Erasmus University Rotterdam

Master Thesis International Economics

Erasmus School of Economics

Asymmetric Effects of Exchange Rate Volatility: Theory and Evidence

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9th July 2019

Abstract

In this paper, I scrutinise the effect of exchange rate volatility on bilateral trade whilst making use of disaggregated positive and negative changes in volatility. The effect of positive and negative changes is shown to be asymmetric. Moreover, a negative effect of squared changes in the coefficient of variation of the exchange rate on trade is found. A combination of isoelastic utility and stochastic volatility provides a theoretical model that explicitly maps the influence of the variance of exchange rate volatility on trade. In this higher order approximation of a CRRA utility function, the notion of risk aversion is sufficient to explain aversion to the variance of volatility. Illustrative numerical examples indicate that the model is qualitatively capable of explaining both the sign and relative magnitudes of the empirical results. The degree of risk aversion and the maturity of the forward market are found to be moderating variables in this model.

Contents

1 Introduction

In an era in which the world witnesses both the increased prevalence of flexible exchange rates and the uprise of the Economic and Monetary Union, an economically extensive currency union, much attention is attributed to exchange rate volatility. More specifically, researchers have concerned themselves with the question whether exchange rate fluctuations lead to changes in the level of trade. Theories on this question often follow the notion of risk aversion. Trade, generally associated with economic welfare, is often considered of great importance by policy makers. Although much has already been written on the potential effect of exchange rate uncertainty on trade, a consensus on the actual effect is far from being reached.

This study aims to add to the ongoing debate on the effect of exchange rate volatility by contributing both on an empirical and a theoretical level. It does so by scrutinising asymmetric effects of exchange rate volatility, an idea introduced by Bahmani-Oskooee and Aftab (2017b). This paper is the first to consider their idea of asymmetric effects of exchange rate volatility on a global scale whilst making use of a plausible approximation of exchange rate volatility. Several empirical specifications are introduced to test the effect of exchange rate volatility, positive and negative changes in exchange rate volatility and fluctuations of exchange rate volatility on bilateral trade. I show that decreases in exchange rate volatility in fact have a detrimental effect on trade, thereby stressing the importance of constant, rather than zero, exchange rate volatility. Moreover, I directly test for a detrimental effect of fluctuations in volatility by showing that squared changes in exchange rate volatility are negatively related to bilateral trade.

After gathering further evidence of asymmetric effects of exchange rate volatility, a theoretical model is derived. This model considers isoelastic utility maximising importers and exporters in a setting with stochastic exchange rate volatility. By obtaining a higher order Taylor approximation of a CRRA utility function, I am capable of introducing explicit terms of exchange rate volatility and the variance of the exchange rate volatility to the model. Due to the introduction of higher moments of the distribution of the exchange rate, the model is capable of explaining aversion to the variance of exchange rate volatility via the notion of temperance. Illustrative numerical examples show that the model is qualitatively capable of generating predictions in line with empirical estimations. The maturity of the forward market and the degree of risk aversion of trading entities are found to be moderating variables to the magnitude of the effects. The contribution of this paper to the current literature is therefore twofold. Besides gathering further evidence of asymmetric effects of exchange rate volatility on a global level, this study is also the first to postulate a theoretical justification of the aforementioned effects.

This paper proceeds by providing a framework based of the existing literature on the effect of exchange rate volatility on trade. Subsequently, the methodology and data of the empirical section of this paper are discussed. After presentation of the empirical results, a theoretical model is derived in section five. Illustrative numerical examples are shown at the end of section five, after which section six concludes.

2 Literature

2.1 Theoretical Literature

After the decay and ultimately collapse of the Bretton Woods system in the seventies of the previous century, the use of flexible exchange rates has become far more prevalent. This rise in fluctuations of the relative currency prices has sparked a great, theoretical and empirical, interest in its potential effect on trade. Various economists have employed sophisticated empirical strategies whilst inspecting the effect of trade on economic welfare (Frankel and Romer, 1999). Assuming that the level of trade is, either positively or negatively, related to welfare implies that exchange rate volatility could indirectly influence economic welfare. This has led to the development of a large strand of literature scrutinising the effect of exchange rate volatility on trade. The earliest theoretical papers in this field suggested a negative association between the two, often making use of the assumption of risk aversion.

The initial expectation was that commodity traders respond negatively to increased uncertainty of the exchange rate. The seminal paper by Ethier (1973) shows that under imperfect information about the revenues of the firm, the level of trade is negatively impacted by exchange rate volatility. The magnitude of this impact is, however, dependent on the strength of the firm's aversity to risk. Demers (1991) shows that the detrimental impact of exchange rate volatility on trade could also be explained whilst considering a risk neutral trader, due to the irreversibility of investments abroad. Broll and Eckwert (1999) explore a positive effect of exchange rate volatility on trade by arguing that exchange rate volatility in fact increases the potential gains from trade. De Grauwe (1988) argues that the response to increased uncertainty is in fact not linear in the risk attitude of the firm. He also states that the introduction of a forward market would not alter his result. Viaene and De Vries (1992) explicitly incorporate a forward market in their analysis and argue that in a mature forward market, the influence of additional exchange rate volatility on a country's trade level is dependent on its trade balance. Their conclusion in the absence of a mature forward market is less ambiguous, the authors find that additional exchange rate uncertainty negatively impacts both the amount of imports and exports in this scenario.

The theoretical model by Viaene and De Vries (1992) will be revisited in more detail later on in this paper. By extending their setting of utility maximising exporters and importers to a fourth order Taylor approximation of an isoelastic utility function, I allow for more complex risk attitudes than mere risk aversion. More specifically, the introduction of the fourth central moment of the distribution facilitates the inclusion of temperance in the decision rules of these trading entities. In the presence of stochastic volatility, analogous to the setting of Gollier (2018), this provides me with an explicit effect of the variance of volatility on bilateral trade.

2.2 Empirical Literature

The question about the actual effect is, however, ultimately an empirical one. A consensus on the true effect is currently far from being reached. Many empirical papers have found small, insignificant, effects of exchange rate volatility on trade (McKenzie, 1999). The papers that do find an effect often differ in the sign of the observed coefficient, thereby leaving the reader with an ambiguous conclusion. The inability of empirical researchers to reconcile their findings with theoretical foundations has been characterised as a paradox, which subsequently stimulated the development of alternative theories (Bahmani-Oskooee and Hegerty, 2007).

Primarily due to the diffusion in observed effects, no dominant theory from the perspective of policy makers has been identified to date (McKenzie, 1999).

The most prominent reason raised for this dispersion in effects is the approximation used for exchange rate volatility (Bahmani-Oskooee and Hegerty, 2007). Examples of differences between these measures include employing the real or nominal currency prices and making use of the variance or standard deviation of the exchange rate. Various GARCH approximations of volatility have also increased in popularity, due to the fact that it is often noted that volatility in exchange rates clusters. Bahmani-Oskooee and Aftab (2017b) nonetheless identify a different complication. The authors show that there may be asymmetric effects of positive and negative changes in exchange rate volatility on bilateral trade. These effects are encountered for several industries when considering trade flows from Malaysia with the United States and the European Union (Bahmani-Oskooee and Aftab, 2017b,a). This asymmetry could indicate that conventional approximations of exchange rate volatility in fact suffer from aggregation bias. The authors, however, do no investigate potential assymmetries on a global level.

Kokken and Wolters (2019) further investigate this point by disaggregating positive and negative changes in exchange rate volatility. Their results indicate the occurrence of this asymmetry on a global level, both positive and negative changes in exchange rate volatility over time seem to have a detrimental effect on bilateral trade. Kokken and Wolters (2019), however, make use of an inappropriate approximation of exchange rate volatility. By using yearly exchange rate observations, they do not allow for intermediate variation. One could nevertheless deduce from the aforementioned papers the conjecture that having stable volatility, regardless of the level, provides a pair of countries with favourable conditions to engage in further trade. When both positive and negative changes in exchange rate volatility lead to a decline in trade, the aggregated effect will merely show differences in the magnitude of the two. If the two magnitudes are relatively similar, an insignificant aggregate effect is likely to be encountered. This could provide a valid explanation as to why it is difficult to find a negative effect of aggregated exchange rate volatility on bilateral trade. This would require a detrimental effect of strong decreases in exchange rate volatility. The evidence provided above introduces the first two hypotheses:

H_1 : There is no aggregated negative effect of exchange rate volatility on bilateral trade.

H_2 : There is a negative effect of decreases in exchange rate volatility on bilateral trade.

From the conjectures presented above, one could deduce a third hypothesis. If both positive and negative changes in volatility reduce trade, it can be stated that general fluctuations in exchange rate volatility lead to falls in trade. This implies that a constant level of volatility, regardless of what this level is, offers the most attractive environment for bilateral trade. A direct test of the effect of dispersion in exchange rate volatility on bilateral trade can be executed by squaring the changes in exchange rate volatility, thereby treating positive and negative changes identical. By doing this, the third hypothesis can be addressed:

H_3 : There is a negative effect of fluctuations in exchange rate volatility on bilateral trade.

3 Methodology

3.1 Gravity Equation

A widely used approach in uncovering the effect of exchange rate volatility on bilateral trade is making use of a gravity equation (Bahmani-Oskooee and Hegerty, 2007). Gravity models include mostly geographical factors as determinants of the level of trade between countries. The combined size of the two economies is also used as an explanatory variable of bilateral trade. Although gravity models have become influential due to their surprisingly good fit to the data, Anderson and Van Wincoop (2003) propose a model in which this peculiarity is reconciled with theory. Their equation takes the form:

$$x_{ij} = \frac{y_i y_j}{y^W} (\frac{t_{ij}}{P_i P_j})^{1-\sigma} \tag{1}$$

where x_{ij} denotes the exports from country i to country j. y_i, y_j and y^W are the gross domestic products of country i, j and the world as a whole respectively. The term t_{ij} captures bilateral resistance in trade, often operationalised as a decreasing function of geographic and cultural proximity. P_i and P_j are multilateral resistance terms of countries i and j and resemble the trade barriers specific to a country faced by all partner countries. Parameter σ denotes the elasticity of substitution in trade and is assumed to carry a value greater than one, thereby ensuring that low bilateral resistance relative to multilateral resistance has a positive effect on bilateral trade. In order to estimate the aforementioned equation whilst using a regression technique, a logarithmic transformation on equation one is often performed. Taking the natural logarithm of both sides of the equation yields the following:

$$\ln x_{ij} = \ln y_i + \ln y_j - \ln y^W + (1-\sigma)\ln t_{ij} - (1-\sigma)\ln P_i - (1-\sigma)\ln P_j \tag{2}$$

Equation two can subsequently be extended to incorporate further variables of interest. Although the other terms in equation two do not by definition function as control variables to mitigate omitted variable bias with regard to the coefficient of the independent variable of interest, they do facilitate precise estimation. In this paper, I will extend equation two with a variable capturing changes in exchange rate volatility over time. Moreover, various control variables will be included in an attempt to uncover a causal effect.

3.2 Exchange Rate Volatility

Although much has been written on the topic, a consensus on the appropriate operationalisation of exchange rate volatility is yet to be reached (Bahmani-Oskooee and Hegerty, 2007). Several possibilities arise when choosing a suitable exchange rate volatility approximation. First of all, one must decide on the definition of exchange rates. I follow the reasoning by Bini-Smaghi (1991) and make use of nominal exchange rates, since price fluctuations form a separate risk to trading entities. Moreover, there are insufficient reliable data available on bilateral exchange rates for all combinations of countries over a long period of time. This problem is, however, easily overcome by taking the cross exchange rate. The use of this cross exchange rate helps denote the bilateral exchange rate via the following formula:

$\frac{Currency_z}{Currency_x} / \frac{Curreny_z}{Currencx_y} = \frac{Currency_z}{Currency_x} * \frac{Currency_y}{Currency_z} = \frac{Currency_y}{Currency_x}$

This indicates that only exchange rates with respect to one common currency need to be obtained to calculate bilateral exchange rates. The most pronounced candidate for this is the U.S. dollar. By obtaining nominal exchange rates for all considered countries relative to the U.S. dollar, all bilateral exchange rates can be computed. One must, however, also consider the frequency of observations. Due to the nature of the panel dataset, which spans yearly trade observations for all country pairs, it is appealing to utilise yearly exchange rates. This, however, provides very modest information on the bilateral rates. Volatility would then either be calculated based on only a few observations, which might not capture intermediate variability, or would have to be an excessively long run concept. An alternative approach would be to match monthly exchange rate observations for all pairs, and subsequently calculate bilateral exchange rate volatility. This is, unfortunately, not feasible for the used trade panel dataset. One would have to create a dataset consisting of nearly 15.5 million observations and perform 1.2 million calculations. There is, nevertheless, an alternate way to calculate bilateral exchange rate volatility without having to create this data structure. In essence, I attempt to calculate the variance of $\frac{Currency_z}{Currency_x} / \frac{Curreny_z}{Currency_y}$, which is simply the variance of $\frac{ExchangeRate_{zx}}{ExchangeRate_{zy}}$, opposed to the variance of $\frac{Currency_y}{Currency_x}$. The variance of a ratio can be approximated by taking a first order Taylor expansion of the variance. Following Seltman (2012), the approximation of the variance of a ratio can be written as: $Var(\frac{R}{S}) \approx \frac{\mu_R^2}{u_S^2}\left[\frac{\sigma_R^2}{\mu_R^2} - 2\frac{Cov(R,S)}{\mu_R\mu_S} + \frac{\sigma_S^2}{\mu_S^2}\right]$. This implies that, specifically for the exchange rate volatility context, I can write:

$$Var(\frac{Currency_z}{Currency_x} / \frac{Currency_z}{Currency_y}) \approx \frac{\overline{(ER_{zx})}^2}{\overline{(ER_{zy})}^2}\left[\frac{Var_{zx}}{\overline{(ER_{zx})}^2} - 2\frac{Cov(ER_{zx},ER_{zy})}{ER_{zx}ER_{zy}} + \frac{Var_{zy}}{\overline{(ER_{zy})}^2}\right]$$

In order to calculate the variance of a bilateral exchange rate, the yearly bilateral trade observations only need to be matched with average exchange rates and yearly measurements of the variance, all calculated based on monthly observations of the exchange rate with the U.S. Dollar. The former are obtained from the International Financial Statistics of the International Monetary Fund (IMF), whilst the latter are computed. The covariance between the bilateral exchange rates are subsequently calculated based on three observations of yearly average exchange rates. All in all, this allows computation of the bilateral variances based on monthly exchange rate data. These variances are squared to collect standard deviations and divided by the bilateral average exchange rate. This provides me with a coefficient of variation, which is a measure of dispersion that is standardised to the scale of the exchange rate. This allows for comparison between the great variety of exchange rates considered in this paper. A limitation of this paper is that the data structure does not allow me to make use of a GARCH calculation of the conditional variance of the exchange rate. I thereby do not allow for clustering in exchange rate volatility.

3.3 Baseline Specifications

After gathering an approximation for exchange rate volatility, a baseline specification is estimated to grasp an idea of the connection between exchange rate volatility and bilateral trade. This specification uses ordinary least squares regression to find an association between exchange rate volatility and the natural logarithm of bilateral trade. Note that the estimated coefficient should, by all means, not be interpreted as causal. This parsimonious specification takes the following form:

$$\ln x_{ijt} = \alpha + \sum_{n=1}^{m} \beta_n X_{ijtn} + \beta_{m+1} Vol_{ijt} + \varepsilon_{ijt} \tag{3}$$

Here, $\ln x_{ijt}$ denotes the natural logarithm of trade. X_n is a set of bilateral resistance terms and the size of the economies. More specifically, the variables included in this and subsequent specifications consist at the minimum of the world gross domestic product, the product of the gross domestic products of the country pair, the bilateral geographic distance and four dummies indicating whether the pair is contiguous, shares a common language, has a colonial history, or participates in a regional trade agreement respectively. Variable Vol_{ij} is the approximation of exchange rate volatility, measured by the coefficient of variation mentioned prior. Newey-West standard errors are used for all models, which are robust to autocorrelation and heteroskedasticity. These standard errors are selected due to serial correlation in the bilateral trade variable, as displayed by the Wooldridge test for autocorrelation in appendix A.

The specification outlined above does not capture the multilateral resistance terms included in equation two. Including approximations of these resistance terms would help in estimating the effect with more precision. Although it is challenging to find variables that suffice in apprehending these terms, making use of fixed effects could resolve this issue. The paired structure of the data allows for the inclusion of country-pair fixed effects. When the assumption is made that these multilateral resistance terms are time-invariant, the fixed effects grasp the full extent of these resistance terms. Note that it would also be possible to introduce year fixed effects to the specifications. One can, however, deduce from equation two that these do not have a theoretical foundation. Inclusion of global GDP already scales the level of trade and mitigates potential omitted variable bias, originating from global declines in economic activity. Capturing a greater variety of the data by including further fixed effects without a theoretical foundation is undersirable from the point of statistical inference, especially since small coefficients are often observed in the literature (McKenzie, 1999).

All in all, this gives rise to the second baseline specification:

$$\ln x_{ijt} - \overline{\ln x_{ij}} = \sum_{n=1}^{m} \beta_n (X_{ijtn} - \overline{X_{ijn}}) + \beta_{m+1}(Vol_{ijt} - \overline{Vol_{ij}}) + \varepsilon_{ijt} \tag{4}$$

To address the second hypothesis, a transformation is made to the exchange rate volatility variable. By taking the first difference, the change in volatility over the duration of one year is obtained. This variable is subsequently disaggregated into two variables. The first variable spans all positive changes and takes the value of zero for all other observations. The second variable gives the first difference for all negative changes and displays zero otherwise. This brings us to the piecewise estimation of the second hypothesis, which takes the following forms for respectively the pooled OLS and fixed effects regression specifications.

$$\ln x_{ijt} = \alpha + \sum_{n=1}^{m} \beta_n X_{ijtn} + \beta_{m+1} \Delta Vol^{+}_{ijt} + \beta_{m+2} \Delta Vol^{-}_{ijt} + \varepsilon_{ijt} \tag{5}$$

$$\ln x_{ijt} - \overline{\ln x_{ij}} = \sum_{n=1}^{m} \beta_n (X_{ijtn} - \overline{X_{ijn}}) + \beta_{m+1}(\Delta Vol^{+}_{ijt} - \overline{\Delta Vol^{+}_{ij}}) + \beta_{m+2}(\Delta Vol^{-}_{ijt} - \overline{\Delta Vol^{-}_{ij}}) + \varepsilon_{ijt} \tag{6}$$

If β_{m+1} or β_{m+2} are significantly different from zero and from each other, asymmetric effects are encountered. Furthermore, a more direct test of the influence of changes in exchange rate volatility on bilateral trade is obtained by using the squared value of the first difference in exchange rate volatility as the variable of interest.

This is captured in the following specifications:

$$\ln x_{ijt} = \alpha + \sum_{n=1}^{m} \beta_n X_{ijtn} + \beta_{m+1} \Delta Vol_{ijt}^{2\pm} + \varepsilon_{ijt} \tag{7}$$

$$\ln x_{ijt} - \overline{\ln x_{ij}} = \sum_{n=1}^{m} \beta_n (X_{ijtn} - \overline{X_{ijn}}) + \beta_{m+1} (\Delta Vol_{ijt}^{2\pm} - \overline{\Delta Vol_{ij}^{2\pm}}) + \varepsilon_{ijt} \tag{8}$$

These baseline specifications suffer from several issues of endogeneity, which may contaminate the estimated effect size. These issues are addressed in the specifications discussed in the following subsections, forming estimations that are preferred over the baseline specifications. Moreover, a robustness exercise is performed.

3.4 Concerns of Endogeneity

3.4.1 Omitted Variable Bias

Although the gravity control variables serve the purpose of obtaining precise estimates, they do not correct for missing variable bias in the baseline specification. The second baseline specification is in this sense superior to the first specification, since country-pair fixed effects mitigate the influence of all time-invariant omitted variables. One would therefore have to tell a story about variables that change over time and influence both the volatility of exchange rates and bilateral trade to identify omitted variables. An evident factor that may indeed influence both variables is whether the country-pair participates in a currency union. Countries participating in a currency union share a common currency, which annihilates all volatility. Moreover, currency unions may lead to a surge in trade between partner countries due to reasons other than volatility, such as stronger cultural affinity or the abolition of trade impediments. If this conjecture is accurate, the bias imposed by omitting this variable leads to underestimation of the causal effect of exchange rate volatility. To overcome this potential missing variable bias, a dummy variable indicating whether a country-pair simultaneously partakes in the same currency union is added to all baseline specifications. These specifications are prefered to the baseline estimations and will therefore be the ones reported in the subsequent section.

Another potential factor that may taint the coefficient found in the baseline specifications is the price level. Inflation or deflation in the partner country could directly influence the domestic demand for foreign goods. Although nominal exchange rates are not explicitly corrected for prices, this does not imply that they are a priori exogenous to them. Three plausible mechanisms that could cause prices and exchange rates to be intertwined come to mind. First of all, the change in prices could influence the amount of goods purchased abroad and thereby also affect the nominal exchange rate. This is, however, an issue of reverse causality rather than omitted variable bias. Reverse causality will be addressed in the subsequent section. The second mechanism is the fact that a change in prices may adjust the expected return on investments in firms, which could lead to an in or outflow of capital, thereby leading to a shift in the bilateral exchange rate. Correcting for variation in prices would alleviate these concerns. The third mechanism, however, introduces ambiguity to the question whether one should make such corrections. In the presence of variability in exchange rates, exporting firms may adjust their prices accordingly. This phenomenon, often referred to as exchange rate pass-through, allows firms to exploit changes in exchange rates to alter their competitive position or profit margin. This behaviour would affect the competitive environment in the destination country. This could make changes in prices an outcome of exchange rate volatility, which would render it a bad control. Due to the ambiguity discussed above, consumer prices will not be present in the main specifications. Nevertheless, a robustness check to the inclusion of consumer price indices will be shown. This will be presented in combination with the inclusion of currency unions.

3.4.2 Reverse Causality

Another concern of endogeneity is reverse causality. Exogenous shocks to bilateral trade flows could also affect the volatility of exchange rates. This is the case if the shock influences the demand and/or supply for one of the currencies to a greater extent than the other. This would introduce reverse causality to the model. Both a positive and negative shock to bilateral trade could lead to a change in the exchange rate, thereby raising exchange rate volatility in all cases. If the effect of exchange rate volatility on bilateral trade is negative, the bias imposed by this reverse causality in baseline specification one and two would induce underestimation. This would imply that merely an upper bound of the absolute value of the causal effect is uncovered. Since our hypothesis stipulates no significant negative impact, this would merely make our hypothesis more difficult to accept. This issue is, moreover, only half as prevalent in the third and fourth baseline specification. Changes in bilateral trade flows are unlikely reasons of decreasing exchange rate volatility. The only example that comes to mind where this would be the case is the complete elimination of bilateral trade and foreign exchange between two countries, which would be quite an extreme case. It can therefore reasonably be assumed that these shocks in trade flows would lead to either a higher increase or a lower decrease in exchange rate volatility. If the hypothesis regarding negative changes in exchange rate volatility is correct, stronger negative changes in exchange rate volatility lead to less trade. This implies that negative volatility changes closer to zero, which are higher, relatively increase trade. Bilateral trade shocks would lead to less negative, and therefore higher, values of negative volatility changes. This positive connection would indicate that I in fact overestimate the causal effect. Since the overestimation is for a negative coefficient, this puts a lower bound on the absolute value of the effect of negative changes on bilateral trade.

Although reverse causality will therefore likely not cause the main coefficient of interest to be more different from zero than the causal effect, it would notwithstanding be advantageous to correct for reverse causality. This is the case because this concern will only allow for the estimation of an upper bound of the effect of positive changes in exchange rate volatility. An Instrumental Variable that has a sufficiently strong first stage with exchange rate volatility and simultaneously satisfies the exclusion restriction would alleviate these concerns. Frankel and Wei (1998) propose the standard deviation of the relative money supply as an instrument. It is, however, likely that changes in the supply of money are driven by factors that also directly affect trade (Tenreyro, 2007), thereby violating the exclusion restriction. Other papers make use of instruments that predict the formation of exchange rate regimes (Broda and Romalis, 2011). These predictions may, however, already have been anticipated prior to the introduction of the actual exchange rate regime. This would also defeat the purpose of the instrument. With no suitable instrument readily available, causal claims regarding effects other than negative changes of exchange rate volatility should be made with precaution.

3.4.3 Measurement Error

A third complication that could affect the estimated coefficient is measurement error. This source of endogeneity could be present for our variable of interest, exchange rate volatility. There are countries where the official exchange rate is not a correct measurement of the actual exchange rate, due to the existence of a black market. These discrepancies between the official and actual exchange rate are often associated with fixed exchange rates, where the official rate is fixed at an unrealistic rate. In these cases, the exchange rate reported by the IMF will be less volatile than the correct measurement of exchange rates. Under the assumption that this measurement error is random, this would bias the estimates of our positive and negative changes of exchange rate volatility towards zero, which is not a great concern. Random measurement error in the independent variable provides lower bounds of the absolute values of the causal effects.

A greater potential disturbance to the identified effect is measurement error in the dependent variable. Of the 1,227,079 total observations, 773,395 are reported as missing values. It is unclear whether these values are in fact supposed to be zeros or are unreported positive values. Making use of a subsample without these missing values could bias the coefficient towards the effect found for countries that have a greater quality of reporting, which could be associated with their development. If the development of a country is also a determinant of their exchange rate volatility, for example via their selected exchange rate regime or the vulnerability of their currency, this would lead to a biased coefficient. I attempt to show robustness of the results to different approaches of coping with these missing values. One way in which these missing values could be handled is by replacing all missing values with zeros. This would, however, not solve the problem in the baseline specifications. This can be deduced from the fact that a logarithmic transformation is performed on the trade variable, prior to estimation. Since the natural logarithm of zero is undefined, these values would still be missing. Simply adding one to all observations would also not alleviate all concerns. As the logarithmic function is non-linear, the effect would not be homogenous for all observations. Moreover, there are also 161.862 trade observations, which are reported in millions, that lie between zero and one. Imposing the value of one on all missing values would therefore cause their trade values to be higher than some actual reported amounts.

The aforementioned issue could be resolved by not making the logarithmic transformation on equation one. To be able to estimate this equation directly with regression technique, a multiplicative form has to be used. Silva and Tenreyro (2006) propose a methodology to do so. With a Poisson pseudo-maximum-likelihood (PPML) regression, this multiplicative form can be estimated directly. PPML regression merely requires the dependent variable to be non-negative, thereby making it suitable for bilateral trade. Moreover, the dependent variable need not be Poisson distributed for the chosen estimator to yield consistent results (Gourieroux et al., 1984). Another large advantage of PPML is the fact that estimating the multiplicative form rather than a log-linearized model is superior in terms of dealing with heteroskedasticity. Heteroskedasticity is well known to be present in gravity equations (Silva and Tenreyro, 2006). The PPML specification therefore provides a way to cope with both zero values of trade and heteroskedasticity. A Poisson pseudo maximum likelihood regression where the missing values of trade have been replaced with zeros will be executed for all specifications. For the purpose of comparison, a PPML regression without the replacement of missing values will also be shown.

3.5 Data

In order to investigate the link between exchange rate volatility and bilateral trade, several variables are required. The dependent variable in all empirical specifications is either trade or the natural logarithm of trade. To obtain data on bilateral trade, an updated version of the dataset used by Glick and Rose (2002) is exploited. In this dataset, 212 countries are paired up with one another to form 24,920 pairs. For each pair, annual trade observations for the period 1957 - 2013 are retrieved from the Direction of Trade (DoT) database assembled by the International Monetary Fund (IMF). The trade variable of the country pair is the simple average of four observations, being the exports from A to B and vice versa, and the imports from A of B and vice versa. Although the exports from A to B and the imports from B of the goods of A should theoretically be identical, this method is used to reduce measurement error in bilateral trade. Trade is reported in units of millions of U.S. dollar and is deflated by Glick and Rose with the consumer price index of the United States. Of the

1,283,868 potential trade measurements, 793,397 records are missing. As discussed prior, this could be either due to a country-pair not engaging in trade over this period of time or due to measurement error. Moreover, the natural logarithm of all trade values is obtained and will function as an important dependent variable. Using the natural logarithm of trade does not only reduce the impact of potential outliers, but also facilitates linear estimation of the gravity equation. Since the natural logarithm of trade is stationary, as shown in appendix A, no further transformations are made.

The main independent variable used in this paper is exchange rate volatility. To address the first, second and third hypothesis, exchange rate volatility, the change in exchange rate volatility over time and the squared changes in exchange rate volatility respectively are used. Volatility is operationalised as a coefficient of variation, by dividing the variance of a bilateral exchange rate with the mean. All coefficients of variation greater than 500% were deleted from the sample since they most likely resemble changes in regimes or redefinitions of the exchange rate, rather than actual volatility. The first difference of the coefficient of variation denotes the change in exchange rate volatility and indicates the increase or decrease of exchange rate volatility over a period of one year. The squared changes over time are the operationalization for the variation in the volatility of the bilateral exchange rate.

To uncover a precise and unbiased causal effect of exchange rate volatility on bilateral trade, various control variables are included. First of all, several variables related to the geographical and cultural distance between the pair of countries are obtained. These variables consist of the natural logarithm of the geographical distance between countries and pair indicators that take the value of one if the country-pair is contiguous, has a colonial relationship, shares a common language or partakes in the same regional trade agreement. These variables are obtained from the World Factbook of the Central Intelligence Agency (CIA), with the exception of the last indicator. Data on trade agreements are employed from the World Trade Organization. The product of the real gross domestic products of the pair of countries is used as a scale factor and is obtained from the World Development Indicators, as published by the World Bank. These distance and scale variables are included due to their predictive power of bilateral trade. This ensures that a large share of the variety in bilateral trade is explained by these determinants, which allows for more precise estimation of the effect of a variable of interest. Since the effect of exchange rate volatility on bilateral trade is often found to be small or insignificant, this property is considered to be desirable.

In order to uncover a causal effect, rather than an association, two further control variables are retrieved. These variables are included for the sole purpose of mitigating omitted variable bias. A dummy indicating whether the pair of countries partake in the same currency union is retained from the original dataset by Glick and Rose (2002), who obtained these data from the Annual Report on Exchange Rate Arrangements and Exchange Restriction published by the IMF. Currency unions are defined transitive, implying that if both country A and B partake in a currency union with country C, a currency union between A and B is also formed. The second variable included with the purpose of controlling for omitted variable bias is the consumer price index (CPI) of both countries. These indices reflect changes in prices and are retrieved from the International Financial Statistics of the IMF. The CPI is available from 1960 and onwards for the lion share of countries and makes use of the year 2010 as its base year. Due to the fact that unit root tests, presented in Appendix A, show that the CPI variables are non-stationary, a transformation is performed on this variable. The first difference of the CPI is taken, which in fact is stationary. This first difference of the CPI will be used as a control variable in the robustness check.

4 Results

4.1 Baseline Results

After estimation of the discussed baseline specifications including a currency union dummy, exploratory results become evident. Frequently, a negative association between increases in exchange rate volatility and bilateral trade is assumed. Yet, for a strand of literature that has been scrutinised for several decades now, there is little compelling evidence of this assertion. Most evidence does not provide an unambiguous conclusion. This study provides no exception. The pooled OLS specification, presented in table 1, displays a negative and significant effect of exchange rate volatility on bilateral trade. All coefficients of the gravity variables show the correct sign, as expected from equation two. Due to the fact that the coefficient of variation is denoted as a factor, where 0.1 resembles 10% variation, we are interested in the effect of an increase of one percentage point in the coefficient of variation. In this case, one additional percentage point of the coefficient of variation would lead to a $(e^{-0.256} - 1) = -0.23\%$ change in trade. With an average bilateral trade value of 299 million U.S. dollar, this implies an average deterioration in bilateral trade of about 675.000 dollar.

The found effect is robust to the inclusion of country-pair fixed effects, which capture multilateral resistance terms to a greater extent. Moreover, the country-pair fixed effects control for all time-invariant omitted variable bias. Although the magnitude of the coefficient changes, a negative and significant effect is encountered. Several gravity variables are expunged from the equation due to their multicollinearity with the fixed effects, they are constant within country-pairs. The remaining gravity variables again show the expected signs. A one percentage point increase in the coefficient of variation now leads to a $(e^{-0.026} - 1) = -0.03\%$ change in trade. On average, this would be equal to a decay in trade of 77.000 dollar.

Subsequently, two Poisson pseudo-maximum-likelihood regressions are executed. The first PPML specification, also reported in table 1 utilises the dataset where missing values are excluded from the analysis and is reported for comparative purposes with regard to the second PPML only. In the first PPML, greater exchange rate volatility is positively associated with bilateral trade. This coefficient is, however, not significant at conventional confidence levels. Note that the magnitude of the effect size should not be directly compared to the coefficients presented in the first two columns, since the dependent variable in the PPML estimation is trade, not the natural logarithm of trade. If the coefficient of variation now increases by one percentage point, the effect on bilateral trade is merely 0.01%, or alternatively 30.000 dollar. Before estimating the second PPML, 773.395 missing values of trade are replaced with the value zero. Since country-pairs that consist of only zeros do not have within-group variation, they are still omitted from the analysis. The increase in the total number of observations is therefore smaller.

The second PPML shows that the sign, magnitude and significance of the effect of changes in exchange rate volatility on trade is robust to the handling of missing values. All in all, ambiguous evidence is encountered. We can therefore not formally accept the first hypothesis, the existence of a negative effect of exchange rate volatility on trade cannot be ruled out within conventional certainty levels.

Table 1: The effect of exchange rate volatility on bilateral trade whilst correcting for currency unions.

	lnTrade		Trade	
	OLS	FE	PPML1	PPML2
World GDP	-0.000***	-0.000***	-0.000***	-0.000***
	(0.000)	(0.000)	(0.000)	(0.000)
Distance	-1.068***			
	(0.007)			
Contiguous	0.132***			
	(0.032)			
Colony	1.224***			
	(0.027)			
Common Language	0.542***			
	(0.012)			
$\prod$ GDP	0.999***	0.647***	0.853***	0.875***
	(0.002)	(0.015)	(0.037)	(0.040)
RTA	1.071***	0.300***	0.166***	0.155
	(0.022)	(0.040)	(0.108)	(0.109)
Currency Union	1.354***	1.0133***	0.390***	0.388***
	(0.039)	(0.117)	(0.053)	(0.053)
Volatility	-0.256***	-0.026**	0.012	0.013
	(0.009)	(0.011)	(0.048)	(0.047)
No. of Observations	175,049	175,049	174,629	269,313
Country-pair FE	No	Yes	Yes	Yes

Notes: Standard errors are presented in parentheses. The effect of exchange rate volatility on bilateral trade is shown for the pooled OLS, fixed effects and PPML specifications. In the PPML2 specification, all missing values are replaced with zeros. Several coefficients for gravity variables are not included due to multicollinearity. In all specifications, a dummy indicating whether the country-pair participated in the same currency union is included. Three, two and one asterisks are used to denote significance at the 1%, 5% and 10% level respectively.

In line with much of the current empirical evidence, no unambiguous association between exchange rate volatility and bilateral trade is observed. I proceed to take the first difference of this variable and subsequently disaggregate exchange rate volatility into positive and negative changes. The same specifications as shown in table 1 are estimated. The pooled OLS specification, displayed in table 2, shows a negative and significant effect of increases of exchange rate volatility on bilateral trade. An increase of one percentage point would lead to a $e^{-0.462} - 1 = -0.37\%$ change in bilateral trade, which corresponds to an average fall in trade of roughly 1.1 million dollar. Moreover, a positive and significant coefficient is found for negative changes in exchange rate volatility. This implies that a higher value of the negative change, which is in fact a smaller decrease, leads to more trade. Although statistically significant different from zero, the size of the coefficient implies that this impact carries no economic significance. The change in average bilateral trade is negligible. This specification nevertheless provides evidence of asymmetric effects of changes in exchange rate volatility on bilateral trade.

The same conclusion is reached when considering the fixed effects specification in table 2. Although increases in volatility do indeed negatively impact bilateral trade, the same may be said about decreases. Both coefficients are significantly different from zero at the 95% confidence level. They, however, greatly differ in economic significance. Where the effect of a negative change has a trivial impact on trade, an increase of the positive change with one percentage point yields a -0.1% change in trade. This change is the equivalent of an average fall in bilateral trade of approximately 270.000 dollar. Although negative changes do not seem to impose substantial effects on trade, the mere lack of an increase in trade itself is interesting.

When shifting attention to the PPML specification, insignificant estimates are observed for the effect of positive changes. This, however, seems to be related to the regression technique rather than the handling of zeros. This can be deduced from the fact that the coefficient in the first and second PPML differ only by a negligible amount. It is therefore unlikely that measurement error in the dependent variable is the cause for this estimate near zero. The estimate of negative changes in exchange rate volatility remains positive and significant, even when considering a 99% confidence level. Once more, no effect of economic importance is encountered. The discussed evidence gives rise to the preliminary conclusion that there are indeed asymmetric effects of changes in exchange rate volatility on trade. In the statistical sense, there indeed seems to be a negative effect of shifts in exchange rate volatility. I therefore, preliminary to a further robustness exercise, accept the second hypothesis.

Table 2: The effect of positive and negative changes in exchange rate volatility on bilateral trade whilst correcting for currency unions.

	lnTrade		Trade	
	OLS	FE	PPML1	PPML2
World GDP	-0.000***	-0.000***	-0.000***	-0.000***
	(0.000)	(0.000)	(0.000)	(0.000)
Distance	-1.070***			
	(0.008)			
Contiguous	0.124***			
	(0.035)			
Colony	1.170***			
	(0.029)			
Common Language	0.542***			
	(0.013)			
$\prod$ GDP	1.003***	0.648***	0.865***	0.889***
	(0.002)	(0.017)	(0.041)	(0.044)
RTA	1.124***	0.287***	0.151	0.139
	(0.024)	(0.044)	(0.122)	(0.124)
Currency Union	1.493***	1.007***	0.511***	0.495***
	(0.044)	(0.129)	(0.111)	(0.107)
Δ^+ Vol	-0.462***	-0.095***	-0.006	-0.008
	(0.024)	(0.018)	(0.065)	(0.064)
Δ^- Vol	0.000**	0.000**	0.000***	0.000***
	(0.000)	(0.000)	(0.000)	(0.000)
No. of Observations	153,319	153,319	152,876	236,072
Country-pair FE	No	Yes	Yes	Yes

Notes: Standard errors are presented in parentheses. Three, two and one asterisks are used to denote significance at the 1%, 5% and 10% level respectively.

A more direct test of the effect of changes in exchange rate volatility on bilateral trade is estimated by using the squared changes of volatility as an explanatory variable. The pooled OLS specification, presented in table 3, reveals a negative and significant association between changes in exchange rate volatility and bilateral trade. This result is robust to the inclusion of fixed effects, as shown in the third column. A likewise result is encountered whilst making use of trade as the dependent variable in the first PPML. Moreover, making use of zeros for all missing values of trade leads to the same conclusion. The effect is nevertheless too small to carry economic importance. In a statistical sense, the third hypothesis is nonetheless preliminary accepted.

Table 3: The effect of squared changes in exchange rate volatility on bilateral trade whilst correcting for currency unions.

	lnTrade		Trade	
	OLS	FE	PPML1	PPML2
World GDP	-0.000***	-0.000***	-0.000***	-0.000***
	(0.000)	(0.000)	(0.000)	(0.000)
Distance	-1.071***			
	(0.008)			
Contiguous	0.118***			
	(0.035)			
Colony	1.175***			
	(0.029)			
Common Language	0.540***			
	(0.013)			
$\prod$ GDP	1.004***	0.645***	0.865***	0.888***
	(0.002)	(0.017)	(0.041)	(0.044)
RTA	1.121***	0.286***	0.150	0.139
	(0.024)	(0.044)	(0.123)	(0.124)
Currency Union	1.519***	1.011***	0.511***	0.495***
	(0.044)	(0.129)	(0.111)	(0.107)
Squared Δ	-0.000**	-0.000**	-0.000***	-0.000***
	(0.000)	(0.000)	(0.000)	(0.000)
No. of Observations	153,319	153,319	152,876	236,072
Country-pair FE	No	Yes	Yes	Yes

Notes: Standard errors are presented in parentheses. Three, two and one asterisks are used to denote significance at the 1%, 5% and 10% level respectively.

4.2 Robustness Exercise

Although the baseline specifications display interesting results, one can question whether they reveal a causal effect. The PPML specifications show that measurement error in the dependent variable is unlikely to be prominent. Moreover, random measurement error in the independent variable would merely bias the observed coefficients towards zero. A more worrisome complication would be omitted variable bias. A further robustness check is therefore the inclusion of the change of the price level of both countries in the piecewise specification. This check, shown in table 4, explores the conjecture that shifts in prices may affect trade levels and exchange rates simultaneously. This control variable is included in the specifications that also make use of a currency union dummy. In these specifications, the result is mixed. Where the pooled OLS shows an effect size of negative changes of the magnitude $e^{0.280} - 1 = 0.3\%$, the other three specifications fail to find effect sizes statistically distinguishable from zero. This could be related to issues of bad controls. For positive changes, the pooled OLS and fixed effects specifications find significant, negative, coefficients.

Table 4: The effect of positive and negative changes in exchange rate volatility on bilateral trade whilst correcting for currency unions and price levels.

	lnTrade		Trade	
	OLS	FE	PPML1	PPML2
World GDP	-0.000***	-0.000***	-0.000***	-0.000***
	(0.000)	(0.000)	(0.000)	(0.000)
Distance	-1.117***			
	(0.009)			
Contiguous	-0.064			
	(0.046)			
Colony	1.090***			
	(0.033)			
Common Language	0.517***			
	(0.015)			
$\prod$ GDP	1.046***	0.643***	0.864***	0.874***
	(0.002)	(0.020)	(0.042)	(0.042)
RTA	1.040***	0.216***	0.111	0.107
	(0.026)	(0.044)	(0.105)	(0.106)
Currency Union	1.261***	0.964***	0.466***	0.467***
	(0.057)	(0.189)	(0.112)	(0.117)
Δ CPI Country 1	-0.054***	0.021***	0.030***	0.031***
	(0.003)	(0.003)	(0.005)	(0.005)
Δ CPI Country 2	-0.005*	0.002	0.021***	0.020***
	(0.002)	(0.002)	(0.005)	(0.006)
Δ^{+} Vol	-0.521***	-0.073***	0.033	0.032
	(0.027)	(0.020)	(0.070)	(0.071)
Δ^{-} Vol	0.280***	0.023	-0.008	-0.008
	(0.024)	(0.017)	(0.058)	(0.058)
No. of Observations	109,822	109,822	109,482	144,945
Country-pair FE	No	Yes	Yes	Yes

Notes: Standard errors are presented in parentheses. Several coefficients for gravity variables are not included due to multicollinearity. In all specifications, a dummy indicating whether the country-pair participated in the same currency union is included as well as both countries CPI. Three, two and one asterisks are used to denote significance at the 1%, 5% and 10% level respectively.

5 Theoretical Model

In 1992, Viaene and De Vries published their theoretical paper on the impact of exchange rate volatility on international trade. Their model distinguishes between a situation with and without a mature forward market (McKenzie, 1999). The authors derive utility maximising decision rules from the perspectives of importers, exporters and speculators. It is shown that, in the absence of well-developed forward markets, higher variances in the exchange rate are detrimental to trade in this setting. This holds for both exporters and importers. When mature forward markets are present, the amount of exports and imports solely relies on the forward rate and demand or supply parameters. This does, however, not indicate that exchange rate volatility bears no influence on trade when forward markets are accessible. The authors explicitly solve for the forward rate to show that exchange rate volatility influences trade via this mechanism. Since exporters and importers are on different sides of the forward market, additional volatility affects these actors opposingly. All in all, one could say that the impact on a nations trade in the presence of a perfect forward market is dependent on their net currency position. Whether the utility of exporters and importers increases or decreases with further volatility also depends on the assumption of their attitudes towards risk.

In this paper, I revisit the seminal paper by Viaene and De Vries (1992) whilst extending their model to allow for stochastic volatility in the exchange rate. Moreover, I make use of an isoelastic utility function in order to show that the result holds under constant relative risk aversion. By making use of a higher order Taylor approximation of a CRRA utility function, I am able to incorporate more sophisticated attitudes towards risk than mere risk aversion. This allows me to include temperance in the preferences of the trading entities.

5.1 The Case of the Importer

In the first case, I consider a domestic firm that imports products from the partner country. The firm merely purchases goods abroad and sells them on the domestic market. Their full purchases are assumed to be denominated in foreign currency. The firm receives a trade credit for one period of time and subsequently fulfils its obligations by paying the vendors at the next period spot rate, denoted as $\widetilde{w}$. The tilde shows the stochastic element of this variable, as the firm does not possess perfect information on the next period spot exchange rate. The inverted demand by domestic consumers for the firm's goods is written in the general sense $S = a - \frac{1}{2}Y$, where a is a demand parameter and Y denotes the number of goods sold. Total revenue of the firm can then be expressed as $aY - \frac{1}{2}Y^2$. Moreover, we could assume that the firm has the option to engage in a forward contract at foreign currency price f, with L as its quantity of foreign currency purchased. The stochastic profits of the firm can consequently be written as:

$$\widetilde{P} = aY - \widetilde{w}Y + (\widetilde{w} - f)L - \frac{1}{2}Y^2 \tag{9}$$

In the adoption of the model made in this paper, the stochastic nature of the exchange rate will be explicitly modelled. In line with Gollier (2018), this stochastic variable is modelled as $w = \overline{w} + \sigma * \eta$. Here, $\overline{w}$ denotes the mean value of a bilateral exchange rate, σ resembles the standard deviation of the exchange rate and η is a random variable that is independently distributed and follows a normal distribution with a mean of zero. All in all, η adds a volatility element to the exchange rate. The standard deviation itself is also stochastic, introducing stochastic volatility. The variance of the exchange rate is modelled as $\sigma^2 = \overline{\sigma}^2 + \rho$. The mean value of the variance is denoted as $\overline{\sigma}^2$, ρ is an independent, normally distributed, random variable with a mean of zero. After substitution, the actual profits of the firm can be expressed as:

$$\widetilde{P} = aY - (\overline{w} + \sigma * \eta)Y + (\overline{w} + \sigma * \eta - f)L - \frac{1}{2}Y^2 \tag{10}$$

5.1.1 Utility Function of the Importer

Derivation of the profit function of the firm allows us to progress to its utility. The firm is expected to display utility maximising behaviour. In order to plausibly parametrise the model, isoelastic utility is assumed. The utility function, which satisfies the Von Neumann-Morgenstern axioms of expected utility, takes the following simplistic form:

$$U(\widetilde{P}) = \frac{\widetilde{P}^{1-\theta} - 1}{1 - \theta} \tag{11}$$

In equation eleven, θ denotes a constant value of relative risk aversion. The scalar one is included to ensure positive, increasing utility for values of θ larger than one. To obtain an expression of expected utility, a Taylor series provides a sensible approximation whilst explicitly modelling further moments in the distribution of the profits. In a likewise fashion from Le Courtois (2012), I take a fourth order Taylor expansion of the utility function around the expected profits. This allows for the inclusion of more sophisticated decision preferences than mere aversion to risk. The incorporation of the third and fourth moment respectively facilitate the inclusion of the notions of prudence and temperance to the model. The utility function presented in equation eleven is well known as a function with constant relative risk aversion.

This can be seen by calculating the Arrow-Pratt coefficient of relative risk aversion, which is $-\frac{u''(\widetilde{P})*\widetilde{P}}{u'(\widetilde{P})}$ (Eeckhoudt, 2012). In the context of equation eleven, the coefficient of relative risk aversion is $-\frac{-\theta\widetilde{P}^{-\theta-1}*\widetilde{P}}{\widetilde{P}^{-\theta}} = \theta$, thereby justifying the adjective constant. Making use of higher order moments of the distribution allows me to also scrutinise relative prudence, which was defined as $-\frac{u'''(\widetilde{P})*\widetilde{P}}{u''(\widetilde{P})}$ by Kimball (1990). For the isoelastic utility curve, the coefficient of relative prudence is $-\frac{(-\theta-1)*-\theta\widetilde{P}^{-\theta-2}*\widetilde{P}}{-\theta\widetilde{P}^{-\theta-1}} = 1+\theta$. The nature of the fourth order Taylor series also enables me to include the concept of temperance in the analysis. The coefficient of temperance can generally be calculated as $-\frac{u''''(\widetilde{P})*\widetilde{P}}{u'''(\widetilde{P})}$, which specifically to our utility function gives a coefficient of $-\frac{(-\theta-2)(-\theta-1)*-\theta\widetilde{P}^{-\theta-3}*\widetilde{P}}{(-\theta-1)*-\theta\widetilde{P}^{-\theta-2}} = 2+\theta$. Note that the utility function presented in equation eleven therefore displays risk aversion, prudence and temperance for values of θ greater than zero.

The approximation of expected utility while making use of a fourth order Taylor series around the expected profits is analogous to the function presented by Le Courtois (2012) and takes the following form:

$$E[U(\widetilde{P})] \simeq U(E[\widetilde{P}]) + E[(\widetilde{P} - E[\widetilde{P}])] * U'(E[\widetilde{P}]) + \frac{E[(\widetilde{P} - E[\widetilde{P}])^2]}{2!} * U''(E[\widetilde{P}]) + \frac{E[(\widetilde{P} - E[\widetilde{P}])^3]}{3!} * U'''(E[\widetilde{P}])$$

$$+ \frac{E[(\widetilde{P} - E[\widetilde{P}])^4]}{4!} * U''''(E[\widetilde{P}]) \simeq U(E[\widetilde{P}]) + \frac{Var(\widetilde{P})}{2} * U''(E[\widetilde{P}]) + \frac{m_3}{6} * U'''(E[\widetilde{P}] + \frac{m_4(\widetilde{P})}{24} * U''''(E[\widetilde{P}]) \quad (12)$$

Where $Var(\widetilde{P})$ denotes the variance of the profit function, which is its second central moment. The third and fourth central moments are respectively denoted as m_3 and m_4 in equation twelve. To address the effect of exchange rate volatility on the amount of imports, it is of importance to calculate explicit expressions of the third and fourth moment of the profit function.

The first step to operationalising the utility function includes deriving the expected profit function of the firm. By taking expectations and noting that η has a mean of zero, one is able to write:

$$E(\widetilde{P}) = aY - \overline{w}Y + (\overline{w} - f)L - \frac{1}{2}Y^2 \tag{13}$$

To obtain the second central moment, the difference between equation ten and thirteen is computed. This gives the following function:

$$\widetilde{P} - E(\widetilde{P}) = (\overline{w} + \sigma * \eta)(-Y + L) - \overline{w}(-Y + L) = (\sigma * \eta)(-Y + L) \tag{14}$$

The second central moment can then derived. In line with Gollier (2018), I make the assumption regarding the value of η that $E[\eta^2] = 1$. This simplifies the analysis and allows me to write:

$$E[(\widetilde{P} - E(\widetilde{P}))^2] = E[((\sigma * \eta)^2(-Y + L))^2] = \sigma^2 * (-Y + L)^2 \tag{15}$$

This shows that the variance of the profits is dependent on the variance of the exchange rate, the size of the forward contract and the number of imports. After calculating the second central moment, I progress to the third central moment. This part of the analysis is greatly simplified by making an assumption analogous to Gollier (2018) that $E[\eta^3] = 0$. This leads to the following equation for the third central moment:

$$E[(\widetilde{P} - E(\widetilde{P}))^3] = E[((\sigma * \eta)^3(-Y + L))^3] = 0 \tag{16}$$

No further consideration of the third central moment will be made. The fourth central moment, however, provides an interesting opportunity to express the utility function as an equation of the variance of exchange rate volatility.

$$E[(\widetilde{P}-E(\widetilde{P}))^4]=E[((\sigma*\eta)^4(-Y+L))^4]=\sigma^4*\eta^4*(-Y+L)^4 \tag{17}$$

This, however, does not provide us with an explicit expression for the variance of the volatility of the bilateral exchange rate. Note, however, that the excess kurtosis of the profit function can be written in the general equation:

$$ExcKurt[\widetilde{P}]=\frac{E[(\widetilde{P}-E(\widetilde{P}))^4]}{(E[(\widetilde{P}-E(\widetilde{P}))^2])^2}-3 \tag{18}$$

In the specific context of our profit function, substitution of equation seventeen and fifteen into equation eighteen yields:

$$ExcKurt[\widetilde{P}]=\frac{E[\sigma^4]*E[\eta^4]*(-Y+L)^4}{(E[\sigma^2])^2*(E[\eta^2])^2*(-Y+L)^4}-3=\frac{E[\sigma^4]*E[\eta^4]}{(E[\sigma^2])^2*(E[\eta^2])^2}-3 \tag{19}$$

We have, however, assumed that random variable η is normally distributed. This implies that its kurtosis is equal to three. Moreover, we have stated that the mean of η is equal to zero. It is therefore possible to write $\frac{E[\eta^4]}{(E[\eta^2])^2}=\frac{E[(\eta-0)^4]}{(E[(\eta-0)^2])^2}=Kurt[\eta]=3$. After substituting this value into equation nineteen, the expression simplifies to:

$$ExcKurt[\widetilde{P}]=\frac{E[\sigma^4]}{(E[\sigma^2])^2}*3-3=3*(\frac{E[\sigma^4]}{(E[\sigma^2])^2}-1) \tag{20}$$

This formula of excess kurtosis can be expressed as an explicit function of the variance of the volatility of the exchange rate by performing the following transformations:

$$ExcKurt[\widetilde{P}]=3(\frac{E[\sigma^4]}{(E[\sigma^2])^2}-\frac{(E[\sigma^2])^2}{(E[\sigma^2])^2})=3(\frac{E[\sigma^4]-(E[\sigma^2])^2}{(E[\sigma^2])^2})=3\frac{Var(\sigma^2)}{(E[\sigma^2])^2}=3\frac{\sigma_\rho^2}{(E[\sigma^2])^2} \tag{21}$$

The last two steps from equation twenty-one are derived in more detail in appendix B. After adding three to the excess kurtosis, we find an equation for the kurtosis, which is the fourth standardised moment. This formula is $3*\frac{\sigma_\rho^2}{(E[\sigma^2])^2}+3$. We are, however, looking for an expression of the fourth central moment, which conveniently is the fourth standardised moment multiplied with $(E[(\widetilde{P} - E(\widetilde{P}))^2])^2$. This multiplication, using terms from formula nineteen, gives the following equation:

$$m_4 = (3\frac{\sigma_\rho^2}{(E[\sigma^2])^2} + 3) * ((E[\sigma^2])^2 * (E[\eta^2])^2 * (-Y + L)^4) \tag{22}$$

Since we stated prior that $E[\eta^2] = 1$, equation twenty-two simplifies to:

$$m_4 = 3 * \sigma_\rho^2 * (-Y + L)^4 + 3 * (E[\sigma^2])^2 * (-Y + L)^4 \tag{23}$$

Aforementioned derivations of the second, third and fourth central moment consequently allow me to write the expected utility function as an explicit function of exchange rate volatility and the variance of exchange rate volatility.

$$E[U(\widetilde{P})] \simeq \frac{(aY - \overline{w}Y + (\overline{w} - f)L - \frac{1}{2}Y^2)^{1-\theta} - 1}{1-\theta} + \frac{\sigma^2 * (-Y + L)^2}{2} * (-\theta * (aY - \overline{w}Y + (\overline{w} - f)L - \frac{1}{2}Y^2)^{-\theta-1}) + 0$$

$$+ \frac{\sigma_\rho^2 * (-Y + L)^4 + (E[\sigma^2])^2 * (-Y + L)^4}{8} * (-\theta * (-\theta - 2)(-\theta - 1) * (aY - \overline{w}Y + (\overline{w} - f)L - \frac{1}{2}Y^2)^{-\theta-3}) \tag{24}$$

5.2 The Case of the Exporter

In this section, we consider the utility function of an exporter rather than an importer. This exporter sells its goods abroad for the numéraire price of one domestic unit. All invoicing occurs in foreign currency and is payable one period in the future, making the total revenue of the exporter equal to $\widetilde{w} * X$. Here, $\widetilde{w}$ has the same interpretation as in the prior section and X denotes the quantity of exports. The size of the forward contract is defined as variable K. Note that we now allow for K to take on negative values, as it resembles the amount of foreign currency purchased. A negative value of K therefore indicates that the exporting firm sells foreign currency on the forward market. Moreover, the firm faces a foreign supply market with price $S = d + \frac{1}{2}X$. The total costs of the exporter are therefore $dX + \frac{1}{2}X^2$. The profit function can subsequently be written as:

$$\widetilde{P} = \widetilde{w}X + (\widetilde{w} - f)K - dX - \frac{1}{2}X^2 \tag{25}$$

We again assume stochastic volatility of the exchange rate. The exchange rate can once more be expressed as $w = \overline{w} + \sigma * \eta$. Moreover, the volatility itself can be formulated as $\sigma^2 = \overline{\sigma}^2 + \rho$. Recall that both η and ρ are independent, normally distributed, variables with a mean of zero. Substituting the expression for the exchange rate into equation twenty-five allows me to write the profits as an explicit function of the volatility.

$$\widetilde{P} = (\overline{w} + \sigma * \eta) * X + (\overline{w} + \sigma * \eta - f)K - dX - \frac{1}{2}X^2 \tag{26}$$

5.2.1 Utility Function of the Exporter

The utility function of the exporter is derived analogous to the utility function of the importer. The utility function of the exporter, which is presented in equation eleven, is again approximated with a fourth order Taylor series. Note that the isoelastic utility function displays risk aversion, prudence and temperance for coefficients of relative risk aversion greater than one. The approximated expected utility function takes the form:

$$E[U(\widetilde{P})] = U(E[\widetilde{P}]) + \frac{Var(\widetilde{P})}{2} * U''(E[\widetilde{P}]) + \frac{m_3}{6} * U'''(E[\widetilde{P}]) + \frac{m_4(\widetilde{P})}{24} * U''''(E[\widetilde{P}]) \tag{27}$$

Again, the second, third and fourth moments of the distribution must be calculated. To do so, the expectation of the profit function is taken. The expected profit function of the exporter is equal to:

$$E(\widetilde{P}) = \overline{w}X + (\overline{w} - f)K - dX - \frac{1}{2}X^2 \tag{28}$$

It is subsequently straight forward to calculate the second moment of the distribution:

$$E[(\widetilde{P} - E(\widetilde{P}))^2] = E[((\sigma * \eta)^2(X + K))^2] = \sigma^2 * (X + K)^2 \tag{29}$$

Which expresses the variance of the profit function as a function of the volatility of the exchange rate, the size of the forward contract and the total value of exports. Note that we have assumed that $E[\eta^2] = 1$ and recall that we allow for negative values of K, showing that a full hedge of the transaction would indicate no variance of the profits. The third central moment of the distribution is trivial due to the assumption that $E[\eta^3] = 0$, in line with the derivation by Gollier (2018). The fourth central moment is again derived whilst making use of the fourth standardised moment, the kurtosis of $\widetilde{P}$. Note that the kurtosis of the normally distributed η is by definition equal to three.

$$E[(\widetilde{P} - E(\widetilde{P}))^4] = \sigma^4 * \eta^4 * (X + K)^4 \tag{30}$$

$$ExcKurt[\widetilde{P}] = \frac{E[(\widetilde{P} - E(\widetilde{P}))^4]}{(E[(\widetilde{P} - E(\widetilde{P}))^2])^2} - 3 = \frac{E[\sigma^4] * E[\eta^4]}{(E[\sigma^2])^2 * (E[\eta^2]^2)} - 3$$

$$ExcKurt[\widetilde{P}] = 3 * (\frac{E[\sigma^4]}{(E[\sigma^2])^2} - 1) = 3(\frac{E[\sigma^4]}{(E[\sigma^2])^2} - \frac{(E[\sigma^2])^2}{(E[\sigma^2])^2}) = 3\frac{\sigma_\rho^2}{(E[\sigma^2])^2}$$

$$m_4 = (3\frac{\sigma_\rho^2}{(E[\sigma^2])^2} + 3) * ((E[\sigma^2])^2 * (E[\eta^2])^2 * (X + K)^4)$$

$$m_4 = 3 * \sigma_\rho^2 * (X + K)^4 + 3 * (E[\sigma^2])^2 * (X + K)^4 \tag{31}$$

After defining the second, third and fourth central moment, substitution into the utility function leads to:

$$E[U(\widetilde{P})] \simeq \frac{(\overline{w}X + (\overline{w} - f)K - dX - \frac{1}{2}X^2)^{1-\theta} - 1}{1 - \theta} + \frac{\sigma^2 * (X + K)^2}{2} * (-\theta * (\overline{w}X + (\overline{w} - f)K - dX - \frac{1}{2}X^2)^{-\theta-1})$$

$$+0 + \frac{\sigma_\rho^2 * (X + K)^4 + (E[\sigma^2])^2 * (X + K)^4}{8} * (-\theta * (-\theta - 2)(-\theta - 1) * (\overline{w}X + (\overline{w} - f)K - dX - \frac{1}{2}X^2)^{-\theta-3}) \tag{32}$$

Here, the assumption to ensure that the firm has the potential to make an operational profit is that $\overline{w} > d$. Equation thirty-two can be decomposed in four terms. The first term, $\frac{(\overline{w}X + (\overline{w} - f)K - dX - \frac{1}{2}X^2)^{1-\theta} - 1}{1-\theta}$, is the utility derived from the expected value of the profits.

This term is independent from exchange rate volatility and the variance of exchange rate volatility. The second term, $\frac{\sigma^2*(X+K)^2}{2}*(-\theta*(\overline{w}X+(\overline{w}-f)K-dX-\frac{1}{2}X^2)^{-\theta-1}$, shows an impact of exchange rate volatility on expected utility. Note that $\frac{\sigma^2*(X+K)^2}{2}$ is always greater than zero. This ensures that for all values of θ greater than zero, a negative effect of exchange rate volatility on expected utility is encountered for term two. This shows risk aversion, increases in uncertainty of the relative price of currencies feed negatively into the utility function. For simplicity, the third term is set equal to zero. Prudence of the exporter therefore does not influence the amount of utility. The fourth term of equation thirty-two is of greater complexity. Both the volatility of the exchange rate and the variance of this volatility influence the level of utility via this term. It is of importance to note that $\frac{\sigma_\rho^2*(X+K)^4+(E[\sigma^2])^2*(X+K)^4}{8}$ is always positive. The term $-\theta*(-\theta-2)(-\theta-1)$ is negative for any θ greater than minus two, except for minus one and zero. Note that we also require a value of θ greater than minus two for the notion of temperance in isoelastic utility functions. Temperance is therefore adequate to ensure that both volatility of the exchange rate and the variance of this volatility feed negatively into the utility function. Recall from the coefficients of relative prudence and temperance that, in a setting of isoelastic utility, a risk averse trading entity by definition also prudent and temperant is. The assumption of risk aversion is therefore sufficient to have the variances feed into the equation with the predicted sign. Note that the case of the importer is identical to the exporter. Although it is evident what effect independent changes in σ_ρ^2 and σ^2 have on the utility function, the more interesting case is when they occur simultaneously. If a decrease in exchange rate volatility leads to an increase in the variance of the exchange rate volatility, it is of importance to know which effect will dominate. Illustrative numerical examples are presented in the subsequent section.

5.3 Solving the Model

Although one can deduce from Żołądek et al. (2000) that all quartic functions can be solved whilst making use of mathematical derivation, solving the derivative of equations twenty-four and thirty-two explicitly for the amount of imports and exports respectively would leave the reader with little additional insights. These functions would be quite complex and would not facilitate simple, unambiguous, interpretation of the effect of additional variance in the volatility of the exchange rate on the value of imports and exports. It is therefore more convenient to obtain the solutions of this model by illustrative numerical examples. Making use of plausible values of the used parameters will allow us to infer whether the predictions of this model can be reconciled with the empirical results presented earlier. We will distinguish between several cases with different maturities of the forward market. Moreover, I assume that the firm has risk preferences similar to a consumer and adopt a coefficient of relative risk aversion of two (Choi and Menezes, 1992). A coefficient of relative risk aversion of four, in line with estimations made by Davies (1981), is also used for purposes of comparative statics. I discuss two examples with relatively similar values of their currency, to have simple interpretation of the level, volatility and variance of the volatility of the exchange rate. Using an exchange rate much smaller than one would lead to less intuitive interpretation and would not leave room for strong decreases in volatility, due to its lower bound of zero. Note that in the presence of an imperfect forward market, the case of the importer and exporter are identical. The examples therefore provide evidence for smaller and greater shocks in both the case of the exporter and the importer. The import (export) relationship could also be viewed from the perspective of the exporter (importer), the only difference this would make is that the values of the exchange rate, the volatility and the variance of the volatility would be scaled to a smaller magnitude.

5.3.1 Optimal Amount of Imports

The first case consists of an importer from Turkey who purchases goods from the United Kingdom in the year 2013. The average nominal exchange rate, denoted as Turkish lira per pound sterling, was equal to 2.98 in 2013. I initially assume that there exists no forward market between the two countries ($L = 0$), which makes the choice of the forward rate irrelevant. The variance of the exchange rate was equal to approximately 0.047. The five-year variance of the exchange rate volatility, which I use as σ_ρ^2 , is equal to 0.00033. Although $(E[\sigma^2])^2$ is in fact not equal to the $\overline{\sigma}^4$ due to Jensen's inequality, I do approximate it by this measure to be able to numerically simulate the model. This value therefore becomes 0.00002. Moreover, for simplicity, I assume that demand parameter a is equal to 5. The condition

$a > \overline{w}$ ensures that it is possible for the firm to generate profits. The optimal amount of imports for a value of θ of two can subsequently be calculated by non-linear optimisation, yielding 1.92 units of imports as the optimal quantity.

Now assume that a large Mean-Preserving Spread (MPS) in the variance of exchange rate volatility takes place, leaving the current and average volatility of the exchange rate unchanged. The variance of the exchange rate volatility increases from 0.00033 to 0.03. As shown in table 5, the optimal quantity of imports now becomes 1.77, which is a 7.8% decrease in trade. The effect is greater for greater parameters of theta, a value of theta of four corresponds to a decrease of the optimal imports of 13.4% when the same MPS is introduced. Under these heavily stylized conditions, we indeed find that an increase in σ_ρ^2 leads to a fall in trade.

It is, however, unlikely that consumers would observe a large change in the variance of the volatility of the exchange rate, without witnessing a change in the actual exchange rate volatility. The 0.02967 increase in the variance of exchange rate volatility would be observed if the variance of the exchange rate in 2014 were equal to 0.45 and σ_ρ^2 were calculated over a five-year rolling window. Albeit not moderate, this increase in volatility is not unrealistic. In the year 2018, the variance of the exchange rate between the Turkish lira and the pound sterling was in fact much larger, 0.99. Although this turmoil should not be viewed as standard, it does show that half its size is far from impossible. Increasing the exchange rate volatility from 0.047 to 0.45 and increasing σ_ρ^2 to 0.03 for a coefficient of relative risk aversion of two leads to a 28.5% decrease in imports. For a coefficient of relative risk aversion of four, this fall in imports is equal to 25.1%. It can be deduced from equation twenty-four that the sign of this effect would be retained for any simultaneous increases in exchange rate volatility and the variance of exchange rate volatility. The qualitative conclusion is therefore not dependent on the magnitude of these shifts.

The interesting case is, however, a decrease in volatility. Suppose that the volatility in the exchange rate of the Turkish lira and the pound sterling became nearly zero in the year 2015, subsequent to its volatility of 0.45 in the year 2014. This value of 0.0001 leads the five-year rolling window variance of exchange rate volatility to increase from 0.00033 to 0.03. Decreasing the value of exchange rate volatility and increasing the value of the variance of the exchange rate volatility in fact leads to a decrease in trade of 5.4% when compared to 2013. For a greater degree of relative risk aversion, a coefficient of four, this effect is larger. The fall in trade then consists of 12.4%.

Hitherto, we have considered scenarios in the absence of a forward market. It is nevertheless also interesting to examine the case with forward markets. I follow Giorgianni (1997) and set the average forward premium equal to 4.5%. In this analysis, I allow for two scenarios. In the first scenario, the transaction is fully hedged. This implies that the value of L is equal to Y. In this case, altering the variance of the exchange rate volatility no longer directly influences the optimal amount of imports. This, however, does not imply that the variance of exchange rate volatility does not influence the export and import decisions in mature forward markets. It could very well be the case that the variance of the volatility of the exchange rate functions as a determinant of the forward rate, which I assume exogenous. As exporters and importers are on opposite sides of the forward market, this would indicate that the aggregate impact of the variance of exchange rate volatility on trade would be dependent on the bilateral trade balance (Viaene and De Vries, 1992). I, however, leave an explicit solution of the forward rate to further research.

In the subsequent scenario, I assume that the size of the forward contract is limited. In this case, the assumption of no forward market is relaxed to an imperfect forward market. The assumption that the forward rate is exogenous and not influenced by trading decisions, for example due to dominance of speculators in the forward market, is retained. In this numeric example, the size of the hedge is set to one unit. The decrease in imports for the MPS increase in the variance of the exchange rate volatility now leads to a 1.9% decrease in the amount of imports when adopting a coefficient of relative risk aversion of two. If the coefficient is four, the decrease is equal to 5.3%. The negative effect of the variance of exchange rate volatility therefore seems to be decreasing in the size of the forward contract.

Table 5: Comparative statics of the baseline results, a MPS increase in σ_ρ^2 and simultaneous changes in σ_ρ^2 and σ^2. Terms are from equation twenty-four.

		$L = 0$				$L = 1$			
		Term 1	Term 3	Term 4	Δ Y%	Term 1	Term 3	Term 4	Δ Y%
$\theta = 2$	Baseline	0.5087	-0.0206	-0.0030		0.4750	-0.0064	-0.0002	
	$\Delta^+\sigma_\rho^2$	0.5023	-0.0182	-0.0291	-7.8%	0.4743	-0.0060	-0.0030	-1.9%
	$\Delta^+\sigma^2\&\Delta^+\sigma_\rho^2$	0.4538	-0.1382	-0.1205	-28.5%	0.4565	-0.0311	-0.0061	-16.0%
	$\Delta^-\sigma^2\&\Delta^+\sigma_\rho^2$	0.5049	-0.0000	-0.0292	-5.4%	0.4750	0.0000	-0.0032	+0.2%
$\theta = 4$	Baseline	0.2931	-0.0094	-0.0031		0.2848	-0.0032	-0.0003	
	$\Delta^+\sigma_\rho^2$	0.2883	-0.0085	-0.0291	-13.4%	0.2837	-0.0027	-0.0026	-5.3%
	$\Delta^+\sigma^2\&\Delta^+\sigma_\rho^2$	0.2792	-0.0823	-0.1800	-25.1%	0.2726	-0.0125	-0.0037	-22.7%
	$\Delta^-\sigma^2\&\Delta^+\sigma_\rho^2$	0.28879	-0.0000	-0.0276	-12.4%	0.2843	-0.0000	-0.0029	-2.89%

All in all, equation twenty-four seems to be capable of generating results that are qualitatively in line with the empirical estimations. It can be seen that the first term of equation twenty-four is merely influenced by the amount of trade. The second term is negatively impacted by the level of exchange rate volatility, whilst the third term is a constant zero. The fourth term is negatively impacted by both exchange rate volatility and the variance of exchange rate volatility. Not only does a MPS in the variance of exchange rate volatility lead to a decrease in trade, this effect also has the magnitude to be able to dominate the positive effect of a decrease in exchange rate volatility. Although the magnitude of the predictions is much larger than found in the data, it is shown that simultaneous increases in volatility and the variance of volatility have greater detrimental effects than a decrease in volatility accompanied with an increase in the variance of exchange rate volatility. We can therefore conclude that the model is qualitatively capable of explaining both the sign and relative magnitudes of the empirical results. Isoelastic utility and temperance seem sufficient to be able to generate this conclusion. Two moderating variables to the magnitudes of these effects are the size of the forward contract and the degree of risk aversion. In the absence of a forward market or when firms do not hedge their transactions, the negative effect of exchange rate volatility and of the variance of volatility is the greatest. Moreover, a larger degree of aversion to risk leads to further decreases in the optimal amount of imports. This applies to both changes in exchange rate volatility and the variance of exchange rate volatility.

5.3.2 Optimal Amount of Exports

Suppose now that a firm from the Czech Republic exports its goods to the United States in 2013. The firm purchases the good for a domestic numéraire price and invoices its customers from the United States in U.S. dollar. The average nominal bilateral exchange rate in the year 2013 consisted of 19.56 Czech Koruna per U.S. dollar. In the base case, I make the assumption of no forward market between the two nations, effectively fixing the value of K to zero. This also allows me to evade making assumptions about the forward rate at this point. The variance of the exchange rate in 2013 was equal to 0.205. The five-year rolling window variance in the exchange rate volatility is equal to 0.718. Moreover, I assume that $(E[\sigma^2])^2$ is equal to the squared exchange rate volatility. I once more follow Choi and Menezes (1992) and adopt a coefficient of relative risk aversion of two, using the value of four from Davies (1981) as a comparative static. To satisfy the condition $\overline{w} > d$, the value of d is fixed to 10.

The optimal quantity of export, as determined by non-linear optimisation, is equal to 9.40 goods. Now suppose a MPS occurs which raises σ_ρ^2 to 1. This causes the optimal amount of exports to decrease to 9.38, which is a 0.3% negative change in the quantity of exports. These results are presented in table 6. If a parameter of relative risk aversion of four is assumed, this decrease is equal to 0.9%. Again, it is shown that an increase in the variance of the volatility of the exchange rate has a detrimental effect on bilateral trade in this model. It is, however, also interesting to inspect a change in the volatility of the exchange rate, which could lead to the conjecture of this increase in the spread of volatility. If the firm calculates σ_ρ^2 by observing the five-year rolling window of the volatility in the exchange rate, the variance in 2014 would need to be equal to 3.0. Although substantial, this value does not seem impossible. The variance of the exchange rate was, for example, equal to 2.6 in 2009. If the volatility of the exchange rate indeed increases from 0.205 to 3.0 and σ_ρ^2 thereby increases to 1, the fall in trade would be as large as 11.7%. For a coefficient of relative risk aversion of four, this decrease is equal to 15.3%.

Assume now that the year 2014 is followed by a year with almost no volatility. In this case, the volatility of the exchange rate decreases to 0.001 in the year 2015. The five-year rolling window variance of exchange rate volatility thereby increases to 1.184. This simultaneous decrease in exchange rate volatility and increase in the variance of exchange rate volatility leads to a 0.4% increase in trade, when compared to 2013. For a coefficient of relative risk aversion of four, a decrease of 0.4% is observed. The effect of the variance in exchange rate volatility on trade thereby seems to be capable of dominating the effect of the volatility of the exchange rate itself when the coefficient of risk aversion is sufficiently large.

Contrary to the previously discussed scenarios, I now relax the assumption of no mature forward market. In the first scenario, I once more follow Giorgianni (1997) and set the average forward premium equal to 4.5%. Moreover, I assume that the exporter fully hedges its transaction. Effectively, this makes the size of the forward contract equal to the value of exports, $X = -K$. As expected, the volatility of the exchange rate or the variance of the volatility of the exchange rate no longer impact the utility maximising quantity of exports. As stated earlier, it could however still be the case that the risk premium on the forward rate is influenced by these factors, thereby making the effect on total trade dependent on a countries bilateral trade balance (Viaene and De Vries, 1992).

It is, however, also possible that the size of the forward contract is limited. In this case, there would not be a perfect forward market. If the amount of the hedge is equal to five, $K = -5$, the effect of the MPS increase to $\sigma_\rho^2 = 1$ would be limited to a 0.03% decrease in exports. For a coefficient of relative risk aversion of four, the decrease would be equal to 0.12%.

Table 6: Comparative statics of the baseline results, a MPS increase in σ_ρ^2 and simultaneous changes in σ_ρ^2 and σ^2. Terms are from equation thirty-two.

		$K = 0$				$K = -5$			
		Term 1	Term 3	Term 4	Δ Y%	Term 1	Term 3	Term 4	Δ Y%
$\theta = 2$	Baseline	0.97811	-0.00019	-0.00009		0.98004	-0.00003	-0.00000	
	$\Delta^+\sigma_\rho^2$	0.97811	-0.00019	-0.00012	-0.28%	0.98004	-0.00003	-0.00000	-0.03%
	$\Delta^+\sigma^2\&\Delta^+\sigma_\rho^2$	0.97773	-0.00223	-0.00078	-11.73%	0.98000	-0.00040	-0.00003	-4.96%
	$\Delta^-\sigma^2\&\Delta^+\sigma_\rho^2$	0.97811	-0.00000	-0.00014	+0.37%	0.98000	0.00000	0.00000	+0.34%
$\theta = 4$	Baseline	0.33333	-1.73E-07	-1.92E-07		0.33333	-1.28E-10	-9.26E-09	
	$\Delta^+\sigma_\rho^2$	0.33333	-1.70E-07	-2.56E-07	-0.91%	0.33333	-2.57E-08	-7.71E-09	-0.12%
	$\Delta^+\sigma^2\&\Delta^+\sigma_\rho^2$	0.33333	-2.16E-06	-1.67E-06	-15.29%	0.33333	-2.61E-07	3.48E-08	-8.78%
	$\Delta^-\sigma^2\&\Delta^+\sigma_\rho^2$	0.33333	-8.39E-10	-2.96E-07	-0.37%	0.33333	-2.58E-08	-5.68E-09	+0.54%

Again, the model is capable of generating predictions with the correct sign. Moreover, the relative impacts are in line with the empirical results. The model, however, seems more sensitive to changes in the variance of exchange rate volatility than empirically observed responses are. It is again evident that detrimental effects of exchange rate volatility and the variance of exchange rate volatility are greater for larger values of aversion towards risk. Moreover, the size of the forward contract functions as a moderating variable to the magnitude of the effect. When a larger proportion of the transaction is hedged, the impact of exchange rate volatility and the variance of exchange rate volatility on trade is greater. As shown in table 6, the variance of the volatility dominates the effect of the volatility when the forward contract is sufficiently small and the coefficient of relative risk aversion is adequately large.

All in all, we can conclude from the cases of the importer and the exporter that this model is, under certain parameterisations, qualitatively capable of generating results in line with empirical specification. Not only the level of the exchange rate is of importance in this model. The distribution of volatility is shown to also affect the decision of the utility maximising trading entity. It could be the case that changes in the exchange rate volatility lead to the expectation of a different distribution of exchange rate volatility. This would reconcile the empirical findings with economic theory under the assumptions of isoelastic utility, stochastic volatility and risk aversion. It is important to note that in this framework, the degree of risk aversion and the size of the hedge moderate the relationship between these shifts in volatility and the variance of volatility, and trade. Relatively low degrees of risk aversion of trading actors and fairly mature forward markets could endogenously explain these empirical results.

6 Conclusion

All in all, this study has contributed to the empirical evidence on asymmetric effects of exchange rate volatility. Inspired by the explorations of Bahmani-Oskooee and Aftab (2017a), Bahmani-Oskooee and Aftab (2017b) and Kokken and Wolters (2019), this paper is the first to consider asymmetric effects on a global scale whilst making use of an appropriate measure of exchange rate volatility. Although parsimonious regression specifications are plagued by concerns of endogeneity, adequate precautions have been taken to overcome these omitted variable bias, reverse causality and measurement error complications. This study provides no exception to the large strand of literature in which no unambiguous effect of exchange rate volatility is observed (McKenzie, 1999). Based on the evidence presented in this paper, we are nonetheless unable to rule out a negative aggregated effect. When disaggregating volatility into positive and negative changes, asymmetric effects are observed. Positive changes in exchange rate volatility are shown to have a detrimental effect on bilateral trade. This effect is both statistically and economically significant. Further decreases in exchange rate volatility also negatively impact bilateral trade. Although these estimates lack economic significance, the mere absence of a positive influence is evidence of asymmetric effects in itself. These effects are found to be robust to corrections for currency unions, thereby ensuring that a positive effect of constant volatility does not stem from a level effect of zero volatility. I subsequently proceed to show that fluctuations in exchange rate volatility have a negative effect on bilateral trade. The observed effect is statistically significant, the economic impact is however negligible. Albeit the magnitude of the effect of negative changes is too modest to provide a sensible resolution to the exchange rate volatility paradox, the encountered asymmetry still provides interesting insights.

Based on the empirical evidence, I develop a novel theoretical foundation of the asymmetry in the effect of exchange rate volatility. To the knowledge of the author, this is the first theoretical model that explicitly captures this asymmetry. Within the exporter and importer framework introduced by Viaene and De Vries (1992), I show that a combination of isoelastic utility and stochastic volatility allows me to explicitly model the effects of the distribution of exchange rate volatility. More specifically, a fourth order Taylor approximation of a constant relative risk aversion utility function and the notion of temperance assure that the variance of exchange rate volatility indeed negatively impacts trade. This result holds for both exporters and importers. Illustrative simulations confirm that, whilst making use of plausible coefficients of relative risk aversion, the model is qualitatively capable of producing similar conclusions to the empirical results in terms of sign and relative magnitudes. The precise magnitude of the predictions, however, seems to be more sensitive than the actual effects. The maturity of the forward market and the degree of risk aversion are found to have moderating effects on the predictions of the model. When firms fully hedge their transactions, direct effects of exchange rate volatility and the variance of exchange rate volatility cease to exist. It would, nonetheless, be myopic to conclude that there would be no effects in a mature forward market. It could very well be the case that exchange rate volatility and the variance of this volatility influence the risk premium on the forward market, thereby affecting the optimal amount of trade. All in all, we can state that the model is capable of generating results in line with the empirical specification for reasonable values of risk aversion and sizes of the forward contracts.

6.1 Limitations

Despite the fact that this study provides an interesting explorative journey in the search for the resolution to the exchange rate volatility paradox, several critical notes must be made. Although a negative effect of squared exchange rate volatility in the statistical sense of the word is observed, the magnitude implies that there is very little actual influence. It is of importance to further investigate the mechanisms behind this, precisely estimated, minor effect. This could be related to the approximation of exchange rate volatility, which is an imperfect measure of exchange rate volatility.

The predicted effects after introducing considerable changes to the model are quantitatively larger than the estimated effects. These predictions are made under strong assumptions of the stochastic structure of exchange rate volatility. More general assumptions on the stochastic process underlying exchange rate volatility would increase the plausibility of the predictions of the model. Moreover, no explicit solution of the forward rate is derived in this paper. I am therefore unable to generalise the predictions to mature forward markets.

6.2 Recommendations

This study provides several interesting pathways for future research. The empirical literature has considered aggregate effects of exchange rate volatility on bilateral trade. It would not only be interesting to disaggregate this into positive and negative changes, but also to test directly for the effect of variety in exchange rate volatility. Rather than considering a level effect of exchange rate volatility, the true impact may lie within the fluctuations of exchange rate volatility. Considering this hypothesis in a greater variety of methodological approaches could prove to lead to new insights. Moreover, it could be relevant to disaggregate the value of trade into exports and imports, thereby allowing for heterogeneity.

The theoretical foundation of this paper could be substantiated by extending this model to allow for more general processes of stochastic volatility. The model could also easily be extended to incorporate utility functions other than isoelastic utility. In general, all utility functions that exhibit risk aversion and temperance could be introduced to this setting.

Both the theoretical and empirical part of this paper could be extended to include further scrutiny to the forward market. Introducing a measure of the maturity of the forward market to empirical specifications of fluctuations in exchange rate volatility could shine light over potential heterogeneity in effects. Moreover, the derivative of the utility function towards the size of the forward contract could be solved for the optimal hedge. Combined with forward market clearing conditions, this could give an explicit solution of the forward rate. Inspecting the effects of the variance of exchange rate volatility on this forward rate would indicate a potential indirect effect. This indirect effect could affect exporters and importers differently.

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Appendix A

Table 7: Wooldridge test for autocorrelation in $\ln trade$ in panel data

F-value	3378.472
P-value	0.0000

Note: H_0: No first order autocorrelation

Table 8: Unit root test for $\ln trade$ in panel data, Fisher-type

$\ln trade$	Statistic	P-value
Inverse X^2	3.17e+04	0.0000

Note: H_0: Panels contain unit roots

Table 9: Unit root test for CPI of country 1 in panel data, Fisher-type

$\ln trade$	Statistic	P-value
Inverse X^2	2.23e+04	1.0000

Note: H_0: Panels contain unit roots

Table 10: Unit root test for CPI of country 2 in panel data, Fisher-type

$\ln trade$	Statistic	P-value
Inverse X^2	2.45e+04	1.0000

Note: H_0: Panels contain unit roots

Table 11: Unit root test for the first difference of CPI of country 1 in panel data, Fisher-type

$\ln trade$	Statistic	P-value
Inverse X^2	8.18e+04	0.0000

Note: H_0: Panels contain unit roots

Table 12: Unit root test for the first difference of CPI of country 2 in panel data, Fisher-type

$\ln trade$	Statistic	P-value
Inverse X^2	1.27e+05	0.0000

Note: H_0: Panels contain unit roots

Appendix B

$E[\sigma^4] - (E[\sigma^2])^2 = E[(\sigma^2)^2] - (E[\sigma^2])^2 = E[(\sigma^2)^2] - 2 * (E[\sigma^2])^2 + 1 * (E[\sigma^2])^2 = E[(\sigma^2)^2] - 2E[\sigma^2] * E[\sigma^2] + E[E[\sigma^2]^2] = E[(\sigma^2)^2] - 2E[\sigma^2 E[\sigma^2]] + E[E[\sigma^2]]^2 = E[(\sigma^2)^2 - 2\sigma^2 E[\sigma^2] + (E[\sigma^2])^2] = E[(\sigma^2 - E[\sigma^2])(\sigma^2 - E[\sigma^2])] = E[(\sigma^2 - E[\sigma^2])^2] = Var(\sigma^2) = E[(\overline{\sigma}^2 + \rho - \overline{\sigma}^2 - 0)^2] = E[(\rho - 0)^2] = E[(\rho - \mu)^2] = \sigma_\rho^2$