0 \in \mathcal{I}_0,\ i_1 \in \mathcal{I}_1 \tag{32i}$$

$$x_{ia} \geq 0 \qquad \forall i \in \mathcal{I},\ a \in \mathcal{A} \tag{32j}$$

$$z_i \in \{0, 1\} \qquad \forall i \in \mathcal{I} \tag{32k}$$

$$y_{i_0} \in \{0, 1\} \qquad \forall i_0 \in \mathcal{I}_0 \tag{32l}$$

$$t_{i_0} \in \mathbb{N} \qquad \forall i_0 \in \mathcal{I}_0 \tag{32m}$$

We will solve this mixed integer program using CPLEX 12.8.0 in Java and the results will be given in Section 4.

3.4 Maintenance policies for multi-component systems

In this section we extend our single component models to multi-component models. In Section 3.4.1 we show how we can extend the state space of the Markov decision process for more components. The optimal policy, which is an age-based maintenance policy, is computed via an LP formulation. The actions for each component depend on the ages of all components in a complex way. In Section 3.4.2 we show that extra constraints and binary variables can be added to obtain an MIP formulation for block-based maintenance policies. A similar approach can be taken to obtain an MIP formulation for the modified block-based maintenance, which is described in Section 3.4.3.

3.4.1 Age-based maintenance

A wind turbine is made out of multiple components. In our model we must thus add components to be able to include maintenance costs of all relevant components. For each component that we add to our model the number of actions doubles, since we can either maintain or not for this extra component. Also an extra dimension must be added to the state space, that gives the age of the component. For n components the state space is $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1 \times \cdots \times \mathcal{I}_n$ and the number of possible actions equals $|\mathcal{A}| = 2^n$, we namely have $\mathcal{A} = \{0,1\}^n$ (i.e we can maintain or not maintain for all n components). We see that the number of decision variables increases exponentially in the number of components, which makes finding the optimal age-based maintenance policy for larger n very time consuming.

For the maintenance of offshore wind turbines a large part of the costs is set-up costs. In the model we split the costs of the maintenance in set-up costs and in addition component specific maintenance costs. We assume the set-up costs are constant over time and we thus generally have the following for the set-up, corrective and preventive maintenance costs.

$$c_s : \{1, 2, \dots, N\} \mapsto \mathbb{R}, \tag{33a}$$
$$c_f^k : \{1, 2, \dots, N\} \mapsto \mathbb{R} \text{ for component } k, \tag{33b}$$
$$c_p^k : \{1, 2, \dots, N\} \mapsto \mathbb{R} \text{ for component } k. \tag{33c}$$

We assume that whenever preventive maintenance actions are combined with each other, that we have to pay set-up costs once only. Also if one component fails during this period, set-up cost are paid once. When we maintain the broken wind turbine, all other components that were planned in this period are also maintained.

Whenever more components fail we do have to pay set-up costs extra for each broken component. When a component fails, we immediately order a new component and when it arrives, we immediately carry out the corrective maintenance, because we wish to avoid extra downtime. In our model we assume that components fail independently and therefore the probability that multiple newly ordered components arrive at the same time is negligible.

The 2-dimensional problem is still solvable in a time in the order of seconds, minutes or hours as long as we discretise time coarsely. The possible actions $a \in \mathcal{A}$ that we can take in the 2-dimensional model are as follows.

$$a = \begin{cases} \{0,0\}, & \text{if we do not maintain,} \\ \{1,0\}, & \text{if we maintain component 1,} \\ \{0,1\}, & \text{if we maintain component 2,} \\ \{1,1\}, & \text{if we maintain both components.} \end{cases} \tag{34}$$

Again when for $i_k = 0$ we have that maintenance of component k indicates corrective maintenance and $i_k \neq 0$ indicates preventive maintenance. For a general number of components we have that $a_k = 1$ if we maintain component k and $a_k = 0$ if we do not maintain.

To be able to come up with a linear program we take a similar approach to the one dimensional setting. Before we can write this linear program we need a finite state space. If the conditions in Theorem 1 are met for all components individually, we know that a finite M exists and we can proceed.

Assume that we save set-up costs whenever preventive maintenance joins a maintenance action. Doing 2 preventive maintenance actions for 2 components and 2 corrective maintenance actions for two other components means that we save set-up costs twice, since the preventive maintenance actions can join the corrective maintenance actions. The total set-up costs paid are then $2c_s(i_0)$. The corrective maintenance actions cannot be combined, since we assume that they need to be performed immediately upon failure. We show the costs for a setting with two components, but this can easily be extended for more components. For each $i = (i_0, i_1, i_2) \in \mathcal{I}$ and $a \in \mathcal{A}$ we obtain the following cost coefficients.

$$c_{i,a} = \begin{cases} 0, & \text{if } a = \{0,0\} \\ c_s(i_0) + c_f^1(i_0), & \text{if } a = \{1,0\}, i_1 = 0, \\ c_s(i_0) + c_f^2(i_0), & \text{if } a = \{0,1\}, i_2 = 0, \\ c_s(i_0) + c_p^1(i_0), & \text{if } a = \{1,0\}, i_1 \neq 0, \\ c_s(i_0) + c_p^2(i_0), & \text{if } a = \{0,1\}, i_2 \neq 0, \\ 2c_s(i_0) + c_f^1(i_0) + c_f^2(i_0), & \text{if } a = \{1,1\}, i_1 = 0, i_2 = 0, \\ c_s(i_0) + c_f^1(i_0) + c_p^2(i_0), & \text{if } a = \{1,1\}, i_1 = 0, i_2 \neq 0, \\ c_s(i_0) + c_p^1(i_0) + c_f^2(i_0), & \text{if } a = \{1,1\}, i_1 \neq 0, i_2 = 0, \\ c_s(i_0) + c_p^1(i_0) + c_p^2(i_0), & \text{if } a = \{1,1\}, i_1 \neq 0, i_2 \neq 0. \end{cases} \tag{35}$$

Using this cost function we include that we save set-up costs whenever we perform maintenance on both components (i.e. action $a = \{1, 1\}$). Note that we do pay set-up costs twice if both components fail during one period, since we need to do the corrective maintenance immediately. If a component fails in the period before the planned preventive maintenance of the other component, the maintenance actions can be combined and set-up costs are paid once only.

In our model we can maintain components when other components fail and then save set-up costs. In the literature this is sometimes referred to as opportunistic maintenance, since we use the opportunity to perform the maintenance. In the literature it is also common to assume that we cannot maintain a component preventively when another component is broken and needs corrective maintenance, see for example Özekici (1988). If we wish, this can also be incorporated in our cost coefficients by adding $c_s(i_0)$ for the case $a = \{1, 1\}$, $i_1 = 0$ and $i_2 \neq 0$ or the case $a = \{1, 1\}$, $i_1 \neq 0$ and $i_2 = 0$. We will however use the cost coefficients of equation (35) throughout the rest of the report. For n components we can now define the following linear program to find the optimal cost policy.

$$\text{(ARP)} \qquad \text{minimise} \quad \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}} c_{i,a} x_{i,a} \tag{36a}$$

such that:

$$\sum_{a\in\mathcal{A}(i)} x_{ia} - \sum_{j\in\mathcal{I}}\sum_{a\in\mathcal{A}(j)} \pi_{ji}(a)x_{ja} = 0 \qquad \forall i \in \mathcal{I} \tag{36b}$$

$$x_{ia} = 0 \qquad \forall k \leq n, i \in \mathcal{I}, a \in \mathcal{A} : i_k \in \{0, M\}, a_k = 0 \tag{36c}$$

$$\sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}(i)} x_{ia} = \frac{1}{N} \qquad \forall i_0 \in \mathcal{I}_0 \tag{36d}$$

$$x_{ia} \geq 0 \qquad \forall i \in \mathcal{I},\ a \in \mathcal{A} \tag{36e}$$

All components fail according to a distribution with failure probabilities p_x^k for component k, where x denotes the age. The transition probabilities can be computed from this and we illustrate this for the two component setting. Then we have $i = (i_0, i_1, i_2) \in \mathcal{I}$, $j = (j_0, j_1, j_2) \in \mathcal{I}$ and $a \in \mathcal{A}$ we can determine the transition probabilities $p_{ij}(a)$.

$$\pi_{ij}(\{0,0\}) = \begin{cases} (1-p_{j_1}^1)(1-p_{j_2}^2) & \text{for } j_0 = i_0+1,\ j_1 = i_1+1,\ j_2 = i_2+1, \\ (1-p_{j_1}^1)p_{j_2}^2 & \text{for } j_0 = i_0+1,\ j_1 = i_1+1,\ j_2 = 0, \\ p_{j_1}^1(1-p_{j_2}^2) & \text{for } j_0 = i_0+1,\ j_1 = 0,\ j_2 = i_2+1, \\ p_{j_1}^1 p_{j_2}^2 & \text{for } j_0 = i_0+1,\ j_1 = 0,\ j_2 = 0, \\ 0 & \text{else.} \end{cases} \tag{37a}$$

$$\pi_{ij}(\{1,0\}) = \begin{cases} (1-p_1^1)(1-p_{j_2}^2) & \text{for } j_0 = i_0+1,\ j_1 = 1,\ j_2 = i_2+1, \\ (1-p_1^1)p_{j_2}^2 & \text{for } j_0 = i_0+1,\ j_1 = 1,\ j_2 = 0, \\ p_1^1(1-p_{j_2}^2) & \text{for } j_0 = i_0+1,\ j_1 = 0,\ j_2 = i_2+1, \\ p_1^1 p_{j_2}^2 & \text{for } j_0 = i_0+1,\ j_1 = 0,\ j_2 = 0, \\ 0 & \text{else.} \end{cases} \tag{37b}$$

$$\pi_{ij}(\{0,1\}) = \begin{cases} (1-p_{j_1}^1)(1-p_1^2) & \text{for } j_0 = i_0+1,\ j_1 = i_1+1,\ j_2 = 1, \\ (1-p_{j_1}^1)p_1^2 & \text{for } j_0 = i_0+1,\ j_1 = i_1+1,\ j_2 = 0, \\ p_{j_1}^1(1-p_1^2) & \text{for } j_0 = i_0+1,\ j_1 = 0,\ j_2 = 1, \\ p_{j_1}^1 p_1^2 & \text{for } j_0 = i_0+1,\ j_1 = 0,\ j_2 = 0, \\ 0 & \text{else.} \end{cases} \tag{37c}$$

$$\pi_{ij}(\{1,1\}) = \begin{cases} (1-p_1^1)(1-p_1^2) & \text{for } j_0 = i_0+1,\ j_1 = 1,\ j_2 = 1, \\ (1-p_1^1)p_1^2 & \text{for } j_0 = i_0+1,\ j_1 = 1,\ j_2 = 0, \\ p_1^1(1-p_1^2) & \text{for } j_0 = i_0+1,\ j_1 = 0,\ j_2 = 1, \\ p_1^1 p_1^2 & \text{for } j_0 = i_0+1,\ j_1 = 0,\ j_2 = 0, \\ 0 & \text{else.} \end{cases} \tag{37d}$$

In the period of maintenance component k can still fail with probability $1 - p_1^k$. Clearly for more than two components this approach becomes cumbersome and time consuming. For two components we implement this method using CPLEX 12.8.0 in Java and we show some results in Section 4. In an optimal policy we can expect the optimal action for a component to depend on the ages of both components. Example 5 illustrates this and considers three different scenarios.

Example 5. *Suppose the cost parameters are independent of the period of the year. We choose* $c_s = 5$, $c_f^k = 25$ *for* $k = 1, 2$ *and* $c_p^k = 10$ *for* $k = 1, 2$ *all in thousands of €. For the Weibull parameters we choose* $\alpha = 25$ *months and* $\beta = 2$. *Then the optimal policy is given in Figure 7. Let us denote* $a = 0$ *by no maintenance and* $a = 3$ *indicates maintenance of both components. Let* $a = 1$ *and* $a = 2$ *denote maintenance of component 1 and 2 only.*

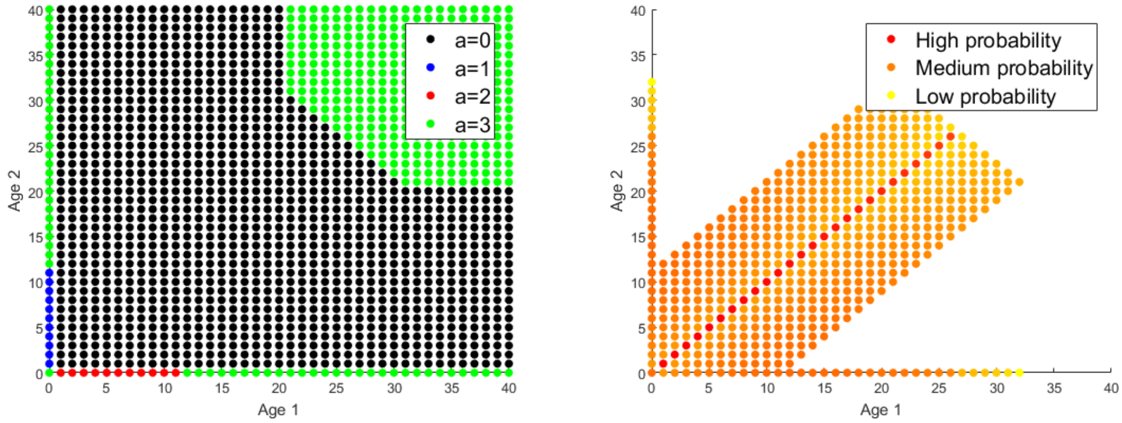


Figure 7: This figure shows the action a that we take for component ages $i_1 \in \mathcal{I}_1$ and $i_2 \in \mathcal{I}_2$. The costs are €29159 per year.

Note that if the age 0 state corresponds to a broken component, meaning that we have to maintain that component correctively. The shape of this figure is different from the figure that Özekici (1988) shows for the continuous case. The difference is that in their model you are not allowed to perform preventive maintenance on one component if the other component fails. Due to the relatively large set-up costs we never maintain one of the components preventively, when we solve the LP. This policy is very dependent on the parameters in the model and for these parameters we never maintain one of the components preventively. This changes if we adjust the set-up costs to $c_s = 1$*, which is shown in Figure 8.*

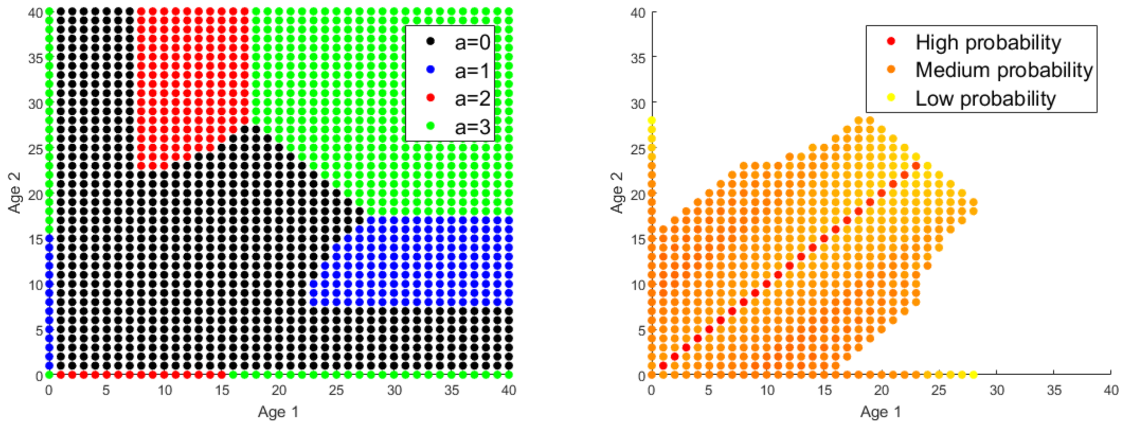


Figure 8: This figure shows the action a that we take for component ages $i_1 \in \mathcal{I}_1$ and $i_2 \in \mathcal{I}_2$. The costs are €25631 per year.

We see that it is sometimes beneficial to maintain one of the components preventively. The lower the set-up costs are, the less beneficial to maintain both components with respect to maintaining one component. If we do not include any set-up costs (i.e. set $c_s = 0$*) we obtain a completely different figure, see Figure 9 below.*

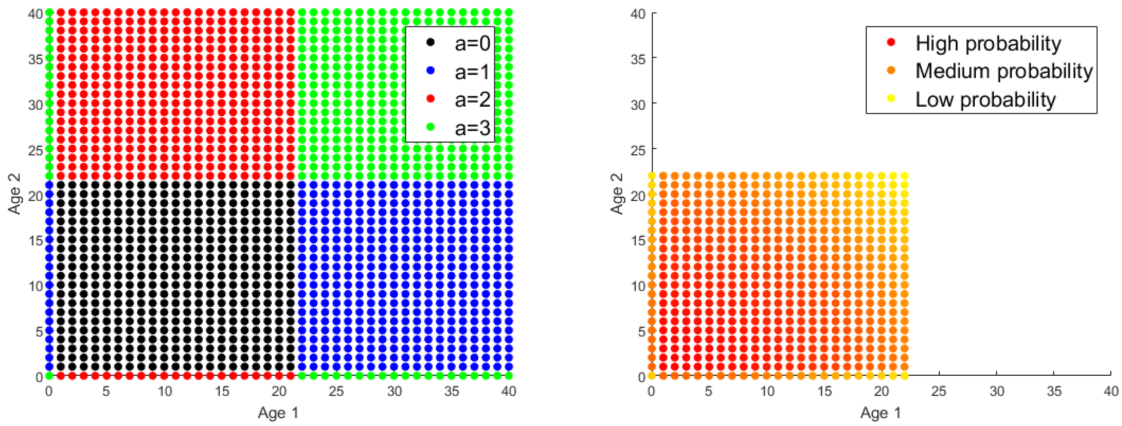


Figure 9: This figure shows the action a that we take for component ages $i_1 \in \mathcal{I}_1$ and $i_2 \in \mathcal{I}_2$. The costs are €24527 per year.

In this figure we see that the decision of maintaining a component only depends on the age of that component. The age of the other component does not influence the decision, since there are no set-up costs to be saved. We can in this case also solve this problem by solving the individual LPs for both components and this gives the same two ages.

In Example 5 we see that for two components we can still visualise when we need to maintain a component, but the decision to maintain a single component depends on the ages of the other components. In a wind park setting in which dozens of components are included in the maintenance model, this becomes hard to track what happens in the optimal policy. The decision whether or not to maintain a blade of a turbine depends on the age of the gearbox of another turbine. As a result, a single failure somewhere in the park completely changes the maintenance decision, which is not desired.

We therefore come up with simplification of this policy in Appendix E. In this simplification the maintenance decision for a component only depends on the age of this component. Since this simplified policy is sub-optimal and takes much more computation time, we will not elaborate any further on this policy and keep using the age-based maintenance policy from Section 3.4.1.

3.4.2 Block-based maintenance

In Section 3.3.2 we added constraints to the LP of the single component age-based maintenance model to obtain an MIP for the block-based maintenance model. A similar approach can be taken for multiple components. In this section we use the same formulation as in Section 3.4.1, but we introduce extra constraints to make sure that we end up with a BRP.

Similarly to Section 3.3.2 we can now introduce variables that help decide in which period we do the preventive maintenance for component k.

$$y_{i_0}^k = \begin{cases} 1, & \text{if we maintain component } k \text{ preventively in period } i_0 \in \mathcal{I}_0, \\ 0, & \text{else.} \end{cases} \tag{38}$$

If we maintain component $k \in \{1, \ldots, n\}$, we are only allowed to take action $a_k = 1$ for this component. The other state action frequencies should thus be set to zero.

The constraints that make sure that we do this are similar to equations (22d) and (22e) and for component $k \leq n$ we have the following.

$$x_{ia} \leq 1 - y_{i_0}^k \qquad \forall i \in \mathcal{I}, a \in \mathcal{A} : i_k > 0,\ a_k = 0,$$
$$x_{ia} \leq y_{i_0}^k \qquad \forall i \in \mathcal{I}, a \in \mathcal{A} : i_k > 0,\ a_k = 1$$

We obtain the following MIP by combining the linear program from Section 3.4.1 with these extra constraints.

$$\text{(BRP)} \qquad \text{minimise} \quad \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}} c_{i,a} x_{i,a} \tag{39a}$$

such that:

$$\sum_{a \in \mathcal{A}(i)} x_{ia} - \sum_{j \in \mathcal{I}} \sum_{a \in \mathcal{A}(j)} \pi_{ji}(a) x_{ja} = 0 \qquad \forall i \in \mathcal{I} \tag{39b}$$
$$x_{ia} = 0 \qquad \forall k \leq n,\ i \in \mathcal{I}, a \in \mathcal{A} : i_k \in \{0, M\}, a_k = 0 \tag{39c}$$
$$x_{ia} + y_{i_0}^k \leq 1 \qquad \forall k \leq n,\ i \in \mathcal{I}, a \in \mathcal{A} : i_k > 0,\ a_k = 0 \tag{39d}$$
$$x_{ia} - y_{i_0}^k \leq 0 \qquad \forall k \leq n,\ i \in \mathcal{I}, a \in \mathcal{A} : i_k > 0,\ a_k = 1 \tag{39e}$$
$$\sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}(i)} x_{ia} = 1 \tag{39f}$$
$$x_{ia} \geq 0 \qquad \forall i \in \mathcal{I},\ a \in \mathcal{A} \tag{39g}$$
$$y_{i_0}^k \in \{0, 1\} \qquad \forall k \leq n,\ i_0 \in \mathcal{I}_0 \tag{39h}$$

This formulation for n components can be used to schedule the maintenance optimally. When we are dealing with identical components, we know that these are scheduled at the same time in the optimal policy. In other words, the maintenance actions must be grouped, which can be forced by additional constraints that speed up computation time. We can namely add the following constraint to the problem.

$$y_{i_0}^{k_1} = y_{i_0}^{k_2} \qquad \forall i_0 \in \mathcal{I}_0, k_1 \leq n, k_2 \leq n. \tag{39i}$$

Solving this MIP is computationally very intensive for more than 2 components. Adding a component namely increases the number of decision variables by a factor of $2N$, since this gives 2 times more possible actions per state and N times more states. Therefore heuristic approaches are needed for problems with more components, such as in a wind turbine or wind park. These methods are introduced in the Section 3.5.

3.4.3 Modified block-based maintenance

In Section 3.3.3 we added constraints to the LP of the single component age-based maintenance model to obtain an MIP for modified age-based maintenance policies. A similar approach can be taken for multiple components. In this section we use the same formulation as in Section 3.4.1, but we introduce extra constraints to make sure that we end up with a modified block-based maintenance policy.

In the literature, see Archibald and Dekker (1996), this policy is discussed for constant costs and maintenance opportunities are at $T, 2T, 3T, \ldots$ for a certain time T. In their model a component is maintained only if the age is above t for $0 \leq t \leq T$. The blocks are thus the same for all components, which can be a problem if some components need to be maintained a lot more than others. Archibald and Dekker (1996) did not encounter this problem, since they focused on identical components only. In our multi-component model we can generalise this for each component k and the maintenance is only performed if the component has reached age t_k, where $0 \leq t_k \leq T$ for all k. The approach can be expected to perform well if components need to be maintained preventively approximately equally often. If this is not the case, we wish the blocks to be different for different components, which can also be included in the MIP.

Below we introduce extra variables that decide for which period and which age we maintain component k.

$$z^k_{i_0 i_k} = \begin{cases} 1, & \text{if we maintain component } k \text{ for age } i_k \in \mathcal{I}_k \text{ in period } i_0 \in \mathcal{I}_0, \\ 0, & \text{else.} \end{cases} \tag{40}$$

First of all, if we maintain component k for age i_k in a certain period i_0 then we must maintain for age $j_k > i_k$ as well in this period. We thus have that:

$$z^k_{i_0 i_k} \leq z^k_{i_0 j_k} \quad \forall i_0 \in \mathcal{I}_0 \text{ and } \forall i_k, j_k \in \mathcal{I}_k : i_k < j_k. \tag{41}$$

Clearly, we must also have that:

$$z^k_{i_0 i_k} \leq y_{i_0} \quad \forall i_0 \in \mathcal{I}_0, i_k \in \mathcal{I}_k, \tag{42}$$

where y_{i_0} is defined in equation (21) and it states whether or not we maintain in period i_0.

Since there is no constraint that makes sure that $t_i \leq T_i - T_{i-1}$, any age-based maintenance policy is now feasible. Now let $t^k_{i_0}$ be the minimum age at which we maintain component k for period i_0. Then the age must satisfy the following constraints for all components k and for all $i_0, j_0 \in \mathcal{I}_0$ with $i_0 \neq j_0$.

$$t^k_{i_0} \leq i_0 - j_0 y_{j_0} + N(1 - y_{j_0}), \qquad \text{if } j_0 < i_0, \tag{43}$$
$$t^k_{i_0} \leq N + i_0 - j_0 y_{j_0}, \qquad \text{if } j_0 > i_0. \tag{44}$$

when $y_{i_0} = 0$, $t^k_{i_0}$ can take any value, since maintenance is never performed in period $i_0 \in \mathcal{I}_0$.

The following constraints must be added to the problem to make sure that we do not maintain for ages under $t^k_{i_0}$.

$$z^k_{i_0, i_k} = \begin{cases} y_{i_0}, & \text{if } i_k \geq t^k_{i_0}, \\ 0, & \text{elsewhere.} \end{cases} \tag{45}$$

Note that this constraint is non-linear since it contains an if-statement. Using the relatively large number M (the maximum age) we can solve this and add the following linear constraints for all k, $i_0 \in \mathcal{I}_0$ and $i_k \in \mathcal{I}_k$.

$$M(y_{i_0} - z^k_{i_0 i_k}) + (1 + i_k - t^k_{i_0}) \leq M, \tag{46}$$
$$M z^k_{i_0 i_k} + (t^k_{i_0} - i_k) \leq M. \tag{47}$$

Combining these extra constraints with the linear program from Section 3.4.1 we obtain the following MIP for n components.

$$\text{(MBRP)} \qquad \text{minimise} \quad \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}} c_{i,a} x_{i,a} \tag{48a}$$

such that:

$$\sum_{a\in\mathcal{A}(i)} x_{ia} - \sum_{j\in\mathcal{I}}\sum_{a\in\mathcal{A}(j)} \pi_{ji}(a)x_{ja} = 0 \qquad \forall i \in \mathcal{I} \tag{48b}$$

$$x_{ia} = 0 \qquad \forall k \leq n,\ i \in \mathcal{I}, a \in \mathcal{A} : i_k \in \{0, M\}, a_k = 0 \tag{48c}$$

$$x_{ia} + z^k_{i_0,i_k} \leq 1 \qquad \forall k \leq n,\ i \in \mathcal{I}, a \in \mathcal{A} : i_k > 0,\ a_k = 0 \tag{48d}$$

$$x_{ia} - z^k_{i_0,i_k} \leq 0 \qquad \forall k \leq n,\ i \in \mathcal{I}, a \in \mathcal{A} : i_k > 0,\ a_k = 1 \tag{48e}$$

$$\sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}(i)} x_{ia} = 1 \tag{48f}$$

$$z^k_{i_0 i_k} - z^k_{i_0 j_k} \leq 0 \qquad \forall k \leq n,\ i_0 \in \mathcal{I}_0 \text{ and } \forall i_k, j_k \in \mathcal{I}_k : i_k < j_k \tag{48g}$$

$$z^k_{i_0 i_k} - y_{i_0} \leq 0 \qquad \forall k \leq n,\ i_0 \in \mathcal{I}_0, i_k \in \mathcal{I}_k \tag{48h}$$

$$t^k_{i_0} + j_0 y_{j_0} + N y_{j_0} \leq N + i_0 \qquad \forall k \leq n,\ i_0, j_0 \in \mathcal{I}_0 : j_0 < i_0 \tag{48i}$$

$$t^k_{i_0} + j_0 y_{j_0} \leq N + i_0 \qquad \forall k \leq n,\ i_0, j_0 \in \mathcal{I}_0 : j_0 > i_0 \tag{48j}$$

$$M(y_{i_0} - z^k_{i_0 i_k}) - t^k_{i_0} \leq M - i_k - 1 \qquad \forall k \leq n,\ i_0 \in \mathcal{I}_0, i_k \in \mathcal{I}_k \tag{48k}$$

$$M z^k_{i_0 i_k} + t^k_{i_0} \leq M + i_k \qquad \forall k \leq n,\ i_0 \in \mathcal{I}_0, i_k \in \mathcal{I}_k \tag{48l}$$

$$x_{ia} \geq 0 \qquad \forall i \in \mathcal{I},\ a \in \mathcal{A} \tag{48m}$$

$$y_{i_0} \in \{0, 1\} \qquad \forall i_0 \in \mathcal{I}_0 \tag{48n}$$

$$z^k_{i_0,i_k} \in \{0, 1\} \qquad \forall k \leq n,\ i_0 \in \mathcal{I}_0, i_k \in \mathcal{I}_k \tag{48o}$$

$$t^k_{i_0} \in \mathbb{N} \qquad \forall k \leq n,\ i_0 \in \mathcal{I}_0 \tag{48p}$$

This approach can give relatively costly solutions if one component individually has to be scheduled much more frequently or less frequently than others. It might then be the case that scheduling components on different blocks is cheaper. We can incorporate this in the model by changing from y_{i_0} to $y^k_{i_0}$. Then $y^k_{i_0}$ equals 1 if preventive maintenance is planned for component k and 0 otherwise. The rest stays the same. We do now not only have binary decision variables that decide at which days we maintain but we also have binary decision variables that decide from which age we have to maintain. As a result, this MIP is even more time-consuming than the MIP for the BRP, which will be shown in the results in Section 4. In the results we consider the MBRP as in MIP formulation (48) and the MBRP alternative in which we allow the maintenance blocks to be different for different components.

3.5 Heuristic approaches for block-based maintenance policies for multiple components

As we have seen in the previous sections, we can formulate our maintenance problem using an LP for age-based maintenance and MIP for (modified) block-based maintenance. The age-based maintenance policies for multiple components are difficult for practitioners, since the decision whether or not to maintain a component depends on the ages of all components and can change if a failure occurs. The block-based methods are very time-consuming when we optimise for more than 2 components, but convenient for maintenance teams. In this section we come up with three heuristic approaches for the block-based maintenance. In Section 3.5.1 we introduce an approach to schedule the maintenance in a sequential manner. We simply schedule the maintenance for the components one after the other. In Sections 3.5.2 and 3.5.3 we show how we can use a genetic and memetic algorithm to find (close to) optimal policies.

3.5.1 Sequential optimisation

In many applications sequential optimisation can be used to come up with a sub-optimal policy. For wind turbine maintenance we can first schedule one of the components, then the next one and proceed until all components are scheduled. Each component can be scheduled with minimal costs, given the earlier scheduled component. This heuristic approach is fast, but does not give a global optimum.

In this heuristic we first schedule the maintenance for one of the components, using formulation (22). Next we schedule a new component given the policy that we have for the other component. We simply set the set-up costs to zero for periods in which maintenance is done already. Due to these lower cost coefficients, it will be more likely that we schedule during these cheaper periods and that we save set-up costs. We proceed till all components are scheduled. For each component we then thus solve an MIP with Nm binary decision variables. m is the number of years considered and N is the number of time periods in a year that follows from the discretisation. We still have to decide in which order we optimise the components and we consider three possible ordering criteria.

(SF) Schedule the item with largest frequency first.

(SR) Schedule the item with smallest frequency first.

(SC) Schedule the most expensive item first.

To determine the sequence of optimisation we thus have to find the optimal maintenance interval for each item individually. We are free to choose whether or not to include set-up costs, but this can influence the sequence and therefore the end result. Since there are typically a lot of components in a wind park, the set-up costs are typically divided over a lot of components and we do not include them in making the sequence.

In the model we assume that we save set-up costs when two or more components are maintained in a period, except if all component are maintained correctively. If more than one component fails in the period of preventive maintenance, we do however save the set-up costs in this computation. We thus underestimate the cost in this calculation. This can be adjusted by adding the long-run fraction of time that this happens multiplied by the set-up costs. Since the deterioration processes of all components are assumed to be independent we can use the long-run fractions of each component. The resulting sequential optimisation is summarised in Algorithm 1.

Algorithm 1 (Sequential optimisation)**.**

Step 0. *Schedule all items individually, using MIP formulation* (22)*.*

Step 1. *List the components according to the ordering criterion.*

Step 2. *Schedule the first component from the list, using MIP formulation* (22)*. Remove the component from the list and update the costs. If there are still components to be scheduled go to Step 3. Otherwise we are finished.*

Step 3. *Set the set-up costs to zero for periods at which preventive maintenance is scheduled, implement this in the MIP formulation for the other components and go to Step 2.*

All three methods will be implemented using CPLEX 12.8.0 in Java and results will be presented in Section 4. We compare the costs with the costs of the optimal age-based and block-based maintenance policies for 2 components. For more components we will only use the heuristic and compare the results of the three different possibilities. We can also compare the results with the genetic and memetic algorithm that are explained in the next two sections.

3.5.2 Genetic algorithm with local search

Genetic algorithms (GA) are metaheuristics inspired by nature, where good individuals (in our algorithm solutions) are combined to create new individuals. In a genetic algorithm we represent solutions by chromosomes and we generate new chromosomes from these chromosomes. Lower cost solutions correspond to fitter individuals, which are more likely to survive and recombine for the next generation. Using these principles we wish to have a pool of a lot of low cost solutions after some generations, potentially with the optimal solution.

In our problem we define the chromosome to be a binary vector of genes of length $l = Nm$. N is the number of time periods in a year and m the number of years after which the policy repeats itself, recall equation (23). Every gene corresponds to a time period and genes can have value 1 and 0 only. Every 1 indicates a maintenance opportunity and every 0 indicates that we are not allowed to perform preventive maintenance for the corresponding time period. For a chromosome $z \in \{0,1\}^l$ we have that

$$z_{i_0} = \begin{cases} 1, & \text{if we have an opportunity of maintaining preventively in period } i_0, \\ 0, & \text{else.} \end{cases} \tag{49}$$

Using the MIP formulation (22) we can schedule the preventive maintenance for each component individually. We simply add the constraint $y_{i_0} = 0$ for all i_0 with $z_{i_0} = 0$ and solve. This prevents components from being scheduled on those zeros. For each component we thus solve the MIP with $\sum_{w=1}^{l} z_w$ binary variables, which is relatively fast.

Let c_k denote the maintenance costs of component k, where we ignore the set-up costs. The total amount of set-up costs that is paid equals $c_s \sum_{i_0=1}^{l} z_{i_0}$ and the resulting costs of this chromosome z is simply the sum of the set-up costs and the individual maintenance costs of the K components.

$$c(z) = \sum_{c=1}^{K} c_k + c_s \sum_{i_0=1}^{l} z_{i_0} + e(z), \tag{50}$$

where $e(z)$ corresponds to a slight underestimation of the costs. The set-up costs are always paid once if preventive maintenance is planned. If more components however fail in the same time period, we do have to pay the set-up costs multiple times. Components breaking down at the same time does not happen often in our model, since components fail independently with low probabilities. Let $b_k(i_0)$ denote the failure probability of component k in period i_0. For two components we have:

$$\begin{aligned} e(z) &= \frac{c_s}{m} \sum_{i_0 \in \mathcal{I}_0} z_{i_0} b_1(i_0) b_2(i_0), \\ &= \frac{c_s}{m} \sum_{i_0 \in \mathcal{I}_0} z_{i_0} \left((1 - b_1(i_0))(1 - b_2(i_0)) - 1 + b_1(i_0) + b_2(i_0) \right). \end{aligned}$$

We might expect that for the three component model we can simply replace $b_1(i_0)b_2(i_0)$ by $b_1(i_0)b_2(i_0) + b_1(i_0)b_3(i_0) + b_2(i_0)b_3(i_0)$. Then we however make an overestimation of the cost. We namely add the set-up costs three times extra for the case in which all three components fail and we should have only added two. For three components we obtain the following.

$$\begin{aligned} e(z) &= \frac{c_s}{m} \sum_{i_0 \in \mathcal{I}_0} z_{i_0} \left(b_1(i_0)b_2(i_0) + b_1(i_0)b_3(i_0) + b_2(i_0)b_3(i_0) - b_1(i_0)b_2(i_0)b_3(i_0) \right), \\ &= \frac{c_s}{m} \sum_{i_0 \in \mathcal{I}_0} z_{i_0} \left((1 - b_1(i_0))(1 - b_2(i_0))(1 - b_3(i_0)) - 1 + b_1(i_0) + b_2(i_0) + b_3(i_0) \right). \end{aligned}$$

Using induction we can show that for a general setting with k components we obtain the following.

$$e(z) = \frac{c_s}{m} \sum_{i_0 \in \mathcal{I}_0} z_{i_0} \left((1 - b_1(i_0)) \cdots (1 - b_k(i_0)) - 1 + b_1(i_0) + \cdots + b_k(i_0) \right). \tag{51}$$

For a chromosome a larger fitness means a larger probability of surviving and being a parent for the next generation and the function that we will use is $f(z) = \frac{1}{c(z)}$. Lower cost solutions then have larger fitness and a larger probability of surviving. There are several ways to choose the parents and we will simply pick the fittest a. We determine a, such that we have enough variety, but computation time is not too large.

To create a chromosome from two parent chromosomes, we use cross-over. We randomly choose a period $w \in \{1, 2, \cdots, l\}$ with equal probabilities. The new chromosomes have the genes of one parent for the first w periods and the genes of the second parent for other periods. Below we give an example of two possible cross-overs from two parents.

Parent 1:	0 0 1 1 1 $\cdots$ 0
Parent 2:	1 0 0 1 0 $\cdots$ 0
Child 1:	0 0 1 1 0 $\cdots$ 0
Child 2:	1 0 0 1 1 $\cdots$ 0

After the cross-over, mutation takes place. For each gene the probability of mutation is the same and we denote this probability by ρ. Below we show the possible chromosomes of the two children after mutation.

Child 1:	0 0 1 1 0 $\cdots$ 0
Child 2:	0 0 1 1 1 $\cdots$ 0

There has only been a mutation in the first gene of child 2. We do this for many combinations and can compute the fitness for each child. The strongest n are selected for the creation of the next generation and this procedure is repeated, until a stopping criterion is met. We will stop the procedure if there is no change in the best objective value for k iterations in a row (we choose $k = 3$). The starting population depends on the cost and distribution parameters and we will explain this in Section 3.5.4.

Given a finite number of iterations, the genetic algorithm does not always find the optimal solution. A local search at the end of the procedure increases the probability of finding an optimal solution. We perform this local search on the 10 best solutions. In this local search we evaluate the fitness for each neighbour of these 10 chromosomes. We define the set of neighbours of a chromosome z to be the following:

$$\mathcal{N}_z = \{z' : \forall i \in \{1, \cdots, l\} \text{ with } z_i = 1 \ \exists! \ j \in \{1, \cdots, l\} \text{ with } |(i - j) \bmod l| \leq 1\}.$$

The neighbours of a chromosome z that has $l = 52$, $z_w = 1$ for period $w = 1$ and period $w = 30$ and are then all the chromosomes that have $z_w = 1$ for period $w = 52$, $w = 1$ or $w = 2$ and period $w = 29$, $w = 30$ or $w = 31$. This thus gives 9 neighbours in total. A chromosome has 3^k neighbours, where k is the number of 1's in the chromosome (as long as a chromosome does not have two maintenance opportunities in two successive periods). Note that solutions that do have two maintenance opportunities in successive periods typically have low fitness so these will not occur in the population very often. If $k \geq 3$ we do not consider all neighbours, since this would become computationally very intensive. We only consider the neighbours for which only one gene or all genes are shifted by one period.

We summarise the procedure in Algorithm 2.

Algorithm 2 (Genetic algorithm)**.**

Step 0. *Determine the starting parents and compute the fitness of every individual.*

Step 1. *Use the cross-over and mutation to create the $A - a$ children.*

Step 2. *Add the children to the population and determine their fitness.*

Step 3. *Select the a fittest individuals to be the parents and go to Step 1. (Go to Step 4 if there is no improvement in fitness for k iterations.)*

Step 4. *Determine the neighbours of the ten fittest individuals and compute their fitness.*

We will show the results of the algorithm in Section 4 and compare to the results from Section 3.5.1.

3.5.3 Memetic algorithm

To improve the solutions obtained from the genetic algorithm or to speed up computation time we introduce a memetic algorithm (MA). This memetic algorithm is a genetic algorithm, where we use the local search step in each iteration of the algorithm instead of at the end only. In this local search we evaluate the fitness for each neighbour of the ten fittest chromosomes.

Algorithm 3 summarises the procedure for the memetic algorithm.

Algorithm 3 (Memetic algorithm)**.**

Step 0. *Determine the starting parents and compute the fitness of every individual.*

Step 1. *Use the cross-over and mutation to create the $A - a$ children.*

Step 2. *Add the children to the population and determine their fitness.*

Step 3. *Select the fittest p children and compute the fitness of their neighbours. For these p fittest childs add the fittest neighbour to the population.*

Step 4. *Select the a fittest individuals to be the parents and go to step 1. (Stop if there is no improvement in fitness for k iterations.)*

We will show the results of the algorithm in Section 4 and compare to the results from Sections 3.5.1 and Section 3.5.2.

3.5.4 Parameter choices in memetic and genetic algorithm

In the MA and GA there are many parameters that we can tune. We choose the mutation rate to be $\frac{0.1}{|\mathcal{I}_0|}$, such that the probability of a mutation in a chromosome is approximately 10%. The population size $A = \min\{300, 4a\}$ in which a equals the number of parents. a is set to be equal to the number of starting parents. The starting parents depend on the problem.

To determine the starting population, we first compute the optimal maintenance times for all single components with and without set-up costs under constant costs. Let t^- and t^+ denote the minimum and maximum of these maintenance times. Consider the problem in which we have l genes. Then the starting population are the solutions with $s^- = \max\{1, \lfloor \frac{l}{t^+} \rfloor\}$ till $s^+ = \lceil \frac{l}{t^-} \rceil$ maintenance opportunities, where the opportunities are equally spaced. In Example 6 this is illustrated.

Example 6 (Starting population)**.** *Suppose that we have four components for which we wish to determine the starting population of the MA and GA with $N = 12$ months and $m = 4$. Suppose we have the following optimal individual maintenance times for the components.*

Table 1: Optimal maintenance time T^* for $\Delta = 0$.

Component	Without set-up costs	With set-up costs
1	29	36
2	59	82
3	20	25
4	32	45

Then $l = Nm = 48$ and we have $s^- = \max\{1, \lfloor \frac{48}{82} \rfloor\} = 1$ and $s^+ = \lceil \frac{48}{20} \rceil = 3$. The starting chromosomes are the 48 chromosomes with 1 maintenance opportunity, the 24 chromosomes with 2 maintenance opportunities every 24 months, and the 16 chromosomes with 3 maintenance opportunities every 16 months. The total starting population consists of $a = 88$ individuals and the total population after each iterations is then $A = 300$. The mutation rate is $\rho = \frac{0.1}{48} = 0.0021$.

3.6 Heuristic approaches for modified block-based maintenance policies for multiple components

In the heuristics we discuss above we optimise for single components given that we know the maintenance opportunities in specific time periods. These opportunities correspond to the 1 genes in the MA and GA. Determining these opportunities is the difficult part, which is solved by evolution.

We could try a similar approach for the MBRP, but encounter a problem. If neither component is maintained, since the critical age is not reached, we still pay the set-up costs in our cost computation. We thus need to subtract the long-run fraction of time this happens multiplied by the set-up costs, otherwise we overestimate the costs under this policy. The problem is that this long-run fraction of time is difficult to compute. This quantity is related to the probability that no component needs maintenance. The individual Markov chains are correlated, so we cannot simply use the individual probabilities to compute the probability of no maintenance. As a result the approach in which we split computing the set-up costs and all the individual maintenance costs does not work. It might be interesting to consider such a method and accept that we make a mistake in the cost computation or use a correlation parameter, that estimates the dependence between failures of different components. Using simulations we could then compute the costs under the MBRP.

Another approach that we could use is scheduling the moments of maintenance using the MA and GA for the BRP. In the results for one and two components we see that the maintenance moments for the optimal BRP and MBRP often coincide, see Section 4. Using the preventive maintenance moments of the optimal BRP could be a good starting step for a MBRP heuristic. After this we can determine the minimal ages after which we perform maintenance. We could for example take this age to be half the block time for all blocks. A simulation can be used to approximate the long-run costs of this resulting MBRP.

4 Results

In this section we present the results of the introduced ARP, BRP and MBRP for time-varying costs. Since we want to make general statements on the performance of the model, we compute the policies and associated costs for many different cost functions. In Section 4.1 we show the results for some single component systems. In Section 4.2 we show the results for some multi-component systems. We also compare the performance of the heuristics with the exact results in the two component setting. At last in Section 4.3 we apply our model to an offshore wind park setting with 10 9.5 MW turbines. In each scenario we compare to the costs of the ARP, BRP and MBRP obtained under the assumption that costs are constant.

4.1 Single component results

In this section we show results of the optimal policies for some single component system. We show the improvement that we make with respect to the standard ARP, BRP and MBRP in our model. In this comparison we compute the percentage cost savings for each policy and for different Δ. The ARPs are compared to the standard ARP from the literature. Similarly the BRPs and MBRPs are compared to the standard BRP and standard MBRP from the literature. To be able to make more general statements on the performance of our model, we first show the results for different cost functions and constant lifetime distributions in Section 4.1.1. In Section 4.1.2 we also vary the lifetime distribution parameters for different cost functions. Some special cost function are considered in Section 4.1.3.

4.1.1 Comparison for different cost functions

In this section we present the results for different cost functions. We do this to get an idea for which scenarios our model saves most costs compared to assuming that maintenance costs are constant. We compute the costs of the optimal ARP, BRP and MBRP. We compare the costs of these policies with the costs of policies obtained from the models that assume constant costs (i.e. the standard ARP, BRP and MBRP). The parameters of the Weibull distribution are chosen to be $\alpha = 1$ year and $\beta = 2$. Using equations (3) and (4) we find for the lifetime distribution that $\mu = 11.13$ months, $\sigma = 5.57$ months, corresponding to a coefficient of variation of $c_T = 0.500$.

We test the model on cosine cost function for the preventive and corrective maintenance. The seasonal pattern in preventive and corrective maintenance costs linearly depends on the seasonal pattern of the power output. Historical data shows that this closely resembles a cosine, see Figure 13. Note that we could also use the historical data directly, but then we might be overfitting.

The costs c_p and c_f then take the following form.

$$c_p(t) = \bar{c}_p + \Delta_p \cos\left(\frac{2\pi t}{N} + \phi\right), \tag{52}$$

$$c_f(t) = \bar{c}_f + \Delta_f \cos\left(\frac{2\pi t}{N} + \phi\right), \tag{53}$$

where t is the time of the year and a phase shift ϕ makes sure that the peak is at the right time of the year. $N = 12$ is the number of decision epochs in the year. Decisions are thus made on a monthly basis to keep computation time limited. We choose $\phi = -\frac{2\pi}{12}$, which ensure that the windy winter months are most expensive and the calmer summer months are cheaper. In this case the most expensive month is January and the cheapest month is July. The exact value of ϕ is not interesting for the results, in the sense that maintenance periods will simply be shifted if ϕ is changed.

The costs of the optimal maintenance policies are given for $\bar{c}_p = 10$ in Tables 2, 3 and 4, where $\bar{c}_f = 20$, $\bar{c}_f = 50$ and $\bar{c}_f = 100$ respectively. The cost fluctuations differ and Δ shows how much cost fluctuations we have. If $\Delta = 50\%$ the minimum and maximum costs deviate 50% from the mean cost. Then there is a factor of 3 ($= \frac{150\%}{50\%}$) between minimum and maximum expected maintenance costs every year. In the Netherlands the expected missed income does not fluctuate this much and so Δ will be lower if we consider a wind turbine example, such as in Section 4.3.

In the tables we give the costs for the optimal ARP, BRP and MBRP and the policies for the BRP and MBRP. The ARP policies are illustrated in figures below the tables, see Figure 10, 11 and 12. For the $\Delta = 0\%$ case, there is no unique solution, since we can shift the maintenance months in the policy without changing the costs. The exact months of maintenance can for this case be ignored, but the time difference between maintenance moments is important. In the Tables we compare the costs with the costs of the standard ARP, MBRP and BRP (i.e. the case $\Delta = 0\%$). Assuming we would have no cost fluctuations this gives us a policy with the same long-run average costs as this $\Delta = 0\%$ case.

We also give the costs and savings of the optimal ARP, BRP and MBRP with $m = 1$ from equation (23). We have considered $m \in \{1, 2, \cdots, 8\}$, but $m = 1$ gives the best solution for the three parameter sets considered in this section. We also give the percentage improvement of our model compared to the standard ARP, BRP and MBRP. Below the tables we give the optimal maintenance age and block for the constant cost ARP, BRP and MBRP. We also compute the costs obtained when no maintenance is performed, using equation (14).

Table 2: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We have $\bar{c}_f = 20$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ and $\alpha = 1$ year, $\beta = 2$.

	ARP		BRP ($m=1$)			MBRP ($m=1$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	21.029		21.554		-	21.167		8	11
10%	20.757	1.29%	21.554	0%	-	20.792	1.77%	8	8
20%	20.303	3.45%	21.554	0%	-	20.323	3.98%	8	7
30%	19.768	5.99%	20.925	2.92%	8	19.776	6.57%	8	6
40%	19.151	8.93%	19.966	7.37%	8	19.151	9.52%	8	5
50%	18.454	12.24%	19.008	11.81%	8	18.454	12.81%	8	4
CPU	< 1 s		< 1 s			< 1 s			

*Optimal age for $\Delta = 0$ is $T^* = 14$ months with costs 21.029.

**There is no optimal block time for $\Delta = 0$.

***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (11, 12)$ with costs 21.167.

****Costs for not maintaining at all is $C(\infty) = 21.554$.

In the BRP ($m = 1$) from Table 2 we always maintain once per year and always in the same month. The cheapest month in the year is July, but maintenance is planned in August to make sure that the probability of failures is lower in the windy winter months. For $\Delta = 30\%,\ 40\%, 50\%$ the maintenance costs increase linearly in Δ, since we maintain in the same month every time. We see that the increase is 4.45% for every time Δ increases with 10%. For the other policies this increase is faster than linear.

We should note that the costs for not maintaining are lower than the costs for the BRP for low cost fluctuations, $\Delta \leq 20\%$. For these parameters the conditions in Theorems 2 or 3 are not satisfied. This indicates that it might be better not to maintain than to come up with a policy in which maintenance is done once a year. We see that an increase to $m = 2, 3, \ldots, 8$ indeed improves the solution for smaller fluctuations, but still not maintaining is cheaper.

The ARP is the cost optimal policy and the BRP the most expensive for all parameters. The MBRP is much closer to the ARP than to the BRP and is even closer to optimal for larger fluctuations. For $\Delta \geq 40\%$ the ARP gives an MBRP, which is therefore optimal as well. In Figure 10 we show how the ARP changes if the fluctuations in costs increase.

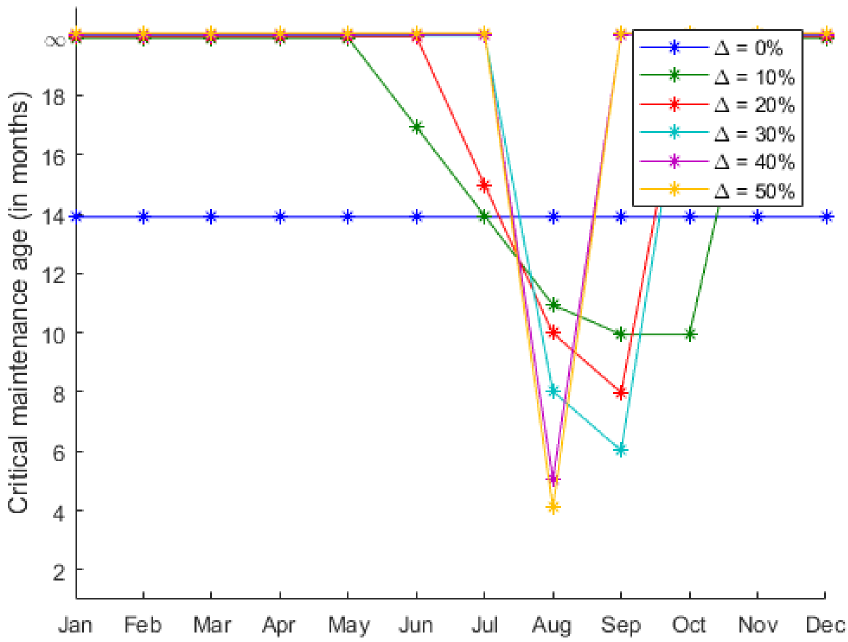


Figure 10: This figure shows the critical maintenance age over the year for the ARP for $\bar{c}_f = 2\bar{c}_f$, $\alpha = 1$ year and $\beta = 2$. ∞ indicates that we do not maintain for any age in this month.

The cheapest month to maintain is July and the most expensive is January. In Figure 10 we see that most preventive maintenance is performed just after this minimum in August, when the maintenance costs are still low, but have started to increase. By doing this we decrease the probability of failure in the more expensive period after this. We see that as the policies shrink in the x-axis towards only a few months as Δ increases, these are the relatively cheap months. The policies extend in the y-axis, since the maintenance ages vary more if Δ increases. Note that we never maintain in the windy winter months for cost fluctuation greater than 10%. After October for $\Delta = 10\%$ and after September for $\Delta = 20, 30\%$ the preventive maintenance is too expensive and it is worth the risk to wait till next year. For $\Delta = 40\%, 50\%$ we maintain only in August if the critical maintenance age is reached, which results in a MBRP policy. The MBRP is thus optimal for these parameters with $\Delta = 40\%$ or $\Delta = 50\%$.

In Table 3 we show the results for the case where $c_f = 5c_p$.

Table 3: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We have $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ.

	ARP		BRP ($m = 1$)			MBRP ($m = 1$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	40.098		41.501		6, 12	40.311		6, 12	4, 4
10%	40.035	0.16%	41.420	0.20%	6, 11	40.263	0.12%	6, 11	4, 4
20%	39.701	0.99%	40.933	1.37%	6, 11	39.855	1.13%	6, 11	4, 4
30%	39.224	2.18%	40.361	2.75%	6, 10	39.338	2.41%	6, 10	5, 3
40%	38.461	4.08%	39.439	4.97%	6, 10	38.556	4.35%	6, 10	5, 3
50%	37.635	6.14%	38.466	7.31%	7, 10	37.773	6.30%	6, 10	5, 3
CPU	< 1 s		< 1 s			3 s			

*Optimal age for $\Delta = 0$ is $t^* = 6$ months with costs 40.098.
**Optimal block for $\Delta = 0$ is $T^* = 6$ months with costs 41.501.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (4, 6)$ with costs 40.311.
***Costs for not maintaining at all is $C(\infty) = 53.885$.

In this table we see that all policies are better than the zero maintenance policy for all cost fluctuations. Preventive maintenance is performed more frequently than in Table 2, since the corrective maintenance is relatively expensive compared to preventive maintenance. We do therefore not maintain preventively only on cheap moments and cost savings are smaller. Again the costs savings are larger when there are larger fluctuations over the year. An increase in m to $m = 2, 3, \ldots, 8$ does not improve the costs and gives the same policy as obtained with $m = 1$. In Figure 11 we show how the ARP changes if the cost fluctuations increase.

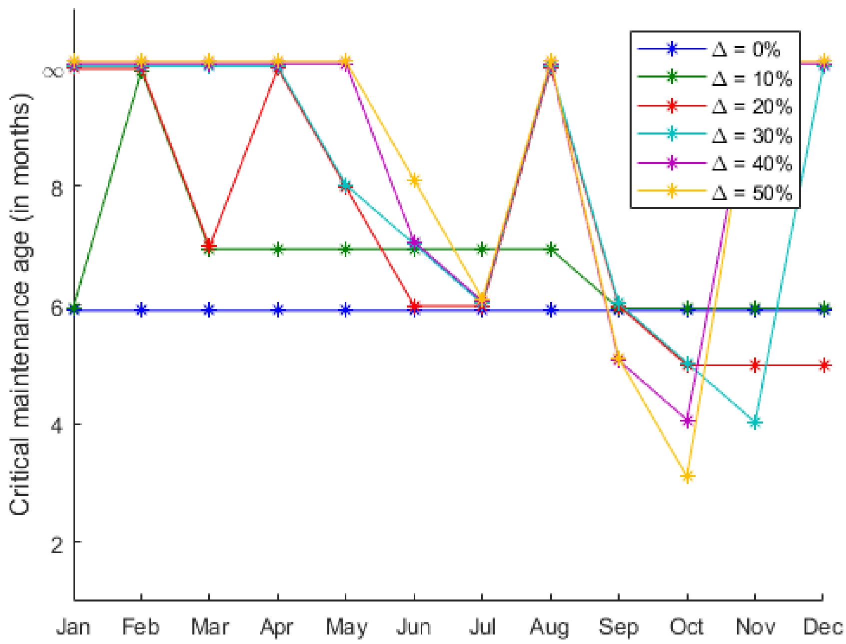


Figure 11: This figure shows the critical maintenance age over the year for the ARP for $\bar{c}_f = 5\bar{c}_f$, $\alpha = 1$ year and $\beta = 2$. ∞ indicates that we do not maintain for any age in this month.

The optimal maintenance age is $t^* = 6$ months for the constant cost case and start to fluctuate more over the year if cost fluctuations increase. This maintenance age of 6 months indicates that we wish to maintain preventively multiple times a year. If the fluctuations increase there are more months that we skip completely for preventive maintennace. For $\Delta = 10\%$ we skip one month. For $\Delta = 20\%, 30\%, 40\%, 50\%$ we skip 3, 6, 8 and 8 months. The the policy does not shrink towards one x-value if Δ increases, but towards two. In Table 17 we show this for $\Delta = 100\%$.

For $\Delta = 50\%$ we maintain only in June, July, September and October, for the ages of 8, 6, 5, 3. We do not maintain in August. If the age is 5 or 6 months in August, we would rather wait till the more expensive September for maintenance, such that a failure in winter is less probable. If a component is maintained in July, it will reach the critical age of 3 in October if it does not fail. It will therefore be maintained again in October to make winter failures less probable. For $\Delta = 10\%$ we maintain for the age of 7 months in spring and summer and for the age of 6 months in the winter months. In February no maintenance is performed.

In Table 4 we show the results for the case where $c_f = 10c_p$.

Table 4: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We have $\bar{c}_f = 100$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ.

	ARP		BRP ($m = 1$)			MBRP ($m = 1$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	59.812		60.549		4, 8, 12	59.987		3, 7, 11	3, 3, 3
10%	59.812	0.00%	60.549	0.00%	3, 7, 11	59.981	0.01%	4, 8, 12	3, 2, 3
20%	59.777	0.06%	60.548	0.00%	3, 7, 11	59.960	0.05%	4, 8, 12	3, 2, 3
30%	59.517	0.49%	60.241	0.51%	1, 6, 9	59.646	0.57%	1, 6, 9	3, 3, 2
40%	59.101	1.19%	59.777	1.28%	1, 6, 9	59.183	1.34%	1, 6, 9	3, 3, 2
50%	58.567	2.08%	59.113	2.37%	6, 9, 12	58.659	2.21%	6, 9, 12	3, 2, 3
CPU	< 1 s		< 1 s			3 s			

*Optimal age for $\Delta = 0$ is $t^* = 4$ months with costs 59.812.
**Optimal block for $\Delta = 0$ is $T^* = 4$ months with costs 60.549.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (3, 4)$ with costs 59.987.
****Costs for not maintaining at all is $C(\infty) = 107.77$.

Again we see that all three policies are always better than the zero maintenance policy. The same behaviour is observed as in Tables 2 and 3, but the cost savings are smallest for Table 4. When there is a big difference between c_f and c_p, we maintain more times in a year, meaning that we must also maintain at more expensive moments in the year, which balances out the savings. For both policies and all cost coefficients we see that the savings become larger when the fluctuations in the costs are larger. If the costs fluctuate more we are more flexible in choosing a cheap moment for preventive maintenance or choosing a moment that makes sure that the probability of failure is low when corrective maintenance is expensive. When the corrective maintenance costs become very large with respect to the preventive maintenance costs the cost savings become smaller. We then namely maintain a lot more during the year and it becomes more difficult to choose cheap maintenance opportunities.

In Figure 12 we show how the ARP changes if the cost fluctuations increase.

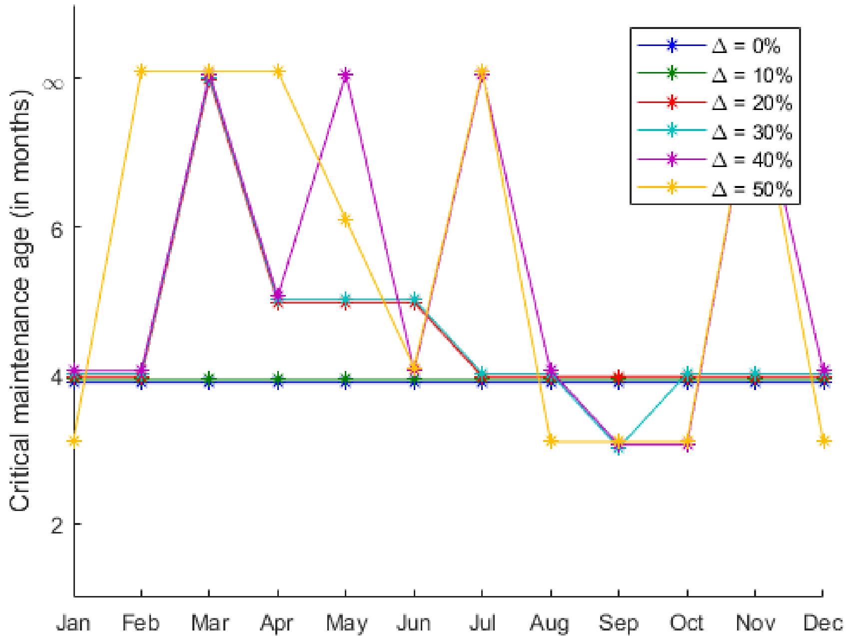


Figure 12: This figure shows the critical maintenance age over the year for the ARP for $\bar{c}_f = 10\bar{c}_f$, $\alpha = 1$ year and $\beta = 2$. ∞ indicates that we do not maintain for any age in this month.

The optimal maintenance age is $t^* = 4$ months for the constant cost case and for the $\Delta = 10\%$ case the policy and costs remain the same. As Δ increase more the maintenacne age starts to fluctuate more over the year. This maintenance age of 4 months indicates that we wish to maintain preventively multiple times a year. If the fluctuations increase there are more months that we skip completely for preventive maintennace. For $\Delta = 10\%$ we skip no months. For $\Delta = 20\%, 30\%, 40\%, 50\%$ we skip 1, 1, 4 and 5 months, respectively. The the policy does not shrink towards one x-value if Δ increases, but towards three. In Table 17 we show this for $\Delta = 100\%$.

4.1.2 Comparison for different lifetime distributions

In this section we also show the effect of the lifetime distribution for different α and β for different cost fluctuations and different preventive versus corrective maintenance cost ratios. First we do this for relatively low, than for medium and at last for relatively high corrective maintenance costs.

Relatively low corrective maintenance costs. In this section we show the results for the case in which the corrective maintenance costs are 2 times larger than the preventive maintenance costs. In Tables 5, 6, 7 and 8 we give the results for $\bar{c}_f = 20$ and $\bar{c}_p = 10$.

Table 5: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We have $\bar{c}_f = 20$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ and $\alpha = 1$ year, $\beta = 2$.

	ARP		BRP ($m = 1$)			MBRP ($m = 1$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	21.029		21.554		-	21.167			
10%	20.757	1.29%	21.554	0%	-	20.792	1.77%	8	8
20%	20.303	3.45%	21.554	0%	-	20.323	3.98%	8	7
30%	19.768	5.99%	20.925	2.92%	8	19.776	6.57%	8	6
40%	19.151	8.93%	19.966	7.37%	8	19.151	9.52%	8	5
50%	18.454	12.24%	19.008	11.81%	8	18.454	12.81%	8	4
CPU	< 1 s		< 1 s			< 1 s			

*Optimal age for $\Delta = 0$ is $T^* = 14$ months with costs 21.029.
**There is no optimal block time for $\Delta = 0$.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (11, 12)$ with costs 21.167.
****Costs for not maintaining at all is $C(\infty) = 21.554$.

Table 6: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We have $\bar{c}_f = 20$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ and $\alpha = 1$ year, $\beta = 3$.

	ARP		BRP ($m = 1$)			MBRP ($m = 3$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	19.245		21.398		-	19.494			
10%	19.007	1.24%	21.031	1.75%	8	19.389	0.54%	4, 12, 21, 31	8, 7, 6, 6
20%	18.401	4.39%	19.856	7.21%	8	18.671	4.22%	8, 20, 32	4
30%	17.634	8.37%	18.680	12.70%	8	17.810	8.64%	8, 20, 32	4
40%	16.760	12.91%	17.504	18.19%	8	16.896	13.32%	8, 20, 32	3
50%	15.866	17.56%	16.328	23.69%	8	15.926	18.30%	8, 20, 32	3
CPU	< 1 s		< 1 s			9 s			

*Optimal age for $\Delta = 0$ is $T^* = 10$ months with costs 19.245.
**There is no finite optimal block time for $\Delta = 0$.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (7, 9)$ with costs 19.494.
****Costs for not maintaining at all is $C(\infty) = 21.398$.

Table 7: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We let $\bar{c}_f = 20$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ and $\alpha = 3$ years, $\beta = 2$.

	ARP		BRP ($m = 2$)			MBRP ($m = -, 3, 2, 2, 2, 2$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	7.183		7.406		-	7.236			
10%	7.042	1.96%	7.406	0%	-	7.114	1.69%	7	23
20%	6.860	4.50%	7.406	0%	-	6.947	3.99%	7	23
30%	6.611	7.96%	7.223	2.47%	7	6.640	8.24%	7	18
40%	6.260	12.85%	6.726	9.18%	7	6.307	12.84%	7	15
50%	5.881	18.13%	6.229	15.89%	7	5.946	17.83%	7	11
CPU	< 1 s		< 1 s			11 s			

*Optimal age for $\Delta = 0$ is $t^* = 40$ months with costs 7.183.
**There is no finite optimal block time for $\Delta = 0$.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (29, 35)$ with costs 7.236.
****Costs for not maintaining at all is $C(\infty) = 7.406$.

Table 8: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We let $\bar{c}_f = 20$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ and $\alpha = 3$ years, $\beta = 3$.

	ARP		BRP ($m = 2$)			MBRP ($m = 2$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	6.516		7.351		-	6.607			
10%	6.280	3.62%	6.956	5.37%	7	6.309	4.51%	7	17
20%	5.885	9.68%	6.450	12.26%	7	5.931	10.23%	7	15
30%	5.477	15.95%	5.944	19.14%	7	5.541	16.13%	7	12
40%	5.057	22.39%	5.438	26.02%	7	5.138	22.23%	7	10
50%	4.623	29.05%	4.932	32.91%	7	4.722	28.53%	7	9
CPU	< 1 s		< 1 s			4 s			

*Optimal age for $\Delta = 0$ is $T^* = 29$ months with costs.
**There is no finite optimal block time for $\Delta = 0$.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (18, 27)$ with costs 6.607.
****Costs for not maintaining at all is $C(\infty) = 7.351$.

If we perform maintenance on a yearly basis, we can simply choose the moment that induces the least costs. If we maintain more frequently we need to maintain at more expensive moments as well, meaning that the cost savings will be less. Therefore, as α increases, so do the cost savings, which we can observe in the tables above. We then also perform less preventive maintenance, since there are less failures for larger α. Clearly the costs itself thus decrease when α increases, which we see in the tables as well.

The total maintenance costs for $\beta = 3$ are lower than for $\beta = 2$ for all cost functions and policies. An increase in β makes the failures more predictable. As a result we can better anticipate failures and perform more preventive maintenance, which decreases the probability of corrective maintenance, making this cheaper. Also the savings are larger for $\beta = 3$ than for $\beta = 2$ in most scenarios.

In all scenarios that we considered above we have that there exists no finite optimal block time for the constant costs BRP, if the fluctuations increase this changes. Still maintenance is not planned a lot and thus we can plan on the relatively cheap moments. The cost saving are therefore relatively large compared to the base case $\Delta = 0$. The MBRP bridges most of the gap between the BRP and ARP, over 75% in most scenarios.

Moderate corrective maintenance costs. In this section we show the results for the case in which the corrective maintenance costs are 5 times larger than the preventive maintenance costs. In Tables 9, 10, 11 and 12 we give the results for $\bar{c}_f = 50$ and $\bar{c}_p = 10$. Then we will maintain more frequently and might also need to maintain on expensive moments, lowering the cost savings.

Table 9: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We let $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ. We let $\alpha = 1$ and $\beta = 2$.

	ARP		BRP ($m = 1$)			MBRP ($m = 1$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	40.098		41.501			40.311			
10%	40.035	0.16%	41.420	0.20%	6, 11	40.263	0.12%	6, 11	4, 4
20%	39.701	0.99%	40.933	1.37%	6, 11	39.855	1.13%	6, 11	4, 4
30%	39.224	2.18%	40.361	2.75%	6, 10	39.338	2.41%	6, 10	5, 3
40%	38.461	4.08%	39.439	4.97%	6, 10	38.556	4.35%	6, 10	5, 3
50%	37.635	6.14%	38.466	7.31%	7, 10	37.773	6.30%	6, 10	5, 3
CPU	< 1 s		< 1 s			3 s			

*Optimal age for $\Delta = 0$ is $t^* = 6$ months with costs 40.098.
**Optimal block for $\Delta = 0$ is $T^* = 6$ months with costs 41.501.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (4, 6)$ with costs 40.311.
*** Costs for not maintaining at all is $C(\infty) = 53.885$.

Table 10: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We let $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ. We let $\alpha = 1$ year and $\beta = 3$.

	ARP		BRP (m=1)			MBRP (m=1)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	30.035		30.837			30.199			
10%	30.035	0.00%	30.837	0.00%	6, 12	30.184	0.05%	2, 8	4, 3
20%	29.730	1.02%	30.465	1.21%	6, 11	29.835	1.21%	6, 11	4, 3
30%	29.232	2.67%	29.901	3.04%	6, 11	29.334	2.86%	6, 11	4, 3
40%	28.671	4.54%	29.337	4.86%	6, 11	28.775	4.72%	6, 10	4, 3
50%	27.736	7.65%	28.323	8.15%	6, 10	27.783	8.00%	6, 10	4, 2
CPU	< 1 s		< 1 s			2 s			

*Optimal age for $\Delta = 0$ is $t^* = 6$ months with costs 30.035.
**Optimal block for $\Delta = 0$ is $T^* = 6$ months with costs 30.837.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (3, 6)$ with costs 30.200.
****Costs for not maintaining at all is $C(\infty) = 53.496$.

Table 11: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We let $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ. We let $\alpha = 3$ year and $\beta = 2$.

	ARP		BRP (m=3)			MBRP (m=3)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	13.530		14.173			13.622			
10%	13.252	2.05%	13.828	2.43%	9, 30	13.338	2.08%	6, 21	11, 9
20%	12.707	6.08%	13.135	7.32%	7, 19, 31	12.707	6.72%	7, 19, 31	12, 12, 12
30%	11.779	12.94%	12.114	14.53%	7, 19, 31	11.779	13.53%	7, 19, 31	10, 10, 10
40%	10.844	19.85%	11.093	21.73%	7, 19, 31	10.844	20.39%	7, 19, 31	8, 8, 8
50%	9.900	26.83%	10.072	28.94%	7, 19, 31	9.900	27.32%	7, 19, 31	7, 7, 7
CPU	< 1 s		2 s			17 s			

*Optimal age for $\Delta = 0$ is $t^* = 19$ months with costs 13.530.
**Optimal block for $\Delta = 0$ is $T^* = 18$ months with costs 14.173.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (11, 18)$ with costs 13.622.
****Costs for not maintaining at all is $C(\infty) = 18.516$.

Table 12: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We let $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ. We let $\alpha = 3$ year and $\beta = 3$.

	ARP		BRP (m=3)			MBRP (m=3)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	10.072		10.487			10.140			
10%	9.864	2.07%	10.261	2.16%	9, 30	9.922	2.15%	9, 30	8, 10
20%	9.395	6.72%	9.659	7.90%	7, 19, 31	9.456	6.75%	8, 30	6, 10
30%	8.472	15.89%	8.619	17.81%	7, 19, 31	8.472	16.45%	7, 19, 31	11, 11, 11
40%	7.461	25.92%	7.579	27.73%	7, 19, 31	7.461	26.42%	7, 19, 31	9, 9, 9
50%	6.449	35.97%	6.539	37.65%	7, 19, 31	6.449	36.40%	7, 19, 31	8, 8, 8
CPU	< 1 s		2 s			12 s			

*Optimal age for $\Delta = 0$ is $t^* = 18$ months with costs 10.072.
**Optimal block for $\Delta = 0$ is $T^* = 18$ months with costs 10.487.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (9, 18)$ with costs 10.140.
****Costs for not maintaining at all is $C(\infty) = 18.378$.

Similar to the case with relatively low corrective maintenance costs, we see that if α is larger, maintenance is done less frequently and the cost savings are larger since we can maintain at cheap moments only. If $\alpha = 1$ year, preventive maintenance is planned more frequently and has to be performed at relatively expensive moments as well. For $\beta = 2$ the failures are less predictable (i.e. the distribution is spread out more) than for $\beta = 3$. The costs are therefore larger for $\beta = 2$ and we see that the savings are too for large Δ. For $\alpha = 3$ years the BRP and MBRP with $m = 3$ gave the best results, increasing to $m = 8$ gave no improvements. For $\alpha = 1$ it was enough to consider $m = 1$ only, increasing to $m = 8$ gave no improvement. Again the MBRP bridges over 75% of the gap between ARP and BRP.

Relatively high corrective maintenance costs. In this section we show the results for the case in which the corrective maintenance costs are 10 times larger than the preventive maintenance costs. In Tables 13, 14, 15 and 16 we give the results for $\bar{c}_f = 100$ and $\bar{c}_p = 10$. We can expect the preventive maintenance to be scheduled more frequently compared to the previous results, since preventive maintenance is relatively cheap.

Table 13: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We let $\bar{c}_f = 100$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ. We let $\alpha = 1$ and $\beta = 2$.

	ARP		BRP ($m = 1$)			MBRP ($m = 1$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	59.812		60.549			59.987			
10%	59.812	0.00%	60.549	0.00%	3, 7, 11	59.981	0.01%	4, 8, 12	3, 2, 3
20%	59.777	0.06%	60.548	0.00%	3, 7, 11	59.960	0.05%	4, 8, 12	3, 2, 3
30%	59.517	0.49%	60.241	0.51%	1, 6, 9	59.646	0.57%	1, 6, 9	3, 3, 2
40%	59.101	1.19%	59.777	1.28%	1, 6, 9	59.183	1.34%	1, 6, 9	3, 3, 2
50%	58.567	2.08%	59.113	2.37%	6, 9, 12	58.659	2.21%	6, 9, 12	3, 2, 3
CPU	< 1 s		2 s			3 s			

*Optimal age for $\Delta = 0$ is $t^* = 4$ months with costs 59.812.
**Optimal block for $\Delta = 0$ is $T^* = 4$ months with costs 60.549.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (3, 4)$ with costs 59.987.
****Costs for not maintaining at all is $C(\infty) = 107.77$.

Table 14: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We let $\overline{c}_f = 100$, $\overline{c}_p = 10$, $\Delta_f = \Delta \cdot \overline{c}_f$ and $\Delta_p = \Delta \cdot \overline{c}_p$ for different Δ. We let $\alpha = 1$ year and $\beta = 3$.

	ARP		$\text{BRP}(m = -, 3, 3, 3, 2, 2)$			$\text{MBRP}(m = -, 3, 3, 3, 2, 2)$			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	39.524		40.978			39.704			
10%	39.485	0.10%	39.891	0.35%	3, 8, 12, 17, 21, 25, 30, 34	39.597	0.37%	4, 8, 12, 17, 21, 26, 31, 35	3, 3, 3, 3, 2, 3, 3, 2
20%	39.299	0.57%	39.664	0.91%	3, 8, 12, 17, 21, 25, 30, 34	39.376	0.92%	3, 8, 12, 17, 21, 25, 30, 34	3, 3, 4, 3, 3, 4, 3, 2
30%	39.072	1.14%	39.437	1.48%	3, 8, 12, 17, 21, 25, 30, 34	39.150	1.49%	3, 8, 12, 17, 21, 25, 30, 34	3, 3, 3, 3, 3, 4, 3, 2
40%	38.738	1.99%	39.147	2.21%	3, 8, 12, 18, 22	38.860	2.22%	3, 8, 12, 18, 22	3, 2, 3, 3, 2
50%	38.235	3.26%	38.621	3.52%	1, 7, 11, 17, 21	38.354	3.49%	1, 7, 11, 17, 21	3, 2, 2, 4, 2
CPU	< 1 s		< 1 s			2 s			

*Optimal age for $\Delta = 0$ is $t^* = 5$ months with costs 39.524.
**Optimal block for $\Delta = 0$ is $T^* = 5$ months with costs 39.978.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (2, 5)$ with costs 39.704.
****Costs for not maintaining at all is $C(\infty) = 106.992$.

Table 15: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We have $\overline{c}_f = 100$, $\overline{c}_p = 10$, $\Delta_f = \Delta \cdot \overline{c}_f$ and $\Delta_p = \Delta \cdot \overline{c}_p$ for different Δ. We have $\alpha = 3$ year and $\beta = 2$.

	ARP		BRP (m=1)			MBRP (m=1)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	20.099		20.519			20.171			
10%	19.135	4.80%	19.439	5.26%	8	19.152	5.05%	8	6
20%	18.119	9.85%	18.359	10.53%	8	18.128	10.13%	8	6
30%	17.093	14.96%	17.280	15.79%	8	17.097	15.24%	8	5
40%	16.062	20.09%	16.200	21.05%	8	16.063	20.37%	8	4
50%	15.020	25.27%	15.120	26.31%	8	15.021	25.53%	8	4
CPU	< 1 s		< 1 s			< 1 s			

*Optimal age for $\Delta = 0$ is $t^* = 12$ months with costs 20.099.
**Optimal block for $\Delta = 0$ is $T^* = 12$ months with costs 20.519.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (7, 12)$ with costs 20.171.
****Costs for not maintaining at all is $C(\infty) = 37.032$.

Table 16: Yearly costs, in thousands of €, savings with respect to constant cost model and maintenance months for ARP, BRP and MBRP. We have $\overline{c}_f = 100$, $\overline{c}_p = 10$, $\Delta_f = \Delta \cdot \overline{c}_f$ and $\Delta_p = \Delta \cdot \overline{c}_p$ for different Δ. We have $\alpha = 3$ year and $\beta = 3$.

	ARP		BRP (m=1)			MBRP (m=1)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	13.142		13.383			13.190			
10%	12.316	6.29%	12.471	6.81%	7	12.320	6.60%	7	7
20%	11.257	14.34%	11.383	14.94%	7	11.258	14.65%	7	7
30%	10.194	22.43%	10.295	23.07%	7	10.195	22.71%	7	6
40%	9.130	30.53%	9.208	31.20%	7	9.131	30.77%	7	6
50%	8.064	38.64%	8.120	39.33%	7	8.064	38.86%	7	5
CPU	< 1 s		< 1 s			< 1 s			

*Optimal age for $\Delta = 0$ is $t^* = 14$ months with costs 13.142.
**Optimal block for $\Delta = 0$ is $T^* = 14$ months with costs 13.383.
***Optimal modified block for $\Delta = 0$ is $(t^*, T^*) = (7, 14)$ with costs 13.190.
****Costs for not maintaining at all is $C(\infty) = 36.756$.

Similar to the case with relatively low and moderate corrective maintenance costs, we see that if α is larger, maintenance is done less frequently and the cost savings are larger since we can maintain at cheap moments only. If $\alpha = 1$ year, preventive maintenance is planned more frequently and has to be performed at relatively expensive moments as well. For $\beta = 2$ the failures are less predictable (i.e. the distribution is spread out more) than for $\beta = 3$. The costs are therefore larger for $\beta = 2$ and we see that the savings are too, except for $\Delta = 10\%$. The MBRP again bridges over 75% of the gap between the ARP and BRP.

Only for the $\alpha = 1$, $\beta = 3$ case considering $m > 1$ improved the solution of the BRP and MBRP. The best m is not the same for all Δ in this scenario. We have that $m = 3$ is best for $\Delta \leq 30\%$, while $m = 2$ is best for $\Delta \geq 40\%$.

4.1.3 Comparison for special cost functions

Suppose that for one of the periods in the year, that preventive maintenance is free. This can for example happen if there is a contract with another party that performs the maintenance in this period, but charges extra fees if maintenance is done in another period. In this scenario our model should choose to maintain in the free period for any block-based policy and age-based policy independent of the age of the component. $\alpha = 1$ year and $\beta = 2$ are chosen to be constant throughout this section.

As a test we will use two combinations of cost functions. First we will use cost functions of the form in equations (52) and (53). We let $\bar{c}_p = \Delta_p = 10$ and test for different $\bar{c}_f = \Delta_f$, which means that maintenance is free of cost in July and very low in June and August. In Table 17 we show the results for these cost functions.

Table 17: Yearly costs (in €) of age-based and block-based maintenance ($m = 1$) model for $\bar{c}_f = \Delta_f$ and $\bar{c}_p = \Delta_p = 10$.

	ARP	BRP ($m = 1$)		MBRP ($m = 1$)		
$\bar{c}_f$	costs	costs	mos.	costs	mos.	ages
20	14.100	14.100	7, 8	14.100	7, 8	1, 1
50	31.396	31.398	7, 9	31.398	7, 9	1, 1
100	51.586	51.682	7, 8, 11	51.590	7, 8, 11	1, 1, 2

* Costs for not maintaining at all are $C(\infty) = 21.554, 53.885, 107.77$.

For $\bar{c}_f = 20$ we have that the age-based maintenance policy maintains in July and August for all ages. For other months maintenance is never performed. This indicates that the ARP coincides with the BRP and MBRP.

For $\bar{c}_f = 50$ we have that the age-based maintenance policy maintains in July for any age and in September and October from the age of 2 months. For other months maintenance is never performed. The MBRP maintains in July and September for all ages and thus coincides with the BRP.

For $\bar{c}_f = 100$ we have that the age-based maintenance policy maintains in July and August for any age and in November and December from the age of 3 months. For other months maintenance is never performed. The MBRP maintains for all ages in July and August and in November from an age of 2 months. All policies are thus different.

In all policies we maintain in the cost-free month. We also maintain in other months, but this depends on the cost parameters. The costs of the ARP, BRP and MBRP are much closer together, than in Section 4.1.1.

In Table 18 we show the results for constant costs of preventive maintenance of $c_p = 10$

and different constant corrective maintenance costs. We adjust the cost function such that we can maintain for free in January.

Table 18: Yearly costs (in €) of age-based and block-based maintenance ($m = 1$) model for constant costs with one free maintenance period.

	ARP	BRP ($m = 1$)		MBRP ($m = 1$)		
c_f	costs	costs	mos.	costs	mos.	ages
20	14.720	14.720	1	14.720	1	1
50	31.555	32.157	1, 7	31.605	1, 7	1, 6
100	50.597	51.003	1, 7	50.642	1, 5, 9	1, 4, 4

* Costs for not maintaining at all are $C(\infty) = 19.758, 49.395, 98.790$.

For $\bar{c}_f = 20$ we have that the age-based maintenance policy maintains in January for any age. For other months maintenance is never performed. This indicates that the ARP coincides with the BRP and MBRP, which is also reflected by the costs.

For $\bar{c}_f = 50$ we have that the age-based maintenance policy maintains in January for any age and in July, August, September from ages 6, 5 and 5 months onwards respectively. For other months maintenance is never performed.

For $\bar{c}_f = 100$ we have that the age-based maintenance policy maintains in January for any age. In May, June, July and September maintenance is performed from an age of 4 and in October from an age of 3. For other months maintenance is never performed.

In all policies we maintain in the cost-free month. We also maintain in other months, but this depends on the cost parameters. The costs of the ARP, BRP and MBRP are much closer together, than in Section 4.1.1.

We see that the ARP costs equal the BRP and MBRP cost for $c_f = 20$. As c_f increases, more preventive maintenance is done to prevent failures that are now more costly. The ARPs take similar shapes if we compare with Figures 10, 11 and 12 except that we maintain in month 1 for all ages.

4.1.4 Conclusions for the single component setting

The maintenance costs decrease for increasing cost fluctuations. This decrease is faster than linear and therefore the savings with respect to the constant cost model increase faster than linear in the cost fluctuations. As failures are more predictable (i.e. a larger β) maintenance is also cheaper, but the savings compared to the constant cost models is not necessarily larger. When the optimal maintenance policy has multiple maintenance moments in a year it is more difficult to plan at cheaper moments. As a result, the savings are smaller in these settings. If α increases we maintain less during a year and therefore the savings become larger. These effects can be observed for all three maintenance policies.

The ARP gives the lowest costs and the BRP is most expensive. The MBRP is closer to the ARP than to the BRP for the considered parameters. As the cost fluctuations increase the three policies come closer together and for extreme cases they can even coincide. As a result, as cost fluctuations increase the savings increase most for the BRP, then for the MBRP and least for the ARP.

4.2 Multi-component results

In this section we show the results for the multi-component problem. First in section 4.2.1 we compute the costs for a problem with two identical components. We use formulation (39) for the exact results of the BRP and we compare the results with the three heuristic approaches that are discussed in Section 3.5. We also give the exact results for the ARP and MBRP and compare these with the BRP as well. The same procedure is used in Section 4.2.2 for different components. Next in Section 4.2.3 we give the results for a setting in which there are four components. We only use the three heuristic approaches in this multi-component setting and compare those, since the exact methods are too time consuming.

4.2.1 Two component model with identical components

Suppose that we have two identical components (i.e. they have the same maintenance cost functions and lifetime distribution). Then we can solve the MIP formulation (39) for the BRP, where we add the constraints that make sure that preventive maintenance of both components is done at the same time, to speed up computation time. We also give the results of the genetic, memetic and sequential optimisation algorithm as described in Algorithms 1, 2 and 3. For the ARP and MBRP we can use formulations (36) and (48). For the MBRP we can make sure that both the block times and minimal ages are the same for both components, again to speed up computation time.

First we consider the setting where $\alpha = 1$ year, $\beta = 2$ and $c_s = 5$. We use equations (52) and (53) with $\bar{c}_p = 5$, $\bar{c}_f = 15$ and use different Δ. The results of the ARP, MBRP, BRP and heuristics for these parameters are given in Table 19. We only give the results for $m = 1$, since no improvement is obtained by considering $m = 2$ or $m = 3$.

Table 19: Yearly costs (in thousands of €) for the block-based maintenance model for $\alpha = 1$ year, $\beta = 2$.

	ARP		MBRP($m = 1$)		BRP($m = 1$)		GA	MA	SC/SF/SR
Δ	costs	savings	costs	savings	costs	savings	costs	costs	costs
0%	37.879		41.197		42.641		42.641	42.641	42.641
10%	37.761	0.31%	40.511	1.67%	41.596	2.45%	41.596	41.596	41.596
20%	37.480	1.05%	39.759	3.49%	40.552	4.90%	40.552	40.552	40.552
30%	37.070	2.14%	38.966	5.42%	39.508	7.35%	39.508	39.508	39.508
40%	36.533	3.55%	38.091	7.54%	38.463	9.80%	38.463	38.463	38.463
50%	35.902	5.22%	37.209	9.68%	37.420	12.24%	37.420	37.420	37.420
CPU	2 s		7 min		12 s		5 s	5 s	< 1 s

*Costs for not maintaining at all is $C(\infty) = 43.108$.

The MBRP maintains once a year in August and the critical maintenance age decreases from 6 months to 3 months for $\Delta = 0\%$ to $\Delta = 50\%$. In the BRP maintenance is also done only once a year in August and all heuristics find the same optimal policy. It is better to maintain for the BRP even though the individual conditions in Theorem 2 is not met. Since we can save set-up costs by combining the maintenance actions, we now have a finite optimal block time.

For the genetic and memetic algorithm the optimal BRP corresponds to one of the chromosomes in the starting population and therefore these terminate relatively fast and always find the optimal BRP. For the sequential scheduling it does not matter in which order we schedule since the items are identical. The optimal maintenance month for one component overlaps with the optimal maintenance month for both components, making the sequential optimisation optimal as well.

It is more interesting to see what happens if one of the components is scheduled more frequently than once a year. Then the optimal solution is not necessarily in the starting population of the GA and MA for $\Delta > 0$. To see what happens we will first increase the corrective maintenance costs of both components with a factor of 3 and keep the rest of the parameters the same. We then have that $\alpha = 1$ year, $\beta = 2$ and $c_s = 5$. We use equations (53) and (52) with $\bar{c}_f = 45$, $\bar{c}_p = 5$ and use different Δ. The results of the heuristics are given in Table 20.

Table 20: Yearly costs (in thousands of €) for the block-based maintenance model for $\alpha = 1$ year, $\beta = 2$.

	ARP		MBRP($m=1$)		BRP($m=1$)		GA	MA	SC/SF/SR
Δ	costs	savings	costs	savings	costs	savings	costs	costs	costs
0%	70.184		72.624		73.046		73.046	73.046	73.046
10%	70.020	0.23%	72.624	0.00%	73.046	0.00%	73.046	73.046	73.046
20%	69.805	0.54%	72.146	0.66%	72.530	0.71%	72.530	72.530	72.530
30%	69.473	1.01%	71.510	1.53%	71.866	1.62%	71.866	71.866	71.866
40%	68.977	1.72%	70.874	2.41%	71.202	2.52%	71.202	71.202	71.202
50%	68.140	2.91%	69.754	3.95%	69.971	4.21%	69.971	69.971	69.971
CPU	2 s		4 min		19 s		9 s	12 s	< 1 s

*Costs for not maintaining at all is $C(\infty) = 107.770$.

Again all heuristics find the optimal BRP. Now maintenance is performed twice a year for the BRP and MBRP. For the MBRP with $\Delta = 0\%$ we maintain every 6 months, for example in May and November, from an age of 2 months. These two months change as Δ increases. For $\Delta = 50\%$ we maintain in July and November for both the BRP and MBRP, where the critical maintenance age is still 2 months.

A more interesting situation arises when it is individually optimal to schedule the components once a year, but combined twice or more due to the savings of the set-up costs. This occurs for example when we change the average corrective maintenance costs to $\bar{c}_f = 25$ and leave the other parameters unchanged. The results are given in Table 21.

Table 21: Yearly costs (in thousands of €) for the block-based maintenance model for $\alpha = 1$ year, $\beta = 2$.

	ARP		MBRP($m=1$)		BRP($m=1$)		GA	MA	SC/SF/SR
Δ	costs	savings	costs	savings	costs	savings	costs	costs	costs
0%	50.685		53.703		54.796		54.796	54.796	57.361
10%	50.615	0.14%	53.694	0.02%	54.796	0.00%	54.796	54.796	56.110
20%	50.364	0.63%	53.308	0.74%	54.320	0.87%	54.320	54.320	54.863
30%	49.917	1.52%	52.900	1.50%	53.615	2.16%	53.615	53.615	53.615
40%	49.308	2.72%	52.124	2.94%	52.366	4.43%	52.366	52.366	52.366
50%	48.608	4.10%	50.933	5.16%	51.116	6.72%	51.116	51.116	51.116
CPU	2 s		6 min.		19 s		10 s	17 s	< 1 s

*Costs for not maintaining at all is $C(\infty) = 64.662$.

Now we see that the sequential optimisation underperforms compared to the GA and MA. The MA and GA schedule maintenance two times a year for $\Delta \leq 20\%$, while the sequential optimisation only schedules one maintenance moment. Since individually it is optimal to schedule maintenance once. In the worst case, for $\Delta = 0\%$, the costs are 6.27% larger for the sequential optimisation. For the GA and MA there is a probability that it does not find the optimal solution, which here does not happen. For $\Delta \geq 30\%$ all three heuristics find the optimal BRP. Due to the larger cost fluctuations it is optimal to perform maintenance only once in August and the expensive winter months are not used for preventive maintenance at all.

Comparison between ARP, MBRP and BRP. In the worst considered scenario the BRP is 12.57% worse than the ARP ($\Delta = 0\%$, $\overline{c}_f = 15$, $\overline{c}_p = 5$). This number decreases if Δ or $\frac{\overline{c}_f}{\overline{c}_p}$ increases. In the best considered scenario ($\Delta = 50\%$, $\overline{c}_f = 45$, $\overline{c}_p = 5$), this difference is 2.69%. The MBRP is closer to the BRP than to the ARP, which it was not for single components. In the MBRP we allow to maintain from a critical maintenance age onwards for each component. The maintenance decision can however not depend on the age of the other component, which is an extra advantage of the ARP. In the MBRP we must for example maintain if one of the components has just reached the critical maintenance age and the other is still very young. It would then possibly be better to not maintain at all. The MBRP thus works very well in single component settings, but loses part of its advantage in two component settings.

4.2.2 Two component model with different components

When the components are different from each other it might be optimal to plan one of the components more frequently than the other component. As a result, the three sequential optimisation methods might give different policies with different costs. Also the problem becomes more difficult and solving the MIP takes more time. In this section we evaluate the performance of the algorithms for two components that differ either in costs or in lifetime distribution. In the tables MBRP denotes the MBRP in which the block times are the same for both components, but the minimal ages can differ. MBRP alternative dentoes the MBRP in which the block times and minimal ages can be different for the two components.

In Tabel 22 we show what happens when we have two components that have corrective maintenance costs $\overline{c}_f^1 = 45$ and $\overline{c}_f^2 = 15$, but the same preventive maintenance costs $\overline{c}_p = 5$. The Weibull parameters $\alpha = 1$ year and $\beta = 2$ are the same for both components and for the set-up costs we have $c_s = 5$. In this scenario it is optimal for component 1 to maintain twice a year and once a year for component 2. As a result SF and SC will give the same results and SR can be different.

Table 22: Yearly costs (in thousands of €) of block-based maintenance model for $\alpha = 1$ year, $\beta = 2$.

	ARP		MBRP		MBRP alternative		BRP		GA	MA	SC / SF	SR
Δ	costs	savings	costs	savings	costs	savings	costs	savings	costs	costs	costs	costs
0%	55.830		58.257		58.257		59.358		59.358	59.358	59.358	60.366
10%	55.802	0.05%	58.244	0.02%	58.244	0.02%	59.358	0.00%	59.358	59.358	59.358	59.828
20%	55.527	0.54%	57.870	0.66%	57.870	0.66%	58.896	0.78%	58.896	58.896	58.928	59.170
30%	55.011	1.47%	57.433	1.41%	57.267	1.70%	58.105	2.11%	58.105	58.105	58.157	58.379
40%	54.356	2.64%	56.904	2.32%	56.540	2.95%	57.313	3.45%	57.313	57.313	57.386	57.543
50%	53.653	3.90%	56.056	3.78%	55.710	4.37%	56.213	5.30%	56.213	56.213	56.386	56.436
CPU	5s		6 min.		1.5 h		12s		10s	15s	1s	1s

*Costs for not maintaining at all is $C(\infty) = 75.439$.

We see that the GA and MA perform best for all Δ and they always find the optimal solution. The sequential optimisation methods are worse except for $\Delta = 0\%$, where the SC and SF give the same policy as the GA and MA. We namely maintain component 1 twice a year and component 2 on one of these opportunities. It does not matter on which of the two possibilities, since the costs are assumed constant. As cost fluctuations increase the savings w.r.t. the constant cost policy of the best method also increase.

In Tabel 23 we show what happens when we have two components that have corrective maintenance costs $\overline{c}_f^1 = 95$ and $\overline{c}_f^2 = 15$, but the same preventive maintenance costs $c_p = 5$. The Weibull parameters $\alpha = 1$ year and $\beta = 2$ are the same for both components and for the set-up costs we have $c_s = 5$. In this scenario it is optimal for component 1 to maintain three times a year and once a year for component 2. As a result SF and SC will give the same results and SR can be different.

Table 23: Yearly costs (in thousands of €) of block-based maintenance model for $\alpha = 1$ year, $\beta = 2$.

	ARP		MBRP		MBRP alternative		BRP		GA	MA	SC / SF	SR
Δ	costs	savings	costs	savings	costs	savings	costs	savings	costs	costs	costs	costs
0%	75.386		79.862		77.631		78.774		78.774	78.774	78.774	79.400
10%	75.358	0.04%	79.861	0.00%	77.280	0.45%	78.251	0.66%	78.251	78.251	78.435	78.878
20%	75.199	0.25%	79.843	0.02%	76.927	0.91%	77.729	1.33%	77.729	77.729	77.896	78.355
30%	74.786	0.80%	79.711	0.19%	76.562	1.38%	77.206	1.99%	77.206	77.206	77.563	77.833
40%	74.169	1.61%	79.244	0.77%	76.056	2.03%	76.557	2.81%	76.557	76.557	76.783	77.093
50%	73.416	2.61%	78.777	1.36%	75.243	3.08%	75.708	3.89%	75.708	75.708	76.005	76.144
CPU	2s		1 min		1 h		23s		16 s	28 s	1 s	1 s

*Costs for not maintaining at all is $C(\infty) = 129.324$.

Again we see that the GA and MA perform best for all Δ and they always find the optimal solution. The sequential optimisation methods are worse except for $\Delta = 0\%$, where the SC and SF give the same policy as the GA and MA. We namely maintain component 1 three times a year and component 2 on one of these opportunities. It does not matter on which of the three possibilities, since the costs are assumed constant. As cost fluctuations increase the savings with respect to the constant cost policy of the best method also increase.

In Tabel 24 we show the results for two components that have corrective maintenance costs $\bar{c}_f^1 = 95$ and $\bar{c}_f^2 = 45$, but the same preventive maintenance costs $c_p = 5$. The Weibull parameters $\alpha = 1$ year and $\beta = 2$ are the same for both components and for the set-up costs we have $c_s = 5$. In this scenario it is optimal for component 1 to maintain three times a year and twice a year for component 2. As a result SF and SC will give the same results and SR can be different.

Table 24: Yearly costs (in thousands of €) of block-based maintenance model for $\alpha = 1$ year, $\beta = 2$.

	ARP		MBRP		MBRP alternative		BRP		GA	MA	SC / SF	SR
Δ	costs	savings	costs	savings	costs	savings	costs	savings	costs	costs	costs	costs
0%	88.603		89.876		89.876		90.172		90.172	90.172	90.172	94.552
10%	88.599	0.00%	89.876	0.00%	89.876	0.00%	90.172	0.00%	90.172	90.172	90.172	94.544
20%	88.475	0.14%	89.876	0.00%	89.876	0.00%	90.172	0.00%	90.172	90.172	90.172	93.484
30%	88.196	0.32%	89.858	0.02%	89.858	0.02%	90.155	0.02%	90.155	90.155	90.172	93.437
40%	87.907	0.33%	89.314	0.61%	89.314	0.61%	89.604	0.61%	89.604	89.604	89.604	93.172
50%	87.417	0.56%	88.766	0.61%	88.766	0.61%	89.052	0.62%	89.052	89.052	89.280	90.333
CPU	1s		12s		16s		3s		5s	5s	1s	1s

*Costs for not maintaining at all is $C(\infty) = 161.655$.

In the BRP and MBRP both components are scheduled 3 times a year for all Δ in Table 24. When components are scheduled for maintenance multiple times a year it is more difficult to choose the cheap moments and for $\Delta \leq 20\%$ maintenance is scheduled every 4 months. The cost savings are therefore relatively low for these parameters in the ARP and there are no savings for the BRP and MBRP. The GA and MA always find the optimal policy. The SC and SF are close to optimal, 0.26% in the worst case scenario. The SR performs worst and is 4.83% more expensive in the worst case scenario.

Comparison between ARP, MBRP and BRP. The optimal BRP is 6.32% more expensive than the optimal ARP in the worst case. Again we see that the MBRP is closer to the BRP than to the ARP. As the components differ a lot, see Table 23, the standard MBRP performs even worse than the BRP. It does not allow different frequencies of maintenance, which we need if one component needs more maintenance than the other component. Allowing different frequencies in the MBRP gives an improvement, but the optimum is still closer to the optimal BRP than to the optimal ARP.

4.2.3 Four component block-based maintenance model

We have seen that the models can be used to compute the optimal costs and that significant savings are obtained compared to the standard ARP, BRP and MBRP for one or two components. We have seen that the performance of the heuristics differs for each situation, but that the GA and MA perform very well for all considered parameters. If we increase the amount of parameters the exact methods are too time consuming and we will only use the heuristics. We might then see bigger differences in the performance for the heuristics, since the problem is more complex for more components.

The number of components that contribute significantly to the maintenance costs is more than two and we are therefore interested in the performance of the heuristics for more components. In the literature different amounts of components are included in maintenance models. Tian et al. (2011) and Shafiee and Finkelstein (2015) include the four components in their model that contribute most to the maintenance costs. These are the rotor (or the three blades), main bearing, gearbox and generator. We therefore consider an example with these 4 different components and test the performance of the three heuristics. Since we consider the three blades individually, we have 6 components in the turbine of which four unique ones. In this section we show the results for these components and for different seasonalities. We let $\Delta = \frac{\Delta_f}{\bar{c}_f} = \frac{\Delta_p}{\bar{c}_p}$ vary from 0% to 50% for all components.

Suppose we have a components with the following parameters in which component 1 represents a blade of which we have three.

Table 25: Parameters of four components of an offshore wind turbine.

Component	α	β
1	1 year	3
2	3 years	2
3	1 year	3
4	3 years	2

(a) Weibull distribution parameters

Component	Corrective	Preventive	Set-up
1	50	10	10
2	50	10	10
3	150	10	10
4	150	10	10

(b) Cost parameters

The individual optimal block maintenance time can be computed for the $\Delta = 0$ setting. We schedule with and without set-up costs and obtain the following.

Table 26: Optimal maintenance time for $\Delta = 0$ and associated costs.

	Without set-up costs		With set-up costs	
Component	T^*	Costs	T^*	Costs
1	6	30.200	7	48.482
2	18	13.622	25	19.682
3	4	45.580	5	72.496
4	10	25.086	13	35.702

In this table we see that the optimal block maintenance time differs per component. For component 2 the costs are lowest and the frequency at which it needs preventive maintenance is lowest. The difference with the more expensive and more frequently scheduled component 3 is large. We can therefore expect that component 2 will skip some of the occasions at which component 3 will be scheduled. We will now compute the combined (sub-)optimal BRP using the three algorithms. We will consider $m = 1, 2, \ldots, 8$ and compute the costs. The $m = 5$ solution always contains the best solution for the genetic and memetic algorithm. We show the results for $m = 5$ in Table 27. It is thus best to consider a maintenanance schedule that repeats itself every 5 years.

Table 27: Yearly costs (in thousands of €) of the BRP($m = 5$) model for different Δ. The savings with respect to the $\Delta = 0$ case are given for the MA.

Δ	GA	MA	SC	SF	SR	savings
0%	211.99	211.99	225.56	212.52	229.24	
10%	211.74	211.74	223.45	212.52	215.68	0.12%
20%	209.32	209.30	221.05	210.83	219.12	1.27%
30%	206.12	206.12	217.76	208.54	216.38	2.77%
40%	201.35	201.35	218.61	204.17	227.08	5.02%
50%	196.58	196.58	211.09	196.58	212.20	7.27%
CPU	1046 s	1133 s	127 s	24 s	30 s	

The memetic algorithm performs best for every Δ. The genetic algorithm finds the same solutions except for the $\Delta = 20\%$ case in which it is only 0.01% more expensive. The sequential optimisation algorithms never find the best solution. The only exception is the SF for $\Delta = 50\%$. The SF is the best of the sequential algorithms and is 1.40% from optimality in the worst case ($\Delta = 40\%$).

For $\Delta = 0$ it is best to maintain components 1 and 3 every 5 months. Then component 4 is scheduled every 10 months and component 2 is scheduled every 20 months.

For $\Delta = 10\%, 20\%$ it is best to maintain component 1 and 3 in total eleven times in 60 months. Component 2 is scheduled at five of these opportunities and component 4 at four of these opportunities.

For $\Delta \geq 30\%$ maintenance is performed twice a year in July and December. Component 1 and 3 are both maintained preventively at these moments. Component 2 and 4 are maintained in July every year.

For wind turbines the components typically have a longer lifetime. Therefore, α is much larger and maintenance is not performed this much during a year. Below we consider an example in which the maintenance has to be performed less often, which might result in larger cost savings.

Table 28: Parameters of four components of an offshore wind turbine.

Component	α	β
1	5 year	3
2	10 years	2
3	5 year	3
4	10 years	2

(a) Weibull distribution parameters

Component	Corrective	Preventive	Set-up
1	50	10	10
2	50	10	10
3	150	10	10
4	150	10	10

(b) Cost parameters

The individual optimal block maintenance time can be computed for the $\Delta = 0$ setting. We schedule with and without set-up costs and obtain the following.

Table 29: Optimal maintenance time for $\Delta = 0$ and associated costs.

	Without set-up costs		With set-up costs	
Component	T^*	Costs	T^*	Costs
1	29	6.092	36	9.803
2	59	4.106	82	5.944
3	20	9.151	25	14.591
4	32	7.540	45	10.745

We will now compute the combined (sub-)optimal BRP using the three algorithms. We consider $m = 1, 2, \dots, 8$ and compute the costs. The $m = 4$ solution always contains the best solution for the genetic and memetic algorithm, which perform best. We show the results for $m = 4$ in Table 30.

Table 30: Yearly costs (in thousands of €) of the BRP($m = 4$) model for different Δ. The savings with respect to the $\Delta = 0$ case are given for the MA/GA.

Δ	GA	MA	SC	SF	SR	savings
0%	47.781	47.781	53.485	48.142	53.908	
10%	45.307	45.307	51.405	45.307	52.082	5.18%
20%	42.472	42.472	49.326	42.472	50.255	11.11%
30%	39.636	39.636	39.636	39.636	40.813	17.05%
40%	36.801	36.801	36.801	36.801	38.227	22.98%
50%	33.966	33.966	33.966	33.966	35.644	28.91%
CPU	434 s	463 s	11 s	5.1 s	5.6 s	

The SF, GA and MA always find the best solution except the SF for $\Delta = 0\%$. For all $\Delta \geq 10\%$ preventive maintenance is planned every 2 years in July. Component 1, 3, and 4 are always maintained at these occasions and component 2 is done every 4 years (i.e. every 2 preventive maintenance moments). The optimal policy for $\Delta = 0$ is maintenance every 26 months for components 1, 3, 4 and maintenance every 52 months for component 2.

Compared to the previous example, see Table 27, the cost savings are much larger. Now the savings are much larger, because maintenance is performed every two years and we can thus choose cheap moments only to perform preventive maintenance. The added value of the GA and MA are lower here, since the SF finds the same solution, except for $\Delta = 0$, and is much faster.

4.2.4 Conclusions for the multi-component setting

The maintenance costs decrease for increasing cost fluctuations. This decrease is faster than linear and therefore the savings with respect to the constant cost model increase faster than linear in the cost fluctuations. Again the savings are small if preventive maintenance is scheduled multiple times in a year. The added value of the model is thus larger for larger α and a larger factor between c_f and c_p. These results are very similar to the single component setting.

The ARP gives the lowest costs and the BRP is most expensive. The MBRP is closer to the BRP than to the ARP for the considered parameters in a multi-component setting. This is a difference with the single component setting. In the MBRP the decision is based on the time of the year and the age of the component itself, but not on the other components. This is an extra advantage that the ARP has. As the cost fluctuations increase the three policies come closer together and for extreme cases they can even coincide. As a result, as cost fluctuations increase the savings increase most for the BRP, then for the MBRP and least for the ARP.

The memetic algorithm performs best for all of the considered scenarios. Due to the local search in each iteration of the algorithm it just outperforms the genetic algorithm. In a finite number of iterations the memetic algorithm does not always give the optimal solution. We always did one run and multiple runs can increase this probability. The sequential optimisation algorithms sometimes perform very well, but are not as robust as the MA and GA that seem to find good solutions in all considered scenarios. The SC and SF are much better than the SR. It is best to schedule costly components and components with smallest individual block time first.

4.3 Offshore wind turbine and wind park example

In this section we will apply the algorithms to a 9.5 MW offshore wind turbine, inspired by the Vestas V164. We apply our model to the following four components of the turbine, the blades, main bearing, gearbox and generator. First we describe the data that we use in Section 4.3.1. This is based on scaling studies and not on actual data for the V164. We also distinguish two scenarios for the preventive maintenance costs that are difficult to determine. We apply the heuristics to obtain a low cost BRP in Section 4.3.3 based on this data for a single wind turbine. The size of the components are not representative for the 9.5 MW, but we assume that the distribution parameters are not so dependent on the size of the turbine. In Section 4.3.4 we apply the heuristics to a wind park setting with 10 of these turbines.

4.3.1 Cost and failure data

The 9.5 MW turbines are one of the newest and the most powerful turbines available. Since the turbines are very new, we do not yet have data that can be used to estimate the lifetime distribution on for the components. We use data from the paper of Tian et al. (2011) for the lifetime distributions, which is based on a 2.0 MW offshore wind turbine. We summarise the distribution parameters in Table 31.

Table 31: Lifetime distribution parameters of the four main components of the 9.5 MW turbine.

#	Weibull Parameters	α (in months)	β
1	Blade	100	3
2	Main Bearing	125	2
3	Gearbox	80	3
4	Generator	110	2

The set-up costs in this paper are 75 thousands of euros and we use these in our model as well. These represent the transportation to the wind turbine park.

The component costs of the 2.0 MW wind turbine are not representative for the costs of a 9.5 MW wind turbine. The data that we use for the 9.5 MW wind turbine is obtained from a scaling study, see Fingersh et al. (2006). The cost for every component in a wind turbine is approximated as a function of the power output and the size of the turbine. For details on the computation of the costs of the components we refer to Appendix C. These costs include material and personnel costs and are used as the corrective maintenance costs excluding the downtime costs. The preventive maintenance costs are more difficult to determine, since many preventive actions can be taken. We simply consider a constant factor between the preventive and corrective maintenance action (both excluding downtime costs). Tian et al. (2011) use a constant factor of four, which we use in Scenario 1 and we also use a constant factor of ten to see what happens in a different setting in Scenario 2. The summarised results are given in Table 32

Table 32: Cost parameters of the four main components (in thousands of euros).

#	Cost Parameter	Corrective	Preventive
1	Blade	513.20	128.30
2	Main Bearing	240.23	60.06
3	Gearbox	592.80	148.20
4	Generator	384.24	96.06

(a) Cost parameters Scenario 1.

#	Cost Parameter	Corrective	Preventive
1	Blade	513.20	51.320
2	Main Bearing	240.23	24.023
3	Gearbox	592.80	59.280
4	Generator	384.24	38.424

(b) Cost parameters Scenario 2.

Note that a wind turbine contains three blades and that the blade cost in Table 32 is given for a single blade. What remains, is to compute the missed income due to downtime of the wind turbine. Tian et al. (2011) state that the lead time of all corrective maintenance actions is approximately a month. The expected missed income for a failure is therefore equal to the expected missed income in the following 30 days plus the number of days it takes to maintain the component.

To compute the missed income we need to compute the expected power output in this month. This expected power output can be computed using historical wind speed data. For each month of the year we compute the average power output, using the power output function and the density function of the wind speed. In Figure 13 we show the average historical realisation of power output from January 1971 until December 2017. The data is obtained from Koninklijk Nederlands Meteorologisch Instituut (2018). We also use Matlab to find the cosine fit with minimised squared error. This is the function that we will use in our model to compute the cost parameters.

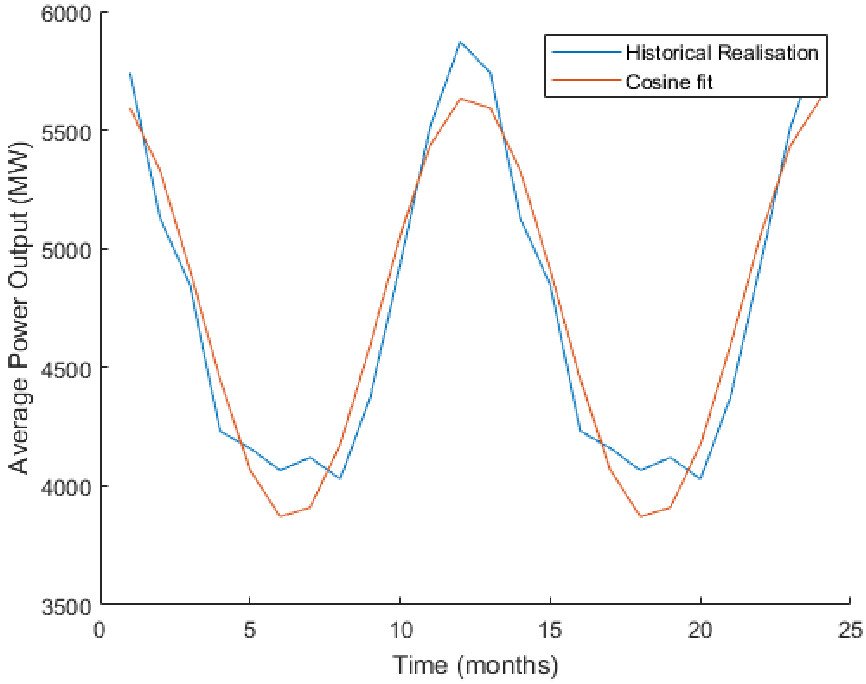


Figure 13: This figure shows the seasonal pattern of the average power output of a Vestas V164-9.5 MW wind turbine in IJmuiden.

The function for the expected power output in kWh takes the following form:

$$P(t) = 4751 + 895\cos\left(\frac{2\pi t}{12} - 0.178\right), \tag{54}$$

where t is the month of the year. Using a constant electricity price of €0.06 per kWh we can compute the missed income. This is the guaranteed price for producers in a 2018 German project, see Ten Brinck (2018). This is given by:

$$c(t) = 6.841 + 1.289\cos\left(\frac{2\pi t}{12} - 0.178\right), \tag{55}$$

where $c(t)$ is the daily missed income in thousands of euros.

Tian et al. (2011) do not state what time it takes to perform the preventive maintenance action. This should also be included since the missed income for the preventive maintenance depends on this downtime. Papatzimos et al. (2018) give the time it takes to maintain rotor blades (12 days), a gearbox (10 days) and generator (3 days) of a 2.3 MW wind turbine. We use these numbers as representative for our 9.5 MW wind turbine. The main bearing is not given but we will use 3 days, since bearings are the least complex and expensive. The missed income due to a preventive maintenance action can be computed from this and are given by:

$$\begin{aligned}
m_p^1(t) &= 82.09 + 15.47\cos\left(\frac{2\pi t}{12} - 0.178\right),\\
m_p^2(t) &= 20.52 + 3.87\cos\left(\frac{2\pi t}{12} - 0.178\right),\\
m_p^3(t) &= 68.41 + 12.89\cos\left(\frac{2\pi t}{12} - 0.178\right),\\
m_p^4(t) &= 20.52 + 3.87\cos\left(\frac{2\pi t}{12} - 0.178\right).
\end{aligned}$$

The missed income due to a corrective maintenance action is larger, since the component needs to be ordered, which takes an extra 30 days.

$$\begin{aligned}
m_f^1(t) &= 287.32 + 54.14\cos\left(\frac{2\pi t}{12} - 0.178\right),\\
m_f^2(t) &= 225.75 + 42.54\cos\left(\frac{2\pi t}{12} - 0.178\right),\\
m_f^3(t) &= 273.64 + 51.56\cos\left(\frac{2\pi t}{12} - 0.178\right),\\
m_f^4(t) &= 225.75 + 42.54\cos\left(\frac{2\pi t}{12} - 0.178\right).
\end{aligned}$$

The resulting cost parameters are given in Table 33.

Table 33: Final cost parameters for the four main components in thousands of euros.

#	Cost Parameter	$\bar{c}_f$	Δ_f	$\frac{\Delta_f}{\bar{c}_f}$	$\bar{c}_p$	Δ_p	$\frac{\Delta_p}{\bar{c}_p}$
1	Blade	800.52	54.14	6.76%	210.39	15.47	7.35%
2	Main Bearing	465.98	42.54	9.13%	80.58	3.87	4.80%
3	Gearbox	866.44	51.56	5.95%	216.61	12.89	5.95%
4	Generator	609.96	42.54	6.97%	116.58	3.87	3.32%

(a) Cost parameters Scenario 1.

#	Cost Parameter	$\bar{c}_f$	Δ_f	$\frac{\Delta_f}{\bar{c}_f}$	$\bar{c}_p$	Δ_p	$\frac{\Delta_p}{\bar{c}_p}$
1	Blade	800.52	54.14	6.76%	133.41	15.47	11.60%
2	Main Bearing	465.98	42.54	9.13%	44.54	3.87	8.69%
3	Gearbox	866.44	51.56	5.95%	127.69	12.89	10.09%
4	Generator	609.96	42.54	6.97%	58.94	3.87	6.57%

(b) Cost parameters Scenario 2.

Using these costs parameters with the distribution parameters we can compute the individual optimal maintenance blocks. Ignoring the cost fluctuations we have the following for the components.

Table 34: Optimal individual BRP excluding set-up costs for $\Delta = 0$ in Scenarios 1 and 2.

#	Component	T^* (in months)	Yearly costs
1	Blade	55	68.809
2	Main Bearing	56	34.594
3	Gearbox	43	90.404
4	Generator	52	53.631

(a) Scenario 1.

#	Component	T^* (in months)	Yearly costs
1	Blade	46	52.416
2	Main Bearing	40	26.604
3	Gearbox	35	65.703
4	Generator	36	39.766

(b) Scenario 2.

In the next section we will schedule the gearbox individually to demonstrate the added value of the single component model. In Section 4.3.3 and 4.3.4 we use these cost parameters as the input for the multi-component model for a single wind turbine and a wind turbine park of ten turbines. The sequential optimisation algorithm will use the information in Table 34 to base the order of optimisation on. The blades will be scheduled first if ordered on costs, since three blades are more expensive than the other items. We will compute the costs under the BRP model using the heuristics and compare to the standard BRP.

4.3.2 Single component example

Suppose we are only interested in optimising the gearbox maintenance for both scenarios. This could be the case if there is a specialised team for the gearbox maintenance, that cannot do other maintenance actions. This could mean that no set-up costs can be saved by combining gearbox maintenance with maintenance of other components and the gearbox can be scheduled on its own. We use the LP and MIP formulations for the ARP, BRP and MBRP to schedule this component and compute the costs. In this computation we do include set-up costs of €75000 for transportation to the wind turbine. As a reference we use the standard model in which we assume that costs are constant and compute the savings with respect to this model.

Table 35: Yearly costs (in €) of the ARP, BRP and MBRP model for the gearbox.

	ARP	BRP ($m = 3$)		MBRP ($m = 3$)		
	costs	costs	mos.	costs	mos.	ages
reference	109.771	118.208		110.914		
our model	107.093	115.043	7	108.266	7	24
savings	2.44%	2.68%		2.39%		

(a) Scenario 1.

	ARP	BRP ($m = 3$)		MBRP ($m = 3$)		
	costs	costs	mos.	costs	mos.	ages
reference	89.307	93.694		89.965		
our model	86.780	90.198	7	87.000	7	23
savings	2.83%	3.73%		3.30%		

(b) Scenario 2.

For the ARP we maintain in July for the age of 44 and in August for the age of 43 in Scenario 1. In the standard ARP we maintain from the age of $t^* = 49$. For the standard BRP we have

$t^* = 46$ and for the standard MBRP $t^* = 26$ and $T^* = 47$.

For the ARP we maintain in June for the age of 43 and in July for the age of 35 in Scenario 2. In the standard ARP we maintain from the age of $t^* = 42$. For the standard BRP we have $t^* = 40$ and for the standard MBRP $t^* = 20$ and $T^* = 40$.

We thus see that the cost savings are largest for Scenario 2. Among the policies the BRP savings are largest. The MBRP bridges 85% of the gap between the BRP and ARP in Scenario 1 and 94% in Scenario 2. The transportation costs have a very large influence on the costs for maintenance in the single component setting. If we include more components in the optimisation, we can expect the savings to be larger, since the constant set-up costs have less influence. In the next section we discuss the results for one complete wind turbine in which we optimise the maintenance costs for the four components.

4.3.3 Wind turbine example

In this section we will give the results for a single wind turbine that has lifetime distribution and cost parameters as in Tables 31 and 33. As we can see in Table 31 the average failure frequency is very low, compared to what we have considered earlier. We should therefore consider large $m > 1$ in equation (23) (i.e. we plan for schedules that repeat themselves over multiple years). We consider $m = 1, 2 \ldots, 8$ and discretise time to months to keep computation time low. In Table 36 we give the results for Scenario 1 for all considered m.

Table 36: Yearly costs (in thousands of €) of block-based maintenance model for different m in Scenario 1.

	GA		MA		SC		SF		SR	
m	costs	time (s)	costs	time (s)	costs	time (s)	costs	time (s)	costs	time (s)
1	1072.39	55	1072.39	61	1072.39	0.66	1072.39	0.37	1072.39	0.33
2	581.00	78	581.00	97	581.00	1.5	581.00	0.8	581.00	0.9
3	454.09	127	454.09	177	454.09	3.8	454.09	3.3	454.09	3.0
4	425.33	262	425.33	304	425.33	24	425.33	16	425.33	18
5	437.12	388	437.12	380	437.12	88	437.12	114	437.12	83
6	453.65	782	453.65	1185	453.65	238	454.71	225	455.19	223
7	437.62	1031	437.16	1070	437.39	875	437.39	873	443.66	821
8	425.33	1290	425.33	1411	425.33	1267	425.33	1455	437.50	1437

*Solving the model for constant costs gives costs of 441.60 for maintenance every $T^* = 50$ months. All components have to be scheduled at the same time for optimality.

We see that the best maintenance policies are obtained for $m = 4$ and $m = 8$ and the resulting policy are policies in which maintenance is done in July every 4 years. During this maintenance, every component is maintained. All algorithms give the same optimal policy and associated costs.

Ignoring the cost fluctuations (i.e. setting $\Delta_f^k = \Delta_p^k = 0$ for all k) gives yearly maintenance costs of 441.60 for $T^* = 50$ months. We thus save 3.68% with respect to this policy by maintaining every 4 years in July.

All algorithms give the same solutions for $m \leq 5$, maintenance once every 5 years. When $m = 6$ it is optimal to schedule some components once a year and some twice. The order of scheduling thus becomes important for the sequential optimisation. Scheduling the largest cost components first gives the best solution, which is the same solution as the solutions found by the GA and MA. For the $m = 7$ policy, maintenance is performed twice in 7 years and the MA finds the best solution. In this solution preventive maintenance is planned in June then 3 years later in July, 4 years later again in June etc. For $m = 8$ all algorithms, except the SR, schedule all components in July every 4 years.

Table 37: Yearly costs (in thousands of €) of block-based maintenance model for different m in Scenario 2.

	GA		MA		SC		SF		SR	
m	costs	time (s)	costs	time (s)	costs	time (s)	costs	time (s)	costs	time (s)
1	659.17	13	659.17	16	659.17	0.58	659.17	0.37	659.17	0.33
2	374.73	45	374.73	59	374.73	1.4	374.73	0.9	374.73	1.0
3	316.89	129	316.89	127	316.89	3.6	316.89	2.8	316.89	2.5
4	322.69	352	322.69	370	322.69	17	322.69	16	322.69	16
5	339.73	825	339.73	803	340.16	64	340.16	60	348.25	61
6	316.89	1296	316.89	1202	316.89	199	316.89	203	332.51	246
7	319.95	1410	319.71	1503	319.95	663	319.70	846	327.17	446
8	322.69	1748	322.69	1847	322.69	845	322.69	941	331.96	995

*Solving the model for constant costs gives costs of 334.30 for maintenance every $T^* = 42$ months. All components have to be scheduled at the same time for optimality.

We see that the best maintenance policies are obtained for $m = 3$ and $m = 6$ and the resulting policy are policies in which maintenance is done in July every 3 years. During this maintenance, every component is maintained. All algorithms give the same optimal policy and associated costs.

Ignoring the cost fluctuations (i.e. setting $\Delta_f^k = \Delta_p^k = 0$ for all k) gives yearly maintenance costs of €334300 for $T^* = 42$ months (i.e. maintenance every three and a half years). We thus save 5.21% with respect to this policy by maintaining every 3 years in July. Compared to Scenario 1, the cost savings are larger. The percentage fluctuations in preventive maintenance costs are namely larger for Scenario 2, since the preventive maintenance costs excluding downtime costs are lower. The downtime costs therefore have a relatively larger impact.

4.3.4 Wind park example

In this section we will give the results for a group of 10 identical wind turbines that have lifetime distribution and cost parameters as in Tables 31 and 33. The cost parameters are thus the same as in previous section. We are only dealing with the maintenance of 30 blades, 10 main bearings, 10 gearboxes and 10 generators. We can use our model in the ideal case that we have enough resources to maintain all components when needed. In a wind park we are typically limited by vessel and personnel constraints. The wind turbines assumed to be close together and set-up costs are associated to going to the wind park and there are no extra set-up costs for maintaining an extra turbine.

The results for Scenario 1 are given in Table 38.

Table 38: Yearly costs (in thousands of €) of block-based maintenance model for different m in Scenario 1.

	GA		MA		SC		SF		SR	
m	costs	time (s)	costs	time (s)	costs	time (s)	costs	time (s)	costs	time (s)
1	10048.85	11	10048.85	11	10048.85	1.5	10048.85	1.6	10048.85	1.6
2	5472.53	44	5472.53	43	5472.53	3.0	5472.53	3.1	5472.53	3.2
3	4315.86	129	4315.86	125	4315.86	6.8	4315.86	11	4315.86	11
4	4084.40	260	4084.40	312	4084.40	25	4084.40	33	4084.40	24
5	4208.16	1011	4208.16	560	4236.02	98	4236.02	140	4236.02	75
6	4311.40	822	4311.40	732	4311.85	264	4322.34	292	4326.69	255
7	4183.32	1008	4178.65	1100	4181.07	791	4178.65	693	4178.65	379
8	4084.40	1521	4084.40	1508	4084.40	926	4084.40	811	4084.40	486

*Solving the model for constant costs gives costs of 4253.82 for maintenance every $T^* = 49$ months. All components have to be scheduled at the same time for optimality.

The components are scheduled at the same moments as for the single turbine. Preventive maintenance is planned every four years in July and we maintain all components. Again all heuristics find the same solution. The resulting costs are lower than 10 times the cost of an individual turbine, since set-up costs are saved. The total savings is 3.98% with respect to the constant cost model, which are larger savings than for a single turbine.

The results for Scenario 2 are given in Table 39.

Table 39: Yearly costs (in thousands of €) of block-based maintenance model for different m in Scenario 2.

	GA		MA		SC		SF		SR	
m	costs	time (s)	costs	time (s)	costs	time (s)	costs	time (s)	costs	time (s)
1	5916.67	12	5916.67	17	5916.67	1.5	5916.67	1.5	5916.67	1.6
2	3409.74	45	3409.74	47	3410.14	2.9	3410.14	3.3	3410.14	3.0
3	2943.78	171	2943.78	137	2943.78	7.4	2943.78	7.2	2943.78	7.2
4	3019.73	372	3019.73	355	3057.60	21	3057.59	23	3057.60	20
5	3130.16	603	3127.26	507	3132.24	85	3131.58	71	3212.43	155
6	2943.78	883	2943.78	984	2943.78	206	2943.78	136	2943.78	211
7	3004.12	1150	3004.12	1336	3004.63	315	3004.12	255	3004.63	479
8	3058.03	1513	3058.03	1806	3058.03	477	3058.03	489	3058.03	577

*Solving the model for constant costs gives costs of 3144.95 for maintenance every $T^* = 40$ months. All components have to be scheduled at the same time for optimality.

The components are scheduled at the same moments as for the single turbine. Preventive maintenance is planned every three years in July and we maintain all components. Again all heuristics find the same solution. The resulting costs are below 10 times the costs for the individual turbine problem, since set-up costs are saved. The total savings is 6.40% with respect to the constant cost model. Again the cost savings are larger compared to the savings for Scenario 1. Also the percentage savings is larger for the park than for a single turbine in Scenario 2.

4.3.5 Conclusions for the wind turbine example

In this section we have seen that we can significantly save maintenance costs by taking the cost fluctuations into account in a wind turbine and wind park example. The results are based on a scaling model and the results are thus indications. Since we use a constant factor between preventive and corrective maintenance costs (excluding downtime costs) and the distribution parameters do not differ a lot, all components have similar optimal maintenance blocks. As a result, it is relatively easy to schedule the components and all algorithms give the same solution. Different factors between the corrective and preventive maintenance costs for different components might give more interesting results, but we have no indications of certain components being relatively cheap or expensive to maintain preventively. In the two scenarios considered for a single turbine we save 3.68% and 5.21% with respect to the constant cost model. For a wind park we can save an even larger amount with respect to the constant cost model, namely 3.98% and 6.40% for the two scenarios. In a twenty year project this means a savings of approximately 2.5 to 4 million euros over the course of the project. It is optimal to perform maintenance every 4 years in July for Scenario 1 and every 3 years in July for Scenario 2. The optimal maintenance moments are the same for the single turbine and the 10 turbines.

5 Discussion

In our model we made a first step into including time-varying costs for maintenance. For future research it would be interesting to see what cost savings can be obtained by combine the approach taken in this thesis with the condition based maintenance (CBM) approach. In current CBM approaches Markov decision processes are also sometimes used to describe the degradation level of a component. The states typically represent a set of 4 or 5 levels, which we can see as a sort of virtual age in our model. Instead of the age always increasing by one after a period the process moves to another degradation level or virtual age with a certain probability. The largest virtual age is the age corresponding to a failure. If this happens we have to maintain. We can then introduce virtual age-based maintenance policies, that base the maintenance decision on the virtual age and the time of the year. The resulting policies will be intractable like the age-based maintenance policies for multi-component settings, which is a downside of this approach. Also enough and reliable measurements of the degradation of the components are required. Block-based maintenance policies do not make sense, since these would not be able to incorporate information on the states of each component. An interesting example of CBM is usage based maintenance for which the virtual age increases less if turbines are shut down.

In our approach we made a long-term maintenance planning. We could also take into account that the expected cost of maintenance, including downtime costs, can be better estimated just before the maintenance action. We could decide to delay the maintenance if the costs of delay are smaller than the difference in expected downtime costs. To be able to model this accurately we should include in the model how wind and its predictions evolve over time. We should also realise that there is intraday correlation in wind speeds. To be able to model the wind speeds we could for example use an auto regressive model, see Ailliot et al. (2006). This would give different cost parameters that still depend on the time of the year. We could feed our model with these parameters and we find a possibly different optimal policy. We can also see the policies that we obtain in our model as a baseline policy on which we can improve by taking these short-term weather forecasts into account.

We did not base the decisions in the wind turbine and wind park example on real data. First of all, the 9.5 MW wind turbine that we considered is the most powerful and one of the newest wind turbines. The first turbines were built in 2014 for testing, but by then it produced only 7 MW, see E & T Magazine (2011). As a result, there is not enough failure data available for the complete support of the failure distribution of the 9.5 MW turbine. Other turbines with the similar size and machine rating (i.e. power output) typically do not have a gearbox and therefore this data is limited. The distribution parameters that we used was based on data for a 2 MW turbine. We used a scaling study based on turbine data of turbines with a rotor radius of maximum 70 m. The 82 m radius that we used for our turbine is above this range and the data is thus based on extrapolations of smaller turbines. It might be the case that the actual component costs are smaller due to technological improvements. The preventive maintenance costs for each component are assumed to be a constant multiplied by the corrective maintenance costs of the same component. This approach is also taken in the literature in for example Tian et al. (2011) and Shafiee and Finkelstein (2015), but it is still a guess. It could be that for some components preventive maintenance is relatively cheap, while for others its cost is closer to corrective maintenance. If more data is available, we can better estimate the cost and distribution parameters. Then the input parameters of the model better resemble the real parameters and thus the model better describes the real situation. The added value of the genetic and memetic algorithm might be larger if we have larger differences between the components.

In our optimisation approach, we did not take into account that there is a limited amount of resources. The number of vessels is finite and the maintenance staff contains a certain number of people that are under contract. Due to the savings of set-up costs, all preventive maintenance actions are grouped and planned at one moment. In practice, this could mean that we do not have enough people to be able to perform every maintenance action. In this case only a certain number of components or turbines can be maintained. Since preventive maintenance is typically performed every 3 or 4 years in our models, we could split the components in 3 or 4 groups and maintain one of the groups every year. Data on the people available and people needed for each action is required to be able to make such a decision.

Current wind park projects are typically scheduled for 20 years, see Staffell and Green (2014). We plan the preventive maintenance activities using an infinite horizon and using a repetition of the schedule every m years. Finite horizon planning changes the problem a bit, but increases the number of decision variables. $m = 8$ is the largest that we considered in the results, which gave decision variables for 96 months. If we would plan with a finite horizon this increase to decision variables for 240 months, which would massively increase computation time if we would also use an MIP to solve. It could on the other hand improve the results.

We can also consider heuristics for the MBRP. Since the optimal block times are typically comparable with the optimal block times for the BRP, we could use the BRP in this heuristic. We could use the MA or GA to find the months at which we maintain and use another approach to determine the minimal age. Choosing the minimal age to be half of the block time could potentially already improve on the BRP. Other methods can also be choosen to determine these ages for all components.

Other heuristic approaches can be considered to be able to find better solutions for the BRP as well. An example could be a discretisation heuristic in which we first discretise time very coarsely and schedule the components. Then we discretise time further and solve the problem, given that the components have to be scheduled somewhere in the time periods that they were scheduled in already. Grouping strategies can also be used in an algorithm. First components with similar maintenance frequencies are grouped together and each group is planned during a certain time period. Typically each group is scheduled without taking other groups into account. We could also do this in a sequential manner and first schedule for example the most costly group. Given these opportunities we schedule the next group, etc. Different groupings lead to different policies and costs. In the wind park example that we considered it would be best to group all components together and then schedule the complete group.

The genetic and memetic algorithms contain a lot of parameters and methods that can be altered to best suit the problem. We choose the parameters with thought, but did not study the influence of all parameters on the performance. A tuning study could be performed to determine good parameters. First of all, the mutation rate can be altered. Given a population size, the number of parents that we pick can be chosen. The selection criterion can be changed to tournament selection or probabilistic selection, instead of picking the fittest only. The number of neighbours that we add to the solution is also important. Also the starting population is very important for fast termination and for termination close to optimality. The best set of parameters and methods depends on the lifetime distributions and cost parameters.

6 Conclusion

In this thesis we introduced new ways to generalise the standard ARP, BRP and MBRP for time-varying maintenance costs. Using a discretisation of time we formulate a discrete time Markov decision process to describe the behaviour of the states of wind turbine components and compute the costs of these policies. Using a discretisation of time shows us that we can find the optimal ARP using a linear programming formulation and the BRP and MBRP using a mixed integer programming formulation. Including the cost fluctuations in the cost functions can save a significant percentage of costs compared to the constant cost policy. We find that under time-varying costs, the optimal ARP is a policy in which the age from which we maintain varies over the year. The optimal BRP is then a policy in which the block time is not constant. Similarly, the optimal MBRP is a policy in which neither the block time nor the minimal age are constant over time. The exact amount that we save using our generalisations of the known policies depends on the failure distributions and the cost parameters.

The ARP is theoretically optimal, but the policy changes every time a component fails, which is not desired and this could lead to extra costs. Also it becomes intractable if the number of components increases, since the action that we take for a certain component depends on the ages of all components. The BRP results in larger theoretical costs, but maintenance actions are planned in advance on fixed intervals and are never cancelled, which is more convenient for the maintenance teams and will not bring extra costs. The MBRP is more expensive than the ARP and less expensive than the BRP. Maintenance is planned in advance, but can be cancelled if the component has not reached a specific age at the preventive maintenance moment. For single component problems the costs of the optimal MBRP is closer to the ARP than to the BRP. For multi-component problems the MBRP is closer to the BRP in costs. Finding the optimal ARP takes the least amount of time, since it is done using an LP. The BRP is slower, since it contains binary variables for the moments of maintenance. The MBRP is slowest since it contains integer variables for the age as well as binary variables for the maintenance moments.

Due to large computation time for finding the optimal multi-component BRP, we considered three heuristic approaches: a genetic algorithm, a memetic algorithm and a sequential optimisation algorithm. Optimality is not guaranteed in any of the heuristics, but they seem to perform quite well. Especially, the memetic and genetic algorithm perform close to optimality for the considered cost and distribution parameters in a two component setting. These two methods are however more time consuming than the sequential optimisation. The methods can also be used for settings with general number of components, but if there are more than two components solving the MIP is computationally very intensive. For general cases we do, thus, not have an optimal policy as a reference and cannot prove that the heurstics are close to optimality. In the setting with more than two components the memetic and genetic algorithms also perform best.

We showed that we could use the algorithms for many different combinations of parameters. Also for a wind park and wind turbine setting with multiple components, the methods find good solutions. We did have to make some assumptions on the cost and distribution parameters and therefore we came with two scenarios and computed the BRPs with the heuristics. The cost savings with respect to the standard BRP solution are 3.68% and 5.21% for a single turbine for the two scenarios. Optimising for a group of 10 turbines saves 3.98% and 6.40% for the two scenarios. The individual optimal maintenance times did not differ a lot between the components, due to the assumptions that were made on the costs. As a result it was optimal to perform all maintenance actions at the same time every 3 or 4 years dependent on the scenario. This simple policy was among the starting population of the GA and MA for $m = 3$ or $m = 4$ and so the algorithms terminate relatively fast. The sequential optimisation also found these optimal solutions. Since the sequential optimisation under performs for other parameter choices, the MA and GA have their added value. The MA and GA seem to be most robust, since these methods always found close to optimal solutions for all parameter choices considered in this thesis.

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A Nomenclature

Below we give important notation and abbreviations used throughout the thesis.

α	scale parameter of the Weibull distribution
β	shape parameter of the Weibull distribution
$N(t)$	number of renewals (break-downs) in t time periods
$m(t)$	renewal density (i.e. expected number of renewals at t for the discrete case)
$M(t)$	renewal function (i.e. expected number of renewals until and including t)
μ	mean interarrival time
$m_f^k(i_0)$	missed income due to corrective maintenance for component k in week i_0
$m_p^k(i_0)$	missed income due to preventive maintenance for component k in week i_0
$c_f^k(i_0)$	corrective maintenance costs for component k in week i_0
$c_p^k(i_0)$	preventive maintenance costs for component k in week i_0
$\bar{c}_f^k$	mean corrective maintenance costs for component k
$\bar{c}_p^k$	mean preventive maintenance costs for component k
$\min\{c_f^k\}$	minimum corrective maintenance costs for component k
$\min\{c_p^k\}$	minimum preventive maintenance costs for component k
$\max\{c_f^k\}$	maximum corrective maintenance costs for component k
$\max\{c_p^k\}$	maximum preventive maintenance costs for component k
Δ_f^k	amplitude of time-varying corrective maintenance costs for component k
Δ_p^k	amplitude of time-varying preventive maintenance costs for component k
$p_{i_k}^k$	the probability of component k breaking down at age $i_k \in \mathcal{I}_k$
$b_k(i_0)$	break-down probability of component k in week i_0
c_T	coefficient of variation of random variable T
$\mathcal{I}$	the state space of the Markov process (for n components we have $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1 \times \cdots \times \mathcal{I}_n$)
$\mathcal{I}_b$	part of the state space that that corresponds to the broken states
$\mathcal{I}_c$	part of the state space that can be reached by the Markov process
$\mathcal{I}_0$	the set of time periods after which the policy is repeated
$\mathcal{I}_i$	the set of ages of component i
$\mathcal{A}$	the set of possible actions
$\mathcal{R}$	the set of possible policies
$\mathcal{R}_s$	the set of possible stationary policies
N	number of decision epochs in a year
M	maximum age for which we must maintain
m	number of years after which policy repeats itself
l	number of binary variables/genes in genetic and memetic algorithm
a	number of parents selected in genetic and memetic algorithm
A	size of population after cross-over in genetic and memetic algorithm
ρ	mutation probability in genetic and memetic algorithm
R	rotor radius
D	rotor diameter
P	power output (machine rating)

ARP	age replacement policy (i.e. age-based maintenance policy)
BRP	block replacement policy (i.e. block-based maintenance policy)
MBRP	modified block replacement policy (i.e. modified block-based maintenance policy)
PM	preventive maintenance/replacement
CM	corrective maintenance/replacement
LP	linear programming
MIP	mixed integer programming
GA	genetic algorithm
MA	memetic algorithm
SC	sequential optimisation with components sorted in decreasing costs
SF	sequential optimisation with components sorted in decreasing frequency
SR	sequential optimisation with components sorted in increasing frequency

B Proofs of theorems

Lemma 1. *Let $C(T)$ denote the long-run maintenance costs for a policy, where we maintain at age $T \in (0, \infty]$ for all $i_0 \in \mathcal{I}_0$. Let $c_p(i_0)$ and $c_f(i_0)$ for $i_0 \in \mathcal{I}_0$ denote the preventive and corrective maintenance costs. Then we have that $C(T) = \overline{C}(T)$, where $\overline{C}(T)$ is the long-run average costs for the problem where the maintenance costs are replaced by the means over the year (i.e. $\overline{c}_p$ and $\overline{c}_f$).*

Proof.
Let $T \in (0, \infty]$ arbitrary. Denote $z_p(i_0, T)$ and $z_c(i_0, T)$ by the long-run fraction of time that we perform preventive and corrective maintenance in period $i_0 \in \mathcal{I}_0$ under ARP with constant critical maintenance age T.

We thus have that the policy is independent of the variable $i_0 \in \mathcal{I}_0$. The resulting equilibrium probabilities for each state are then also independent of $i_0 \in \mathcal{I}_0$ and equal to the long-run fraction of time that we are in this state.

We thus have that $z_p(i_0, t) = z_p(j_0, t)$ and $z_c(i_0, t) = z_c(j_0, t)$ for all $i_0, j_0 \in \mathcal{I}_0$ and $t \in \mathbb{N}$. Then we can derive the following.

$$\begin{aligned}
C(T) &= \sum_{i_0 \in \mathcal{I}_0} \left(c_p(i_0) z_p(i_0, T) + c_f(i_0) z_c(i_0, T) \right), \\
&= \sum_{i_0 \in \mathcal{I}_0} \left(c_p(i_0) z_p(1, T) + c_f(i_0) z_c(1, T) \right), \\
&= z_p(1, T) \sum_{i_0 \in \mathcal{I}_0} c_p(i_0) + z_c(1, T) \sum_{i_0 \in \mathcal{I}_0} c_f(i_0), \\
&= z_p(1, T) \left| \mathcal{I}_0 \right| \overline{c}_p + z_c(1, T) \left| \mathcal{I}_0 \right| \overline{c}_f, \\
&= \sum_{i_0 \in \mathcal{I}_0} \left(\overline{c}_p z_p(i_0, T) + \overline{c}_f z_c(i_0, T) \right), \\
&= \overline{C}(T).
\end{aligned}$$

This proofs that the equality $C(T) = \overline{C}(T)$ must hold. □

Theorem 1. *Let p_x denote the failure probabilities of a component between the ages $x - 1$ and x and let $p_\infty = \lim_{x \to \infty} p_x > \frac{\Delta t}{\mathbb{E}(T)} \frac{\overline{c}_f}{\overline{c}_f - \overline{c}_p}$. Then the optimal ARP has a finite maintenance age for at least one period.*

Proof.
Let $\overline{C}(T)$ denote the long-run average costs for the problem where the maintenance costs are replaced by the means over the year (i.e. $\overline{c}_p$ and $\overline{c}_f$). According to Nakagawa and Osaki (1977) we have that there exists a $T \in (0, \infty)$ such that:

$$\overline{C}(T) < \overline{C}(\infty).$$

From Lemma 1 we have that:

$$\begin{aligned}
C(T) &= \overline{C}(T), \\
C(\infty) &= \overline{C}(\infty).
\end{aligned}$$

It follows that there exists a $T \in (0, \infty)$ such that:

$$C(T) = \overline{C}(T) < \overline{C}(\infty) = C(\infty).$$

This shows that there exists a policy with finite optimal preventive maintenance age that has lower costs than the policy where we do not maintain preventively at all. □

Lemma 2. *For 1-arithmetic lifetime distributions any ARP induces a Markov chain that contains one recurrent class and is therefore unichain.*

Proof. Suppose we have an ARP with critical maintenance ages t_{i_0} for $i_0 \in \mathcal{I}_0$.

Then the state $i = \{1, 0\}$ can be reached directly from any state with $j = \{j_0, j_1\}$ with $j_0 = N$ (i.e. any state corresponding to period N). States outside period N will eventually reach period N (within $N - 1$ periods) and thus has a positive probability of reaching $i = \{1, 0\}$ (within N periods). i is thus accessible from any other state.

Now consider the set:

$$\mathcal{I}_c = \{\{i_0, i_1\} \in \mathcal{I} : i_1 < t_{j_0} + 1 + (i_0 - j_0 - 1) \mod N \ \forall j_0 \in \mathcal{I}_0\}.$$

Now let $k = \{k_0, k_1\} \in \mathcal{I} \backslash \mathcal{I}_c$ arbitrarily. From the definition of $\mathcal{I}_c$ we have that there exists a j_0 such that $k_1 \geq t_{j_0} + 1 + (k_0 - j_0 - 1) \mod N$.

Then the maximum age at period j_0 is $t_{j_0} - 1$ and $1 + (k_0 - j_0 - 1) \mod N$ days later we are in period k_0. Then the maximum age that can be reached in period k_0 is $t_{j_0} - 1 + 1 + (k_0 - j_0 - 1) \mod N = t_{j_0} + (k_0 - j_0 - 1) \mod N$. Since $k_1 > t_{j_0} + (k_0 - j_0 - 1) \mod N$, k cannot be reached. All points in $\mathcal{I} \backslash \mathcal{I}_c$ are not accessible from points in $\mathcal{I}_c$.

Clearly $\{0, 1\} \in \mathcal{I}_c$. Now we show that any point in $\mathcal{I}_c$ is accessible from $\{0, 1\}$. This implies that $\mathcal{I}_c$ is a recurrent class, since i is accessible from any point in $\mathcal{I}_c$.

Let $l = \{l_0, l_1\}$ be arbitrarily other than $\{1, 0\}$. We distinguish two cases.

1. $l_1 = 0$.
2. $l_1 > 0$.

Suppose $l_1 = 0$.
Then l can be reached from $\{1 + (l_0 - 2) \mod N, 0\}$ (i.e. the broken-down state of the previous period). This state can again be reached from $\{2 + (l_0 - 3) \mod N, 0\}$ the broken-down state of the period before. By induction we see that we can reach the state l from any state $\{i_0, 0\}$ with $i_0 \in \mathcal{I}_0$ and so it is accessible from $\{1, 0\}$.

Suppose $l_1 > 0$.
Then we have that $l_1 < t_{j_0} + 1 + (l_0 - j_0 - 1) \mod N \ \forall j_0 \in \mathcal{I}_0$. Then also $\forall j_0 \in \mathcal{I}_0$:

$$\begin{aligned}
l_1 - 1 &< t_{j_0} + (l_0 - j_0 - 1) \mod N, \\
&\leq t_{j_0} + 1 + (l_0 - j_0 - 2) \mod N, \\
&= t_{j_0} + 1 + ((l_0 - 2) \mod N - j_0) \mod N, \\
&= t_{j_0} + 1 + ((1 + (l_0 - 2) \mod N) - j_0 - 1) \mod N.
\end{aligned}$$

which shows that l can be reached from $\{1 + (l_0 - 2) \mod N, i_1 - 1\} \in \mathcal{I}_c$ (i.e. the state corresponding to the previous period and the component being one time-unit younger).

If $i_1 - 1 > 0$ this state can be reached from $\{2 + (l_0 - 3) \mod N, i_1 - 2\} \in \mathcal{I}_c$. By induction we can reach the state $\{i_0, 0\}$ for some $i_0 \in \mathcal{I}_0$ in i_1 steps. The state $\{i_0, 0\}$ again can be reached from $\{1, 0\}$ within $N - 1$ steps.

This shows that any state $\{l_0, l_1\}$ can be reached from $\{1, 0\}$ within $N - 1 + l_1$ steps, so any state in $\mathcal{I}_c$ is accessible from $\{1,0\}$ and $\{1,0\}$ is accessible from any state in $\mathcal{I}_c$. So $\mathcal{I}_c$ is a recurrent class.

Points outside $\mathcal{I}_c$ cannot be accessed from $\mathcal{I}_c$ and are thus transient. There is thus one recurrent class and the corresponding Markov chain is unichain. □

Lemma 3. *Let the lifetime distribution of components T be distributed such that $\frac{c_p}{c_f} < \frac{1}{2}(1-c_T^2)$, where c_f and c_p are the constant corrective and preventive maintenance costs. Then the optimal block-based maintenance time is finite.*

Proof.
Suppose this is not the case. We will derive a contradiction.
Assume that $\frac{c_p}{c_f} < \frac{1}{2}(1-c_T^2)$.

Using the renewal-reward theorem we can derive that the long-run costs are

$$\frac{c_f}{\mu} = \frac{c_f}{\mathbb{E}(T)},$$

if we do not perform any maintenance. The long-run costs of doing maintenance every t time units is given by

$$\frac{c_p + c_f M(t)}{t},$$

where $M(t)$ is the renewal function. Since we assumed that not maintaining is optimal we must have that:

$$\frac{c_p + c_f M(t)}{t} \geq \frac{c_f}{\mu} \quad \forall t \in \mathbb{N},$$

For the renewal function we have that:

$$\lim_{t\to\infty}\left(M(t) - \frac{t}{\mu}\right) = \frac{1}{2}(c_T^2 - 1).$$

Now let $\epsilon > 0$ arbitrarily. Then there exists a $t \in \mathbb{N}$ such that:

$$M(t) < \frac{t}{\mu} + \frac{1}{2}(c_T^2 - 1) + \epsilon.$$

So we have

$$\begin{aligned} \frac{c_f}{\mu} &\leq \frac{c_p + c_f M(t)}{t}, \\ &< \frac{c_p}{t} + \frac{c_f}{\mu} + \frac{c_f(c_T^2-1)}{2t} + \frac{c_f\epsilon}{t}. \end{aligned}$$

So for $\epsilon > 0$ arbitrarily we have that:

$$\frac{1}{2}(1-c_T^2) - \epsilon < \frac{c_p}{c_f}.$$

So

$$\frac{1}{2}(1-c_T^2) = \lim_{\epsilon\downarrow 0}\left[\frac{1}{2}(1-c_T^2) - \epsilon\right] \leq \lim_{\epsilon\downarrow 0}\frac{c_p}{c_f} = \frac{c_p}{c_f},$$

which is in contradiction with the assumption $\frac{c_p}{c_f} < \frac{1}{2}(1-c_T^2)$. The optimal block time is thus finite and thus the theorem holds. □

Theorem 2. *Let the lifetime distribution of components T be distributed such that $\frac{\overline{c}_p}{\overline{c}_f} < \frac{1}{2}(1-c_T^2)$, where c_f and c_p are the time-varying corrective and preventive maintenance costs with yearly means $\overline{c}_p$, $\overline{c}_f$. Then the optimal block-based maintenance time is finite.*

Proof.
From Lemma 3 we know that the problem where the maintenance costs are replaced by the means, has a finite optimal block-based maintenance time $t^* \in \mathbb{N}$. We will now consider the policy in which maintenance is done every t^* time periods. Our long-run expected costs could now depend on the period of the year in which we start.

Let $C(t, t_0)$ denote the long-run average yearly costs when we maintain preventively at $t_0 + nt$ for all $n \in \mathbb{N}$. Let $C(t)$ denote the long-run costs for the policy in which we maintain preventively every t periods and the policy is started uniformly at period $t_0 \in \{1, 2, \dots, N\}$. Then we have that $C(t) = \frac{1}{N}\sum_{t_0=1}^{N} C(t, t_0)$.

Now consider the policy P in which we maintain every t^* periods and let the first maintenance time be at t_0, where t_0 is chosen uniformly from $\{1, 2, \dots, N\}$. Consider regeneration epochs at intervals of length Nt^*, starting from t_0 and use the renewal reward theorem to compute the long-run costs of policy P. The expected preventive maintenance costs during a cycle are:

$$\sum_{i_0=1}^{N} \frac{1}{N} \sum_{n=1}^{N} c_p(i_0 + nt^*) = \sum_{n=1}^{N} \frac{1}{N} \sum_{i_0=1}^{N} c_p(i_0 + nt^*) = \sum_{n=1}^{N} \overline{c}_p = N\overline{c}_p.$$

The expected corrective maintenance costs during a cycle are:

$$\sum_{i_0=1}^{N} c_f(i_0) M'(Nt^*, i_0),$$

where $M'(x, i_0)$ is the expected number of break-downs in period i_0 under this policy during x time periods. Since t_0 is uniform on $\{1, 2, \dots, N\}$, the expected number of break-downs is equal for all time periods $i_0 \in \{1, 2, \dots, N\}$ as long as $x \in N \cdot \mathbb{N}$. Then

$$\begin{aligned}
\sum_{i_0=1}^{N} c_f(i_0) M'(Nt^*, i_0) &= \sum_{i_0=1}^{N} c_f(i_0) M'(Nt^*, 1), \\
&= N\overline{c}_f M'(Nt^*, 1), \\
&= \overline{c}_f M'(Nt^*), \\
&= N\overline{c}_f M'(t^*).
\end{aligned}$$

Combining these costs and using a cycle length of Nt^* in the renewal reward theorem gives the following.

$$C(t^*) = \frac{N\overline{c}_p + N\overline{c}_f M(t^*)}{Nt^*} = \frac{\overline{c}_p + \overline{c}_f M(t^*)}{t^*},$$

which are the minimal costs for the constant cost model with costs $\overline{c}_p$ and $\overline{c}_f$.

Applying Lemma 3 gives the following:

$$C(t^*) = \frac{\overline{c}_p + \overline{c}_f M(t^*)}{t^*} < \frac{\overline{c}_f(i_0)}{\mu} = C(\infty),$$

where we use equation (14). We thus see that we have costs lower than $C(\infty)$ if we use policy P.

Now let $C(t^*, t_0)$ denote the long-run average costs of the policy in which we maintain at $t_0 + nt^*$ for all $n \in \mathbb{N}$. Using the linearity of the expectation gives:

$$\sum_{t_0=1}^{N} \frac{1}{N} C(t^*, t_0) = C(t^*),$$

so there exists a t_0 for which

$$C(t^*, t_0) \leq C(t^*) < C(\infty).$$

So the optimal block-based maintenance policy is a policy with finite maintenance times. □

Theorem 3. *Let the lifetime distribution of components T be distributed such that $\frac{\min\{c_p\}}{\bar{c}_f} + b < \frac{1}{2}(1 - c_T^2)$, where $b = \limsup_{n\to\infty} \left\{\sum_{t=1}^{nN} m(t) \left(\frac{c_f(t_{min}+t)}{\bar{c}_f} - 1\right)\right\}$. Then the optimal block-based maintenance time is finite.*

Proof.
Suppose this is not the case. We will derive a contradiction.

Using the renewal-reward theorem we can derive that the long-run costs are

$$\frac{\bar{c}_f}{\mu} = \frac{\bar{c}_f}{\mathbb{E}(T)},$$

if we do not perform any maintenance. The long-run costs of doing maintenance every n years is given by

$$\frac{\min\{c_p\} + \sum_{t=1}^{nN} m(t) c_f(t + t_{min})}{nN}.$$

Here $m(t)$ is the expected number of break-downs at t. Since we assumed that not maintaining is optimal we must have that for arbitrary $n \in \mathbb{N}$:

$$\begin{aligned}
\frac{\bar{c}_f}{\mu} &\leq \frac{\min\{c_p\} + \sum_{t=1}^{nN} m(t) c_f(t + t_{min})}{nN}, \\
&= \frac{\min\{c_p\} + \sum_{t=1}^{nN} m(t) \left(c_f(t + t_{min}) - \bar{c}_f\right) + \sum_{t=1}^{nN} m(t)\bar{c}_f}{nN}, \\
&= \frac{\min\{c_p\} + \bar{c}_f \sum_{t=1}^{nN} m(t) \left(\frac{c_f(t+t_{min})}{\bar{c}_f} - 1\right) + \bar{c}_f M(nN)}{nN}, \\
&\leq \frac{\min\{c_p\} + \bar{c}_f b + \bar{c}_f M(N)}{nN}.
\end{aligned}$$

For the renewal function we have that:

$$M(t) \to \frac{t}{\mu} + \frac{1}{2}(c_T^2 - 1) \quad \text{for } t \to \infty.$$

Now let $\epsilon > 0$ arbitrarily. Then there exists an $n \in \mathbb{N}$ such that:

$$M(nN) < \frac{nN}{\mu} + \frac{1}{2}(c_T^2 - 1) + \epsilon.$$

Therefore

$$\begin{aligned}
\frac{\bar{c}_f}{\mu} &\leq \frac{\min\{c_p\} + \bar{c}_f b + \bar{c}_f M(nN)}{nN}, \\
&\leq \frac{\min\{c_p\} + \bar{c}_f b}{nN} + \frac{\bar{c}_f}{\mu} + \frac{\bar{c}_f(c_T^2 - 1)}{2nN} + \frac{\bar{c}_f \epsilon}{nN}.
\end{aligned}$$

So we have that:

$$\frac{1}{2}(1-c_T^2) \leq \frac{\min\{c_p\}}{\bar{c}_f} + b + \epsilon \quad \forall \epsilon,$$

which is in contradiction with the assumption $\frac{\min\{c_p\}}{\bar{c}_f} + b < \frac{1}{2}(1-c_T^2)$. The optimal block-based maintenance time is thus finite. □

Theorem 4. *Let p_x denote the failure probabilities of a component between the ages $x-1$ and x and let $p_\infty = \lim_{x\to\infty} p_x > \frac{\Delta t}{\mathbb{E}(T)} \frac{c_f}{c_f - c_p}$. Then the optimal MBRP under has a finite optimal maintenance age t and block T.*

Proof.
Since condition (11) is met, we know that there exists a finite optimal age for the ARP. Denote this optimal age by T^*. Every time the maintenance age reaches T^*, we in expectation save $\Delta C > 0$ by maintaining, compared to never maintaining any more.

Now consider the MBRP in which we maintain at nT^* for all $n \in \mathbb{N}$ if the age exceeds T^*. Denote the long-run costs of this policy by $C(T^*, T^*)$. The MBRP has the same decision rule as the zero maintenance policy for $T \notin \{nT^* : n \in \mathbb{N}\}$ and the same decision rule as the optimal ARP for $T \in \{nT^* : n \in \mathbb{N}\}$. Suppose that a component reaches an age $T \geq T^*$ at time period nT^* for some $n \in \mathbb{N}$. Then it is optimal to perform preventive maintenance and we save at least ΔC compared to not maintaining any more.

Now define

$$\begin{aligned} a(t) &:= \text{ The age at time } t. \\ Q(n) &:= \text{ Number of times that we maintain until time } nT^*. \\ 1_M(n) &:= \begin{cases} 1, & \text{if we maintain preventively at } nT^*, \\ 0, & \text{else.} \end{cases} \end{aligned}$$

The total amount that we save until nT^* by maintaining at these moments is at least $\Delta C Q(n)$. Then for the long-run average costs per time period:

$$\begin{aligned} C(\infty) - C(T^*, T^*) &\geq \lim_{n\to\infty} \Delta C \frac{Q(n)}{nT^*}, \\ &= \frac{\Delta C}{T^*} \lim_{n\to\infty} \frac{Q(n)}{n}, \\ &= \frac{\Delta C}{T^*} \lim_{n\to\infty} \mathbb{P}(\mathbf{1}_M(n) = 1). \end{aligned}$$

Denote P_x by the probability of not breaking down till the next opportunity given that the age is x at the previous opportunity. For any age the probability of not breaking down in T^* time units is greater than zero, since the Weibull distribution has infinite support and so $P_x > 0$ for all $x \geq 0$.

$$\begin{aligned} \mathbb{P}(1_M(n) = 1) &= \sum_{x=0}^{2T^*-1} \mathbb{P}(1_M(n) = 1 | a((n-1)T^*) = x) \cdot \mathbb{P}(a((n-1)T^*) = x), \\ &= \sum_{x=0}^{2T^*-1} P_x \cdot \mathbb{P}(a((n-1)T^*) = x), \\ &\geq \min_x P_x \sum_{x=0}^{2T^*-1} \mathbb{P}(a((n-1)T^*) = x), \\ &= \min_x P_x. \end{aligned}$$

Then also

$$C(\infty) - C(T^*, T^*) \geq \frac{\Delta C}{T^*} \min_x P_x > 0.$$

and so the (T^*, T^*) policy is better than the zero maintenance policy and thus the optimal t and T are finite. □

Note that we did the analysis for a maintenance block $t = T^*$ and minimum age $T = T^*$. We can also do the analysis for $t = kN$ and $T = kN$ as long as $kN \geq T^*$, since an ARP with maintenance age $T \geq T^*$ is better than not maintaining at all. We can in fact show that any MBRP with $t^* \geq T^*$, with T^* the optimal ARP age, is better than no maintenance.

Theorem 5. *Let p_x denote the failure probabilities of a component between the ages $x-1$ and x and let $p_\infty = \lim_{x\to\infty} p_x > \frac{\Delta t}{\mathbb{E}(T)} \frac{\overline{c}_f}{\overline{c}_f - \overline{c}_p}$. Then the optimal MBRP has finite block times and minimal ages.*

Proof.
From Theorem 4 we know that the problem where the maintenance costs are replaced by the means, has a finite optimal maintenance block $T^* \in \mathbb{N}$ and minimal age $t^* \in \mathbb{N}$. We will now consider the MBRP with $t = kN$ and $T = kN$ for some k such that $kN \geq T^*$ for the time-varying maintenance cost problem. Our long-run average costs could now depend on the period of the year in which we start denoted by t_0.

Let $C(t, T, t_0)$ denote the long-run costs when we maintain preventively at $t_0 + nT$ for all $n \in \mathbb{N}$ if the age reaches t. Let $C(t, T)$ denote the long-run costs per time period for the policy in which we maintain preventively every T periods if the age reaches t and the policy is started uniformly at period $t_0 \in \{1, 2, \ldots, N\}$. Then we have that $C(t, T) = \frac{1}{N} \sum_{t_0=1}^{N} C(t, T, t_0)$.

Now let $\overline{C}(t, T)$ denote the long-run costs per unit of time when we maintain every T time units from age t onwards, where we pay constant costs $\overline{c}_p$ and $\overline{c}_f$ for prevent and corrective maintenance respectively. Using the renewal reward theorem with regeneration epoch at every preventive maintenance action, we have that:

$$\overline{C}(kN, kN) = \frac{\overline{c}_p + \overline{c}_f \mathbb{E}(N_f)}{\mathbb{E}(L)},$$

where N_f denotes the number of corrective maintenance moments during a cycle. L denotes the length of the cycle.

Similarly we have for the MBRP with starting period t_0, $t = kN$ and $T = kN$.

$$C(kN, kN, t_0) = \frac{c_p(t_0) + \mathbb{E}(C_f(t_0))}{\mathbb{E}(L)},$$

where $C_f(t_0)$ is the corrective maintenance costs during a cycle that is started at t_0. $C(kN, kN, t_0)$ is different for t_0 since we maintain at different moments, resulting in different costs. The expected number of break-downs is still the same, since the statistical process of break-downs is independent of the time of the year. Starting uniformly over the year means that break-downs are also uniform over the year. The average costs that are paid for corrective maintenance are thus $\overline{c}_f$, so we can use that $\sum_{t_0=1}^{N} C_f(t_0) = \overline{c}_f \cdot \mathbb{E}(N_f)$.

$$
\begin{aligned}
C(kN,kN) &= \frac{1}{N}\sum_{t_0=1}^{N} C(kN,kN,t_0), \\
&= \frac{1}{N}\sum_{t_0=1}^{N} \frac{c_p(t_0)+\mathbb{E}(C_f(t_0))}{\mathbb{E}(L)}, \\
&= \frac{1}{N\mathbb{E}(L)}\sum_{t_0=1}^{N} \left(c_p(t_0)+\mathbb{E}(C_f(t_0))\right), \\
&= \frac{1}{N\mathbb{E}(L)}\left(N\overline{c}_p+\sum_{t_0=1}^{N}\mathbb{E}(C_f(t_0))\right), \\
&= \frac{1}{N\mathbb{E}(L)}\left(N\overline{c}_p+N\overline{c}_f\mathbb{E}(N_f)\right), \\
&= \frac{\overline{c}_p+\overline{c}_f\mathbb{E}(N_f)}{\mathbb{E}(L)}, \\
&= \overline{C}(kN,kN), \\
&< C(\infty),
\end{aligned}
$$

where the last inequality follows from Theorem 4. The zero maintenance policy is thus suboptimal. The optimal MBRP is thus a policy in which preventive maintenance is performed. □

C Cost data

The data used for the wind turbine example is obtained from a scaling study, see Fingersh et al. (2006). The cost for every component in a wind turbine is discussed. The costs for the four components that we consider are a function of the radius R (or diameter D) of the rotor in meters and the machine rating P in kW. We give the four costs below in equation (56).

$$c_{blade} = \frac{0.4019R^3 - 21051 + 2.7445R^{2.5025}}{0.72} \tag{56a}$$

$$c_{bearing} = 0.0184D^{2.5}\left(\frac{D}{75} - 0.033\right) \tag{56b}$$

$$c_{gearbox} = 74.1P \tag{56c}$$

$$c_{generator} = 48.03P \tag{56d}$$

Using $R = 82$ m, $D = 164$ m, $P = 8000$ kW we get the following for the costs.

Table 40: Costs for corrective maintenance

#	Cost Parameter	Corrective
1	Blade	513.20
2	Main Bearing	240.23
3	Gearbox	592.80
4	Generator	384.24

Note that we use $P = 8$ MW, since the design of the blade, main bearing, gearbox and generator are unchanged in the upgrade from 8 MW to 9.5 MW, see Modern Power Systems (2017). There are three blades in each turbine and in the maintenance planning we must thus consider three times component 1 in the optimisation.

D Alternative ways to compute the standard ARP, BRP and MBRP

Given that there are no cost fluctuations our methods return a standard ARP, BRP and MBRP (i.e. the ages from which we maintain and blocks times are constant in time). Alternative ways to compute the optimal policy in a single component setting are given in Section D.1. In Section D.2 we do this for multi-component settings.

D.1 Single component systems

In this section we show an alternative way to compute the optimal policy given that maintenance costs are constant in the single component setting. Let c_f and c_p denote the constant corrective and preventive maintenance costs. Furthermore let p_t denote the probability of a failure at age t. For Examples 1, 2 and 4 we can use these methods to compute the optimal ages and block times. We also use these methods for computing the optimal policy if costs are constant in Section 4.1.

Given a policy we can compute the costs using a Markov process in which the state space consists of the ages of the component only, generally we have $\mathcal{I} = \mathbb{N}$ as state space. The time of the year is thus not included in the state space. If we maintain from age t we will never reach states $s > t$. The state space can be reduced to $\mathcal{I} = \{0, \ldots, t\}$.

Given a maintenance age t we can compute the steady state probabilities of reaching each state by setting the inflow equal to the outflow. Let x_s denote this long-run probability of moving to age s under the ARP with maintenance age t. We have that $x_s = 0$ for $s > t$. Solving the following set of equations can give us these long-run probabilities x_s for $s \leq t$.

$$\begin{aligned}
x_0 &= p_0 x_t + \sum_{s=0}^{t-1} p_s x_s \\
x_1 &= (1 - p_0)(x_0 + x_t) \\
x_2 &= (1 - p_1)x_1 \\
&\vdots \\
x_t &= (1 - p_{t-1})x_{t-1} \\
\sum_{s=0}^{t} x_s &= 1
\end{aligned}$$

Given these long-run probabilities we can compute the costs using the renewal reward theorem. Then we have that the costs per time period are given by:

$$c_f x_0 + c_p x_t.$$

Finding the optimal solution can also be done by solving LP formulation (16), where we use constant costs. In this formulation we can also add constraints to compute the costs of a specific ARP with maintenance age t.

A similar approach can be taken for the BRP. The state space should be expanded. If we maintain every T time periods, the age cannot exceed T and we have $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1$ with $\mathcal{I}_0 = \{1, \ldots, T\}$ and $\mathcal{I}_0 = \{0, \ldots, T\}$. The maintenance decision depends on the time that passed and the state should contain information about the time that passed till the last maintenance. Given a maintenance block time T we can compute the steady state probabilities of reaching each state by setting the inflow equal to the outflow.

For simplicity we use formulation (22), with constant costs. The state space is changed to $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1$ with $\mathcal{I}_0 = \{1, \dots, T\}$ and $\mathcal{I}_0 = \{0, \dots, T\}$ and $M = T$. Furthermore, we set $y_s = 0$ for $s \leq T - 1$ and $y_T = 1$.

A similar approach can be taken for the MBRP. If we maintain every T time periods if the age is at least t, the age cannot exceed $T + t$ and we have $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1$ with $\mathcal{I}_0 = \{1, \dots, T\}$ and $\mathcal{I}_0 = \{0, \dots, T + t\}$. The maintenance decision depends on the time that passed and the state should contain information about the time that passed till the last maintenance. Given a maintenance block time T and minimal age t we can compute the steady state probabilities of reaching each state by setting the inflow equal to the outflow.

For simplicity we use formulation (32), with constant costs. The state space is changed to $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1$ with $\mathcal{I}_0 = \{1, \dots, T\}$ and $\mathcal{I}_0 = \{0, \dots, T + t\}$ and $M = T$. Furthermore, we set $y_s = 0$ for $s \leq T - 1$ and $y_T = 1$. In this formulation we also set $t_T = t$ to make sure that we maintain for ages above t only.

D.2 Multi-component systems

For multi-component systems we can also compute the standard ARP, BRP and MBRP for constant maintenance costs. Assume we have constant preventive and corrective maintenance costs c_p^i and c_f^i respectively for component i. If we are dealing with 2 components we can simply use formulation (36).

For the BRP with 2 components we can use formulation (39). We should however adjust the state space. For a certain BRP with maintenance block times T_1 and T_2 we can compute the optimal policy using formulation (39), where we replace $|\mathcal{I}_0| = Nm$ by $\text{lcm}(T_1, T_2)$ (i.e. the least common multiple of T_1 and T_2). Furthermore we add constraints such that we maintain every T_i periods for component i. If we wish to find the optimal BRP, we should consider all possibilities for $|\mathcal{I}_0|$. The optimal value for $|\mathcal{I}_0|$ can be any natural number, so it can take much time to find the optimal $|\mathcal{I}_0|$.

The heuristics can also be used to compute multi-component BRPs. In the MA, GA and sequential optimisation we can set $l = Nm$ to be any natural number and the algorithms will find solutions that repeat themselves every l periods. It can however be the case that we should consider many l before we find a good solution.

For the MBRP with 2 components we can use formulation (48). We should similarly to the BRP adjust the state space. For a certain BRP with maintenance block times T_1 and T_2 and minimal ages t_1 and t_2 we can compute the optimal policy using formulation (39), where we replace $|\mathcal{I}_0| = Nm$ by $\text{lcm}(T_1, T_2)$ (i.e. the least common multiple of T_1 and T_2). Furthermore we add constraints such that we maintain every T_i periods from age t_i for component i. If we are interested in finding the optimal MBRP we might need to consider many values for $|\mathcal{I}_0|$ before we find the optimal solutions.

E Simplified age-based maintenance

In this section we restrict ourselves to the class of policies in which the action that we take on a component only depends on the age of that component. This class might not contain the optimal policy, but it is much easier to interpret the results and the policy is simply given by the critical maintenance ages for each period of the year.

We can implement this in our model by adding binary decision variable to the formulation of Section 3.4.1 that indicate whether or not we maintain preventively for each age and each period of the year.

$$y^k_{i_0,i_k} = \begin{cases} 1, & \text{if we maintain component } k \text{ preventively at the start of period } i_0 \in \mathcal{I}_0 \text{ at age } i_k, \\ 0, & \text{else.} \end{cases}$$

If we maintain component $k \leq n$, we are only allowed to take action $a_k = 1$. The other state action frequencies should thus be set to zero. We add the following constraints for all $k \leq n$ and $i \in \mathcal{I}$ with $i_1 \neq 0$ $i_2 \neq 0$.

$$\begin{aligned} x_{i,a} &\leq 1 - y^k_{i_0,i_k} && \text{for } a \in \mathcal{A} : a_k = 0, \\ x_{i,a} &\leq y^k_{i_0,i_k} && \text{for } a \in \mathcal{A} : a_k = 1, \end{aligned}$$

Furthermore, if we maintain at age t we must maintain at age $t+1$ as well. The following constraints make sure that this is indeed the case.

$$y^k_{i_0,i_k} \leq y^k_{i_0,i_k+1} \qquad \forall k \leq n,\ i_k \in \{1, 2, \ldots, M-1\}.$$

Adding these constraints to the original problem gives the following MIP formulation. This is computationally thus more intensive than the non-simplified version, which was an LP formulation. The policy thus seems easier, but it is harder to solve.

$$\text{(SARP)} \qquad \text{minimise} \quad \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}} c_{i,a} x_{i,a} \tag{57a}$$

such that:

$$\sum_{a \in \mathcal{A}(i)} x_{ia} - \sum_{j \in \mathcal{I}} \sum_{a \in \mathcal{A}(j)} \pi_{ji}(a) x_{ja} = 0 \qquad \forall i \in \mathcal{I} \tag{57b}$$

$$x_{ia} = 0 \qquad \forall k \leq n, i \in \mathcal{I}, a \in \mathcal{A} : i_k \in \{0, M\}, a_k = 0 \tag{57c}$$

$$x_{i,a} + y^k_{i_0,i_k} \leq 1 \qquad \forall k \leq n, i \in \mathcal{I}, a \in \mathcal{A} :\ a_k = 0 \tag{57d}$$

$$x_{i,a} - y^k_{i_0,i_k} \leq 0 \qquad \forall k \leq n, i \in \mathcal{I}, a \in \mathcal{A} :\ a_k = 1 \tag{57e}$$

$$y^k_{i_0,i_k} - y^k_{i_0,i_k+1} \leq 0 \qquad \forall k \leq n,\ i_k \in \mathcal{I}_k,\ i_0 \in \mathcal{I}_0. \tag{57f}$$

$$\sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}(i)} x_{ia} = 1 \tag{57g}$$

$$x_{ia} \geq 0 \qquad \forall i \in \mathcal{I},\ a \in \mathcal{A} \tag{57h}$$

$$y^k_{i_0,i_k} \in \{0, 1\} \qquad \forall k \leq n,\ i_0 \in \mathcal{I}_0,\ i_k \in \mathcal{I}_k \tag{57i}$$

In Figure 14 we show a possible policy that we obtain from this simplification. This is the policy that we obtain if we have the same cost parameters as in Figure 7.

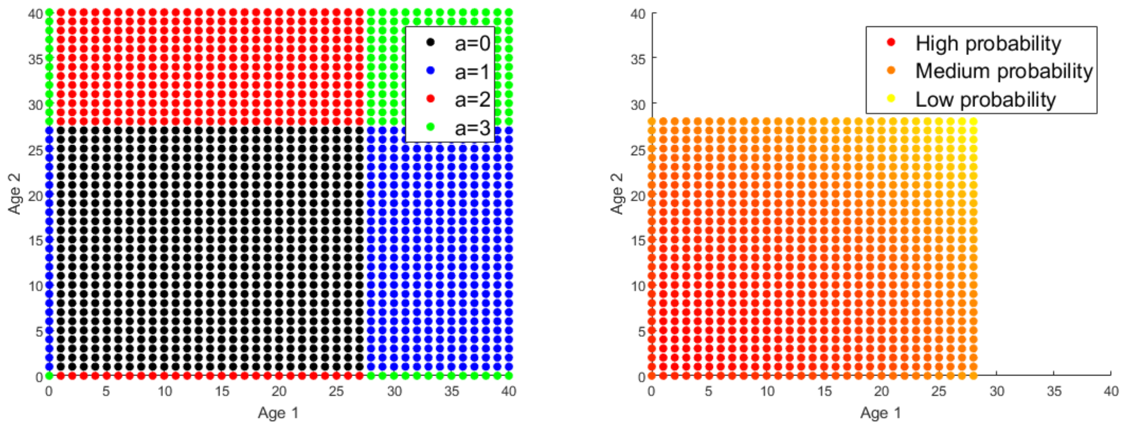

Figure 14: This figure shows the action a that we take for component ages $i_1 \in \mathcal{I}_1$ and $i_2 \in \mathcal{I}_2$. The costs are €30795 per year.

We see that we maintain both components from an age of 27 onwards. The costs of this policy are 5.61% larger than the costs of the optimal policy from Figure 7.

This policy can easily be extended to multiple dimensions. For n dimensions the policy is given by a tuple of size n that contains the critical ages of all components. Note that the number of binary decision variables increases with a factor M for every component (M is the dimension to the state space) that we add. This massively increases computation time and a heuristic approach might be needed for larger n.